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The CERN Accelerator School

CAS Advanced Accelerator Physics

Collective effects

Part 1: Introduction – multiparticle systems and dynamics

Kevin Li and Giovanni Rumolo

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Outline

In this introductory part, we will provide a qualitative description of **collective effects** and their **impact on particle beams**.

We will introduce **multiparticle systems** and investigate **multiparticle effects**. This will be the first step towards a more involved understanding of collective effects and their effect (next lectures).

- **Part 1: Introduction – multiparticle systems and dynamics**
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

What are collective effects?

- We will study the dynamics of **charged particle beams** in a **particle accelerator environment**, taking into account the **beam self-induced electromagnetic fields**, i.e. not only the **impact of the machine onto the beam** but also the **impact of the beam onto the machine**.

What are collective effects?

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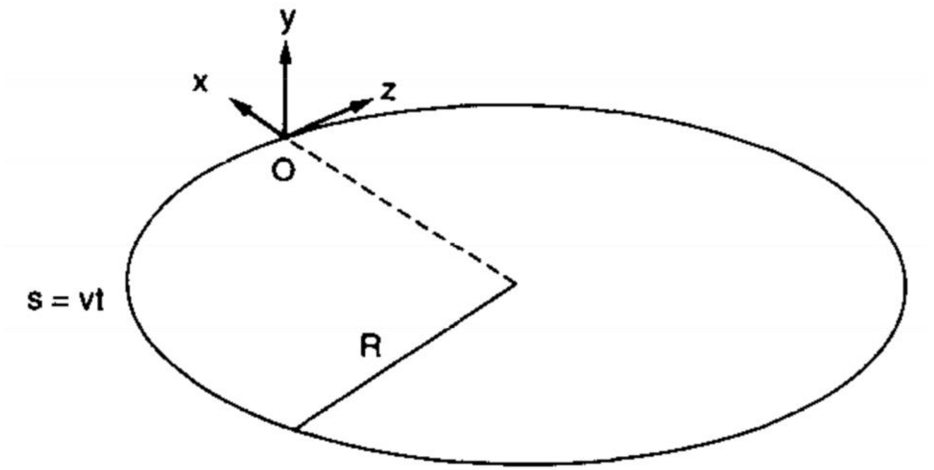
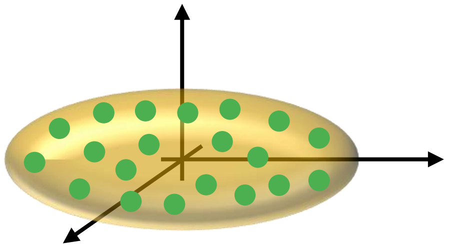
- First step → **Coordinates system** we will use throughout this set of lectures
  - The origin O is moving along with the "synchronous particle", i.e. a reference particle that has the design momentum and follows the design orbit
  - Transverse coordinates x (Horizontal) and y (Vertical) relative to reference particle (x,y << R), typically x is in the plane of the bending
  - Longitudinal coordinate z relative to reference particle
  - Position along accelerator is described by independent variable s = vt



What are collective effects?

- A charged particle beam is generally described as a **multiparticle system** via the **coordinates** and the **canonically conjugate momenta** of all of its particles – this makes up a distribution in the 6-dimensional phase space which can be described by a **particle distribution function**.
- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function)**:**



$$\frac{\partial}{\partial s}\psi\left(x, x', y, y', z, \delta, s\right)$$

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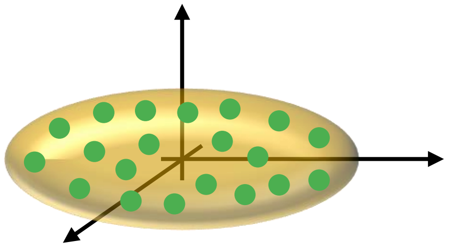
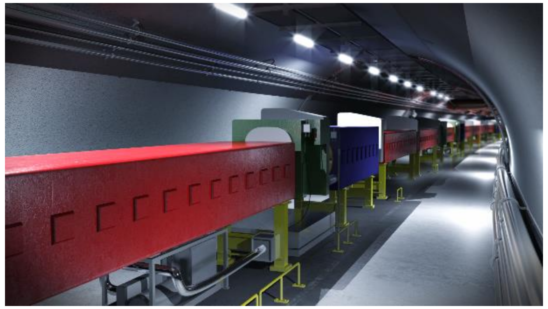
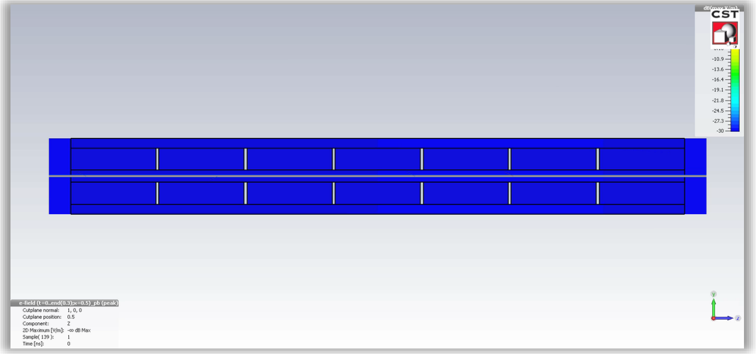
- Hence, we will study the **evolution of the phase space** occupied by this particle distribution (and described by its particle distribution function)**:**
    - Optics defined by the machine lattice provides the **external force fields** (magnets, electrostatic fields, RF fields), e.g. for guidance and focusing
    - Collective effects add to this **distribution dependent force fields** (space charge, wake fields)







$$\frac{\partial}{\partial s}\psi\left(x, x', y, y', z, \delta, s\right) \propto f\left(F_{\text{extern}} + F_{\text{coll}}\left(\psi\right)\right)$$

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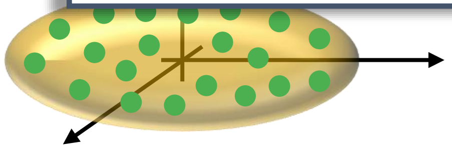
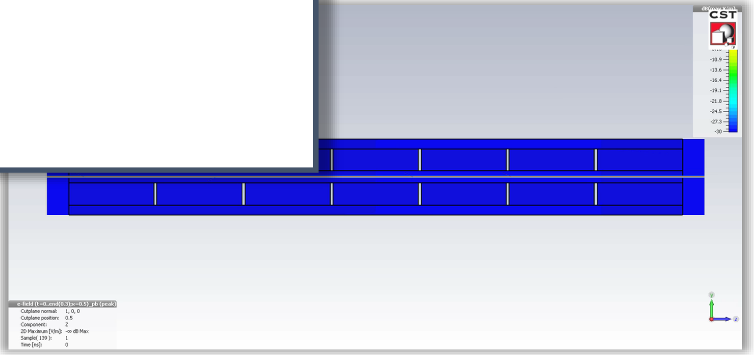
- Hence, we will … ution (and described by its particle distribut…
  - Optics define … lds, RF fields), e.g. for guidance and focusing …
  - Collective eff…

- We will see later how **multiparticle dynamics** can be modeled and solved
  - Can be just the description of the evolution of a set of particles without mutual interactions (linear dynamics & matching, nonlinear dynamics and incoherent effects)
  - Can include mutual interactions among particles (coherent and incoherent effects)

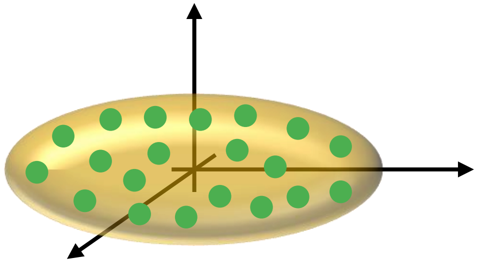




$$\frac{\partial}{\partial s}\psi\left(x, x', y, y', z, \delta, s\right) \propto f\left(F_{\text{extern}} + F_{\text{coll}}\left(\psi\right)\right)$$

What is a beam coherent instability?

- A beam becomes unstable when a **moment of its distribution** exhibits an **exponential growth** (e.g. mean positions, standard deviations, etc.), resulting into beam loss or emittance growth!



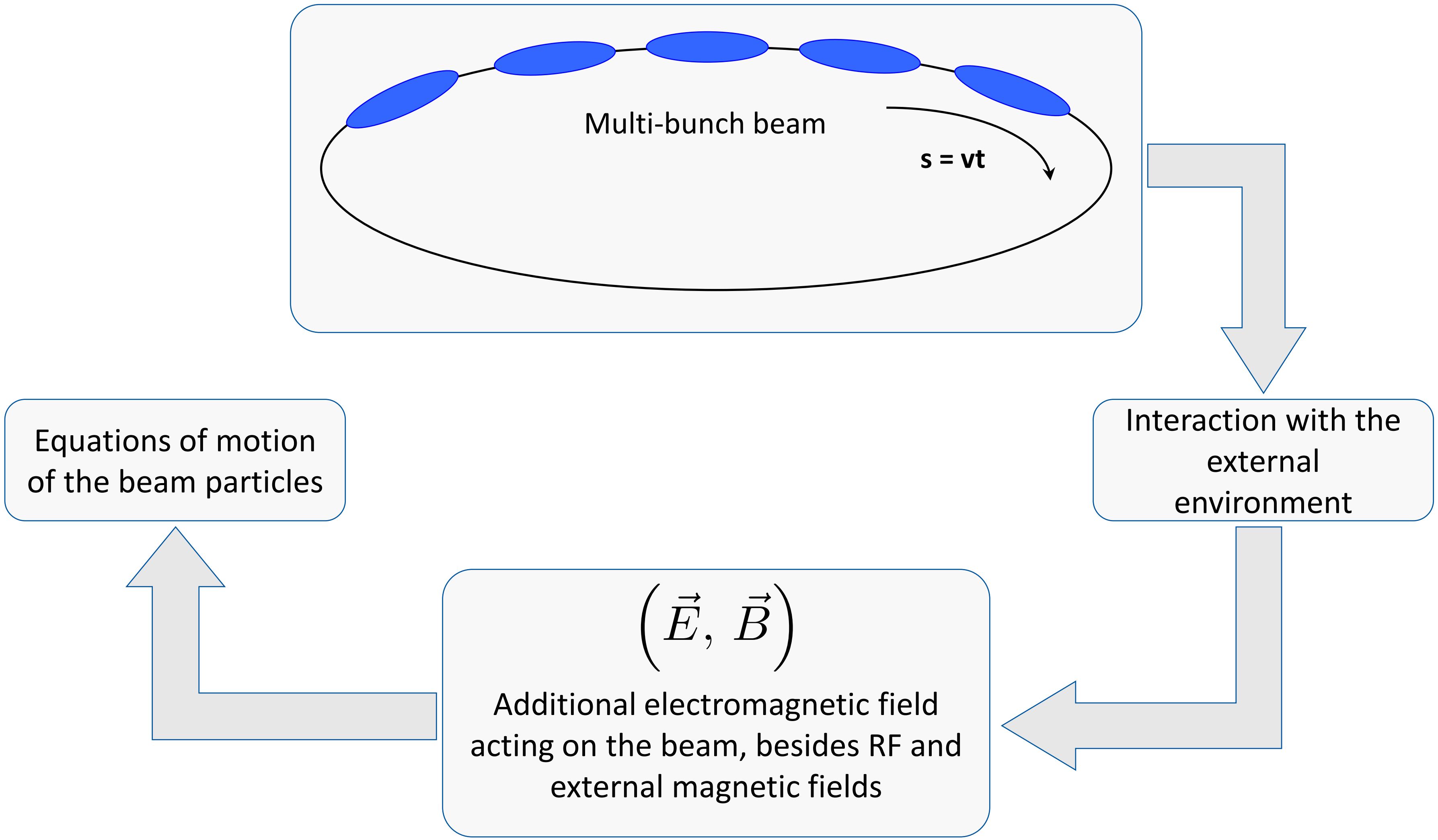
$$N = \int \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

$$\langle x \rangle = \frac{1}{N} \int x \cdot \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

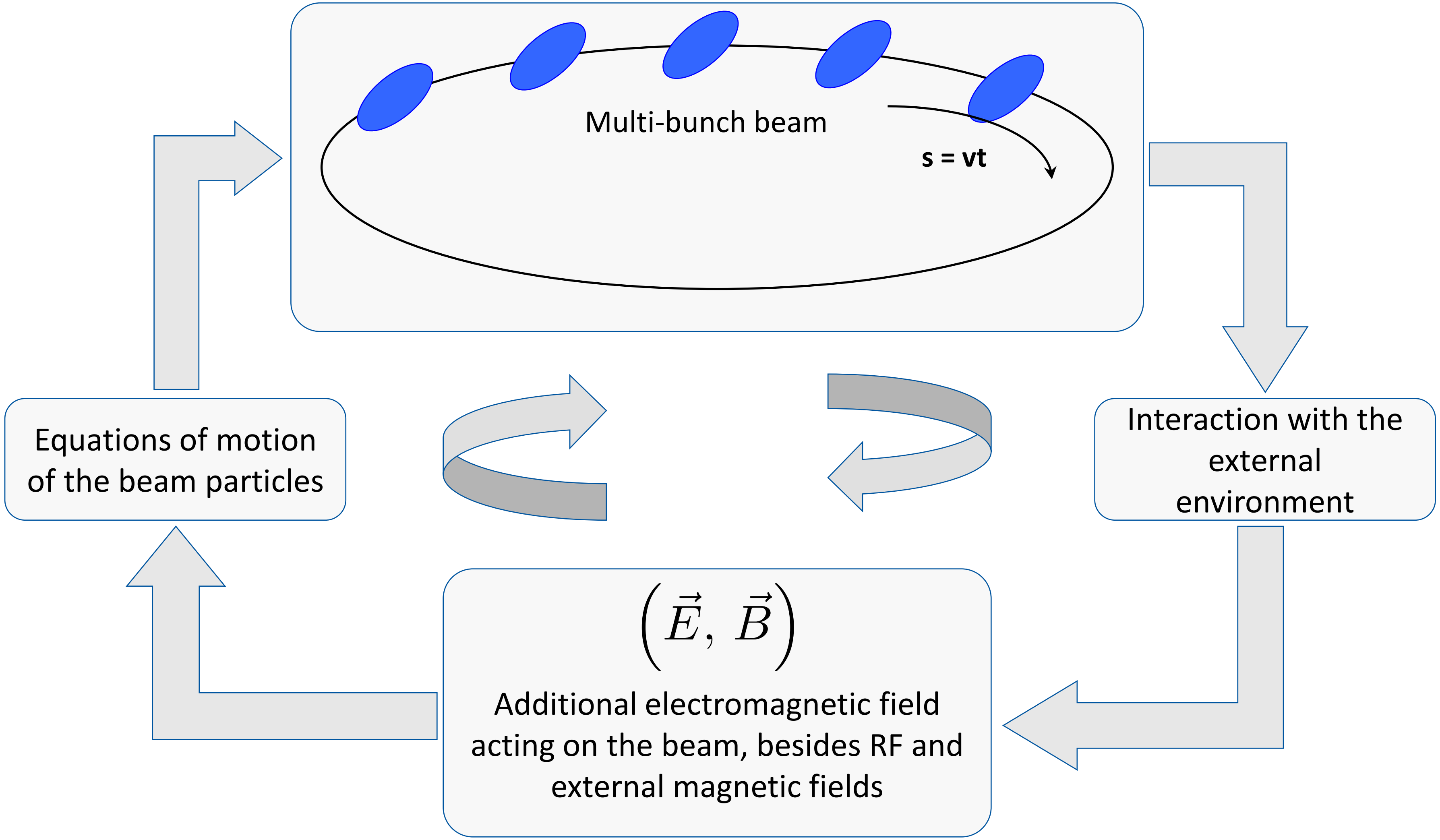
$$\sigma_x^2 = \frac{1}{N} \int \left(x - \langle x \rangle\right)^2 \cdot \psi\left(x, x', y, y', z, \delta\right)\, dx dx' dy dy' dz d\delta$$

and similar definitions for $\langle y \rangle, \sigma_y, \langle z \rangle, \sigma_z$

The instability loop

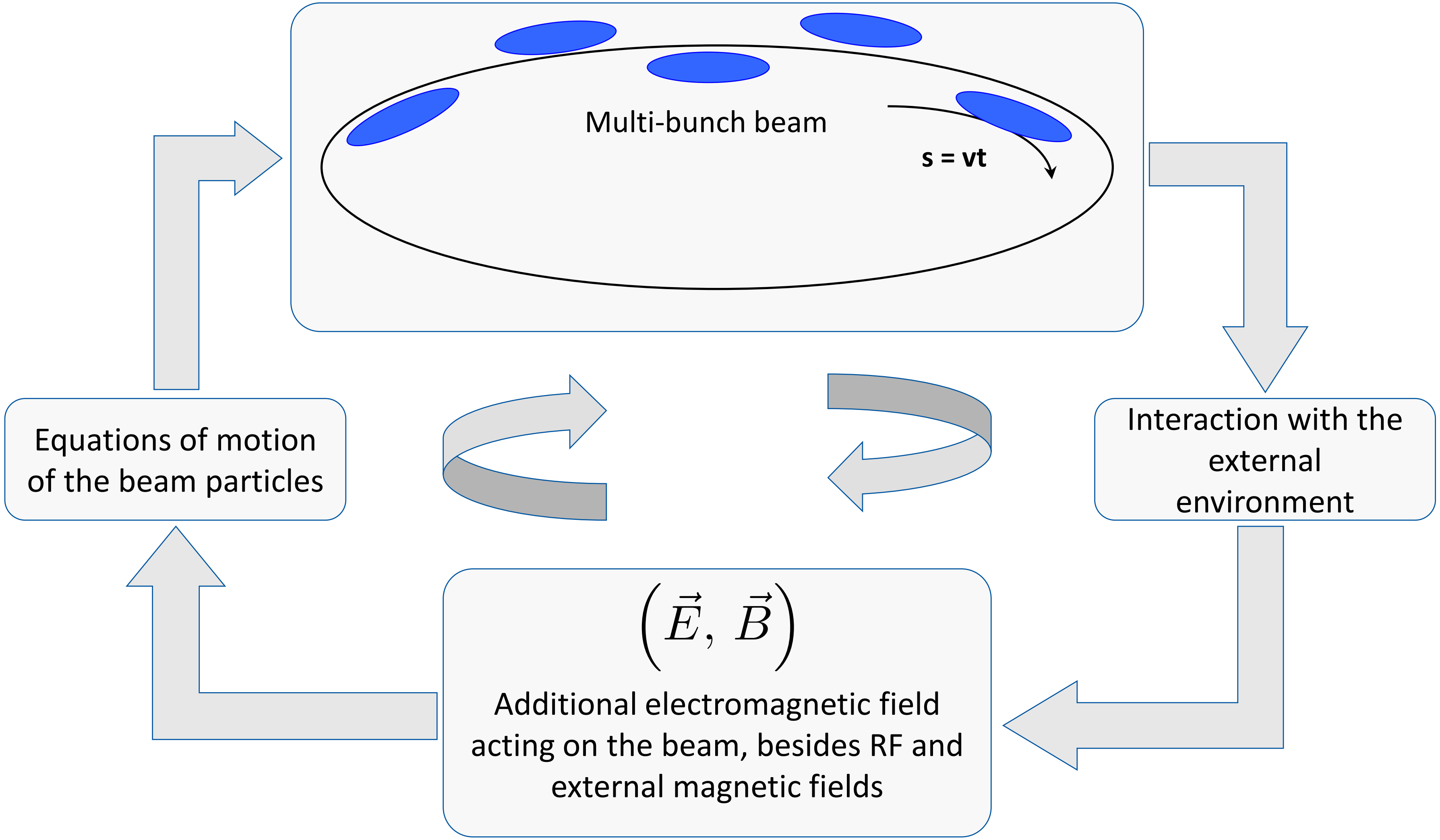


The instability loop



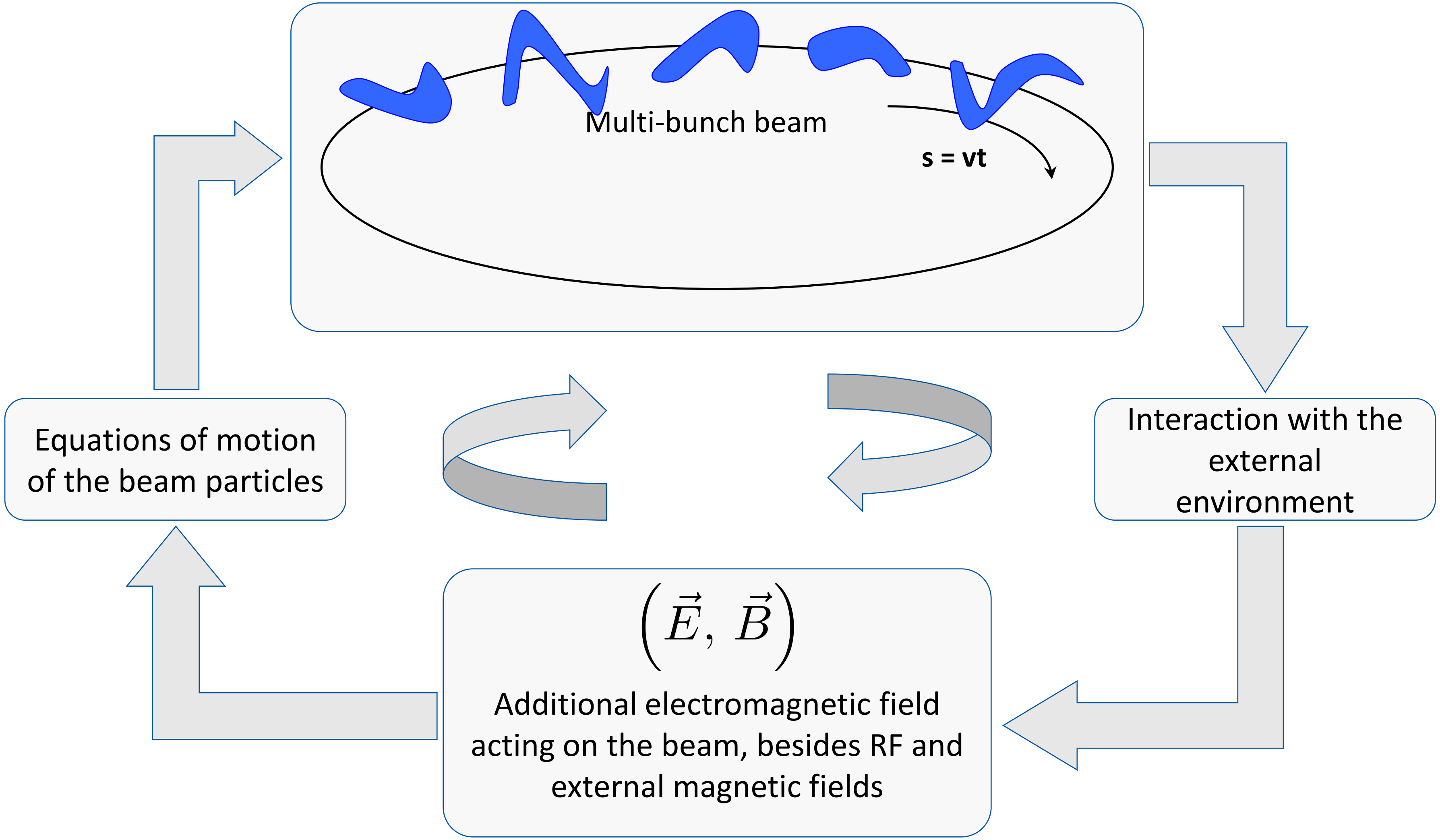
… when the loop closes, either the beam will find a new stable equilibrium configuration

The instability loop



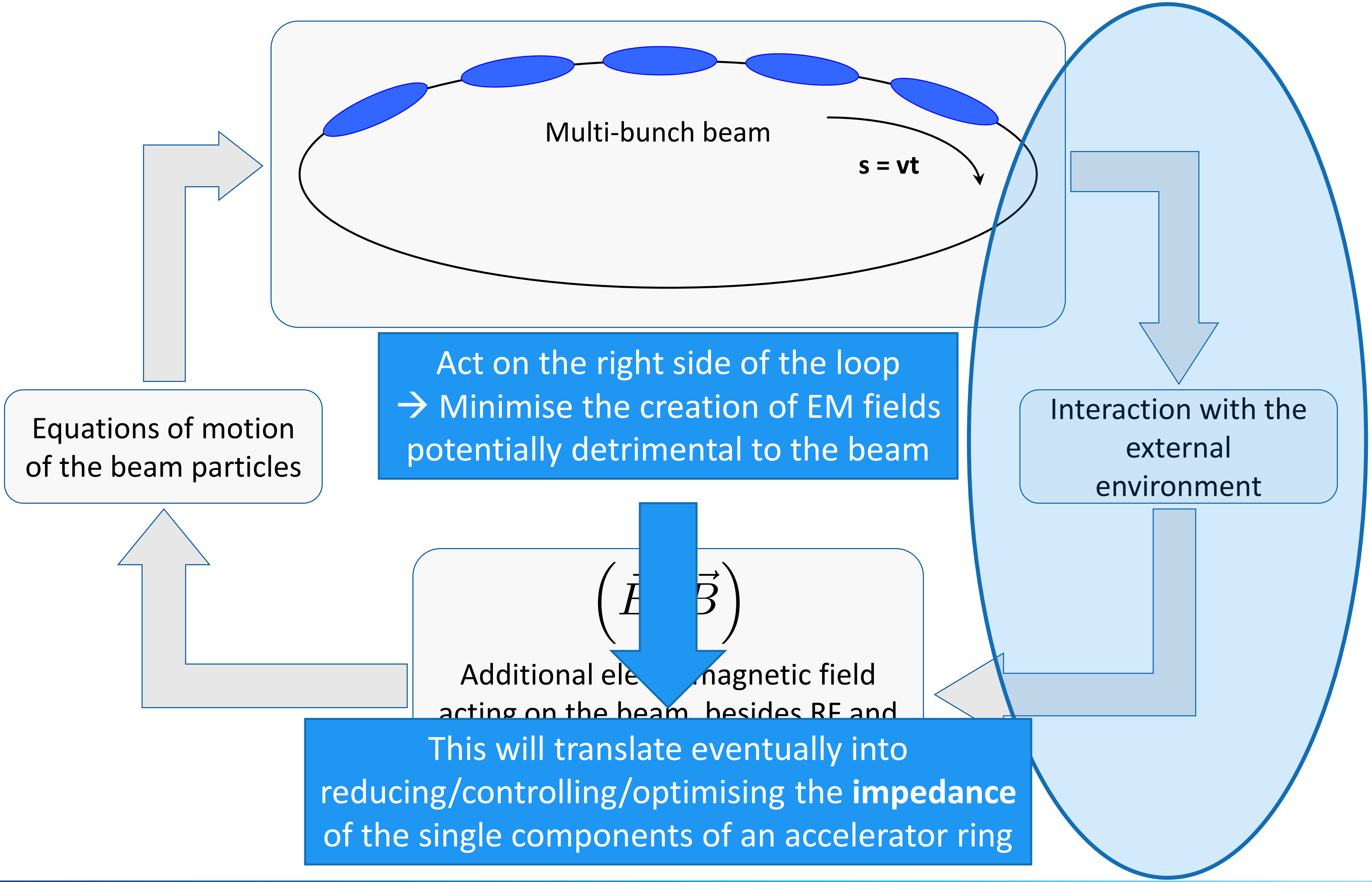
… or it might develop an instability along the bunch train

The instability loop



… or also an instability affecting different bunches independently of each other

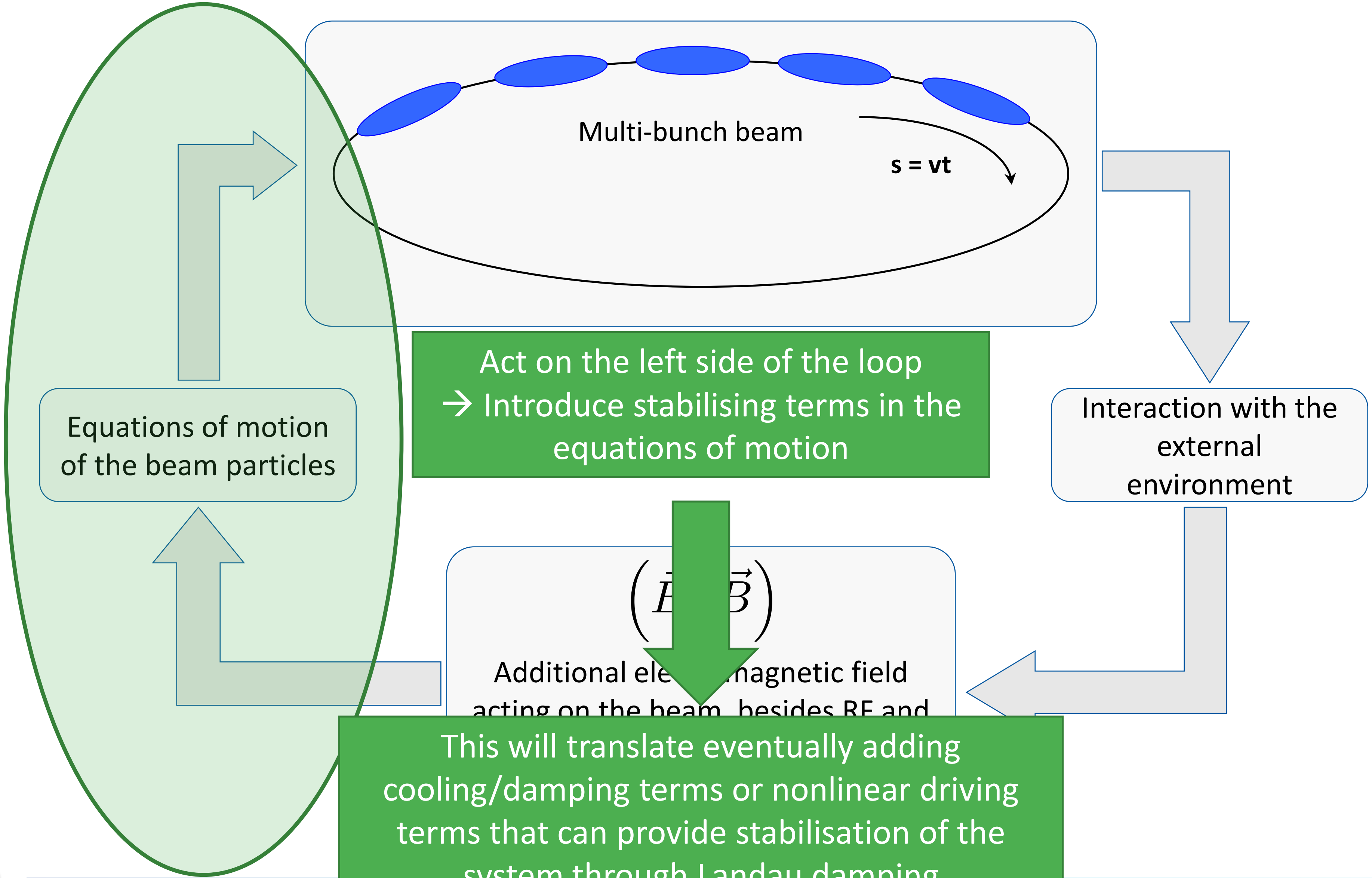
The instability loop



Act on the right side of the loop
→ Minimise the creation of EM fields potentially detrimental to the beam

This will translate eventually into reducing/controlling/optimising the **impedance** of the single components of an accelerator ring

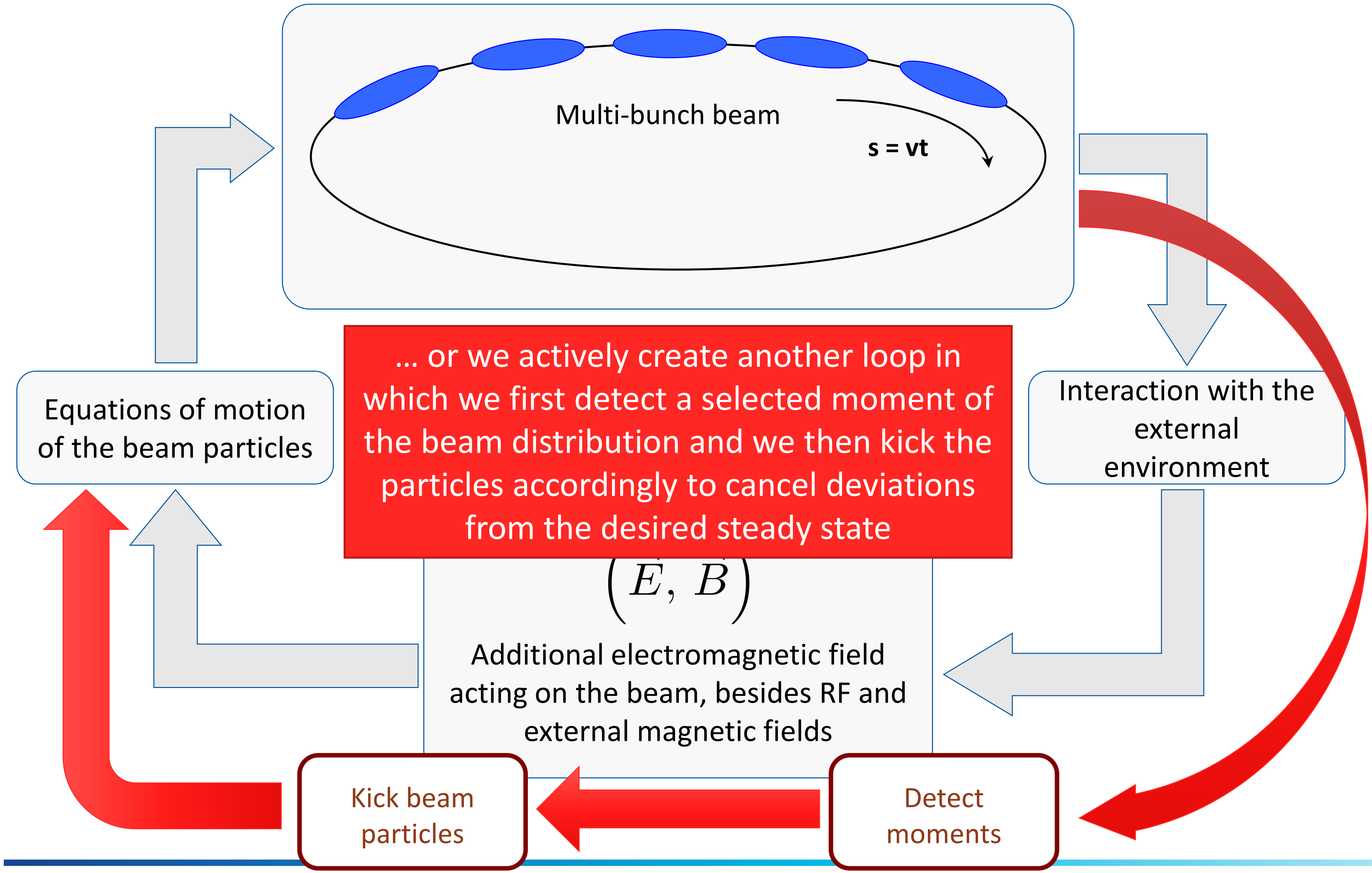
The instability loop



Act on the left side of the loop
→ Introduce stabilising terms in the equations of motion

This will translate eventually adding cooling/damping terms or nonlinear driving terms that can provide stabilisation of the system through Landau damping

The instability loop



Formal description of collective beam motion

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- Formally, instead of investigating the full set of equations for a multiparticle system, we typically instead describe the latter by a **particle distribution function**:

$$\psi = \psi(x, x', y, y', z, \delta, s)$$

where

$$dN(s) = \psi(x, x', y, y', z, \delta, s)\, dx dx' dy dy' dz d\delta$$

- The accelerator environment together with the multiparticle system forms a **Hamiltonian system** for which the **Hamilton equations of motion** hold:

$$\frac{\partial x}{\partial s} = \frac{\partial H}{\partial x'}, \qquad \frac{\partial x'}{\partial s} = -\frac{\partial H}{\partial x}$$

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Formal description of collective beam motion

- Formally, inst … nstead describe the latter by a …

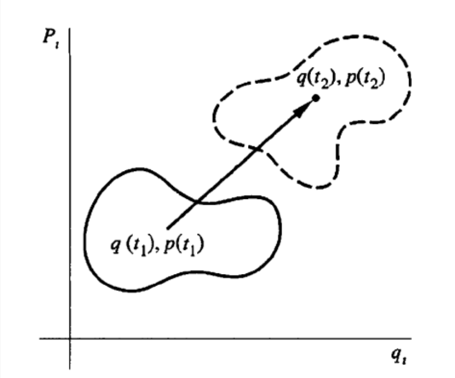
where

We can now derive the Vlasov equation which forms the foundation of the theoretical treatment of beam dynamics with collective effects:

- Consider an infinitesimal volume element dΩ containing a finite number of particles dN in phase space which evolve in time
  - dN is conserved as no particles can enter or leave the area (Picard-Lindelöf)
  - dΩ is conserved by means of the Hamilton equations of motion

- It follows that:

$$\frac{d}{ds}\psi = \frac{\partial \psi}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial \psi}{\partial x'}\frac{\partial x'}{\partial s} + \frac{\partial}{\partial s}\psi$$
$$= \underbrace{\frac{\partial \psi}{\partial x}\frac{\partial H}{\partial x'} - \frac{\partial \psi}{\partial x'}\frac{\partial H}{\partial x}}_{[\psi, H]:\ \text{Poisson bracket}} + \frac{\partial}{\partial s}\psi = 0$$



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Formal description of collective beam motion

- The evolution of a **multiparticle system** is given by the evolution of its **particle distribution function**

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi]$$

- With the Hamiltonian composed of **an external** and **a collective part**, and the particle distribution function decomposed into **an unperturbed part** and **a small perturbation** one can write

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H_0} + \boldsymbol{H_1}, \psi_0 + \psi_1]$$

- This becomes to **first order**

$$\underbrace{\frac{\partial}{\partial s}\psi_1 = [\boldsymbol{H_0}, \psi_1] + [\boldsymbol{H_1}(\psi_0 + \psi_1), \psi_0]}_{\text{Linearization in } \psi_1: \quad \ldots \propto \widehat{\boldsymbol{\Lambda}}\psi_1}$$

Spatial component | Temporal component

$$\Longrightarrow \quad \psi_1 = \sum_k a_k\, \boldsymbol{v_k} \exp\left(\frac{i\Omega_k}{\beta c}s\right)$$

We are looking for the EV of the evolution
→ **becomes an EV problem**!

Formal description of collective beam motion

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Linearization in ψ_1 ... $\propto \hat{\boldsymbol{A}}\psi_1$

We call these distinct eigenvalues ψ_k **the coherent k-mode**.

The mode and thus for example also an instability is fully characterized by a single number:

the complex tune shift Ω_k

Spatial component | Temporal component

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$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi]$$

- With the Ham … ribution function decomposed in…

- This becomes t…

Remark:

- The stationary distribution ψ_0 is the distribution where

$$\frac{\partial}{\partial s}\psi_0 = [\boldsymbol{H_0}, \psi_0] = 0$$

- In particular, a distribution is always stationary if

$$\psi_0 = \psi_0(\boldsymbol{H_0}), \quad \text{as} \quad [\boldsymbol{H_0}, \psi_0(\boldsymbol{H_0})] = 0$$

Solving for or finding the stationary solution for a given H_0 will be later referred to as **matching**.

…tion

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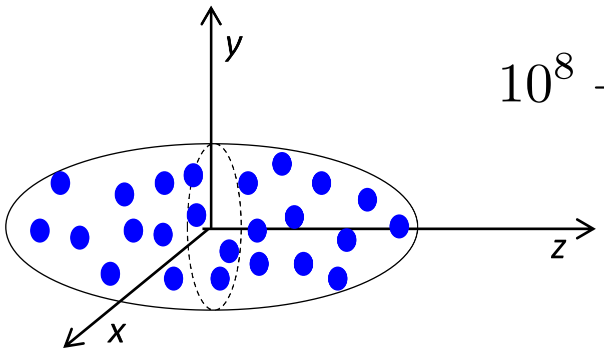
Formal description of collective beam motion

- The evolution of a **multiparticle system** can also be studied via direct **macro-particle simulation**
  - Number of macro-particles needs to be chosen to have results statistically significant but in reasonable execution times within the available computing power
- Here we need to solve numerically a set of equations of motion corresponding to macro-particles representing the beam
  - The driving terms of these equations are the EM fields externally applied as well as the EM fields generated by the macro-particle distribution itself
  - Therefore, we typically need to couple with an EM solver



$$10^8 - 10^{11} \text{ particles} \longrightarrow 10^4 - 10^6 \text{ macroparticles}$$

$$\frac{d\vec{p}_{\mathrm{mp}}}{dt} = q\left(\vec{E} + \vec{v}_{\mathrm{mp}} \times \vec{B}\right)$$

$$\begin{cases} \vec{E} = \vec{E}_{\mathrm{ext}} + \vec{E}(\psi_{\mathrm{mp}}) \\ \vec{B} = \vec{B}_{\mathrm{ext}} + \vec{B}(\psi_{\mathrm{mp}}) \end{cases}$$

Solutions of Maxwell's equations

Signpost

We have seen the difference between **external forces** and **self-induced forces** and how these lead to **collective effects**.

We have seen schematically how these collective effects can induce **coherent beam instabilities and some knobs to avoid them**.

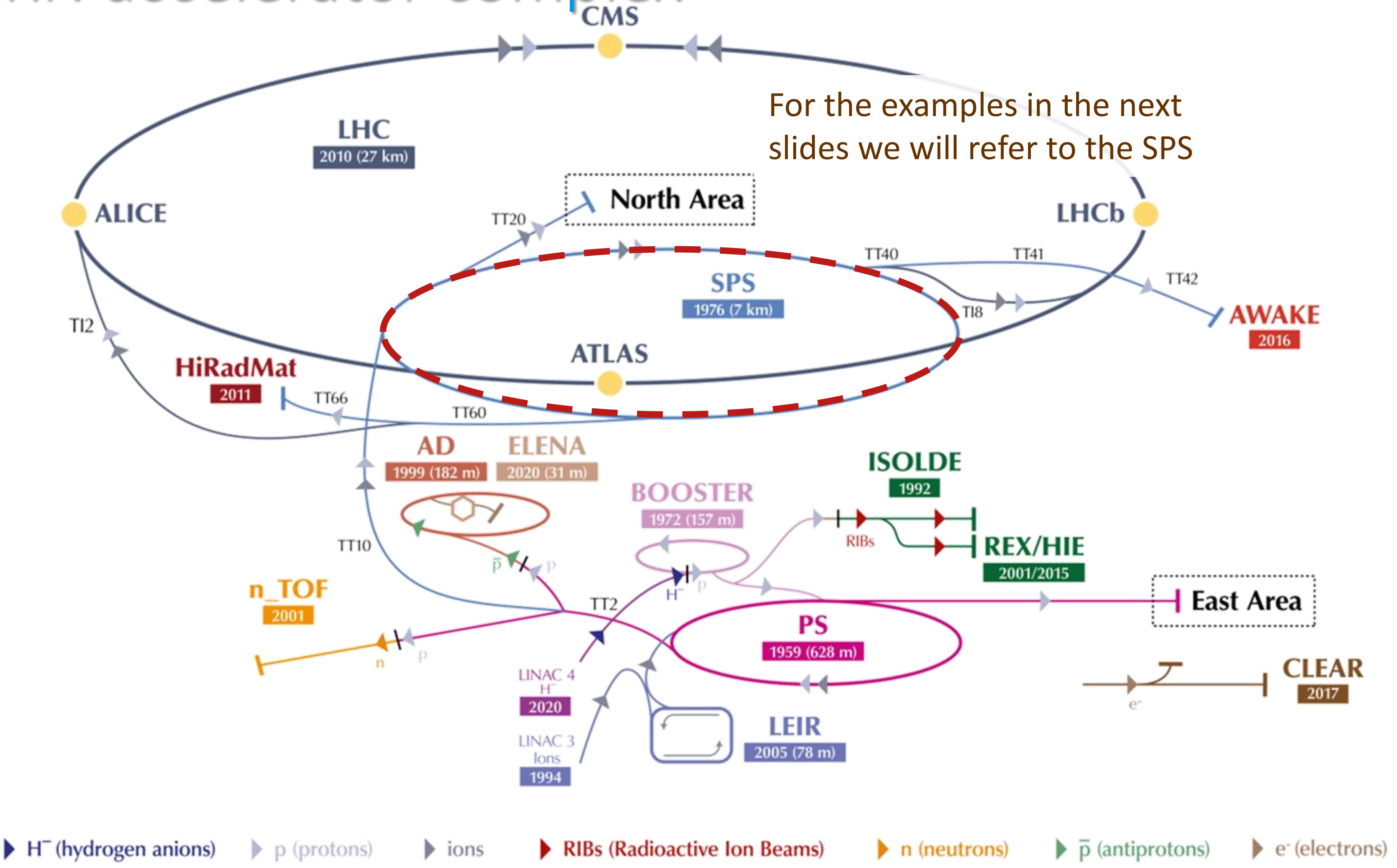
We have briefly sketched the **theoretical framework** within which the beam dynamics of collective effects is usually treated – we have encountered the Vlasov equation, bunch / beam eigenmodes and the complex tune shift.

- **Part 1: Introduction – multiparticle systems and dynamics**
    - Introduction to beam instabilities
    - Instabilities examples
    - Basic concepts
        - Beam matching
        - Multiparticle effects – filamentation and decoherence

Why worry about beam instabilities?

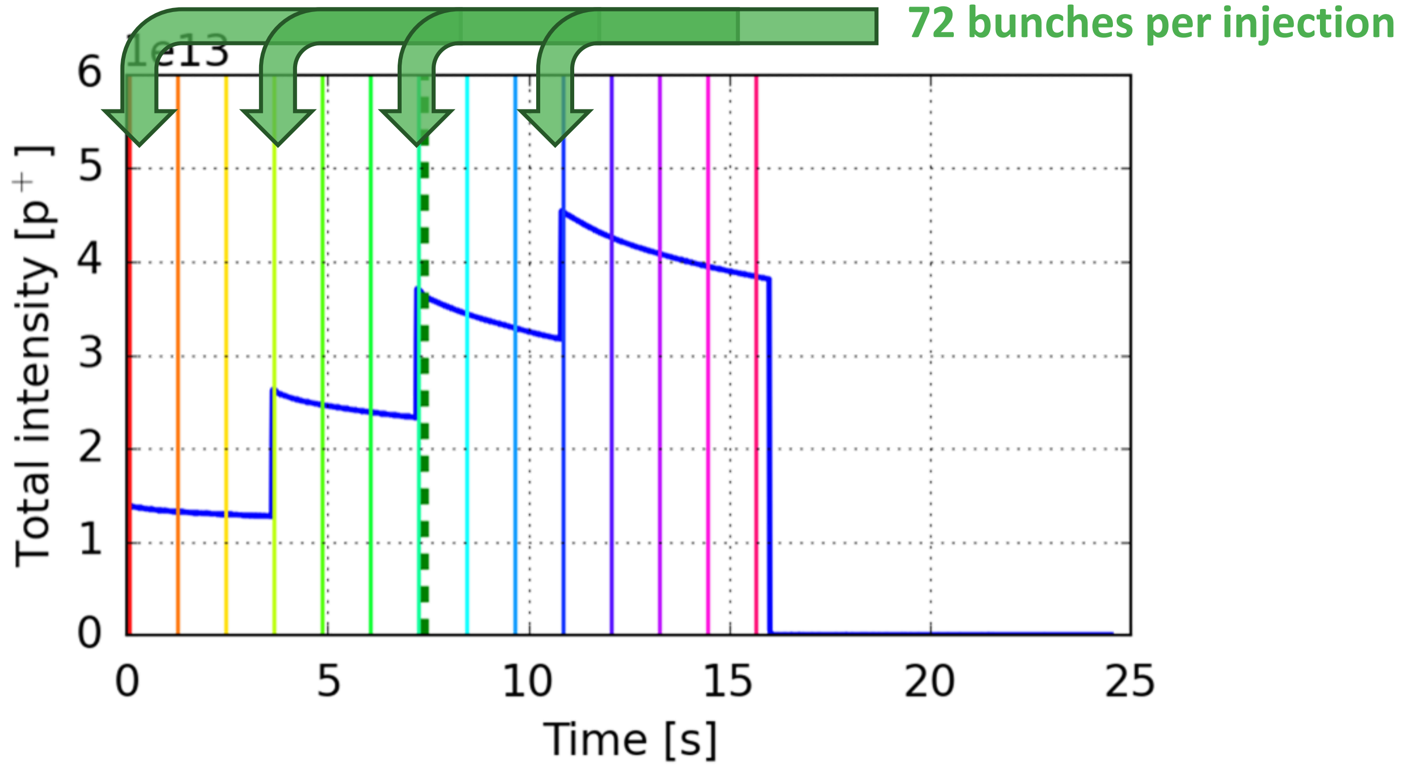
- Why study beam instabilities?
  - The onset of a beam instability usually determines the maximum beam intensity that a machine can store/accelerate (performance limitation)
  - Understanding the type of instability limiting the performance, and its underlying mechanism, is essential because it:
    - Allows identifying the source and possible measures to mitigate/suppress the effect
    - Allows dimensioning an active feedback system to prevent the instability

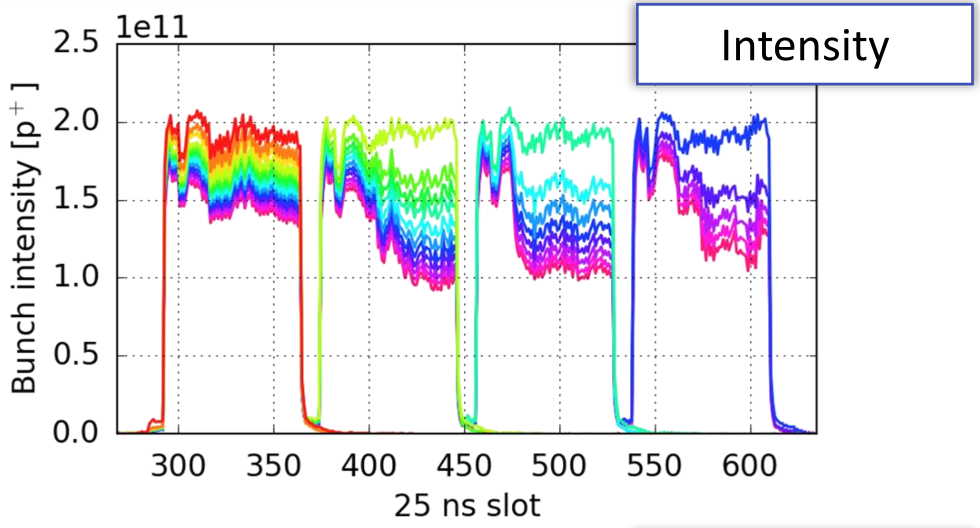
The CERN accelerator complex

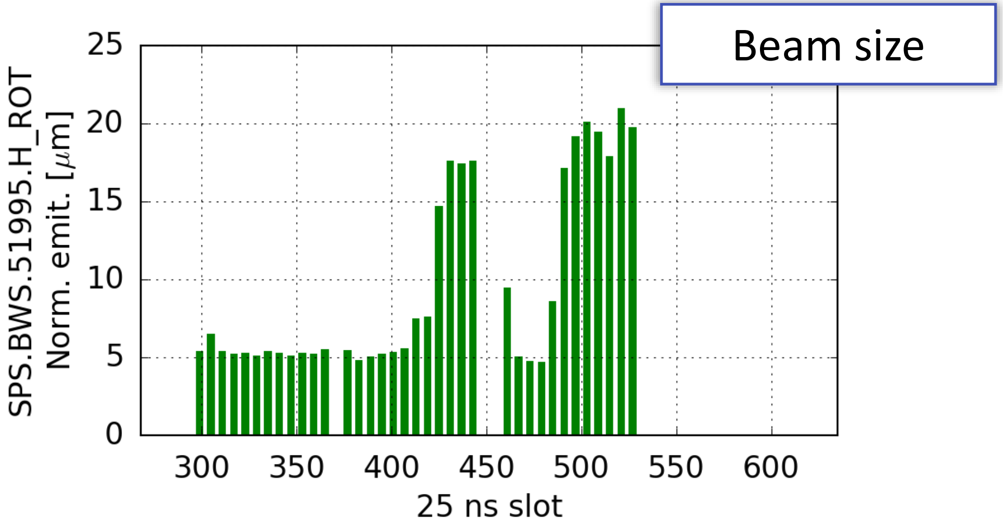


For the examples in the next slides we will refer to the SPS

Coupled bunch instability in the SPS





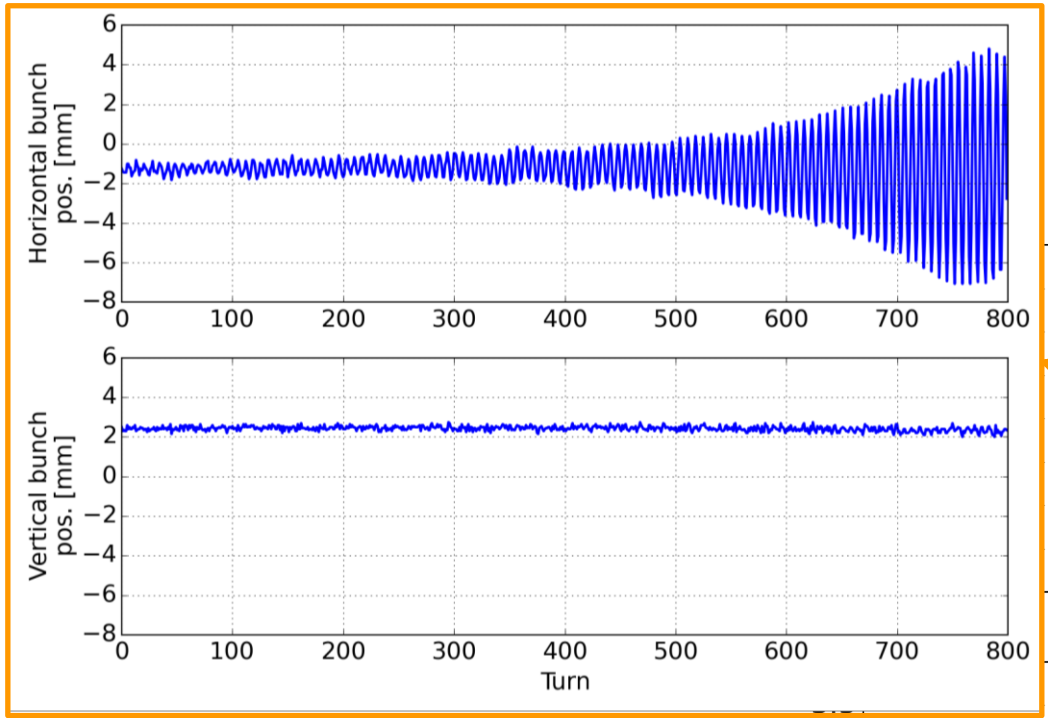


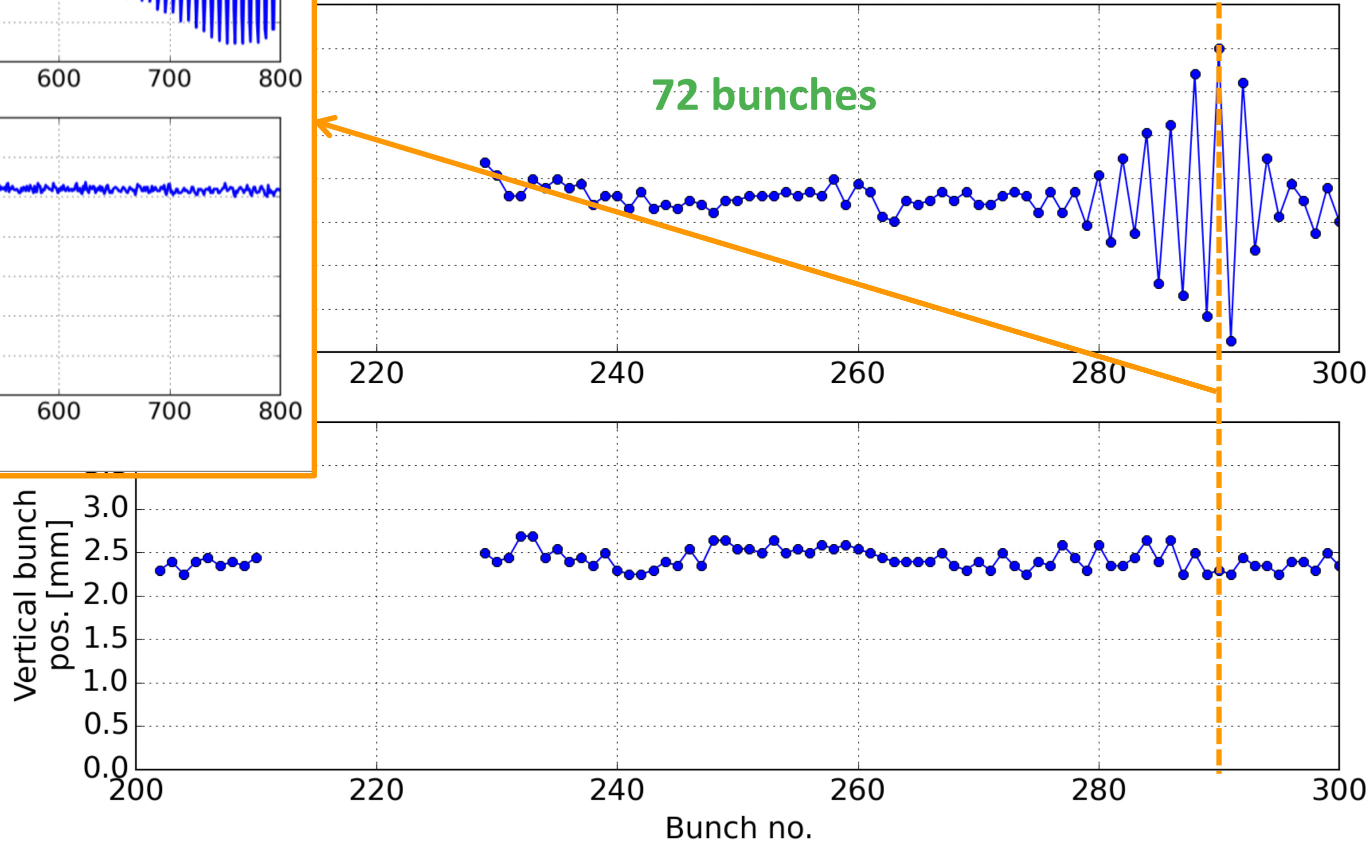
- Injection of 4 batches of 72 bunch trains into the SPS
- Later trains feature **strong losses (intensity)** and **large blow-up (emittance)** – this leads to a **strong loss of beam brightness**

Coupled bunch instability in the SPS

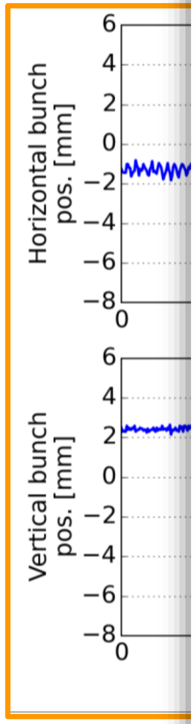
- Injection of 4 batches of 72 bunch trains into the SPS
- A closer look into one train exhibits **strong coherent coupled bunch oscillations**

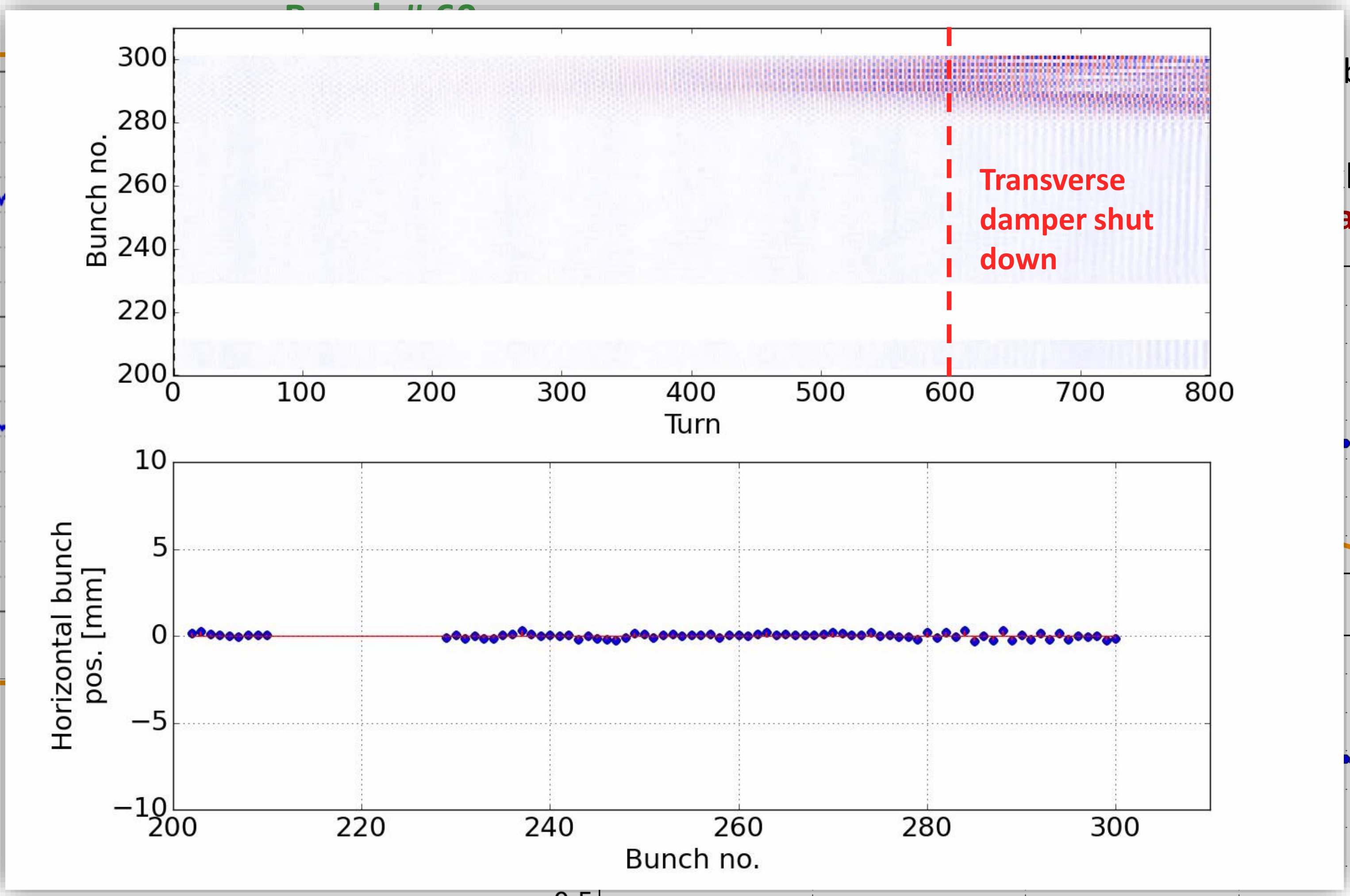
Bunch # 60





Coupled bunch instability in the SPS

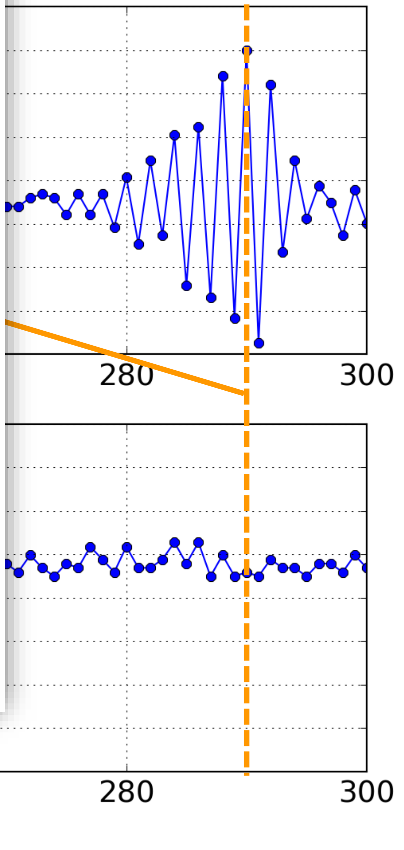




bunch trains

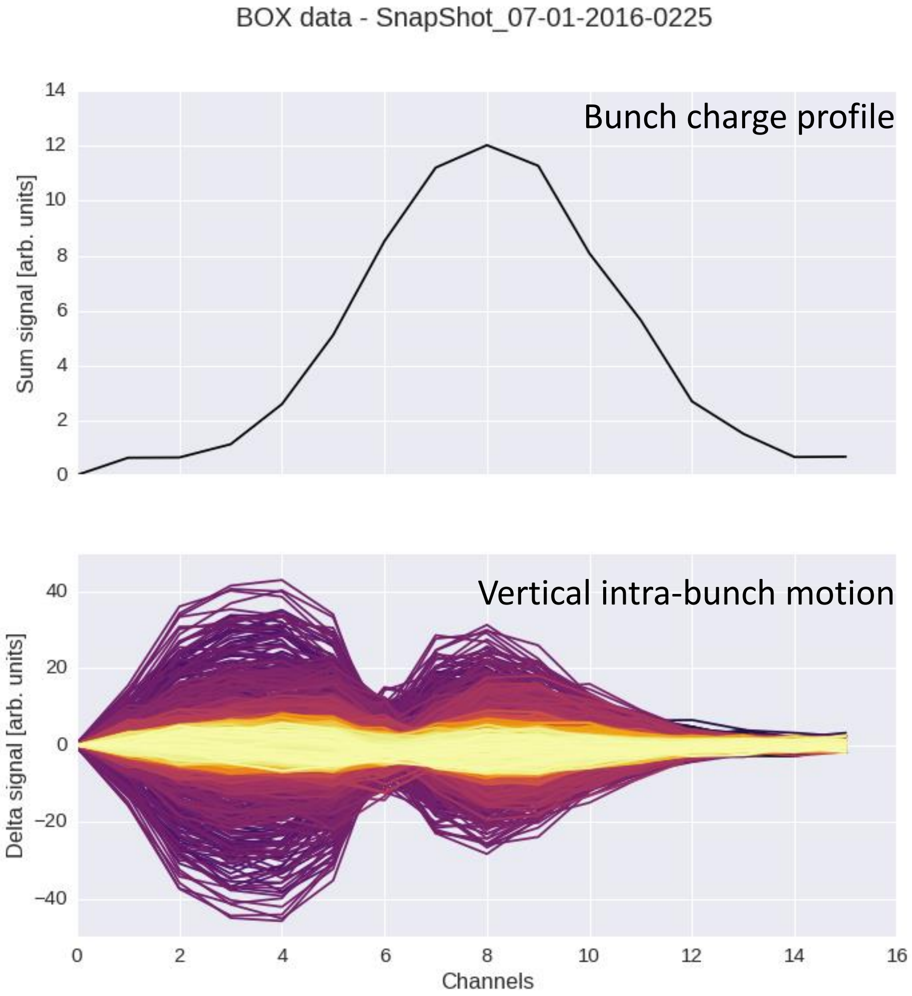
hibits **strong**

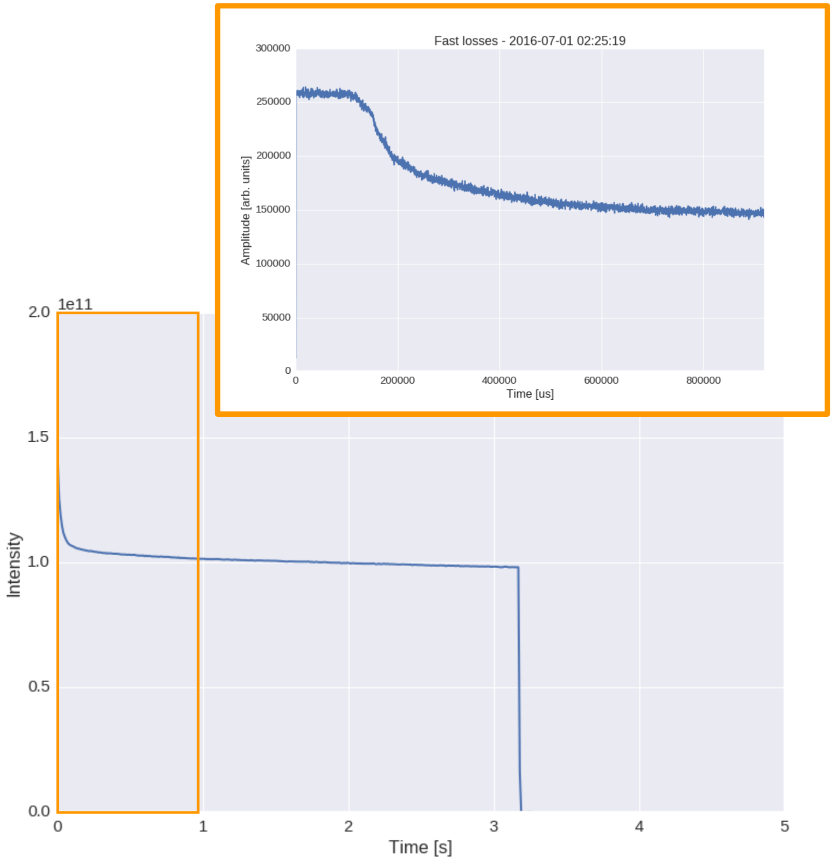
ations



Single bunch instability in the SPS

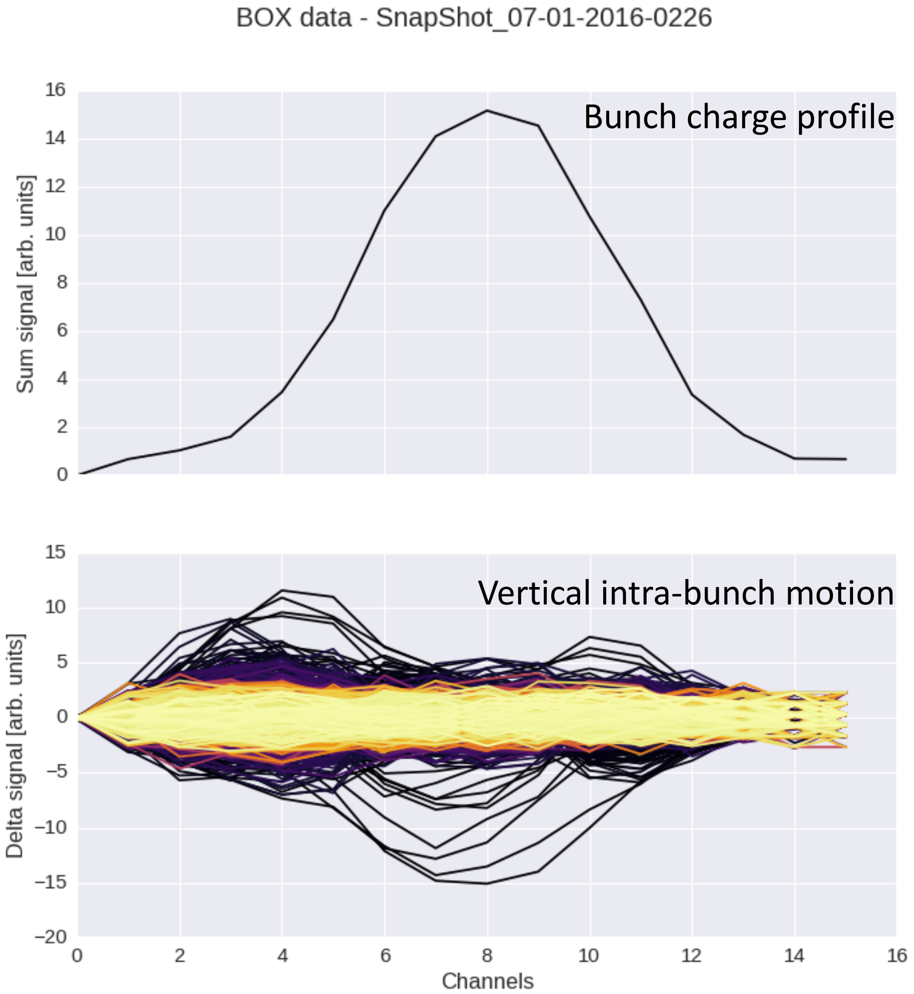
Open loop



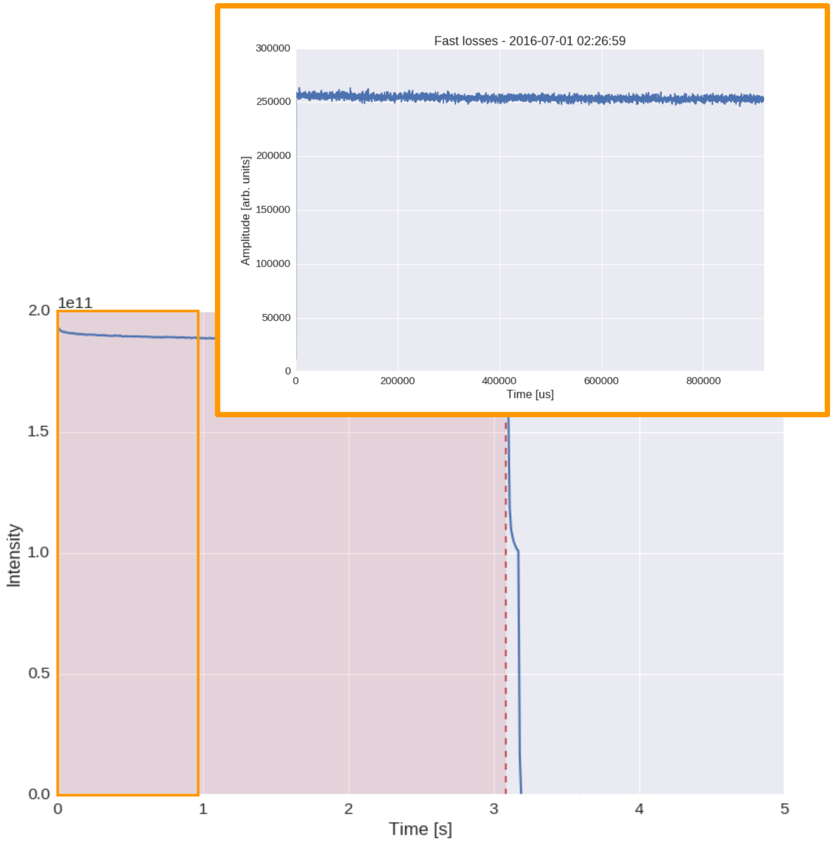


- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)**

Single bunch instability in the SPS



Closed loop



- Loss of more than 30% of the bunch intensity due to a **slow transverse mode coupling instability (TMCI)** → can be mitigated by a **wideband feedback system**.

Signpost

We now understand that collective effects can have a **huge detrimental impact** on the machine performance and why, therefore, the study and the understanding of instabilities is important.

We have encountered some **real world examples** of instabilities observed throughout the CERN accelerator chain.

Before moving on to a more detail view of collective effects, we will have a quick look at some **distinct characteristics of multi-particle beam dynamics**.

- **Part 1: Introduction – dynamics of multiparticle systems**
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    - Beam matching
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Beam matching

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- As seen earlier, given a particle distribution function and a machine (described by a Hamiltonian H) the stationary solution is given by:

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi] = 0$$

  and can be constructed via matching…

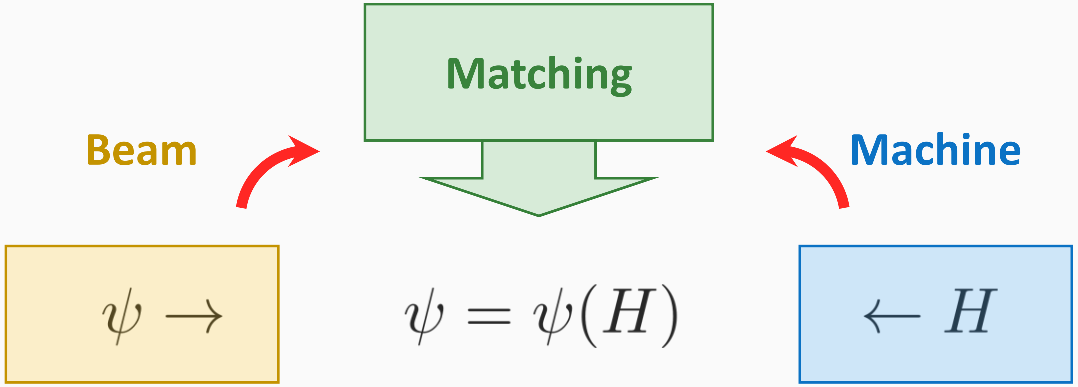
Beam matching

- As seen earlier, given a particle distribution function and a machine (described by a Hamiltonian H) the stationary solution is given by:

$$\frac{\partial}{\partial s}\psi = [\boldsymbol{H}, \psi] = 0$$

and can be constructed via matching…

- In real life, an injected beam ought to be **matched to the machine** for best performance.
- Given a **particle distribution function** and a **machine optics** locally described by a Hamiltonian we ensure matching by targeting for:



Matching examples

We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

- Betatron motion

$$H = \frac{1}{2}x'^2 + \left(\frac{Q_x}{R}\right)^2 x^2$$

$$H_0 = \sigma_{x'}^2 = \left(\frac{Q_x}{R}\right)^2 \sigma_x^2 \implies \boxed{\frac{\sigma_x}{\sigma_{x'}} = \frac{R}{Q_x} = \beta_x}$$

- Synchrotron motion - linear

$$H(z,\delta) = -\frac{1}{2}\eta\beta c\,\delta^2 + \frac{eVh}{4\pi R^2 p_0}z^2$$

$$H_0 = \eta\beta c\,\sigma_\delta^2 = \frac{eVh}{2\pi R^2 p_0}\sigma_z^2 \implies \boxed{\frac{\sigma_z}{\sigma_\delta} = R\eta\sqrt{\frac{2\pi\beta^2 E_0}{eV\eta h}} = \frac{R\eta}{Q_s}\sigma_\delta = \beta_z}$$

Matching examples

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We take the example of Gaussian distribution functions

$$\psi(H) = \exp\left(\frac{H}{H_0}\right)$$

- Beta

In reality the synchrotron motion is described by the Hamiltonian:

$$H(z,\delta) = -\frac{1}{2}\eta\beta c\,\delta^2 + \frac{eV}{2\pi h p_0}\left(\cos\left(\frac{hz}{R}\right) - \cos\left(\frac{hz_c}{R}\right) + \frac{\Delta E}{eV}\left(\frac{hz}{R} - \frac{hz_c}{R}\right)\right)$$

This leads to **nonlinear equations** and the matching procedure becomes more involved.

- Syn

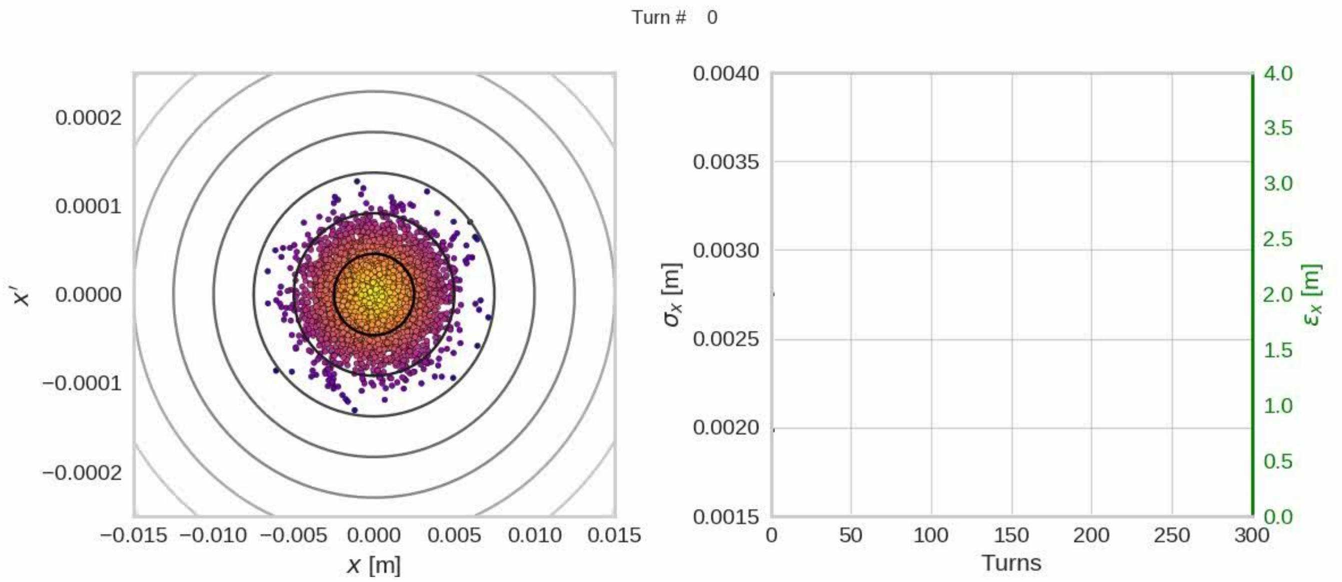
$$H_0 = \eta\beta c\,\sigma_\delta^2 = \frac{eVh}{2\pi R^2 p_0}\,\sigma_z^2 \implies \boxed{\frac{\sigma_z}{\sigma_\delta} = R\eta\sqrt{\frac{2\pi\beta^2 E_0}{eV\eta h}} = \frac{R\eta}{Q_s}\,\sigma_\delta = \beta_z}$$

Matching illustration – matched beams

- Betatron motion – **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

Matched beams **maintain their beam moments** and their shape in phase space



Matching illustration – matched beams

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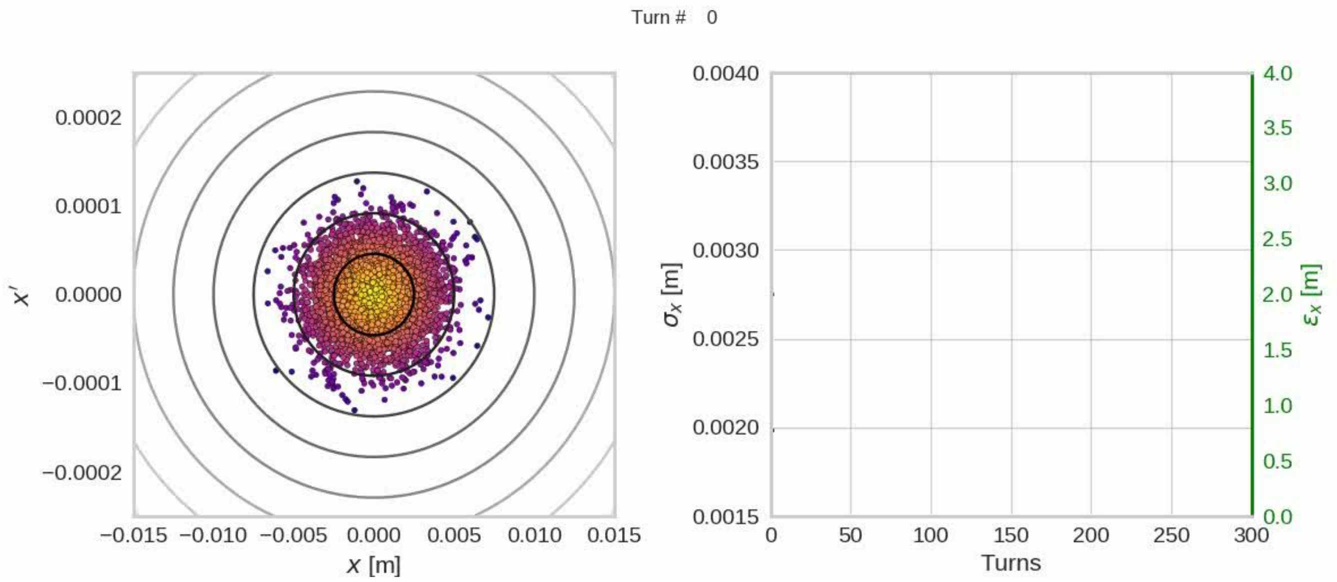
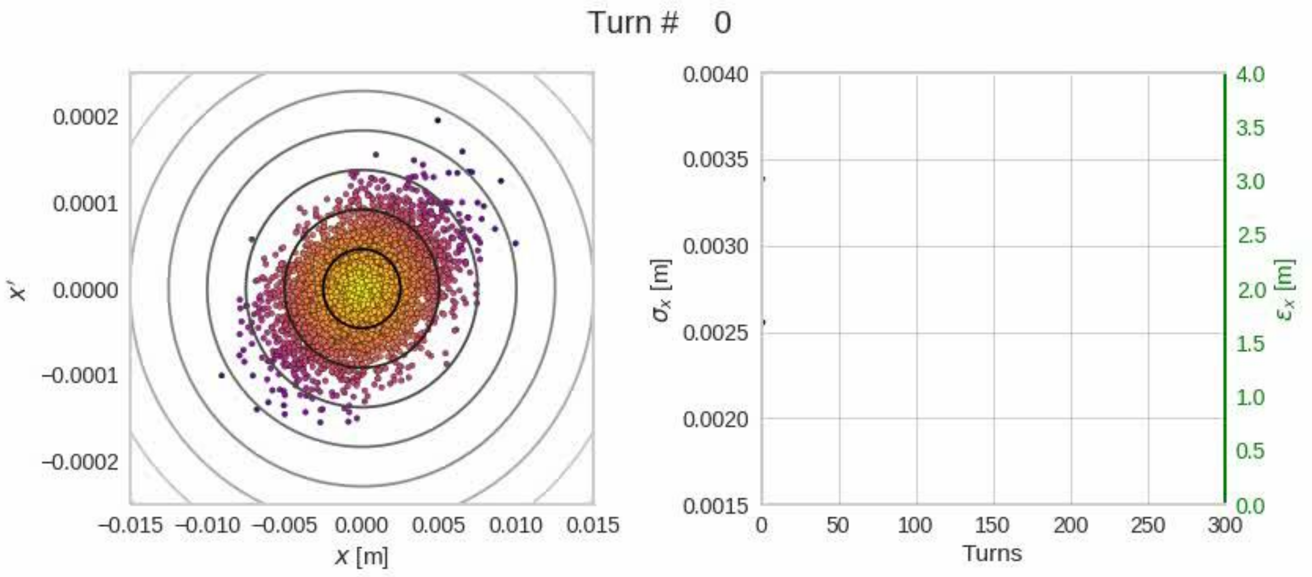


Matching illustration – mismatched beams

- Betatron motion – **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

Mismatched beams show **oscillations in their beam moments** and may **change their shape due to filamentation**

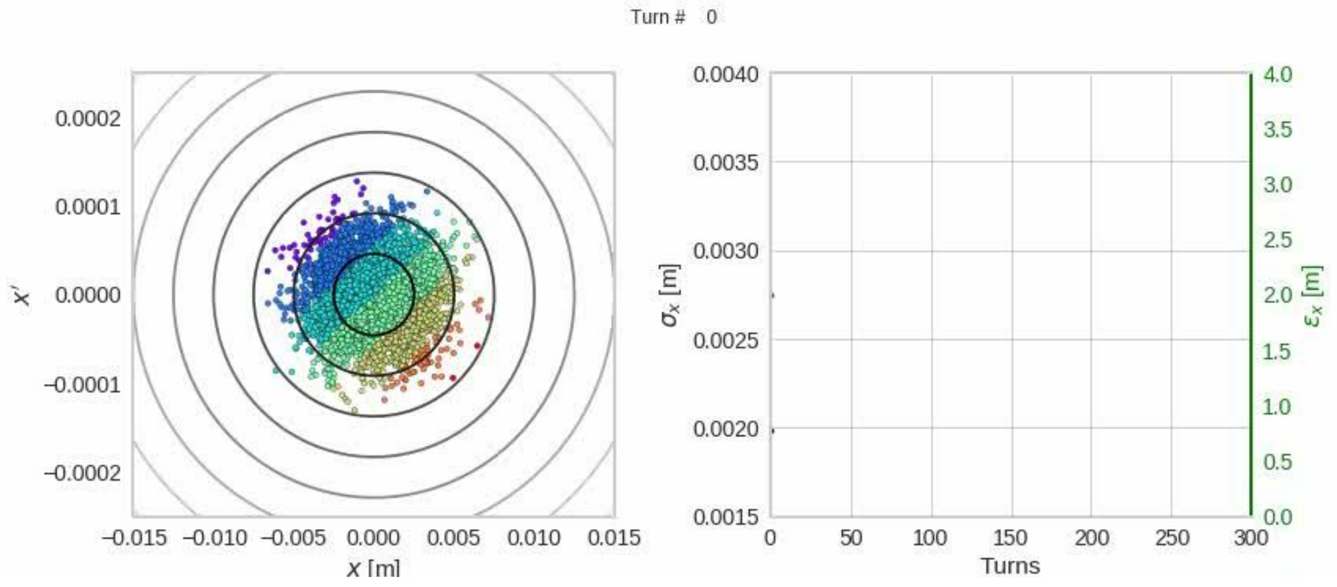


Matching illustration – linear vs. nonlinear

- Betatron motion – **linear**

$$\frac{\sigma_x}{\sigma_{x'}} = \beta_x$$

Nonlinearities lead to **detuning with amplitude**. This is visible as the **characteristic spiraling** of larger amplitude particles.



Signpost

We have learned about the **meaning of matching** a beam to the machine optics.

We have seen how to **formally match a beam** to a given description of a machine.

We have seen **examples of matched and mismatched beams** and have seen the difference between **linear and non-linear motion**.

- **Part 1: Introduction – dynamics of multiparticle systems**
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

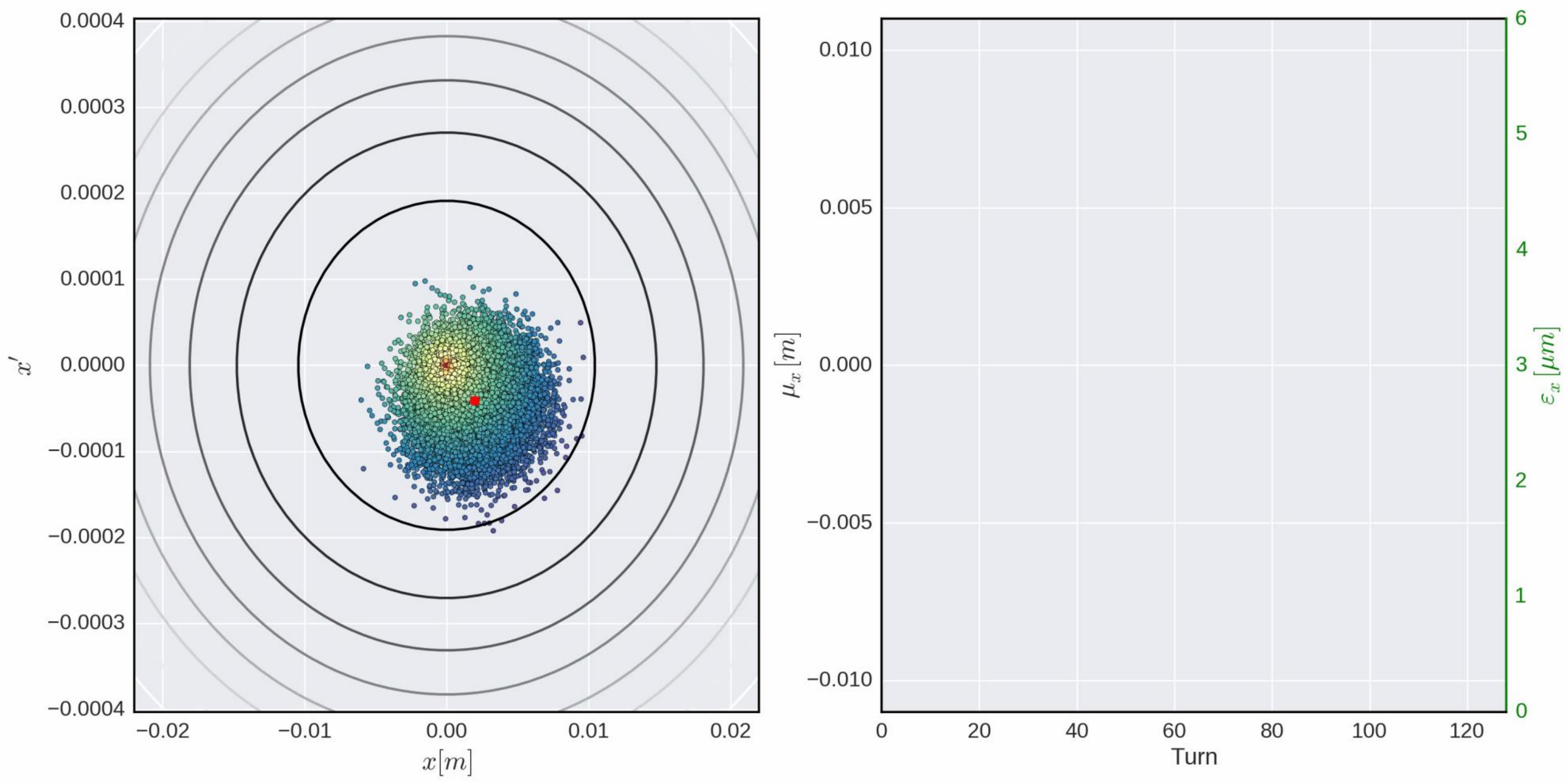
Sources and impact of transverse nonlinearities

- We have learned or we may know from operational experience that there are a set of **crucial machine parameters to influence beam stability** – among them **chromaticity and amplitude detuning**

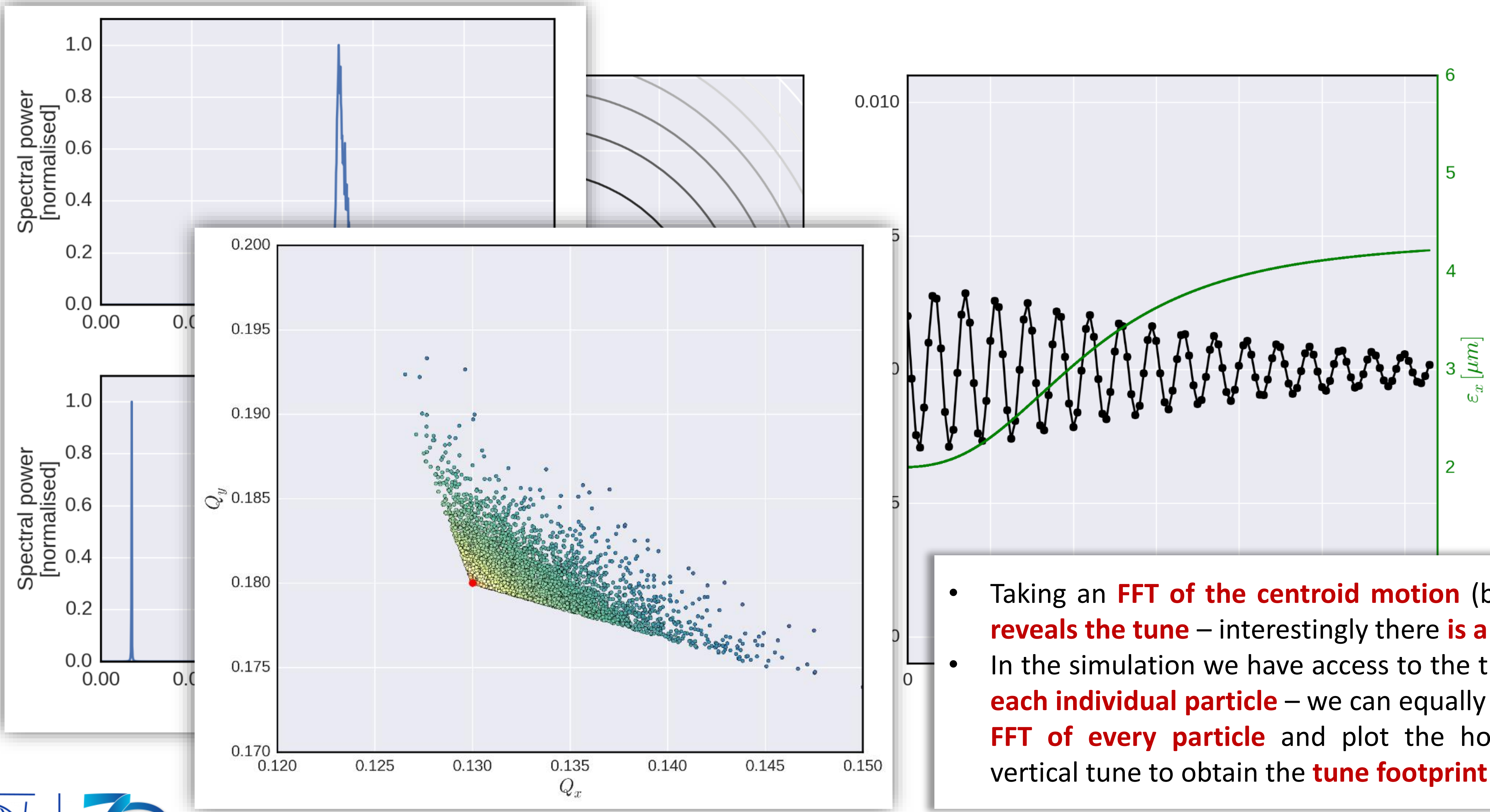
- Chromaticity
  - Controlled with sextupoles – provides **chromatic shift** of bunch spectrum wrt. impedance
  - Changes interaction of beam with impedance
  - Damping or excitation of **headtail modes**

- Amplitude detuning
  - Controlled with octupoles – provides (incoherent) **tune spread**
  - Leads to absorption of coherent power into the incoherent spectrum → **Landau damping**

Example: filamentation as result of detuning

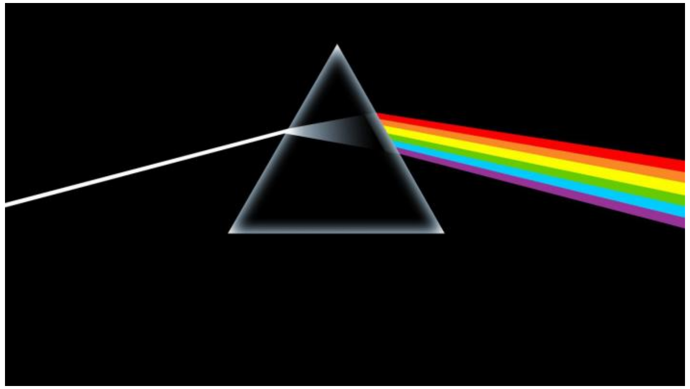


Example: filamentation as result of detuning



- Taking an **FFT of the centroid motion** (black curve) **reveals the tune** – interestingly there **is a spread**
- In the simulation we have access to the trajectory of **each individual particle** – we can equally perform an **FFT of every particle** and plot the horizontal vs. vertical tune to obtain the **tune footprint**

Example: chromaticity – de- & recoherence



- Chromatic detuning:

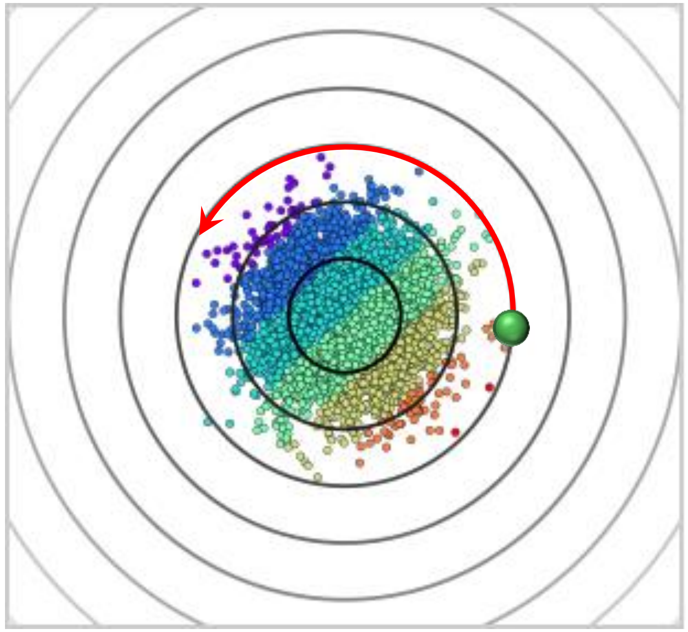
$$\Delta Q_x = Q'_x \, \delta$$

$$\delta = \hat{\delta} \, \sin(\varphi)$$

- Consider a particle in 6d phase space performing both betatron and synchrotron oscillations
- The accumulated betatron detuning after one half, resp. one full synchrotron period reads

$$\Delta Q_{x,\,\mathrm{acc}}\Big|_{T_s/2} = \hat{\delta} \int_0^{\pi} \sin(\varphi)\, d\varphi = 2\hat{\delta}$$

$$\Delta Q_{x,\,\mathrm{acc}}\Big|_{T_s} = \hat{\delta} \int_0^{2\pi} \sin(\varphi)\, d\varphi = 0$$



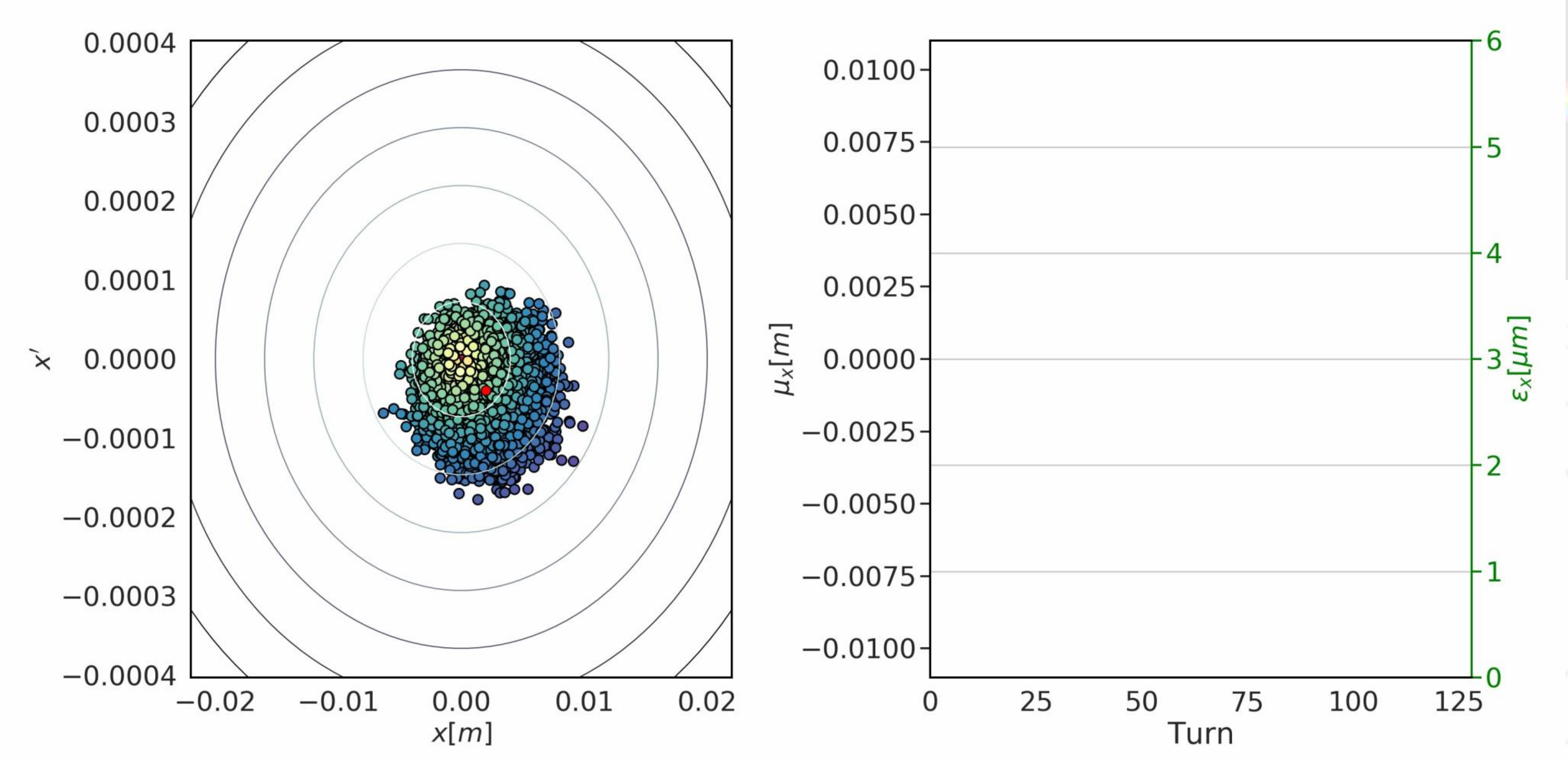
- After **one full synchrotron period** all tune shifts have vanished (i.e., also the tune spread has vanished – the **beam has re-cohered**)

Example: chromaticity – de- & recoherence

- Chromatic detuning:

- Conside
  synchro

- The ac
  synchro



- After o
  also the tune spread has vanished – the **beam has re-cohered**)

Signpost

Sources for transverse nonlinearities are, e.g., **chromaticity** and **detuning with amplitude** from octupoles.

Transverse nonlinearities can lead to **decoherence** and **emittance blow-up**.

The effects seen so far are **chacteristics for multiparticle systems** but are **not collective effects**.

- **Part 1: Introduction – dynamics of multiparticle systems**
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

Signpost

We have learned about some of the peculiarities of **collective effects**. We have also introduced **multi-particle systems** and have seen how these can be described and treated theoretically.

We have seen some **real-world example of collective effects** manifesting themselves as coherent beam instabilities.

We have looked at some specific **features of multi-particle beam dynamics** such as matching, decoherence and emittance blow-up due to filamentation. These are not to be confused with collective effects.

- **Part 1: Introduction – dynamics of multiparticle systems**
  - Introduction to beam instabilities
  - Instabilities examples
  - Basic concepts
    - Beam matching
    - Multiparticle effects – filamentation and decoherence

End part 1

CERN

Backup

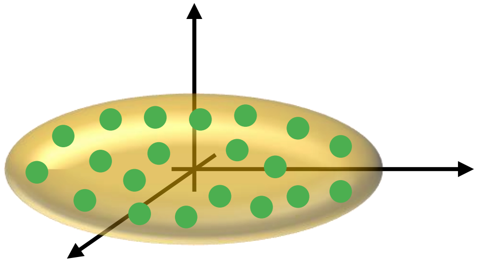
Signpost

- We have learned about the **particle description** of a beam.
- We have seen **macroparticles** and **macroparticle models**.
- We have seen how **macroparticle models** are **mapped and represented in a computational environment**.

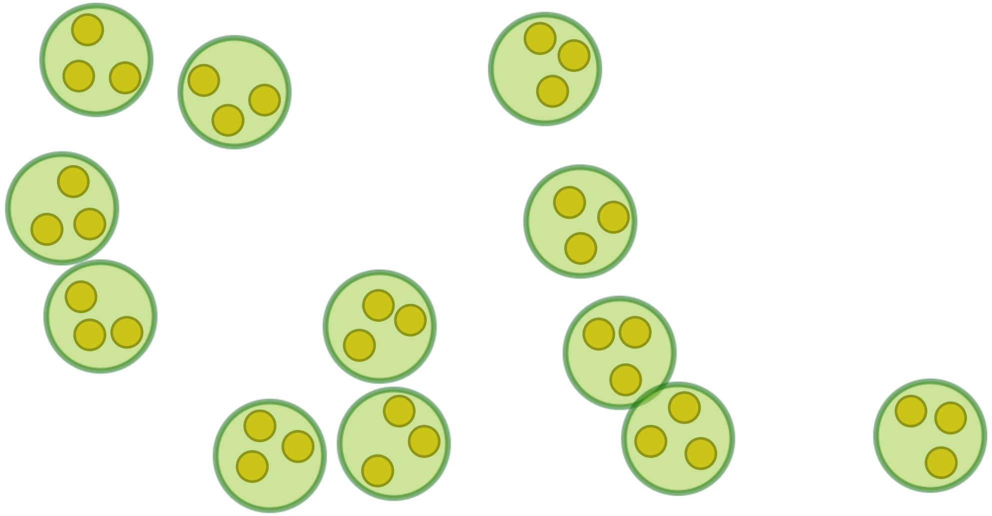
- Part 1: Introduction – multiparticle systems, macroparticle models and wake functions
  - Introduction to beam instabilities
  - Basic concepts
    - Particles and macroparticles – macroparticle distributions
    - Beam matching
    - Multiparticle effects – filamentation and decoherence
    - Wakefields as sources of collective effects

The particle description

- As seen earlier, and especially for the analytical treatment, we can represent a charged particle beam via a **particle distribution function**.
- In computer simulations, a charged particle beam is still represented as a multiparticle system. However, to be **compatible with computational resources**, we need to rely on **macroparticle models**.
- A **macroparticle** is a numerical **representation** of a **cluster of neighbouring physical particles**.
- Thus, instead of solving the system for the **N ($\sim 10^{11}$)** physical particles one can significantly **reduce the number of degrees of freedom** to **N_{MP} ($\sim 10^6$)**. At the same time one must be aware that this **increases of the granularity** of the system which gives rise to numerical noise.



$$\Psi\left(x, x', y, y', z, \delta\right)$$



Macroparticle representation of the beam

- Macroparticle models permit a **seamless mapping** of realistic systems into a **computational environment** – they are fairly easy to implement

Beam:

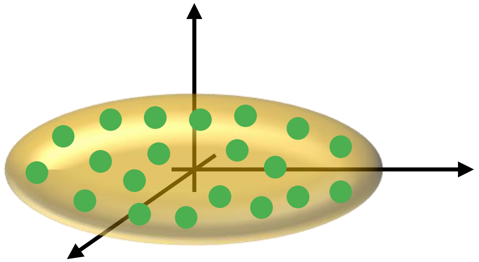
$$\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \quad \begin{pmatrix} q_i \\ m_i \end{pmatrix}, \quad i = 1, \ldots, N$$

Macroparticlenumber

$$\begin{pmatrix} y_i \\ y'_i \end{pmatrix}$$

Canonically conjugate coordinates and momenta

$$\begin{pmatrix} z_i \\ \delta_i \end{pmatrix}$$



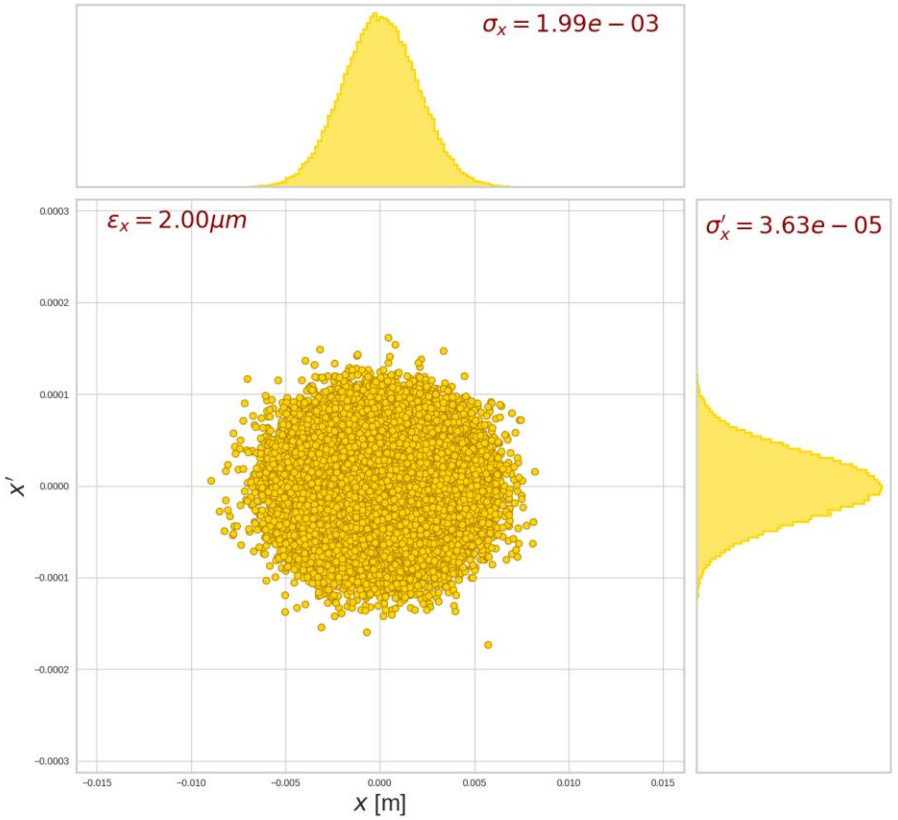
$$\Psi\left(x, x', y, y', z, \delta\right)$$

```
In [6]: df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
        df
```

Out[6]:

	dp	x	xp	y	yp	z
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-06	0.353427
1	0.001978	0.000370	1.954404e-05	-0.000359	5.543904e-05	0.159670
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.627340e-05	-0.251489
3	0.002195	-0.001668	-2.317633e-05	0.001878	-1.870926e-05	-0.038597
4	0.000572	0.000990	5.493907e-05	0.000152	-1.951051e-05	0.492968
5	-0.000418	0.001088	4.778027e-05	0.003320	-7.716856e-06	0.415582
6	-0.000114	-0.000194	1.065400e-05	0.001798	-4.984276e-07	-0.349064
7	0.001100	-0.001257	-6.873217e-05	-0.002374	5.657645e-06	-0.023157
8	0.002706	0.005351	-1.867898e-07	-0.000765	3.012523e-05	-0.291095
9	0.003508	0.000499	1.865768e-05	-0.001032	-5.363820e-05	0.211726
10	-0.001711	-0.003168	4.372560e-05	-0.001933	-2.151020e-05	-0.145358
11	-0.002150	-0.000565	-1.853825e-05	-0.003895	-6.192450e-06	0.072499
12	0.002059	0.003453	-3.808703e-05	0.000118	3.179588e-05	-0.001816
13	0.002709	0.000241	-3.457535e-05	0.000474	5.057865e-05	-0.005464
14	-0.001593	0.000711	-1.667091e-05	-0.002523	-3.804168e-05	[illegible]
15	-0.000830	-0.000393	-7.473946e-05	[illegible]		
16	-0.001743	[illegible]				

Macroparticle representation of the beam




```
In [6]: df = pd.DataFrame(bunch.get_coords_n_momenta_dict())
        df
```

Out[6]:

	dp	x	xp	y	yp
0	0.001590	0.000566	-2.285393e-05	-0.001980	4.283152e-0
1	0.001978	0.000370	1.954404e-05	-0.000359	5.54390
2	0.003492	-0.000829	-2.773707e-05	0.000291	6.62
3	0.002195	-0.001668	-2.317633e-05	0.001878	

- Initial conditions of the beam/particles

Profile	Size	Matching
Gaussian	Emittance	Optics
Parabolic		
Flat		
…		

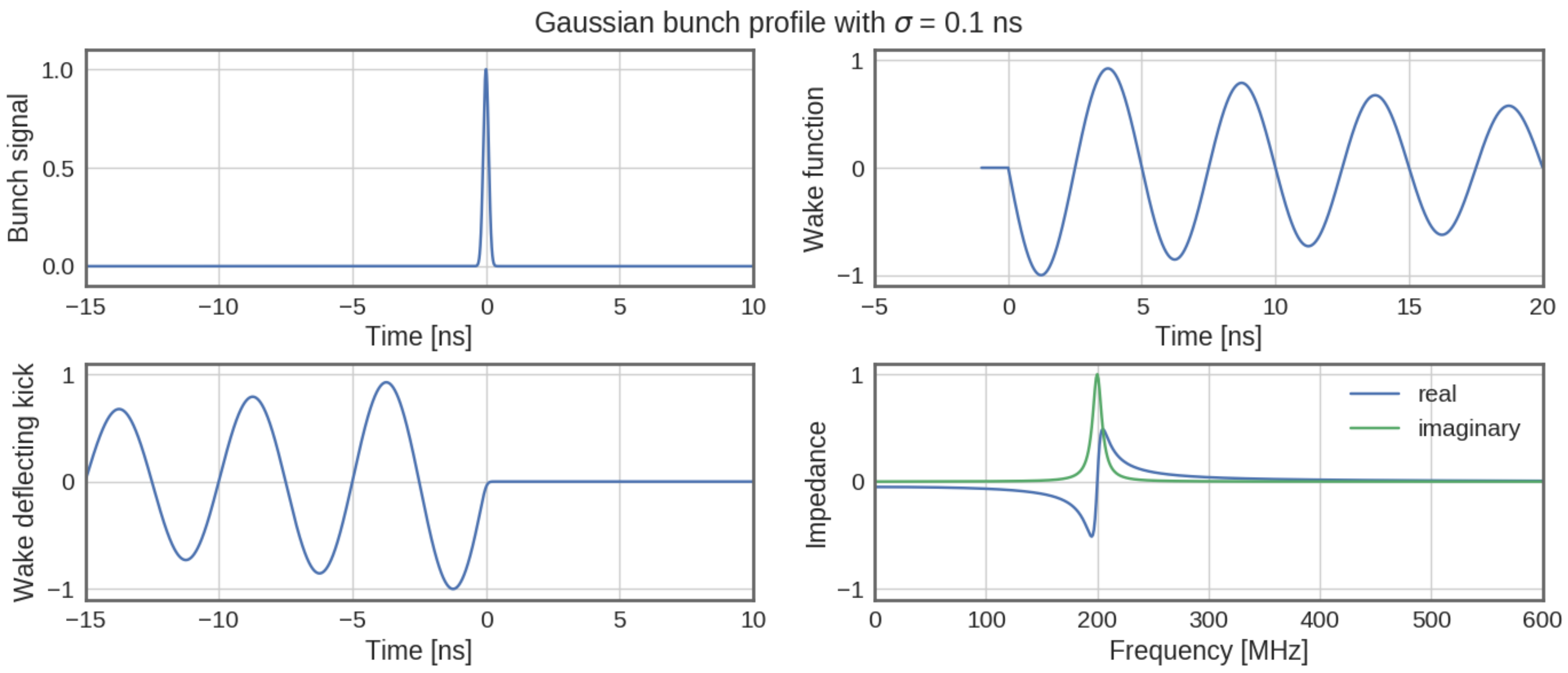
- We use **random number generators** to obtain **random distributions of coordinates and momenta**
- Example transverse Gaussian beam in the SPS with normalized emittance of 2 um (0.35 eVs longitudinal)

$$\varepsilon_\perp = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2} = \beta\gamma\sigma_x\sigma_{x'}$$

$$\varepsilon_\parallel = 4\pi\sigma_z\sigma_\delta\frac{p_0}{e}$$

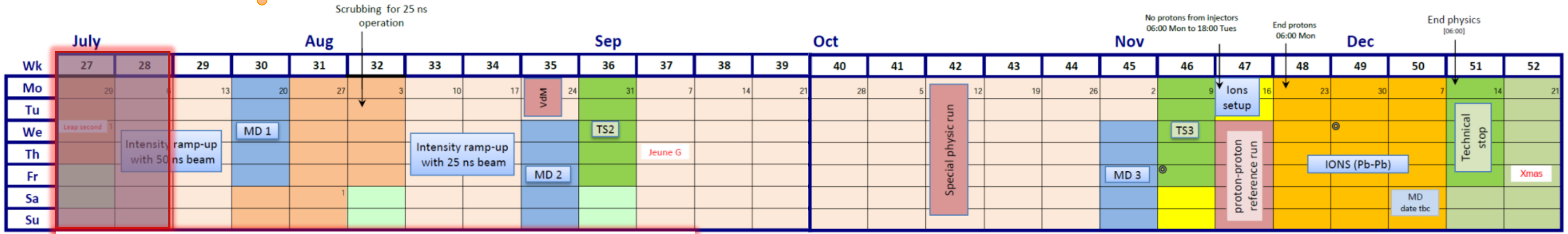
Wake fields illustrative examples

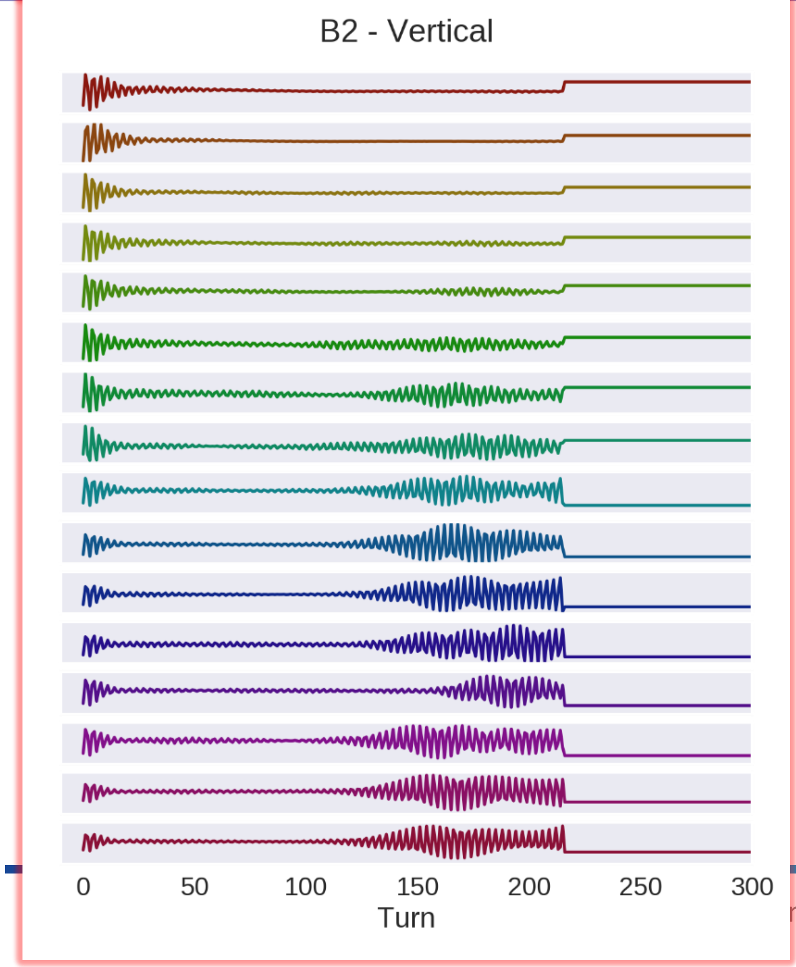
- Resonator wake: fr = 200 MHz, Q = 20 – Gaussian bunch charge profile
- The plots show how the bunch moments and the wake function **convolve into an integrated deflecting kick** at the different positions along the bunch



E-cloud instabilities in the LHC

Scrubbing run in 2015 – early stage





Head of batch

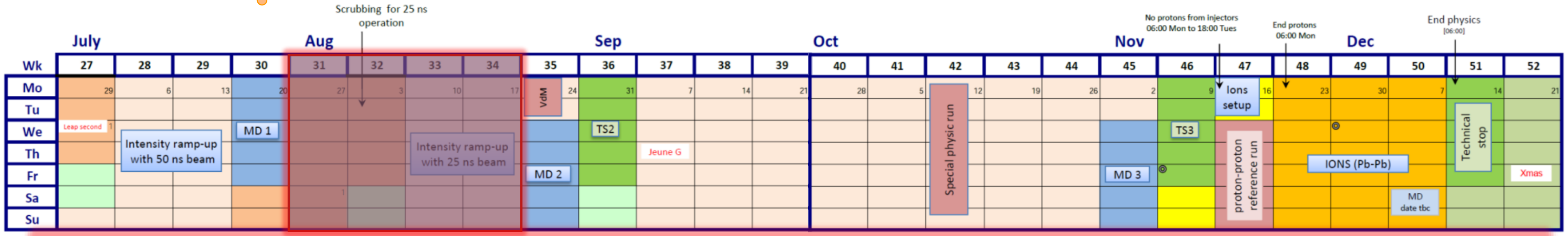
every 4th bunch just after injection

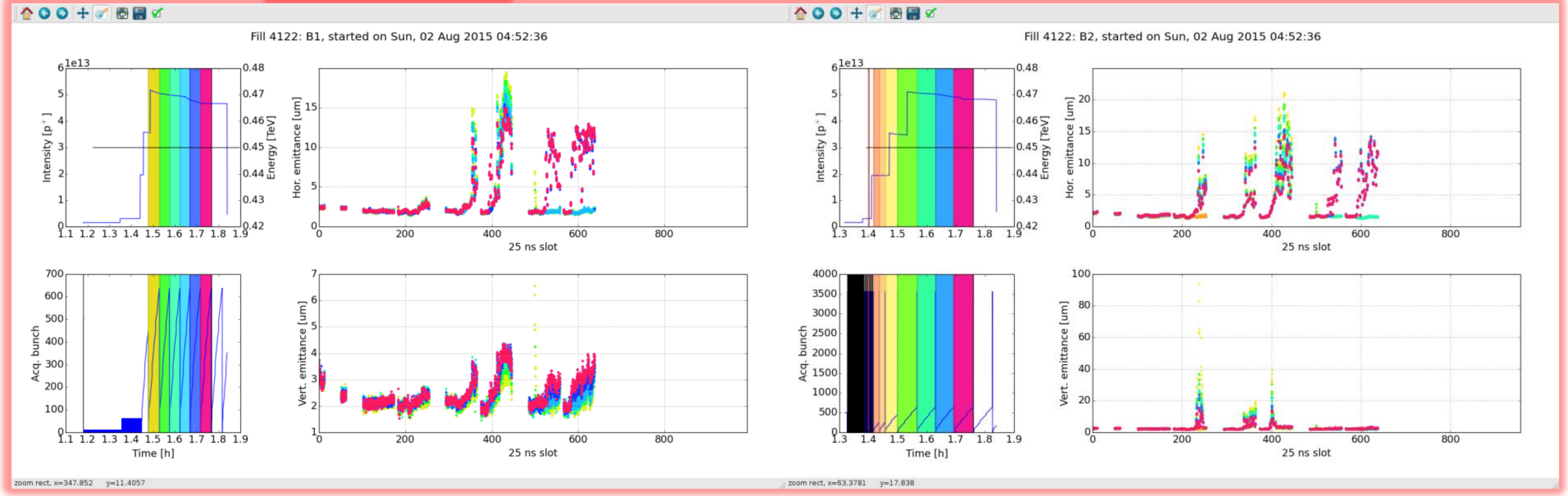
Tail of batch

- Injection of multiple bunch batches from the SPS into the LHC.
- Violent **instabilities during initial stages of scrubbing** – clear e-cloud signature
- Very hard to control in the beginning – **slow and staged ramp-up of intensity** (24 → 36 → 48 → 60 → 72 → 144 bpi)

E-cloud instabilities in the LHC

Scrubbing run in 2015 – second stage

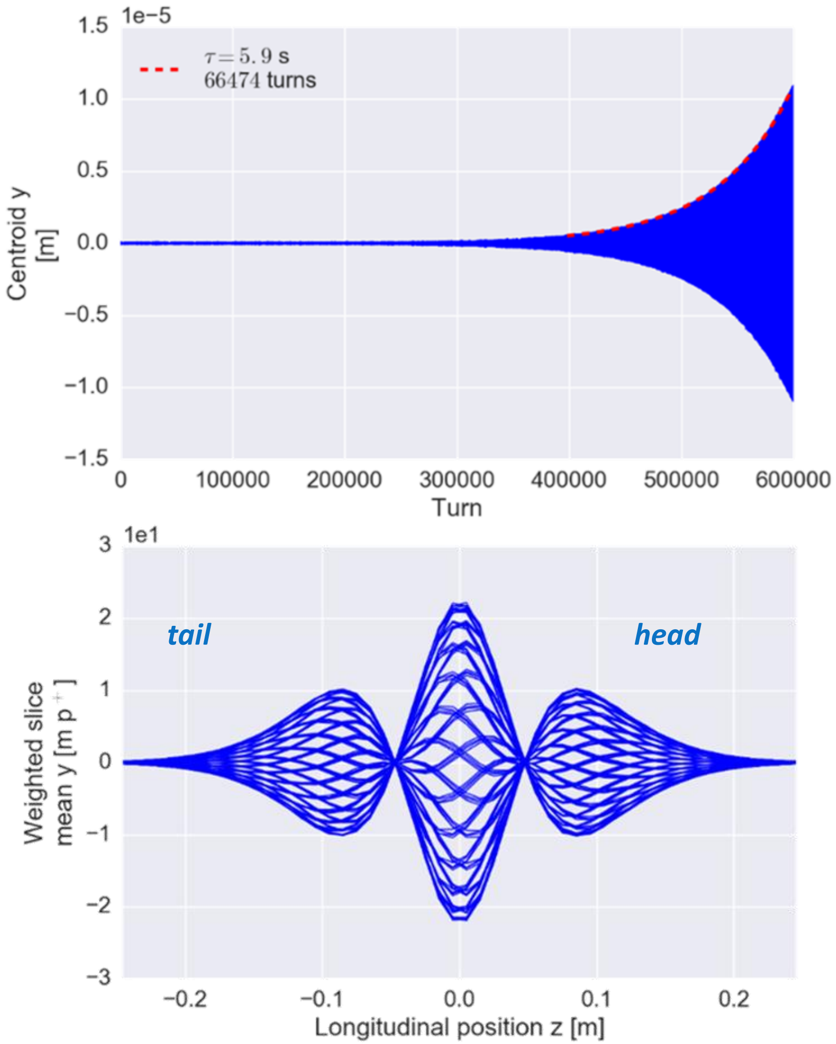




- At later stages dumps under control but still **emittance blow-up and serious beam quality degradation**.
- Beam and e-cloud induced h**eating of kickers and collimators**.

Headtail instabilities in the LHC

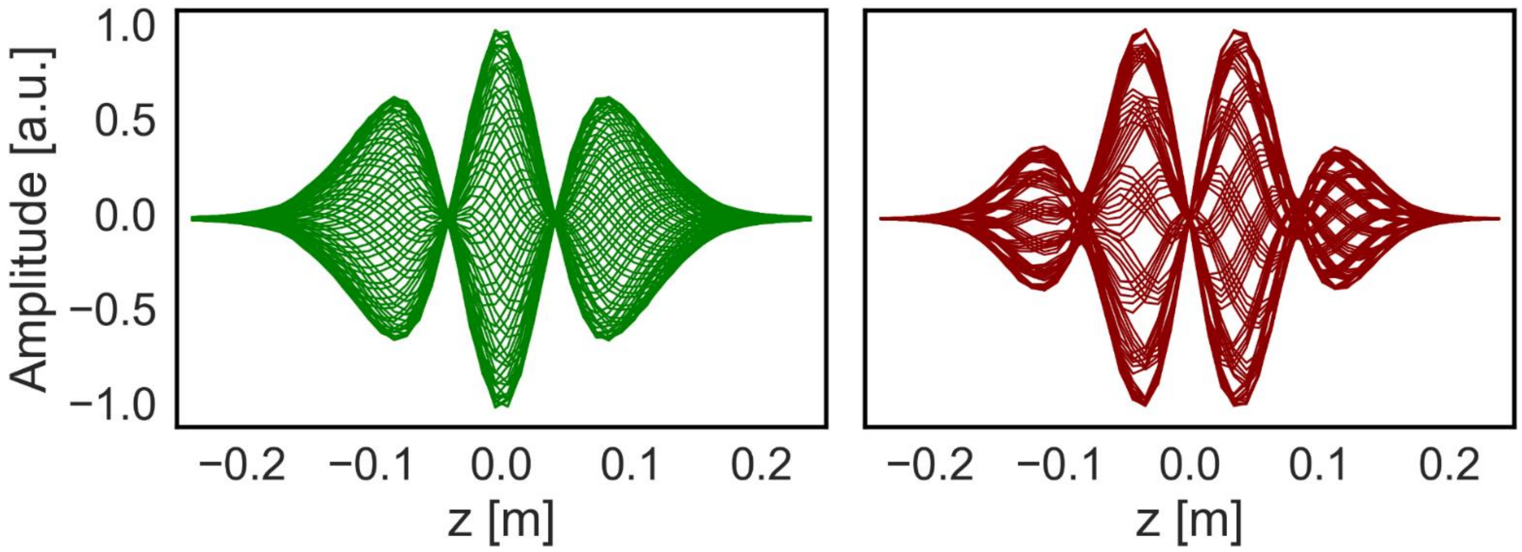
- The **impedance in the LHC** can give rise to coupled and single bunch instabilities which, when left untreated, can lead to **beam degradation and beam loss**.
- As an example, **headtail instabilities** are predicted from **macroparticle simulations** using the LHC impedance model.
- These simulations help to understand and to predict unstable modes which are observed in the real machine.



Headtail instabilities in the LHC

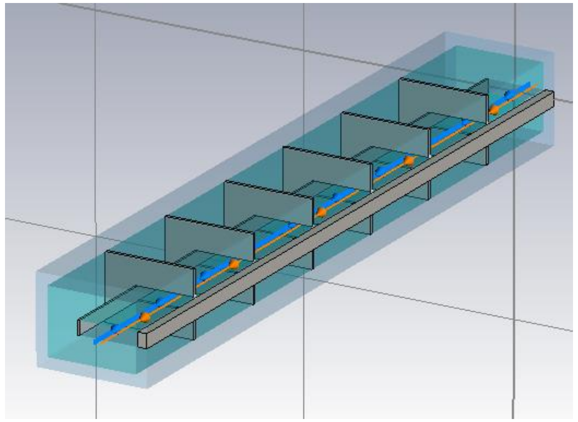
m = 0 m = -1

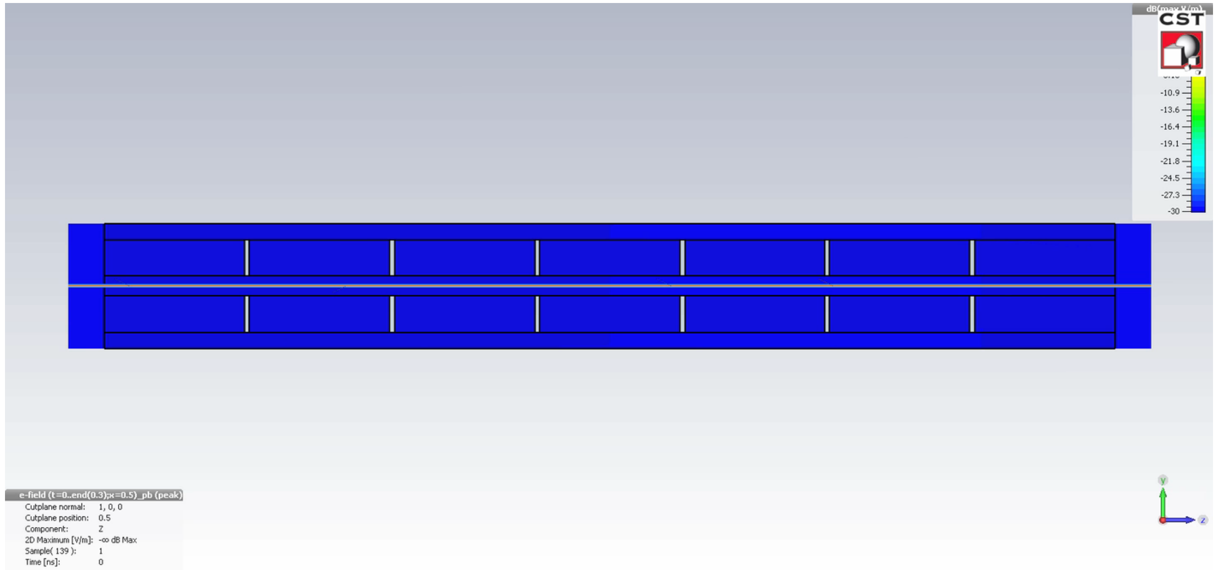
Macroparticle simulations (PyHEADTAIL)



- These simulations help to understand and to **predict instabilities** which are **observed in the real machine**.

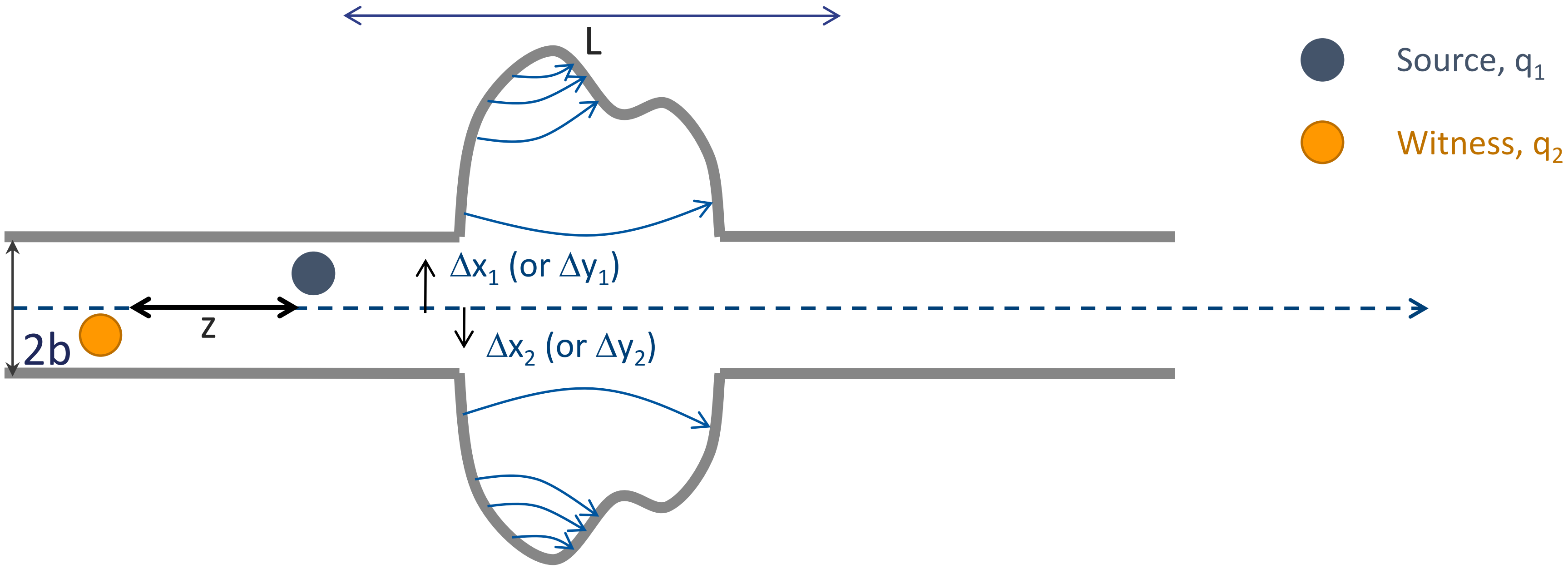
Wakefields as sources of collective effects





- The **wake function** is the **electromagnetic response** of an object to a charge pulse. It is an intrinsic property of any such object.
- The wake function **couples two charge distributions** as a function of the distance between them.
- The response depends on the boundary conditions and can occur e.g. due to **finite conductivity** (resistive wall) or more or less sudden **changes in the geometry** (e.g. resonator) of a structure.

Wake functions in general



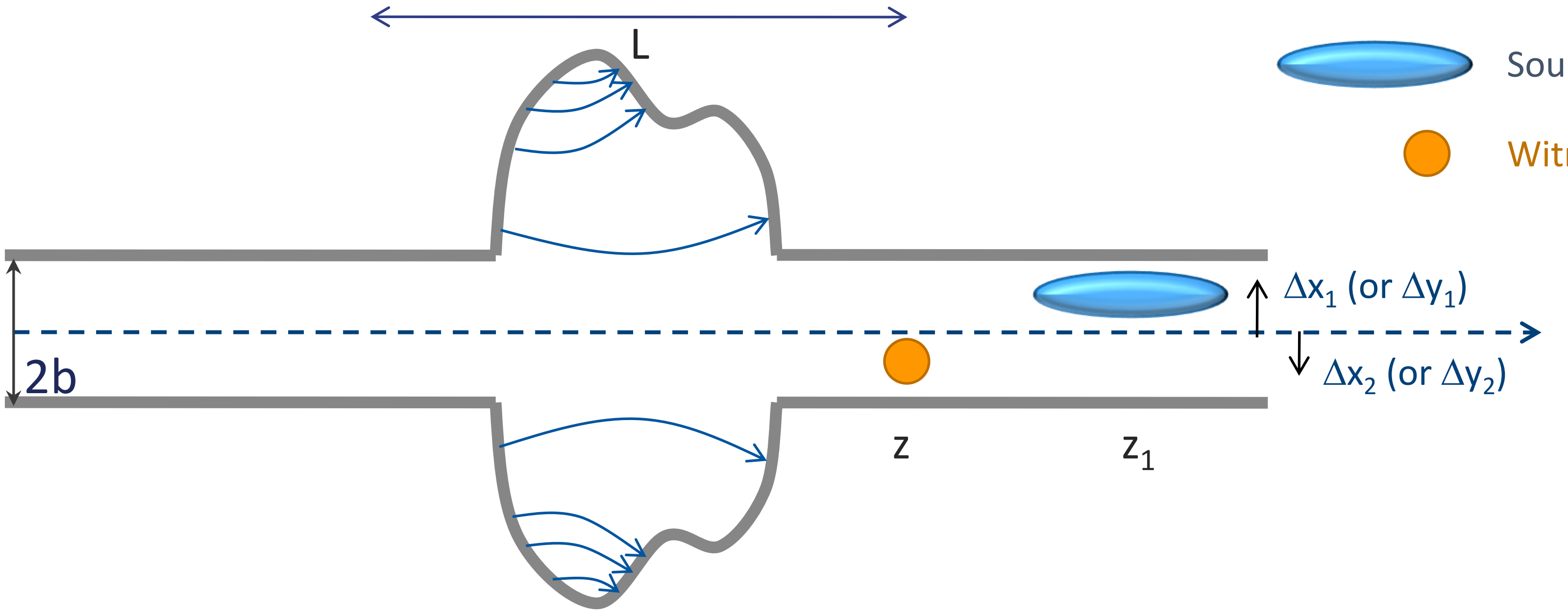
Definition as the **integrated force** associated to a change in energy:

- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

w is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

Wake potential for a distribution of particles



Definition as the **integrated force** associated to a change in energy:

- For an extended particle distribution this becomes

$$\Delta E_2(z) \propto \int \boxed{\lambda_1(x_1, z_1)}\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

Forces become dependent on the **particle distribution function**

Wake fields – impact on the equations of motion

$$\Delta E_2(z) \propto \int \lambda_1(x_1, z_1)\, \boldsymbol{w(x_1, x_2, z - z_1)}\, dx_1 dz_1$$

- We include the impact of wake field into the standard Hamiltonian for linear betatron (or synchrotron motion):

$$H = \frac{1}{2} x' + \frac{1}{2}\left(\frac{Q_x}{R}\right)^2 x^2 + \frac{e^2}{\beta^2 EC}\int \lambda_1(x_1, z_1)\, w(x_1, x, z - z_1)\, dx_1 dz_1 dx$$

- The equations of motion become:

$$x'' + \left(\frac{Q_x}{R}\right)^2 x + \frac{e^2}{\beta^2 EC}\int \boxed{\lambda_1(x_1, z_1)\, w(x_1, x, z - z_1)}\, dx_1 dz_1 = 0$$

The presence of wake fields adds an **additional excitation** which depends on

1. The **moments of the beam distribution**
2. The **shape and the order** of the wake function

How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage (e.g. resistive wall for axisymmetric chambers)
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances

- **Numerical approach**
  - Different codes have been developed over the years to solve numerically Maxwell's equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the ICFA mini-Workshop on "Electromagnetic wake fields and impedances in particle accelerators", Erice, Sicily, 23-28 April, 2014

- **Bench measurements** based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances, usefulness mainly lies in that they can be used for validating 3D EM models for simulations