

# $\Lambda$ polarization in heavy ion collisions

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Che Ming Ko**

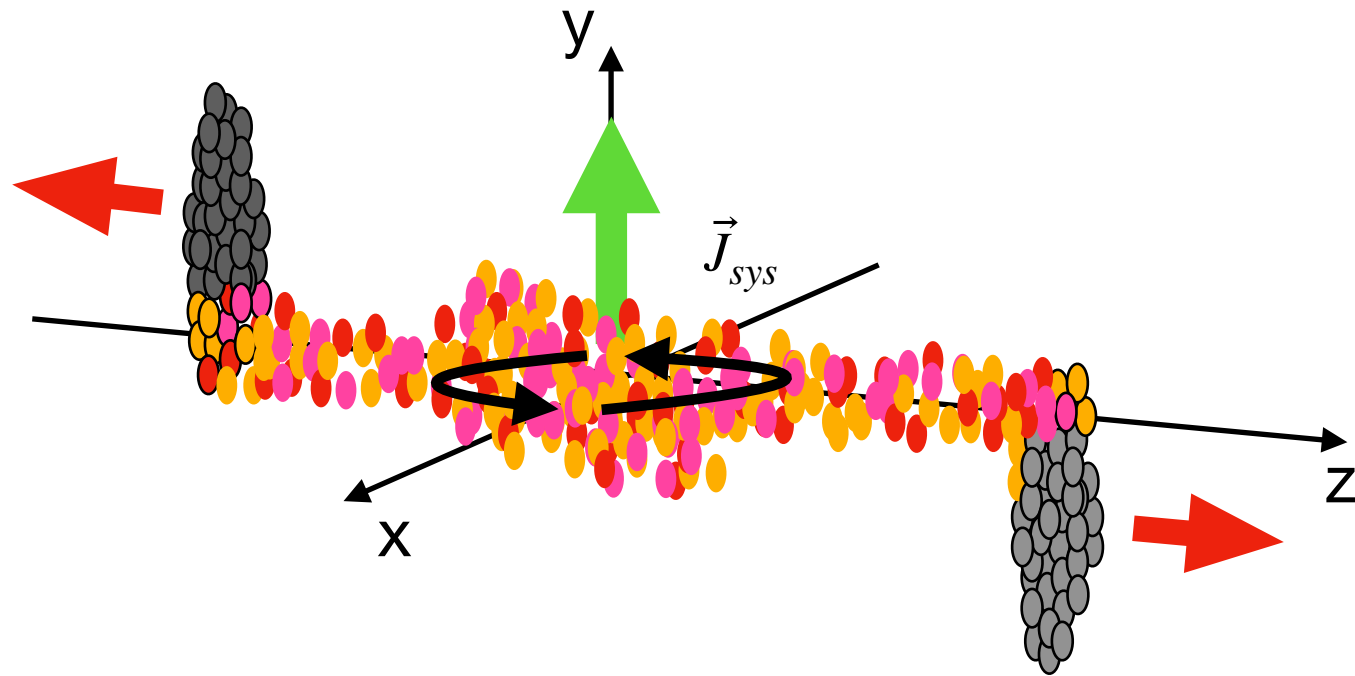
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- **Introduction**
  - Global and local polarization in heavy ion collision
  - Positive global  $\Lambda$  polarization in STAR experiment
- **$\Lambda+\pi$  cross section**
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# Introduction

## Global and local polarization



- The vorticity  $\omega$  represents local mechanical rotation of a fluid.
- The vorticity acts as a spin-current source.

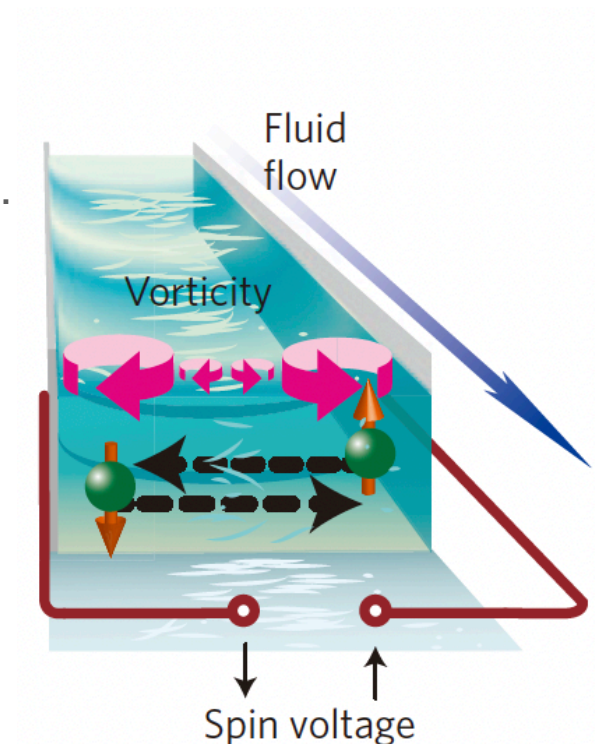
Takahashi. R, et al. Spin hydrodynamic generation. *Nature Phys* **12**, 52–56 (2016).

$$\nabla^2 \mu^s = \frac{1}{\lambda^2} \mu^s - \frac{4e^2}{\sigma_0 \hbar} \xi \omega$$

$$\mu_s = \mu_{\uparrow} - \mu_{\downarrow}$$

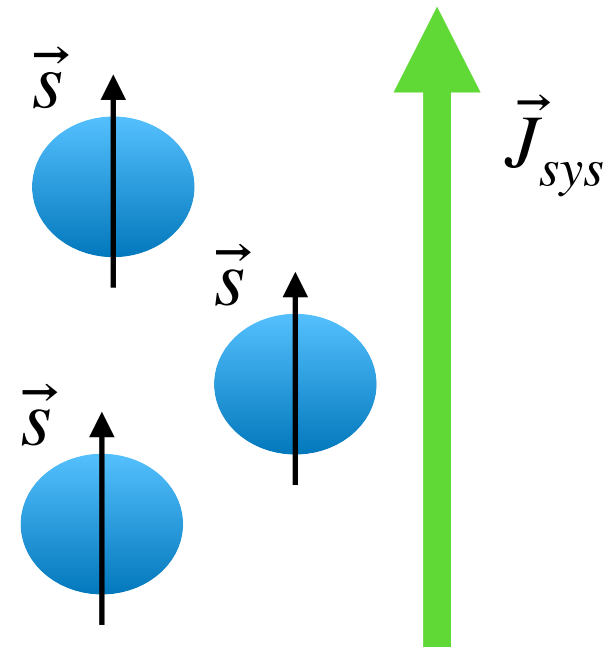
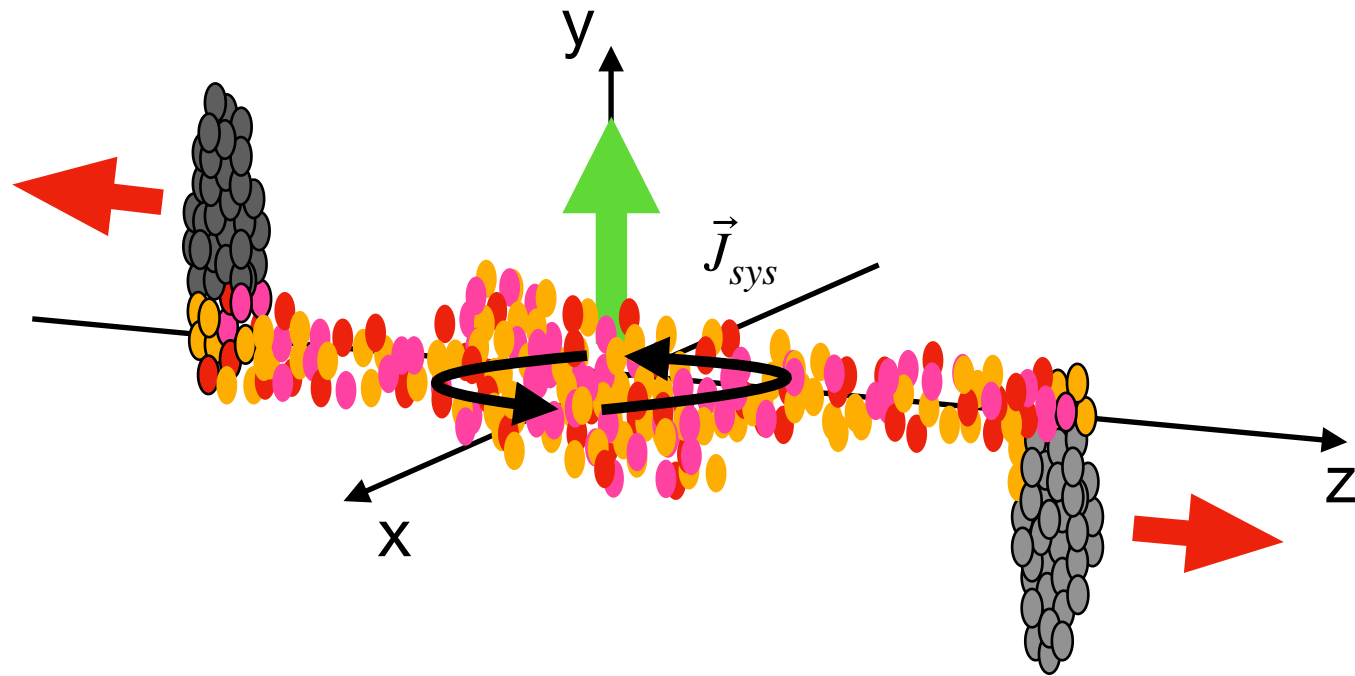
$$\text{vorticity } \omega = \frac{1}{2} \nabla \times v$$

v: fluid velocity



# Introduction

## Global and local polarization



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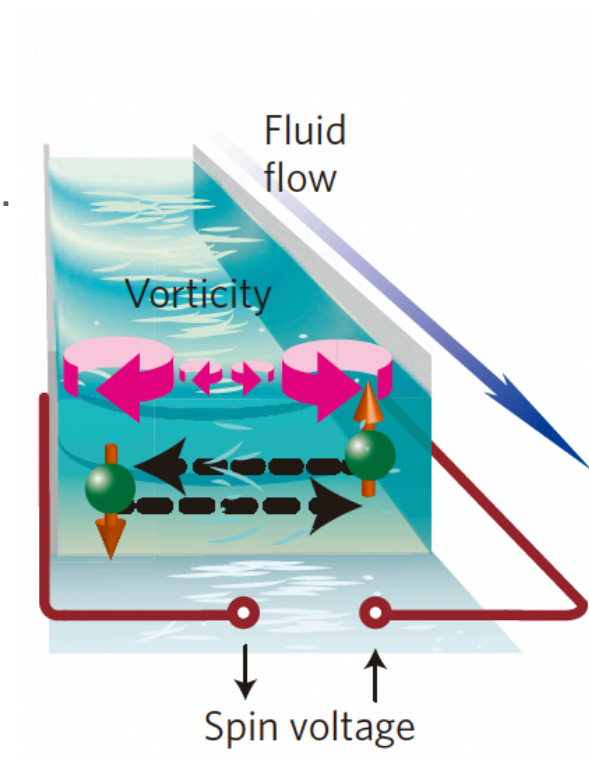
Takahashi. R, et al. Spin hydrodynamic generation. *Nature Phys* 12, 52–56 (2016).

$$\nabla^2 \mu^s = \frac{1}{\lambda^2} \mu^s - \frac{4e^2}{\sigma_0 \hbar} \xi \omega$$

$$\mu_s = \mu_{\uparrow} - \mu_{\downarrow}$$

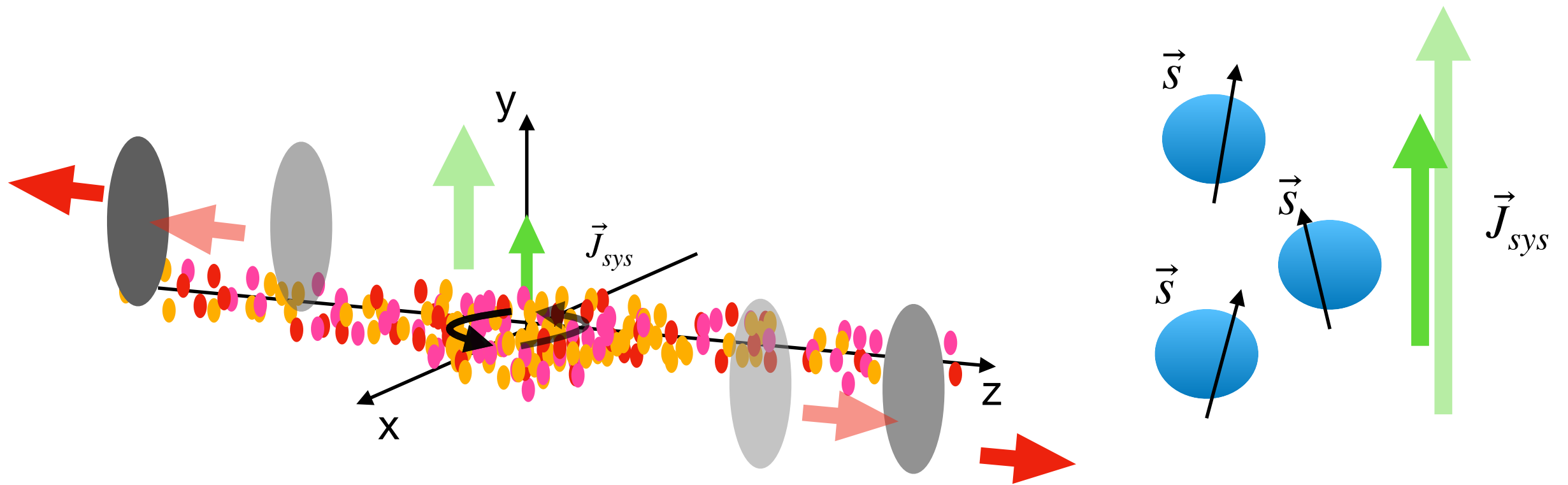
$$\text{vorticity } \omega = \frac{1}{2} \nabla \times v$$

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# Introduction

## Global and local polarization

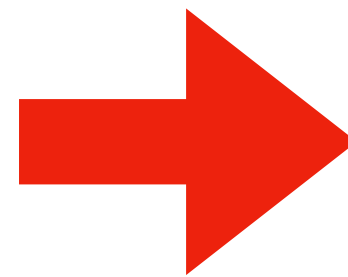
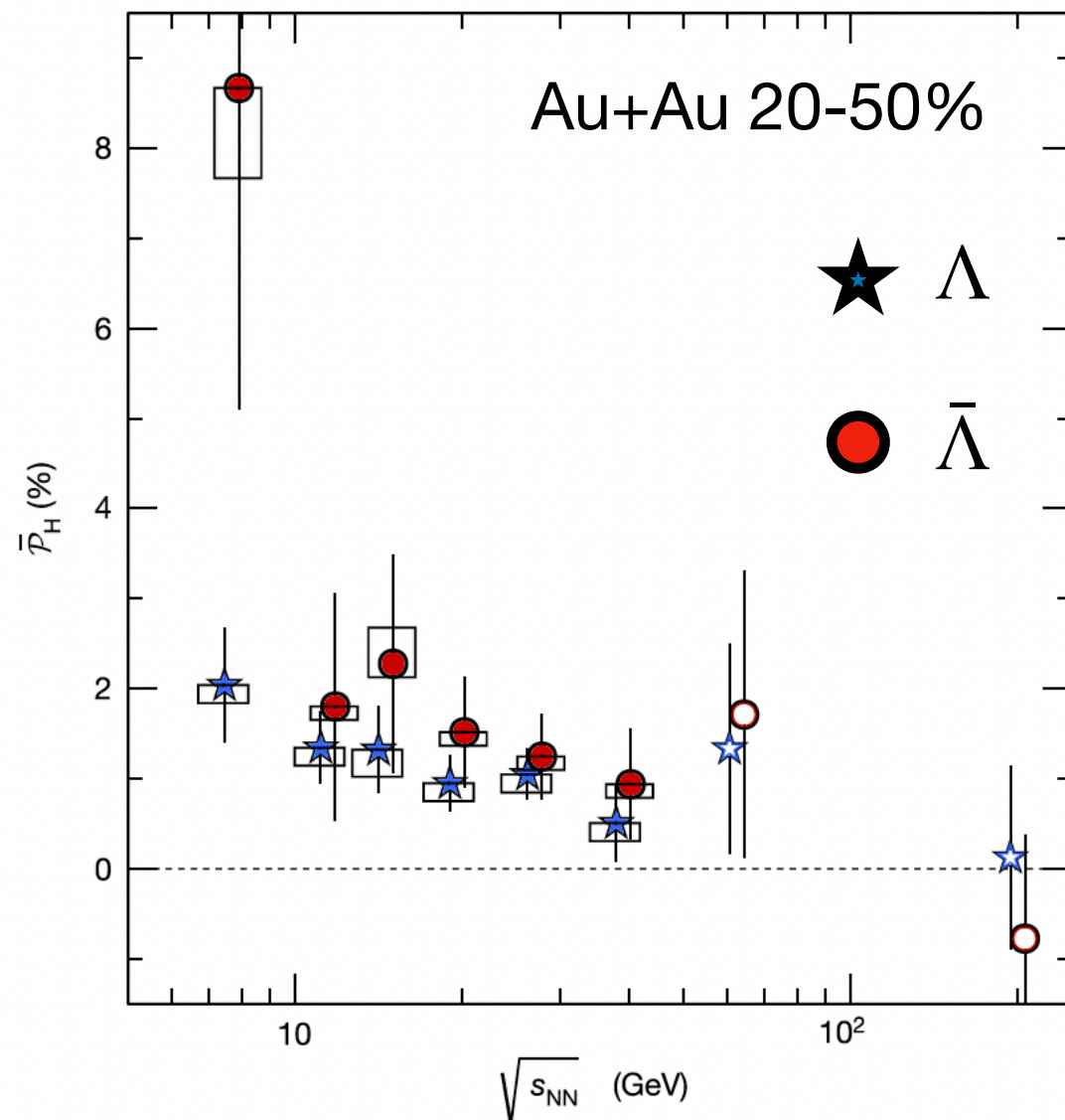


- Vorticity is getting weak with expanding of system
- We thought that collisions with other hadrons in the hadronic phase make the initial polarization to disappear.
- We expected that initial alignment of particle disappear in the final state.

# Introduction

## Global and local polarization

L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

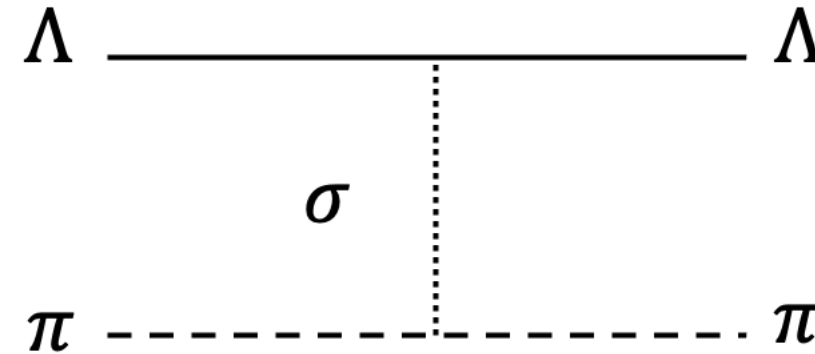
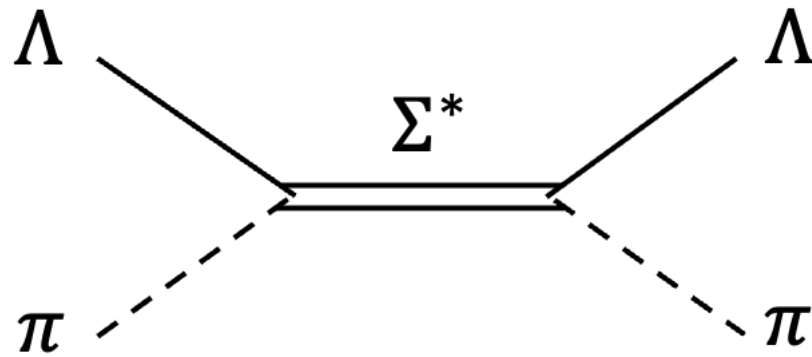


Non-zero global  $\Lambda$  polarization in Au+Au collisions

- Lambda polarization in early stage of system remains after scattering during hadron phase
- Estimate the  $\Lambda$  polarization during hadron phase

# Kinetic equation of $\Lambda$

## Scattering channel



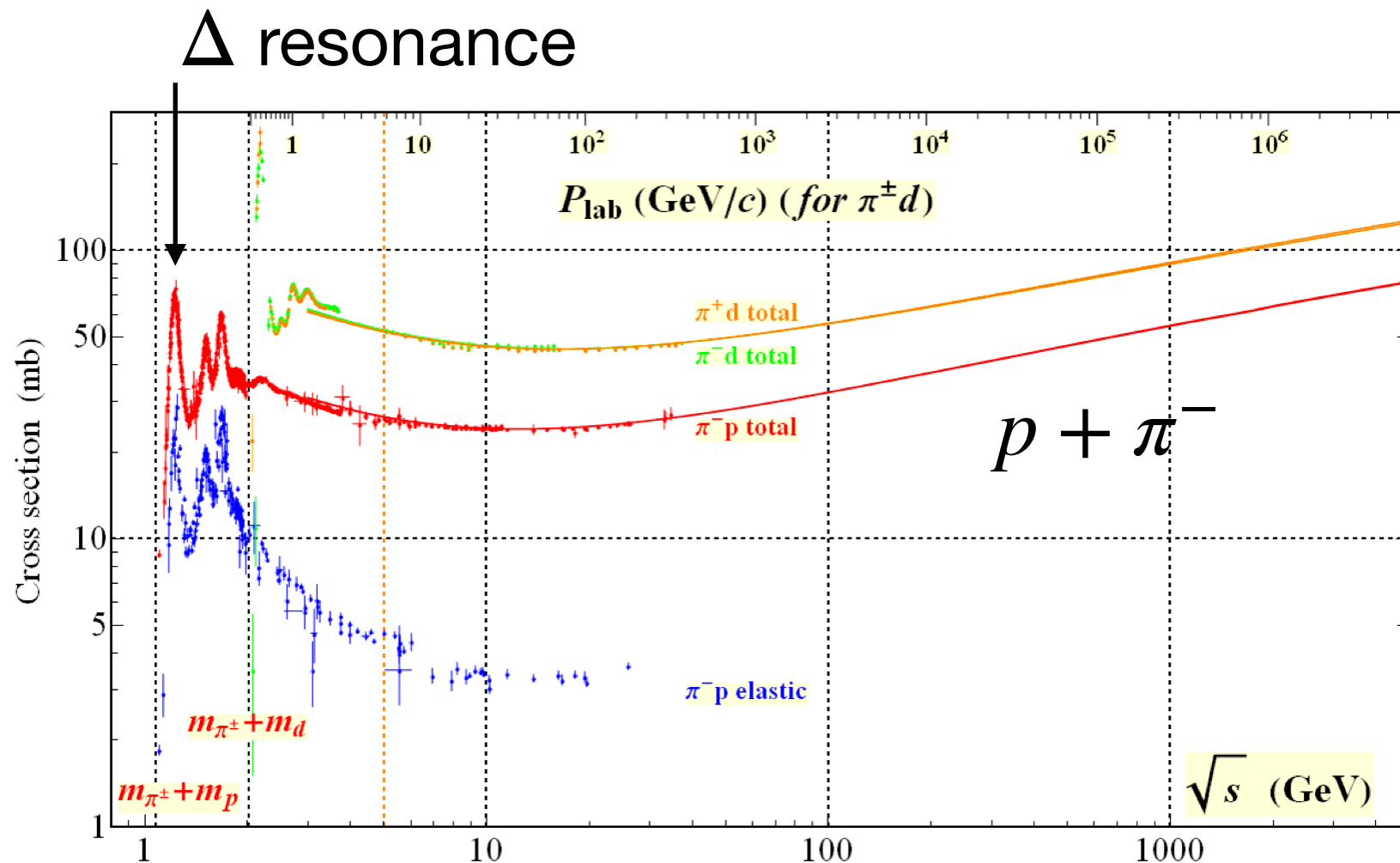
## Kinetic equation

$$\frac{dN_{\Lambda\uparrow}}{d\tau} = -\langle\sigma_{\Lambda\uparrow\pi\rightarrow\Lambda\downarrow\pi}v\rangle n_{\pi}N_{\Lambda\uparrow} + \langle\sigma_{\Lambda\downarrow\pi\rightarrow\Lambda\uparrow\pi}v\rangle n_{\pi}N_{\Lambda\downarrow}, \quad (20)$$

$$\frac{dN_{\Lambda\downarrow}}{d\tau} = \langle\sigma_{\Lambda\uparrow\pi\rightarrow\Lambda\downarrow\pi}v\rangle n_{\pi}N_{\Lambda\uparrow} - \langle\sigma_{\Lambda\downarrow\pi\rightarrow\Lambda\uparrow\pi}v\rangle n_{\pi}N_{\Lambda\downarrow}, \quad (21)$$

# $\Lambda$ s-channel cross section

$\Sigma^*$  resonance dominance

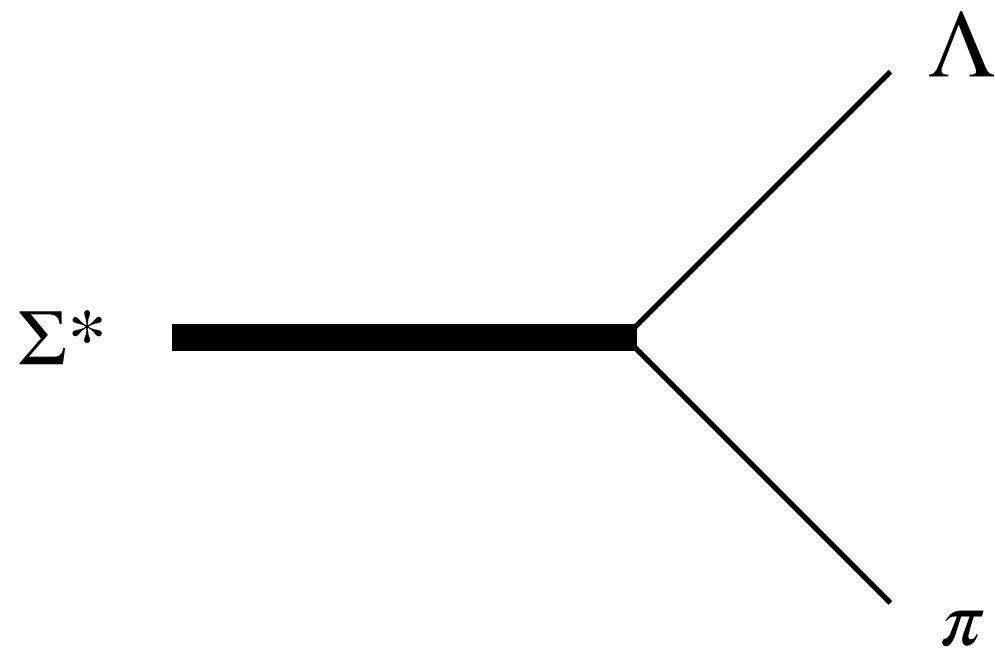


- $p + \pi^-$  is p-wave,  $\Delta(s=3/2)$  resonance is dominance
- $\Rightarrow$  In the case of  $\Lambda + \pi$ ,  $\Sigma^*(3/2)$  will be dominance



# $\Lambda$ s-channel cross section

## $\Sigma^*$ decay width and coupling constant



$$P^{\mu\nu} = -(\not{q} + M) \left[ g_{\mu\nu} - \frac{\gamma_\mu \gamma_\nu}{3} - \frac{2q_\mu q_\nu}{3M^2} + \frac{q_\mu \gamma_\nu - q_\nu \gamma_\mu}{3M} \right]$$

$$i\mathcal{M} = \bar{u}_s(p_1) \underset{\mathbf{a}}{(ig)} (ip_{2,\mu}) \underset{\mathbf{ab}}{u_r^\mu(k)}$$

$$\begin{aligned} |\mathcal{M}|^2 &= g^2 p_{2\mu} p_{2\nu} \bar{u}_s(p_1) \underset{\mathbf{a}}{u_r^\mu(k)} \underset{\mathbf{ab}}{\bar{u}_r^\nu(k)} \underset{\mathbf{cd}}{u_s(p_1)} \underset{\mathbf{d}}{\bar{u}_s(p_1)} \\ &= g^2 p_{2\mu} p_{2\nu} \frac{1}{4} \sum_{r,s} [u_r^\mu(k) \bar{u}_r^\nu(k)]_{ad} [u_s(p_1) \bar{u}_s(p_1)]_{da} \end{aligned}$$

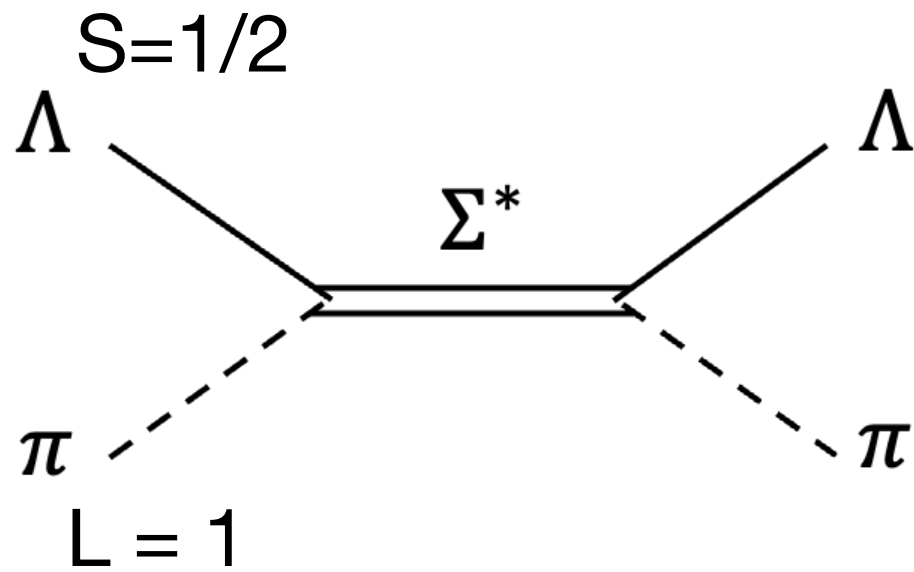
$$|\mathcal{M}|^2 = -\frac{2}{3} g^2 k^2 (Mm_1 + \sqrt{k^2 + m_1^2} \sqrt{k^2 + m_2^2} + m_1^2 + k^2)$$

Decay width = 36MeV

$$\Rightarrow g_{\Sigma^* \Lambda \pi} = 9.83$$

# $\Lambda + \pi$ cross section

## Spin non-flip cross section



$$\sigma(s) = \frac{8\pi}{k^2} \frac{s\Gamma^2(s)}{(s - m_{\Sigma^*}^2)^2 + s\Gamma^2(s)}$$

$$\langle j_1, m_1; j_2, m_2 | S | j_1, m_1; j_2, m_2 \rangle$$

$$\left\langle 1, 1; \frac{1}{2}, \frac{1}{2} \middle| S \middle| 1, 1; \frac{1}{2}, \frac{1}{2} \right\rangle = a_{3/2},$$

$$\left\langle 1, 1; \frac{1}{2}, -\frac{1}{2} \middle| S \middle| 1, 0; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{\sqrt{2}}{3} (a_{3/2} - a_{1/2}),$$

$$\left\langle 1, 0; \frac{1}{2}, \frac{1}{2} \middle| S \middle| 1, 0; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{1}{3} (2a_{3/2} + a_{1/2}),$$

$$\left\langle 1, 0; \frac{1}{2}, -\frac{1}{2} \middle| S \middle| 1, -1; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{\sqrt{2}}{3} (a_{3/2} - a_{1/2}),$$

$$\left\langle 1, -1; \frac{1}{2}, \frac{1}{2} \middle| S \middle| 1, -1; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{1}{3} (a_{3/2} - 2a_{1/2}).$$

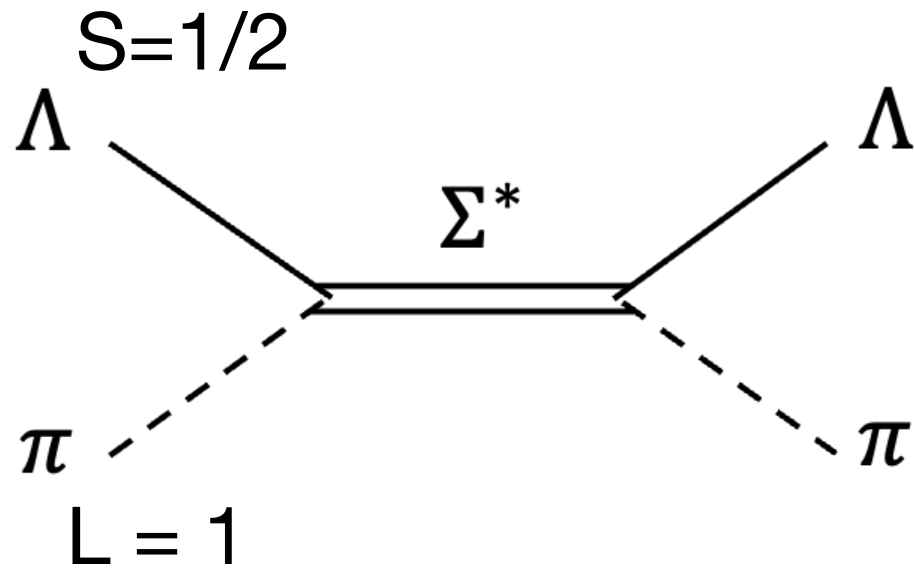
### 1. Spin non-flip cross section

$$\sigma_{\uparrow\uparrow} = \bar{\sigma} \left( 1 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right) = \frac{14}{9} \bar{\sigma}$$

$$\begin{aligned} \sigma_{\uparrow\uparrow} &= \bar{\sigma} \sum_{m, M} \left| \left\langle \frac{1}{2} \frac{1}{2} 1 m \middle| \frac{3}{2} M \right\rangle \right|^4 \\ &= \bar{\sigma} \left[ \left| \left\langle \frac{1}{2} \frac{1}{2} 1 1 \middle| \frac{3}{2} \frac{3}{2} \right\rangle \right|^4 + \left| \left\langle \frac{1}{2} \frac{1}{2} 1 0 \middle| \frac{3}{2} \frac{1}{2} \right\rangle \right|^4 \right. \\ &\quad \left. + \left| \left\langle \frac{1}{2} \frac{1}{2} 1 -1 \middle| \frac{3}{2} -\frac{1}{2} \right\rangle \right|^4 \right] \\ &= \bar{\sigma} \left[ 1 + \left(\frac{\sqrt{2}}{3}\right)^4 + \left(\frac{\sqrt{1}}{3}\right)^4 \right] = \frac{14}{9} \bar{\sigma}. \end{aligned}$$

# $\Lambda + \pi$ cross section

## Spin flip cross section



$$\sigma(s) = \frac{8\pi}{k^2} \frac{s\Gamma^2(s)}{(s - m_{\Sigma^*}^2)^2 + s\Gamma^2(s)}$$

$$\langle j_1, m_1; j_2, m_2 | S | j_1, m_1; j_2, m_2 \rangle$$

$$\left\langle 1, 1; \frac{1}{2}, \frac{1}{2} | S | 1, 1; \frac{1}{2}, \frac{1}{2} \right\rangle = a_{3/2},$$

$$\left\langle 1, 1; \frac{1}{2}, -\frac{1}{2} | S | 1, 0; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{\sqrt{2}}{3} (a_{3/2} - a_{1/2}),$$

$$\left\langle 1, 0; \frac{1}{2}, \frac{1}{2} | S | 1, 0; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{1}{3} (2a_{3/2} + a_{1/2}),$$

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$$\left\langle 1, -1; \frac{1}{2}, \frac{1}{2} | S | 1, -1; \frac{1}{2}, \frac{1}{2} \right\rangle = \frac{1}{3} (a_{3/2} + 2a_{1/2}).$$

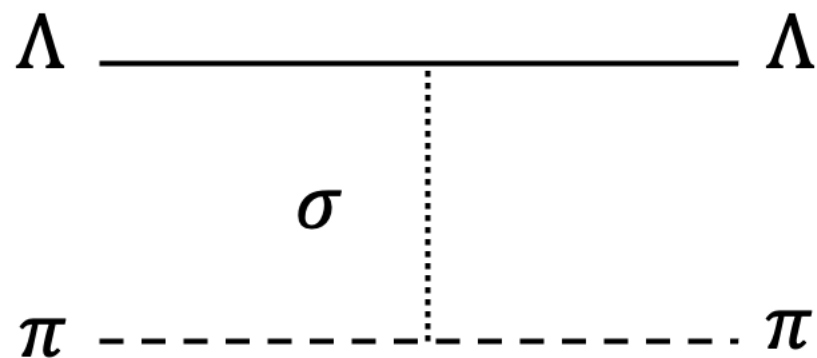
## 2. Spin flip cross section

$$\sigma_{\uparrow\downarrow} = \bar{\sigma} \left( \left( \frac{\sqrt{2}}{3} \right)^2 + \left( \frac{\sqrt{2}}{3} \right)^2 \right) = \frac{4}{9} \bar{\sigma}$$

$$\begin{aligned} \sigma_{\uparrow\downarrow} &= \bar{\sigma} \sum_{m, M} \left| \left\langle \frac{1}{2} - \frac{1}{2} 1 m \middle| \frac{3}{2} M \right\rangle \left\langle \frac{3}{2} M \middle| \frac{1}{2} \frac{1}{2} 1 m \right\rangle \right|^2 \\ &= \bar{\sigma} \left[ \left| \left\langle \frac{1}{2} - \frac{1}{2} 1 1 \middle| \frac{3}{2} \frac{1}{2} \right\rangle \left\langle \frac{3}{2} \frac{1}{2} \middle| \frac{1}{2} \frac{1}{2} 1 0 \right\rangle \right|^2 \right. \\ &\quad \left. + \left| \left\langle \frac{1}{2} - \frac{1}{2} 1 0 \middle| \frac{3}{2} - \frac{1}{2} \right\rangle \left\langle \frac{3}{2} - \frac{1}{2} \middle| \frac{1}{2} \frac{1}{2} 1 - 1 \right\rangle \right|^2 \right] \\ &= \bar{\sigma} \left[ \left( \sqrt{\frac{1}{3}} \sqrt{\frac{2}{3}} \right)^2 + \left( \sqrt{\frac{2}{3}} \sqrt{\frac{1}{3}} \right)^2 \right] = \frac{4}{9} \bar{\sigma} \quad (4) \end{aligned}$$

# $\Lambda + \pi$ cross section

## t-channel with $\sigma$

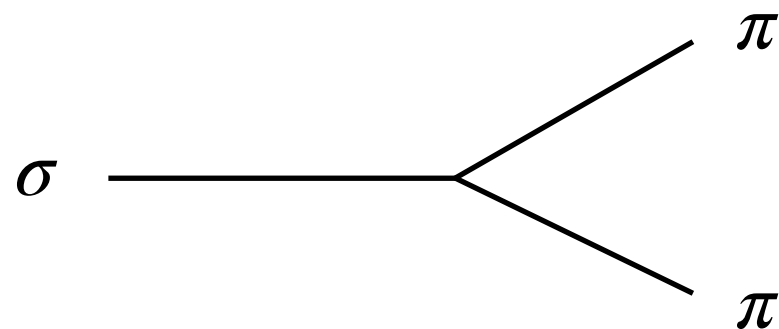


For quark counting rule,

$$g_{\Lambda\Lambda\sigma} = g_{NN\sigma} * 2/3 = 10.6 * 2/3 = 7.07$$

Advanced in Nuclear Physics Vol19, (p.226), Machleidt

## $\sigma$ decay width and coupling constant



$$i\mathcal{M} = ig_2$$

$$\mathcal{L}_{\sigma\Lambda\Lambda} = g_{\sigma\Lambda\Lambda}\bar{\Lambda}\Lambda\sigma$$

$$\mathcal{L}_{\sigma\pi\pi} = g_{\sigma\pi\pi}\sigma\pi\pi.$$

$$\Gamma = \frac{g_2^2}{8\pi s} |\vec{p}_{cm}|$$

$$= \frac{g_2^2}{8\pi s} \frac{[(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)]^{1/2}}{2\sqrt{s}}$$

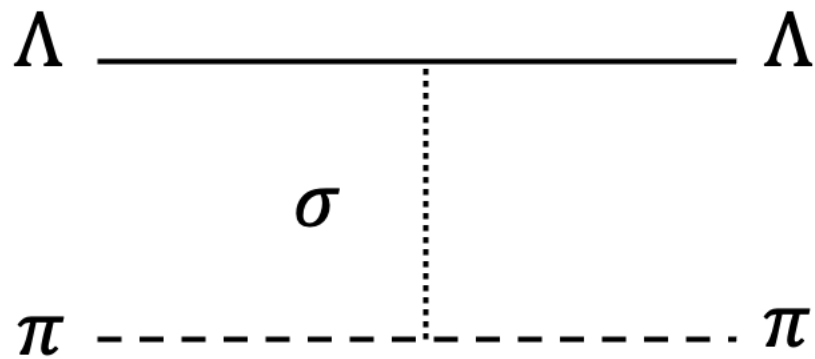
$$= \frac{g_2^2}{8\pi m_\sigma^2} \frac{\sqrt{m_\sigma^2 - 4m_\pi^2}}{2}$$

In Fig. 64.3 we read the range of pole positions for the  $f_0(500)$ , namely,

$$\sqrt{s_{\text{Pole}}^\sigma} = (400 - 550) - i(200 - 350) \text{ MeV}.$$

# $\Lambda + \pi$ cross section

## Spin averaged t-channel cross section



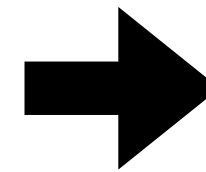
$$\mathcal{L}_{\sigma\Lambda\Lambda} = g_{\sigma\Lambda\Lambda} \bar{\Lambda} \Lambda \sigma$$

$$\mathcal{L}_{\sigma\pi\pi} = g_{\sigma\pi\pi} \sigma \pi \pi.$$

$$F(q^2) = \frac{\Lambda^2 + t}{\Lambda^2 + m_\sigma^2}, \quad \text{with } \Lambda = 1.8 \text{ GeV}$$

$$i\mathcal{M}_t = g_{\Lambda\Lambda\sigma} g_{\sigma\pi\pi} \bar{u}(p_3) \frac{i}{t - m_\sigma^2 + im_\sigma \Gamma_\sigma(s)} u(p_1),$$

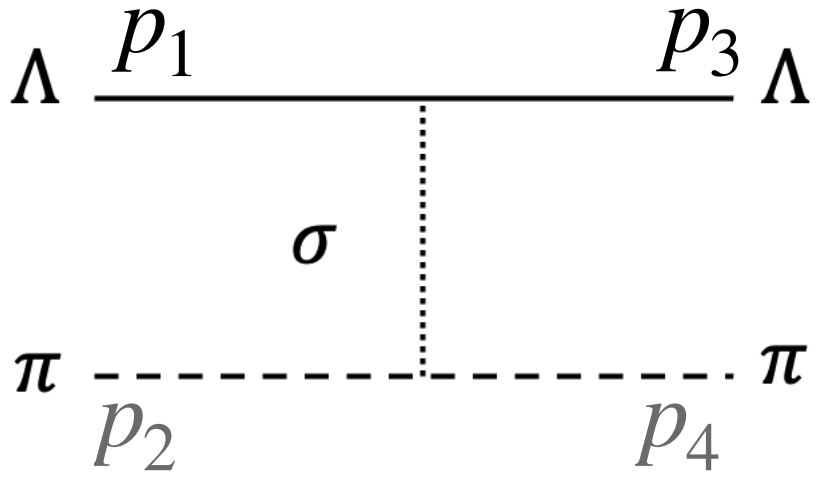
$$\begin{aligned} |\overline{\mathcal{M}_t}|^2 &= \frac{g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2}{2} \frac{\text{Tr} \left[ (\not{p}_3 + m_\Lambda)(\not{p}_1 + m_\Lambda) \right]}{(t - m_\sigma^2)^2 + m_\sigma^2 \Gamma_\sigma(s)^2}, \\ &= 2g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2 \frac{p_1 \cdot p_3 + m_\Lambda^2}{(t - m_\sigma^2)^2 + m_\sigma^2 \Gamma_\sigma(s)^2}, \\ &= g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2 \frac{4m_\Lambda^2 - t}{(t - m_\sigma^2)^2 + m_\sigma^2 \Gamma_\sigma(s)^2}. \end{aligned}$$



$$\frac{d\sigma}{d\Omega} = \frac{1}{63\pi^2 s} |\overline{\mathcal{M}_t}|^2 F^4(t).$$

# $\Lambda + \pi$ cross section

## spin flip & non-flip cross section



$$i\mathcal{M}_t = g_{\Lambda\Lambda\sigma} g_{\sigma\pi\pi} \bar{u}(p_3) \frac{i}{t - m_\sigma^2 + im_\sigma\Gamma_\sigma(s)} u(p_1),$$

### 1. Spin non-flip cross section

$$\begin{aligned} \bar{u}_{s_f}(p_3) u_{s_i}(p_1) &= (E_\Lambda + m_\Lambda) \left( \chi_{s_f}^\dagger, \chi_{s_f}^\dagger \frac{\mathbf{p}_3 \cdot \boldsymbol{\sigma}}{E_\Lambda + m_\Lambda} \right) \\ &\quad \times \left( -\frac{\chi_{s_i}}{E_\Lambda + m_\Lambda}, \frac{\mathbf{p}_1 \cdot \boldsymbol{\sigma}}{E_\Lambda + m_\Lambda} \chi_{s_i} \right) \\ &= (E_\Lambda + m_\Lambda) \left\{ \left[ 1 - \frac{\mathbf{p}_3 \cdot \mathbf{p}_1}{(E_\Lambda + m_\Lambda)^2} \right] \chi_{s_f}^\dagger \chi_{s_i} \right. \\ &\quad \left. - i \frac{\mathbf{p}_3 \times \mathbf{p}_1}{(E_\Lambda + m_\Lambda)^2} \chi_{s_f}^\dagger \boldsymbol{\sigma} \chi_{s_i} \right\}. \end{aligned} \quad (13)$$

$$\chi_+ = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \end{pmatrix}, \quad \chi_- = \begin{pmatrix} \sin \frac{\theta}{2} \\ -e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix}.$$

Using these expressions and  $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ , one then has

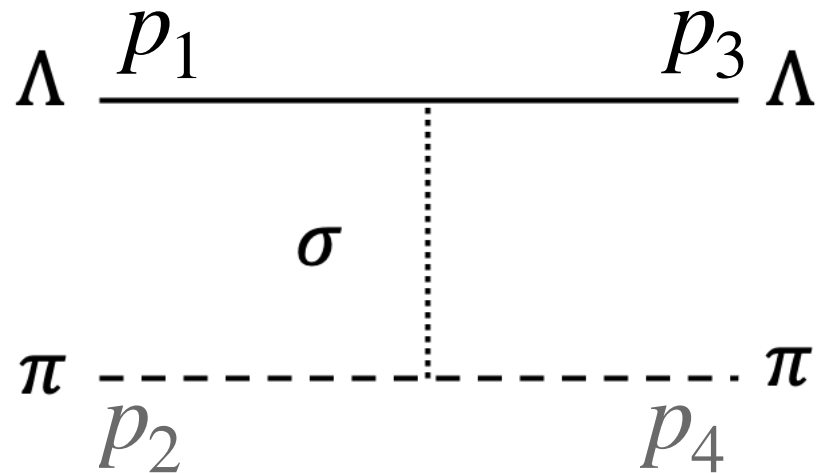
$$\chi_+^\dagger \sigma_y \chi_+ = 0, \quad \chi_-^\dagger \sigma_y \chi_+ = -ie^{i\phi}, \quad (15)$$

**➔** 
$$\bar{u}_+(p_3) u_+(p_1) = (E_\Lambda + m_\Lambda) \left[ 1 - \frac{p^2 \cos \theta}{(E_\Lambda + m_\Lambda)^2} \right],$$



# $\Lambda + \pi$ cross section

## spin flip & non-flip cross section



$$i\mathcal{M}_t = g_{\Lambda\Lambda\sigma} g_{\sigma\pi\pi} \bar{u}(p_3) \frac{i}{t - m_\sigma^2 + im_\sigma\Gamma_\sigma(s)} u(p_1),$$

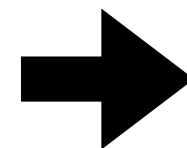
## 2. Spin flip cross section

$$\begin{aligned} \bar{u}_{s_f}(p_3) u_{s_i}(p_1) &= (E_\Lambda + m_\Lambda) \left( \chi_{s_f}^\dagger, \chi_{s_f}^\dagger \frac{\mathbf{p}_3 \cdot \boldsymbol{\sigma}}{E_\Lambda + m_\Lambda} \right) \\ &\quad \times \left( -\frac{\chi_{s_i}}{E_\Lambda + m_\Lambda}, \frac{\mathbf{p}_1 \cdot \boldsymbol{\sigma}}{E_\Lambda + m_\Lambda} \chi_{s_i} \right) \\ &= (E_\Lambda + m_\Lambda) \left\{ \left[ 1 - \frac{\mathbf{p}_3 \cdot \mathbf{p}_1}{(E_\Lambda + m_\Lambda)^2} \right] \chi_{s_f}^\dagger \chi_{s_i} \right. \\ &\quad \left. - i \frac{\mathbf{p}_3 \times \mathbf{p}_1}{(E_\Lambda + m_\Lambda)^2} \cdot \chi_{s_f}^\dagger \boldsymbol{\sigma} \chi_{s_i} \right\}. \end{aligned} \quad (13)$$

$$\chi_+ = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \end{pmatrix}, \quad \chi_- = \begin{pmatrix} \sin \frac{\theta}{2} \\ -e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix}.$$

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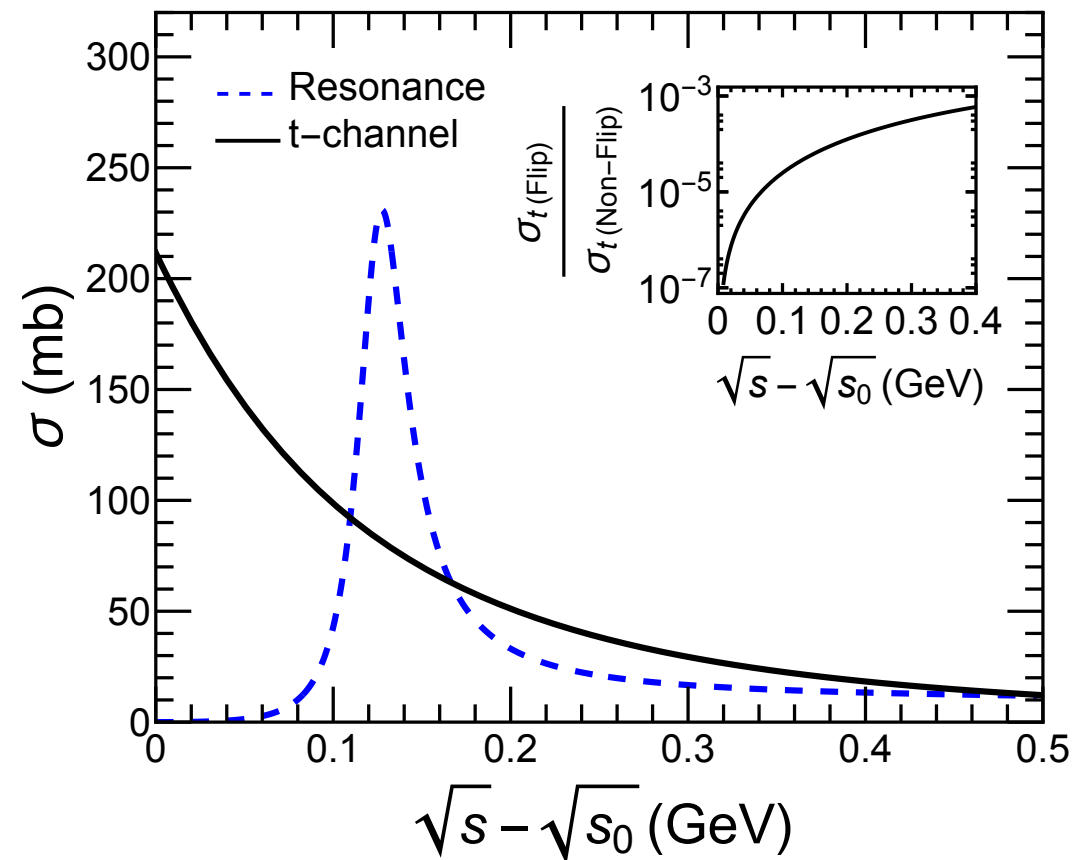
$$\chi_+^\dagger \sigma_y \chi_+ = 0, \quad \chi_-^\dagger \sigma_y \chi_+ = -ie^{i\phi}, \quad (15)$$



$$\bar{u}_-(p_3) u_+(p_1) = \frac{e^{i\phi} p^2 \sin \theta}{E_\Lambda + m_\Lambda}.$$

# $\Lambda + \pi$ cross section

## spin flip cross section of t-channel

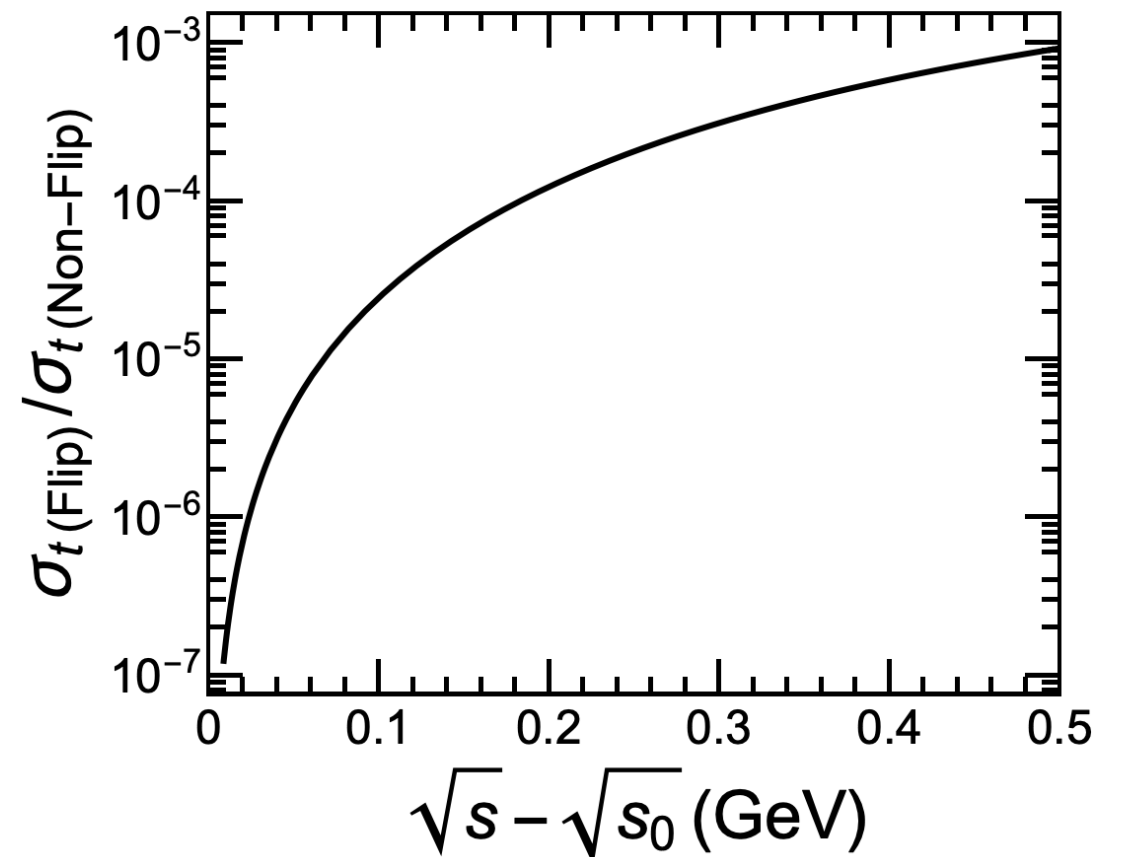


Resonance cross section is the majority of  $\Lambda$  spin flip cross section.  
t-channel is negligible for spin flip case

## Spin flip & non-flip amplitude

$$|\mathcal{M}_{t++}|^2 = \frac{g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2}{(t - m_\sigma^2)^2 + m_\sigma^2 \Gamma_\sigma(s)^2} \times (E_\Lambda + m_\Lambda)^2 \left[ 1 - \frac{p^2 \cos \theta}{(E_\Lambda + m_\Lambda)^2} \right]^2. \quad (17)$$

$$|\mathcal{M}_{t+-}|^2 = \frac{g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2}{(t - m_\sigma^2)^2 + m_\sigma^2 \Gamma_\sigma(s)^2} \frac{p^4 \sin^2 \theta}{(E_\Lambda + m_\Lambda)^2} \quad (18)$$





# Result

## Kinetic equation

$$\frac{dN_{\Lambda\uparrow}}{d\tau} = -\langle\sigma_{\Lambda\uparrow\pi\rightarrow\Lambda\downarrow\pi}v\rangle n_{\pi}N_{\Lambda\uparrow} + \langle\sigma_{\Lambda\downarrow\pi\rightarrow\Lambda\uparrow\pi}v\rangle n_{\pi}N_{\Lambda\downarrow}, \quad (20)$$

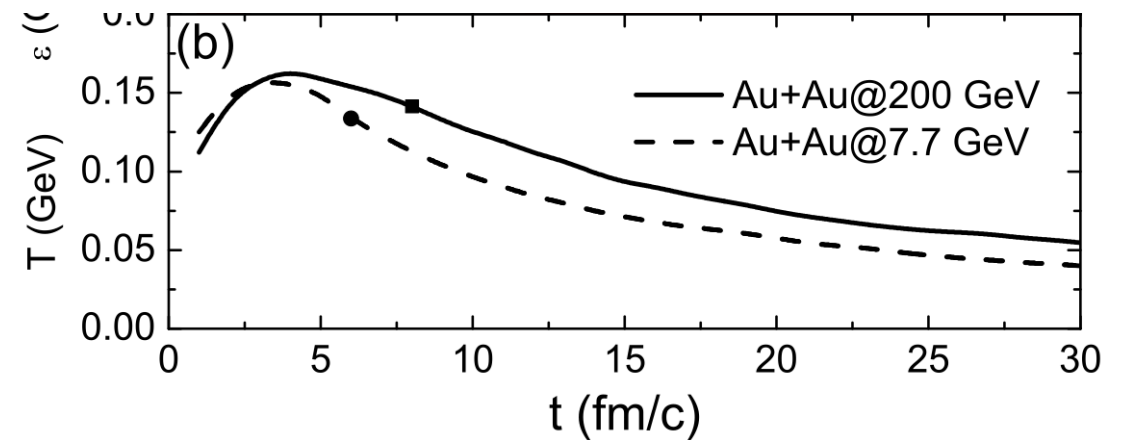
$$\frac{dN_{\Lambda\downarrow}}{d\tau} = \langle\sigma_{\Lambda\uparrow\pi\rightarrow\Lambda\downarrow\pi}v\rangle n_{\pi}N_{\Lambda\uparrow} - \langle\sigma_{\Lambda\downarrow\pi\rightarrow\Lambda\uparrow\pi}v\rangle n_{\pi}N_{\Lambda\downarrow}, \quad (21)$$

$$\Delta N(\tau) = N_{\Lambda\uparrow}(\tau) - N_{\Lambda\downarrow}(\tau)$$

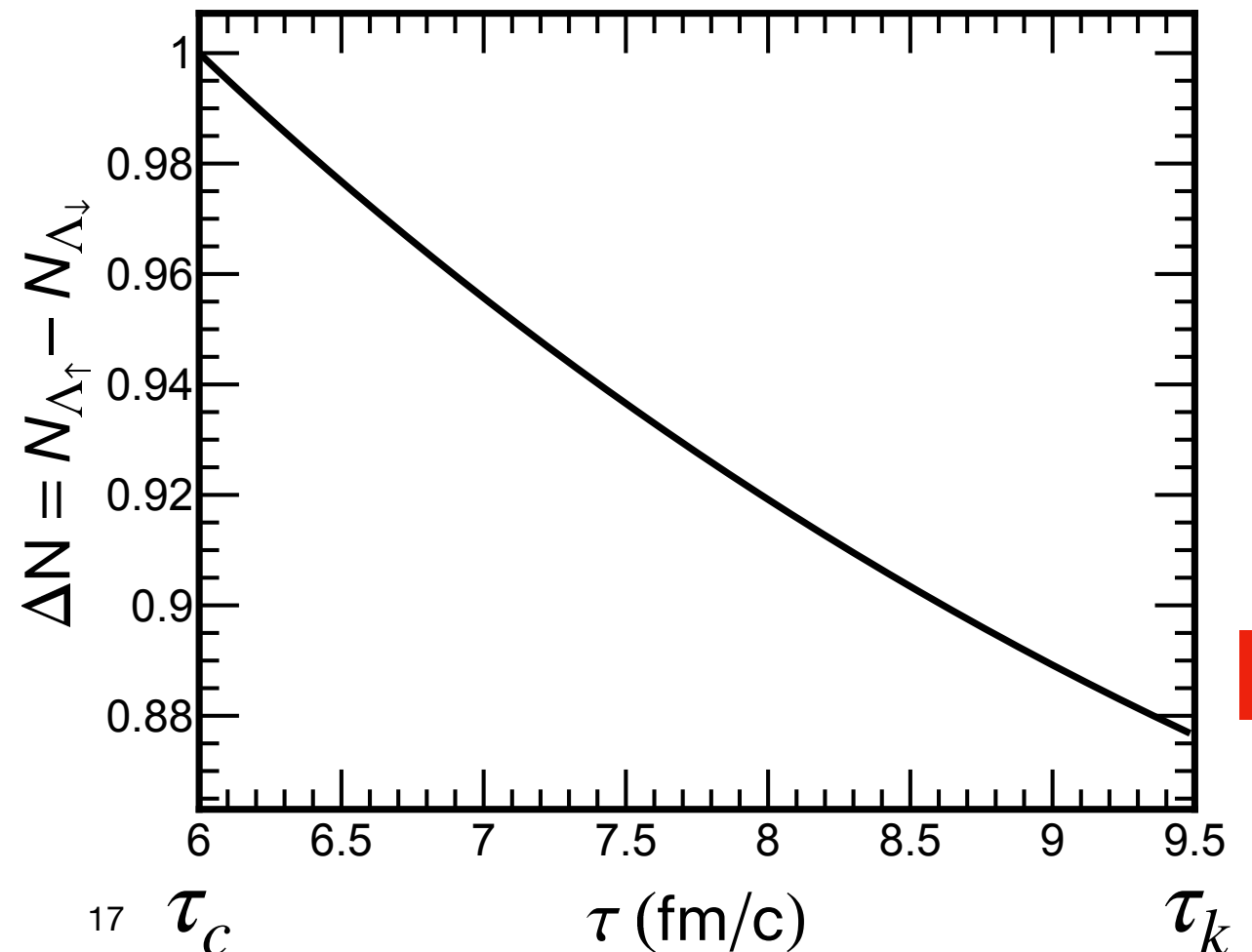
$$\frac{d\Delta N}{d\tau} = -2 \langle\sigma v\rangle \Delta N$$

Result show that  $\Lambda$  scattering during hadronic phase do not much affect to disappear the initial polarization

J. Xu and C. M. Ko, Phys. Lett. B 772, 290 (2017), arXiv:1704.04934 [nucl-th]



Au+Au 7.7 GeV



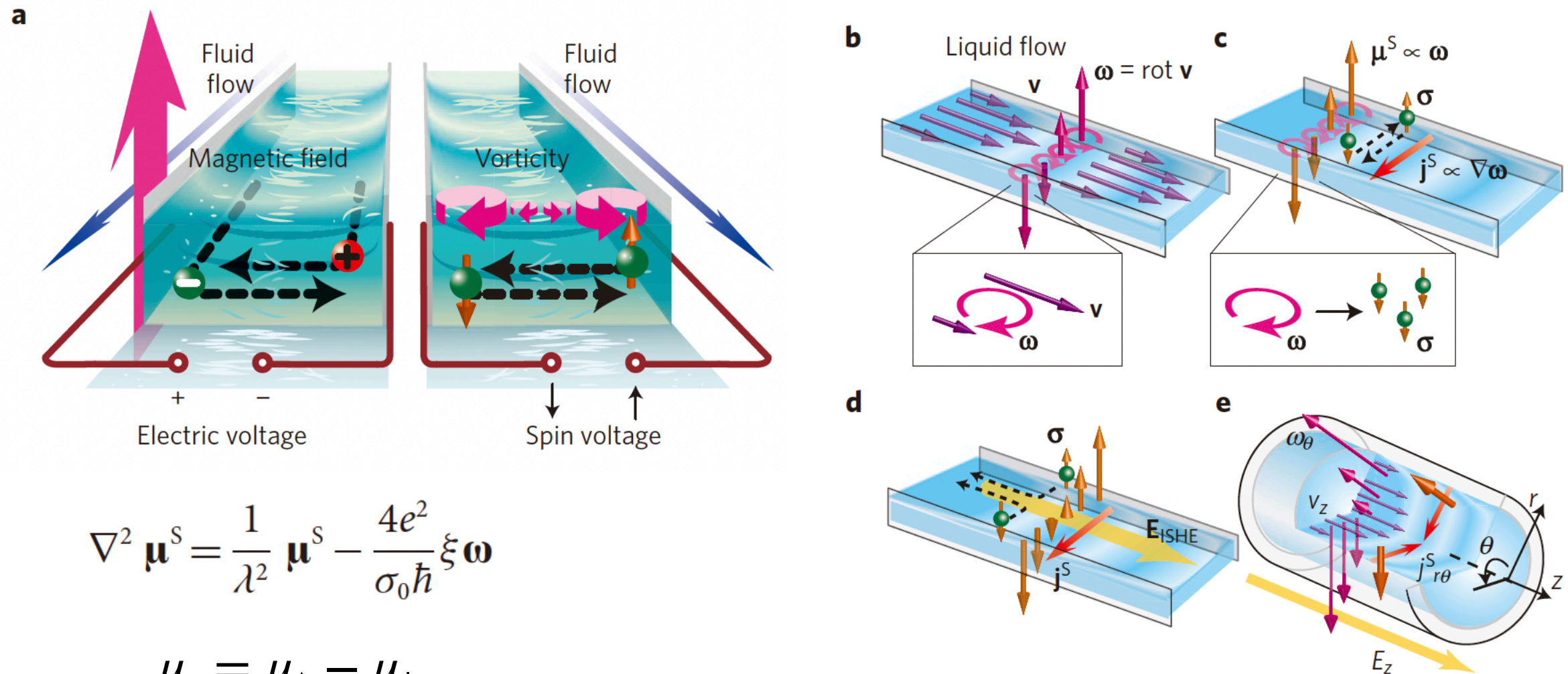
# Summary

- $\Sigma^*$  resonance cross section is important for estimating the spin non-flip  $\Lambda$
- The  $\Lambda$  spin polarization decrease by only about 12% during the hadronic stage of heavy ion collisions (Au+Au at  $\sqrt{s} = 7.7$  GeV)

# Appendix

# Vorticity & Chiral Magnetic Effect

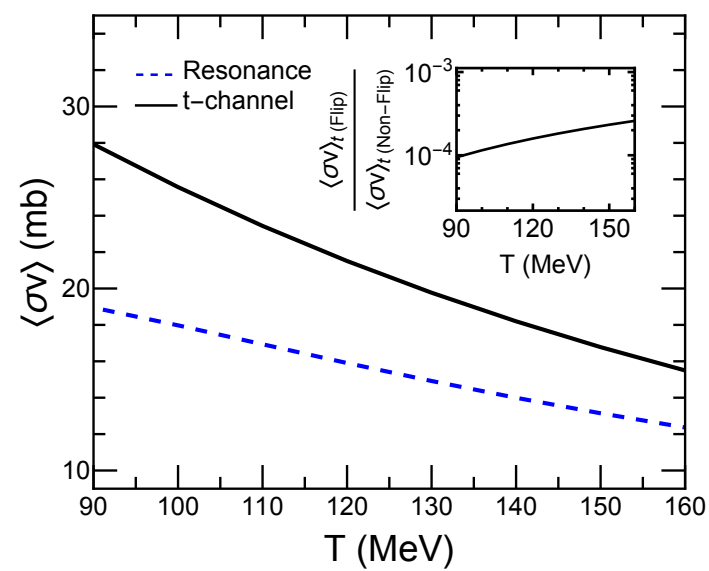
Takahashi, R., Matsuo, M., Ono, M. *et al.* Spin hydrodynamic generation. *Nature Phys* **12**, 52–56 (2016). <https://doi.org/10.1038/nphys3526>



The vorticity  $\boldsymbol{\omega} = \text{rot } \mathbf{v}$  represents local mechanical rotation of a fluid, where  $\mathbf{v}$  is the fluid velocity.

The vorticity acts as a spin-current source

- The effect of  $\Lambda$  scattering by nucleons, whose number is only about a factor of three smaller than the pion number in AuAu at 7.7 GeV



# Spinor 1/2

## 1. Weyl representation (Peskin, Schwartz)

$$\gamma_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, u(0) = \sqrt{m} \begin{bmatrix} \xi \\ \xi \end{bmatrix}$$

$$u(p) = \begin{bmatrix} \sqrt{p \cdot \sigma} \xi \\ \sqrt{p \cdot \bar{\sigma}} \xi \end{bmatrix}$$

$$\begin{aligned} \sum_{s=1,2} u^s(p) \bar{u}^s(p) &= \sum_s \begin{pmatrix} \sqrt{p \cdot \sigma} \lambda^s \\ \sqrt{p \cdot \bar{\sigma}} \lambda^s \end{pmatrix} (\lambda^{s\dagger} \sqrt{p \cdot \bar{\sigma}}, \lambda^{s\dagger} \sqrt{p \cdot \sigma}) \\ &= \begin{pmatrix} \sqrt{p \cdot \sigma} \sqrt{p \cdot \bar{\sigma}} & \sqrt{p \cdot \sigma} \sqrt{p \cdot \sigma} \\ \sqrt{p \cdot \bar{\sigma}} \sqrt{p \cdot \bar{\sigma}} & \sqrt{p \cdot \bar{\sigma}} \sqrt{p \cdot \sigma} \end{pmatrix} \\ &= \begin{pmatrix} m & p \cdot \sigma \\ p \cdot \bar{\sigma} & m \end{pmatrix} \\ &= \not{\partial} \cdot p + m \end{aligned}$$

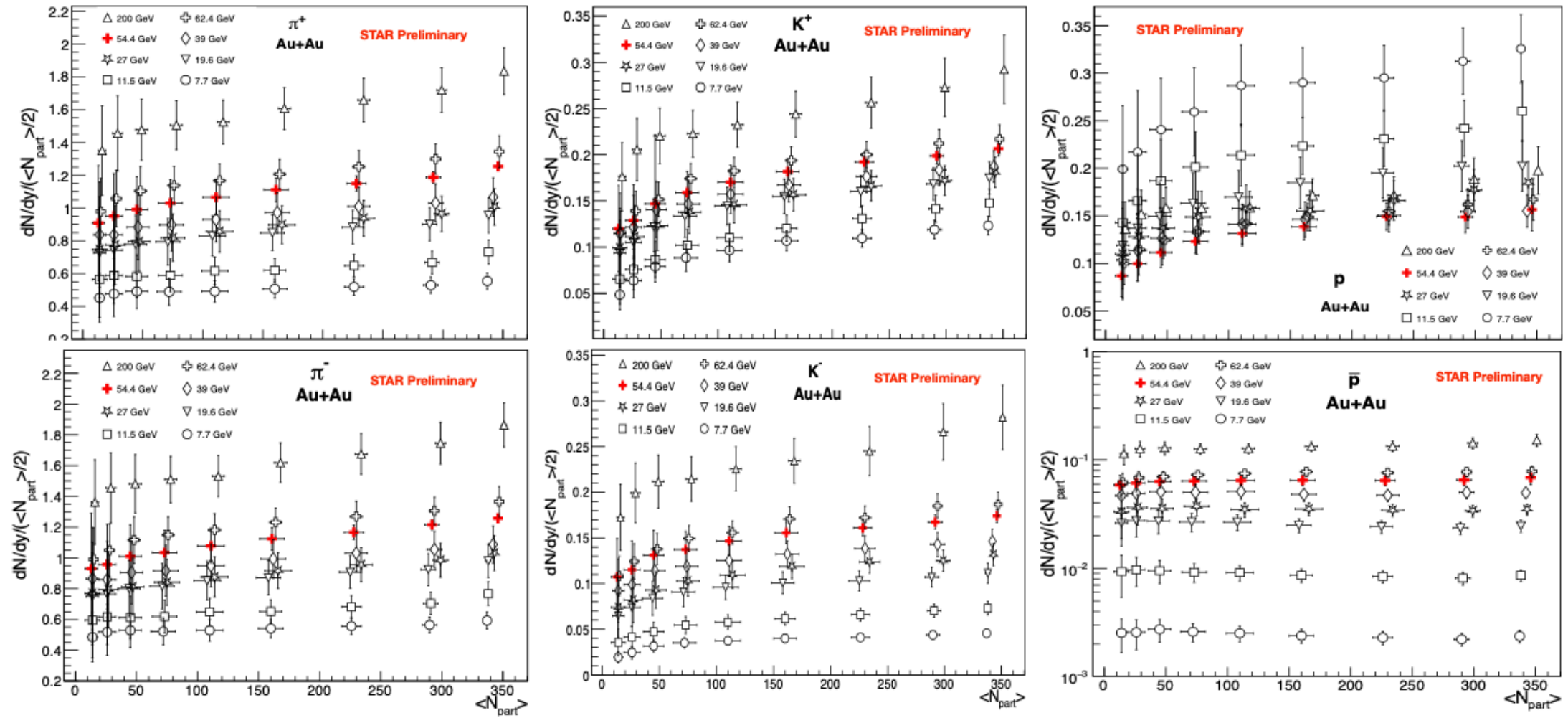
## 2. Dirac representation (Sakurai)

$$\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, u(0) = \begin{bmatrix} \xi \\ 0 \end{bmatrix}$$

$$u(p) = \sqrt{E+m} \begin{bmatrix} \xi \\ \frac{\vec{p} \cdot \vec{\sigma}}{E+m} \xi \end{bmatrix}$$

$$\begin{aligned} \sum_s u^s(p) \bar{u}^s(p) &= \begin{pmatrix} 1 \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \end{pmatrix} \begin{pmatrix} 1 & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & E+m \end{pmatrix} \times (E+m) \\ &= \begin{pmatrix} 1 & -\frac{\vec{\sigma} \cdot \vec{p}}{E+m} \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} & -\frac{|\vec{p}|^2}{(E+m)^2} \end{pmatrix} \times (E+m) = \begin{pmatrix} E+m & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & -\frac{|\vec{p}|^2}{(E+m)} \end{pmatrix} \\ &= \begin{pmatrix} E+m & -\vec{\sigma} \cdot \vec{p} \\ \vec{\sigma} \cdot \vec{p} & -E+m \end{pmatrix} \\ &= \not{\partial} \cdot p + m \end{aligned}$$

$\not{\partial} \cdot p + m = E \gamma^0 - \vec{p} \cdot \vec{\gamma} + m \mathbb{1}$   
 $= \begin{pmatrix} E+m & -\vec{p} \cdot \vec{\sigma} \\ \vec{p} \cdot \vec{\sigma} & -E+m \end{pmatrix}$



**Figure 2.** Centrality dependence of  $dN/dy$  normalized by  $\langle N_{\text{part}} \rangle / 2$  for  $\pi^\pm$ ,  $K^\pm$ , and  $p(\bar{p})$  at mid-rapidity ( $|y| < 0.1$ ) in Au+Au collisions at  $\sqrt{s_{NN}} = 54.4$  GeV. Errors shown are quadrature sums of statistical and systematic uncertainties.



# Nature 548, 62 (2017)

## Global $\Lambda$ hyperon polarization in nuclear collision, STAR

- Heavy ion collisions in non-central collisions make fluid with strong vortical structure, which is needed to understand fluid properly.
- The vortical structure is also of particular interest because the **restoration of fundamental symmetries of quantum chromodynamics** is expected to produce novel **physical effects** in the presence of strong vorticity.
  - \*No experimental indication of fluid vorticity in heavy ion collisions. Since **vorticity** represent **local rotational structure**, **S-L coupling** can lead to preferential orientation of particle spins along the angular momentum.

=> Measurement of an alignment between the global angular momentum and the spin of the emitted particles( $\Lambda$ ). => few cent of positive polarization

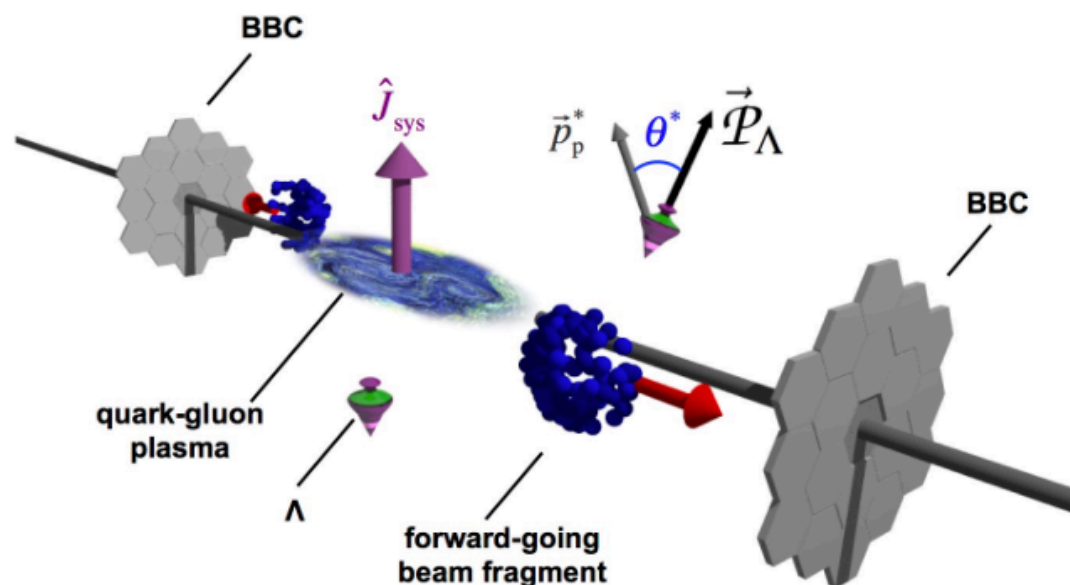


# Global $\Lambda$ polarization

L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

<https://www.star.bnl.gov/central/focus/LdbPola/>

- Due to the parity-violating nature of their weak decay, **Lambdas reveal** the direction of their spin by preferentially **emitting the daughter proton** along that direction
- the polarization of emitted particles is directly related to the vorticity - the curl of the flow field - of the fluid
- The coupling between mechanical rotation of a system and quantum spin of a particle



$\Lambda$  polarization:

new tool to study QGP and

relativistic Quantum fluid Vorticity

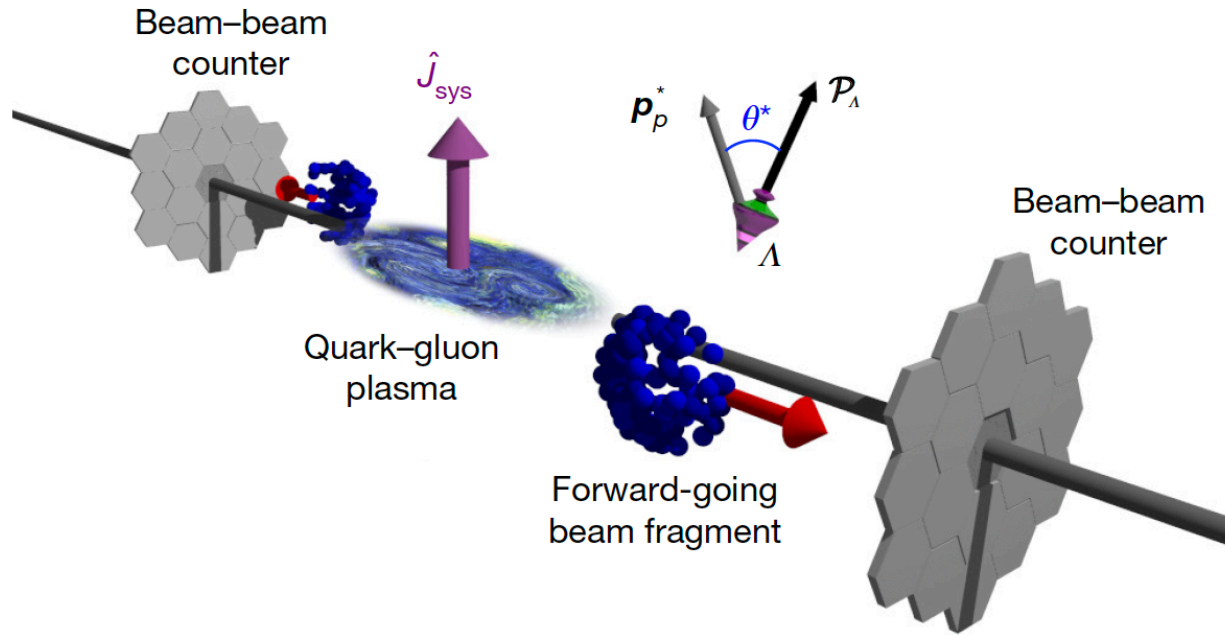
# Nature 548, 62 (2017)

## Global $\Lambda$ hyperon polarization in nuclear collision, STAR

- fluid  $\rightarrow$  low viscosity  $\rightarrow$  hydrodynamic theory
- RHIC  $\rightarrow$  quark confinement, the origin of hadron mass
- \*shear force introducing vorticity to the fluid
- The **vorticity** is a key ingredient in theories that predict observable effects associated with **chiral symmetry restoration**
- Spin-orbit coupling  $\rightarrow$  spin alignment along the vorticity in the local fluid cell, which, when averaged over the entire system, is parallel to  $\hat{J}_{sys}$ .
- **self-analyzing**: hyperon's decay process inherently contains information about its polarization. e.x.  $\Lambda \rightarrow p + \pi^-$ , the proton tends to be emitted along the spin direction of the parent  $\Lambda$ .

# Nature 548, 62 (2017)

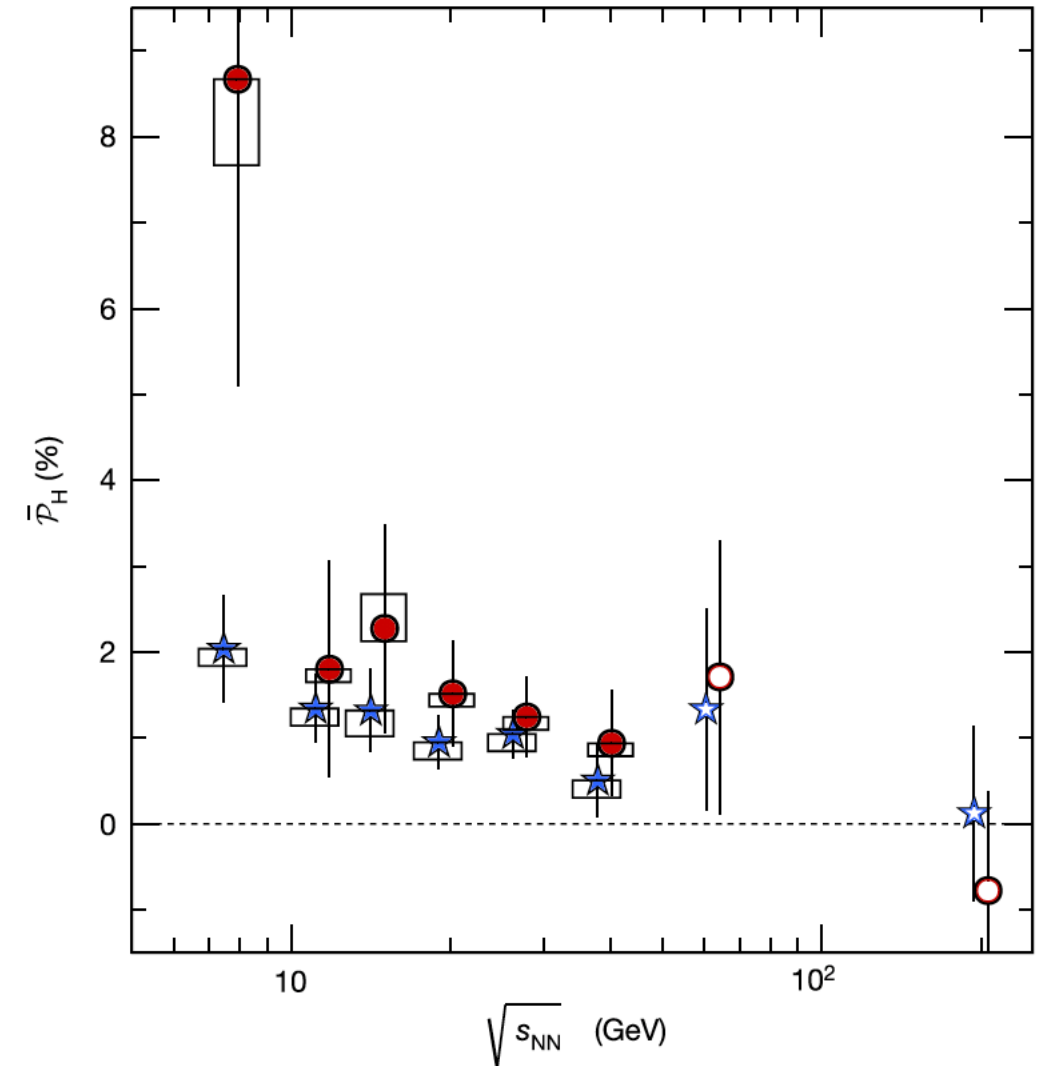
## Global $\Lambda$ hyperon polarization in nuclear collision, STAR



the hyperon rest frame

$$\frac{dN}{d \cos \theta^*} = \frac{1}{2} (1 + \alpha_H |\mathcal{P}_H| \cos \theta^*)$$

$\mathcal{P}_H$ : polarization vector, parallel to  $\hat{J}_{sys}$



**Figure 4 | The hyperon average polarization in Au + Au collisions.** The average polarization for  $\Lambda$  (blue stars) and  $\bar{\Lambda}$  (red circles) from 20–50% central collisions are plotted as a function of collision energy. Error bars represent statistical uncertainties only, while boxes represent systematic uncertainties. The results of the present study ( $\sqrt{s_{NN}} < 1$  GeV), indicated by filled symbols, are shown together with those reported earlier<sup>7</sup> for 62.4 GeV and 200 GeV collisions, indicated by open symbols and for which only statistical errors are plotted.

# Nature 548, 62 (2017)

## Global $\Lambda$ hyperon polarization in nuclear collision, STAR

- $\Lambda'$ ,  $\bar{\Lambda}'$ : 'primary' hyperons emitted directly from the fluid
- However, most of  $\Lambda$  and  $\bar{\Lambda}$  hyperons at these collisions are not primary, but also decay products from heavier particles (e.x.  $\Sigma^{*,+} \rightarrow \Lambda + \pi^+$ ), which themselves would be polarized by the fluid
- RHIC are expected to produce intense magnetic fields parallel to  $\hat{J}_{sys}$ . Coupling between the field and the intrinsic magnetic moments of emitted particles may induce a larger polarization for  $\bar{\Lambda}$  than for  $\Lambda$ . This is not consistent with our observable.

# Eur. Phys. J. C (2017) 77:213, Iu. Karpenko

Study of  $\Lambda$  polarization in relativistic nuclear collisions at  $\sqrt{s_{NN}} = 7.7 - 200$  GeV

- Hydrodynamic theory
- abstract: UrQMD+vHLE, the mean polarization of  $\Lambda$  in the out of plane direction is predicted to decrease rapidly with collision energy from a top value of about 2% at the lowest energy examined. polarization signal-thermal vorticity- estimate the feed-down contribution to  $\Lambda$  polarization due to the decay of higher mass hyperons.
- polarized quarks  $\rightarrow$  polarized hadrons