CERN

CAS Advanced Accelerator Physics

Collective effects

Part 2: Longitudinal wake fields – impact on machine elements and beam dynamics

Kevin Li and Giovanni Rumolo

Outline

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Last lecture: Introduction to **multi-particle effects**, concept of **particle distributions, peculiarities of multiparticle dynamics** in accelerators, decoherence, filamentation.

This lecture:

- ➢ Basic **concept of wake fields** and how these can be characterized as a **collective effect** in that they depend on the particle distribution.
- ➢ Multiparticle systems and wakefields and **impact of these** in the longitudinal and transverse planes.

- **Part 2: Multiparticle dynamics with wake fields – impact on machine elements and longitudinal beam dynamics**
  - General introduction to wake fields
  - Longitudinal wake fields and the longitudinal wake function
  - Energy loss – beam induced heating and stable phase shift
  - Potential well distortion, bunch lengthning and microwave instability

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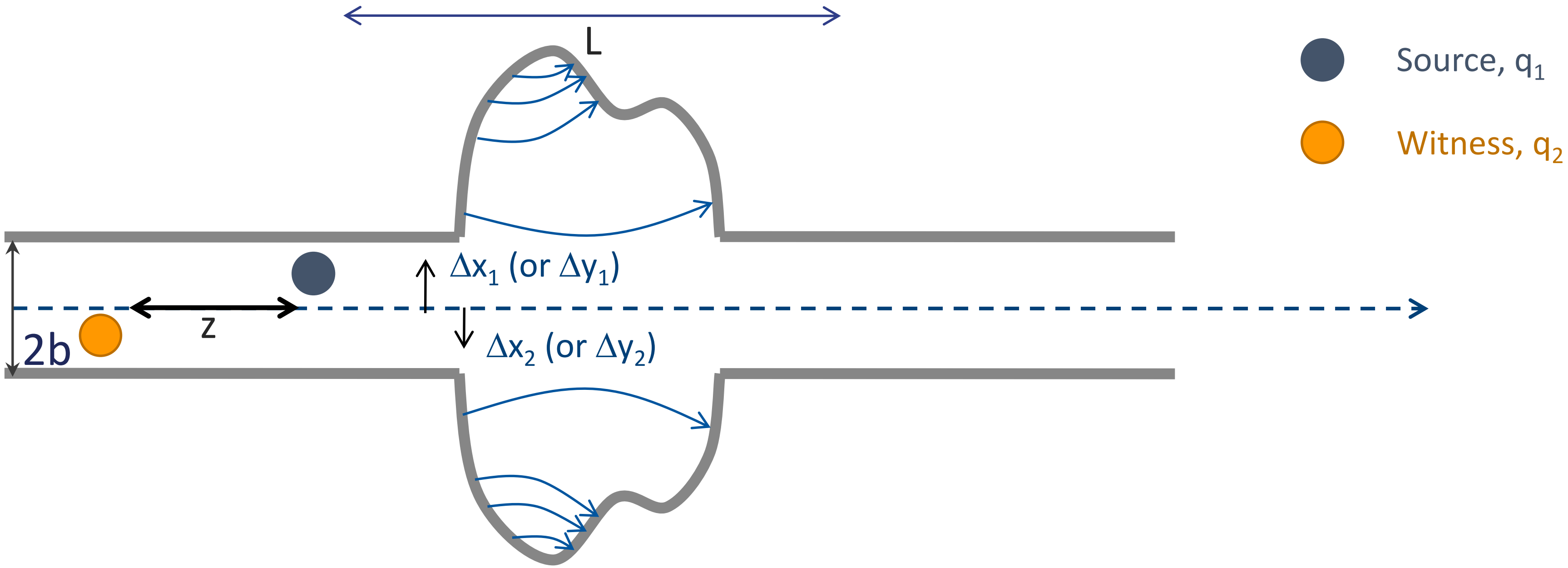
Wake functions in general



Definition as the **integrated force** felt by a witness charge following a source charge ('energy kick'):

- In general, for two point-like particles, we have

$$\Delta E_2 = \int F(x_1, x_2, z, s)\, ds = -q_1 q_2\, \boldsymbol{w(x_1, x_2, z)}$$

w is typically expanded in the transverse offsets of source and witness particles. This yields the different types of wake fields (dipole, quadrupole, coupling wakes)

Wakefields as sources of collective effects

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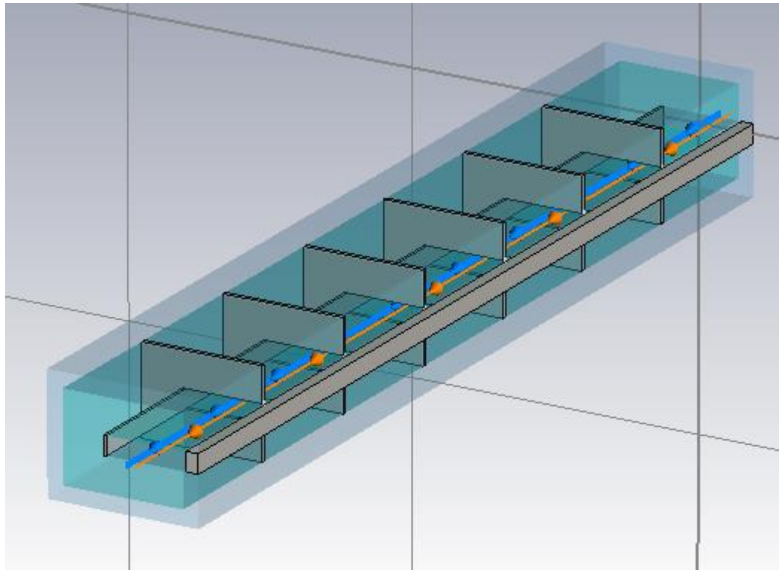
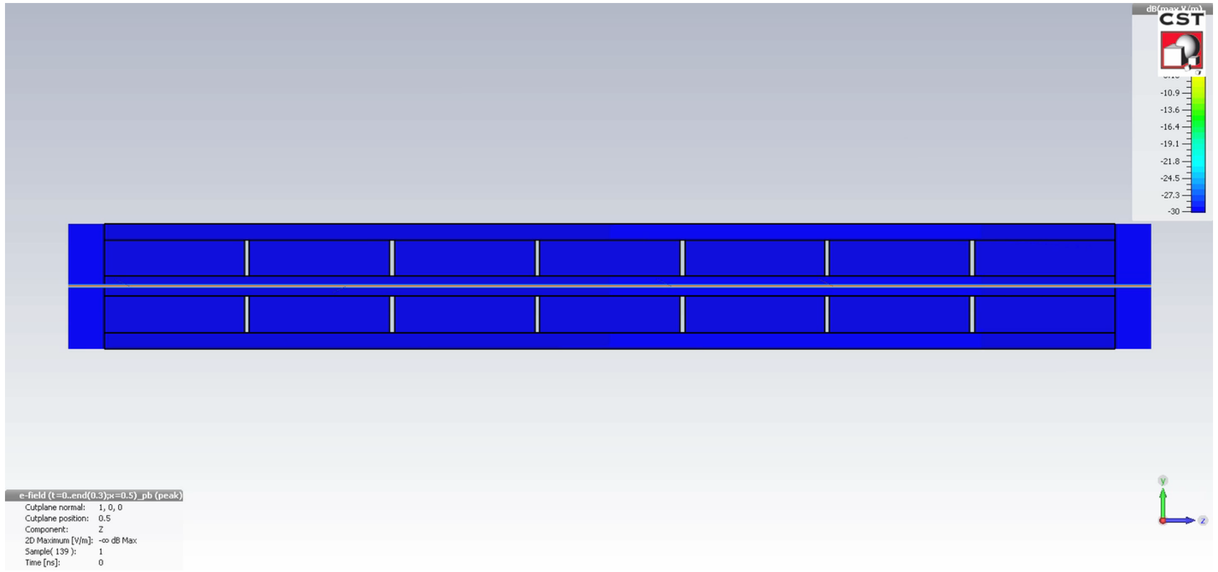
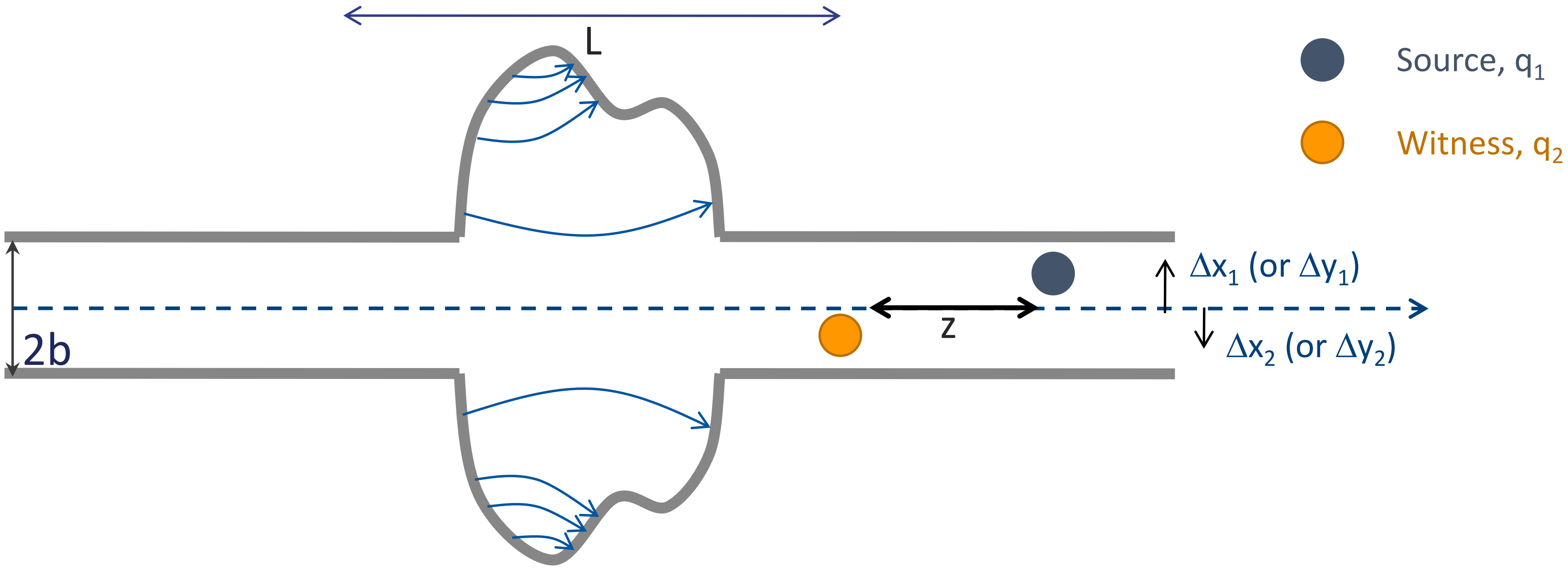




- The **wake function** is a type of **electromagnetic response** of a device to a charge pulse. It is an intrinsic property of this device and depends on
  - The device's **geometry** (transitions, cavities, etc.)
  - The **electromagnetic properties** of the materials exposed to the beam (e.g. PEC, finite conductivity, lossy materials, metamaterials, etc.)
- The wake function describes the **electromagnetic coupling between two point charges** as a function of the distance between them

Longitudinal wake function



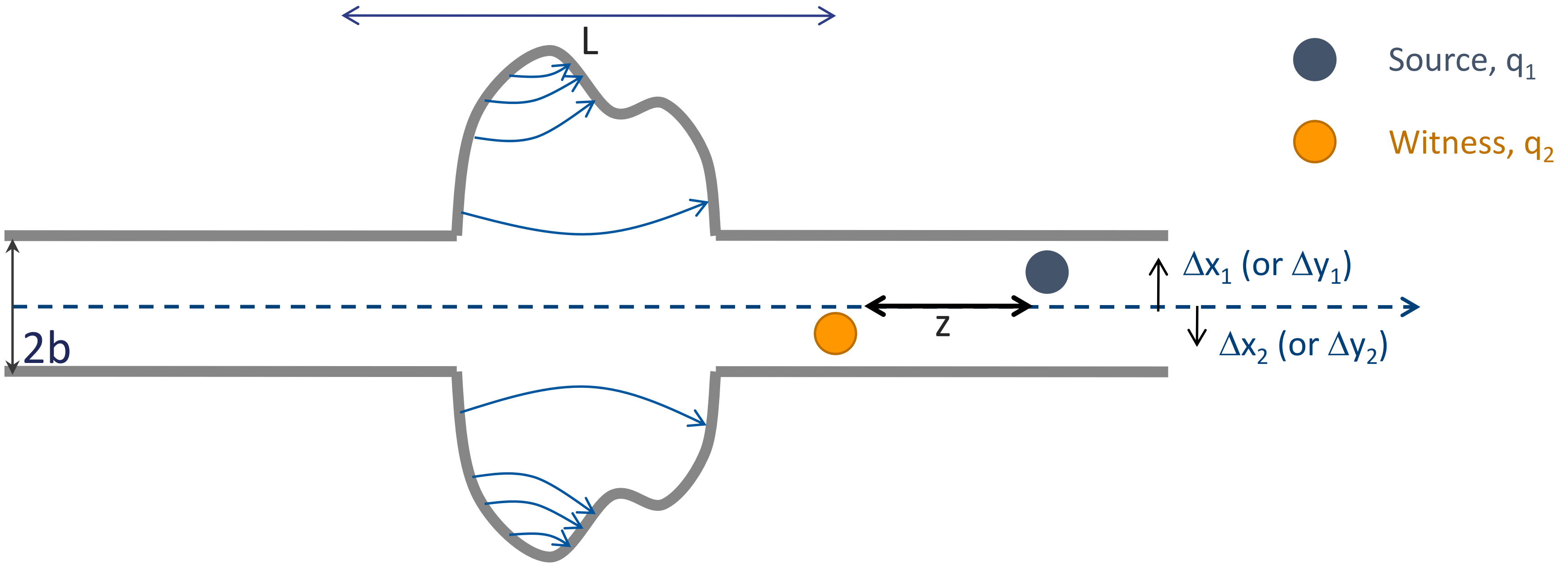
- Longitudinal wake fields

$$\int F_z(x_1, x_2, z, s)\, ds = -q_1 q_2 \left(\boxed{W_{\|}(z)} + \boxed{O(\Delta x_1) + O(\Delta x_2)} \right)$$

Zeroth order with source and test centred
usually dominant

Higher order terms
Usually negligible for small offsets

Longitudinal wake function



- Longitudinal wake fields

$$\Delta E_2 = \int F_z(z,s)\,ds = -q_1 q_2\, W_{||}(z)$$

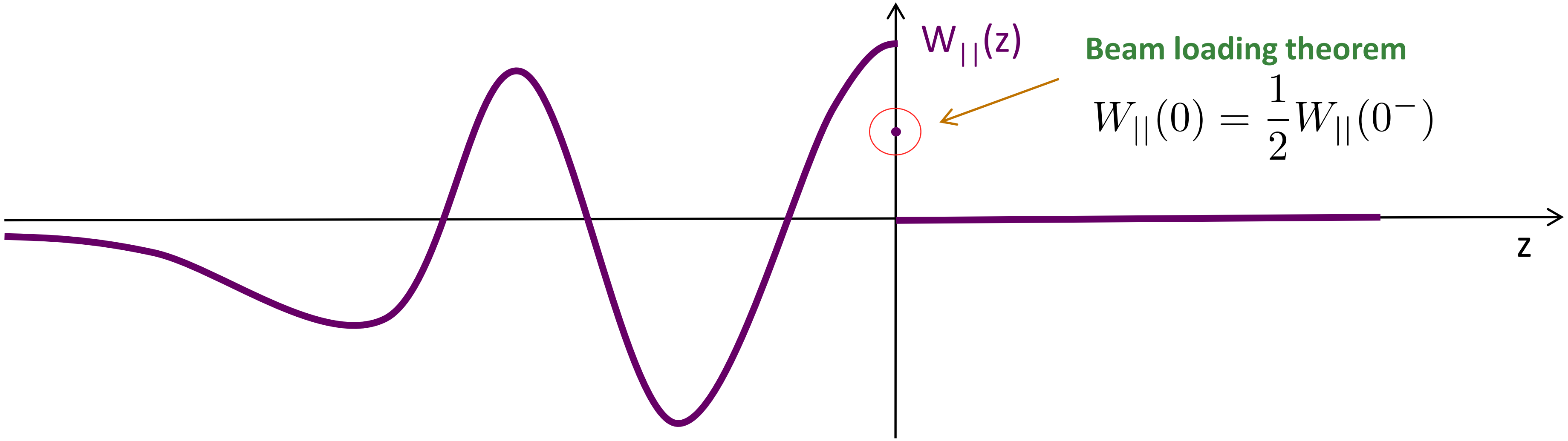
$$\longrightarrow \frac{\Delta E_2}{E_0} = \left(\frac{\gamma^2 - 1}{\gamma}\right)\frac{\Delta p_2}{p_0}$$

Energy kick of the witness particle from longitudinal wakes

Longitudinal wake function

$$W_{||}(z) = -\frac{\Delta E_2}{q_1 q_2} \xrightarrow[q_2 \to q_1]{z \to 0} W_{||}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The value of the wake function in z=0 is related to the **energy lost by the source particle** in the creation of the wake
- ***$W_{||}(0)>0$*** since ***$\Delta E_1<0$***
- ***$W_{||}(z)$*** is discontinuous in z=0 and it vanishes for all z>0 because of the ultra-relativistic approximation



Beam loading theorem

$$W_{||}(0) = \frac{1}{2} W_{||}(0^-)$$

Longitudinal impedance

$$W_{\|}(z) = -\frac{\Delta E_2}{q_1 q_2} \quad \xrightarrow[q_2 \to q_1]{z \to 0} \quad W_{\|}(0) = -\frac{\Delta E_1}{q_1^2}$$

- The **wake function** of an accelerator component is basically its **Green function in time domain** (i.e., its response to a pulse excitation)
  - → Very useful for macroparticle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion!

- We can also describe it as a **transfer function in frequency domain**
  - → This is the definition of **longitudinal beam coupling impedance** of the element under study

$$\boxed{Z_{\|}(\omega)} = \int_{-\infty}^{\infty} \boxed{W_{\|}(z)} \exp\left(-\frac{i\omega z}{c}\right) \frac{dz}{c}$$

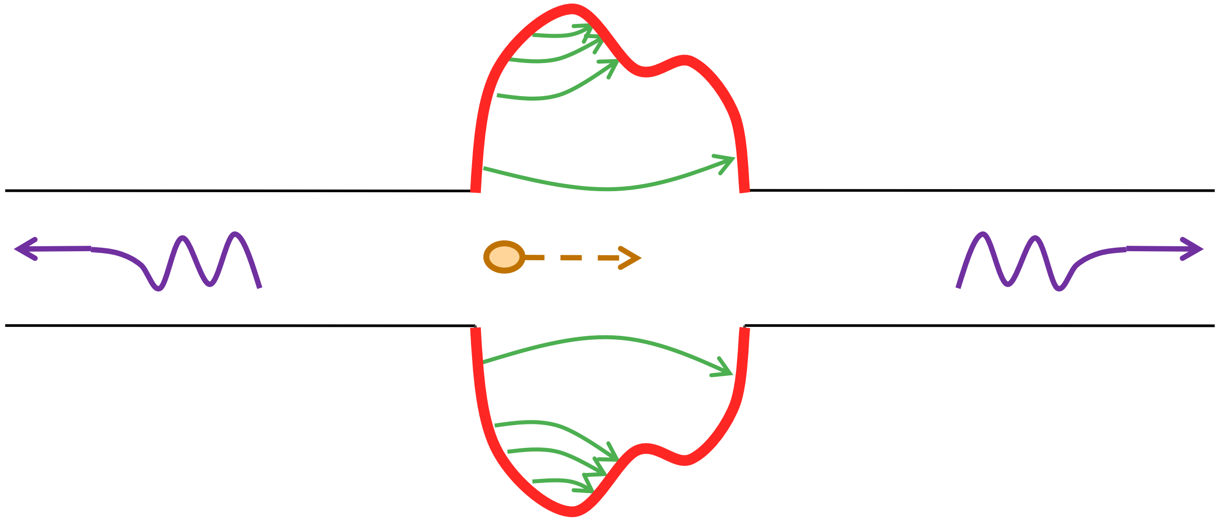
$[\Omega]$ $\qquad\qquad$ $[\Omega/s]$

The energy balance

$$W_{||}(0) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left(Z_{||}(\omega)\right)\, d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

- In the global energy balance, the energy lost by the source splits into:
  - Electromagnetic energy of the **modes that remain trapped** in the object
    - → Partly dissipated on **lossy walls** or into purposely designed inserts or HOM absorbers
    - → Partly transferred to **following particles** (or the same particle over successive turns), possibly feeding into an instability!
  - Electromagnetic energy of **modes that propagate** down the beam chamber (above cut-off), eventually lost on surrounding lossy materials



The energy balance

$$W_{\parallel}(0) = \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left(Z_{\parallel}(\omega)\right)\, d\omega = -\frac{\Delta E_1}{q_1^2}$$

What happens to the energy lost by the source?

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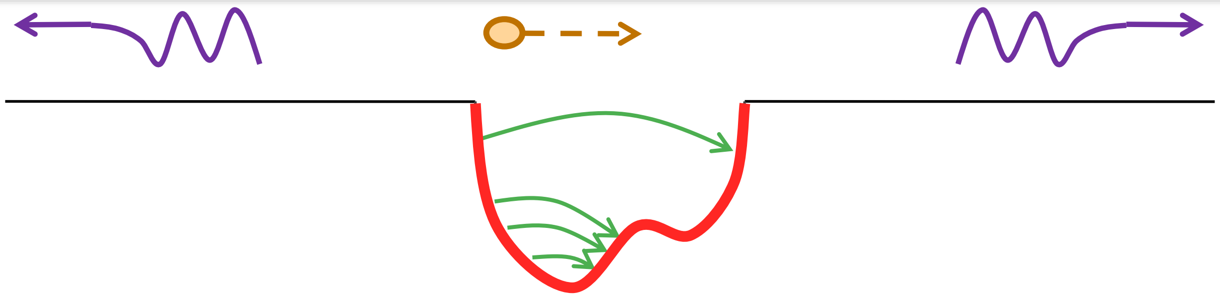
    - → Partly dissipated
    - → Partly transferre ... eding into an instability!
  - Electromagnetic e ... ut-off), eventually lost on surrounding lossy

The energy loss of a particle bunch

⇒ causes **beam induced heating** of the machine elements (damage, outgassing) or **sparking** due to high field

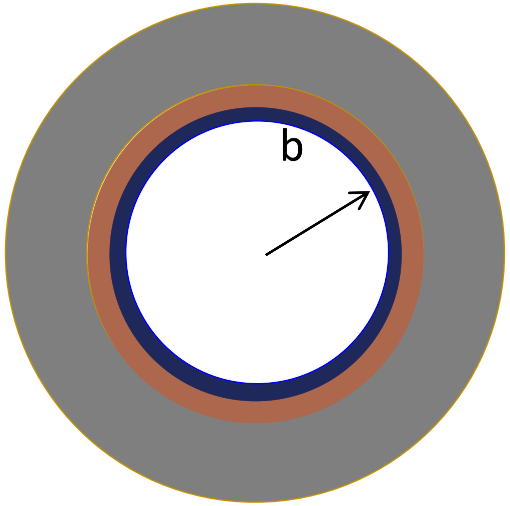
⇒ feeds into both **longitudinal and transverse instabilities** through the associated EM fields

⇒ is compensated by the RF system determining a **stable phase shift**



How are wakes and impedances computed?

- **Analytical or semi-analytical** approach, when geometry is simple (or simplified)
  - Solve Maxwell's equations with the correct source terms, geometries and boundary conditions up to an advanced stage
  - Find closed expressions or execute the last steps numerically to derive wakes and impedances



→ An example: **axisymmetric beam chamber** with several layers with different EM properties

$$\nabla \times \vec{E} = -i\omega \vec{B} \qquad \nabla \cdot \vec{E} = \frac{\tilde{\rho}}{\epsilon_0 \epsilon_1(\omega)}$$

$$\nabla \times \vec{B} = \mu_0 \mu_1(\omega) \vec{J} + i\omega \frac{\mu_1(\omega)\epsilon_1(\omega)}{c^2} \vec{E}$$

$$\nabla \cdot \vec{B} = 0$$

+ Boundary conditions

$$\tilde{\rho}(r, \theta, s, \omega) = \frac{q_1}{r_1 v} \delta(r - r_1) \delta_P(\theta) \exp\left(-\frac{i\omega s}{v}\right)$$

$$\vec{J}(r, \theta, s, \omega) = \tilde{\rho}(r, \theta, s, \omega) \vec{v}$$

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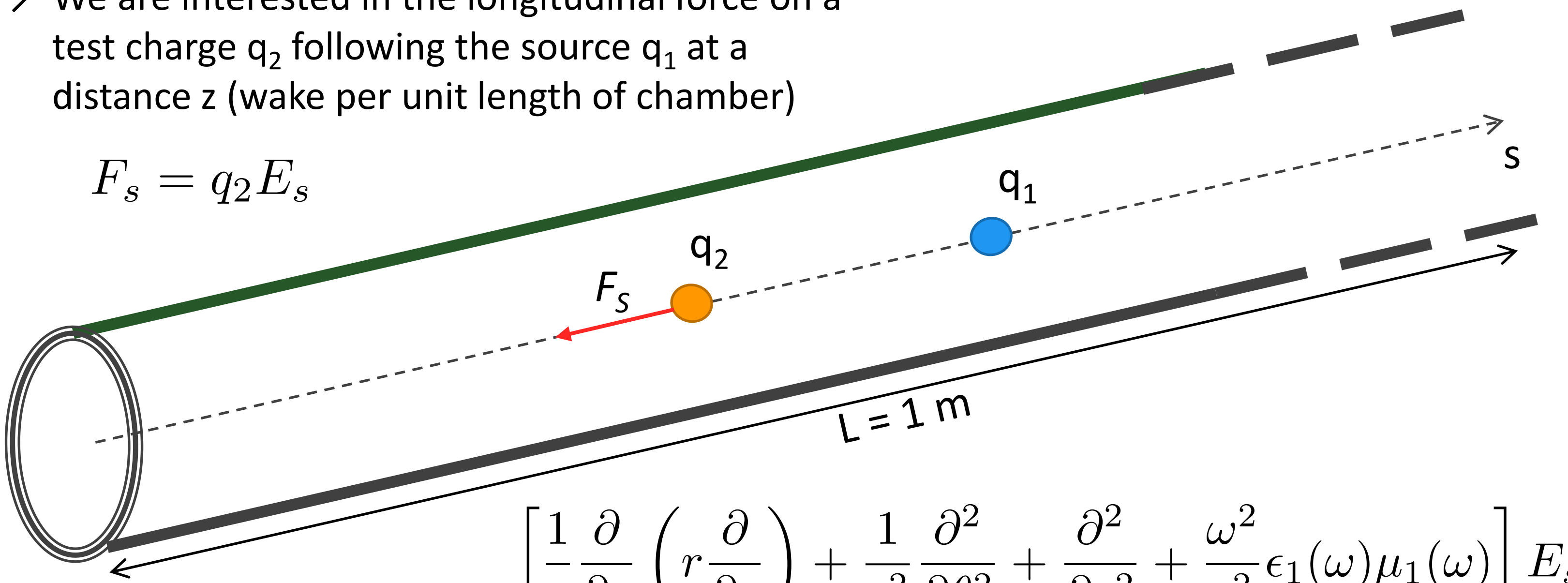
→ We are interested in the longitudinal force on a test charge q_2 following the source q_1 at a distance z (wake per unit length of chamber)

$$F_s = q_2 E_s$$



$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial s^2} + \frac{\omega^2}{c^2}\epsilon_1(\omega)\mu_1(\omega)\right] E_s =$$

$$= \frac{1}{\epsilon_0 \epsilon_1(\omega)}\frac{\partial \tilde{\rho}}{\partial s} + i\omega\mu_0\mu_1(\omega)\tilde{\rho}v$$

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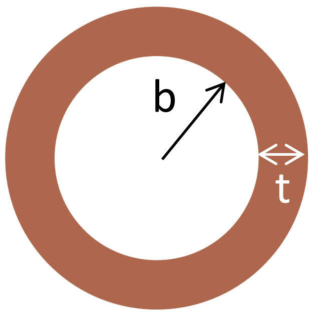
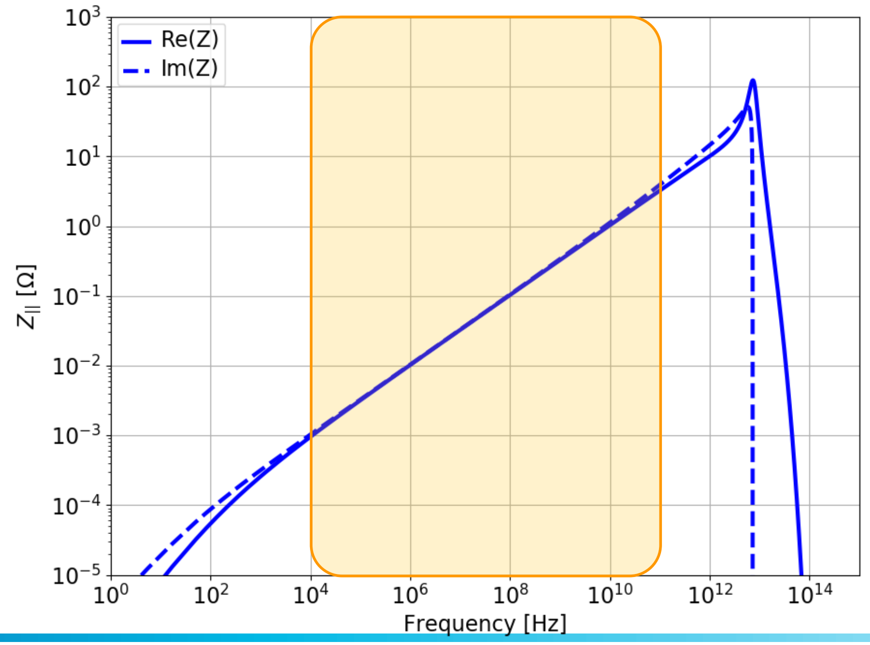
→ An example: a 1 m long Cu pipe with radius b=2 cm and thickness t = 4 mm in vacuum





- Highlighted region shows the typical $\omega^{1/2}$ scaling
- Scaling is with respect to b:
  - Longitudinal impedance $\sim b^{-1}$

How are wakes and impedances computed?

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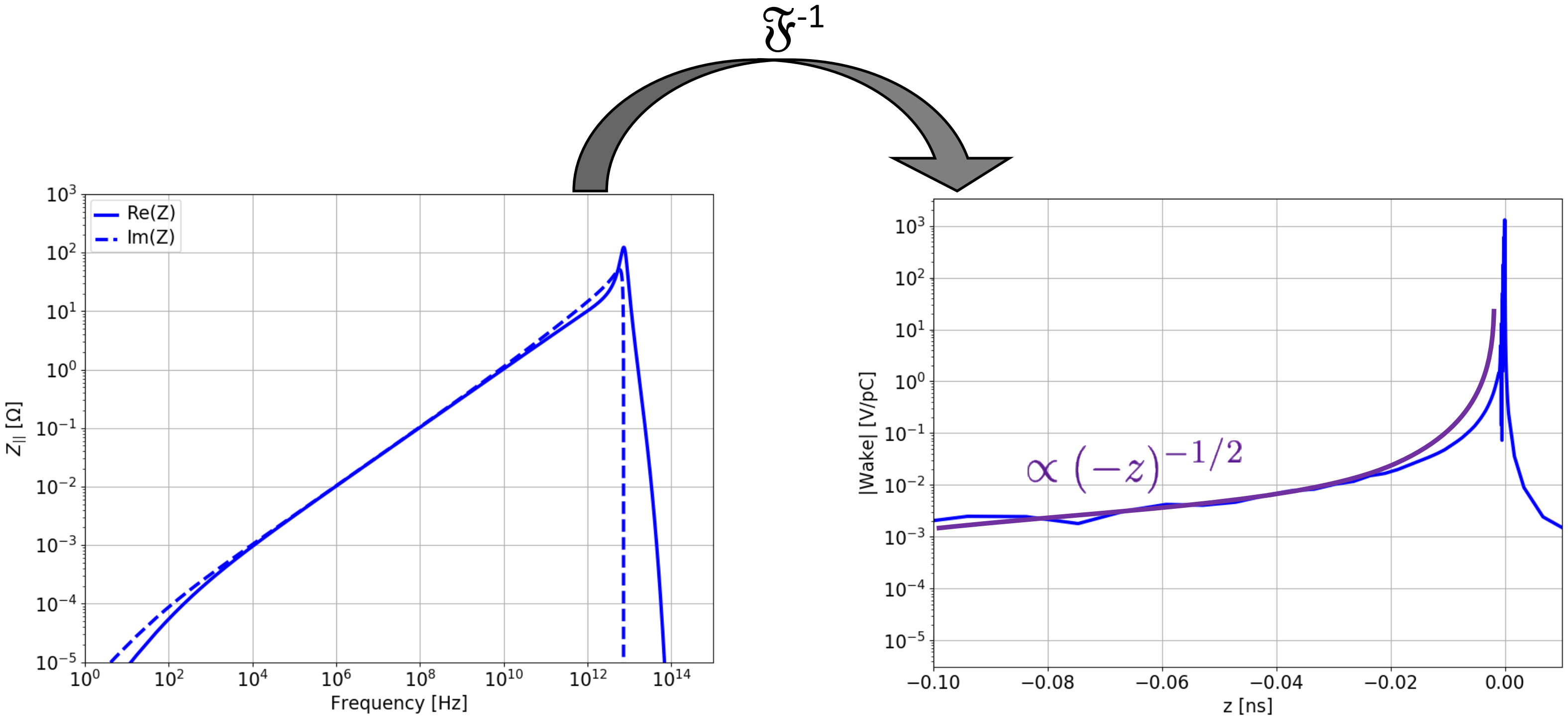

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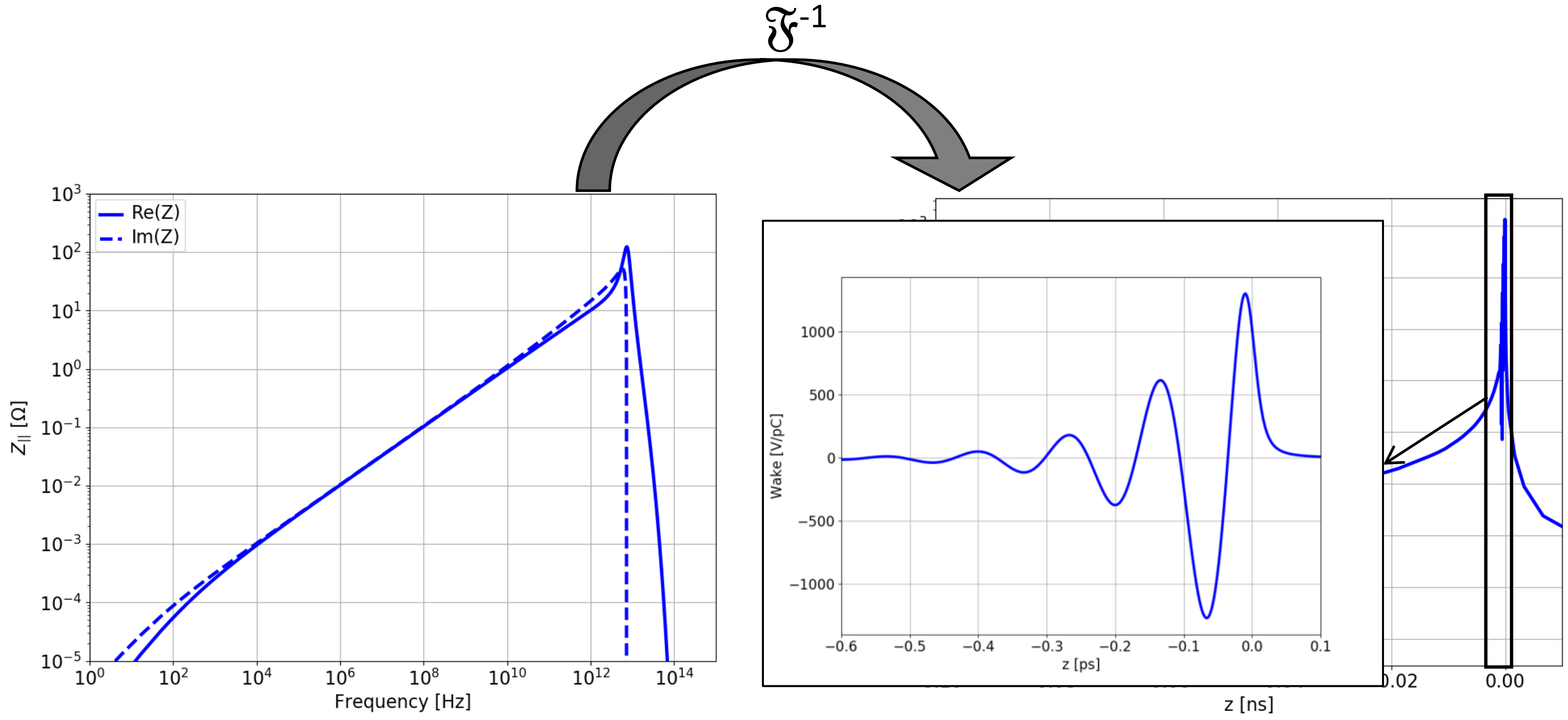
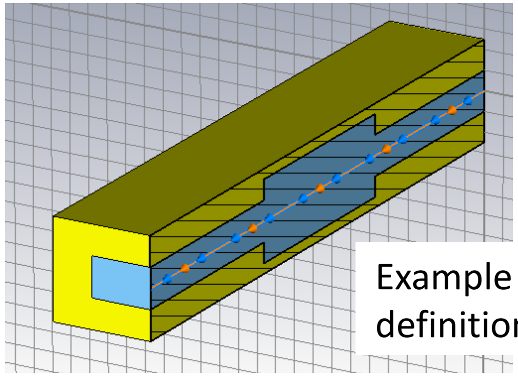
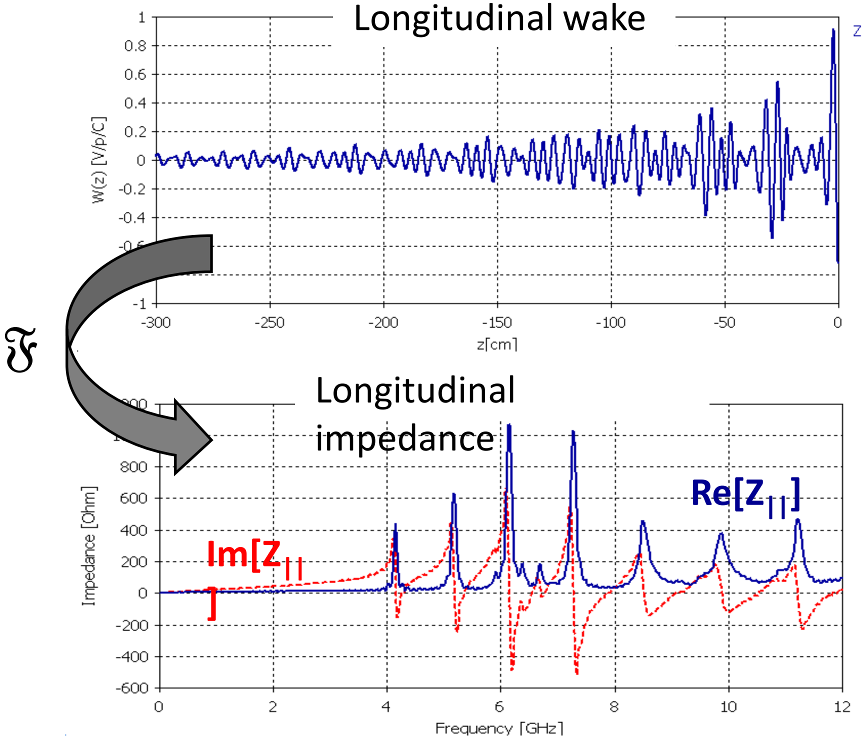


How are wakes and impedances computed?

- **Numerical approach**
  - Different codes have been developed over the years to solve numerically Maxwell’s equations in arbitrarily complicated structures
  - Examples are CST Studio Suite (Particle Studio, Microwave Studio), ABCI, GdFidL, HFSS, ECHO2(3)D. Exhaustive list can be found from the program of the [ICFA mini-Workshop on “Electromagnetic wake fields and impedances in particle accelerators”](), Erice, Sicily, 23-28 April, 2014
  - Computations can become very **challenging** if high frequency resolution (long wake) or knowledge of impedance spectrum at high frequency (short excitation) are required, especially for large/complicated geometries

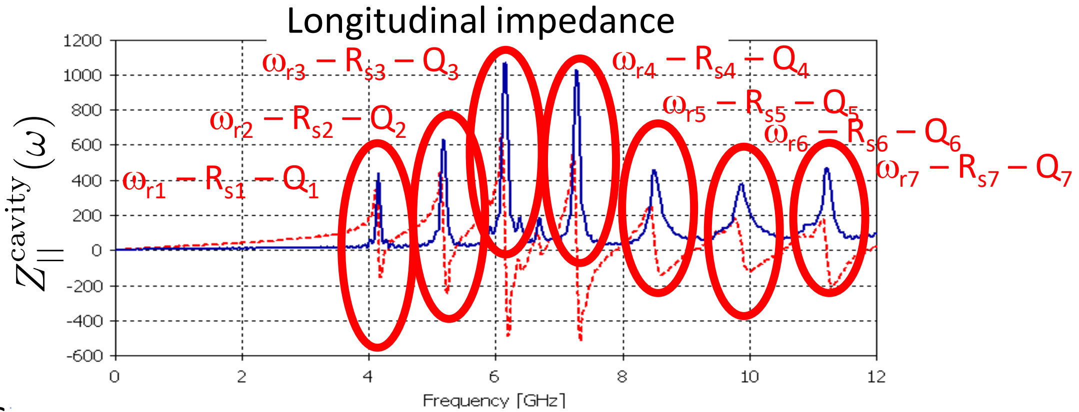


Example of a cavity definition in CST



How are wakes and impedances computed?

- **Numerical approach**
  - To limit numerical noise, in cases with many resonances, the resonances are first characterized through their frequencies (ω_{ri}), shunt impedances (R_{si}) and quality factors (Q_i)



  - Then analytical formulae for resonators are used in computations

$$Z_{||}^{\text{Res}}(\omega) = \frac{R_{s||}}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

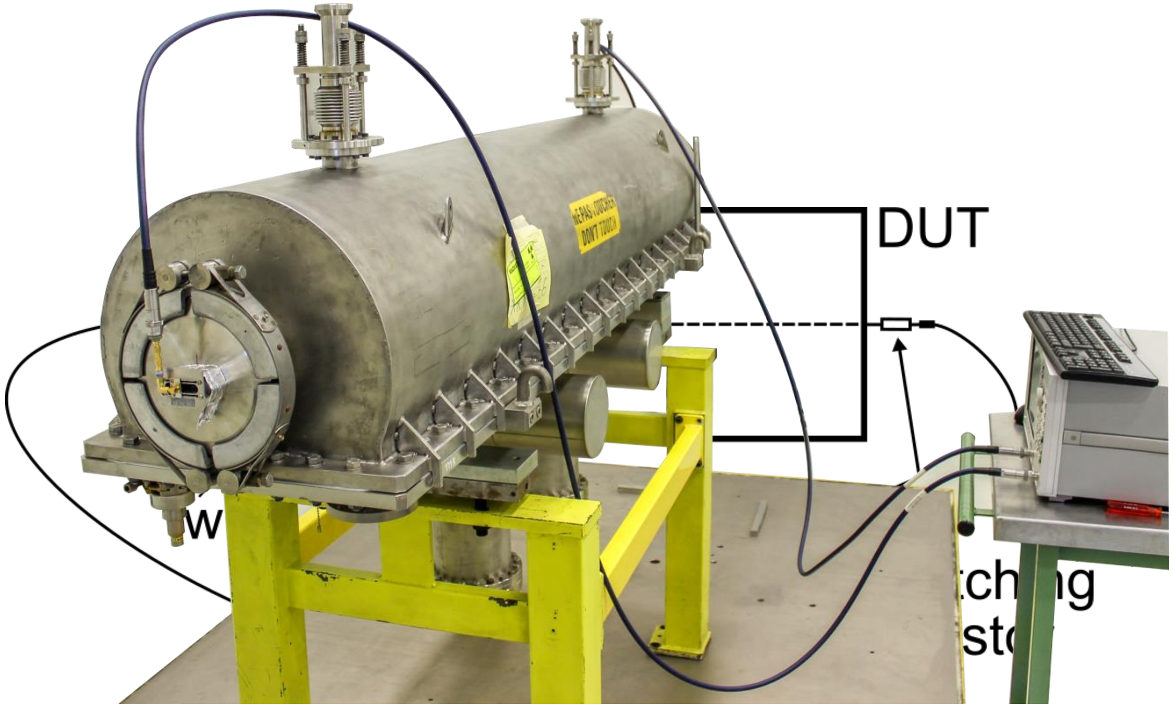
$$W_{||}^{\text{Res}}(z) = \begin{cases} 2\alpha_z R_{s||} \exp\left(\frac{\alpha_z z}{c}\right)\left[\cos\left(\frac{\bar{\omega} z}{c}\right) + \frac{\alpha_z}{\bar{\omega}} \sin\left(\frac{\bar{\omega} z}{c}\right)\right] & \text{if} \quad z < 0 \\ \alpha_z R_{s||} \quad \text{if} \quad z = 0 \\ 0 \quad \text{if} \quad z < 0 \end{cases}$$

$$\alpha_z = \frac{\omega_r}{2Q} \qquad \bar{\omega} = \sqrt{\omega_r^2 - \alpha_z^2}$$

$$Z_{||}^{\text{cavity}}(\omega) = \sum Z_{||i}^{\text{Res}}(\omega)$$

How are wakes and impedances computed?

- **Bench measurements** based on transmission/reflection measurements with stretched wires
  - Seldom used independently to assess impedances due to the perturbation introduced by the measurement set up (flanging, presence of wire)
  - Usefulness mainly lies in that they can be used for validating 3D EM models for simulations
  - New wireless methods being developed for direct impedance measurements minimizing perturbation



- A **wire** is stretched in the middle of the device to simulate the beam
- **Reflection and transmission coefficients** are measured via a VNA
- The impedance can be calculated by plugging the measured scattering parameters into the **LOG formula**

$$Z_{||} = 2Z_L \ln(S_{21})$$

Signpost

We have learnt what is a wake function and how it is defined in the **longitudinal plane**. We have introduced the **longitudinal impedance.**

We have seen how longitudinal wake functions are related to the **energy loss** of the source particles.

We have discussed the **energy balance** which contains all the **fundamental underlying mechanisms** for collective effects related to wake fields and impedances.

We have shown how wake functions and impedances **can be computed.**

- **Part 2: Multiparticle dynamics with wake fields – impact on machine elements and longitudinal beam dynamics**
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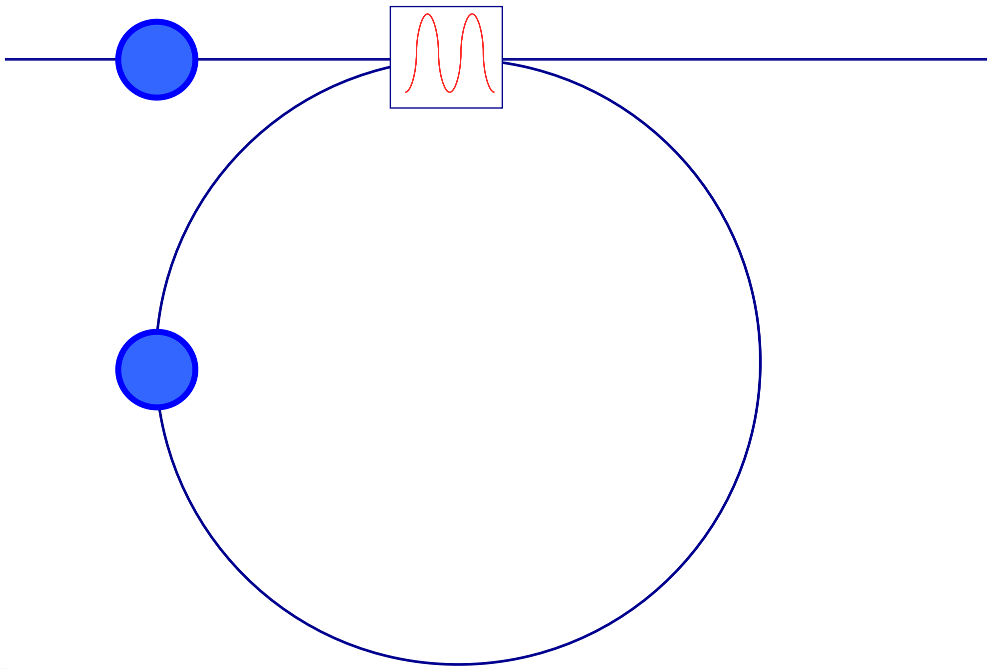
Bunch energy loss per turn

- Single traversal of a bunch through an impedance source
  - We assume a single bunch of particles that goes only once through a known (characterized) wake/impedance source, representing both
    - Single passage (e.g. in a line)
    - Energy loss per turn if the bunch passes every turn but the wake fully decays between subsequent turns
  - Our goal is to calculate how much energy the bunch loses in this passage due to the electromagnetic interaction



Bunch energy loss per turn

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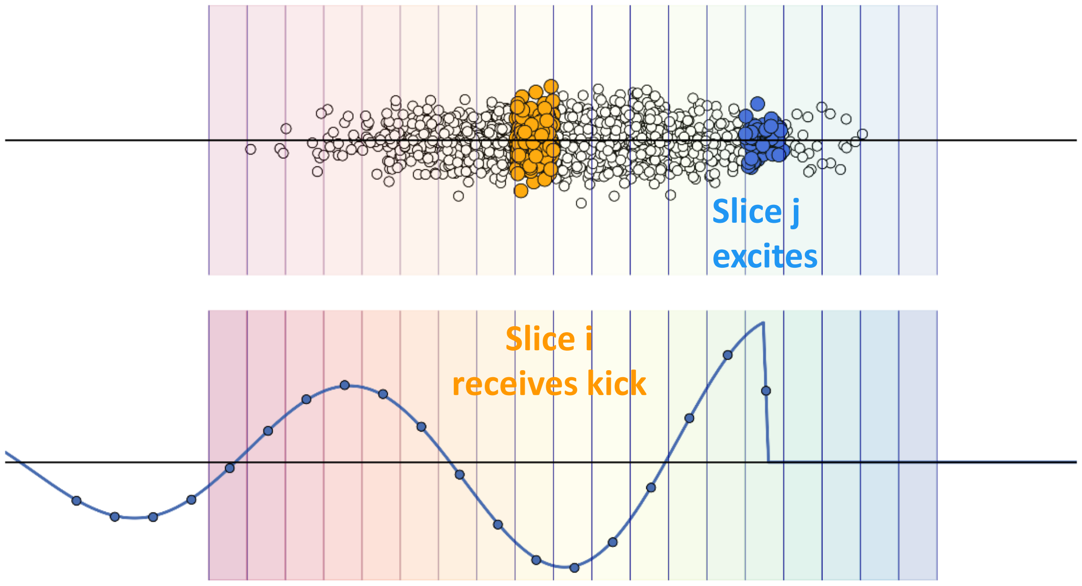

$$\Delta E_{ij} = -e^2 W_{||}(z_{ij})$$

$$\Delta E_{bunch} = -e^2 \sum_{j=1}^{N_b} \sum_{i=1}^{N_b} W_{||}(z_{ij})$$

$$\Delta E_{ij} = -e^2 N[j] N[i] W_{||}[(i-j)\Delta z]$$

$$\Delta E_i = -e^2 N[i] \sum_{j=0}^{i} N[j] W_{||}[(i-j)\Delta z]$$

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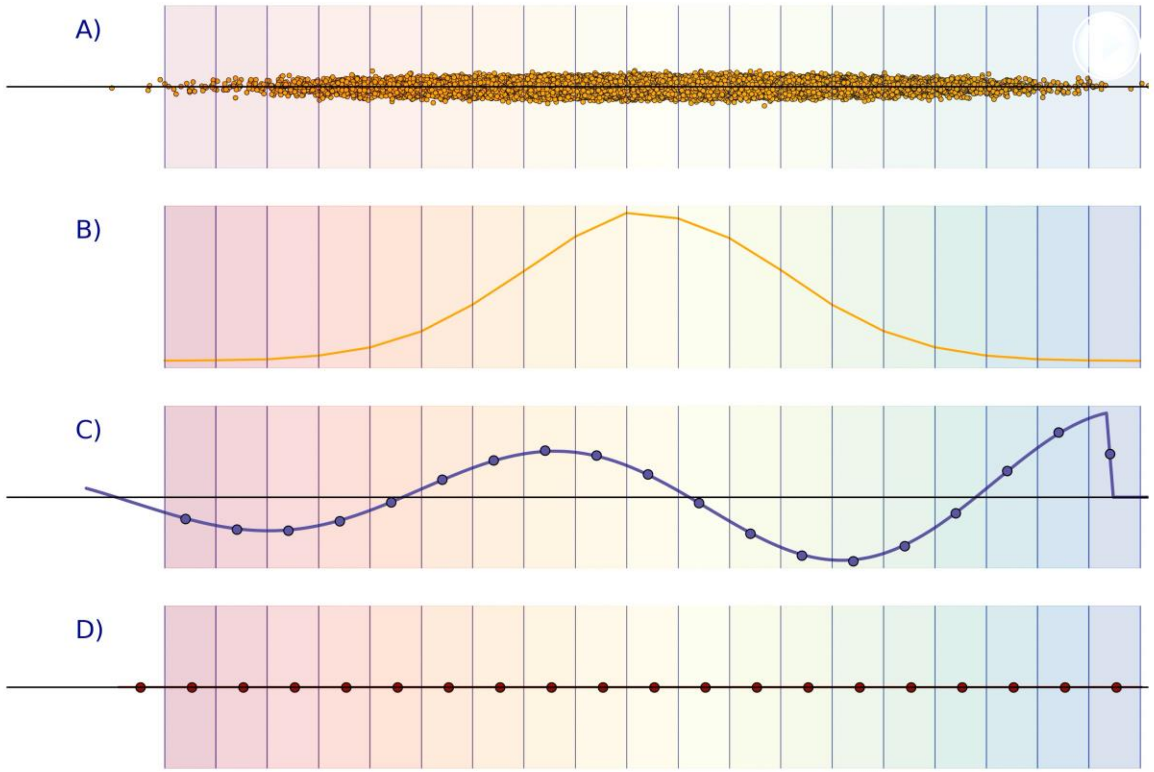

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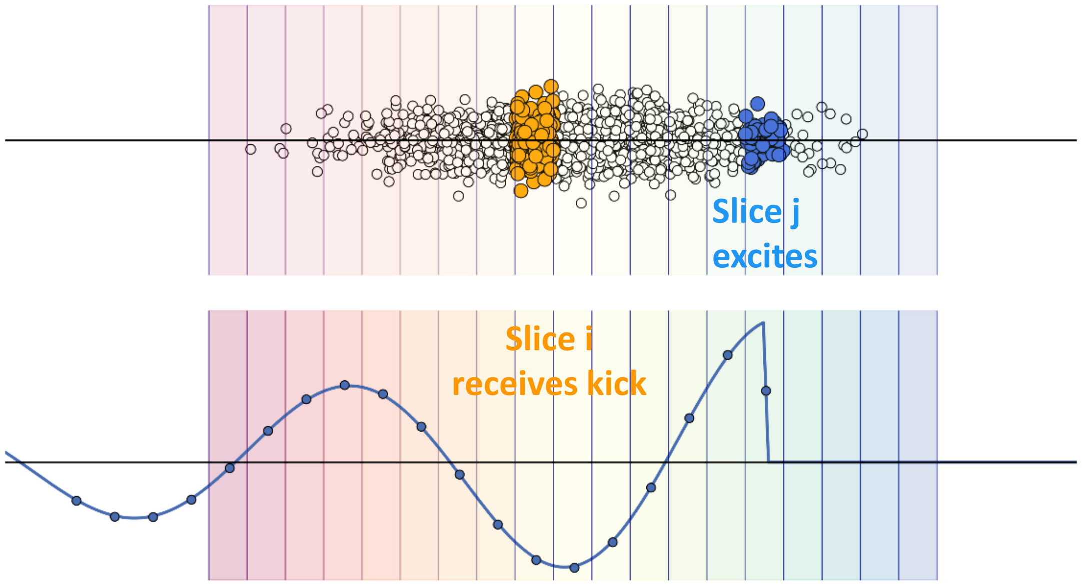

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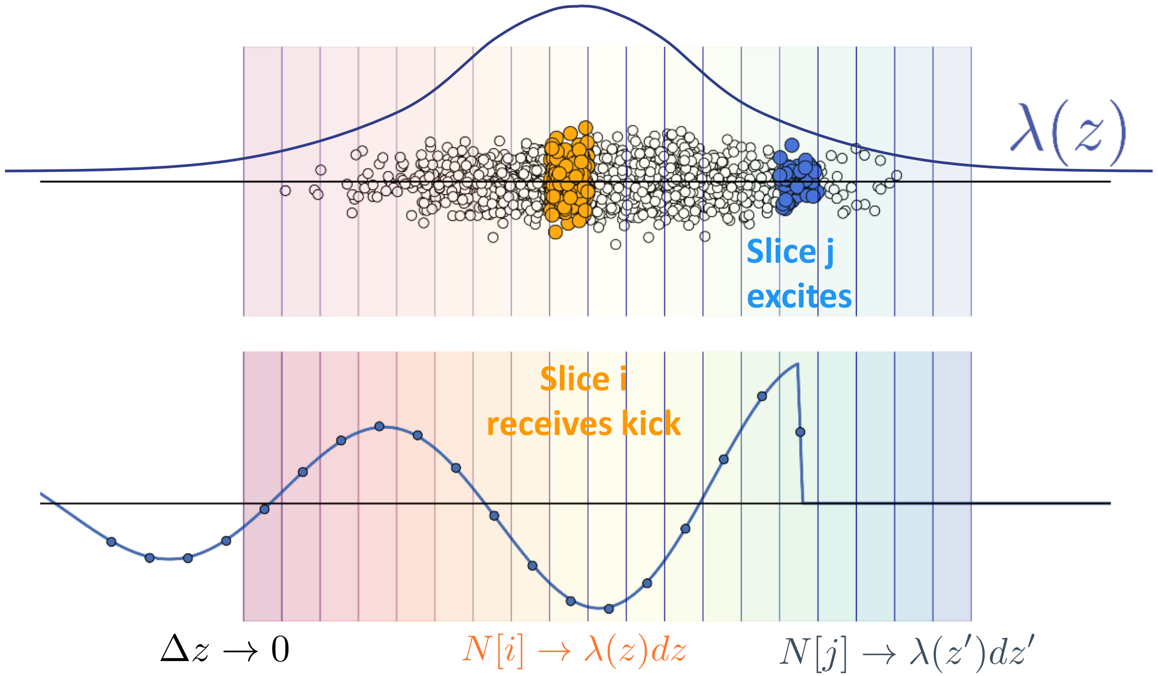

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$$\Delta E_{bunch} = -e^2 \int \lambda(z) dz \int \lambda(z') W_{||}(z-z') dz'$$

$$\boxed{\Delta E_{bunch} = -\frac{e^2}{2\pi} \int \left|\hat{\lambda}(\omega)\right|^2 \mathrm{Re}\left[Z_{||}(\omega)\right]}$$

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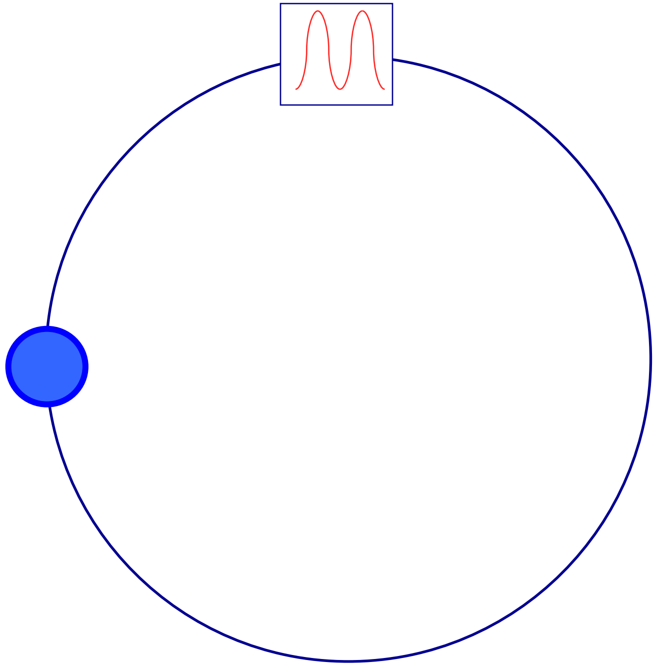
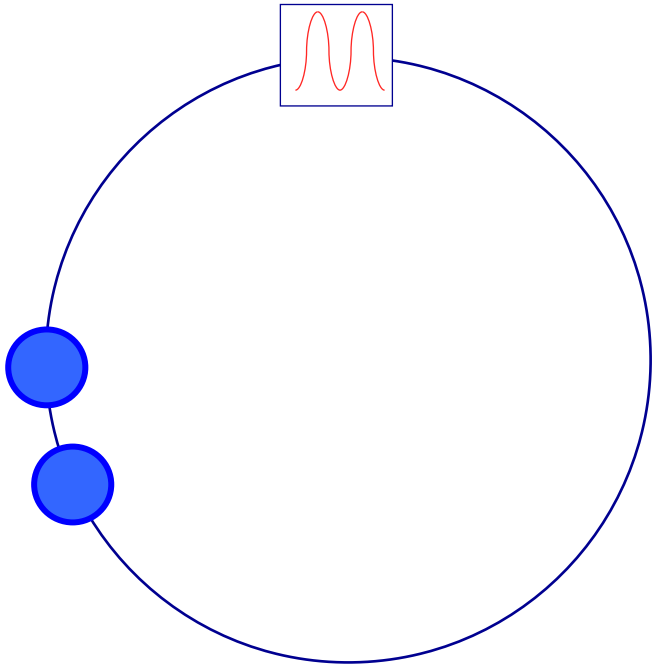


Beam energy loss per turn

- Not one bunch but a train of bunches …



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Bunch energy loss per turn

$$\Delta E = -\frac{e^2}{2\pi}\int_{-\infty}^{\infty}\lambda(z)dz\int_{-\infty}^{\infty} \underbrace{dz'\lambda(z')}\sum_{k=-\infty}^{\infty} W_{||}(kC+z-z')dz'$$

λ(z' + kC) = λ(z'), i.e. assuming that the distribution doesn't change from turn to turn

$$\sum_{k=-\infty}^{\infty} W_{||}(kC+z-z') = \frac{c}{C}\sum_{p=-\infty}^{\infty} Z_{||}(p\omega_0)\exp\left[-\frac{ip\omega_0(z-z')}{c}\right]$$

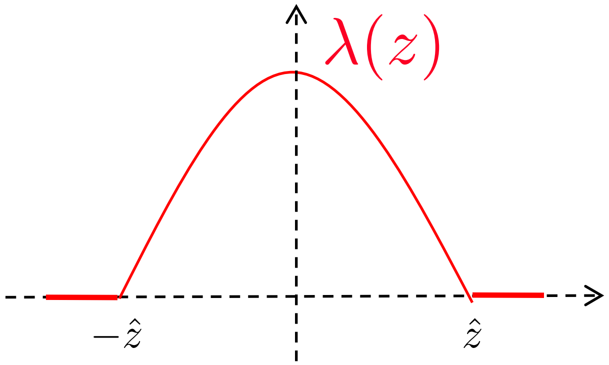
$$\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty} Z_{||}(p\omega_0)\underbrace{\int_{-\infty}^{\infty}\lambda(z)\exp\left(\frac{-ip\omega_0 z}{c}\right)dz}_{\hat{\lambda}(p\omega_0)}\underbrace{\int_{-\infty}^{\infty}\lambda(z')\exp\left(\frac{ip\omega_0 z'}{c}\right)dz'}_{\hat{\lambda}^*(p\omega_0)}$$

$$\boxed{\Delta E = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{\infty}\left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left[Z_{||}(p\omega_0)\right]}$$

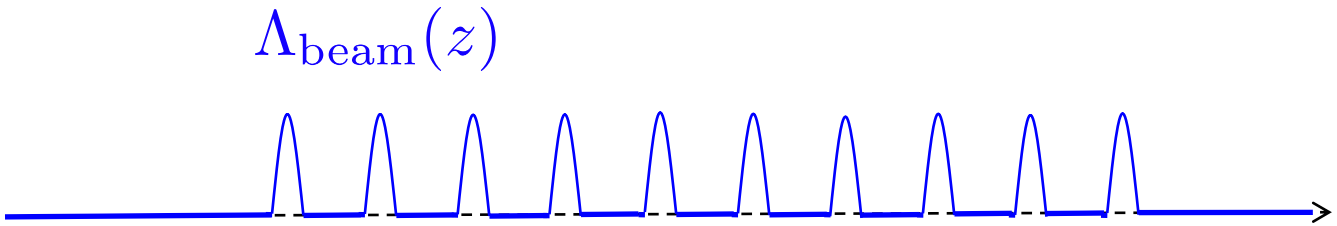
Beam energy loss per turn

Replacing the **bunch spectrum with the beam spectrum**, we can calculate the energy loss from a beam

Bunch profile and spectrum



Beam profile and spectrum



Beam energy loss per turn

Replacing the **bunch spectrum with the beam spectrum**, we can calculate the energy loss from a beam

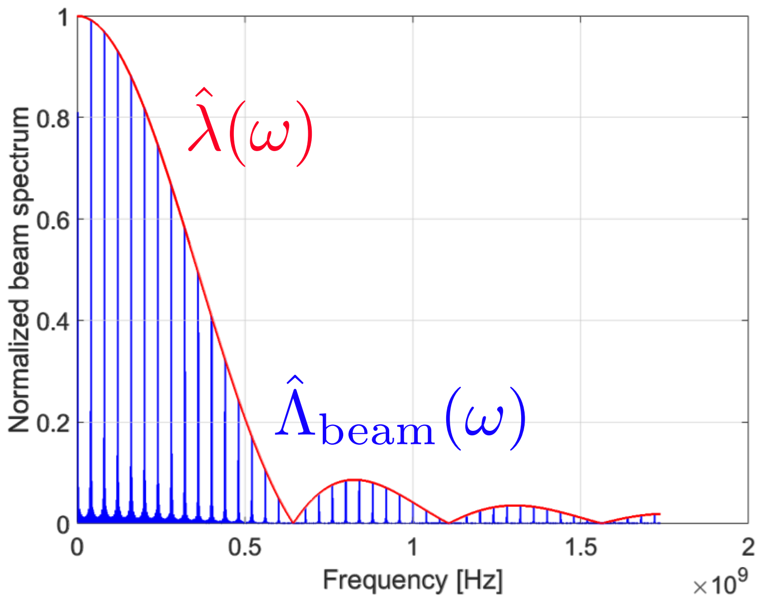
Bunch profile and spectrum

$$\lambda(z) \leftrightarrow \hat{\lambda}(\omega)$$

Beam profile and spectrum

$$\Lambda_{\text{beam}}(z)$$

Ex. parabolic, as shown in the previous slide



$$\Delta E_{\text{beam}} = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{+\infty}|\Lambda_{\text{beam}}(p\omega_0)|^2\,\mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

Beam energy loss per turn

Replacing the **bunch spectrum with the beam spectrum**, we can calculate the energy loss from a beam

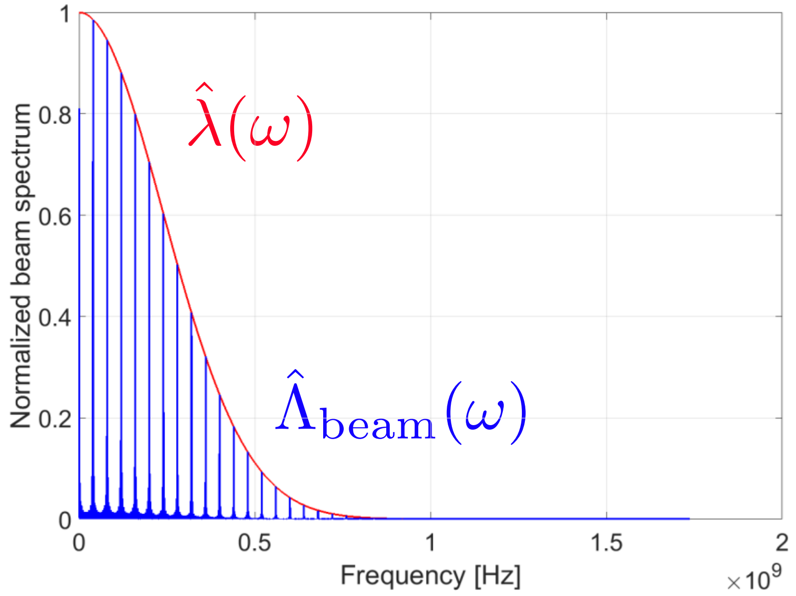
Bunch profile and spectrum

$$\lambda(z) \leftrightarrow \hat{\lambda}(\omega)$$

Beam profile and spectrum

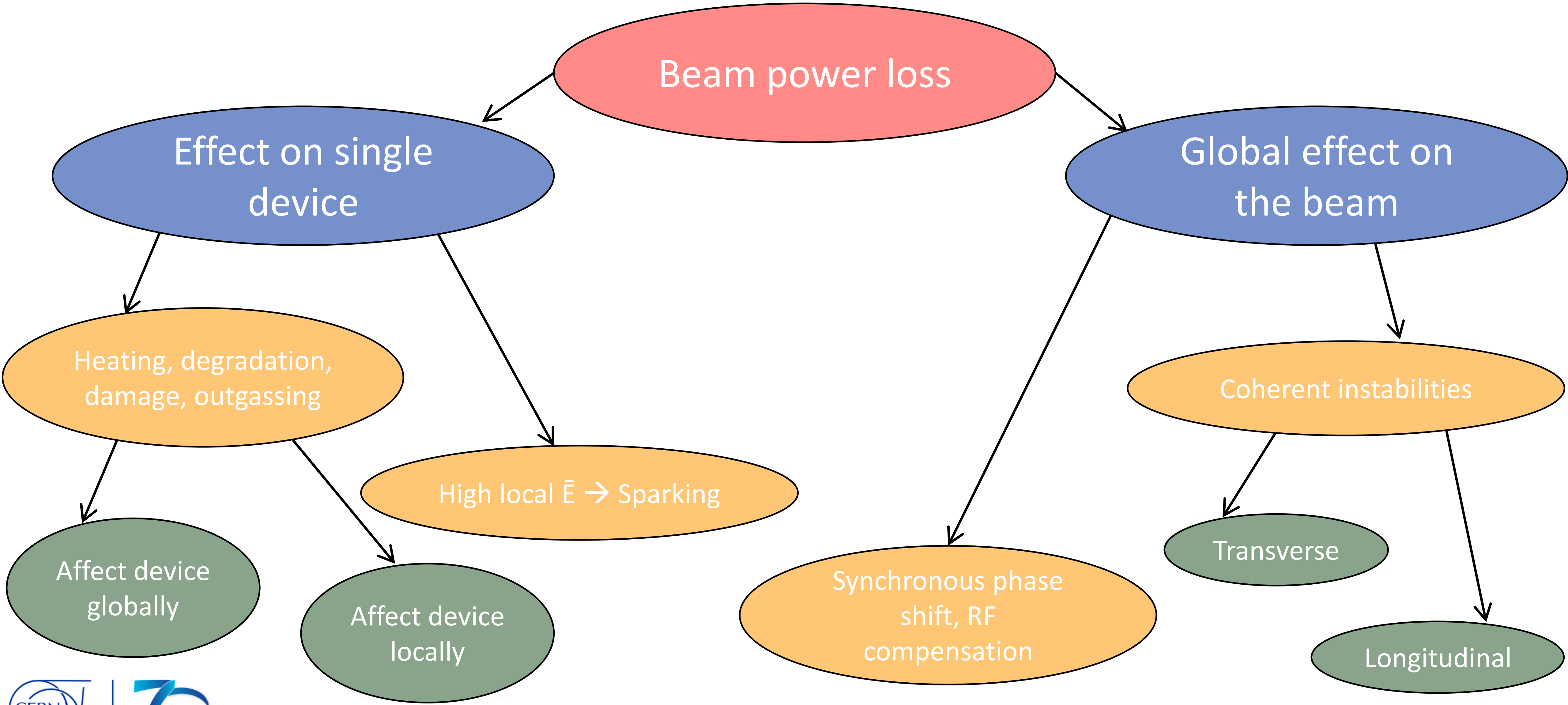
$$\Lambda_{\rm beam}(z) \leftrightarrow \hat{\Lambda}_{\rm beam}(\omega)$$

Or for a train of Gaussian bunches

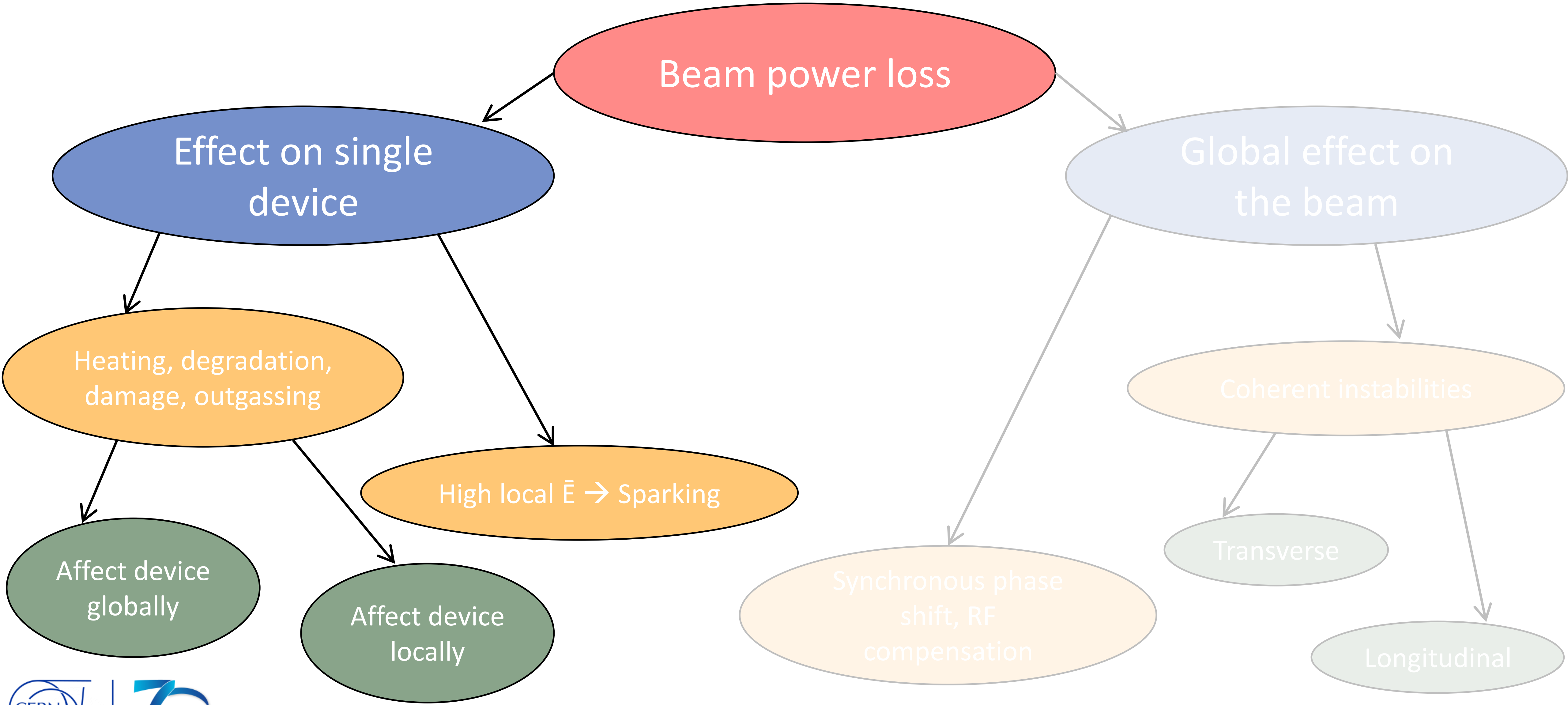


$$\Delta E_{\rm beam} = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{+\infty}|\Lambda_{\rm beam}(p\omega_0)|^2\,\mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

Impact of beam power loss

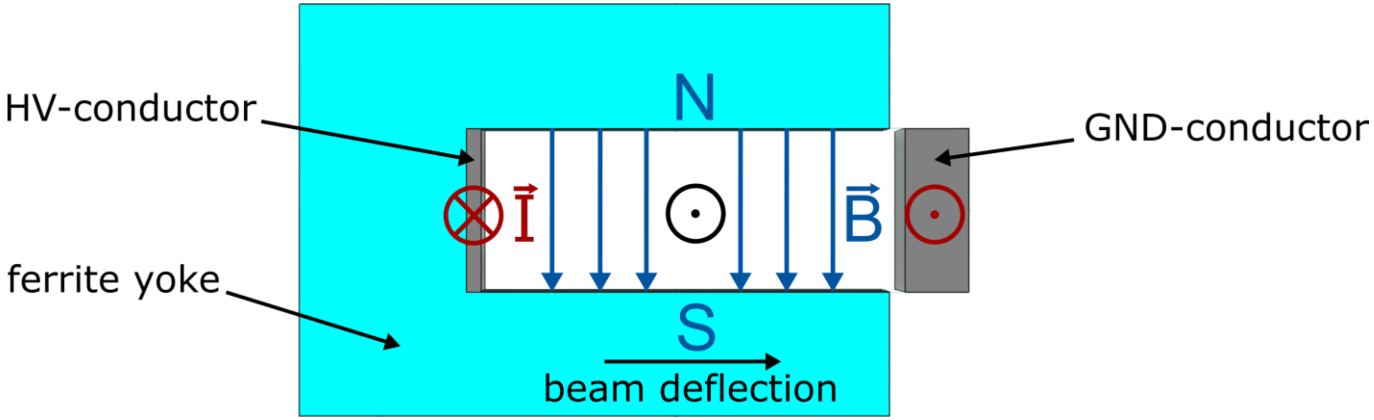


Impact of beam power loss



Case of the SPS extraction kickers

- Problem with SPS extraction kickers (MKE)
  - Extraction elements through which the beam passes every turn
    - Based on a fast pulsed magnet capable of deflecting the whole beam over one turn
    - Active only on turn in which beam has to be extracted, otherwise passive but with all its elements (ferrite, conductors) exposed to the beam



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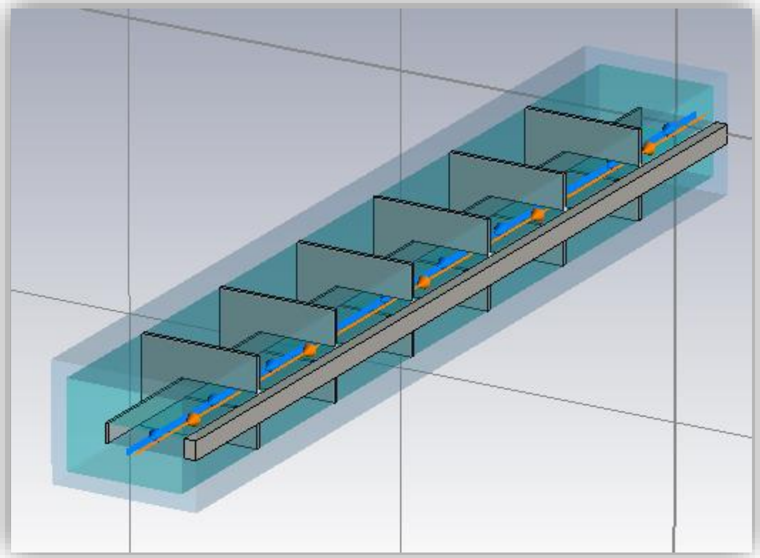
    - Based on a fast pulsed magnet capable of deflecting the whole beam over one turn
    - Active only on turn in which beam has to be extracted, otherwise passive but with all its elements (ferrite, conductors) exposed to the beam
  - Use of beam for LHC filling (4x 200-ns spaced trains of 72x 25-ns spaced bunches) led to inacceptable heating of these elements
    - Heating above Curie temperature leads to ferrite degradation → Beam cannot be extracted anymore from the SPS
    - Heating causes outgassing and strong pressure rise in the kicker sector, with consequent beam interlocking due to poor vacuum



Case of the SPS extraction kickers

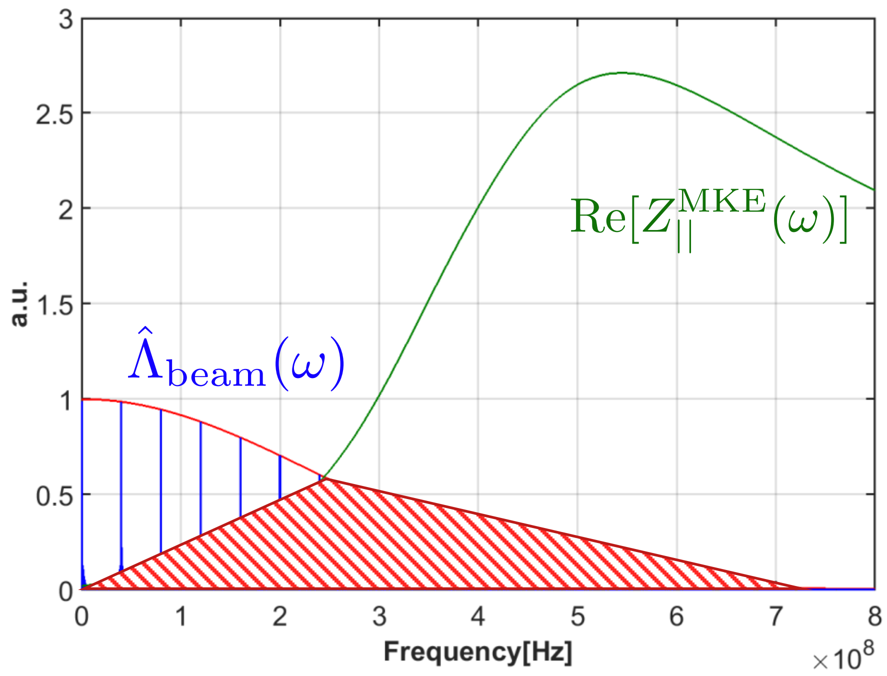
- We need to calculate the power loss in the kicker
  - Kicker impedance can be evaluated semi-analytically or via simulations
  - Then we apply the energy loss formula

$$\Delta E_{\mathrm{beam}} = -\frac{e^2\omega_0}{2\pi}\sum_{p=-\infty}^{+\infty}\left|\Lambda_{\mathrm{beam}}(p\omega_0)\right|^2 \mathrm{Re}\left[Z_{||}(p\omega_0)\right]$$

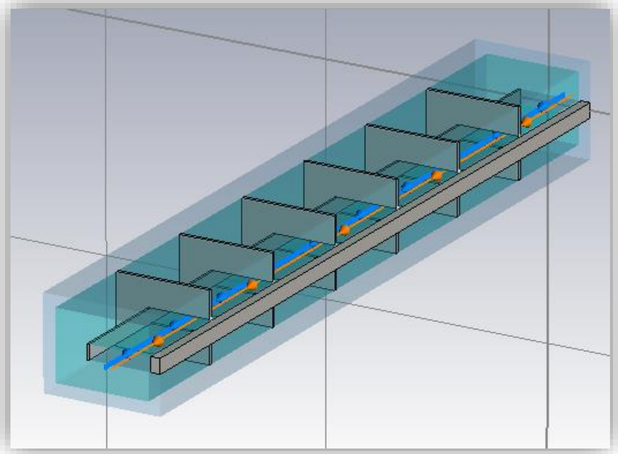
$$\Delta W = \frac{\Delta E_{\mathrm{beam}}}{T_0}$$

Case of the SPS extraction kickers

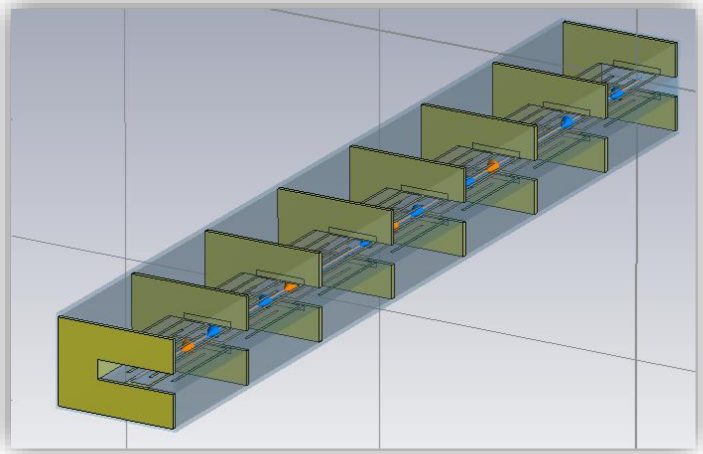
- We need to calculate the power loss in the kicker
  - Kicker impedance can be evaluated semi-analytically or via simulations
  - Then we apply the energy loss formula
- Kicker impedance already becomes significant at frequencies for which the beam spectrum has not fully decayed, causing the undesired heating
- We need to lower the kicker impedance → Impedance dominated by losses in ferrite → Ferrite shielding

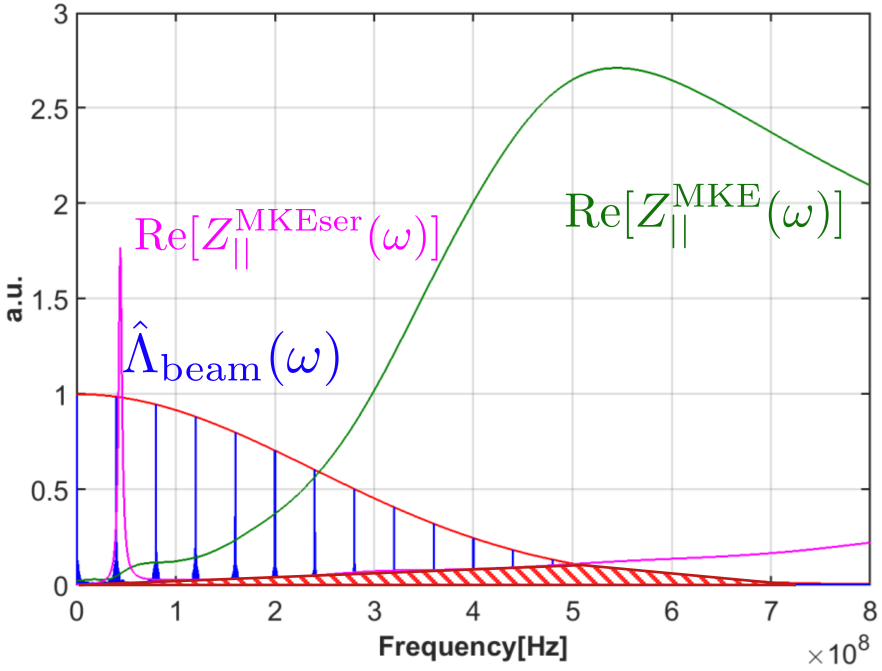


Case of the SPS extraction kickers



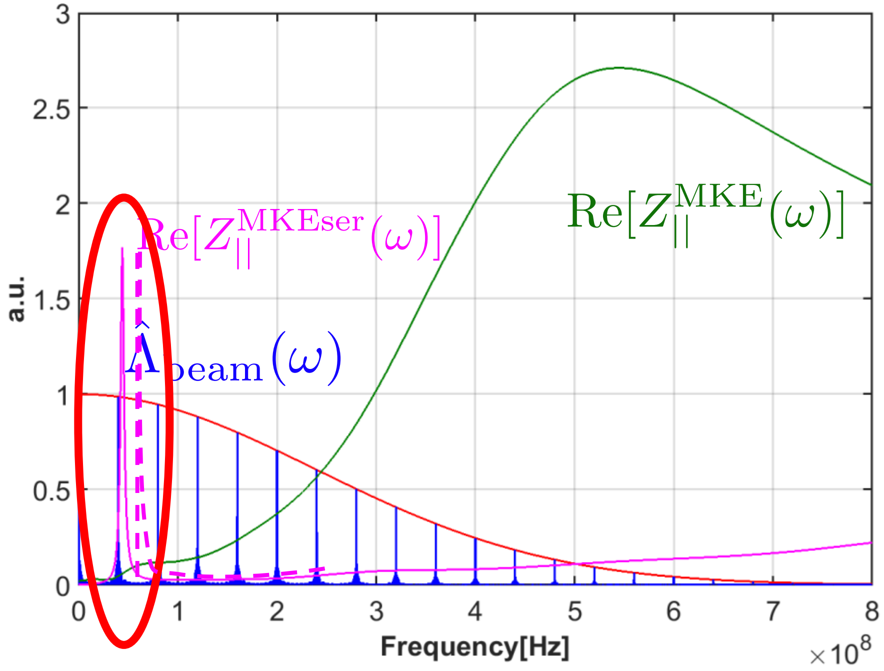
Print striped pattern of good conductor on ferrite (serigraphy)





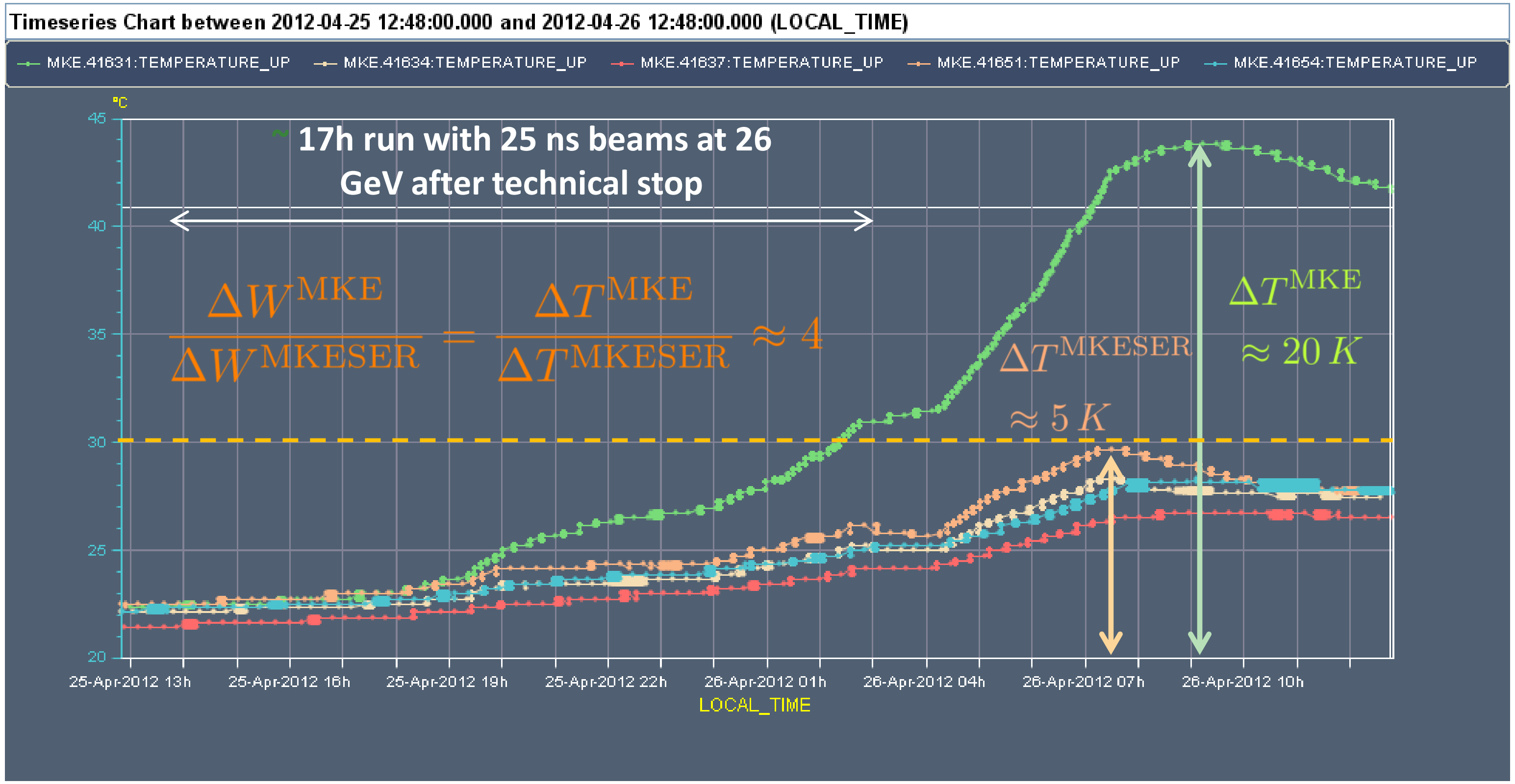
Case of the SPS extraction kickers

- This almost suppresses the impedance over the bunch spectrum
- It however introduces a low frequency peak, which needs to be kept far from beam spectral lines
  - Define serigraphy geometry such as to separate impedance peak from beam spectrum as much as possible



Case of the SPS extraction kickers

- This almost suppresses the impedance over the bunch spectrum
- It however introduces a low frequency peak, which needs to be kept far from beam spectral lines
- Factor 4 less heating measured for 25-ns LHC-type beam at 26 GeV!!



Signpost

We have further looked into the mechanism of energy loss and have seen the **impact of longitudinal impedances on machine elements** as these lead to **beam induced heating**.

We have found that beam induced heating depends on the overlap of the **beam power spectrum** and the **impedance** of a given object.

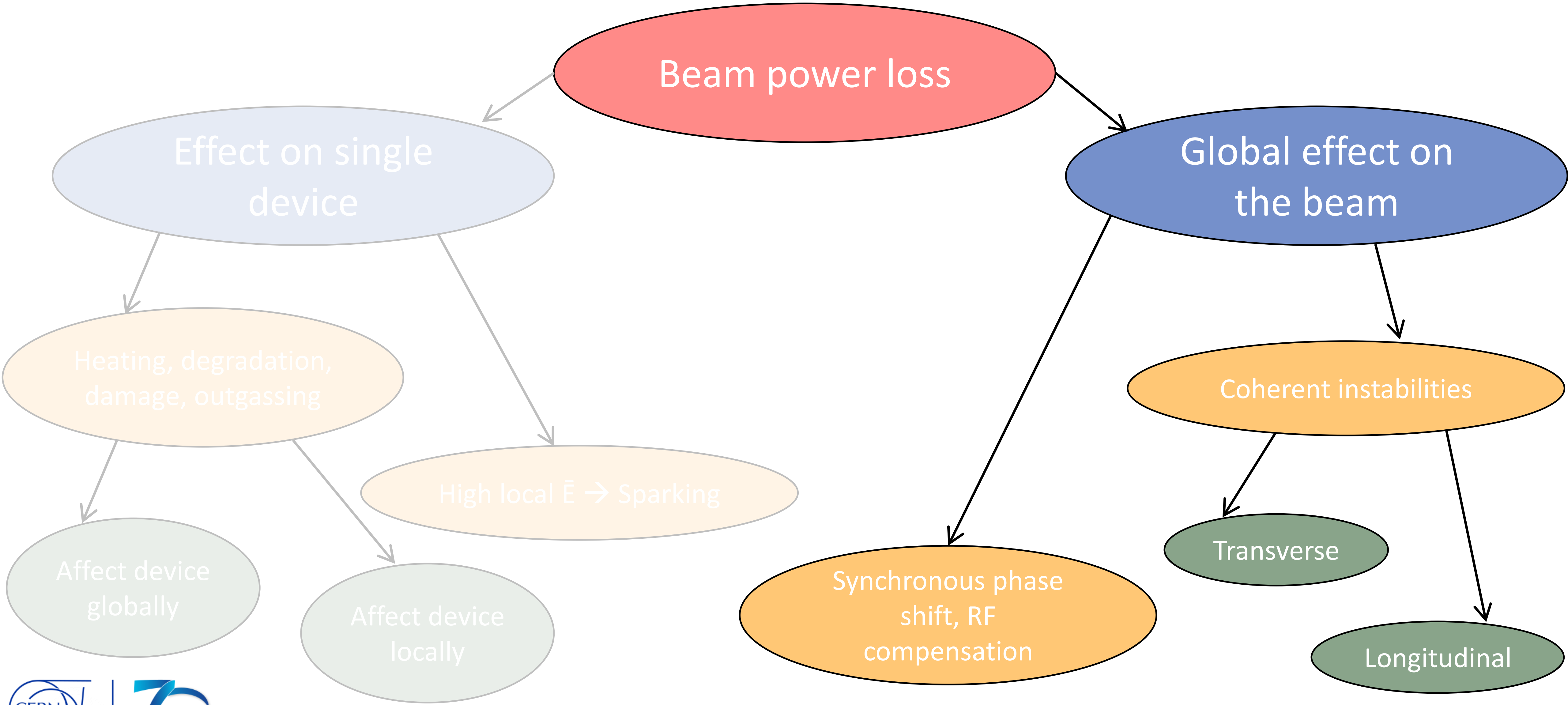
We have seen a **real world example** of the impact of an objects impedance on the beam induced heating.

Part 2: Multiparticle dynamics with wake fields –

impact on machine elements and longitudinal beam dynamics

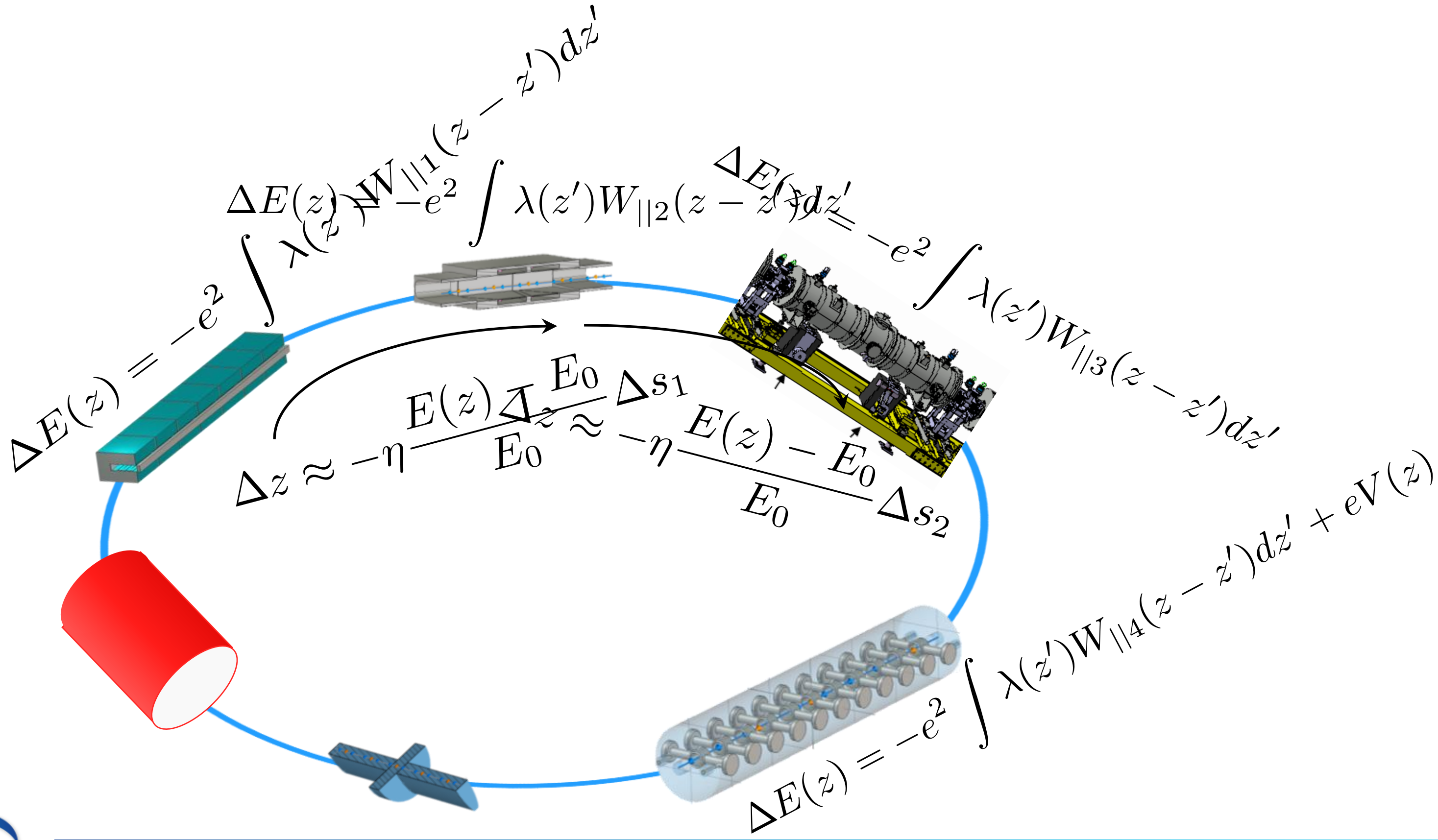
- General introduction to wake fields
- Longitudinal wake fields and the longitudinal wake function
- Energy loss – beam induced heating and stable phase shift
- Potential well distortion, bunch lengthening and microwave instability

Impact of beam power loss



Longitudinal wakes in beam dynamics

- The effect of each localised wake/impedance on each particle in a beam can be described as an energy kick
- The accelerator is made of many components, each giving a small kick to the beam particles, which drift freely in between



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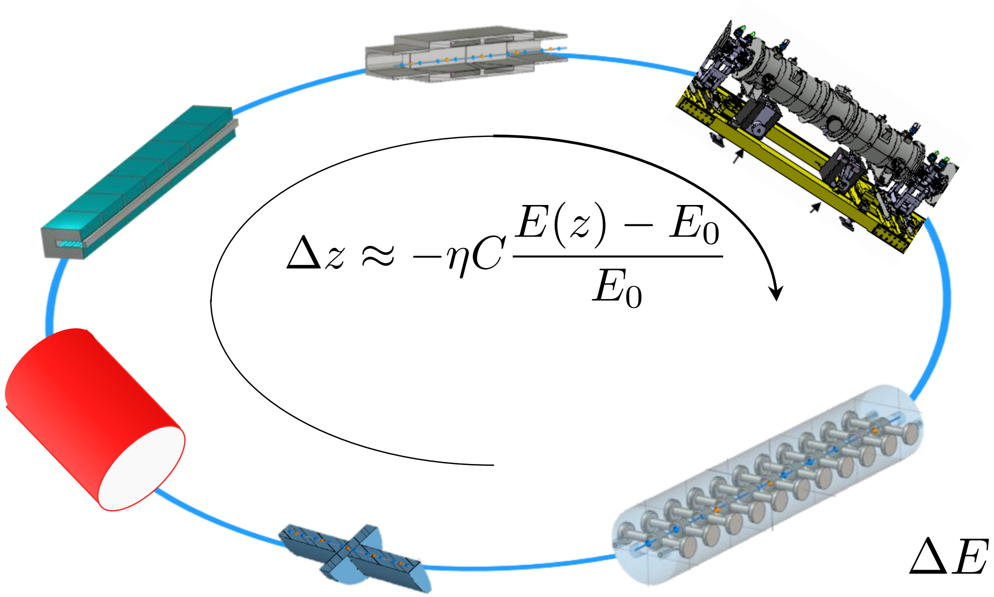
- For simulations, the impedance is lumped in one place and kicks to beam particles are applied once per turn, with free drift over one turn



$$\Delta z \approx -\eta C \frac{E(z) - E_0}{E_0}$$

$$W_{||}^{\text{Ring}}(z) = \sum W_{||i}(z)$$

$$Z_{||}^{\text{Ring}}(\omega) = \sum Z_{||i}(\omega)$$

$$\Delta E(z) = -e^2 \int \lambda(z') W_{||}^{\text{Ring}}(z - z') dz'$$

Longitudinal wakes in beam dynamics

- The effect of each localised wake/impedance on each particle in a beam can be described as an energy kick
- The accelerator is made of many components, each giving a small kick to the beam particles, which drift freely in between
- For simulations, the impedance is lumped in one place and kicks to beam particles are applied once per turn, with free drift over one turn
- For analytical calculations, both global impedance and RF are smeared over the ring

$$\begin{cases} \dfrac{dz}{ds} = -\eta\delta \\ \dfrac{d\delta}{ds} = \dfrac{e}{m_0\gamma c C}\left[V_{\mathrm{rf}}(z) - e\sum_k \int \lambda(z' + kC) W_{||}^{\mathrm{Ring}}(z - z' - kC)dz'\right] \end{cases}$$

$$H = -\frac{1}{2}\eta\delta^2 + \frac{e}{\beta^2 EC}U_{\mathrm{rf}}(z) +$$
$$+\frac{e^2}{\beta^2 EC}\int_{-\infty}^{z} dz'' \sum_k \int \lambda(z' + kC) W_{||}^{\mathrm{Ring}}(z'' - z' - kC)dz'$$

Longitudinal wakes in beam dynamics

- For a bunch under the effect of longitudinal wake fields, two different regimes can be found:
  - Regime of **potential well distortion**, i.e. due to the impedance a new equilibrium distribution can be found for the bunch
    - Stable phase shift
    - Synchrotron frequency shift
    - Different matching (→ bunch lengthening for lepton machines)
  - Regime of **longitudinal instability**, i.e. no equilibrium distribution can be found under the effect of the impedance, a perturbation grows exponentially
    - Dipole mode instabilities
    - Coupled bunch instabilities
    - Microwave instability (longitudinal mode coupling)

Potential well distortion and Haissinki equation

The CERN Accelerator School

- The **equilibrium distribution** in the presence of a longitudinal wake field can be found analytically. The (linearized) **longitudinal Hamiltonian** with longitudinal wake fields is given as:

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_\parallel(z''-z'-kC)$$

- We assume a Gaussian matched beam distribution, hence:

$$\psi = \psi(H) = \frac{A}{\sqrt{2\pi}\sigma_\delta}\exp\left(\frac{H}{H_0}\right) \Longrightarrow \psi(z,\delta) = \frac{A}{\sqrt{2\pi}\sigma_\delta}\exp\left(-\frac{\eta\,\delta^2}{2\,H_0}\right)\exp\left(\frac{-V(z)}{2\,H_0}\right)$$

- Now, choosing for H_0 and consider for λ that:

$$H_0 = \eta\,\sigma_\delta^2\,, \;\text{ and }\; \lambda(z) = \int \psi(z,\delta)\,d\delta\,,$$

- The equilibrium (matched) line charge density is then given by the self-consistency equation (**Haissinski equation**):

$$\lambda(z) = A\,\exp\left(-\frac{1}{2}\left(\frac{\omega_s z}{\eta\sigma_\delta\beta c}\right)^2 + \frac{e^2}{\eta\sigma_\delta^2\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_\parallel(z''-z'-kC)\right)$$

Potential well distortion and Haissinki equation

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- We assume a G

- Now, choosing

A simple Taylor expansion in z already qualitatively reveals some of the effects of the longitudinal wake fields onto the beam:

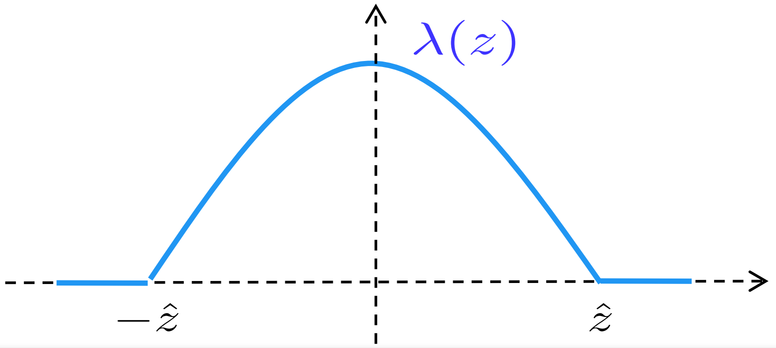
1. First order:
   shift in the mean position (**stable phase shift**)
2. Second order:
   change in bunch length accompanied by an (incoherent) **synchrotron tune shift**

- The equilibrium (matched) line charge density is then given by the self-consistency equation (**Haissinski equation**):

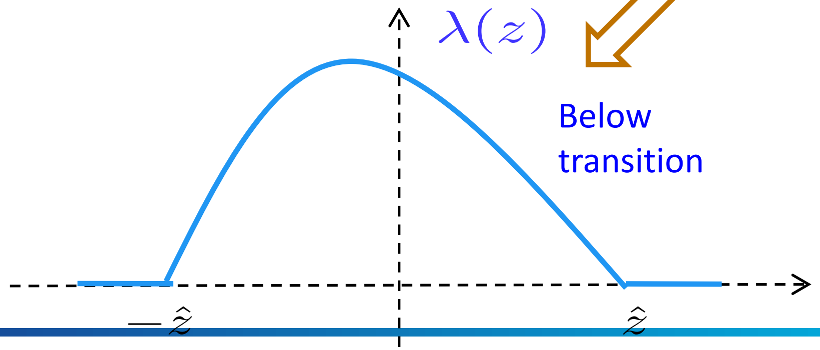
$$\lambda(z) = A\,\exp\left(-\frac{1}{2}\left(\frac{\omega_s z}{\eta\sigma_\delta \beta c}\right)^2 + \frac{e^2}{\eta\sigma_\delta^2\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)W_{\|}(z''-z'-kC)\right)$$

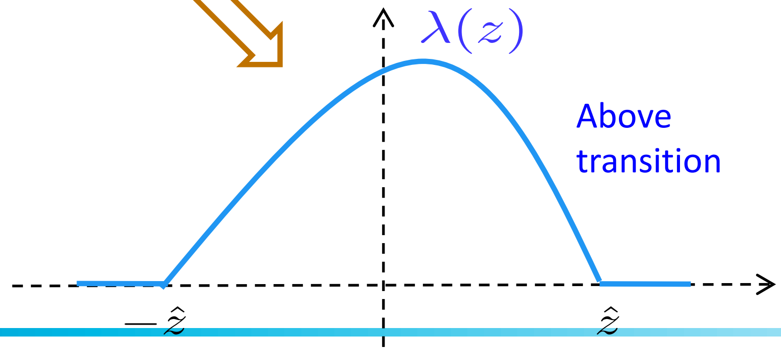
Bunch energy loss per turn and stable phase

- The **RF system compensates** for the energy loss by imparting a net acceleration to the bunch
- Therefore, the bunch readjusts to a **new equilibrium distribution** in the bucket and moves to an average synchronous angle $\Delta\Phi_s$



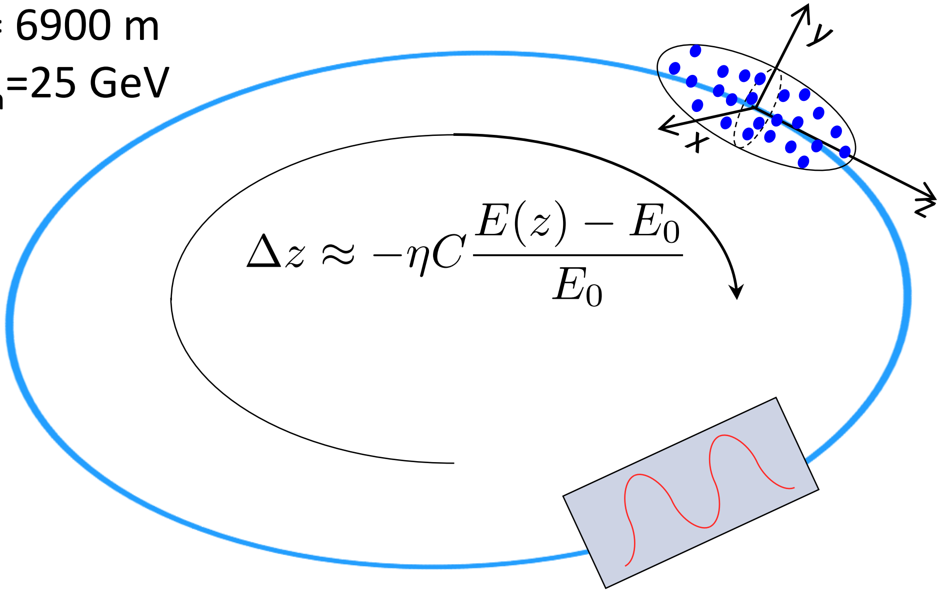
$$\boxed{\sin\Delta\Phi_s = \frac{\Delta E_{\mathrm{turn}}}{NeV_m} = -\frac{e\omega_0}{2\pi N V_m}\sum_{p=-\infty}^{\infty}\left|\hat{\lambda}(p\omega_0)\right|^2 \mathrm{Re}\left(Z_{||}(p\omega_0)\right)}$$





Bunch lengthening and μW instability

SPS ring
C = 6900 m
E_{kin}=25 GeV



$$\Delta z \approx -\eta C \frac{E(z) - E_0}{E_0}$$

Single Gaussian bunch
σ_z = 0.2 m (0.67 ns)

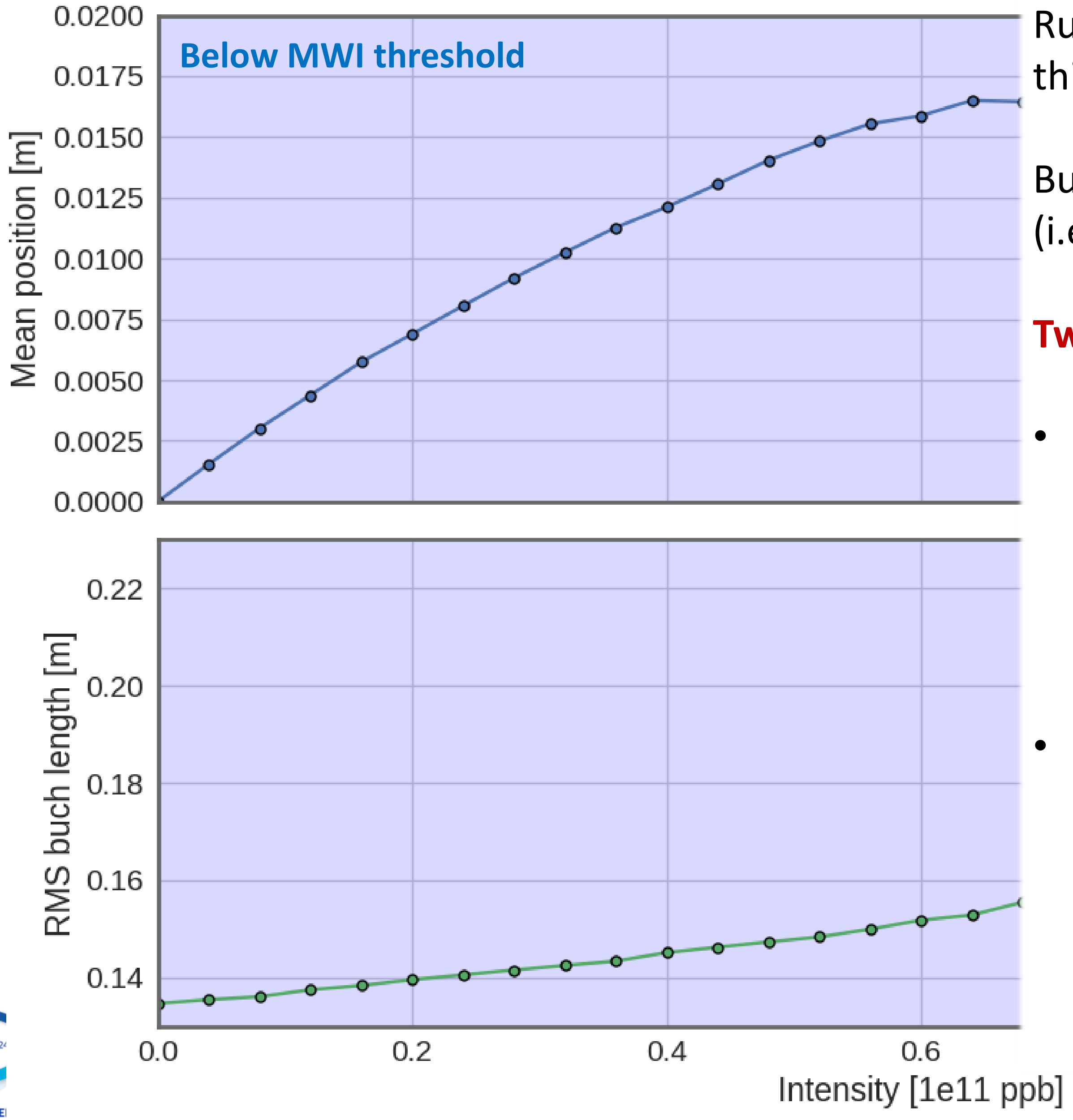
Ring impedance modeled as broad band resonator with
ω_r = 700 MHz
Q=1
R_s=

Single RF system
ω_{rf} = 200 MHz
V_{rf}^{max} = 3 MV

$$\Delta E(z) = -e^2 \int \lambda(z') W_{||}^{\mathrm{Res}}(z - z') dz' + e V_{\mathrm{rf}}(z)$$

$$Z_{||}^{\mathrm{Res}}(\omega) = \frac{R_{s||}}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Bunch lengthening and μW instability



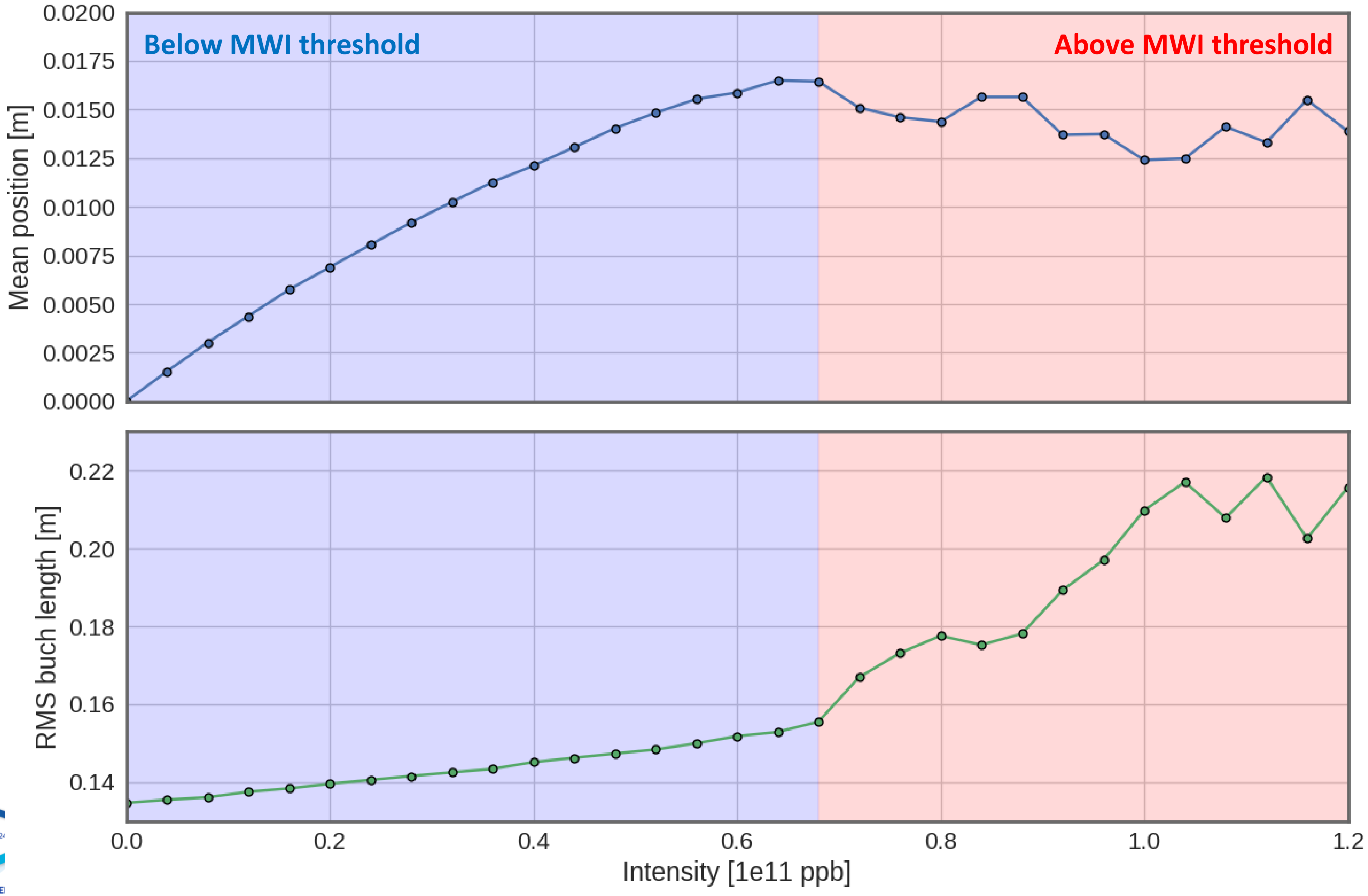
Running the numerical simulation for this case:

Bunch is matched at low intensity (i.e. without impedance)

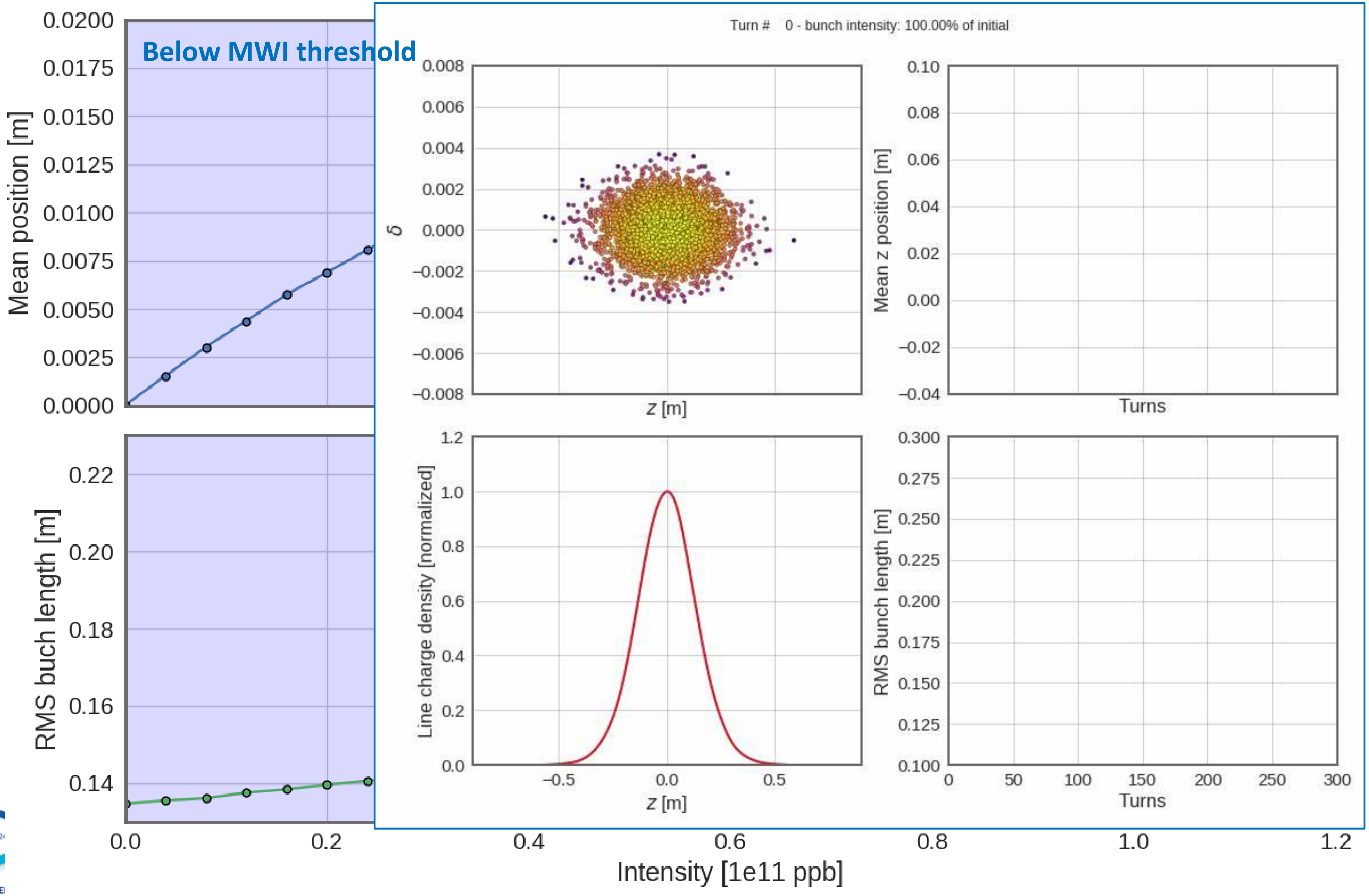
Two regimes are found:

- Bunch lengthening/emittance blow up regime with roughly linear increase of the **synchronous phase** and **bunch length** with intensity
- Unstable regime (**turbulent bunch lengthening**)

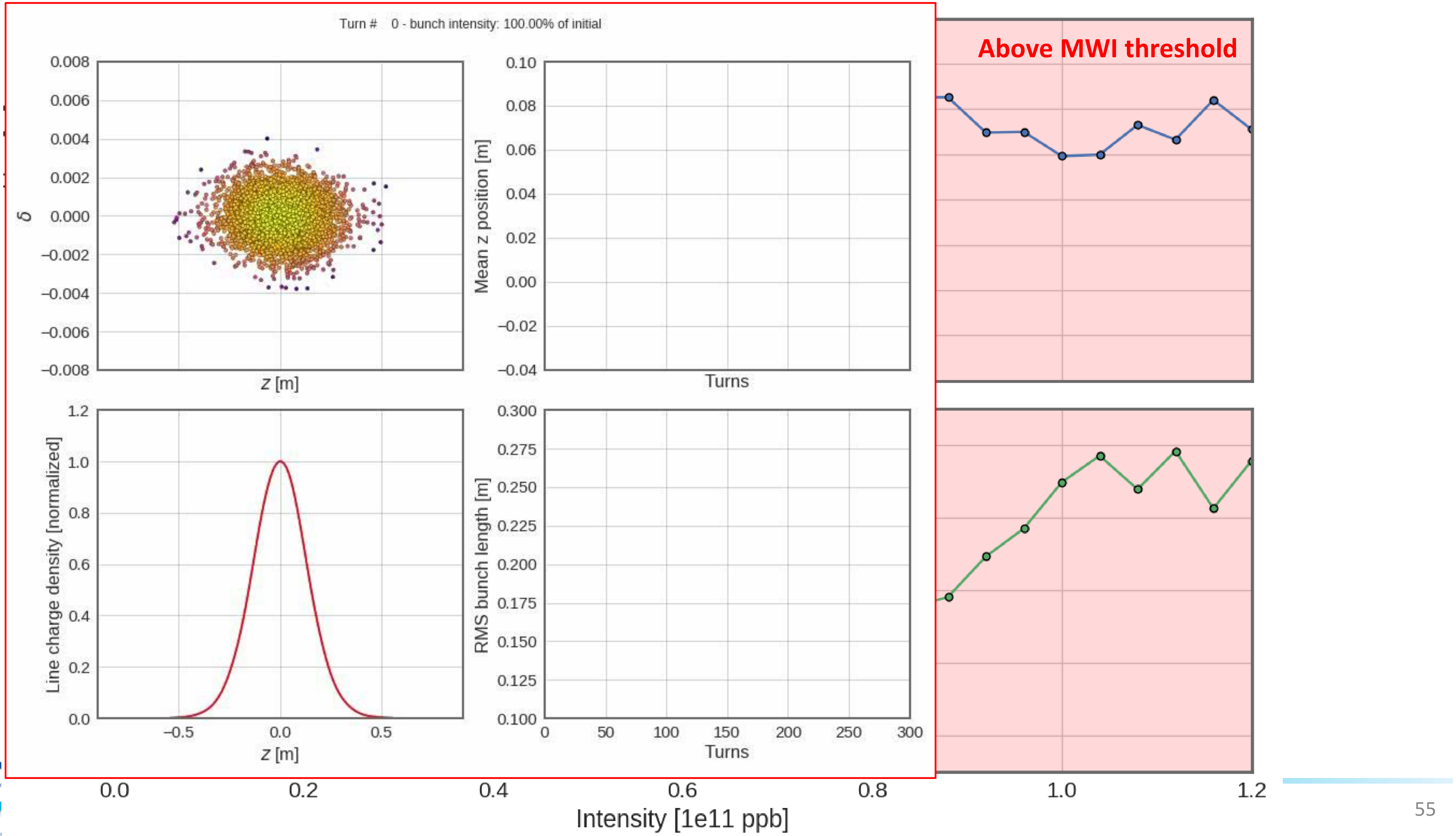
Bunch lengthening and μW instability



Bunch lengthening and μW instability



Bunch lengthening and µW instability



Signpost

We have **discussed longitudinal wake fields** and impedances and examples of their impact on both the machine as well as the beam.

We have learned about **beam induced heating** and how it is related to the beam power spectrum and the machine impedance.

We have discussed the effects of **potential well distortion** (stable phase and synchrotron tune shifts, bunch lengthening and shortening).

We have seen one example of **longitudinal instability** (microwave).

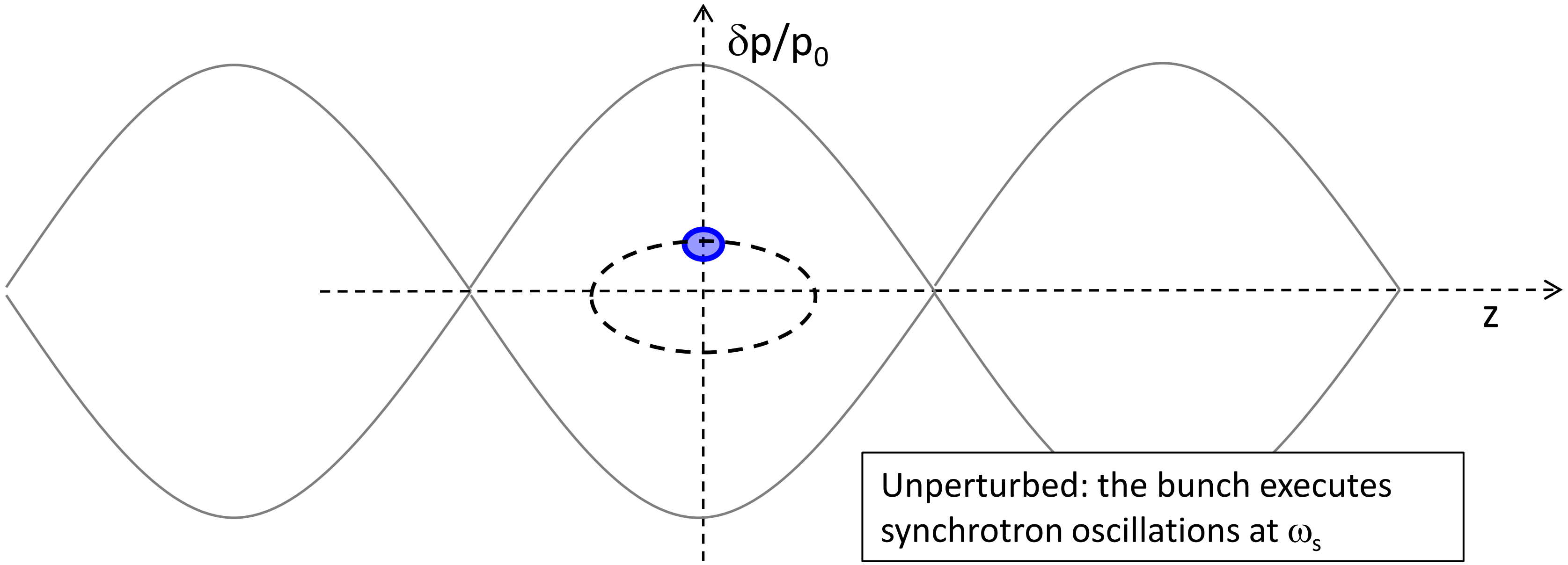
Tomorrow Part 3

→ Transverse wake fields and impedances and their effects on the beam

End part 2

The Robinson instability

- To illustrate the Robinson instability we will use some simplifications:
  - The bunch is **point-like** and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - The bunch additionally feels the effect of a **multi-turn wake**



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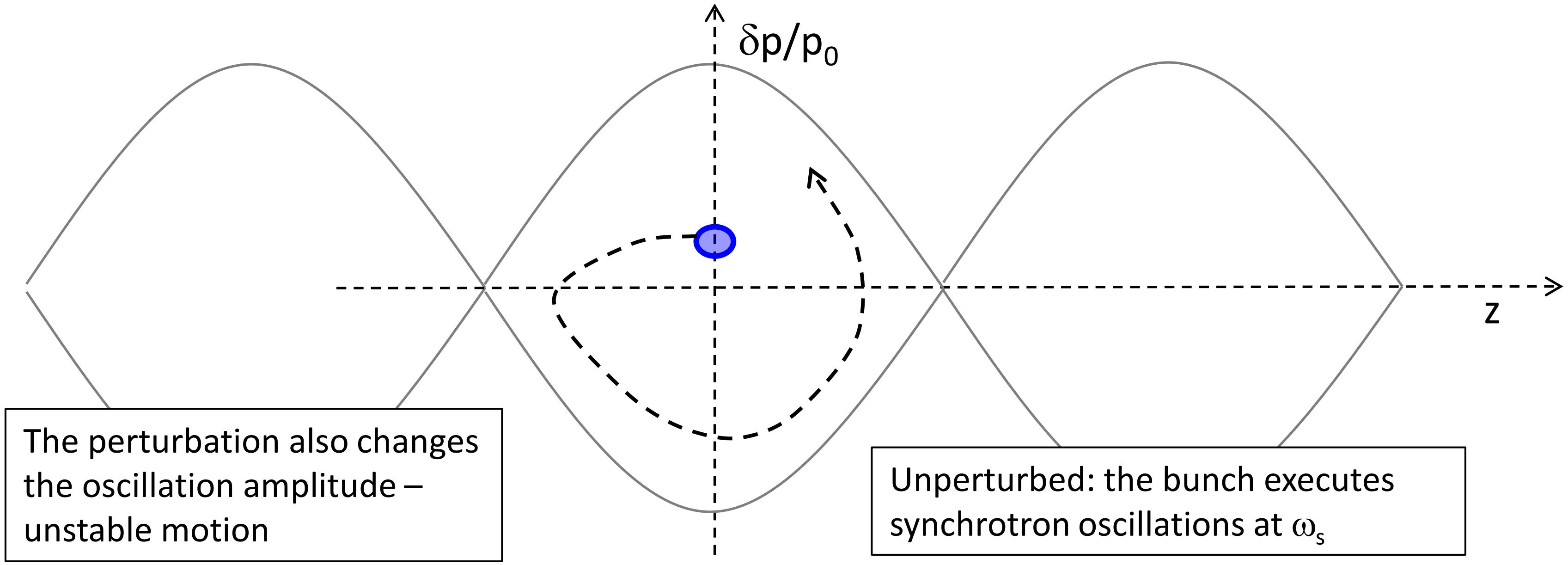

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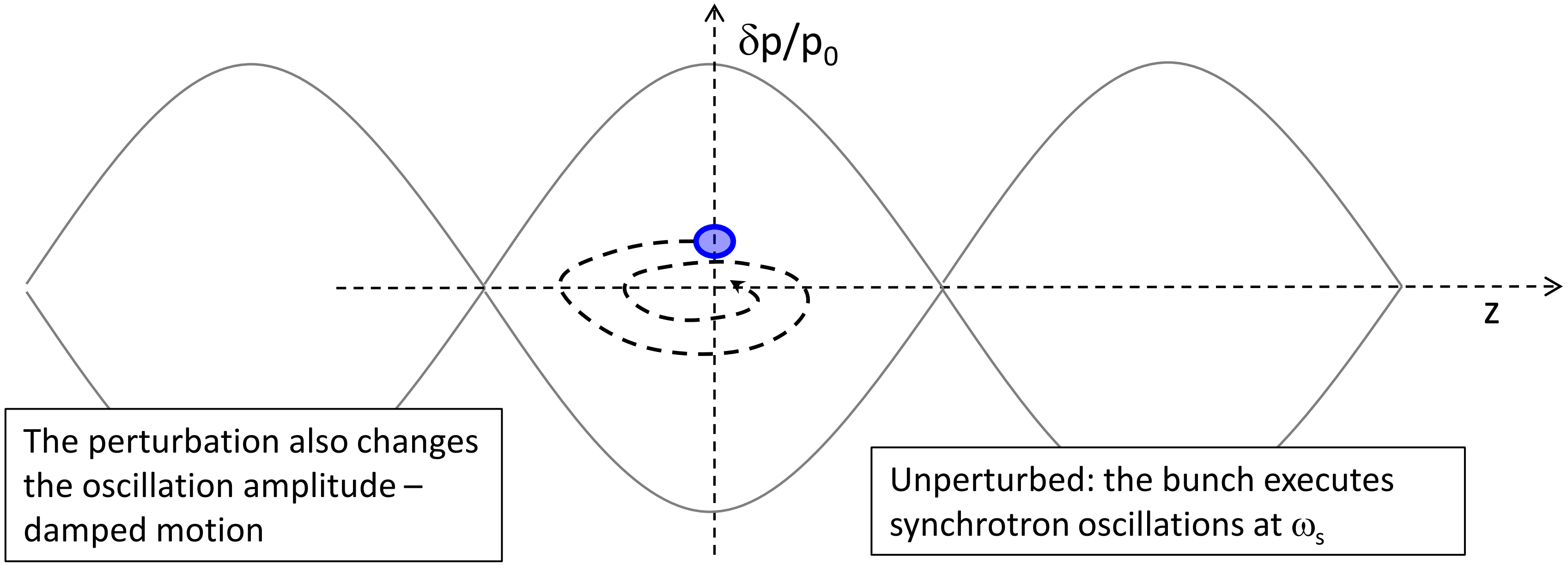

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  - The bunch is **point-like** and feels an external linear force (i.e. it would execute linear synchrotron oscillations in absence of the wake forces)
  - The bunch additionally feels the effect of a **multi-turn wake**

- Longitudinal Hamiltonian

$$H = -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{e^2}{\beta^2 EC}\sum_k \int_0^z dz'' \int_{z''}^{\infty} dz'\,\lambda(z'+kC)\,W_{||}(z''-z'-kC)$$

$$= -\frac{1}{2}\eta\,\delta^2 - \frac{1}{2\eta}\left(\frac{\omega_s}{\beta c}\right)^2 z^2 + \frac{Ne^2}{\beta^2 EC}\sum_k \int_0^z dz''\,W_{||}\Big(z(t)-z(t-kT_0)-kC\Big)$$

- Expansion of wake field (we assume that the wake can be linearized on the scale of a synchrotron oscillation)

$$W_{||}(z(t)-z(t-kT_0)-kC) \approx W_{||}(kC) + W_{||}'(kC)\Big(z(t)-z(t-kT_0)\Big)$$

$$\approx W_{||}(kC) + W_{||}'(kC)\,kT_0\,\frac{dz(t)}{dt}$$

The Robinson instability

- The **first term** only contributes as a constant term in the solution of the equation of motion, i.e. the synchrotron oscillation will be executed around a certain z0 and not around 0. This term represents the **stable phase shift** that compensates for the energy loss
- The **second term** is a dynamic term introduced as a **“friction” term** in the equation of the oscillator, which can **lead to instability**!

- Equations of motion

$$\frac{d^2 z}{dt^2} + \omega_s^2\, z^2 = \frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} \cancel{W_{\|}(kC)} + W_{\|}'(kC)\, kT_0\, \frac{dz}{dt}$$

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$$\frac{d^2 z}{dt^2} + \omega_s^2 z^2 = \frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} \cancel{W_{\parallel}(kC)} + W_{\parallel}'(kC)\, kT_0 \frac{dz}{dt}$$

- Ansatz

$$z(t) \propto \exp\left(-i\Omega t\right)$$

$$\boxed{\frac{i}{C} \sum_{p=-\infty}^{\infty} \Big(p\omega_0\, Z_{\parallel}\left(p\omega_0\right) - \left(p\omega_0 + \Omega\right)\, Z_{\parallel}\left(p\omega_0 + \Omega\right)\Big)}$$

Expressed in terms of impedance

- Solution

$$\left(\Omega^2 - \omega_s^2\right) = -\frac{Ne^2\eta}{Cm_0\gamma} \sum_{k=-\infty}^{\infty} \boxed{\Big(1 - \exp\left(-ik\Omega T_0\right)\Big) W_{\parallel}'(kC)}$$

The Robinson instability

- We assume a small deviation from the synchrotron tune:
  - Re(Ω - ω_s) → **Synchrotron tune shift**
  - Im(Ω - ω_s) → **Growth/damping rate**, only depends on the dynamic term, if it is positive there is an instability!

- **Solution:**

$$\left(\Omega^2 - \omega_s^2\right) = -\frac{iNe^2\eta}{C^2 m_0 \gamma} \sum_{p=-\infty}^{\infty} \Big(p\omega_0 \, Z_{\|} \left(p\omega_0\right) - \left(p\omega_0 + \Omega\right) \, Z_{\|} \left(p\omega_0 + \Omega\right) \Big)$$

$$\approx 2\omega_s \, \left(\Omega - \omega_s\right)$$

- **Tune shift:**

$$\Delta\omega_s = \mathrm{Re}\left(\Omega - \omega_s\right) = \frac{e^2}{m_0 c^2} \frac{N\eta}{2\omega_s \gamma T_0^2}$$

$$\sum_{p=-\infty}^{\infty} \Big(p\omega_0 \ \mathrm{Im}\left[Z_{\|}\right] \left(p\omega_0\right) - \left(p\omega_0 + \omega_s\right) \ \mathrm{Im}\left[Z_{\|}\right] \left(p\omega_0 + \omega_s\right) \Big)$$

- **Growth rate:**

$$\tau^{-1} = \mathrm{Im}\left[\Omega - \omega_s\right] = \frac{e^2}{m_0 c^2} \frac{N\eta}{2\omega_s \gamma T_0^2} \sum_{p=-\infty}^{\infty} \Big(\left(p\omega_0 + \omega_s\right) \ \mathrm{Re}\left[Z_{\|}\right] \left(p\omega_0 + \omega_s\right) \Big)$$

The Robinson instability

- We assume the impedance to be peaked at a frequency ω_r close to $h\omega_0 \gg \omega_s$ (e.g. RF cavity fundamental mode or HOM)
- Only two dominant terms are left in the summation at the RHS of the equation for the growth rate
- Stability requires that η and $\Delta\,\mathrm{Re}\left[Z_{\|}\right](p\omega_0)$ have different signs

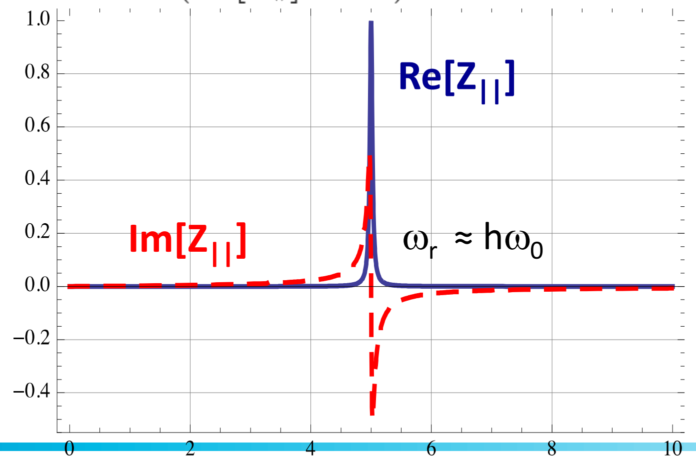
- **Solution:**

$$\tau^{-1} = \mathrm{Im}\left(\Omega - \omega_s\right) = \frac{e^2}{m_0 c^2}\frac{N\eta}{2\omega_s \gamma T_0^2}\sum_{p=-\infty}^{\infty}\left((p\omega_0+\omega_s)\ \mathrm{Re}(Z)_{\|}(p\omega_0+\omega_s)\right)$$

$$= \frac{e^2}{m_0 c^2}\frac{N\eta h\omega_0}{2\omega_s \gamma T_0^2}\underbrace{\left(\mathrm{Re}\left[Z_{\|}\right](h\omega_0+\omega_s) - \mathrm{Re}\left[Z_{\|}\right](h\omega_0-\omega_s)\right)}_{\Delta\left(\mathrm{Re}\left[Z_{\|}\right](h\omega_0)\right)}$$

- **Stability criterion:**

$$\eta\cdot\Delta\left(\mathrm{Re}\left[Z_{\|}\right](h\omega_0)\right) < 0$$



The Robinson instability

- **Stability criterion:** $\eta \cdot \Delta\Big(\mathrm{Re}\left[Z_{\parallel}\right](h\omega_0)\Big) < 0$

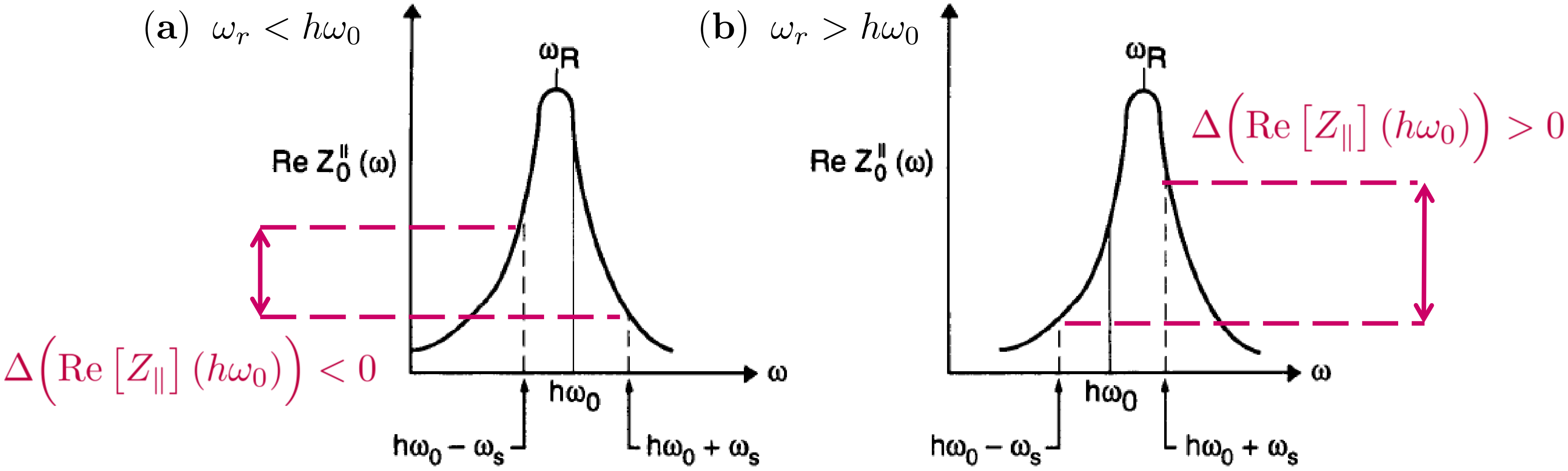
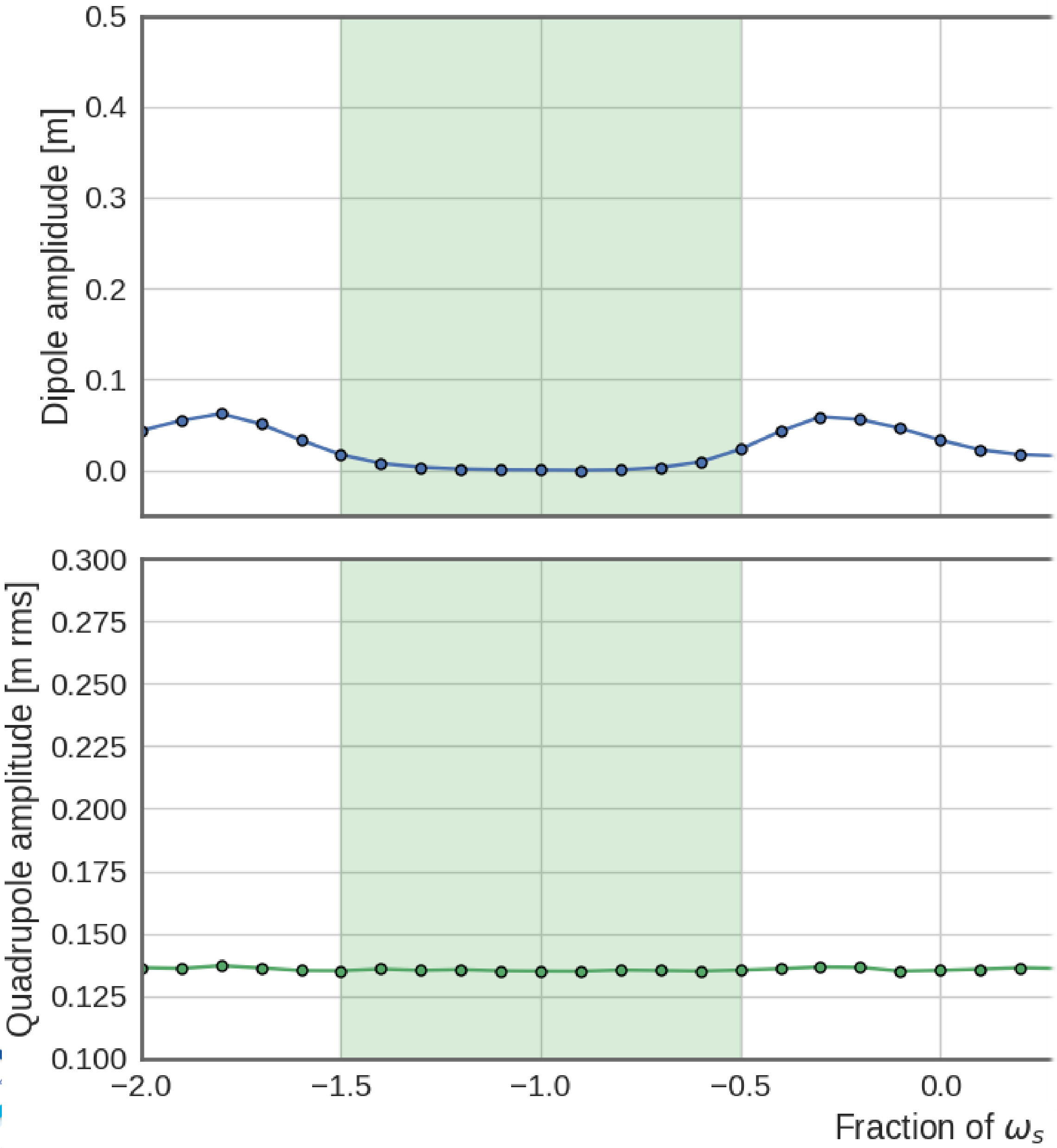


Figure 4.4. Illustration of the Robinson stability criterion. The rf fundamental mode is detuned so that ω_R is (a) slightly below $h\omega_0$ and (b) slightly above $h\omega_0$. (a) is Robinson damped above transition and antidamped below transition. (b) is antidamped above transition and damped below transition.

	$\omega_r < h\omega_0$	$\omega_r > h\omega_0$
Above transition ($\eta > 0$)	stable	unstable
Below transition ($\eta < 0$)	unstable	stable

Robinson damping and instability

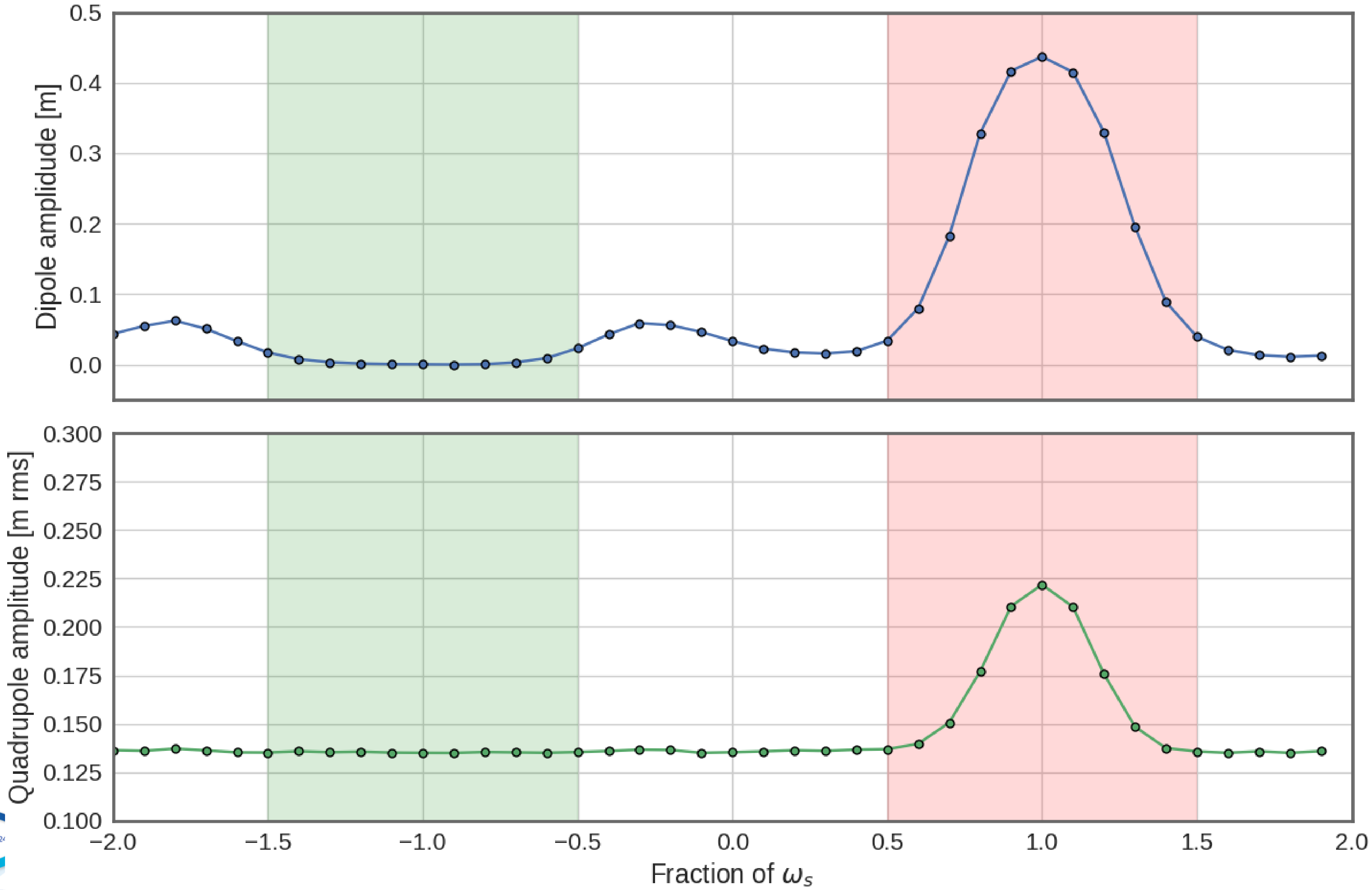


Examples of numerical simulations – SPS bunch with **single narrow-band resonator** wake:

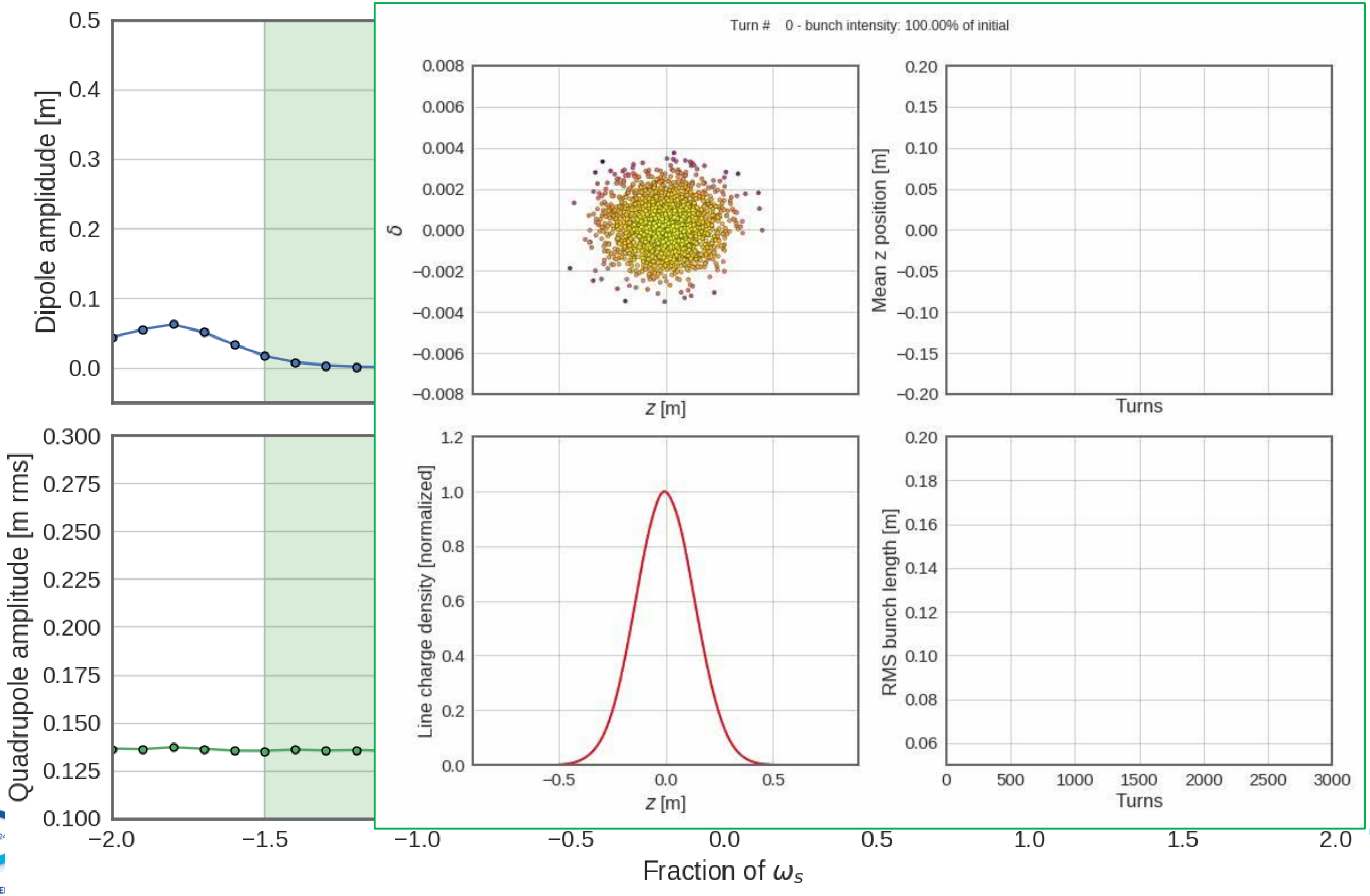
Initializing an otherwise matched bunch with a slight momentum error, **two regimes are found**:

- Regime of **Robinson damping** when the resonator is **detuned to $h\omega_0 - \omega_s$**. Initial dipole oscillations are damped.
- Regime of **Robinson instability** when the resonator is **detuned to $h\omega_0 + \omega_s$**. Initial dipole oscillations start to are grow exponentially.

Robinson damping and instability



Robinson damping and instability



Robinson damping and instability