

Beam Loading



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CERN



Advanced Accelerator Physics

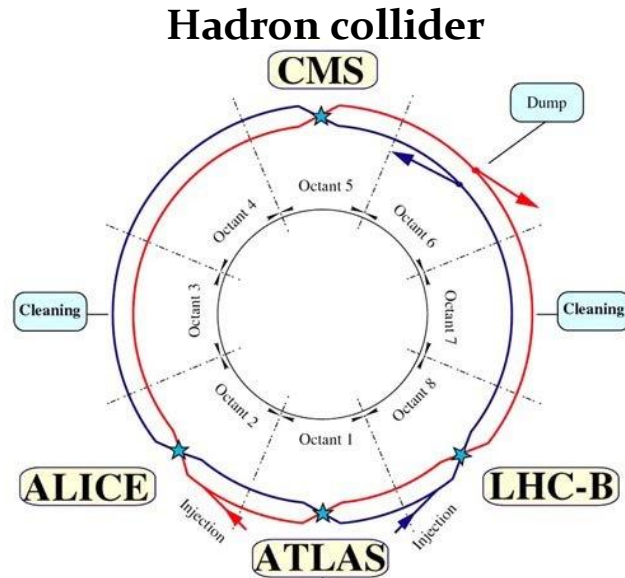
16 November 2024

Outline

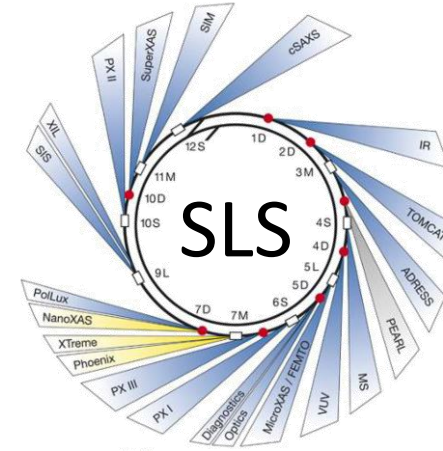
- **Introduction**
- **RF cavity parameters**
 - Shunt impedance, beam loading, power coupling
- **Fundamental theorem of beam loading**
- **Passage of a bunches through a cavity**
 - Single passage or bunches with large spacing
 - Multiple bunch passages
- **Steady state beam loading and partial filling**
 - Few bunches with large spacing
- **Summary**

Introduction

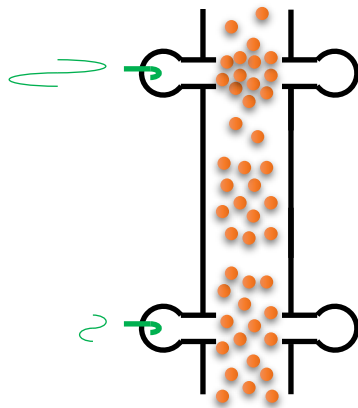
What do these devices in common?



Electron storage ring



Klystron amplifier



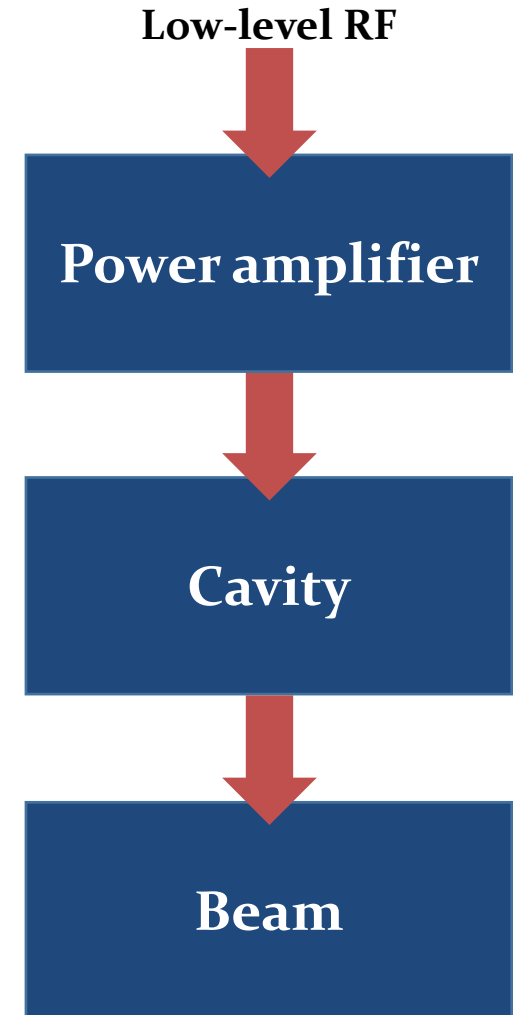
Microwave oven



→ They all **suffer** from or **make use** of beam loading

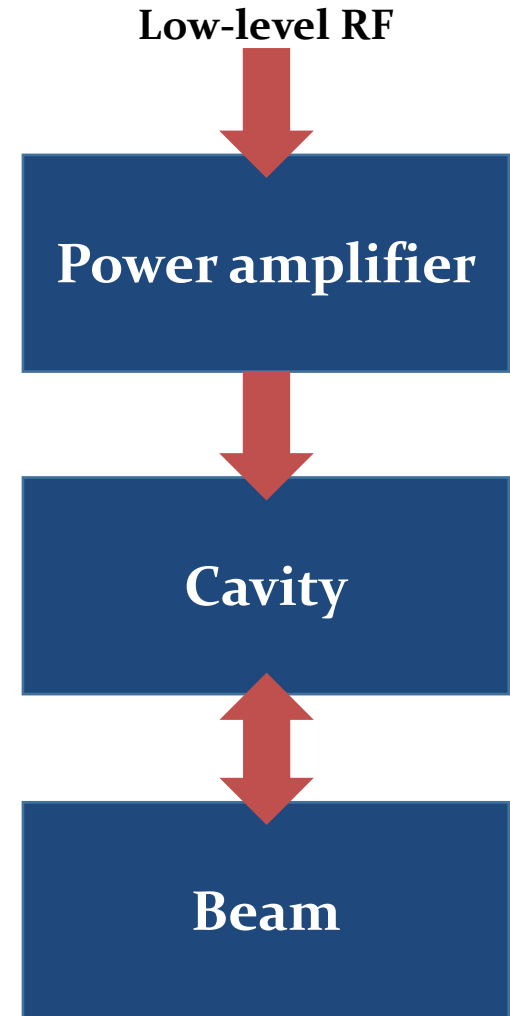
Introduction

- The **radiofrequency (RF)** system should provide
 - Energy to the beam
 - Longitudinal focusing
- Intended energy flow usually from cavity to beam
- But beam also likes to influence the field in the cavity



Introduction

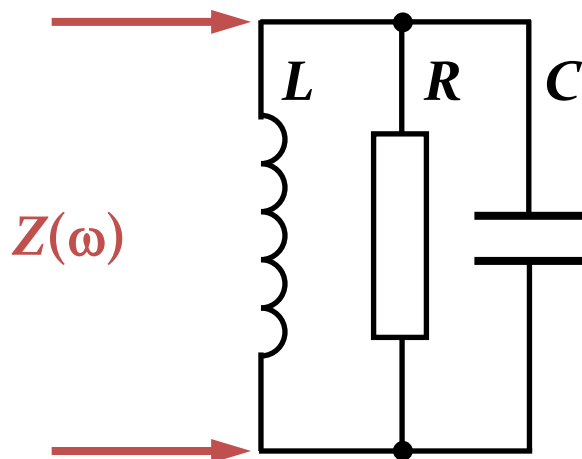
- The **radiofrequency (RF)** system should provide
 - Energy to the beam
 - Longitudinal focusing
 - Intended energy flow usually from cavity to beam
 - But beam also likes to influence the field in the cavity
- Beam loading



RF cavity

Cavity parameters

- The resonance of a cavity can be understood as simple parallel resonant circuit described by R, L, C



$$\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$$

with $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q = \omega_0 \frac{\text{Stored energy}}{\text{Average power loss}} = \frac{\omega_0 E}{P}$$

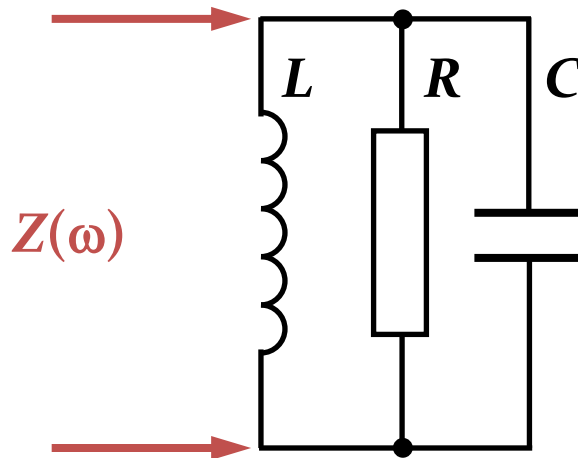
$$E = \frac{1}{2} CV^2 = \frac{1}{2} LI^2$$

$$P = \frac{1}{2} \frac{U^2}{R} = \frac{1}{2} I^2 R$$

$$Q = \omega_0 RC = \frac{R}{\omega_0 L}$$

Cavity parameters

- The resonance of a cavity can be understood as simple parallel resonant circuit described by R, L, C



$$\frac{1}{Z(\omega)} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C$$

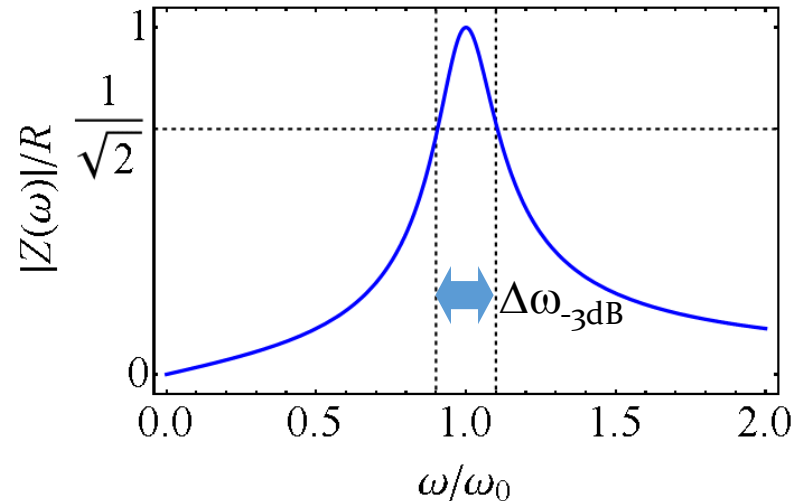
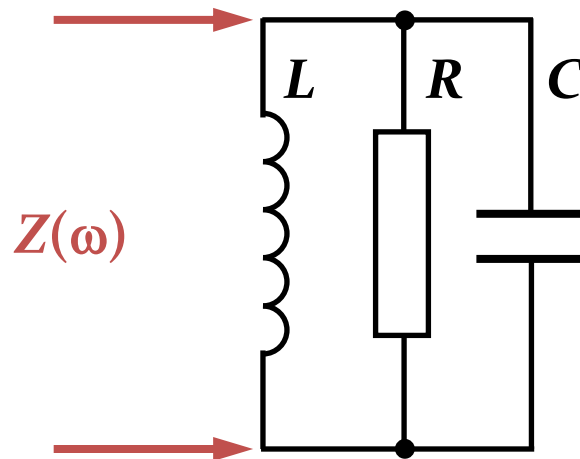
with $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q = \omega_0 RC = \frac{R}{\omega_0 L} \quad Z(\omega) = \frac{R}{1 + iQ \left(\frac{\omega^2 - \omega_0^2}{\omega\omega_0} \right)} \approx \frac{R}{1 + 2iQ \frac{\Delta\omega}{\omega_0}}$$

- Resonant circuit can also be described by $R, R/Q, \omega_0$ or any other set of three parameters

Cavity parameters

- The resonance of a cavity can be understood as simple parallel resonant circuit described by R , L , C



$$Q = \omega_0 RC = \frac{R}{\omega_0 L} \quad Z(\omega) = \frac{R}{1 + iQ \left(\frac{\omega^2 - \omega_0^2}{\omega \omega_0} \right)} \approx \frac{R}{1 + 2iQ \frac{\Delta\omega}{\omega_0}}$$

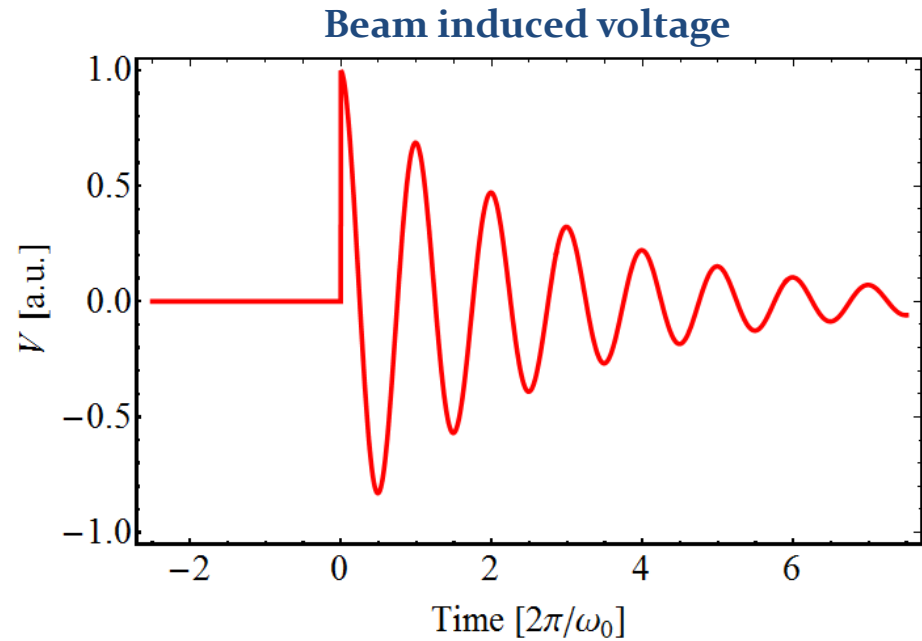
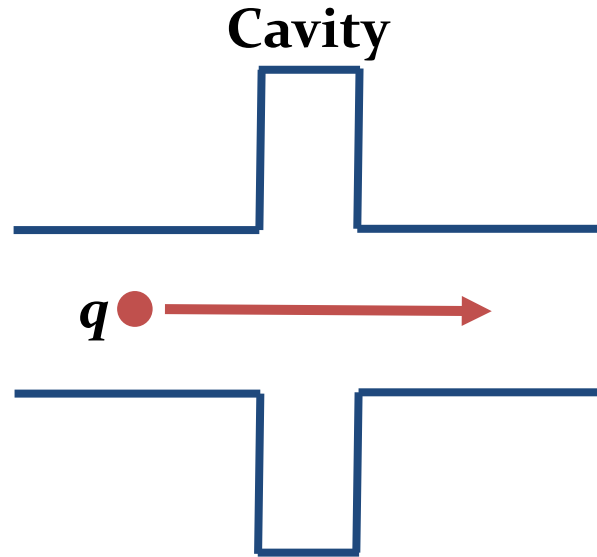
- Resonant circuit can also be described by R , R/Q , ω_0 or any other set of three parameters

Cavity parameters

- Most common choice by cavity designers ω_0 , R , R/Q – why?
- **Resonance frequency, ω_0**
 - Exactly defined for given application, e.g. $h\omega_{\text{rev}}$
- **Shunt impedance, R**
 - Power required to produce a given voltage **without beam**
- **“R-upon-Q”, R/Q**
 - Defined only by the cavity geometry
 - Criterion to optimize a geometry
 - Detuning with beam proportional to R/Q

Why R/Q ?

→ Charged particle experiences cavity gap as capacitor



$$q = V_{b0}C$$

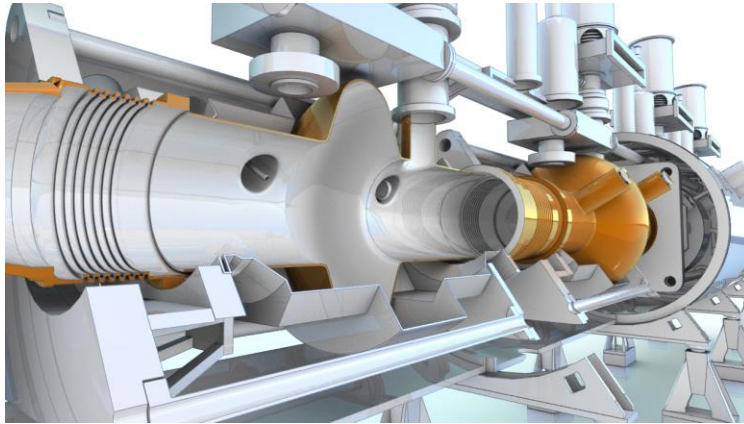
$$Q = \omega_0 RC \quad \rightarrow \quad \frac{1}{C} = \left(\frac{R}{Q}\right) \omega_0$$

$$V_{b0} = \frac{q}{C} \propto \frac{R}{Q}$$

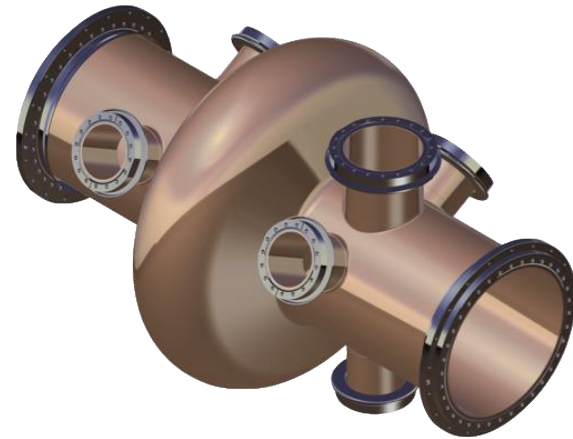
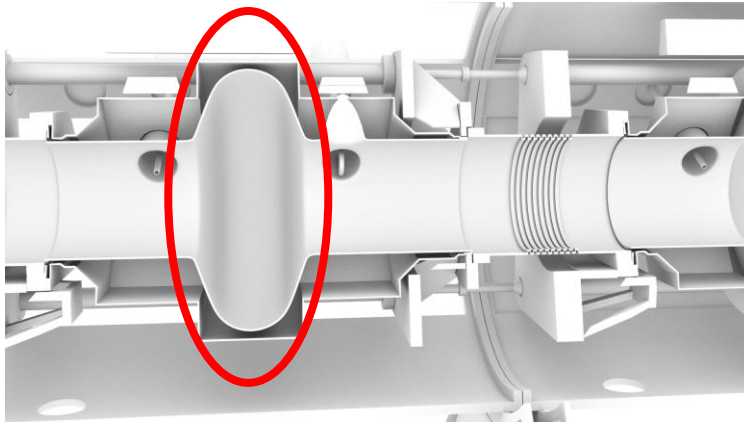
→ Cavity geometry with small R/Q to reduce beam loading

Example: 400 MHz cavities in LHC

- Reduce beam loading in RF cavities
- Shunt impedance, R , low for small R/Q with normal conducting cavities → superconducting cavities in LHC



Bell shape: $R/Q \sim 44 \Omega$, 400 MHz



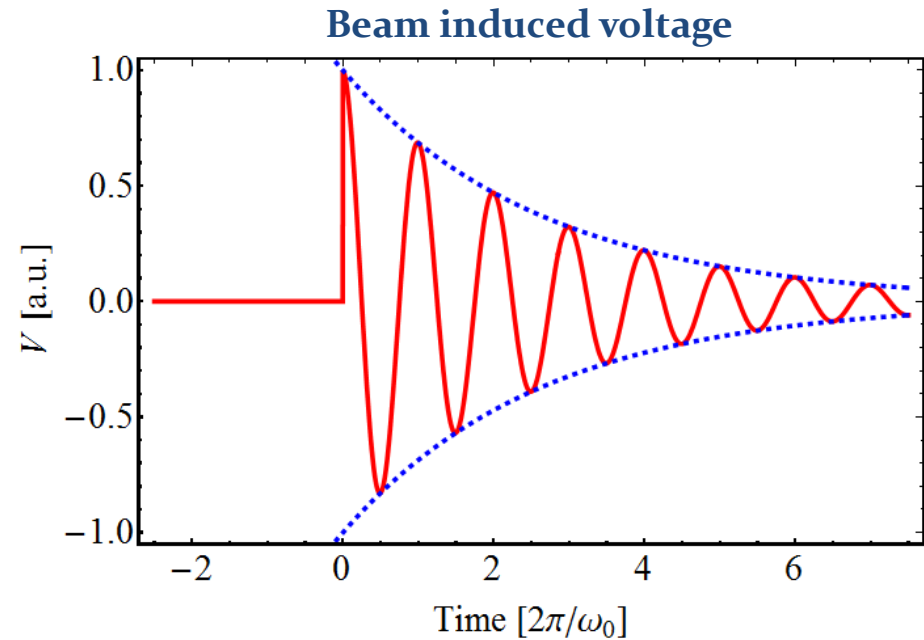
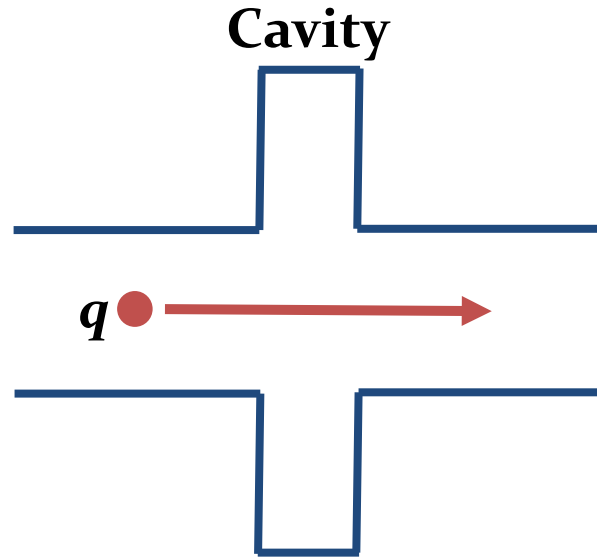
→ 2×8 cavities, 5.3 MV/m

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}}$$

~~$\frac{1}{Q_0}$~~ ^{~0}

Field decay in cavity

→ After passage of charge: energy and fields decay exponentially



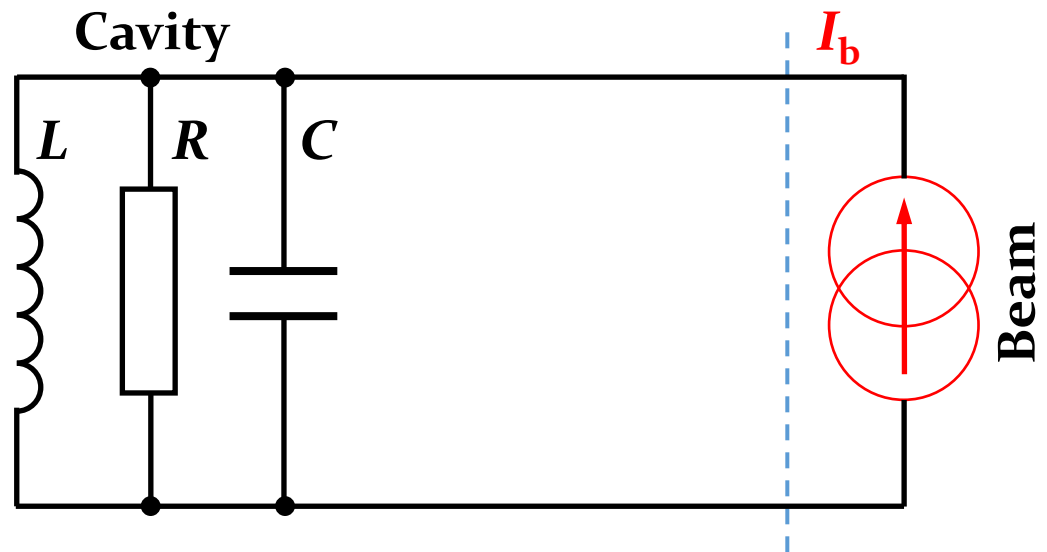
→ **Energy:** $W(t) = W_0 e^{-\frac{\omega_0}{Q}t}$

→ **Voltage:** $V(t) = V_{b0} e^{-\frac{\omega_0}{2Q}t} = V_{b0} e^{-t/T_f}$ **and**

→ **Filling time:** $T_f = \frac{2Q}{\omega_0}$

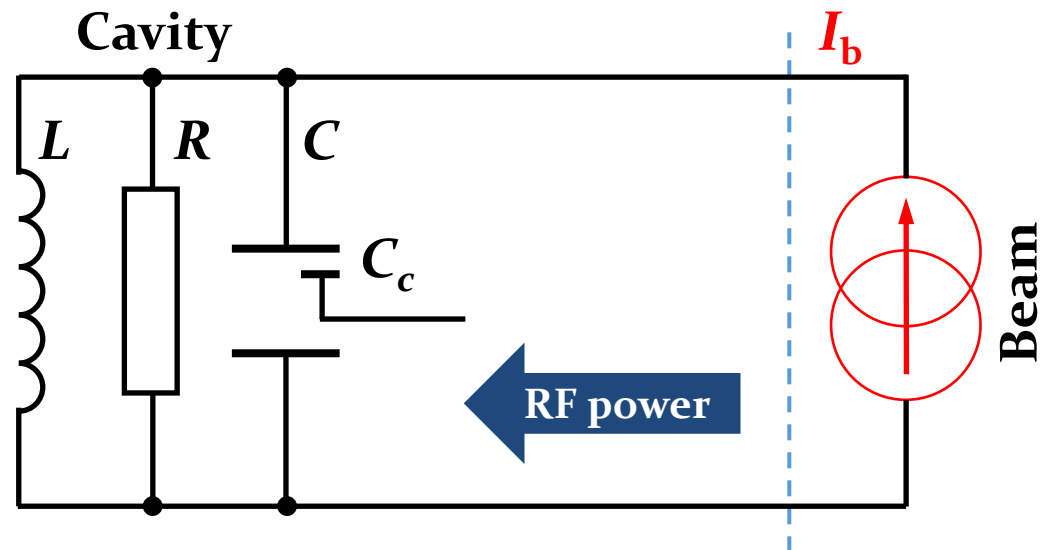
Coupling power into a cavity

- **Connection of cavity to power amplifier**
 - **Capacitive:** Capacitor coupling electrically to the gap
 - **Inductive:** Coupling loop in region of large magnetic field



Coupling power into a cavity

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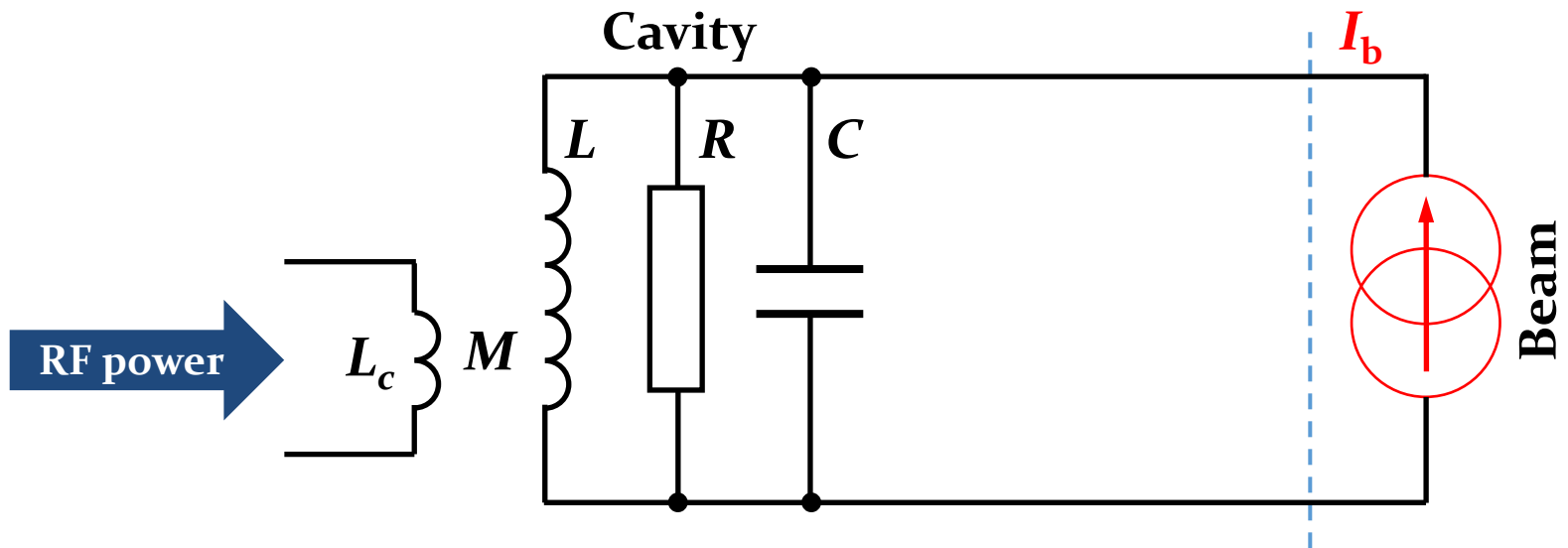


Capacitive coupler
of CERN PS 40 MHz



Coupling power into a cavity

- **Connection of cavity to power amplifier**
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 - **Inductive:** Coupling loop in region of large magnetic field



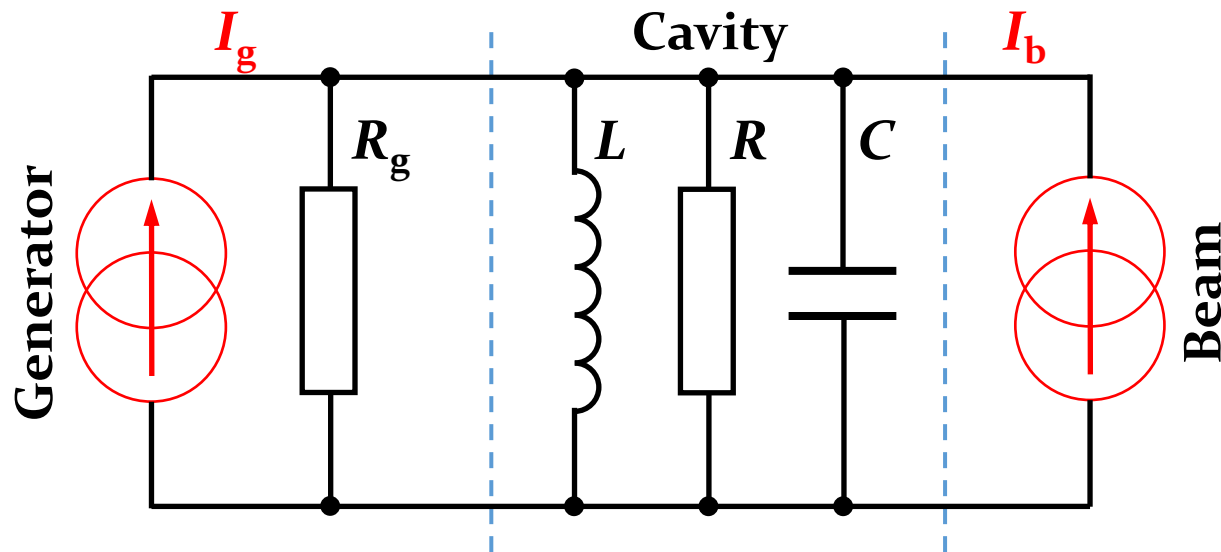
Main coupler
PSI cyclotron



L. Stingelin

Coupling power into a cavity

- Output impedance loads the resonant circuit: $R_g \parallel R$
- Reduction of quality factor: $Q_o \rightarrow Q_L$
- Coupling coefficient, β , defines coupling ratio



$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} = \frac{1}{Q_0} + \frac{\beta}{Q_0}$$

$$\frac{1}{R_L} = \frac{1}{R} + \frac{1}{R_g} = \frac{1}{R} + \frac{\beta}{R}$$

$$Q_L = Q_0 \frac{1}{1 + \beta}$$

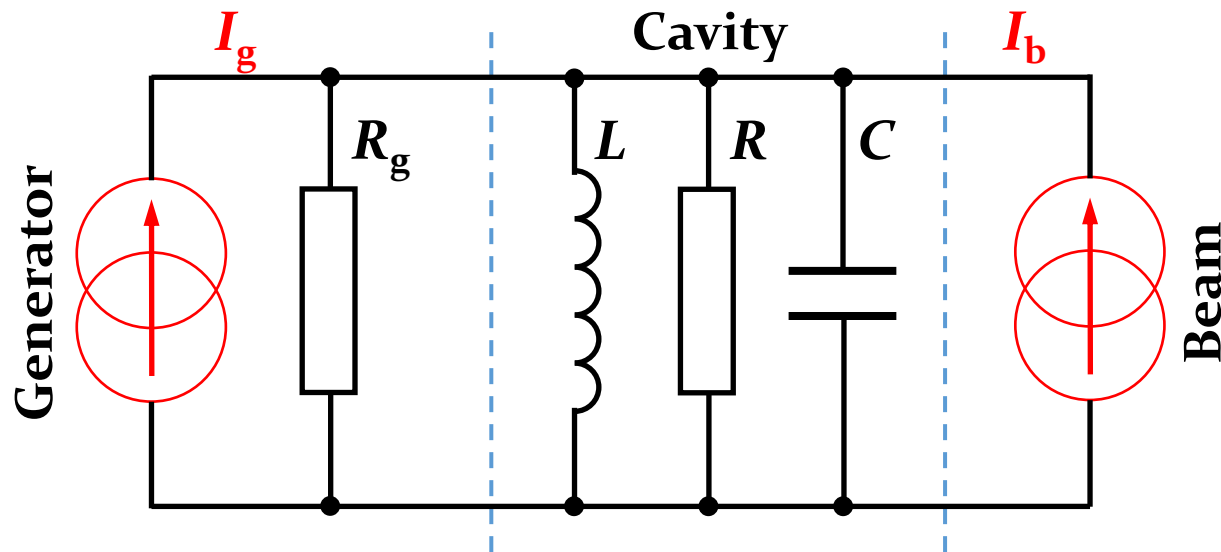
$$R_L = R \frac{1}{1 + \beta}$$

$$Q_{\text{ext}} = \frac{Q_0}{\beta}$$

$$R_g = \frac{R}{\beta}$$

Coupling power into a cavity

- Output impedance loads the resonant circuit: $R_g \parallel R$
- Reduction of quality factor: $Q_o \rightarrow Q_L$
- Coupling coefficient, β , defines coupling ratio



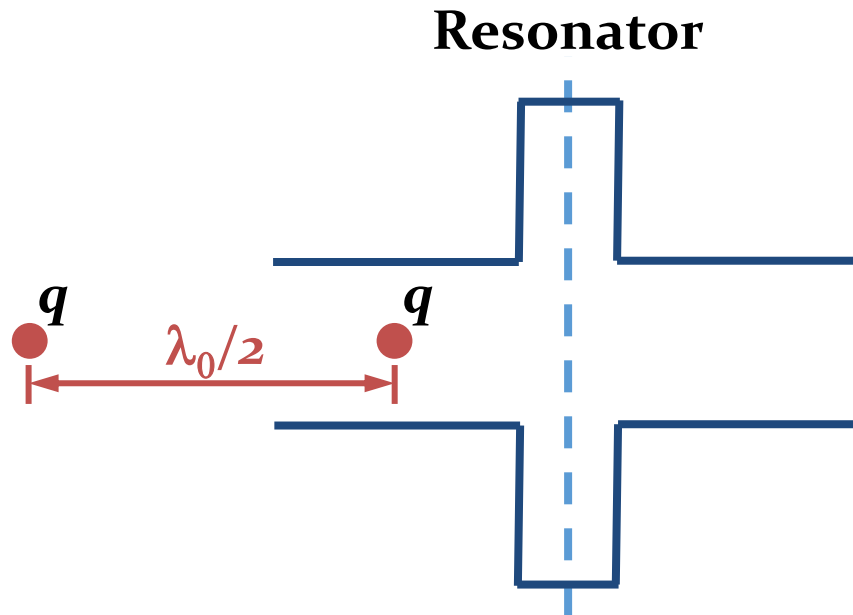
1. **Generator output impedance is not a physical resistor**
 - Generator does not experience own output impedance
2. **Beam experiences output impedance of generator as resistor**
 - $R_g \parallel R$ relevant for beam loading

Fundamental theorem of beam loading

Initially empty cavity

Which fraction does a charge experience of its induced voltage?

- Equal charges passing through cavity at distance $\lambda_0/2 = \pi c_0/\omega_0$
- Principles: **energy conservation** and **superposition**



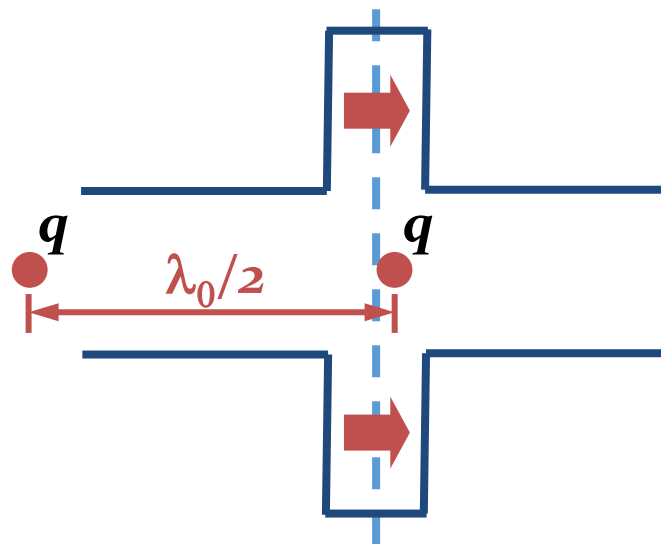
→ $W = 0$ and $V_c = 0$

After passage of first charge

- 1st charge passes through the cavity and induces voltage
- Fraction, r describes part of induced voltage affecting itself:

$$\Delta U_1 = r \cdot qV_{b1}$$

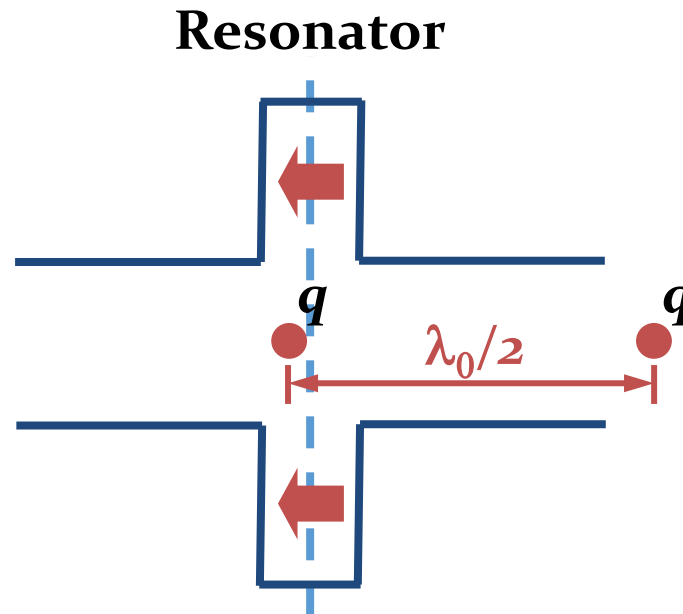
Resonator



→ $W = \Delta U_1$ and $V_c = V_{b1}$

Before passage of 2nd charge

- 2nd charge passes through the cavity
- Affected by induced field of 1st charge



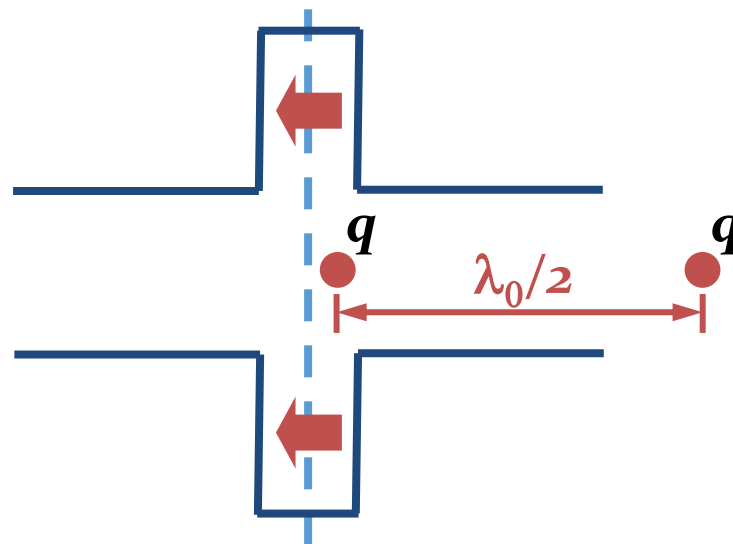
→ $W = \Delta U_1$ **and** $V_c = -V_{b1}$

Passage of 2nd charge

- 2nd charge passes through the cavity
- Affected by induced field of 1st charge and its own induced

$$\Delta U_2 = -qV_{b1} + r \cdot qV_{b2}$$

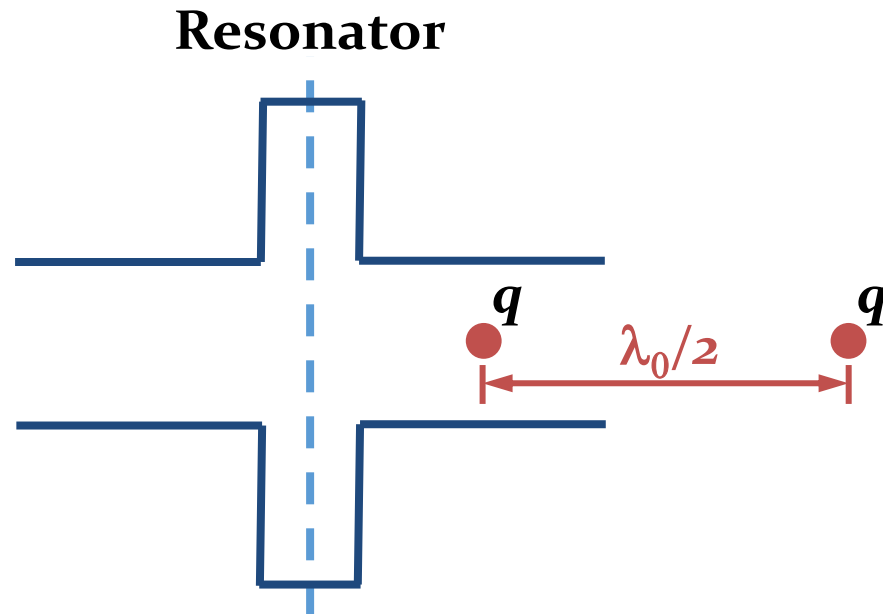
Resonator



→ $W = \Delta U_1$ and $V_c = -V_{b1}$

After passage of first bunch

- After passage of 2nd charge through the cavity
- Takes the same energy as brought into cavity by 1st charge



→ $W = 0$ and $V_c = -V_{b1} + V_{b2} = 0$

Ratio of induced field

→ Total energy brought in and taken out of cavity **must be zero**

$$\Delta U_1 + \Delta U_2 = 0$$

$$r \cdot qV_{b1} - qV_{b1} + r \cdot qV_{b2} = 0$$

$$r (V_{b1} + V_{b2}) = V_{b1}$$

$$2r \cdot V_{b0} = V_{b0}$$

$$\rightarrow r = \frac{1}{2}$$

→ Fundamental theorem of beam loading:

**Charge passing through a resonator sees
 $\frac{1}{2}$ of its induced voltage: $V_b = \frac{1}{2} V_{b0}$**

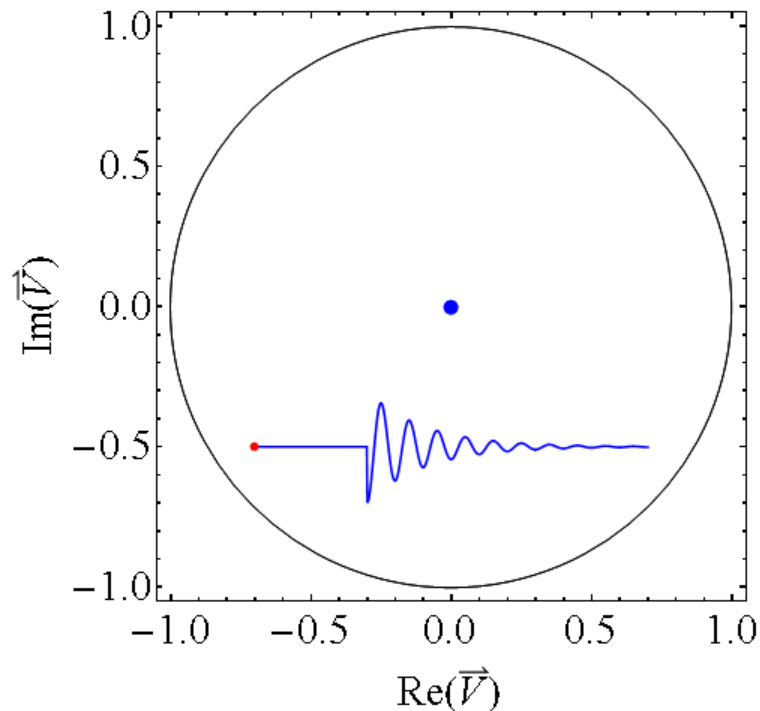
Single passage through a cavity

Vector representation

- Passing charge induces voltage
- Voltage vector rotates with resonance frequency of cavity

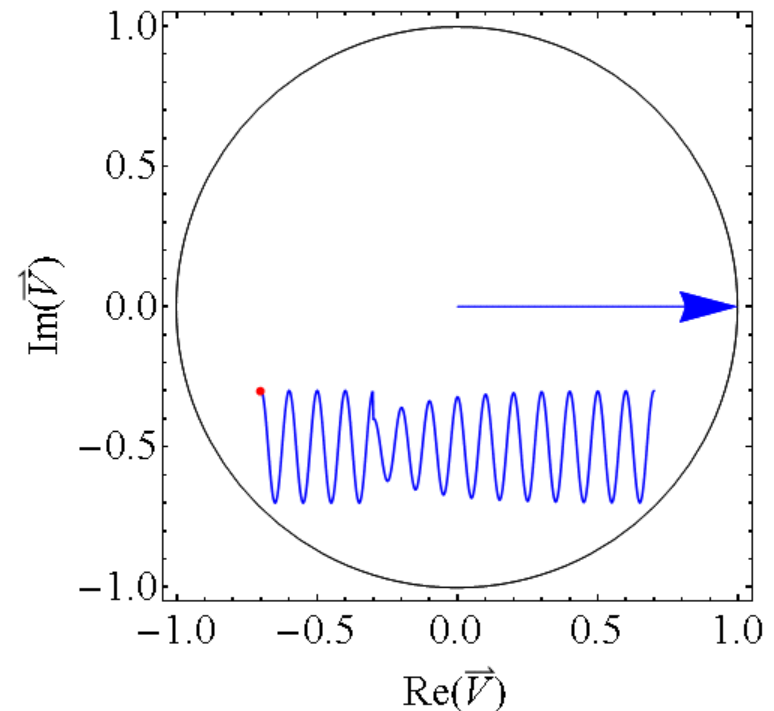
Cavity without external drive

Time $[2\pi/\omega_0]$: -4.00



Cavity driven by external source

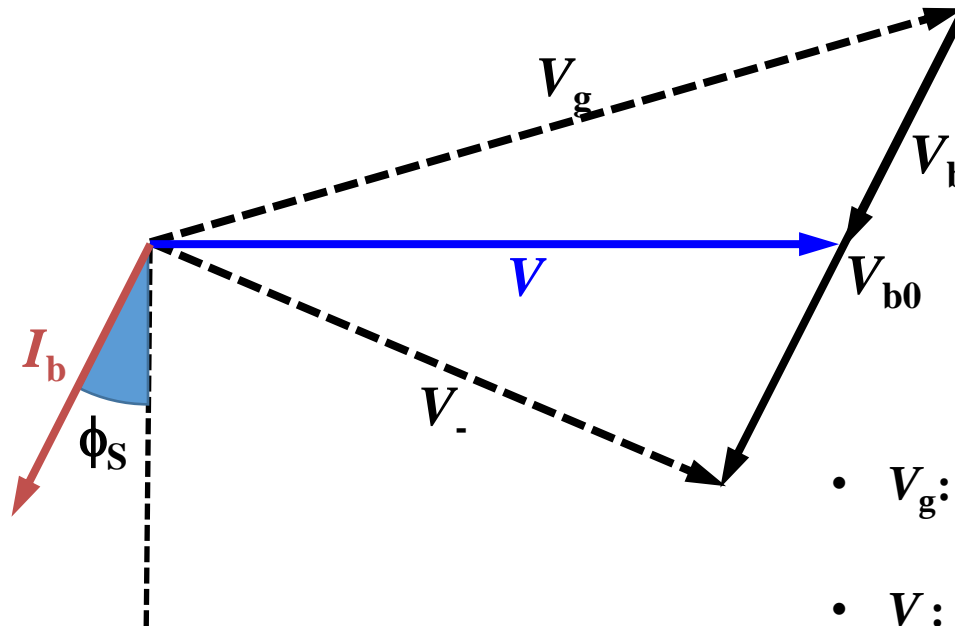
Time $[2\pi/\omega_0]$: -4.00



- Vector rotation with ω_0 not relevant
- Need cavity voltage at arrival of next charge

Single passage

- Vector diagram at the instant of the bunch passage:



- V_g : Generator driven voltage before bunch passage
- V_- : Voltage after bunch passage
- V : Net voltage seen by beam

→ Vector sum:
$$\vec{V} = \vec{V}_g + \vec{V}_b = \vec{V}_g + \frac{1}{2}\vec{V}_{b0}$$

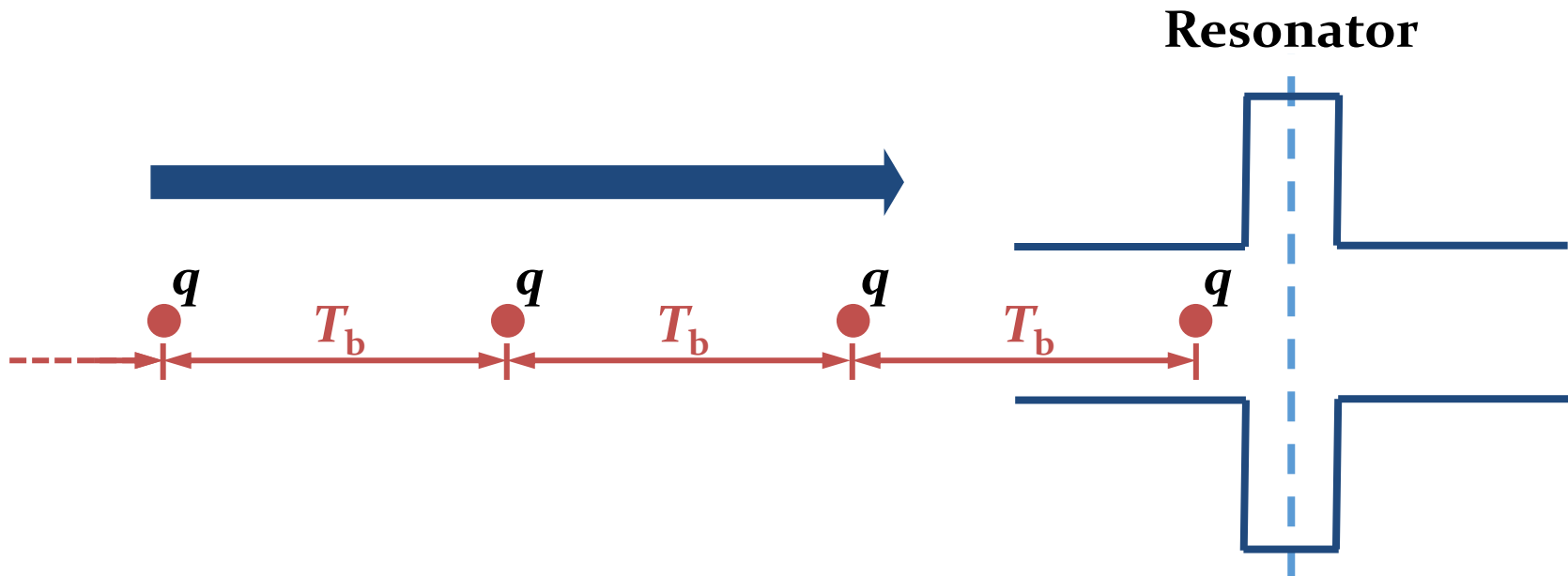
→ Induced voltage changes cavity phase: **detuning**

→ **De-phase** generator to obtain expected net voltage

Multiple passages through a cavity

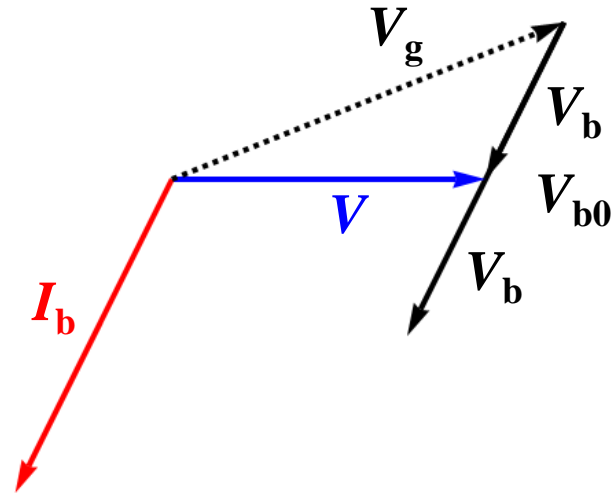
Multiple passage of bunches

- Resonator excited by chain of charges or particle bunches
 1. Fields in resonator decay from one charge to the next
→ Single passage case
 2. Field from previous still present
→ Accumulation of induced voltages



Multiple passages

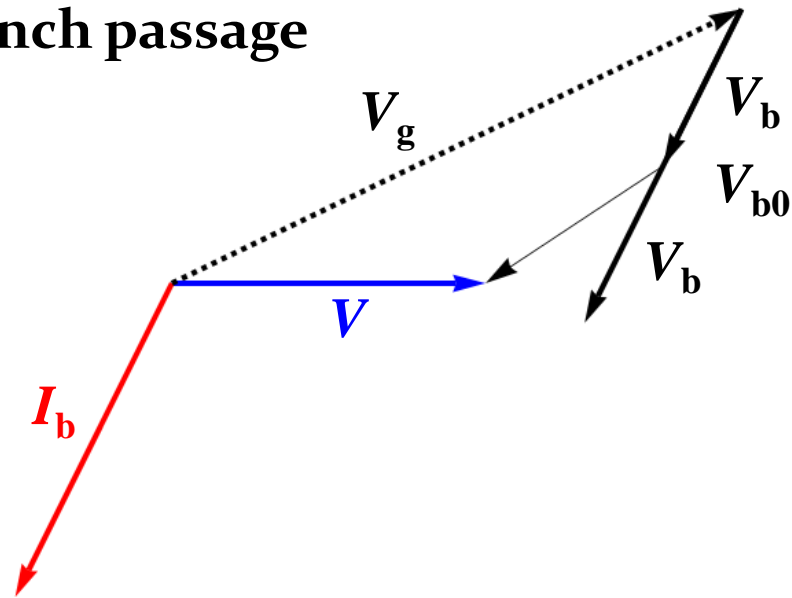
- Arrange generator phase and voltage for real net voltage
- After 1st bunch passage



$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0}$$

Multiple passages

- Arrange generator phase and voltage for real net voltage
- After 2nd bunch passage



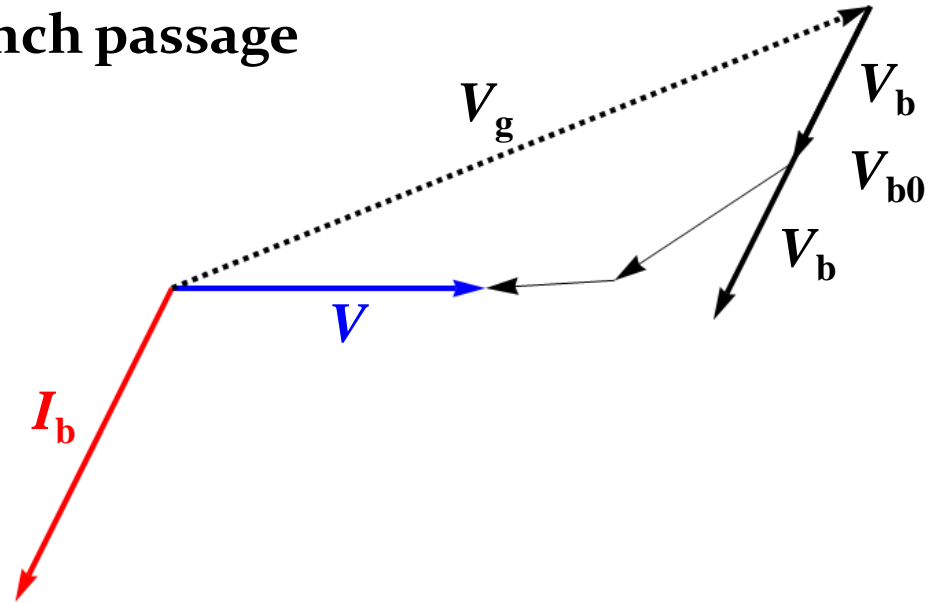
$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0}e^{-\delta}e^{i\Psi}$$

$$\rightarrow \text{Induced voltage of 1st passage decayed: } e^{-\delta} \quad \text{with} \quad \delta = \frac{T_b}{T_f}$$

$$\rightarrow \text{Phase advance between two bunches: } e^{-i\Psi} \quad \text{with} \quad \Psi = \omega_0 T_b - 2\pi h_b$$

Multiple passages

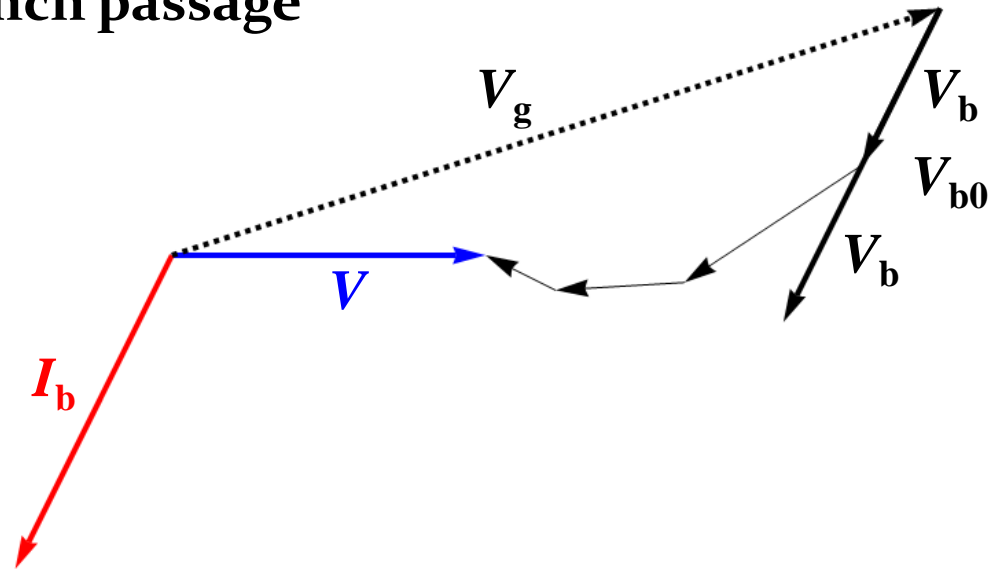
- Arrange generator phase and voltage for real net voltage
- After 3rd bunch passage



$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0}e^{-\delta}e^{i\Psi} + \vec{V}_{b0}e^{-2\delta}e^{2i\Psi}$$

Multiple passages

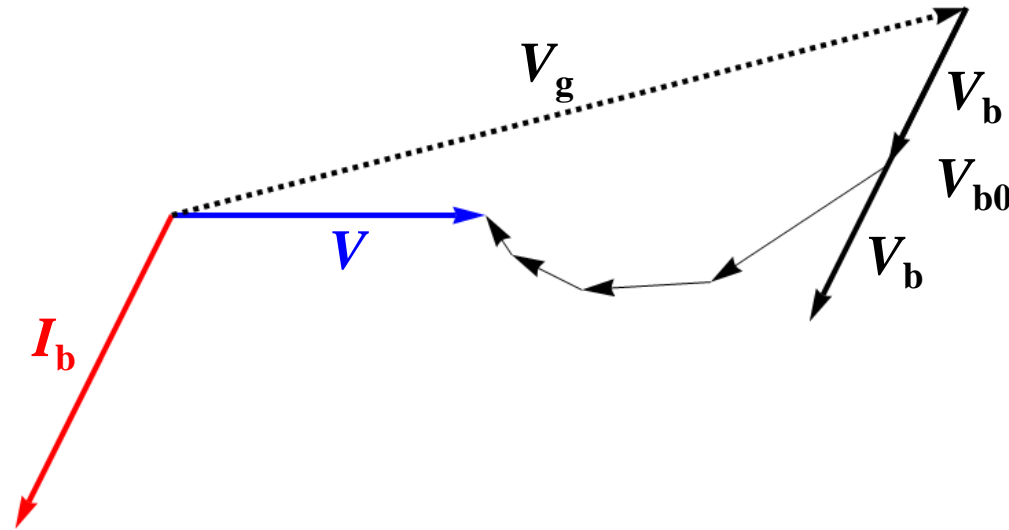
- Arrange generator phase and voltage for real net voltage
- After 4th bunch passage



$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0} (e^{-\delta}e^{i\Psi} + e^{-2\delta}e^{2i\Psi} + \dots)$$

Multiple passages

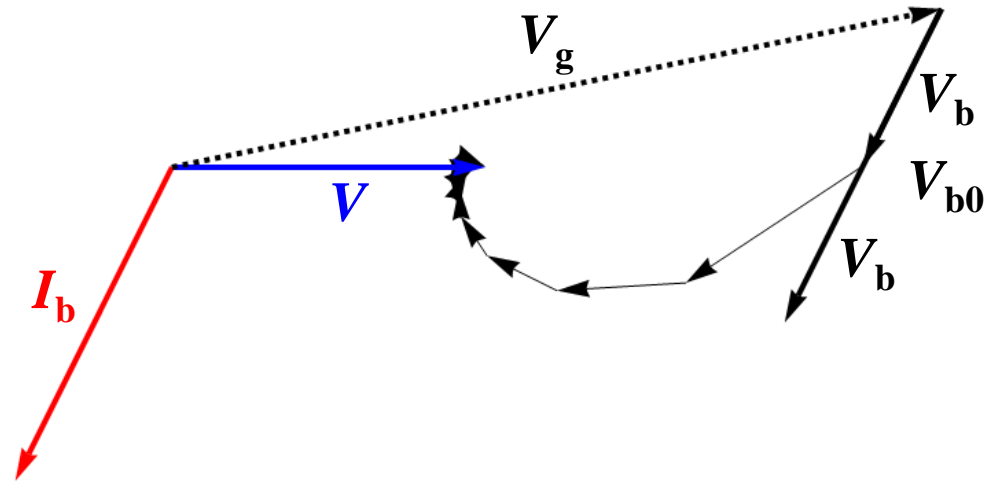
- Arrange generator phase and voltage for real net voltage
- After 5th bunch passage



$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0} (e^{-\delta} e^{i\Psi} + e^{-2\delta} e^{2i\Psi} + \dots)$$

Multiple passages

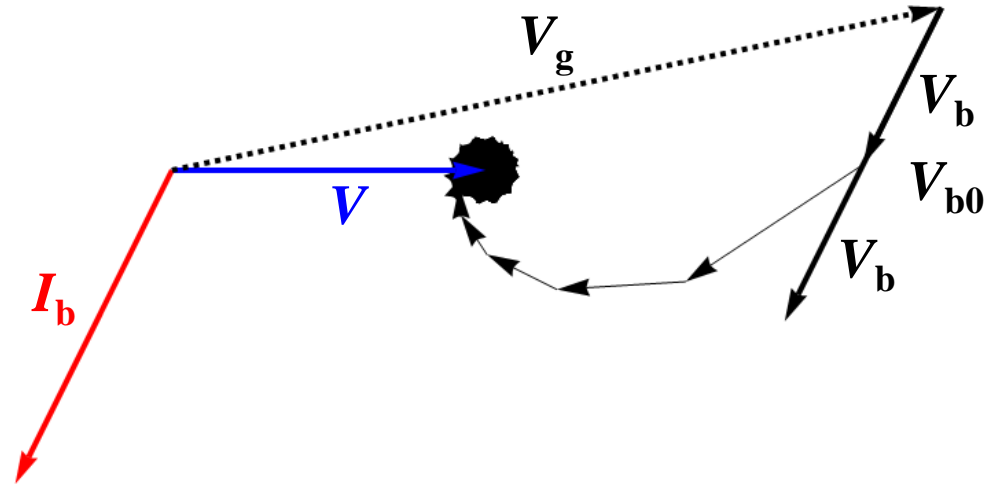
- Arrange generator phase and voltage for real net voltage
- After **10th** bunch passage



$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0} (e^{-\delta} e^{i\Psi} + e^{-2\delta} e^{2i\Psi} + \dots)$$

Multiple passages

- Arrange generator phase and voltage for real net voltage
- After **100th** bunch passage



$$\rightarrow \vec{V} = \vec{V}_g + \frac{1}{2}\vec{V}_{b0} + \vec{V}_{b0} (e^{-\delta} e^{i\Psi} + e^{-2\delta} e^{2i\Psi} + \dots)$$

$$\rightarrow \text{Infinite passages: } 1 + e^{-\delta} e^{i\Psi} + e^{-2\delta} e^{2i\Psi} + \dots = \frac{1}{1 - e^{-\delta} e^{i\Psi}}$$

General beam induced voltage

$$\vec{V}_b = \vec{V}_{b0} \left(\frac{1}{1 - e^{-\delta} e^{i\Psi}} - \frac{1}{2} \right)$$

Separate **real** and
imaginary part:



$$V_b = V_{b0} [F_1(\delta, \Psi) + iF_2(\delta, \Psi)]$$

$$F_1(\delta, \Psi) = \frac{1 - e^{-2\delta}}{2(1 - 2e^{-\delta} \cos \Psi + e^{-2\delta})}$$

$$F_2(\delta, \Psi) = \frac{e^{-\delta} \sin \Psi}{1 - 2e^{-\delta} \cos \Psi + e^{-2\delta}}$$

Change of variables

- Variables for damping, δ , and bunch-by-bunch phase advance, Ψ , **not very practical**
- New variables with RF system parameters:

1. Coupling coefficient, β

$$1 + \beta = \frac{Q_0}{Q_L}$$

2. Cavity tuning angle, ϕ_c

$$\tan \phi_c = 2Q_L \frac{\omega_0 - \omega}{\omega_0}$$

$$Z_L(\omega) = \frac{R}{1 + 2iQ_L \frac{\Delta\omega}{\omega_0}}$$

$$Z_L(\phi_c) = \frac{R}{1 - i \tan \phi_c}$$

$$\delta = \delta_0 (1 + \beta)$$

$$\Psi = \delta_0 (1 + \beta) \tan \phi_c$$

$$\delta_0 = \frac{T_b}{T_{f0}} = \frac{2Q_0}{\omega_0}$$

Beam induced voltage in new variables

$$V_b = I_b R \delta_0 [F_1(\beta, \phi_c) + iF_2(\beta, \phi_c)]$$

$$F_1(\beta, \phi_c) = \frac{1 - e^{-2\delta_0(1+\beta)}}{2D}$$

$$F_2(\beta, \phi_c) = \frac{e^{-\delta_0(1+\beta)} \sin [\delta_0(1 + \beta) \tan \phi_c]}{D}$$

with denominator

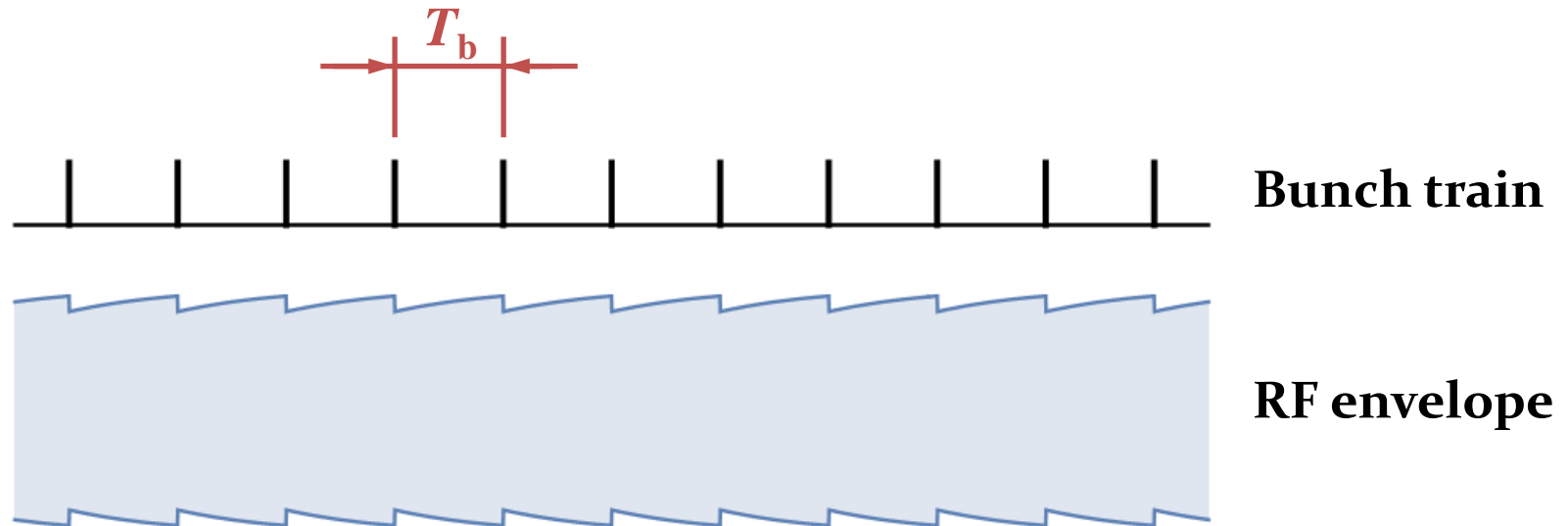
$$D = 1 - 2e^{-\delta_0(1+\beta)} \cos [\delta_0(1 + \beta) \tan \phi_c] + e^{-2\delta_0(1+\beta)}$$

→ **Numerical computations required for analysis**

→ **Let us look at a particularly relevant approximation: $\delta_0 \simeq 0$**

Approximation

- Bunch distance short compared filling time: $\delta_0 \simeq 0$
- Approximate terms including $\mathcal{O}(\delta_0^2)$



Approximation

- Bunch distance short compared filling time: $\delta_0 \simeq 0$
- Approximate terms including $\mathcal{O}(\delta_0^2)$

$$V_b = I_b R \delta_0 [F_1(\beta, \phi_c) + iF_2(\beta, \phi_c)]$$

$$F_1(\beta, \phi_c) \simeq \frac{1}{\delta_0(1 + \beta)(\tan^2 \phi_c + 1)}$$

$$F_2(\beta, \phi_c) \simeq \frac{\tan \phi_c}{\delta_0(1 + \beta)(\tan^2 \phi_c + 1)}$$

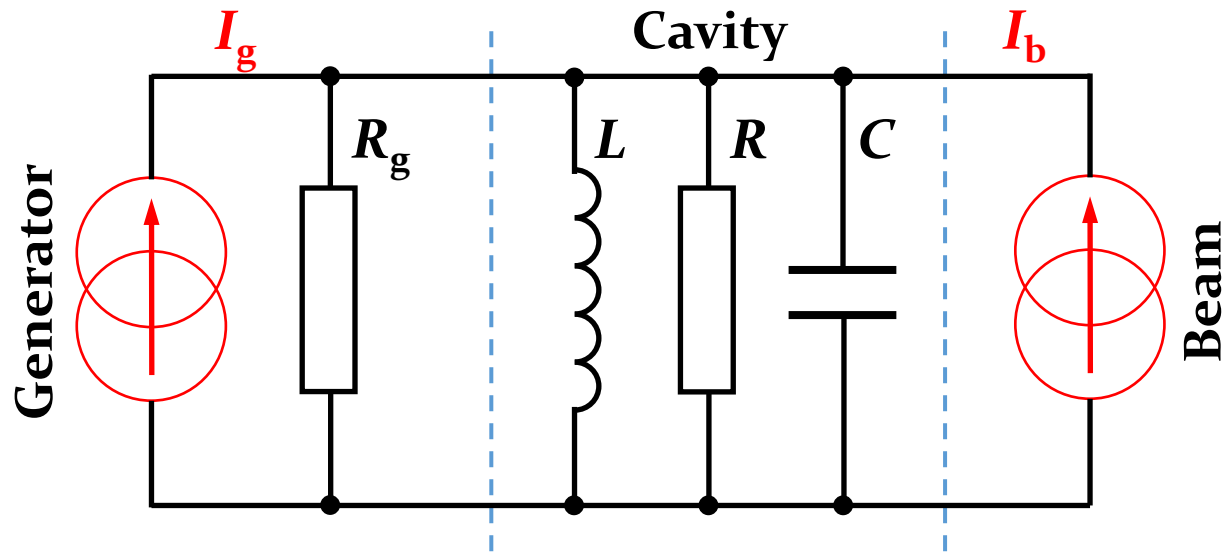
$$V_b \simeq \frac{I_b}{(1 + \beta)} \frac{R}{1 - i \tan \phi_c} = I_b \frac{Q_L}{Q_0} Z_L(\phi_c)$$

- Ohm's law for the loaded cavity impedance: steady state case

Steady state beam loading

Equivalent circuit model

- Lumped element circuit model for steady state case

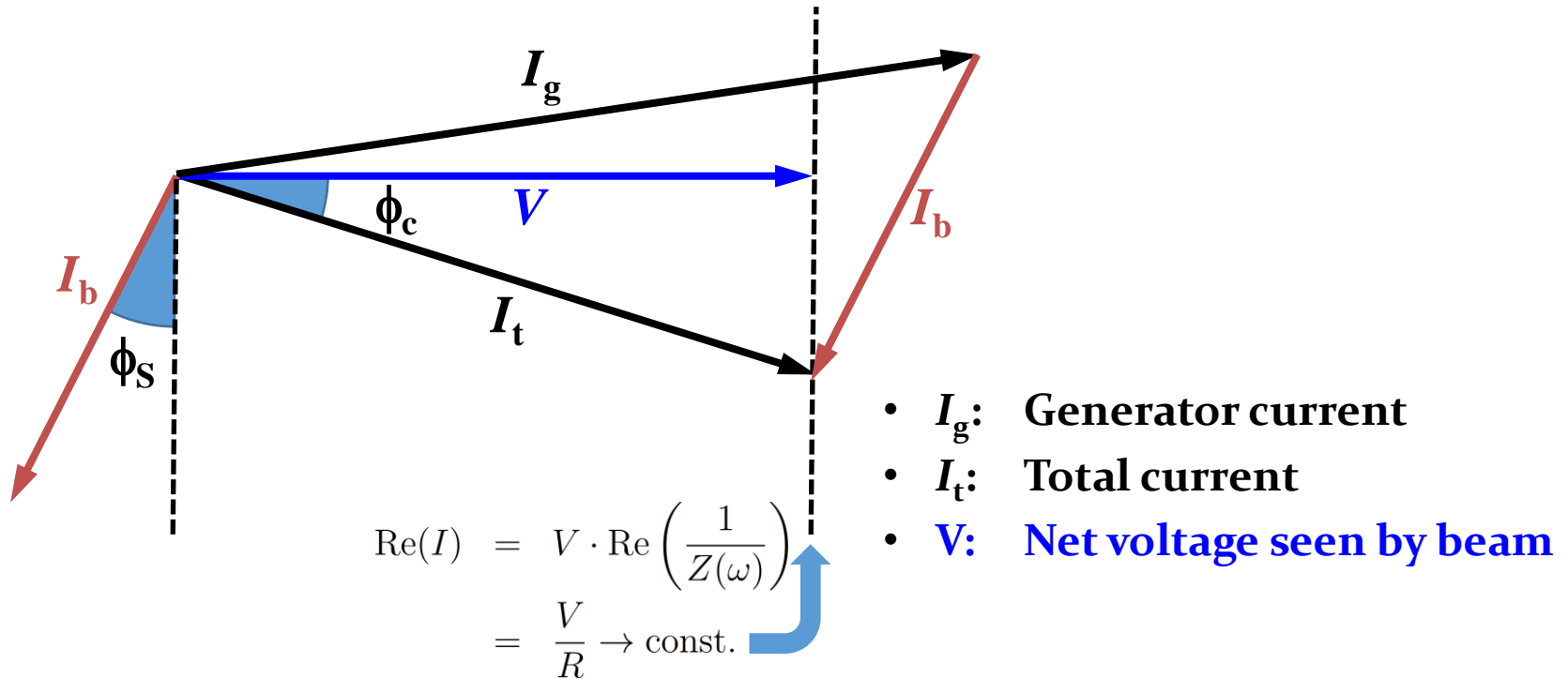


→ Total current: $\vec{I}_t = \vec{I}_g + \vec{I}_b$

→ Power required from generator: $P_g = \frac{1}{2} R_g I_g^2$

Steady state

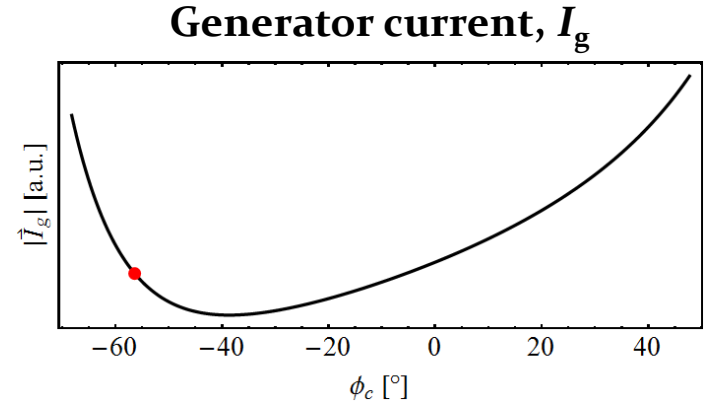
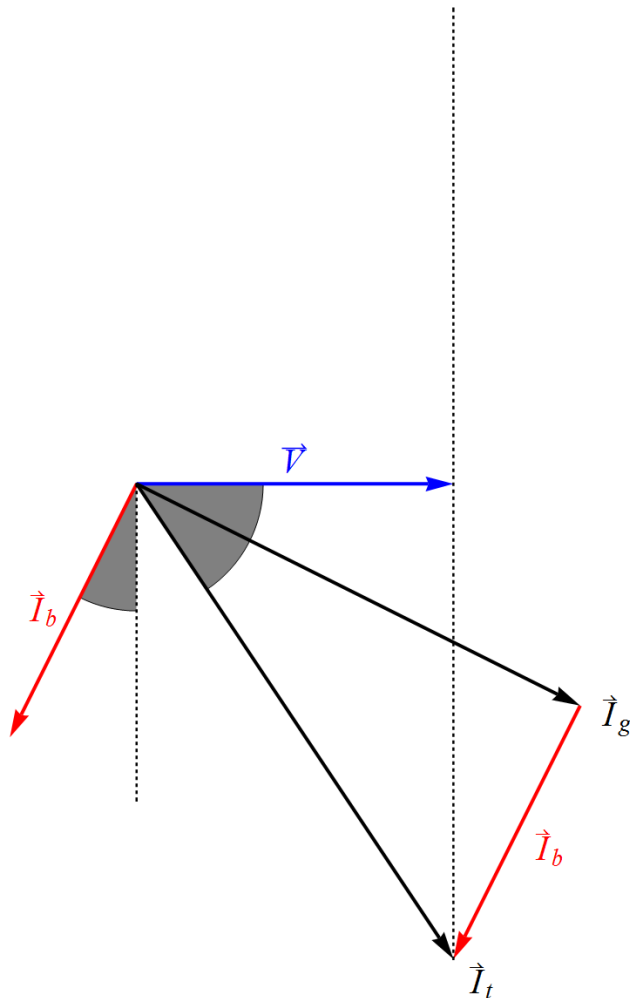
- Vector diagram for passage of continuous bunch train



→ Parameters to achieve minimum generator current?

Steady state: minimum generator current

- Vector diagram for passage of continuous bunch train



- I_g : Generator current
- I_t : Total current
- V : Net voltage seen by beam

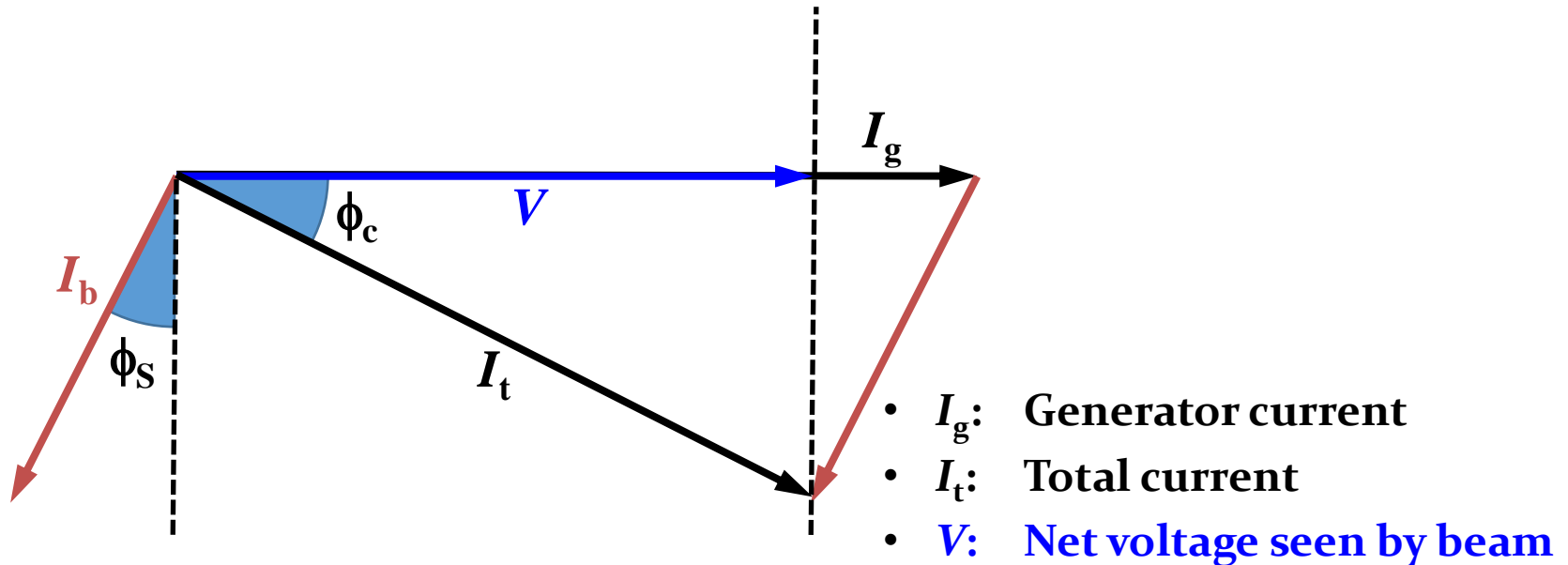
Lowest power (current I_g)

→ Generator current, I_g in phase with voltage

→ Resistive load with beam

Steady state: minimum generator current

- Vector diagram for passage of continuous bunch train

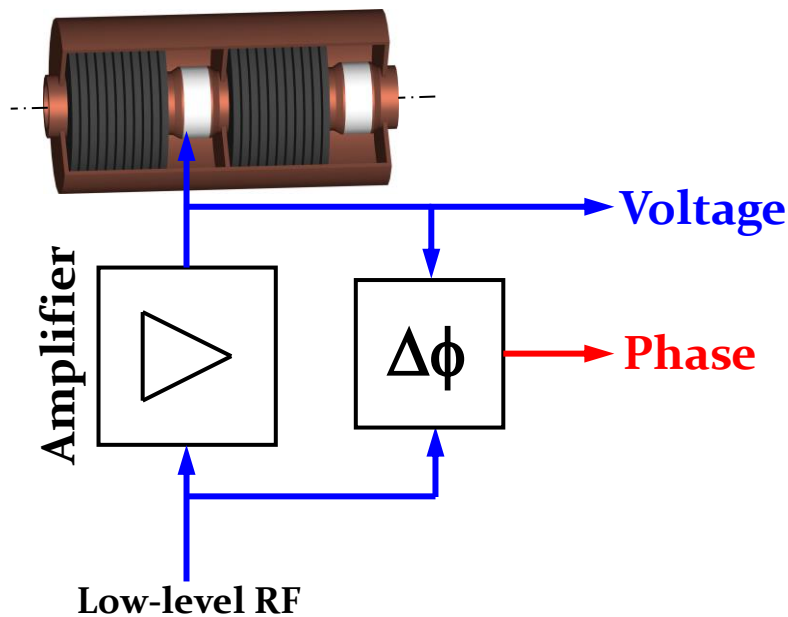


$$\begin{aligned} \rightarrow \tan \phi_c &= -\frac{I_b}{V} \frac{R}{1 + \beta} \cos \phi_s \\ &= -\frac{I_b}{V} Q_L \left(\frac{R}{Q_0} \right) \cos \phi_s \\ \beta &= 1 + I_b \frac{R}{V} \sin \phi_s \end{aligned}$$

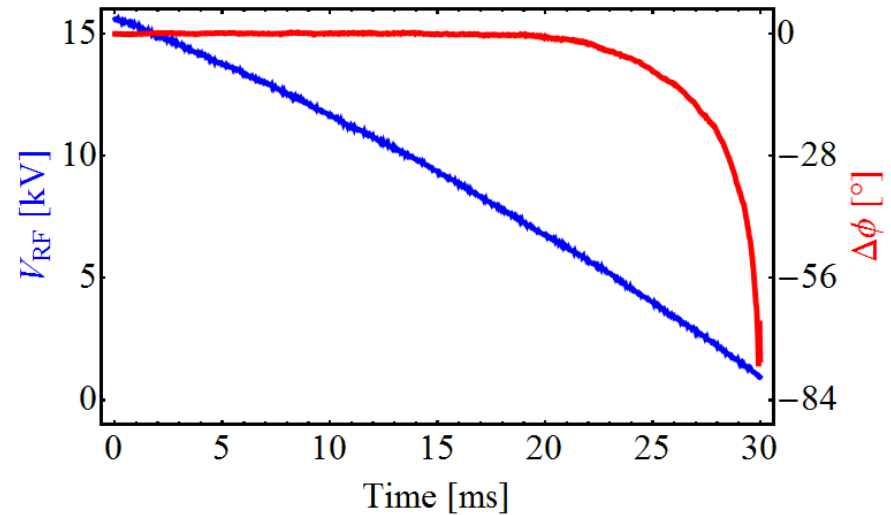
Example: Cavity dephasing in PS

- Voltage descent with beam:

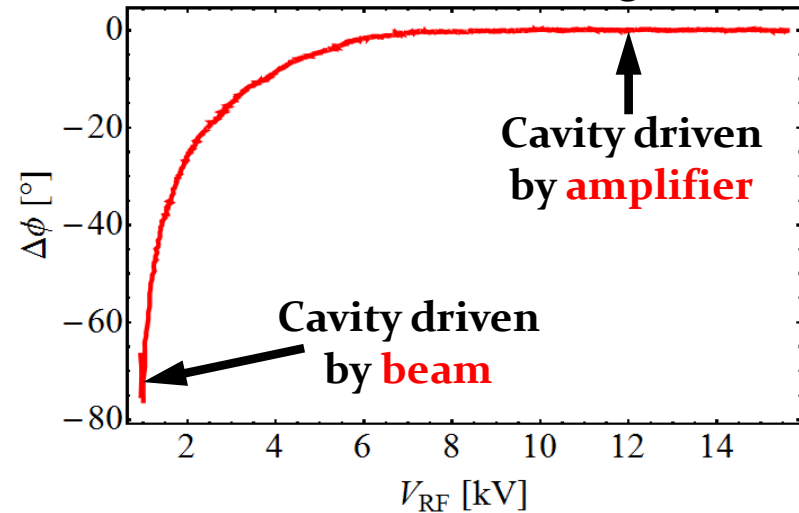
$$\frac{\Delta\omega}{\omega_0} \propto I_b \cdot \frac{1}{V}$$



Voltage and phase vs. time



Phase versus voltage



- Tuning loop recovers cavity resonance frequency
- Dephasing at low RF voltage

Steady state: minimum generator current

- **Minimum power:** $P_g = \frac{V^2}{2R} + V I_{b,DC} \sin \phi_S$
 ($I_b \simeq 2I_{b,DC}$)
 (short bunches)

Resistive losses
in cavity



Power delivered
to beam



1. **Optimum detuning:** $\frac{\omega - \omega_0}{\omega_0} = \frac{\Delta\omega}{\omega_0} = \frac{1}{2} \frac{I_b}{V} \left(\frac{R}{Q_0} \right) \cos \phi_S$

→ Cavity and beam appear as resistive load to generator

→ Automatically adjusted by cavity tuning loop

2. **Optimum coupling:** $\beta = 1 + I_b \frac{R}{V} \sin \phi_S$

→ Usually mechanically fixed by construction

Example: LHC power coupler

- Control of both **cavity resonance frequency and coupling**
- Optimize quality through Q_{ext} for injection and storage

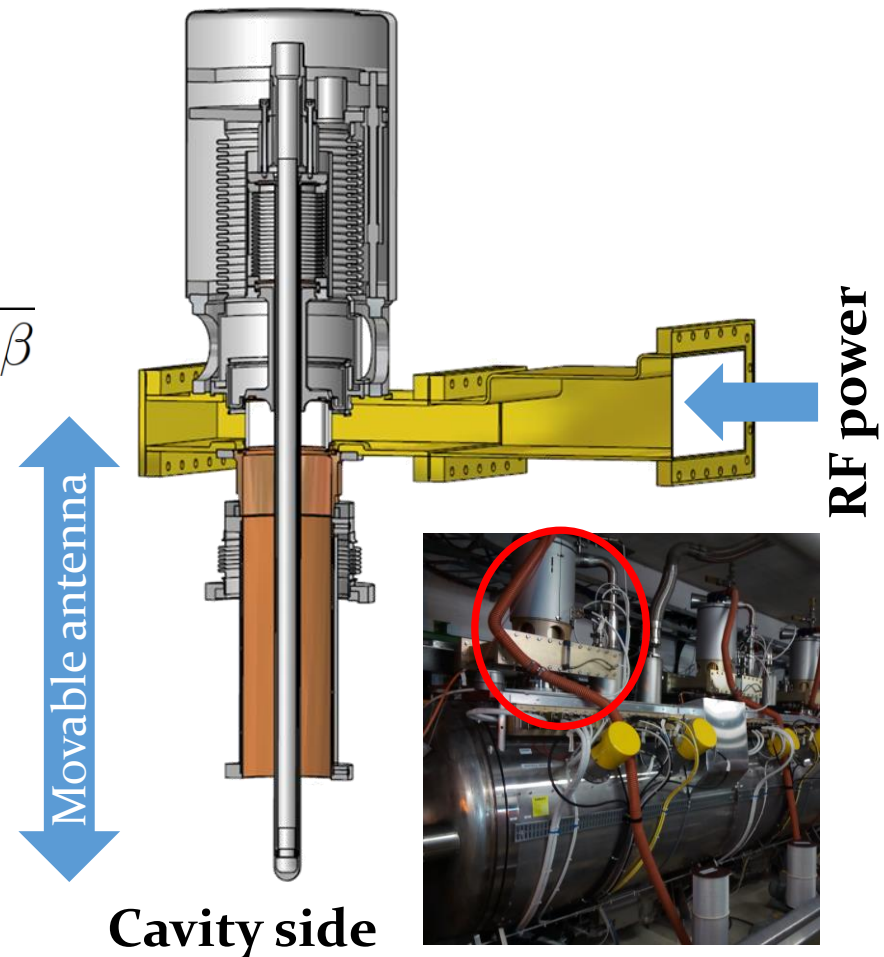
$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}}$$

(Note: In the original image, the $1/Q_0$ term is crossed out with a red line and a red arrow points to a ~ 0 above it.)

$$Q_L = Q_0 \frac{1}{1 + \beta}$$

Loaded quality factor:

Mode	Q_L, Q_{ext}	Comment
Injection	$\sim 2 \cdot 10^4$	Suppress transients
Collision	$\sim 6 \cdot 10^4$	Maximum voltage



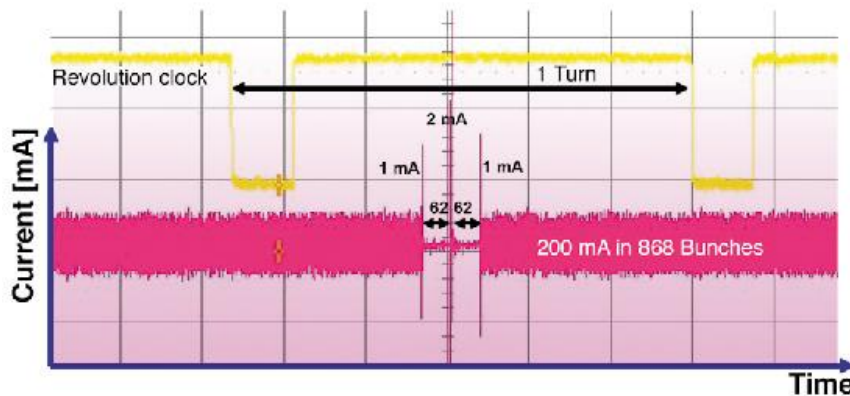
Filling pattern with gaps

Why leaving a gap and not filling full ring?

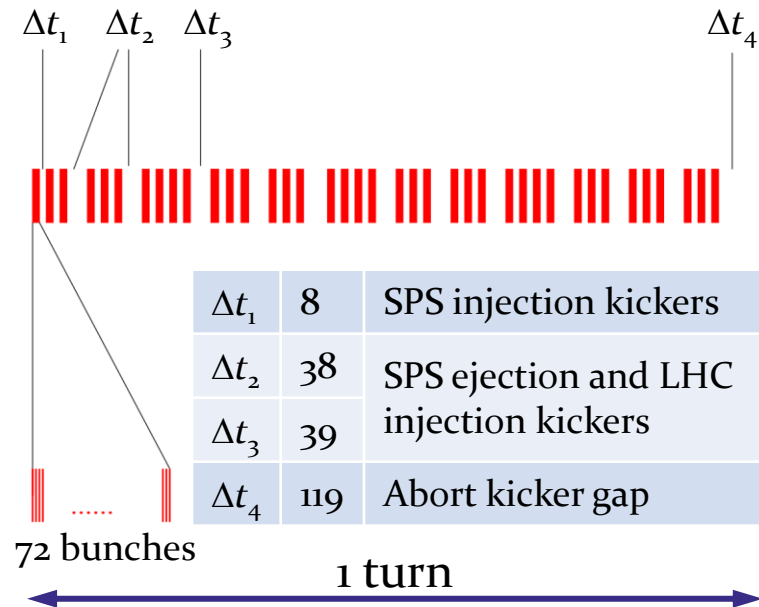
- **Electron storage rings:** Clear ions attracted by electron beam
- **Hadron accelerator:** Leave gap for kicker magnets at injection/ejection

ESRF: 7/8 + 1 filling mode

868	23 uA/b	200 mA in 7/8 train
2	1 mA/b	Marker bunches
1	2 mA	Single bunch
2×62	<2 pA/b	Gap

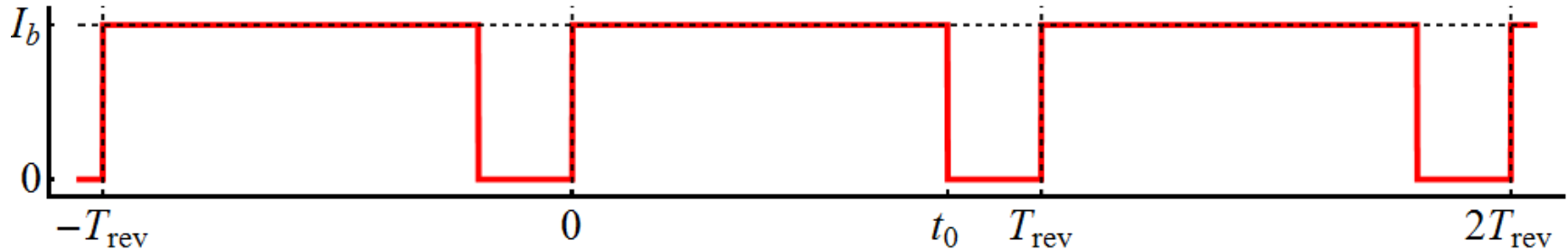


LHC: original nominal

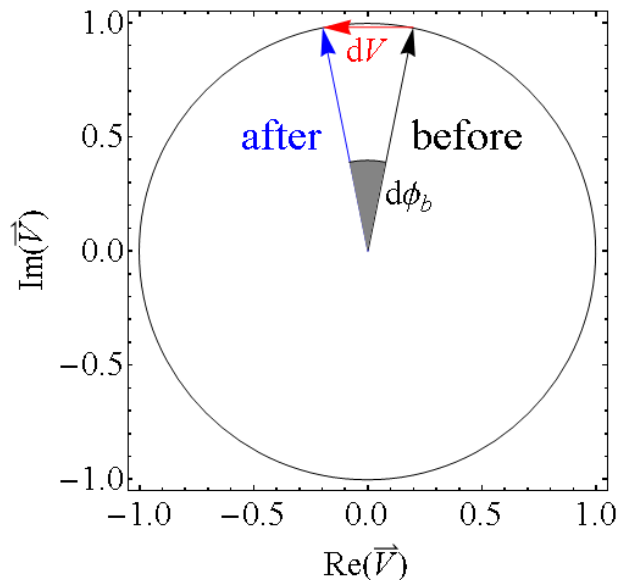


Beam loading with gaps

- **Limitations:** $\delta_0 \simeq 0$, no acceleration, lossless cavity



- **Phase change due to cavity detuning:** $d\phi_a = \Delta\omega dt$
- **Phase change due to induced voltage:** $d\phi_b = \frac{1}{V} dV$ with



$$dV = \frac{1}{2} \left(\frac{R}{Q_0} \right) \omega_0 I_b(t) dt$$

→ **Total phase advance:**

$$d\phi = \left[\Delta\omega - \frac{1}{2} \left(\frac{R}{Q_0} \right) \frac{\omega_0}{V} I_b(t) \right] dt$$

Beam loading with gaps

→ **Periodicity condition** $\int_{1\text{turn}} d\phi = 0$ **to get average detuning**

$$\Delta\omega_0 = \frac{1}{2} \left(\frac{R}{Q_0} \right) \frac{\omega_0}{V} \frac{1}{T_{\text{rev}}} \int_0^{T_{\text{rev}}} I_b(t) dt = \frac{1}{2} \left(\frac{R}{Q_0} \right) \frac{\omega_0}{V} \bar{I}_b$$

→ **and phase along the circumference**

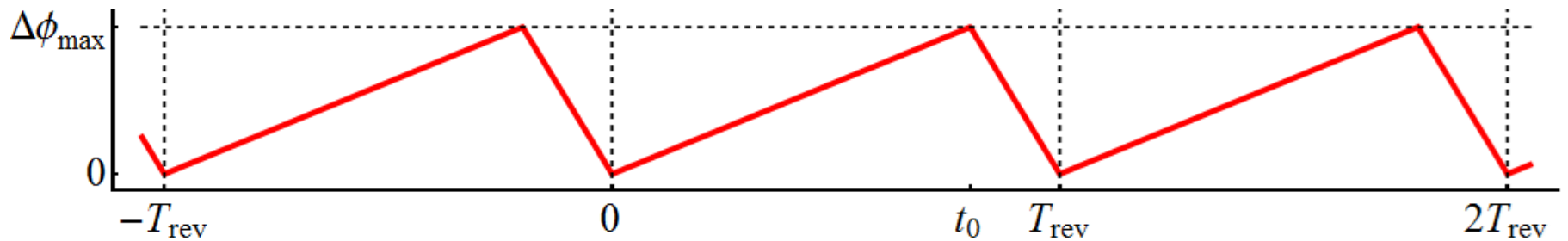
$$\phi(t) = \int_0^t d\phi = \frac{1}{2} \left(\frac{R}{Q_0} \right) \frac{\omega_0}{V} \int_0^t [\bar{I}_b - I_b(t)] dt$$

→ **Phase changes linearly for $I_b(t) = \text{const.}$ during beam region**

Maximum phase excursion

- **Maximum phase excursion**

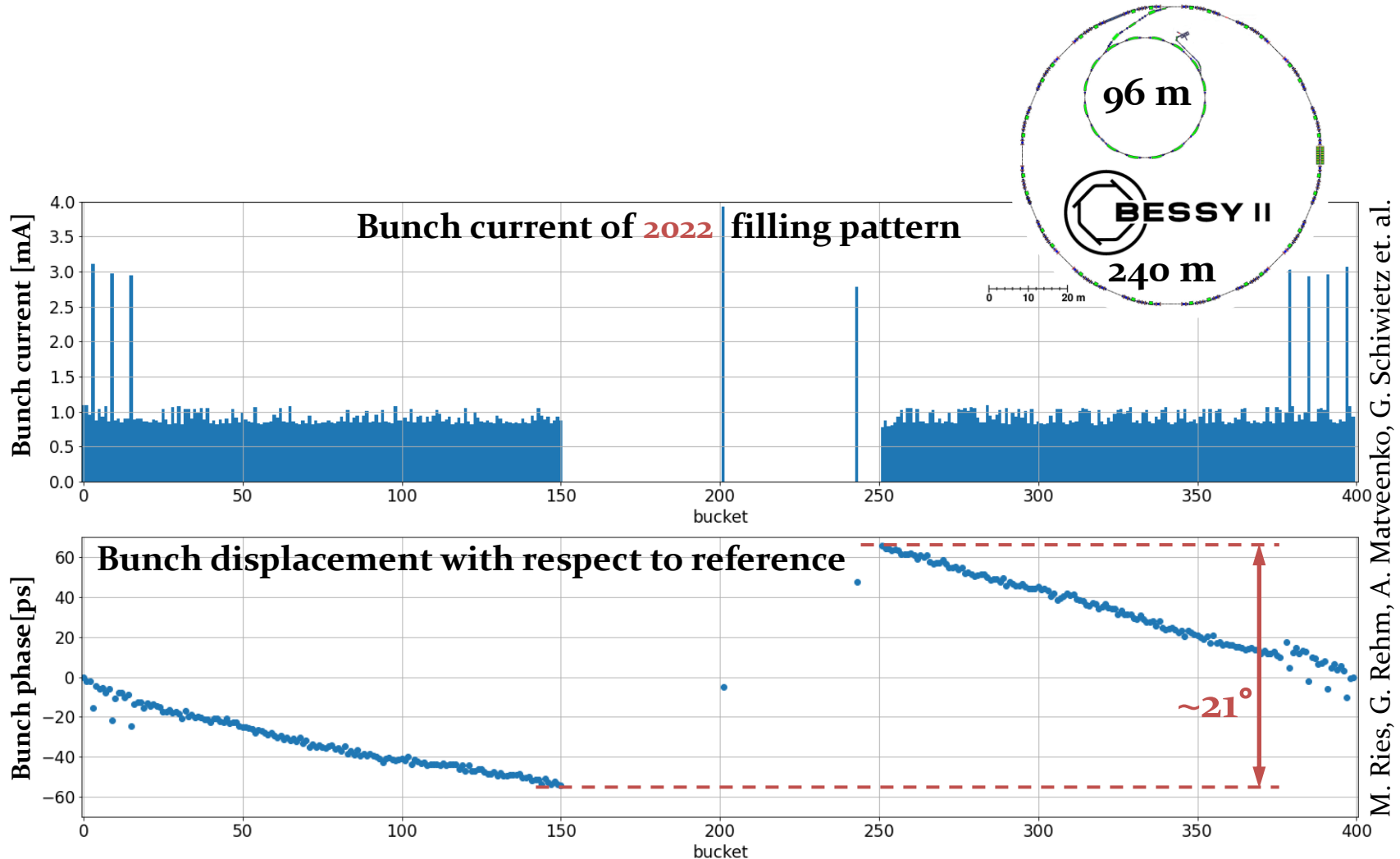
$$\Delta\phi_{\max} = \frac{1}{2} \left(\frac{R}{Q_0} \right) \frac{\omega_0}{V} \bar{I}_b (T_{\text{rev}} - t_0) = \Delta\omega_0 (T_{\text{rev}} - t_0)$$



- **Displaces timing** of synchrotron radiation pulses
- **Longitudinally moves collision point** in collider
- **Compromise** between RF power and collision point

Example: Electron storage ring

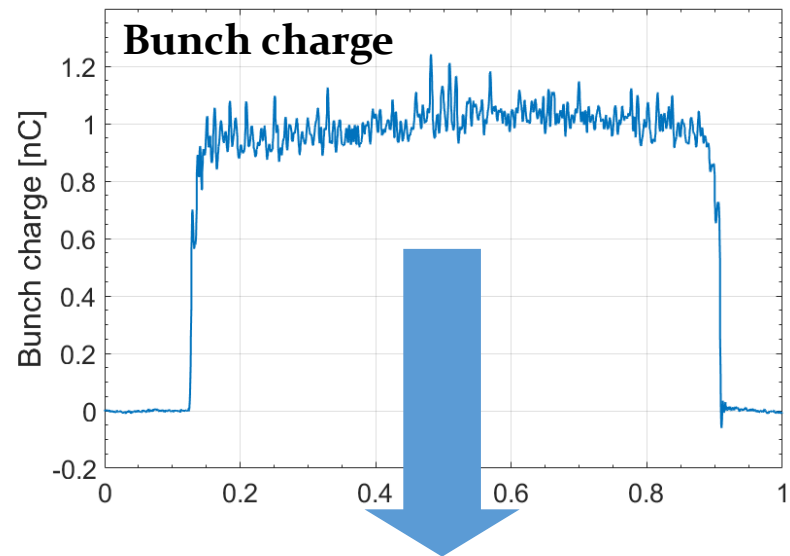
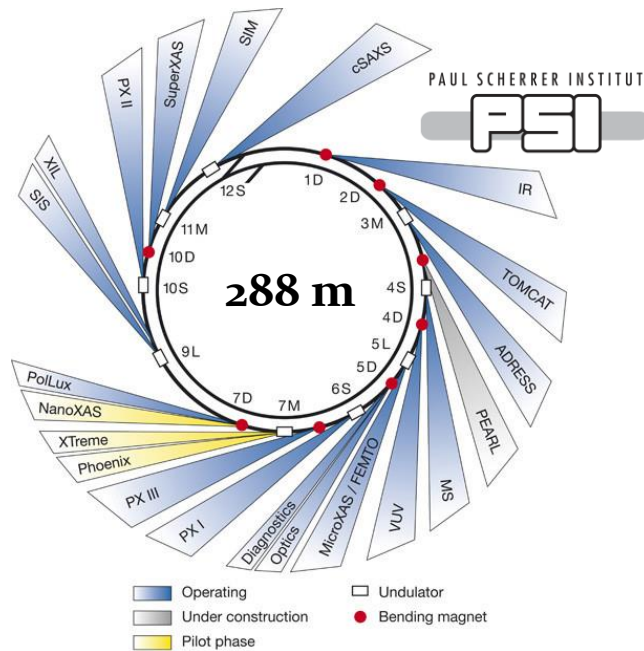
- Transient beam loading in **electron storage ring BESSY II**



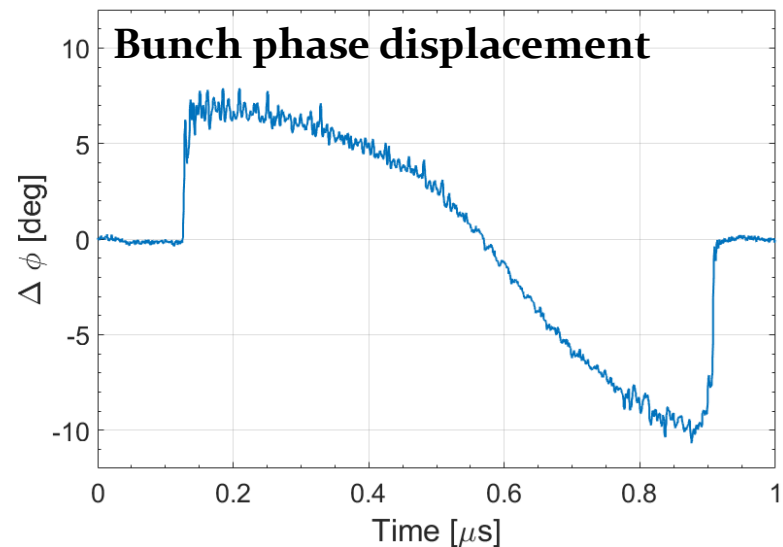
→ Synchrotron radiation light pulses slightly shifted in time

Example: Electron storage ring

- **Transient beam loading in electron storage ring SLS**

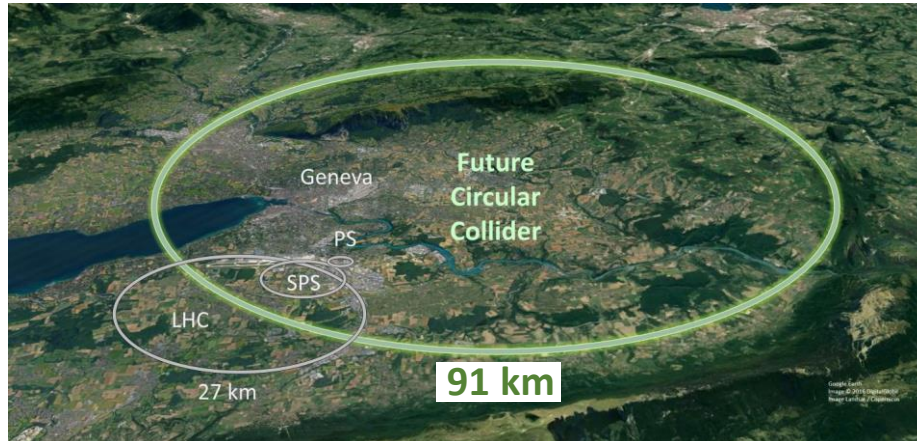


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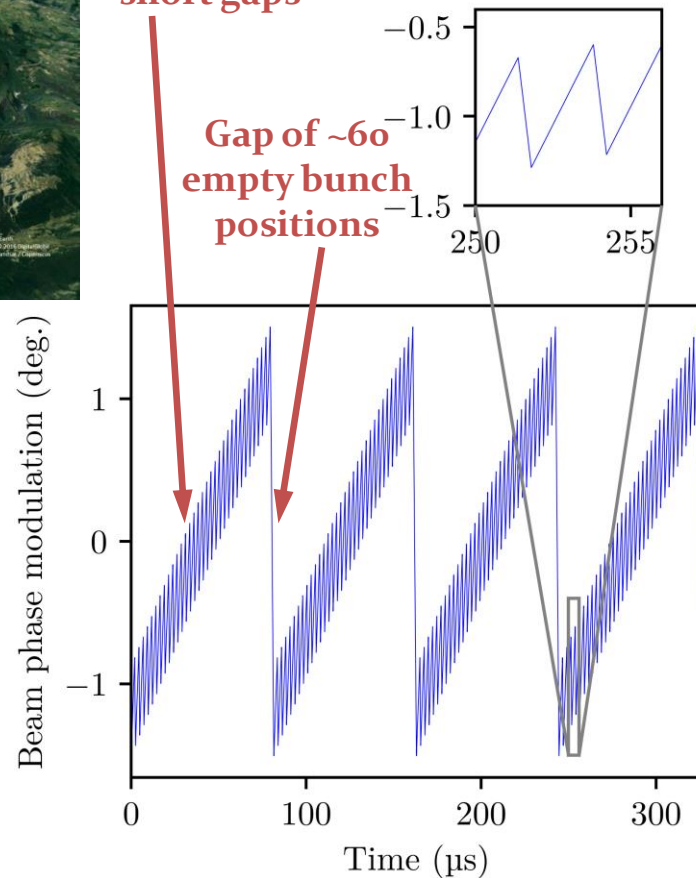
Example: FCC-hh (hadron-hadron)

- Proposed future circular collider



Batch of 2600 bunches with short gaps

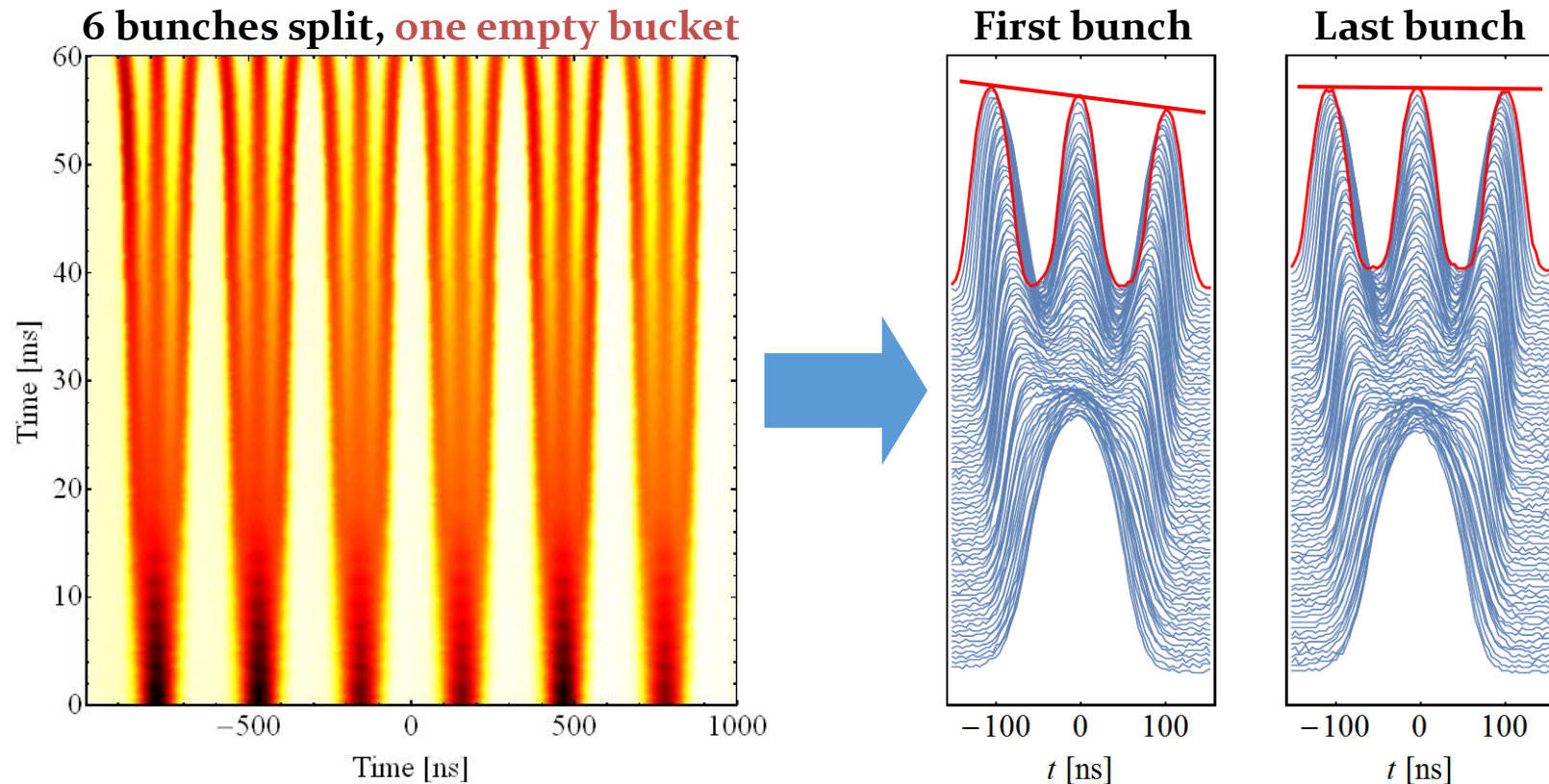
Gap of ~60 empty bunch positions



- Machine protection requires
 - Four batches per turn
 - Gaps of $\sim 1.5 \mu\text{s}$
- Full-detuning causes a bunch phase modulation of $\sim 2^\circ$
- Position of collision point modulated

Transient beam loading between RF systems ⁵⁹

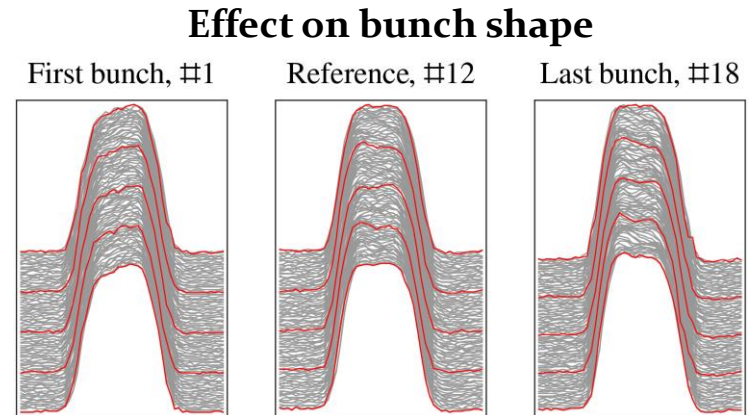
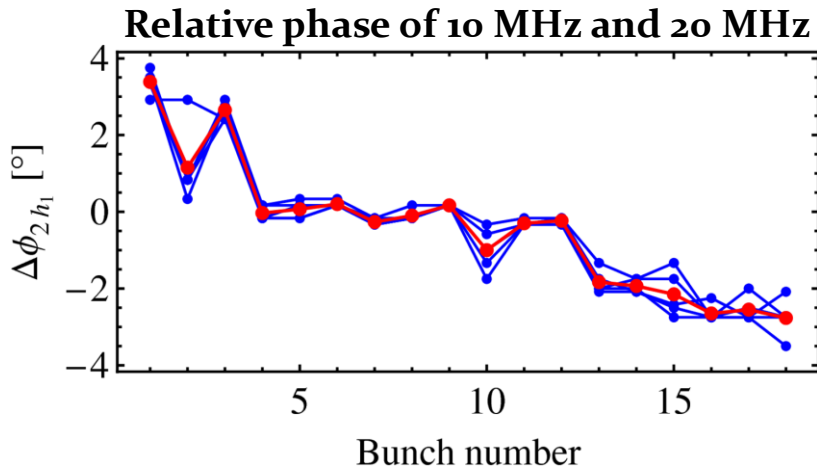
- Triple splitting of LHC-type beams in CERN PS requires three RF systems ($h = 7, 14$ and 21) in phase at degree level



- Transient beam loading: relative phases different for 1st bunch
- Bunch-by-bunch intensity variations in LHC

Transient beam loading between RF systems ⁶⁰

→ Fast phase measurement to directly observe relative changes

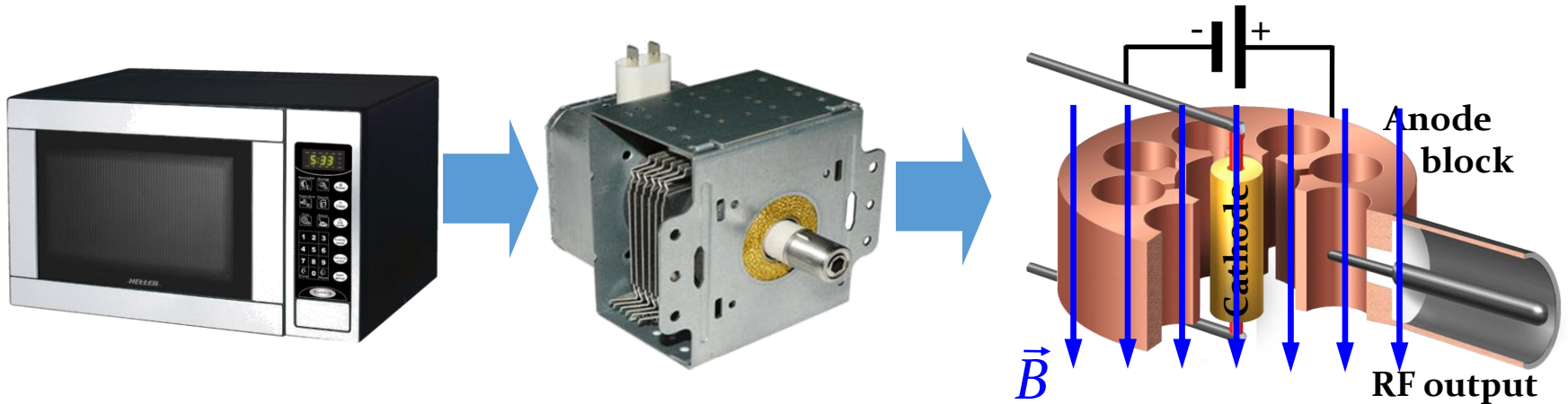


- **Cavity detuning not an option**
 - **Would even enhance phase modulation along batch**
- **Feedback systems**
 - **Counteract beam loading with additional RF power**
 - **Stabilize phase**

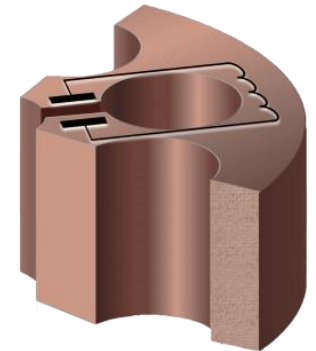
**Beam loading in
microwave oven?**

Beam loading in microwave oven?

- Microwave ovens use magnetrons as RF power source

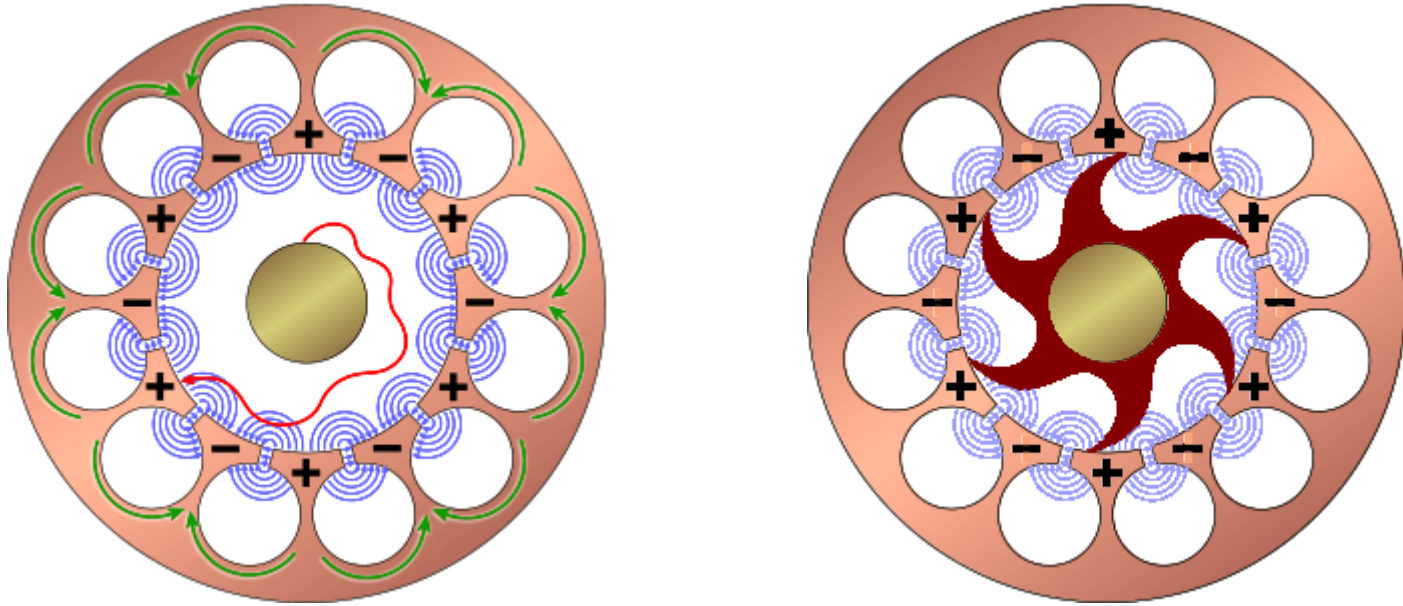


- Anode block consists of ring of cavity resonators
- Electrons from the cathode accelerated towards anode (cavities)
- Perpendicular magnetic field causes cyclotron motion



Beam loading in microwave oven?

- Magnetron as RF power source



- **Electron flow** from cathode to anode **self-bunched** under influence of oscillating fields in anode resonators
- **Bunched electrons excite RF fields** → **beam loading!**
- **Food gets heated**

Summary

- **RF cavity parameters**
 - **System of cavity, coupling and amplifier**
- **Single and multi-passage of bunches through a cavity**
 - **Fundamental theorem of beam loading**
 - **Multiple passages limiting case of steady state**
- **Steady state beam loading**
 - **Minimize RF power by detuning and coupling**
- **Partial filling**
 - **Modulation of bunch phase and RF voltage**
- **Magnetron principle**
 - **Heating food with beam loading**

A big Thank You

to all colleagues providing support, material and feedback

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**Thank you very much
for your attention!**

References

- P. B. Wilson, Beams Loading in High-Energy Storage Rings, HEACC1974, 1974, p. 57-62, http://inspirehep.net/record/92909/files/HEACC74_78-83.pdf
- P. B. Wilson, Transient Beam Loading in Electron-positron Storage Rings, CERN-ISR-TH-78-23-rev, 1978, <http://cds.cern.ch/record/119634/files/cer-000030126.pdf>
- P. B. Wilson, J. E. Griffin, High Energy Electron Linacs; Application to Storage Ring RF Systems and Linear Colliders, AIP Conf. Proc. 87 (1982) 564-582, <https://aip.scitation.org/doi/pdf/10.1063/1.33620>
- D. Boussard, Beam Loading, CERN-87-03-V-2, 1987, <http://cds.cern.ch/record/1246443/files/p626.pdf>
- D. Boussard, Design of a Ring RF System, CERN SL/91-2 (RFS, rev.), 1991, <http://cds.cern.ch/record/1023436/files/CM-P00065157.pdf>
- R. Garoby, Beam Loading in RF cavities, Lect. Notes Phys. 400 (1992) 509-541, <http://cds.cern.ch/record/220916/files/CM-P00059475.pdf>
- D. Boussard, Beam Loading, CERN-95-06, 1995, <http://cds.cern.ch/record/302480/files/p415.pdf>
- J. Tückmantel, Optimum Cavity Detuning under Reactive Beam Loading with Accelerated Beams, AB-Note-2005-040 (RF), 2005, <http://cds.cern.ch/record/910371/files/ab-note-2005-040.pdf>
- J. Tückmantel, Cavity-Beam-Transmitter Interaction Formula Collection with Derivation, CERN-ATS-Note-2011-002 TECH, 2011, <http://cds.cern.ch/record/1323893/files/CERN-ATS-Note-2011-002%20TECH.pdf>

Approximations

→ **2nd order Taylor expansion for $\delta_0 \simeq 0$**

$$e^{-\delta_0(1+\beta)} \simeq 1 - \delta_0(1 + \beta) + \frac{1}{2}\delta_0^2(1 + \beta)^2$$

$$e^{\delta_0(1+\beta)} \simeq 1 + \delta_0(1 + \beta) + \frac{1}{2}\delta_0^2(1 + \beta)^2$$

$$\cos [\delta_0(1 + \beta) \tan \phi_c] \simeq 1 - \frac{1}{2}\delta_0^2(1 + \beta)^2 \tan^2 \phi_c$$

$$\sin [\delta_0(1 + \beta) \tan \phi_c] \simeq \delta_0(1 + \beta) \tan \phi_c$$

Approximations: F_1

→ **Simplification of real part $F_1(\beta, \phi_c)$ for $\delta_0 \simeq 0$**

$$\begin{aligned}
 F_1 &= \frac{1 - e^{-2\delta_0(1+\beta)}}{2\{1 - 2e^{-\delta_0(1+\beta)} \cos[\delta_0(1+\beta) \tan \phi_c] + e^{-2\delta_0(1+\beta)}\}} \\
 &= \frac{e^{\delta_0(1+\beta)} - e^{-\delta_0(1+\beta)}}{2\{e^{\delta_0(1+\beta)} - 2 \cos[\delta_0(1+\beta) \tan \phi_c] + e^{-\delta_0(1+\beta)}\}} \\
 &\simeq \frac{1 + \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2 - 1 + \delta_0(1+\beta) - \frac{1}{2}\delta_0^2(1+\beta)^2}{2\{1 + \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2 - 2[1 - \frac{1}{2}\delta_0^2(1+\beta)^2 \tan^2 \phi_c] + 1 - \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2\}} \\
 &= \frac{2\delta_0(1+\beta)}{2\{2 + \delta_0^2(1+\beta)^2 - 2 + \delta_0^2(1+\beta)^2 \tan^2 \phi_c\}} \\
 &= \frac{\delta_0(1+\beta)}{\delta_0^2(1+\beta)^2 + \delta_0^2(1+\beta)^2 \tan^2 \phi_c} \\
 &= \frac{1}{\delta_0(1+\beta)(\tan^2 \phi_c + 1)}
 \end{aligned}$$

Approximations: F_2

→ **Simplification of real part $F_2(\beta, \phi_c)$ for $\delta_0 \simeq 0$**

$$\begin{aligned}
 F_2 &= \frac{e^{-\delta_0(1+\beta)} \sin [\delta_0(1+\beta) \tan \phi_c]}{1 - 2e^{-\delta_0(1+\beta)} \cos [\delta_0(1+\beta) \tan \phi_c] + e^{-2\delta_0(1+\beta)}} \\
 &= \frac{\sin [\delta_0(1+\beta) \tan \phi_c]}{e^{\delta_0(1+\beta)} - 2 \cos [\delta_0(1+\beta) \tan \phi_c] + e^{-\delta_0(1+\beta)}} \\
 &\simeq \frac{\delta_0(1+\beta) \tan \phi_c}{1 + \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2 - 2 \left[1 - \frac{1}{2}\delta_0^2(1+\beta)^2 \tan^2 \phi_c \right] + 1 - \delta_0(1+\beta) + \frac{1}{2}\delta_0^2(1+\beta)^2} \\
 &= \frac{\delta_0(1+\beta) \tan \phi_c}{2 + \delta_0^2(1+\beta)^2 - 2 + \delta_0^2(1+\beta)^2 \tan^2 \phi_c} \\
 &= \frac{\delta_0(1+\beta) \tan \phi_c}{\delta_0^2(1+\beta)^2 + \delta_0^2(1+\beta)^2 \tan^2 \phi_c} \\
 &= \frac{\tan \phi_c}{\delta_0(1+\beta)(\tan^2 \phi_c + 1)}
 \end{aligned}$$

Frequency and wavelength ranges



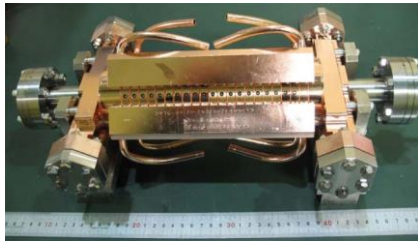
PS longitudinal damper



PS main RF system



SPS 200 MHz



CLIC 12 GHz

100 kHz
3 km

1 MHz
300 m

10 MHz
30 m

100 MHz
3 m

1 GHz
30 cm

10 GHz
3 cm

100 GHz
3 mm



Long wave

**Medium/
short wave**



VHF



**Microwave
links**

