# RF Manipulations I & II

*· Longitudinal Regulation of Beams in Synchrotrons ·*


Helga Timko
CERN, SY-RF Group

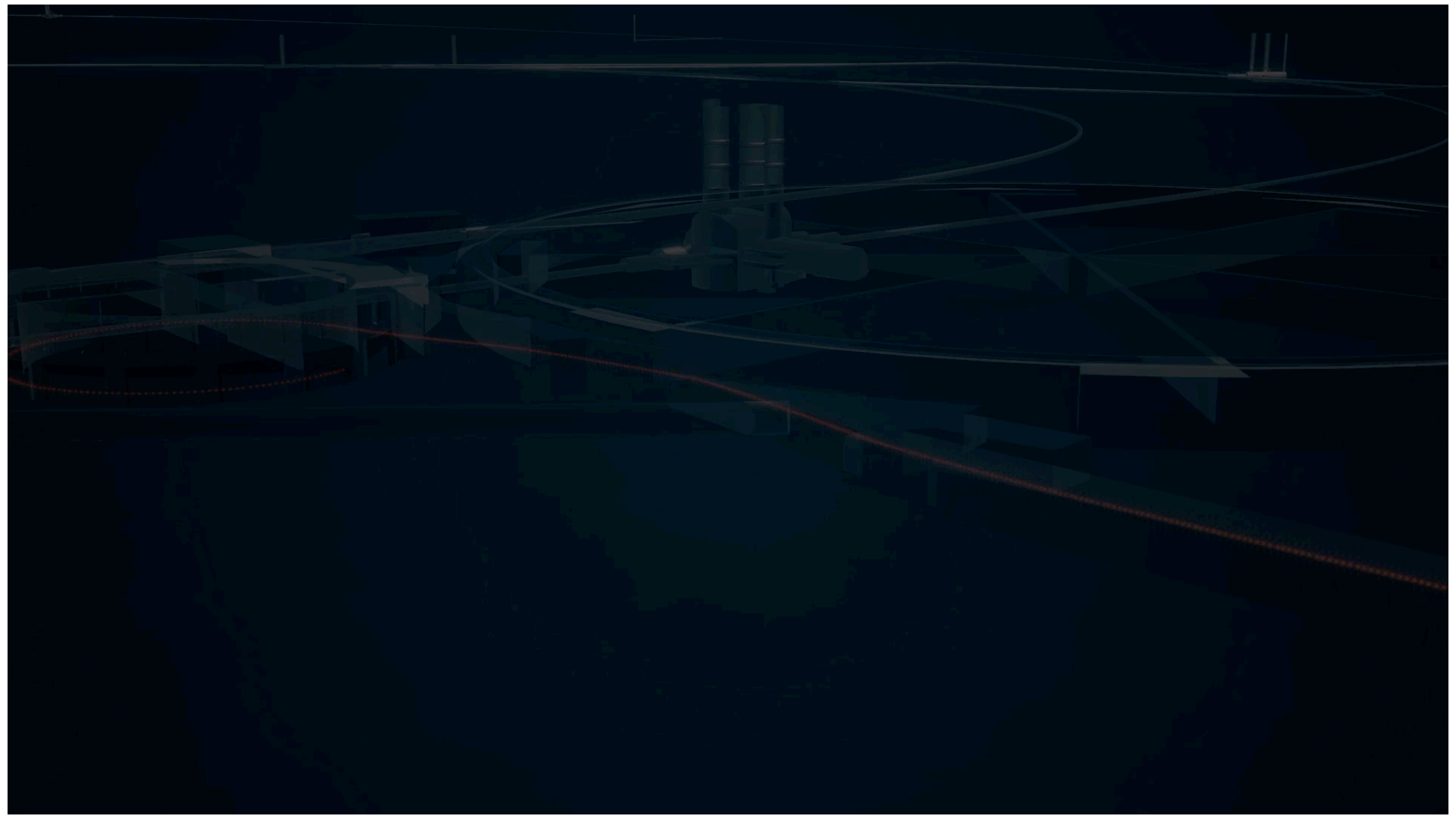

Advanced CAS, Spa, Belgium, 13th November 2024

*With many thanks to S. Albright, T. Argyropoulos, H. Damerau, B. Karlsen-Baeck, A. Lasheen, N. Gallou, and I. Karpov*

# What are RF manipulations and why do them?

**RF manipulations ('gymnastics') = manipulations done by changing the RF voltage, phase, frequency, harmonic…**

**…for regulating the beam longitudinally**



Example: CERN proton injector chain

| | Proton Synchrotron Booster (PSB) → | Proton Synchrotron (PS) → | Super Proton Synchrotron (SPS) → | Large Hadron Collider (LHC) |
|---|---|---|---|---|
| **bunches** | 1 bunch/ring (4 rings) | 2 injections,<br>6 → 36/48/72 bunches | 3-5 injections,<br>up to 288 bunches | ≳20 injections,<br>up to 2808 bunches |

***How to make so many bunches out of one?***

| | | | | |
|---|---|---|---|---|
| **bucket length** | >500 ns | >25 ns | 5 ns | 2.5 ns |

***How to make these bunches fit into the bucket?***

# Contents

## Bunch length regulation

- Adiabatic changes
- Rotation
- Splitting, merging

## Phase space regulation

- Diffusion, noise injection
- Resonant excitation
- Longitudinal painting
- Debunching

## Advanced manipulations

- Momentum slip stacking
- Barrier bucket

## Integration in an RF system

- Beam loading
- RF voltage/power limitations
- Designing RF parameters

# Contents

# Coordinates & notations

**Single RF system, no intensity effects**

- Frequently used conjugate variables are [1] $(\varphi, \Delta E/\omega_{\mathrm{rev}})$
  - Kick and drift equations

$$\dot{\varphi} = \frac{h\omega_{\mathrm{rev}}^2\eta}{\beta_d^2 E_d}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = h\omega_{\mathrm{rev}}\eta\delta$$

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{\beta_d^2 E_d}{\omega_{\mathrm{rev}}}\dot{\delta} = \frac{eV}{2\pi}(\sin\varphi - \sin\varphi_d)$$

  - Hamiltonian

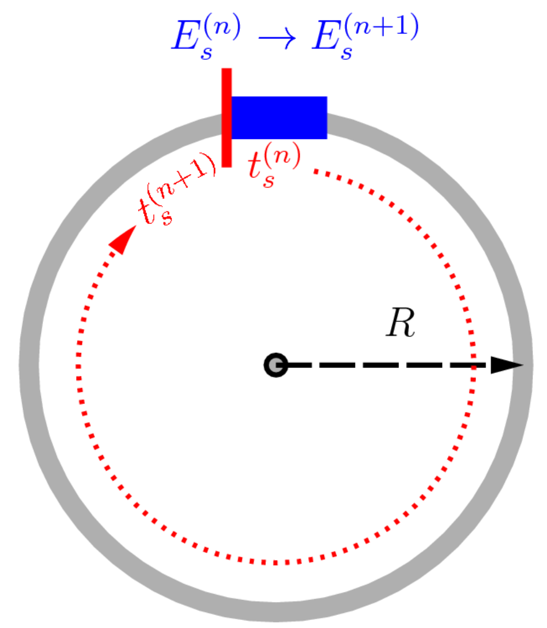
$$H\left(\varphi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{\beta_d^2 E_d}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 + \frac{eV}{2\pi}\{\cos\varphi - \cos\varphi_d + (\varphi - \varphi_d)\sin\varphi_d\}$$

- Subscript ‘d’ denotes the design particle
  - Following the design energy and frequency ramp
  - In simple cases, corresponds to the synchronous particle

| | |
|---|---|
| $\beta_d$ | relativistic beta from design momentum |
| $\Delta E = E - E_d$ | relative particle energy |
| $\delta = \dfrac{\Delta p}{p_d} \simeq \dfrac{\Delta E}{\beta_d^2 E_d}$ | relative momentum offset |
| $\eta = \eta(\delta) = \eta_0 + \eta_1\delta + \eta_2\delta^2 + \ldots$ | slippage factor |
| $\varphi$ | RF voltage phase at particle arrival |
| $\varphi_d$ | phase of design particle |
| $\omega_{\mathrm{rev}}$ | angular revolution frequency |
| $e$ | elementary charge |
| $E$ | particle energy |
| $E_d$ | energy of design particle |
| $E$ | particle energy |
| $h$ | harmonic number |
| $V$ | RF voltage amplitude |

[1] S. Y. Lee: ‘Accelerator Physics’, World Scientific, 3rd Ed. (2012).

# Coordinates & notations




## General kick and drift equations

- For tracking, more convenient conjugate variables are [2] $(\Delta t, \Delta E)$
  - Coordinate transformation: $\varphi = \omega_{\mathrm{rf}} \Delta t + \varphi_{\mathrm{rf}}$
  - Kick equation (discrete)

$$\Delta E_{(n+1)} = \Delta E_{(n)} + \sum_{k=1}^{n_{\mathrm{rf}}} q V_{k,(n)} \sin(\omega_{\mathrm{rf},k,(n)} \Delta t_{(n)} + \varphi_{\mathrm{rf},k,(n)}) - (E_{d,(n+1)} - E_{d,(n)}) + E_{\mathrm{other},(n)}$$

Multiple RF systems in one location

Change of design energy (magnetic field)

All other effects on energy (e.g. induced voltage)

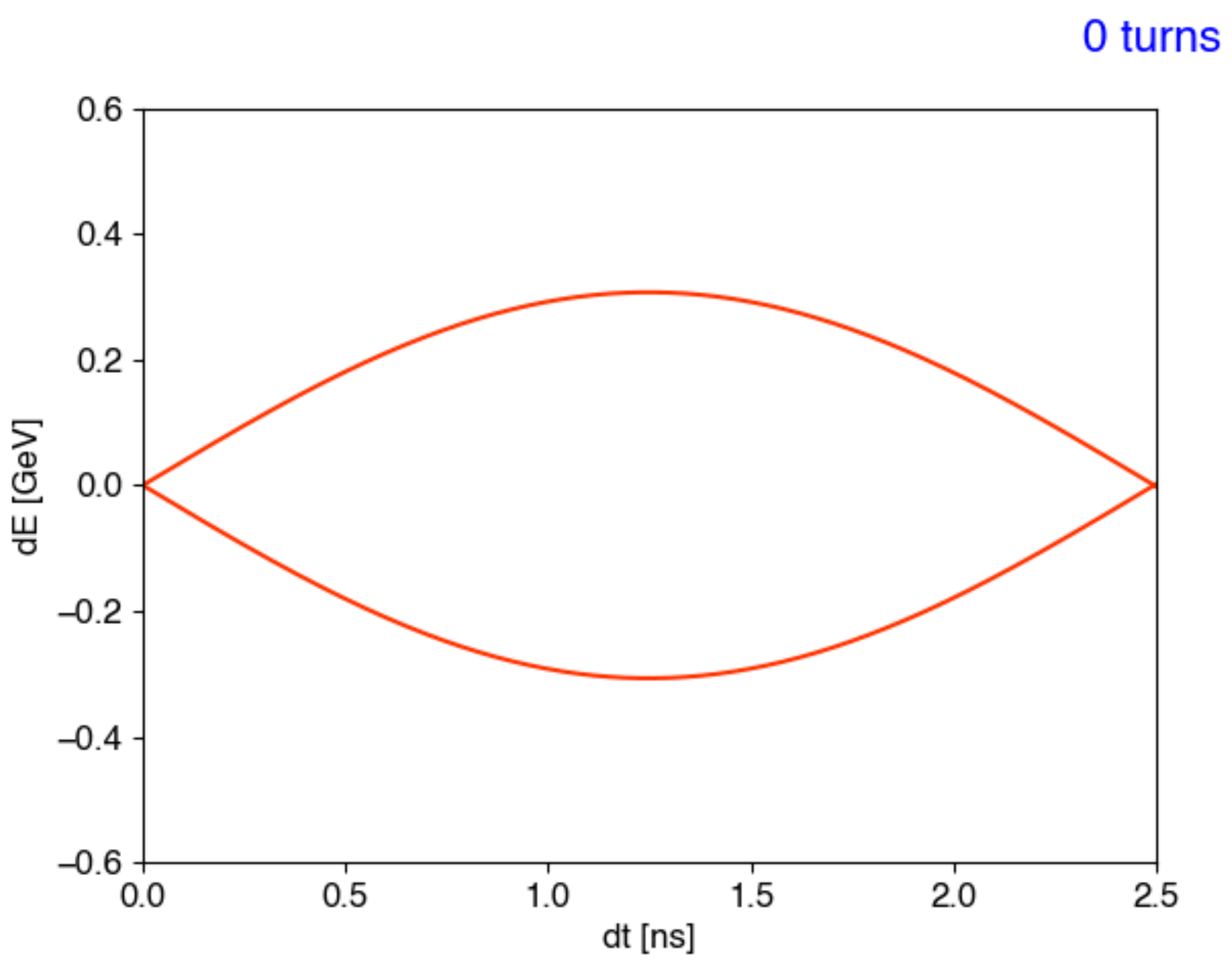
  - Drift equation (discrete)

$$\Delta t_{(n+1)} = \Delta t_{(n)} + T_{\mathrm{rev},(n+1)} \frac{\eta_{0,(n+1)} \Delta E_{(n+1)}}{\beta_{d,(n+1)}^2 E_{d,(n+1)}} + \mathcal{O}(\delta^2)$$

  - Hamiltonian (continuous)

$$H(\Delta t, \Delta E) = \frac{\eta_0}{2\beta_d^2 E_d} (\Delta E)^2 + \sum_{k=0}^{n_{\mathrm{rf}}-1} \frac{q V_k}{T_{\mathrm{rev}} \omega_{\mathrm{rf}}^k} \left( \cos(\omega_{\mathrm{rf}}^k \Delta t + \varphi_{\mathrm{rf}}^k) - \cos(\omega_{\mathrm{rf}}^k \Delta t_d + \varphi_{\mathrm{rf}}^k) \right) +$$
$$+ \dot{E}_d (\Delta t - \Delta t_d) + \frac{E_{\mathrm{other}}}{T_{\mathrm{rev}}} (\Delta t - \Delta t_d) \,.$$

| | |
|---|---|
| $\Delta t = t - t_d$ | relative particle arrival time |
| $t_{d,(n)} \equiv \sum_{k=1}^{n} T_{\mathrm{rev},(k)}$ | design turn-by-turn arrival time |




*Example: synchrotron motion of a single particle, single RF, no acceleration, no intensity effects*



[2] H. Timko *et al.*: 'Beam Longitudinal Dynamics simulation suite BLonD', Phys. Rev. Accel. Beams **26**, 114602 (2023).

QUIZ

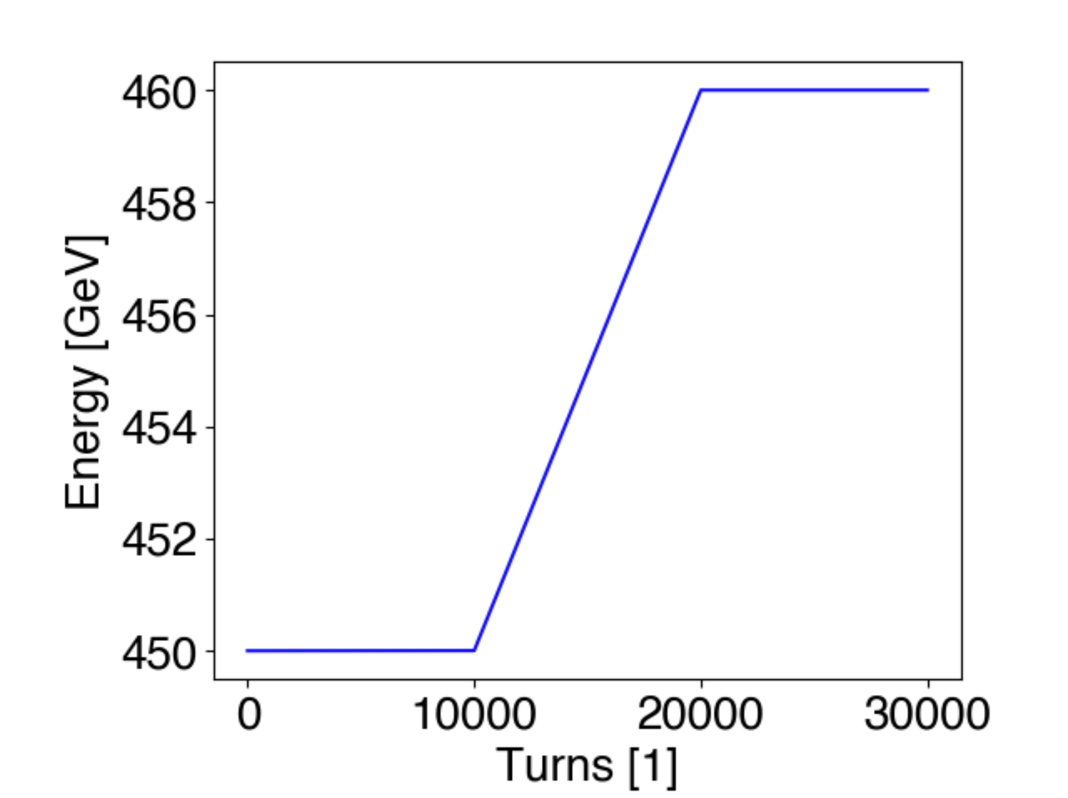
## What happens to the synchronous phase when the energy is first constant, then increased linearly (everything else remains constant)?

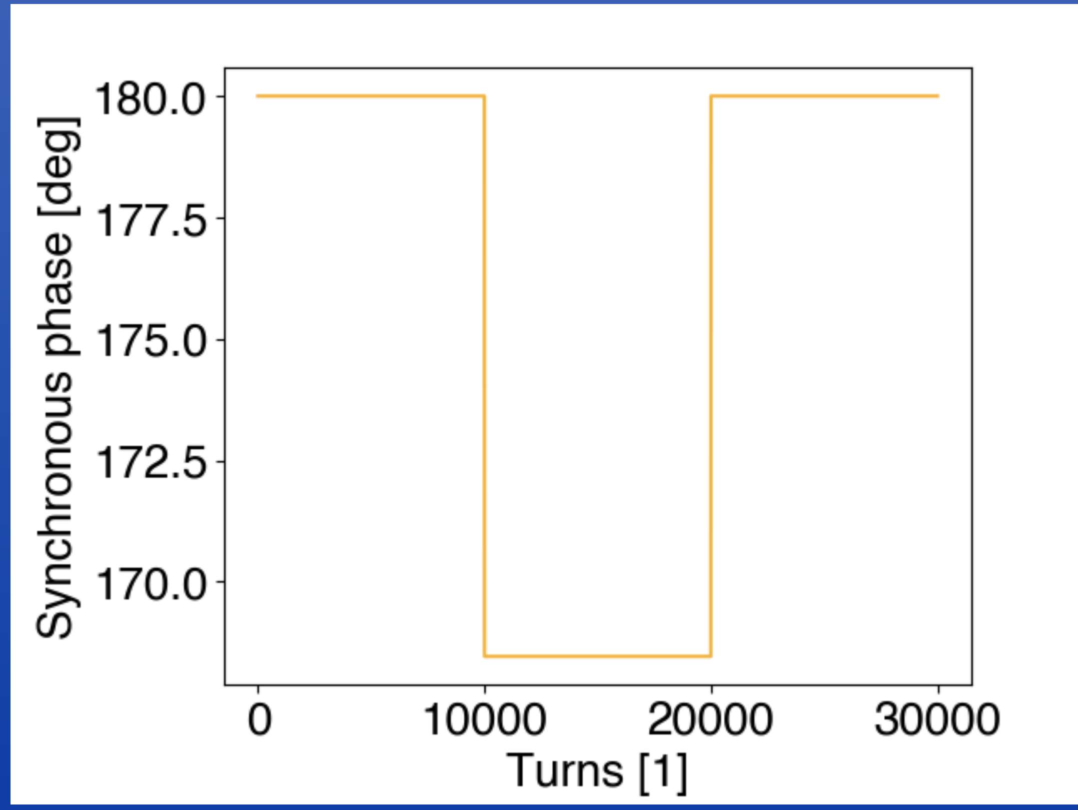
TIPS

*For a single RF system, and in the absence of intensity effects,* $\sin \varphi_d = \frac{\dot{E}_d T_{\text{rev}}}{qV}$

**Solution**

- There will be discontinuity in the synchronous phase



# Beam observables

**Beam profile**

- Gaussian: $\lambda(\Delta t) = \lambda_0 e^{\frac{-\Delta t^2}{\sigma_t^2}}$
- Binomial: $\lambda(\Delta t) = \lambda_0 \left(1 - 4\left(\Delta t/\tau_{\mathrm{full}}\right)^2\right)^{\mu+\frac{1}{2}}$

**Bunch length**

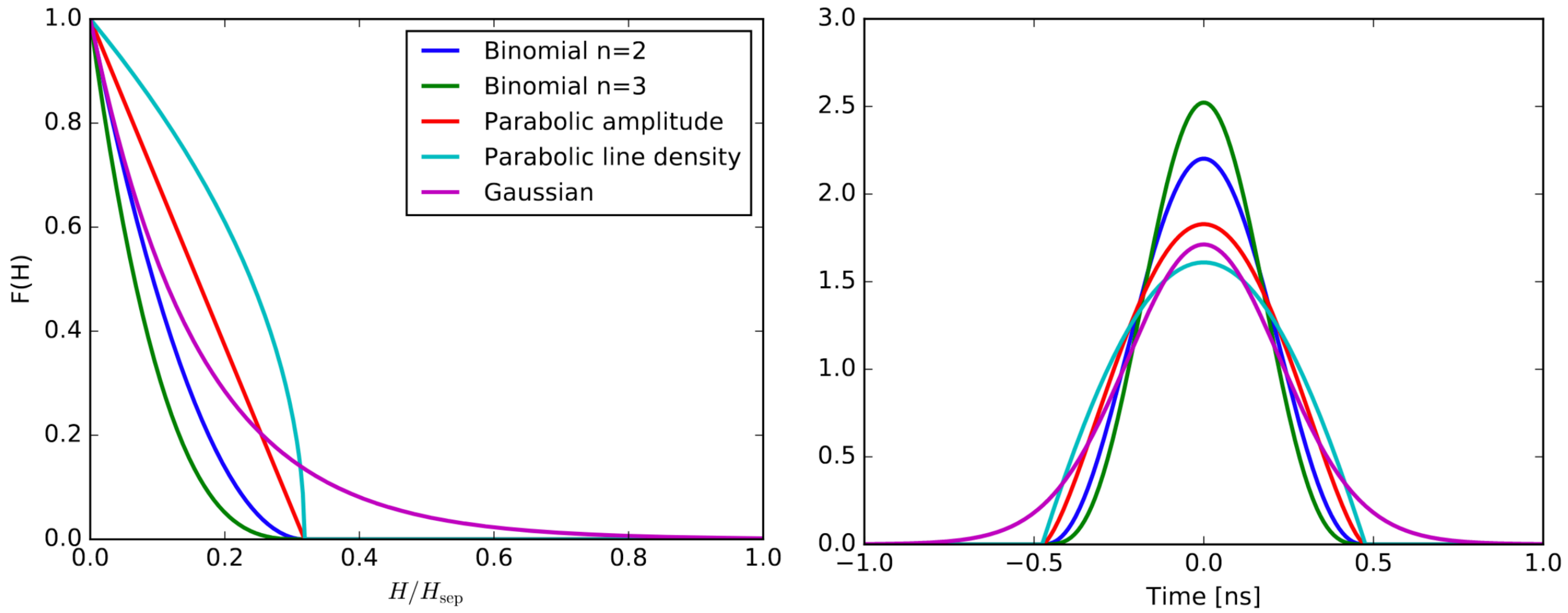
- Full bunch length $\tau_{\mathrm{full}}$
- R.m.s. bunch length $\sigma_t$
- Four-sigma equivalent FWHM bunch length:

$$\tau_{4\sigma} \equiv \frac{2}{\sqrt{2 \ln 2}} \tau_{\mathrm{FWHM}}$$

**Bunch emittance**

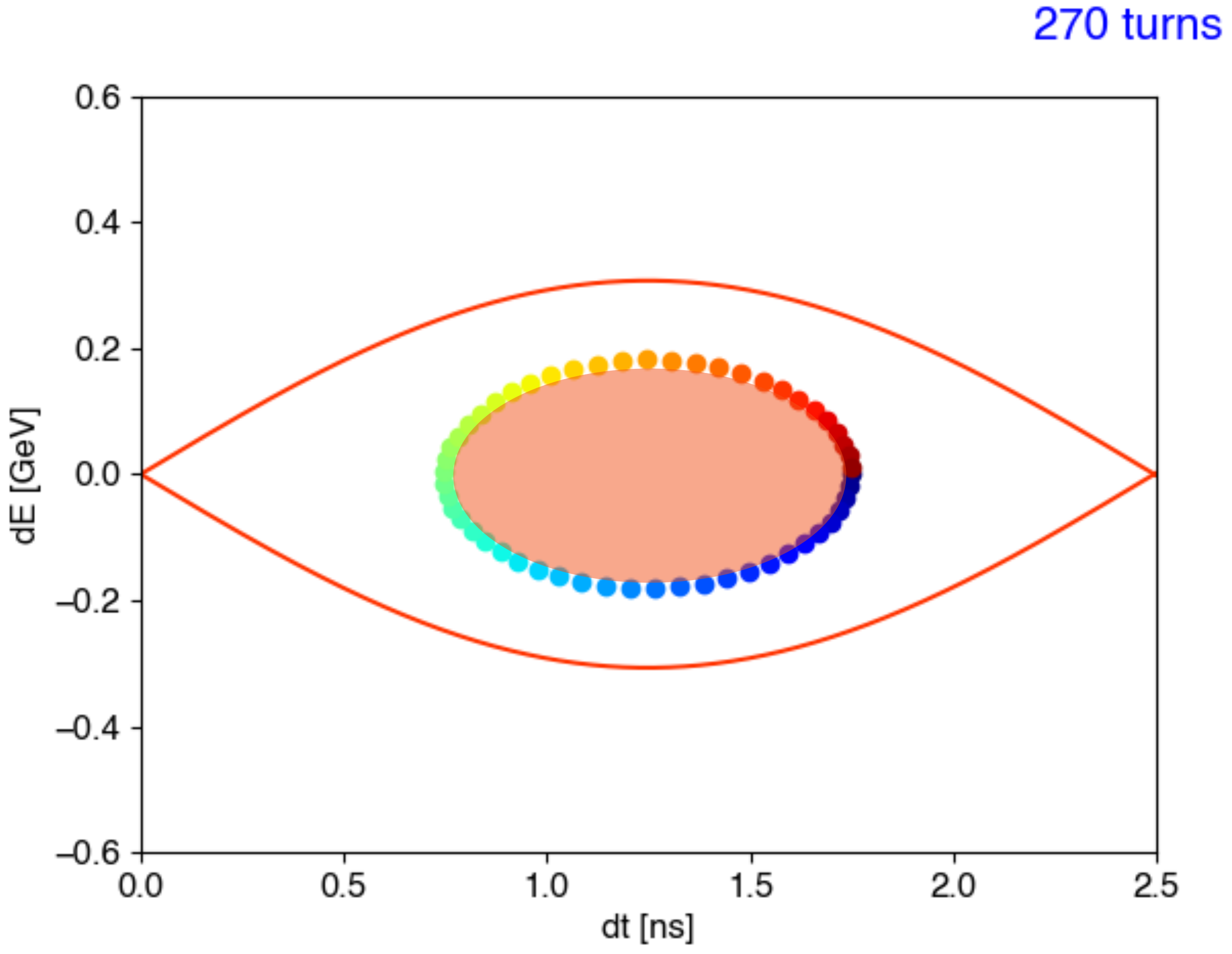
- The area enclosed by the phase-space trajectory at a given bunch length

$$\varepsilon \equiv \oint_{\Delta t = \tau/2} \Delta E(\Delta t)\, d(\Delta t)$$




*Distribution functions (left) and line densities (right) from [3]*

*Emittance determined by a given contour*






[3] J. Esteban Müller: 'Longitudinal intensity effects in the CERN Large Hadron Collider', CERN-THESIS-2016-066 (2016).

# Adiabatic change of RF voltage

**How to do it?**

- Slowly in(de)crease the voltage to de(in)crease the bunch length

$$\frac{\tau'}{\tau} = \sqrt[4]{\frac{V}{V'}}$$

  - Drawback: already a small decrease requires a large voltage change

**What is adiabatic?**

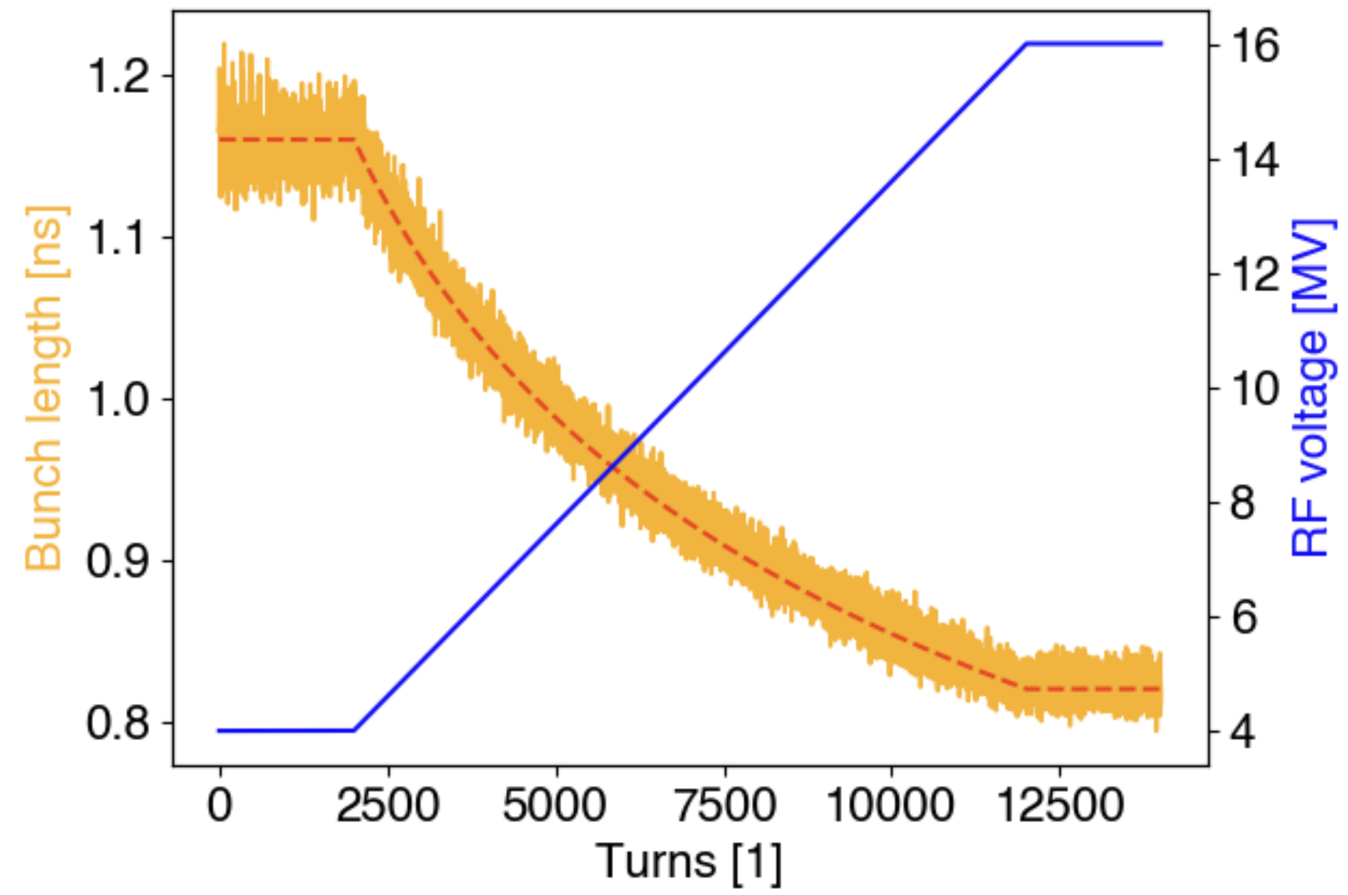
- Has to be slow w.r.t. synchrotron period, adiabaticity parameter:

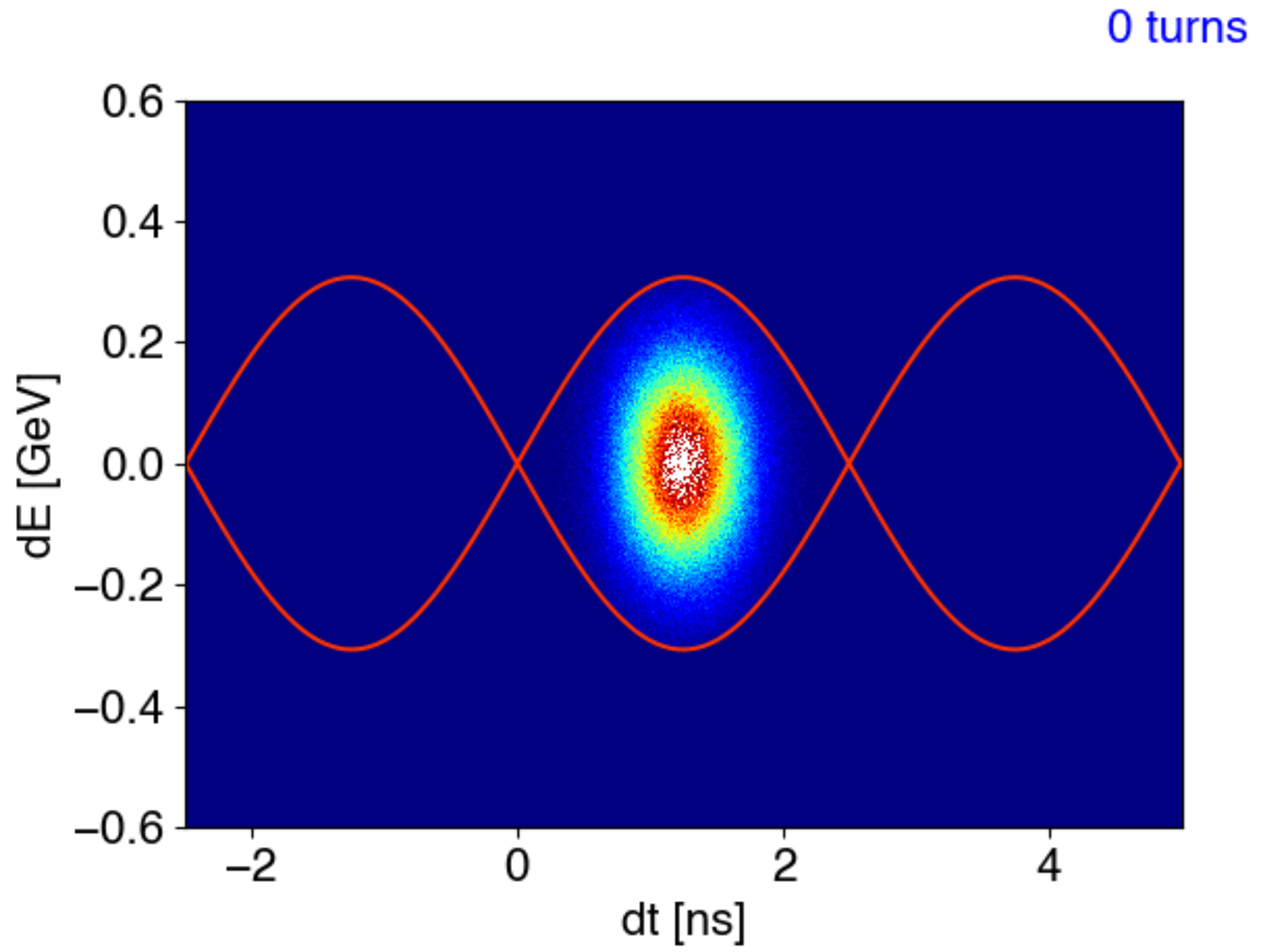
$$\alpha \equiv \frac{1}{\omega_s^2} \left| \frac{d\omega_s}{dt} \right| \ll 1, \ (\mathcal{O}(0.1)) \qquad \omega_{s,0} = \omega_{\text{rev}} \sqrt{\frac{heV|\eta_0 \cos\varphi_d|}{2\pi\beta_d^2 E_d}}$$

- In our example, the change happens over 10'000 turns, while the synchrotron period decreases from 240 to 120 turns

**When is it applied?**

- Voltage increase: typically in the acceleration ramp, also for beam stability reasons
- Voltage decrease: e.g. to create continuous beam for fixed target experiments



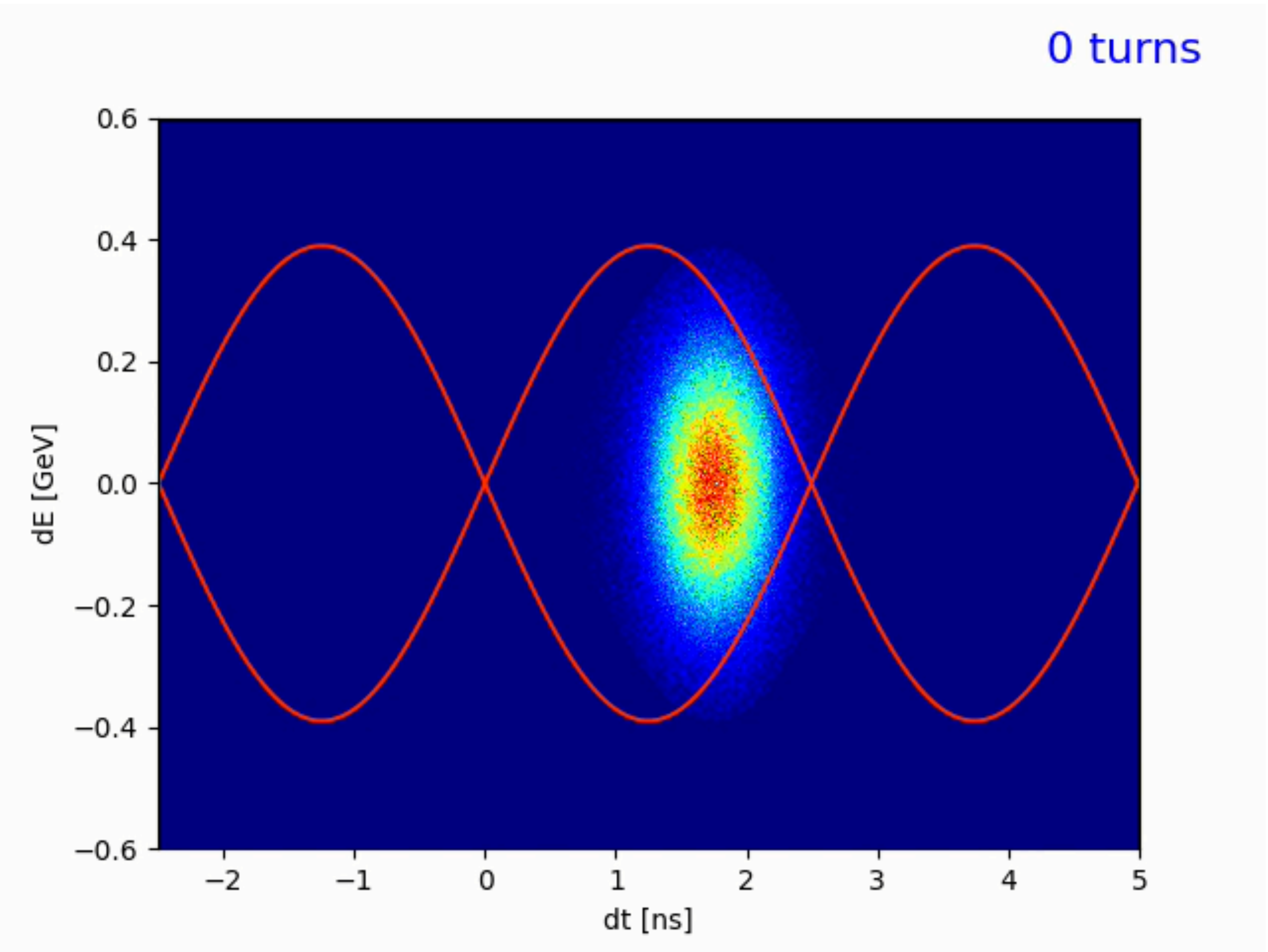




*Example: LHC flat bottom*
*4 MV → 16 MV results in 1.16 ns → 0.82 ns*

# Injection errors

**Phase and energy errors**

- Usually undesired
  - Use beam phase and frequency loops to counteract it
- Causes beam losses and filamentation
- On rare occasions, can also be used for lengthening the bunch!




*Filamentation after injection with phase error*

# Separatrix and bucket

**Unstable fixed point and turning point**

- Unstable fixed point (UFP): $(\pi - \varphi_d, 0)$
- Turning point: $(\varphi_u, 0)$
  - Determined by the equation

$$H(\pi - \varphi_d, 0) = H(\varphi_u, 0)$$

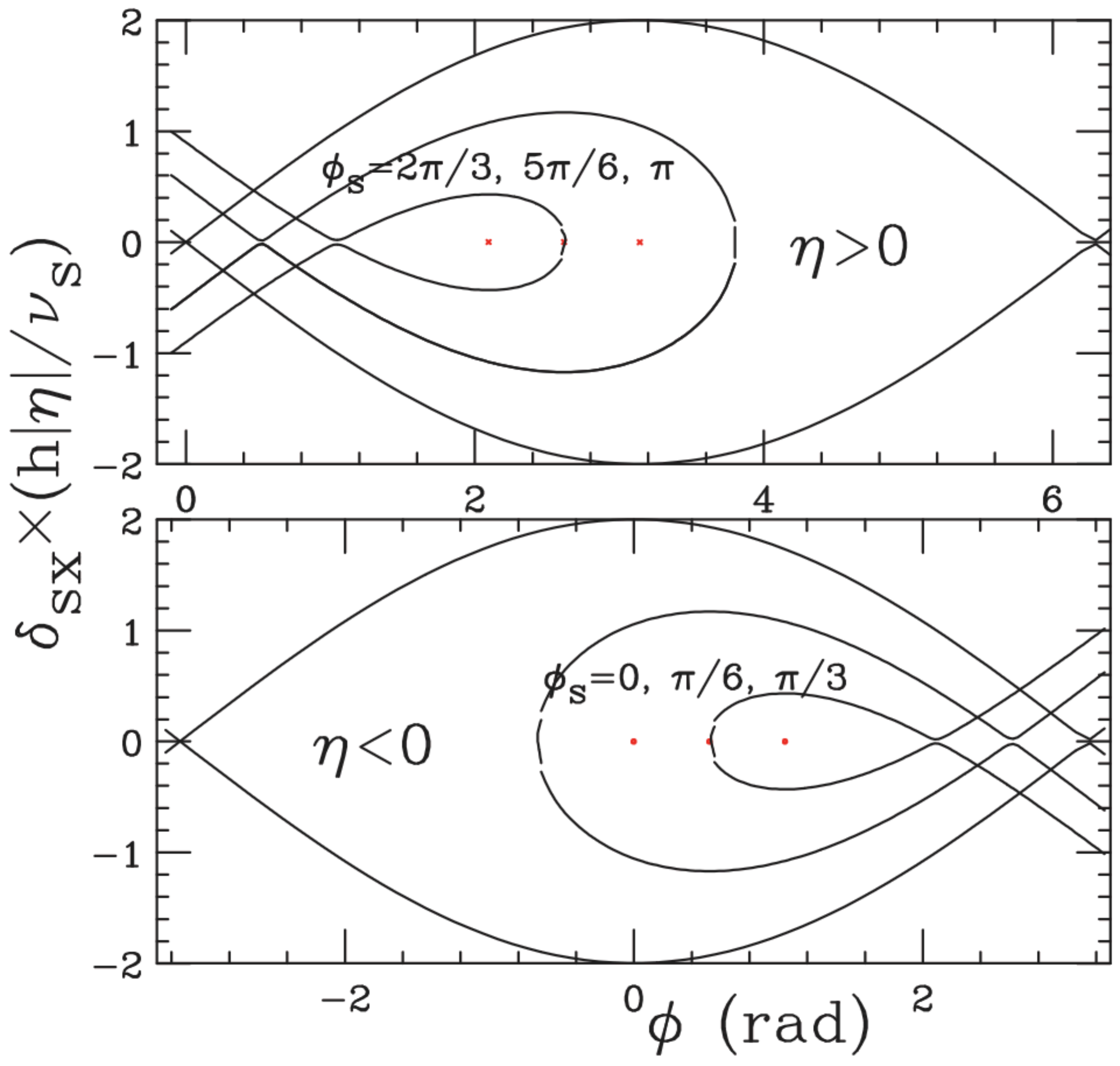
**Separatrix**

- Determined by the equation $H(\pi - \varphi_d, 0) = H(0, \Delta E_{\text{sep}})$
- No intensity effects:

$$\Delta E_{\text{sep}} = \pm \sqrt{\frac{2\beta_d^2 E_d}{\eta_0} \left\{ \sum_{k=0}^{n_{\text{rf}}-1} \frac{qV_k}{T_{\text{rev}}\omega_{\text{rf}}^k} \left[ \cos(\omega_{\text{rf}}^k \Delta t_{\text{ufp}} + \varphi_{\text{rf}}^k) - \cos(\omega_{\text{rf}}^k \Delta t_{\text{sep}} + \varphi_{\text{rf}}^k) \right] + \dot{E}_d(\Delta t_{\text{ufp}} - \Delta t_{\text{sep}}) \right\}}$$

- Using $\varphi_{\text{ufp}} = \pi - \varphi_s$ and $\dot{E}_d = qV \sin \varphi_s / T_{\text{rev}}$

$$\Delta E_{\text{sep}} = \pm \sqrt{\frac{2\beta_d^2 E_d qV}{2\pi h \eta_0} \{ \cos(\pi - \varphi_d) - \cos \varphi + (\pi - \varphi_d - \varphi) \sin \varphi_d \}}$$




*Separatrices for different stable phase values [1]*

*Top: above transition energy*
*Bottom: below transition energy*

[1] S. Y. Lee: 'Accelerator Physics', World Scientific, 3rd Ed. (2012).

# Adiabatic change of bucket area

**Magnetic ramp**

- To avoid beam losses, the bucket area should not decrease
- Make adiabatic changes, avoiding discontinuities in $\dot{E}_d$
  - Simplest model: parabolic - linear - parabolic energy ramp
- Bucket area in [eVs]:

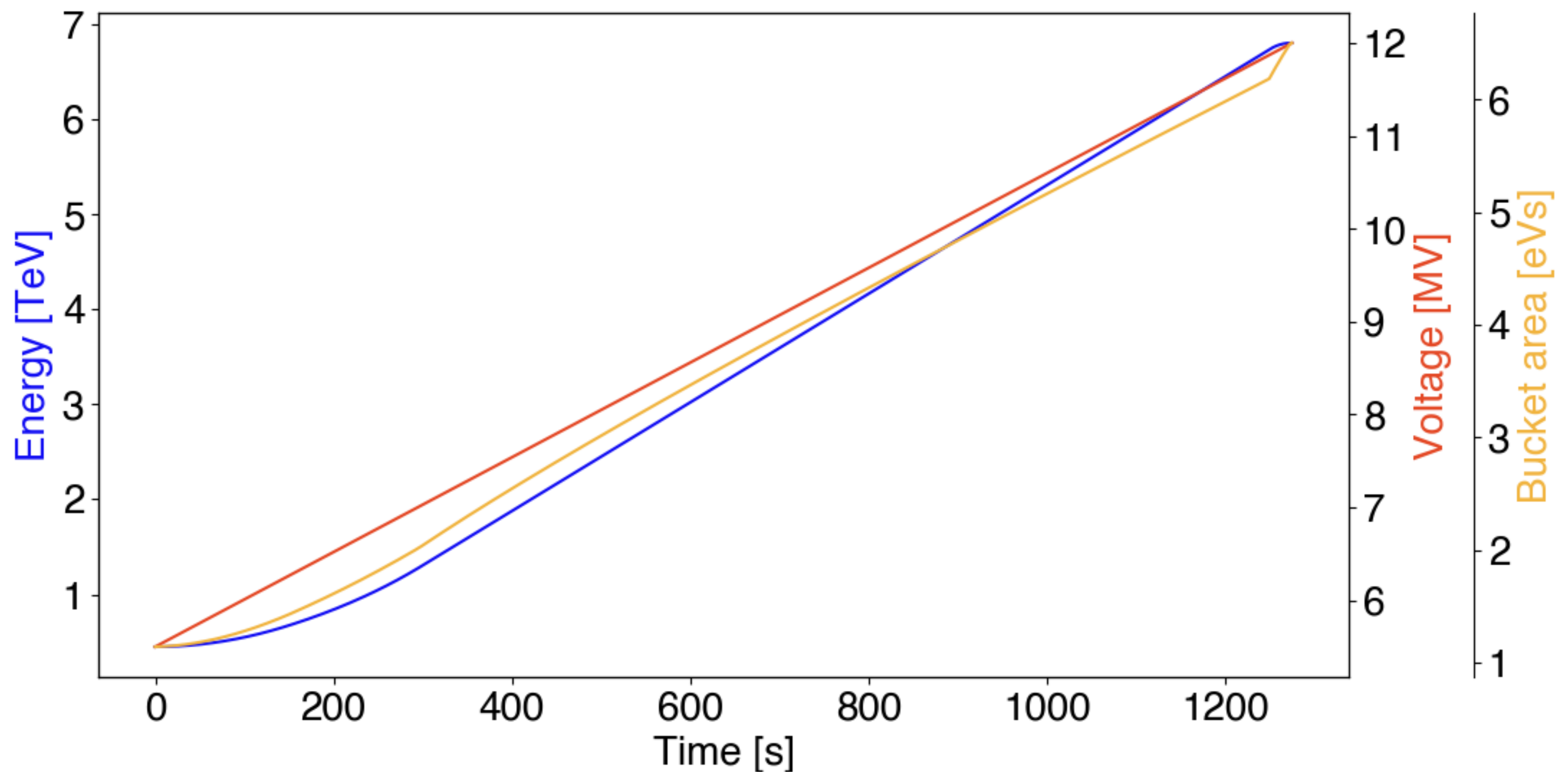
$$A_b = \oint d(\Delta t_{\mathrm{sep}}) \Delta E_{\mathrm{sep}}(\Delta t_{\mathrm{sep}})$$

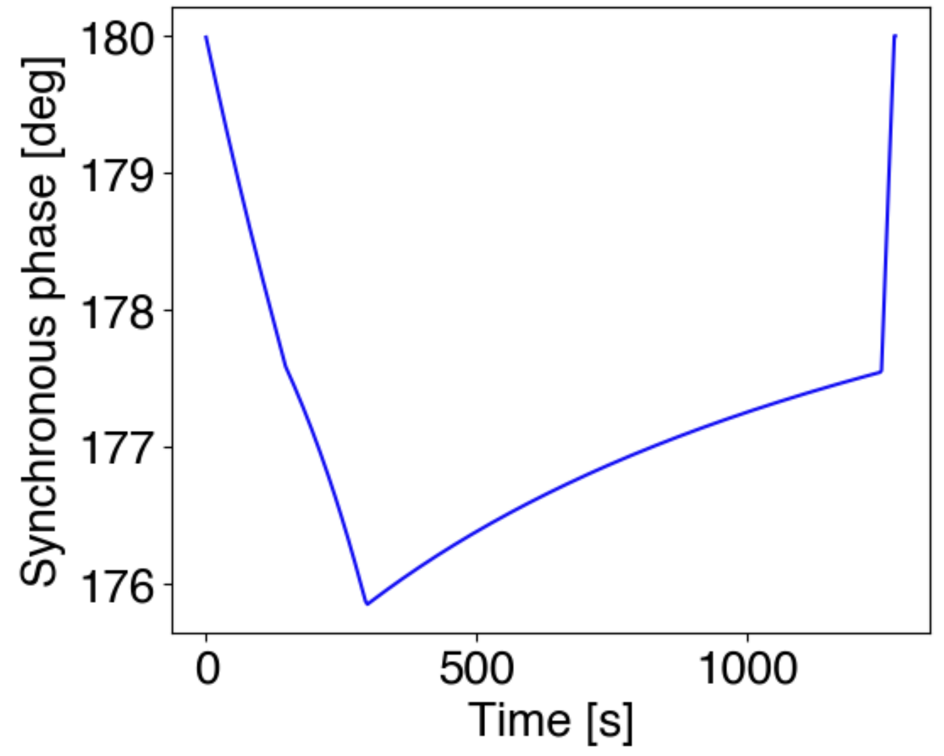
- Separatrix for single RF system, no intensity effects:

$$\Delta E_{\mathrm{sep}}(\Delta t_{\mathrm{sep}}) = \pm \sqrt{\frac{\beta_d^2 E_d e V_{\mathrm{rf}}}{\pi h |\eta_0|} \left\{ -\cos(\omega_{\mathrm{rf}} \Delta t_{\mathrm{sep}} + \varphi_{\mathrm{rf}}) - (\omega_{\mathrm{rf}} \Delta t_{\mathrm{sep}} + \varphi_{\mathrm{rf}}) \sin\varphi_s - \cos\varphi_s + (\pi - \varphi_s)\sin\varphi_s \right\}}$$

  - Bucket area in this case:

$$A_b = \frac{2}{\omega_{\mathrm{rf}}} \sqrt{\frac{\beta_d^2 E_d e V_{\mathrm{rf}}}{\pi h |\eta_0|}} \int_{\pi - \varphi_d}^{\varphi_u} d\varphi \sqrt{\cos(\pi - \varphi_d) - \cos\varphi + (\pi - \varphi_d)\sin\varphi_d - \varphi \sin\varphi_d}$$




*Example: LHC parabolic-exponential-linear-parabolic ramp*

QUIZ

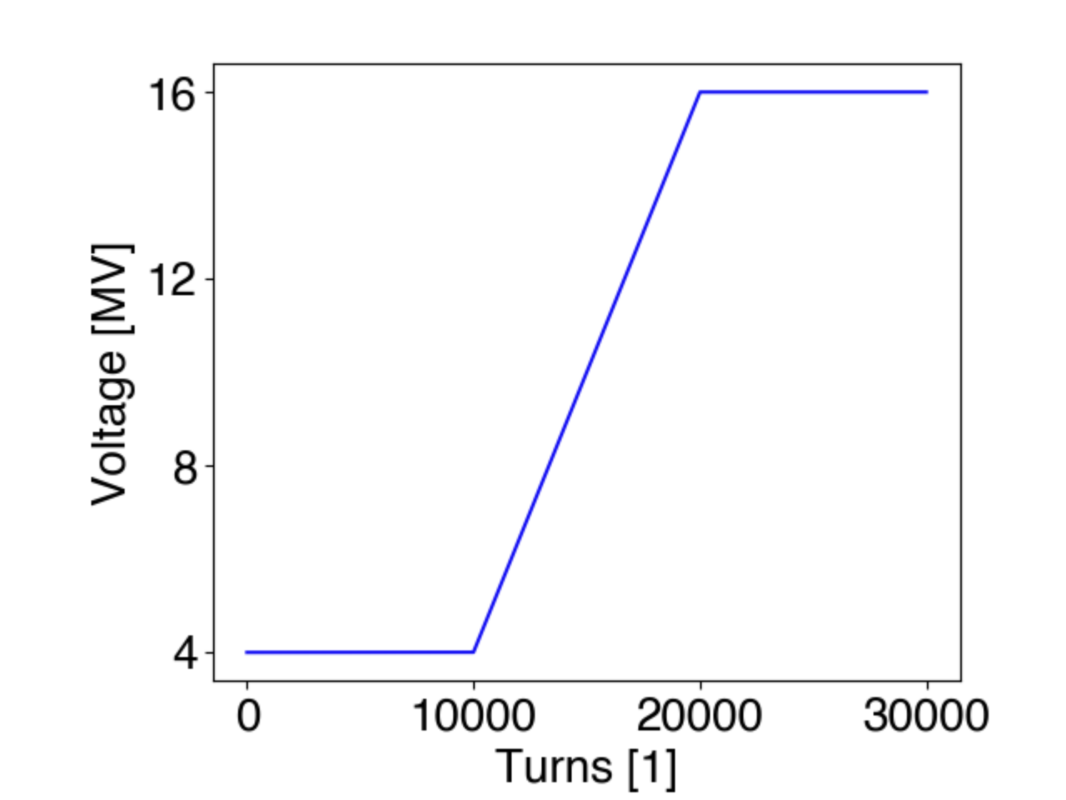
## What happens to the bucket area when the voltage is increased linearly (everything else remains constant)?

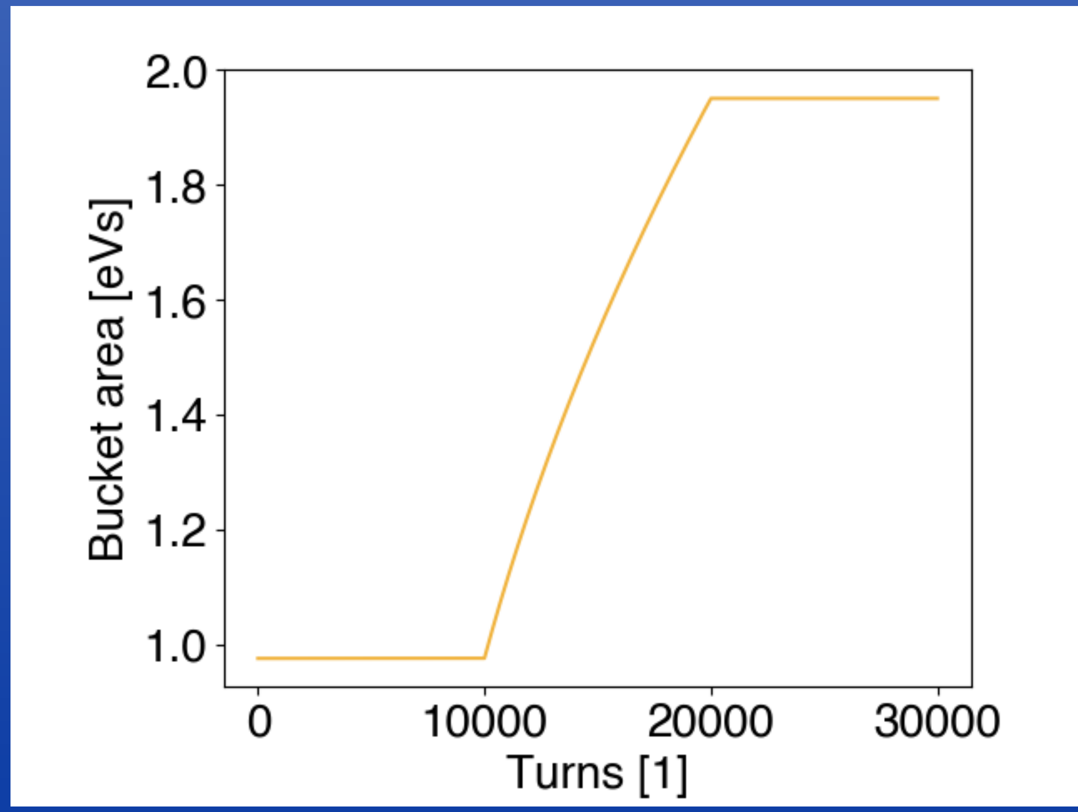
TIPS

*For a single RF system, and in the absence of intensity effects,* $A_b \propto \sqrt{V_{\rm rf}}$

**Solution**

- Assuming phase-space coordinates $(\Delta t, \Delta E)$, the bucket area is $A_b \approx \dfrac{16}{\omega_{\rm rf}}\sqrt{\dfrac{\beta^2 E_d\, eV_{\rm rf}}{2\pi h\,|\eta_0|}}$



# Bunch rotation

**What is it?**

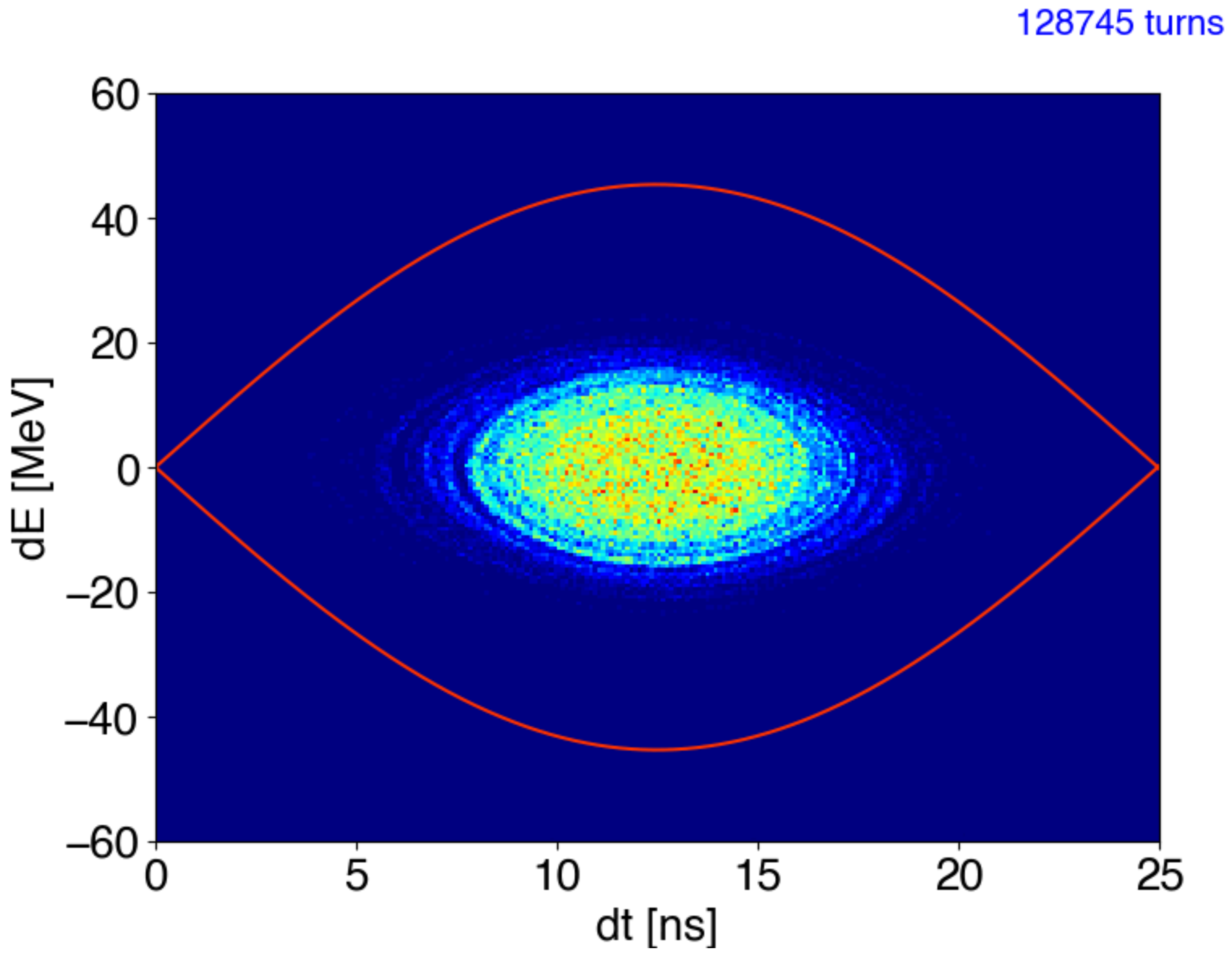
- Bunch rotation is an "exchange" of the longitudinal conjugate variables $\Delta t$ and $\Delta E$

**How is it done?**

- Non-adiabatic, large increase in RF voltage, then extract or recapture the beam
- Exchange of energy spread and bunch length happens for the bunch core after $1/4T_s$
  - The larger the tails, the more distortion there is in the bunch halo
  - The optimum extraction time depends on the initial bunch length

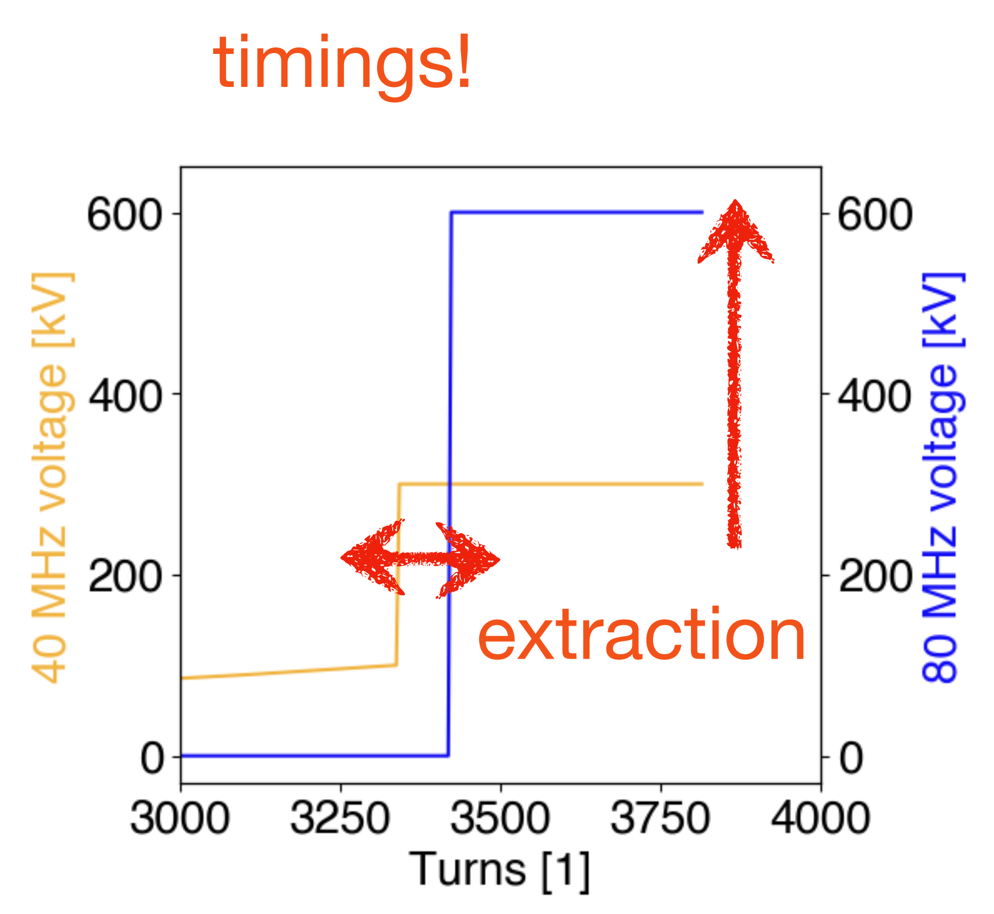
**Why use it?**

- To significantly shorten/lengthen the bunch length/energy spread



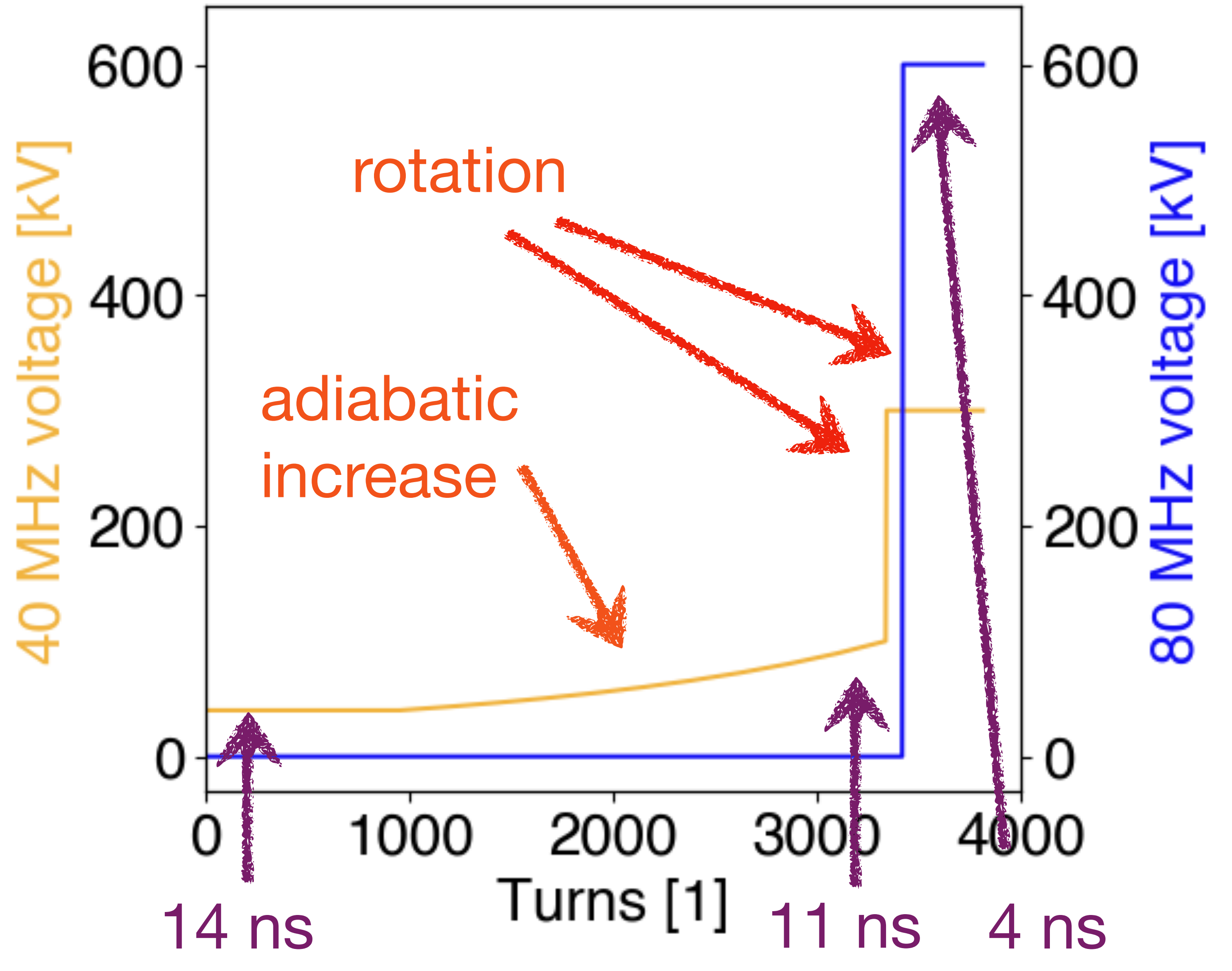

*Example: PS rotation in double-harmonic RF, before extraction to the SPS*

# Bunch rotation




**Where is it applied?**

- The LHC-type proton beam is rotated in the PS before extraction to the SPS
  - To fit the long PS bunches (~4 ns after rotation) into the short (5 ns) SPS bucket
  - Thus, to limit the capture losses upon PS-to-SPS bunch-to-bucket transfer
  - Fine-tuning of timings requires modelling with intensity effects




**What does it require from the hardware?**

- Capability to significantly increase the voltage within a few machine turns
- Capability to time extraction/recapture within a few machine turns
- Large peak voltage and power during $1/4T_s$
  - In the PS, the 80 MHz cavities cannot capture the beam, but can rotate it

*Example: PS rotation in double-harmonic RF, before extraction to the SPS*

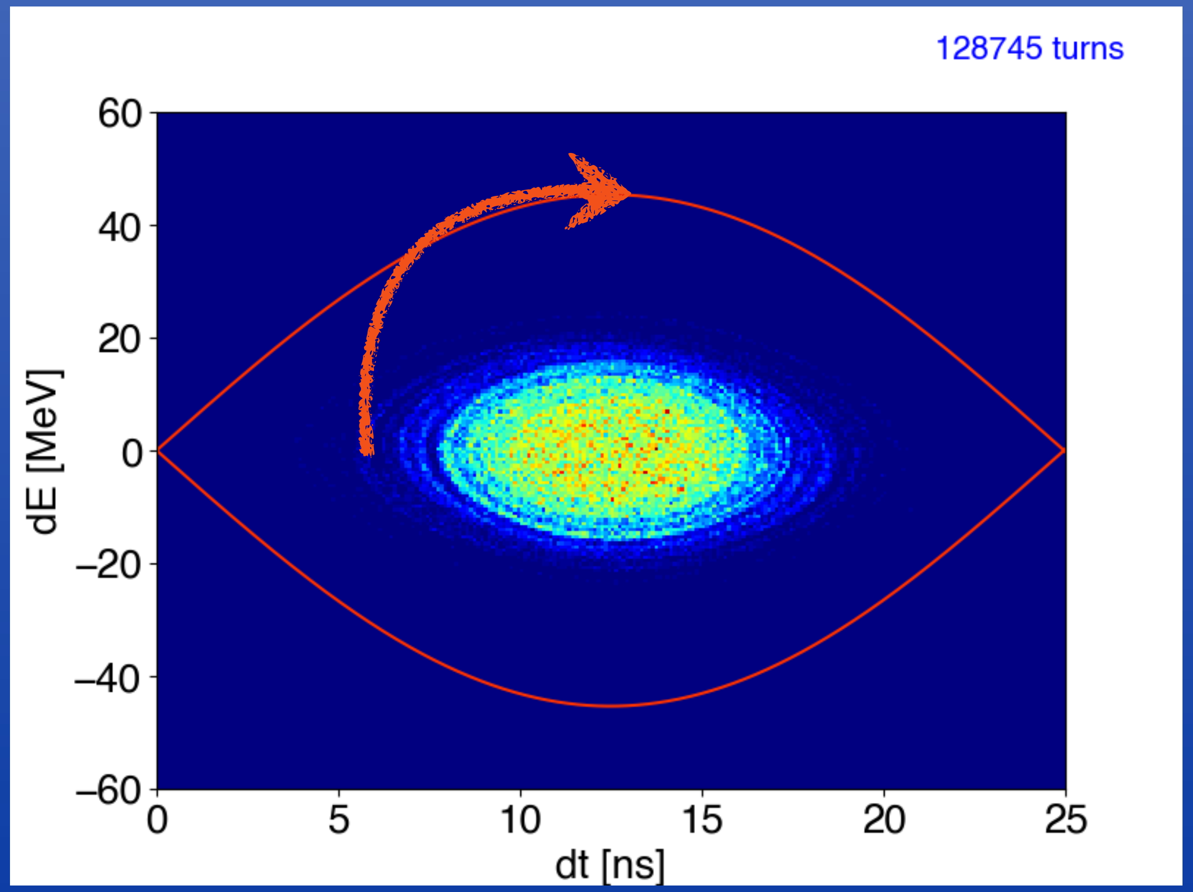



[4] H. Timko *et al.*, 'Longitudinal transfer of rotated bunches in the CERN injectors', Phys. Rev. ST Accel. Beams **16**, 051004 (2013).
[5] A. Lasheen *et al.*, 'Improvement of the longitudinal beam transfer from PS to SPS at CERN', Proc. IPAC'18, Vancouver, Canada (2018).

QUIZ

# Can you imagine any other way to achieve bunch rotation?


*You want to move the particles with a large longitudinal coordinate close to the separatrix in energy*



# Phase jump

**How is it done?**

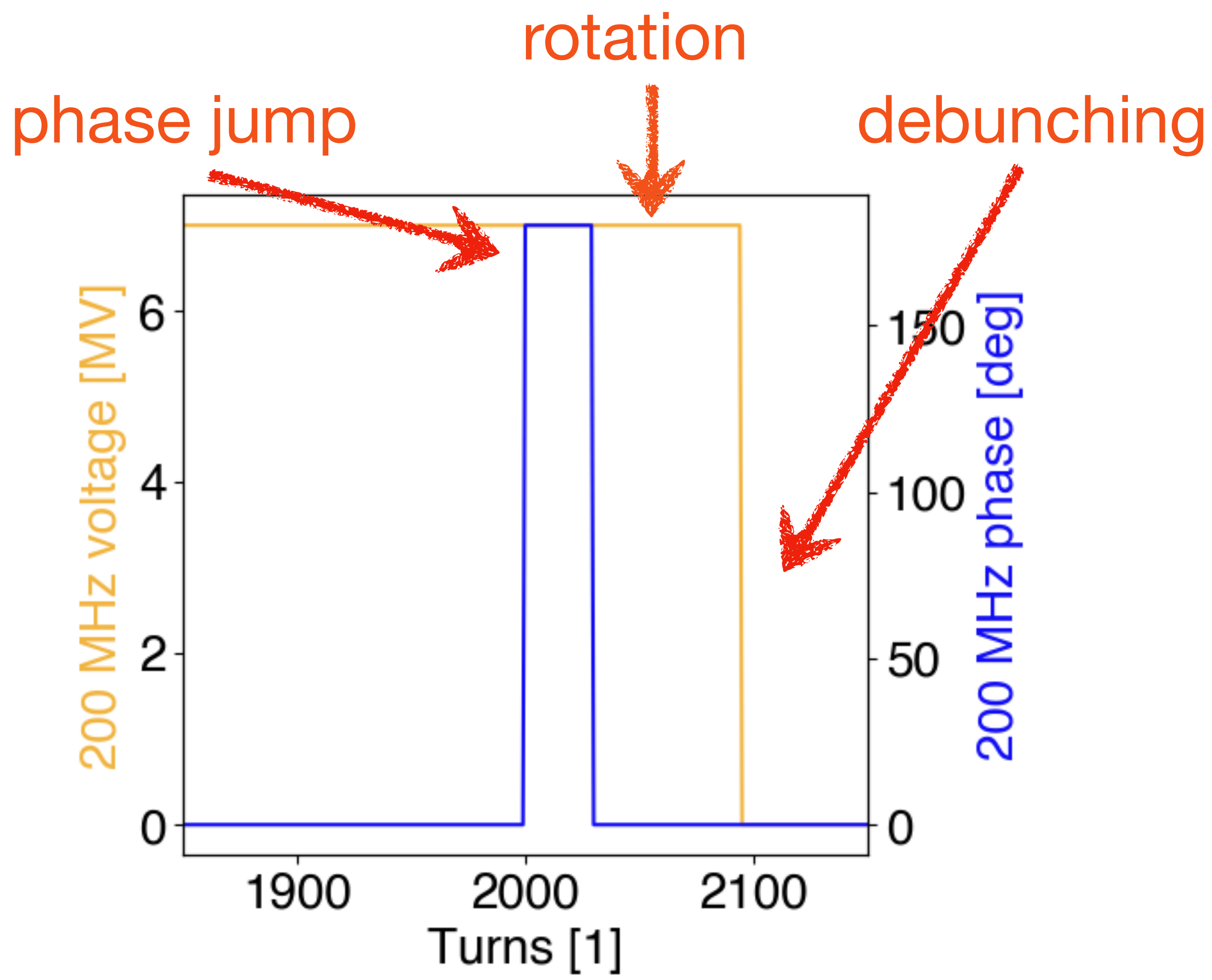
- Jump by 180° to unstable phase and stay for a few turns
  - Duration determines how much the bunch is stretched
- Jump back to stable phase and recapture
  - Need to have sufficient voltage compared to the amount of stretch
  - Let the bunch rotate till momentum spread is maximum $> 1/4T_s$
- Extract or switch RF off for debunching

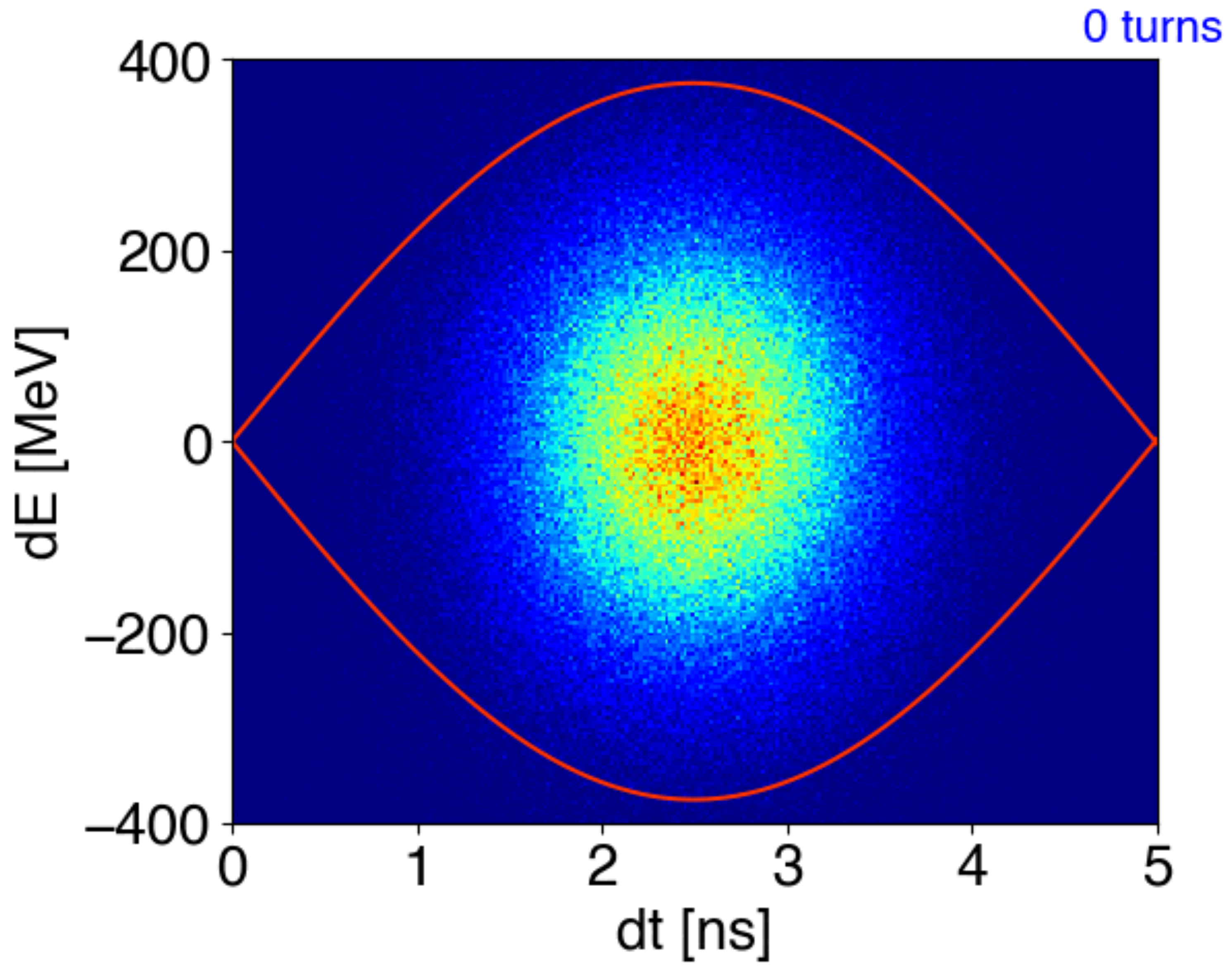
**Where is it used?**

- For SPS slow extraction, to create a coasting beam with large $dp/p$
  - This is then extracted slowly by slices of $dp/p$

**Pros and cons**

- Does not require large peak power
- Rotation is $> 1/4T_s$ and increases halo distortion in the distribution




*Example: SPS phase jump on flattop for the slow extraction of fixed-target beams*

# Bunch splitting

**What is it?**

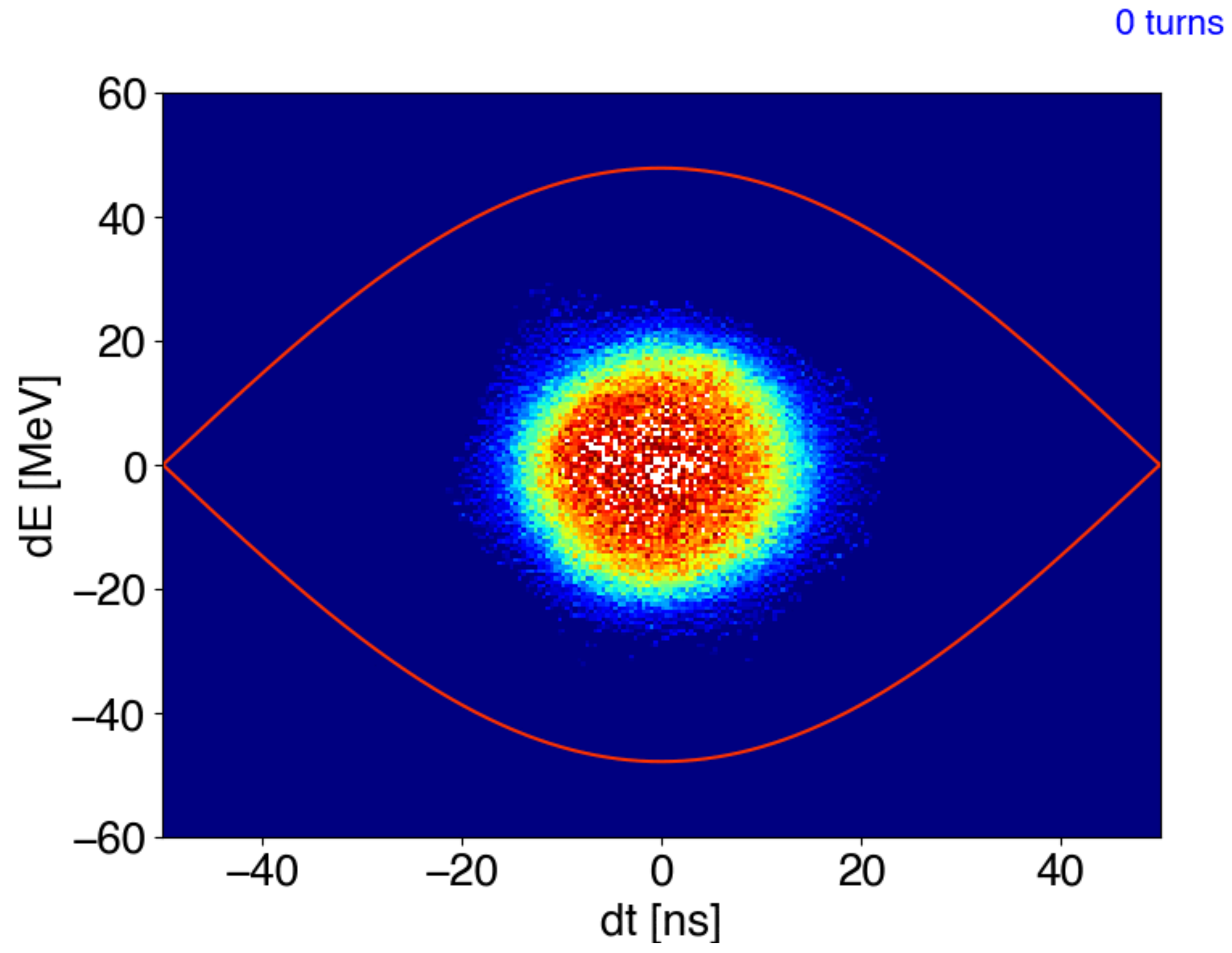
- Split one bunch into two or more bunches
  - Double splitting: 1 → 2 bunches, triple splitting: 1 → 3 bunches, etc.

**How is it done?**

- Have the bunch fully captured in one RF system
  - For better splitting efficiency, lower the voltage to lengthen the bunch
- Adiabatically increase the voltage of a higher-harmonic system
  - Use the second harmonic for double splitting
- Once split, adiabatically decrease the voltage of the lower-harmonic system

**Why use it?**

- To increase the number of bunches
- To divide the longitudinal emittance
  - In principle, preserving the total emittance and phase-space distribution




*Example: two double splittings at PS flattop*

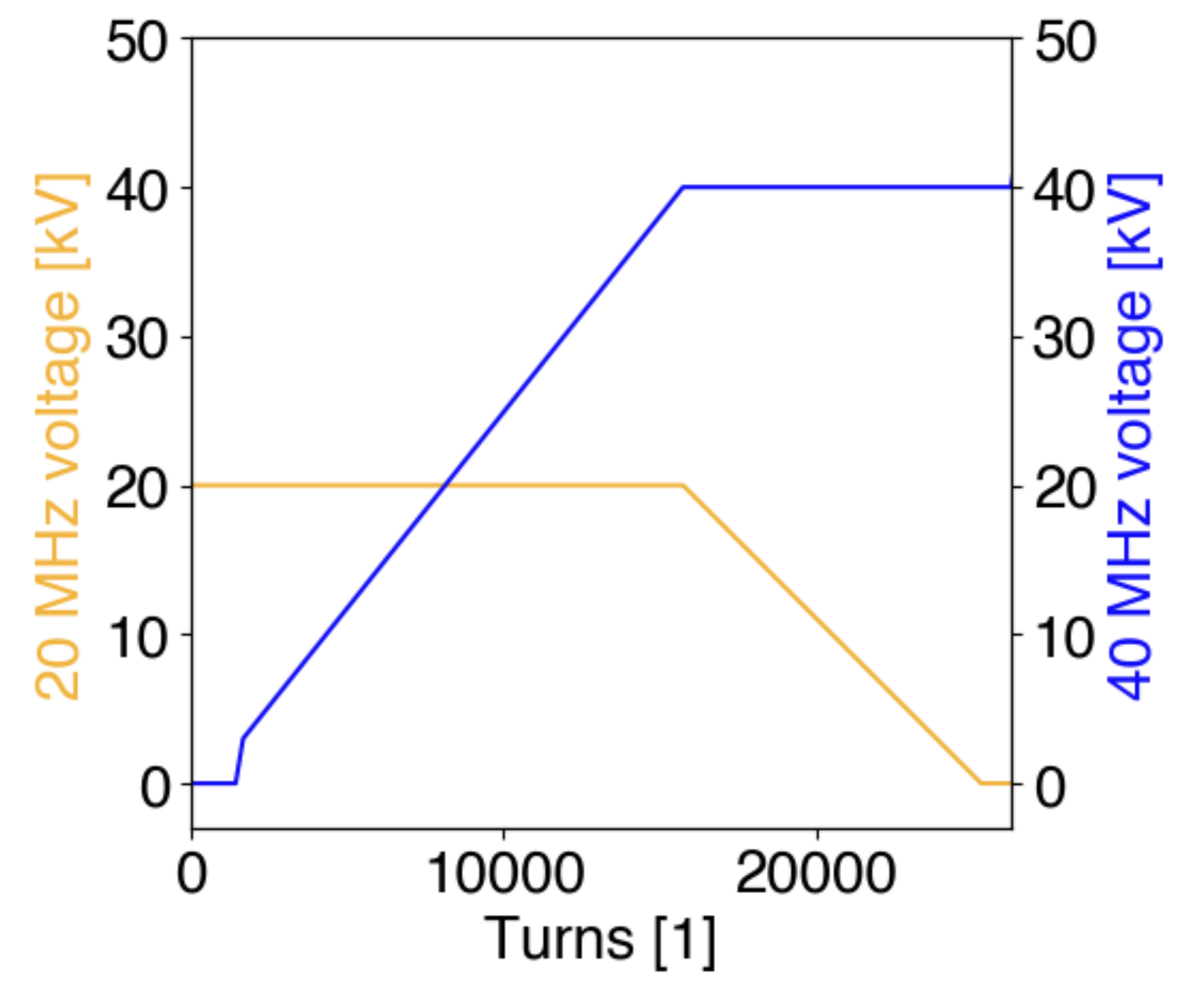
# Bunch splitting

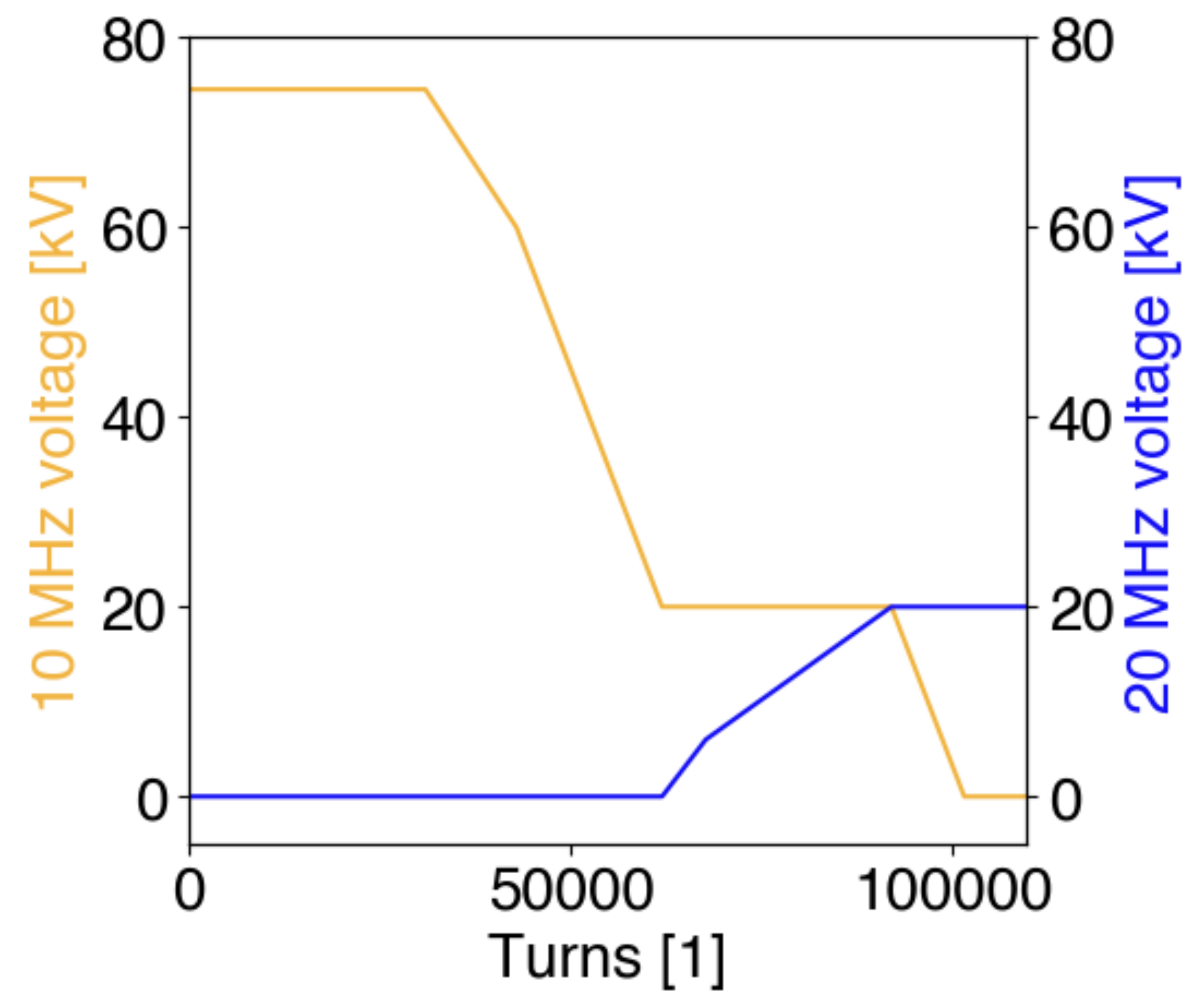
**Where is it applied?**

- In the PS at flattop, the LHC-type beam is double split twice
  - From a 10 MHz RF system, it is transferred to a 40 MHz RF system
  - Creating the ‘25 ns beam’ for the SPS and LHC

**What does it require from the hardware?**

- Sufficient voltage and power in the higher-harmonic RF system to fully capture the beam without the lower-harmonic system
- Adiabatic voltage changes
- Good control of the relative RF phase between the two systems
  - Stable RF phase for both systems
  - Second RF in bunch lengthening mode, i.e. in phase




*Example: two double splittings at PS flattop*

QUIZ

# How to split one bunch into three?


*You want to divide the bucket length into three equal pieces*

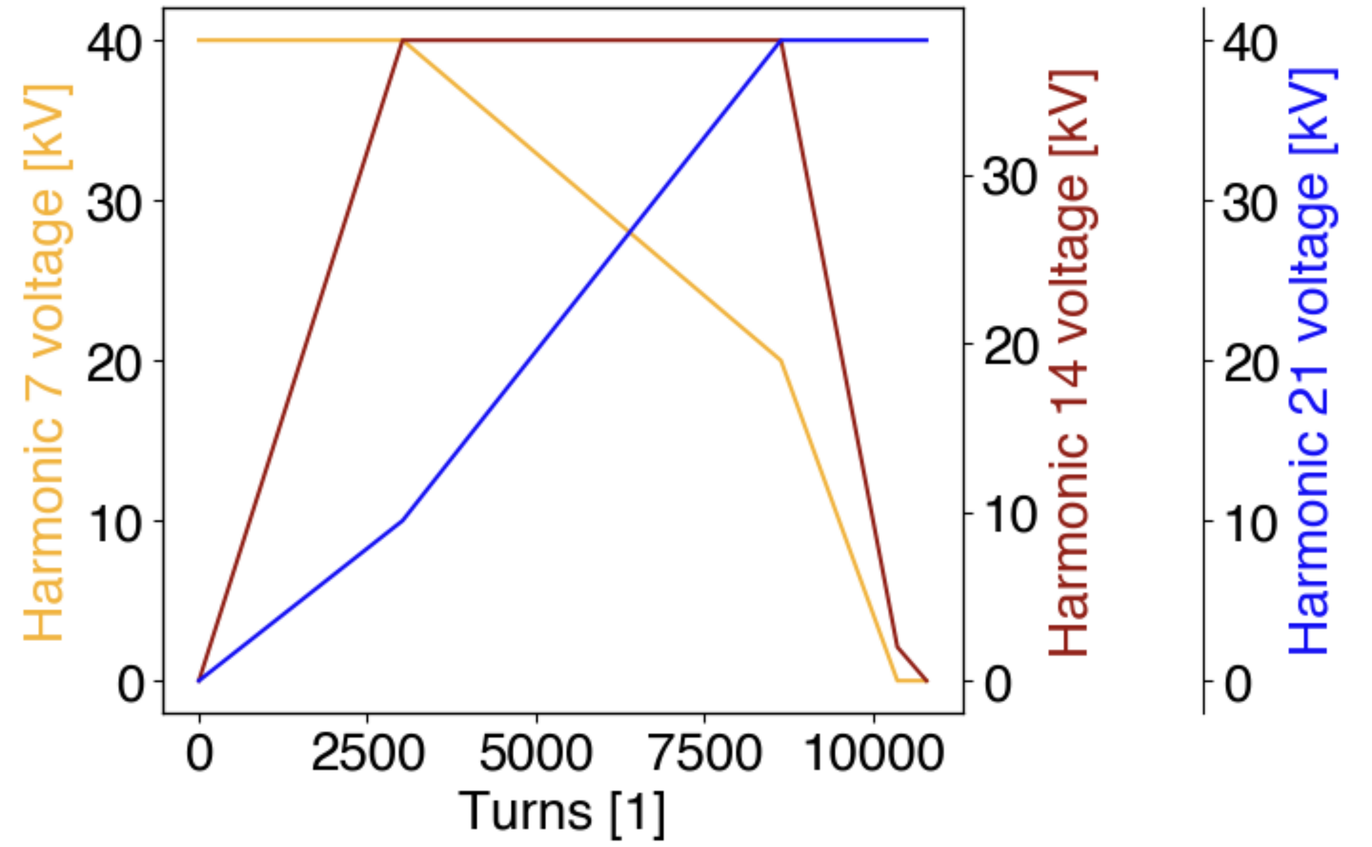
# Triple splitting

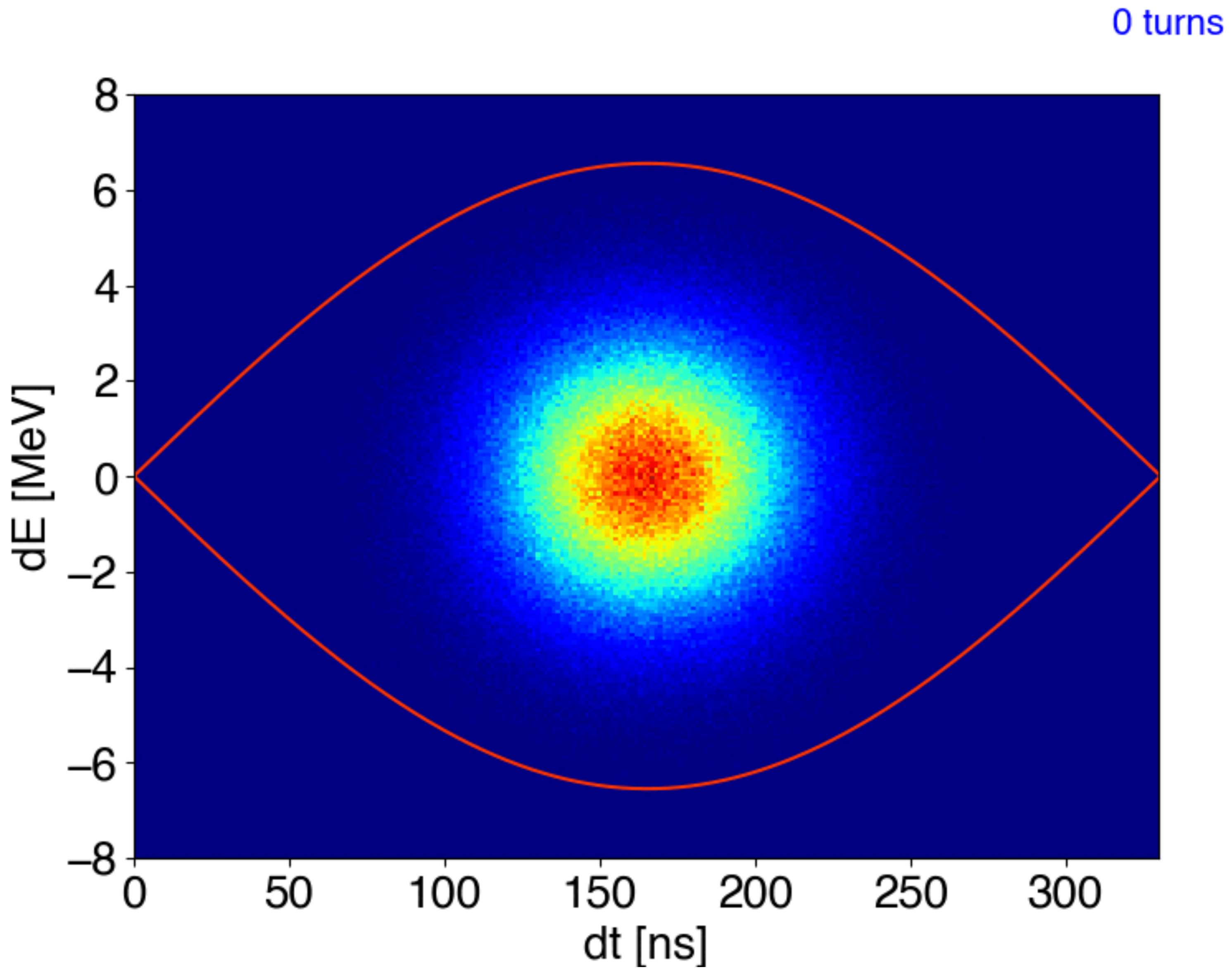
**Where is it applied?**

- In the PS at flat bottom, the LHC-type beam is triple split
  - First one batch of 4 bunches, then one batch of 2 bunches is injected on $h = 7$
  - After the splitting, 18 bunches are obtained on $h = 21$

**How is it implemented?**

- Requires the presence of double and triple harmonic RF systems
  - In the PS, all is done on the broad-band 2.8-10 MHz cavities
  - These are grouped into 3 different groups and controlled at $h = 7$, $h = 14$, $h = 21$; all groups in phase
  - After the splitting, all cavities are tuned on $h = 21$ to accelerate the beam with the full RF voltage
- Requires the fine-adjustment of the timings and voltages to get three equal distributions




*Example: triple splitting at PS flat bottom*



[6] D. Grier 'The PS 10 MHz cavity and power amplifier', CERN PS-RF-NOTE-2002-073 (2002).
[7] R. Garoby *et al.*: 'Demonstration of bunch triple splitting in the CERN PS', Proc. EPAC (2000).

# Bunch merging

**What is it?**

- The inverse process of bunch splitting; RF cavities in phase

**Why use it?**

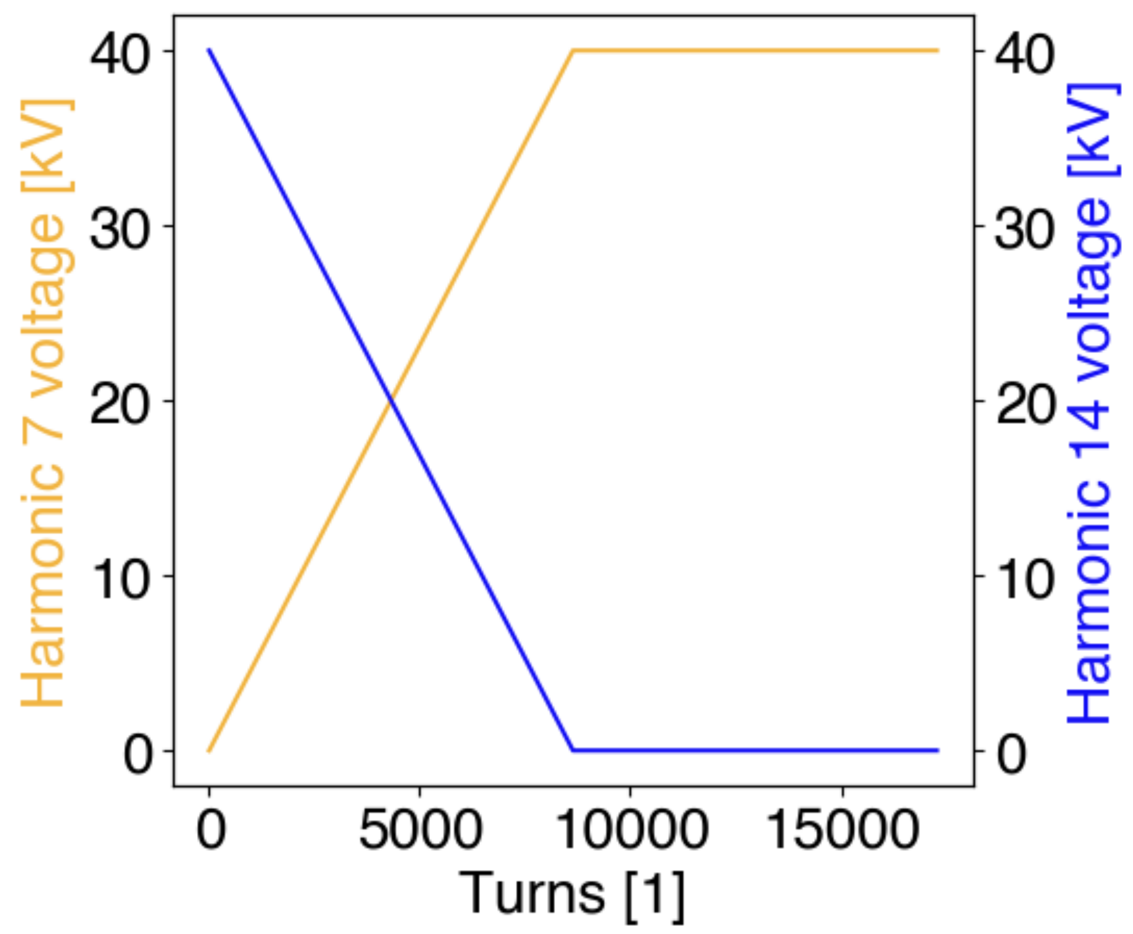
- To add up the bunch intensities into one bunch
  - As the transverse emittance is not affected, the beam brightness increases

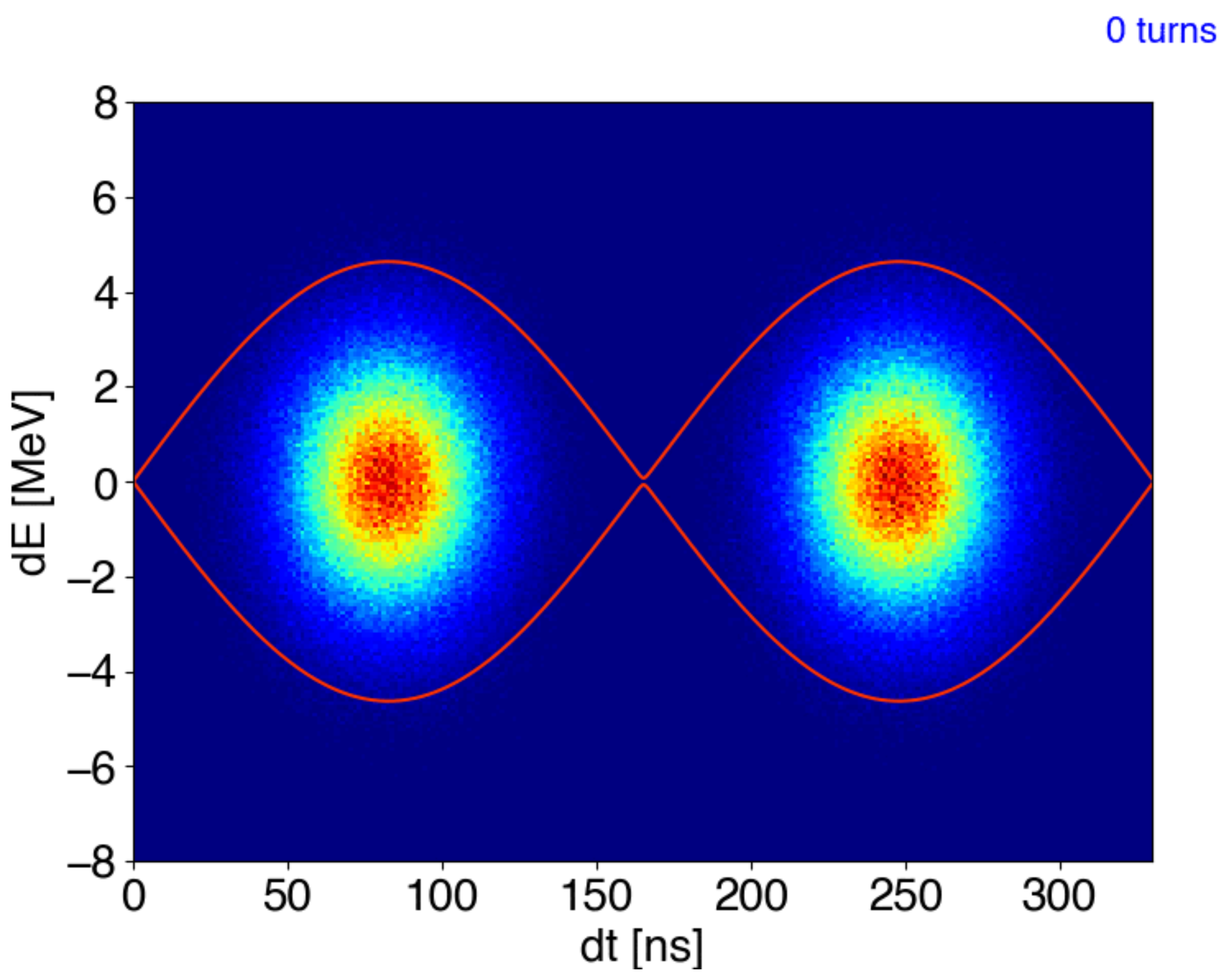
**What does it require?**

- Bunches with equal longitudinal emittance in neighbouring buckets
- Good control of the relative RF phase between the two systems

**Where is it applied?**

- In the PS for the production of the high-brightness LHC-type beam called ‘BCMS’ = batch compression, merging, and splitting
  - We’ll come back to it




*Example: triple splitting at PS flat bottom*

# Batch compression

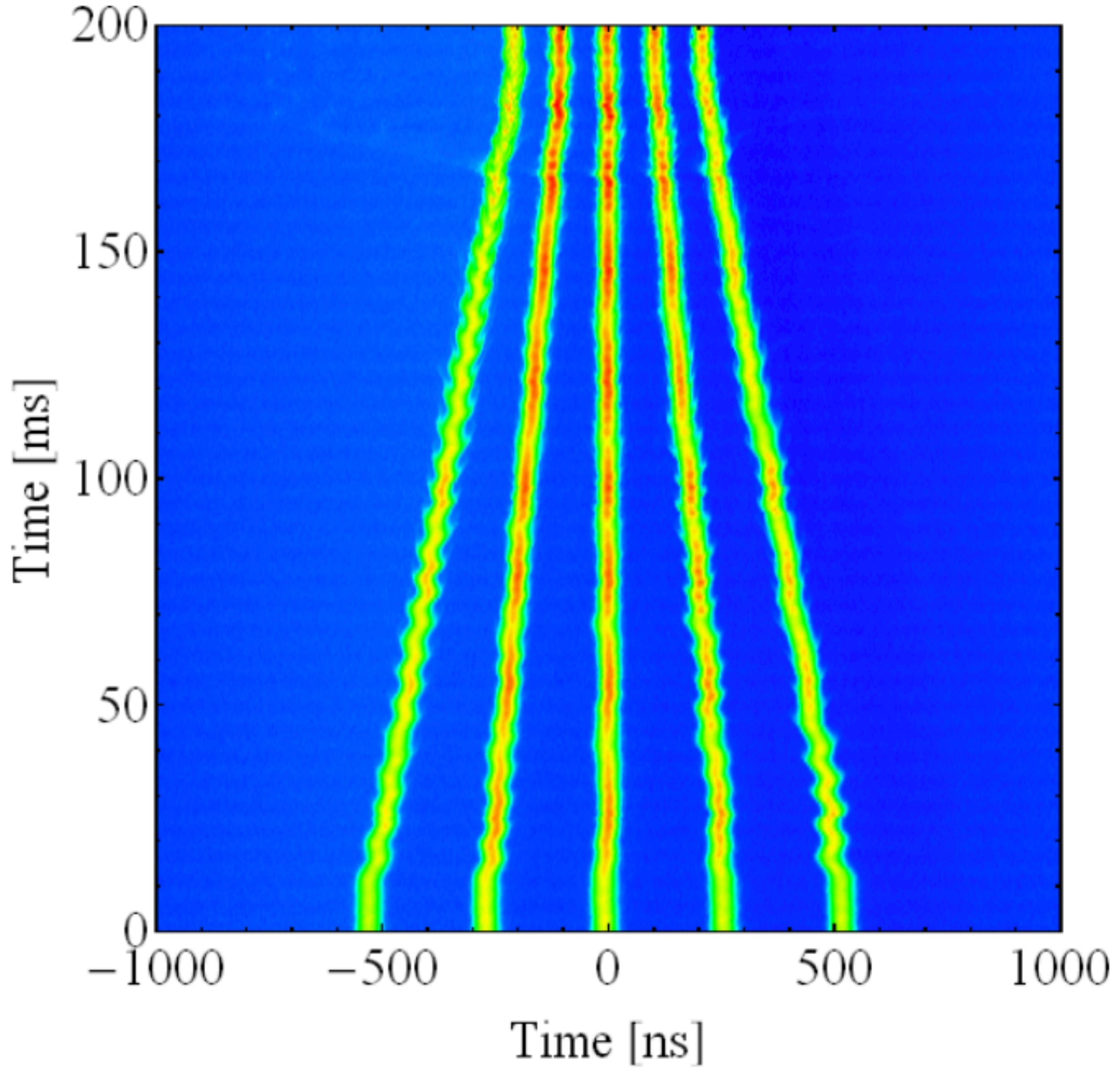
**What is it?**

- Batch compression is continuous, adiabatic reduction of the bunch spacing

**How is it done?**

- Step-by-step RF frequency change corresponding to a harmonic change

  $\omega_{\mathrm{rf}} = h\omega_{\mathrm{rev}}$
- A continuous change in RF frequency would result in a phase slippage and therefore in a bucket slippage
  - Instead, the RF voltages of nearby harmonics are ramped up/down



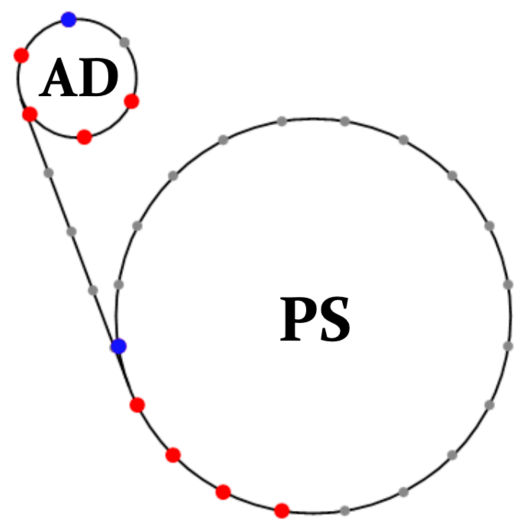

*PS batch compression from [8]*

**Why use it?**

- To bring adjacent bunches closer to each other
  - E.g. in order to match the spacing required in the downstream machine
  - E.g. in order to merge the bunches


[8] C. Lombard *et al.*: 'Improved antiproton production beam at CERN', Proc. IPAC'23, TUPM075, Venice, Italy (2023).

# Batch compression

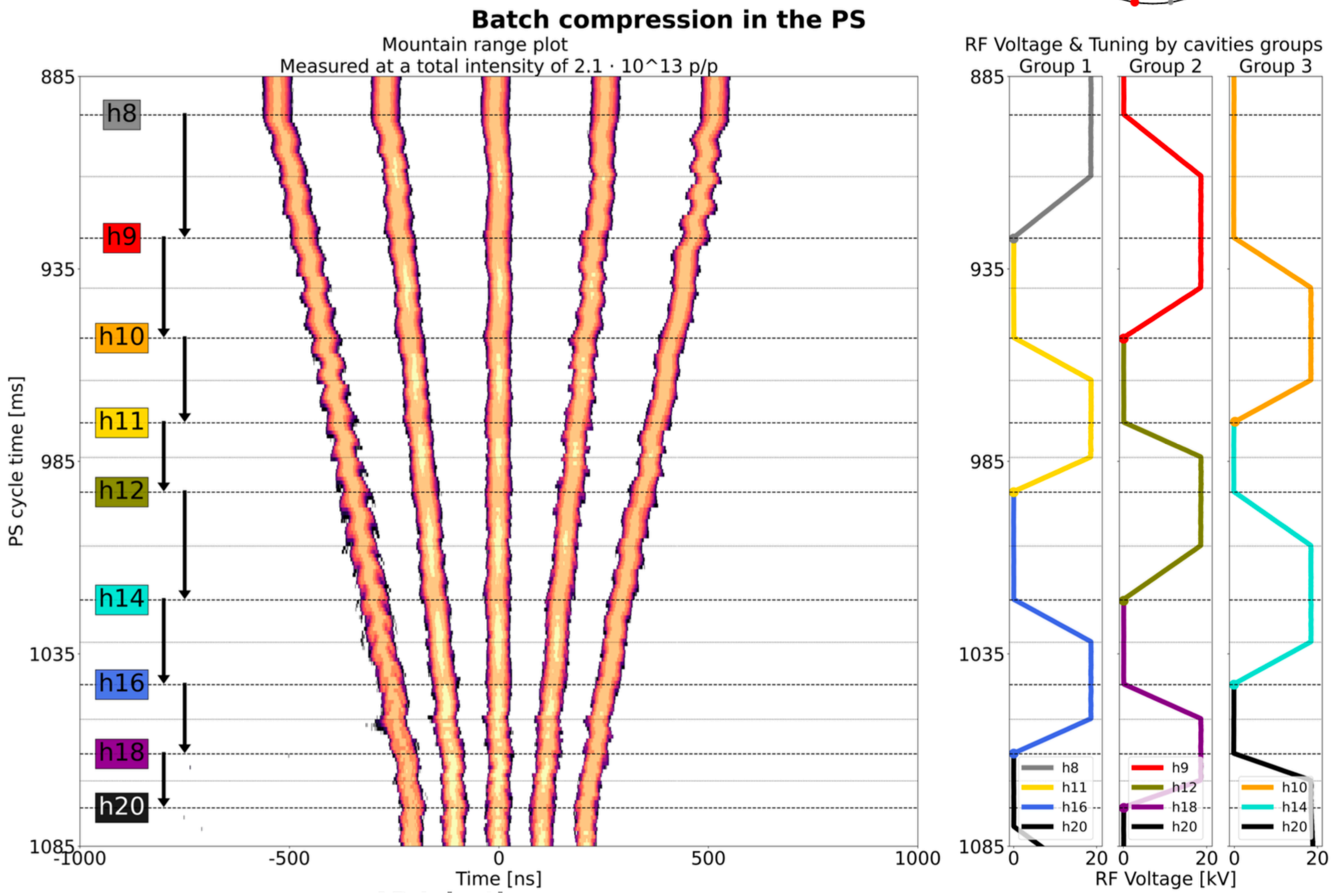



## Where is it applied?

- In the PS, to produce the proton beam for the Antiproton Decelerator (AD)
  - This beam is then shot on a target to generate antiprotons
  - The AD is 3x smaller in circumference than the PS, and needs therefore batch compression

## What does it require from the hardware?

- Broad-band cavities or many cavities at different harmonics
- E.g. PS main RF system is tuneable from 2.8-10 MHz
  - Nine out of ten cavities are grouped in three groups
  - Two groups are active at a time, while the third is tuned to the next harmonic
  - After compression, 5 bunches are condensed to 1/4 of the circumference




*PS batch compression for the AD beam from [8]*

$$h_{PS} = 8 \rightarrow 9 \rightarrow 10 \rightarrow 11 \rightarrow 12 \rightarrow 14 \rightarrow 16 \rightarrow 18 \rightarrow 20$$

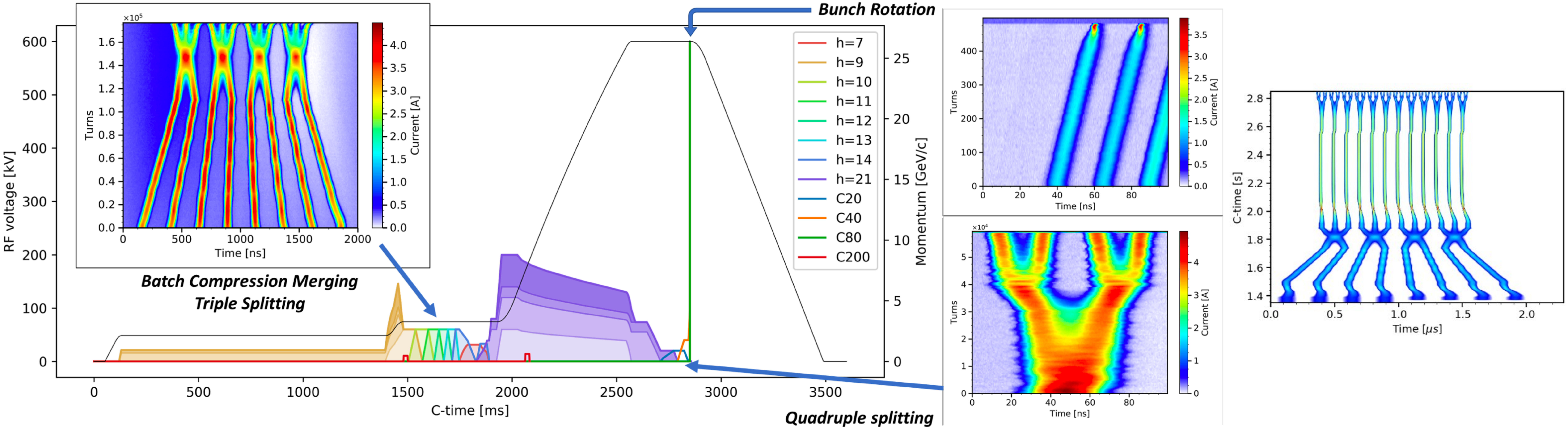



[8] C. Lombard *et al.*: 'Improved antiproton production beam at CERN', Proc. IPAC'23, TUPM075, Venice, Italy (2023).

# PS BCMS beam

**Batch compression, merging, and splitting**

- High-brightness beam for LHC proton physics production




*PS momentum and voltage programmes for the BCMS cycle, with the bunch profile evolution as simulated in BLonD, from [9]*


[9] A. Lasheen *et al.*: 'End-to-end longitudinal simulations in the CERN PS', Proc. HB2021, MOP17, Batavia, USA (2021).

# Contents

## Bunch length regulation

- Adiabatic changes
- Rotation
- Splitting, merging

## Phase space regulation

- Diffusion, noise injection
- Resonant excitation
- Longitudinal painting
- Debunching

## Advanced manipulations

- Momentum slip stacking
- Barrier bucket

## Integration in an RF system

- Beam loading
- RF voltage/power limitations
- Designing RF parameters

QUIZ

## Intermezzo: how can we generate diffusion in phase space?

TIPS

*You can make use of the cavity voltage vector*

# Diffusion in phase space

**Diffusion equation in action space**

- Hamiltonian of conjugate pair with a perturbation term:

$$\mathscr{H}(\vartheta, \vartheta', t) = H(\vartheta, \vartheta') + \Delta H(\vartheta, t)$$

$$\Delta H(\vartheta, t) = -\frac{\omega_{s,0}}{hV_{\mathrm{acc}} \sin \varphi_s} \int \Delta V(\tilde{\vartheta}, t) d\tilde{\vartheta}$$

- We describe the phase-space distribution in action space $F(J, \psi, t)$
  - The equations of motion in action-angle space are

$$\frac{d\psi}{dt} = \frac{\partial \epsilon}{\partial J} = \omega_s(\epsilon)$$

$$\frac{dJ}{dt} = -\frac{\partial \epsilon}{\partial \psi} = 0$$

- RF noise introduces diffusion described by the diffusion coefficient $D$
  - Evolution of the phase-space distribution is stochastic, only described for the ensemble average

$$\partial_t < F > (J, t) = \partial_J \left\{ D \partial_J F(J, t) \right\}$$

$$D(J) = \frac{1}{2} \left( \frac{\omega_{s,0}^2}{hV_{\mathrm{acc}} \sin \varphi_s} \right)^2 \Re \left\{ \sum_{m=-\infty}^{\infty} m^2 \sum_{k,l=-\infty}^{\infty} \frac{I^*_{mk}(J)}{k} \frac{I_{ml}(J)}{l} 2 \int_0^{\infty} < \Delta V_k(t) \Delta V_l^*(t-\tau) > e^{im\omega_s(J)\tau} d\tau \right\}$$

| | |
|---|---|
| $V_{\mathrm{acc}}$ | Nominal accelerating voltage |
| $\Delta V(\vartheta, t)$ | A given realisation of random noise |
| $K(x)$ | Complete elliptic integral of 1st kind |
| $E(x)$ | Complete elliptic integral of 2nd kind |

In action-angle variables,

$$J_{\mathrm{sep}} = \frac{8\omega_{s,0}}{\pi h^2}$$

$$J(x) = J_{\mathrm{sep}} \left( E(x) - (1 - x^2) K(x) \right)$$

$$\omega_s(x) = \omega_{s,0} \frac{\pi}{2K(x)}$$

Normalisation

$$\frac{1}{2\pi} \int \int_{\mathrm{sep}} dJ d\psi F(J, \psi, t = 0) = 1$$

Fourier expansion coefficients for a plane wave in multipole oscillations:

$$I^*_{m,k}(J) \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} d\psi e^{ik\varphi(J,\psi) - im\psi}$$



[10] S. Krinsky, J.M. Wang: 'Bunch diffusion due to RF noise', Part. Accel. **12**, 107–117 (1982).
[11] G. Dôme: 'Diffusion due to RF noise', CERN Accelerator School '85, Oxford, UK, 370–401 (1985).

# Diffusion for small bunches

**Semi-analytic solver on discrete grid [12]**

- Matrix iteration over time step $\Delta t$ $\left(\mathbf{A}+\frac{1}{2}\Delta t\mathbf{B}\right)\vec{F}_{n+1}=\left(\mathbf{A}-\frac{1}{2}\Delta t\mathbf{B}\right)\vec{F}_{n}$
- Diffusion coefficient for amplitude $a$ and phase noise $\varphi$

$$D^{(a,\varphi)}(J)=\frac{1}{2}\left(\frac{\omega_{s,0}^2}{hV_{\mathrm{acc}}}\right)^2\sum_{-\infty}^{\infty}W_m^{(a,\varphi)}P^{(a,\varphi)}(m\omega_s(J))$$

$$W_m^{(a,\varphi)}\equiv m^2\,|\,I_{mq}(J)\,|^2\frac{1}{4}(1\pm(-1)^m)^2$$

where $m^{(a)}=2j$ and $m^{(\varphi)}=2j+1$

- Distribution and diffusion coefficient discretised over the grid $l\in[0,L-1]$

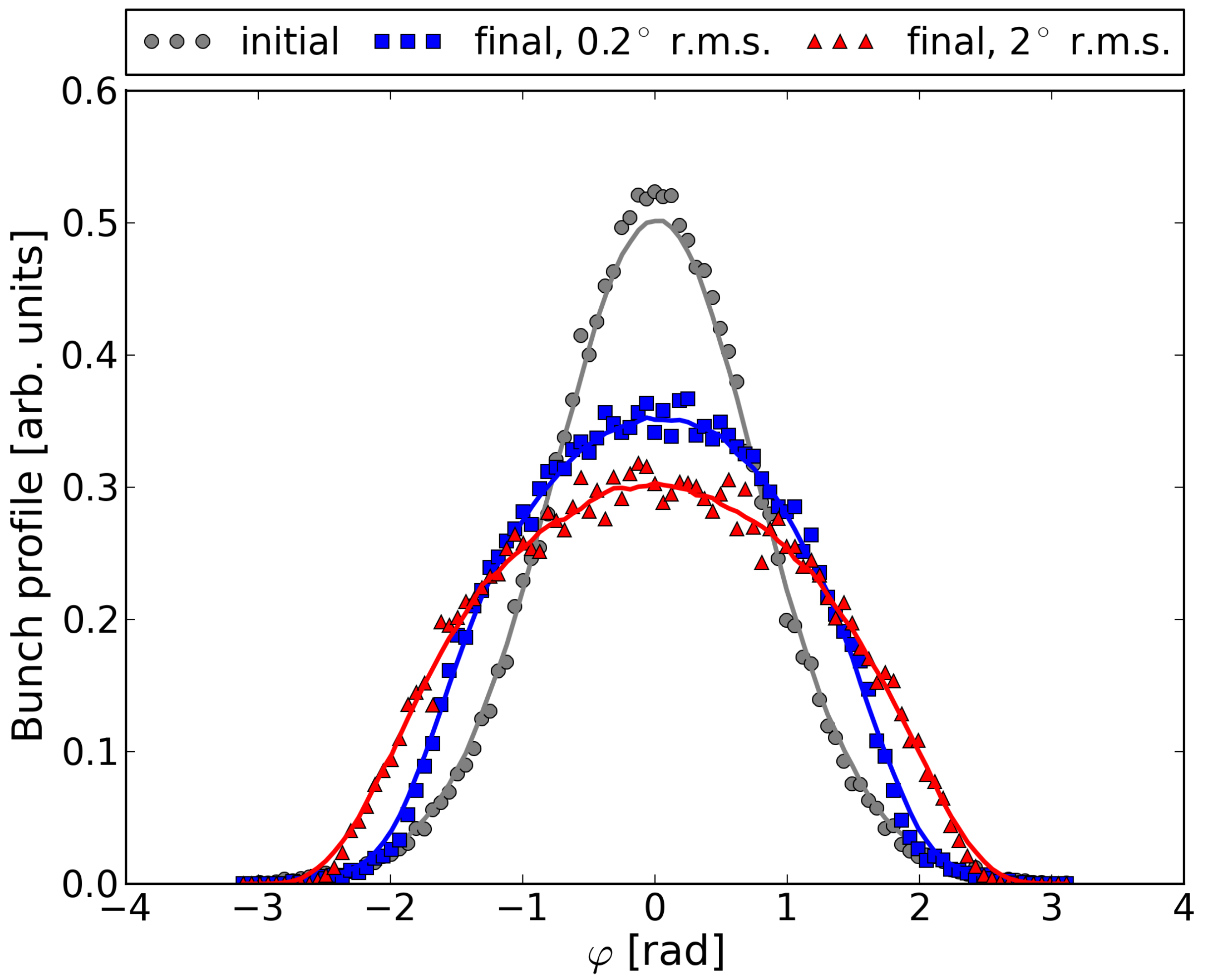
Average over grid $\overline{D}_l=\frac{1}{\Delta J_l}\int_{l-1}^{l}dJ\,D(J)$

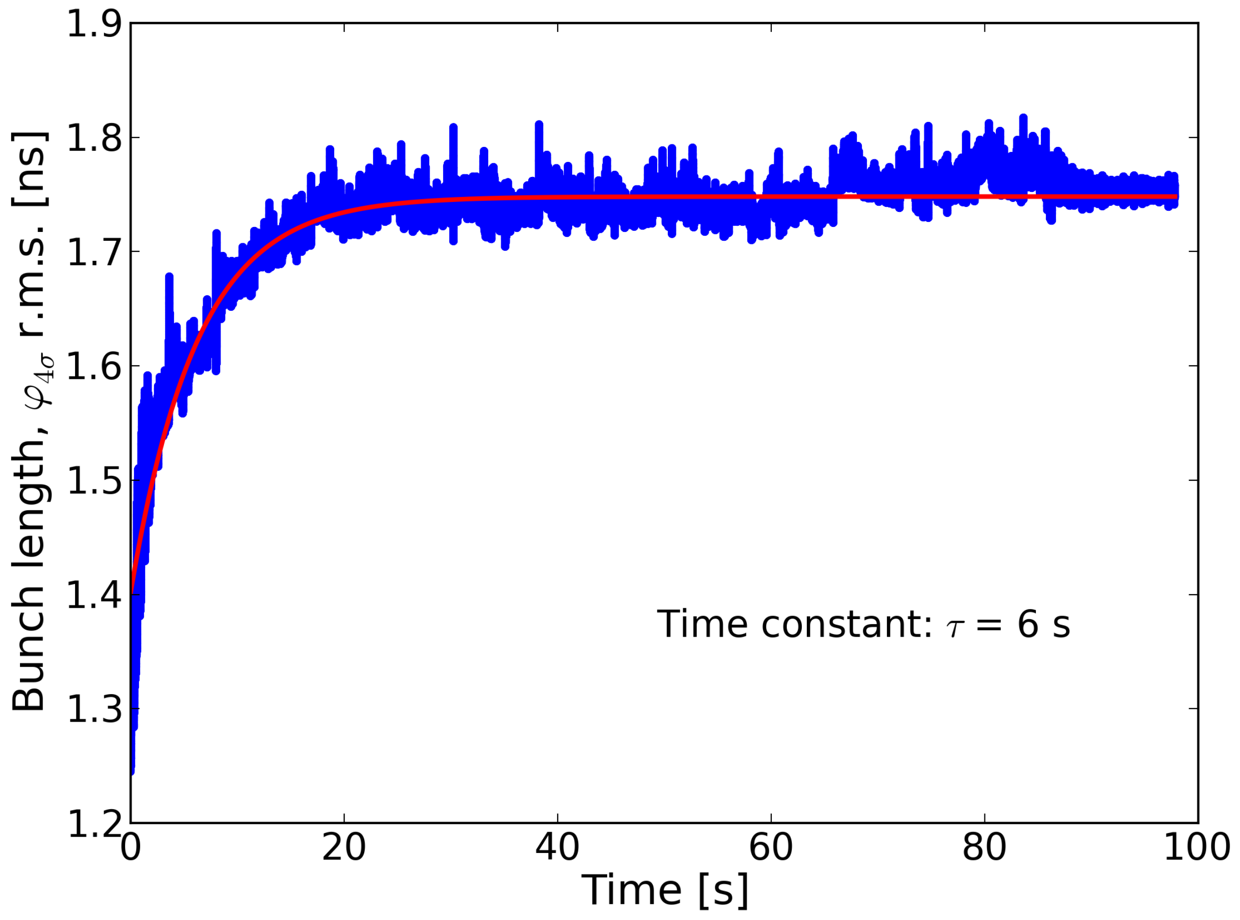
$$\vec{F}=\begin{pmatrix}F_0\\F_1\\\vdots\\F_{L-1}\end{pmatrix}\quad A=\begin{pmatrix}\frac{1}{3}&\frac{1}{6}&\cdots&&\\&\frac{1}{6}&\frac{2}{3}&\frac{1}{6}&\cdots\\\vdots&&&&\\&&\cdots&\frac{1}{6}&\frac{1}{3}\end{pmatrix}\Delta J\quad B=\begin{pmatrix}\overline{D}_0&-\overline{D}_0&&\cdots&\\&-\overline{D}_{l-1}&\overline{D}_{l-1}+\overline{D}_l&-\overline{D}_l&\cdots\\\vdots&&&&\\&&\cdots&-\overline{D}_{L-1}&\overline{D}_{L-1}\end{pmatrix}\frac{1}{\Delta J}$$

**Bunch length evolution in short-bunch approximation**

- For a given double-sided power spectral density $P_{DS}=\left[\frac{\mathrm{rad}^2}{\mathrm{Hz}}\right],\sigma=[\mathrm{rad}]$

$$\sigma(t)=\sqrt{\sigma_0^2+\omega_{s,0}^2P_{DS}(\omega_{s,0})t}$$




*Diffusion due to band-limited white noise*

[12] S.V. Ivanov: 'Longitudinal diffusion of a proton bunch under external noise', IHEP Preprint 92-43, Protvino (1992).

# Controlled emittance blow-up

**What is it?**

- Controlled increase of the phase-space area through noise injection

**How is it done?**

- Injection of RF (phase) noise, applied on the main RF system
- RF (phase) modulation on a higher-harmonic RF system

**Why use it?**

- To regulate the bunch length to a desired value
  - E.g. for extraction requirements or RF heating of sensitive equipment
- To counteract loss of Landau damping (LLD) in the energy ramp
  - At injection, there might not be enough bucket area to have a large enough emittance that would be stable at flattop
  - During the ramp, the bucket area increases as the energy (and voltage) increase
    - The bunch length shrinks adiabatically if nothing is done

Intensity threshold for LLD

$$N_{p,\mathrm{th}} = -\frac{\pi V_{\mathrm{rf}} \cos\varphi_{s,0} \varphi_{\max}^5}{32 q h^2 \omega_{\mathrm{rev}} \mu(\mu+1) \chi(y_{\max}, \mu) \Im\left(\frac{Z}{n}\right)}$$

Binomial distribution

$$F(H) = F_0 \left(1 - \frac{H}{H_0}\right)^{\mu}$$

Maximum phase amplitude in distribution

Contains a hypergeometric function ${}_2F_3$

*A recent model of loss of Landau damping threshold for a binomial distribution and an effective impedance*

[13] I. Karpov *et al.*: ‘Thresholds for loss of Landau damping in longitudinal plane’, Phys. Rev. Accel. Beams **24**, 011002 (2021).

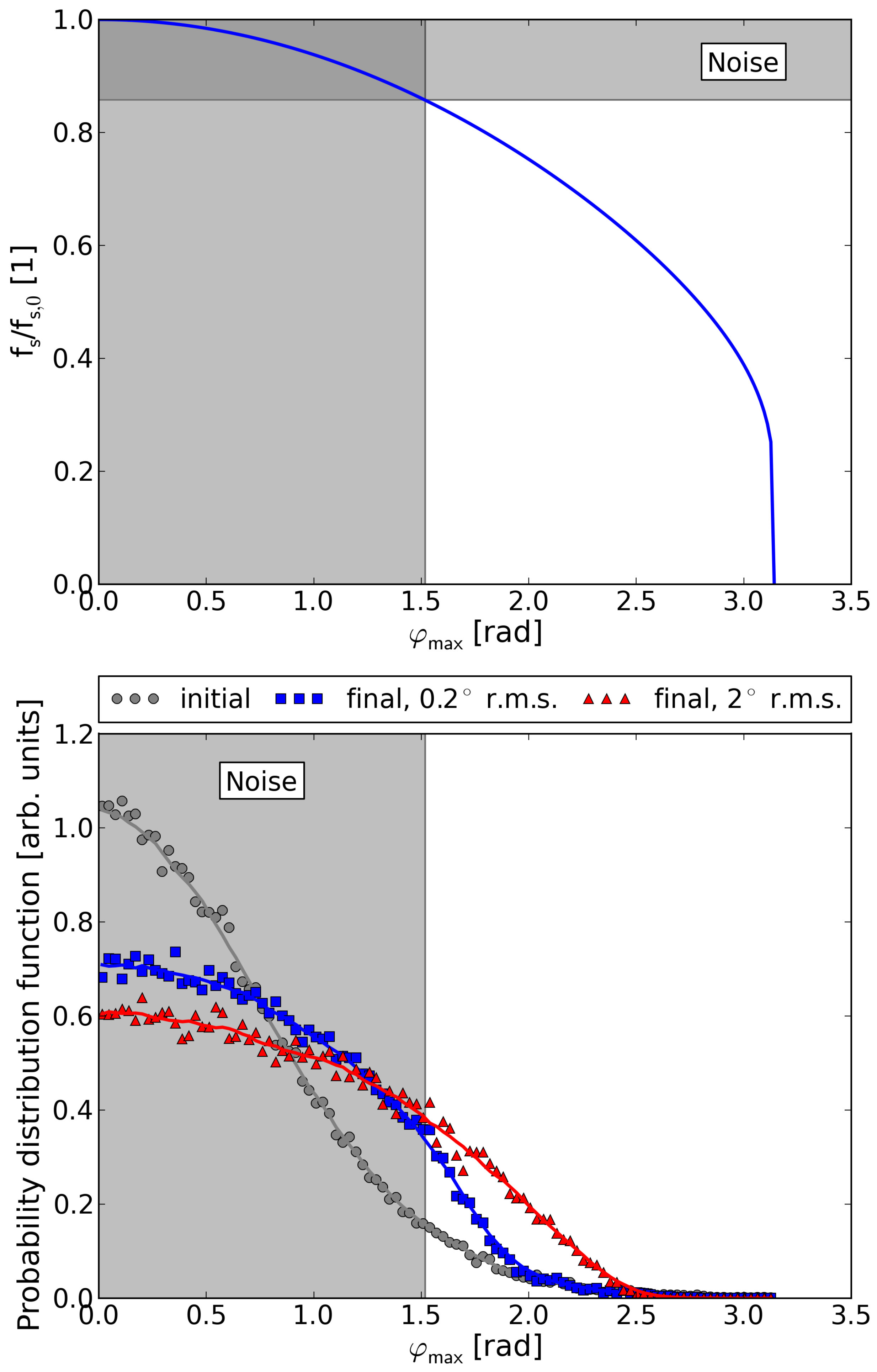
# Controlled emittance blow-up

**Where is it applied?**

- PSB, SPS, and LHC: on main harmonic RF
  - Band-limited phase noise in a frequency band close to the central synchrotron frequency
- PS: on a dedicated 200 MHz RF system at a fixed frequency

**What does it require from the hardware?**

- Turn-by-turn modulation of the RF phase
  - Can be implemented globally through the beam phase loop
  - Or locally on a given cavity controller
- In the LHC, x6 emittance blow-up is needed (cf. injectors 10-40 %)
  - Noise amplitude is regulated with a bunch length feedback




*Diffusion at constant energy due to band-limited phase noise*

[14] T.Toyama: 'Uniform bunch formation by RF voltage modulation with a band-limited white signal', NIM A, 447, 317–327 (2000).

# RF phase noise generation

**Generation in time domain by colouring white noise**

- Implementation for the SPS and LHC [15]
  - Turn-by-turn injection: $\Delta t = T_{\mathrm{rev}}$
  - Using a complex Fourier transform: $f_{\max} = 1/\Delta t = f_{\mathrm{rev}}$
  - Generate white (carrier) noise in time domain:

    $w_k(t) = e^{2\pi i \mathrm{RAND}_{1,k}} \sqrt{-2 \ln \mathrm{RAND}_{2,k}}$
  - Transform the generated white noise to frequency domain

    $W_l(f) = \mathrm{FFT}[w_k(t)] = \sum_{k=1}^{N} w_k(t) e^{-2\pi i \frac{kl}{N}}$
  - Colour the spectrum with the desired noise probability density [rad]:

    $\Phi_l(f) = s_l(f) W_l(f)$, where $s_l(f) = \sqrt{A S_l^{\mathrm{DB}} f_{\max}}$
  - Transform back to time domain to obtain the RF phase noise

    $\varphi_k(t) = \mathrm{IFFT}[\Phi_l(f)]$

Considerations

- Frequency is changing along the ramp
  - Need to readjust the spectrum every turn
  - Need to make sure that the final sequence is continuous and has no jumps
- LHC parameters
  - Synchrotron frequency: ~20-50 Hz
  - Noise band ~0.15 $f_{s,0}$
  - Resolution needed: ~0.01 Hz (30 points)
  - Revolution frequency: 11245 Hz
  - Need at least 1.1 M points in the FFT for every single phase generated!



[15] J. Tückmantel: 'Digital generation of noise-signals with arbitrary constant or time-varying spectra', LHC-PROJECT-Report-1055 (2008).

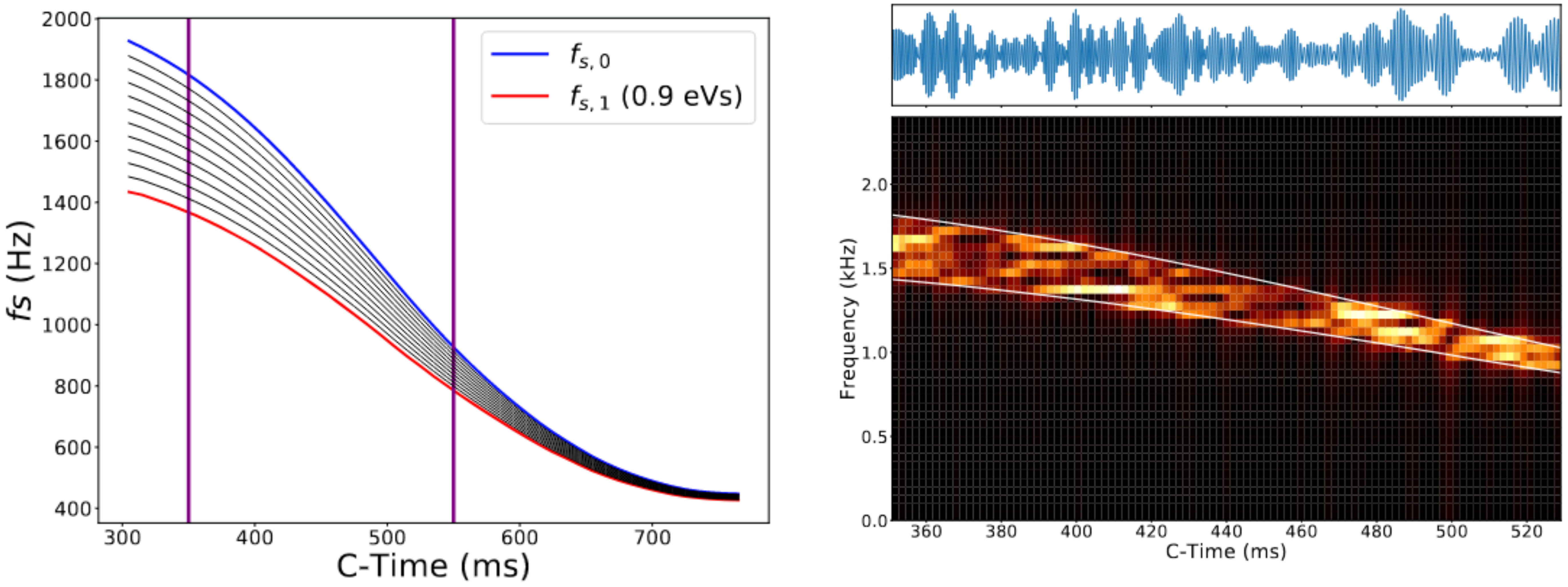
# RF phase noise generation

**Generation in frequency domain by summing single frequencies**

- Sum of RF phase modulation from single sine-waves at different, close frequencies in the desired band

$$\Delta\varphi_{\mathrm{rf},(n)} = A \sin\left(2\pi \sum_{k=0}^{n} f_{\mathrm{mod},(k)} T_{\mathrm{rev},(k)}\right) + \varphi_{\mathrm{off},(n)}$$

- Used in the PSB operationally, gives a noise spectrum equivalent to the time-domain implementation




*Phase noise generation in frequency domain:*
*a sum of single-frequency modulations applied in the PSB [16]*



[16] S. Albright *et al.*: 'Time Varying RF Phase Noise for Longitudinal Emittance Blow-up', J. Phys.: Conf. Ser. **1350**, 012144 (2019).

# LHC implementation

## Injection through the beam phase loop

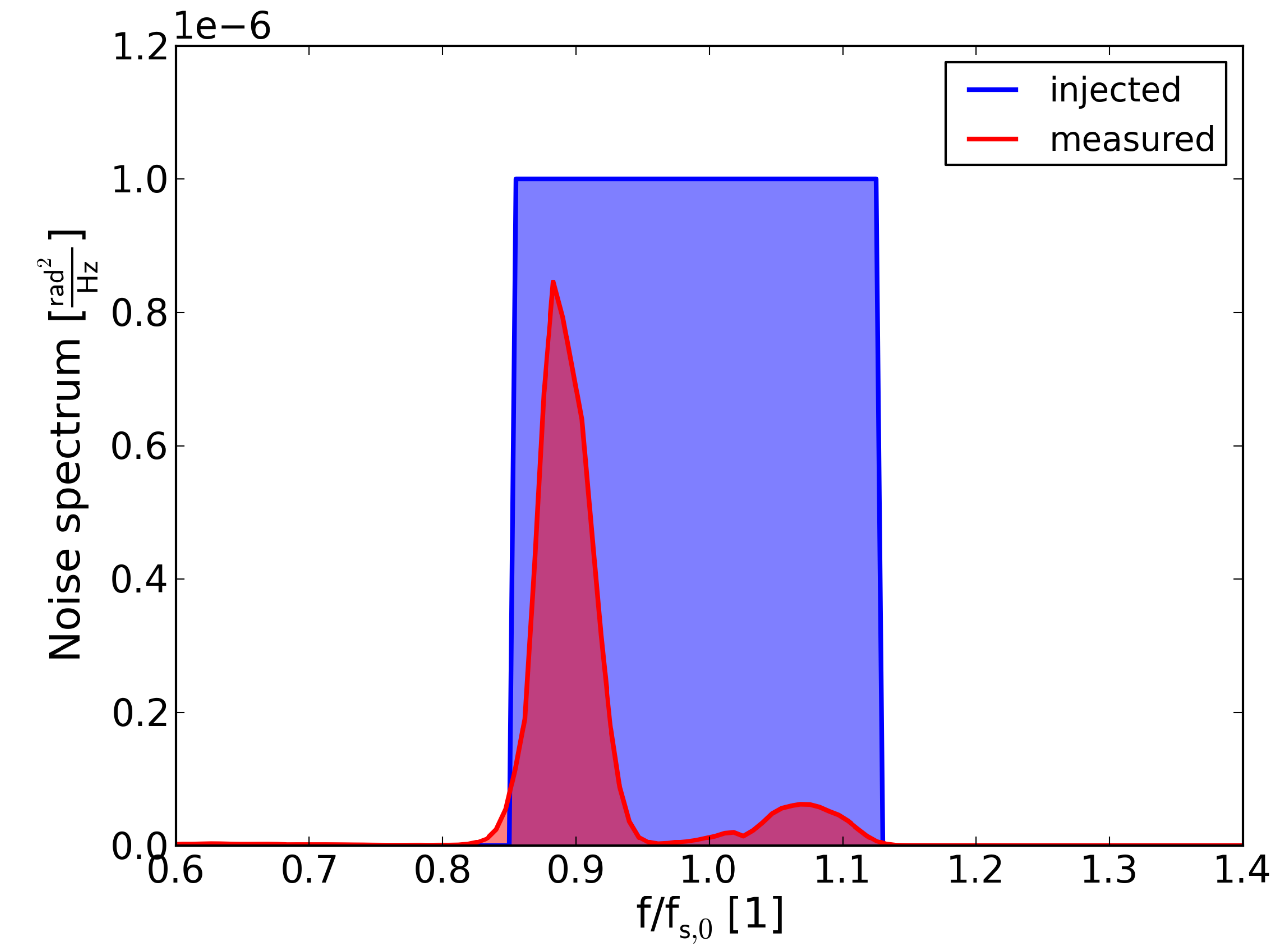
- Designed to damp background RF noise at the central synchrotron frequency
  - To increase beam lifetime from 10s minutes to 10s hours
- Heavily distorts the spectrum of the injected noise
  - In the LHC, the noise is shaped with the beam transfer function (BTF) of a parabolic function to the beam phase loop
    - Assumes a constant 1.25 ns bunch length!

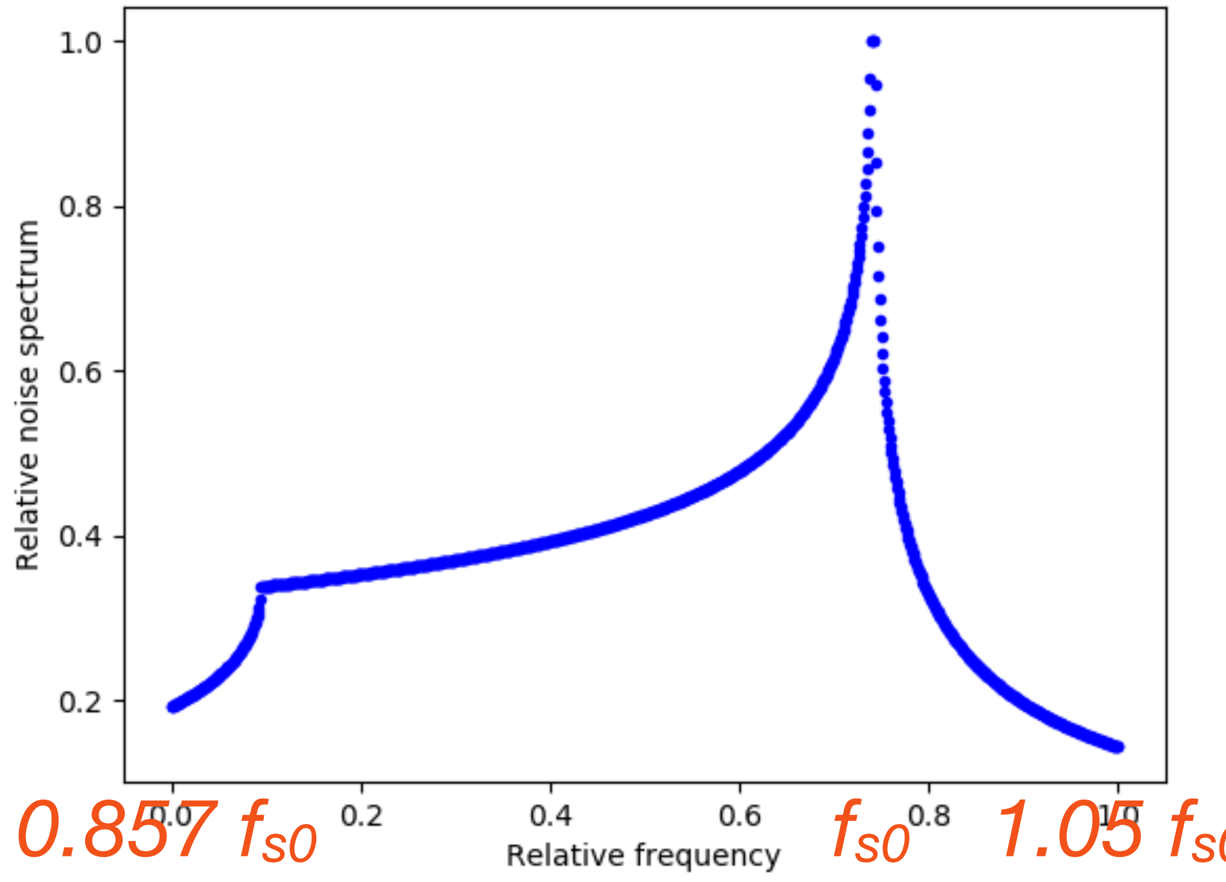
## Noise amplitude regulation through bunch length feedback

- Bunch length acquired every 2 s, $\tau_{\text{meas}} \equiv 2/\sqrt{2\ln 2}\,\tau_{\text{FWHM}}$
- Noise amplitude iterated through a lossy low-pass filter:

$$x_{n+1} = a x_n + g(t)(\tau_{\text{targ}} - \tau_{\text{meas}})$$

  - Memory factor: $a = 0.87$
  - Gain: $g(t) = 0.2\,\text{ns}^{-1}[f_{s,0}(t=0)/f_{s,0}(t)]^2$




*Top: injected vs measured noise spectrum [17]*
*Bottom: operational noise spectrum*

[17] P. Baudrenghien *et al.*: 'Longitudinal emittance blow-up in the large hadron collider', NIM A **726**, 181190 (2013).

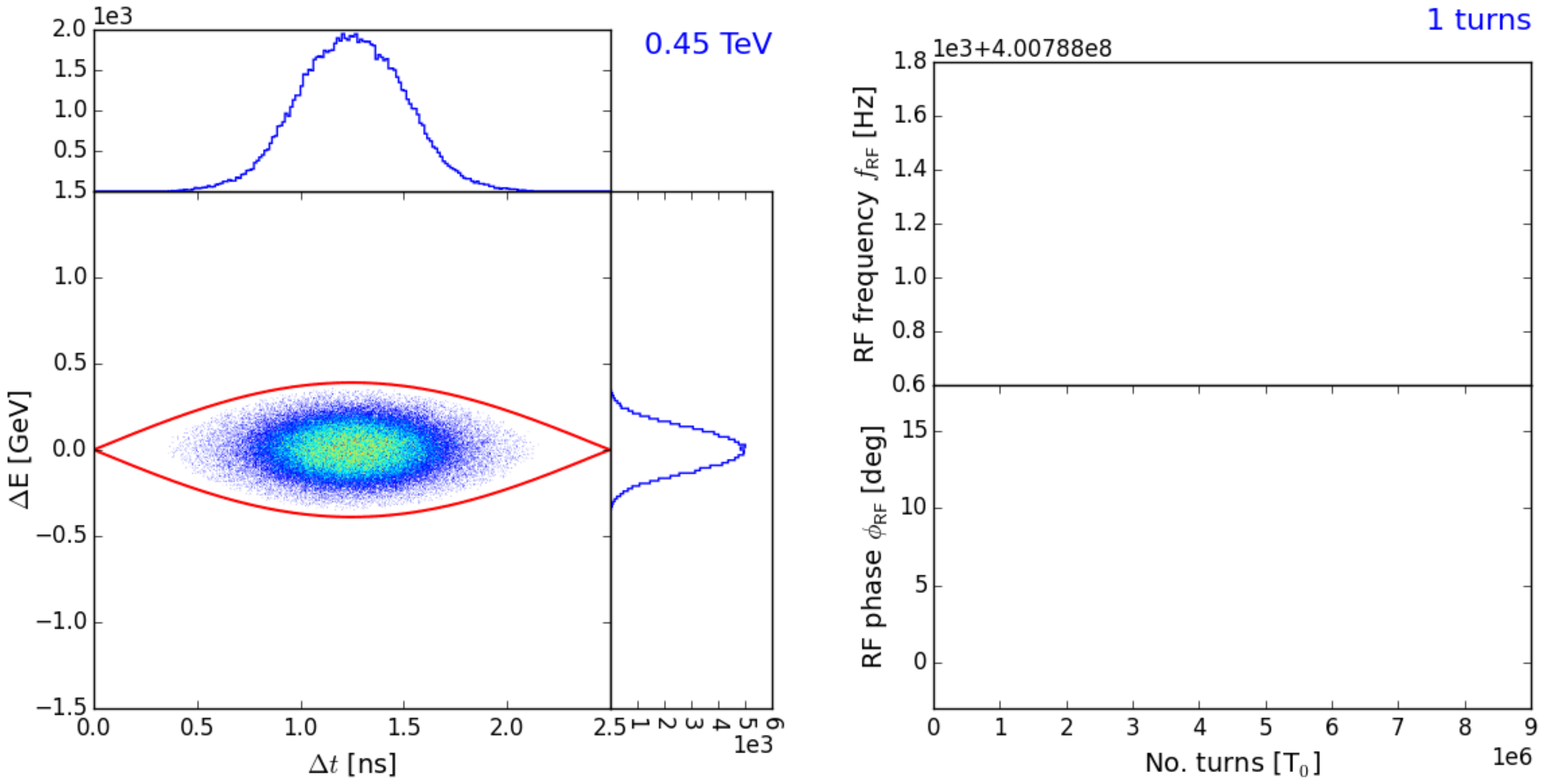
# LHC bunch length and shape evolution

**Shape transition in the first part of the ramp**

- Exact evolution and outcome of each ramp is statistical
  - Depends heavily on initial conditions
    - Bunch length, bunch intensity, phase-space distribution…
- Only average behaviour can be compared to

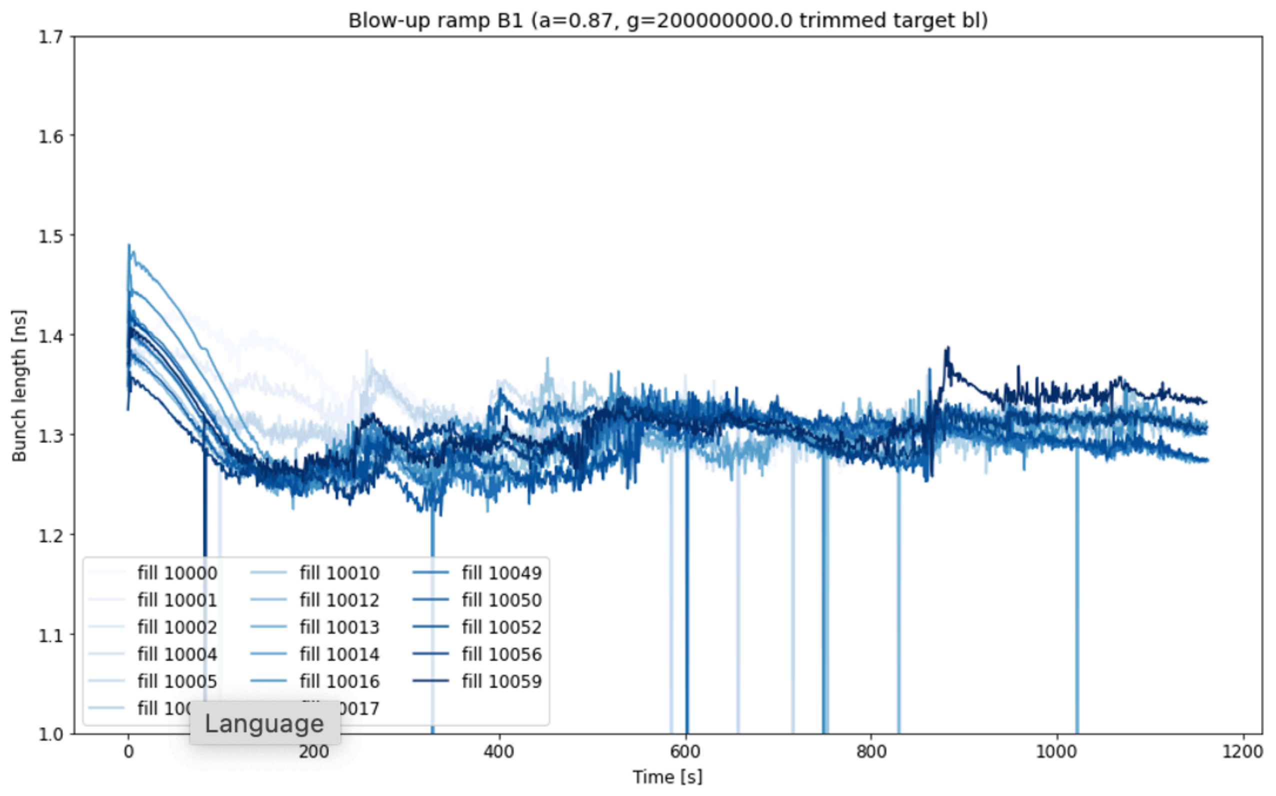
**Modelling challenges**

- Ramp is 20 minutes, >14 M turns
  - Simulation requires control loops and intensity effects
- Limited to a few bunches
  - In the machine, the noise is regulated through the average bunch length of >2000 bunches!
- Would require ensemble averages over large scans to predict average behaviour




*Left: phase-space distribution and longitudinal profile*

*Right: RF frequency and phase evolution*




*Bunch length throughout the ramp*

*Courtesy N. Gallou*


[18] H. Timko *et al.*: 'Studies on controlled RF noise for the LHC', Proc. HB'14, THO4LR03, East-Lansing, USA (2014).

QUIZ

## Intermezzo: how to shrink the longitudinal emittance?


*You can make use of a physical phenomenon*

# Shrinking emittance - synchrotron radiation

**In high-energy machines, synchrotron radiation & quantum excitation become significant**

- Synchrotron radiation shrinks, quantum excitation blows up the bunch emittance
- Energy loss per turn [19]

$$E_{\mathrm{other},(n)} = -U_0 - \frac{2}{\tau_z}\Delta E_{(n)} + 2\frac{\sigma_{\Delta E}}{\sqrt{\tau_z}}E_{d,(n)}\mathtt{RANDN}$$

Average energy loss ($U_0$); Difference in energy loss for each particle ($\frac{2}{\tau_z}\Delta E_{(n)}$); Quantum excitation ($2\frac{\sigma_{\Delta E}}{\sqrt{\tau_z}}E_{d,(n)}\mathtt{RANDN}$)

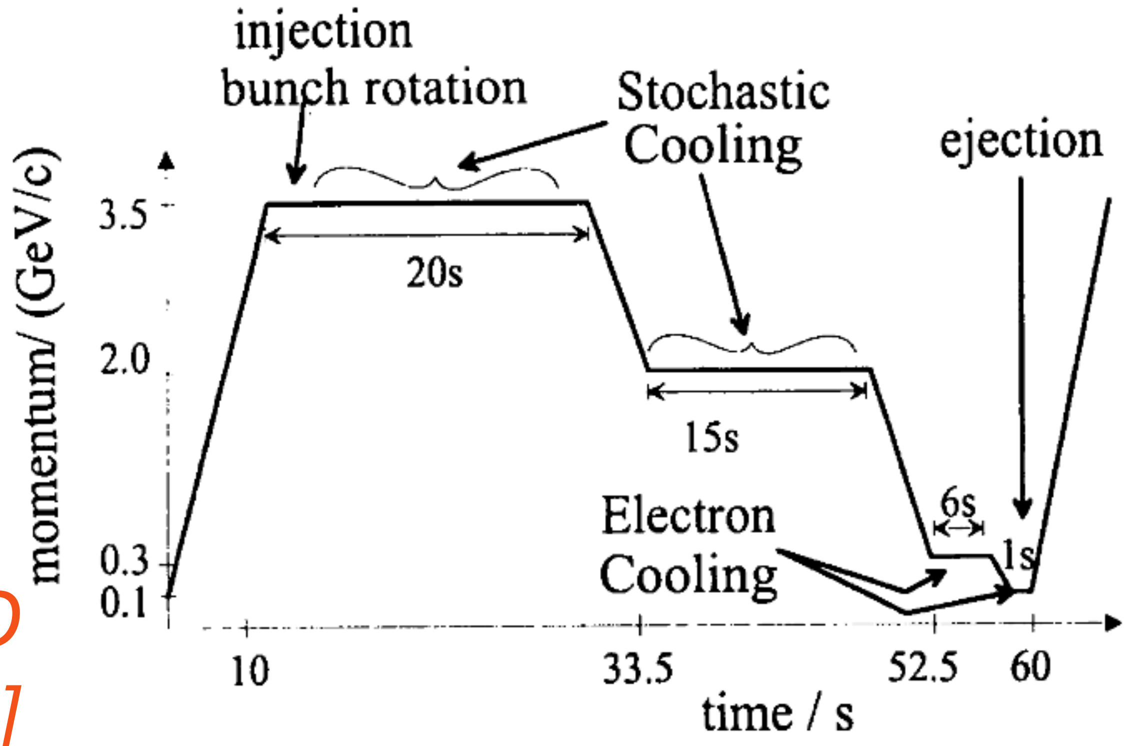
  - Where the average energy loss is:

$$U_0 = \frac{4\pi}{3}\frac{r_{\mathrm{cl}}}{m_p^3 c^6}\frac{1}{\rho}E_{d,(n)}^4\frac{R}{C}$$

| | |
|---|---|
| $\rho$ | magnet bending radius |
| $\sigma_{\Delta E}$ | equilibrium energy spread |
| $\tau_z$ | damping time [turns] |
| $m_p$ | particle mass |
| $\mathtt{RANDN}$ | normal random number in (0,1) |
| $r_{\mathrm{cl}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{m_p c^2}$ | classical particle radius |

**Shrinking emittance in a controlled manner**

- Stochastic cooling
  - Cooling with a pick-up and kicker feedback system
- Electron cooling
  - Thermal exchange of electron-ion plasma
- Laser cooling, ionisation cooling, etc.




*Design cycle for the AD deceleration from [20]*



[19] J. Esteban Müller: 'Modification of the simulation code BLonD for lepton rings', Tech. Note, (2017)
[20] C. Carli *et al.*: 'Stochasitic cooling at the CERN Antiproton Decelerator', Tech. Note CERN-PS-2000-024 (2000).

# Resonant excitation

**What is it?**

- Single sine-wave modulation injected on the RF phase

$$\Delta\varphi_{\mathrm{rf},(n)} = A_{\mathrm{mod}} \sin\left(2\pi f_{\mathrm{mod}} \sum_{k=0}^{n} T_{\mathrm{rev},(k)}\right) + \varphi_{\mathrm{off},(n)}$$

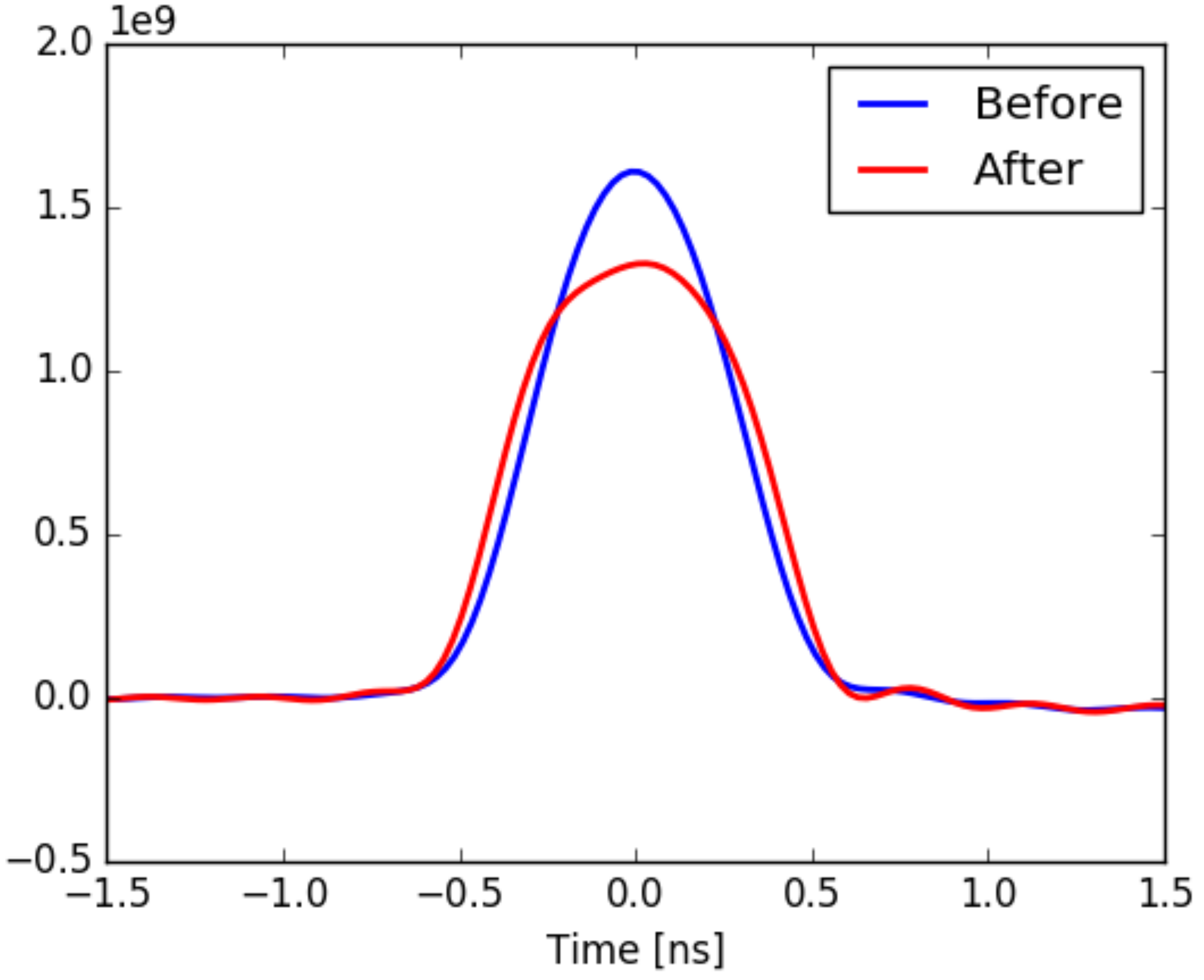
- Results in a resonant change of the bunch profile

**How is it done?**

- The modulation frequency determines the final bunch length and bunch shape
- The modulation amplitude has to be above a given threshold, but does not influence the bunch length

**Why use it?**

- Shapes the bunch without generating tails
- A round core can be used to reduce space charge, IBS, etc.




*Measured bunch profile in the LHC before and after flattening*

*Modulation at $0.98 f_{s0}$ with $0.6°$, resulting in 100-150 ps increase for ~1.2 ns bunches*



[1] S. Y. Lee: 'Accelerator Physics', World Scientific, 3rd Ed. (2012).
[21] C. Y. Tan *et al.*: 'Phase modulation of the bucket stops bunch oscillations at the Fermilab Tevatron', PRAB **15**, 044401 (2012).

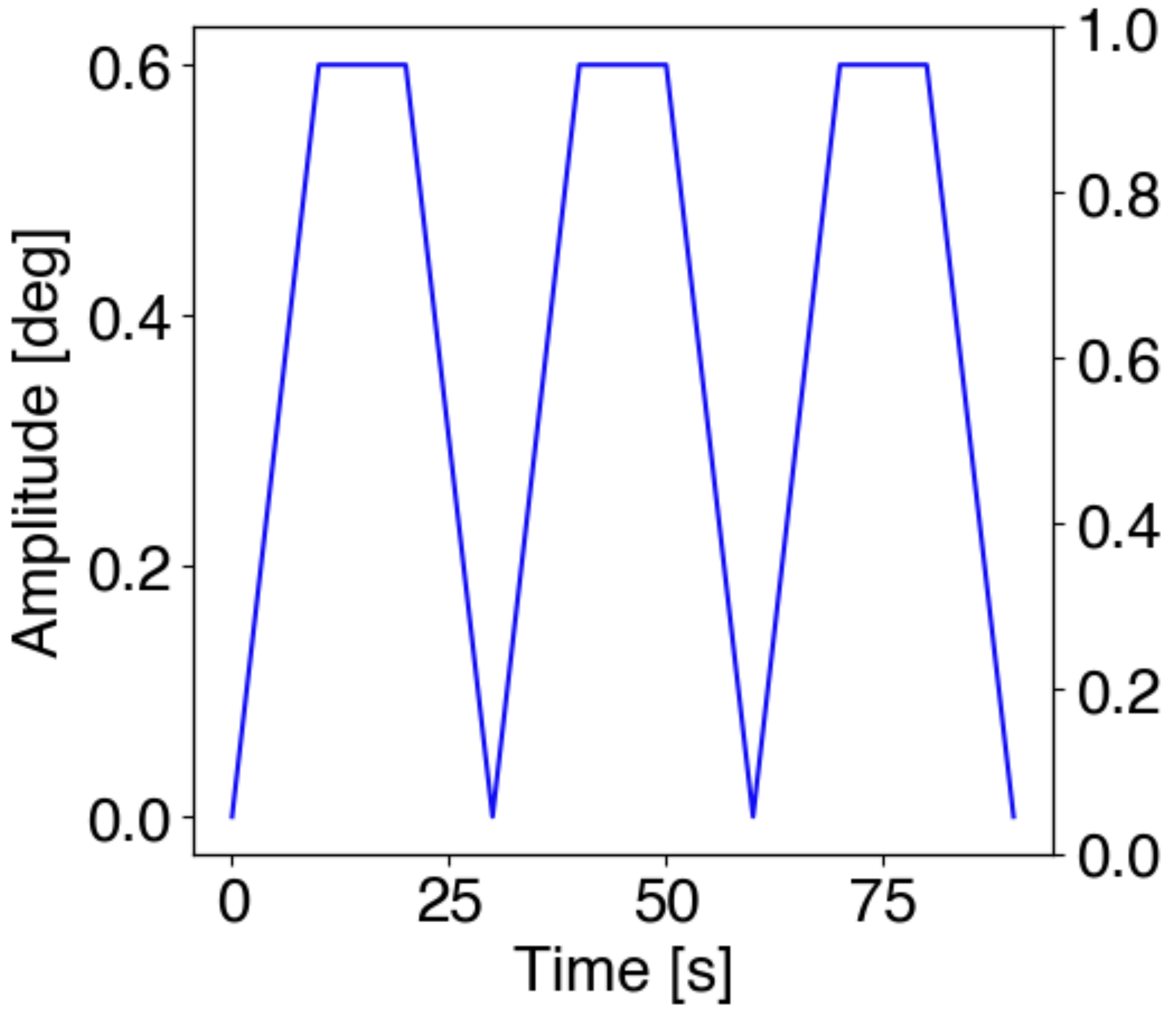
# Resonant excitation

**Where is it applied?**

- In the LHC, during stable beams with protons
  - At 6.8 TeV (flattop), the bunch length is shrinking significantly due to synchrotron radiation
  - With bunch flattening, the luminous region in the experiments is kept about constant

**What does it require from the hardware?**

- Opening the beam phase loop while injecting the modulation
  - If done too often, or too long, can impact beam lifetime
- Experimentally, the best configuration found is to do three trapezoids in amplitude
  - Once the reshaping happened, subsequent applications of the modulation do not affect the beam anymore



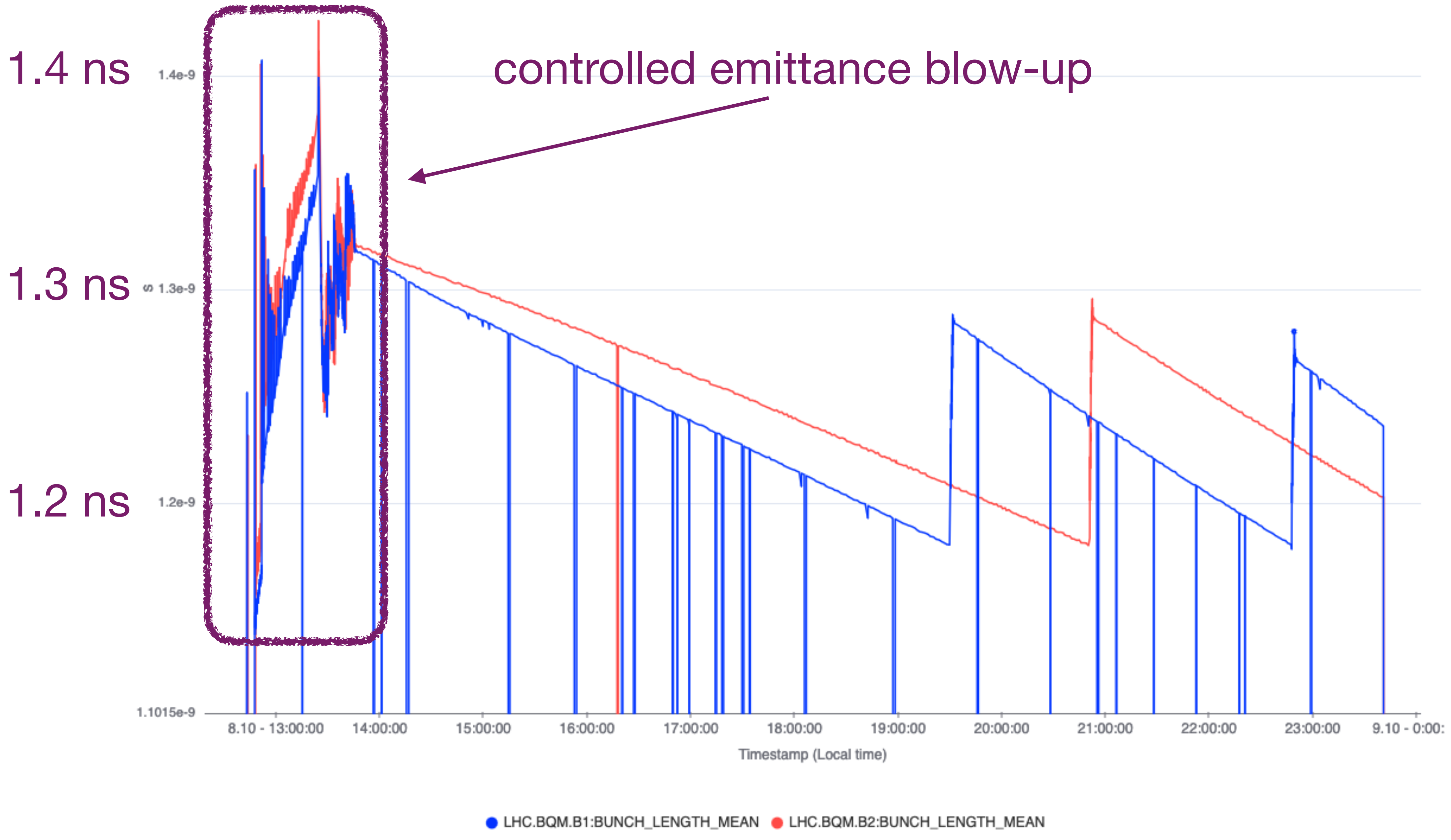

*Amplitude function used in the LHC*

[22] E. Shaposhnikova *et al.*: 'Flat Bunches in the LHC', Proc. IPAC'14, Dresden, Germany (2014).

# Regulating the luminous region

**When the bunch length decreases**

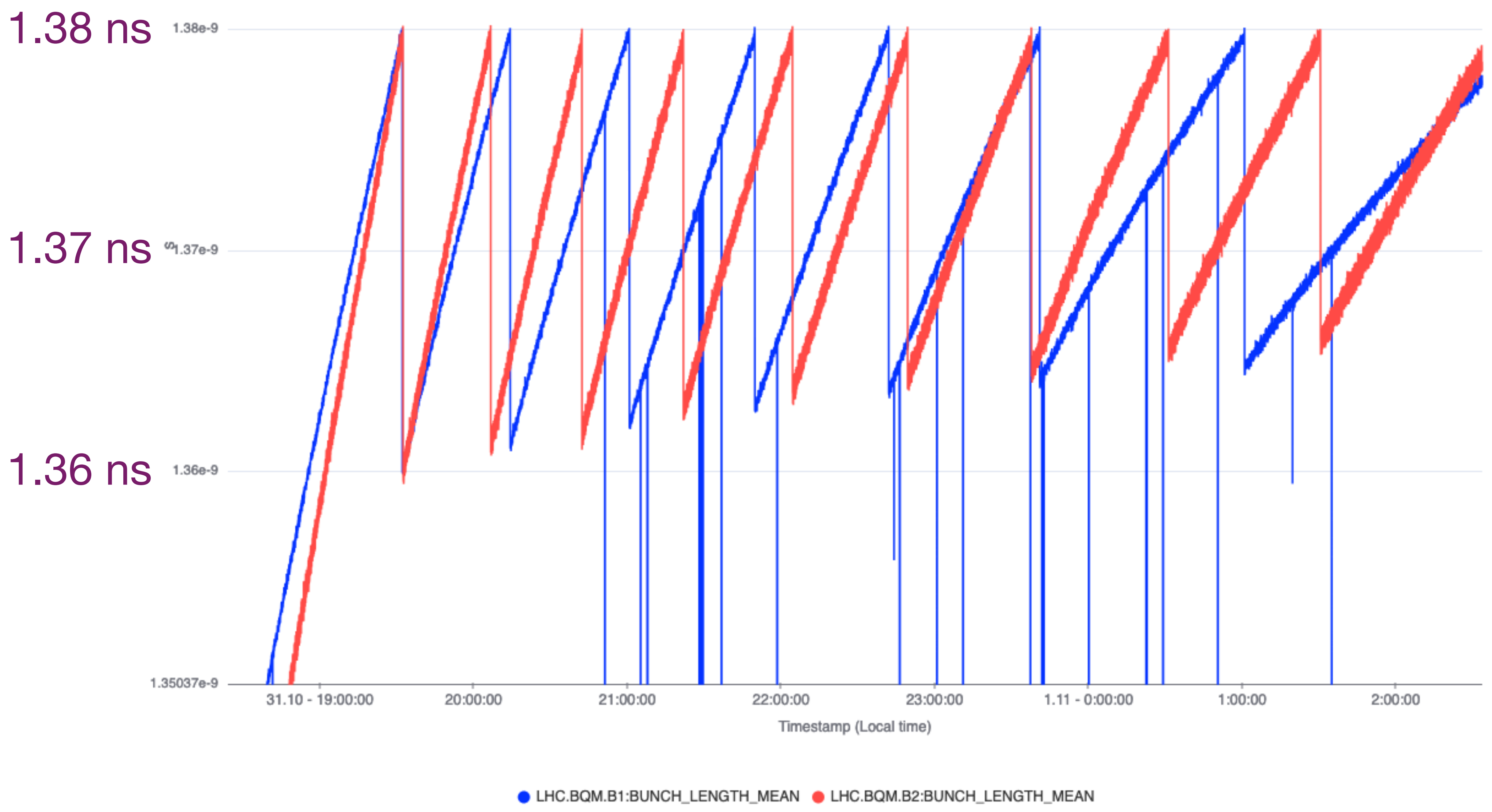
- With resonant excitation
- E.g. LHC protons at 6.8 TeV




*Bunch length regulation in the range of 1.18-1.29 ns, with resonant exctitation*

**When the bunch length increases**

- With adiabatic voltage steps
- E.g. LHC protons at 2.68 TeV




*Bunch length regulation in the range of 1.36-1.38 ns, with steps of 0.5 MV*

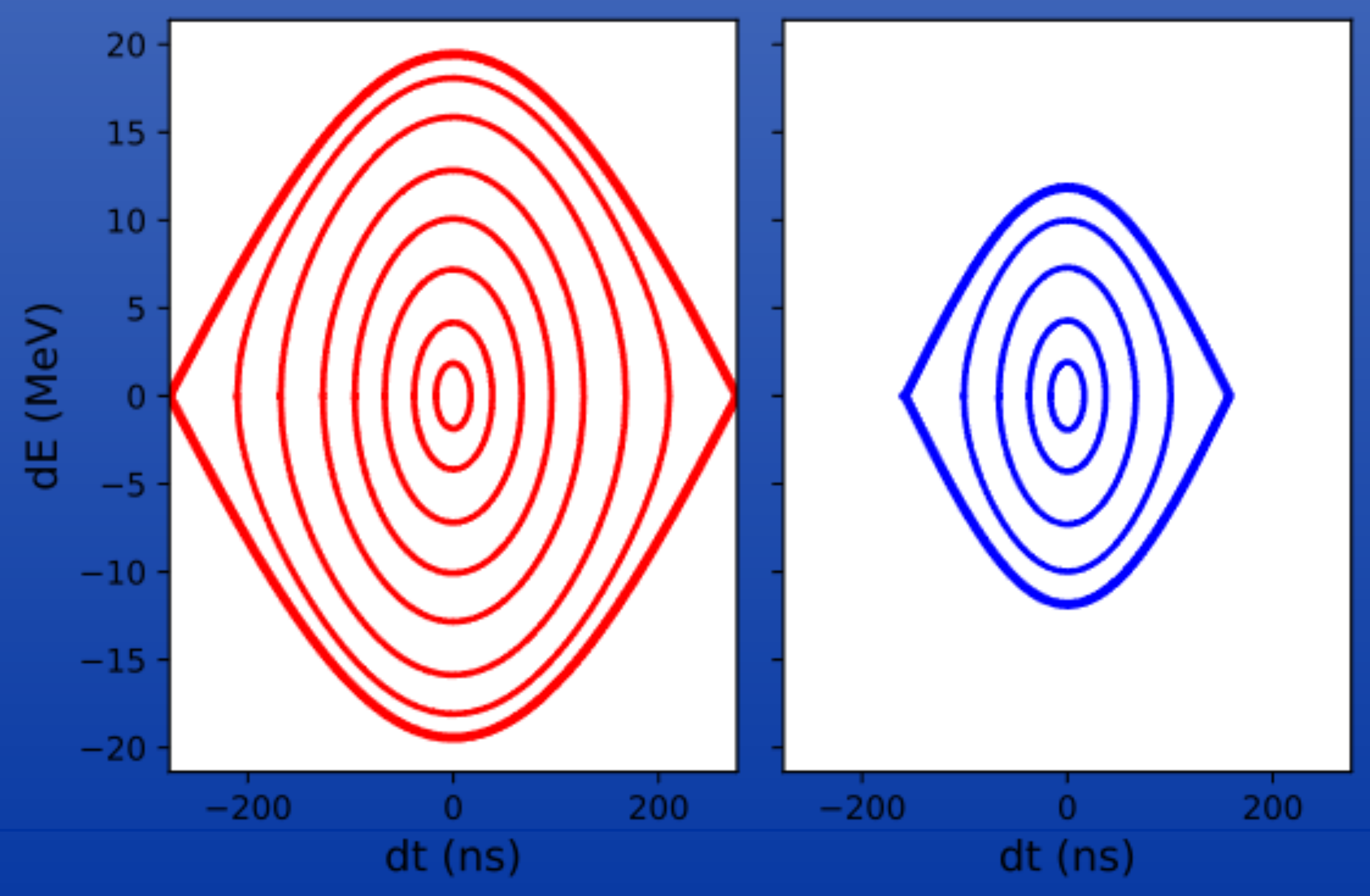
QUIZ

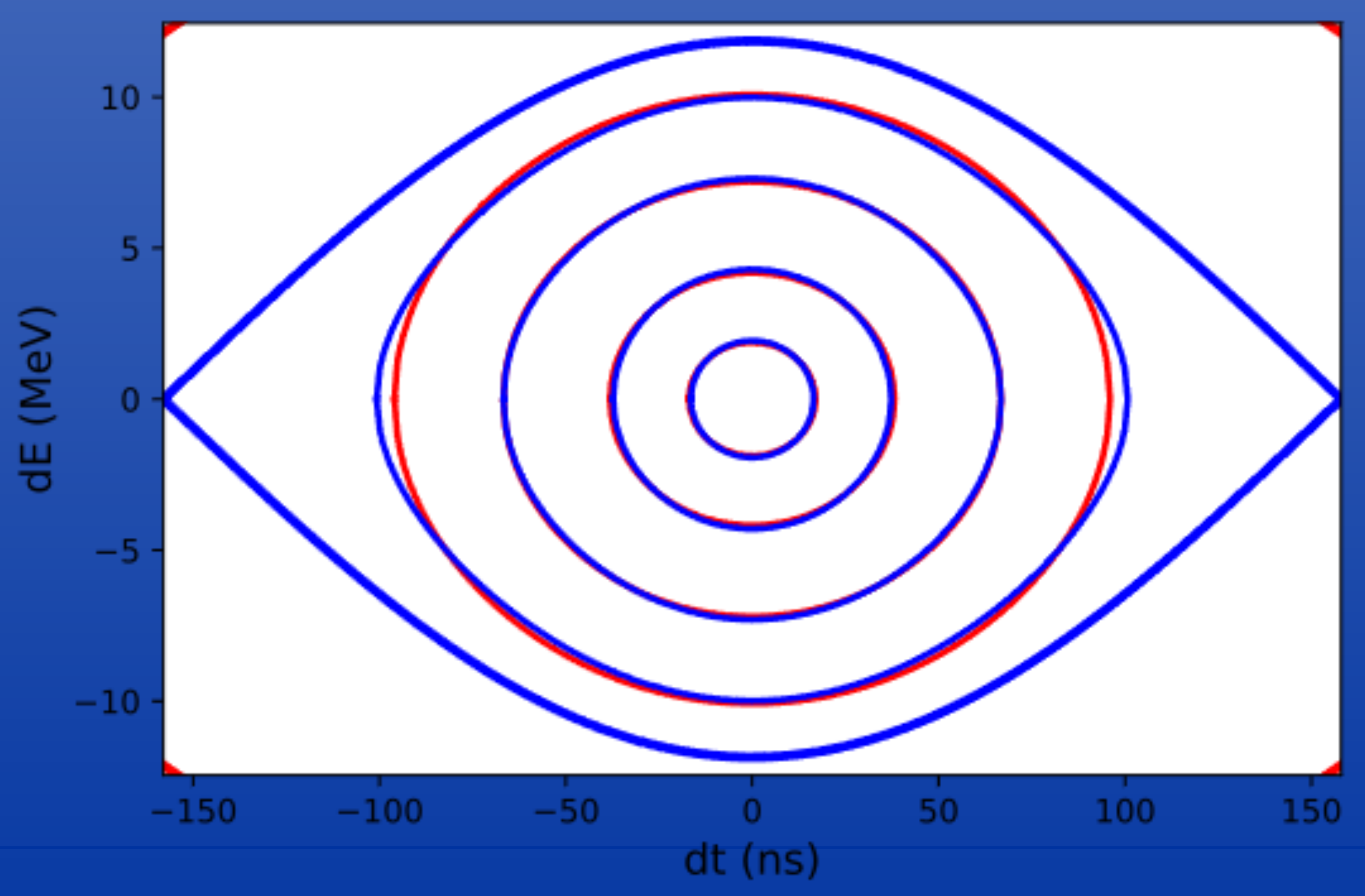
# Bunch-to-bucket transfer: how do I know whether an extracted bunch is matched for a given injection bucket?


*A matched bunch does not exhibit oscillations*

**Example:**

**PSB-to-PS transfer**



# Longitudinal painting

**What is it?**

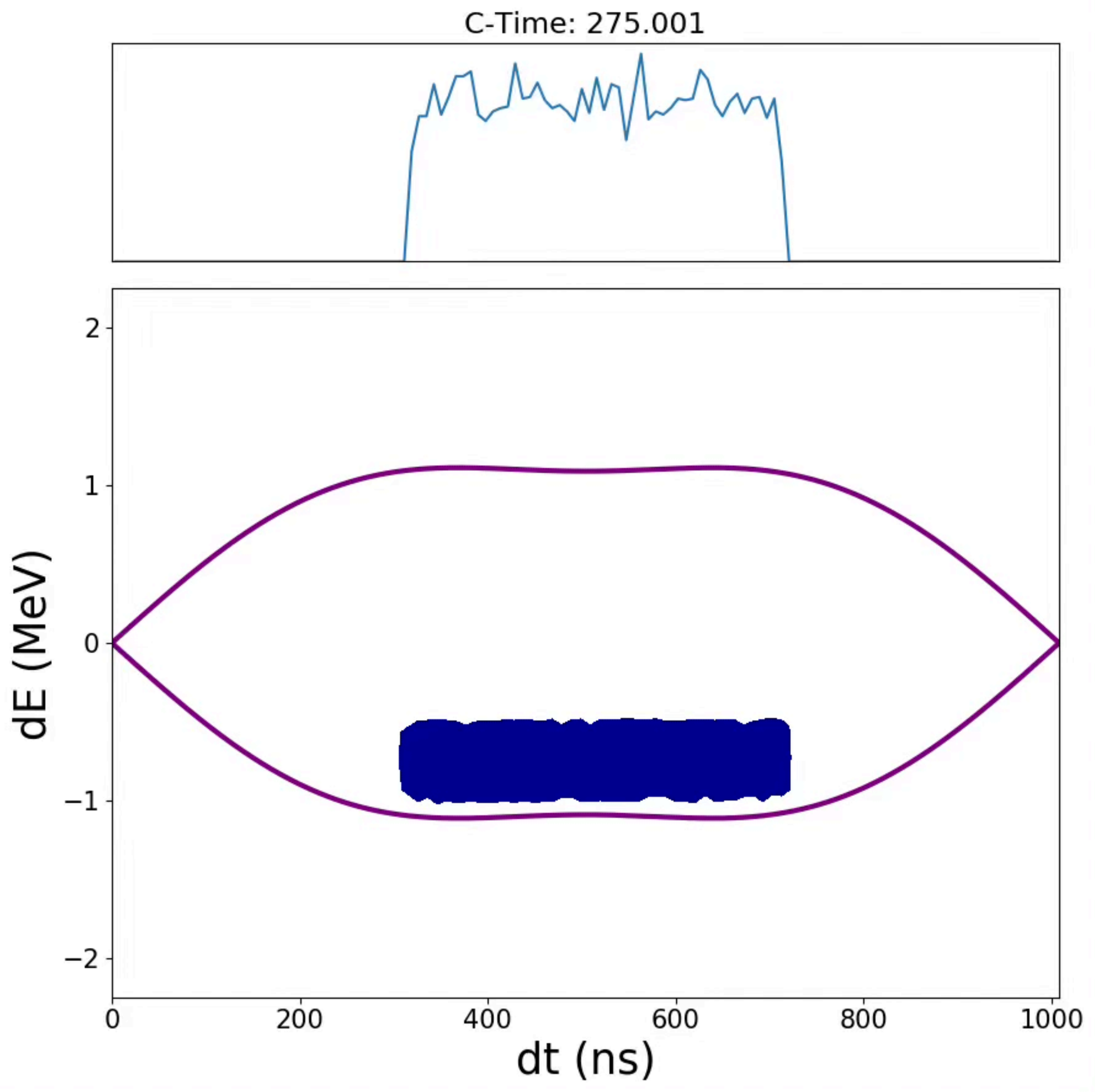
- Filling out of the longitudinal phase space by injection of many micro-bunches
  - Injection over several turns, on different time and energy positions in phase space
  - Filamentation process thereafter

**How is it done?**

- Injection over tens or hundreds of turns
- Sweeping the energy offset and the length of the arriving batch

**Why use it?**

- To increase the bunch intensity
- To match the bunch into the injection bucket
- To fill out the phase space for longitudinal stability
  - Reduce space charge effects, for instance




*Multi-turn injection and longitudinal painting*

*Courtesy of S. Albright*

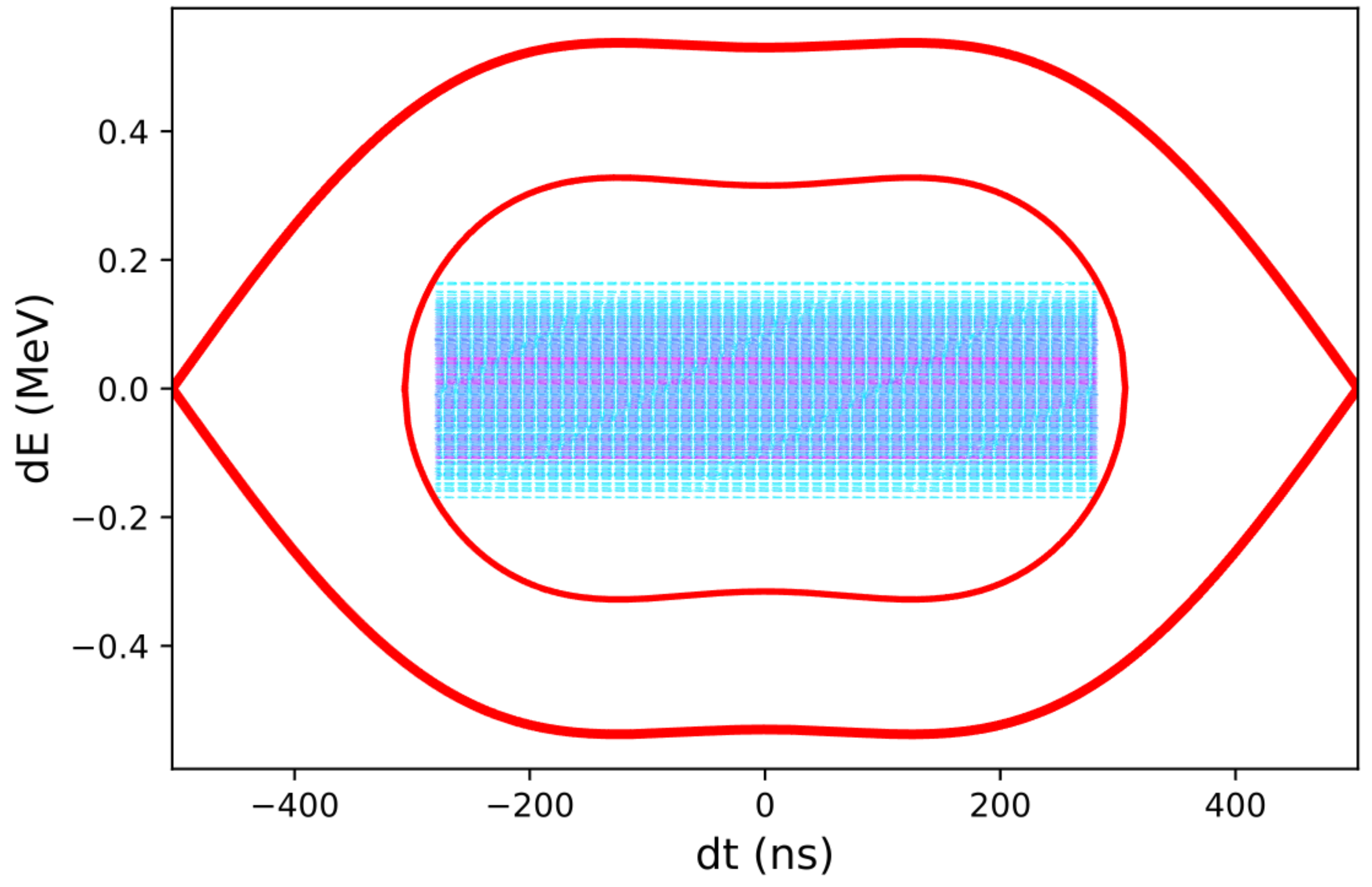
# Longitudinal painting

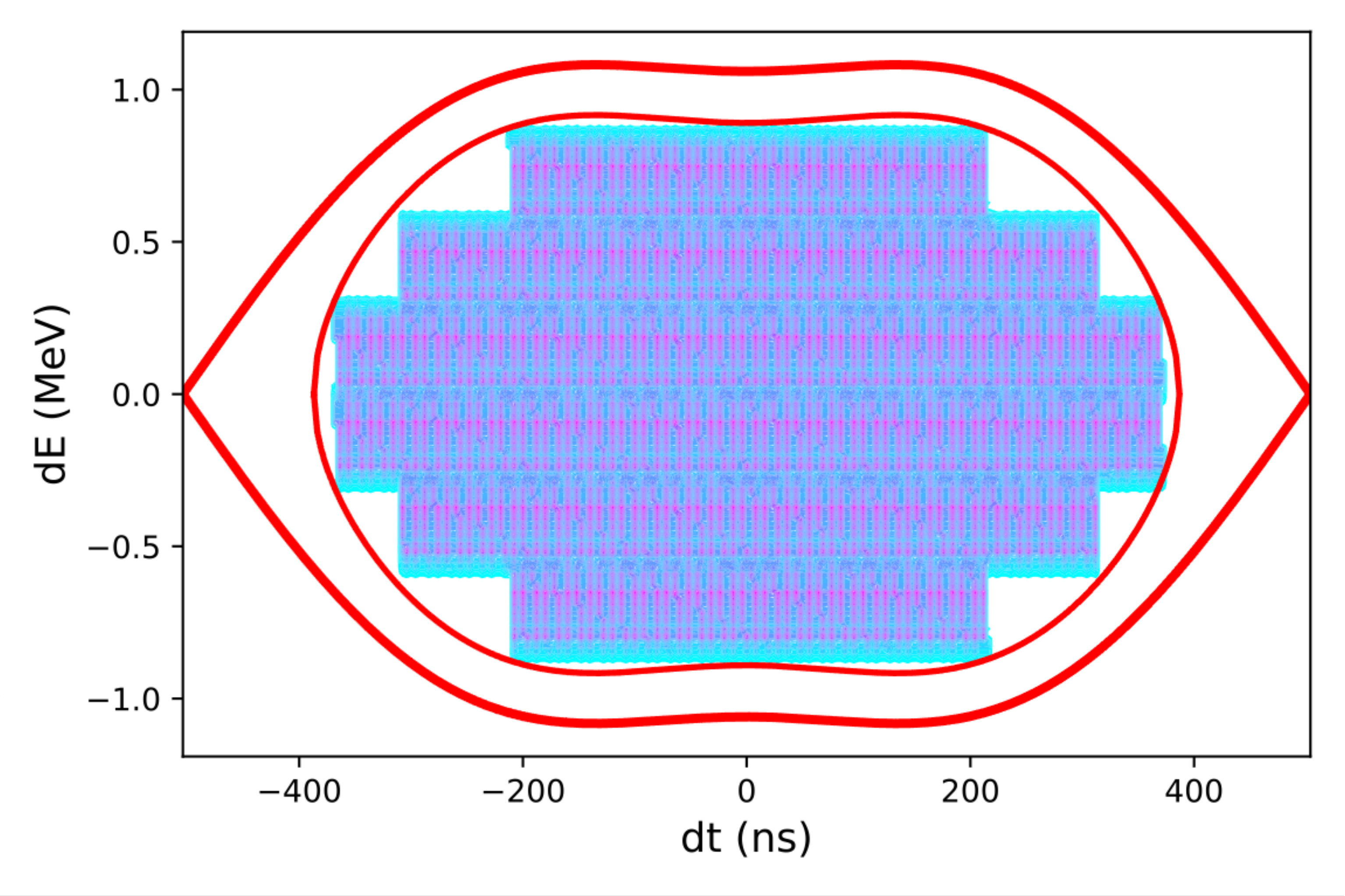
**Where is it applied?**

- In the PSB at injection for ISOLDE and TOF (highest beam currents)
  - Linac4: 352 MHz vs PSB 1 MHz, so matching is not possible
  - Injecting a large number of micro-bunches

**What does it require from the hardware?**

- On the Linac4 side
  - Being able to modulate the extraction energy and calibrate it
    - In addition, need to phase the bunches at different energies
  - Being able to chop the bunches as a function of energy offset
- On the PSB side
  - Capture in double harmonic RF
  - Fill all four rings one after the other




*The principle of the PSB painting with Linac 4 bunches, from [23]*

[23] S. Albright: 'Implementation of longitudinal painting at PSB injection', CERN SY-RF group seminar (2024).

# Debunching

**What is it?**

- Creation of continuous beam from bunched beam

**How is it done?**

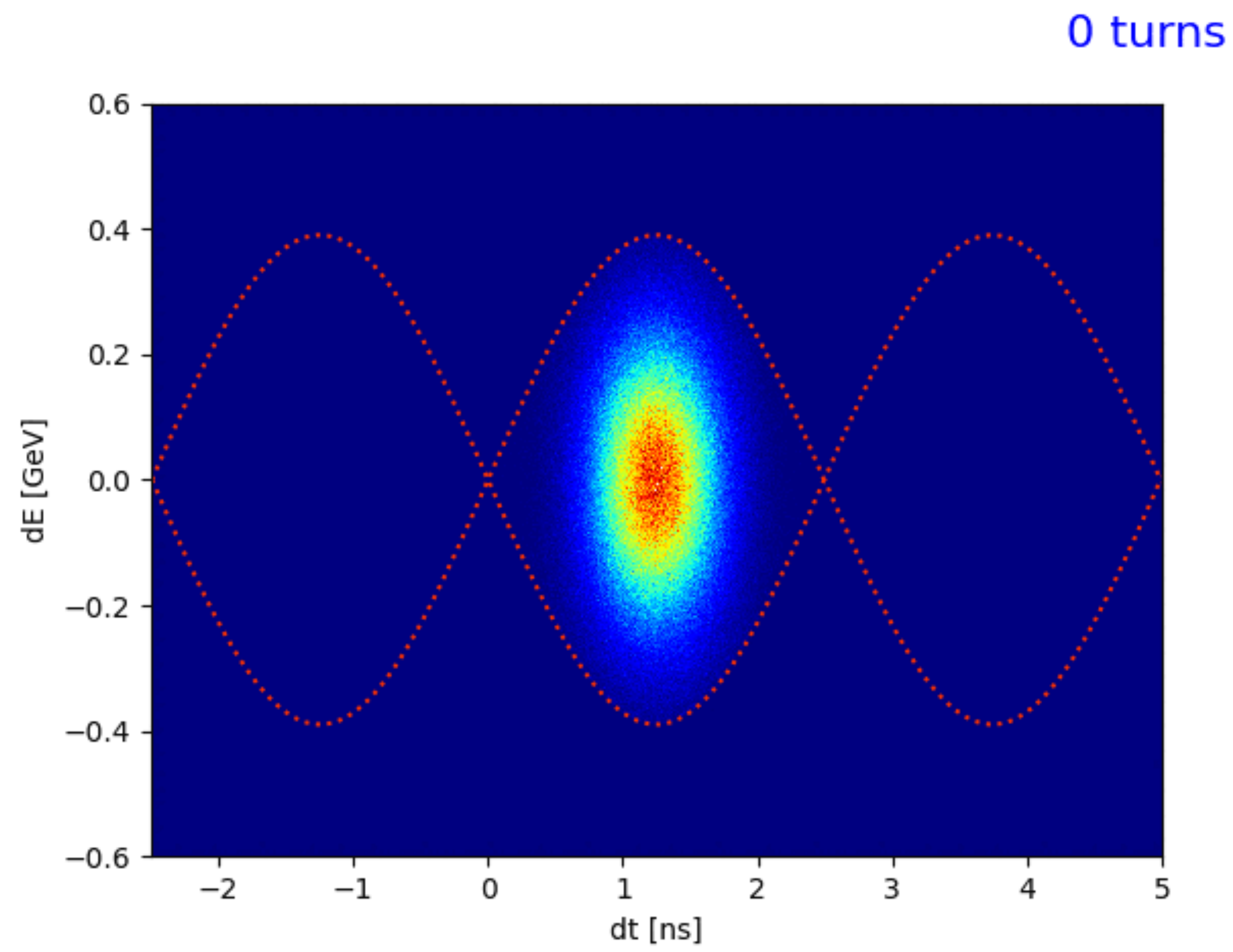
- Switching off the RF voltage
  - Alternative: use low voltage and counter-phase cavities
  - Debunching time: $$t_{\mathrm{deb}} = \frac{2\pi}{h\omega_{\mathrm{rev}}|\eta_0|\delta_{\max}}$$

**Why use it?**

- For experiments that want a continuous beam (e.g. fixed target)
- Combined with re-bunching, to perform a change of harmonic number

**Where is it applied?**

- SPS fixed-target experiments
- AD change of harmonic number
  - Need good control of RF frequency for capture




*How debunching in the LHC would look like over 2000 turns…*

# (Re-)bunching

**What is it?**

- Creation of bunched beam from continuous (coasting) beam

**How is it done?**

- Slowly switching on the RF voltage
  - Adiabaticity factor

$$\alpha = 2\pi \frac{1}{\omega_s^2} \frac{d\omega_s}{dt}$$

  - Voltage function with constant adiabaticity:

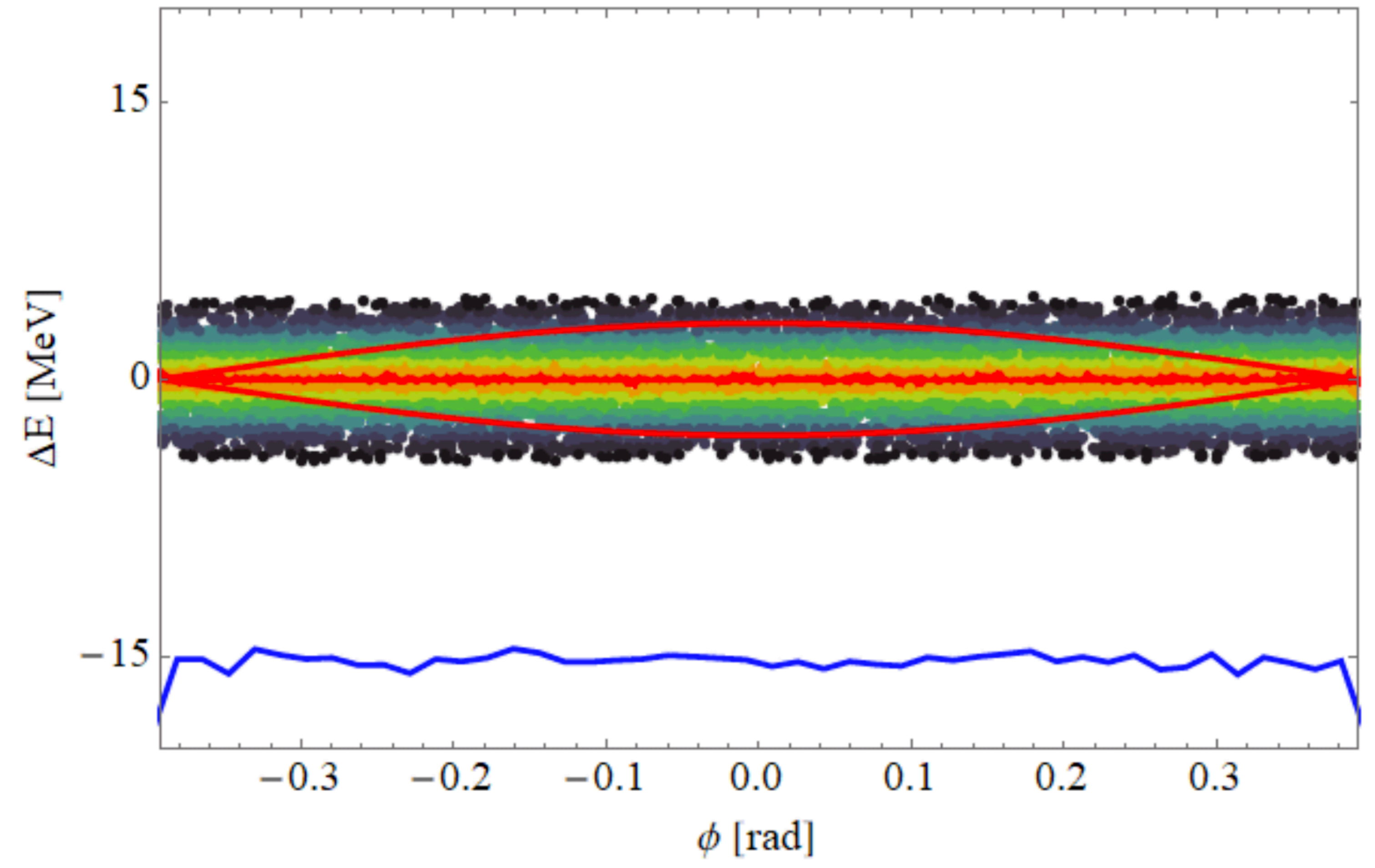
$$V_{\mathrm{rf}}(t) = \frac{V_i}{\left(1 - \frac{t}{\tau} \frac{\sqrt{V_f} - \sqrt{V_i}}{\sqrt{V_f}}\right)^2}$$

**Why use it?**

- To capture coasting beam

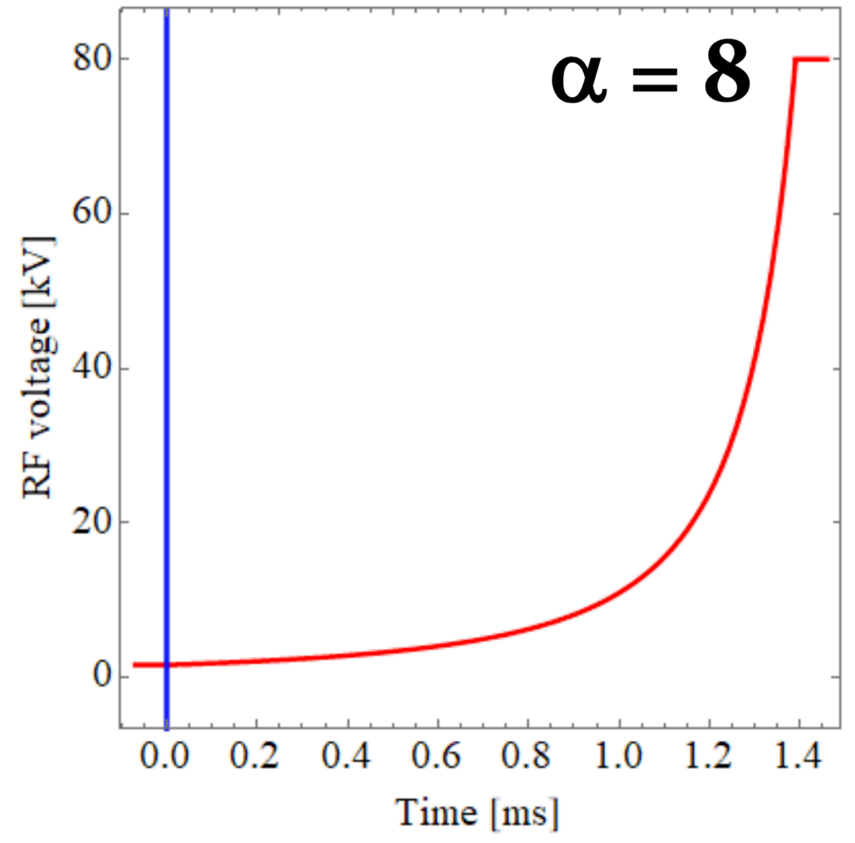
**Where is it applied?**

- AD beam capture




*RF capture of coasting beam*

*Courtesy of H. Damerau*



# Contents

## Bunch length regulation

- Adiabatic changes
- Rotation
- Splitting, merging

## Phase space regulation

- Diffusion, noise injection
- Resonant excitation
- Longitudinal painting
- Debunching

## Advanced manipulations

- Momentum slip stacking
- Barrier bucket

## Integration in an RF system

- Beam loading
- RF voltage/power limitations
- Designing RF parameters


QUIZ

# What is slip stacking?

**A.) A particle accelerator configuration used to store two particle beams with different momenta in the same ring**

**B.) A process of combining two bunched beams in a synchrotron into a single beam**

**C.) An accumulation technique used at Fermilab to nearly double proton intensity**

# Momentum slip stacking

**What is it?**

- Azimuthal slippage of two batches in opposite direction
  - In the same beam pipe, via slightly different momenta

**How is it done?**

- Capture two beams with two RF systems of slightly different frequency

$$V_{\rm rf} = V_{\rm rf,1} \sin(\omega_{\rm rf,1} t + \varphi_{\rm rf,1}) + V_{\rm rf,2} \sin(\omega_{\rm rf,2} t - \varphi_{\rm rf,2})$$

- The small frequency difference results in a phase error

$$\Delta\varphi_{\rm rf} = \frac{2\pi h \Delta\omega_{\rm rf}}{\omega_{\rm rf,d}}$$

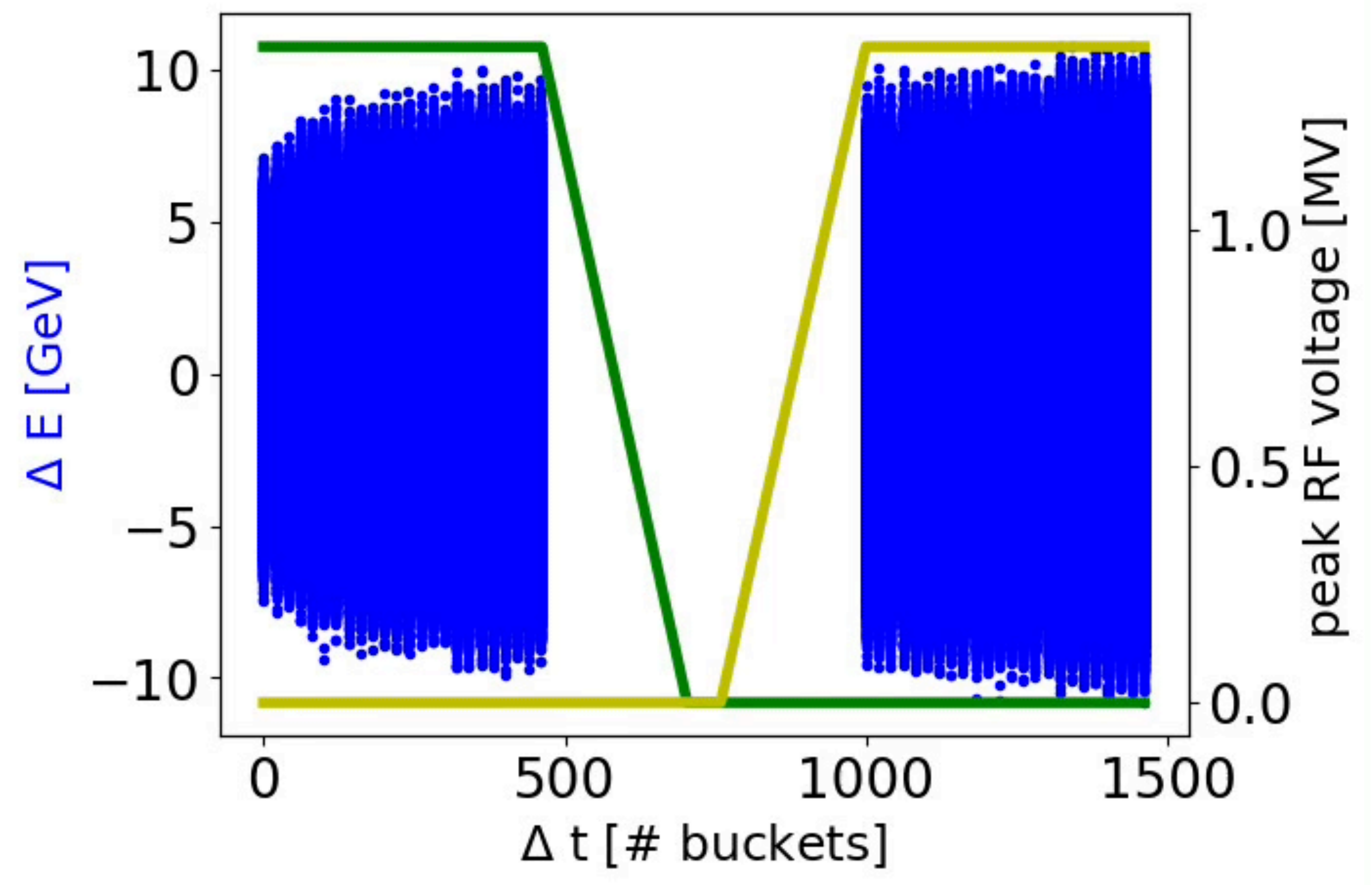
- And at constant magnetic field it translates to a slippage (drift) of

$$\frac{\Delta\omega_{\rm rf}}{\omega_{\rm rf,d}} = -\eta_0 \frac{\Delta p}{p_{\rm d}}$$

- Recapture with the full RF system at the desired longitudinal position

**Why use it?**

- To interleave the batches, i.e. reduce batch spacing
- To merge the batches, i.e. increase the intensity




*Simulated momentum slip stacking for ion beams in the SPS*

*Courtesy of D. Quartullo*



[24] D. Boussard *et al.*: 'Production of beams with high line-density by azimuthal combination…', CERN-SPS-ARF-79-11 (1979).
[25] K. Seiya *et al.*: 'Finalizing the Roadmap for the Upgrade of the CERN & GSI Accelerator Complex', Proc. BEAM'07 (2007).

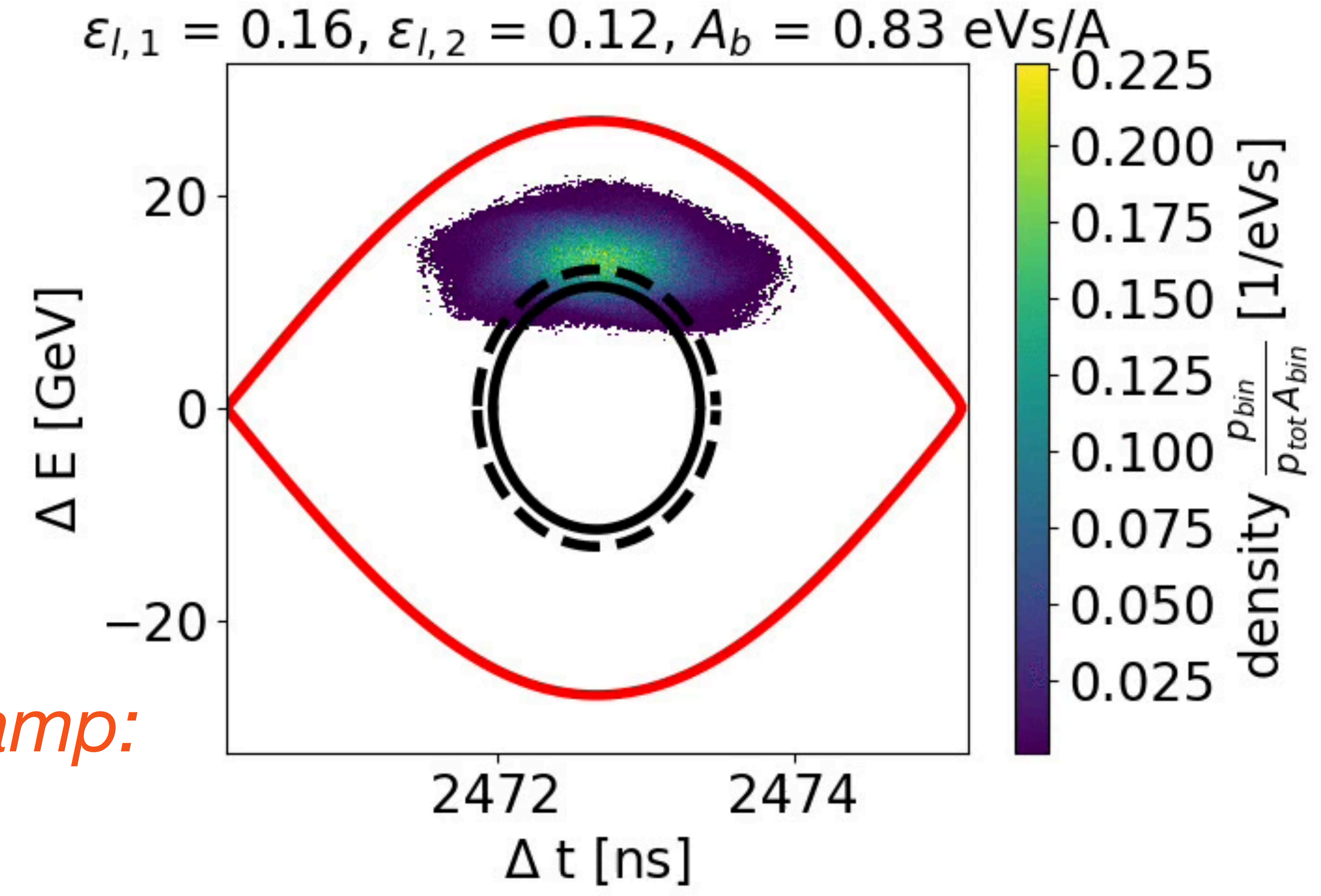
# Momentum slip stacking

**Where is it applied?**

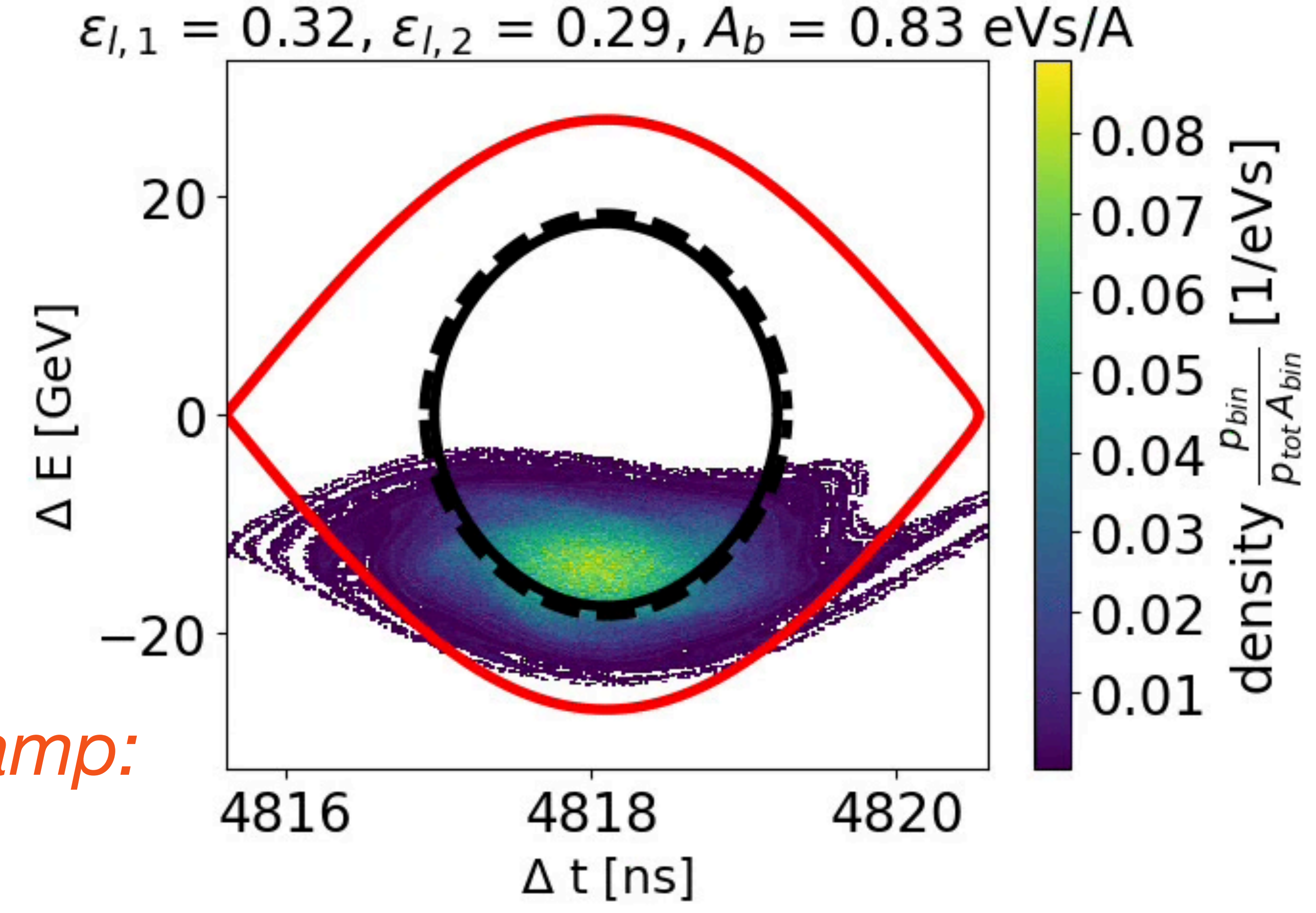
- In the SPS for the 50 ns spaced ion beam production
  - Two 100 ns spaced batches are interleaved on an intermediate momentum plateau

**What does it require from the hardware?**

- The ability to control the RF cavities in two groups
  - In voltage, phase, and frequency
  - For LIU-SPS, implemented in the Long Shutdown 2
- Sufficient voltage in the two groups during slippage
- Sufficient total voltage for recapture
- Sufficient aperture in the beam pipe
- The timings for the exact voltage and frequency programs are very intricate
  - For the SPS, designed with particle tracking simulation scans




*Re-capture and ramp: first bunch*




*Re-capture and ramp: last bunch*

[26] J. Coupard, *et al.*: 'LHC Injectors Upgrade', Technical Design Report, CERN-ACC-2016-0041 (2016).

# Coalescing

**What is it?**

- A bunch rotation of many bunches, merging them together

**How is it done?**

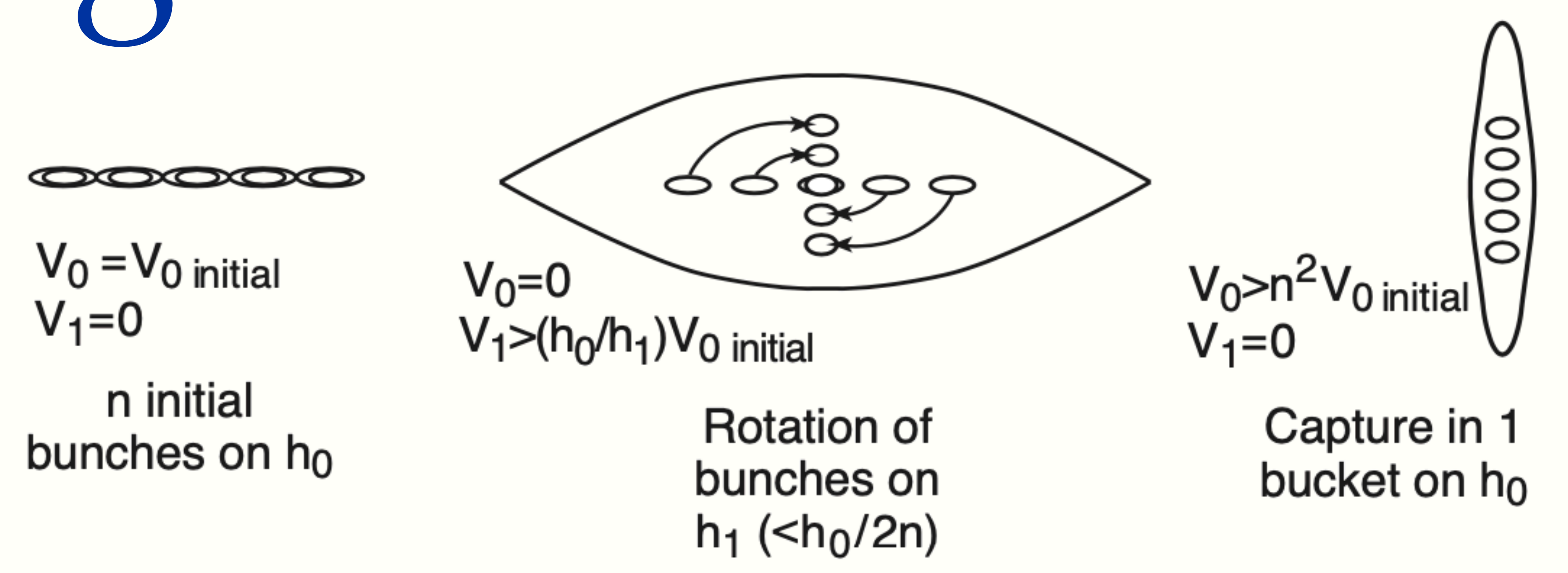
- Via a non-adiabatic voltage increase at a harmonic that captures all bunches to be merged
  - Finally, the bunches are recaptured on the initial harmonic, with a much higher voltage

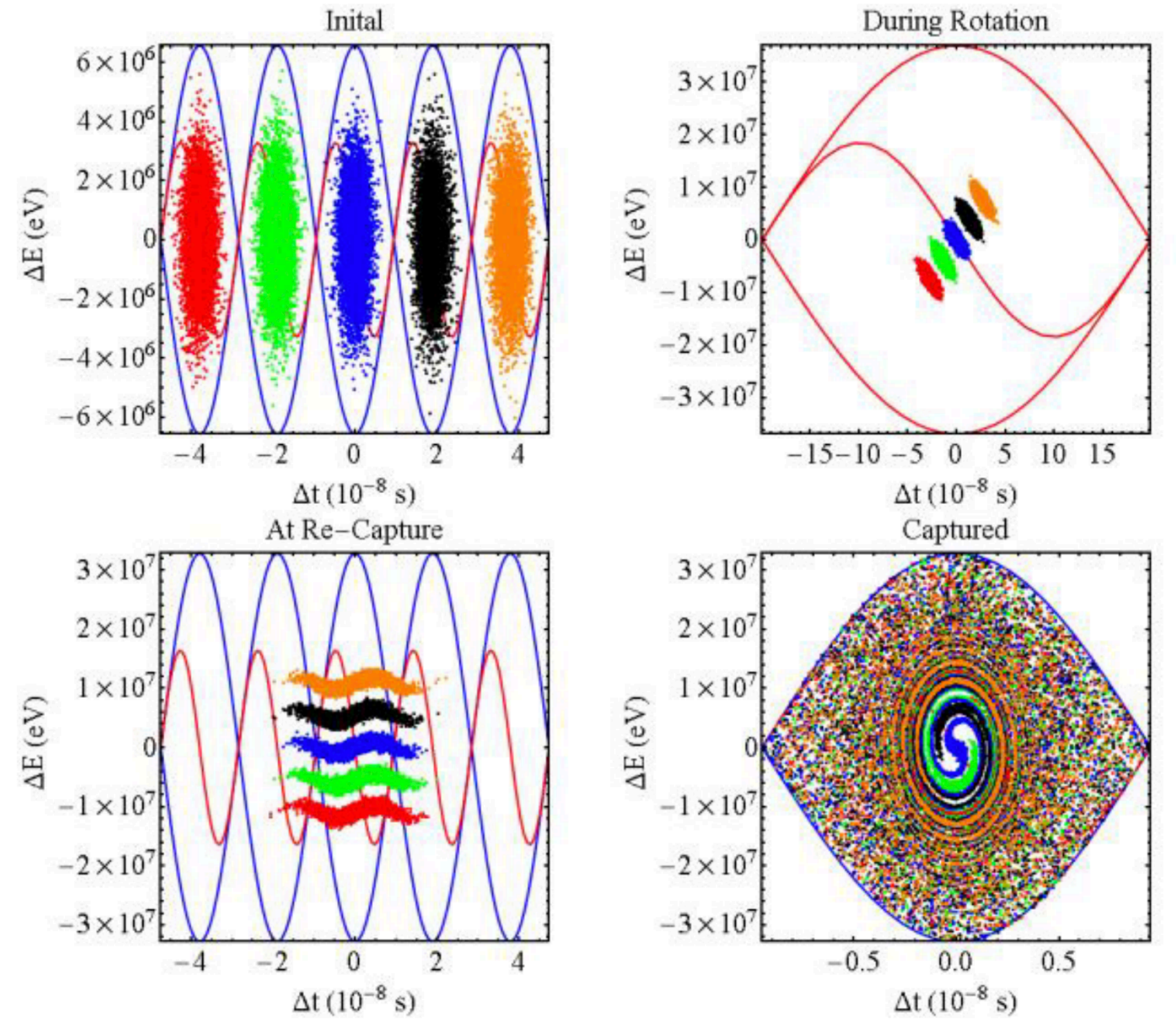
**Why use it?**

- Can significantly increase the bunch intensity
  - Also increases the longitudinal emittance

**Where is it applied?**

- At the Fermilab main injector with protons




*Top: sketch of coalescing from [27]*

*Bottom: coalescing in Fermilab from [28]*


[27] A. Chao *et al.*, 'Handbook of Accelerator Physics and Engineering', 2nd Ed., World Scientific, (2013).
[28] D. J. Scott *et al.*, 'Coalescing at 8 GeV in the Fermilab Main Injector', Proc. IPAC'12, New Orleans, USA, (2012).

# Barrier bucket

**What is it?**

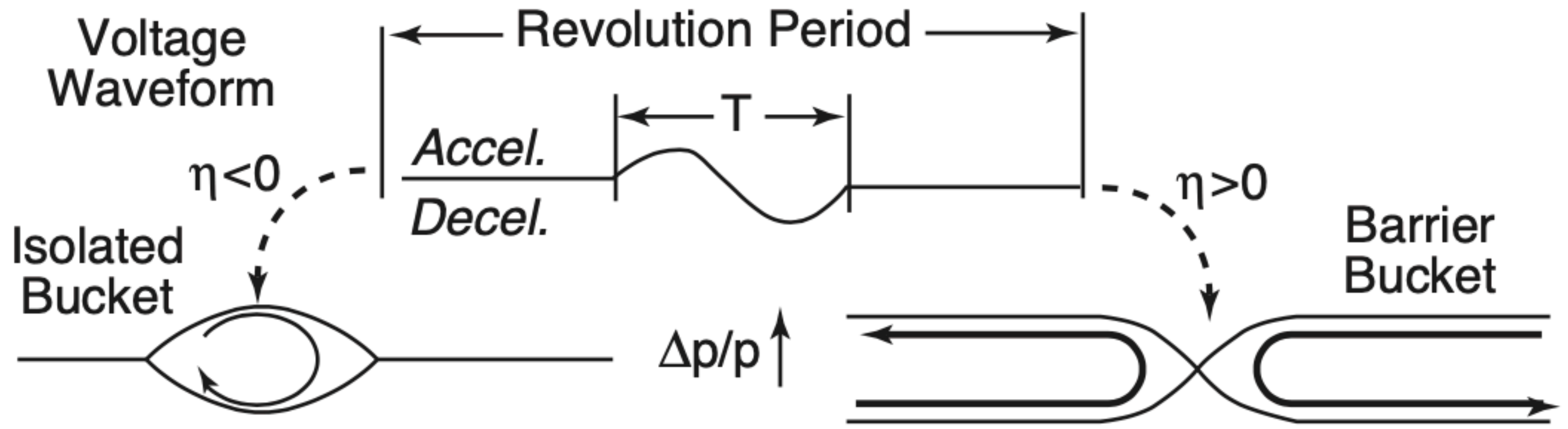
- A non-sinusoidal RF bucket that is stretched longitudinally

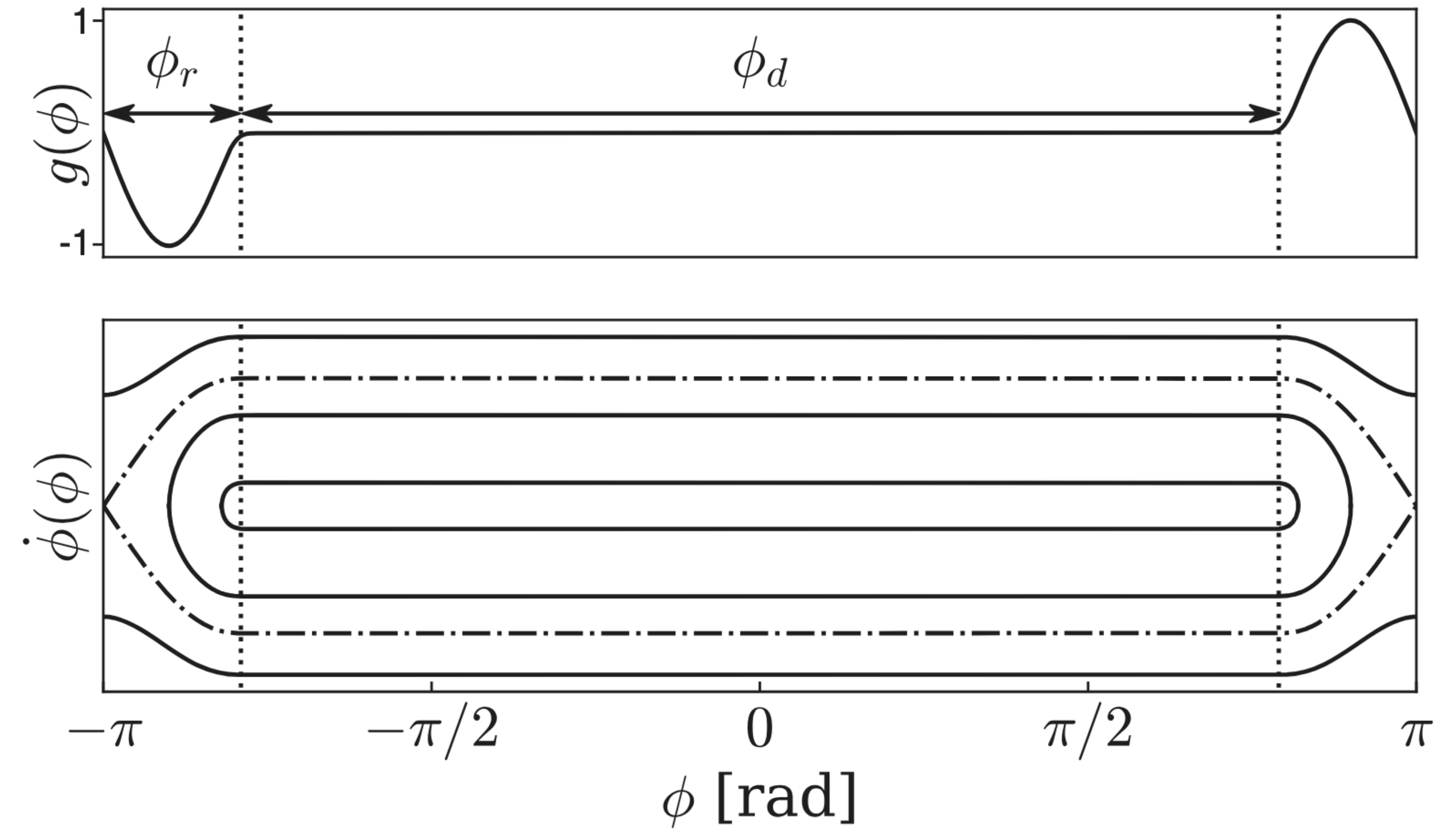
**How is it done?**

- Single voltage pulse per turn, with same or opposite polarity
  - Results in an isolated or barrier bucket

**Why use it?**

- To create coasting beam within the potential well
- Can stretch or compress the bunch by moving the RF phase




*Top: sketch of isolated and barrier bucket from [27]*

*Bottom: potential well for barrier bucket from [29]*



[27] A. Chao *et al.*, 'Handbook of Accelerator Physics and Engineering', 2nd Ed., World Scientific, (2013).
[29] M. Vadai *et al.*, 'Barrier bucket gymnastics and transversely split proton beams', Phys. Rev. Accel. Beams **25**, 050101, (2022).

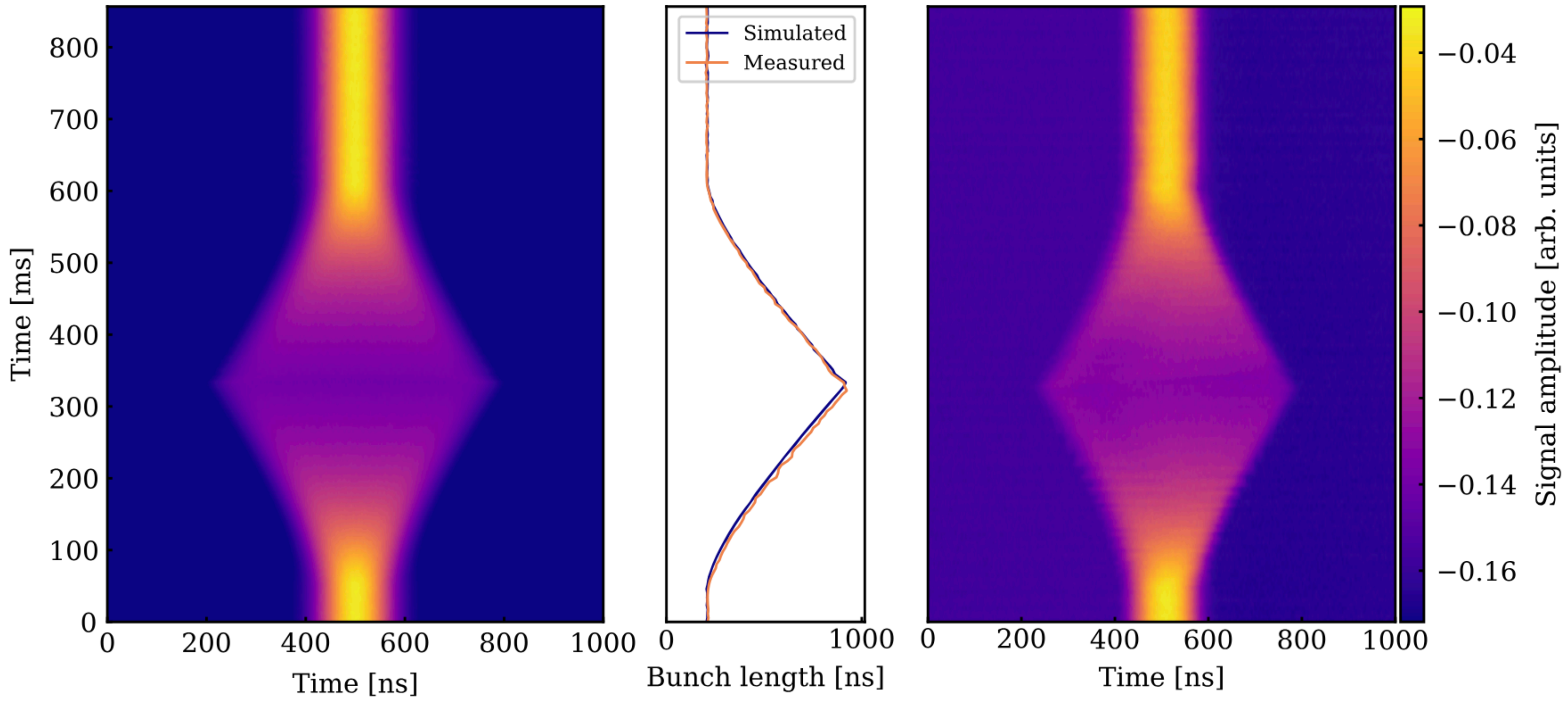
# Barrier bucket

**Where is it applied?**

- In the PS, to generate beams for SPS fixed target use
  - Accelerated through the SPS
- Advantage w.r.t. coasting beam: leaving a kicker gap with no beam
  - Significantly reduces beam losses and irradiation

**What does it require from the hardware?**

- A broad-band RF systems
  - Intrinsically, the peak voltage is limited
  - In the PS, Finemet® cavities that are usable from 400 kHz -10 MHz




*Slowly stretching and compressing the batch with the barrier-bucket mechanism [29]*



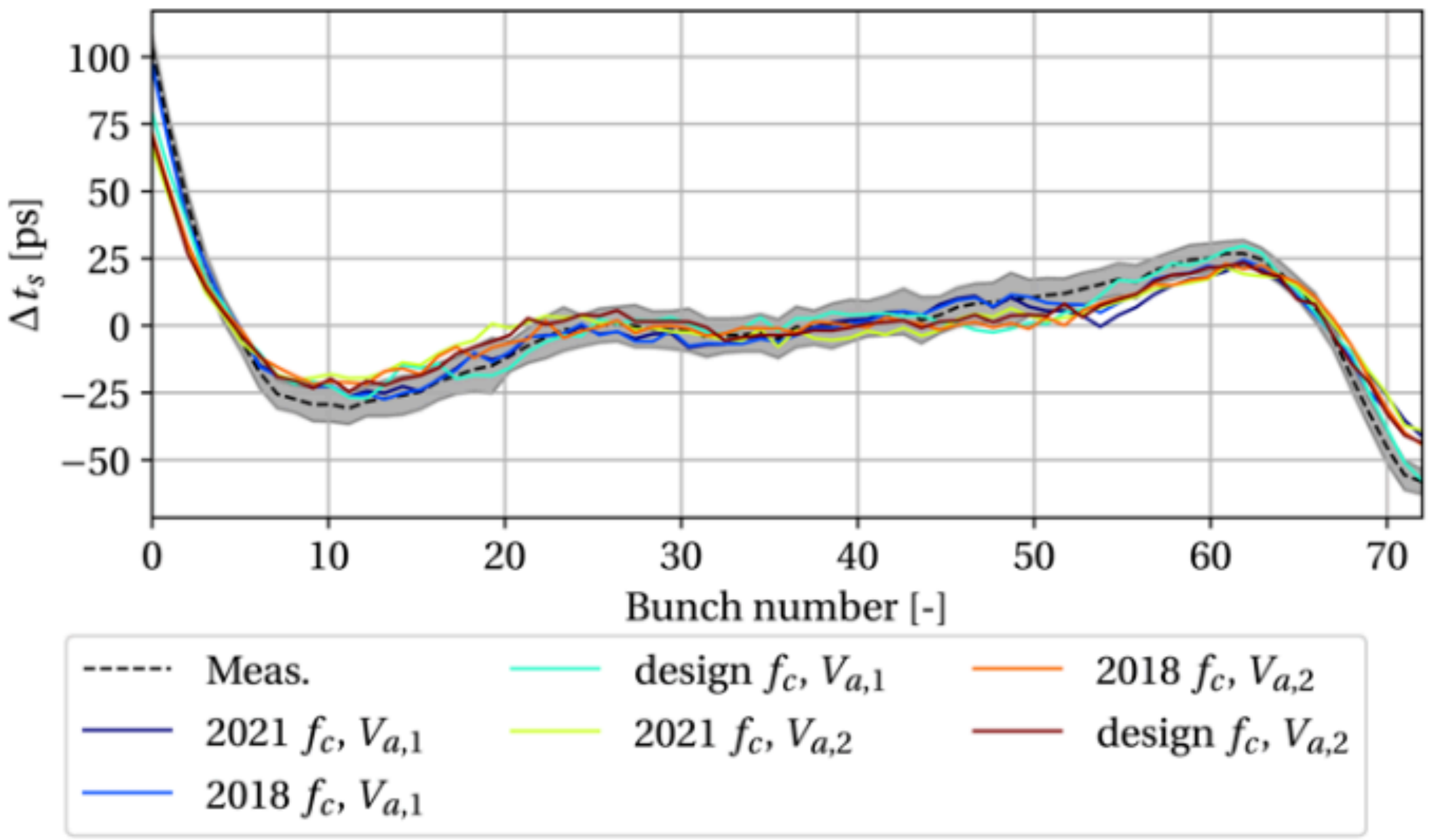
[29] M. Vadai *et al.*, 'Barrier bucket gymnastics and transversely split proton beams', Phys. Rev. Accel. Beams **25**, 050101, (2022).

# Contents

# Beam-loading compensation

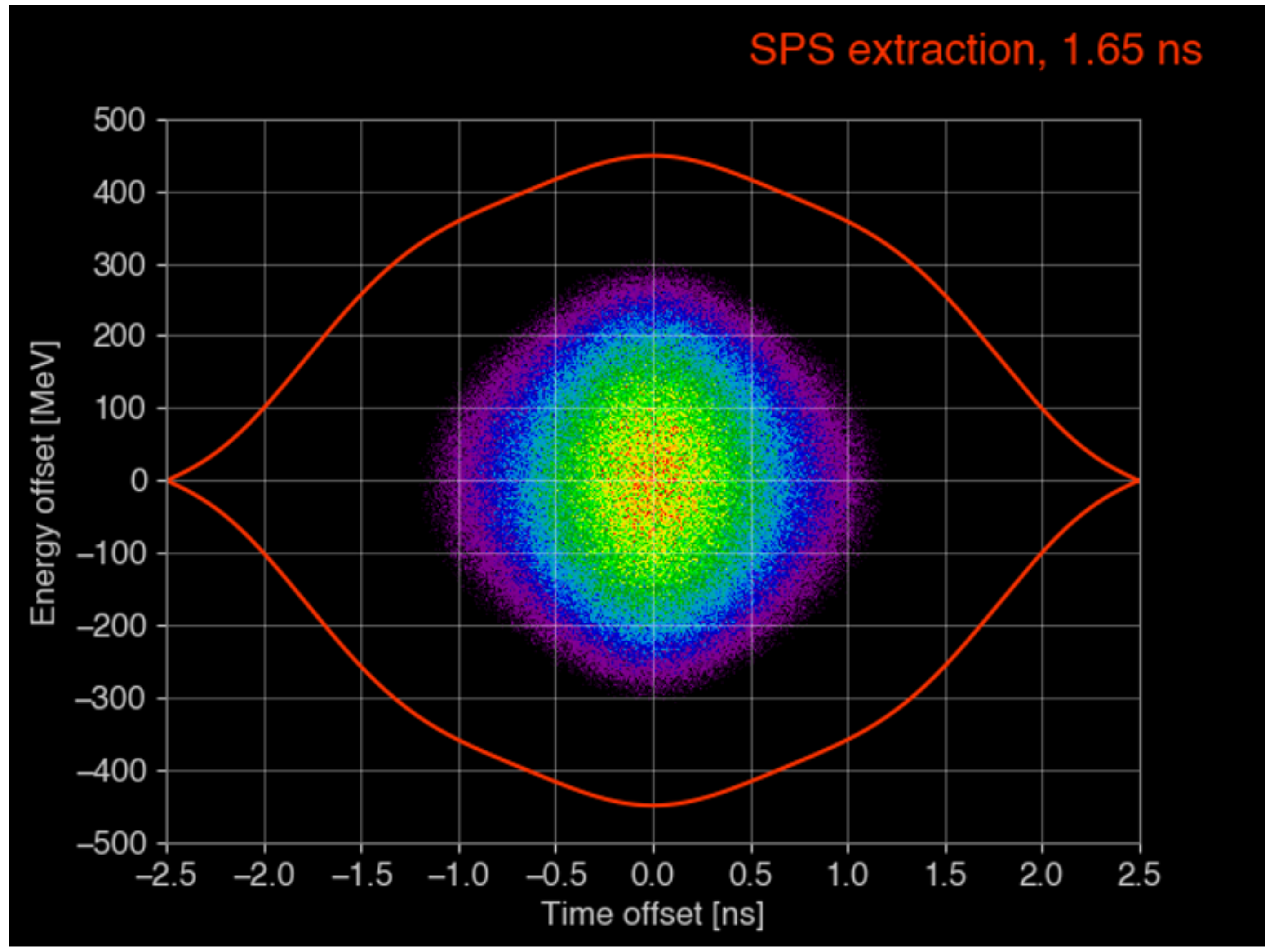
**SPS case: normal-conducting, travelling-wave cavities at 200 MHz and 800 MHz**

- The cavity impedance is reduced by a one-turn delay feedback (OTFB)
  - Strong beam-loading pattern
- Beam stability required controlled emittance blow-up to maximum acceptable emittance
  - Maximum bunch length determined by the LHC bucket (half the size); LIU design for HL-LHC: (1.65 ± 0.15) ns
  - Maximum momentum spread determined by flattop RF voltages (10+2 MV) in the presence of beam loading: $5.32 \times 10^{-4}$




*Bunch-by-bunch longitudinal offset from beam loading*

*Courtesy of B. Karlsen-Bæck*




*LIU baseline at SPS extraction*

*$V_{200}$ = 10 MV*
*$V_{800}$ = 2 MV*
*$\tau$ = 1.65 ns*

# Beam-loading compensation

**LHC case: super-conducting, standing-wave cavities at 400 MHz**

- Eight cavities per beam
- One klystron of 300 kW generating the RF power per cavity
  - For HL-LHC, these klystrons will be upgraded to 350 kW klystrons

**The cavity impedance is reduced by a direct RF feedback**

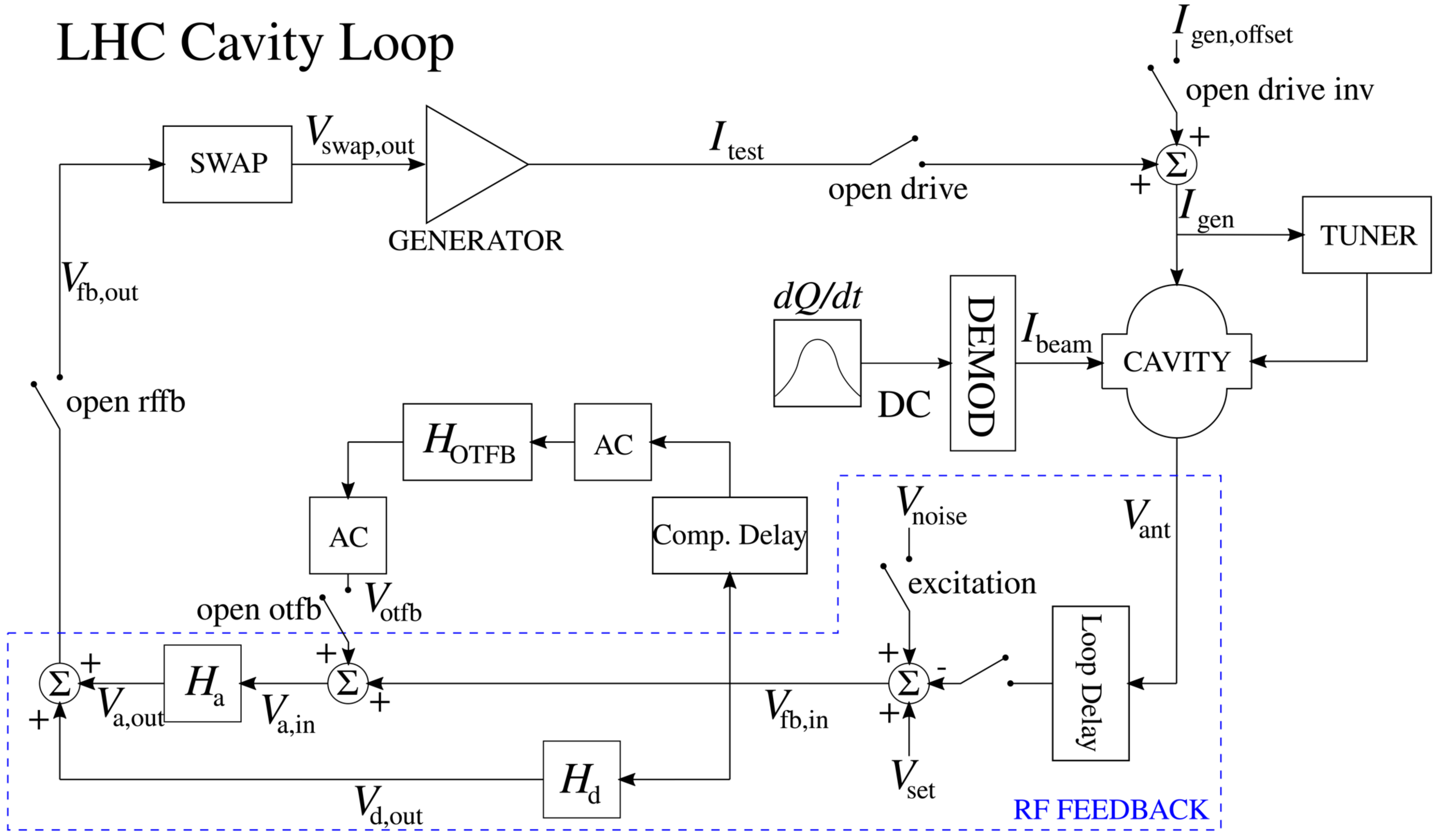
- Analog high-pass branch: gain $G_a$, delay $\tau_a$

$$y^{(n)} = \left[1 - \frac{T_s}{\tau_a}\right] y^{(n-1)} + G_a(x^{(n)} - x^{(n-1)})$$

- Digital low-pass branch: gain $G_d$, delay $\tau_d$

$$y^{(n)} = \left[1 - \frac{T_s}{\tau_d}\right] y^{(n-1)} + G_a G_d e^{i\Delta\varphi_{\mathrm{ad}}} \frac{T_s}{\tau_d} x^{(n-1)}$$

- One-turn delay feedback (comb filter) boosts the analog gain
- Tuning and clamping loops



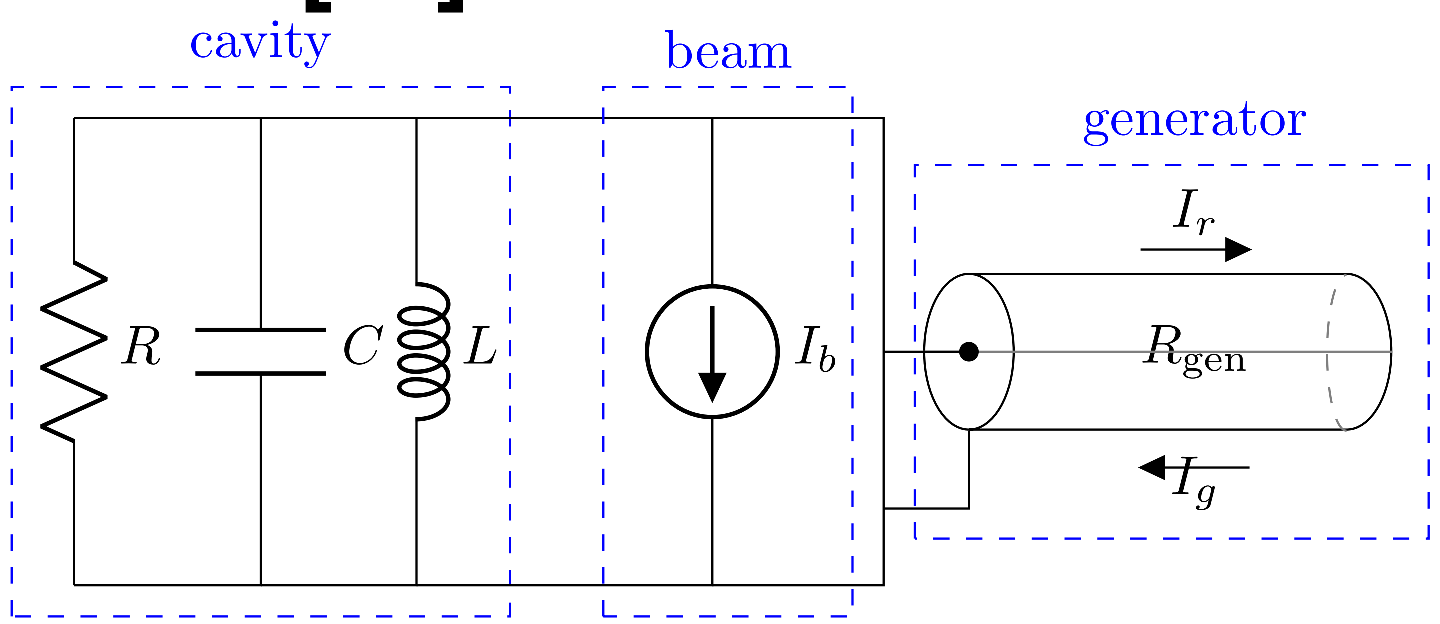

*Model of the LHC cavity controller as in [30]*

[30] J. Holma: 'The model and simulations of the LHC 400 MHz cavity controller', CERN-AB-Note-2007-012 (2007).

# Half-detuning scheme

**Cavity-transmitter-beam interaction can be described using a circuit model [31]**

- Cavity: RLC circuit, beam: current source, generator: transmission line




$$I_{\mathrm{gen}}(t) = \frac{V_{\mathrm{ant}}(t)}{2R/Q}\left(\frac{1}{Q_L} - 2i\frac{\Delta\omega}{\omega}\right) + \frac{dV_{\mathrm{ant}}(t)}{dt}\frac{1}{\omega R/Q} + \frac{1}{2}I_{\mathrm{b,rf}}(t)$$

**Cavity tune chosen usually to minimise the RF power**

- Without beam, and for a given cavity $Q_L$, the frequency is tuned to the centre of the resonance $\omega_r$
- At LHC injection, we require the $\overrightarrow{V}_{\mathrm{ant}} = \mathrm{const.}$
  - Then the power becomes:

$$P_{\mathrm{gen}} = \frac{1}{8}R/QQ_L\left(\frac{V_{\mathrm{ant}}}{R/Q}\frac{1}{Q_L} + \cancelto{0}{\Re(I_{b,\mathrm{rf}})}\right)^2 + \frac{1}{8}R/QQ_L\left(-2\frac{V_{\mathrm{ant}}}{R/Q}\frac{\Delta\omega}{\omega_r} + \Im(I_{b,\mathrm{rf}})\right)^2$$

  - And its minimum average value is [32]:

$$P_{\mathrm{gen}} = \frac{1}{8}\frac{V_{\mathrm{ant}}^2}{R/QQ_L} + \frac{1}{32}R/QQ_L I_{b,\mathrm{rf}}^2 = \frac{1}{8}V_{\mathrm{ant}}I_{b,\mathrm{rf}}$$, with the optimum detuning of $\Delta\omega_{\mathrm{HD}} = \frac{1}{4}R/Q\frac{I_{b,\mathrm{rf}}}{V_{\mathrm{ant}}}\omega_r$



[31] J. Tückmantel: 'Cavity-beam-transmitter interaction formula collection with derivation', CERN-ATS-Note-2011-002 TECH (2011).
[32] D. Boussard: 'RF power requirements for a high intensity proton collider, parts 1 and 2', CERN-SL-91-20-DI-16 (1991).

# Full-detuning scheme

**Once the beam is in the machine, we can let the phase slip and only keep $|\vec{V}_{\mathrm{ant}}| = $ const.**

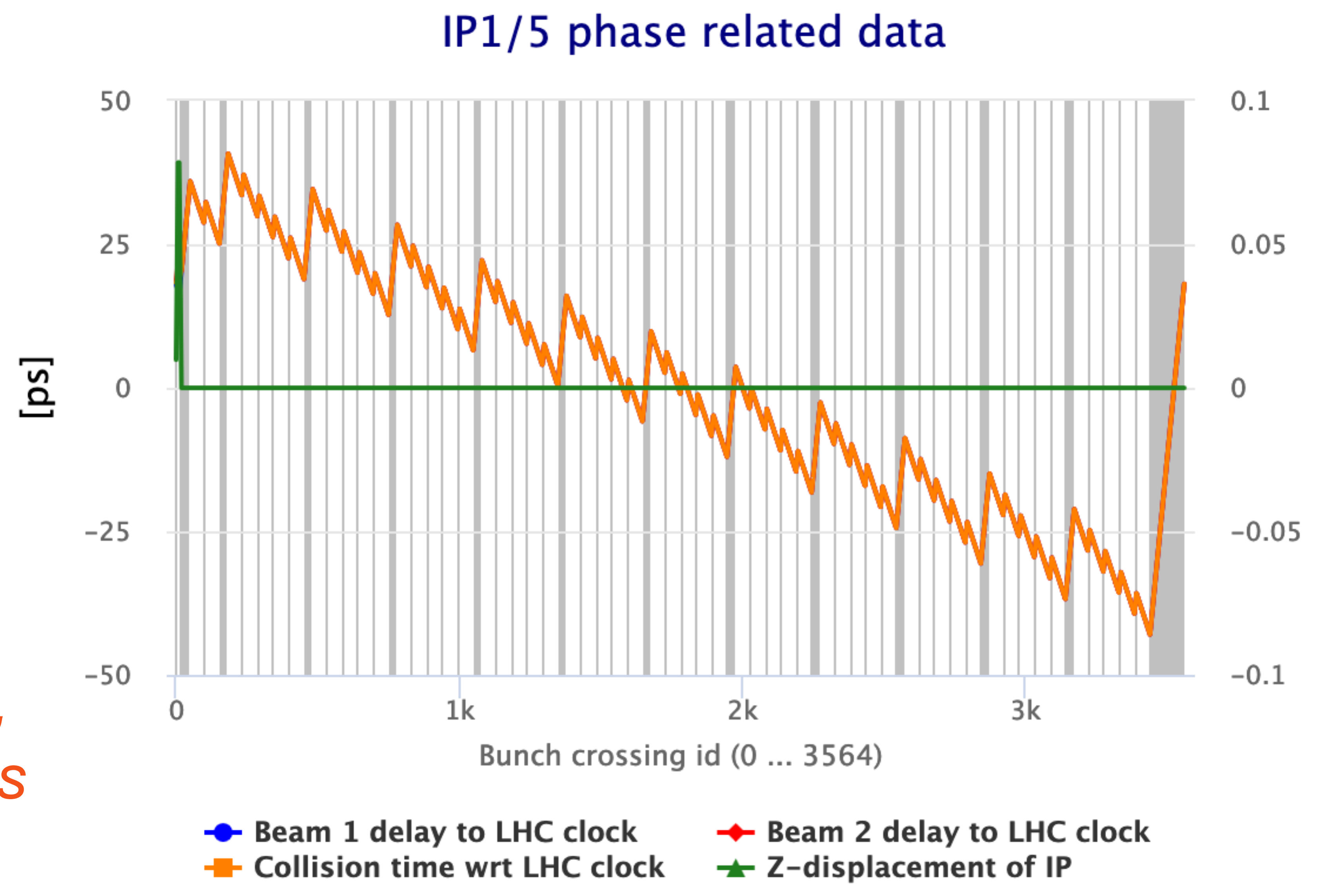
$$P_{\mathrm{gen}} = \frac{1}{8}\frac{V_{\mathrm{ant}}^2}{R/QQ_L} + \frac{1}{2}R/QQ_L\left(\frac{V_{\mathrm{ant}}}{R/Q}\frac{\dot{\varphi}}{\omega_r} - \frac{V_{\mathrm{ant}}}{R/Q}\frac{\Delta\omega}{\omega_r} + \frac{1}{2}\Im(e^{-j\varphi}I_{b,\mathrm{rf}})\right)^2$$

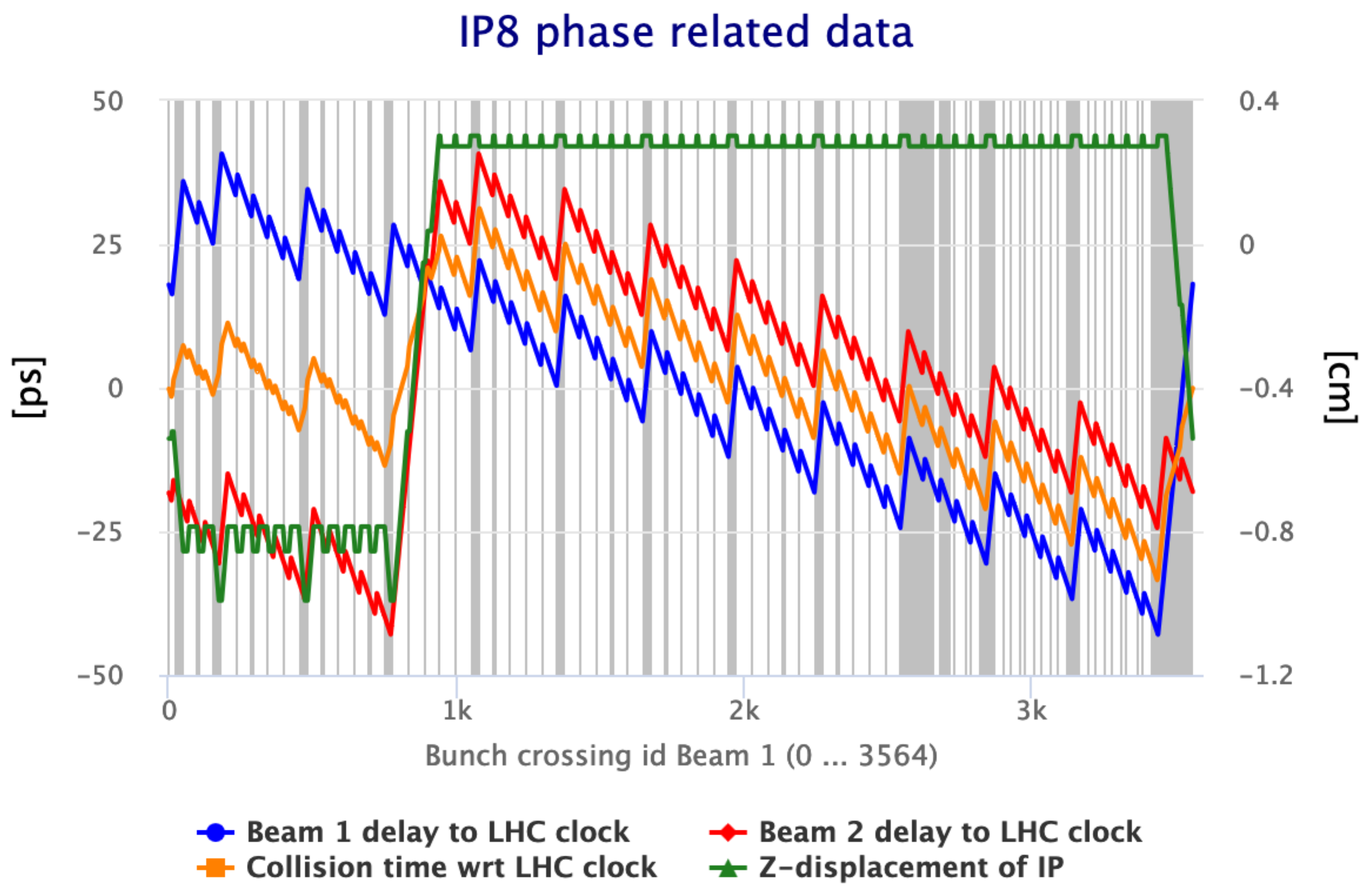
- Now we have a “knob” to compensate the bunch-by-bunch variation of the beam current along the ring via $\dot{\varphi}$
- Abracadabra: the minimum voltage becomes the same as if there was no beam loading!

$$P_{\mathrm{gen}} \approx \frac{1}{8}\frac{V_{\mathrm{ant}}^2}{R/QQ_L}$$

*Impact of full detuning on the LHC experiments:*

*Left: delay in collision time in IP1/5, Right: displacement of the luminous spot in IP2/8*



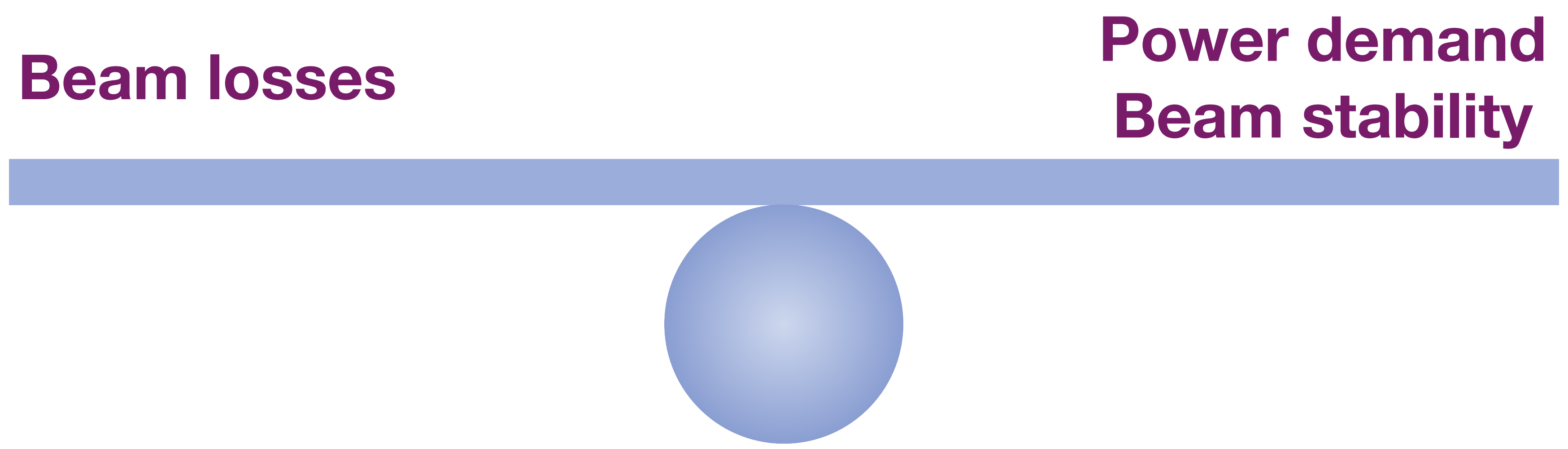



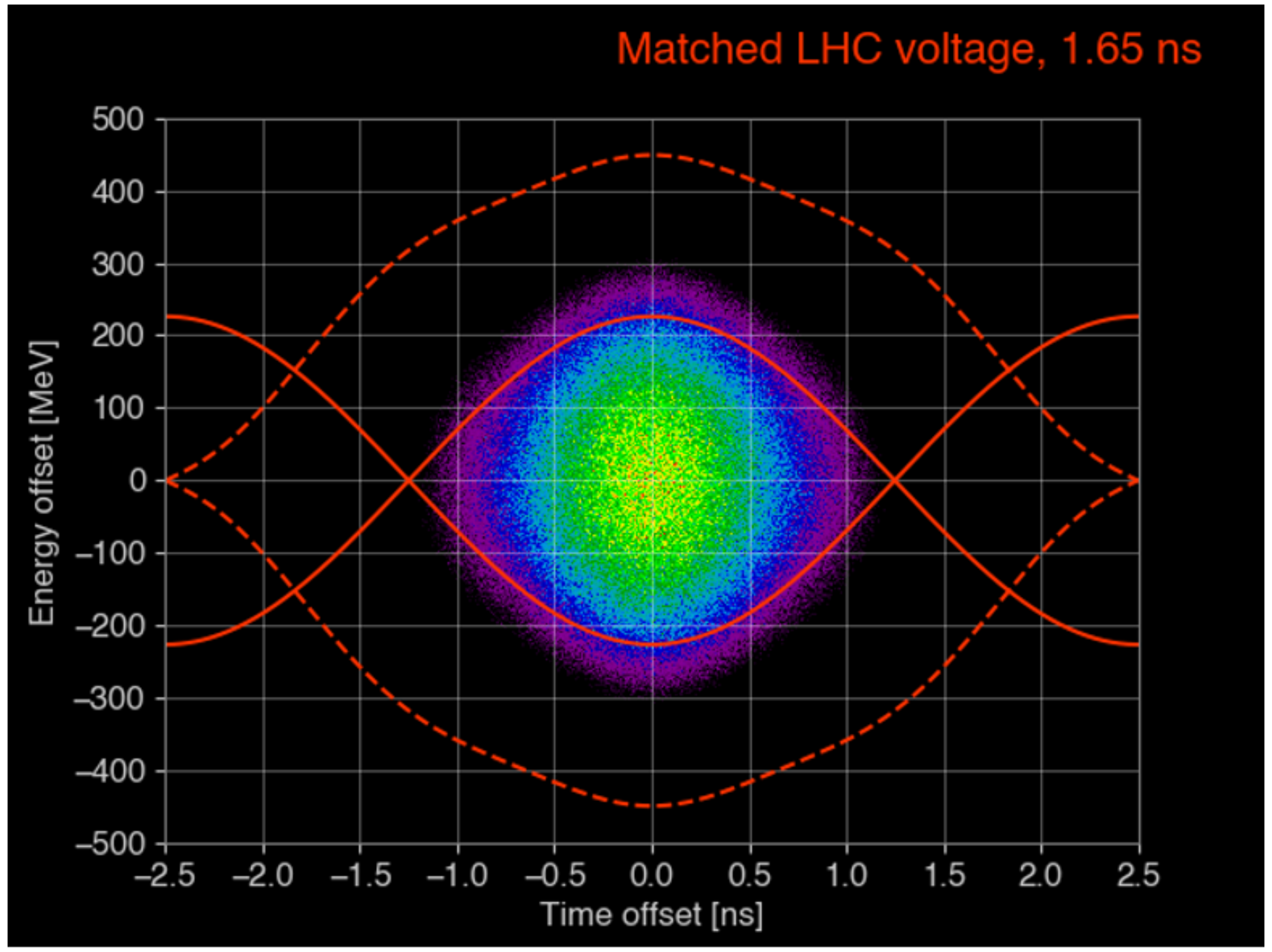

[33] P. Baudrenghien *et al.*: ‘Proposal for an RF roadmap towards ultimate intensity in the LHC’, Proc. IPAC’12 (2012).

# RF voltage and power limitations

**HL-LHC optimum voltage at injection?**

- The SPS bunches are long for the LHC bucket
  - Using a matched voltage is detrimental for capture losses
- In addition, the SPS beam loading pattern introduced a bunch-by-bunch phase offset, making the situation worse
  - Injection phase and energy errors are kept to a minimum (±60 MeV, ±10°)
- To reduce beam losses, a larger-than-matched voltage is used
- On the other hand, we need to limit the capture voltage
  - Most power is used for beam-loading compensation (> 60-70 %)
  - Mismatch not beneficial for undamped oscillations observed at flat bottom



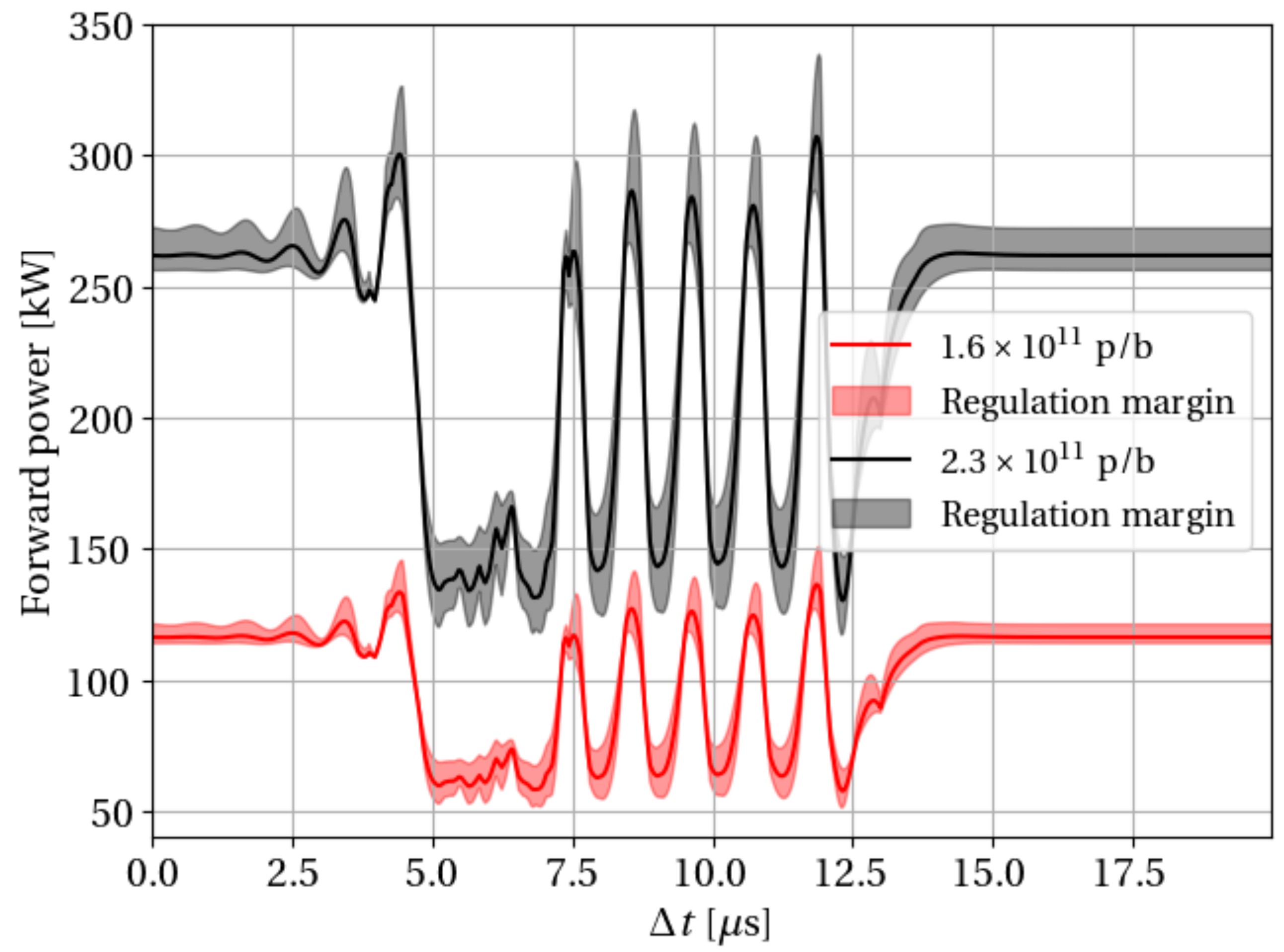




*Why using a matched LHC capture voltage doesn't work for SPS bunches*

# RF voltage and power limitations

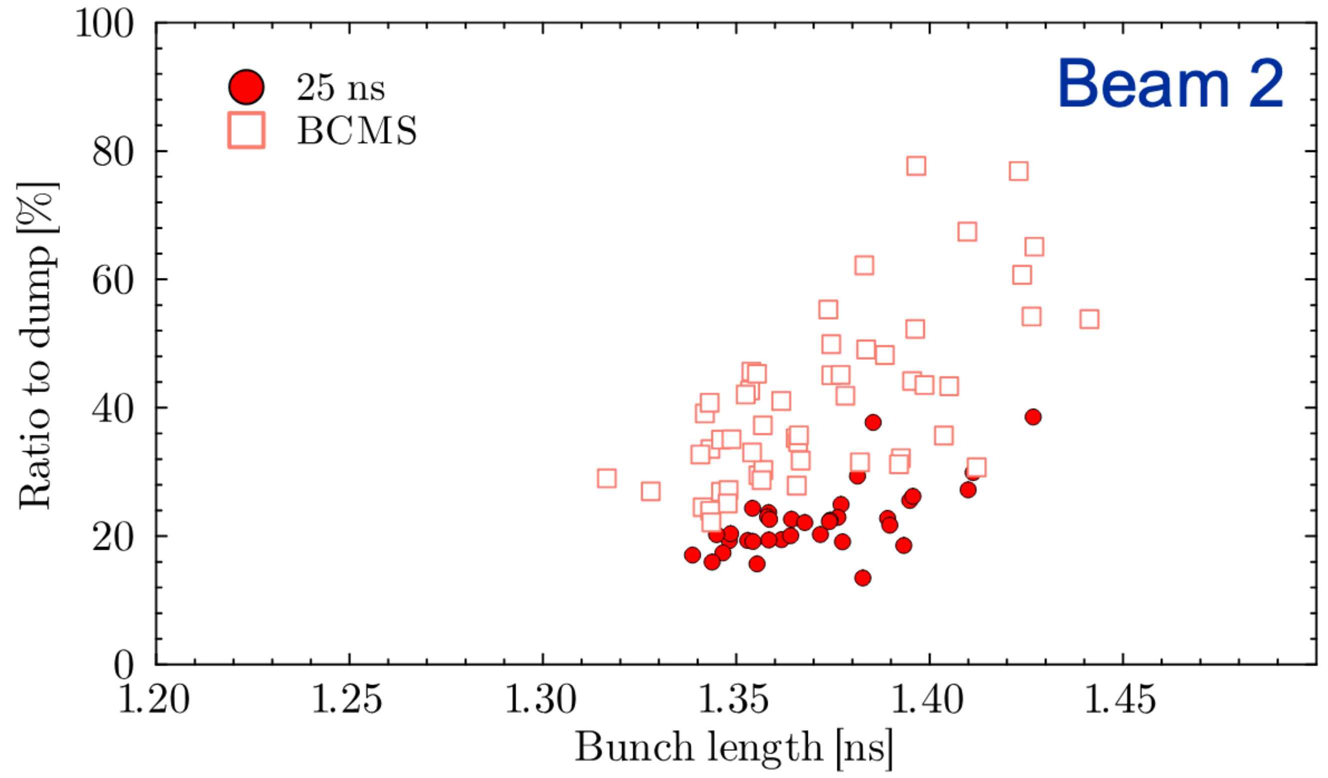
## HL-LHC beam loading at injection

- Equally spaced bunches arriving from the SPS
  - Require using the half-detuning scheme
  - For HL-LHC beam currents, $I_{\mathrm{b,rf}} = 2.2$ A, the average power is close to 300 kW, and the peak power exceeds 315 kW in the best case
  - Need of high-efficiency klystrons




## The complexity of beam losses

- Immediate capture losses determined by SPS-to-LHC transfer
- Blow-up along the flat bottom due to intra-beam scattering and RF noise
  - Debunching from the halo population
- Filling 15-20 batches, some batches spend only 5 minutes on flat bottom, while others may spend 1 hour
  - Injection and abort gap cleaning on
  - Bottleneck for HL-LHC: start-of-ramp losses




*Top: simulated power*
*Bottom: start-of-ramp losses*

*Courtesy of B. Karlsen-Baeck*

QUIZ

**If you were given the momentum programme of a machine, how would you design the RF voltage programme?**

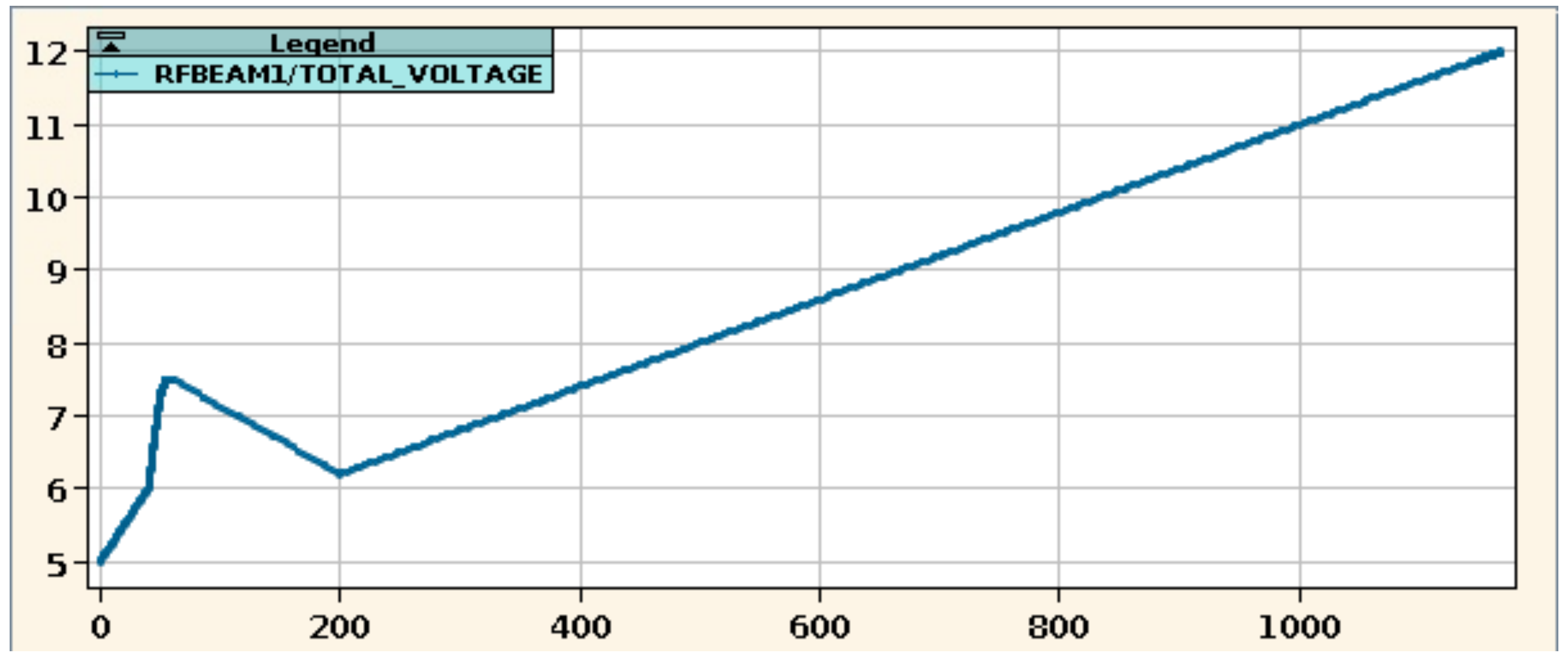
TIPS

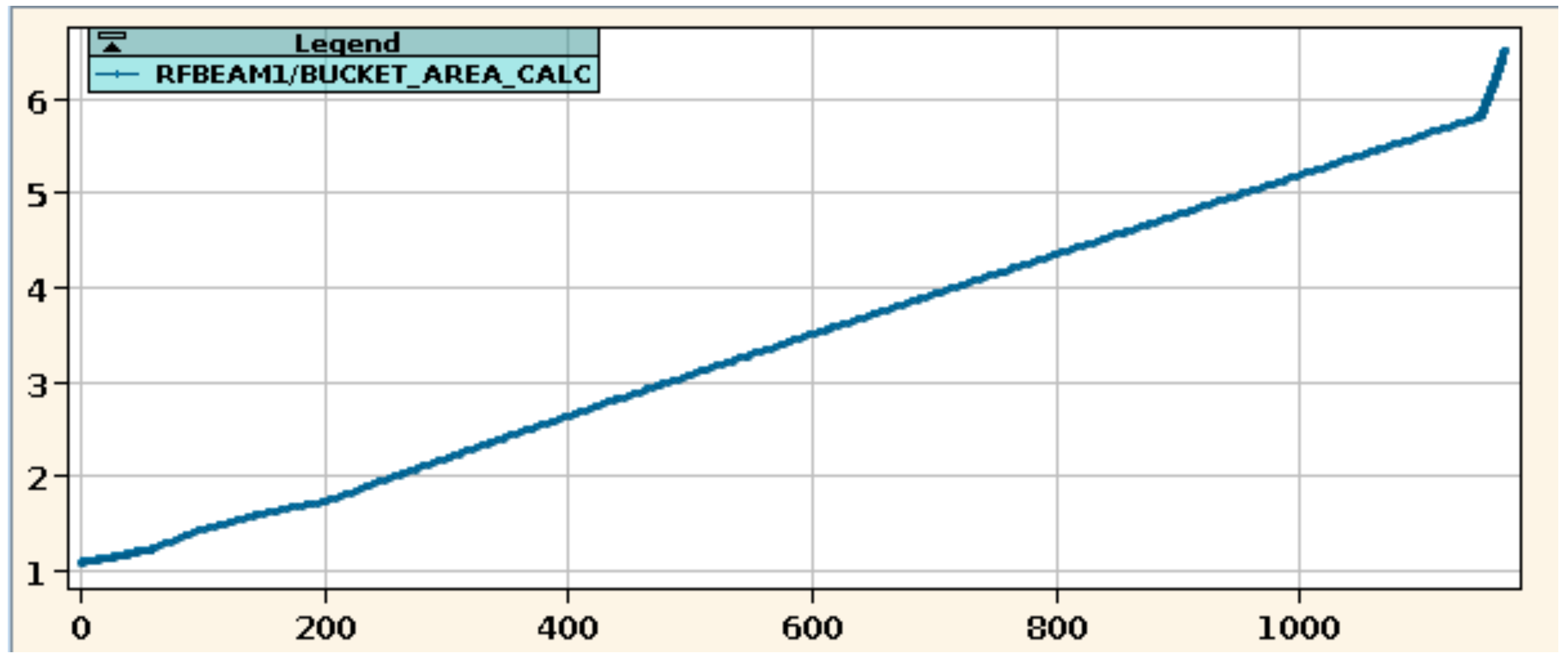
*For different machines, different criteria should be considered*

# Designing the RF cycle

**Example: LHC voltage design**

- Operational momentum programme: Parabolic-Exponential-Linear-Parabolic (PELP)
  - Ramp rate of superconducting magnets limited, overall ramp takes ~20 minutes
  - Using a linear voltage ramp ensures a monotonic increase in bucket area
  - The blow-up with a constant target bunch length ensures beam stability through a constant filling factor
- Experimental ramp: Parabolic-Parabolic-Linear-Parabolic (PPLP)
  - Gain: 110 s
  - Using a linear voltage ramp would decrease the bucket area at the start of ramp and thus create losses
  - Voltage shaping applied to keep a monotonic increase in bucket area




*PPLP voltage (top) and bucket area (bottom)*

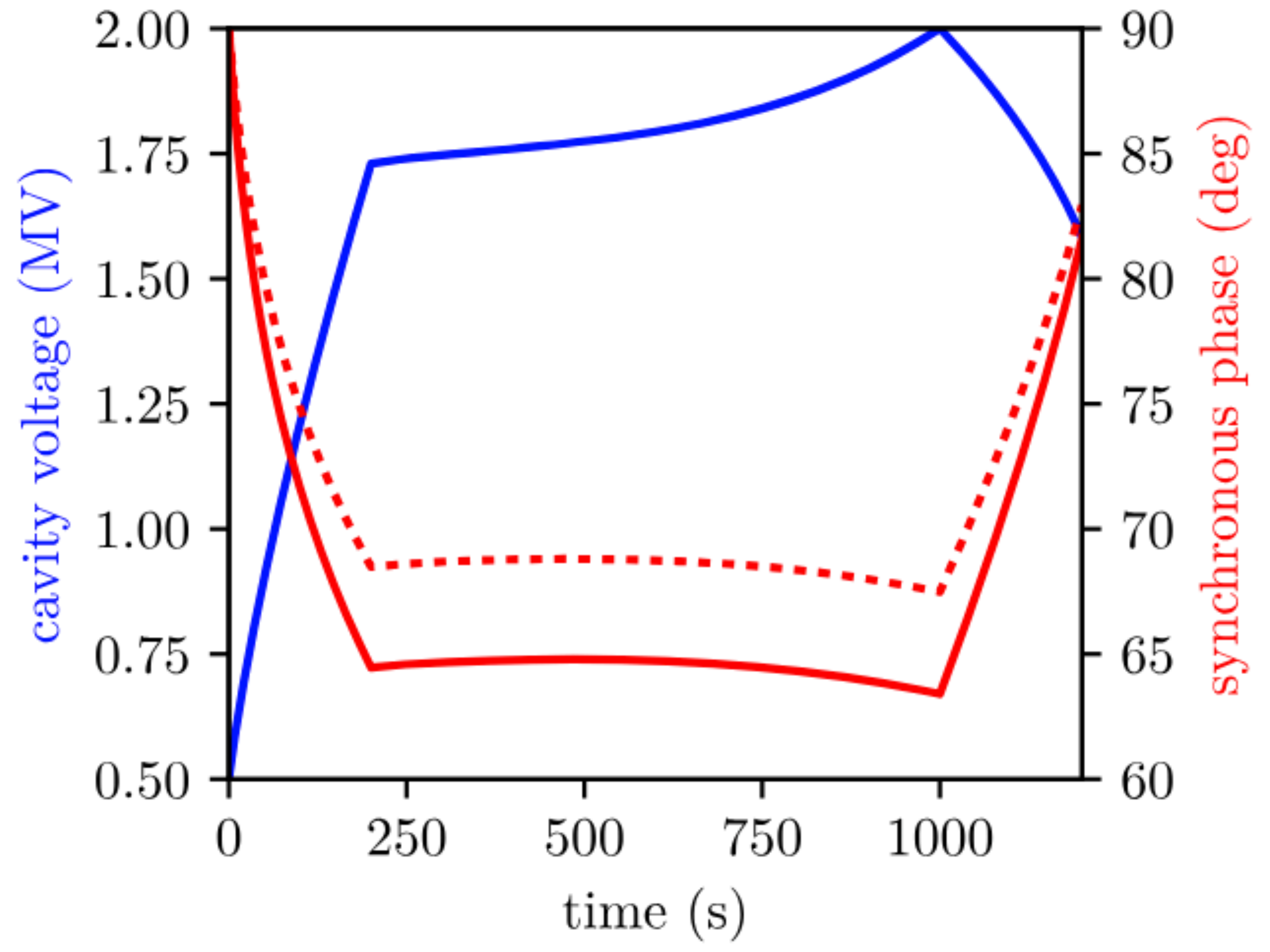
Bucket area

$$A_b = \frac{2}{\omega_{\mathrm{rf}}}\sqrt{\frac{\beta_d^2 E_d e V_{\mathrm{rf}}}{\pi h |\eta_0|}} \int_{\pi-\varphi_d}^{\varphi_u} d\varphi \sqrt{\cos(\pi-\varphi_d) - \cos\varphi + (\pi-\varphi_d)\sin\varphi_d - \varphi\sin\varphi_d}$$

# Designing the RF system

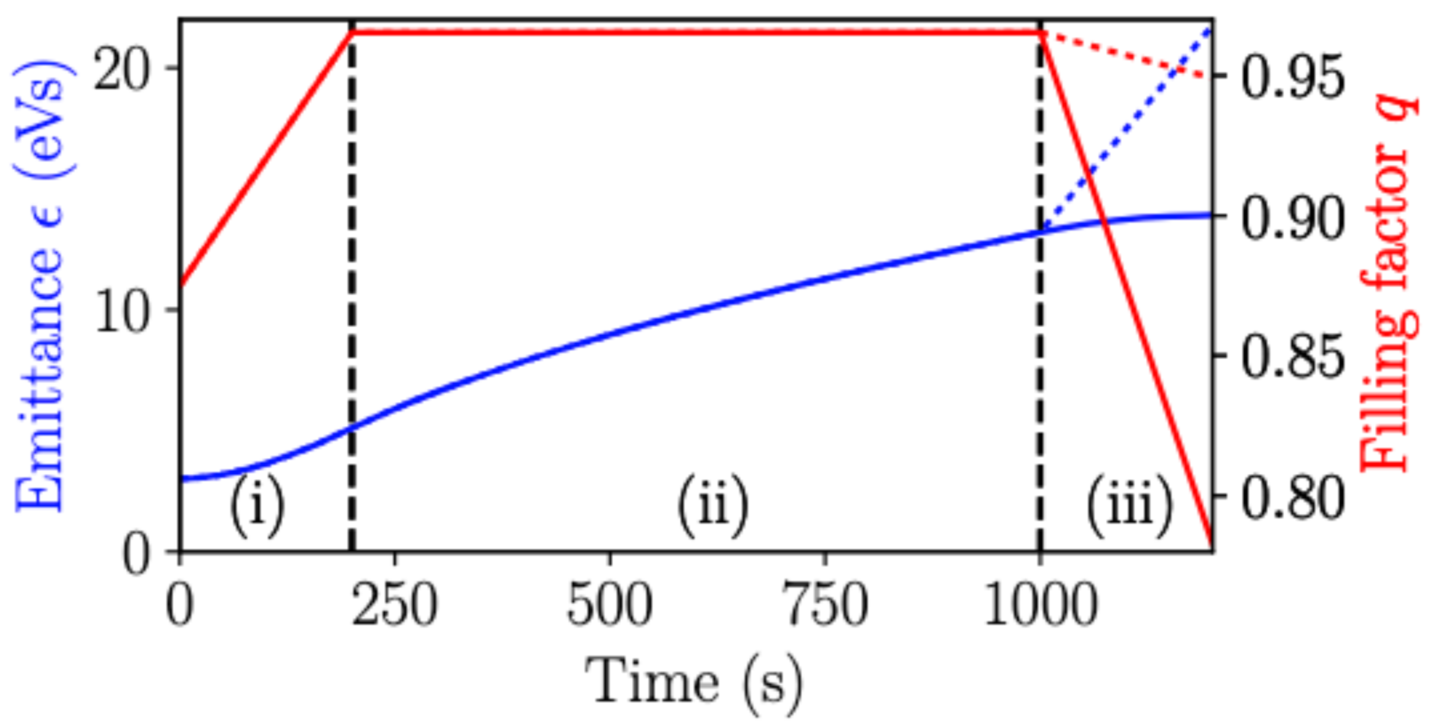
**What are the considerations to be taken into account e.g. for the FCC?**

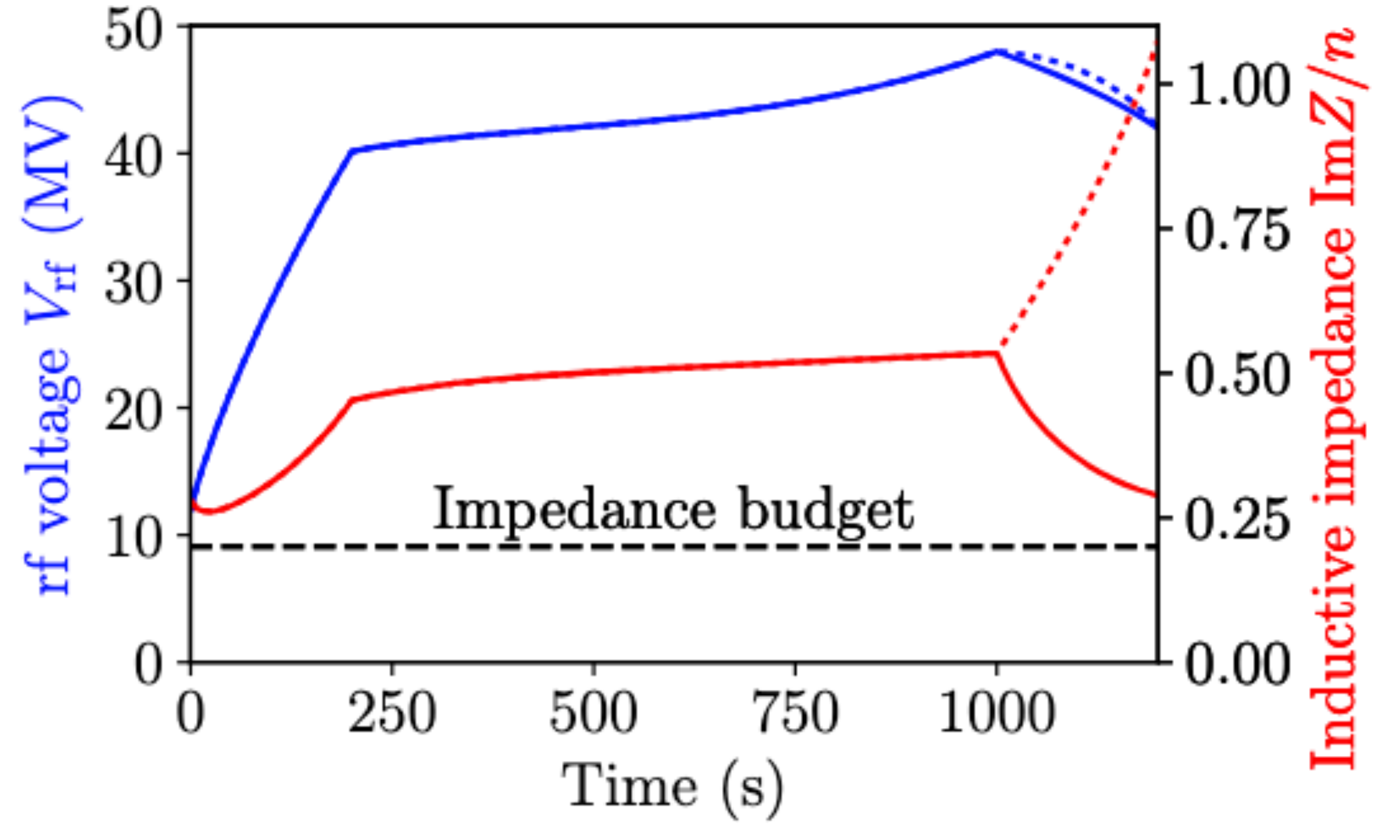
- Keep ~constant filling factor to counteract loss of Landau damping
  - Similar to LHC, controlled emittance blow-up keeping constant bunch length
- At flattop, need an extra blow-up to counteract the fast SR damping
- Cavities similar to LHC design, with 2 MV maximum field
  - Maximum is actually not reached at flattop but during the ramp




*RF voltage design for FCC from [34]*

*Synchronous phase of the beam (red solid line) and the synchronous particle (red dotted line)*




*Cycle design for FCC from [35]*

*Top: emittance and filling factor*

*Bottom: total voltage and impedance budget for Landau damping*




[34] I. Karpov *et al.*: 'Transient beam loading and rf power evaluation for future circular colliders', Phys. Rev. Accel. Beams **22**, 081002 (2019).
[35] I. Karpov *et al.*: 'Longitudinal coupled-bunch instability evaluation for FCC-hh', Proc. IPAC'19, Melbourne, Australia (2019).

# Summary

## Bunch length regulation

- Adiabatic changes
- Rotation
- Splitting, merging

## Advanced manipulations

- Momentum slip stacking
- Barrier bucket

## Phase space regulation

- Diffusion, noise injection
- Resonant excitation
- Longitudinal painting
- Debunching

## Integration in an RF system

- Beam loading
- RF voltage/power limitations
- Designing RF parameters

*Thank you for your attention!*