The Interface of HEP and Quantum Sensing

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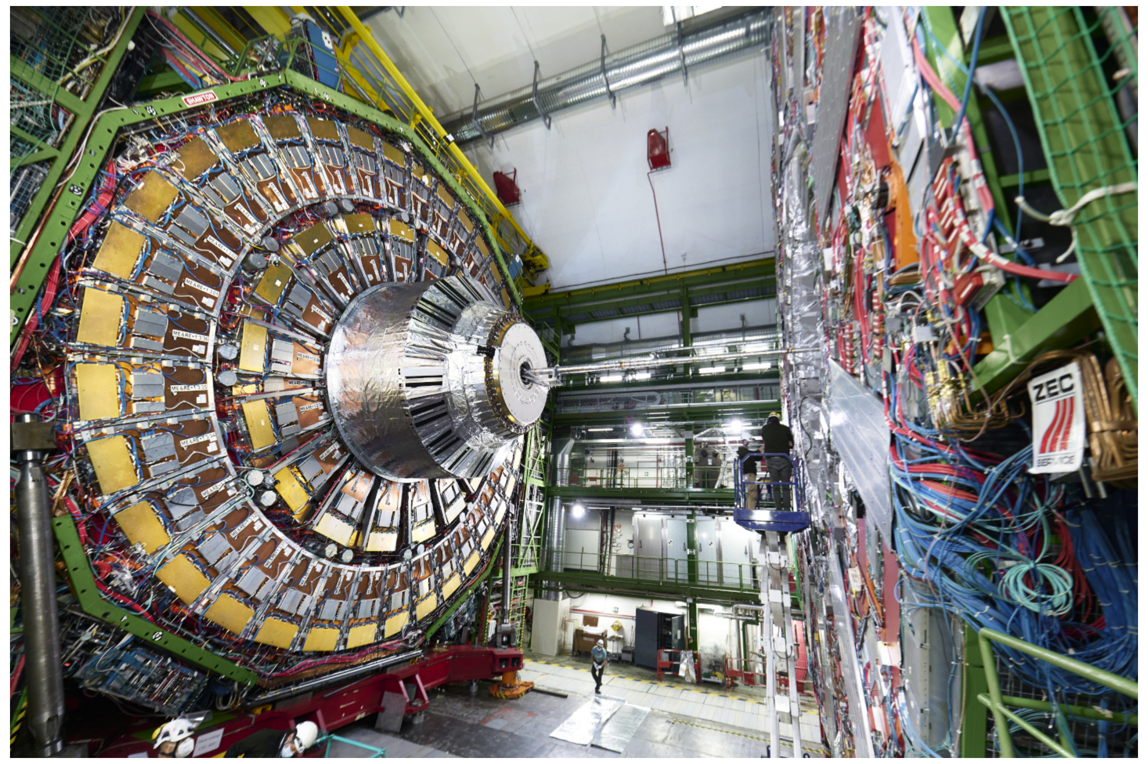
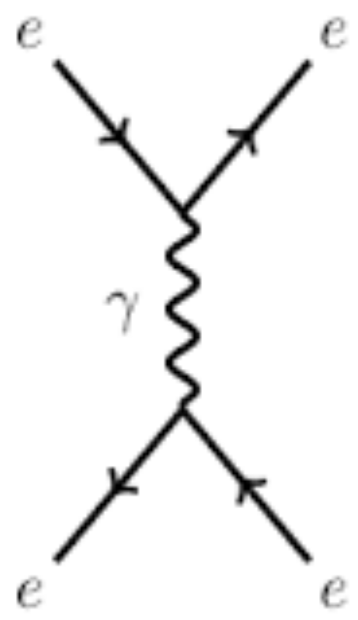
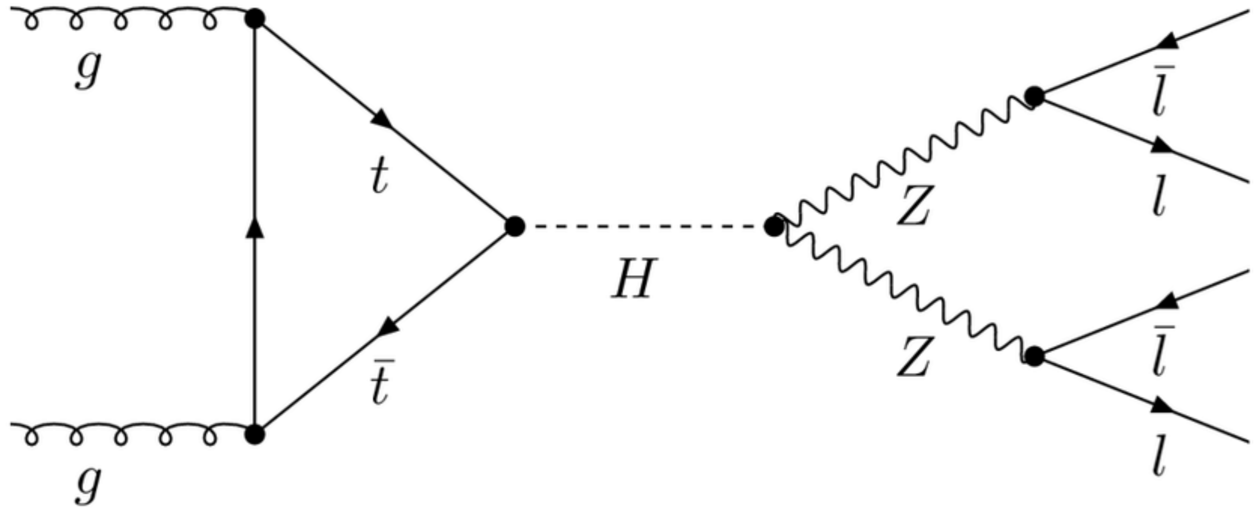
SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

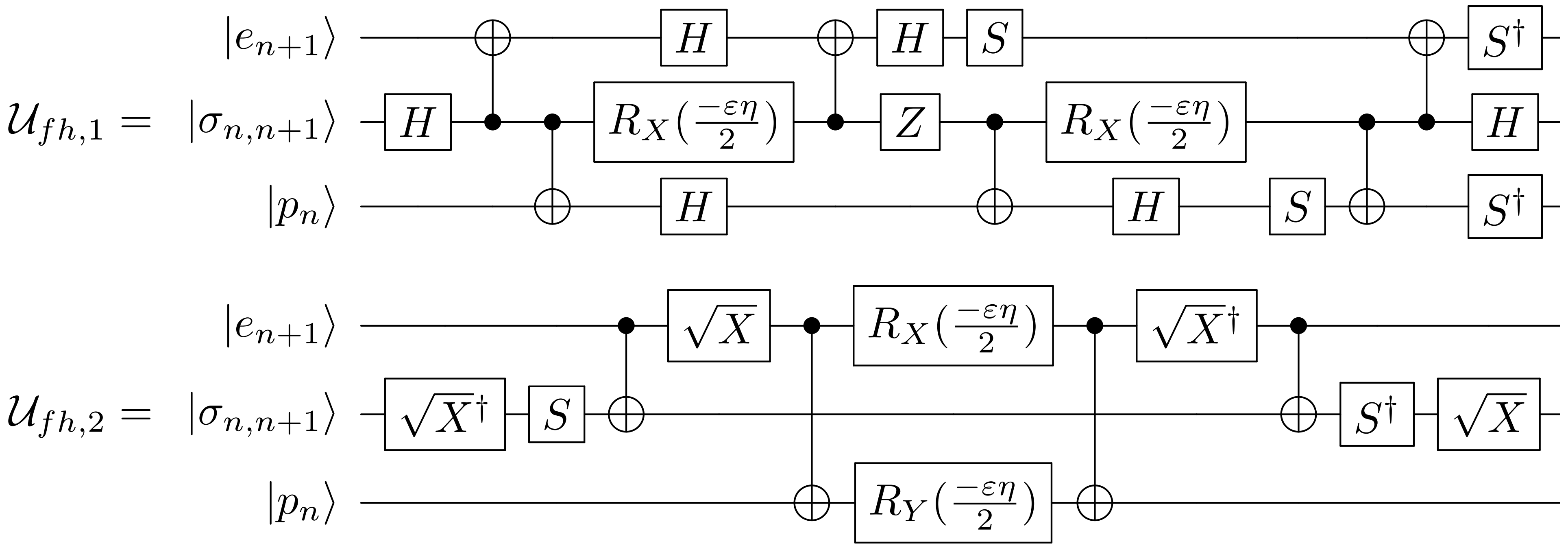
Particle - Quantum Interface

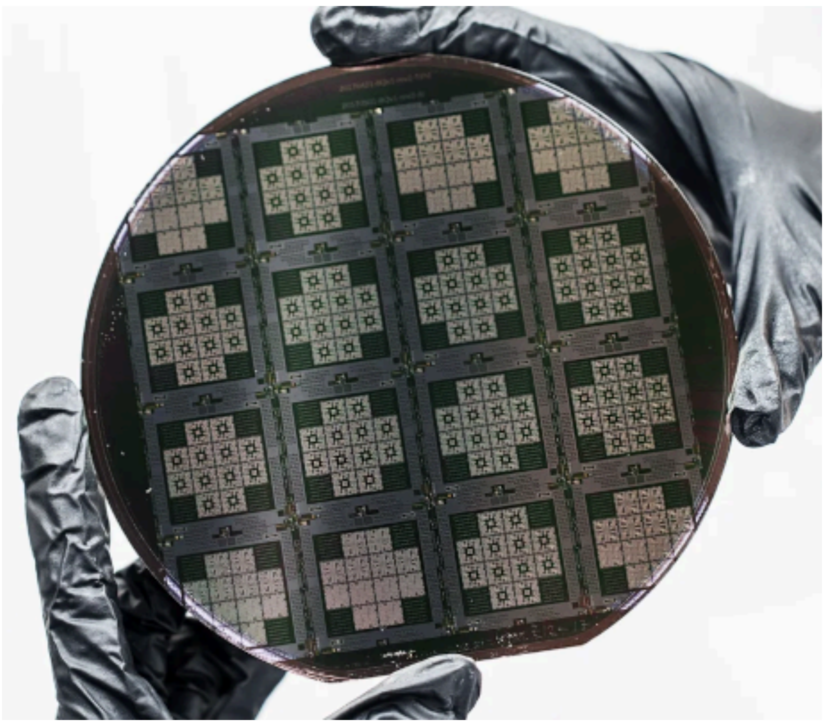
- Particle physics was always inherently quantum. Duh.
- A new field of Quantum physics is rapidly emerging, QIS.
- The interface is still (too) small -

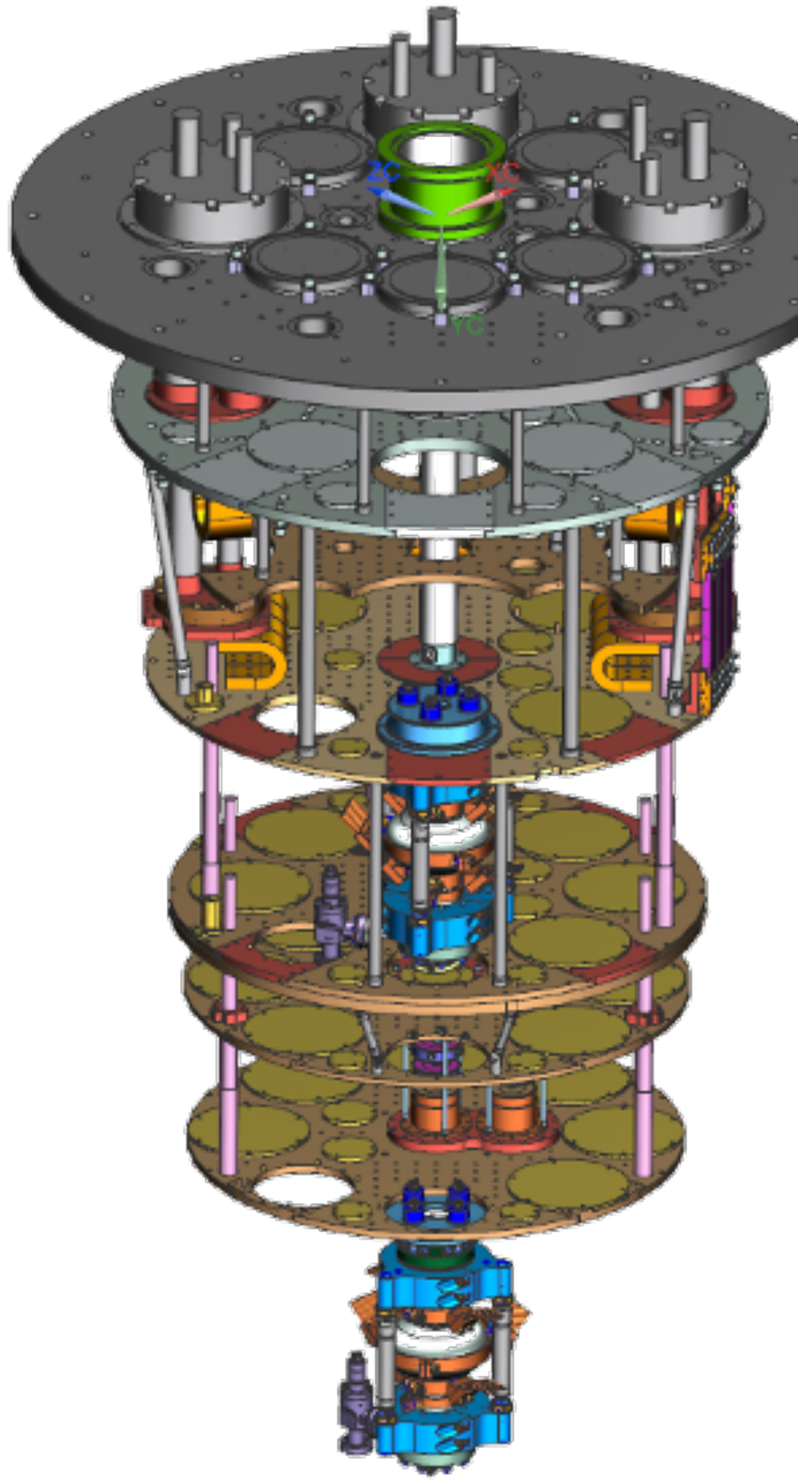












Overview

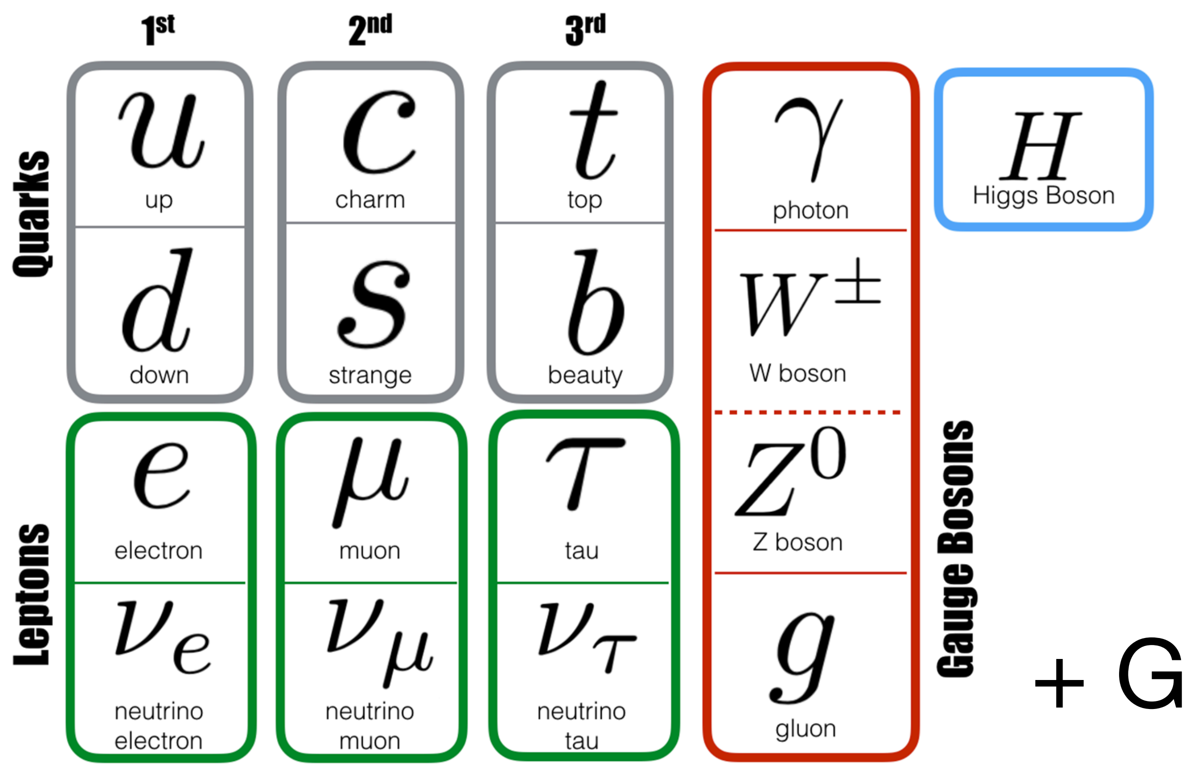
- At SQMS and in the new quantum sensing area we are bridging the divide!

As in intro to this day of quantum sensing for fundamental physics:

- Talk about quantum devices in HEP language.
- Talk about BSM models in a QIS language.

HEP - Quantum Fields in a Big Universe

- The instant recipe for particle physics:



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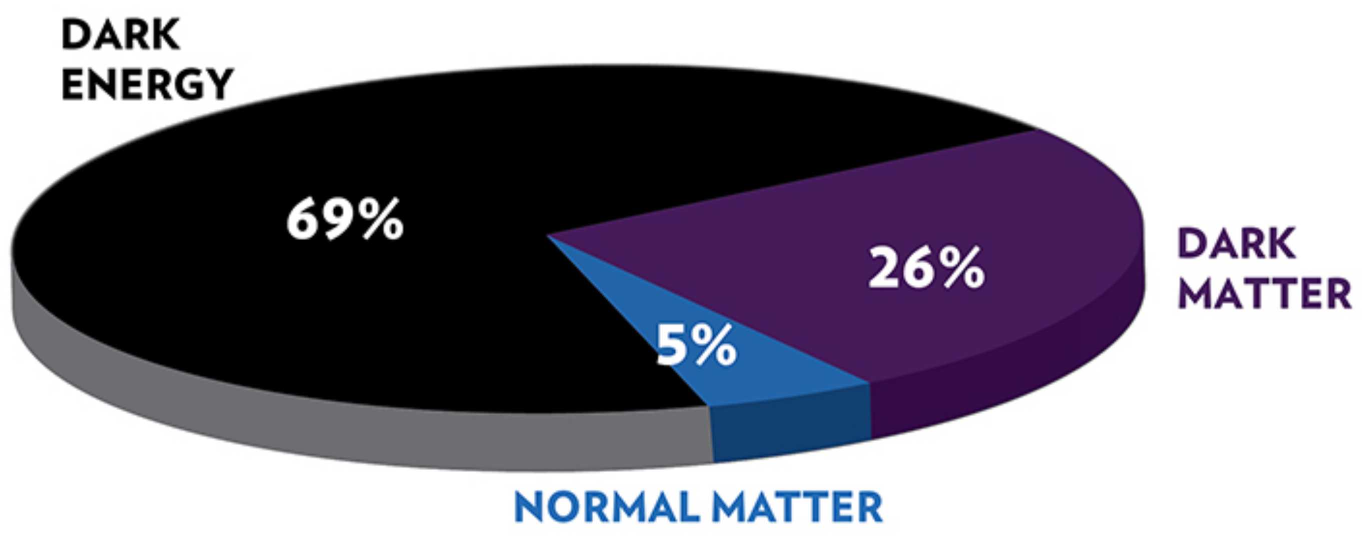

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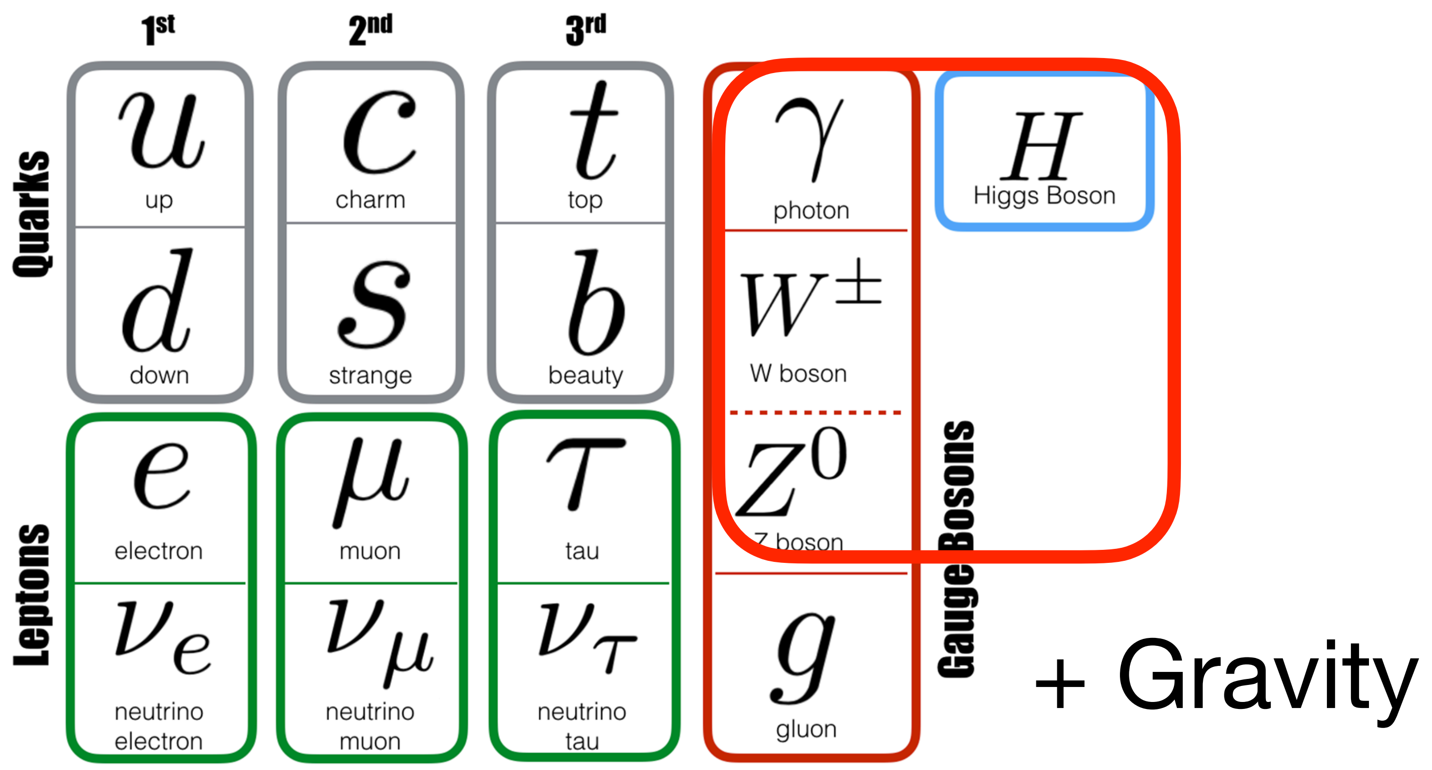

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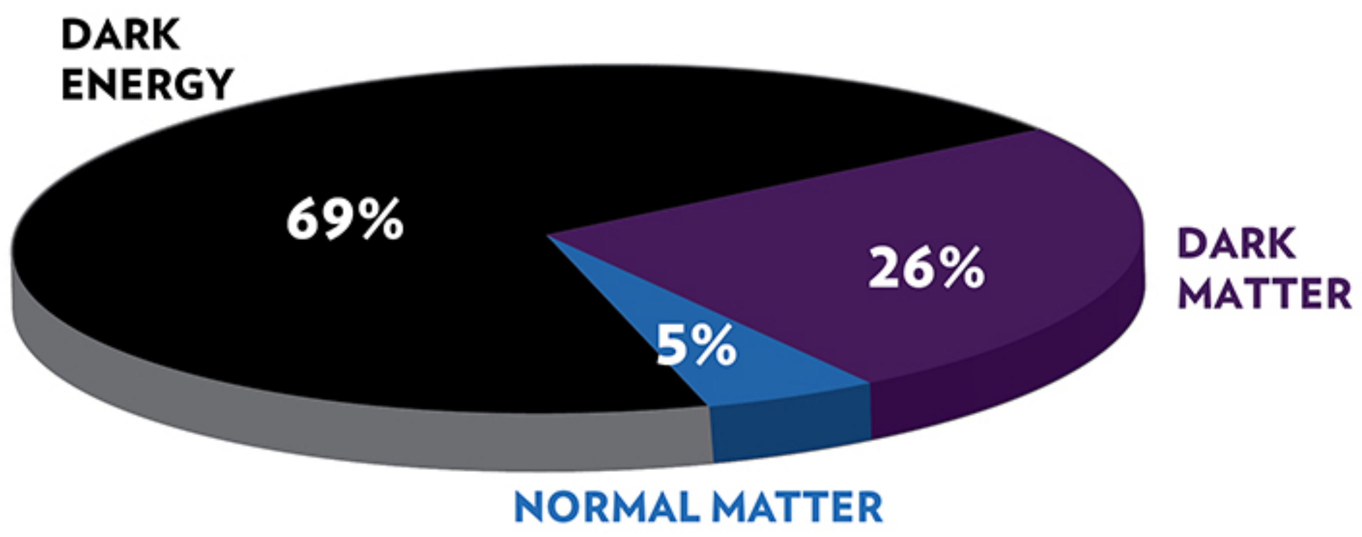

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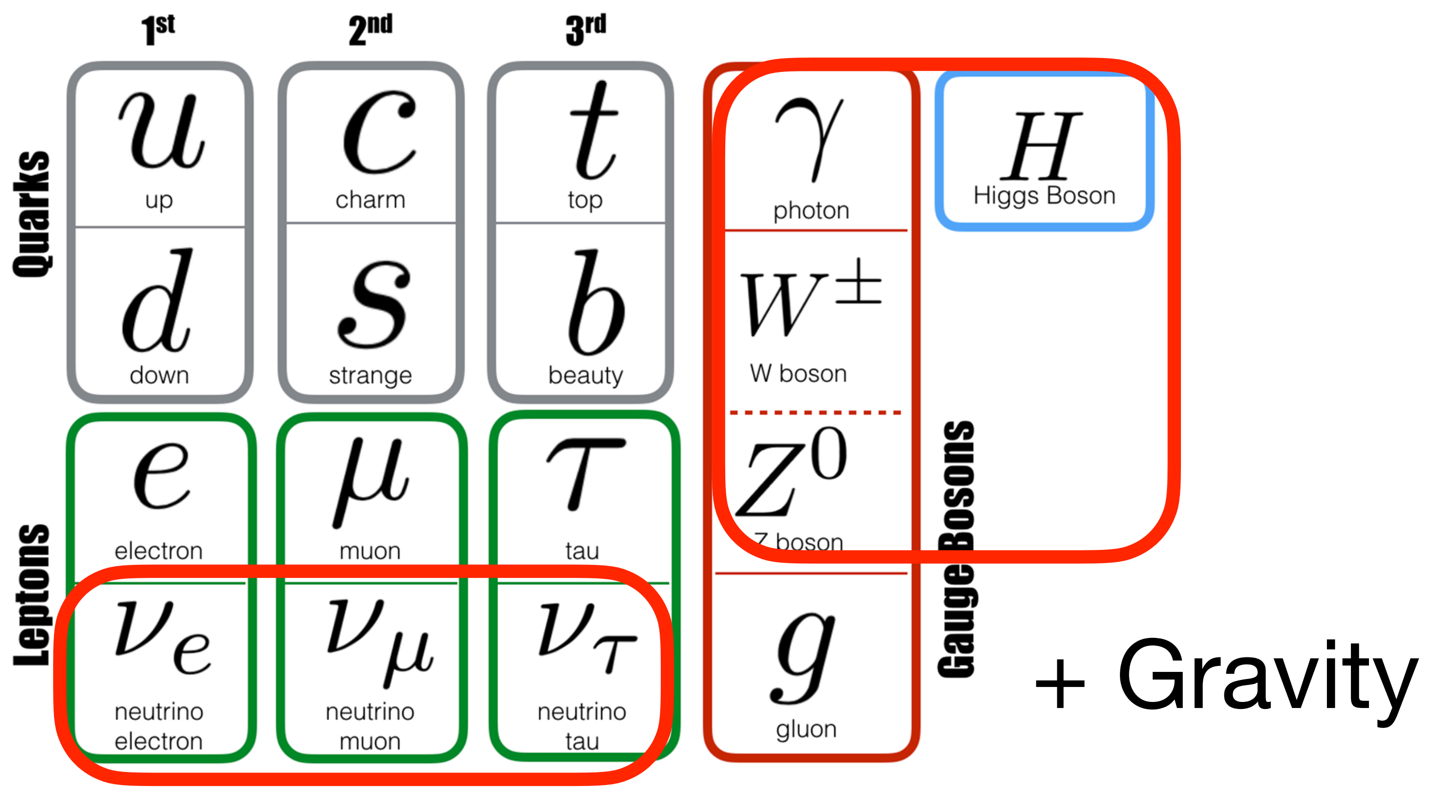

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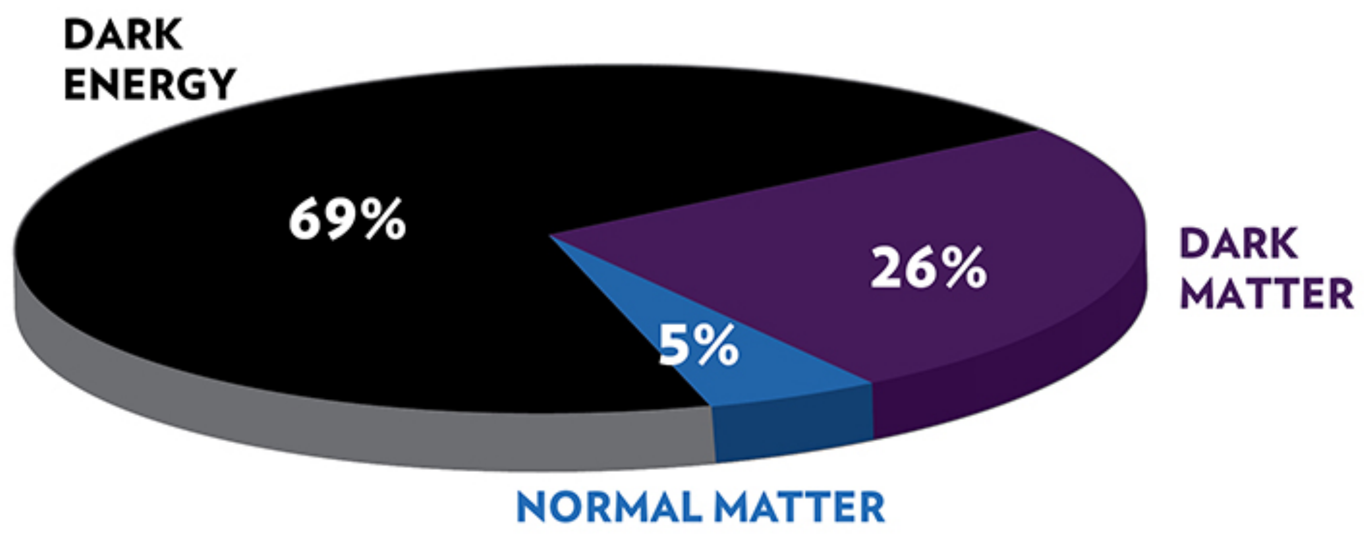

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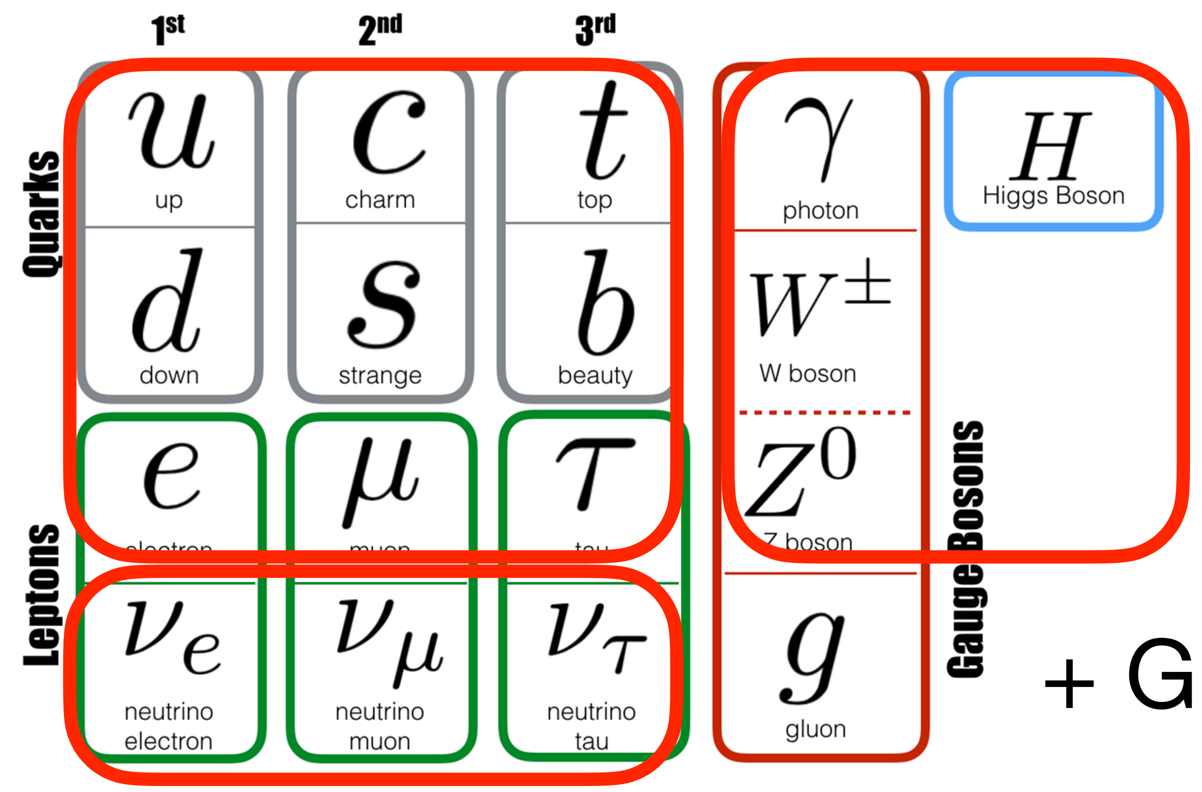

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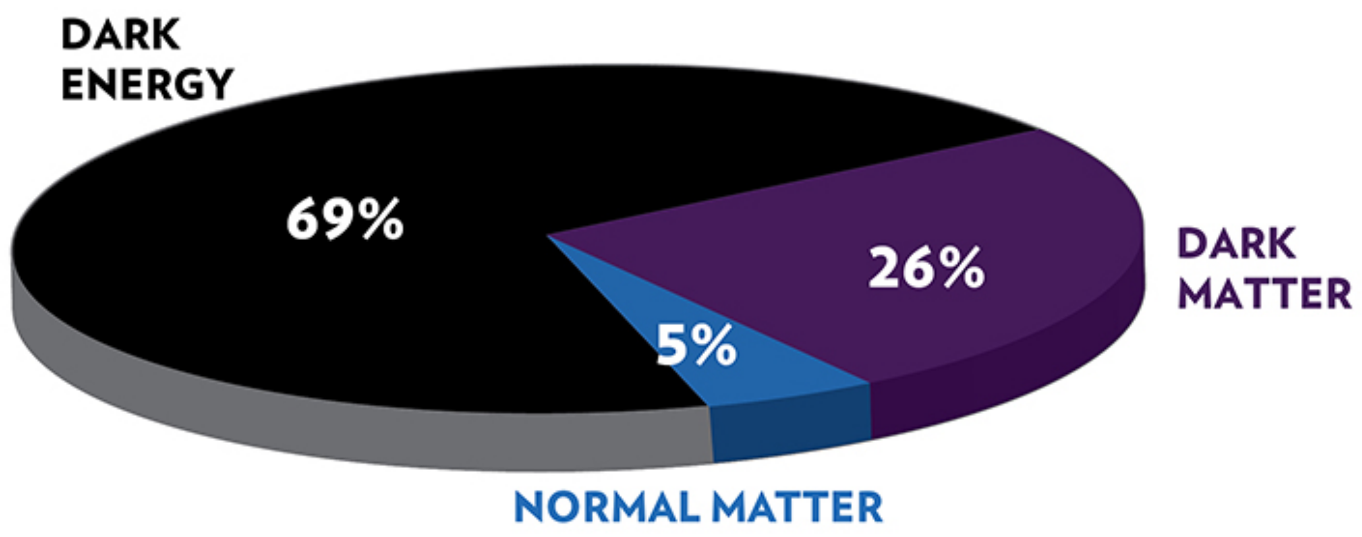

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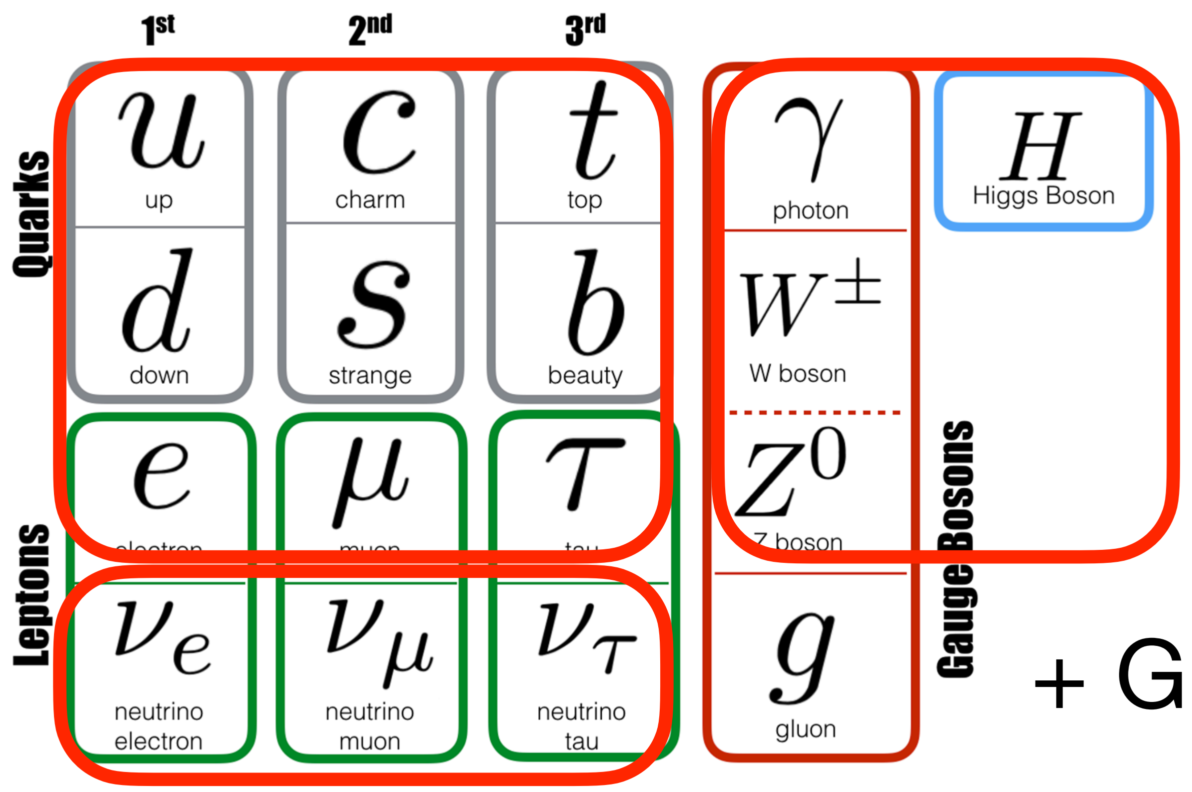

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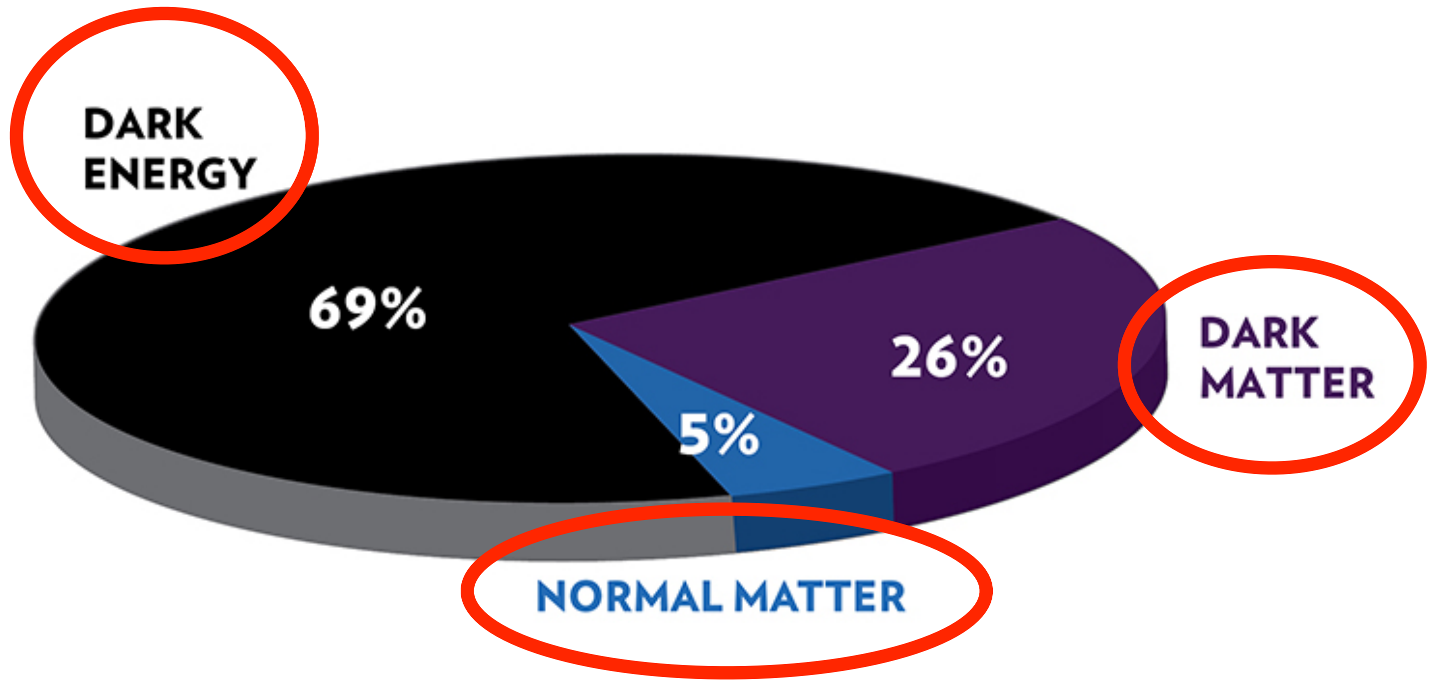

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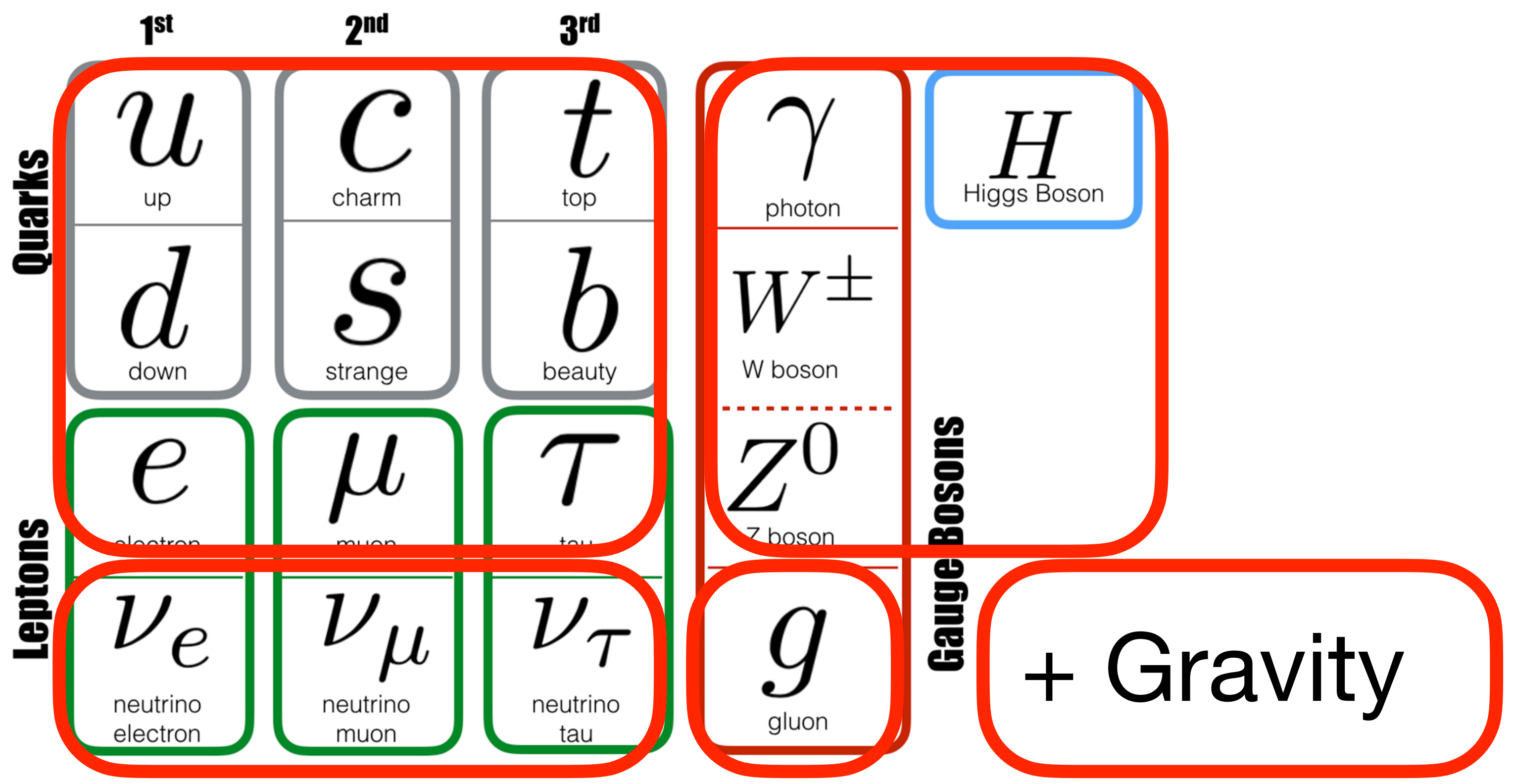

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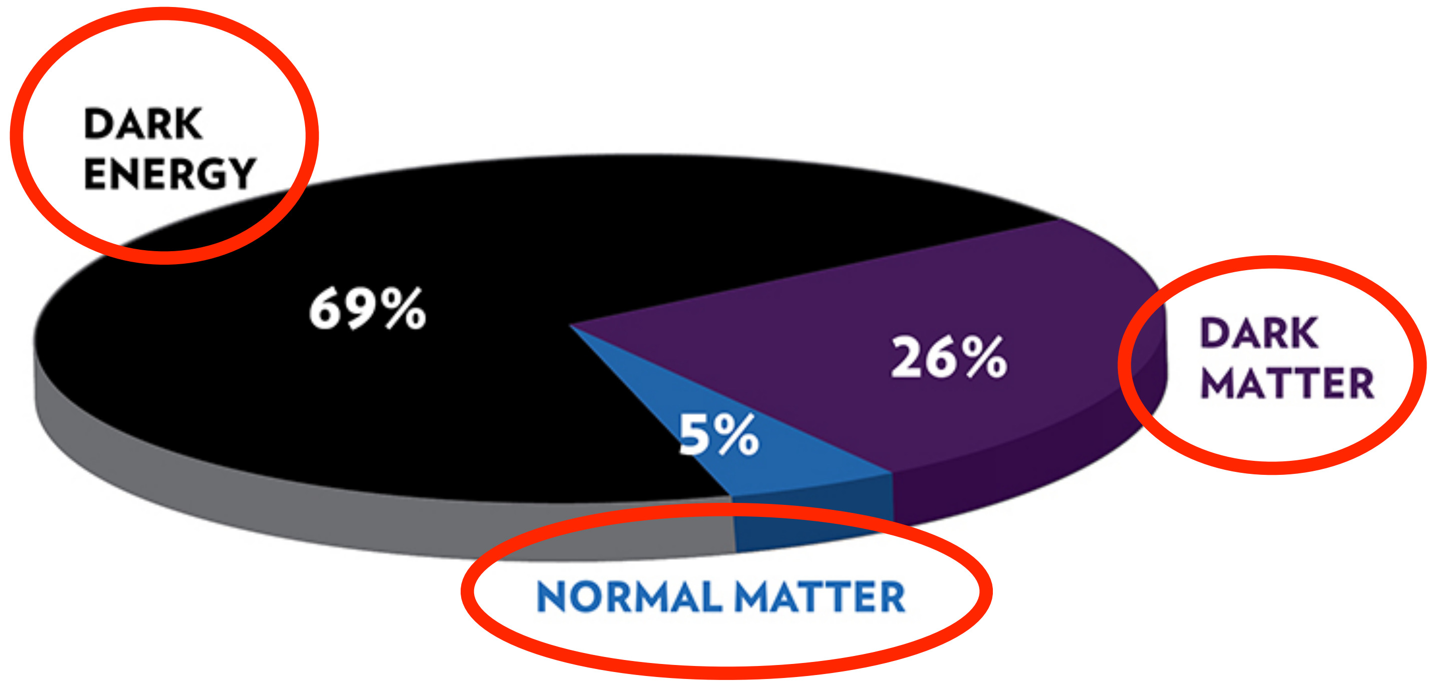

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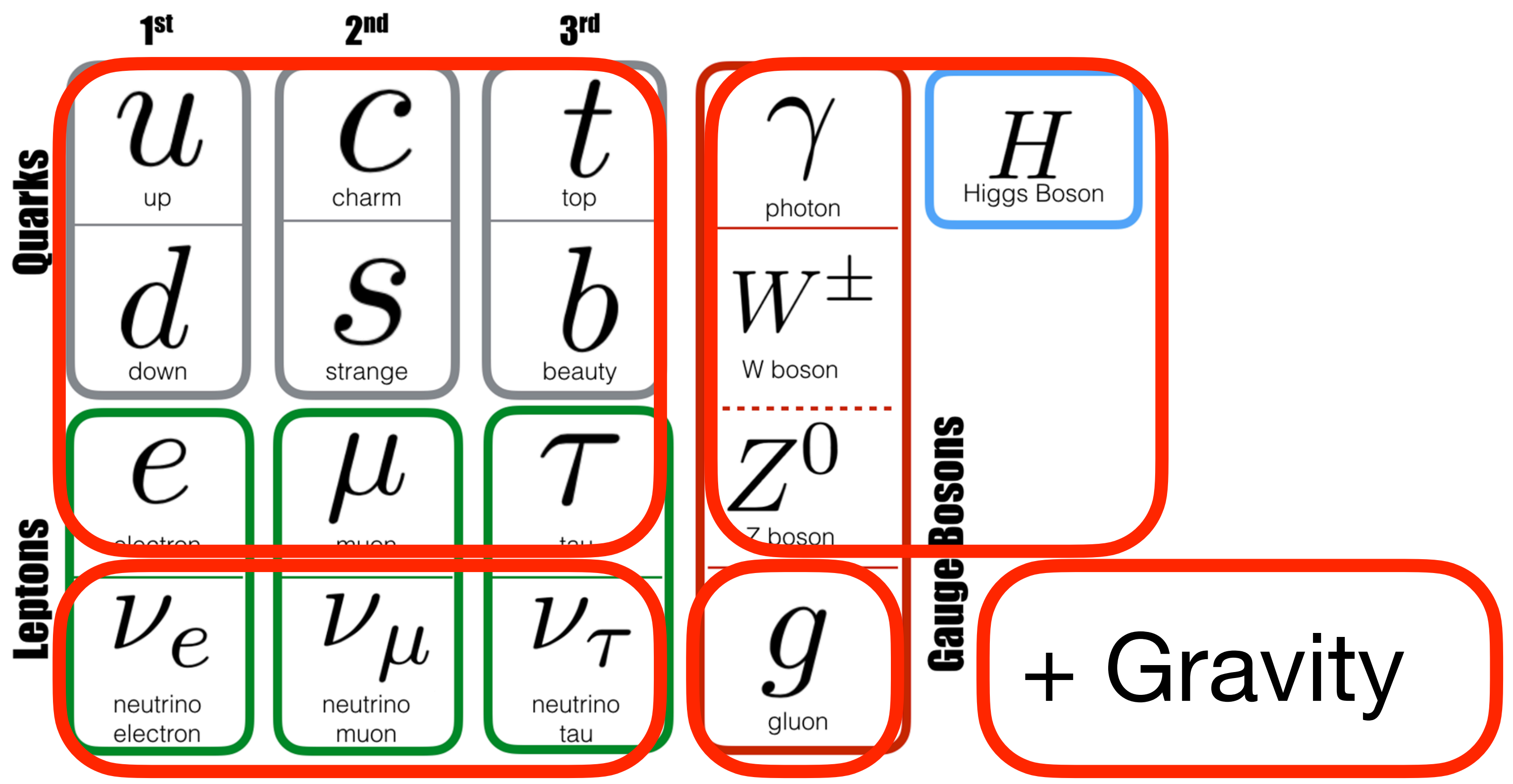

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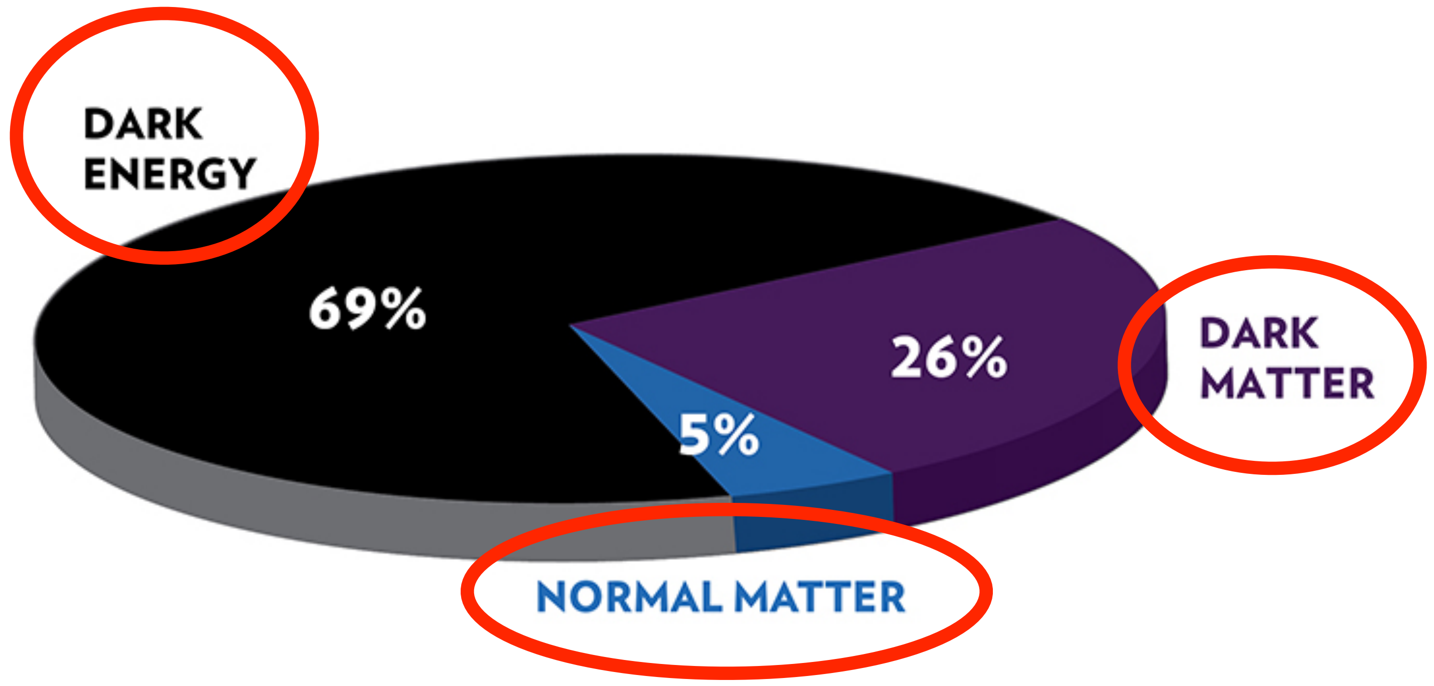


There is more. **BSM**. More fields! We'll get back to that!

HEP - Quantum Fields in a Big Universe

- The instant recipe for particle physics:

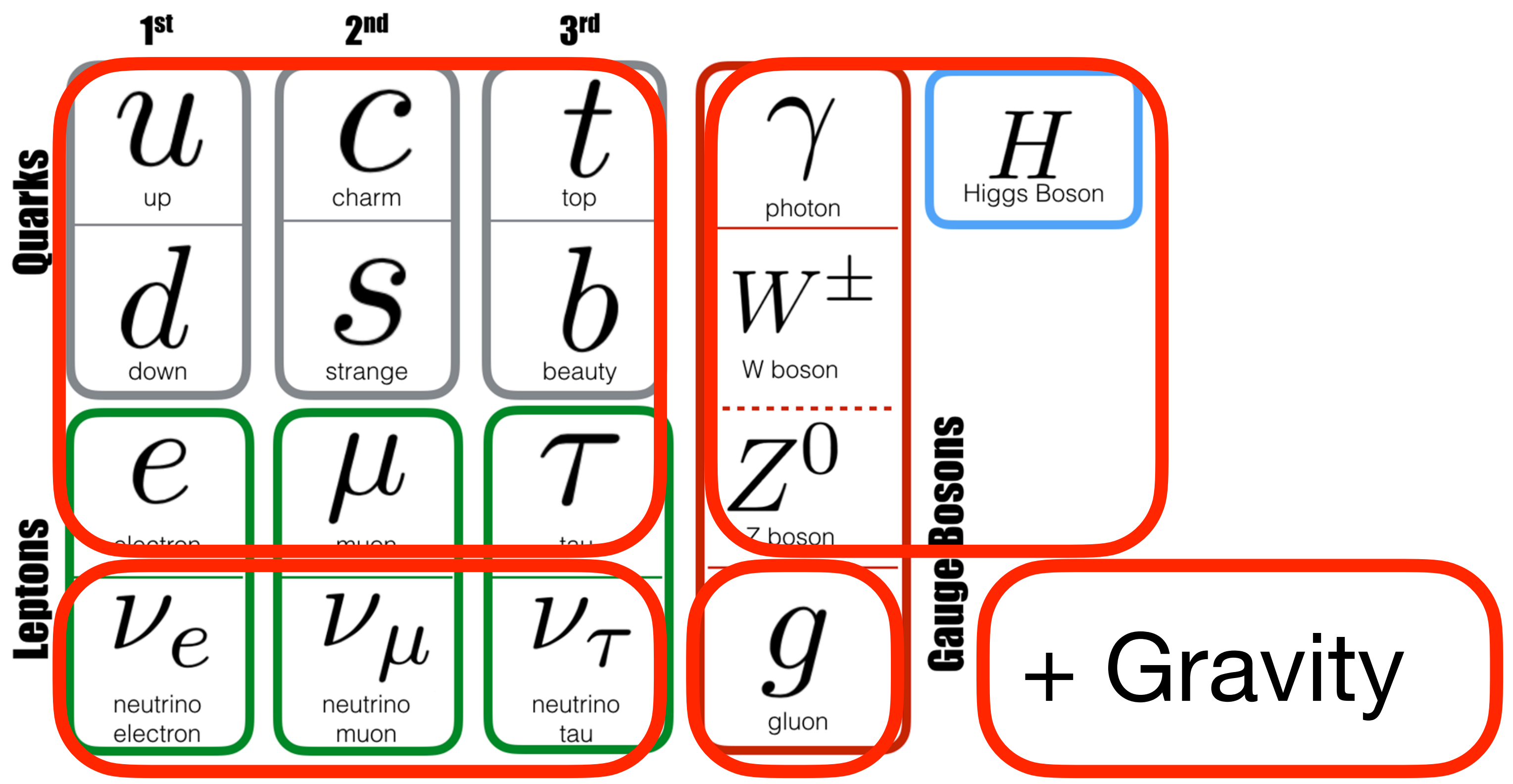
QFT:

$$|\psi\rangle = \psi|\Omega\rangle$$

$$\psi \sim \sum a_k e^{ikx} + h.c.$$

$$\mathcal{L} = \cdots$$

+



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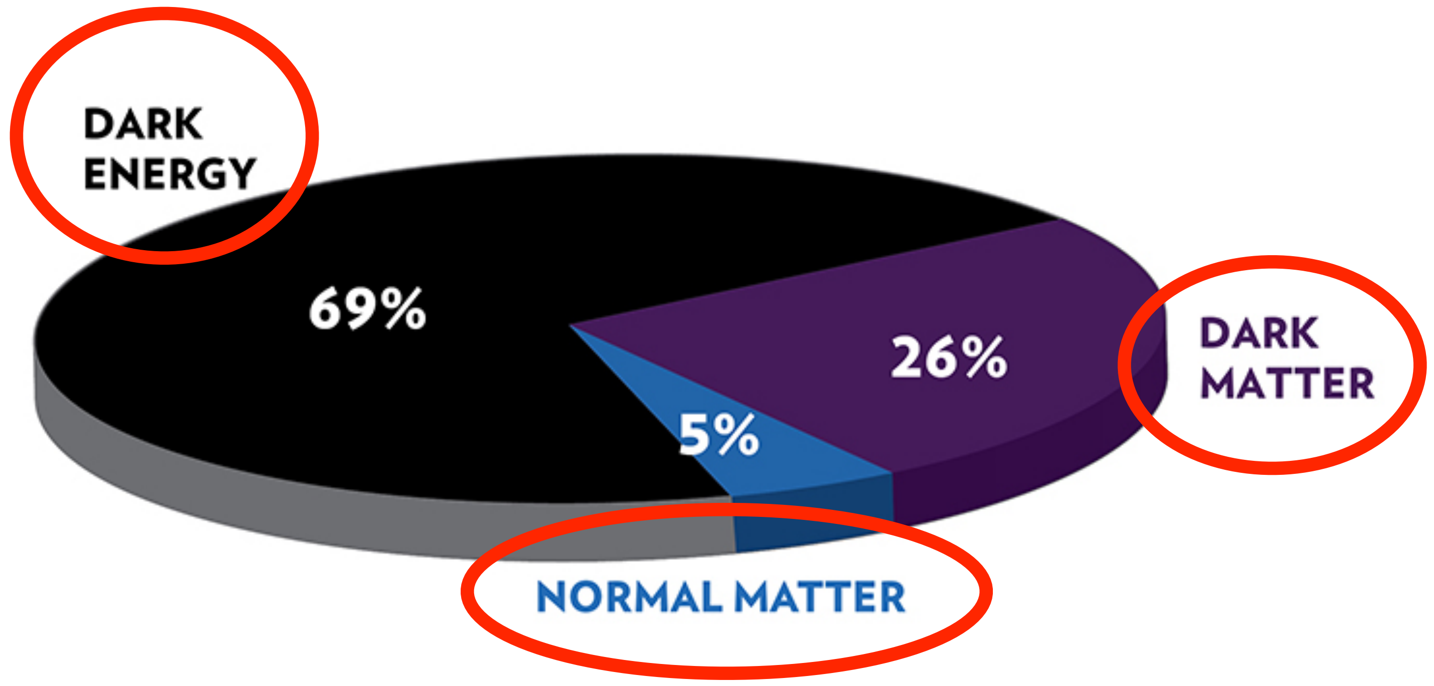

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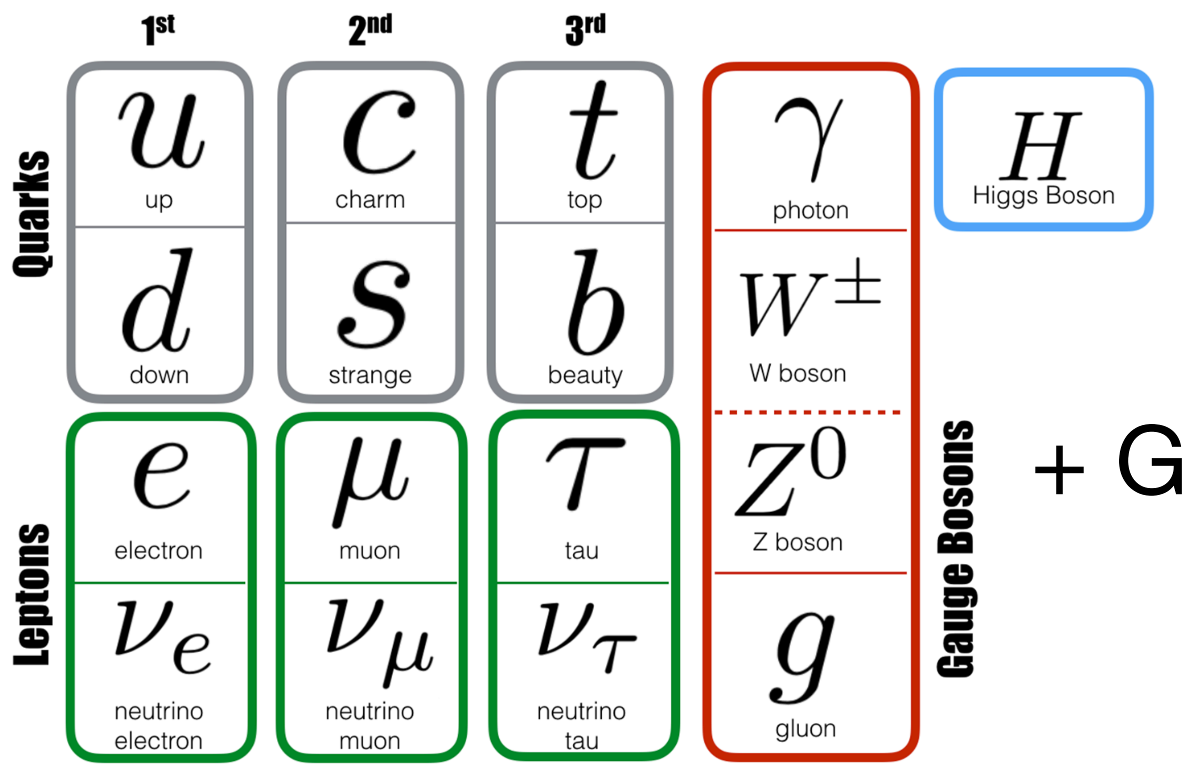

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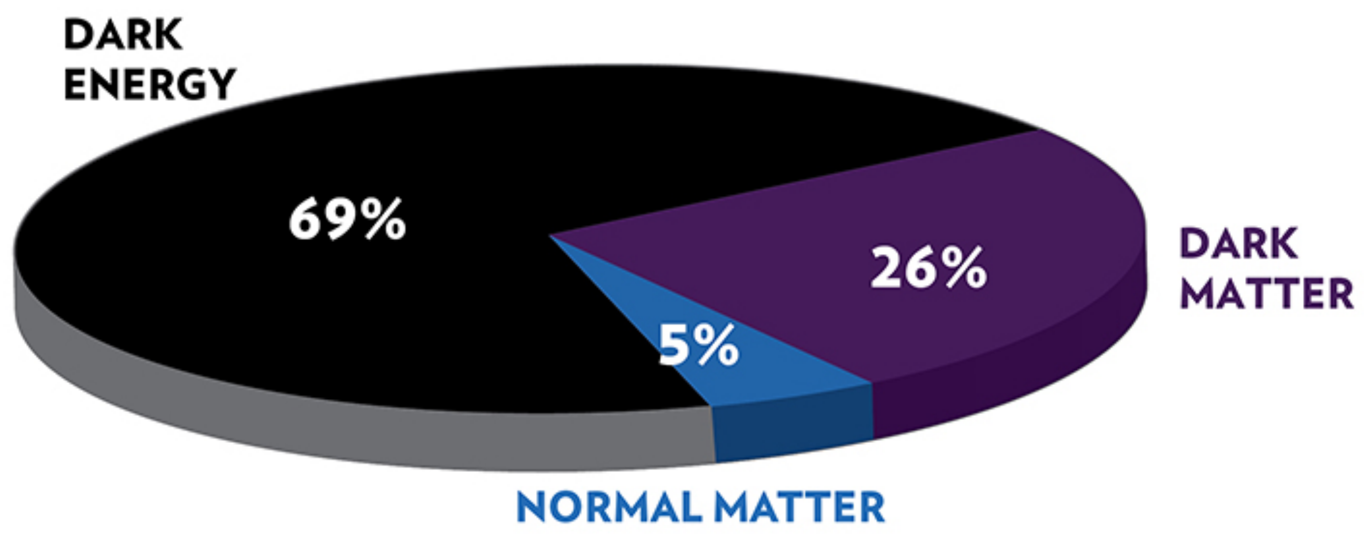

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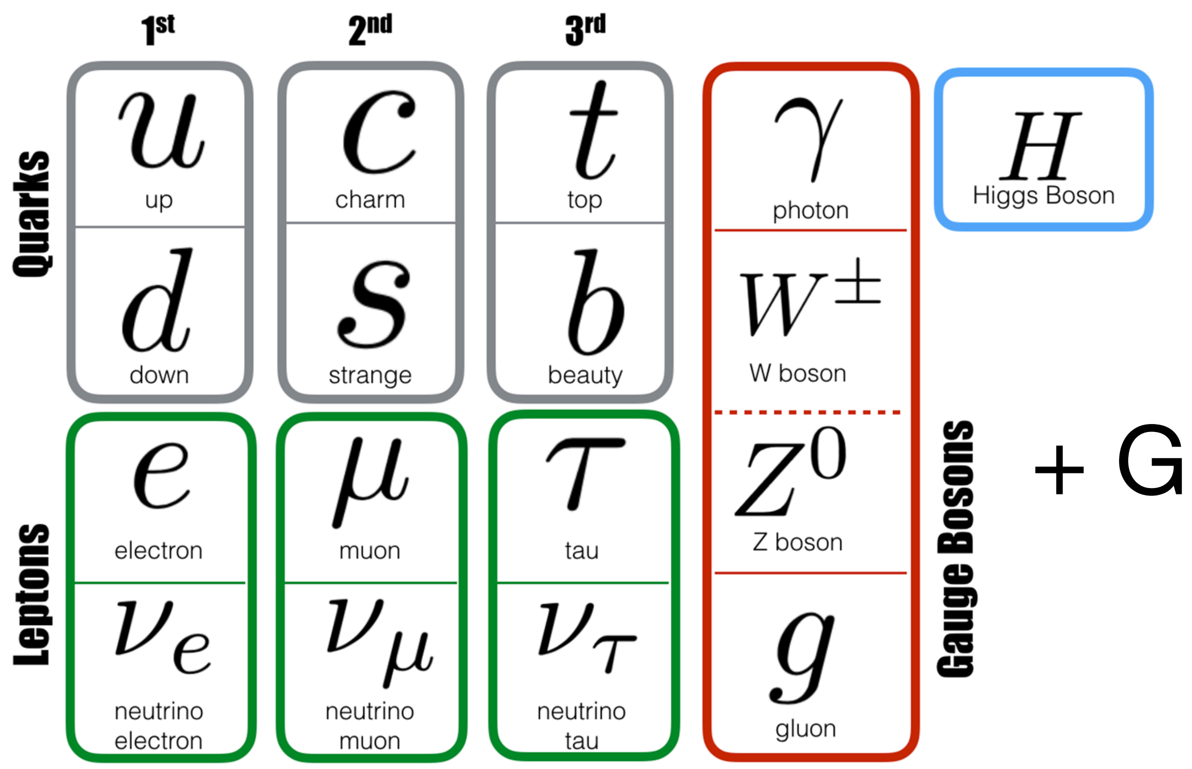

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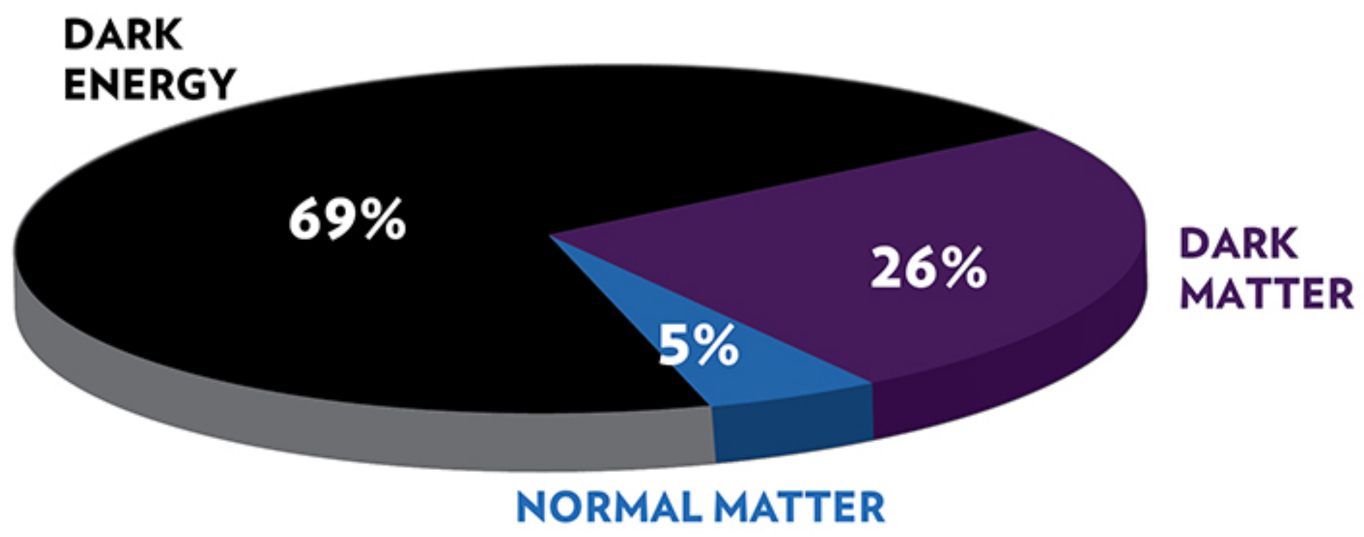


QFT in a big Universe.

A continuum of interacting oscillators. All frequencies.

Quantum Fields

- At the heart of QFT is a mode expansion. We get to pick the modes. Something like -

$$\phi(x_\mu) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} \left(a_{\vec{k}} u_{\vec{k}}(\vec{x}) e^{i\omega t} + a^\dagger_{\vec{k}} u_{\vec{k}}{}^*(\vec{x})(e^{-i\omega t} \right)$$

Just a Fourier decomposition of a function in an infinite space-time.
BUT, the coefficients of every mode are **creation/annihilation operators.**

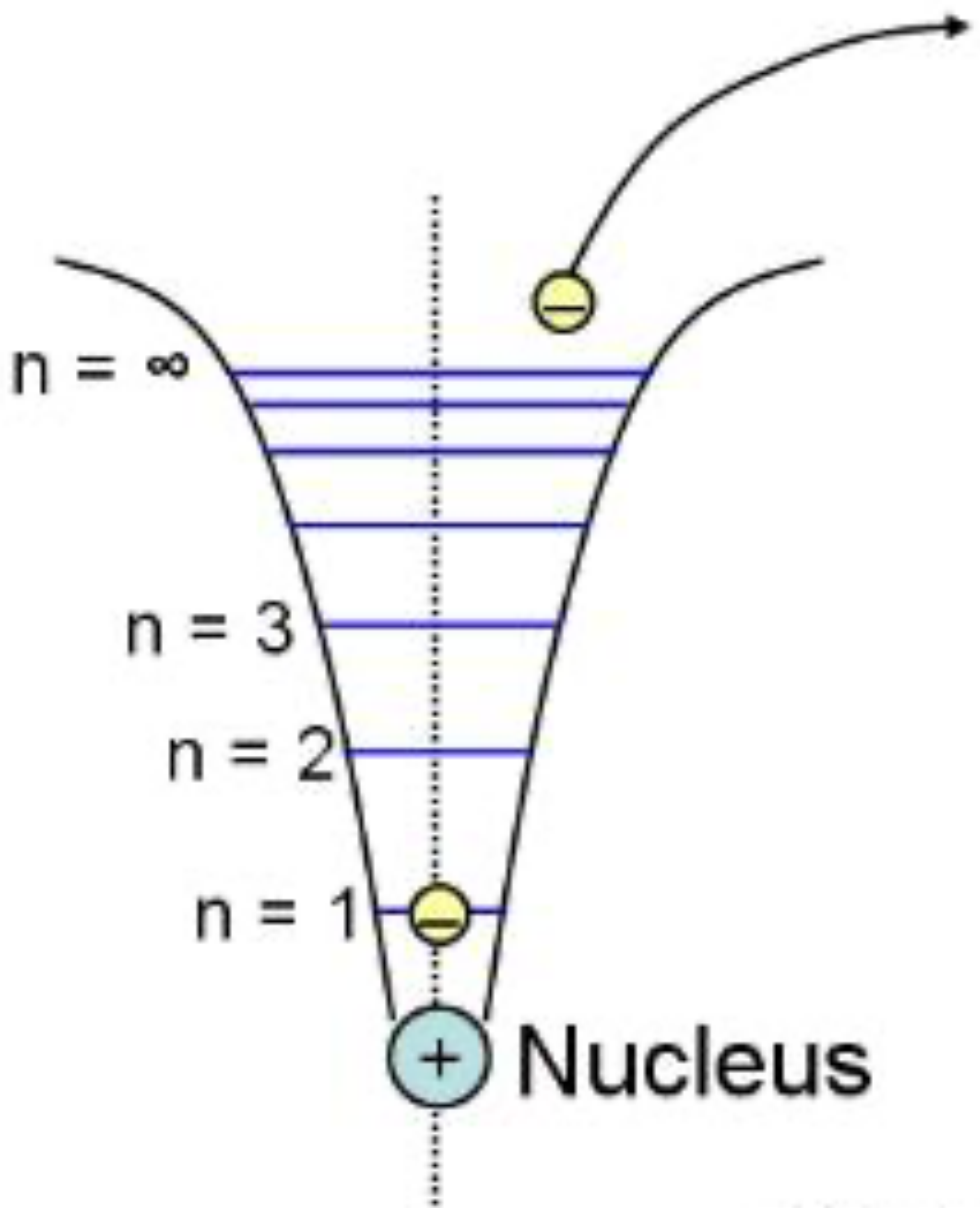
Particle wave duality, $[a, a^\dagger] = 1$, and all that.

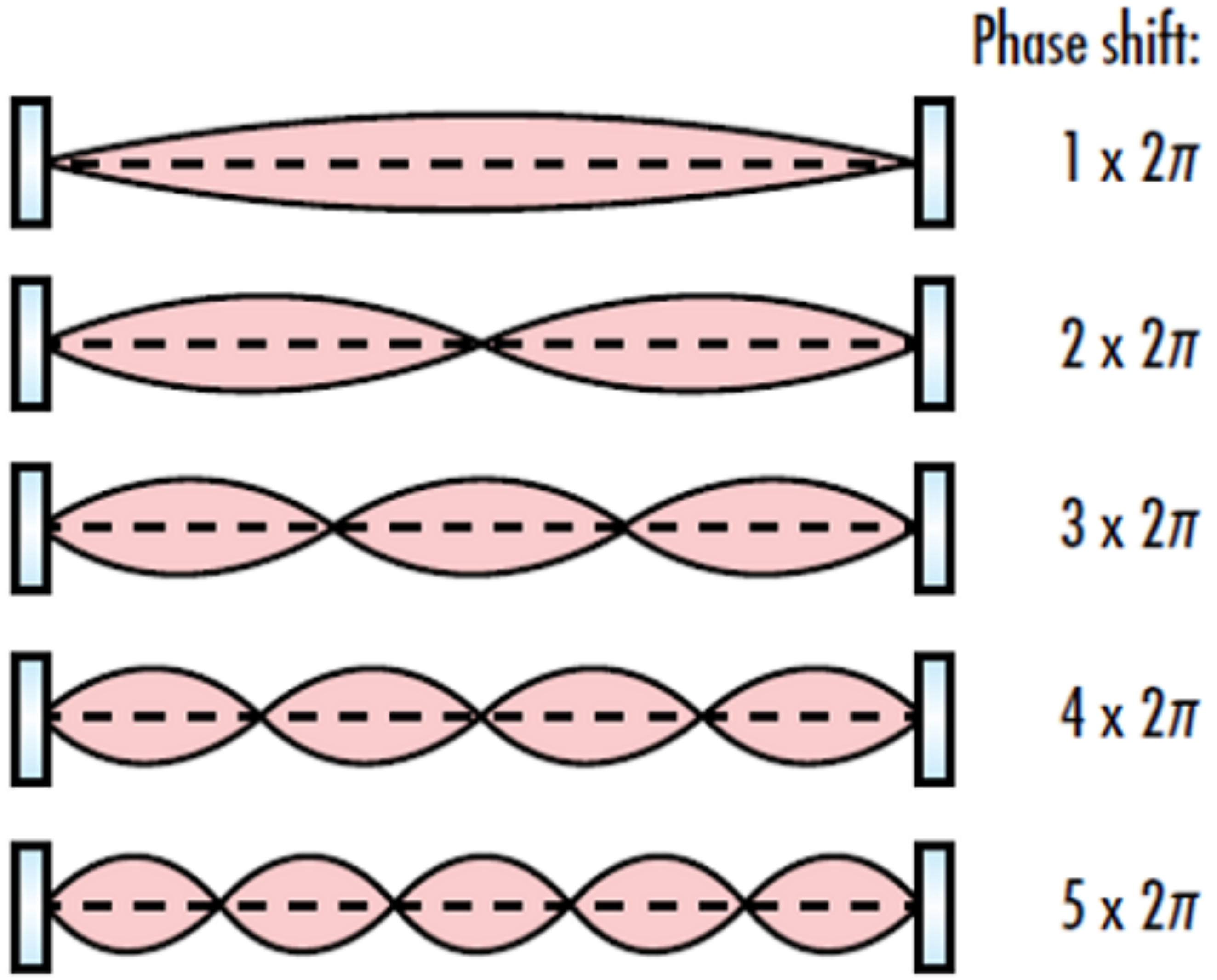
(This is sometimes called "second" quantization)

Quantum Fields in Small Devices

- In this big Universe, fields sometimes get localized to a finite regions. Either "naturally" or in a lab.

$$\phi(x_\mu) \underset{\sim}{=} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega}} \left(a_{\vec{k}} u_{\vec{k}}(\vec{x}) e^{i\omega t} + a^\dagger_{\vec{k}} u_{\vec{k}}{}^*(\vec{x})(e^{-i\omega t} \right)$$





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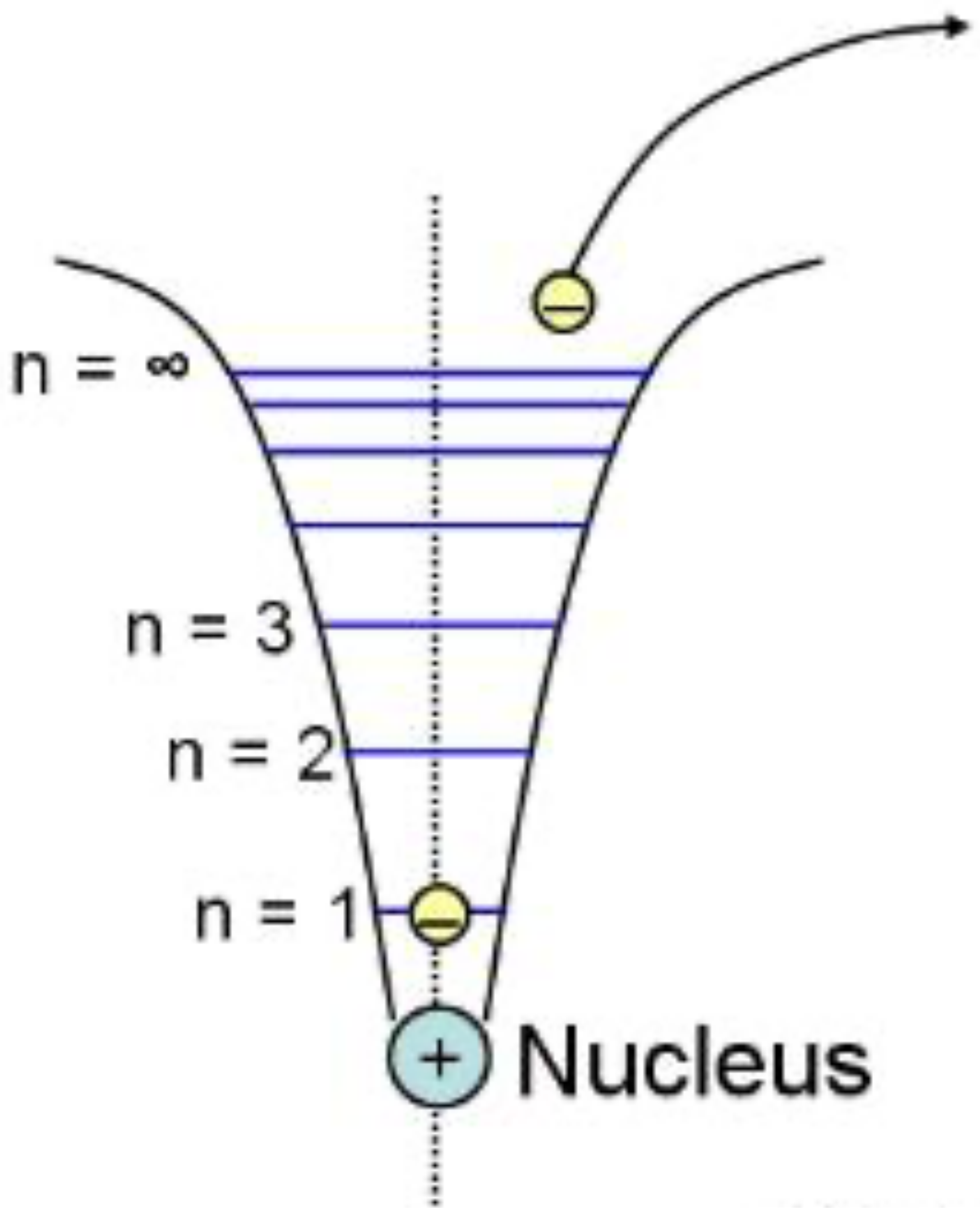
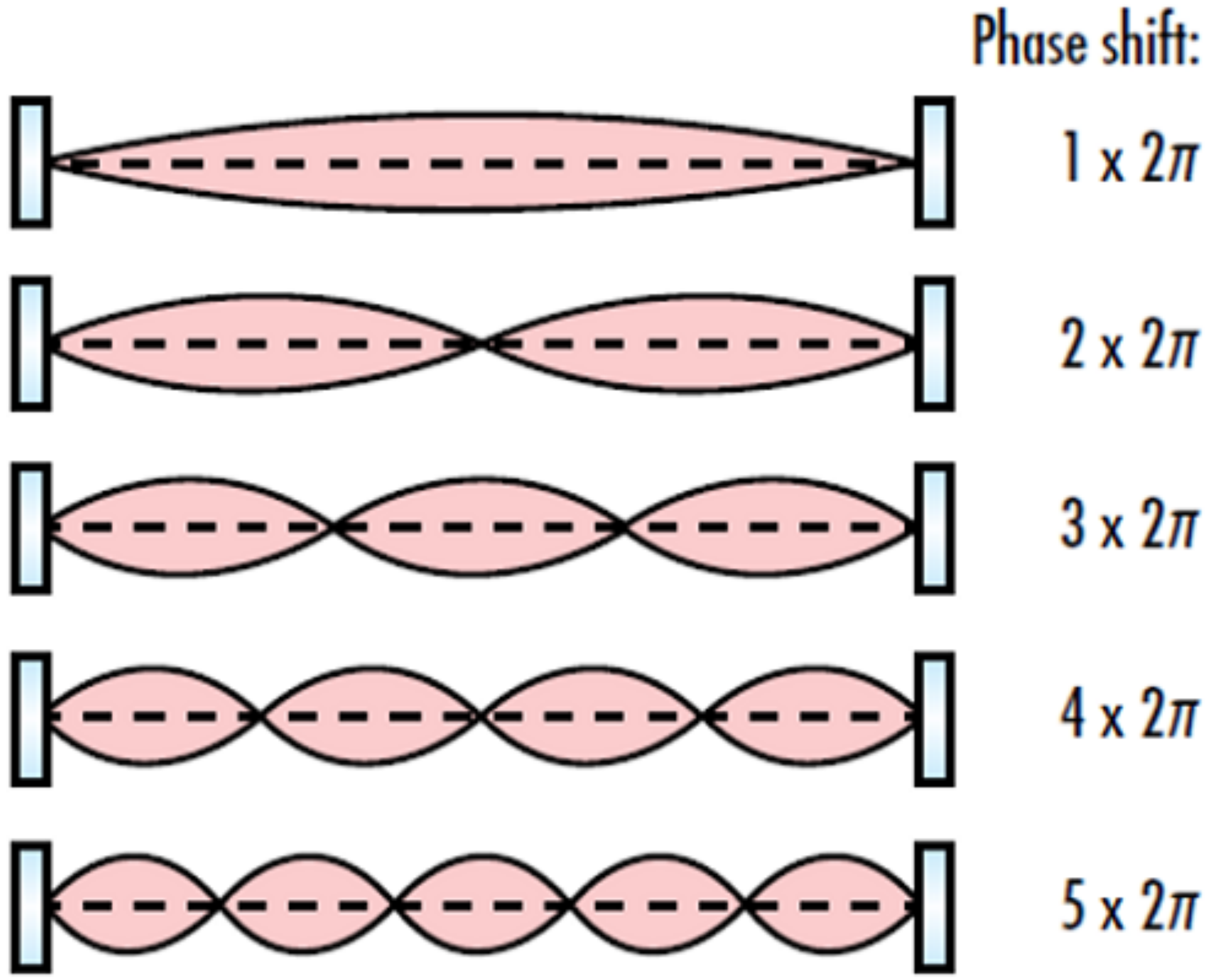
$$+\sum_j \frac{1}{\sqrt{2\omega}}\left(a_j u_j(\vec{x})e^{i\omega t} + a^\dagger_j u_j{}^*(\vec{x})(e^{-i\omega t}\right)$$

Only a discretum satisfies boundary conditions.



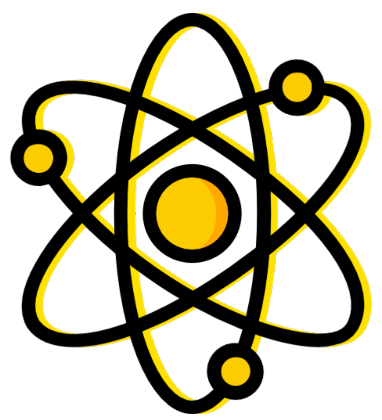


Quantum Fields in Small Devices

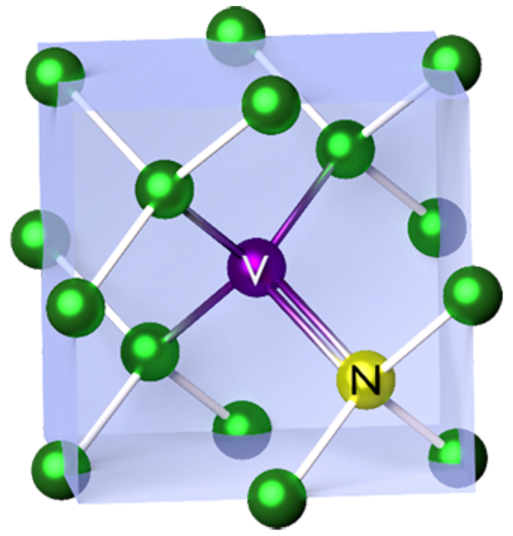
- Consider the low energy EFT of the discretum. Often in terms of a, a†

$$\phi_j(x_\mu) = \frac{1}{\sqrt{2\omega}} \left(a_j u_j(\vec{x}) e^{i\omega t} + a_j^\dagger u_j{}^*(\vec{x})(e^{-i\omega t} \right)$$

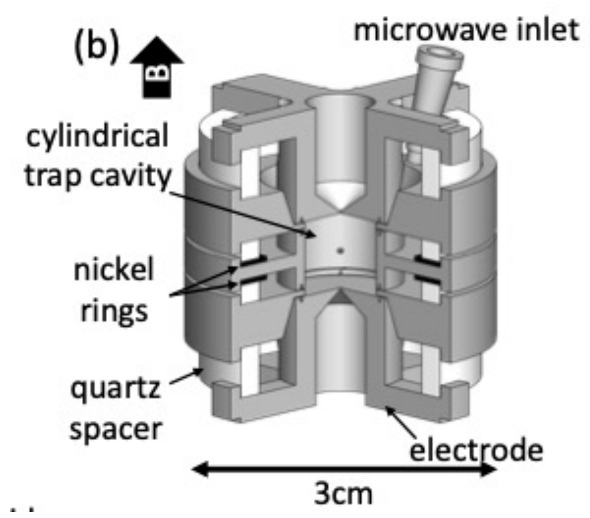
- In these EFTs, modes separate from the continuum, Quantum Mechanics shines:



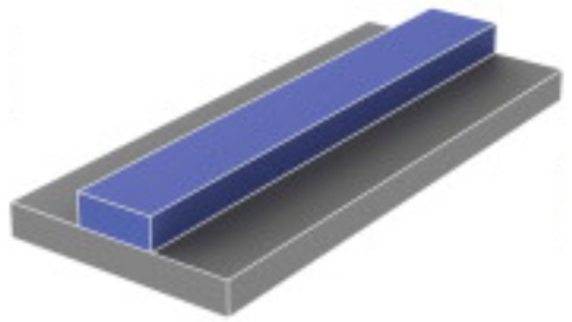
Atoms



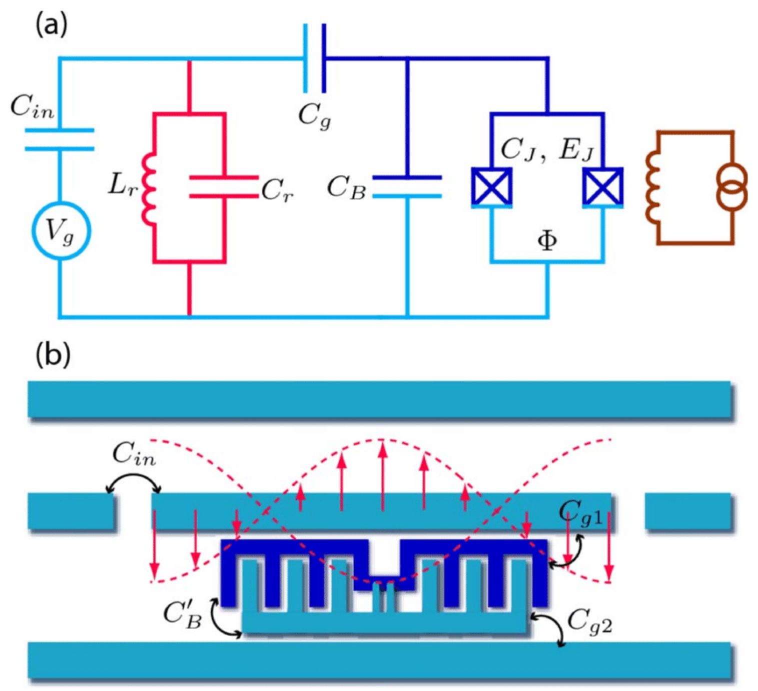
Defects



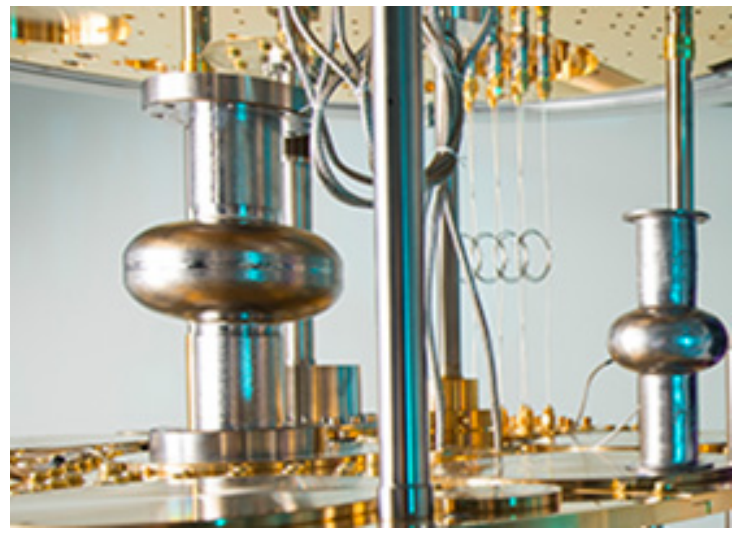
Artificial Atoms (particle in trap)



Optical waveguide



Superconducting circuits

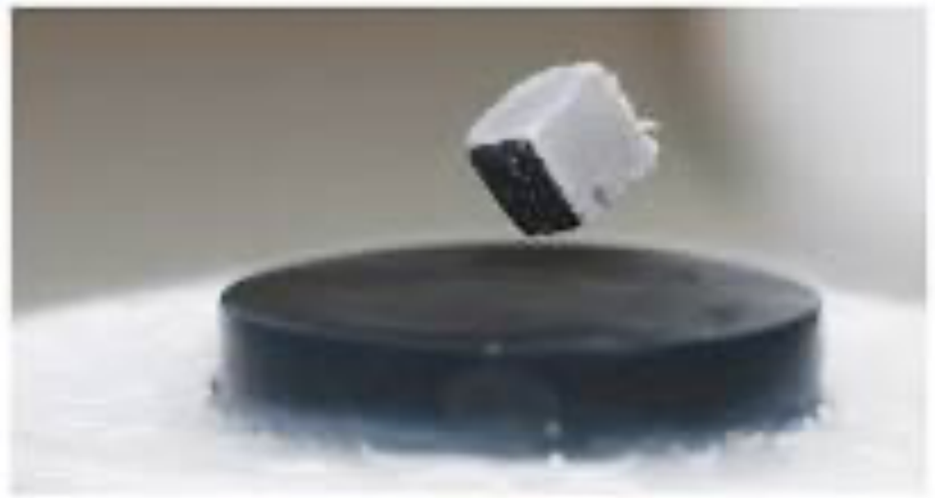
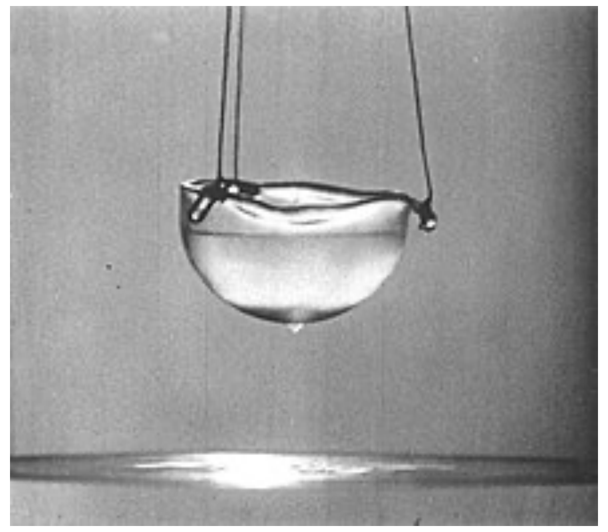
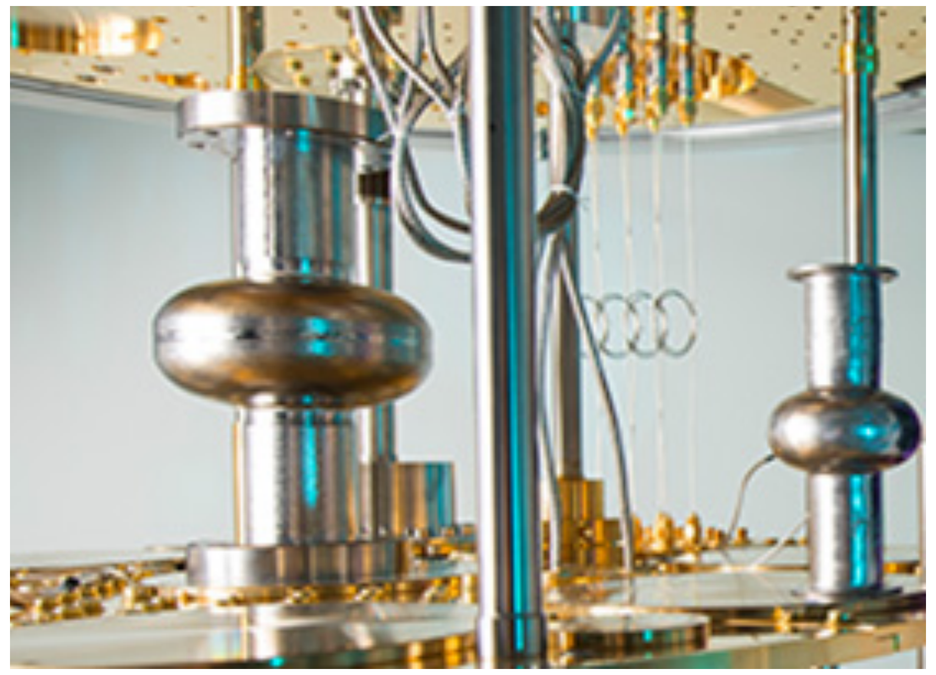
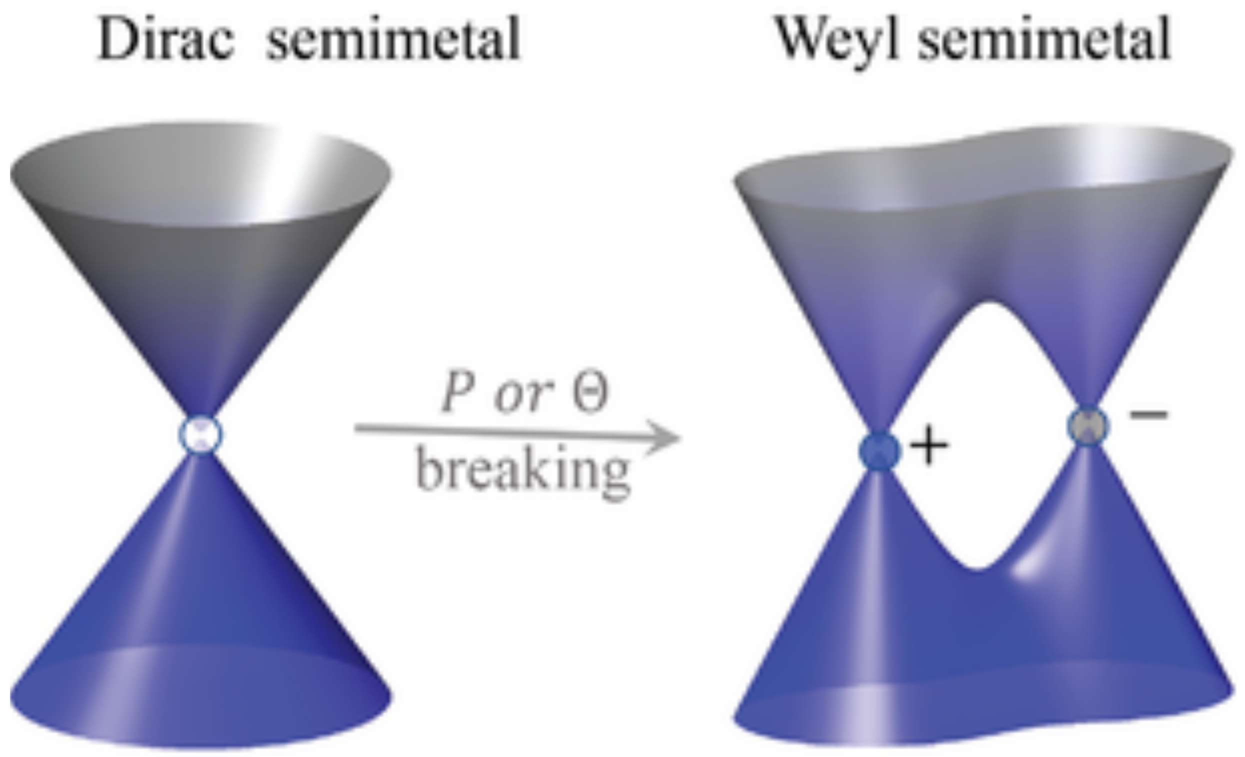


Electromagnetic Cavities

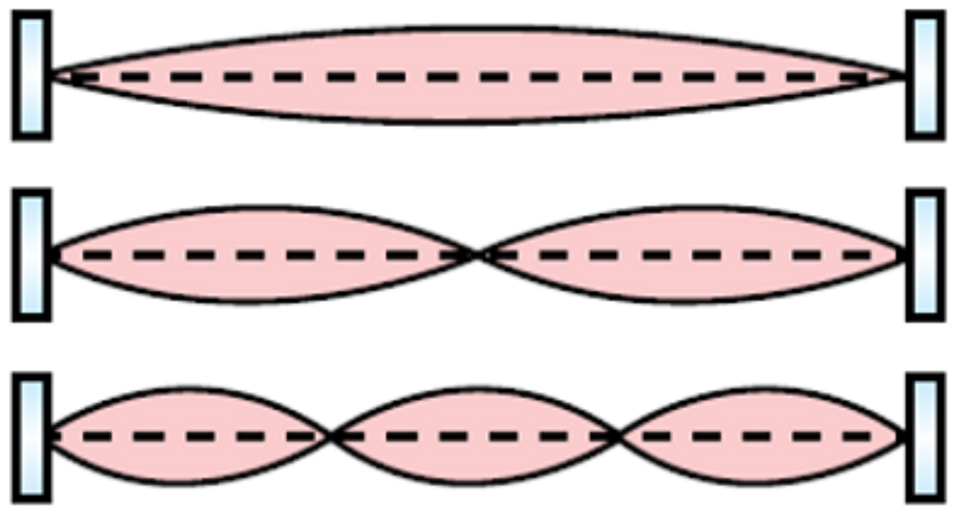
New phases

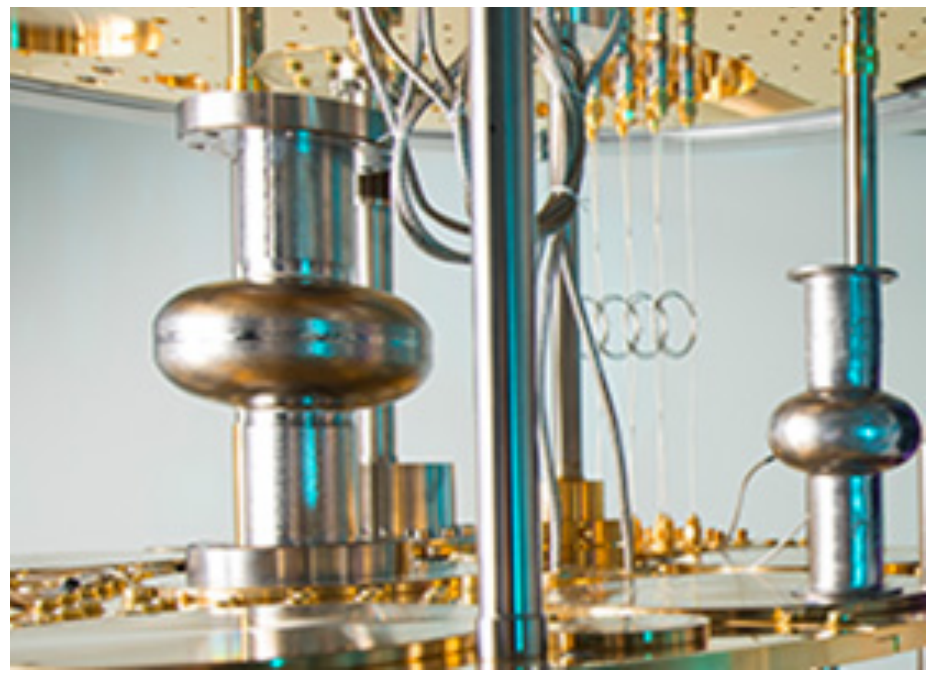
- As an aside: interesting quantum effects sometimes happen even without boundary conditions

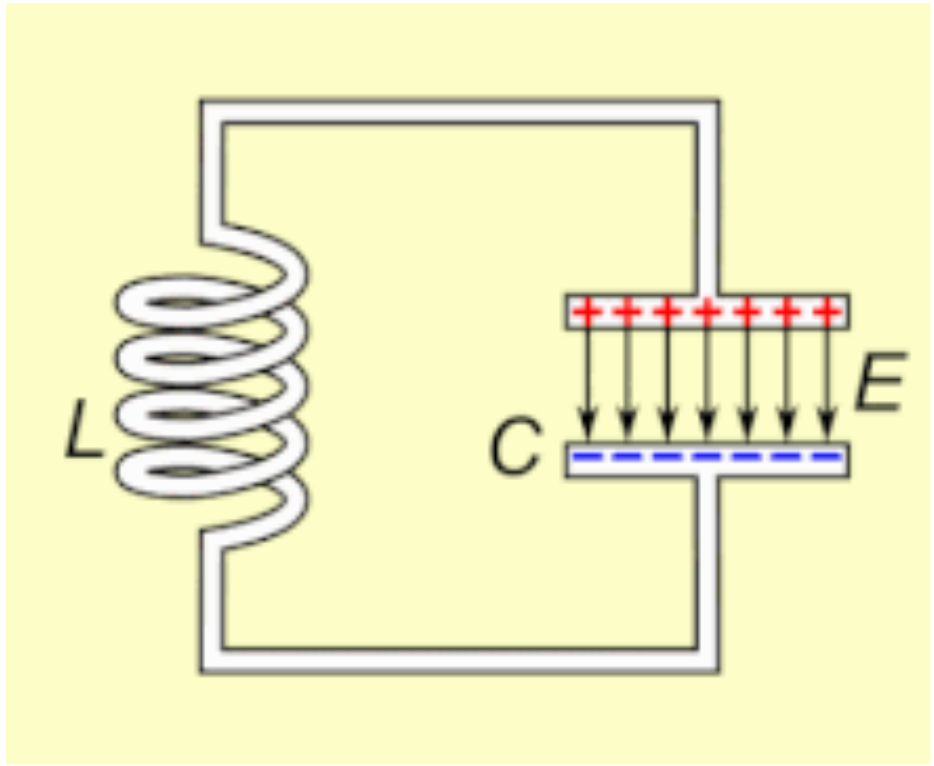
- New phases, gaps, ⋯ :
  - Superconductors
  - Superfluids
  - Semiconductors
  - Semi-metals
  - ⋯

Cavities & Circuits







- <u>Cavities</u>: Light in a box. A discretum of states.
- Separation from the continuum is parametrized by Q.

  $Q \sim 10^{10}$ is now routine. (Thank you accelerators!)

- <u>LC Circuits</u>: periodic current/quantized flux.
- Control frequency with L & C.

Both are harmonic. Equally spaced levels.

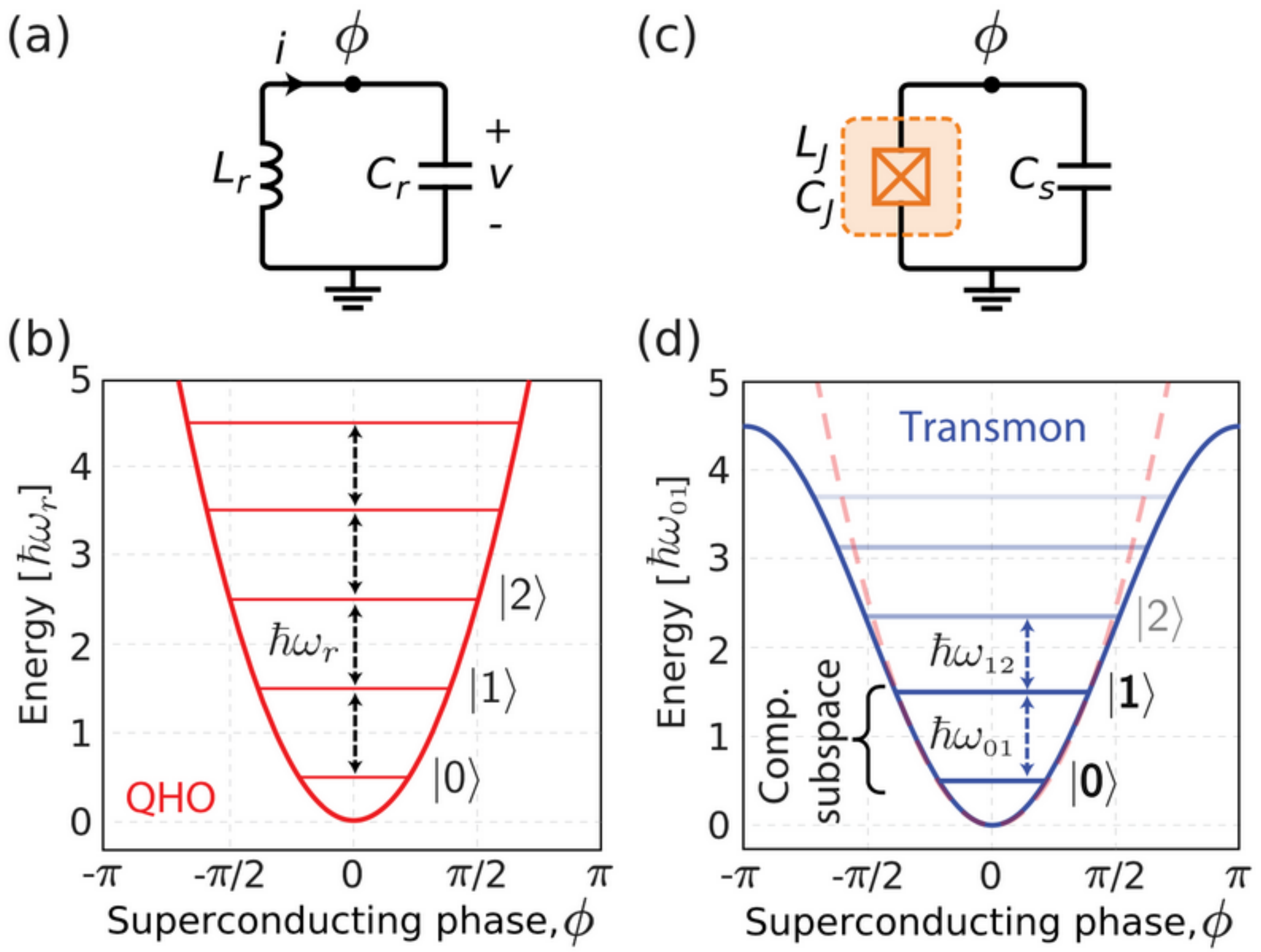
$$H_{mode} = \hbar\omega(a^\dagger a + 1/2)$$

Nonlinear Devices

- Like any EFT, in a quantum device there is a UV cutoff.
- We can add higher dim operators, e.g. making L a function of $a^\dagger a$.

$$H = \hbar\omega(a^\dagger a + 1/2) + \kappa(a^\dagger a)^2$$

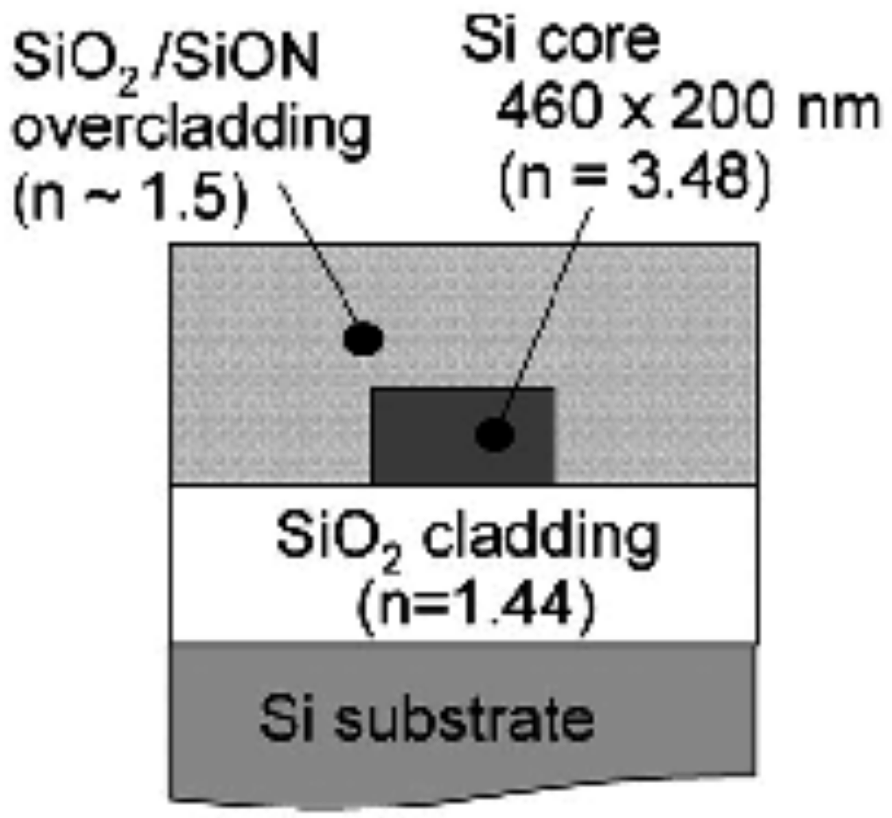
- Level spacing is **nonuniform.**
- This allows for control of individual levels of a given mode!

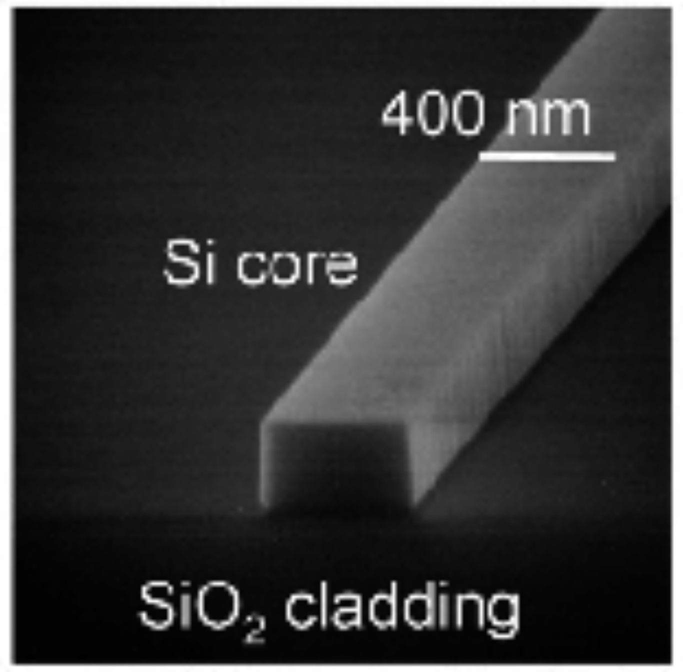


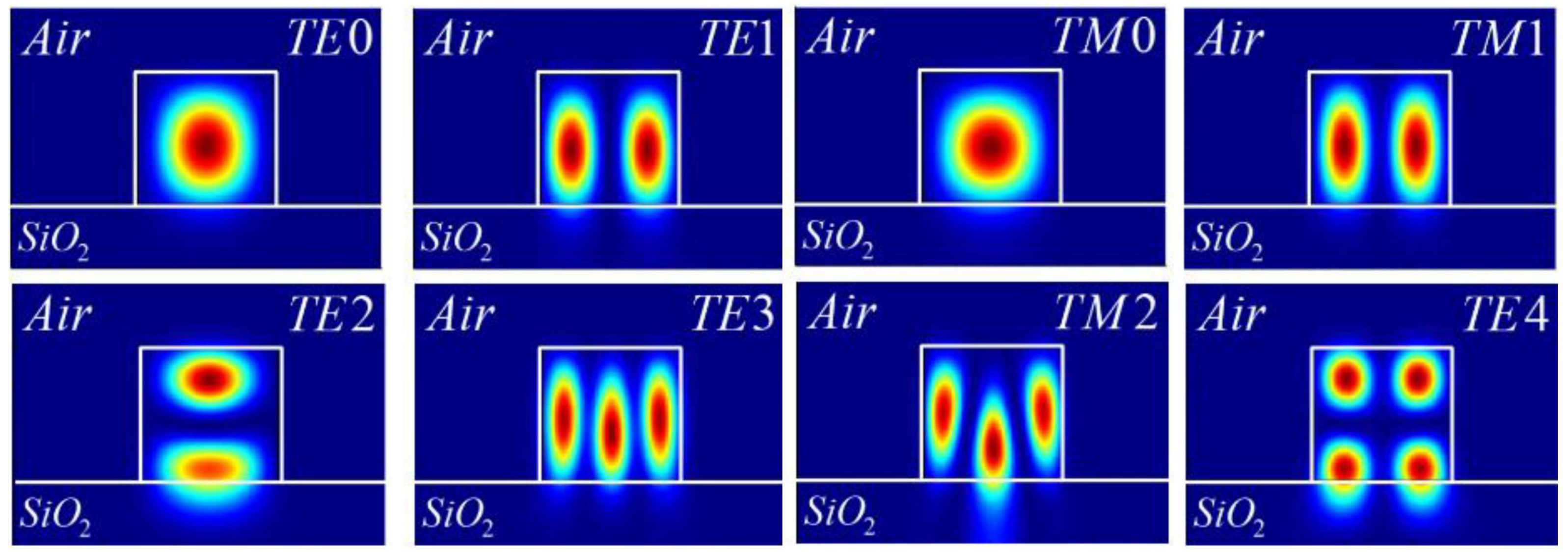
Optical Devices

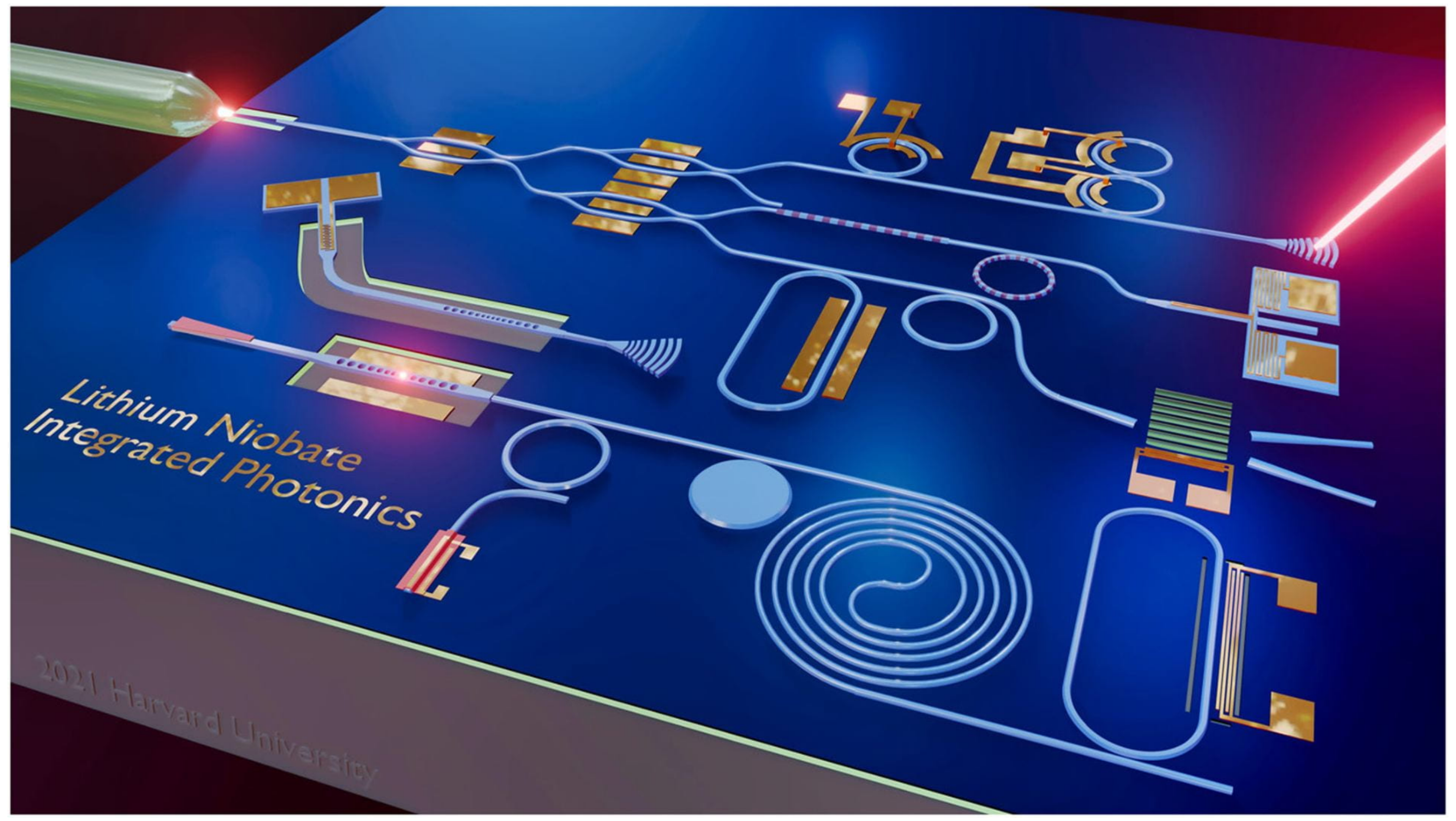
- Optics is the low energy EFT of light in matter.
- We can control the dispersion relation: $k = n\omega$.
  Useful for localization.
- A waveguide admits a 1D EFT w/ modes quantized in transverse direction.
- Transverse wave function affects longitudinal dispersion relation (a la KK modes!)

Linear Optics: $H = E^2 + B^2 = \sum \hbar\omega(a^\dagger a + \frac{1}{2})$









"Integrated photonics"

Nonlinear devices

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Dim-6: $$H_{\text{SPDC}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(2)}_{jkl} E_j E_k E_l \right)$$

Dim-8: $$H_{\text{4-wave}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(3)}_{jkl} E_j E_k E_l E_m \right)$$

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Estimate χ's in naive dimensional analysis:

When the field is set to that in an atom, we set (Dim-4 ~ Dim-6 ~ Dim-8):

$$E_{\text{atom}} \sim e/4\pi a_0^2$$

$$\chi^{(2)} \sim \frac{\sqrt{4\pi}}{\alpha^{5/2} m_e^2}$$

$$\chi^{(3)} \sim \frac{4\pi}{\alpha^5 m_e^4}$$

(by comparison, in vacuum $\chi^{(2)} = 0$, $\chi^{(3)} = \dfrac{2\alpha^2}{45 m_e^4}$)

etc

Many interesting "device EFTs":

Phonons

Magnons

Atoms (trapped or free)

Ions

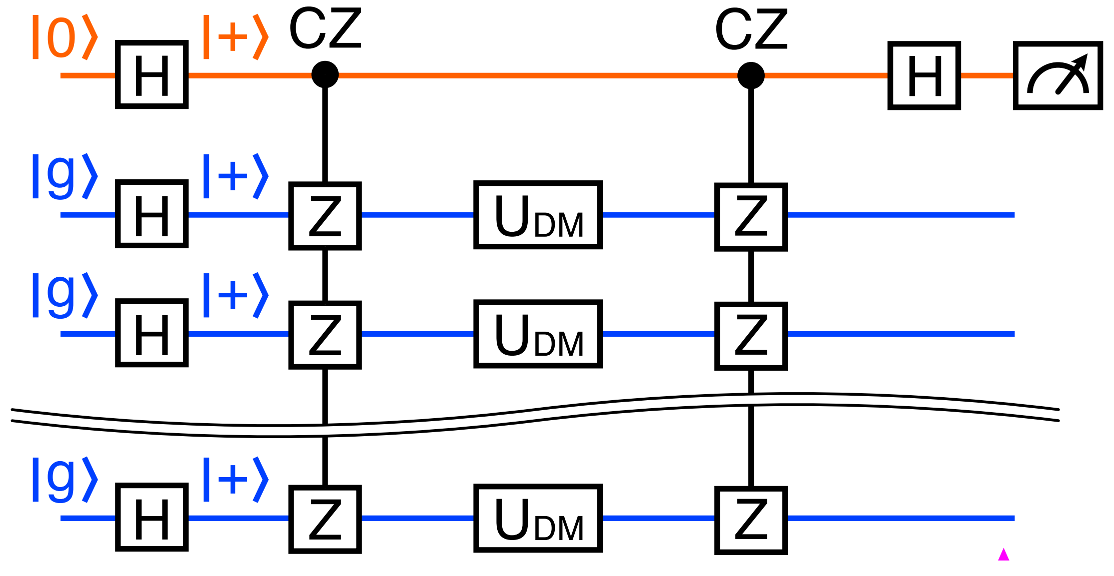
Mechanical modes

Modes of electron/ion in a trap

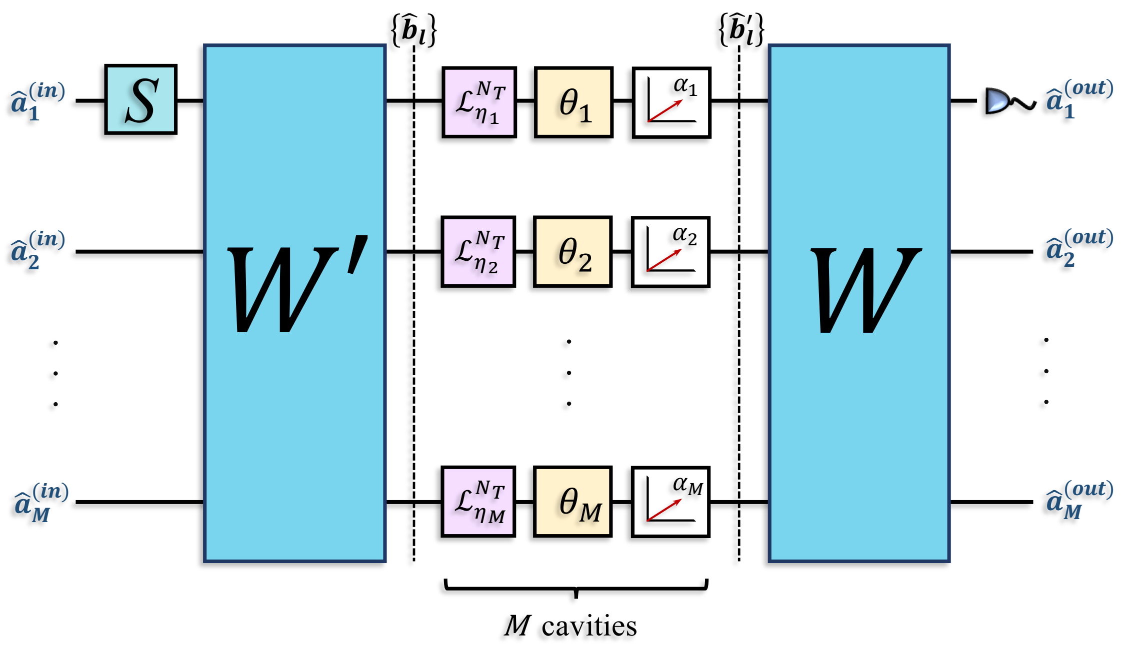
Cooper pairs

Quantum Sensing

- The isolation of modes, and the ability to control them enables feeble effects to lead to dramatic consequences:
  - Appearance of mode occupation (Haloscope, light shining though wall, phonon detection)
  - Removal of mode occupation (e.g. TES, Nanowires: SC to normal)
  - Time evolution of ultra sensitive states (Atomic clocks, squeezed states, spin precession, photon counting, entangled qubit states, etc)



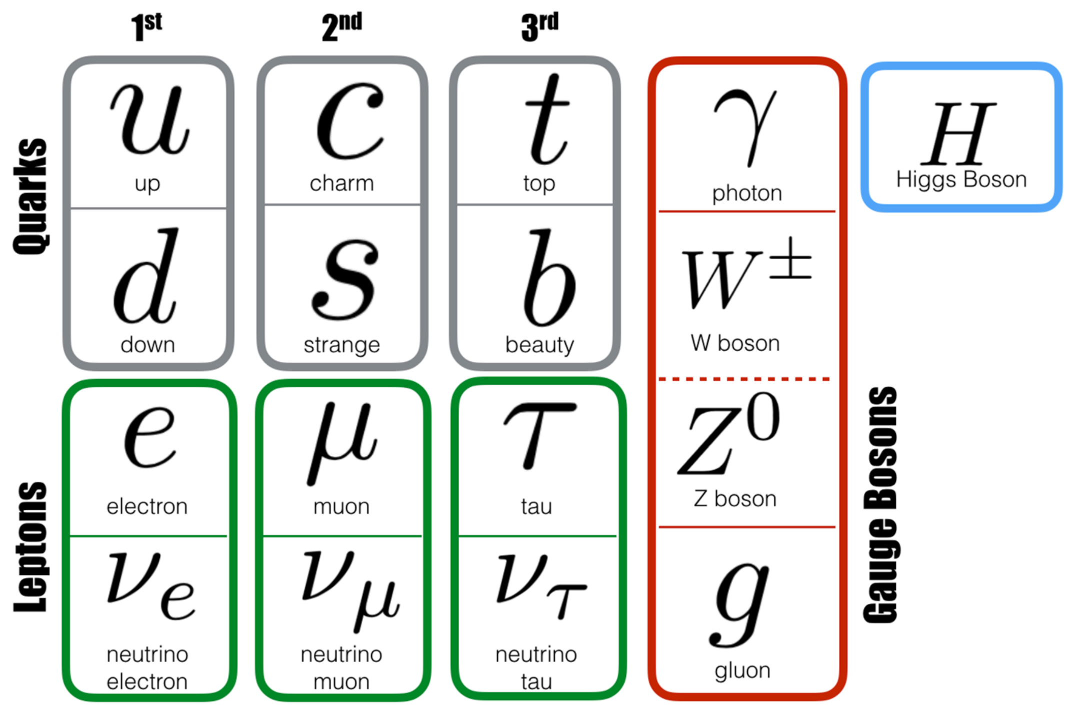
Qubits:
Chen et al, 2311.10413, Ito et al 2311.11632



Distributed squeezing:
Brady et al, *PRX Quantum* 3 (2022) 3, 030333

BSM - for Quantum Mechanics

New Physics ⟶ New Fields



+ Something new.

Ok. For concreteness,

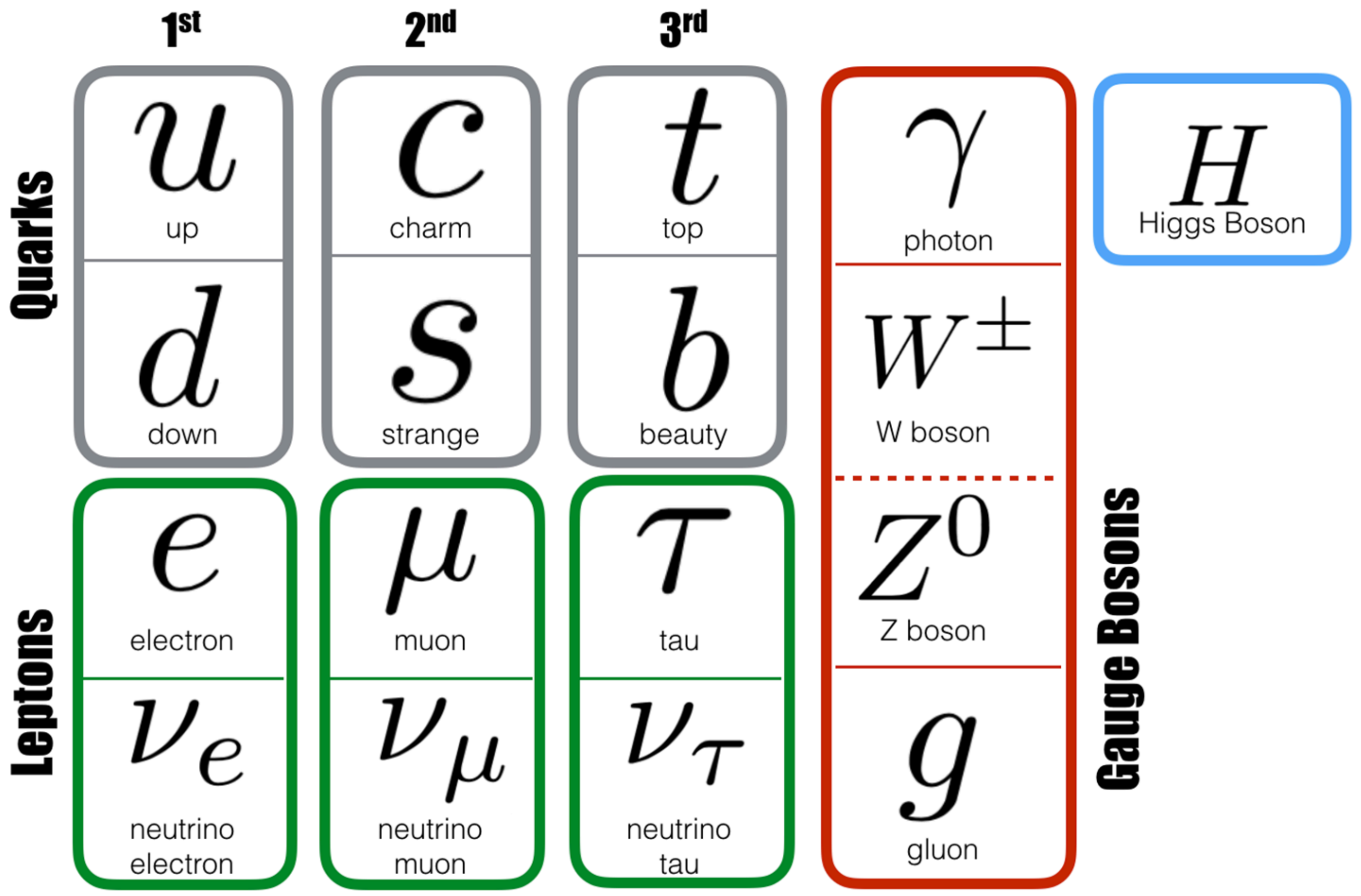
(and because QIS is often about controlling light)

lets assume the new field couples to photons.

Linear or nonlinear?

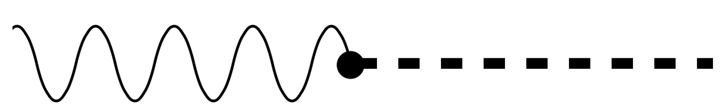
Dark Photons - a Linear Extension

- If something mixes linearly with the photon, it must have the same quantum numbers:



- The dark Photon effective Hamiltonian:

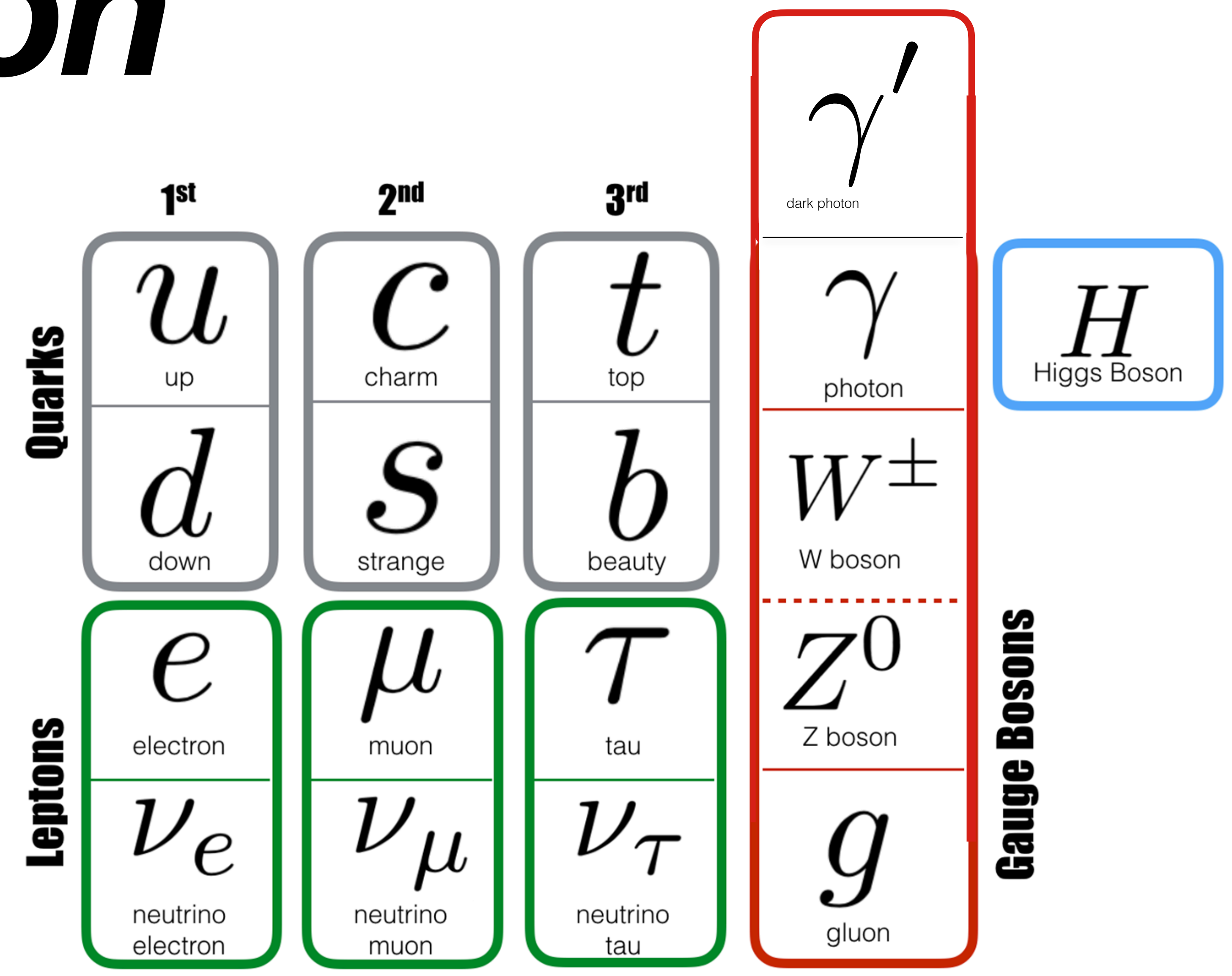
$$\mathcal{H} \supset \mathcal{H}_{QED} + \varepsilon\, \vec{E}\cdot\vec{E}' + \vec{B}\cdot\vec{B}'$$



(and dark photon also has a mass, and a longitudinal polarization!)

- A dark photon, if it exists, would teach us profound lessons. New force of nature. Grand Unification, etc.

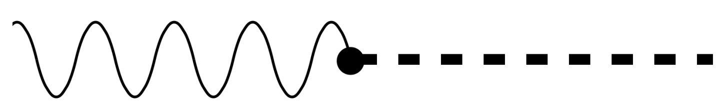
Dark Photons - a Linear Extension



- If something mixes linearly with the photon, it must have the same quantum numbers:

- The dark Photon effective Hamiltonian:

$$\mathcal{H} \supset \mathcal{H}_{QED} + \varepsilon\, \vec{E}\cdot\vec{E}' + \vec{B}\cdot\vec{B}'$$



(and dark photon also has a mass, and a longitudinal polarization!)

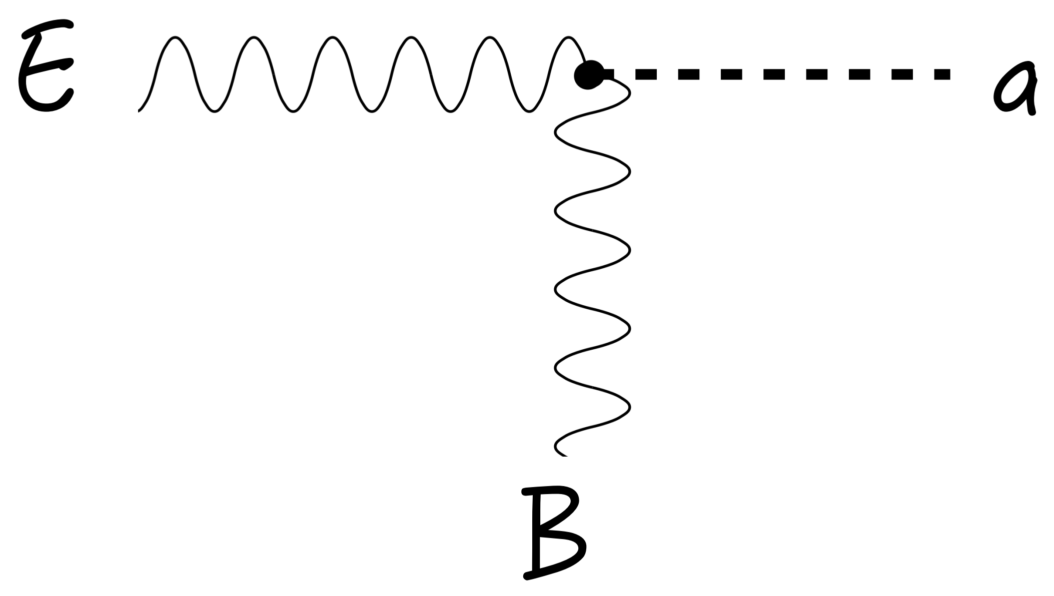
- A dark photon, if it exists, would teach us profound lessons. New force of nature. Grand Unification, etc.

Pecci and Quinn (77)

Axions - A nonlinear extension of QED

- A nonlinear interaction, naively, would involve 2 photons & 1 new field.

$$\mathcal{L} \supset \frac{a}{f} F^{\mu\nu}\tilde{F}_{\mu\nu} = \frac{a}{f} \vec{E}\cdot\vec{B}$$



- Axion phenomenology:
  - Axion mixing w/ photons polarized along background B field. [Sikivie]
  - Axion can be absorbed by photon -> up conversion. [Berlin et al (2019)] [Gao, RH (2020)]
  - Axion exchange -> photon nonlinearity in vacuum. [PVLAS] [e.g. Bogorad, Hook, Kahn, Soreq (2020)]
- Of course, the discovery of an axion will be a profound insight.

[Talks by Sebastian and Bianca]

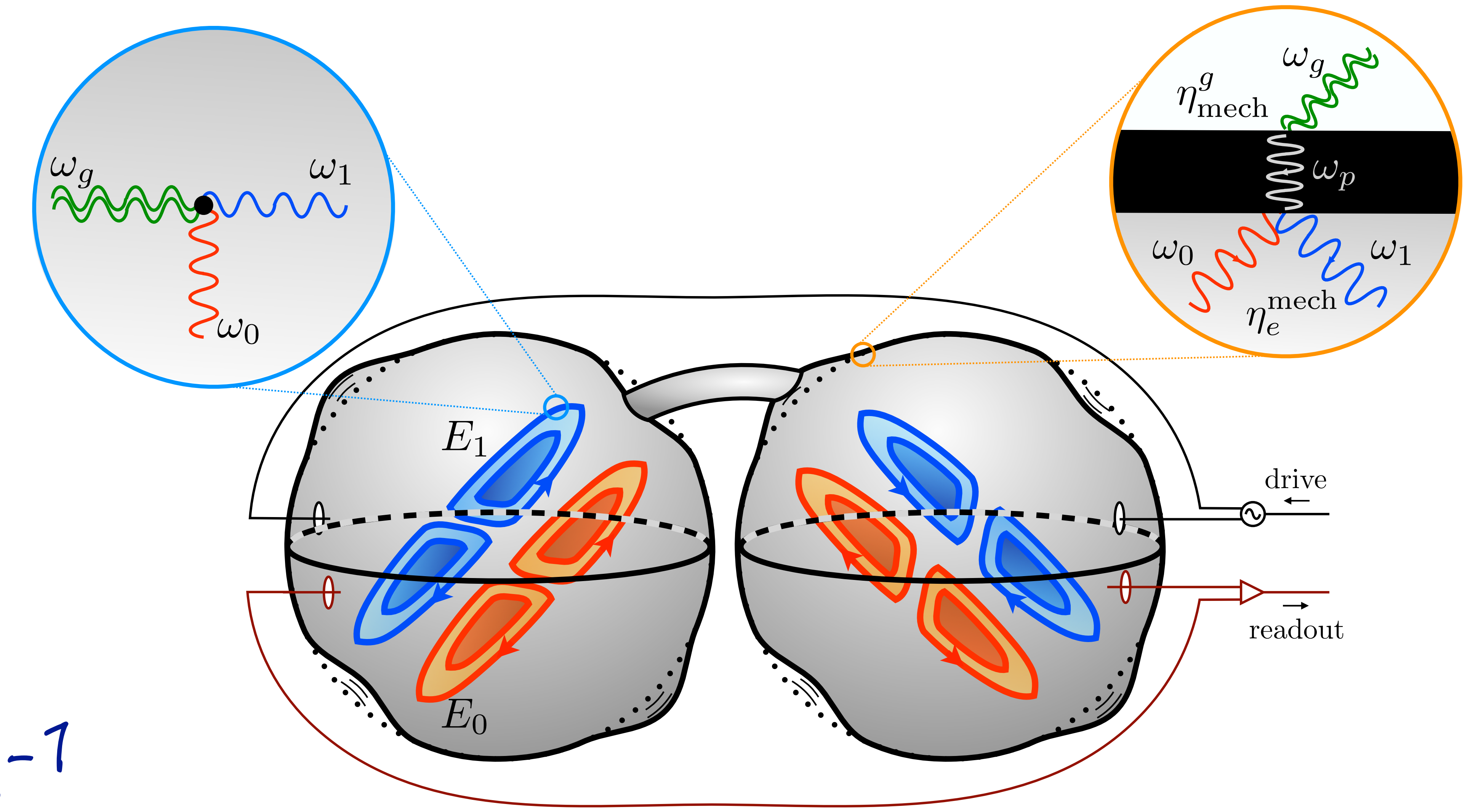
Gravity Waves - A nonlinear extension

- A gravity wave also interacts w/ two photons

$$\mathcal{L} \supset F^{\mu\nu}F_{\mu\nu} \sim h(B\cdot B + E\cdot E) + \cdots$$

- But often more important:

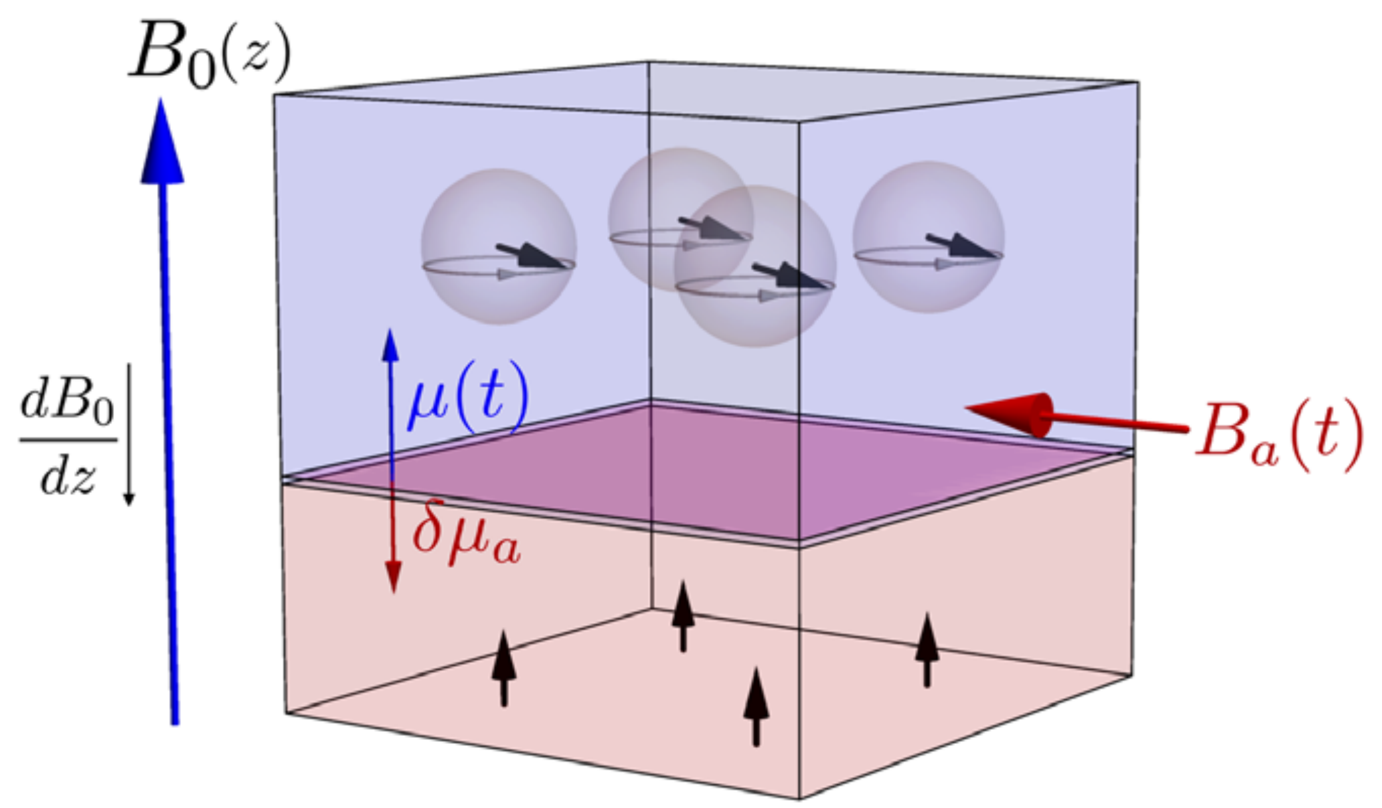
$$H = \hbar\omega(a^\dagger a + 1/2) \quad \text{with} \quad \omega \sim (1+h)L^{-1}$$



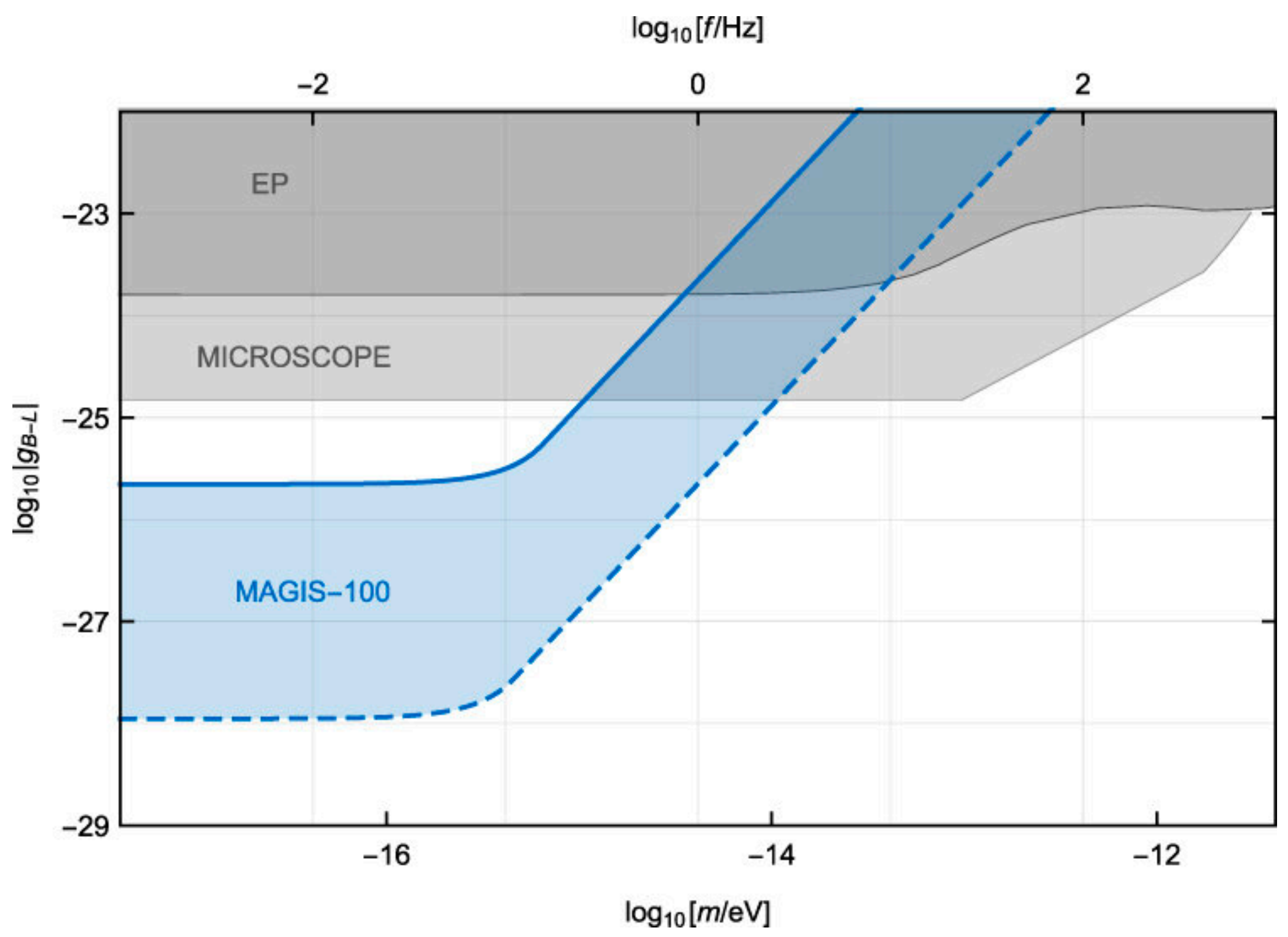
- Axion-like phenomenology:
  - GW mixing w/ photons in background B field.
  - GW can be absorbed by photon -> up conversion.

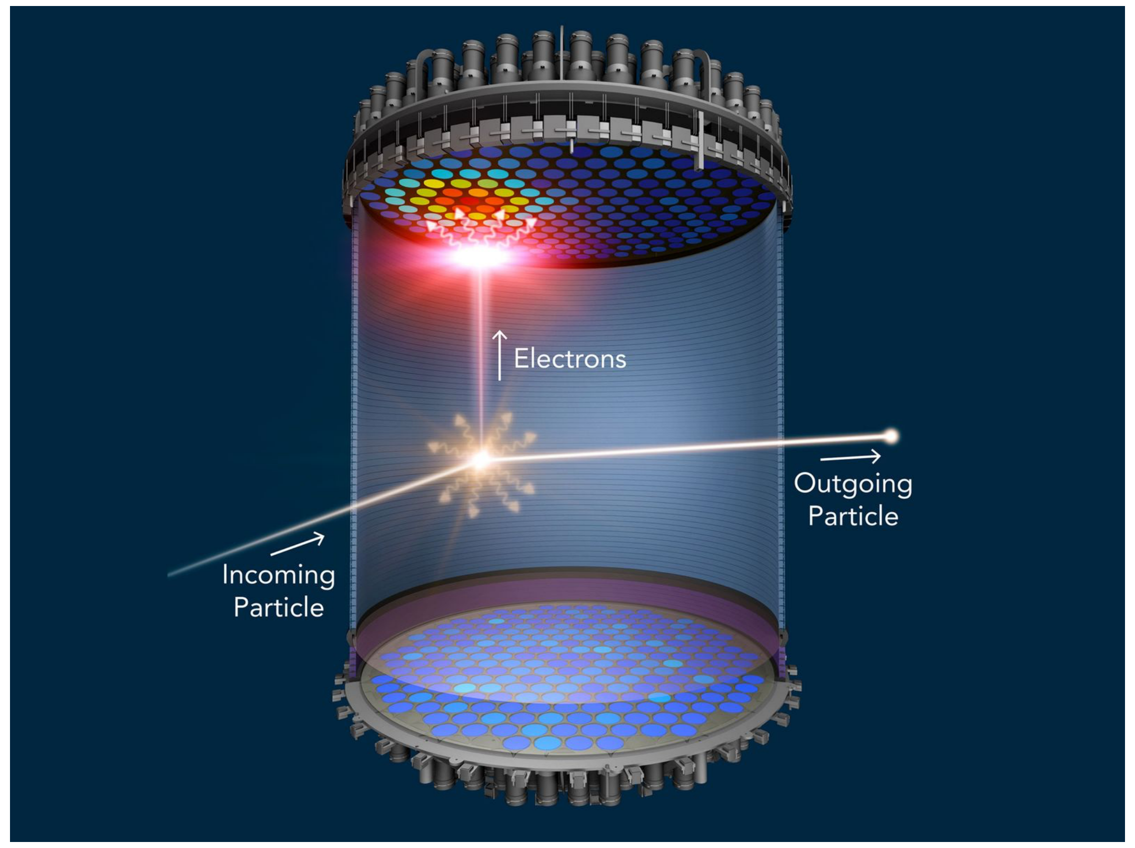
[Talks by Sebastian and Bianca]

Interaction with Matter



- New particles may interact with electrons on nuclei.
- Linear:
  - Spin precision (e.g. CASPER, spin qubits, [Talk by Josh])
  - Forces (accelerometers, e.g. [Talk by Tim])
- Non-liner:
  - Scattering (direct detection, e.g. [Talk by Andrew])

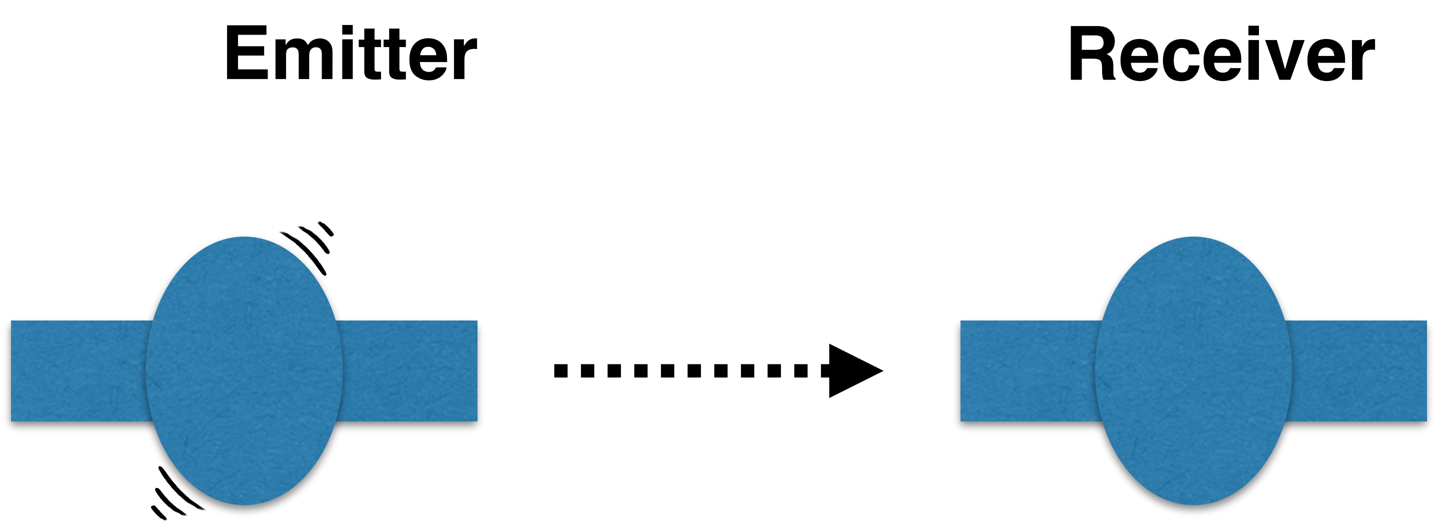




Signal Frequency

- What is the frequency of signal?
  - Narrowband:

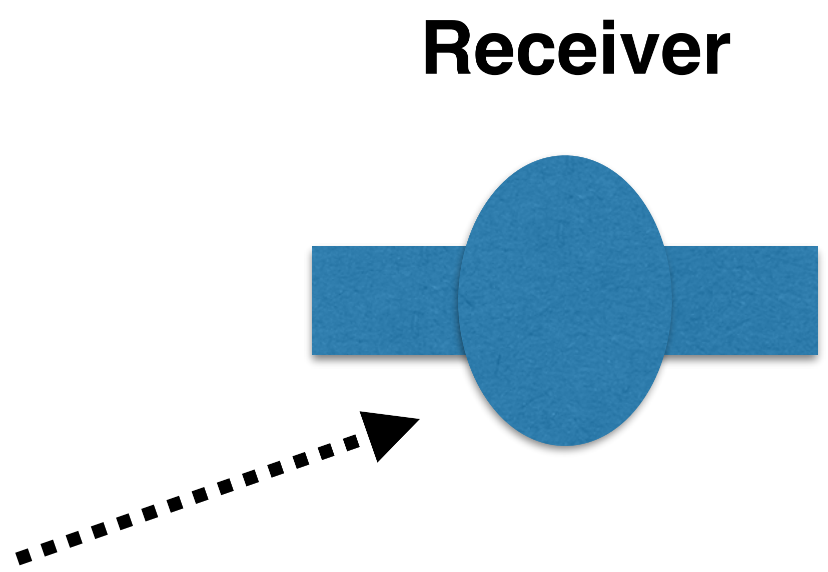
Light Shining through wall:



Looking for a new particle
Known frequency

[Talk by Bianca]

A dark matter search:



Looking for a new particle that's DM.
Unknown frequency (Scan!)

[Talks by Raphael and Ed]

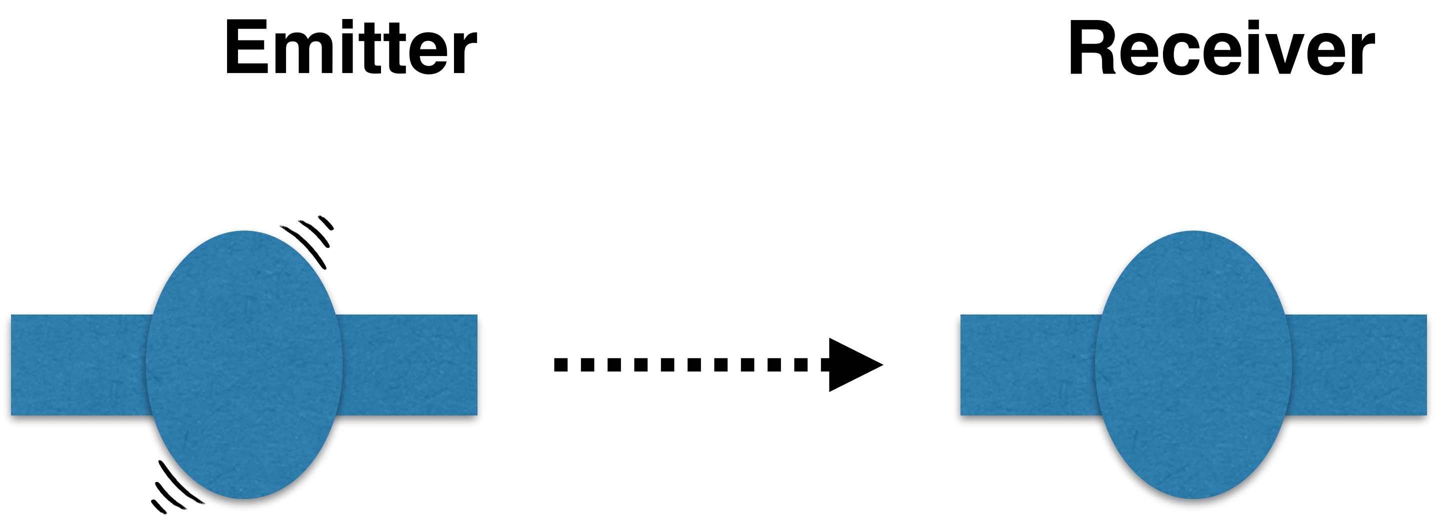
- Boradband - "impulse"
- None of the above (e.g. LIGO)

Signal Frequency

- What is the frequency of signal?
  - Narrowband:

    High Quality detectors needed.

Light Shining through wall:



Looking for a new particle

Known frequency

[Talk by Bianca]

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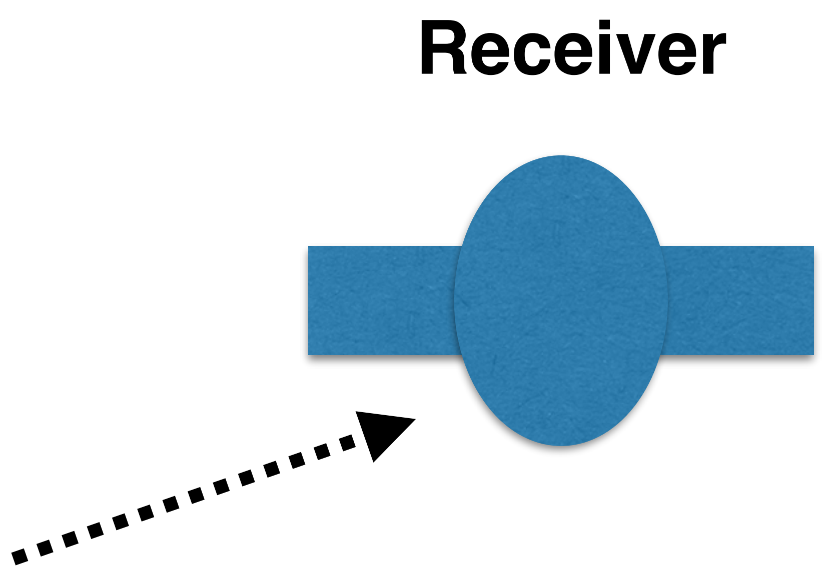


Looking for a new particle that's DM.

Unknown frequency (Scan!)

[Talks by Raphael and Ed]

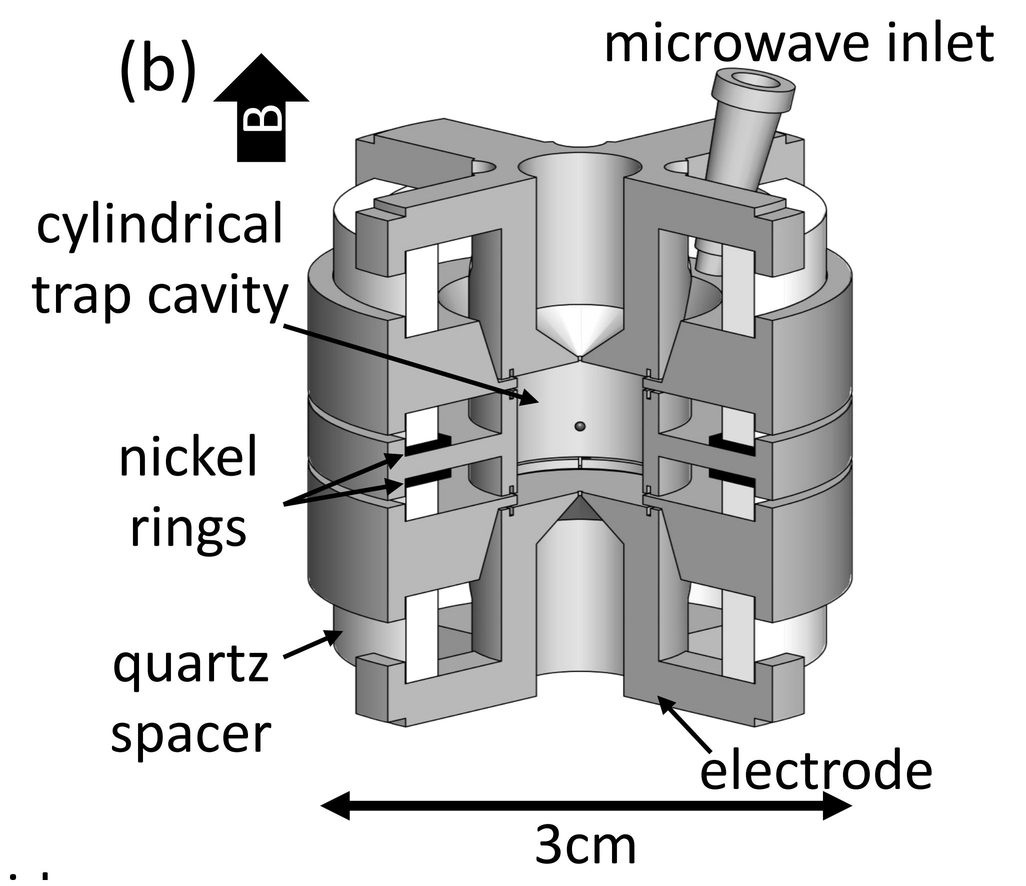
- Boradband - "impulse"
- None of the above (e.g. LIGO)

Examples

Today's schedule!

SRF Cavities
L^3He
Atom Interferometry

Single Particle Qubit



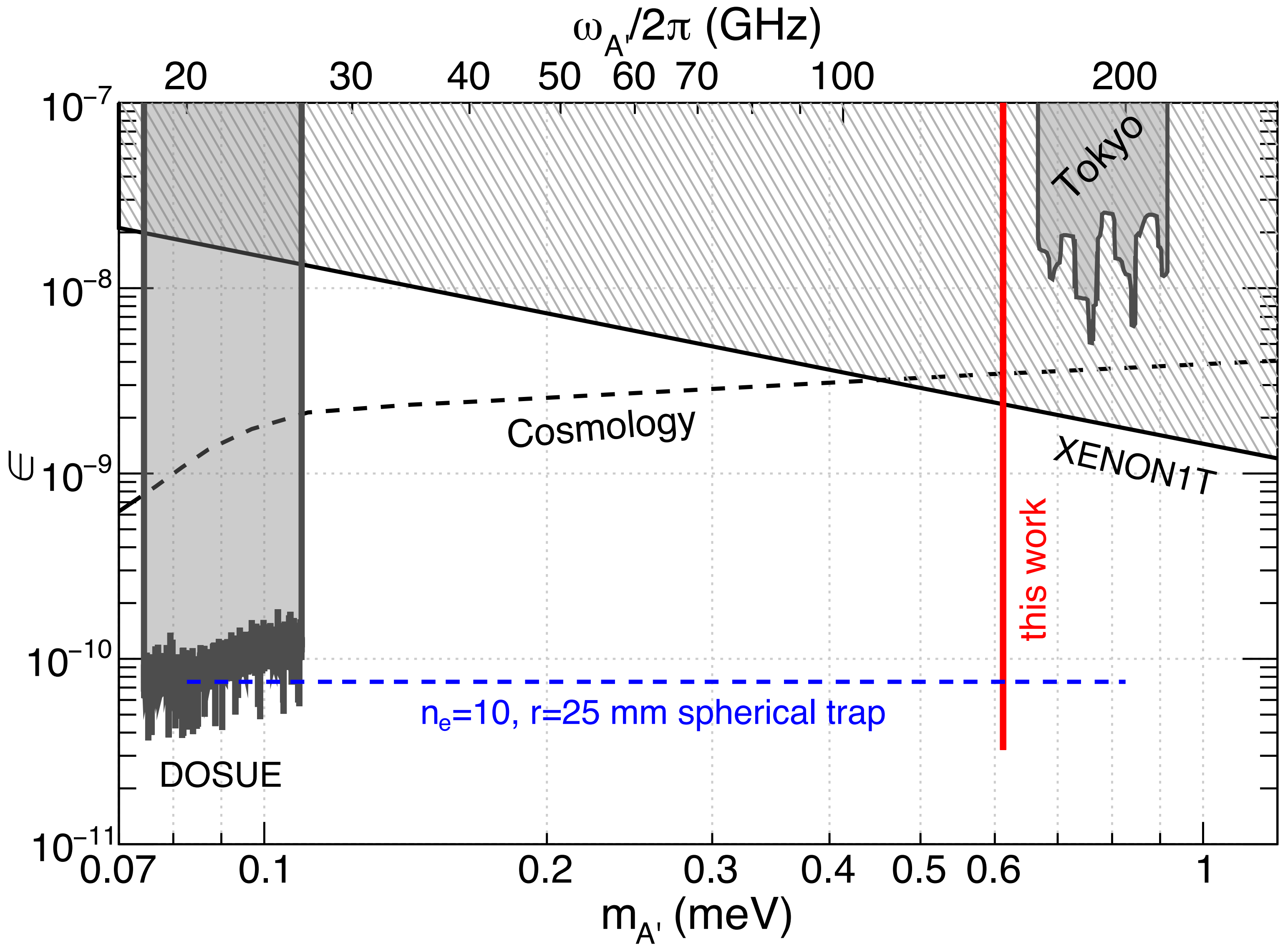
- The most precise theory-experiment comparison in physics:

  Electron magnetic moment (g-2)$_e$:
  The quantum state of a single electron in a trap is monitored via a **QND measurement**.

$$-\frac{\mu}{\mu_B} = \frac{g}{2} = 1.001\,159\,652\,180\,59\,(13) \quad [0.13 \text{ ppt}]$$

Phys. Rev. Lett. 130, 071801 (2023)
Editors choice!

- SQMS joined the effort, contributed to understanding loss sources.



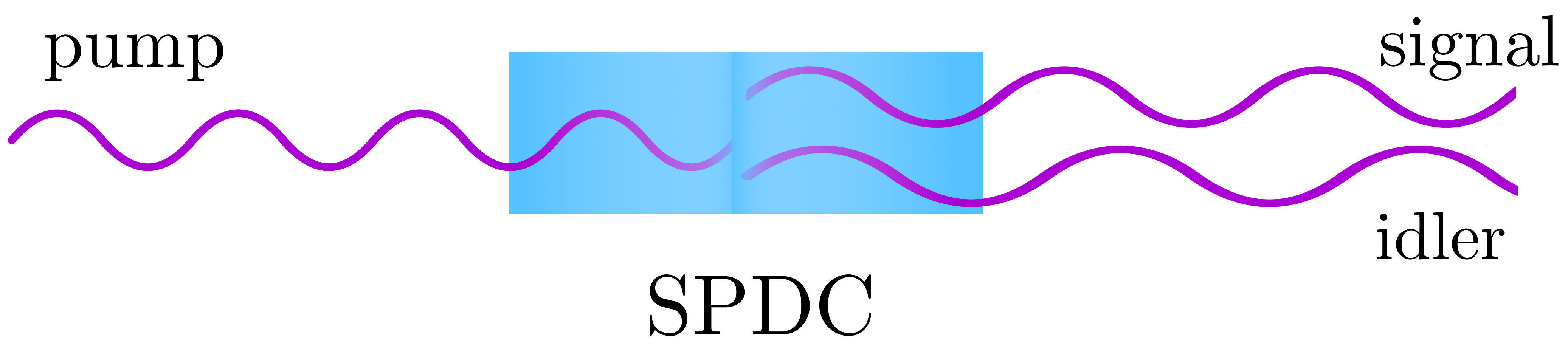
- **SQMS bonus:** We also found that a single-electron qubit is a sensitive DM search in a challenging frequency range!
- Theory + proof-of-concept!

Phys.Rev.Lett. **129 (2022) 26, 261801**
(a new NU-Stanford-Fermilab collaboration)

Estrada et al. *PRX Quantum* 2 (2021) 3, 030340

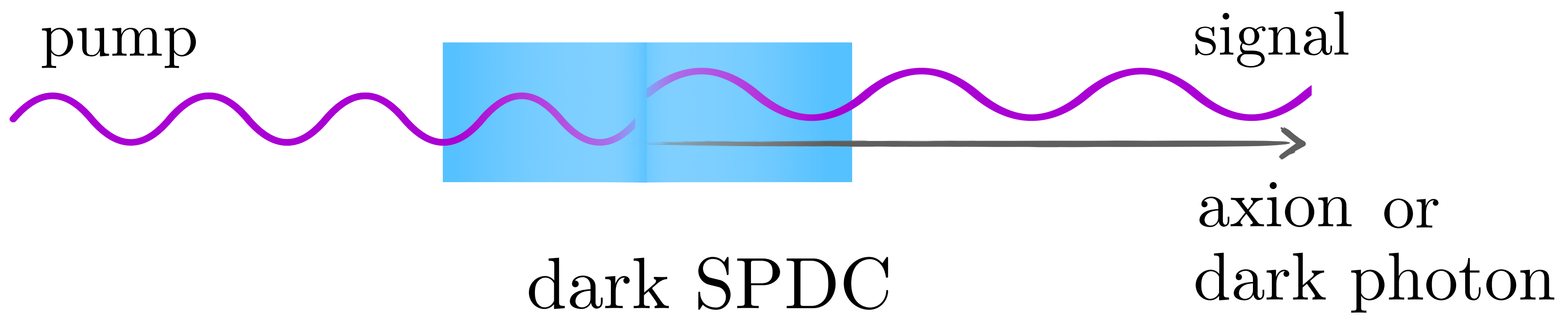
Nonlinear Optics with Dark States

- SPDC: a workhorse in quantum optics.
- Pump -> signal + idler (a "decay")
- Presence of idler is **inferred**. Might as well be invisible!

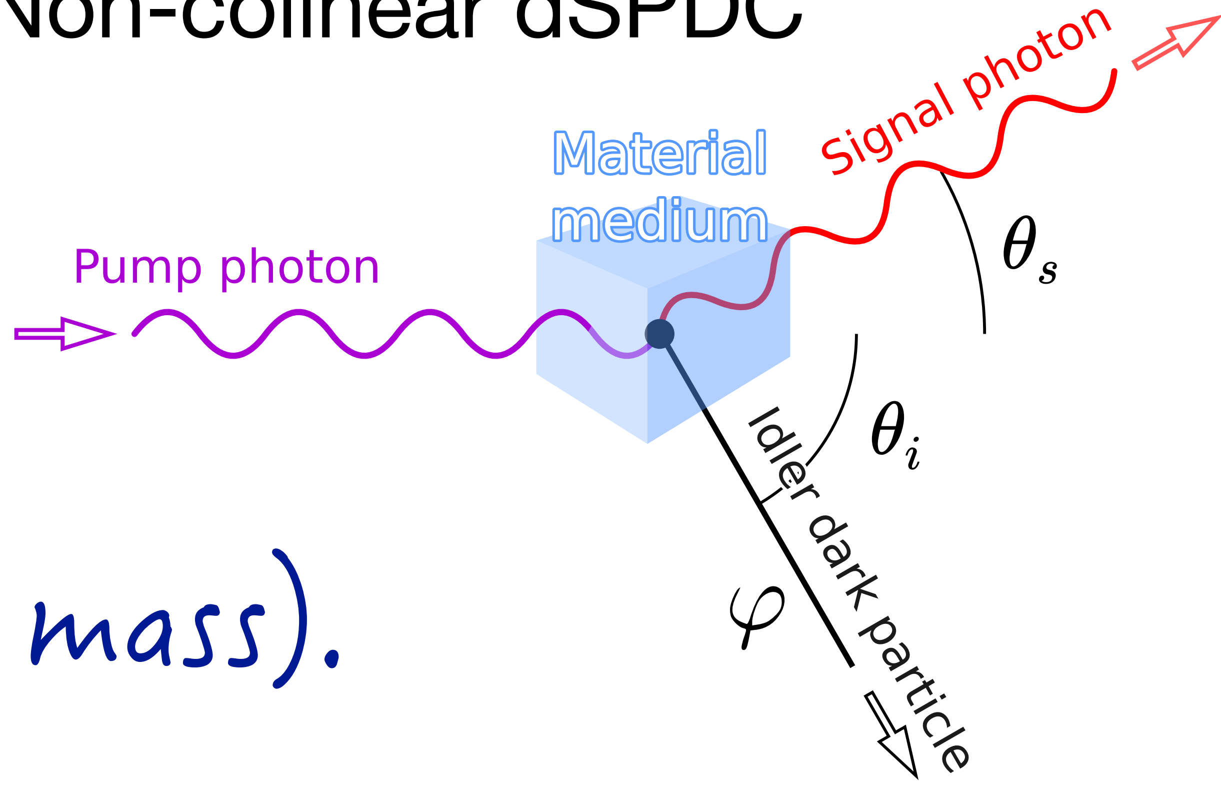


- Dark SPDC: Pump -> signal + axion or dark photon. Rate $\propto \varepsilon^2$ (vs ε^4 for LSW)

colinear dSPDC



Non-colinear dSPDC



Note: the axion or dark photon have index of refraction of 1 (and a mass).
dSPDC has significantly different phase matching conditions.

Summary

- The interface of HEP and QIS is growing!
  - Quantum simulation of HEP (exciting, but not today's topic)
  - Quantum sensing
- Quantum Field theory extends into today's quantum devices!
- Quantum senstors can probe new hypotheses in HEP

Deleted Scenes

Examples

Cavity based Searches @ SQMS SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

Optics based searches @ our imagination so far

Estrada et al. *PRX Quantum* 2 (2021) 3, 030340

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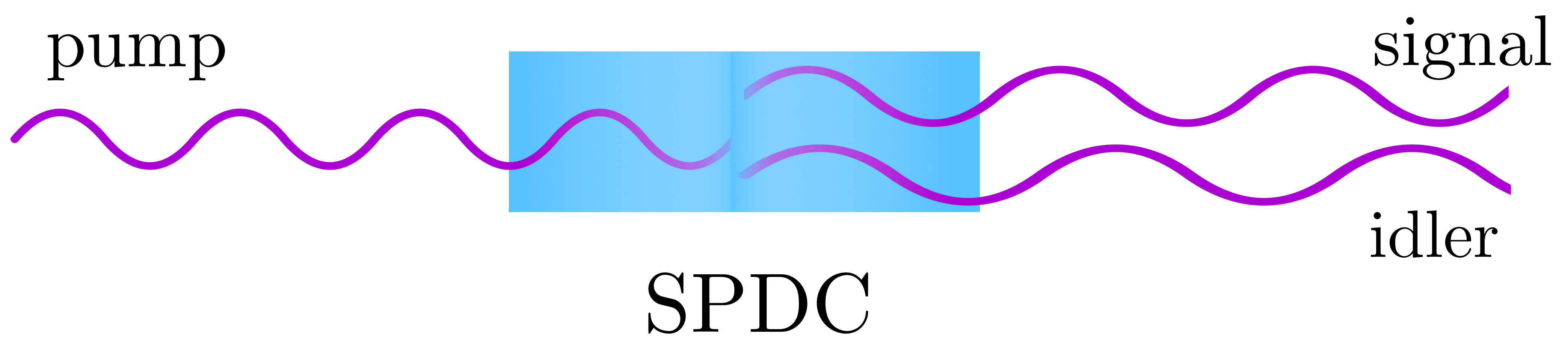

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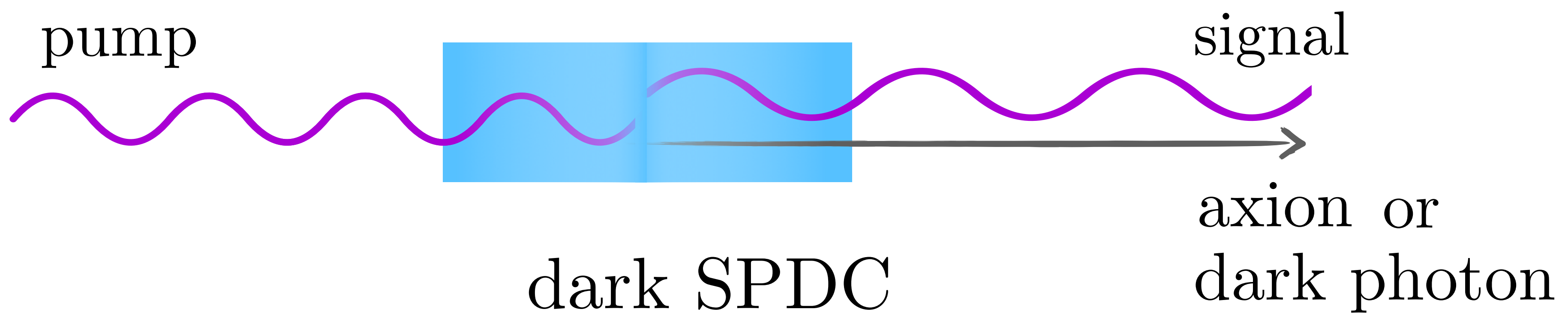
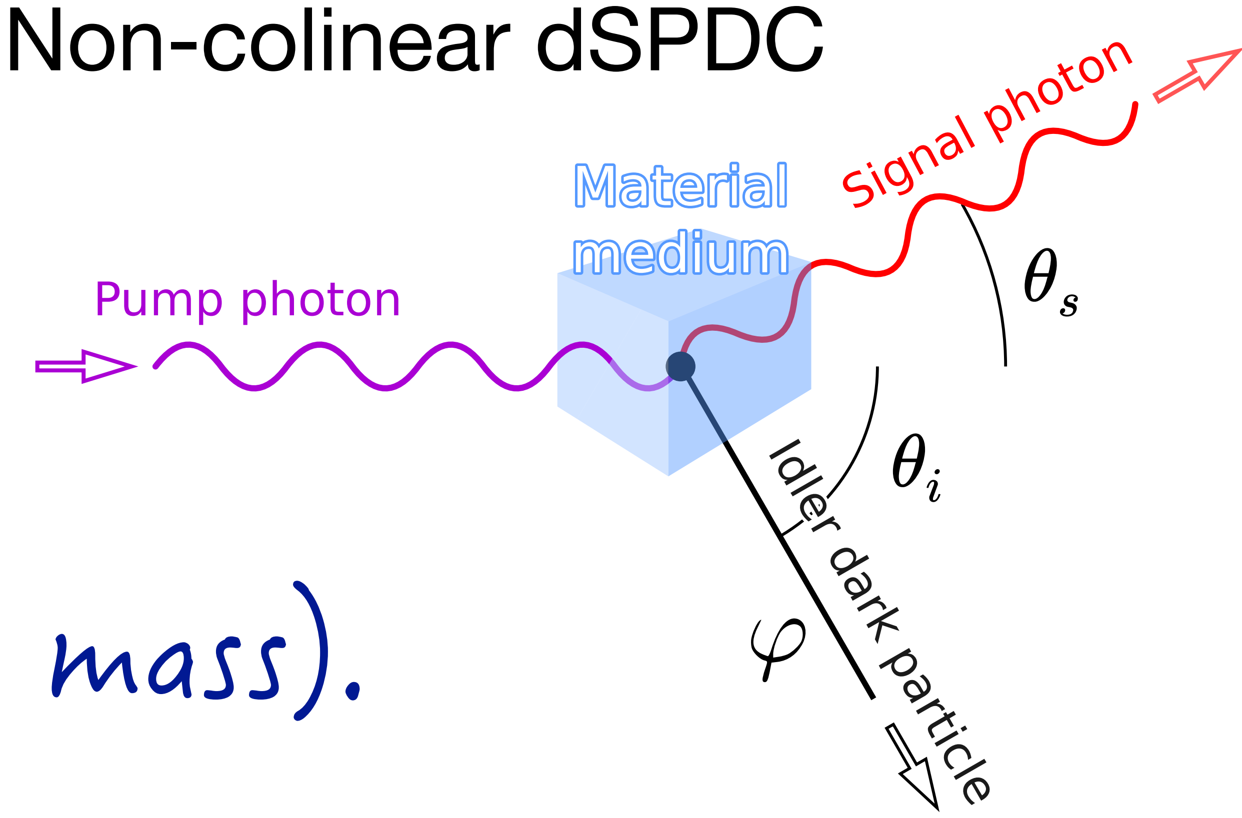


Non-colinear dSPDC

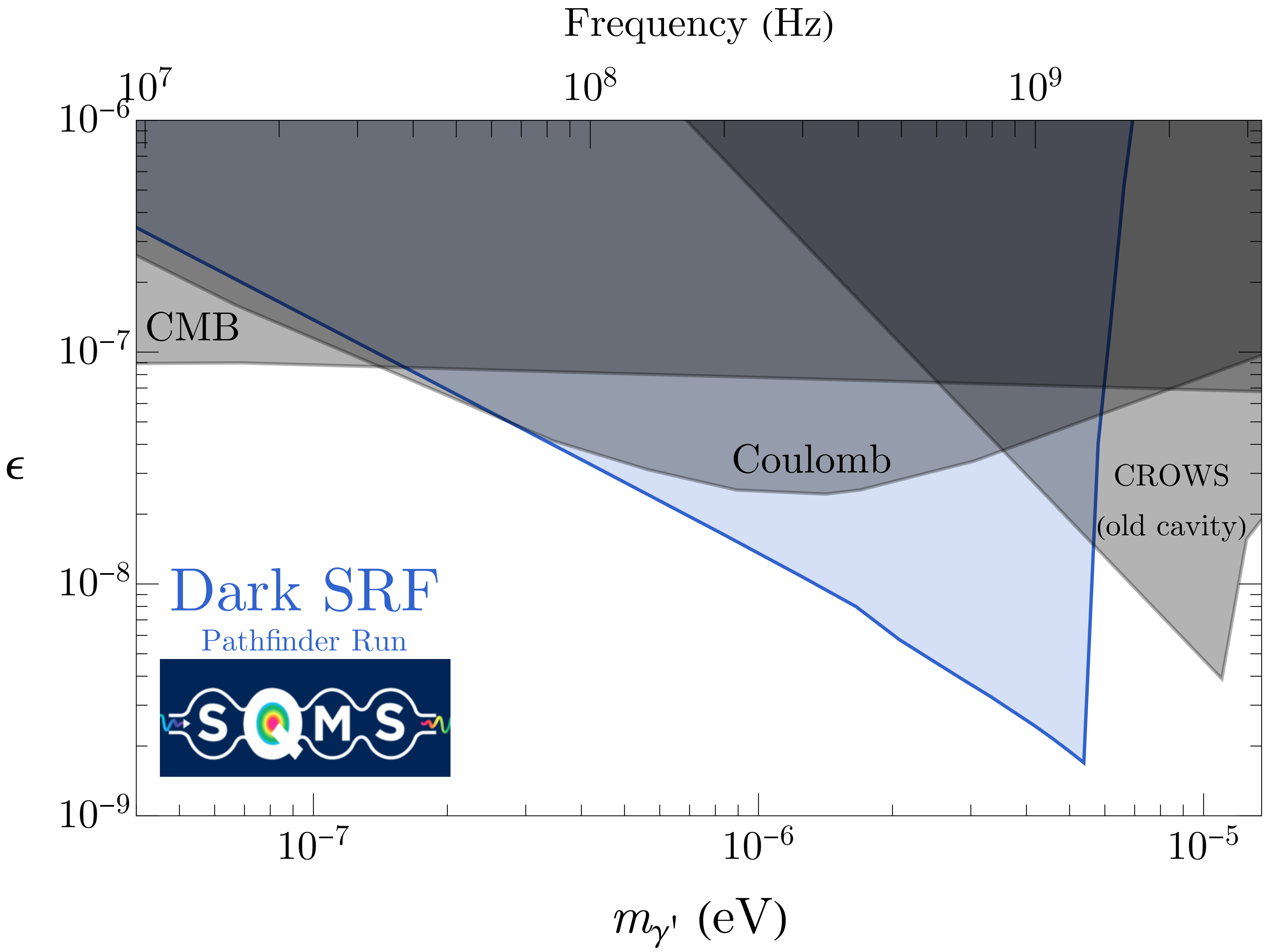


Note: the axion or dark photon have index of refraction of 1 (and a mass).
dSPDC has significantly different phase matching conditions.

Dark SRF: cavity-based search for the Dark Photon

A light-shining-through-wall experiment.

Phase 1: Pathfinder run in LHe. Demonstrated enormous potential for SRF based searches.

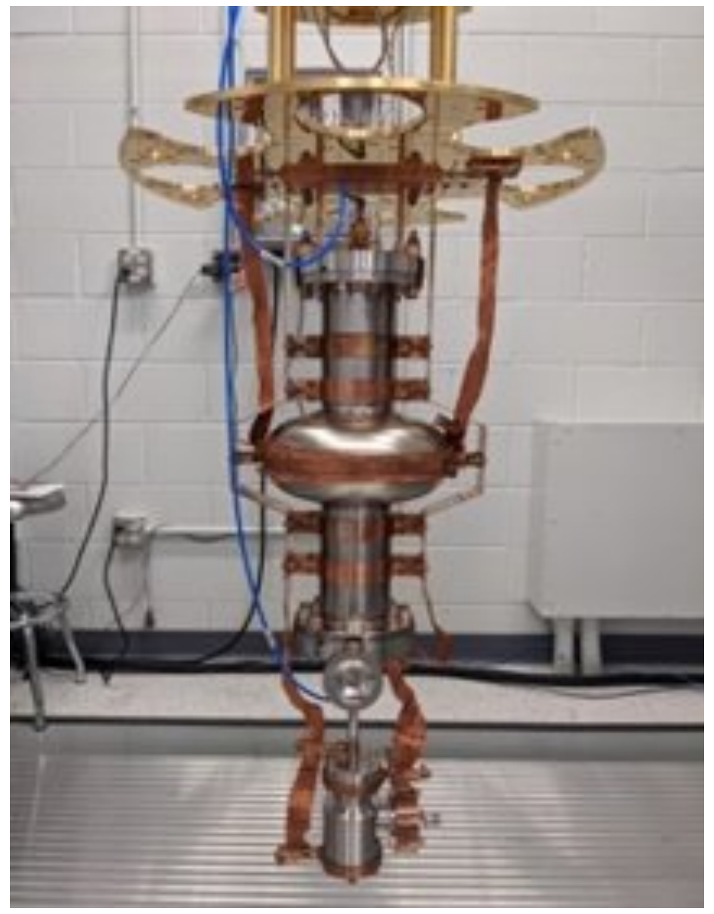


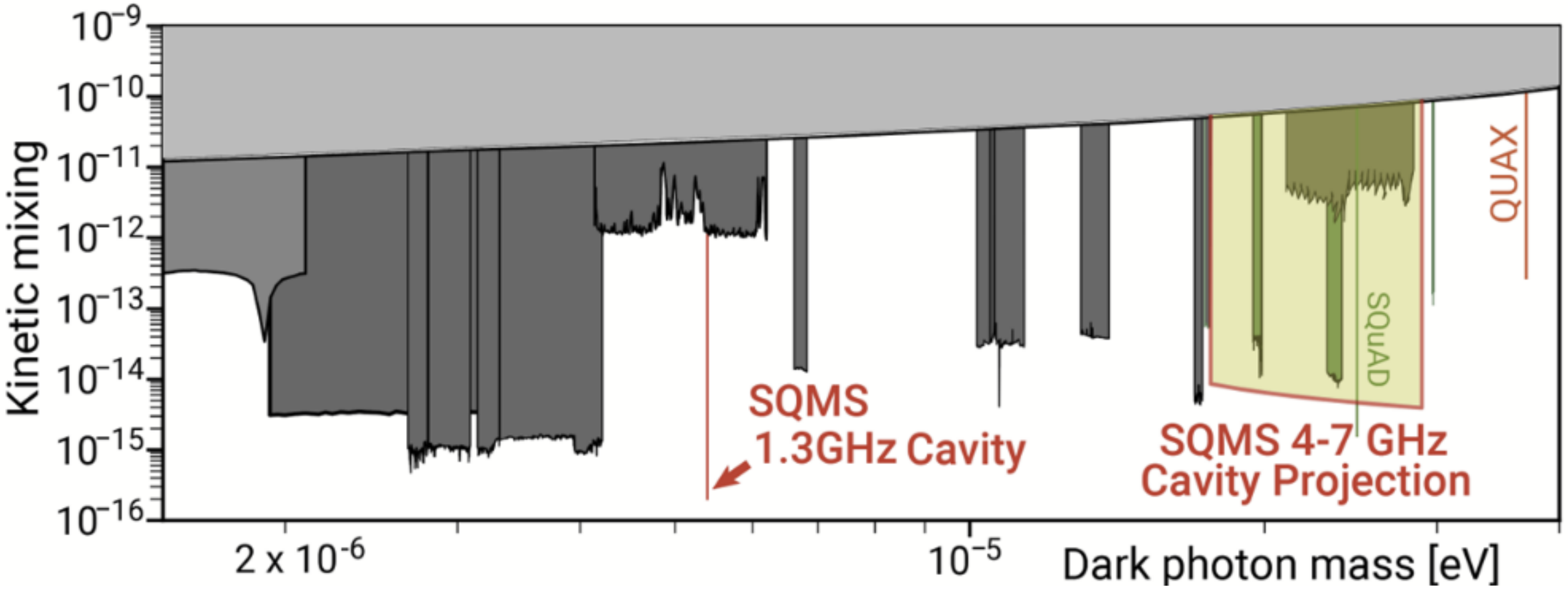
Phase 2: in DR, receiver at ~mk, in quantum regime. Improved frequency stability. Phase sensitive readout.

Will increase the search reach.

Romanenko et al., *Phys.Rev.Lett.* 130 (2023) 26, 261801

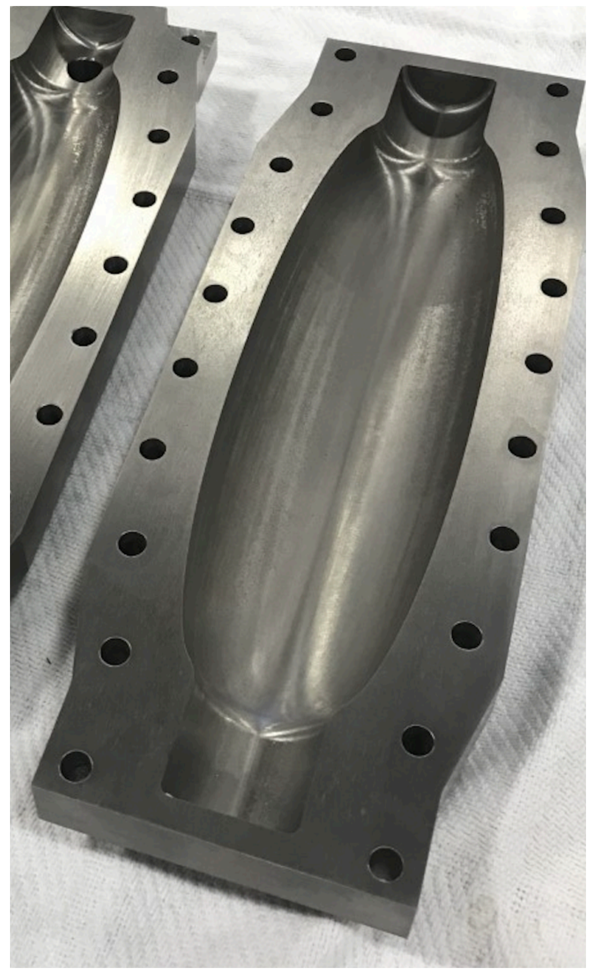
Ultrahigh Q for Dark Matter

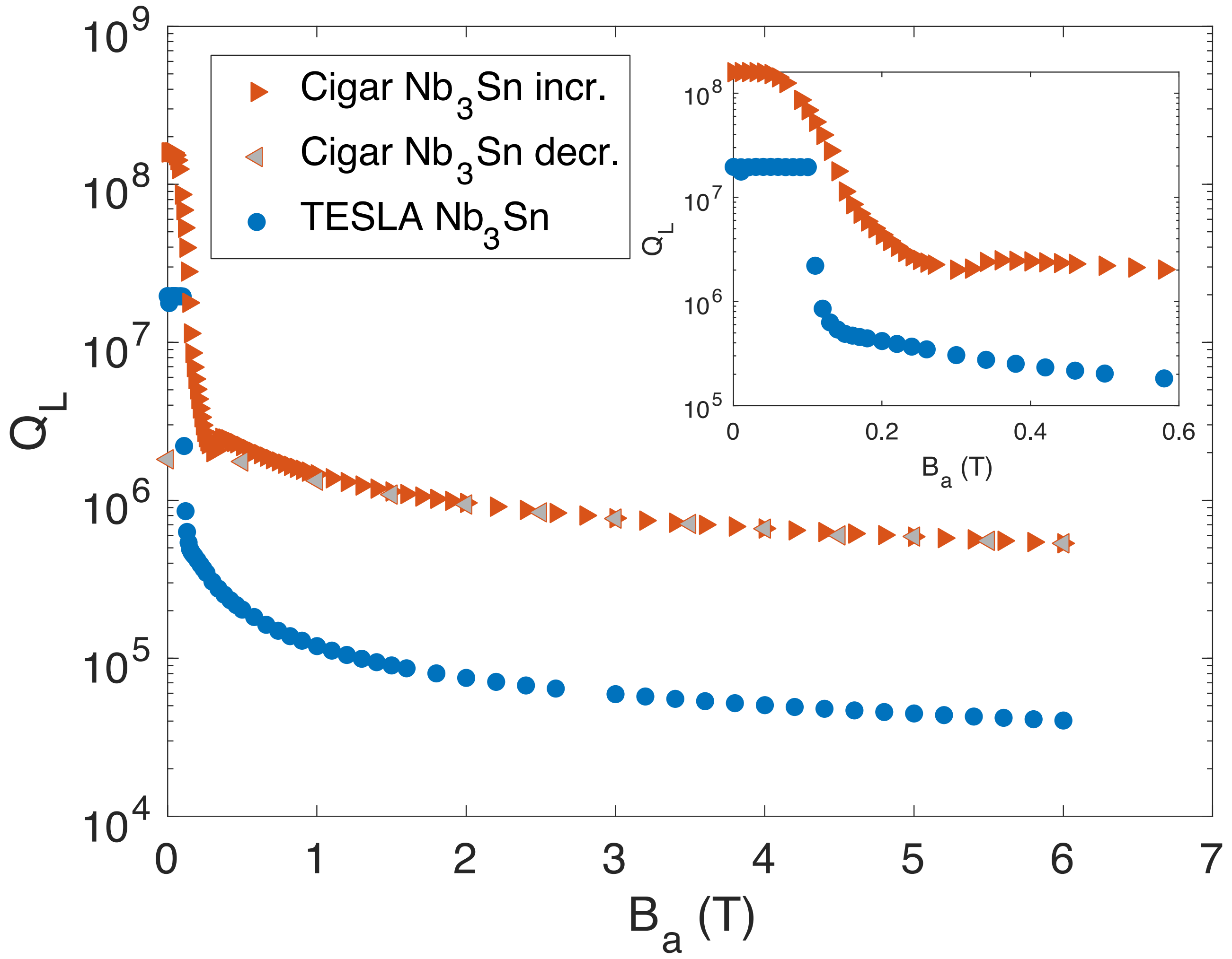




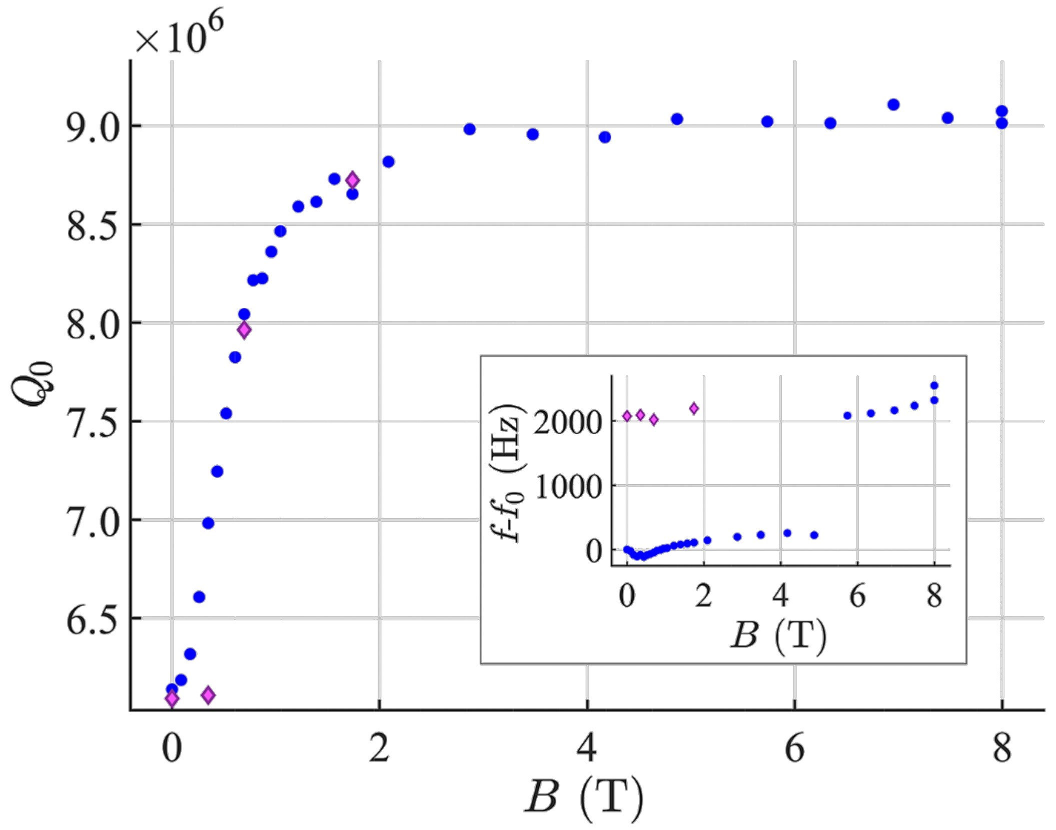
Cervantes et al., arxiv:2208.03183, in review in Phys. Rev. Lett.

No B-Field: $Q > 10^{10}$





Superconducting Nb_3Sn cavity (FNAL): Posen et al., arxiv:22014.10733, in review in Phys Rev Applied



Hybrid copper-dielectric cavity (INFN): Di Vora et al., PhysRevApplied.17.054013

With B-Field: $Q \geq 10^{5\text{-}7}$

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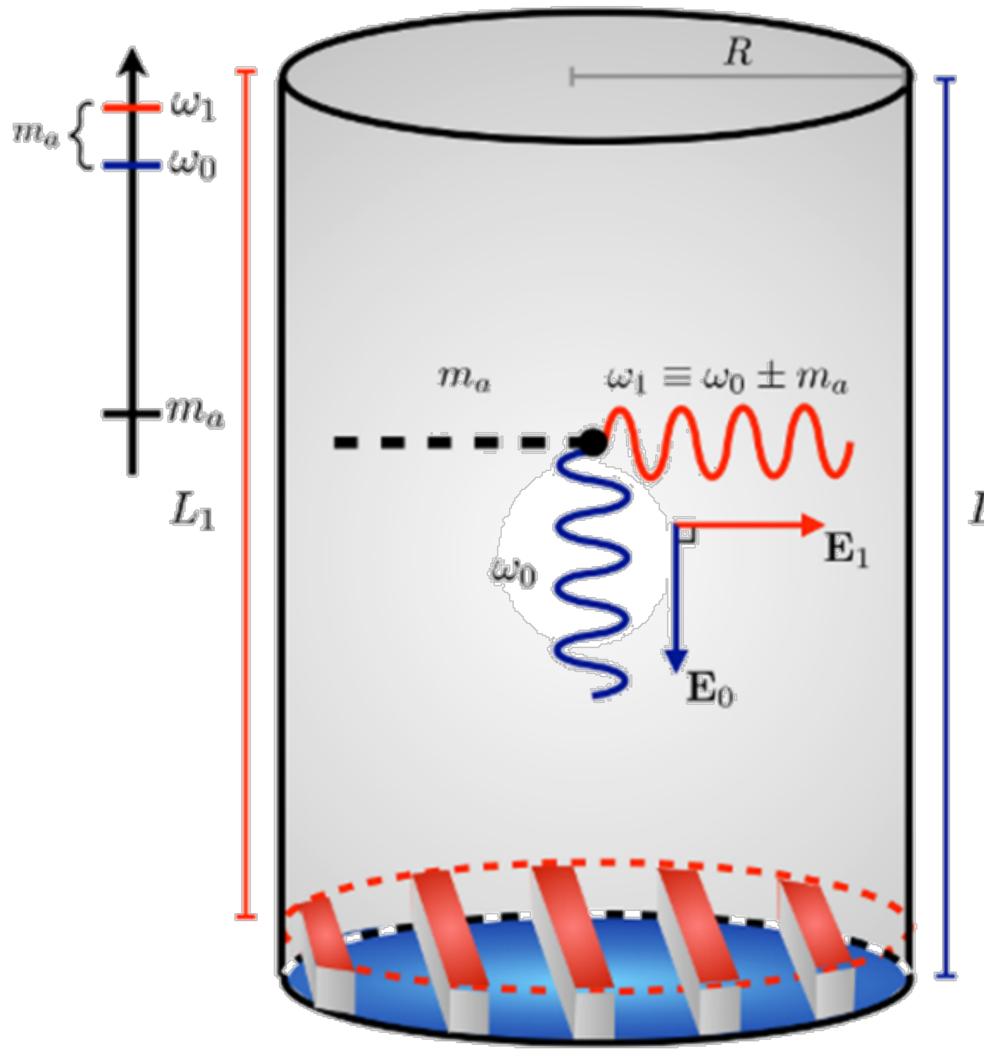
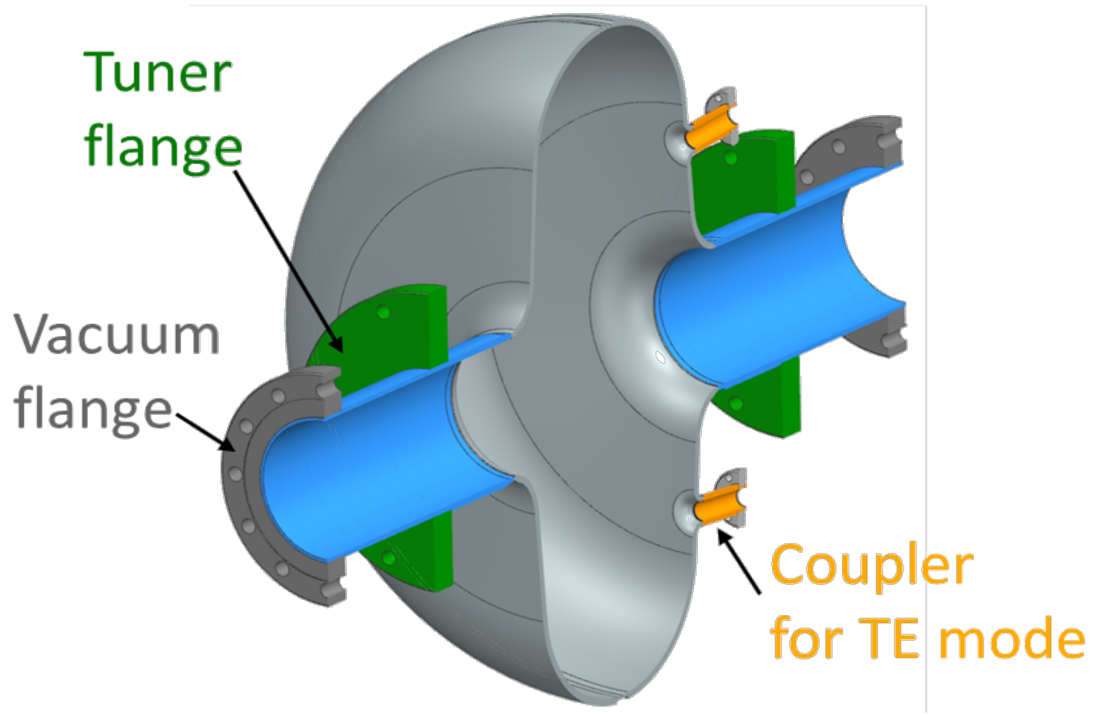
Multimode searches

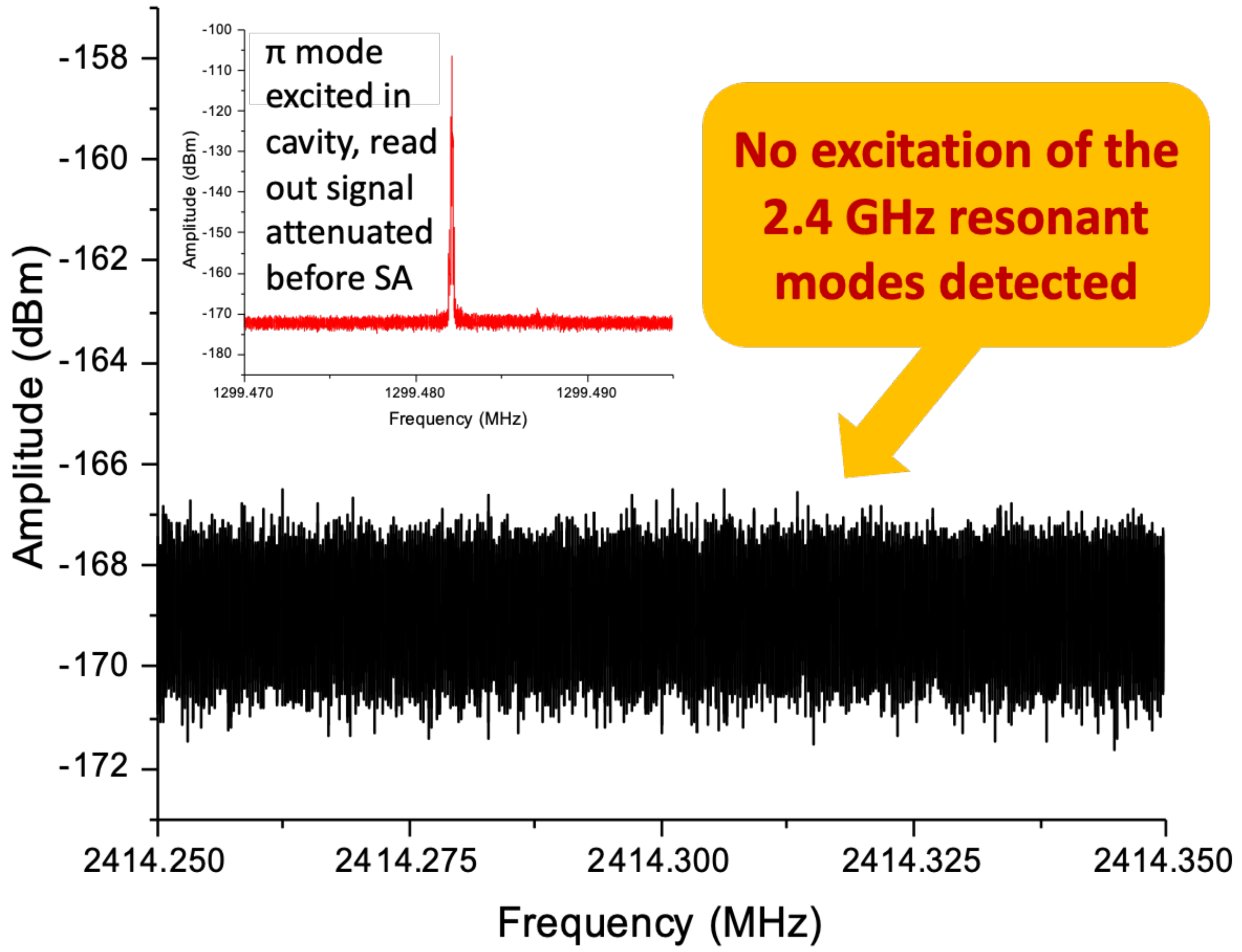
Bogorad, et al., PRL, DOI:10.1103/PhysRevLett.123.021801
Berlin, et al., JHEP, DOI:10.1007/JHEP07 (2020) 088
Gao & Harnik, JHEP, DOI:10.1007/JHEP07 (2021) 053
Berlin, et al., arXiv:2203.12714, Snowmass WP (2022)
Sauls, PTEP, DOI:10.1093/ptep/ptac034 (2022)
Giaccone, et al., arXiv:2207.11346 (2022)



Axion DM search concept



Experimental design for MHz Axion search

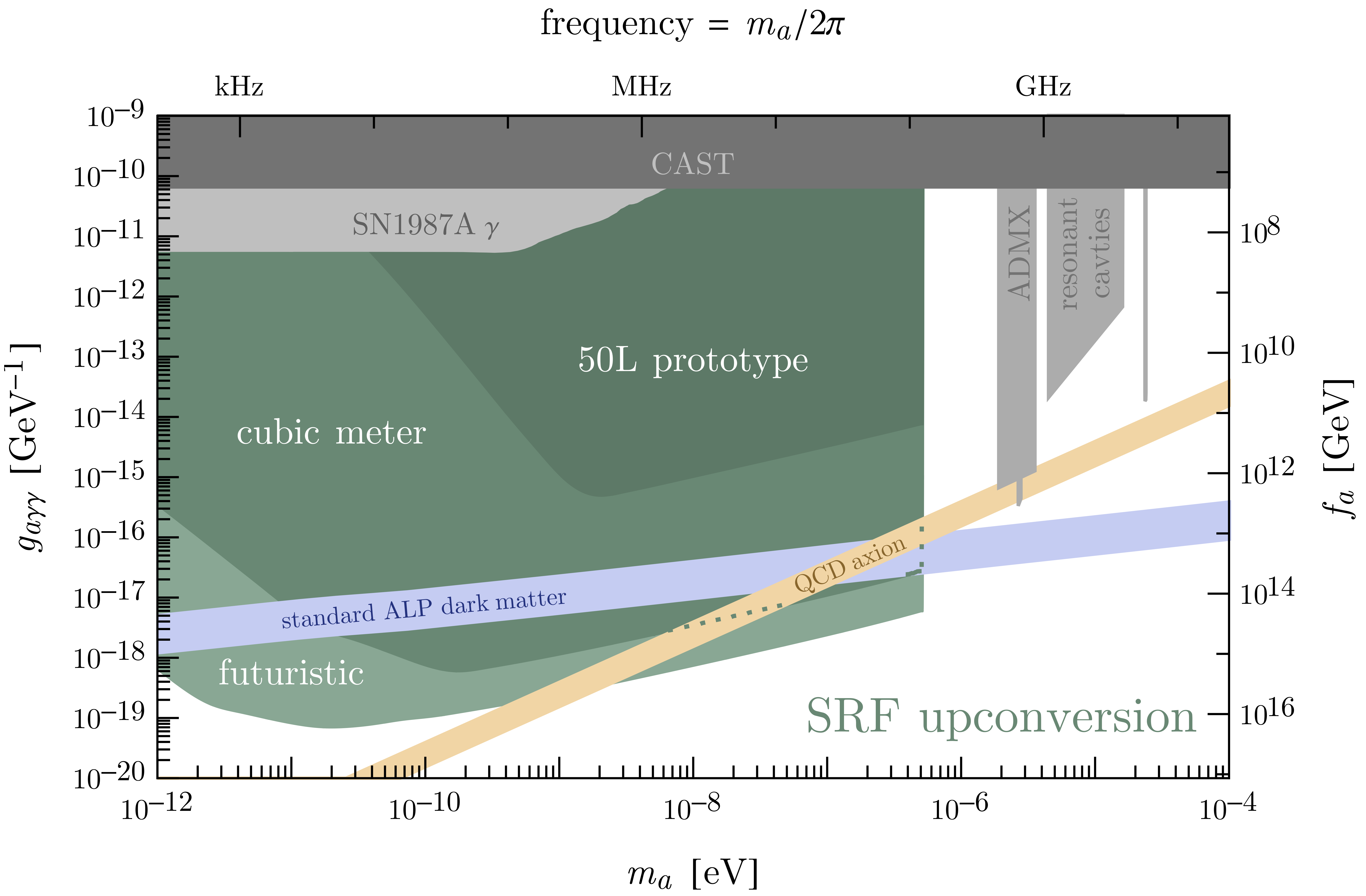


- **Axion DM** search based on the heterodyne detection scheme: cavity design is finalized, contract for cavity fabrication placed (cavity arrival: Fall 2023)
- In preparation for search:
  - Working on RF experimental set up and read out system
  - Addressing experimental challenges such as passive dampening of vibrations in LHe facility
  - Multimode feasibility study

Contacts: Asher Berlin, Bianca Giacone

Multimode searches

Berlin, et al., JHEP, DOI:10.1007/JHEP07 (2020) 088
Berlin, et al., arXiv:2203.12714, Snowmass WP (2022)
Giaccone, et al., arXiv:2207.11346 (2022)

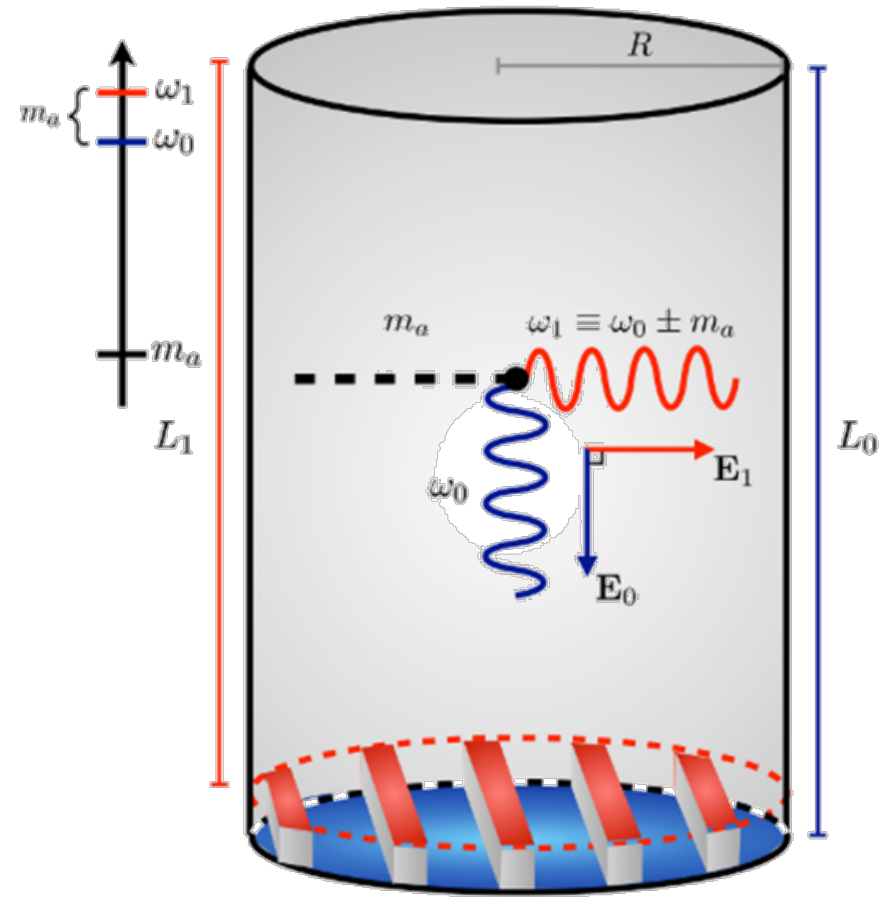


Snowmass name:

SRF-m³

Asher's proposal:

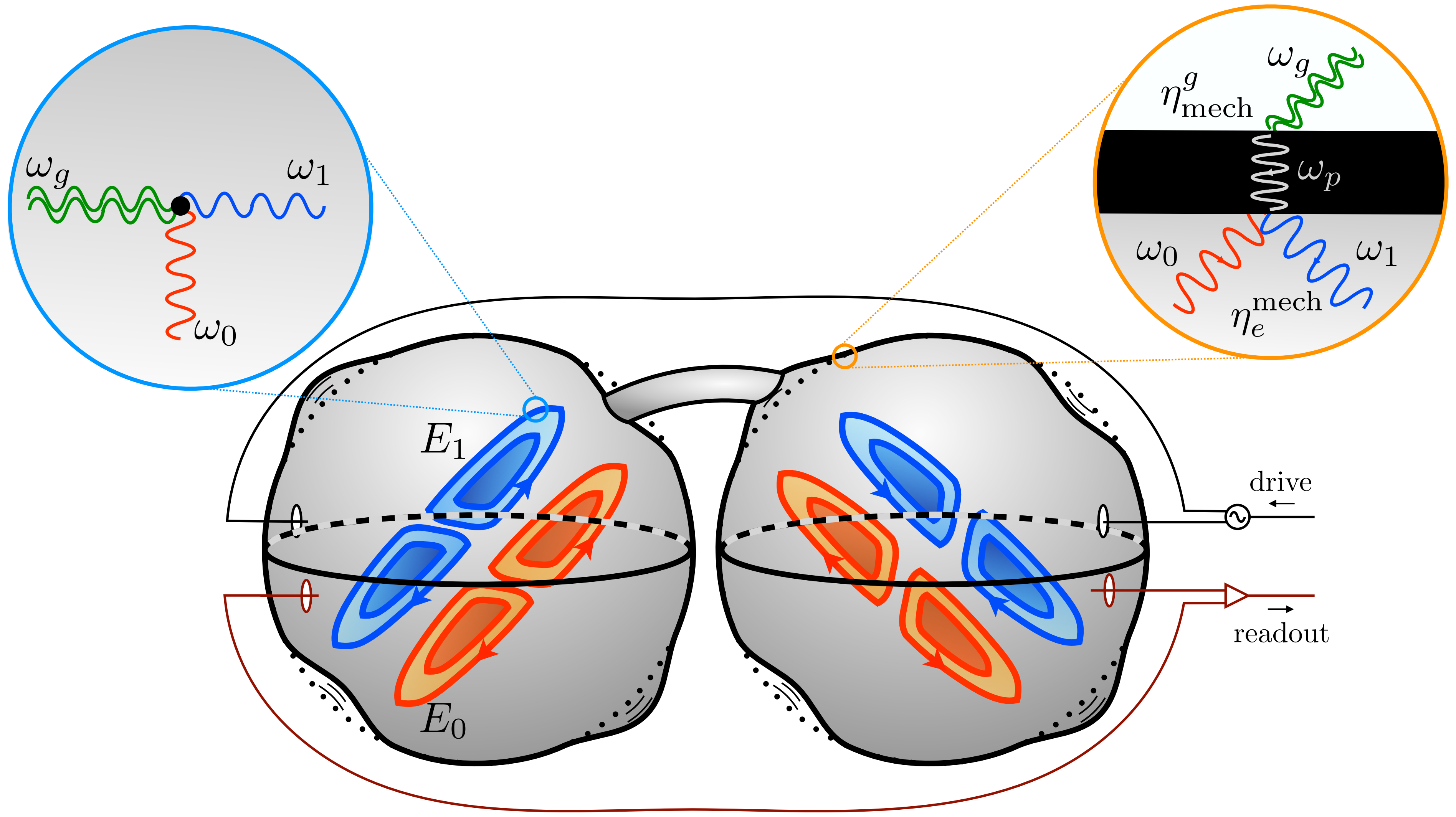
SuperRAD



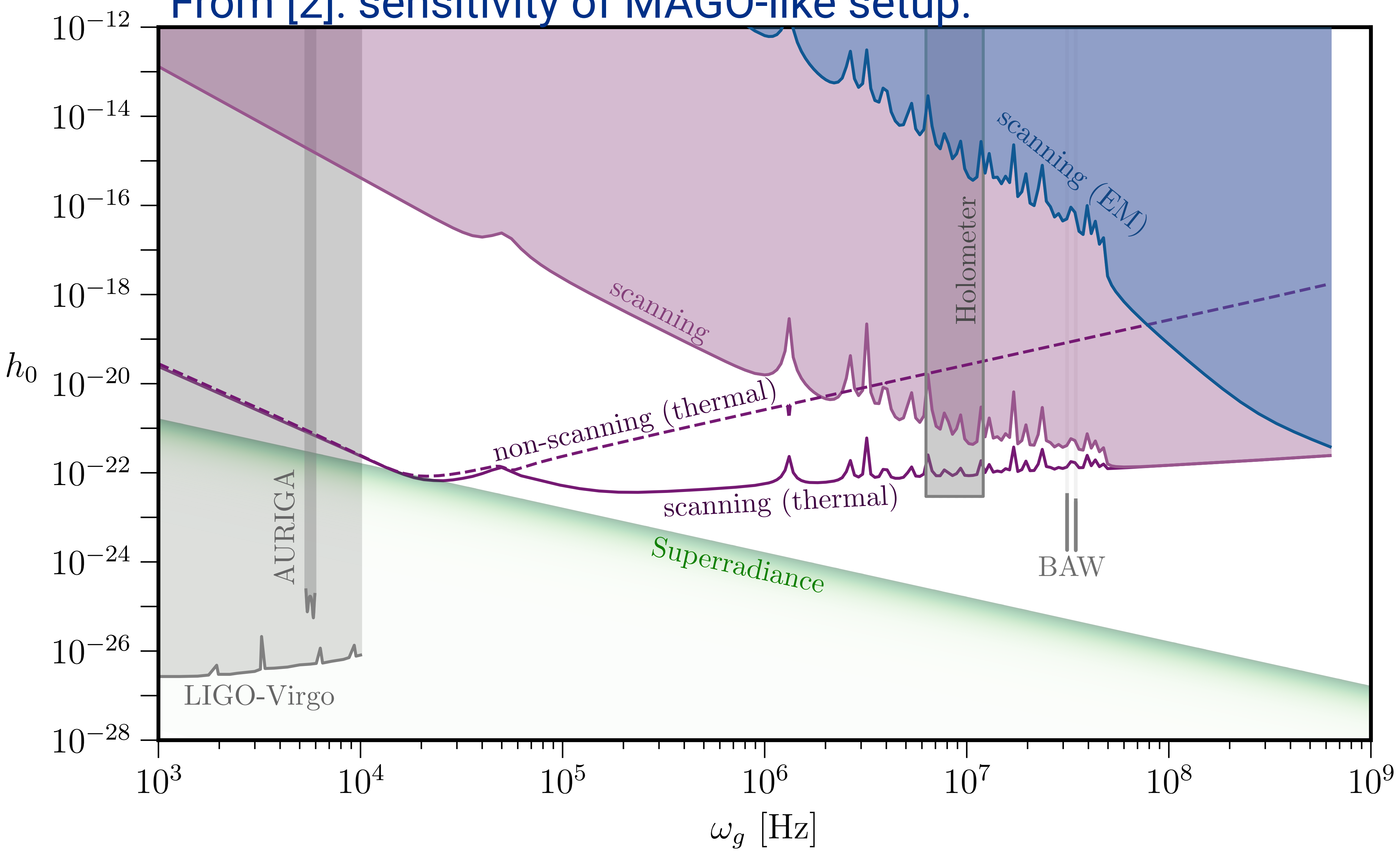
Contacts: Asher Berlin, Bianca Giacone

Gravitational waves

- Photon up-conversion due to GW.
- Current axion experiments have sensitivity to GHz Gravity waves [1].
- A dedicated cavity experiment, e.g. MAGO, has significant reach at MHz [2].
- MAGO traveled from INFN to DESY to Fermilab for testing
- **A Fermilab KEK collaboration to design new dedicated broadband cavity.**



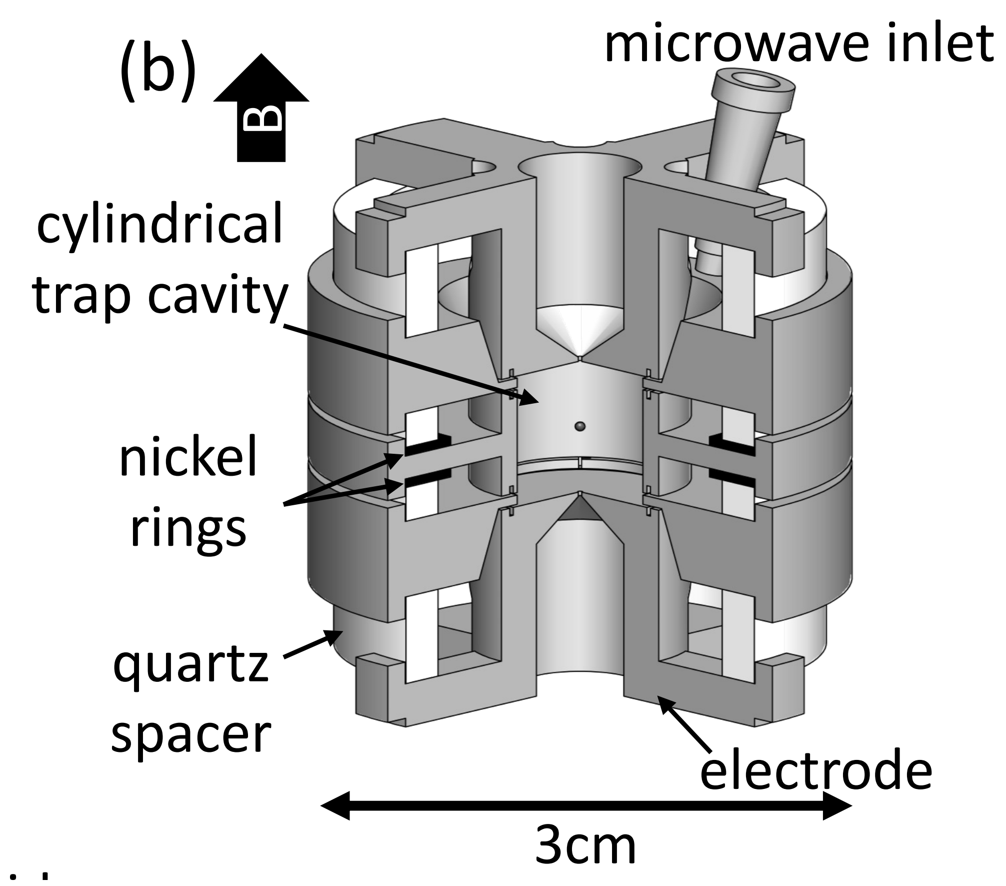
From [2]: sensitivity of MAGO-like setup.



MAGO (INFN)

[1] *Phys.Rev.D* 105 (2022) 11, 116011
[2] *Phys.Rev.D* 108 (2023) 8, 084058

Single Particle Qubit



- The most precise theory-experiment comparison in physics:

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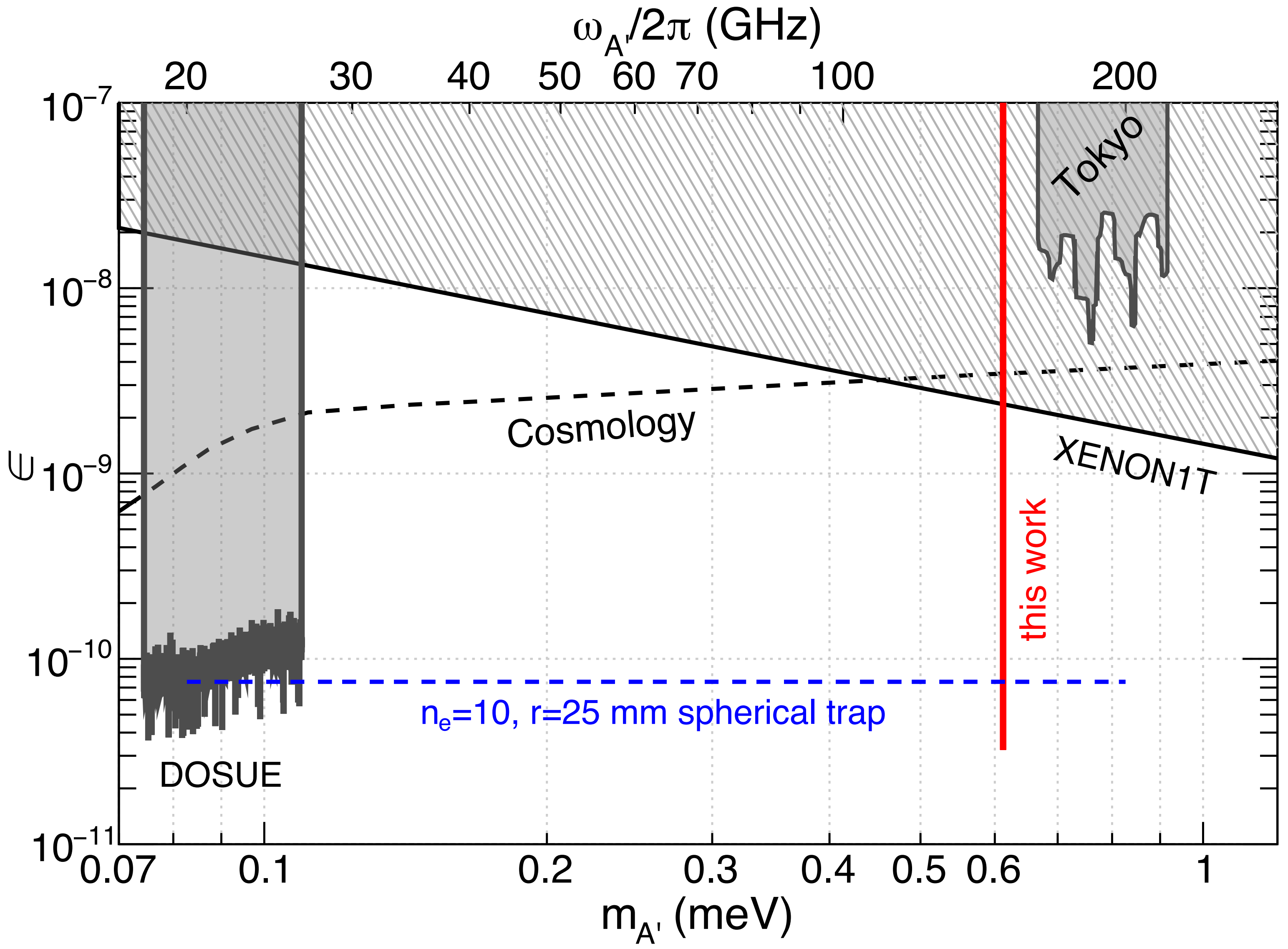

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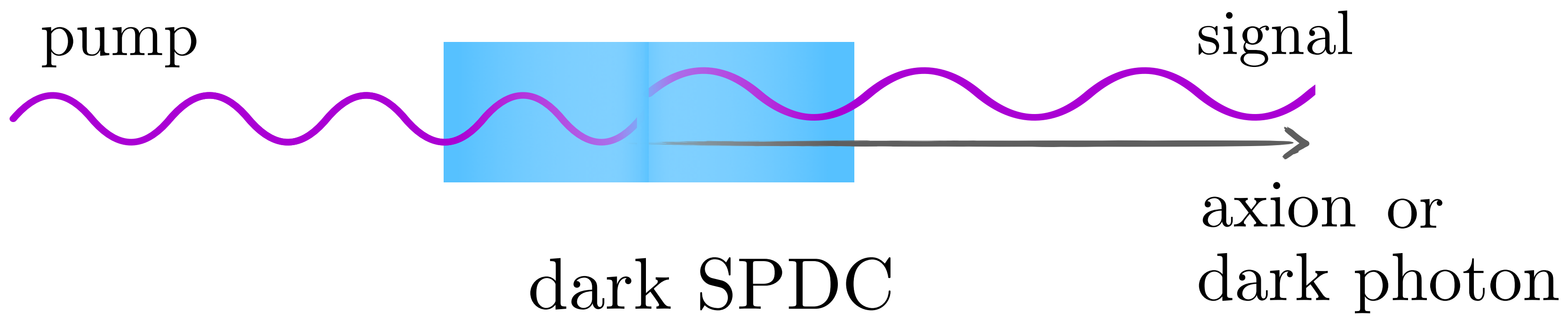
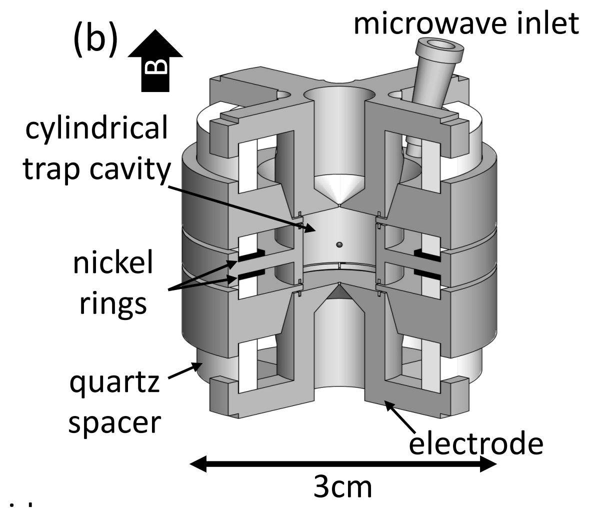
- Theory + proof-of-concept!

***Phys.Rev.Lett.* 129 (2022) 26, 261801**
(a new NU-Stanford-Fermilab collaboration)

To Summarize

- QIS relies on extending the QFT formalism to devices that impose boundary conditions onto quantum fields.
- The low energy EFT of devices. For me, its is particle physics on small scales.
- BSM extensions of optics are simple well motivated targets, both linear and nonlinear.
- Several ongoing and proposed efforts at SQMS and Fermilab.
  It is great to see the complementarity!



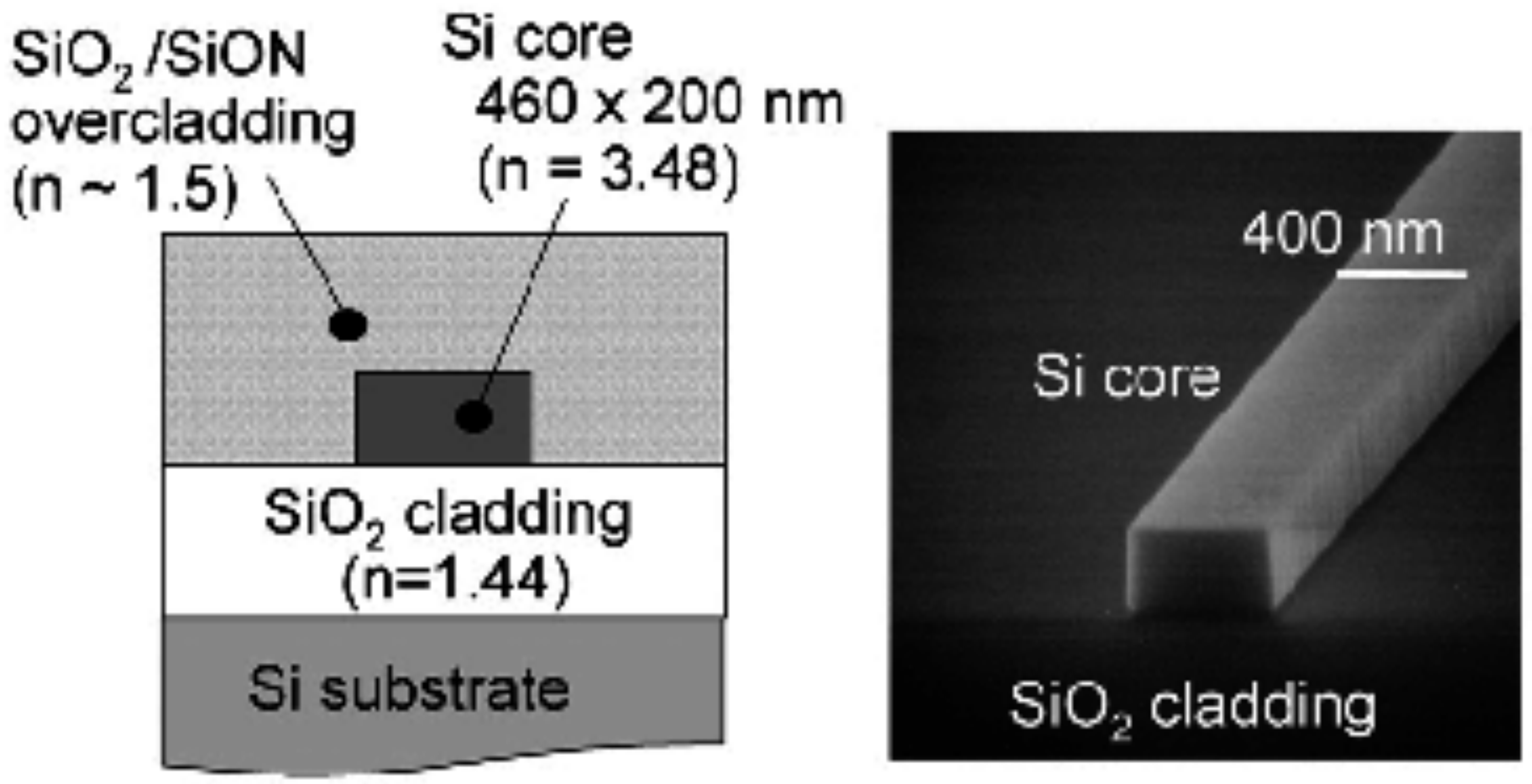


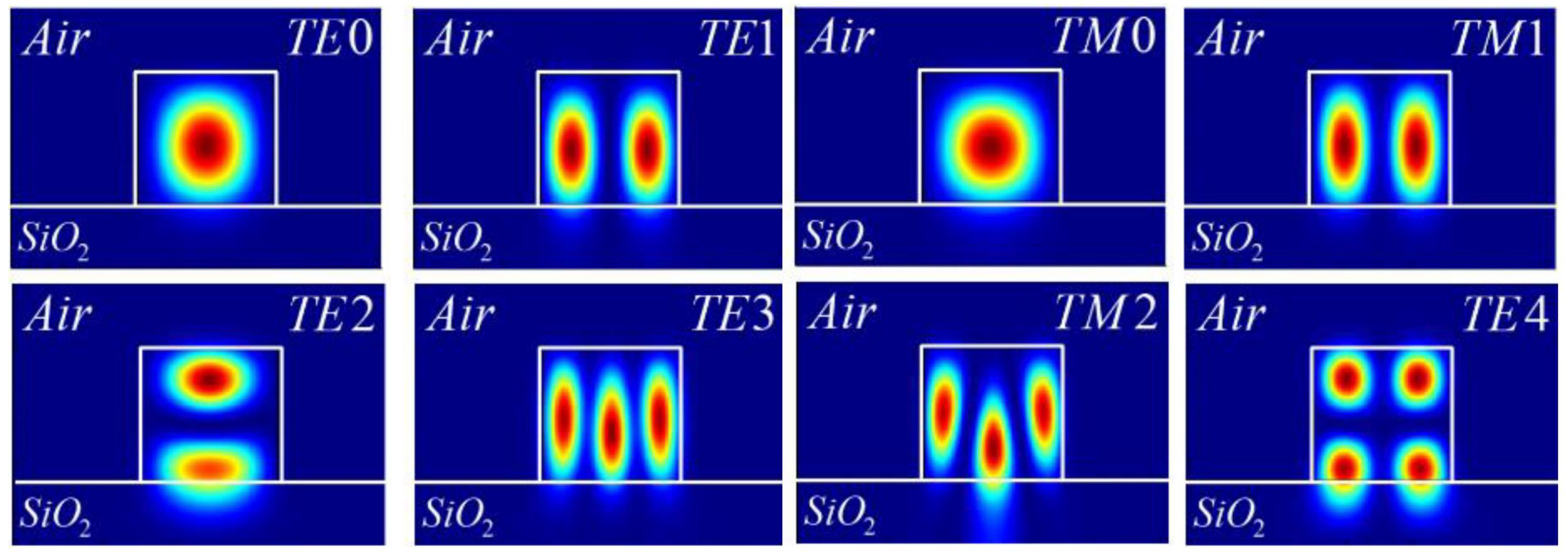
Deleted Scenes

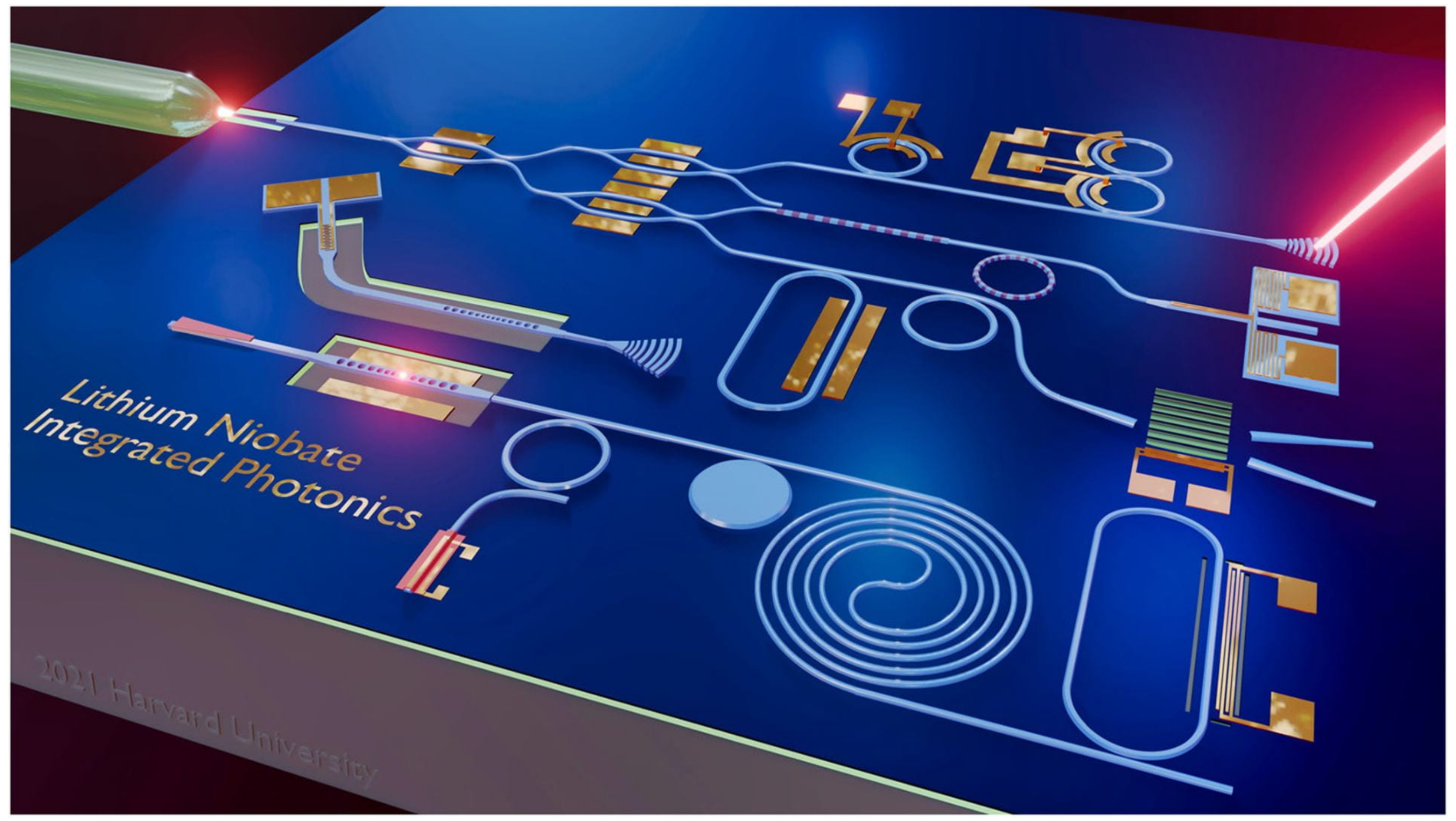
Optical Devices

- Optics is the low energy EFT of light in matter.
- We can control the dispersion relation: $k = n\omega$. Useful for localization.
- A waveguide admits a 1D EFT w/ modes quantized in transverse direction.
- Transverse wave function affects longitudinal dispersion relation (a la KK modes!)

Linear Optics: $H = E^2 + B^2 = \sum \hbar\omega(a^\dagger a + \tfrac{1}{2})$







"Integrated photonics"

Nonlinear devices

- Like any EFT, in a quantum device there is a UV cutoff.
- We can add higher dim operators. For example, in optics

Dim-6: $$H_{\text{SPDC}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(2)}_{jkl} E_j E_k E_l \right)$$

Dim-8: $$H_{\text{4-wave}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(3)}_{jkl} E_j E_k E_l E_m \right)$$

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Dim-8: $$H_{\text{4-wave}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(3)}_{jkl} E_j E_k E_l E_m\right)$$

Estimate χ's in naive dimensional analysis:

When the field is set to that in an atom, we set (Dim-4 ~ Dim-6 ~ Dim-8):

$$E_{\text{atom}} \sim e/4\pi a_0^2$$

$$\chi^{(2)} \sim \frac{\sqrt{4\pi}}{\alpha^{5/2} m_e^2} \qquad \chi^{(3)} \sim \frac{4\pi}{\alpha^5 m_e^4}$$

(by comparison, in vacuum $\chi^{(2)} = 0$, $\chi^{(3)} = \dfrac{2\alpha^2}{45 m_e^4}$)

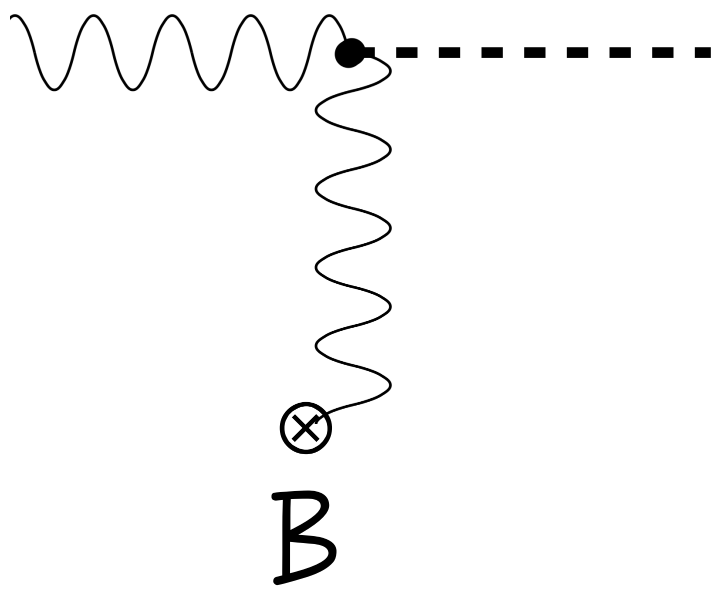
Axions - A nonlinear extension of QED

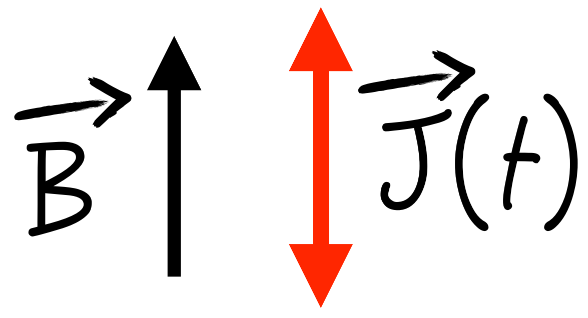
Introduce a field: $\mathcal{L} \supset \frac{a}{f} G^{\mu\nu}\tilde{G}_{\mu\nu} = \frac{a}{f} \vec{E}_G \cdot \vec{B}_G$ $\langle a \rangle \to 0$ dynamically.

- Naturally, one would also expect: $\mathcal{L} \supset \frac{a}{f} F^{\mu\nu}\tilde{F}_{\mu\nu} = \frac{a}{f} \vec{E} \cdot \vec{B}$

- Axion phenomenology w/ background B field is similar to dark photon. Mixing:

$$\mathcal{L} \supset \frac{a}{f} F^{\mu\nu}\tilde{F}_{\mu\nu} = \frac{a}{f} \vec{E} \cdot \vec{B}$$





Photons polarized along a B field can mix with axions.

Optics

- The EFT of light traveling through a medium, made of atoms: $N_{atoms} \sim 10^{23}$!!

$$\omega_{atoms} \sim \alpha^2 m_e$$

$$\delta x_{atoms} \sim a_{Bohr} \sim (\alpha\, m_e)^{-1}$$

Optics

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Optics

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$\omega_{atoms} \sim \alpha^2 m_e$

$\delta x_{atoms} \sim a_{Bohr} \sim (\alpha m_e)^{-1}$

$\delta x_{atoms} \ll \lambda_{light} \rightarrow$ atoms react collectively!

Optics

The hierarchy of scales, $\delta x_{atoms} \ll \lambda_{light}$, has several implications:

- Collective (coherent) back reaction:
  - Amplitude for forward scattering off an atom may be small, $O(\alpha)$.
  - Amplitude for forward scattering off of the medium can be $O(1)$.
- The effect of the medium can be described as mean field(s) in a derivative expansion:

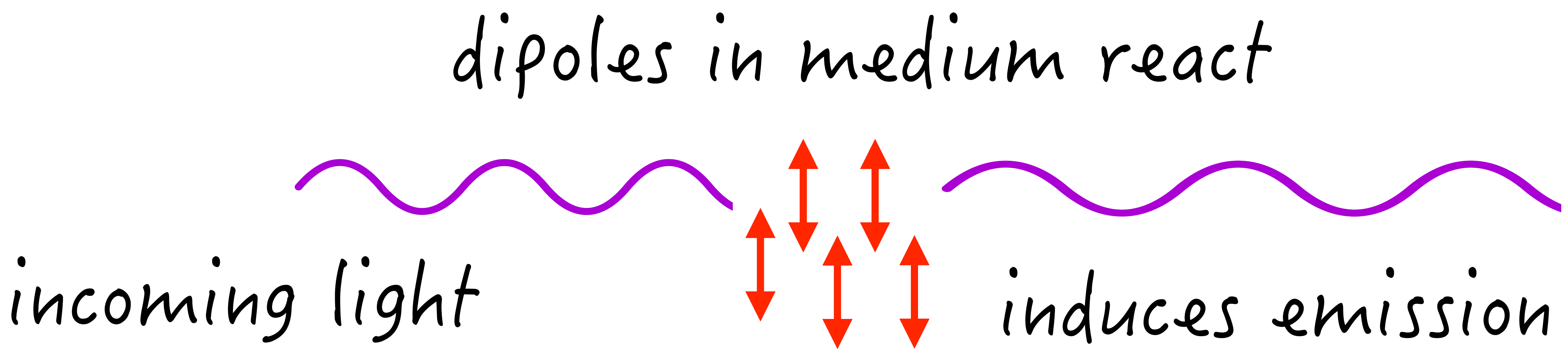
Polarization and magnetization densities $\vec{P}$ and $\vec{M}$.

Optics

The hierarchy of scales, $\delta x_{atoms} \ll \lambda_{light}$, has several implications:

- Collective (coherent) back reaction:
  - Amplitude for forward scattering off an atom may be small, $O(\alpha)$.
  - Amplitude for forward scattering off of the medium can be $O(1)$.
- The effect of the medium can be described as mean field(s) in a derivative expansion:

  Polarization and magnetization densities $\vec{P}$ and $\vec{M}$.



Interference in the forward direction
→ **index of refraction!**

$$\boxed{\vec{P} = \chi \vec{E}}$$

Apologies, in this talk c=1

Indices of Refraction

- Index of refraction is a correction to dispersion relation

$$(\nabla^2 - n^2 \partial_t^2)E = 0 \quad \rightarrow \quad k = n\,\omega$$

- n can depend on frequency, propagation direction, and polarization!

- For example: In the EFT of a birefringent medium, two polarizations of the photon are literally two different "**flavors**" of particles!

Analogy: in the SM EFT, **e** and **μ** are two degrees of freedom with different dispersion relations (different mass). (Important in kinematics, $\mu \rightarrow e+\nu+\bar{\nu}$, etc)

- Indeed, in (nonlinear) optics a photon can "decay" to photons! (and what else?!)

Apologies, in this talk c=1

Indices of Refraction

- Index of refraction appears at the "renormaliable" level of the EFT.
- As in any EFT, we can include higher order terms, surpassed by the cutoff

Dim-6: $$H_{\text{SPDC}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(2)}_{jkl} E_j E_k E_l\right)$$

Dim-8: $$H_{\text{4-wave}} = \int_{\text{crystal}} d^3\vec{x} \left(\chi^{(3)}_{jkl} E_j E_k E_l E_m\right)$$

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<u>Estimate χ's in naive dimensional analysis:</u>

When the field is set to that in an atom, we set (Dim-4 ~ Dim-6 ~ Dim-8):

$$E_{\text{atom}} \sim e/4\pi a_0^2$$

$$\chi^{(2)} \sim \frac{\sqrt{4\pi}}{\alpha^{5/2} m_e^2} \qquad \chi^{(3)} \sim \frac{4\pi}{\alpha^5 m_e^4}$$

(by comparison, in vacuum $\chi^{(2)} = 0$, $\chi^{(3)} = \dfrac{2\alpha^2}{45 m_e^4}$)