Hadron–hadron interaction femtoscopy through correlation analysis using the CATS framework

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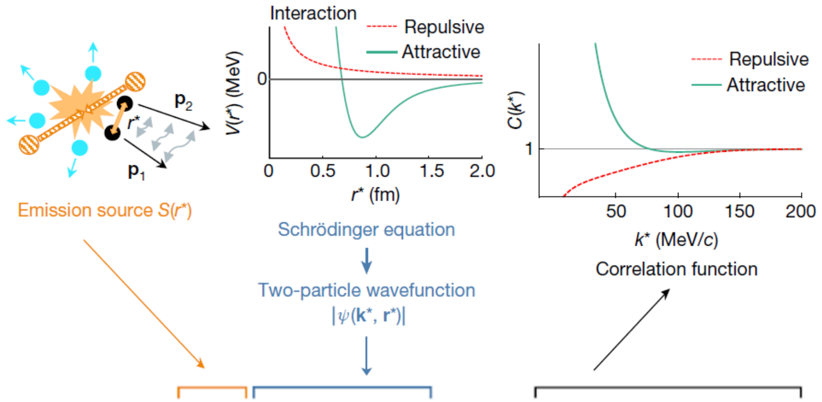
Effective interactions between two hadrons

- Theories & Phenomenology
  - QCD: the fundamental theory
  - Argonne V18 potential (for NN interaction)
  - Boson exchange models
  - Extended-soft-core model
  - Lattice QCD calculation
- Experiments
  - Scattering experiments
    - **Difficult or impossible for unstable hadrons**
    - HQ measurements exist only for hadrons containing u & d quarks
  - **Correlation function analysis** in nuclear collisions
    - Energy and colliding particle dependence of hadron production

Femtoscopy via correlation analysis



$$C(k^*) = \int S(r^*)|\psi(\mathbf{k}^*, \mathbf{r}^*)|^2 d^3r^* = \xi(k^*)\frac{N_{same}(k^*)}{N_{mixed}(k^*)}$$

* ALICE Collaboration, *Nature* **588**, 232 - 238 (2020) (DOI:10.1038/s41586-020-3001-6)

CATS framework

- CATS: Correlation Analysis Tool using Schrödinger equation
- Useful for femtoscopic analysis in non-relativistic regions (approx. $m > 500\ \mathrm{MeV}/c$)

* D. L. Mihaylov et al., *Eur. Phys. J. C* (2018) 78:394 (DOI: 10.1140/epjc/s10052-018-5859-0)
* Software: https://www.ph.nat.tum.de/denseandstrange/publications/software/

Two-particle correlation function

- Definition of the two-particle correlation function

$$C(\mathbf{p}_1, \mathbf{p}_2) \equiv \frac{P(\mathbf{p}_1, \mathbf{p}_2)}{P(\mathbf{p}_1)P(\mathbf{p}_2)}$$

  - $P(\mathbf{p}_1, \mathbf{p}_2)$, $P(\mathbf{p}_{1,2})$: Lorentz invariant spectra

- Correlation in the pair rest frame from approximation

$$C(\mathbf{k}^*) = \int S(\mathbf{r}^*)|\psi(\mathbf{k}^*, \mathbf{r}^*)|^2 \, d^3r$$

  - $S(\mathbf{r}^*)$: source function, $\psi(\mathbf{k}^*, \mathbf{r}^*)$: relative wave function

- Reformulate into experimentally accessible quantities

$$C(k^*) = \xi(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

  - $N(k^*)$: k^* distribution of hadron pairs produced in the same or different collisions
  - $\xi(k^*)$: corrections for experimental effect

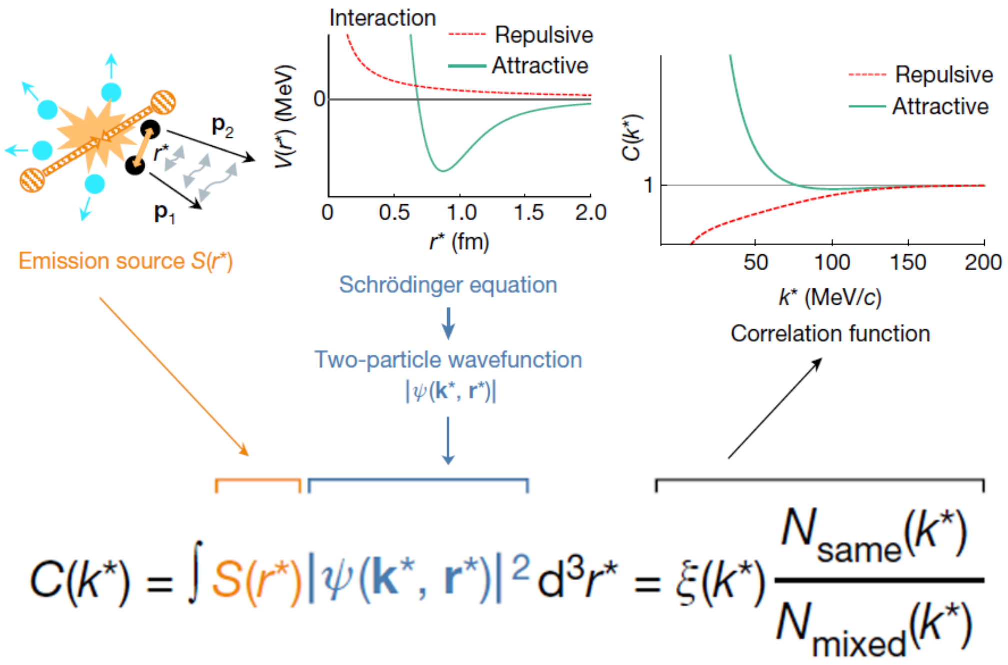
→ $C(k^*) = \int S(\mathbf{r}^*)|\psi(\mathbf{k}^*, \mathbf{r}^*)|^2 \, d^3r = \xi(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$

Femtoscopy via correlation analysis

- From the $C(k^*)$ formula, if
  - Source $S(r^*)$ size is small enough ($r_0 \approx 1$ fm),
  - Model is accurate in short-range ($r^* = 0 - 2$ fm),

  we can determine in detail the short-range interaction.

→ **Femtoscopy** (range of 1 fm scale)



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CATS framework

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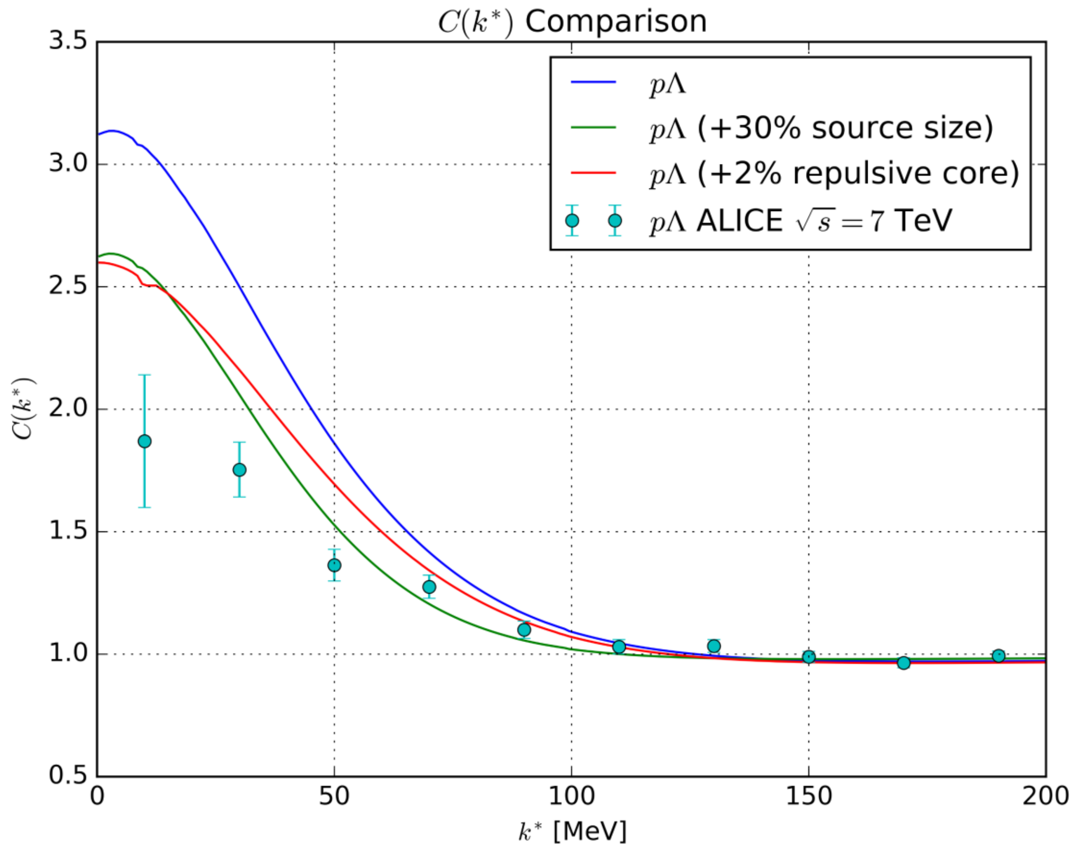
  - Software: https://www.ph.nat.tum.de/denseandstrange/publications/software/
- Can be used for:
  - The various potentials in analytic form
  - Both of the analytic & transport sources
  - Approximately non-relativistic regions
    - Lower limit of $m > 500$ MeV, light mesons are off-limit
- Easy to use (instruction included in examples codes)

```
//you can define any potential function you want and pass it a
//the trick is to leave the first 2 parameters as placeholders (u
//in this example the potential function does not get any para
//N.B. the array you pass to CATS should always have a min. si:
double PotPars[3];
//the 0,0 means that we set the 0th channel, l=0 (1S0)
Kitty.SetShortRangePotential(0,0,ReidPotential1S0,PotPars);
//the 1,1 means that we set the 1st channel, l=1 (3PX)
Kitty.SetShortRangePotential(1,1,ReidPotential3P,PotPars);

//this is where the magic happens - we run CATS and all releva
Kitty.KillTheCat();
```

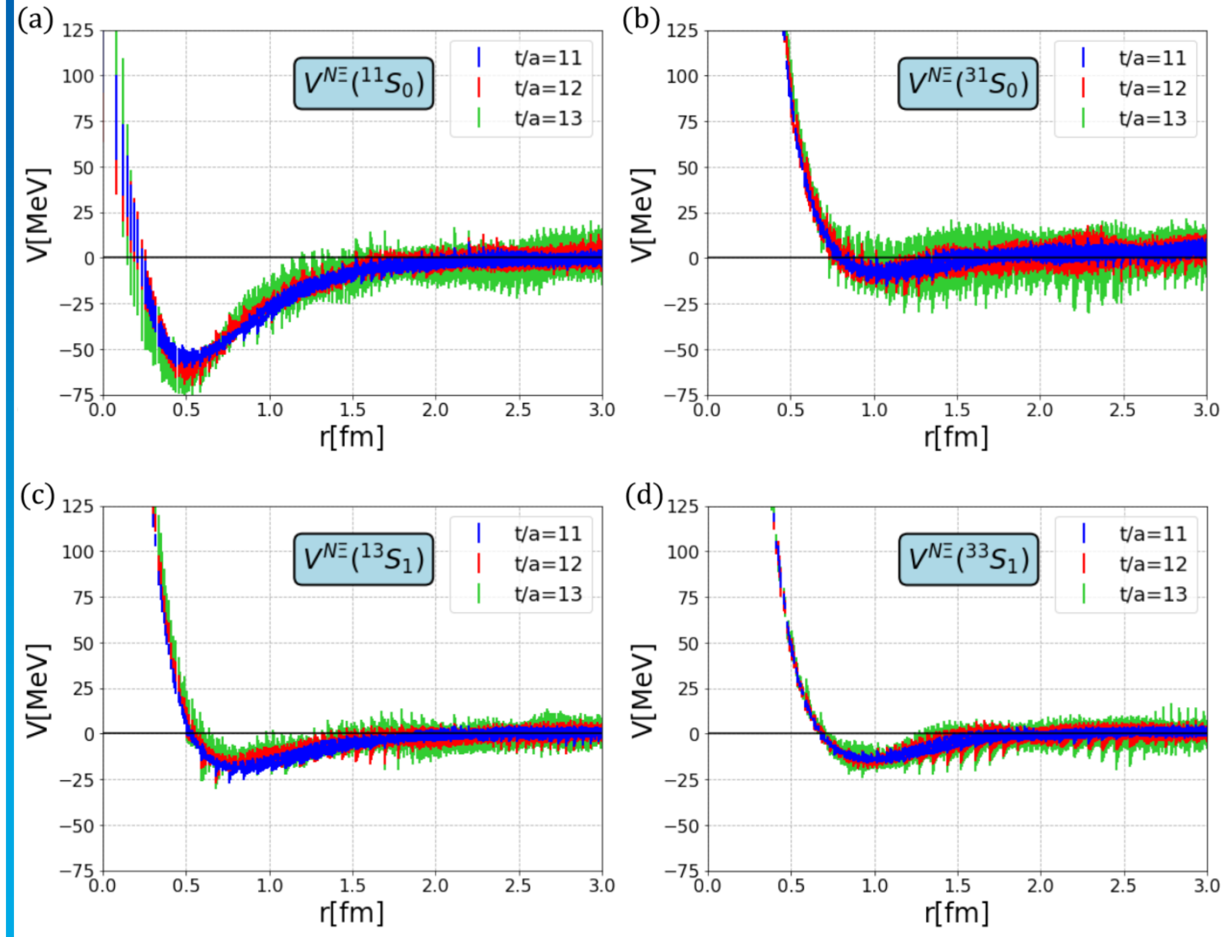
$C(k^*)$ from example code & comparison

- In example code, they used Usmani potential for $p - \Lambda$



HAL QCD – S-wave NΞ potentials

- Channels C: ${}^{11}S_0$, ${}^{31}S_0$, ${}^{13}S_1$, ${}^{33}S_1$



* K. Sasaki et al., *Nuclear Physics A* 998 (2020), 121737 (DOI: 10.1016/j.nuclphysa.2020.121737)

Analytic form with fitting

- The analytic form (fit function) of $V^{N\Xi}(C)(r)$

$$V^{N\Xi}(C)(r) = \sum_{i=1}^{3} \alpha_i(C) e^{-\frac{r^2}{\beta_i^2}} + \lambda_1(C)\mathcal{Y}(\rho_1, m_\pi, r) + \lambda_2(C)[\mathcal{Y}(\rho_2, m_\pi, r)]^2$$

$$\mathcal{Y}(\rho, m, r) \equiv \left(1 - e^{-r^2/\rho^2}\right)\frac{e^{-mr}}{r}, \qquad m_\pi = 146 \text{ MeV} \ \text{(fixed)}.$$

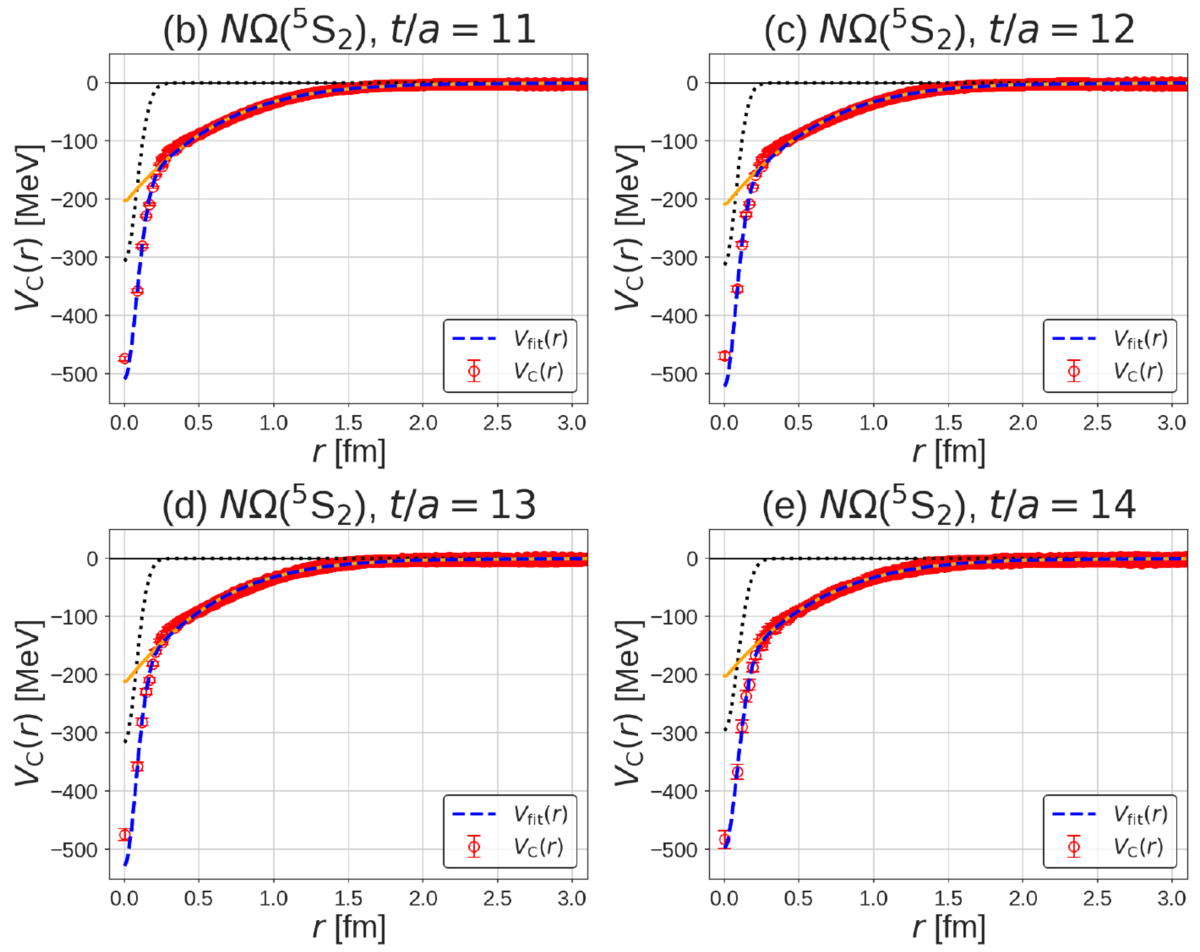
- Fitted parameters for $V^{N\Xi}$ with $t/a = 12$

$t/a = 12$	Gauss-1 α_1	Gauss-2 α_2	Gauss-3 α_3	Yukawa λ_1	$[\text{Yukawa}]^2$ λ_2
${}^{11}S_0$	-81.3(54.3)	171.1(59.1)	4.9(27.3)	-12.8(2.2)	-97.3(9.6)
${}^{31}S_0$	1677.2(90.1)	991.3(62.7)	290.8(43.2)	4.3(7)	-97.3(9.6)
${}^{13}S_1$	449.2(52.5)	348.9(31.8)	110.3(22.3)	4.3(7)	-97.3(9.6)
${}^{33}S_1$	849.5(53.4)	653.9(32.7)	210.8(35.9)	-1.4(2)	-97.3(9.6)
	β_1	β_2	β_3	ρ_1	ρ_2
	0.124(3)	0.241(12)	0.533(22)	0.136(22)	0.603(48)

* K. Sasaki et al., *Nuclear Physics A* 998 (2020), 121737 (DOI: 10.1016/j.nuclphysa.2020.121737)

HAL QCD - S-wave $N\Omega$ potentials

- $^5S_2$ channel only (elastic)



* T. Iritani et al., *Physics Letters B* 792 (2019), 284-289 (DOI: 10.1016/j.physletb.2019.03.050)

Analytic form with fitting

- The analytic form (fit function) of $V^{N\Omega}(\,^5S_2)(r)$

$$V^{N\Omega}(\,^5S_2)(r) = b_1 e^{-b_2 r^2} + b_3\left(1 - e^{-b_4 r^2}\right)\left(\frac{e^{-m_\pi r}}{r}\right)^2,$$

$$m_\pi = 146\ \text{MeV}\ \ (\text{fixed}).$$

- Strong coupling to the octet-octet channels for the $^3S_1$ channel
  → Consider $^5S_2 + \,^3S_1$ as Inelastic case

- Fitted parameters for $V^{N\Omega}(\,^5S_2)$

t/a	11	12	13	14
b_1 [MeV]	−306.5(5.5)	−313.0(5.3)	−316.7(9.4)	−296(18)
b_2 [fm^{-2}]	73.9(4.4)	81.7(5.4)	81.9(8.4)	64(16)
b_3 [MeV·fm^2]	−266(32)	−252(27)	−237(43)	−272(109)
b_4 [fm^{-2}]	0.78(11)	0.85(10)	0.91(18)	0.76(34)

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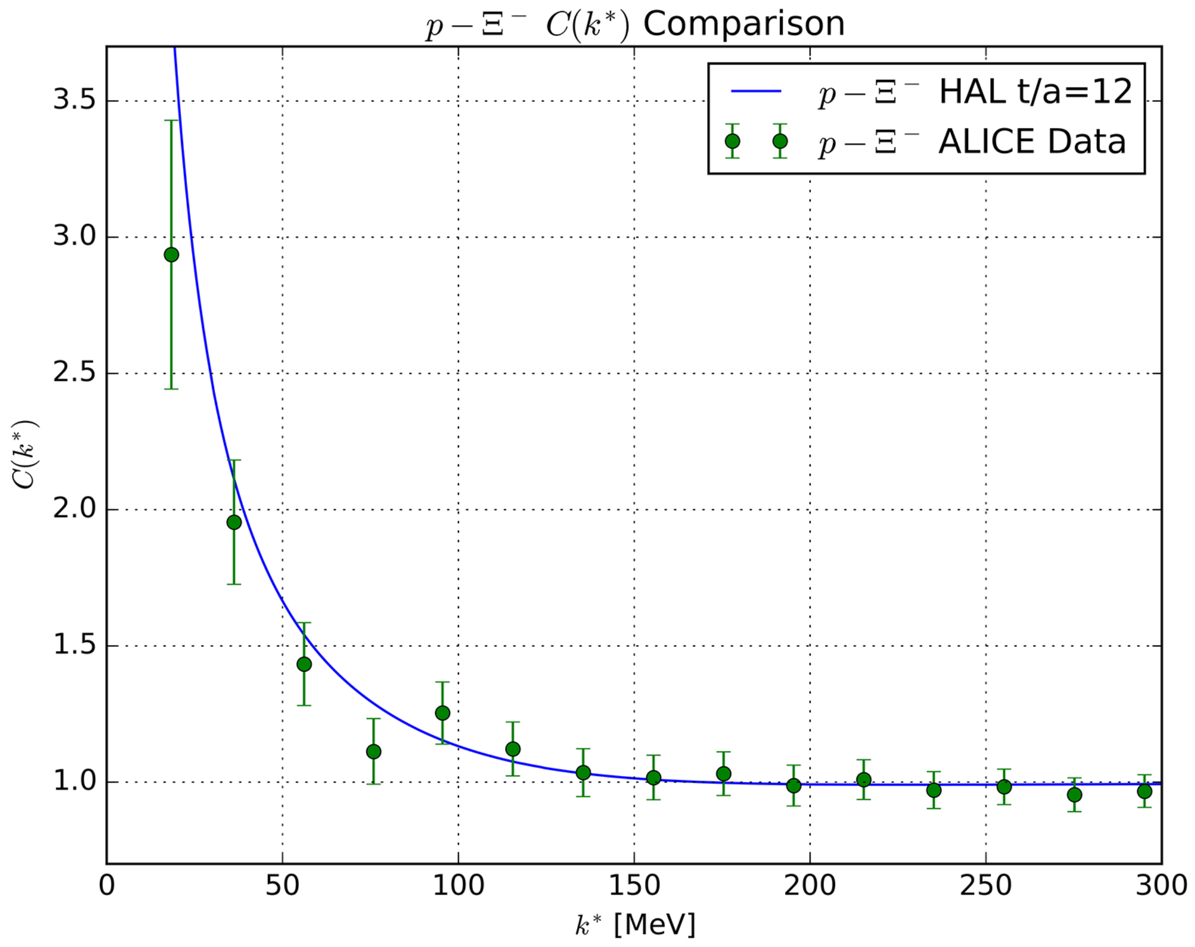
Correlations for NY Interactions

$C(k^*)$ for $p\Xi^-$ (reproduced)



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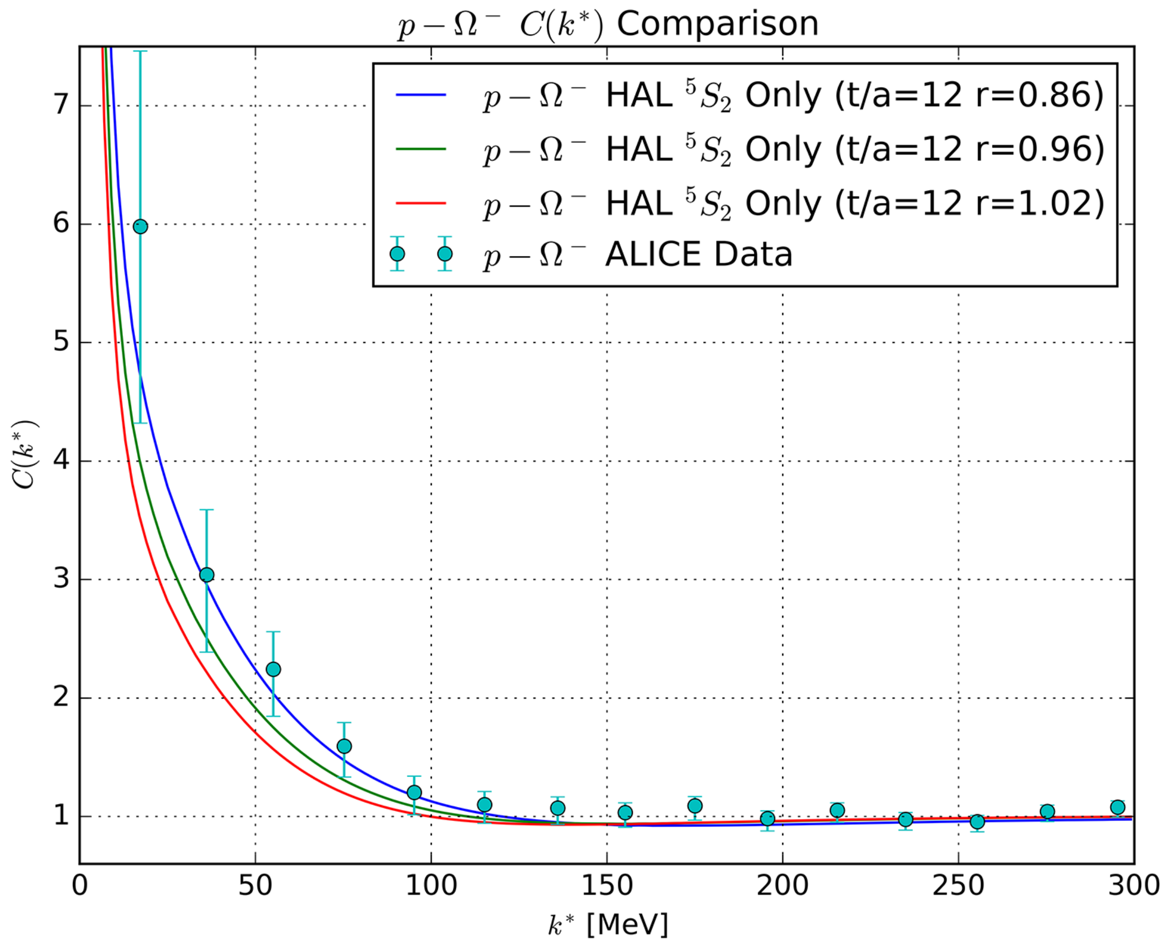

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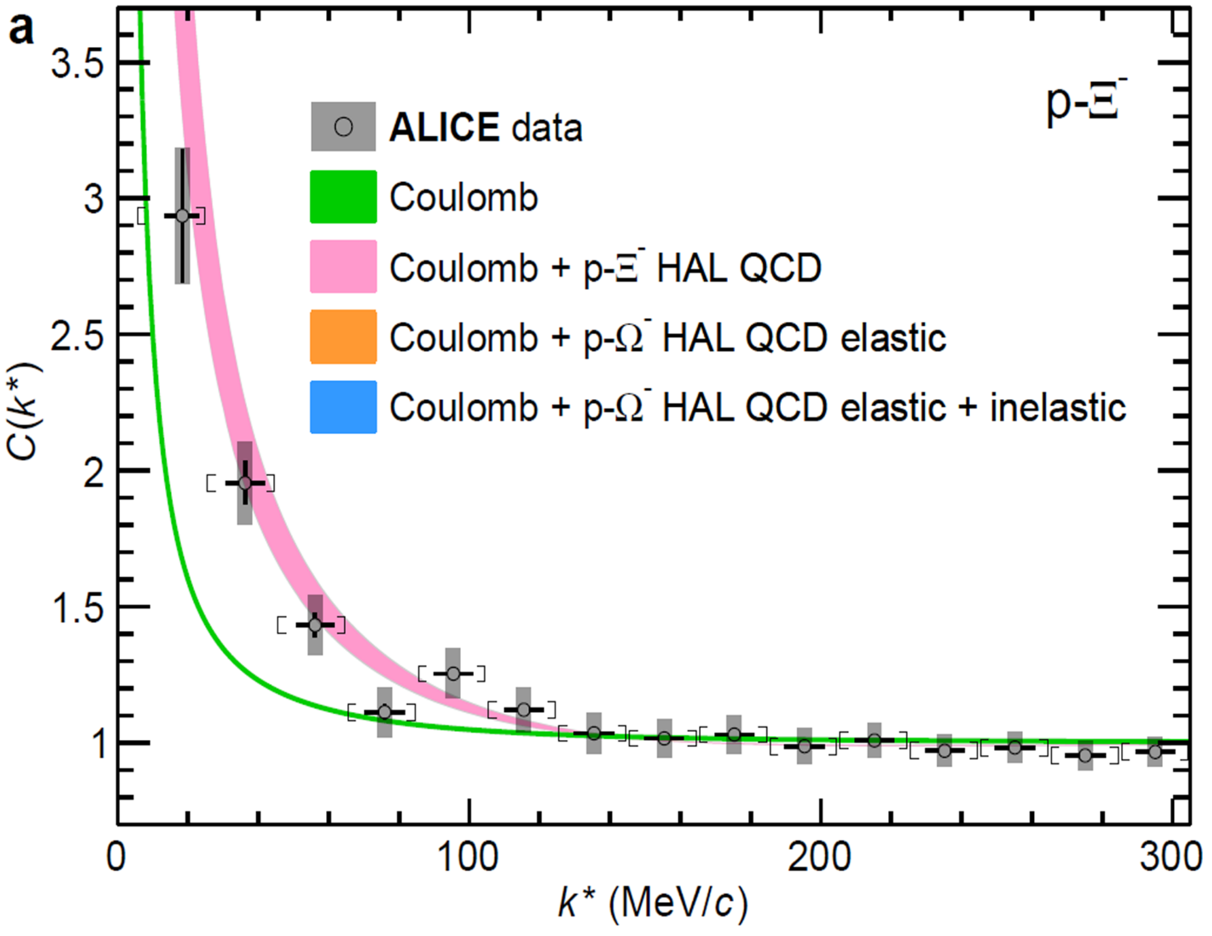
$C(k^*)$ for $p\Xi^-$ (reference)



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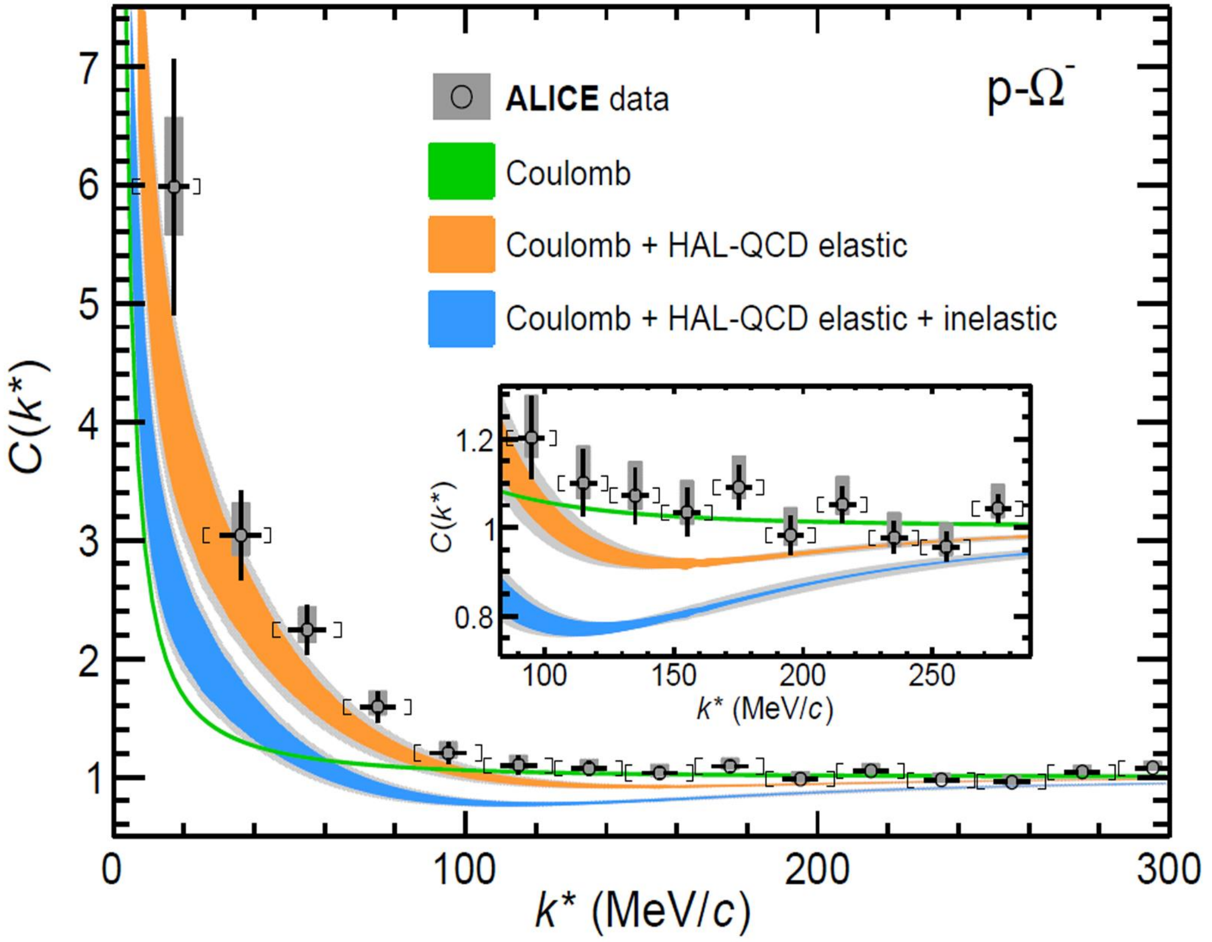

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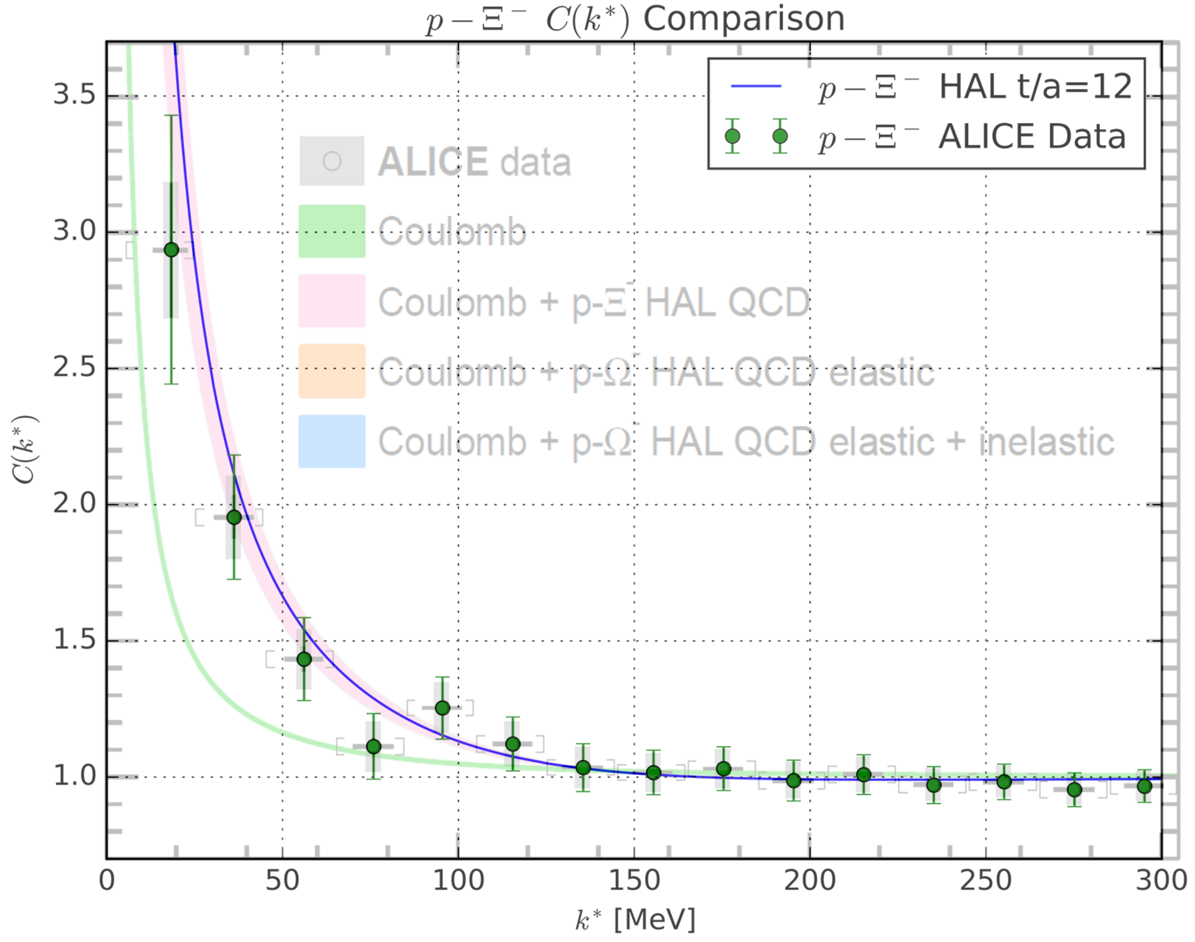
Correlations for NY Interactions

$C(k^*)$ for $p\Xi^-$ (overlaid)



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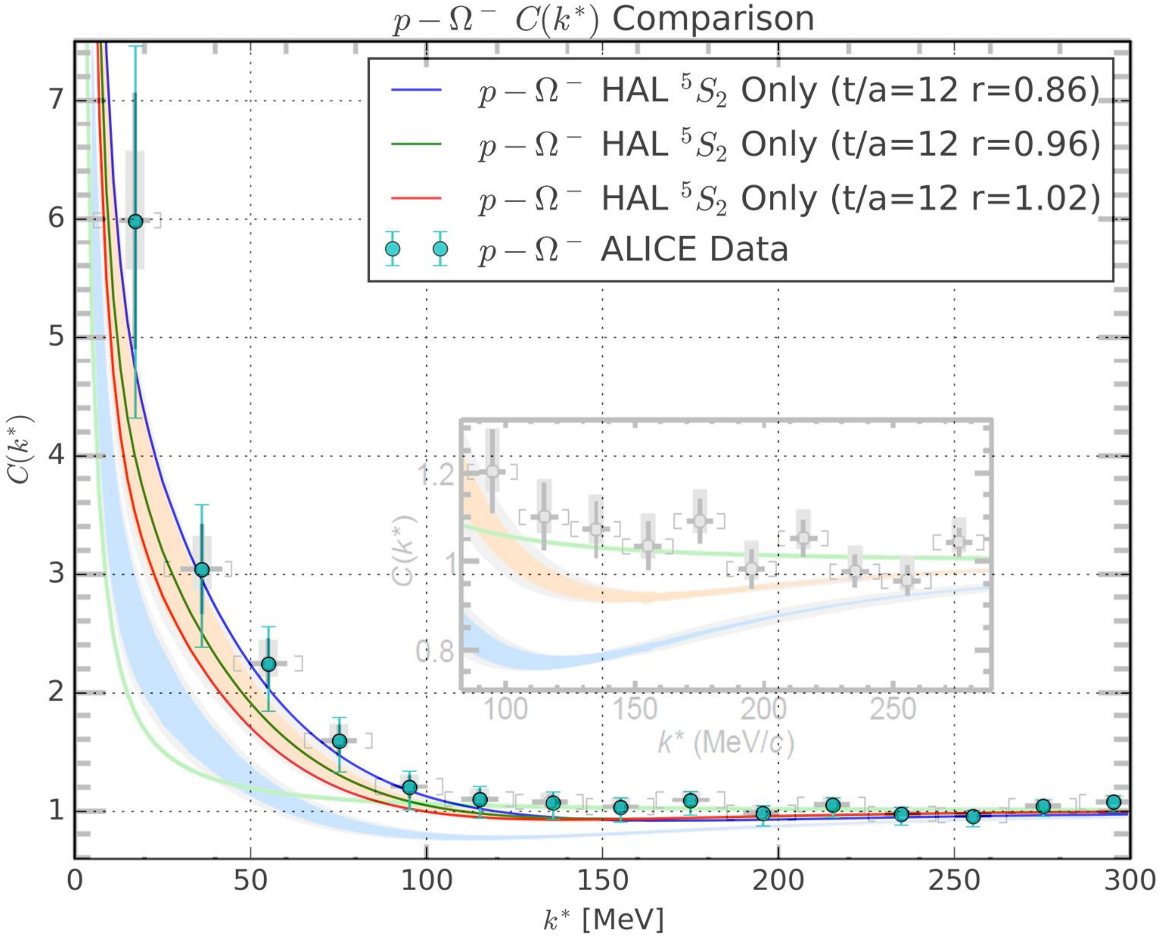
$C(k^*)$ for $p\Omega^-$ (overlaid)



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Model evaluation process

Simple example: Yukawa potential

- The form of the Yukawa potential

$$V_{\text{Yukawa}} = -g^2 \frac{e^{-x}}{x}$$

where

$$x = \mu r, \qquad \mu = \frac{m_\pi c}{\hbar}$$

and g^2 treated as a parameter

- If we consider both of isospin and spin, the central term

$$V_C = \frac{g^2}{3}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\frac{e^{-x}}{x}$$

where

$$\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = \begin{cases} -3, & I = 0 \\ +1, & I = 1 \end{cases}, \qquad \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = \begin{cases} -3, & S = 0 \\ +1, & S = 1 \end{cases}$$

Fitting process for g^2 in Yukawa potential

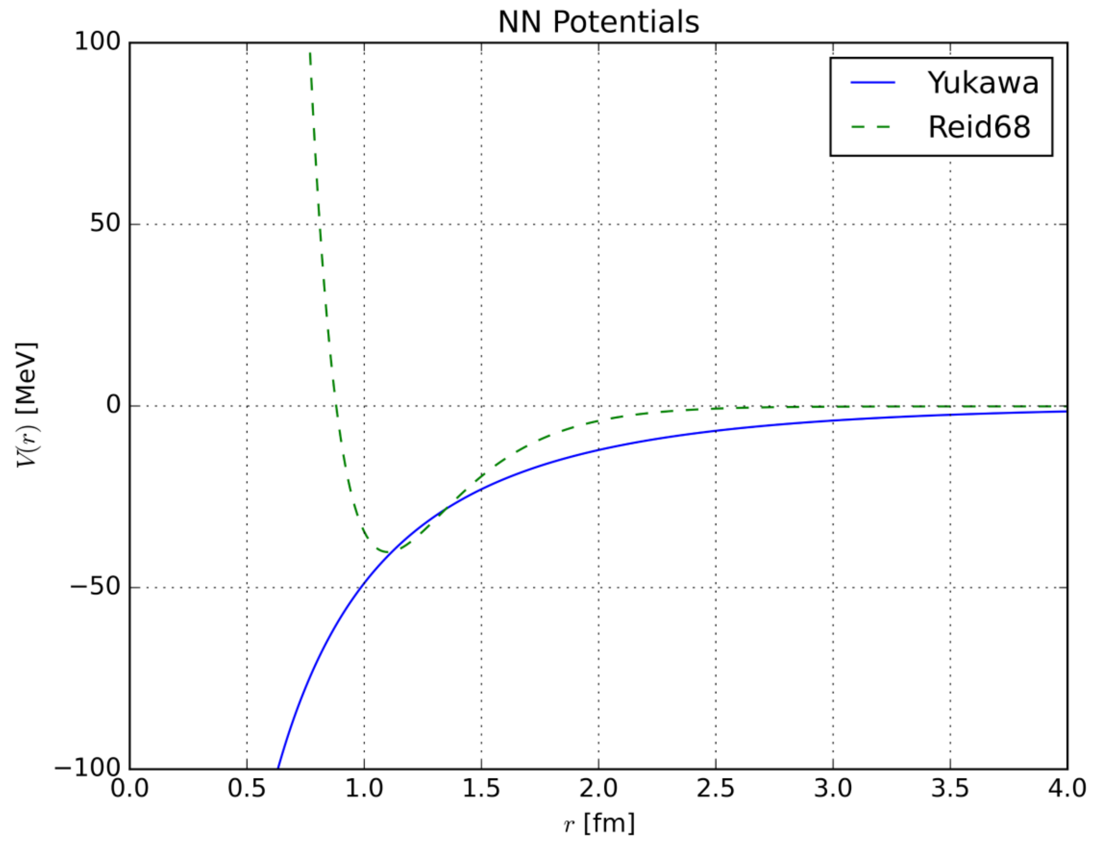
- Assumption: The interaction between two nucleons in a deuteron is of the form Yukawa potential.
- Set g so that the ground state energy for this potential becomes the actual ground state energy.
- In this process, interactions except for the central term were not considered. (e.g. tensor term $V_T S_{12}$)

```
g=68.773000        r_div: 37.424   46.396   53.647   61.743   71.575
Ground state energy for g=68.773000: -2.223991394043 MeV

g=68.774000        r_div: 37.448   45.954   54.504   62.257   69.926
Ground state energy for g=68.774000: -2.224206924438 MeV

g=68.775000        r_div: 37.473   45.570   55.716   63.218   71.118
Ground state energy for g=68.775000: -2.224421501160 MeV

g=68.776000        r_div: 37.498   45.279   57.813   64.667   73.598
Ground state energy for g=68.776000: -2.224637031555 MeV

g=68.777000        r_div: 37.523   45.629   67.672   67.672   77.798
Ground state energy for g=68.777000: -2.224852561951 MeV
```

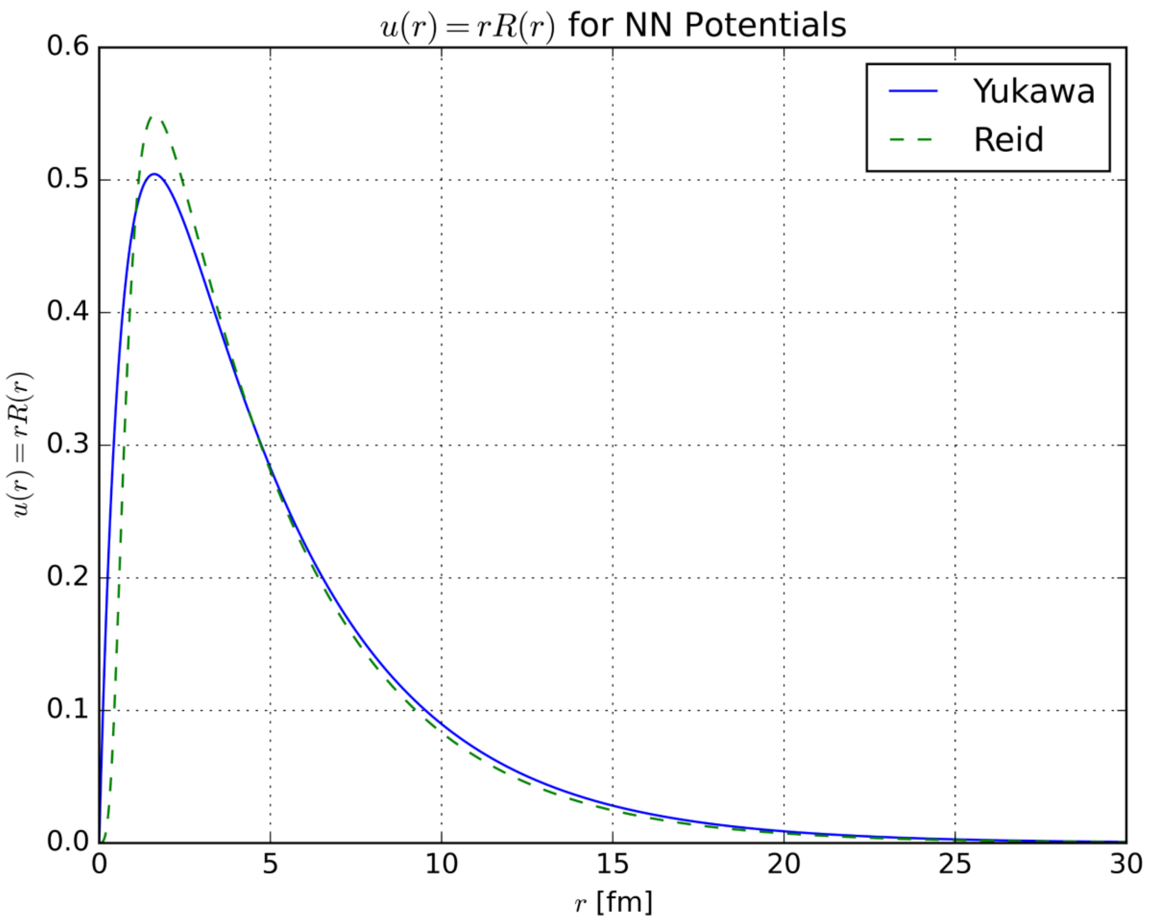
Model evaluation process

Comparison of V_C for ${}^3S_1$ channel

- For Yukawa, $g^2 = 68.7755$
- For Reid68, also central term only



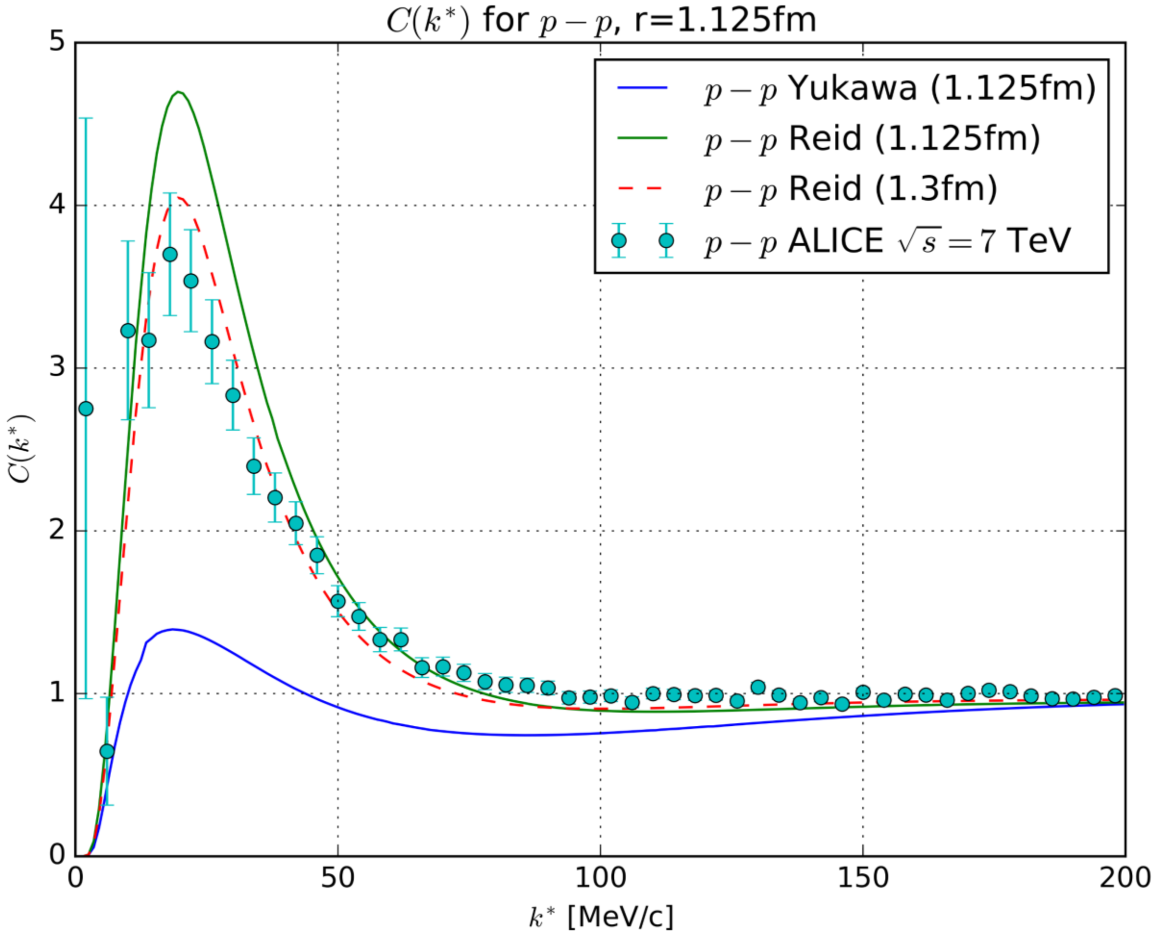
Comparison of eigenfunctions (${}^3S_1$)



Model evaluation process

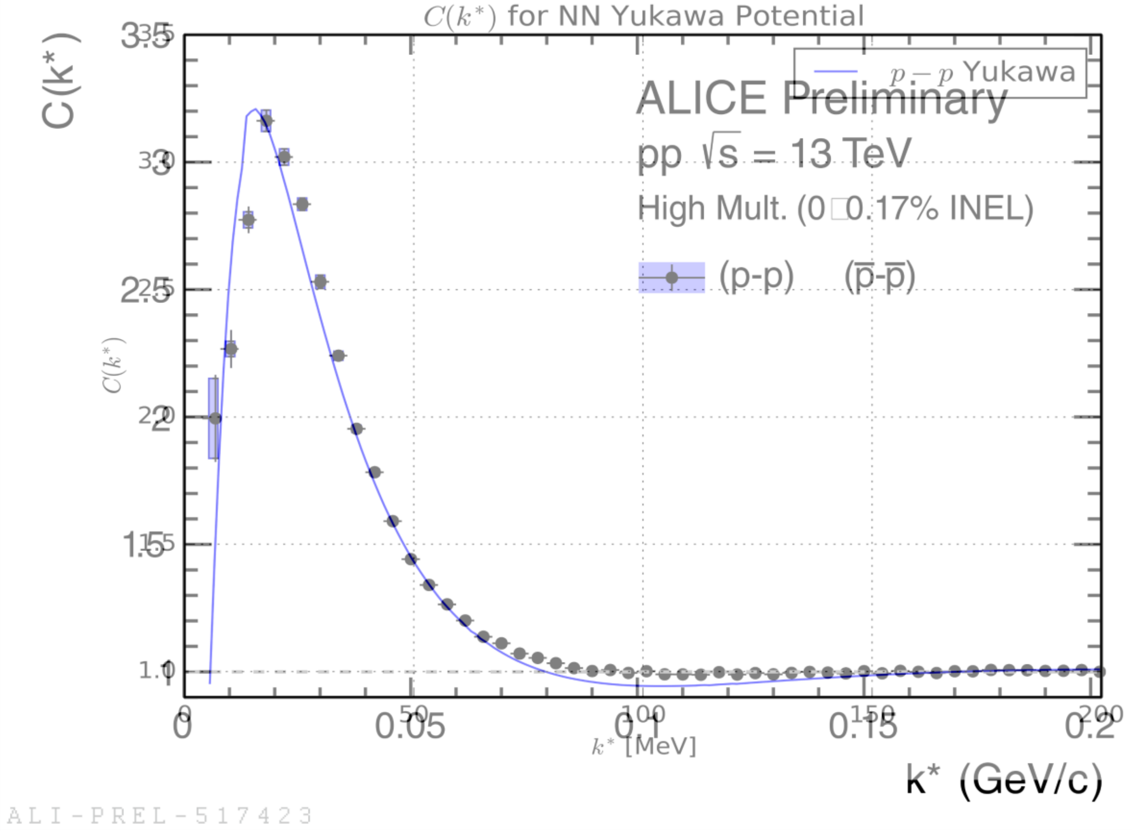
$C(k^*)$ for $p - p$ & comparison

- For Yukawa,



$C(k^*)$ in some variation in g^2 and r_0

- Source size $r_0 = 0.96$ fm, $g^2_{\text{modified}} = 0.94g^2$
- E_0 for deuteron: -2.224 MeV $\rightarrow -1.427$ MeV



Summary

Conclusion

- From correlation function analysis, the interaction of fm scale could be confirmed.
- For the $p\Xi$ and $p\Omega$, reproduction of correlation using HAL QCD potentials was successful.
- CATS framework for femtoscopic correlation analysis was convenient to use, and it enables us to evaluate the various models.
- If we analyze interactions between two light mesons, we need to modify CATS or make a new analysis tool.

Plans for study

- Studying and the various hadron-hadron interactions (focusing on $D - D^*$ or $\overline{D} - D^*$ for studying T_{cc} or $X(3872)$)
- Comparison of research results with future experimental results

Thanks for listening!