

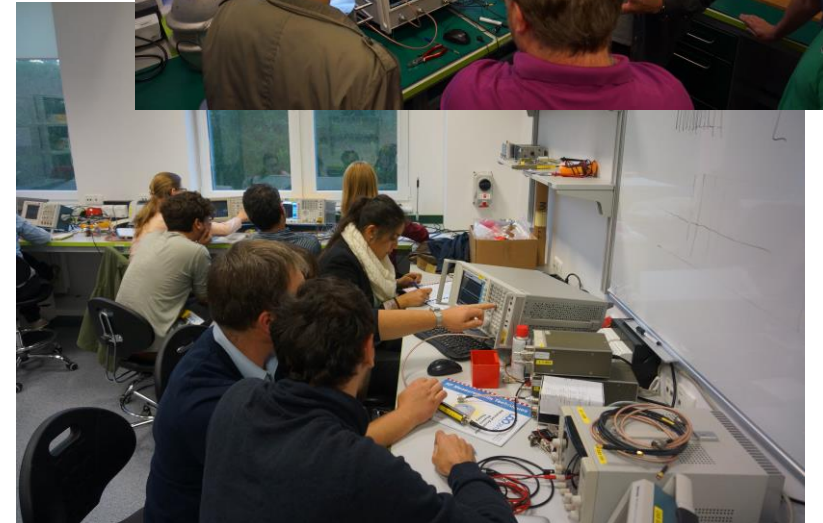
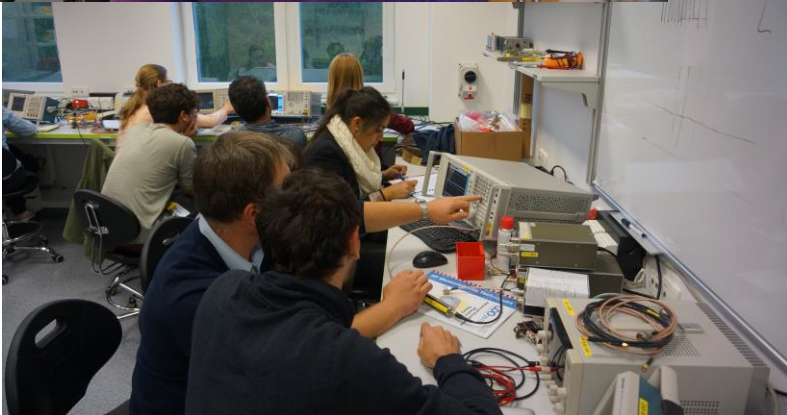
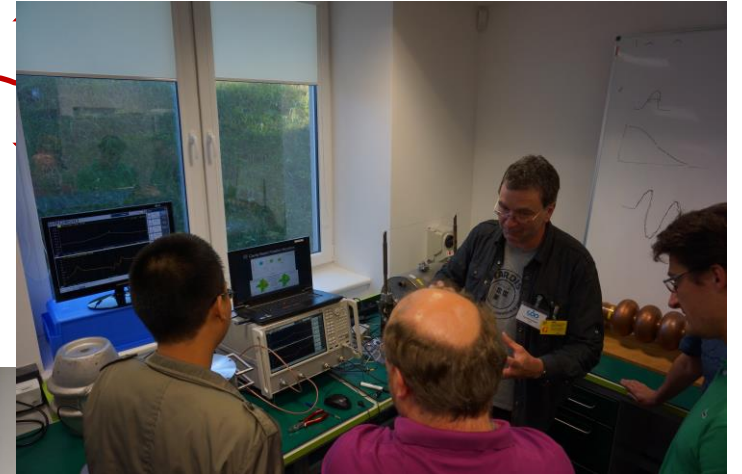
2024 CAS course on “RF for Accelerators”

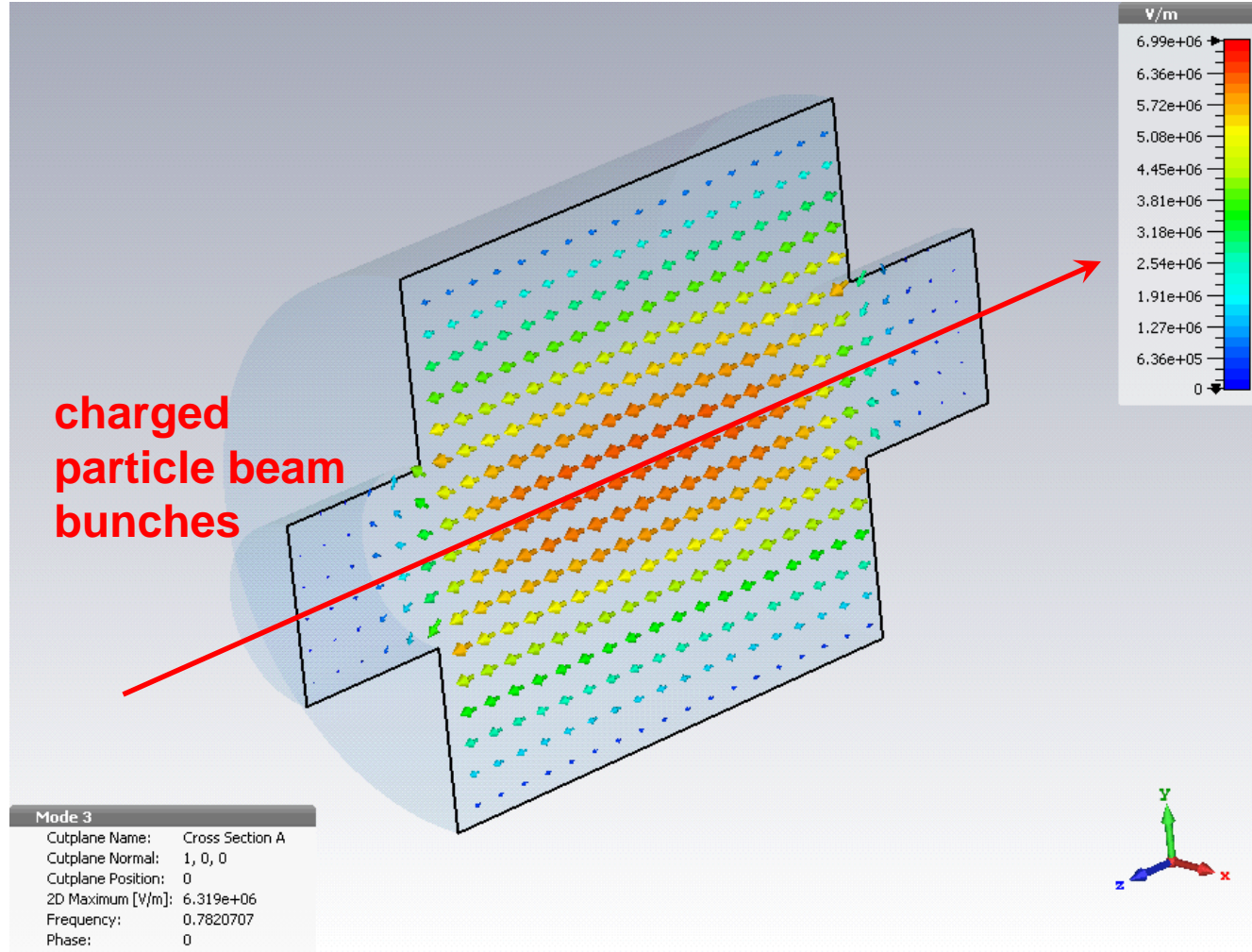
RF Measurements

– *An Introduction* –

Manfred Wendt – CERN

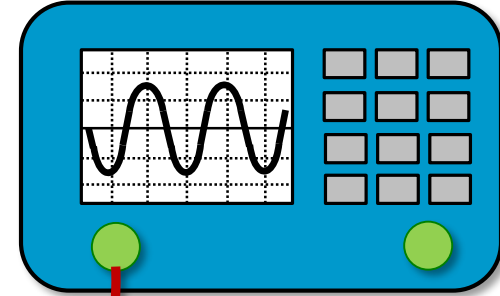
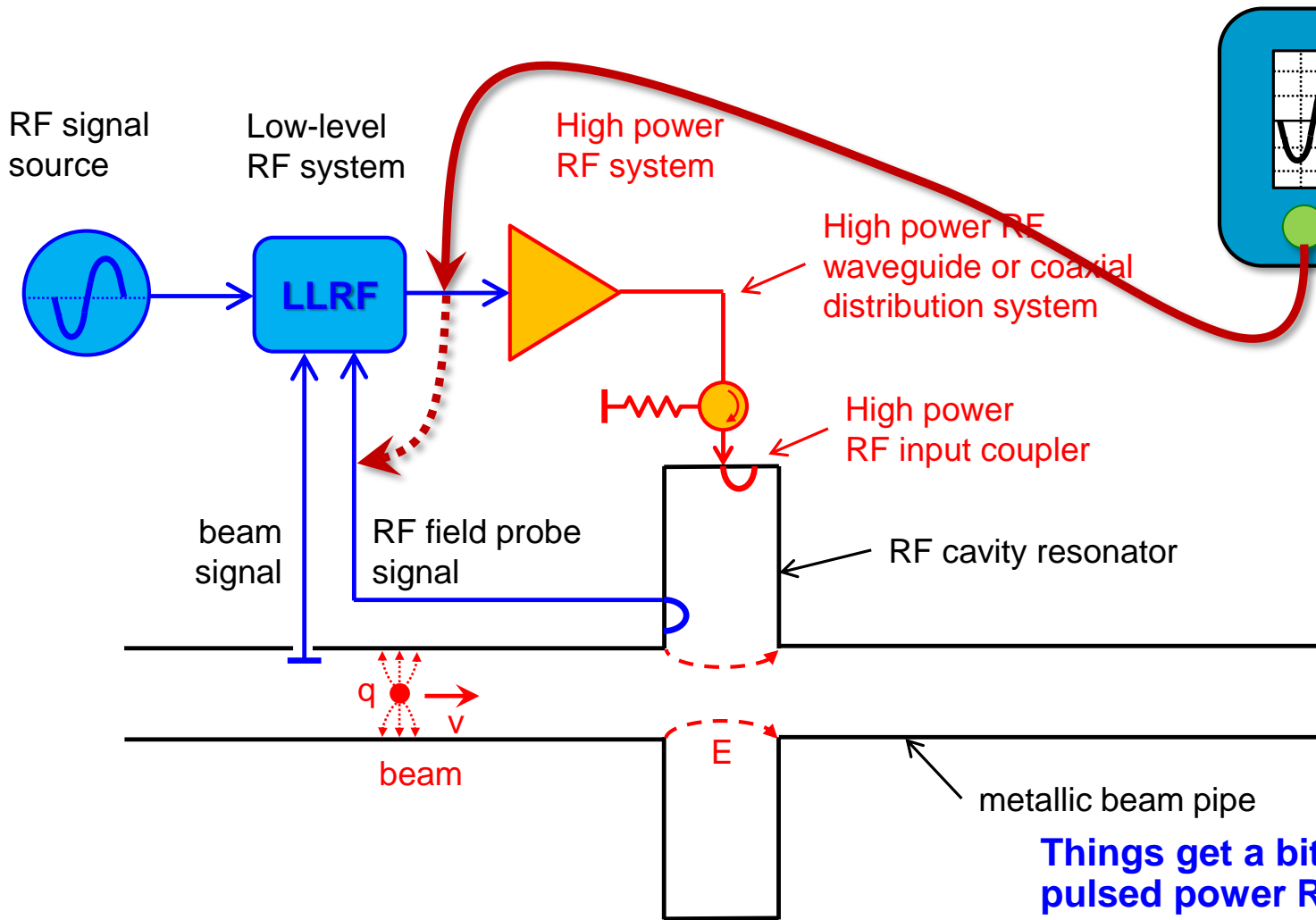
- Introduction
- Part I: Signals and reflections on transmission-lines
- Part II: The *Smith* chart
- ~~Part III: Scattering parameters~~
- Part IV: RF measurement methods
- Backup slides:
If you want to know more...





- EM-fields, RF technology, material science, etc.
 - High acceleration gradient, up to 100MV/m
- Beam dynamics
 - Particle interaction with EM-fields
 - HOMs, wakefields
 - f_{res} , harmonic number h
 - V_{RF} defines a stable RF bucket (potential well)
 - ...and therefore, f_s , etc.
 - Transit time

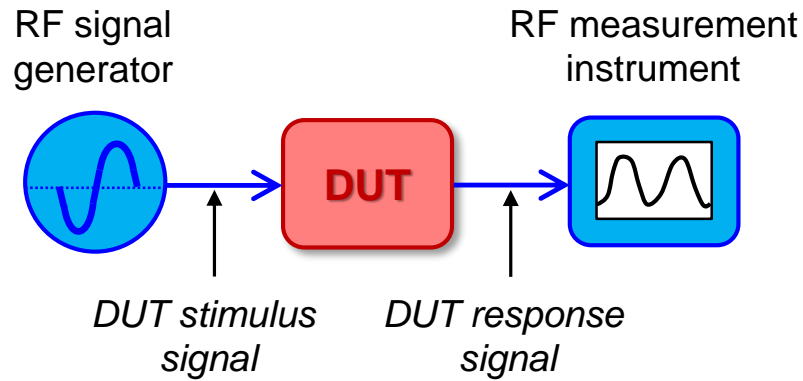
Introduction – A simple RF system



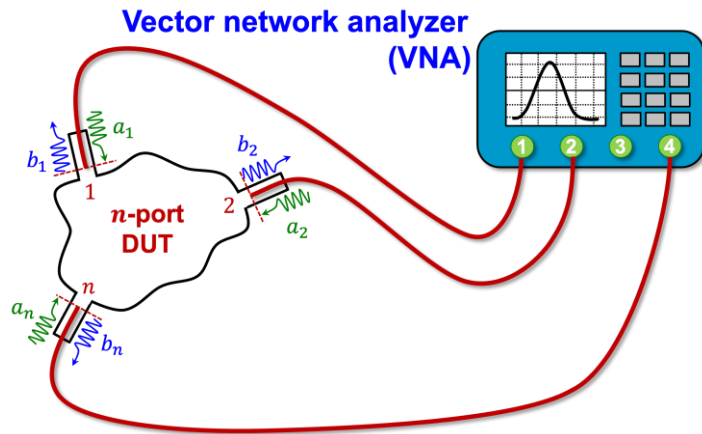
RF measurement instrument, e.g.:
spectrum analyzer,
oscilloscope,
diode detector, etc.

- **Please note:**
RF (measurement) techniques are not limited to accelerator RF
 - **Beam / bunch instrumentation and diagnostics**
 - **Injection / extraction elements**
 - **Pulsed accelerator systems, etc.**

Things get a bit more complicated in the real world:
pulsed power RF, multi-cell resonators or traveling wave structures, non-relativistic beams, HOM's, SRF, etc.

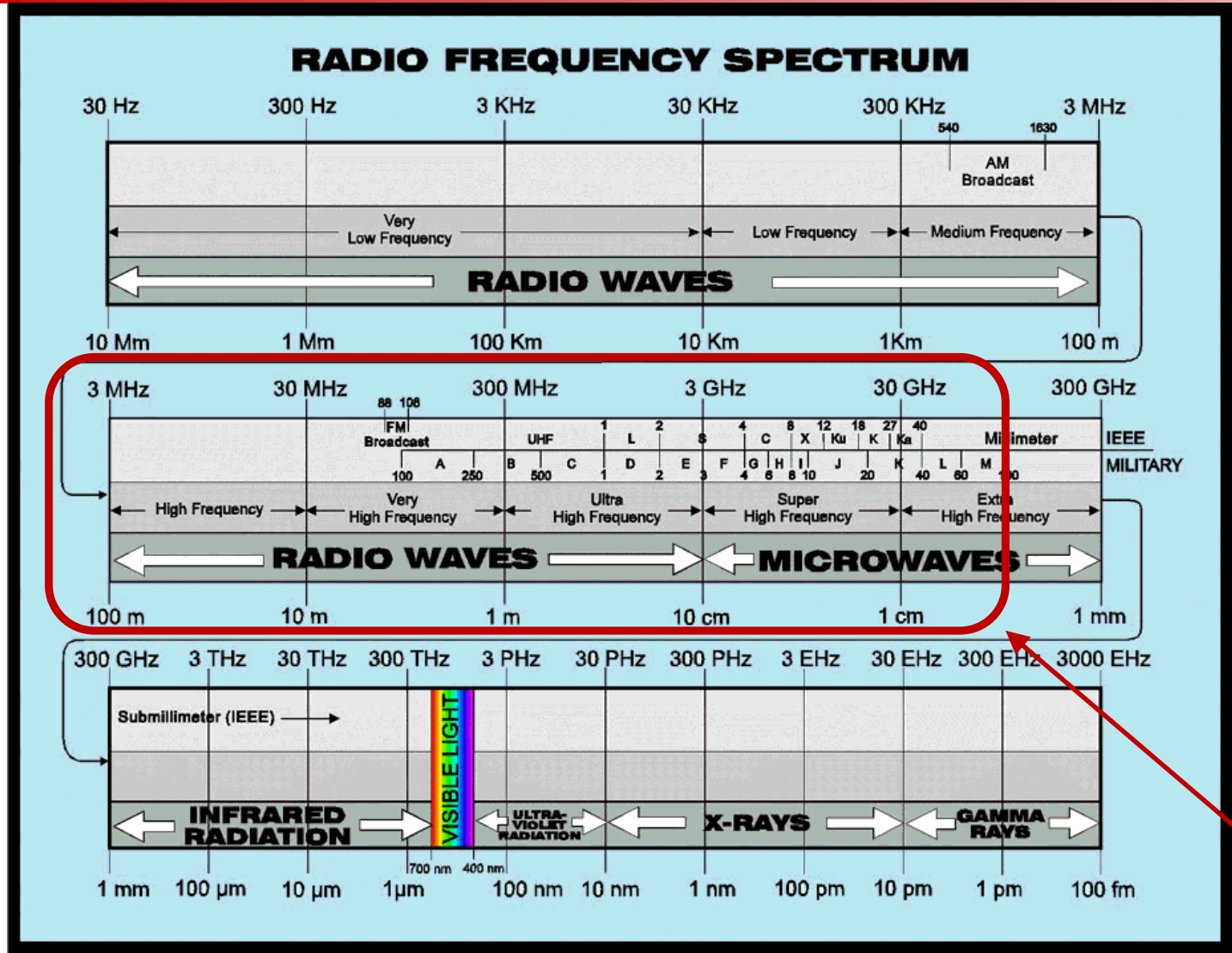


DUT: Device Under Test



- **Characterization of RF components and subsystems on the test bench**
 - **Again: not limited to accelerator RF**
 - also used in beam instrumentation,
 - fast kickers and other beam manipulation devices,
 - beam-coupling impedance measurements of accelerator vacuum components
- **Most popular:**
Vector Network Analyzer (VNA) measurement characterization

Introduction – A simple RF system



Free space wavelength:

$$\lambda = \frac{c}{f}$$

We care about RF concepts if the physical dimensions of an apparatus is $> \lambda/10$

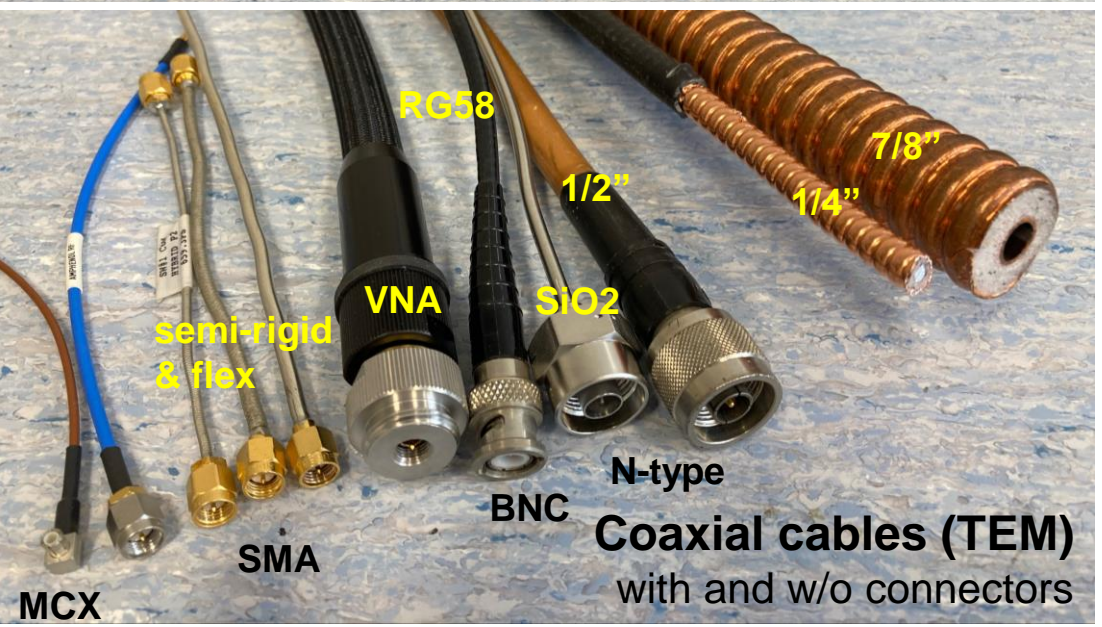
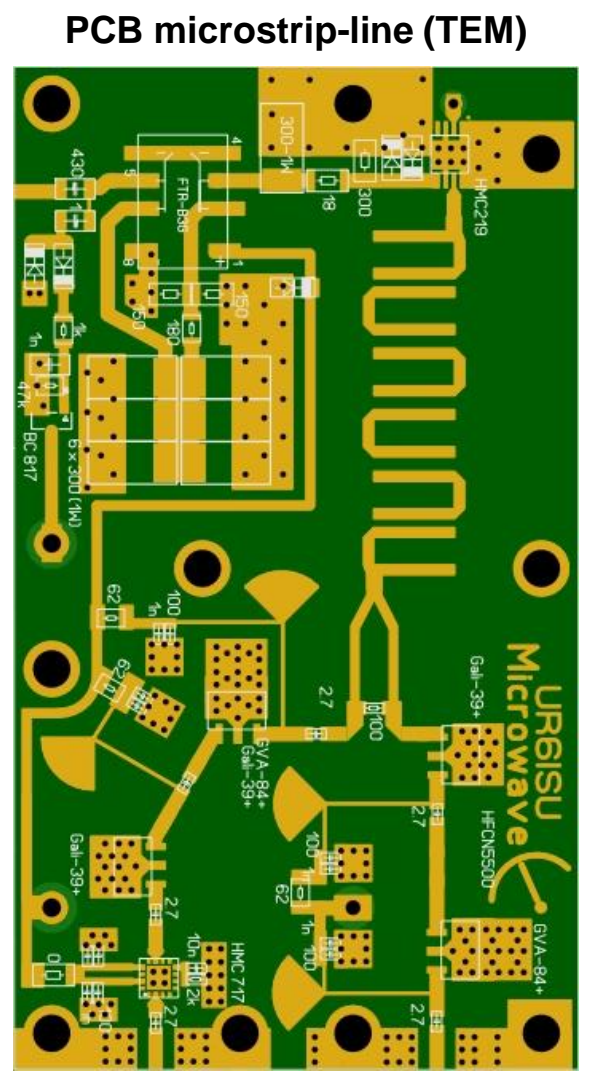
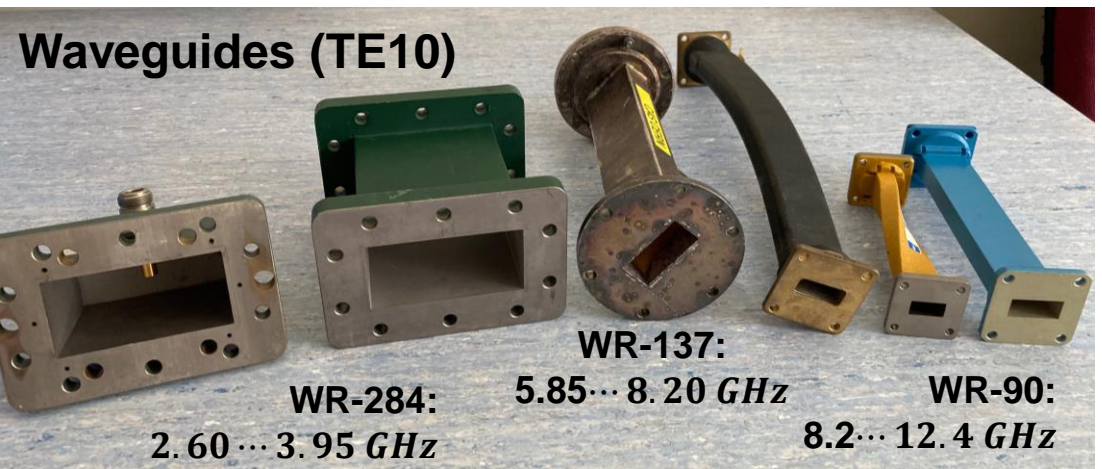
RF frequencies typically utilized in accelerator applications



- **Low-power RF signals can be processed digitally**
 - **RF System-on-Chip: processors, FPGA, ADCs and DACs**
 - 8x ADC (5 GBPS, 12-bit, 70 dB dynamic range), 8x DAC (9.8 GBPS), incl. up-down converters
 - Multi-core processors, on-chip memory, many I/O options
 - **For frequencies <2 GHz, operation in the 1st Nyquist passband**
- **Not part of this lecture...**

- **Outline and Learning objectives**
 - **Introduction and some minimum theoretical background**
 - **Reflections effects of pulse signals on transmission-lines due to characteristic-termination impedance mismatch**
 - **Standing waves on transmission-lines for continuous wave (CW) sinusoidal signals, definition of the reflection coefficient**
 - **Relation between reflection coefficient, standing wave ration and return loss**

Transmission-lines (1)



- **Transmission-lines transport RF energy (waves) between components and sub-systems and exist in a large variety. Most popular are**
 - **Coaxial cables, always in TEM operation below the TE₁₁ cut-off frequency**
 - Popular connectors are BNC (*Bayonet Neill–Concelman*), N-type and SMA (Sub-Miniature version A)
 - There exist a large variety of coaxial cables and connectors for HOM-free operation up to 110 GHz
 - **PCB (printed circuit board)-based planar quasi-TEM transmission-lines**
 - Popular are microstrip, stripline and coplanar-waveguide structures
 - **Rectangular waveguides, usually in TE₁₀-mode operation**
 - For low-loss (high-power), high-frequency RF signal transmission
 - Clumsy and expensive, however, for some applications the only solution
- **Strictly speaking: Transmission-lines are 2-dimensional objects with infinite length**
 - The cross-section geometry defines the characteristic impedance Z_0
- **TEM transmission-lines (coaxial cables, PCB planar lines) operate from DC (direct current: 0 Hz) to a frequency given by the 1st higher-order mode**
 - Various frequency depended losses contribute, therefore, high insertion losses at high frequencies
- **Rectangular waveguides do not operate at DC or low frequencies!**
 - Instead utilize the TE₁₀ mode for the signal transmission

- **TEM: coaxial cable, PCB stripline, micro-stripline, co-planar waveguide, etc.**
 - but also, **TE / TM: waveguides** (rectangular, circular, elliptical)
- **Transport RF energy (EM waves) from a RF source to a load.**
- **Physical length ℓ becomes relevant for**

$$\ell \gtrsim \frac{\lambda_g}{10}, \quad \text{with guide wavelength: } \lambda_g = \frac{v_p}{f}$$

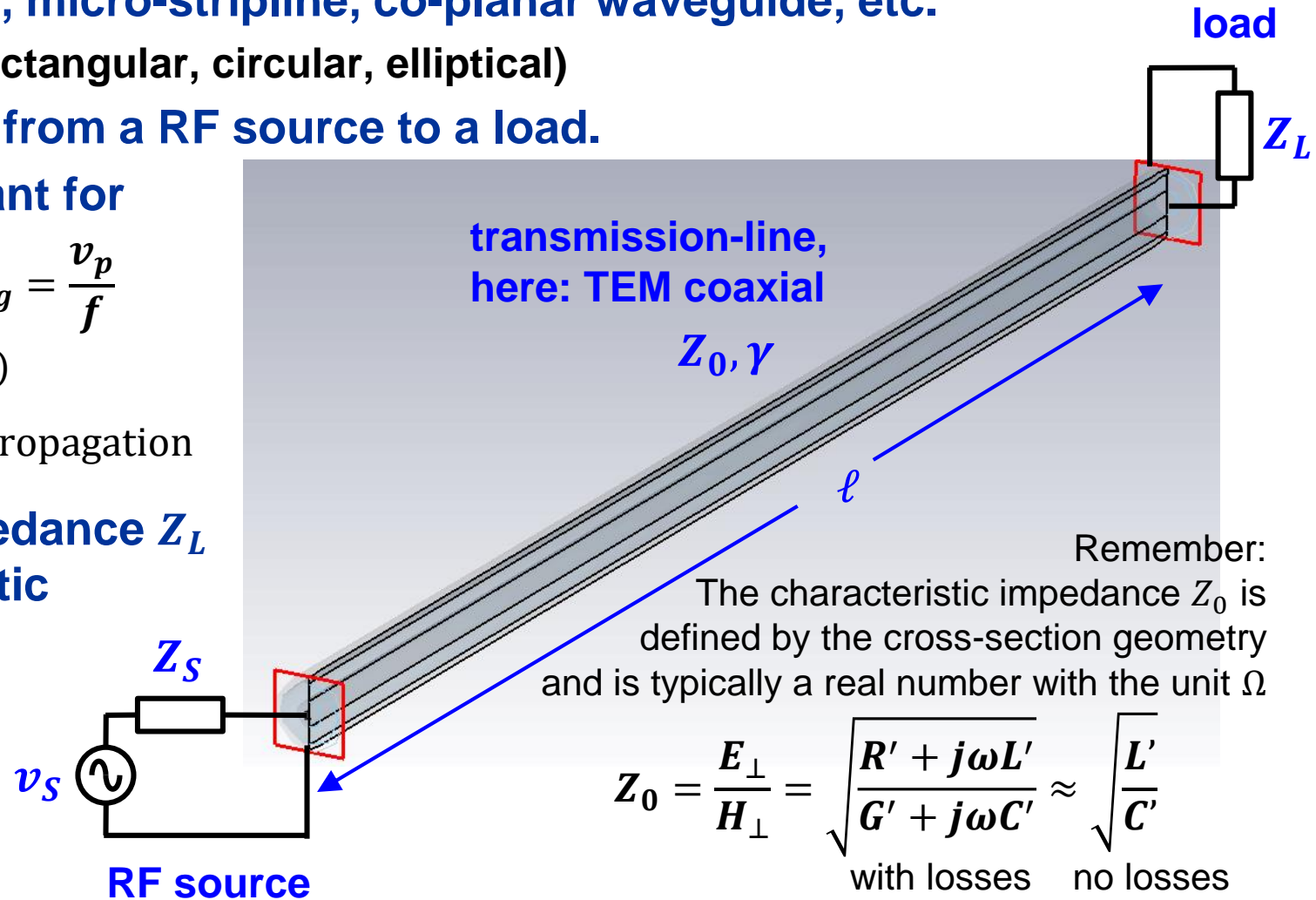
f : operating frequency (max.)

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{c}{\sqrt{\epsilon_r}} \text{ velocity of propagation}$$

- **Reflections occur if the load impedance Z_L is not matched to the characteristic impedance Z_0 of the TL**

$$Z_L = Z_0 \Rightarrow \Gamma = 0: \text{no reflections} \text{ 😊}$$

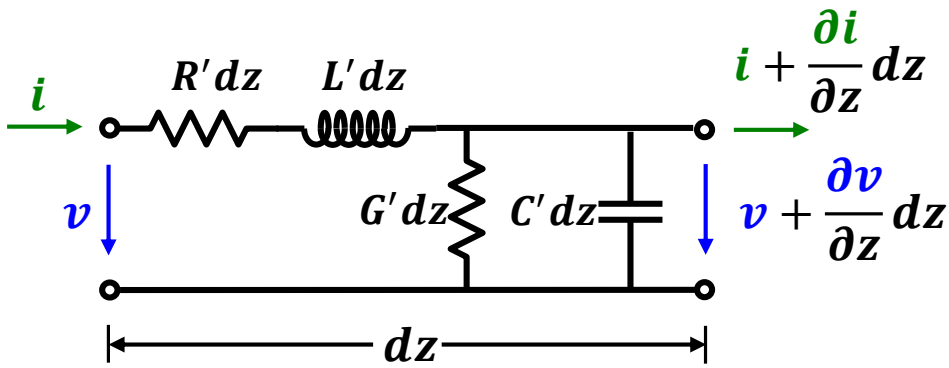
$$Z_L \neq Z_0 \Rightarrow \Gamma \neq 0: \text{reflections!} \text{ 😞}$$



A more general approach:

$$\frac{\partial v(z, t)}{\partial z} = - \left(R' + L' \frac{\partial}{\partial t} \right) i(z, t)$$

$$\frac{\partial i(z, t)}{\partial z} = - \left(G' + C' \frac{\partial}{\partial t} \right) v(z, t)$$



phase velocity

$$v_p = \frac{\lambda}{T} = \frac{\omega}{\beta}$$

in steady state:

$$\frac{dV}{dz} = -(R' + j\omega L')I$$

$$\frac{dI}{dz} = -(G' + j\omega C')V$$

voltage and current along a transmission-line:

$$V(z) = V_0 \cosh \gamma z - Z_0 I_0 \sinh \gamma z$$

$$I(z) = I_0 \cosh \gamma z - V_0 / Z_0 \sinh \gamma z$$

V_0, I_0 : voltage and current at the beginning of the line ($z = 0$)

wave number

$$k = \frac{2\pi}{\lambda} = \beta$$

propagation constant

attenuation constant

phase constant

characteristic impedance

$$\frac{d^2 V}{dz^2} = \gamma^2 V$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

$$Z_0 \cong \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

- The characteristic impedance is defined by the cross-section geometry:

$$Z_0 = \frac{\eta_0}{2\pi\sqrt{\epsilon_r}} \ln\left(\frac{D}{d}\right) \approx \frac{60 \Omega}{\sqrt{\epsilon_r}} \ln\left(\frac{D}{d}\right) \quad [\Omega]$$

- The attenuation losses in the propagation constant γ are dominated by the material properties:

– Propagation constant: $\gamma = \alpha + j\beta$

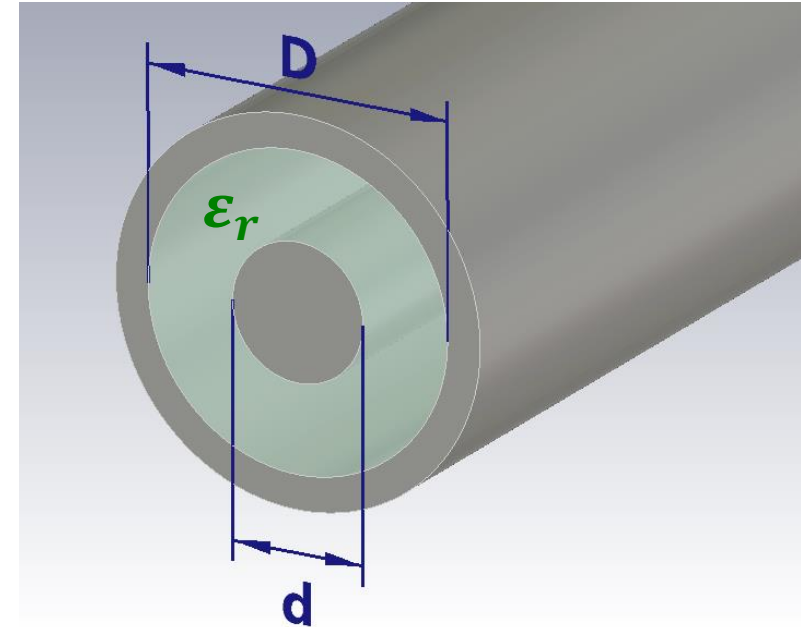
– Attenuation constant: $\alpha = \alpha_c + \alpha_d + \alpha_r + \alpha_l$

➤ The attenuation losses are dominated by conductor and dielectric losses

– Conductor losses: $\alpha_c = \underbrace{\sqrt{\frac{\mu_0 \mu_r \pi f}{\sigma}}}_{\text{(surface resistance)} = R_s} \frac{\sqrt{\epsilon_r}}{\eta_0 \ln\left(\frac{D}{d}\right)} \left(\frac{1}{D} + \frac{1}{d}\right) \quad [Np/m]$

– Dielectric losses: $\alpha_d = \pi f \sqrt{\mu_0 \epsilon_0 \epsilon_r} \tan \delta_\epsilon \quad [Np/m]$

➤ with: resistivity: $\rho = \frac{1}{\sigma} \quad [\Omega m]$; loss tangent: $\tan \delta_\epsilon = \frac{\epsilon''}{\epsilon'}$; permittivity: $\epsilon = \epsilon' - j\epsilon'' \quad [F/m]$; permeability: $\mu = \mu' - j\mu'' \quad [H/m]$



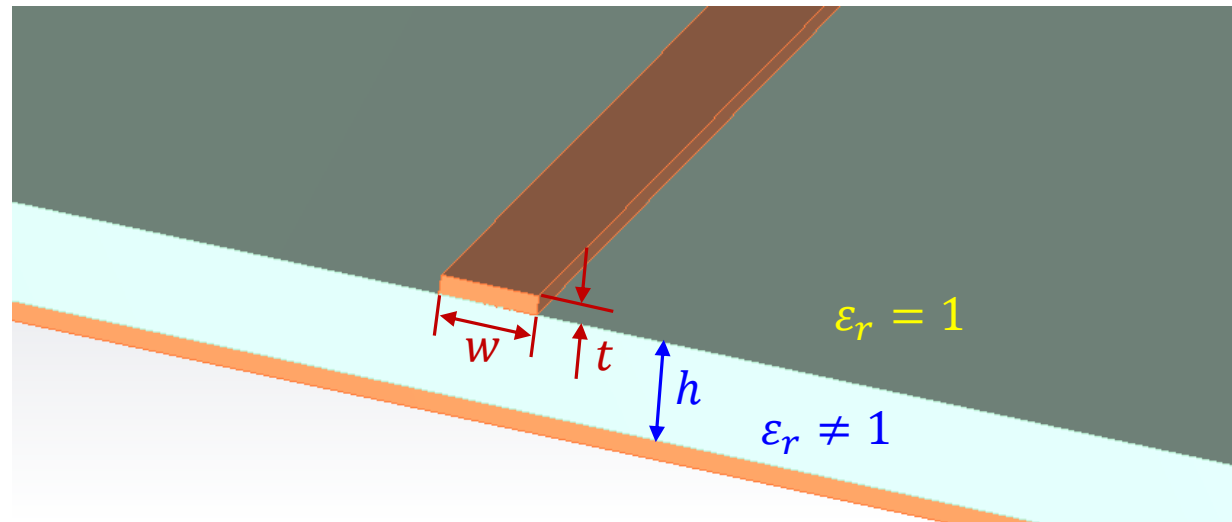
For inner and outer conductor in copper:

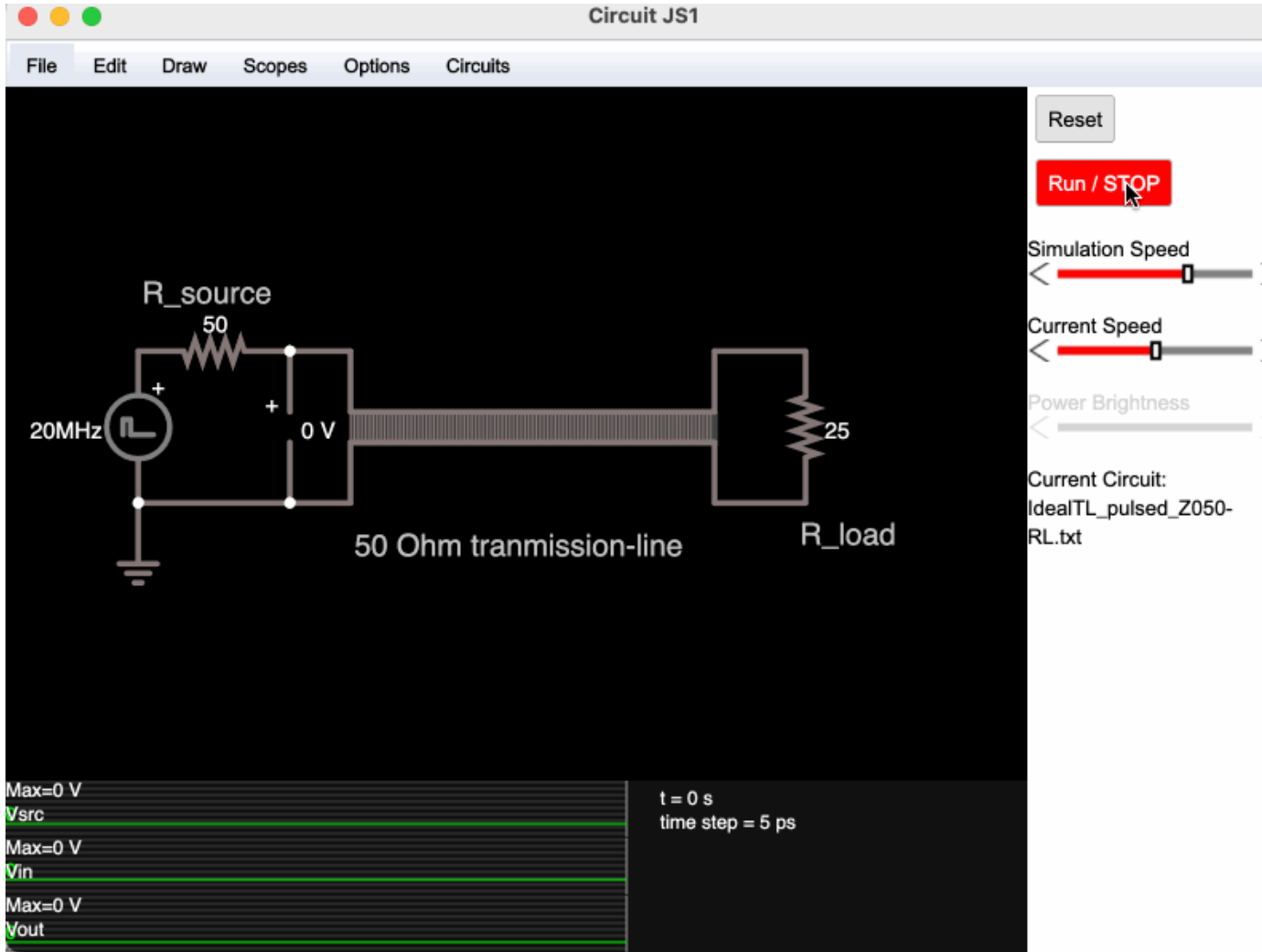
$$\mu_{rD} = \mu_{rd} = \mu_{rcu} = 1; \quad \rho_D = \rho_d = \rho_{cu} = 1.72 \cdot 10^{-8} \Omega m$$

$$\alpha_c = 6 \cdot 10^{-9} \sqrt{f \epsilon_r} \frac{D + d}{dD \ln(D/d)} \quad [dB/m]$$

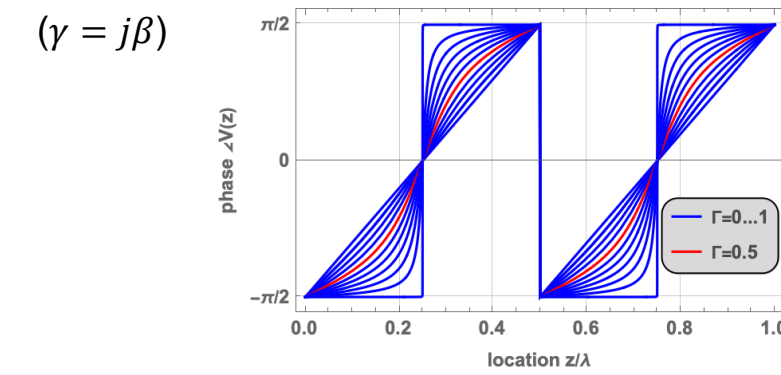
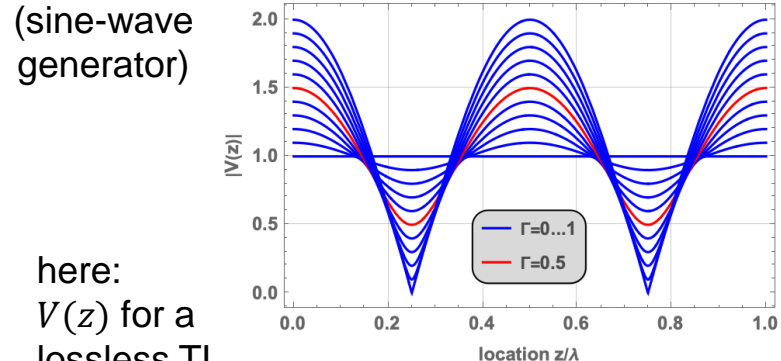
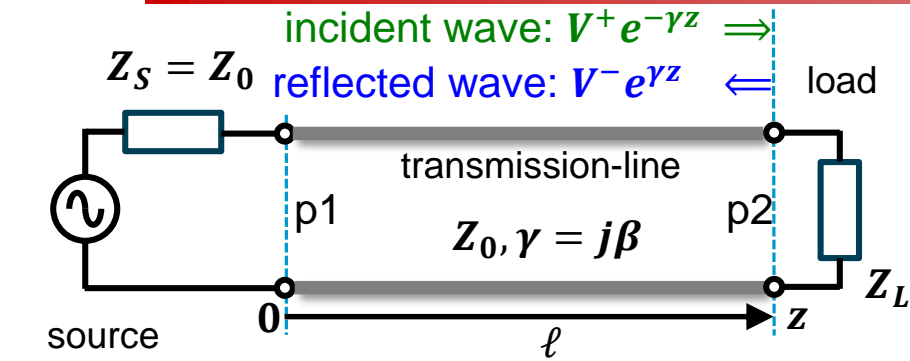
$$\alpha_d = 91 \cdot 10^{-9} f \sqrt{\epsilon_r} \tan \delta \quad [dB/m]$$

- **Microstrip-line:**
 - **Metallic strip conductor of width w and thickness t on a dielectric substrate of height h over a conductive ground plane.**
 - Typically, the dielectric layer is a low-loss printed circuit board substrate
 - Quasi-TEM field as the EM-field propagates in two medias of different ϵ_r
 - Complicated analytical approximations to calculate the properties (Z_0 , ϵ_{eff} , losses, high-order modes, etc.)
- **Other popular planar structures are coplanar waveguides and striplines**





- **Circuit simulator applet:**
<https://www.falstad.com/circuit/>
 - **Load file:**
IdealTL_DCswitched_Z050-RL.txt
 - Change the load resistor value:
 $RL = 50, 100, 25 \Omega$
 - Operate the switch and observe the signals at the beginning, and at the end of the transmission-line.
 - **Load file:**
IdealTL_pulsed_Z050-RL.txt
 - Change the load resistor value:
 $RL = 50, 100, 25 \Omega$
 - Observe the signal waveforms!
 Can you predict the values?!
 - (Press *Run/STOP* and hover with the mouse over the waveform)



here:
 $V(z)$ for a
 lossless TL
 $(\gamma = j\beta)$

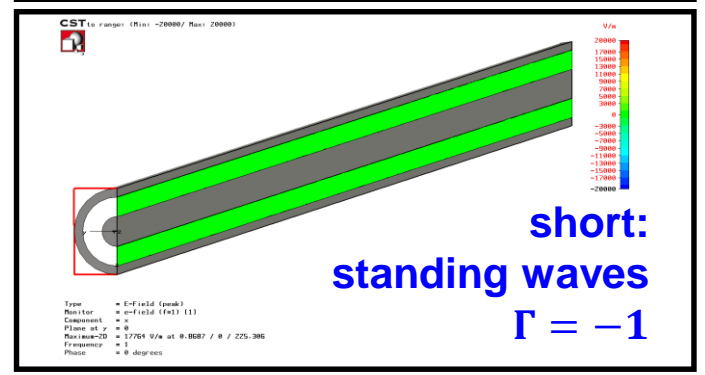
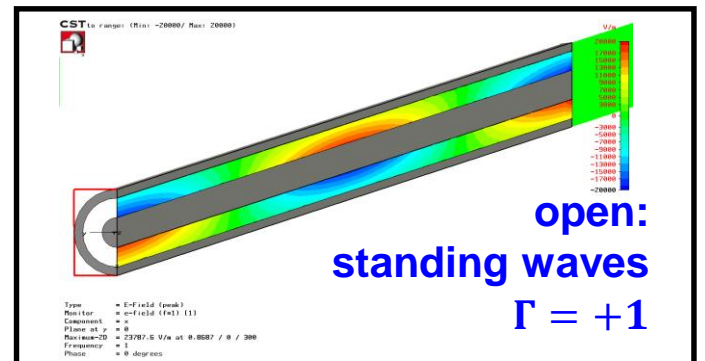
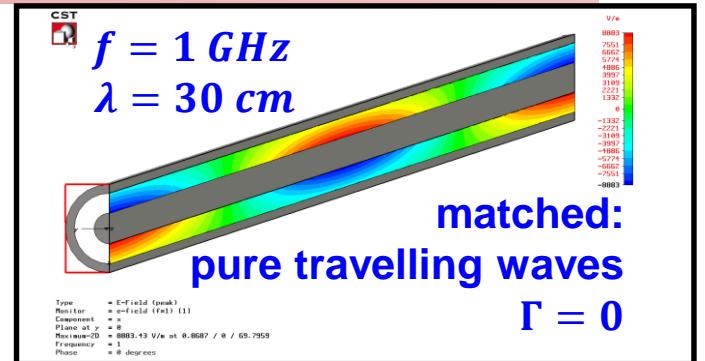
- For a single frequency, continuous wave (CW) signal on a transmission-line
 - Superposition of forward a, E^{inc}, V^+ and backward b, E^{refl}, V^- traveling waves
 - \Rightarrow **standing waves**

Reflection coefficient Γ

$$\Gamma = \frac{b}{a} = \frac{E^{refl}}{E^{inc}} = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

waves E-fields voltages impedances

- Γ is related to the impedance ratio: Z_L/Z_0
 - \triangleright The load impedance Z_L can be complex
- Γ is a complex number
- Reflections also originate due to discontinuities of the TL along z
 - Same Z_0 does not prevent reflections! EM-field effect!



- The voltage standing wave ratio (VSWR) expresses the ratio between the maximum and minimum voltage of a standing wave along a transmission-line

$$VSWR = \frac{|V_{max}|}{|V_{min}|} = \frac{|a| + |b|}{|a| - |b|} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \left| \frac{Z_L}{Z_0} \right|$$

with: $\begin{cases} |V_{max}| = |V^+| + |V^-| \\ |V_{min}| = |V^+| - |V^-| \end{cases}$

symbols: **incident (forward) wave: a, X^+**
reflected (backward) wave: b, X^-

– The VSWR is a function of the frequency.

- The return loss (RL) is another way to express reflection effects

$$RL [dB] = 10 \log_{10} \frac{P^+}{P^-} = -20 \log_{10} |\Gamma|$$

Γ	$VSWR = Z_L/Z_0$	Return Loss [dB]	Refl. Power $ \Gamma ^2$	Inc. Power $1 - \Gamma ^2$
0.0	1.00	∞	0.00	1.00
0.1	1.22	20.0	0.01	0.99
0.2	1.50	14.0	0.04	0.96
0.3	1.87	10.5	0.09	0.91
0.4	2.33	8.0	0.16	0.84
0.5	3.00	6.0	0.25	0.75
0.6	4.00	4.4	0.36	0.64
0.7	5.67	3.1	0.49	0.51
0.8	9.00	1.9	0.64	0.36
0.9	19.00	0.9	0.81	0.19
1.0	∞	0	1.00	0.00

- **dezi-Bel: 1 dB = 0.1 B (Bel)**

- Logarithmic scaling to compare large, e.g., power ratios:

$$P_{dB} = 10 \log_{10} \left(\frac{P_1}{P_2} \right)$$

- or large ratios of other quantities, e.g.:

voltage ratio: $V_{dB} = 20 \log_{10} \left(\frac{V_1}{V_2} \right)$

current ratio: $I_{dB} = 20 \log_{10} \left(\frac{I_1}{I_2} \right)$

<i>dB ratio</i>	P_1/P_2	V_1/V_2
$n \times dB$	10^n	$10^{\frac{n}{2}}$
40 dB	10000	100
20 dB	100	10
10 dB	10	~3.16
6 dB	~4	~2
3 dB	~2	~1.41
0 dB	1	1
<i>-3 dB</i>	<i>~0.5</i>	<i>~0.71</i>
-20 dB	0.01	0.1

$$\frac{P_1}{P_2} = 10^{\left(\frac{P_{dB}}{10}\right)} \quad \frac{V_1}{V_2} = 10^{\left(\frac{V_{dB}}{20}\right)}$$

The 3 dB ratio (half power) is a common specification for the bandwidth

“dB” is not “dBm”

- ***dBm*** is defined as a logarithmic power unit
 - based on dB and $P_{ref} = 1 \text{ mW}$
- ***dBm*** can also be used as logarithmic voltage unit
 - **Only for sinusoidal waveforms!**
 - e.g., for $Z_0 = 50 \Omega$: $V_{ref} = 0.2236 \text{ V}$

$$P_{dBm} = 10 \log_{10} \left(\frac{P}{P_{ref}} \right)$$

$$V_{dBm} = 20 \log_{10} \left(\frac{V}{V_{ref}} \right)$$

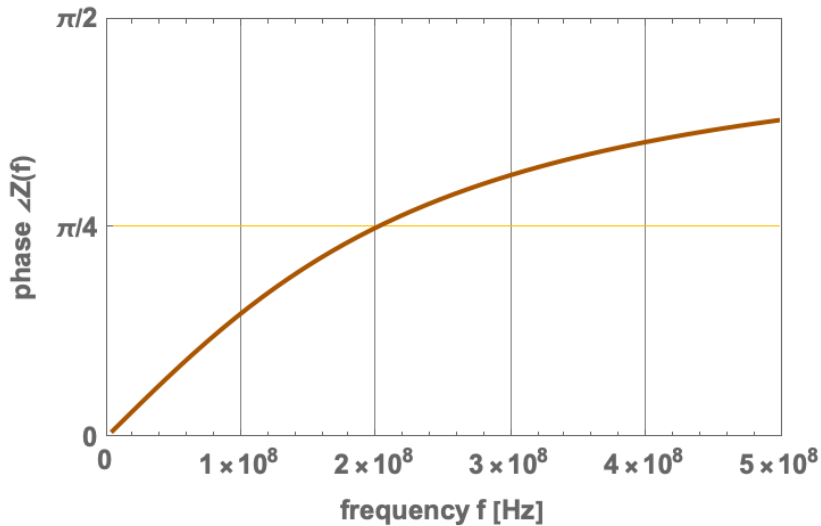
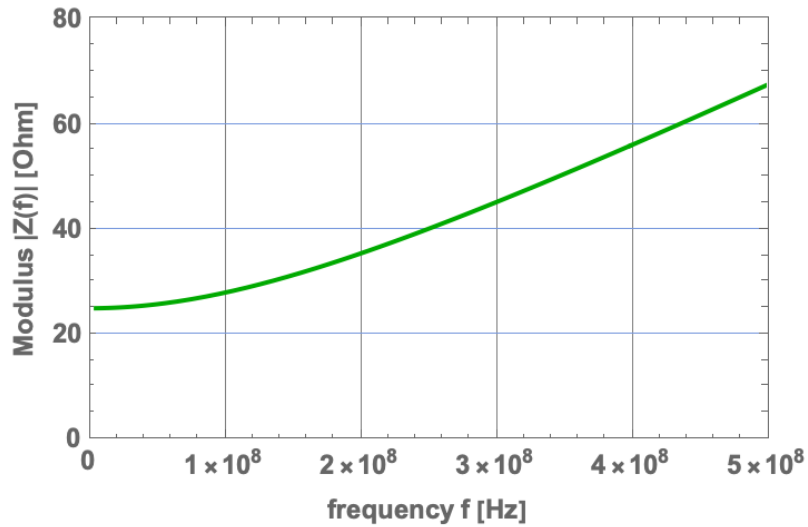
<i>dBm</i>	<i>P</i>	<i>V (RMS)</i>
+30 <i>dBm</i>	1 W	7.07 V
+20 <i>dBm</i>	100 mW	2.24 V
+10 <i>dBm</i>	10 mW	707 mV
+6 <i>dBm</i>	4 mW	446 mV
0 <i>dBm</i>	1 mW	224 mV
−20 <i>dBm</i>	10 μW	22.4 mV
−60 <i>dBm</i>	1 nW	224 μV
−120 <i>dBm</i>	1 fW	224 nV
−174 <i>dBm</i>	4 × 10^{−21} W	0.446 nV

$$P = P_{ref} 10^{\left(\frac{P_{dBm}}{10}\right)}$$

$$V = V_{ref} 10^{\left(\frac{V_{dBm}}{20}\right)}$$

← noise power in a bandwidth $BW = 1 \text{ Hz}$ at room temperature

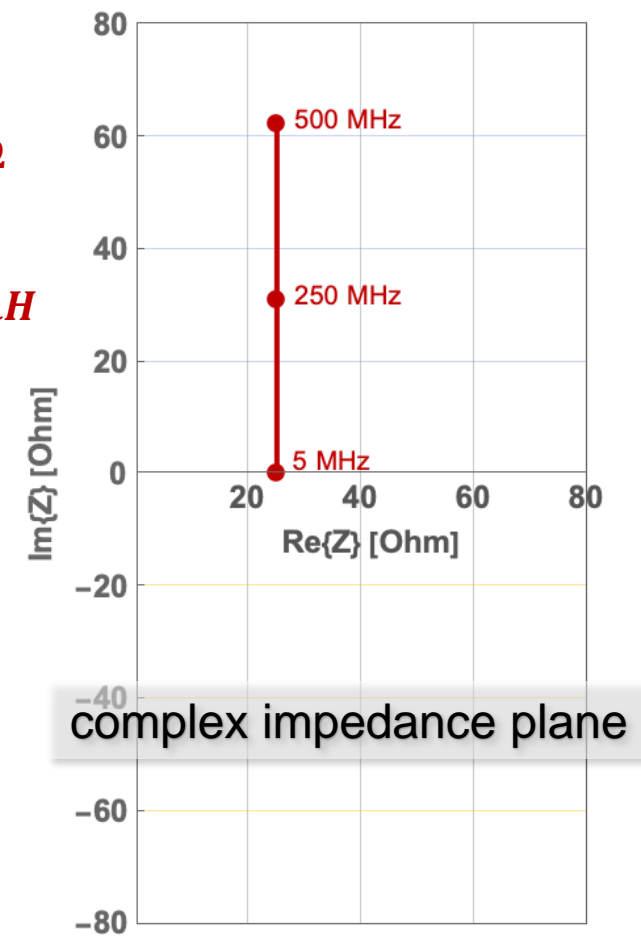
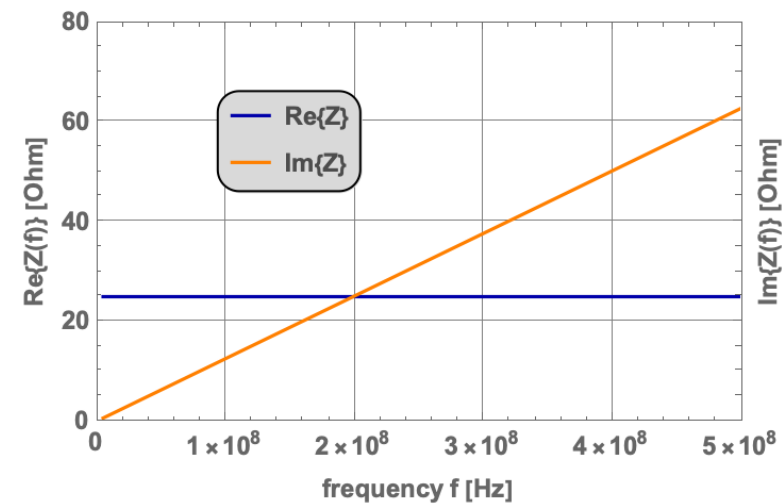
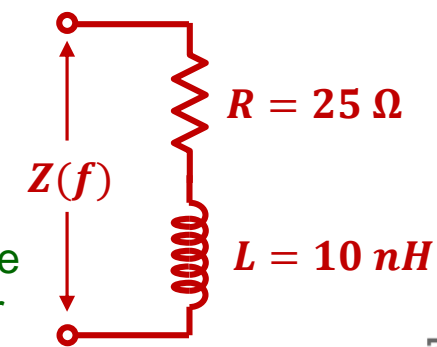
- **Outline and Learning objectives**
 - **Refresher: Visualization of a complex impedance in the frequency domain**
 - **Definition of the *Smith* chart, mapping the complex impedance / admittance plane with the complex reflection coefficient**
 - **Basic facts and important points on the *Smith* chart**
 - **Simple example**
 - More examples and information in the backup section:
 - Examples for a RL and RC series circuit, and for a transmission-line terminated with a RL series circuit.
 - Operation of a $\lambda/4$ -transformer based on a transmission-line as a (normalized) impedance inverter



- Different ways to visualize $Z(f)$

- magnitude / phase
- real / imaginary
- complex Z-plane

➤ parametric graph in the complex plane with f as parameter



complex impedance plane

conductor, with:
conductance G [S]



resistor, with:
resistance R [Ω]



inductor, with:
inductance L [H]
reactance $X_L = \omega L$ [Ω]

susceptance $B_L = 1/\omega L$ [S]



capacitor, with:
capacitance C [F]
susceptance $B_C = \omega C$ [S]

reactance $X_C = -1/\omega C$ [Ω]

- Resistance, impedance, reactance are inverse proportional to conductance, susceptance, admittance**

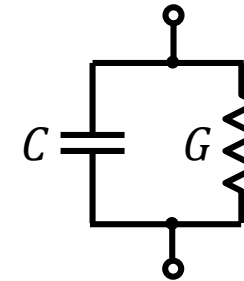
$$R = \frac{1}{G}$$

$$Z = \frac{1}{Y}$$

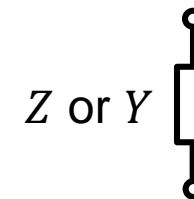
$$X = \frac{-1}{B}$$



complex impedance example:
 $Z = R + j\omega L$ [Ω]

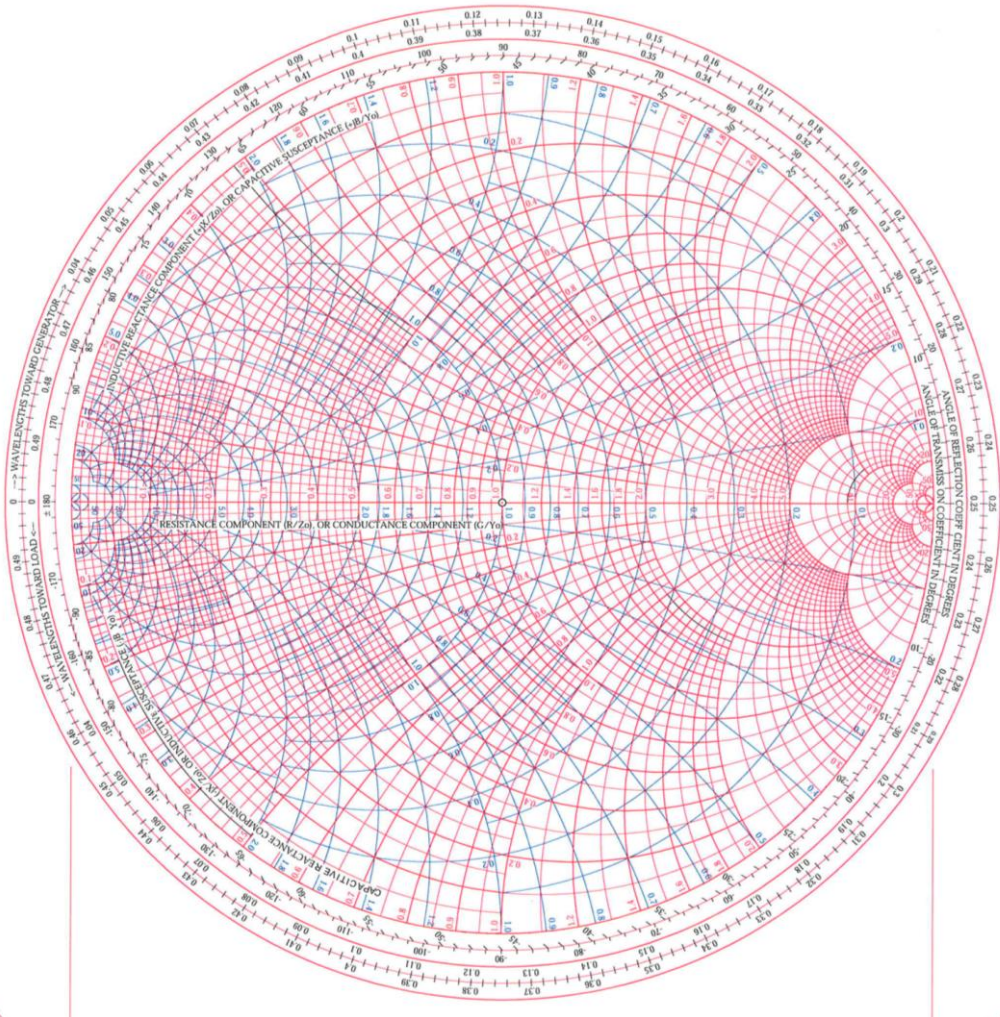


complex admittance example:
 $Y = G + j\omega C$ [S]

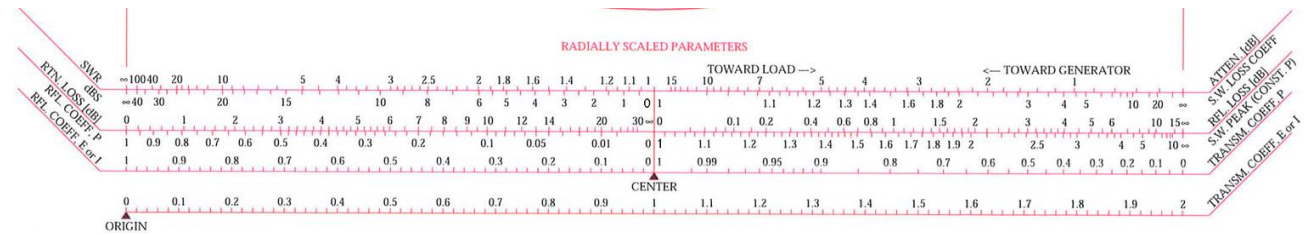


complex impedance Z
or
complex admittance Y

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



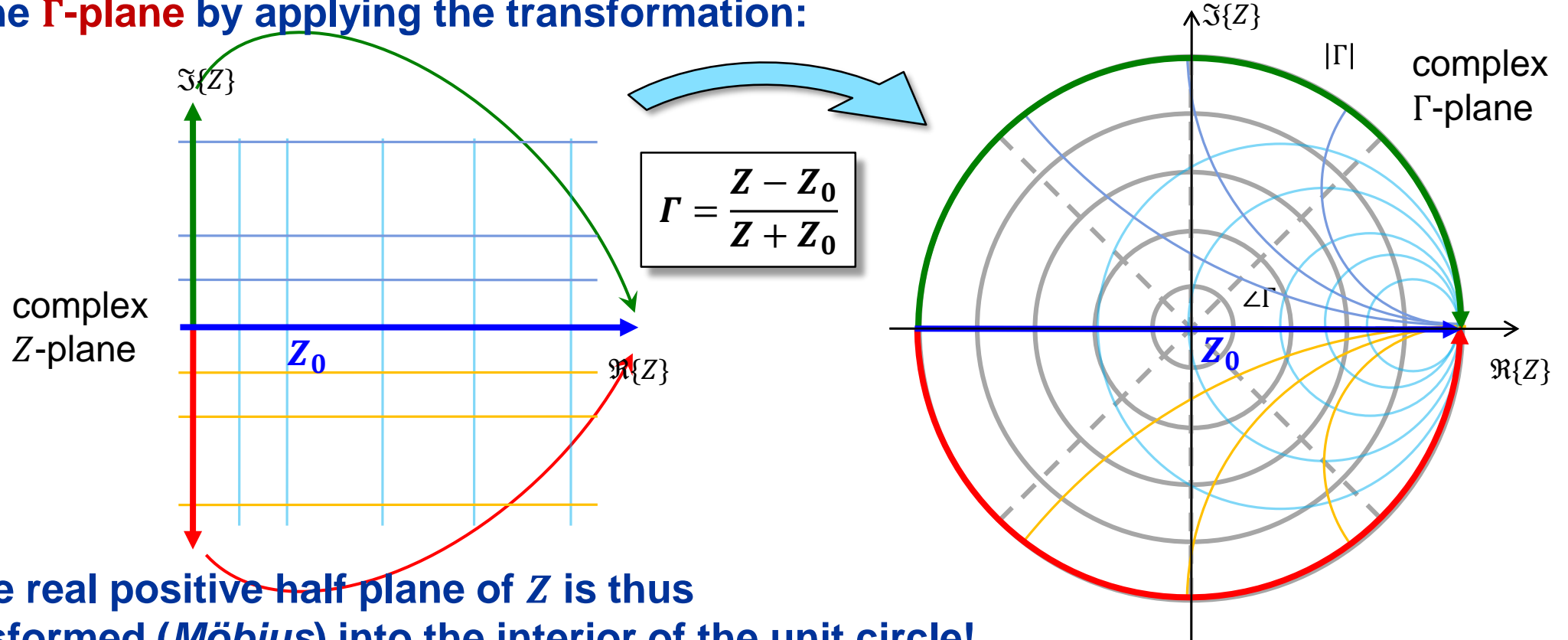
- The *Smith* chart is a calculation tool, divided in 2 parts:
 - A transformation of the complex, normalized **impedance z** and **admittance y -planes** on a circle.
 - A set of “rulers” below, for additional computations
 - VSWR, return and reflection loss, etc.



- At a 1st look the Smith chart is quite overwhelming
 - In this introduction the focus is on the **complex z -plane**

The *Smith Chart* (2)

- The **Smith chart** (in impedance / admittance coordinates) represents the **complex Γ -plane** (in polar coordinates) within the unit circle.
- It is a conformal mapping of the **complex Z -plane** on the **Γ -plane** by applying the transformation:



- \Rightarrow the real positive half plane of Z is thus transformed (**Möbius**) into the interior of the unit circle!

- In the classical paper *Smith* chart the impedance Z is **normalized**:

$$z = \frac{Z}{Z_0}$$

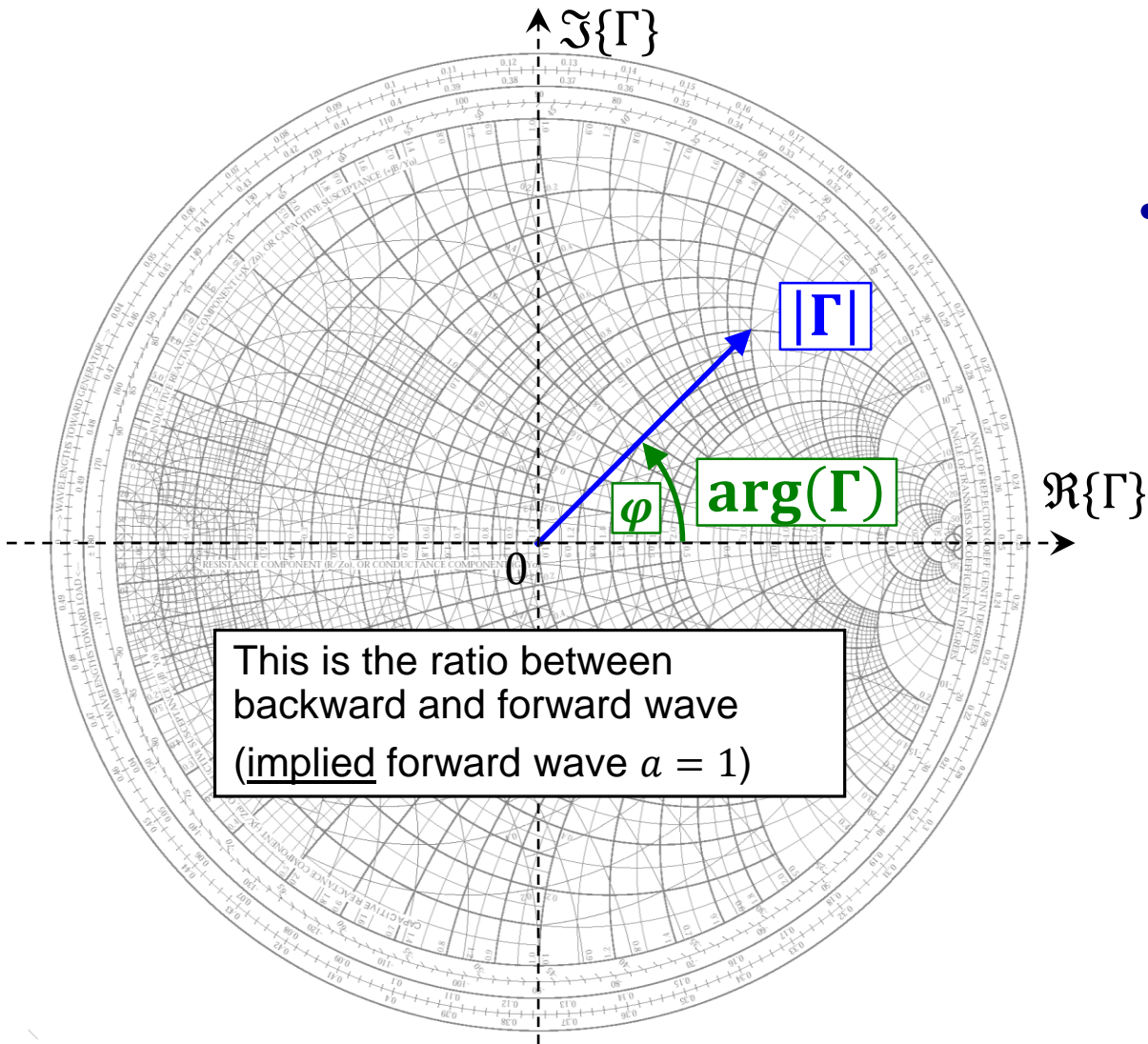
to a **reference impedance** Z_0 ,

typically, to the characteristic impedance of the coaxial cable transmission-lines used in RF / microwave engineering: $Z_0 = 50 \Omega$.

- The normalized form of the transformation follows then as:

$$\Gamma = \frac{z - 1}{z + 1} \Rightarrow \frac{Z}{Z_0} = z = \frac{1 + \Gamma}{1 - \Gamma}$$

- The *Smith* chart is a **parametric graph**
 - with the **frequency** f as parameter
 - and the **normalized, complex impedance** z and **complex reflection coefficient** Γ as variables
 - also, the normalized, complex admittance $y = 1/z$ is mapped and can be used as variable.
- In the past
 - The *Smith* chart was used as a calculation tool for impedance matching, e.g., antennas to transmitters or receivers, amplifier input / output stages, couplers of accelerating cavities, etc.
- At presence, the *Smith* chart is still popular
 - for visualization purposes, e.g., vector network analyzer (VNA) measurement of input / output impedances (display of the Snn scattering parameters)
 - for the optimization of the coupling between RF source and a cavity resonator.
 - ...



- In the *Smith* chart, the **complex reflection factor**

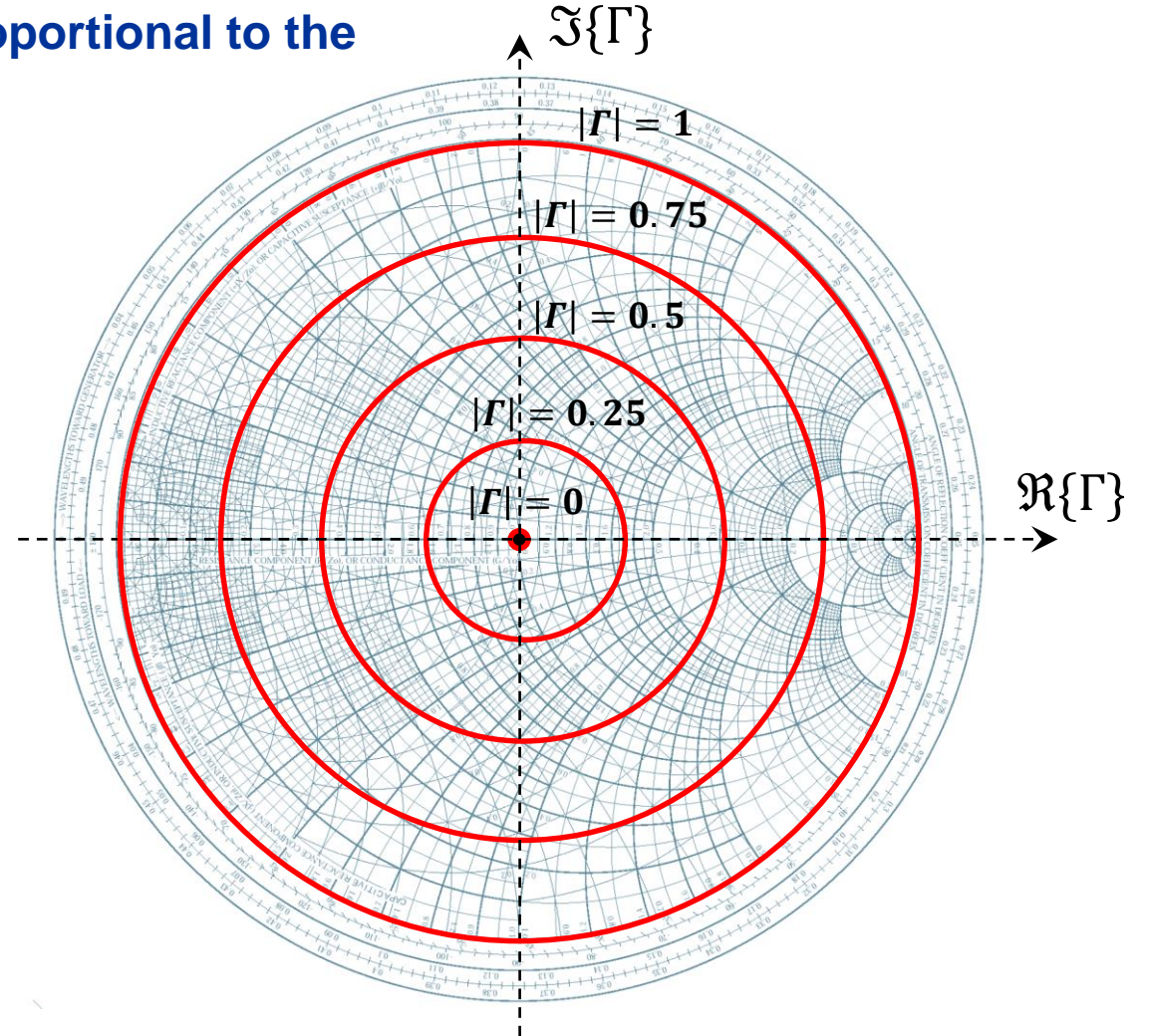
$$\Gamma = |\Gamma| e^{j\varphi} = \frac{b}{a}$$

is expressed in linear **polar coordinates**, representing the ratio of backward b vs. forward a traveling waves.

- The distance from the center of the diagram proportional to the **magnitude of the reflection factor $|\Gamma|$** and permits an easy visualization of the **matching performance**.
 - In particular, the perimeter of the diagram represents total reflection: $|\Gamma| = 1$.
 - (power dissipated in the load) = (forward power) – (reflected power)

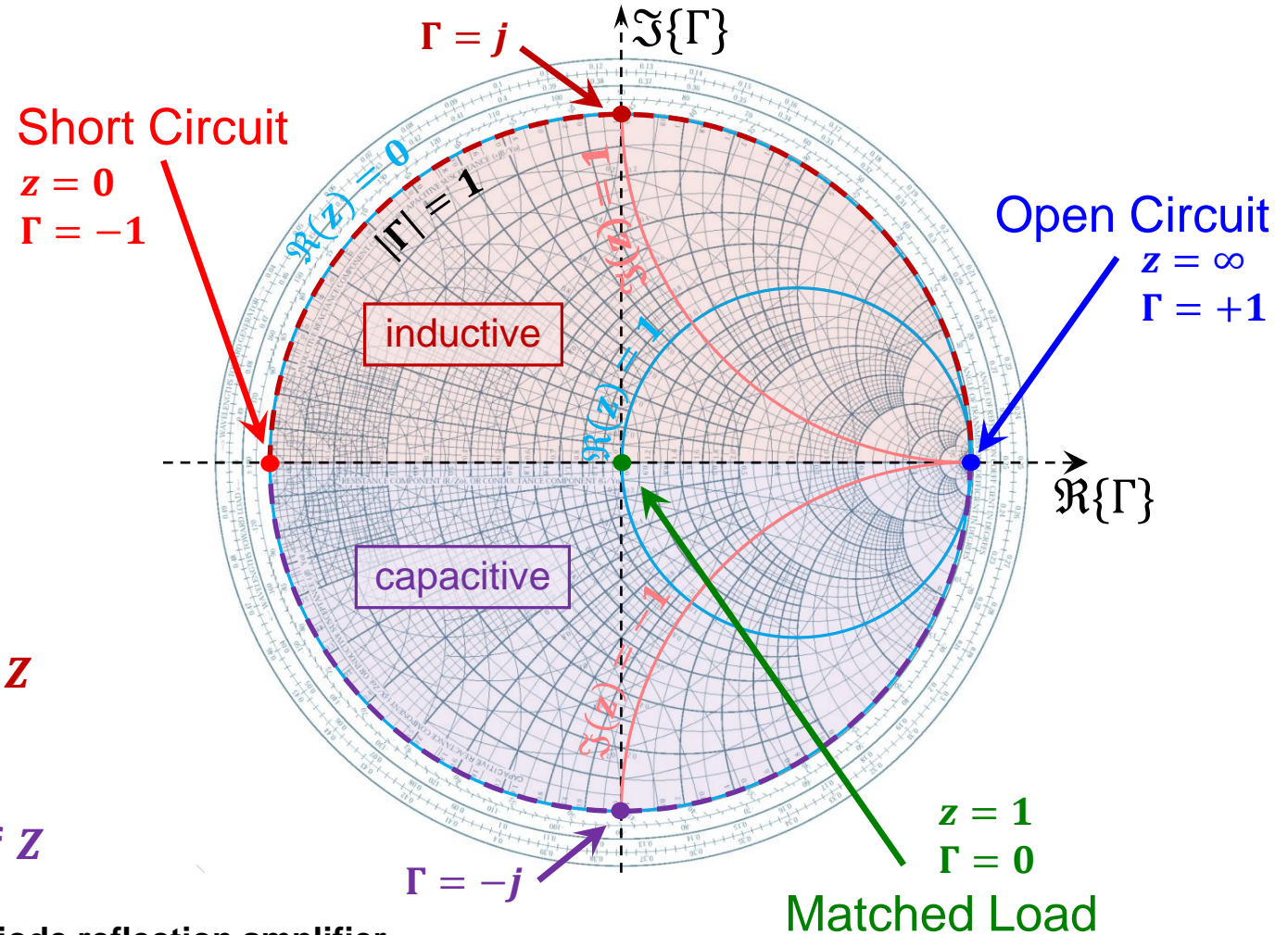
$$\begin{aligned}
 P &= |a|^2 - |b|^2 \\
 &= |a|^2 (1 - |\Gamma|^2)
 \end{aligned}$$

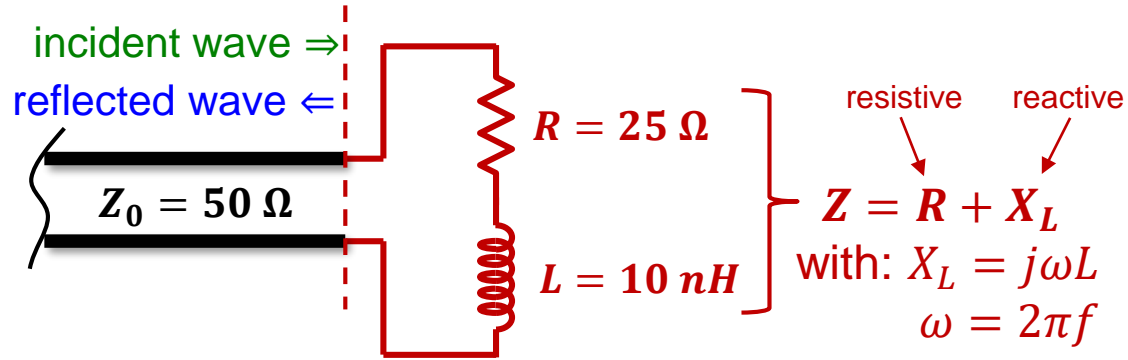
↑ available source power ↑ mismatch losses



Important Points:

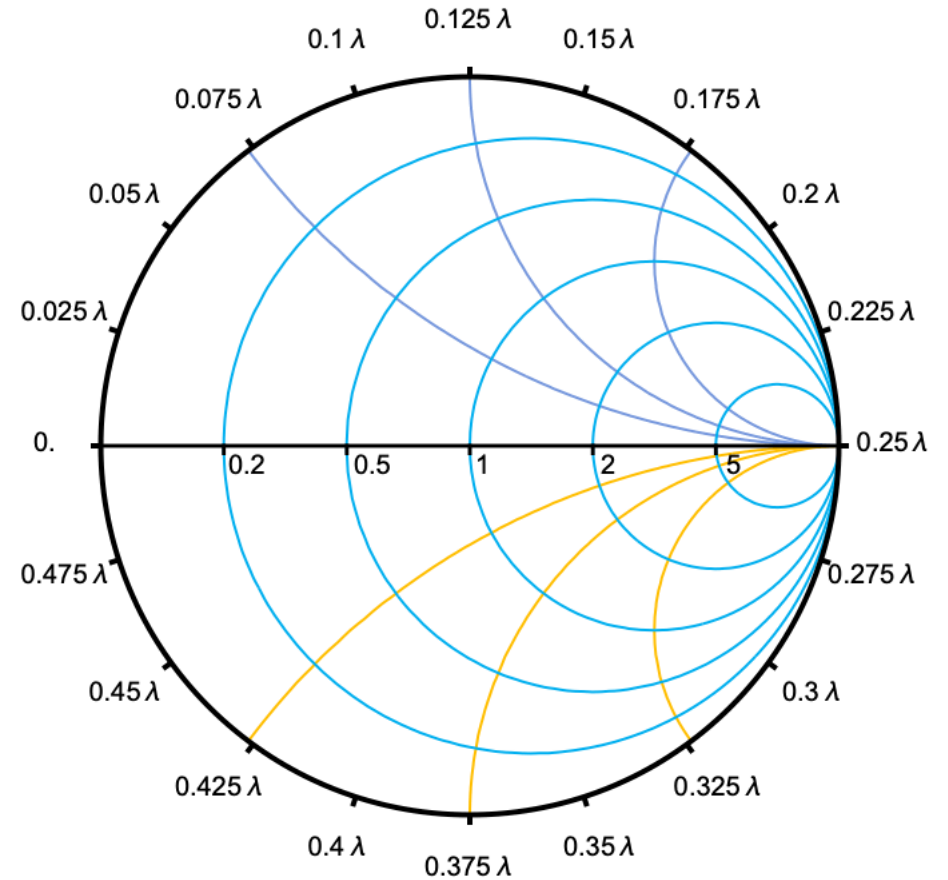
- **Short Circuit**
 $\Gamma = -1, z = 0$
- **Open Circuit**
 $\Gamma = +1, z \rightarrow \infty$
- **Matched Load**
 $\Gamma = 0, z = 1$
- **On the circle $|\Gamma| = 1$:**
lossless element
- **Upper half:**
”inductive” =
positive imaginary part of Z
- **Lower half:**
”capacitive” =
negative imaginary part of Z
 - Outside the circle, $\Gamma > 1$:
active element, e.g., tunnel diode reflection amplifier





Complex impedance based on lumped element components

- Calculate Z for a given frequency, e.g., $f = 50 \text{ MHz}$: $Z = (25 + j6.28) \Omega$
- Calculate the normalized impedance $z = Z/Z_0 = 0.5 + j0.126$
 - Locate z in the Smith chart
 - Retrieve $\Gamma = 0.34 \angle 161^\circ = 0.34e^{j2.81}$
- Repeat for other frequencies...



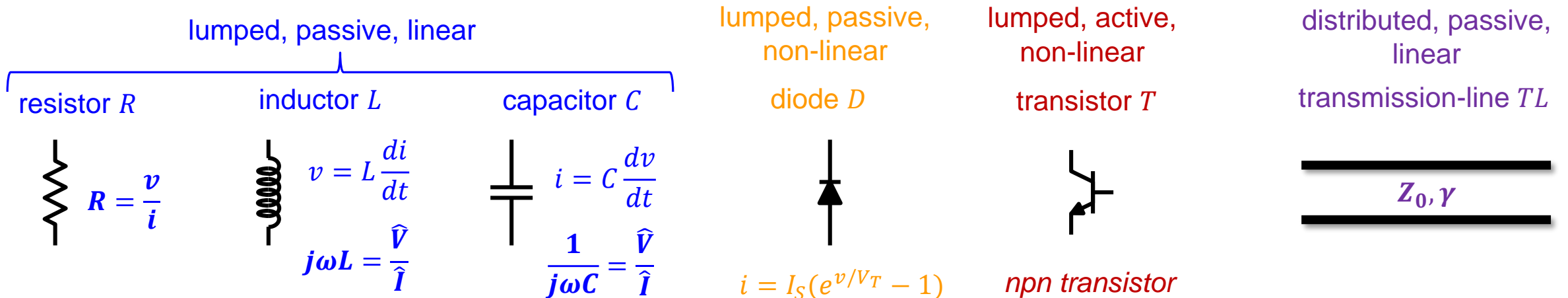
- The *Smith* chart is a special type of a parametric plot for the complex impedance or admittance, mapped to the plane of the complex reflection coefficient.
- The use of the *Smith* chart can be viewed in two ways:
 - As a calculation and impedance matching tool as originally envisioned.
 - However, today this will happen rather rarely!
 - Please notice: All the examples presented (in the backup slides) are based on the paper-style *Smith* chart, which is always based on an unitless, normalized impedance $z = Z/Z_0$
 - As a visualization tool for the complex impedance, along with the reflection coefficient
 - Still very popular and useful for displaying and analyzing S_{ii} on a vector network analyzer, also used in data-sheets, and RF simulation and education software.
 - Here the *Smith* chart utilizes the actual complex impedance Z in units of Ω ! Markers on the parametric trace give all relevant information, including the element values of a selected equivalent circuit.
- More information and basics examples are found in the backup slides
- Old, but excellent information on transmission-lines and standing waves:
 - <https://www.youtube.com/watch?v=l9m2w4DgeVk>
 - <https://www.youtube.com/watch?v=DovunOxIY1k&t=38s>
- *Smith* chart education software (only for MS-Windows):
 - <https://www.fritz.dellsperger.net/smith.html>

- **A brief recap on electrical networks**
 - A simplified way to describe electrical, electronics and RF circuits
 - Electrical network composed out of lumped and distributed elements
 - Two-port RC-filter example using admittance (Y) parameters
- **Introduction to scattering (S) parameters**
 - General concept of incident / reflected waves scattered at the ports of an RF network
 - Reference impedance Z_0
 - 1-port and 2-port S-parameters
 - Properties of the S-matrix: reciprocity, symmetry, losses
 - with more examples in the backup slides
 - A few S-matrix examples for RF networks with 1-, 2-, 3-, and 4-ports
 - S-parameters in practice: the SnP Touchstone file format
 - General n -ports

- The electromagnetic behavior of RF circuits and systems, like any other electrical / electronics circuit or system can be described by Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon}, \quad \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{c^2 \partial t} = \mu \mathbf{J}, \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

- These equations need to be solved, taking all the boundaries and materials into account
- **However, this is far too complicated and inconvenient for most practical situations!**
 - **simplified electrical network description based on approximative lumped or distributed elements**
 - With given characteristics and values of each circuit element represented by a symbol in an electrical network, following the laws of *Ohm* and *Kirchhoff*. Here some examples:



- **Complex electrical, electronics and RF systems are divided into functional blocks, i.e., networks, with n -ports, each port has two terminals**
 - The two-port network is most popular, typical examples for two-port networks are filters, attenuators, amplifiers, etc.



Y- parameters

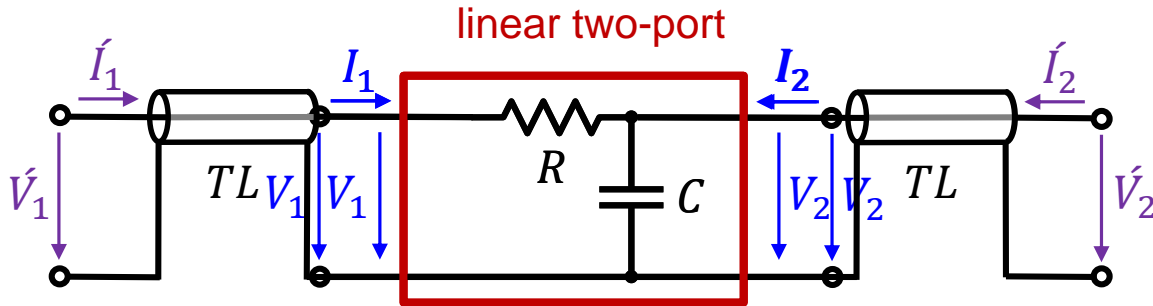
$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} \quad Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

I_1, I_2 : dependent
 V_1, V_2 : independent

- **The characteristic behavior of the n -port network is defined by a set of $2n$ parameters, linked to their ports.**
 - A two-port network has four parameters as port voltages (V_1, V_2) and port currents (I_1, I_2), two are independent, the other two are dependent parameters.
 - Various combinations of dependent and independent port voltages and currents exist, accordingly various $n \times n$ matrix definitions for linear n -port networks exist, known as Y-, Z-, h- and g- parameters.

- It is more convenient to express the parameters for **linear networks in the frequency domain**
 - Avoids solving differential equations! Example: Simple RC two-port network
 - Only for time-invariant and linear networks!



Y- parameters:

$$Y = \frac{1}{R} \begin{bmatrix} 1 & -1 \\ -1 & 1 + j\omega RC \end{bmatrix}$$

Z- parameters:

$$Z = \frac{1}{j\omega C} \begin{bmatrix} 1 + j\omega RC & 1 \\ 1 & 1 \end{bmatrix}$$

- **Voltage/current-based network parameters fail at high frequencies!**
 - Now voltages and currents are a function of frequency (or time) AND space: $V(\omega, z), I(\omega, z)$
 - Originating from time/space varying EM-fields
 - Example: RC two-port network embedded between transmission-lines
 - While it is still possible to solve the network problem, it becomes complicated and cumbersome based on V and I
 - The circuit may become unstable or might be damaged, when operating on a short or open end for characterizing the network parameters
 - Due to parasitic effects, a reliable measurement of V and I becomes almost impossible.
- **Resolution: RF scattering parameters based on power-waves for linear networks which include distributed elements**
 - The magnitude of a traveling wave is independent of the location z in a lossless transmission-line

- **Analogy to optical waves**
 - **Light falls on a car window**
 - Some parts of the incident light is reflected (you see the mirror image)
 - Other parts of the light is transmitted through the window (you can still see objects inside the car)
 - **Optical reflection and transmission coefficients of the window glass define the ratio between reflected and transmitted light.**

- **Similar in RF networks:**
The scattering (S)-parameters of an n -port RF network (DUT) is characterized by incident and reflected / transmitted (power) waves.



- for an arbitrary n -port microwave or RF network are defined by a set of normalized complex voltage waves:

incident wave at port i :

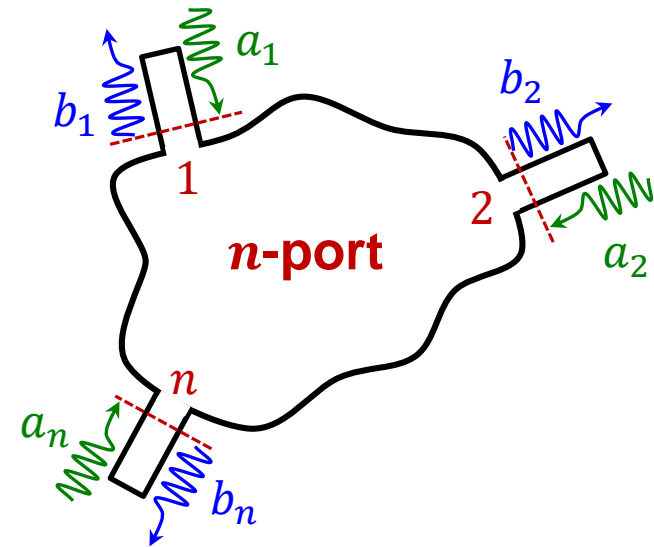
$$a_i = \frac{V_i + Z_i I_i}{2\sqrt{\Re\{Z_i\}}} = \frac{V_i^{inc}}{\sqrt{\Re\{Z_i\}}}$$

reflected wave at port i :

$$b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{\Re\{Z_i\}}} = \frac{V_i^{refl}}{\sqrt{\Re\{Z_i\}}}$$

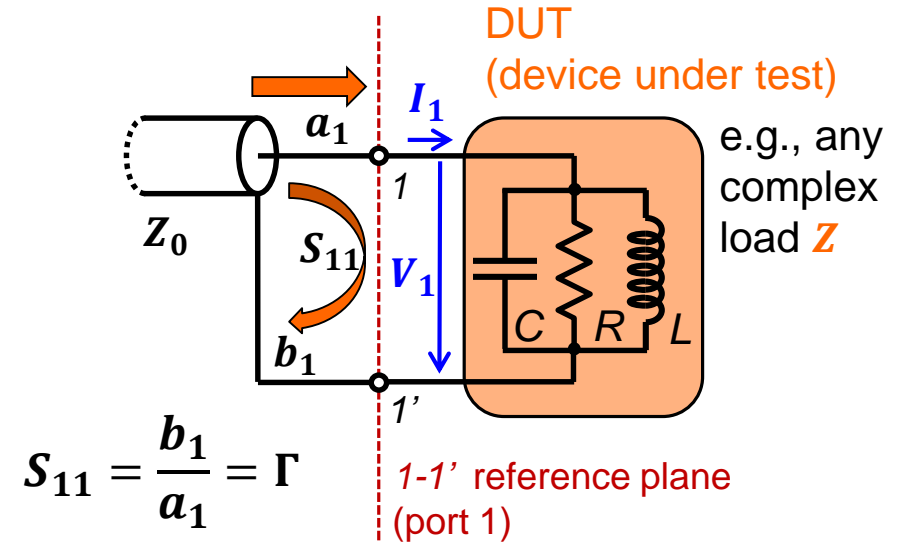
Z_i^* :
conjugate
complex

- as **incident** a_i and **reflected / transmitted** b_i **power waves** at the i^{th} port of the network, defined by the terminal voltage V_i and current I_i , and an arbitrary **reference impedance** Z_i
 - Please note the complex notation implies linear, time-invariant networks describe in the frequency-domain

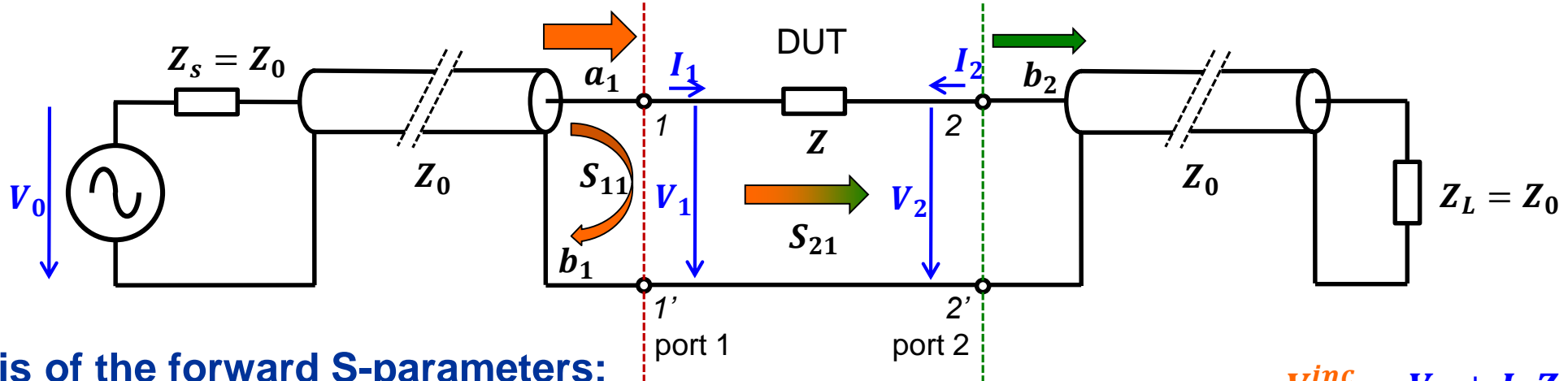


- Today, for most practical cases the **RF network**, also called “**device under test**” (DUT) is characterized by a **vector network analyzer (VNA)**, connected with coaxial cables (transmission-lines) with a **characteristic impedance of $Z_0 = 50 \Omega$** to the ports.
 - Usually the S-parameters are defined for a port reference impedance: $Z_i = Z_0 = 50 \Omega$
 - Some VNAs with a physical reference impedance of $Z_0 = 50 \Omega$ allow a mathematical port impedance conversion to adapt to a port reference impedance $Z_i \neq 50 \Omega$

- **Electrical / electronics networks**
 - 1 ... n -port electronics circuits
 - Defined by **voltages** $V_i(\omega)$ or $v_i(t)$ and **currents** $I_i(\omega)$ or $i_i(t)$ at the port terminals
 - Characterized by circuit matrices, e.g., Z , Y , h , etc.
- **RF / microwave networks**
 - 1 ... n -port RF DUT circuit or subsystem, e.g., filter, amplifier, transmission-line, hybrid, circulator, resonator, etc., which may include distributed elements
 - Defined by **incident** $a_i(\omega)$ and **reflected / transmitted waves** $b_i(\omega)$ at a **reference plane** s (physical position) at the ports.
 - Characterized by a scattering parameter (S-parameter) matrix of the reflected and transmitted power waves, typically as a function of the frequency $f = 2\pi/\omega$
 - Normalized to a **reference impedance** of typically $Z_0 = 50 \Omega$



- 1-port RF network (DUT) example**
- S-Parameters allow to characterize the DUT with the measurement equipment located at some physical distance
 - All high frequency effects of distributed elements are included with respect to the reference plane



• Analysis of the forward S-parameters:

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \equiv \text{input reflection coefficient}$$

$(Z_L = Z_0 \Rightarrow a_2 = 0)$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \equiv \text{forward transmission gain}$$

- 3 +-port networks still can be fully characterized with a 2-port VNA, but always remember:
- **Terminated unused ports in their characteristic impedance!**

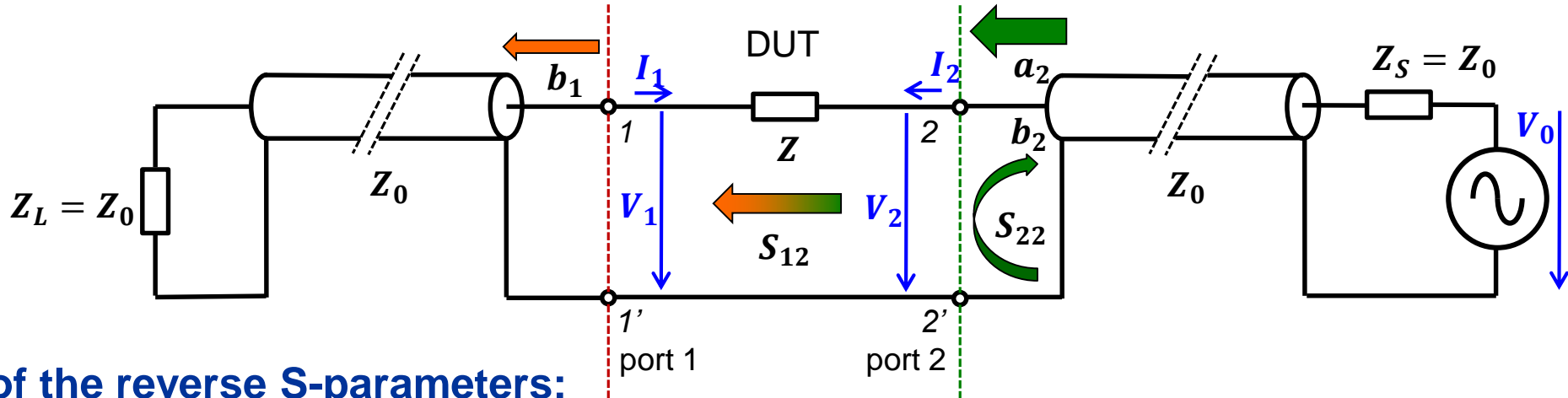
- Independent parameters:

$$a_1 = \frac{V_1^{inc}}{\sqrt{Z_0}} = \frac{V_1 + I_1 Z_0}{2\sqrt{Z_0}}$$

- Dependent parameters:

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$



• Analysis of the reverse S-parameters:

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \equiv \text{output reflection coefficient}$$

$(Z_L = Z_0 \Rightarrow a_1 = 0)$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \equiv \text{reverse transmission gain}$$

- Examples of 2-ports DUT: filters, amplifiers, attenuators, transmission-lines (cables), etc.
- **ALL ports ALWAYS need to be terminated in their characteristic impedance!**

– Independent parameters:

– Dependent parameters:

$$a_2 = \frac{V_2^{inc}}{\sqrt{Z_0}} = \frac{V_2 + I_2 Z_0}{2\sqrt{Z_0}}$$

$$b_1 = \frac{V_1^{refl}}{\sqrt{Z_0}} = \frac{V_1 - I_1 Z_0}{2\sqrt{Z_0}}$$

$$b_2 = \frac{V_2^{refl}}{\sqrt{Z_0}} = \frac{V_2 - I_2 Z_0}{2\sqrt{Z_0}}$$

- Linear equations for a 2-port network (DUT):

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

– with:

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \equiv \text{input reflection coefficient}$$

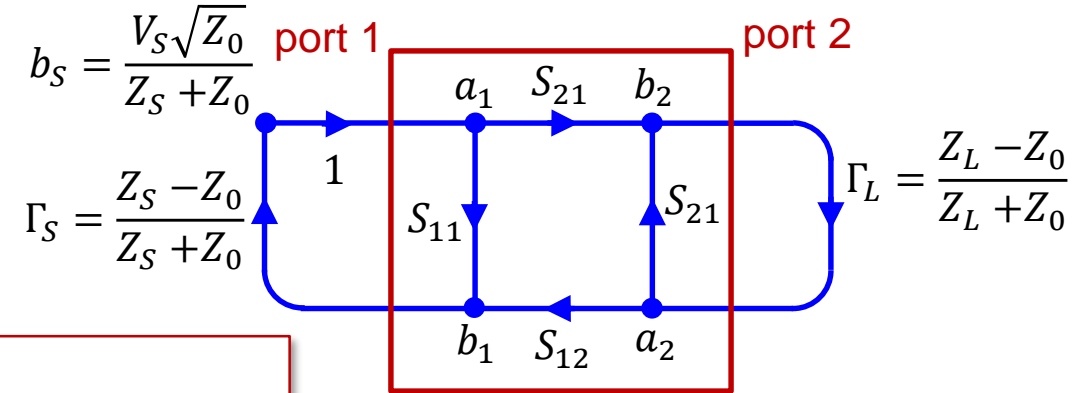
$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \equiv \text{output reflection coefficient}$$

} impedance measurements

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \equiv \text{forward transmission (insertion) gain}$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \equiv \text{reverse transmission (insertion) gain}$$

} transmission (insertion) measurements



signal flow graph (SFG) of a 2-port network

- port 1: V_S with $Z_S \neq Z_0$
- port 2: $Z_L \neq Z_0$

- Reflection coefficient and impedance at the i^{th} -port of a RF network (DUT):

$$S_{ii} = \frac{b_i}{a_i} = \frac{\frac{V_i}{I_i} - Z_0}{\frac{V_i}{I_i} + Z_0} = \frac{Z_i - Z_0}{Z_i + Z_0} = \Gamma_i$$

$$Z_i = Z_0 \frac{1 + S_{ii}}{1 - S_{ii}} \text{ with } Z_i = \frac{V_i}{I_i} \text{ being the input impedance at the } i^{th} \text{ port}$$

- Power reflection and transmission for a n -port network (DUT):

$$|S_{ii}|^2 = \frac{\text{power reflected from port } i}{\text{power incident on port } i}$$

$$|S_{ij}|^2 = \text{transmitted power between ports } i \text{ and } j$$

with all ports terminated in their characteristic impedance Z_0
and $Z_s = Z_0$

Here the US notion is used,
where power = $|a_i|^2$.
European notation (often):
power = $|a_1|^2/2$
These conventions have no
impact on the S-parameters,
they are only relevant for
absolute power calculations

The Scattering Matrix (1)

- Waves traveling towards the n -port: $(\mathbf{a}) = (a_1, a_2, a_2, \dots, a_n)$
- Waves traveling away from the n -port: $(\mathbf{b}) = (b_1, b_2, b_2, \dots, b_n)$
- The relation between a_i and b_i ($i = 1 \dots n$) can be written as a system of n linear equations
(a_i = the independent variable, b_i = the dependent variable)

one-port	$b_1 = S_{11}a_1$	$+ S_{12}a_2$	$+ S_{13}a_3$	$+ S_{14}a_4$	$+ \dots$
two-port	$b_2 = S_{21}a_1$	$+ S_{22}a_2$	$+ S_{23}a_3$	$+ S_{24}a_4$	$+ \dots$
three-port	$b_3 = S_{31}a_1$	$+ S_{32}a_2$	$+ S_{33}a_3$	$+ S_{34}a_4$	$+ \dots$
four-port	$b_4 = S_{41}a_1$	$+ S_{42}a_2$	$+ S_{43}a_3$	$+ S_{44}a_4$	$+ \dots$

– in compact matrix form follows

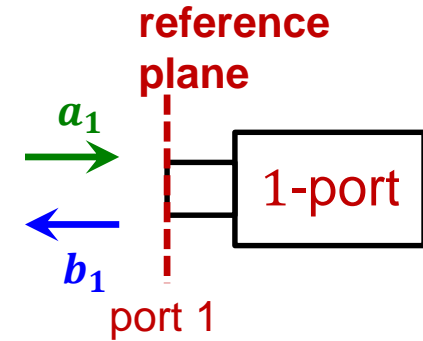
$$(\mathbf{b}) = (\mathbf{S})(\mathbf{a})$$

- Its simplest form is for a passive **1-port network**:

$$(S) = S_{11} \Rightarrow b_1 = S_{11}a_1$$

- with the **reflection coefficient**:

$$\Gamma = S_{11} = \frac{b_1}{a_1}$$

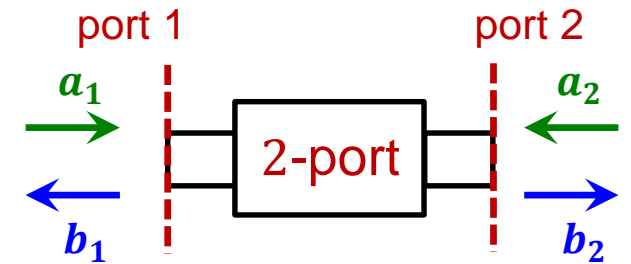


- Most popular is the **2-port network**:

$$(S) = \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix} \Rightarrow \begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

- An unmatched load, present at port 2 with a **reflection coefficient** Γ_{load} transfers to the input port as:

$$\Gamma_{in} = S_{11} + \frac{S_{21}\Gamma_{load}S_{12}}{1 - S_{22}\Gamma_{load}}$$



- **A port of the network is matched if:** $S_{ii} = 0$
 - i.e., no reflections!
- **A n -port is reciprocal if:** $(S)^T = (S) \Rightarrow S_{ij} = S_{ji} \quad \forall i, j$ $(S)^T$: transpose Matrix symmetry
 - **Most passive components are reciprocal, e.g., resistor, capacitor, inductor, transformer, etc.**
 - But not components with inhomogeneous material properties, e.g., magnetized ferrites, plasma, etc.
 - **Active components, like amplifiers are non-reciprocal**
- **A n -port is symmetric if:** $S_{ij} = S_{ji} \wedge S_{ii} = S_{jj}$ Matrix symmetry and electrical symmetry
 - It **needs to be reciprocal**, and input and output reflection coefficient need to be equal.
- **A n -port is passive and lossless if the matrix (S) is unitary:** $(S)^\dagger (S) = (S)^T (S)^* = (I)$ $(S)^\dagger = (S^*)^T$: conjugate transpose
 (I) : identity matrix
 - **Example: passive, lossless 2-port:**

$$(S^*)^T (S) = \begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{aligned} \angle S_{11} - \angle S_{12} &= \angle S_{21} - \angle S_{22} - \pi \\ |S_{11}| &= |S_{22}|, \quad |S_{12}| = |S_{21}| \\ |S_{11}| &= \sqrt{1 - |S_{12}|^2} \end{aligned}$$

- **Examples for 1-port S-matrices are any simple, passive (complex) impedances Z**
 - Any R, L, C, RL, RC, LC and RLC circuit or any combinations of those elements leading to a single port network, which of course also may include distributed (transmission-line) elements
 - “Special” cases are:
 - $Z = Z_0 \Rightarrow S_{11} = 0$ (matched, ideal termination)
 - $Z = 0 \Rightarrow S_{11} = -1$ (ideal short)
 - $Z = \infty \Rightarrow S_{11} = +1$ (ideal open)
 - If $|S_{11}| > 1$ an active element is involved, e.g., a reflection amplifier
- $$(S) = S_{11} = Z$$
- **Strictly speaking, a simple RF resonator, e.g., a “pill-box” cavity, is a 3-port**
 - One coaxial or waveguide port as RF power coupler, plus two beam (waveguide) ports.
 - **However, for many practical cases it can be treated as 1-port**
 - The mode of interest, e.g., TM₀₁₀, is trapped with no or negligible fields contribution near the beam-ports
 - We consider only a single coupler to characterize, e.g., the TM₀₁₀ mode in terms of a 1-port S-parameter measurement
 - Typically applying an RLC-parallel equivalent circuit

- **Ideal (matched: $Z = Z_0$) transmission-line of length ℓ**

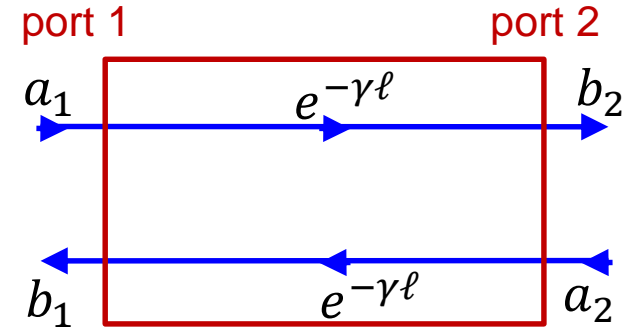
$$(S) = \begin{pmatrix} 0 & e^{-\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{pmatrix}$$

$\gamma = \alpha + j\beta$: propagation constant
 α : attenuation constant in [Np/m]
 $\beta = 2\pi/\lambda$: phase constant [rad/m]

– For a lossless transmission-line: $\alpha = 0 \Rightarrow |S_{21}| = |S_{12}| = 1$

– For a lossless line of length $\ell = \lambda/4$: $(S) = \begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}$

signal flow graph (SFG):



- **Ideal attenuator**

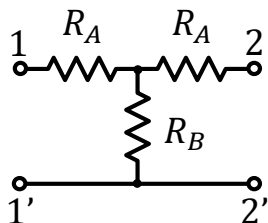
$$(S) = \begin{pmatrix} 0 & k \\ k & 0 \end{pmatrix}$$

$k = V_2/V_1 = 10^{-(\Delta dB/20)}$: attenuation $k < 1, k \in \mathbb{R}$
 $\Delta dB = 20 \log_{10} V_1/V_2$: attenuation in dB
 $\alpha = -\ln k$: attenuation in neper

T-attenuator:

$$R_A = \frac{1-k}{1+k} Z_0$$

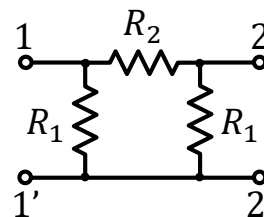
$$R_B = \frac{2k}{1-k^2} Z_0$$



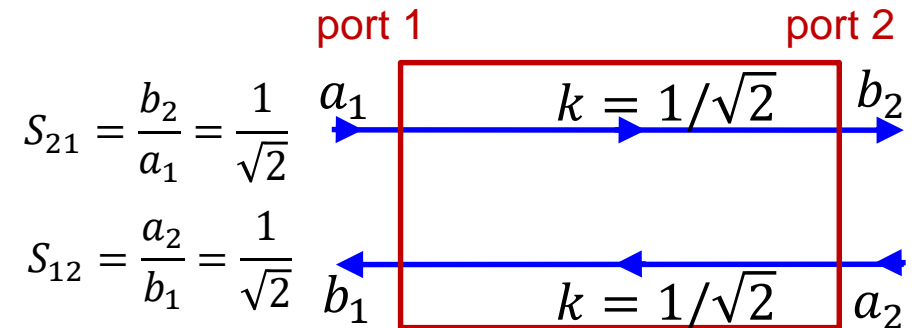
π -attenuator:

$$R_1 = \frac{1-k}{1+k} Z_0$$

$$R_2 = \frac{1-k^2}{2k} Z_0$$



SFG example: 3 dB attenuator



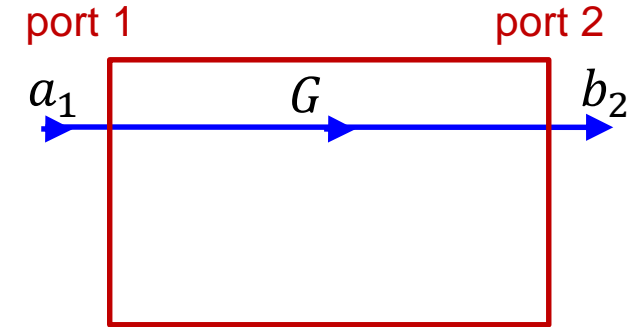
$$S_{21} = \frac{b_2}{a_1} = \frac{1}{\sqrt{2}}$$

$$S_{12} = \frac{a_2}{b_1} = \frac{1}{\sqrt{2}}$$

- **Ideal amplifier (gain stage)**

$$(S) = \begin{pmatrix} 0 & 0 \\ G & 0 \end{pmatrix}$$

$G = V_{out}/V_{in} = 10^{g/20}$: voltage gain $G > 1$
 $g = 20 \log_{10} V_{out}/V_{in}$: voltage gain in dB



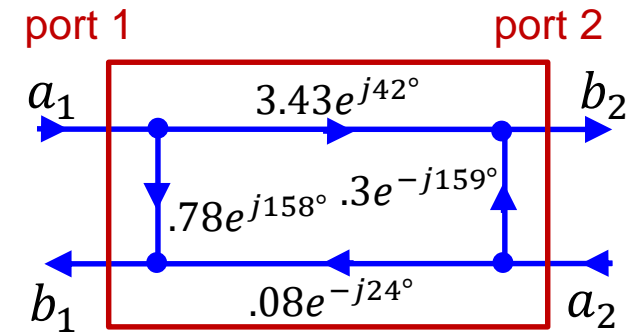
- **Low-noise RF transistor**

$$(S) = \begin{pmatrix} 0.78e^{j158^\circ} & 0.08e^{-j24^\circ} \\ 3.43e^{j42^\circ} & 0.3e^{-j159^\circ} \end{pmatrix}$$

Datasheet Avago VMMK-1218:
 $f = 10 \text{ GHz}$, $Z_0 = 50\Omega$, $T_A = 25^\circ\text{C}$,
 $V_{ds} = 2\text{V}$, $I_{ds} = 20\text{mA}$

- Avago VMMK-1218
- E-pHEMT GaAs FET

- The S-parameters are different at other frequencies and operational conditions
- The transistor requires impedance matching networks at in- and output



- 3-port resistive power divider

$$(S) = \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$b_1 = \frac{1}{2}(a_2 + a_3)$$

$$b_2 = \frac{1}{2}(a_1 + a_3)$$

$$b_3 = \frac{1}{2}(a_1 + a_2)$$

– The transfer-loss between *ij*-ports is 6 dB.

- Ideal circulator

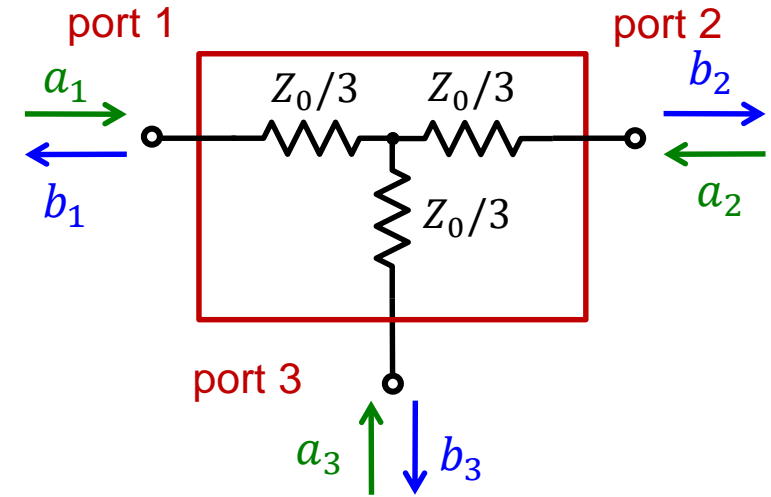
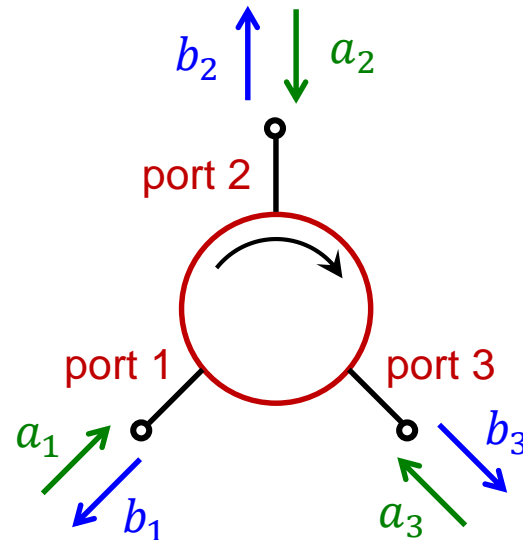
$$(S) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b_1 = a_3$$

$$b_2 = a_1$$

$$b_3 = a_2$$

– Matched, but not reciprocal

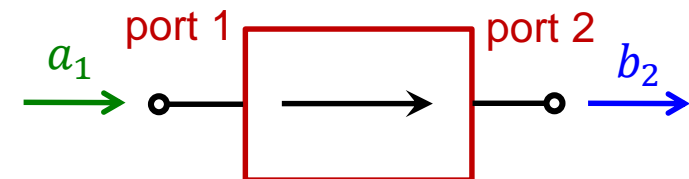


- Isolator, based on the circulator

– Terminating, e.g., port 3 internally results in a 2-port, called **isolator**

$$(S) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

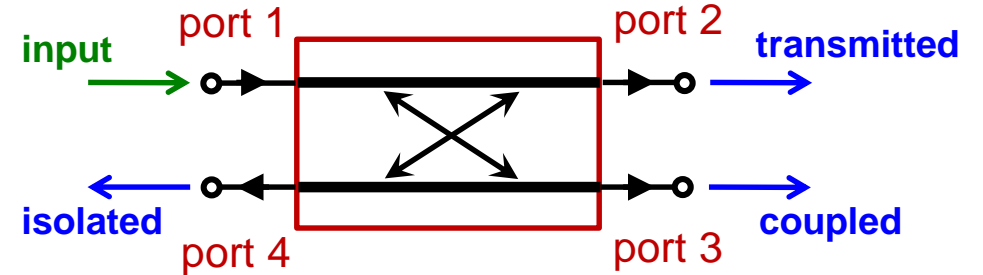
$$b_2 = a_1$$



- **Ideal directional coupler**

coupling coefficient: $k = \left| \frac{b_2}{a_1} \right|$; $\kappa dB = 20 \log_{10} k$

$$(S) = \begin{pmatrix} 0 & -j\sqrt{1-k^2} & k & 0 \\ -j\sqrt{1-k^2} & 0 & 0 & k \\ k & 0 & 0 & -j\sqrt{1-k^2} \\ 0 & k & -j\sqrt{1-k^2} & 0 \end{pmatrix}$$



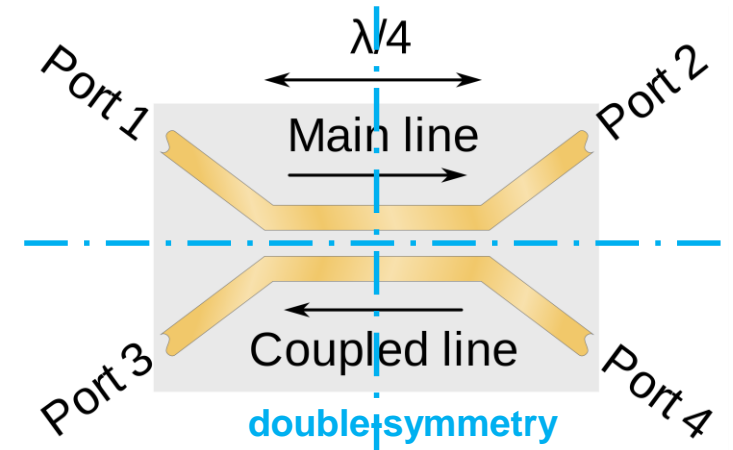
- **Operating at the center frequency**

- **Figures of merit (ideal, lossless):**

- **Coupling factor** $C_{3,1} = \kappa dB = 10 \log_{10}(P_3/P_1)$ [dB]
 - **Insertion loss** $L_{i2,1} = -10 \log_{10}(P_2/P_1)$ [dB]
 - **Coupling loss** $L_{c2,1} = -10 \log_{10}[1 - (P_3/P_1)]$ [dB]
- } no losses:
 $L_{i2,1} = L_{c2,1}$

- **Coupler with losses, imperfections, etc.**

- **Isolation** $I_{4,1} = -10 \log_{10}(P_4/P_1)$ [dB]; $I_{3,2} = -10 \log_{10}(P_3/P_2)$ [dB]
- **Directivity** $D_{3,4} = -10 \log_{10}(P_3/P_4) = -10 \log_{10}(P_4/P_1) + 10 \log_{10}(P_3/P_1)$ [dB]



https://en.wikipedia.org/wiki/Power_dividers_and_directional_couplers

- In practice, **S-parameters are a function of the frequency: $S(f)$**
 - Some instruments or applications can also provide time-domain S-parameters
- In most real-world practical situations, S-parameters are acquired by a measurement, e.g., characterization of a RF component or sub-system by a VNA.
 - By characterizing the DUT over a range of frequencies, $f_{min} < f < f_{max}$ in steps of Δf
- Also, numerical RF analysis tools (Qucs, ADS, Microwave Office, etc.) generate S-parameters through linear RF circuit / systems simulations.
 - Numerical EM software tools (CST, HFSS, etc.) and PCB tools (Cadence Allegro) can also generate S-parameters
- Both application types, VNA measurements and RF/EM simulation software exchange S-parameters on a file basis
 - The *SnP Touchstone* ASCII file format is de-facto the industry standard for S-parameters
 - Example *Touchstone s2p* file:


```

BRSTM_refline_2-4GHz_20001.s2p
!Keysight Technologies,P5024A,MY58100247,A.15.20.07
!Date: Wednesday, October 06, 2021 16:19:17
!Correction: S11(C 2-Port )
!S21(C 2-Port )
!S12(C 2-Port )
!S22(C 2-Port )
!S2P File: Measurements: S11, S21, S12, S22:
# Hz S dB R 50 # format
2000000000 -2.5430779 -88.497566 -18.274168 38.763039 -18.26178 38.742687 -2.4251425 -85.792152
2000100000 -2.5365531 -88.473915 -18.266272 38.606499 -18.269154 38.716461 -2.4215624 -85.861908
2000200000 -2.5314419 -88.471634 -18.280306 38.559021 -18.258684 38.624985 -2.4253747 -85.806236
2000300000 -2.5216722 -88.617905 -18.269596 38.352692 -18.266785 38.342678 -2.4210978 -85.8368
2000400000 -2.5178108 -88.521271 -18.257862 38.298622 -18.275055 38.266792 -2.4244151 -85.914787
2000500000 -2.5327342 -88.484985 -18.263821 38.31945 -18.27046 38.20409 -2.4263382 -85.85775
2000600000 -2.5191193 -88.462044 -18.262426 38.195797 -18.246525 38.250874 -2.3989511 -85.885635
2000700000 -2.5219827 -88.44445 -18.253748 38.216946 -18.245417 38.138355 -2.4155877 -85.859711
2000800000 -2.5198817 -88.588783 -18.255999 37.902744 -18.257757 38.035915 -2.415241 -85.887199
2000900000 -2.5370498 -88.496101 -18.256392 38.004234 -18.258446 37.872906 -2.4170656 -85.854294
2001000000 -2.5363033 -88.544846 -18.252661 37.830475 -18.259445 37.925297 -2.4102871 -85.901245
2001100000 -2.5417418 -88.47406 -18.270754 37.731613 -18.252403 37.74762 -2.4158244 -85.822517
  
```

2-port VNA file

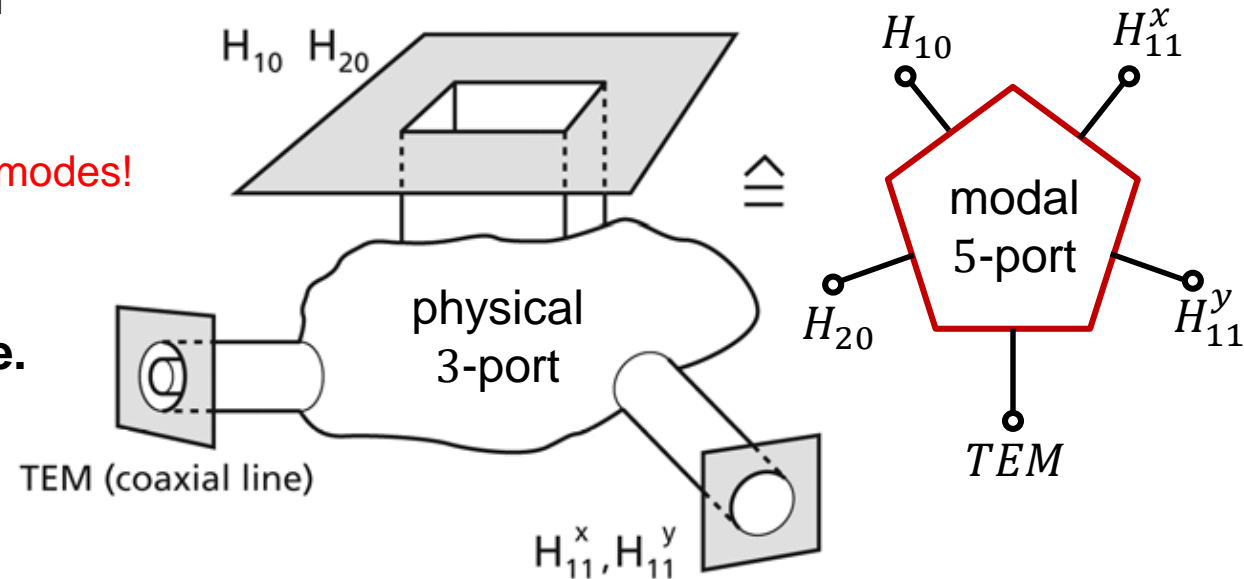
Touchstone v1.1 example file

- v2.0 is different, file ext. *.ts

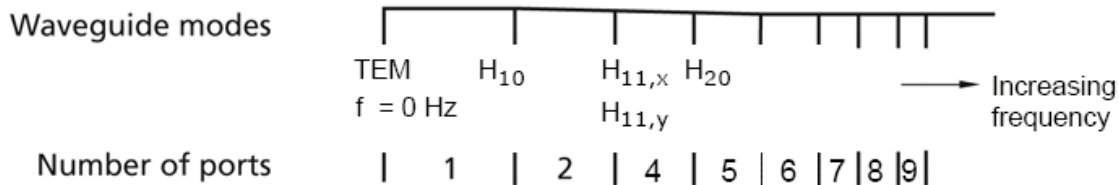
frequency f $|S_{11}|$ [dB] $\angle S_{11}$ [deg]

- The file name extension specifies the number n of ports
 - Attention: NOT equal to the number of columns! The carriage return (CR) is different between s1p, s2p and s3p, s4p files!
- The comment header (!) includes general information, e.g., type of instrument, measurement time, etc.
- The format line (#) defines the format (mag[dB],angle[deg], mag/angle, real/imag), stimulus units and the reference impedance
- The column delimiter varies, e.g., space, comma, semicolon, etc.
 - Column order in the example file: f $S11dB$ $S11a$ $S21dB$ $S21a$ $S12dB$ $S12a$ $S22dB$ $S22a$

- A general n -port may include ports of different technologies, i.e., waveguides, as well as TEM transmission-lines, such as coaxial lines, microstrip lines etc.
 - In the frequency range of interest different modes may propagate at each physical port, e.g., several waveguide modes in a rectangular waveguide and/or higher order modes in a coaxial line..
 - Each EM-mode must then be represented by a distinct modal port.
 - This is very important in EM-simulation to ensure the absorption of the energy for all modes!
 - The number of modal ports needed generally, increases with frequency, as more waveguide modes can propagate.



H_{11}^x, H_{11}^y : x, y -polarization of the E_{11} circular mode



- The scattering (**S**) parameters are based on incident and reflected normalized complex voltage waves (power waves), defined at the ports of a RF network.
- **S-Parameters** are used to characterize a linear, time-invariant RF component, circuit or sub-system as function of frequency under realistic operational conditions
 - The **S-parameters** are given in a matrix notation, and have complex values
- The characteristic of the **S-matrix** may provide additional details about the network, such as reciprocity, symmetry, losses.
- Typically, the **S-parameters** matrix of a RF network is acquired by measurement characterization with a vector network analyzer (VNA), or by a numerical analysis, e.g., circuit analysis or electromagnetic simulation software
- The **S-parameter** matrices of a set of networks can be converted to transfer (**T**) parameter matrices to enable a simple cascading of those networks
- The number of logical, modal ports might be higher than the number of physical ports for a general RF network utilizing various transmission-line technologies.

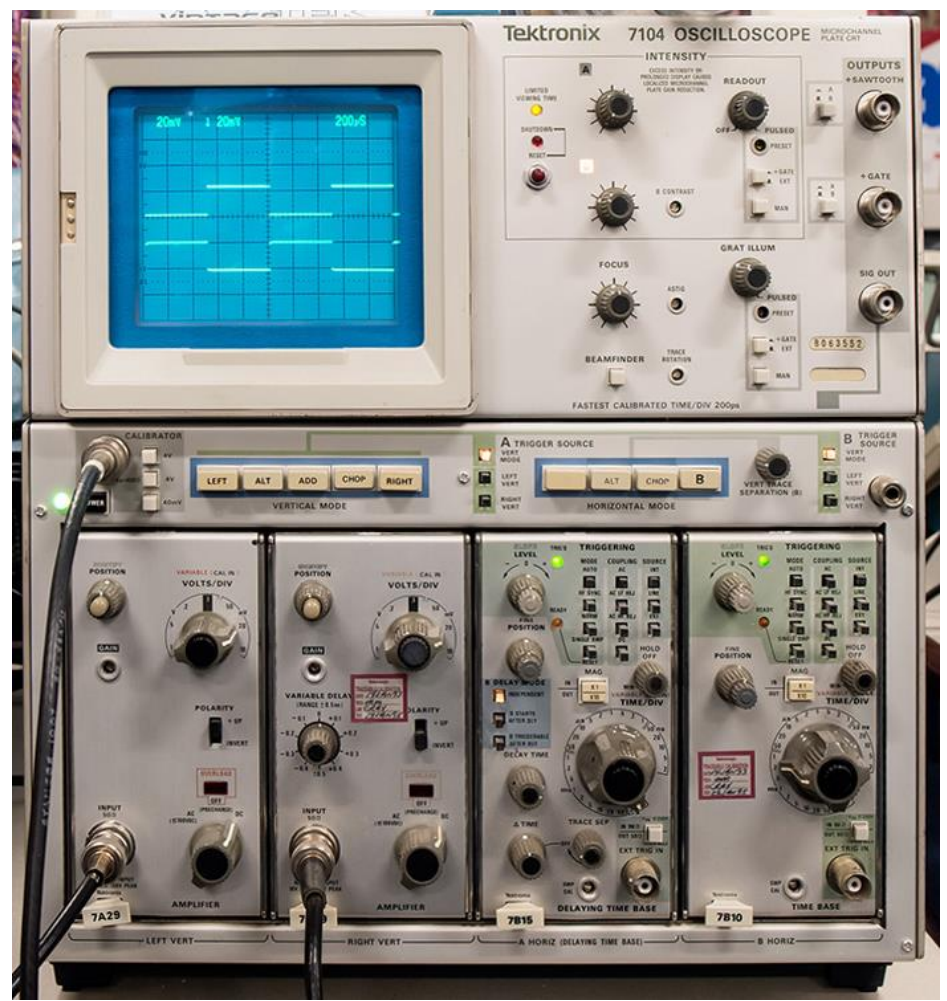
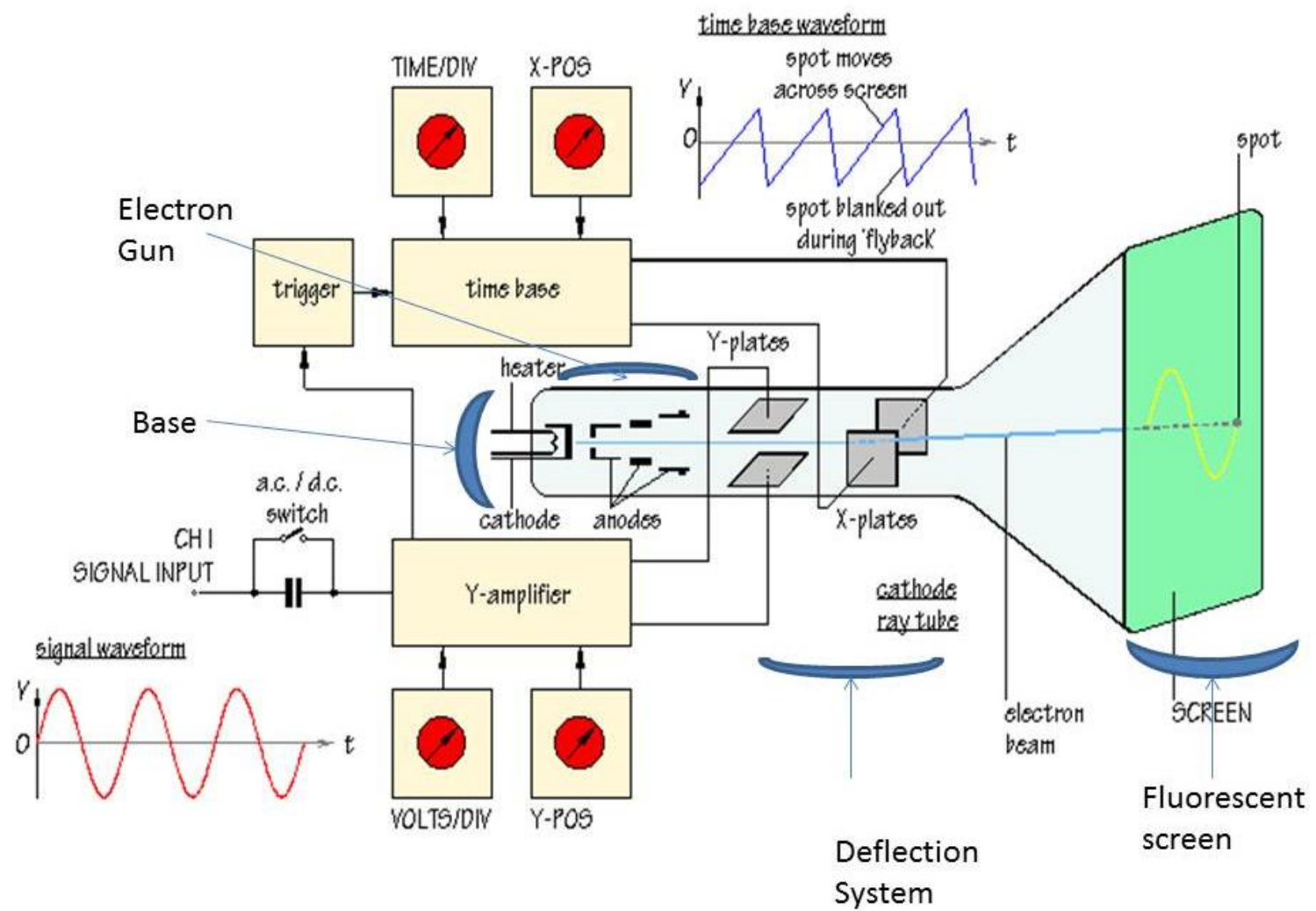
- **Overview of RF measurement instruments**
 - Oscilloscope, spectrum analyzer (SA), signal (FFT) analyzer, slotted measurement line, vector network analyzer (VNA)
- **The super-heterodyne receiver principle**
 - Modulation, down-conversion, mixer, spectrum analyzer block schematics
- **Reflection measurement with the slotted coaxial air-line**
- **S-parameter measurements**
 - Simple measurement setup, VNA block schematics
 - VNA calibration
 - Features of modern RF measurement equipment
 - Synthetic pulse measurements with the VNA
 - **Measurement example: pillbox resonator characterization**
 - Equivalent circuit parameters, Q -factor measurement in the *Smith*-chart, R/Q measurement
 - **Measurement of the beam-coupling impedance with a stretched-wire**

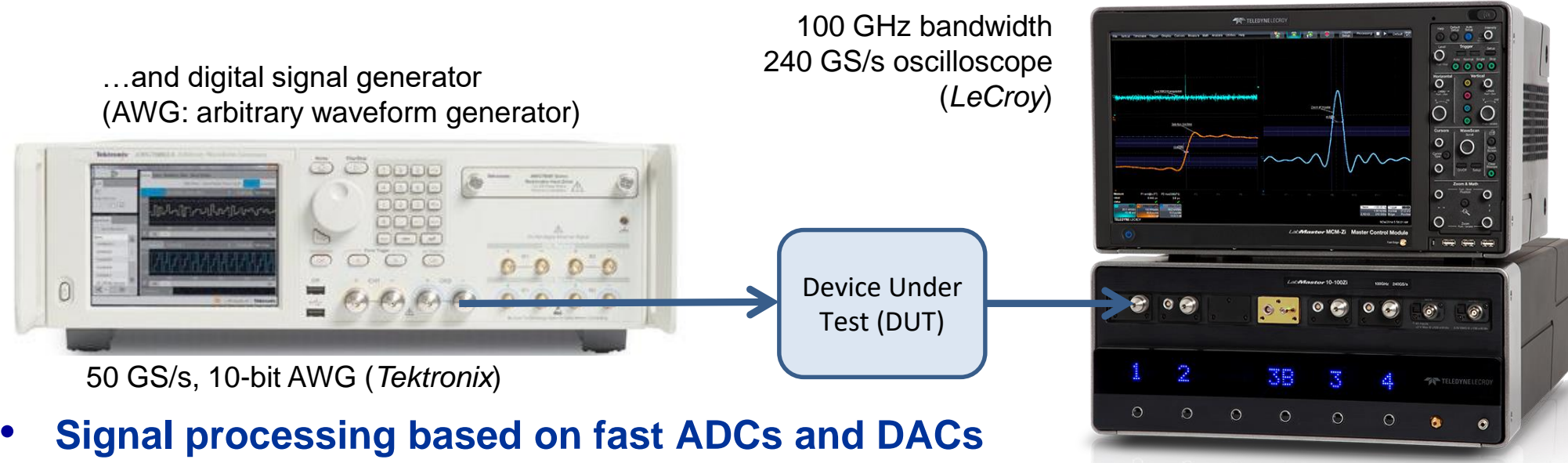
There are different options to observe RF signals

Here some typical measurement tools:

- **Oscilloscope:** to observe signals in **time-domain**
 - periodic signals
 - burst and transient signals with arbitrary waveforms
 - application: direct observation of signals from a beam pick-up, from a test generator, or from other sources
 - visualizes the shape of a waveform, etc.
 - limited performance for the evaluation of non-linear effects.

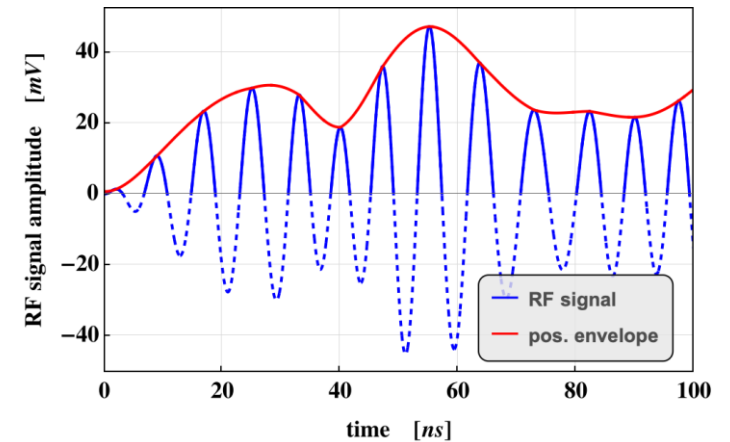
Cathode Ray Tube (CRT) Oscilloscope

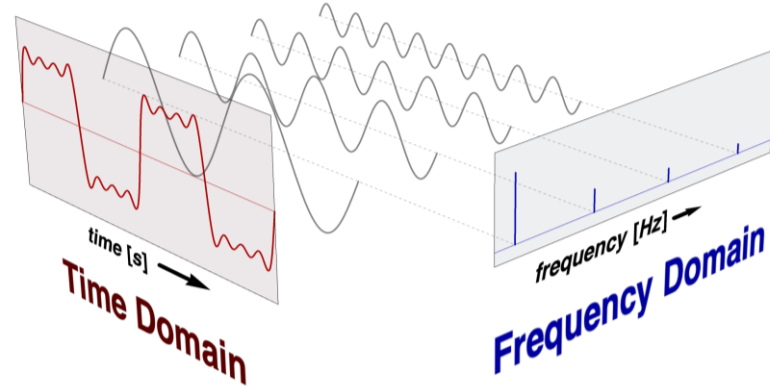




- **Signal processing based on fast ADCs and DACs**
 - **Similar “look and feel” as analog oscilloscopes, but better performance**
 - 8...12-bit multi-GS/s ADCs, still, be aware of aliasing effects!
 - Fast sampling oscilloscope require sufficient memory resources.
- **AWG or pulse generator & digital oscilloscope:**
Time-domain (TD) test setup
 - **Device under test (DUT) characterization and trouble shooting**
 - Impulse, step, or arbitrary waveform (e.g., beam signal) as stimulus signal
 - High impedance probe for measurements on the printed circuit board (PCB)

- **RF detection (Schottky) diode (RF power meter)**
 - Supplies a rectified (video) output signal proportional
 - to the RF signal level
 - Delivers no frequency or phase information but operates over a very broad frequency range few MHz to many GHz, and up to ~60 dB dynamic range.
- **Spectrum analyzer: to observe signals in a “frequency-domain like” fashion**
 - sweeps in equidistant steps through a given frequency range
 - application: observation of spectrum from the beam, or from a signal generator or RF source, or the spectrum emitted from an antenna to locate EMI issues in the accelerator tunnel, etc.
 - Also, DUT characterization in the laboratory, e.g., noise figure measurement on amplifiers (requires a noise source), intermodulation measurements on amplifiers (requires two RF generators).
 - Requires periodic signals
 - Assumes **time-invariance** of the measurement object (DUT) throughout the frequency sweep
 - **Large dynamic range!**



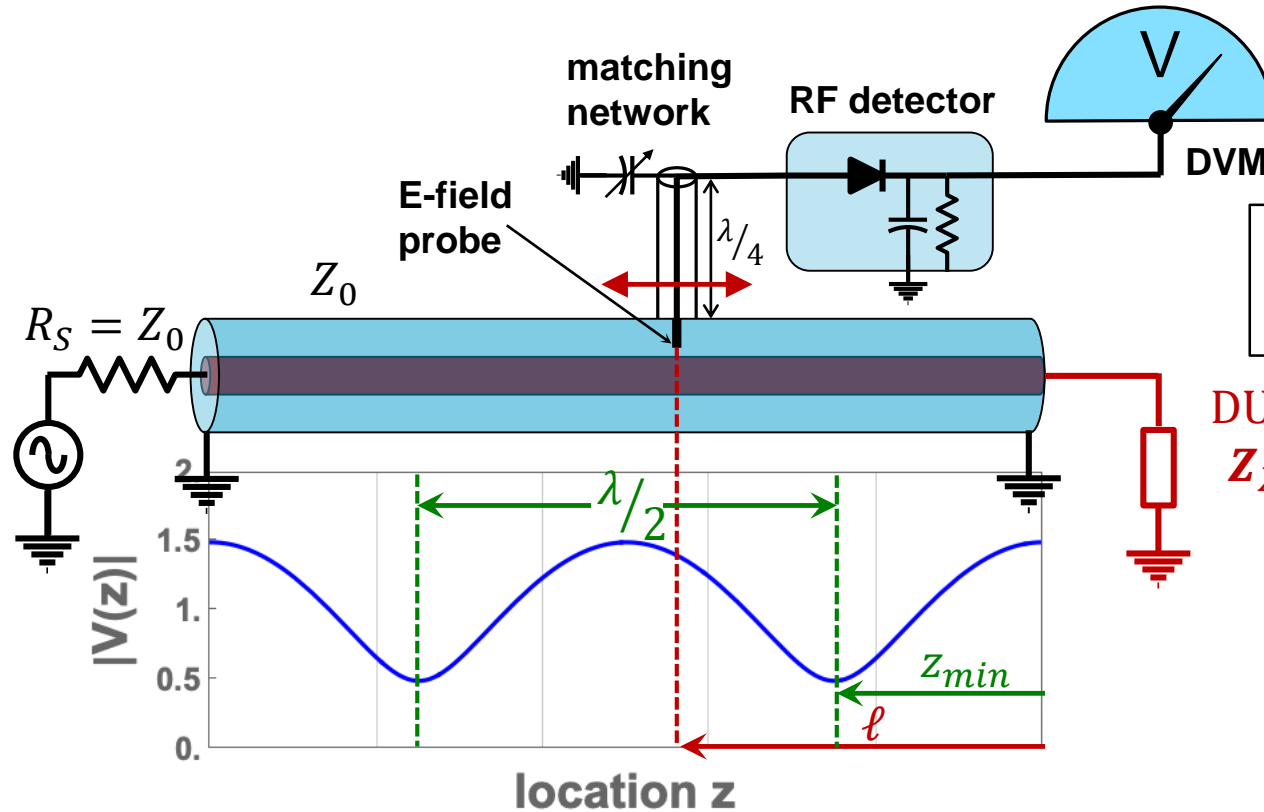
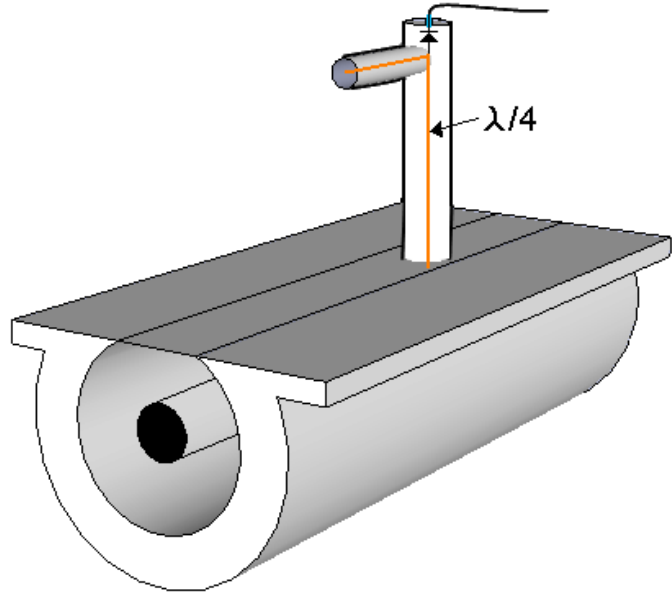


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 - Requires periodic signals
 - Assumes **time-invariance** of the measurement object (DUT) throughout the frequency sweep
 - **Large dynamic range!**

- **Vector signal analyzer (VSA), sometimes called FFT analyzer**
 - Acquires the RF signal, after down-conversion to an intermediate (IF) signal, in time-domain by fast sampling
 - Further numerical treatment in digital signal processors (DSPs)
 - Spectrum calculated using Fast Fourier Transform (FFT)
 - Combines **features of an oscilloscope and a spectrum analyzer**: Signals can be observed directly in time-domain, or in a frequency-domain like fashion
 - Contrary to the SA, also the spectrum of non-periodic signals and transients can be measured
 - Application: Observation of tune sidebands, transient behavior of a phase locked loop, single pass beam signal spectrum, etc.
 - **Digital oscilloscopes** and **FFT analyzers** share similar technologies, i.e., fast sampling and digital signal processing, and therefore can provide similar measurement options
 - The digital oscilloscope directly digitizes the RF signal
 - limited dynamic range, large instantaneous bandwidth
 - The FFT analyzer digitizes the down-converted IF signal
 - large dynamic range, but (still) limited instantaneous bandwidth

Tools to characterize RF components and sub-systems:

- **Slotted coaxial (or waveguide) measurement transmission-line**
 - For study and illustration purposes only – not anymore used in today's RF laboratory environment.
- **Vector Network Analyzer (VNA)**
 - Combines the functions of a vector spectrum analyzer (FFT analyzer), a RF sweep generator, and a S-parameter test set (directional coupler)
 - Excites a *Device Under Test* (DUT, e.g., circuit, antenna, amplifier, etc.) network at a given sinusoidal *continuous wave* (CW) frequency, and measures the response in magnitude and phase => **determines the S-parameters**
 - Covers a selectable frequency range by measuring step-by-step at subsequent frequency points (like a spectrum analyzer, again requires the DUT to be time-invariant!)
 - Applications: characterization of passive and active RF components, *Time Domain Reflectometry* (TDR) by Fourier transformation of the reflection response, etc.
 - Also, power sweep measurements (1 dB compression point), 4-port VNAs enable virtual ports: e.g., single-ended / differential port DUT characterization.
 - **The VNA is the most versatile and comprehensive tool in the RF laboratory!**



$$VSWR = \frac{|V_{max}|}{|V_{min}|}$$

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

$$\angle \Gamma = \frac{4\pi}{\lambda} Z_{min} - \pi$$

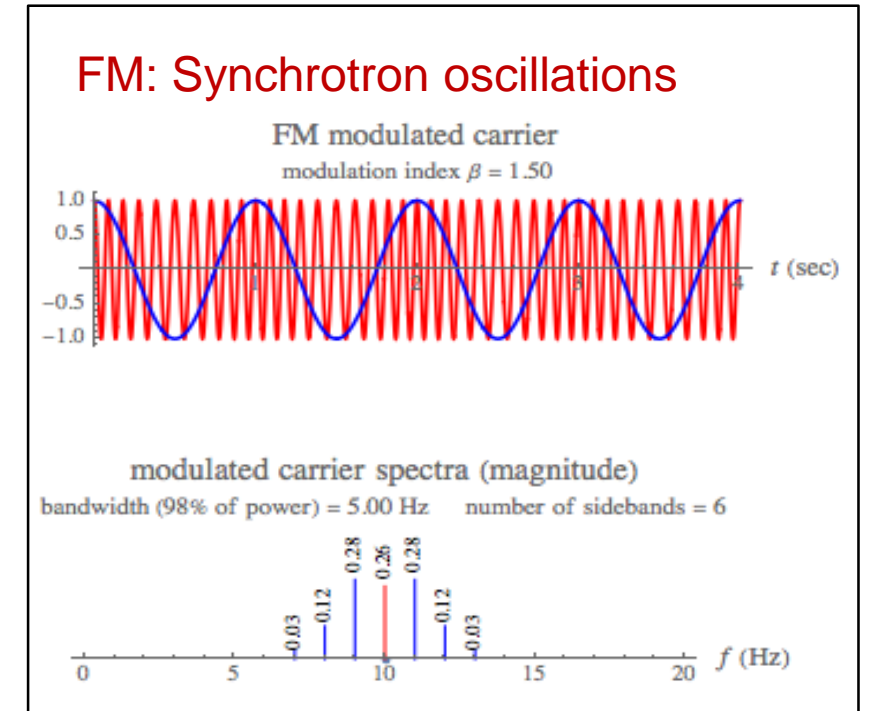
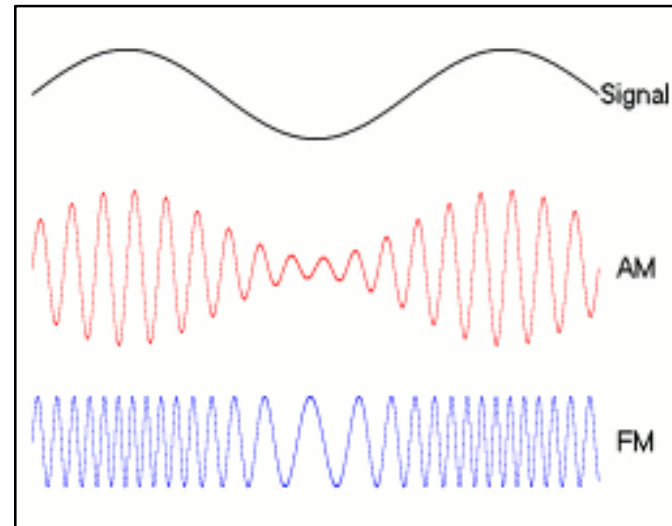
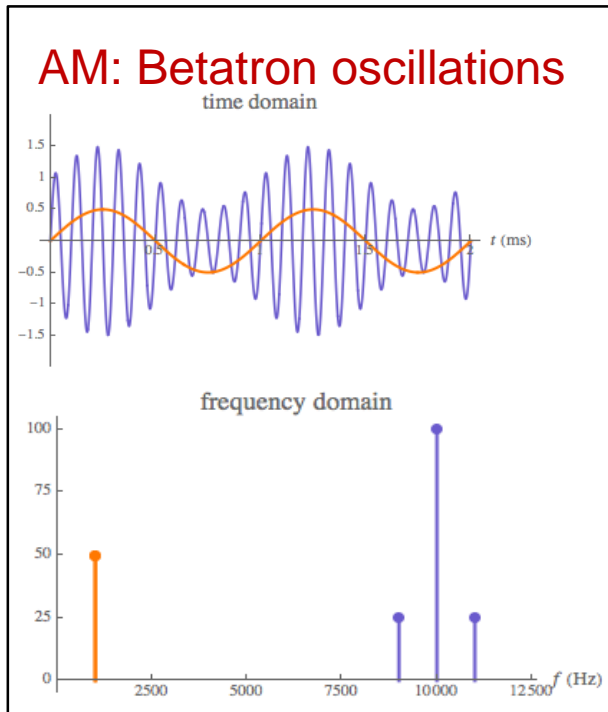
$$Z_x = Z_0 \frac{1 - \Gamma}{1 + \Gamma}$$

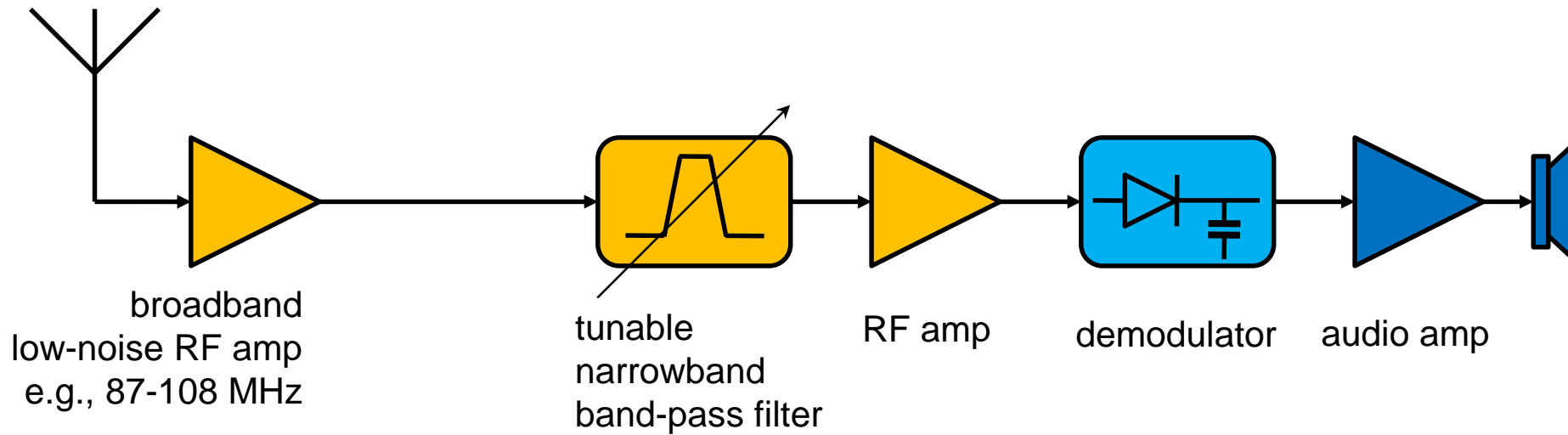
- Slotted coaxial air-line is used as standing wave detector**

- Probes the radial **electric field** along the slotted line.
- Measurement of E-field **minima's** E_{min} and **maxima's** E_{max} with a diode detector, thus detect $|V_{min}|$ and $|V_{max}|$ along the line.
- Evaluate the **reflection coefficient** Γ of a **DUT of unknown** Z_X at the end of the line

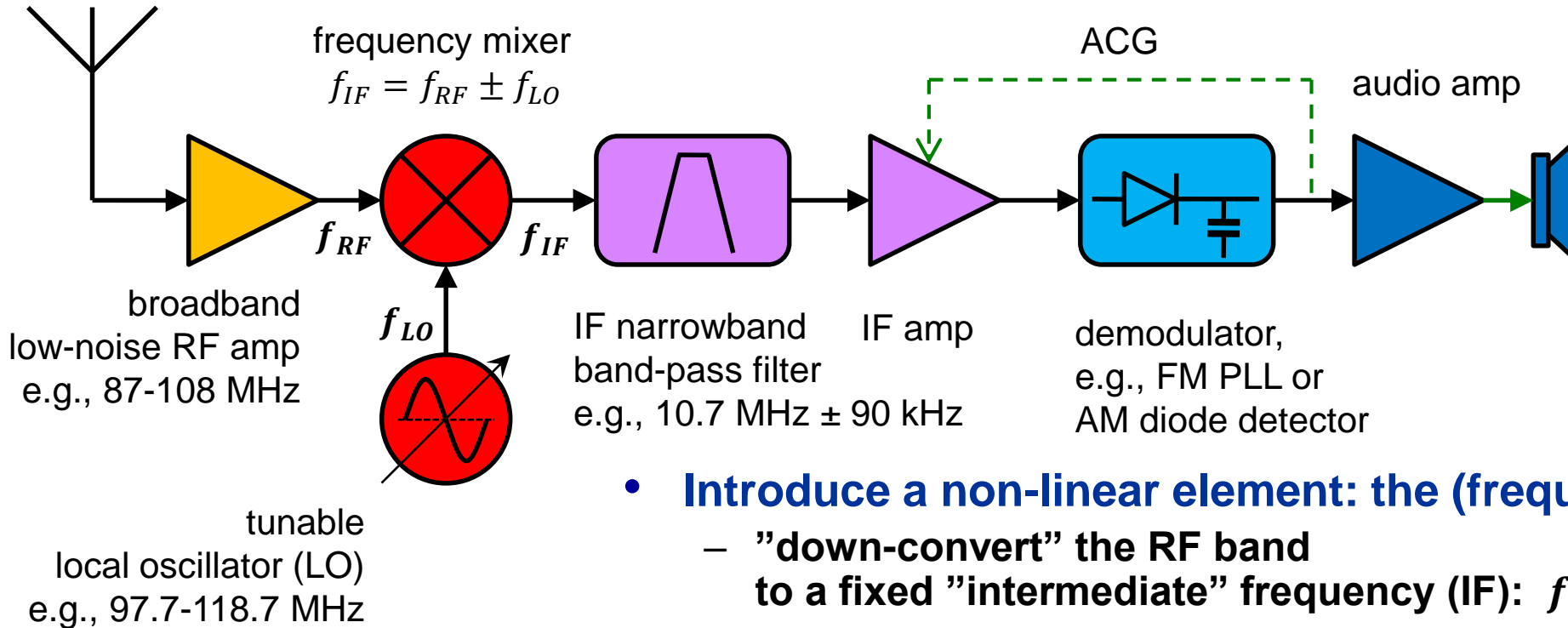
RF Signals & Modulation, without Math!

- RF signals are continuous wave (CW), sinusoidal signals
 - Often, a high frequency carrier is **modulated** with low frequency information
 - Modulation appears “naturally” in ring accelerators as:
 - Modulation is also provided through the LLRF system to the accelerating structures

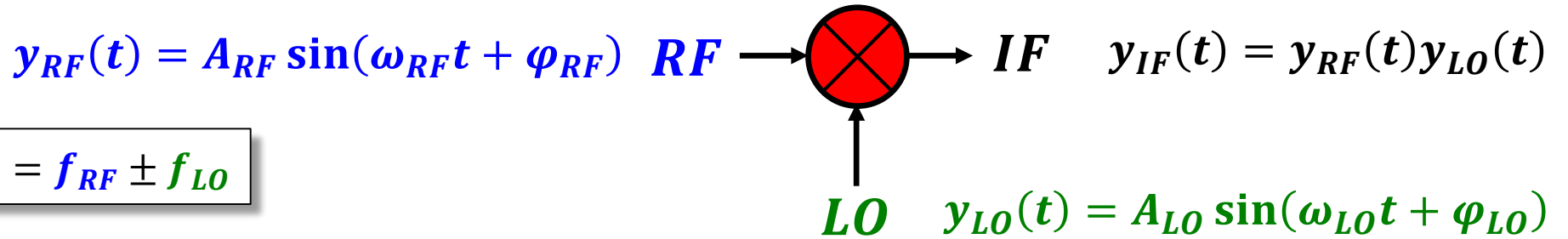




- **...or: How does a "traditional" analog radio works?**
 - It was, and still is, difficult to make precisely tunable narrowband, band-pass filters for high frequencies (~100 MHz)!!
 - high frequency low-noise amplifiers are expensive!
 - high frequency demodulators are not trivial.
 - **direct detection of radio and RF signals is challenging!**



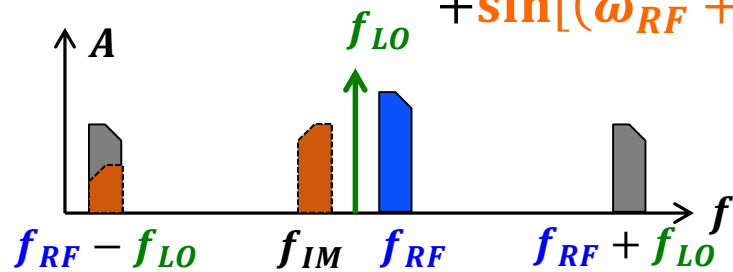
- **Introduce a non-linear element: the (frequency) mixer!**
 - "down-convert" the RF band to a fixed "intermediate" frequency (IF): $f_{IF} = f_{RF} \pm f_{LO}$
 - requires a tunable local oscillator (LO)
 - well manageable IF section:
 - narrowband band-pass filter(s) (BPF) and amplifier(s)
 - RF telecommunication standard
 - Often multiple mixing stages are used in modern RF instruments, e.g., spectrum and network analyzers



- Ideal mixer:

$$f_{IF} = f_{RF} \pm f_{LO}$$

$$y_{IF}(t) = \frac{1}{2} A_{LO} A_{RF} \left\{ \sin[(\omega_{RF} - \omega_{LO})t + (\varphi_{RF} - \varphi_{LO})] \text{ upper sideband} \right. \\ \left. + \sin[(\omega_{RF} + \omega_{LO})t + (\varphi_{RF} + \varphi_{LO})] \text{ lower sideband (also called "base-band")} \right\}$$

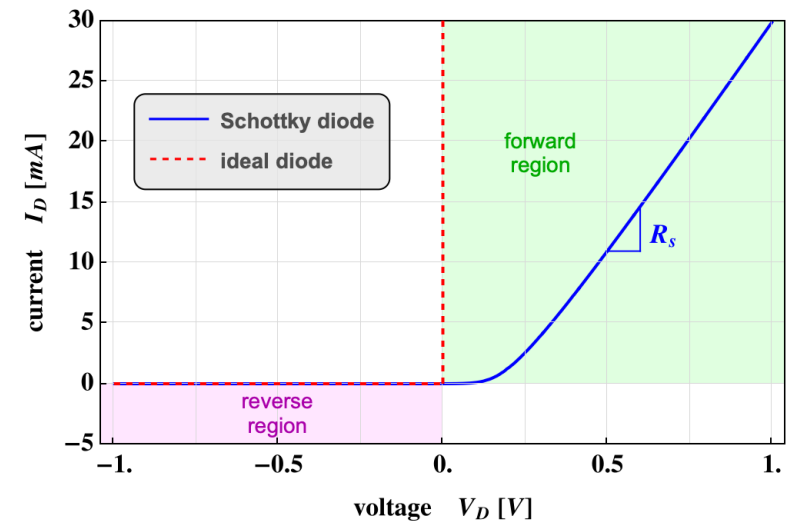


- The mixer is based on non-linear circuit elements

– e.g., the diode

$$I_D = I_S \left(e^{\frac{V_D - I_D R_S}{nV_T}} - 1 \right)$$

➤ popular is the double-balanced mixer



- Frequency conversion

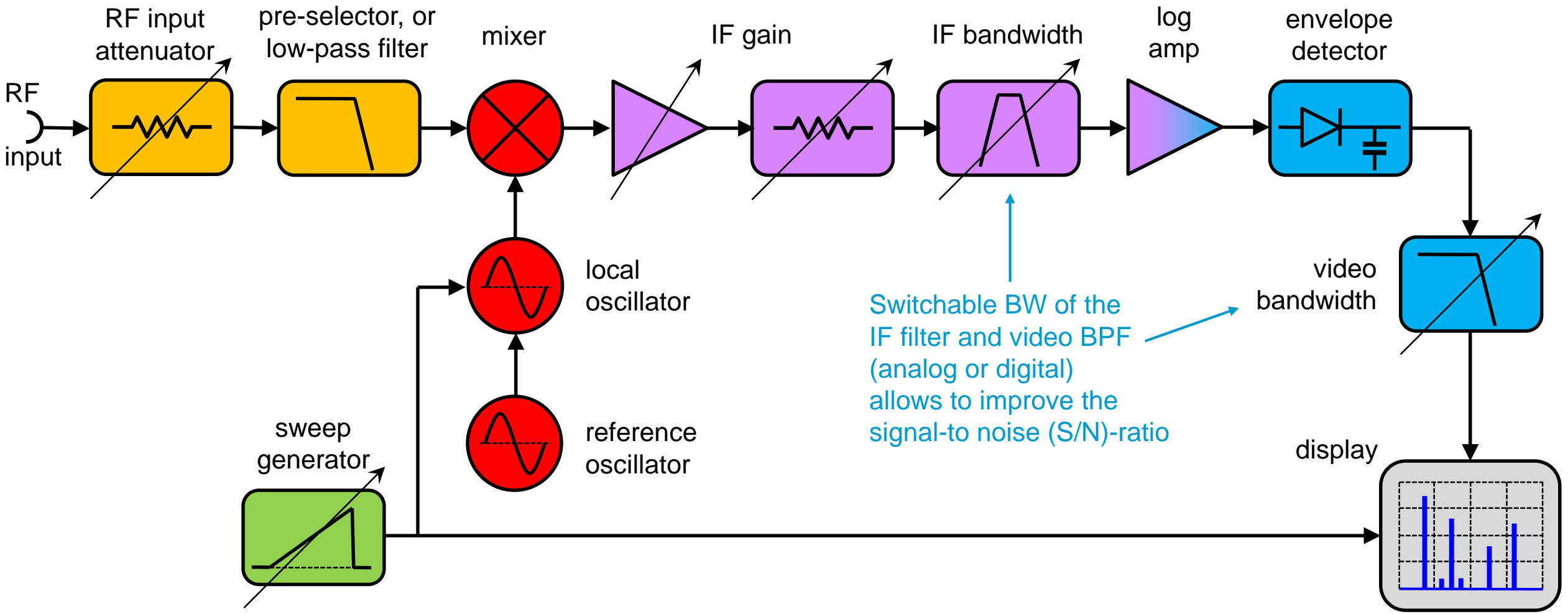
- $f_{RF} \neq f_{LO}$: heterodyne receiver
- $f_{RF} = f_{LO}$: homodyne, demodulator

- Real-world mixer: $f_{IF} = m f_{RF} \pm n f_{LO}$

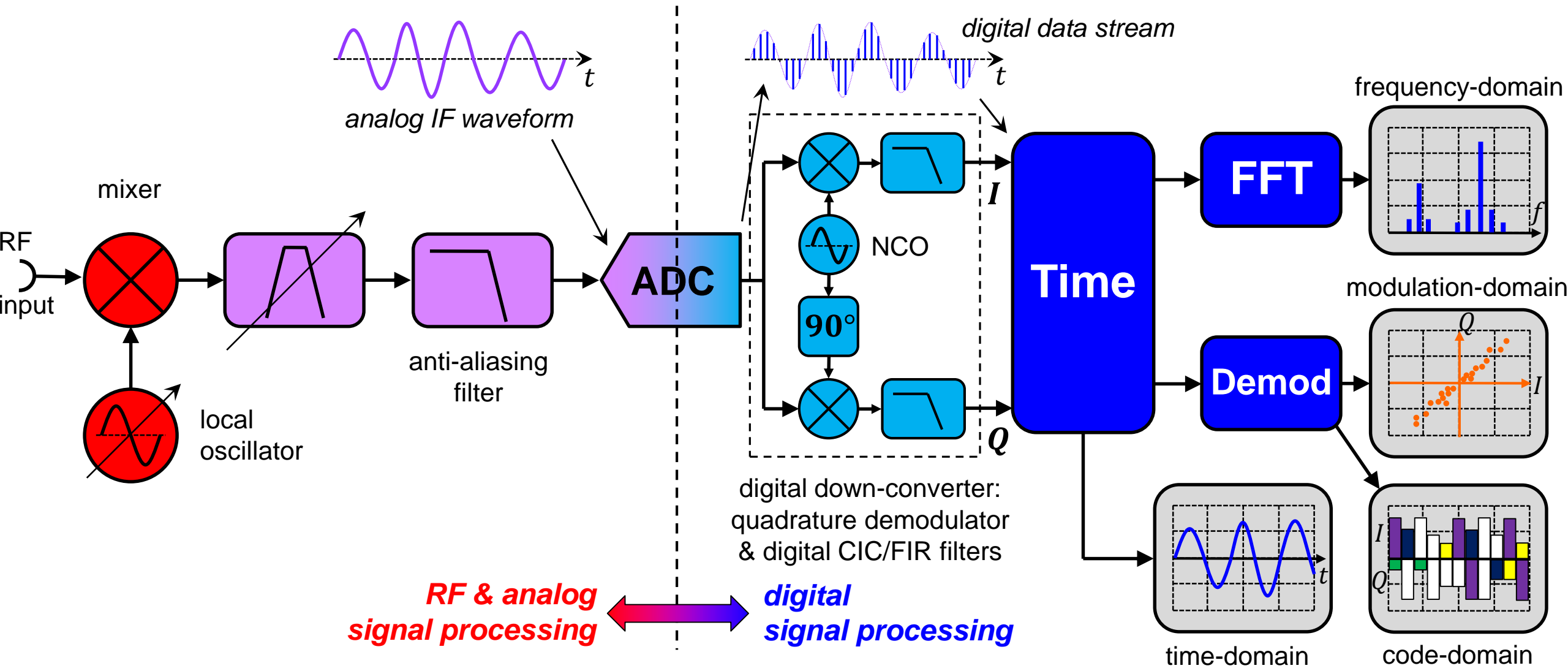
- Image frequency: $f_{IM} = f_{LO} - f_{IF}$

Simplified Spectrum Analyzer

- based on the super-heterodyne principle



Vector Signal Analyzer (VSA) or FFT Analyzer

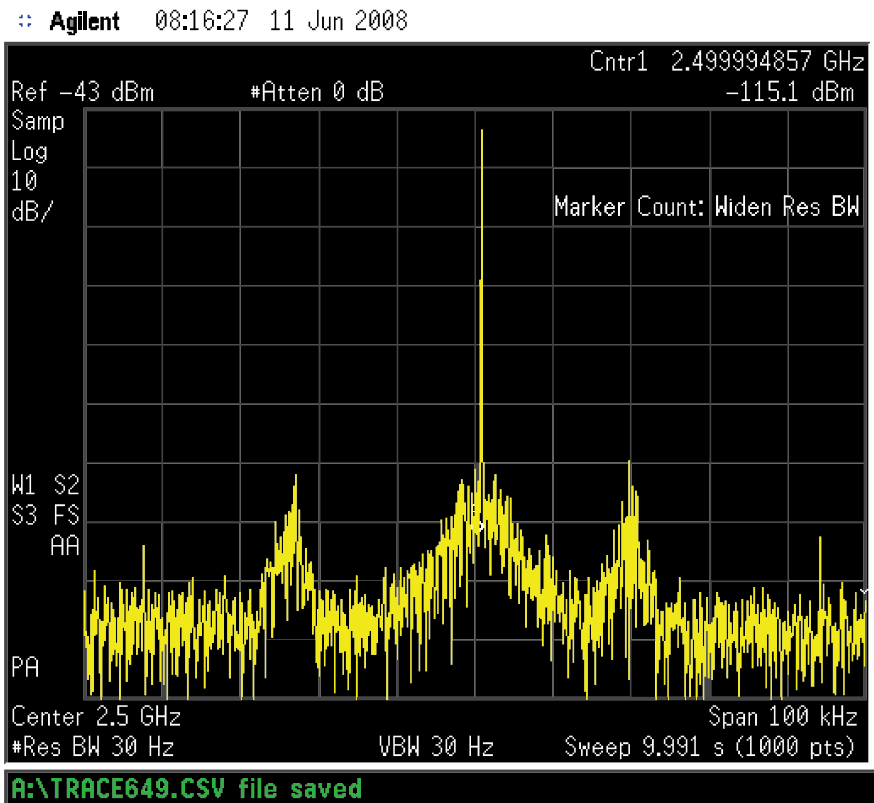


Modern Spectrum (RF Signal) Analyzer

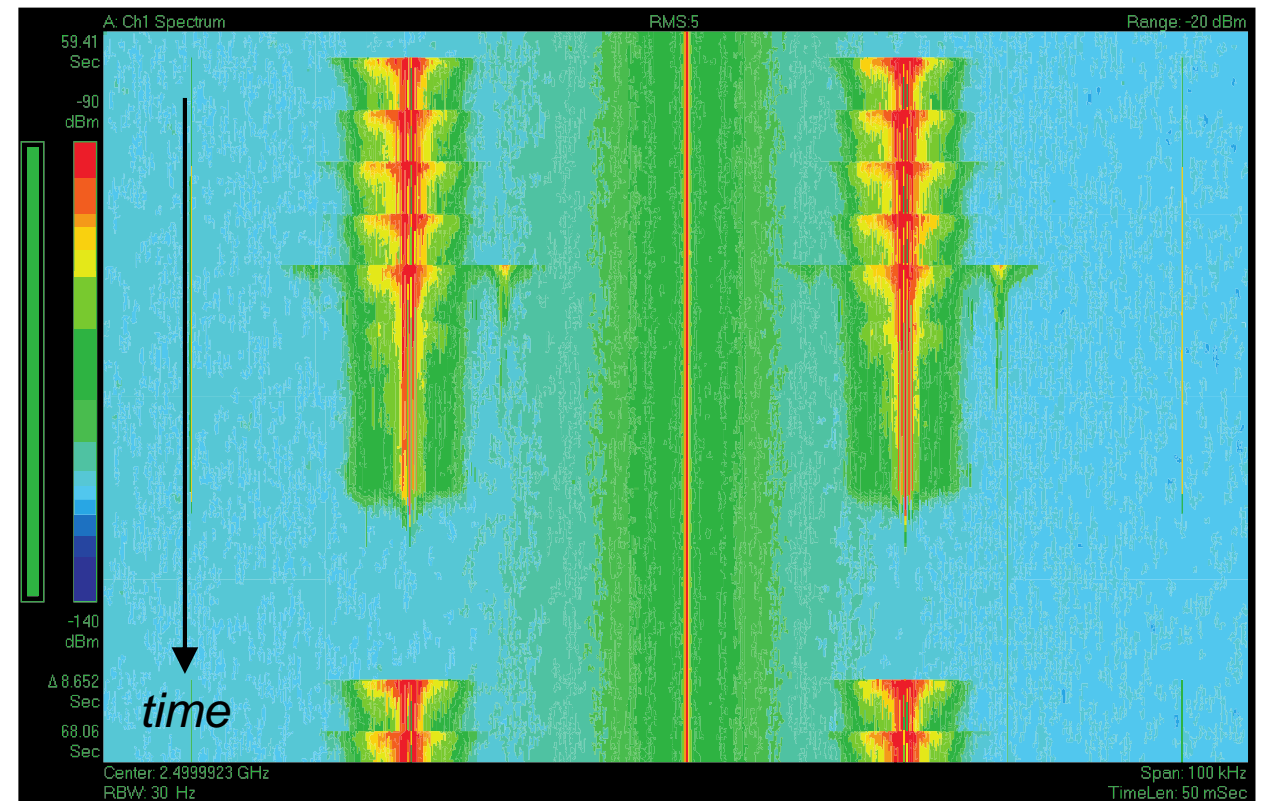


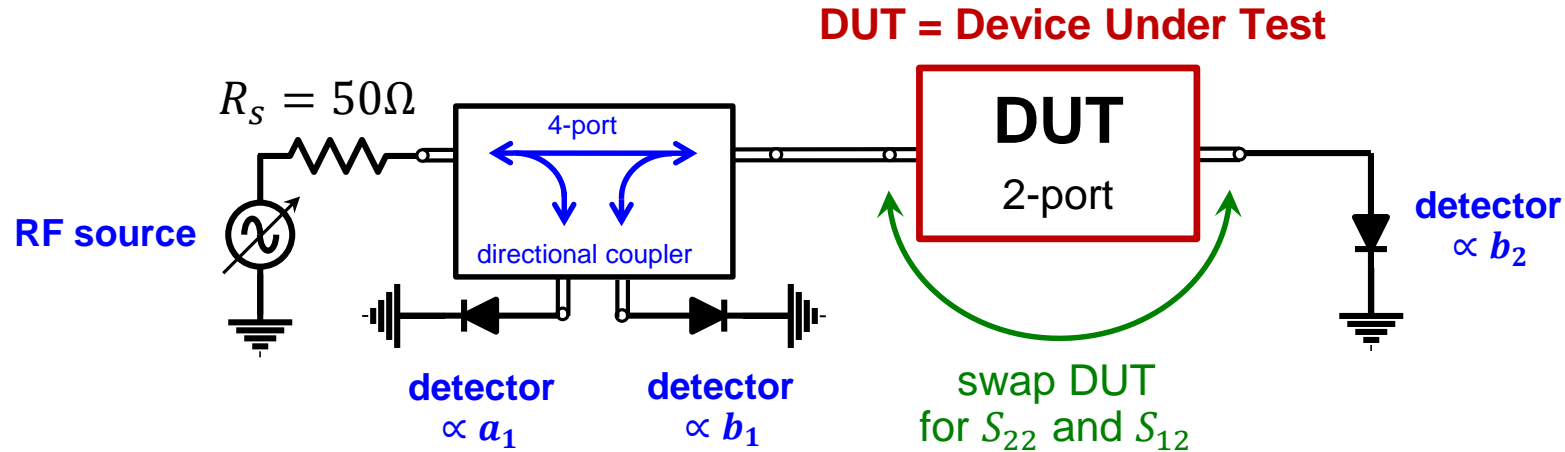
- **Electron cloud beam studies in the SPS (CERN)** – courtesy *F. Caspers*

high-resolution measurement
of the detected beam signal



color-coded spectrogram display
of 200 measurement traces



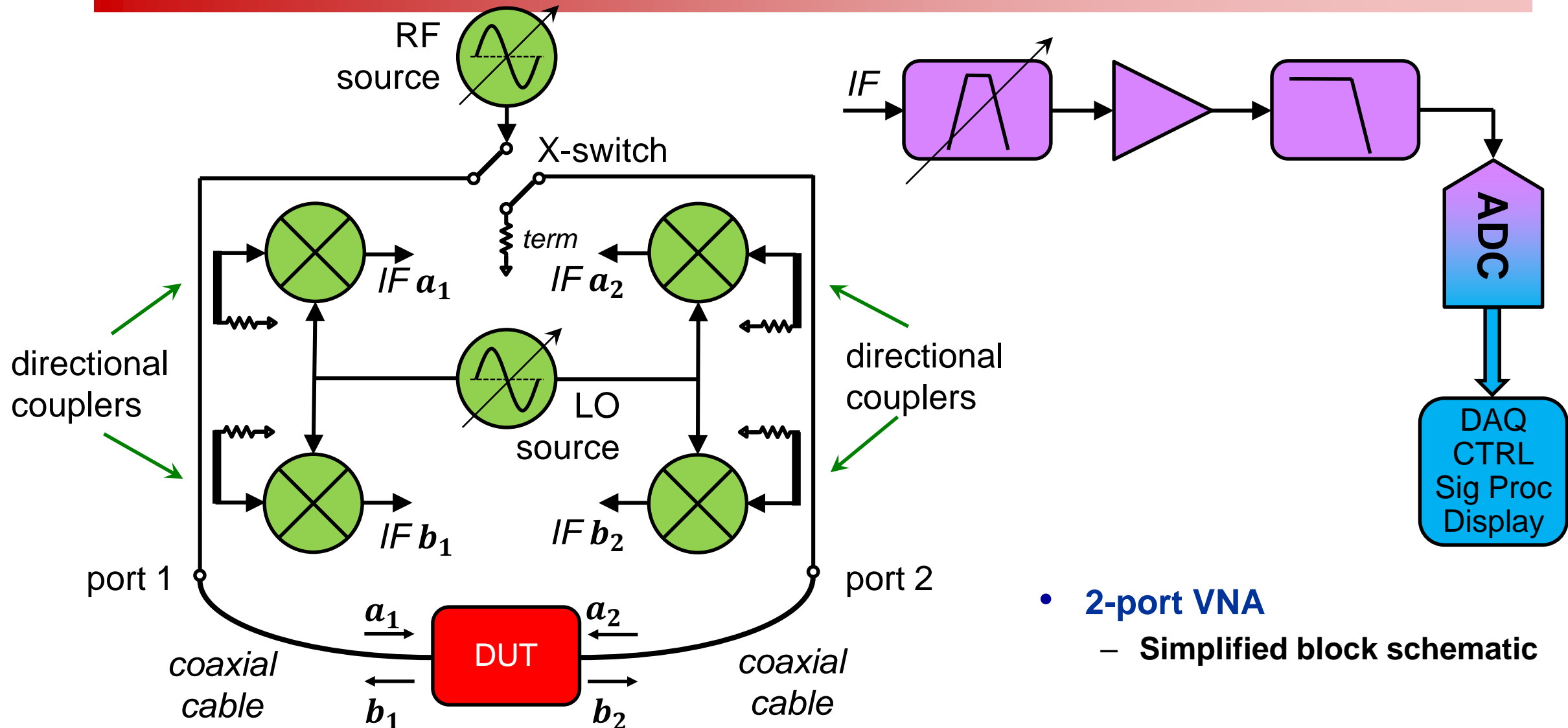


- **Performed in the “frequency domain”**
 - Single or swept frequency generator, stand-alone or as part of a VNA or SA
 - Requires a **directional coupler** and RF detector(s) or receiver(s)
- **Evaluate S_{11} and S_{21} of a 2-port DUT**
 - Ensure $a_2 = 0$, i.e., the detector at port 2 offers a well-matched impedance
 - Measure incident wave a_1 and reflected wave b_1 at the directional coupler ports and compute for each frequency
 - Measure transmitted wave b_2 at DUT port 2 and compute
- **Evaluate S_{22} and S_{12} of the 2-port DUT**
 - Perform the same methodology as above by exchanging the measurement equipment on the DUT ports

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0}$$

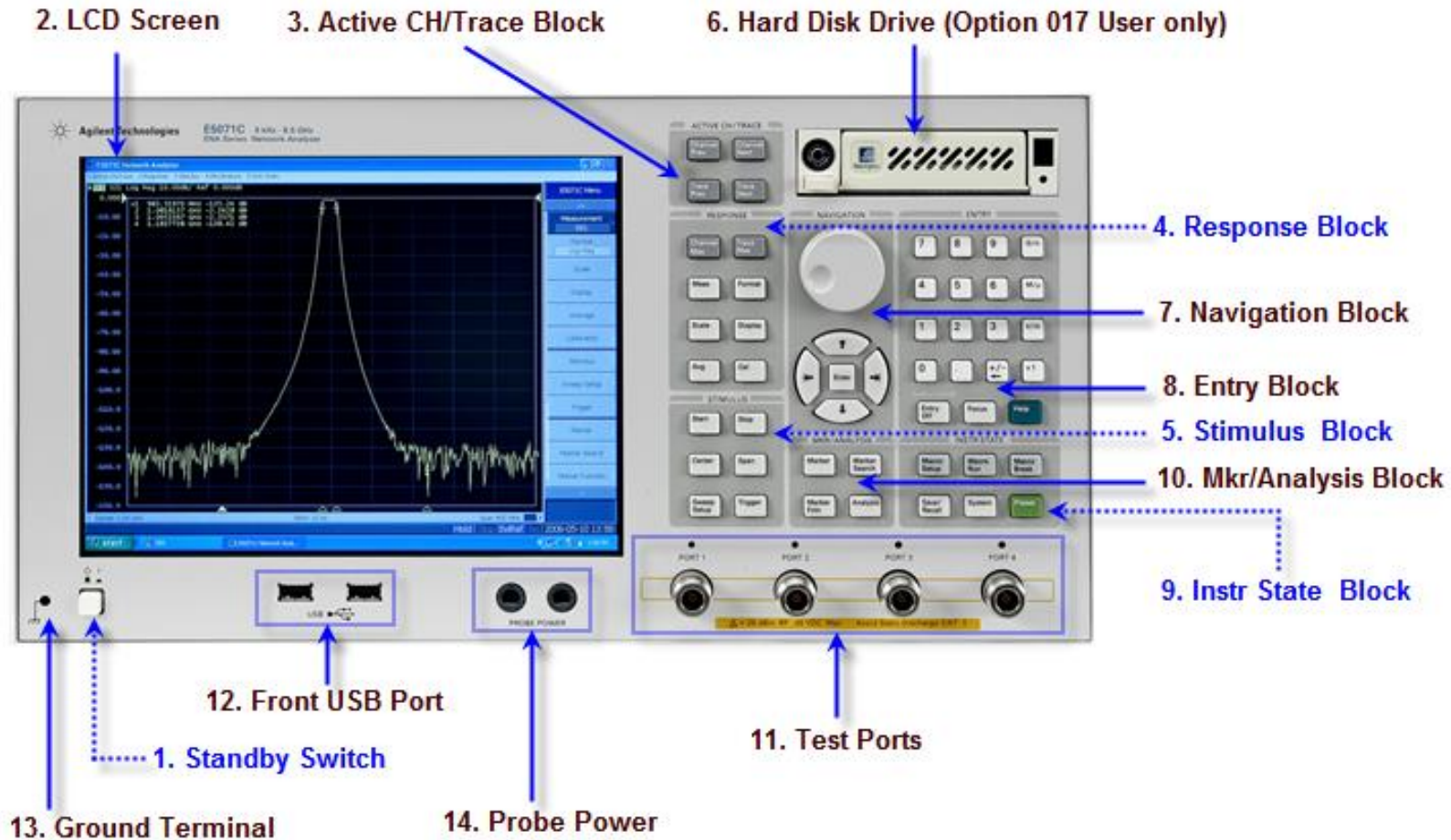
$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0}$$

The Vector Network Analyzer (VNA)



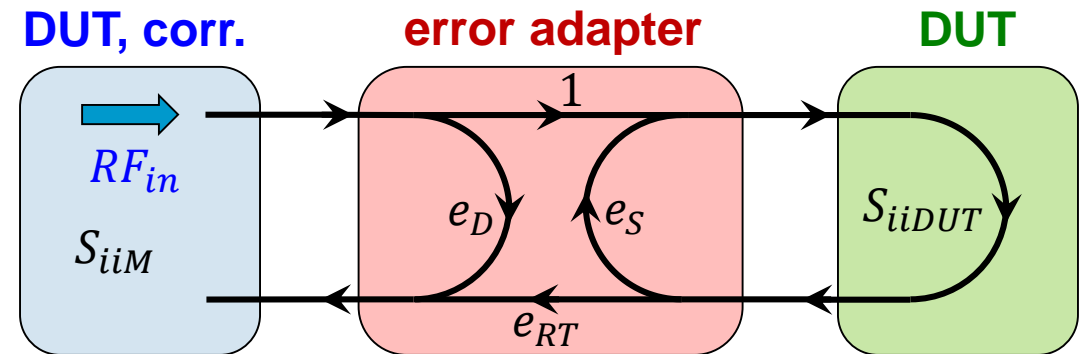
- **2-port VNA**
 - Simplified block schematic

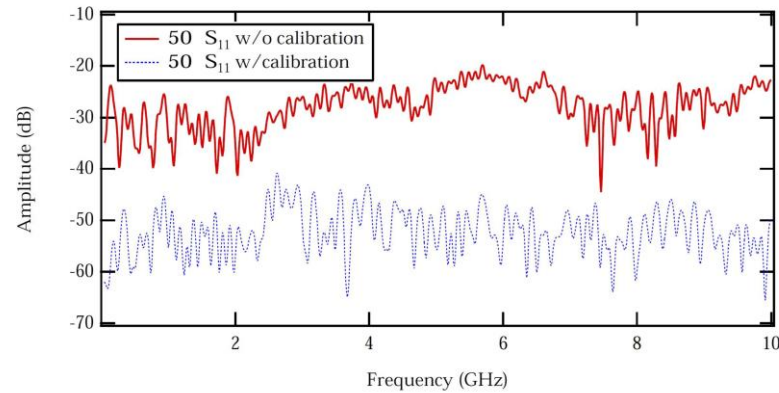
Fun with the VNA!



- The “look and feel” between VNAs vary between manufacturers and models
 - Concepts and operation is still very similar

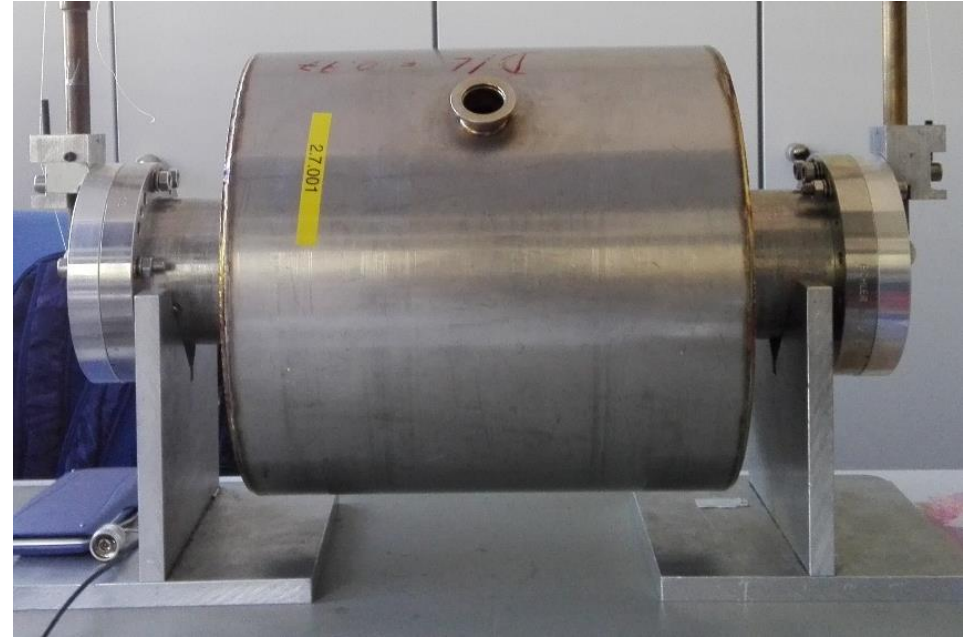
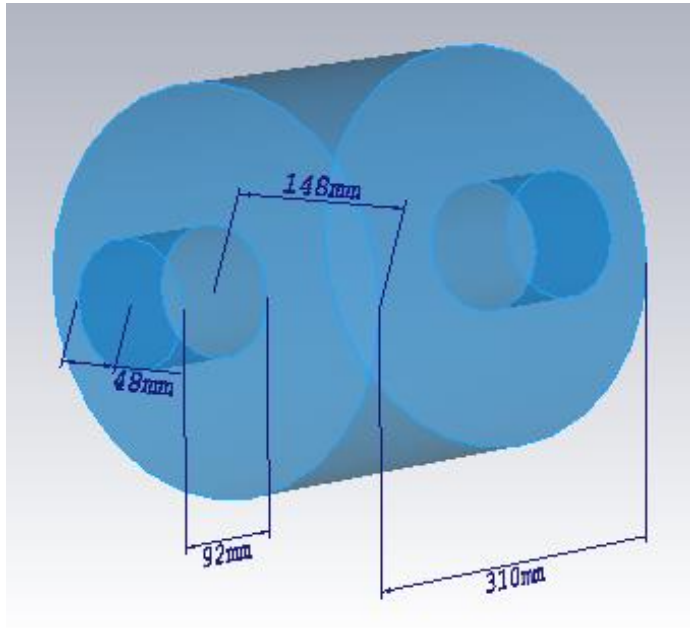
- Calibration is not necessary for pure frequency or phase measurements
- Before calibrating the VNA measurement setup, perform a brief measurement and chose appropriate VNA settings:
 - Frequency range (center, span or start, stop)
 - Number of frequency points
 - Can be sometimes increased by rearranging the VNA memory (# of channels)
 - IF filter bandwidth
 - Output power level
- **Calibrate** the setup, preferable with an **electronic calibration system** if more than 2 ports are used!
 - Each port and combination needs to be calibrated, with the cables attached
 - Choose the appropriate connector type and sex
 - The instrument establishes a correction matrix and displays the "CAL" status.





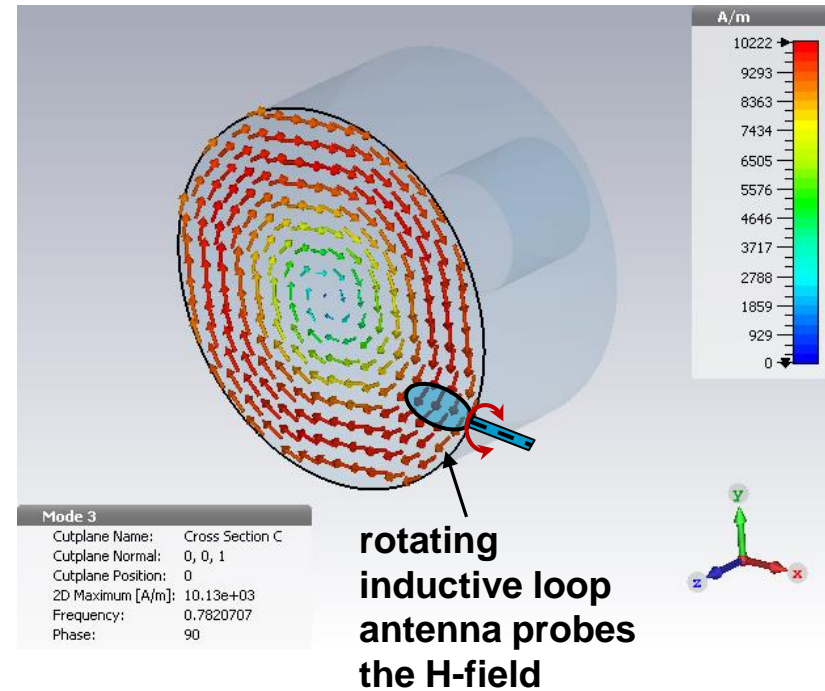
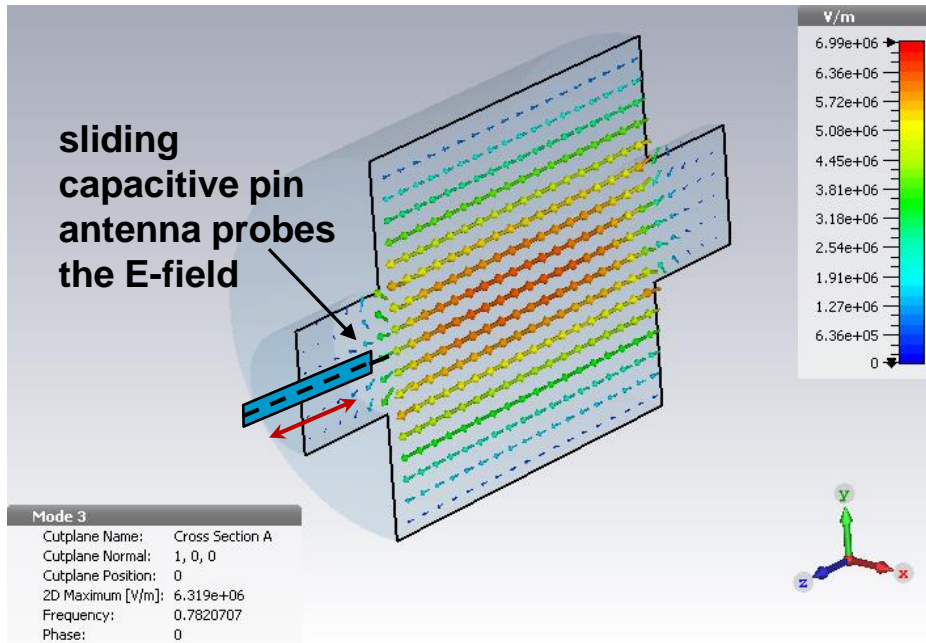
- **Calibration improves the measurement performance**
 - Return loss improvement by typically 20 dB. Enables mdB accuracy measurements!
 - Full 2-port or 4-port calibration with manual calibration kits is prone to errors, better use electronic calibration systems.
 - **Changing VNA settings after calibration** will cause the instrument to inter- and extrapolate, and the **calibration status becomes uncertain!**
- **Cables need to be included in the calibration!**
 - However, changing coaxial connector types usually not!
 - Special VNA cables allows the adaption of different connector types and sex, without requiring a re-calibration of the setup!

- **Modern VNAs (SAs, oscilloscopes, etc. as well) have many “features”**
- **Hardware features, e.g.**
 - Automatic calibration system, down to DC
 - 4 and more ports
 - Additional 2nd source, for downconverter / mixer measurements
 - Integrated spectrum analyzer function
- **Software, control and data post processing options, e.g.**
 - Far too many to list all
 - Sweep options, e.g., lin., log., segmented, in frequency or power
 - iDFT (or iFFT), gating
 - TDR, TDT for BP or LP step or impulse, segmented (advanced) TDR
 - Only for linear, time-invariant systems!
 - Port extension, virtual ports (4-port VNA), Z_{0e} , Z_{0o} characterization, virtual baluns, etc.
 - Data transformations, e.g., $\Gamma \Rightarrow Z$
 - Noise figure measurements
 - Measurements following telecommunication standards

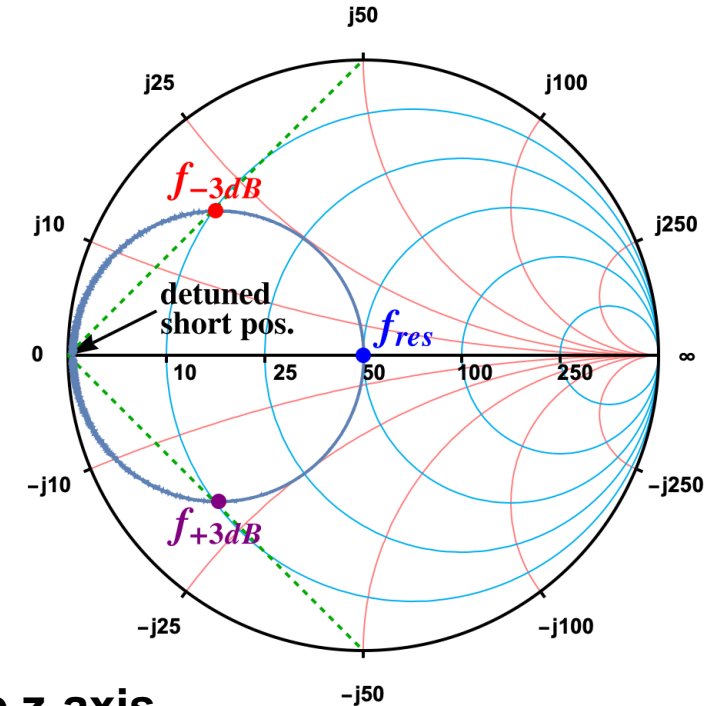
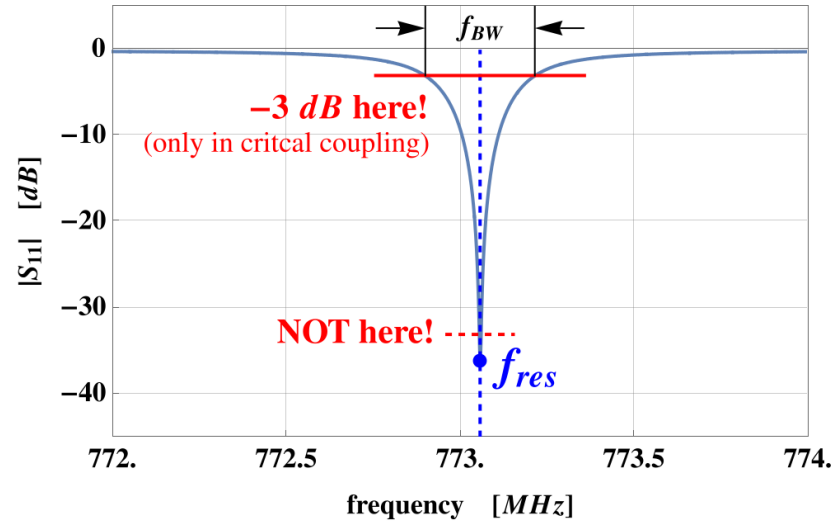
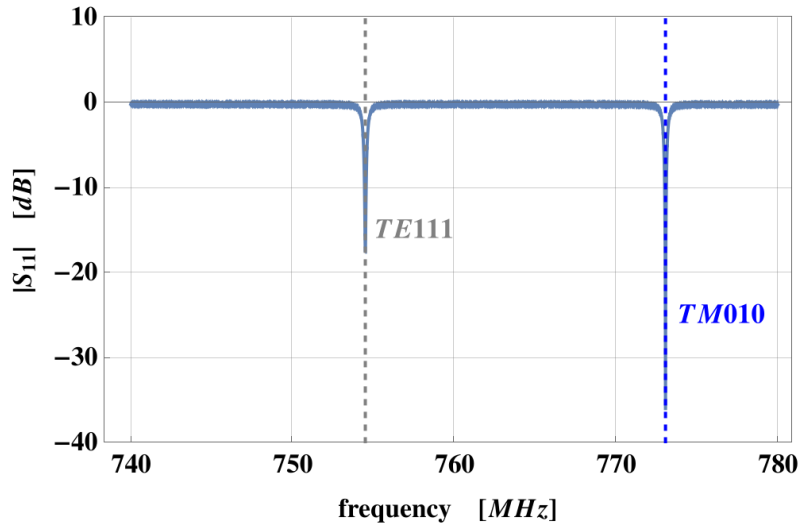


- Characterize the accelerating TM_{010} mode of a cylindrical cavity with beam ports
 - The TM_{010} does not have to be the lowest frequency mode
- Compare the measured values of f_{res} , Q_0 and R/Q
 - with an analytical analysis of a perfect cylinder (no beam ports)
 - with a numerical analysis

* normal conducting!

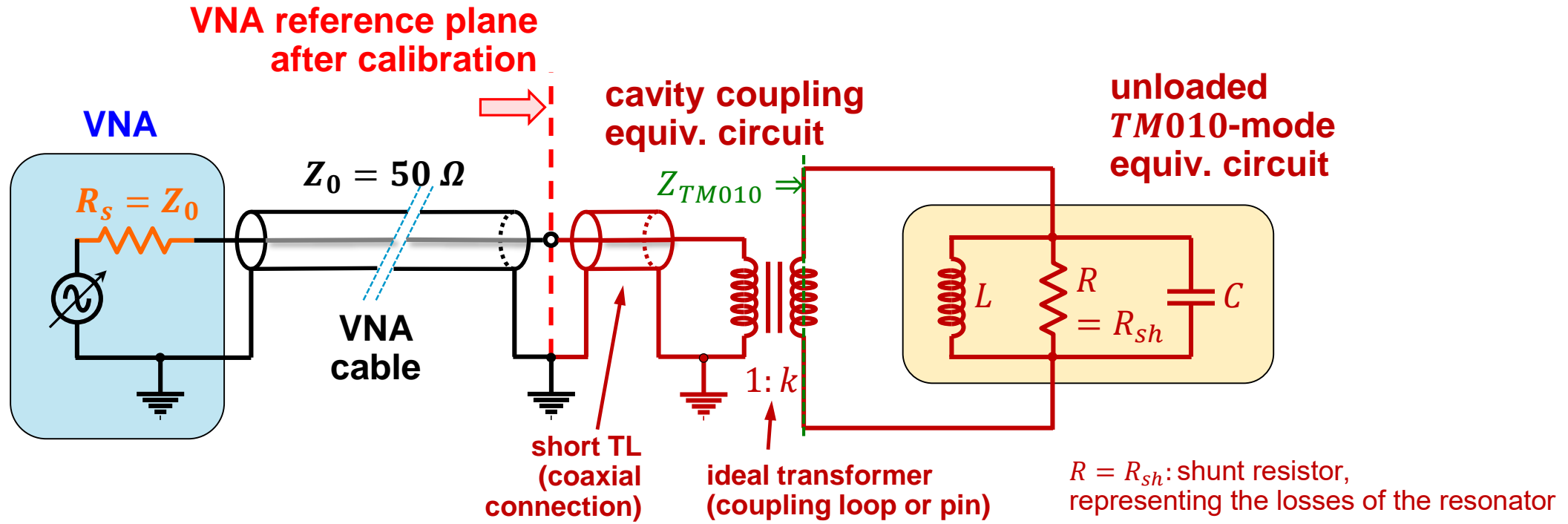


- S_{11} measurement with tunable coupling antenna
 - E-field on z-axis using a capacitive coupling pin
 - Center pin, e.g., of semi-rigid coaxial cable
 - H-field on the cavity rim using an inductive coupling loop
 - Bend the center conductor to a closed loop connected to ground



- **Identify the correct (TM_{010}) mode frequency**
 - Introduce a small perturbation, e.g., metallic rod or wire on the z-axis, and observe the shift of the mode frequencies
- **Calibrate the VNA and measure S_{11}**
 - Tune the coupling loop for critical coupling
 - Display the resonant circle in the *Smith* chart using enough points!

The Equivalent Circuit of a Resonant Mode



we have resonance condition, when:

$$\omega L = \frac{1}{\omega C}$$

at resonance: $Z_{TM010} = R$

→ resonance frequency:

$$\omega_{res} = 2\pi f_{res} = \frac{1}{LC} \Rightarrow f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

Useful Formulas of the Equivalent Circuit

- Characteristic impedance "R over Q"
- Stored energy at resonance
- Dissipated power
- Q-factor
- Shunt impedance (circuit definition)
- Tuning sensitivity
- Coupling parameter (shunt impedance over generator or feeder impedance)

$$X = \frac{R}{Q} = \omega_{res} L = \frac{1}{\omega_{res} C} = \sqrt{L/C}$$

$$U = U_e + U_m = \frac{1}{4} |V_C|^2 C + \frac{1}{4} |I_L|^2 L$$

V_C ... Voltage at the capacitor
 I_L ... Current in the inductor

$$P = \frac{V^2}{2R}$$

$$Q = \frac{R}{X} = \frac{\omega_{res} U}{P}$$

U ... stored energy
 P ... dissipated power over 1 period

$$R = \frac{V^2}{2P} \quad \frac{R}{Q} = \frac{V^2}{2\omega_{res} U}$$

$$\frac{\Delta f}{f} = \frac{1}{2} \frac{\Delta C}{C} = -\frac{1}{2} \frac{\Delta L}{L}$$

$$k^2 = \frac{R}{R_{input}}$$

tune for critical coupling

- The quality (Q) factor of a resonant circuit is defined as ratio of the stored energy U over the energy dissipated P in one oscillation cycle:

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated in 1 cycle}} = \frac{\omega_{res}U}{P}$$

- The Q -factor of an impedance loaded resonator:

- Q_0 : unloaded Q-value of the unperturbed system
- Q_L : loaded Q-value, e.g., measured with the impedance of the connected generator
- Q_{ext} : external Q-factor, representing the effects of the external circuit (generator and coupling circuit)

- Q-factor and bandwidth**

- This is how we actually "measure" the Q-factor!

$$Q = \frac{f_{res}}{f_{BW}}$$

with: $f_{BW} = f_{+3dB} - f_{-3dB}$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

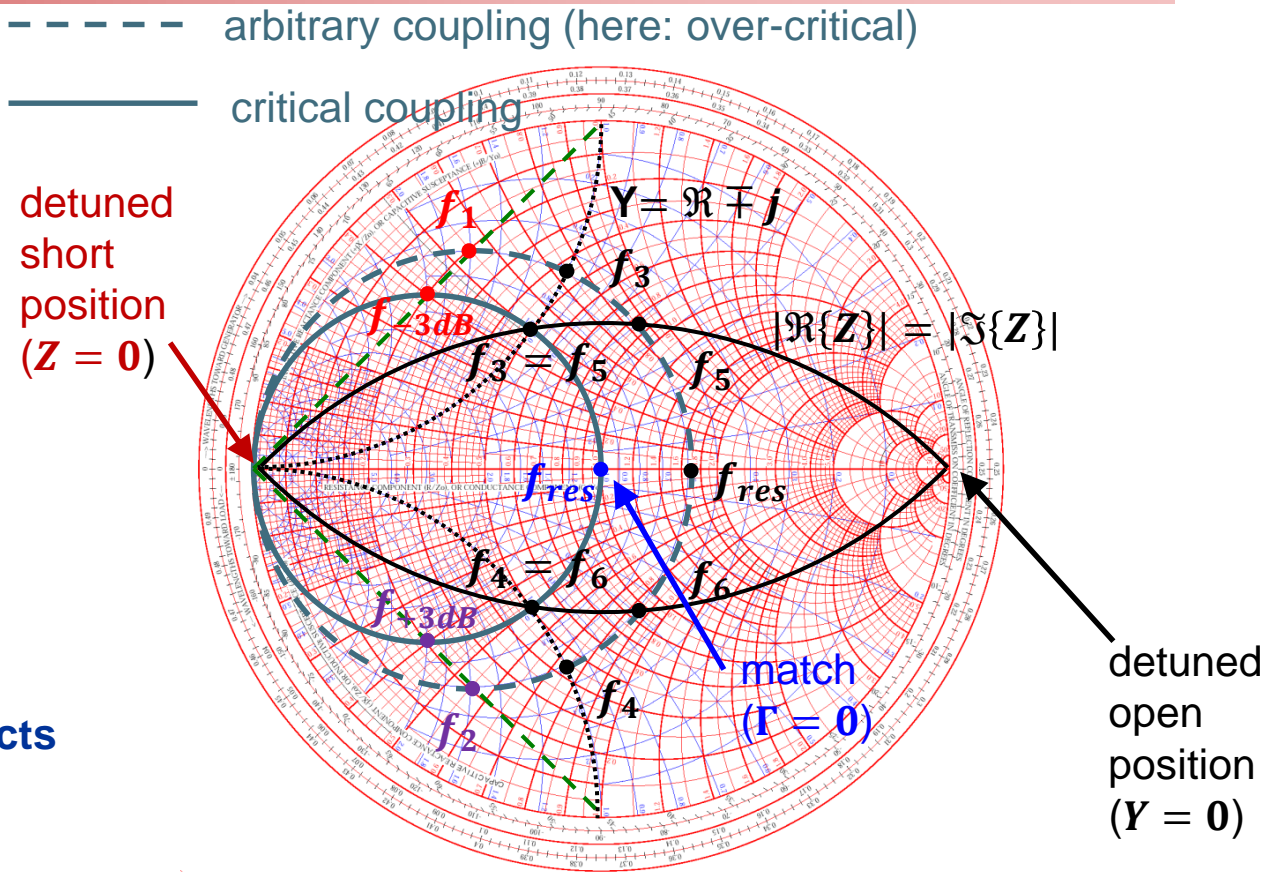
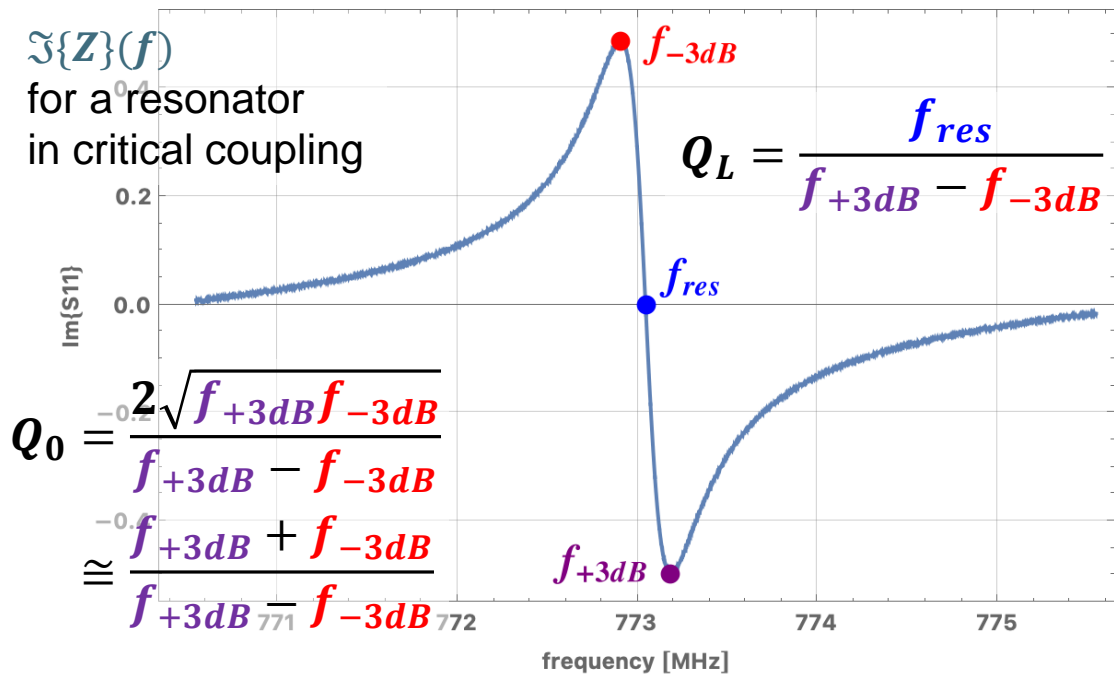
tune k for critical coupling:

$$Q_0 = Q_{ext}$$

$$\Rightarrow Q_0 = 2 Q_L$$

with Q_L being our measured Q-value

Q-factor from S_{11} Measurement



- Correct for the uncompensated transmission-line effects between calibration reference and the coupling loop
 - Electrical length adjustment: "straight" $\Im\{Z\}(f)$
- Adjust the locus circle to the detuned short location
 - Phase offset
- Verify no evanescent fields penetrating outside the beam ports
 - i.e., no frequency shifts if the boundaries at the beam ports are altered

Frequency marker points in the *Smith* chart:
 $f_{1,2}$ (f_{-3dB}, f_{+3dB}): $|\Im\{S_{11}\}| = \mathbf{max}$. to calculate Q_L
 $f_{3,4}$: $Y = \Re + j$ to calculate Q_{ext}
 $f_{5,6}$: $|\Re\{Z\}| = |\Im\{Z\}|$ to calculate Q_0

- Remember from the equivalent circuit:

$$\frac{R}{Q} = \frac{V_{acc}^2 / 2P_d}{\omega_{res} U / P_d} = \frac{V_{acc}^2}{2\omega_{res} U} \quad \text{with:} \quad V_{acc} = \left| \int E_z(z) \cos\left(\frac{\omega z}{\beta c_0}\right) dz \right|$$

transit time related

- V_{acc} is based on the integrated longitudinal E-field component E_z along the z-axis ($x = y = 0$)
- Based on Slater's perturbation theorem:

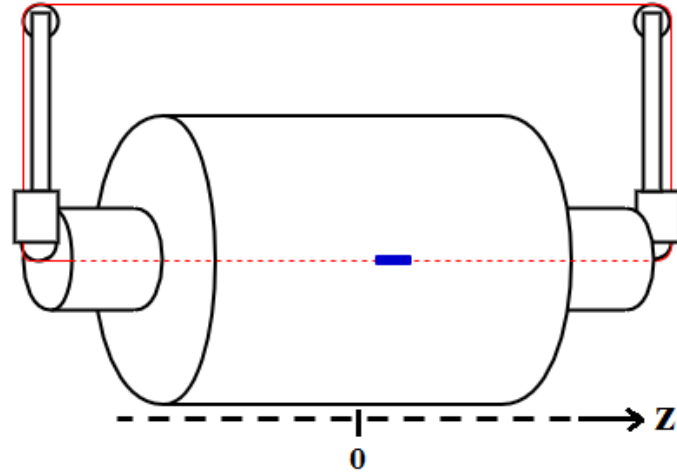
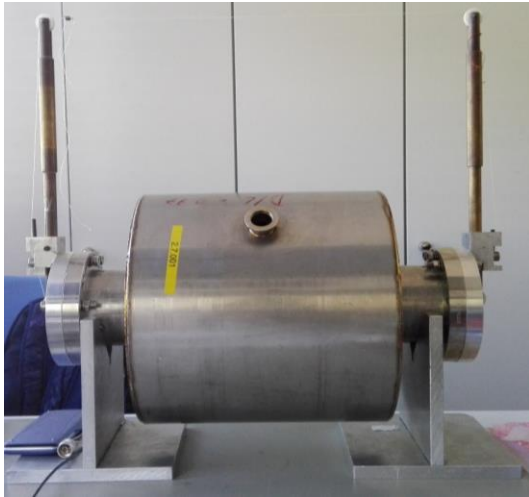
$$\frac{\Delta f}{f_{res}} = \frac{1}{U} \left[\mu_0 \left(k_{\parallel}^H |H_{\parallel}|^2 + k_{\perp}^H |H_{\perp}|^2 \right) - \epsilon_0 \left(k_{\parallel}^E |E_{\parallel}|^2 + k_{\perp}^E |E_{\perp}|^2 \right) \right]$$

- Resonance frequency shift due to a small perturbation object, expressed in longitudinal and transverse E and H field components
- k : coefficients proportional to the electric or magnetic polarizability of the perturbation object (here: only k_{\parallel}^E for a longitudinal metallic object)

- E-field characterization along the z-axis

$$E(z) = E_{\parallel}(z) = \sqrt{U \frac{\Delta f(z)}{f_{res}} \cdot \frac{-1}{k_{\parallel}^E \epsilon_0}}$$

with: $k_{\parallel}^E = \frac{\pi}{3} l^3 \left[\sinh^{-1} \left(\frac{2l}{3\pi a} \right) \right]^{-1}$
 (metallic ellipsoid, e.g., syringe needle of half length l and radius a)



- **E-field characterization by evaluating**

- The frequency shift Δf (S_{11} reflection measurement with a single probe) or
- The phase shift ϕ at f_{res} (S_{21} transmission measurement with 2 probes)

$$\frac{\Delta f}{f_{res}} = \frac{1}{2 Q_0} \tan \phi$$

- **Exercise with a manual bead-pull through a known cavity**

- requires: fishing wire, syringe needle, ruler and VNA
- Compare the measured E_z at the maximum f or ϕ shift (in the center of the cavity) with the theoretical estimation (e.g., numerical computed value)

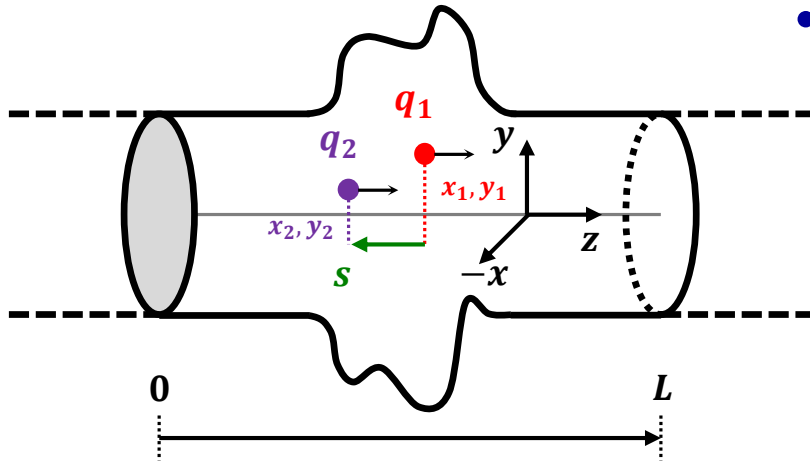
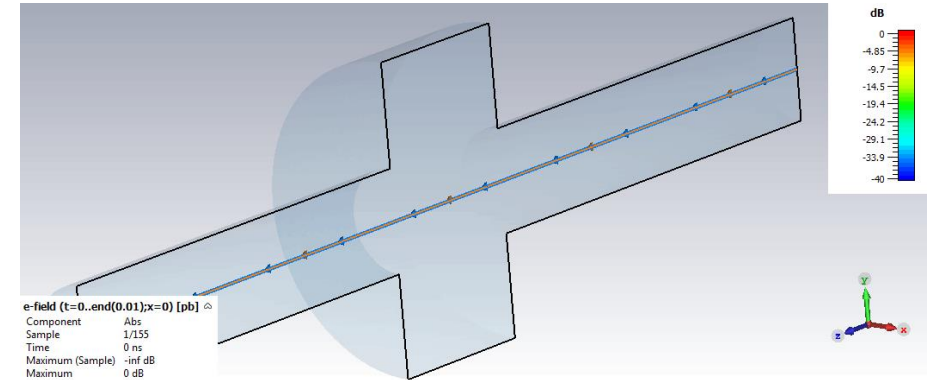
- **The wake potential**

- Lorenz force on q_2 by the wake field of q_1 :

$$\vec{F} = \frac{d\vec{p}}{dt} = q_2(\vec{E} + c_0\vec{e}_z \times \vec{B})$$

- Wake potential of a structure, e.g., a discontinuity driven by q_1

$$\vec{w}(x_1, y_1, x_2, y_2, s) = \frac{1}{q_1} \int_{-\infty \text{ (or } 0)}^{+\infty \text{ (or } L)} dz [\vec{E}(x_2, y_2, z, t) + c_0\vec{e}_z \times \vec{B}(x_2, y_2, z, t)]_{t=(s+z)/c}$$



- **Beam coupling impedance**

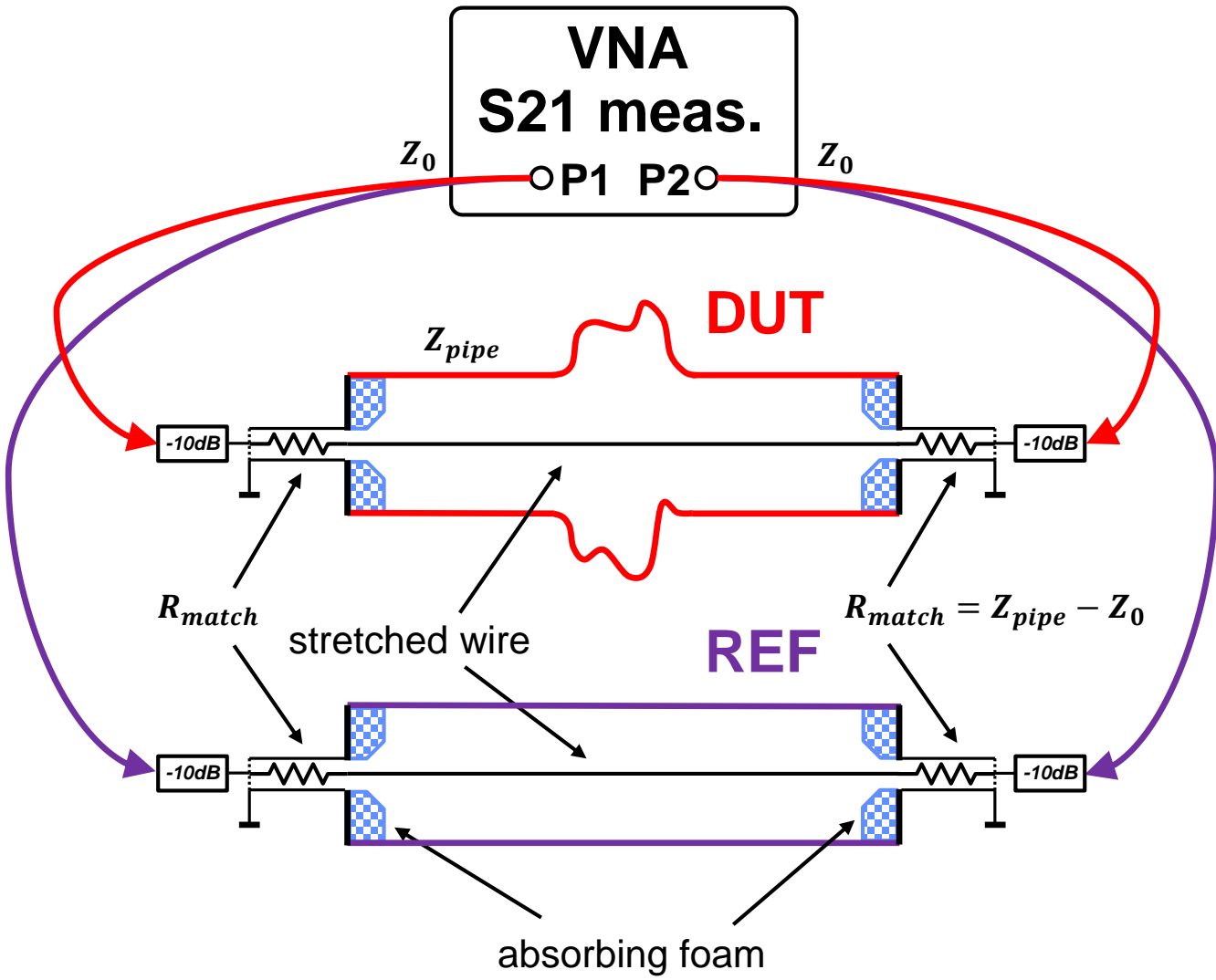
- Frequency domain representation of the wake potential

$$Z(x_1, y_1, x_2, y_2, \omega) = -\frac{1}{c_0} \int_{-\infty}^{+\infty} ds \vec{w}(x_1, y_1, x_2, y_2, s) e^{-j\omega s/c_0}$$

- Can be decomposed in **longitudinal Z_{\parallel}** and **transverse Z_{\perp}** components (*Panofsky-Wenzel* theorem)

- Resonant structures, i^{th} mode: $R_{sh,i} = Z_{\parallel,i} = \frac{2k_{loss,i}Q_i}{\omega_i}$

Stretched-Wire Z_{\parallel} Measurement



- Formulas:**

- Normalized electrical length: $\theta = 2\pi \frac{L}{\lambda}$

- Lumped impedance formula

$$Z_{\parallel} = 2Z_{pipe} \frac{1 - S_{21}}{S_{21}} \quad \theta \leq 1, L < D_{pipe}$$

- Log formula

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21}$$

- Improved log formula

$$Z_{\parallel} = -2Z_{pipe} \ln S_{21} \left(1 + j \frac{\ln S_{21}}{2\theta} \right)$$

- Transmission coefficient

$$S_{21} = \frac{S_{21,DUT}}{S_{21,REF}}$$

- Circular beam pipe impedance

$$Z_{pipe} = \frac{\eta_0}{2\pi\sqrt{\epsilon_r}} \ln \frac{D}{d} \cong 60 \Omega \ln \frac{D_{pipe}}{d_{wire}}$$

- No summary, just thank you for listening!
- Also, a big THANK YOU for all the help from Christine and Piotr!
- More THANKS to *Dassault / CST, SIMUSERV (Frank)* and *Computer Controls / Keysight*
- THANK YOU to my colleagues at CERN, in particular *Joel D.!*

Thank you!



Backup Slides



- **Characteristic impedance**

- for a TEM transmission-line

- with losses

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad [\Omega]$$

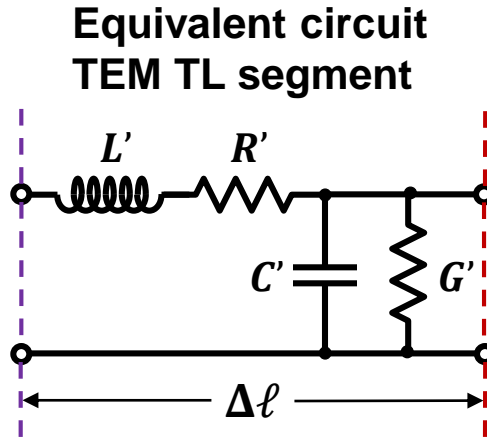
- lossless, non-magnetic media ($\mu_r = 1$)

$$Z_0 \cong \sqrt{\frac{L'}{C'}} \quad [\Omega]$$

$$Z_0 \cong \frac{\sqrt{\mu_0 \epsilon_0 \epsilon_r}}{c C'} = \frac{\sqrt{\epsilon_r}}{c C'} = \frac{1}{v_p C'} \quad [\Omega]$$

The characteristic impedance can be calculated from 2D electrostatic equations

$$Z_0 \cong \frac{c L'}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} = \frac{c L'}{\sqrt{\epsilon_r}} = v_p L' \quad [\Omega]$$



- **Propagation constant**

- for a TEM transmission-line

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

- attenuation constant

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} - 1 \right)} \quad \left[\frac{Np}{m} \right]$$

- phase constant

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon^2}} + 1 \right)} \quad \left[\frac{rad}{m} \right]$$

$$\beta = \frac{2\pi}{\lambda_g} \quad \left[\frac{rad}{m} \right]$$

$$\beta = \omega\sqrt{L'C'}$$

- Wave impedance**

in media

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \quad [\Omega]$$

Characteristic impedance of free space

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \cong 120\pi \quad [\Omega] \cong 377 \quad [\Omega]$$

- Speed of light**

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \cong 2.997925 \cdot 10^8 \quad \left[\frac{m}{s}\right]$$

- Phase velocity**

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \quad \left[\frac{m}{s}\right]$$

– non-magnetic media ($\mu_r = 1$)

$$v_p \cong \frac{c}{\sqrt{\epsilon_r}} \quad \left[\frac{m}{s}\right]$$

- Wavelength**

in free space

$$\lambda_0 = \lambda = \frac{c}{f} \quad [m]$$

guide wavelength (in media)

$$\lambda_g = \frac{c}{f\sqrt{\mu_r\epsilon_r}} = \frac{\lambda_0}{\sqrt{\mu_r\epsilon_r}} \quad [m]$$

– non-magnetic media: ($\mu_r = 1$)

$$\lambda_g \cong \frac{c}{f\sqrt{\epsilon_r}} \quad [m]$$

- Electrical length**

– for a TEM line of physical length ℓ

$$\theta = \beta\ell = 2\pi \frac{\ell}{\lambda_g} \quad [rad]$$

$$\theta = \beta\ell = 360 \frac{\ell}{\lambda_g} \quad [deg]$$

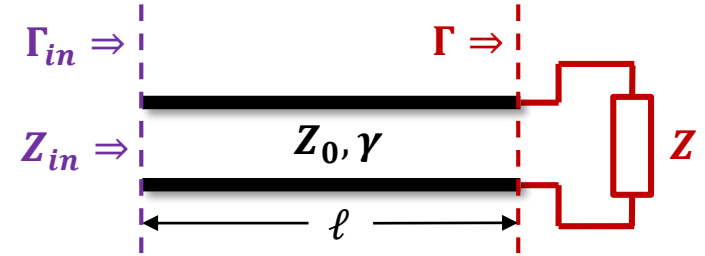
- Permeability:** $\mu = \mu_0\mu_r$ $\mu_0 \cong 4\pi \cdot 10^{-7} \quad [H/m]$

- Permittivity:** $\epsilon = \epsilon_0\epsilon_r$ $\epsilon_0 \cong 8.854 \cdot 10^{-12} \quad [F/m]$

- **Transmission-line terminated with an arbitrary impedance Z :**

$$Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} \quad \text{with: } \Gamma_{in} = \Gamma e^{-2\gamma\ell} \quad \text{and: } \Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$Z_{in} = Z_0 \left(\frac{Z \cosh \gamma\ell + Z_0 \sinh \gamma\ell}{Z_0 \cosh \gamma\ell + Z \sinh \gamma\ell} \right) = Z_0 \frac{Z + Z_0 \tanh \gamma\ell}{Z_0 + Z \tanh \gamma\ell} \quad [\Omega]$$



- **Lossless transmission-line:**

$$\alpha = 0 \Rightarrow$$

$$Z_{in} = Z_0 \left(\frac{Z \cos \beta\ell + jZ_0 \sin \beta\ell}{Z_0 \cos \beta\ell + jZ \sin \beta\ell} \right) \quad [\Omega]$$

– Popular applications

➤ Quarter-wave line: $\ell = \frac{\lambda}{4} \Rightarrow \theta = \frac{\pi}{2} \Rightarrow Z_{in} = \frac{Z_0^2}{Z}$

➤ Terminated (matched) line: $Z = Z_0 \Rightarrow Z_{in} = Z_0$

➤ Open line: $Z \rightarrow \infty \Rightarrow Z_{in} = -jZ_0 \cot \beta\ell$

➤ Shorted line: $Z = 0 \Rightarrow Z_{in} = jZ_0 \tan \beta\ell$

$$Z_{in,open} = -j Z_0 \cot \beta \ell$$

$$Z_{in,short} = j Z_0 \tan \beta \ell$$

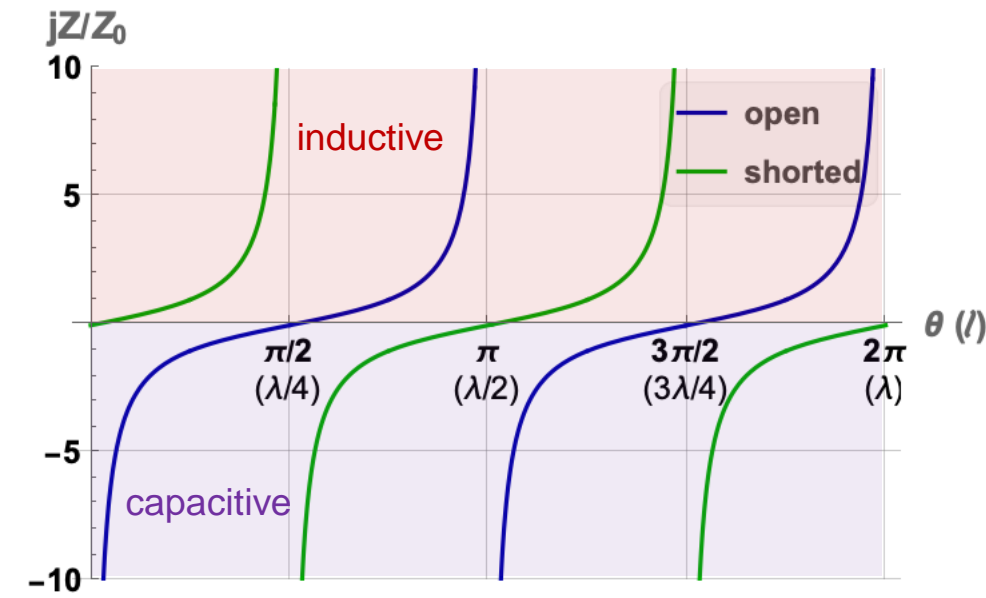
physical length ℓ electrical length $\theta = \beta \ell$	$0 < \ell < \lambda_g/4$ $0 < \theta < \pi/2$	$\lambda_g/4 < \ell < \lambda_g/2$ $\pi/2 < \theta < \pi$	$\lambda_g/2 < \ell < 3\lambda_g/4$ $\pi < \theta < 3\pi/2$	$3\lambda_g/4 < \ell < \lambda_g$ $3\pi/2 < \theta < 2\pi$
lossless TL open	“capacitive”	“inductive”	“capacitive”	“inductive”
lossless TL shorted	“inductive”	“capacitive”	“inductive”	“capacitive”

- A lossless TL with open ($Z = 0$) or shorted ($Z \rightarrow \infty$) termination can approximate a lumped reactive element (capacitor or inductor)

– A “capacitive” element has the form: $Z_C = \frac{1}{j\omega C} = -j \frac{1}{\omega C}$

– An “inductive” element has the form: $Z_L = j\omega L$

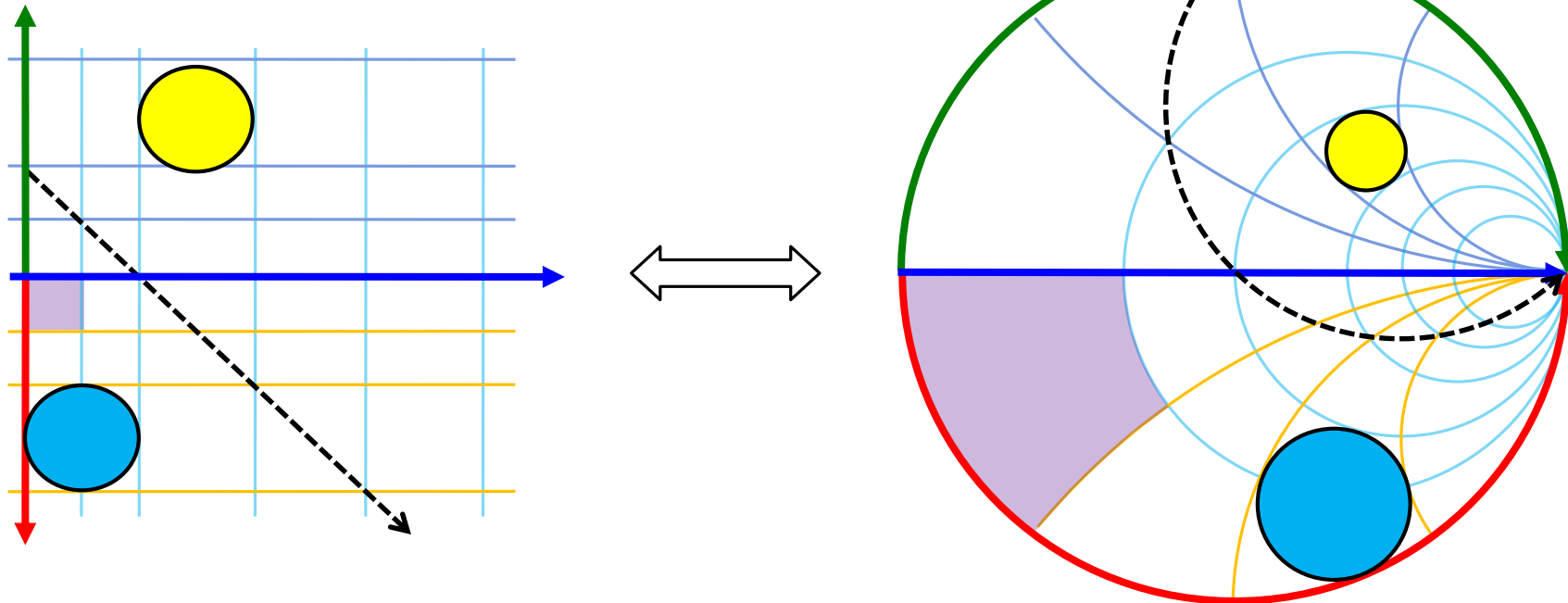
- A more precise method follows a “T” or “π” LCL or CLC equivalent circuit of the lossless TL.
- In case of $\theta \ll 1$, we can simplify $\tan \theta \cong \theta$, etc.
- In practice it is useful to select a low Z_0 for a capacitive, or a high Z_0 for an inductive semi-lumped element approximation



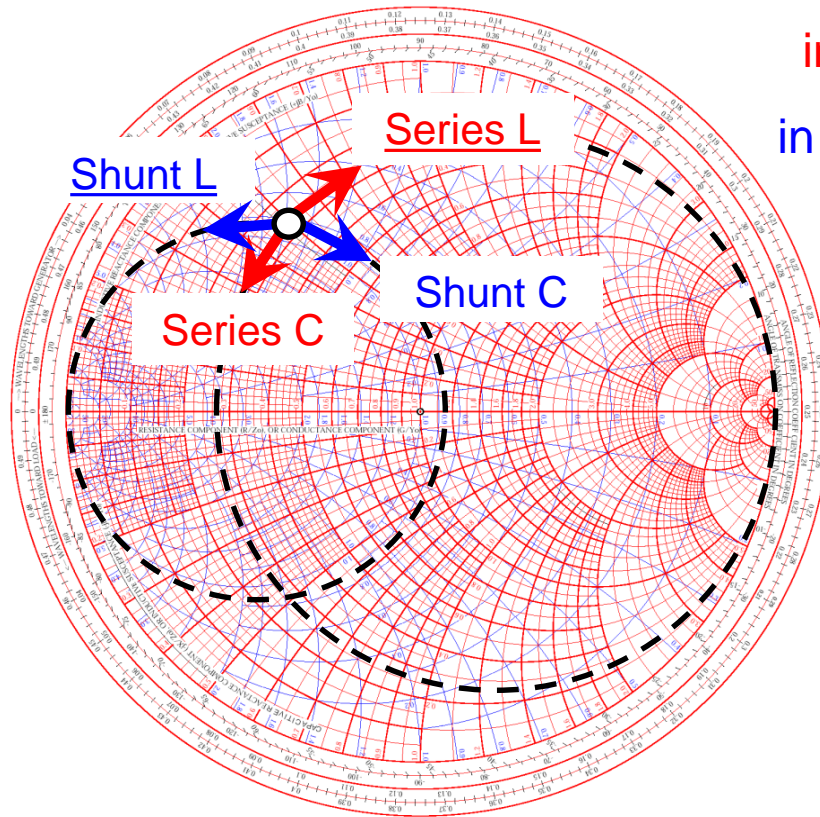
This is a “bilinear” transformation with the following properties:

- **Generalized circles are transformed into generalized circles**
 - **circle** → **circle**
 - **straight line** → **circle**
 - **circle** → **straight line**
 - **straight line** → **straight line**
- **Angles are preserved locally**

- a straight line is equivalent to a circle with infinite radius
- a circle is defined by 3 points
- a straight line is defined by 2 points

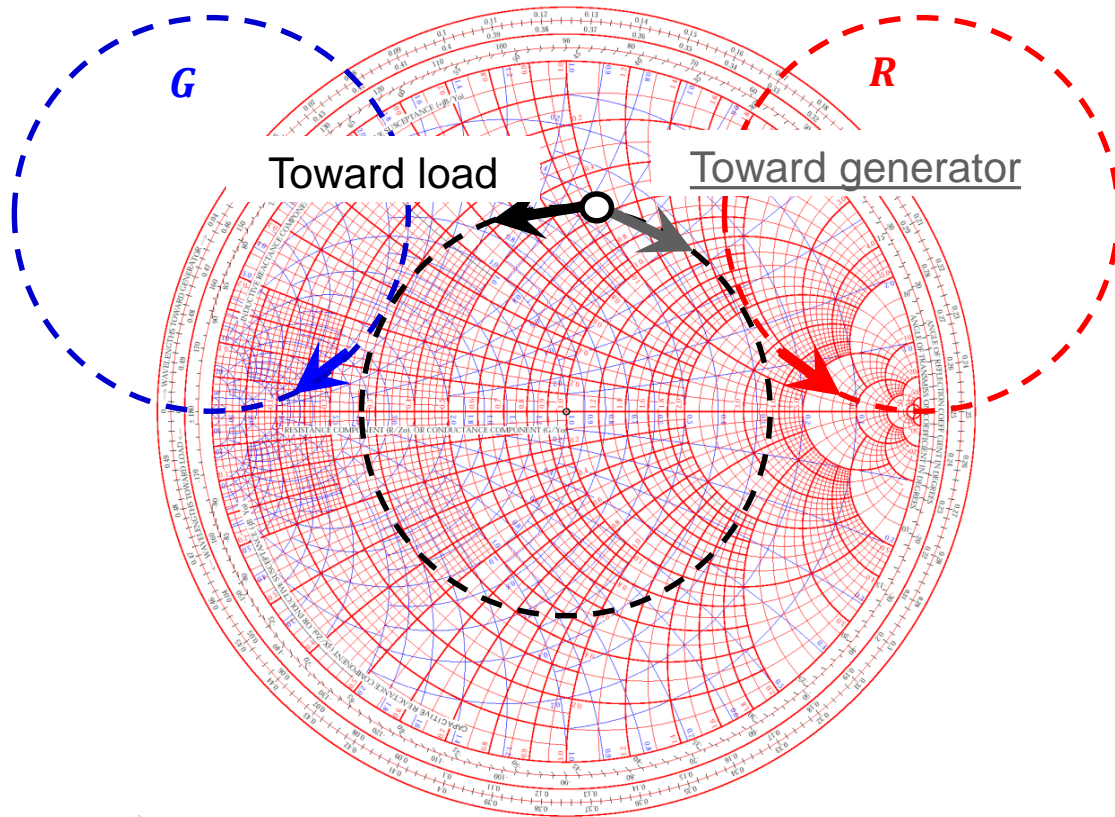


Navigation in the *Smith Chart* (2)

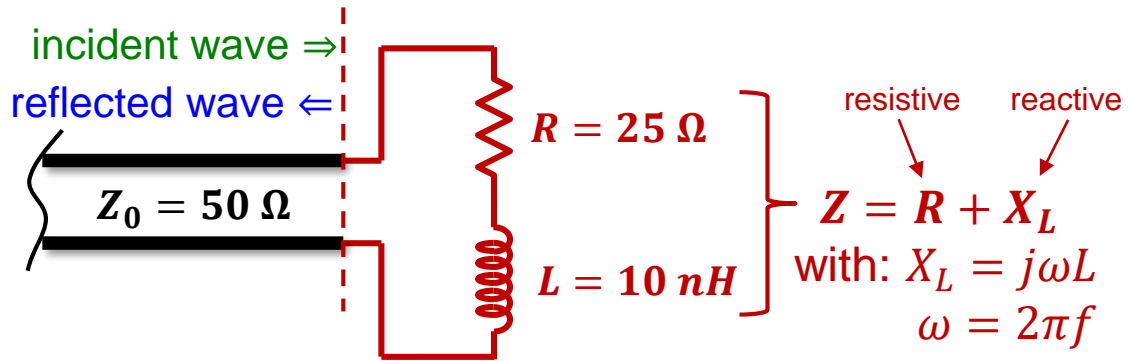


in red: impedance plane (= z)
 in blue: admittance plane (= y)

	<u>Up</u>	Down
Red circles	<u>Series L</u>	Series C
Blue circles	<u>Shunt L</u>	Shunt C

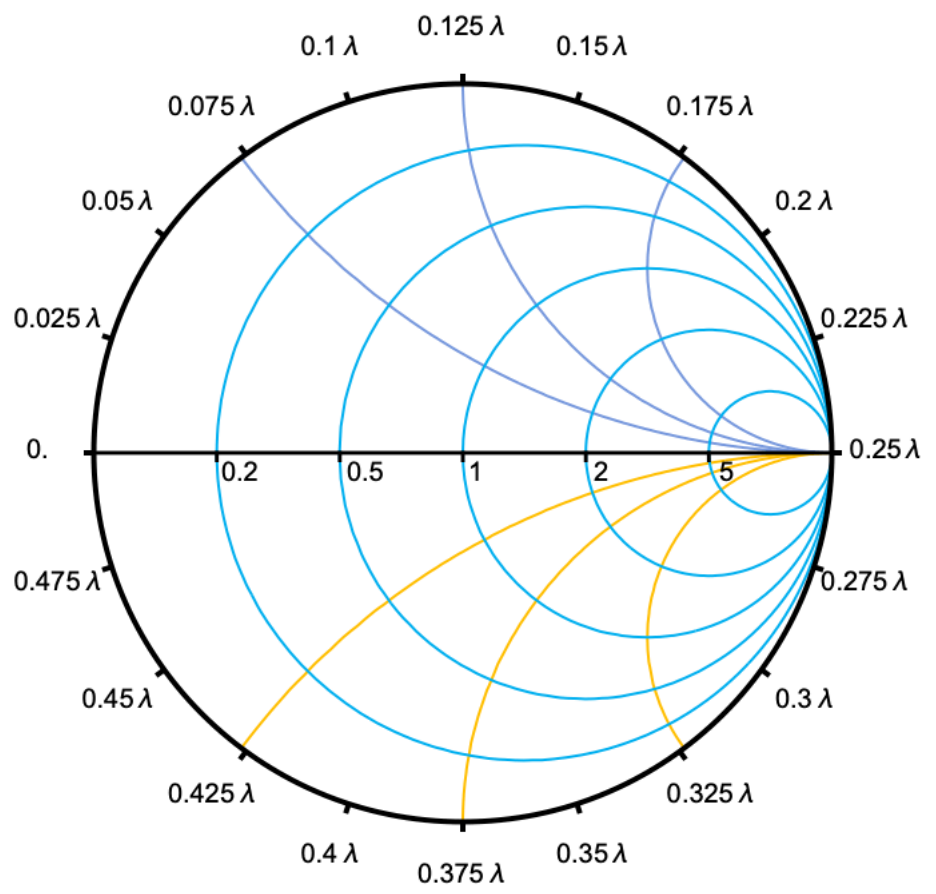


Red arcs	Resistance R
Blue arcs	Conductance G
Con-centric circle	Transmission line going Toward load <u>Toward generator</u>

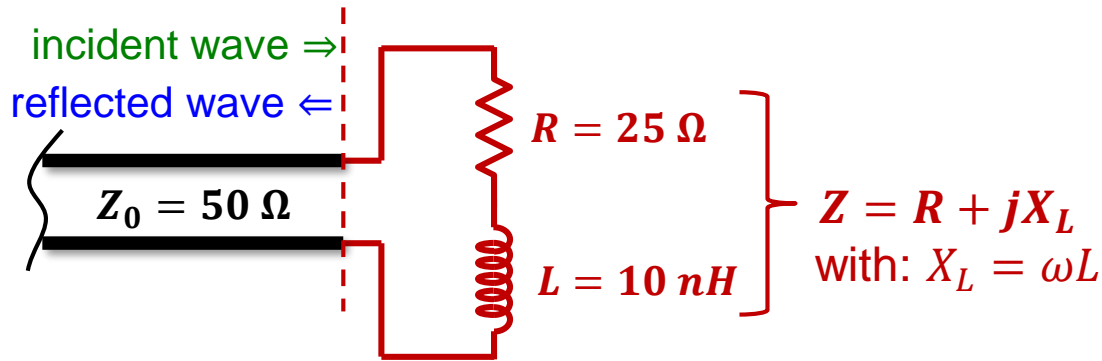


Complex impedance based on lumped element components

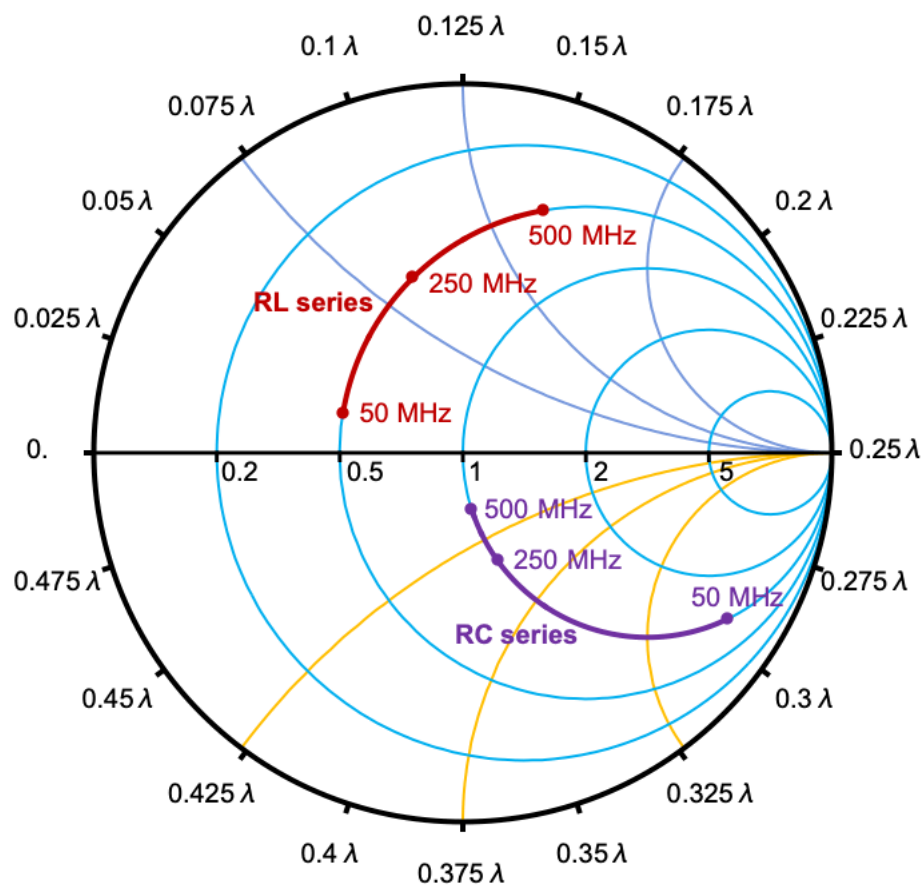
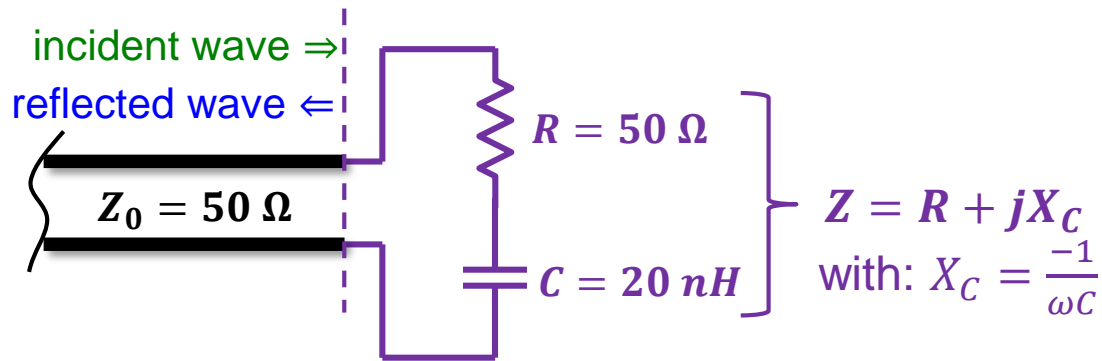
- Calculate Z for a given frequency, e.g., $f = 50 \text{ MHz}$: $Z = (25 + j6.28) \Omega$
- Calculate the normalized impedance $z = Z/Z_0 = 0.5 + j0.126$
 - Locate z in the Smith chart
 - Retrieve $\Gamma = 0.34 \angle 161^\circ = 0.34e^{j2.81}$
- Repeat for other frequencies...

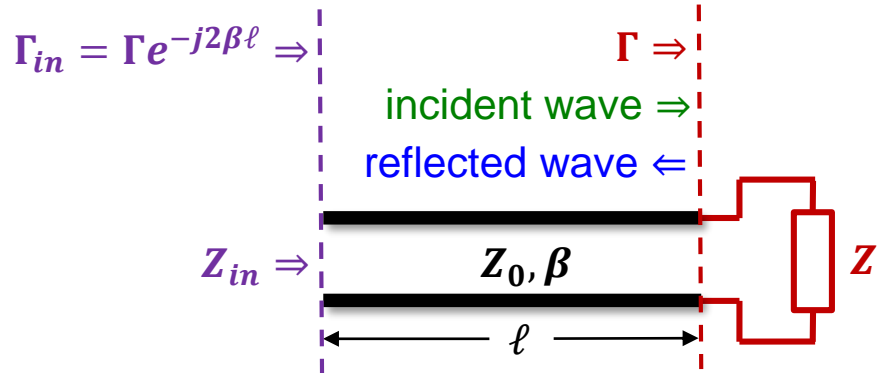


The *Smith Chart* – Basic Example (2)



- ...and for different component values and circuit combinations





- **S-parameter of a lossless transmission-line:**

$$S = \begin{bmatrix} 0 & e^{-j\beta\ell} \\ e^{-j\beta\ell} & 0 \end{bmatrix}$$

forward transmission coefficient S21

backward transmission coefficient S12

– **Phase delay (electrical length) of**

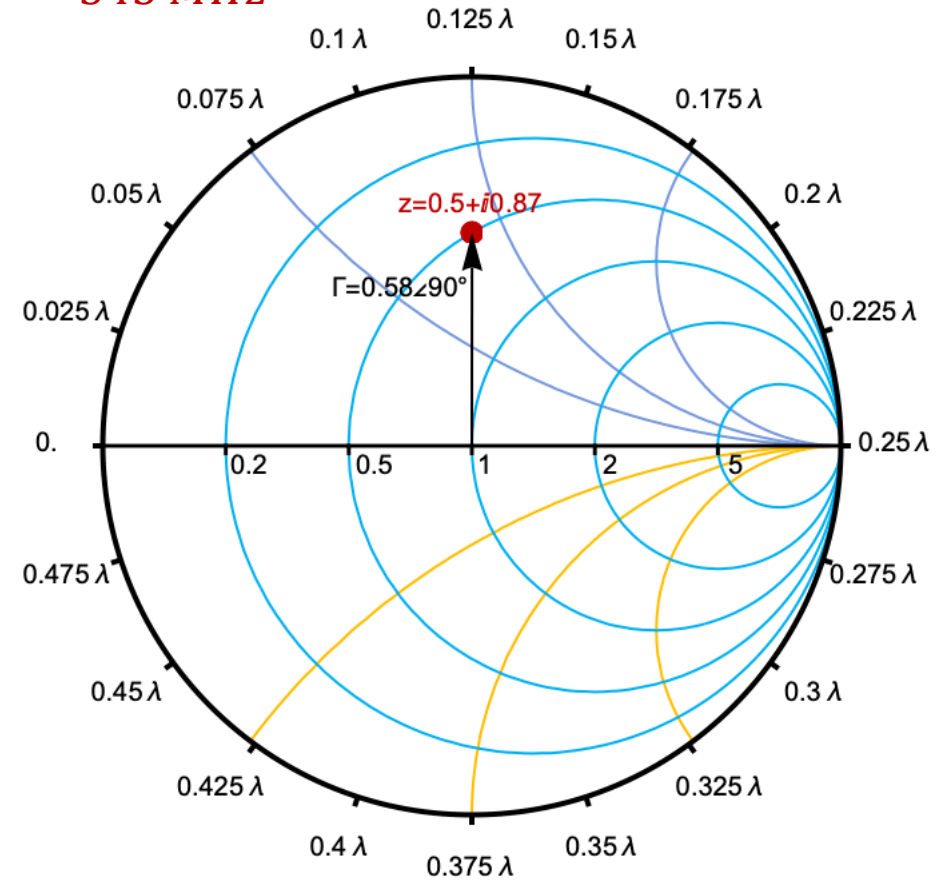
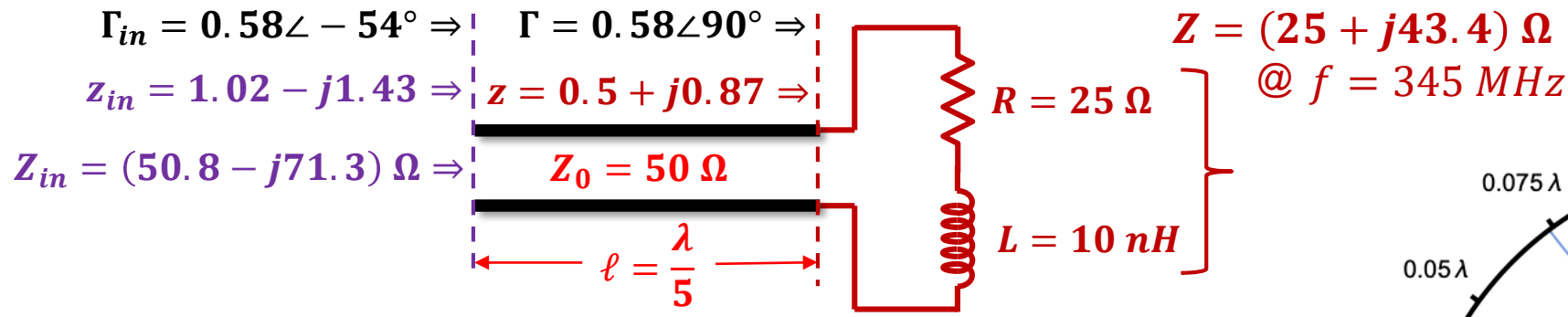
$$\theta = \beta\ell$$

with: $\beta = \frac{2\pi}{\lambda_g}$

- The lossless transmission-line adds a phase delay of $2\beta\ell$, seen at its input, to the reflection coefficient at its output:

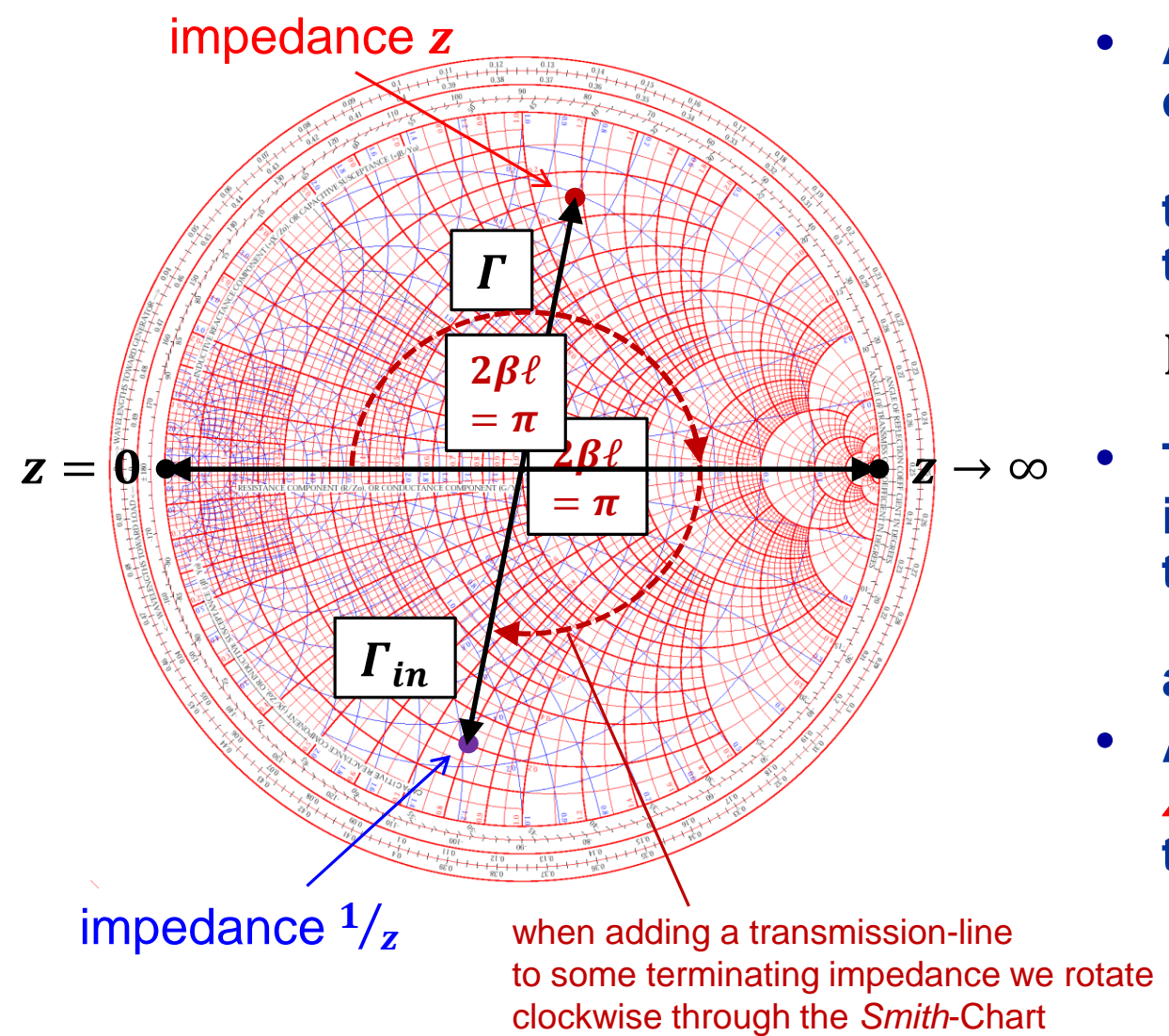
$$\Gamma_{in} = \Gamma e^{-j2\beta\ell}$$

- This results in a transformation of the impedance Z at the end of the line to a different impedance Z_{in} at the input of the line
 - The Smith chart offers an effective, simple graphical way to calculate this transmission-line based impedance transformation



Based on the previous example

- Calculate the normalized impedance z for $f = 345 \text{ MHz}$ and locate the point in the Smith chart.
 - Notice the corresponding value of Γ , and read the λ -length value on the rim
- Add $\ell = \lambda/5$ by rotating Γ by $2\beta\ell = 4\pi/5 \equiv 144^\circ$
 - From 0.125λ to 0.325λ
 - The phase is subtracted, therefore clockwise rotation!
- Notice the value of Γ_{in} and read the corresponding value of the normalized impedance Z_{in}
 - Calculate the transformed impedance $Z_{in}@f = 345 \text{ MHz}$
 - What is the equivalent circuit, and what are the compon
 - ...and what is the physical length ℓ of the transmission-line?
 - assuming a coaxial cable as transmission-line with a dielectric constant of $\epsilon_r = 2.1$



- A transmission-line of length

$$\ell = \lambda/4 \equiv \beta\ell = \frac{\pi}{2}$$

transforms a reflection Γ at the end of the line to its input as

$$\Gamma_{in} = \Gamma e^{-j2\beta\ell} = \Gamma e^{-j\pi} = -\Gamma$$

- This results the unitless, normalized impedance z at the end of the line to be transformed into:

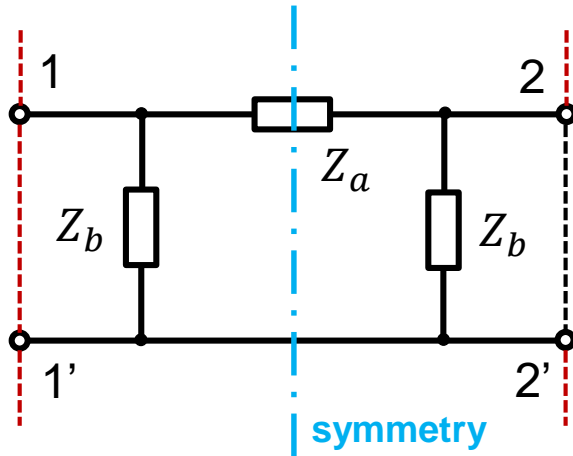
$$z_{in} = 1/z$$

at the beginning of the line

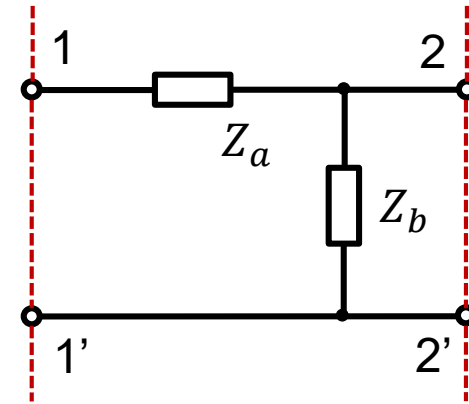
- A short circuit at the end of the **$\lambda/4$ -transformer** is transformed to an open, and vice versa
 - This is the principle of the $\lambda/4$ -resonator.

Examples for Symmetry and Reciprocity

π -network



divider-network



$$(S_{\pi}) = \frac{1}{\Delta} \begin{pmatrix} Z_a Z_b^2 - Z_0^2 (Z_a + 2Z_b) & 2Z_0 Z_b^2 \\ 2Z_0 Z_b^2 & Z_a Z_b^2 - Z_0^2 (Z_a + 2Z_b) \end{pmatrix}$$

with: $\Delta = (Z_a + Z_b)[Z_a Z_b + Z_0(Z_a + 2Z_b)]$

$S_{12} = S_{21} \wedge S_{11} = S_{22} \Rightarrow$ reciprocal and symmetric

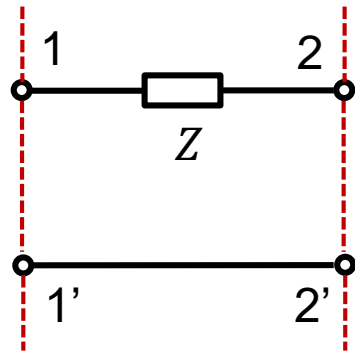
$$(S_{div}) = \frac{1}{\Delta} \begin{pmatrix} Z_a Z_b - Z_0(Z_0 - Z_a) & 2Z_0 Z_b \\ 2Z_0 Z_b & Z_a Z_b - Z_0(Z_0 + Z_a) \end{pmatrix}$$

with: $\Delta = Z_0(Z_0 + Z_a) + Z_b(2Z_0 + Z_a)$

$S_{12} = S_{21} \wedge S_{11} \neq S_{22} \Rightarrow$ reciprocal, but not symmetric

- Without prof: The S-matrix is always symmetric for reciprocal networks.

2-port series-network



$$(S_{ser}) = \frac{1}{2Z_0 + Z} \begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

$$Z = j\omega L = j10$$

$$Z = R = 10$$

$$|S_{11}| = \sqrt{1 - |S_{12}|^2}$$

$$\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$$

$$\frac{1}{11} \neq \sqrt{1 - \left(\frac{10}{11}\right)^2}$$

$$\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$$

$$\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan^{-1}\frac{1}{10} - \tan^{-1}(10) - \pi$$

$$0 - 0 \neq 0 - 0 - \pi$$

⇒ lossless

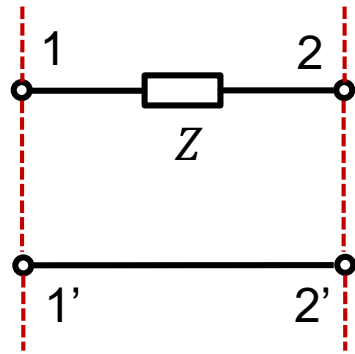
⇒ lossy

- It is evident, this ideal 4-port coupler is symmetric and reciprocal

$$S_{ij} = S_{ji} \wedge S_{ii} = S_{jj}$$

- It also is matched: $S_{ii} = 0$
- But is it lossless or lossy?

2-port series-network



$$(S_{ser}) = \frac{1}{2Z_0 + Z} \begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

$$Z = j\omega L = j10$$

$$Z = R = 10$$

$$|S_{11}| = \sqrt{1 - |S_{12}|^2}$$

$$\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$$

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$$\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan^{-1}\frac{1}{10} - \tan^{-1}(10) - \pi$$

$$0 - 0 \neq 0 - 0 - \pi$$

⇒ lossless

⇒ lossy

4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

$$(S)^\dagger(S) = (I) \Rightarrow \sum_{k=1}^N S_{ki}S_{ki}^* = 1 \wedge \sum_{k=1}^N S_{ki}S_{kj}^* = 0 \forall i \neq j$$

- Multiply matrix columns by itself with the conjugate complex
 - and test for = 1

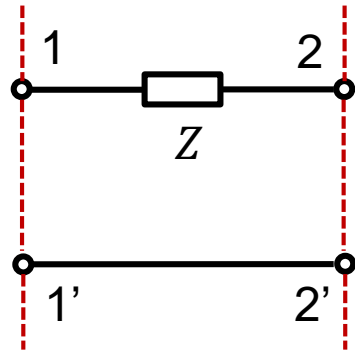
$$S_{11}S_{11}^* + S_{21}S_{21}^* + S_{31}S_{31}^* + S_{41}S_{41}^* = (0 \cdot 0 + \sqrt{3} \cdot \sqrt{3} + j \cdot (-j) + 0 \cdot 0)/2^2 = 1$$

$$S_{12}S_{12}^* + S_{22}S_{22}^* + S_{32}S_{32}^* + S_{42}S_{42}^* = (\sqrt{3} \cdot \sqrt{3} + 0 \cdot 0 + 0 \cdot 0 + j \cdot (-j))/2^2 = 1$$

$$S_{13}S_{13}^* + S_{23}S_{23}^* + S_{33}S_{33}^* + S_{43}S_{43}^* = (j \cdot (-j) + 0 \cdot 0 + 0 \cdot 0 + \sqrt{3} \cdot \sqrt{3})/2^2 = 1$$

$$S_{14}S_{14}^* + S_{24}S_{24}^* + S_{34}S_{34}^* + S_{44}S_{44}^* = (0 \cdot 0 + j \cdot (-j) + \sqrt{3} \cdot \sqrt{3} + 0 \cdot 0)/2^2 = 1$$

2-port series-network



$$(S_{ser}) = \frac{1}{2Z_0 + Z} \begin{pmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{pmatrix}$$

$$S_{11} = S_{22} \wedge S_{12} = S_{21}$$

$$Z = j\omega L = j10$$

$$Z = R = 10$$

$$|S_{11}| = \sqrt{1 - |S_{12}|^2}$$

$$\frac{1}{\sqrt{101}} = \sqrt{1 - \left(\frac{10}{\sqrt{101}}\right)^2}$$

$$\frac{1}{11} \neq \sqrt{1 - \left(\frac{10}{11}\right)^2}$$

$$\angle S_{11} - \angle S_{12} = \angle S_{21} - \angle S_{22} - \pi$$

$$\tan^{-1}(10) + \tan^{-1}\frac{1}{10} = -\tan^{-1}\frac{1}{10} - \tan^{-1}(10) - \pi$$

$$0 - 0 \neq 0 - 0 - \pi$$

⇒ **lossless**

⇒ **lossy**

4-port ideal directional coupler

$$(S_{dc}) = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{3} & j & 0 \\ \sqrt{3} & 0 & 0 & j \\ j & 0 & 0 & \sqrt{3} \\ 0 & j & \sqrt{3} & 0 \end{pmatrix}$$

$$(S)^\dagger(S) = (I) \Rightarrow \sum_{k=1}^N S_{ki}S_{ki}^* = 1 \wedge \sum_{k=1}^N S_{ki}S_{kj}^* = 0 \forall i \neq j$$

- **Multiply all different matrix columns with the conjugate complex**
 - and test for = 0

$$S_{11}S_{12}^* + S_{21}S_{22}^* + S_{31}S_{32}^* + S_{41}S_{42}^* = (0 \cdot \sqrt{3} + \sqrt{3} \cdot 0 + j \cdot 0 + 0 \cdot (-j))/2^2 = 0$$

$$S_{11}S_{13}^* + S_{21}S_{23}^* + S_{31}S_{33}^* + S_{41}S_{43}^* = (0 \cdot (-j) + \sqrt{3} \cdot 0 + j \cdot 0 + 0 \cdot \sqrt{3})/2^2 = 0$$

$$S_{11}S_{14}^* + S_{21}S_{24}^* + S_{31}S_{34}^* + S_{41}S_{44}^* = (0 \cdot 0 + \sqrt{3} \cdot (-j) + j \cdot \sqrt{3} + 0 \cdot 0)/2^2 = 0$$

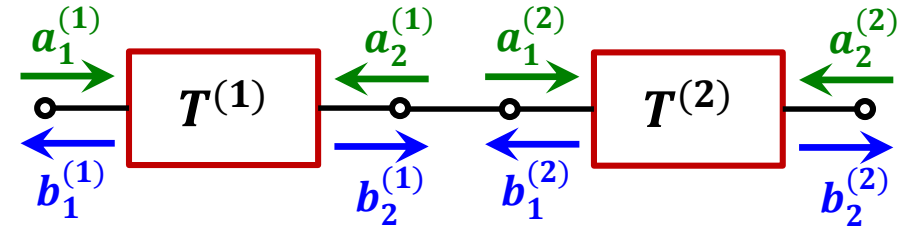
$$S_{12}S_{13}^* + S_{22}S_{23}^* + S_{32}S_{33}^* + S_{42}S_{43}^* = (\sqrt{3} \cdot (-j) + 0 \cdot 0 + 0 \cdot 0 + j \cdot \sqrt{3})/2^2 = 0$$

$$S_{12}S_{14}^* + S_{22}S_{24}^* + S_{32}S_{34}^* + S_{42}S_{44}^* = (\sqrt{3} \cdot 0 + 0 \cdot (-j) + 0 \cdot \sqrt{3} + j \cdot 0)/2^2 = 0$$

$$S_{13}S_{14}^* + S_{23}S_{24}^* + S_{33}S_{34}^* + S_{43}S_{44}^* = (j \cdot 0 + 0 \cdot (-j) + 0 \cdot \sqrt{3} + \sqrt{3} \cdot 0)/2^2 = 0$$

- Cascading e.g., 2-port S-parameter files is important to characterize a larger RF system.
 - Solution: Transfer (T) parameters, which directly relates the waves at input and output

$$(T) = \begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix} \Rightarrow \begin{aligned} b_1 &= T_{11}a_2 + T_{12}b_2 \\ a_1 &= T_{21}a_2 + T_{22}b_2 \end{aligned}$$



- T-parameters enable cascaded 2-port networks by simply multiplying their matrices:

$$(T) = (T^{(1)})(T^{(2)}) \dots (T^{(N)}) = \prod_{i=1}^N (T^{(i)})$$

- Relation between 2-port **T-parameters** and **S-parameters**:

$$(T) = \frac{1}{S_{21}} \begin{pmatrix} -\det(S) & S_{11} \\ -S_{22} & 1 \end{pmatrix}$$

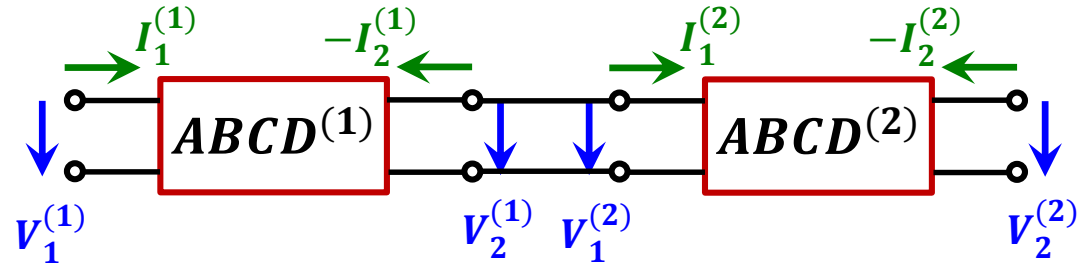
with: $\det(S) = S_{11}S_{22} - S_{12}S_{21}$

$$(S) = \frac{1}{T_{22}} \begin{pmatrix} T_{12} & \det(T) \\ 1 & -T_{21} \end{pmatrix}$$

with: $\det(T) = T_{11}T_{22} - T_{12}T_{21}$

- Also called “chain” parameters, used for cascading networks based on V and I
 - Useful for chaining a mix of lumped elements and transmission-lines

$$(ABCD) = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \Rightarrow \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$



- Chaining 2-port ABCD-parameter networks:

$$(ABCD) = (ABCD^{(1)})(ABCD^{(2)}) \dots (ABCD^{(N)}) = \prod_{i=1}^N (ABCD^{(i)})$$

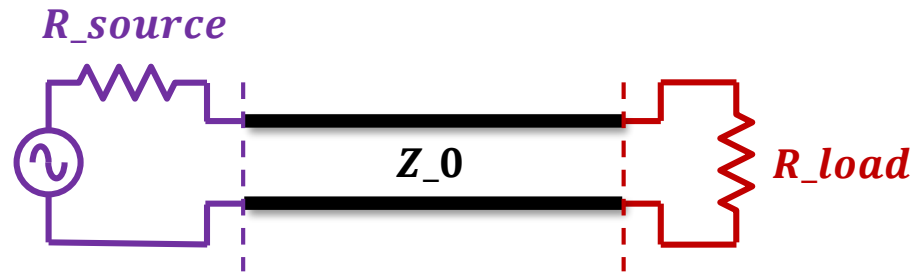
- Relation between 2-port **ABCD-parameters** and **S-parameters**:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$$

$$S_{12} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D}$$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D}$$

$$S_{22} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D}$$



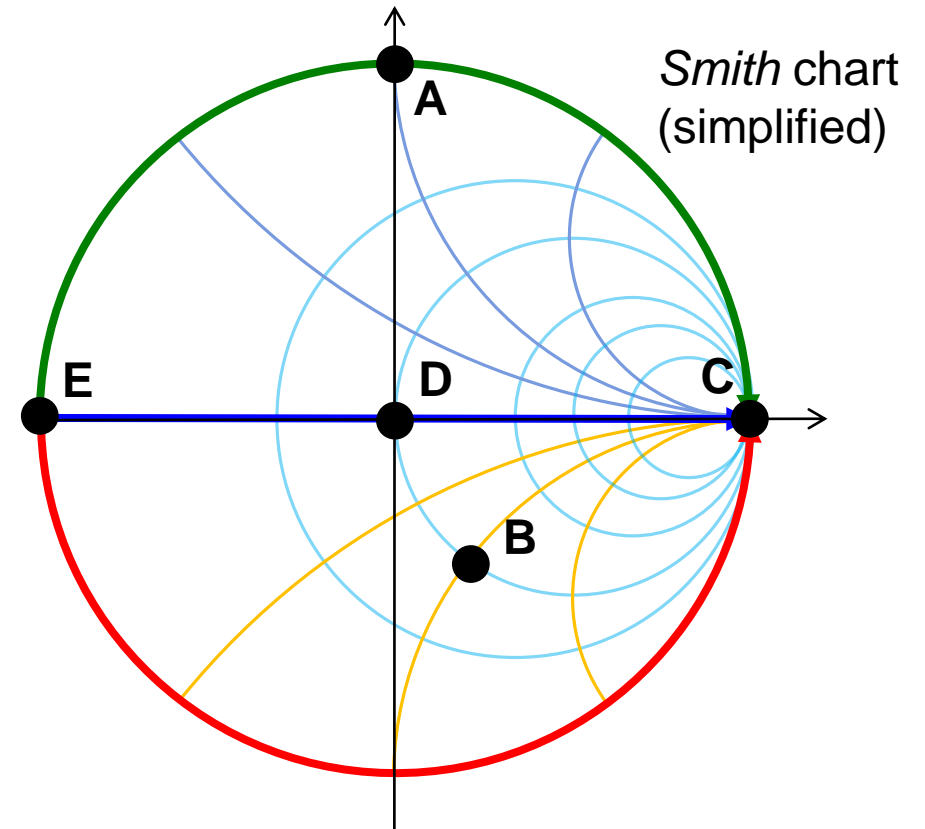
1. When do no signal reflections occur at the end of a transmission-line?

- $R_source = R_load$
- $R_source = Z_0$
- $Z_0 = R_load$
- $R_source = Z_0 = R_load$

2. The Smith chart transforms the complex impedance plane onto the complex Gamma (reflection coefficient) plane within the unit circle.

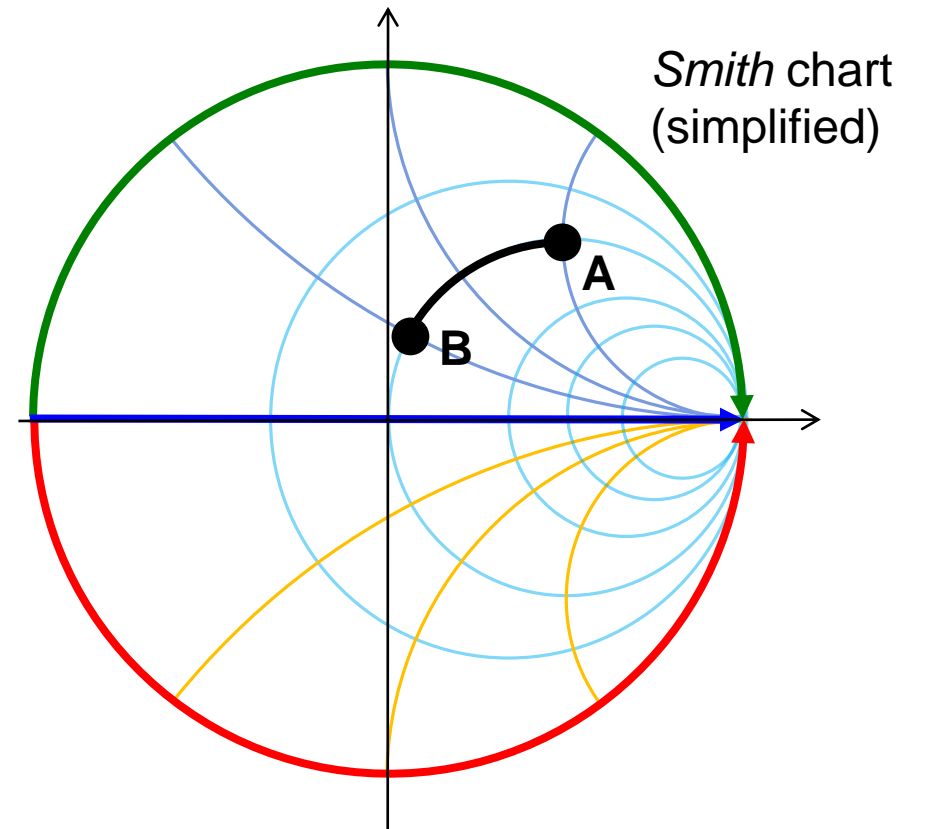
Quiz (2)

Prompts		Possible Answers
A. Point A	A5	1. Gamma = +1, z -> infinity
B. Point B	B4	2. Gamma = -j
C. Point C	C1	3. Gamma = 0, z = 1, match
D. Point D	D3	4. Point in the capacitive half plane
E. Point E	E6	5. Gamma = +j
		6. Gamma = -1, z = 0
		7. Point in the inductive half plane



4. Trace with marker points in the simplified Smith chart for an RL series impedance

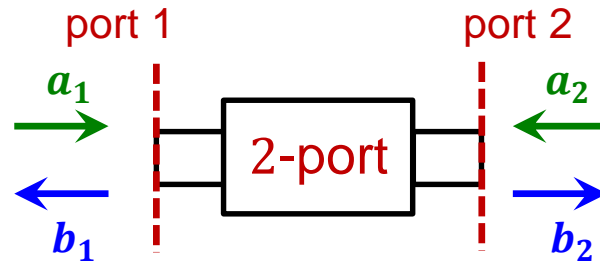
- Frequency f at point B $>$ frequency f at point A
- Frequency f at point B $<$ frequency f at point A
- There is no frequency related to points A and B
- Frequency f at point A = frequency f at point B



1. Select all correct answers

- Y- and Z-parameters of electrical networks require a reference impedance Z_0
- Scattering parameters of RF networks are based on normalize complex voltage waves incident and reflected / transmitted at their ports
- DUT stands for “Device Under Test”, as acronym for the RF network to be characterized
- S-parameters are only defined for a reference impedance of $Z_0 = 50 \text{ Ohm}$.
- Unused ports in a S-parameter measurement setup always need to be terminated in their characteristic port impedance

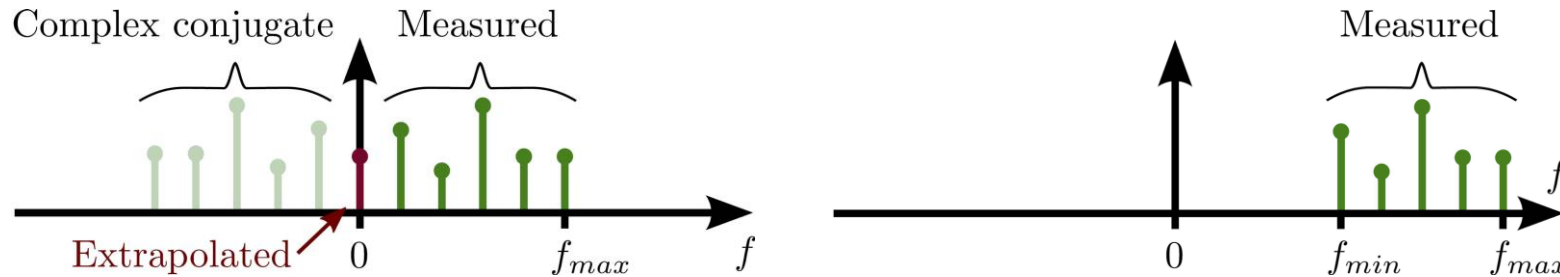
Prompts		Possible Answers
A. matched	A4	1. $S_{ii} = S_{ij}$
B. symmetric	B3	2. $(S^*)^T = (i)$
C. reciprocal	C5	3. $S_{ij} = S_{ji}$ and $S_{ii} = S_{jj}$
D. passive and lossless	D2	4. $S_{ii} = 0$
		5. $\Gamma = +j$
		6. $S_{ij} = S_{ji}$



3. Mark all correct answers for the S-parameters of a 2-port RF network

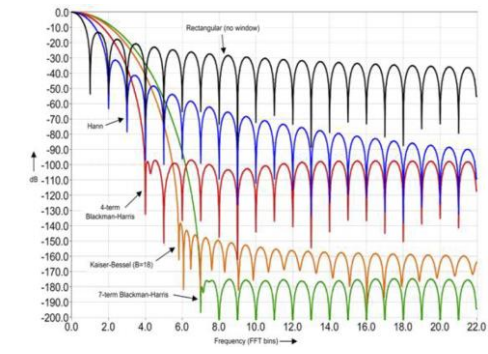
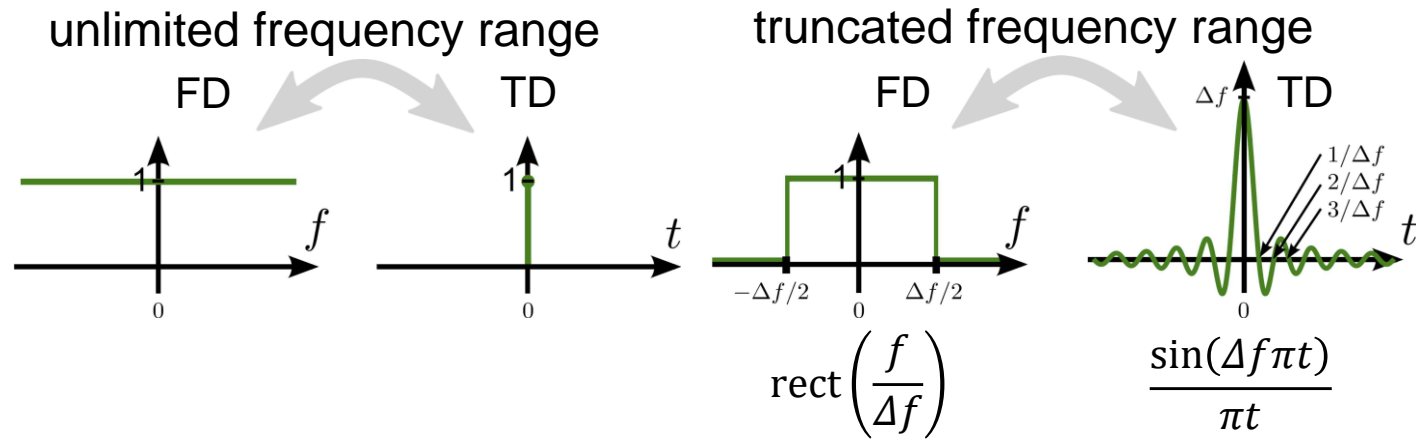
- a_1 and b_1 are independent parameters
- S_{11} is the input reflection coefficient
- a_1 and a_2 are the incident waves at port 1 and port 2, respectively.
- b_1 and b_2 are the transmitted waves between port 1 and port 2, and vice versa.
- S_{21} and S_{12} are the forward and reverse transmission gains.
- To characterize the S-parameters at port 2, port 1 needs to be terminated in its characteristic port impedance.

- Based on an inverse discrete *Fourier* transformation (iDFT) option in the VNA



- **Low-pass mode: Impulse or step response, relying on equidistant samples over the extrapolated (to DC) frequency range.**
 - **The VNA does not measure at DC!**
 - Manually match frequency range and # of points for DC extrapolation, e.g., 1...1000 MHz -> 1001 points, to enable extrapolation exactly to DC, or let the instrument chose the extrapolation settings automatically
- **Enables time-domain reflectometry (TDR)**
 - Very useful on portable VNAs, troubleshooting RF cable problems
- **Band-pass response (no DC extrapolation)**
- **Allows time-domain gating and de-embedding of non-resonant sub-systems, e.g., measurements on a PCB**
- **Limited to linear systems**
- **Select the "real" format for S_{11} or S_{21} for time-domain transformations (*Keysight* instruments)!**
 - or dB magnitude to detect small reflections in TDR analysis

Synthetic Pulse TD Measurements (2)



smoothing window functions

