Low Emittance Lattices

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Lecture 1 summary

In Lecture 1, we:

- Discussed the effects of synchrotron radiation on the (linear) motion of particles in storage rings
- Derived expressions for the damping times of the longitudinal, vertical and horizontal emittances
- Discussed the effects of quantum excitation
- Derived expressions for the equilibrium horizontal and longitudinal emittances in an electron storage ring in terms of the lattice functions, dipole bend radii and gradient and the beam energy

Lecture 2 objectives: emittance and lattice design

In this lecture we shall:

- Derive expressions for the natural (horizontal) emittance in four types of lattice:
  - Focussing-Defocussing (FODO)
  - Double-bend achromat (DBA)
  - Theoretical minimum emittance (TME)
  - Multi-bend achromat (MBA)

- Discuss some practical aspects of low emittance lattices:
  - Transverse gradient bends, longitudinally-varying bends, reverse bends
  - Examples of modern low-emittance lattice designs

Calculating the natural emittance in a lattice

Our first goal is to calculate the natural emittance in a lattice with magnets of given strengths, lengths and positions.

In lecture 1 we showed that the natural emittance in a storage ring is given by:

$$\varepsilon_0 = C_q \gamma^2 \frac{I_5}{j_x I_2} \qquad (1)$$

Where C_q is the quantum radiation constant, γ is the relativistic factor and I_2 and I_5 are the second and fifth synchrotron radiation integrals. The horizontal damping partition number j_x is:

$$j_x = 1 - \frac{I_4}{I_2} \qquad (2)$$

Note that j_x, I_2 and I_5 are all functions of the lattice and are independent of the beam energy.

Calculating the natural emittance in a lattice

In most storage rings, if the bends have no quadrupole component, the horizontal damping partition number $j_x \approx 1$.

In this case, to find the natural emittance we just need to evaluate the two synchrotron radiation integrals:

$$I_2 = \oint \frac{1}{\rho^2} ds, \qquad I_5 = \oint \frac{\mathcal{H}_x}{|\rho^3|} ds \tag{3}$$

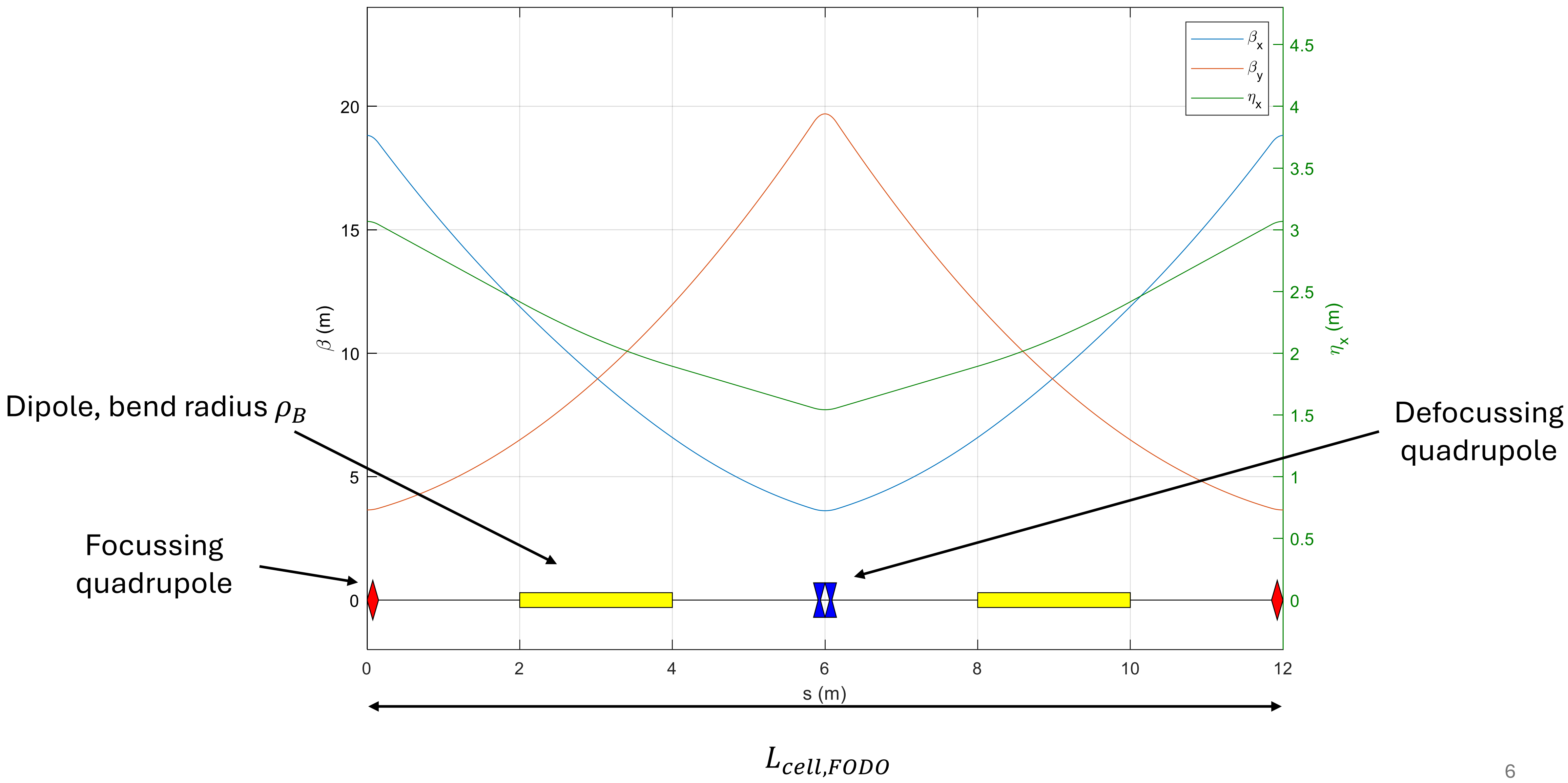
If we know the strength and length of all the dipoles in the lattice it is straightforward to calculate I_2. For example, if all the bends are identical, then in a complete ring (total bending angle = 2π):

$$I_2 = \oint \frac{1}{\rho^2} ds = \oint \frac{B}{B\rho} \frac{1}{\rho} ds = 2\pi \frac{B}{B\rho} \approx 2\pi \frac{cB}{E/e} \tag{4}$$

where E is the beam energy. I_5 is more complicated, it depends on the lattice functions in the bending magnets:

$$\mathcal{H}_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_{px} + \beta_x \eta_{px}^2 \tag{5}$$

Case 1: natural emittance in a FODO lattice



Case 1: natural emittance in a FODO lattice

Let us consider the case of a FODO lattice. To simplify the system, we use the following approximations:

- The quadrupoles are represented as thin lenses ($f = 1/K_Q L_Q$)
- The space between the quadrupoles is completely filled by the dipoles

With these approximations, the lattice functions (Courant-Snyder parameters and dispersion) are completely determined by the following parameters:

- The focal length of a quadrupole, f
- The bend radius of the dipole, ρ_B
- The length of the dipole, L_B (where $L_B = L_{cell,FODO}/2$ in this example)

Case 1: natural emittance in a FODO lattice

From the evolution of the lattice functions through a given FODO cell, we can find an (approximate) expression for I_5/I_2 (see Appendix A in ref. [1]):

For small θ, and if $\rho_B \gg 2f$ (which is often the case), we find:

$$\frac{I_5}{I_2} \approx \left(1 - \frac{L_B^2}{16f^2}\right)\left(\frac{2f}{\rho_B}\right)^3 \tag{6}$$

This result can be further simplified if $4f \gg L_B$ (which is not always the case):

$$\frac{I_5}{I_2} \approx \left(\frac{2f}{\rho_B}\right)^3 \tag{7}$$

Case 1: natural emittance in a FODO lattice

Making the approximation $j_x \approx 1$ (valid for cases where there is no gradient in the dipole), and writing $\rho_B = L_B/\theta$, we have:

$$\varepsilon_0 \approx C_q \gamma^2 \left(\frac{2f}{L_B}\right)^3 \theta^3 \quad (8)$$

Notice how the emittance scales with the beam and lattice parameters. The emittance:

- Is proportional to the square of the beam energy
- Is proportional to the cube of the bending angle: increasing the number of cells in a storage ring reduces the dipole bending angle and reduces the emittance
- Is proportional to the cube of the quadrupole focal length: stronger quads means lower emittance
- Is inversely proportional to the cube of the cell length

Case 1: natural emittance in a FODO lattice

The phase advance in a FODO cell is given by:

$$\cos(\mu_x) = 1 - \frac{L_B^2}{2f^2} \qquad (9)$$

This means that a stable lattice must have:

$$\frac{f}{L_B} \geq \frac{1}{2} \qquad (10)$$

In the limiting case, $\mu_x = 180°$, and f has the minimum value $f = L_B/2$. Using the approximation (8), we have:

$$\varepsilon_0 \approx C_q \gamma^2 \left(\frac{2f}{L_B}\right)^3 \theta^3$$

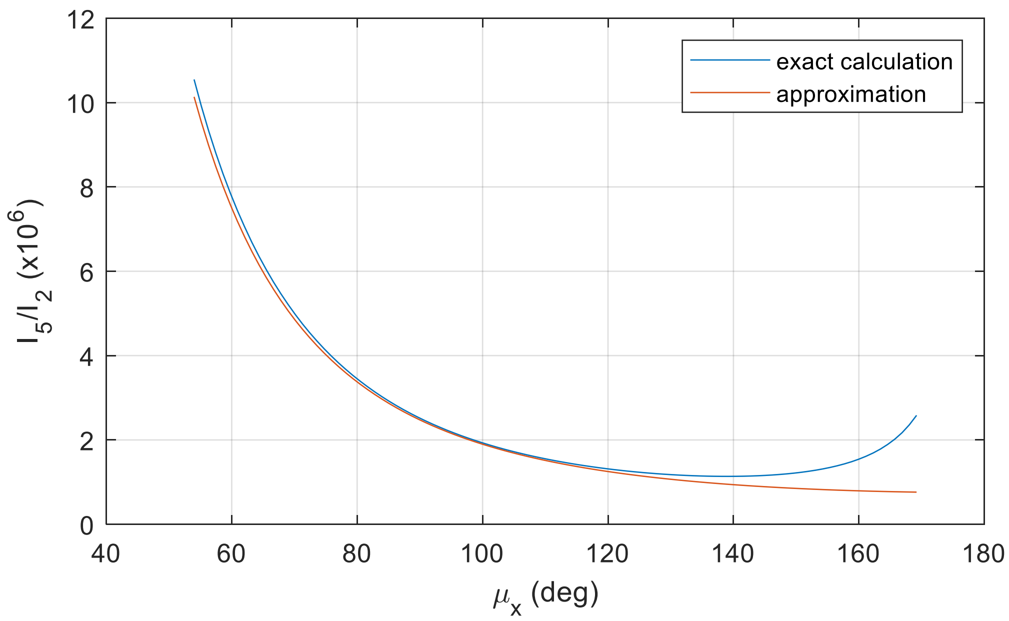
The minimum emittance in a FODO cell is expected to be:

$$\varepsilon_0 \approx C_q \gamma^2 \theta^3 \qquad (11)$$

However, as we increased the focussing strength, the approximations we used to obtains the simple expression for ε_0 start to break down ...

Case 1: natural emittance in a FODO lattice

Plotting the exact formula for I_5/I_2 as a function of the phase advance for our example, we find that there is a minimum in the natural emittance, at $\mu_x \approx 137°$.



Assumptions:

- $L_B = 1.0\ m$
- $\rho_B = 100\ m$
- Thin lens quadrupoles
- Dipoles fill the space between quadrupoles

It turns out that the minimum value of the natural emittance in a FODO lattice is given by:

$$\varepsilon_{0,FODO,min} \approx 1.2 C_q \gamma^2 \theta^3 \qquad (12)$$

<u>Case 1: natural emittance in a FODO lattice</u>

Using (8) we estimate that a storage ring constructed of 16 FODO cells (32 dipoles) with $90°$ phase advance per cell ($f = L_B/\sqrt{2}$), and storing beam at 2 GeV would have a natural emittance of around 125 nm.

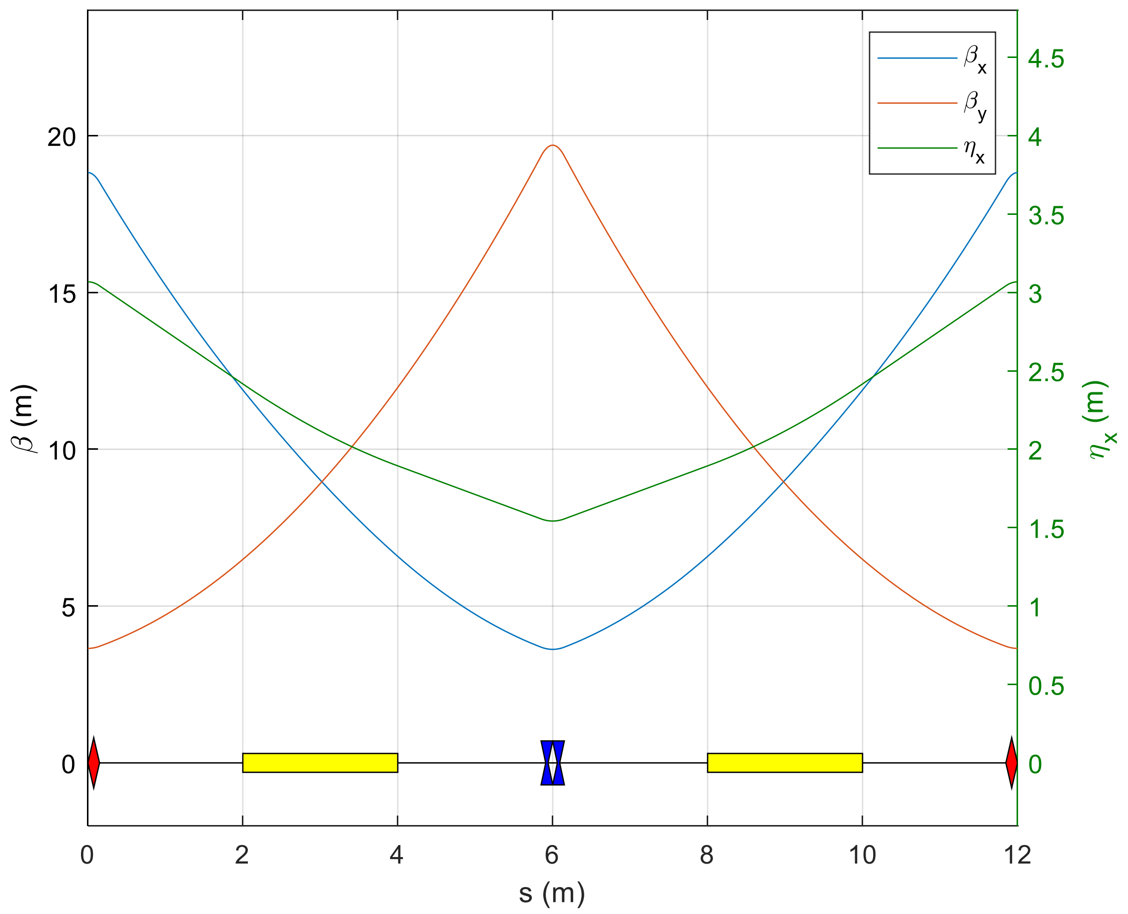
Many modern applications (including synchrotron light sources) demand emittances smaller by two to three orders of magnitude.

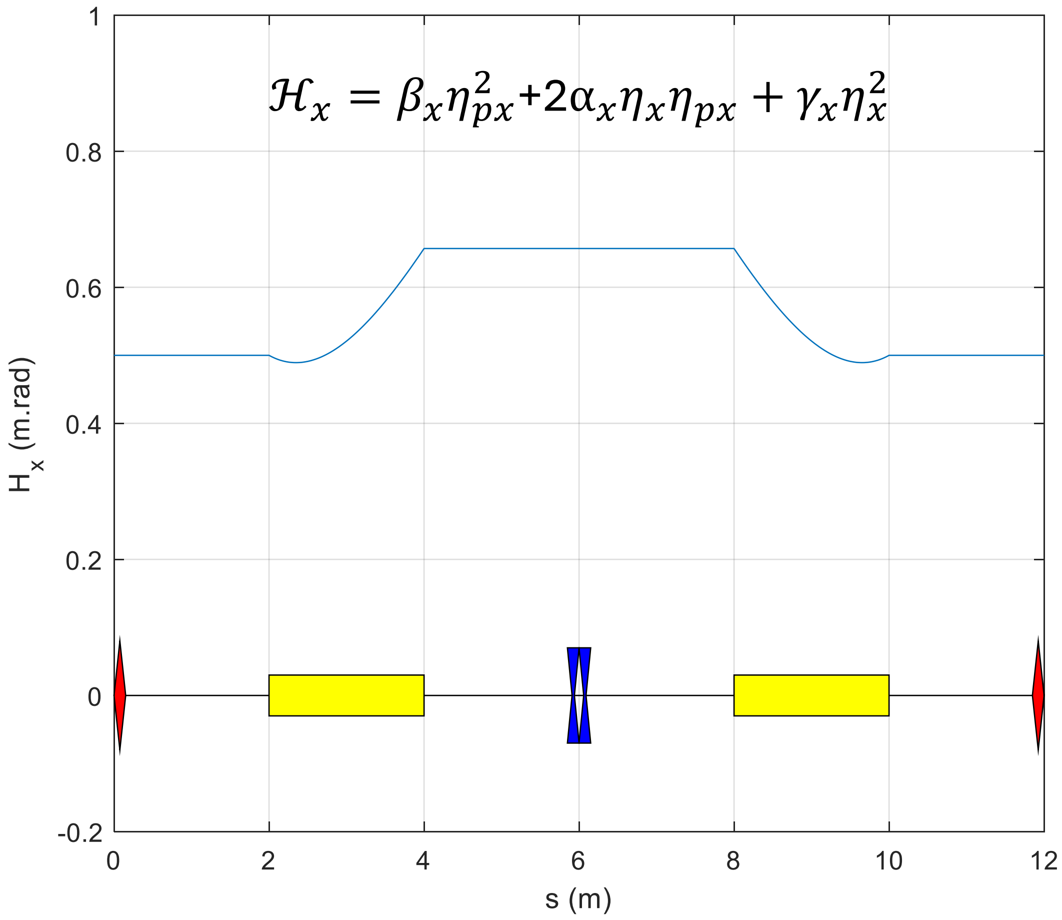
How can we design a lattice with a smaller natural emittance?

Looking at the chromatic invariant $\mathcal{H}_x$ (‘curly-H’) in a FODO cell provides a clue ...

Case 1: natural emittance in a FODO lattice

The function $\mathcal{H}_x$ remains at a relatively constant value throughout the lattice:

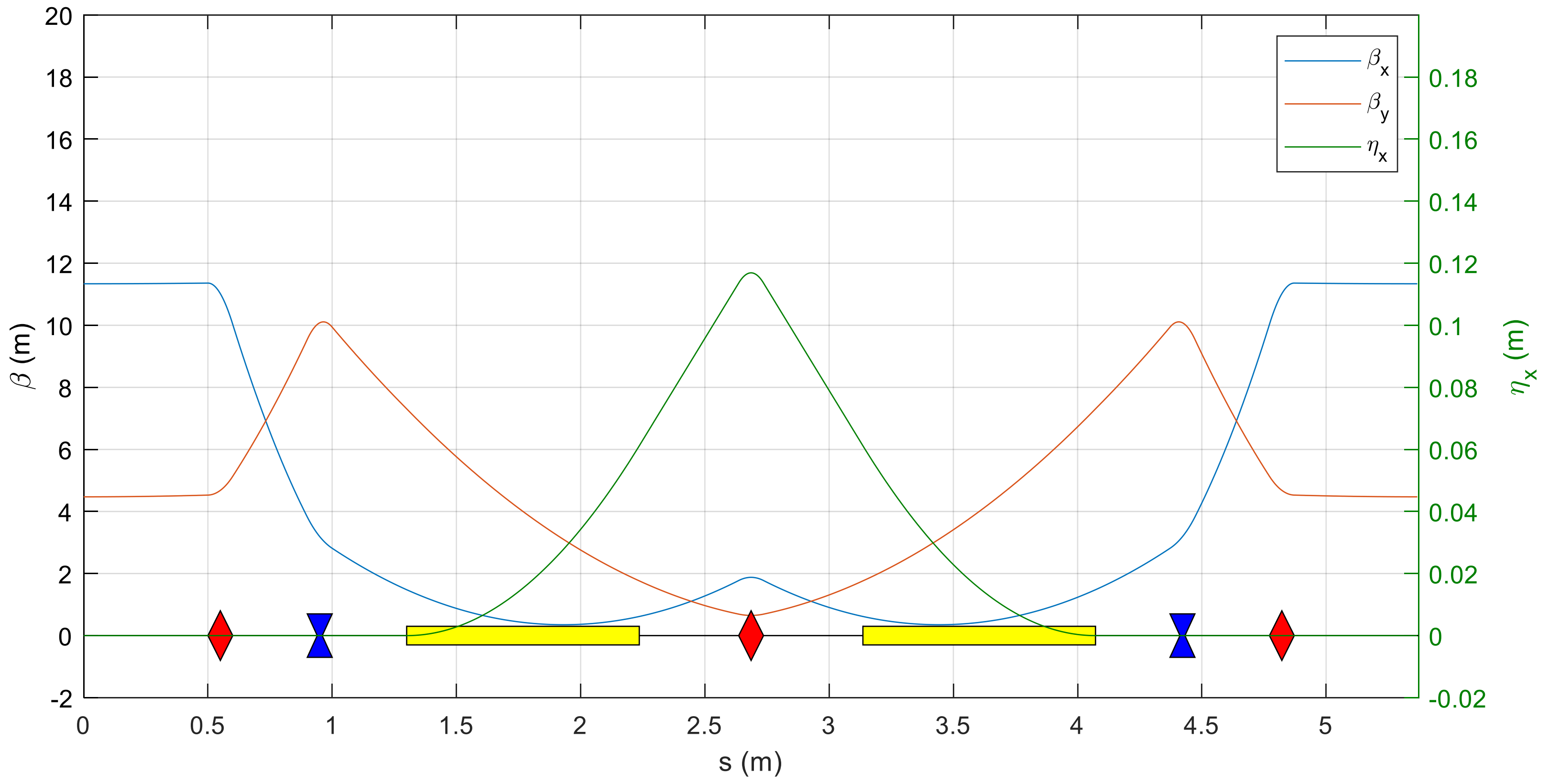




Case 2: natural emittance in a DBA lattice

As a first attempt at reducing the natural emittance, we can try reducing the function $\mathcal{H}_x$ in the dipoles by designing a lattice that has zero dispersion at either ends of a dipole pair.

The result is known as a double-bend achromat (DBA) cell:



Case 2: natural emittance in a DBA lattice

DBA cells have a dispersion function that is mirror-symmetric about the centre with zero dispersion away from the dipole pairs.

To solve this problem, let us begin by considering how the dispersion needs to change through the cell.

- Assume that the dispersion is zero at the start of the cell.

- Dispersion grows in amplitude through the first dipole.

- Place a quadrupole midway between the dipoles, to reverse the gradient of the dispersion.

By symmetry, the dispersion at the exit of the cell will be zero.

Case 2: natural emittance in a DBA lattice

In the thin lens approximation, the required focal length of the quadrupole to reverse the gradient can be found from:

$$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} \eta_x \\ \eta_{px} \end{pmatrix} = \begin{pmatrix} \eta_x \\ \eta_{px} - \eta_x/f \end{pmatrix} = \begin{pmatrix} \eta_x \\ -\eta_{px} \end{pmatrix} \tag{13}$$

Where η_x and η_{px} are the dispersion and gradient of the dispersion at the quadrupole.

Hence, the central quadrupole must have focal length:

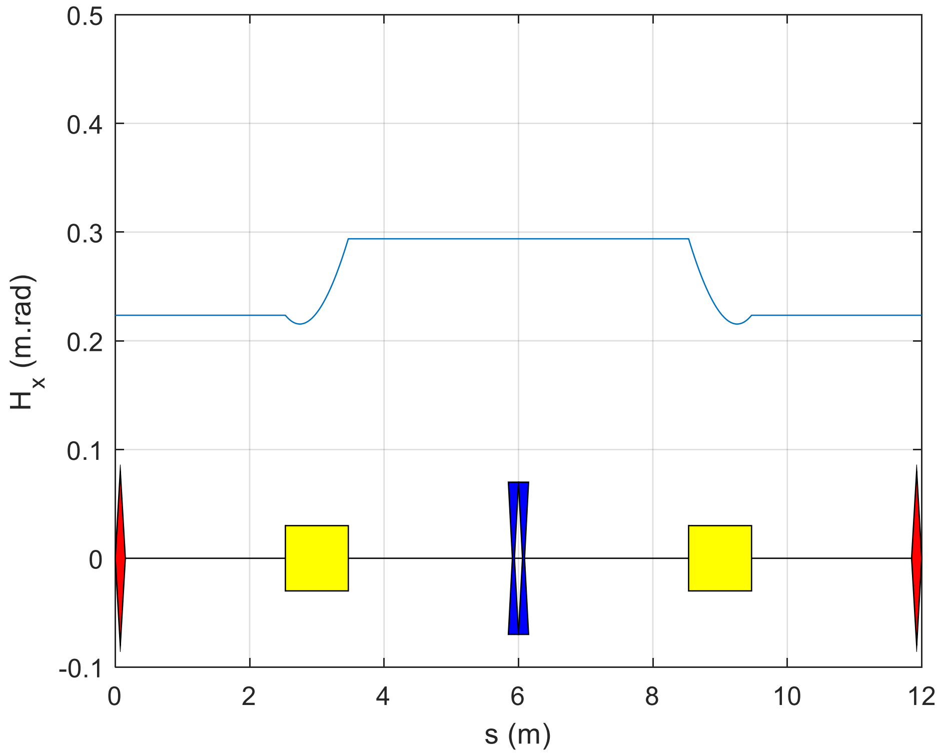
$$f = \frac{\eta_x}{2\eta_{px}} \tag{14}$$

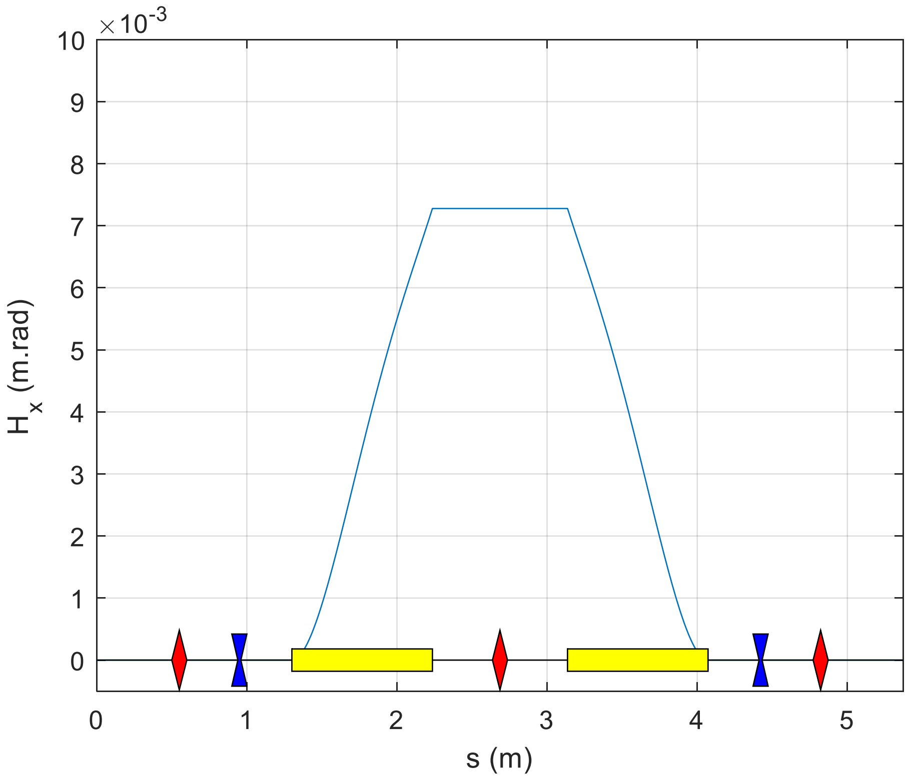
The dispersion is determined by the dipole bending angle θ, the bending radius ρ and the drift length L_D:

$$\eta_x = \rho(1 - \cos(\theta)) + L_D \sin(\theta), \qquad \eta_{px} = \sin(\theta) \tag{15}$$

Is this style of lattice likely to have a lower natural emittance than a FODO lattice? We can get some idea by looking at the function $\mathcal{H}_x$ …

Case 2: natural emittance in a DBA lattice





The function $\mathcal{H}_x$ is much smaller in the DBA lattice (right) than in the FODO lattice (left).

Note that we use the same dipoles (bending angle and length) in both cases.

Case 2: natural emittance in a DBA lattice

Let us calculate the minimum natural emittance of a DBA lattice for a given bending radius ρ and bending angle θ in the dipoles.

To do this, we need to calculate the minimum value of:

$$I_5 = \int \frac{\mathcal{H}_x}{\rho^3} ds \quad (16)$$

in one dipole, subject to the constraints:

$$\eta_{x,0} = \eta_{px,0} = 0 \quad (17)$$

Where $\eta_{x,0}$ and $\eta_{px,0}$ are the dispersion and gradient of the dispersion at the entrance of a dipole.

Case 2: natural emittance in a DBA lattice

We know how the dispersion and Courant-Snyder parameters evolve through the dipole, so we can calculate I_5 for one dipole for given initial values of the Courant-Snyder parameters $\alpha_{x,0}$ and $\beta_{x,0}$.

Then we simply have to minimise the value of I_5 with respect to $\alpha_{x,0}$ and $\beta_{x,0}$.

Again, the algebra is rather formidable, and the full expression for I_5 is not especially enlightening.

Therefore, we just quote the significant results.

Case 2: natural emittance in a DBA lattice

We find that, for given ρ and θ and with the initial dispersion constraints:

$$\eta_{x,0} = \eta_{px,0} = 0 \tag{18}$$

The minimum value of I_5 is given by:

$$I_{5,min} = \frac{1}{4\sqrt{15}} \frac{\theta^4}{\rho} + \mathcal{O}(\theta^6) \tag{19}$$

This minimum occurs for values of the Courant-Snyder parameters at the entrance of the dipole given by:

$$\beta_{x,0} = \sqrt{\frac{12}{5}} L + \mathcal{O}(\theta^3), \qquad \alpha_{x,0} = \sqrt{15} + \mathcal{O}(\theta^2) \tag{20}$$

Where $L = \rho\theta$ is the length of the dipole.

Case 2: natural emittance in a DBA lattice

Since we know that I_2 in a single dipole is given by:

$$I_2 = \int \frac{1}{\rho^2} ds = \frac{\theta}{\rho} \tag{21}$$

we can now write down an expression for the minimum emittance in a DBA lattice:

$$\varepsilon_{0,DBA,min} = C_q \gamma^2 \frac{I_{5,min}}{j_x I_2} \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3 \tag{22}$$

The approximation is valid for small θ. Note that we have again assumed that there is no quadrupole component in the dipole so that $j_x \approx 1$.

Case 2: natural emittance in a DBA lattice

Compare the expression for a DBA lattice to the one we found for the minimum emittance in a FODO lattice (12):

$$\varepsilon_{0,FODO,min} \approx 1.2 C_q \gamma^2 \theta^3 \quad (23)$$

We see that in both cases (FODO and DBA) the emittance scales with the square of the beam energy and with the cube of the bending angle.

However, the emittance in a DBA lattice is smaller than that in a FODO lattice (for a given beam energy and dipole bending angle) by a factor of roughly $4\sqrt{15} \approx 15.5$.

This is a significant improvement, but can we do even better?

Case 3: natural emittance in a TME lattice

For a DBA lattice, we imposed the constraints

$$\eta_{x,0} = \eta_{px,0} = 0 \tag{24}$$

To get a lower emittance, we can consider relaxing these constraints.

To derive the conditions for a 'theoretical minimum emittance' (TME) lattice, we write down an expression for:

$$I_5 = \int \frac{\mathcal{H}_x}{\rho^3} ds \tag{25}$$

With arbitrary dispersion $\eta_{x,0}$, $\eta_{px,0}$ and Courant-Snyder parameters $\alpha_{x,0}$ and $\beta_{x,0}$ in a dipole with given bending radius ρ and angle θ.

Then, we minimise I_5 with respect to $\eta_{x,0}$, $\eta_{px,0}$, $\alpha_{x,0}$ and $\beta_{x,0}$

Case 3: natural emittance in a TME lattice

The result is:

$$\varepsilon_{0,TME,min} \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3 \tag{26}$$

The minimum emittance is obtained with dispersion at the entrance to the dipole given by:

$$\eta_{x,0} = \frac{1}{6} L\theta + \mathcal{O}(\theta^3), \qquad \eta_{px,0} = -\frac{\theta}{2} + \mathcal{O}(\theta^3) \tag{27}$$

and with Courant-Snyder functions at the entrance:

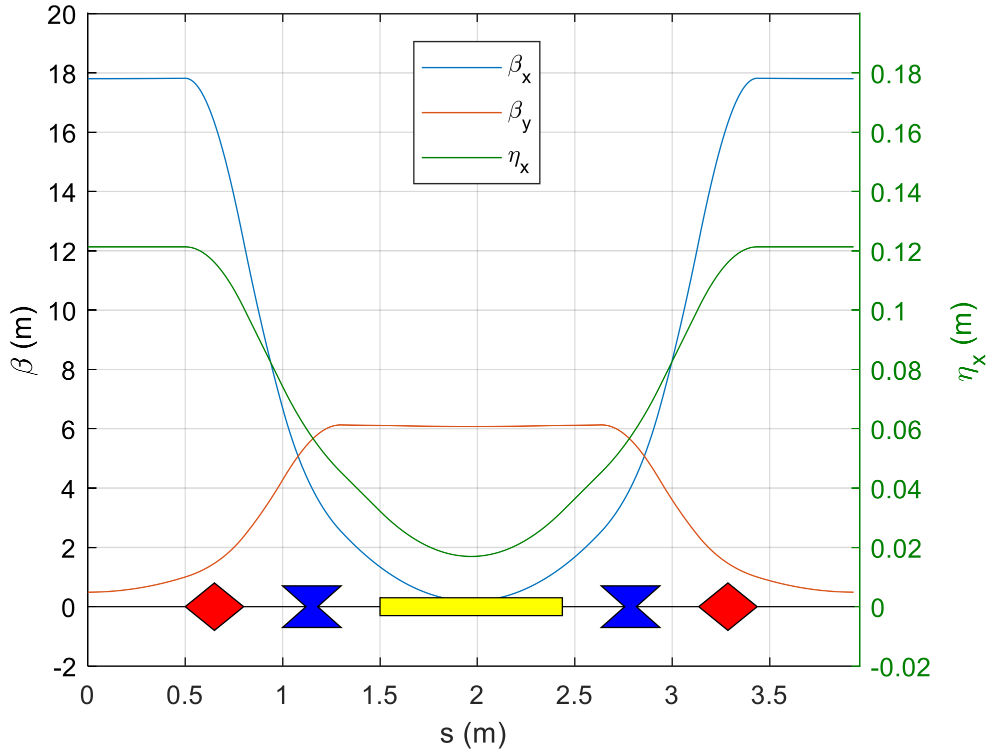
$$\beta_{x,0} = \frac{8}{\sqrt{15}} L + \mathcal{O}(\theta^2), \qquad \alpha_{x,0} = \sqrt{15} + \mathcal{O}(\theta^2) \tag{28}$$

The dispersion and beta function reach minimum values in the centre of the dipole:

$$\eta_{x,min} = \rho\left(1 - \frac{2}{\theta}\sin\left(\frac{\theta}{2}\right)\right) = \frac{L\theta}{24} + \mathcal{O}(\theta^3), \qquad \beta_{x,min} = \frac{L}{2\sqrt{15}} + \mathcal{O}(\theta^2) \tag{29}$$

Case 3: natural emittance in a TME lattice

By symmetry, we can consider a single TME cell to contain a single dipole, rather than a pair of dipoles as was necessary for the FODO and DBA cells.



Outside the dipole, the dispersion is relatively large.

This is not ideal for a light source, since insertion devices at locations with large dispersion will blow up the emittance.

Note that the cell shown here does not achieve the exact conditions for a TME lattice: a more complicated design would be needed for this.

Summary: natural emittance in FODO, DBA and TME lattices

Lattice Style	Minimum Emittance	Conditions / Comments
90° FODO	$\varepsilon_0 \approx 2\sqrt{2}C_q\gamma^2\theta^3$	$\frac{f}{L} = \frac{1}{\sqrt{2}}$
137° FODO	$\varepsilon_0 \approx 1.2C_q\gamma^2\theta^3$	Minimum emittance FODO
DBA	$\varepsilon_0 \approx \frac{1}{4\sqrt{15}}C_q\gamma^2\theta^3$	$\eta_{x,0} = \eta_{px,0} = 0$ $\beta_{x,0} \approx \sqrt{12/5}L, \ \ \alpha_{x,0} \approx \sqrt{15}$
TME	$\varepsilon_0 \approx \frac{1}{12\sqrt{15}}C_q\gamma^2\theta^3$	$\eta_{x,min} \approx \frac{L\theta}{24}, \quad \beta_{x,min} \approx \frac{L}{2\sqrt{15}}$

Comments on lattice design for low emittance

The results we have derived have been for ‘ideal’ lattices that perfectly achieve the stated conditions in each case.

Practical lattice designs rarely achieve the ideal conditions. In particular, the beta function in an achromat is usually not optimal for low emittance, and it is difficult to tune the dispersion for the ideal TME conditions.

The main reasons for this are:

- Beam dynamics phenomena (chromaticity, nonlinearities, collective effects, ...) generally impose a number of strong constraints on the design

- Matching the lattice functions to achieve maximum brightness for the insertion device photon beams

- Optimising the lattice functions while meeting all the various constraints can require complex configurations of quadrupoles; space becomes a problem!

Comments on lattice design for low emittance

A particular challenging constraint on design of a low-emittance lattice is the dynamic aperture.

Storage rings require a large dynamic aperture in order to achieve good injection efficiency and good beam lifetime.

However, a low emittance lattice generally needs low dispersion and beta-functions and hence require strong quadrupoles. As a result, the natural chromaticity can be large and must be corrected using strong sextupoles.

Strong sextupoles lead to strongly nonlinear motion and limit the dynamic aperture (the trajectory of particles at large betatron amplitudes or large energy deviations become unstable).

Further options and issues

There are many other options beside FODO, DBA and TME for the style of the lattice.

Here we discuss (briefly):

- The use of the DBA lattice in third generation light sources
- A ‘detuning’ of the DBA lattice to reduce the emittance
- The use of multi-bend achromats
- Allowing the bending magnet field to vary (transverse gradient bends, longitudinally-varying bends, reverse bends)

Practical DBA lattices

For light sources, the storage ring design has to provide straight sections (insertions) where the undulators / wigglers can be located, as well as deliver a low emittance.

A good figure-of-merit in this regard is the ratio between the total length of the straights to the overall circumference.

Although having the dispersion at a waist in the centre of the bending magnet (like for the TME solution) leads to a lower emittance than matching it to zero at the edges (DBA), there are many reasons why the achromatic solution is desirable:

- It avoids increasing the horizontal beam size due to the energy spread
- Beam energy fluctuations do not translate to position offsets in the straights
- Provides a good location for the RF cavities and injection elements
- Decouples the chromatic and harmonic sextupoles

‘Detuning’ a DBA

If an insertion device (undulator or wiggler) is placed in a storage ring at a location with large dispersion, then the dipole fields in the device can make a significant contribution to the quantum excitation (I_5).

The insertion device can then increase the natural emittance of the storage ring.

A DBA lattice provides dispersion-free straights in which undulators and wigglers can be placed without blowing up the natural emittance.

However, there is some tolerance. It is often possible to ‘detune’ a lattice from the strict DBA conditions, allowing some reduction in natural emittance at the cost of some dispersion in the straights.

The insertion devices will then contribute to the quantum excitation, but depending on the lattice and insertion devices there can still be a net benefit.

<u>Practical DBA lattices</u>

Many third-generation light sources use a DBA structure, with cells of magnets interspersed with the straight sections that house the insertion devices:

ESRF (France)

ELETTRA (Italy)

APS (USA)

SPRING8 (Japan)

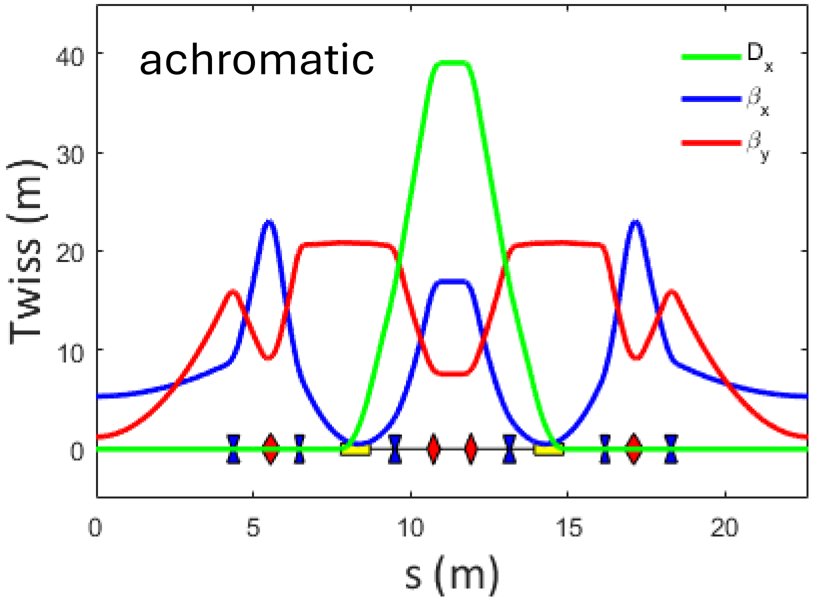
Diamond (UK)

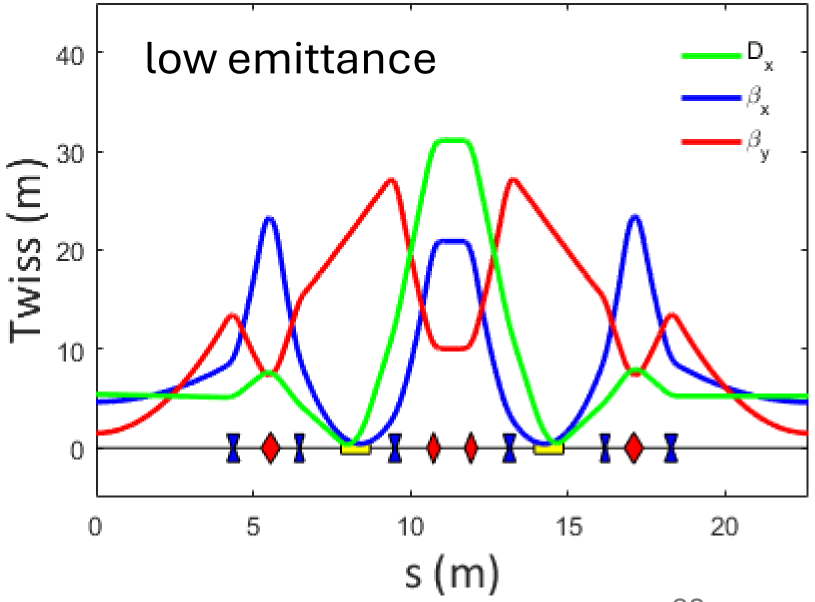
SSRF (China)

...

Some light sources that were originally designed with zero-dispersion straight sections take advantage of tuning flexibility to operate with non-zero dispersion in the straights.

This provides a lower natural emittance and better output for users.





Multi-bend achromats

In principle, it is possible to combine the DBA and TME lattices by having an arc cell consisting of more than two dipoles.

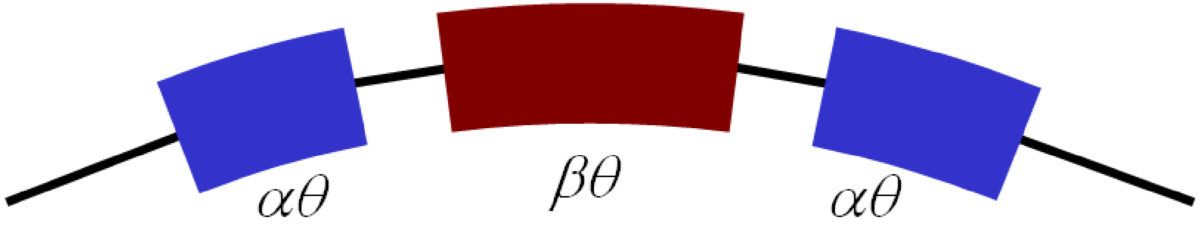
The dispersion is zero at the entrance and exit of each cell, thus satisfying the ‘achromat’ condition and providing dispersion-free straight sections.

In the dipoles within the body of the cell, the Courant-Snyder parameters and dispersion satisfy the TME conditions.

This kind of lattice is known as a ‘multi-bend achromat’ (MBA)

Multi-bend achromats

Suppose that the dipoles all have the same bending radius (i.e. the same field strength) but can have different lengths.



In this case, the minimum natural emittance in an M-bend achromat is given by:

$$\varepsilon_0 \approx \frac{1}{12\sqrt{15}}\left(\frac{M+1}{M-1}\right) C_q \gamma^2 \theta^3, \qquad 2 < M < \infty \tag{30}$$

Where θ is the average bending angle per dipole.

The minimum emittance is achieved when the central dipoles are longer than the outer dipoles by a factor $\sqrt[3]{3}$.

Triple-Bend Achromat (TBA) lattices

The simplest form of an MBA lattice is the triple-bend achromat (TBA):

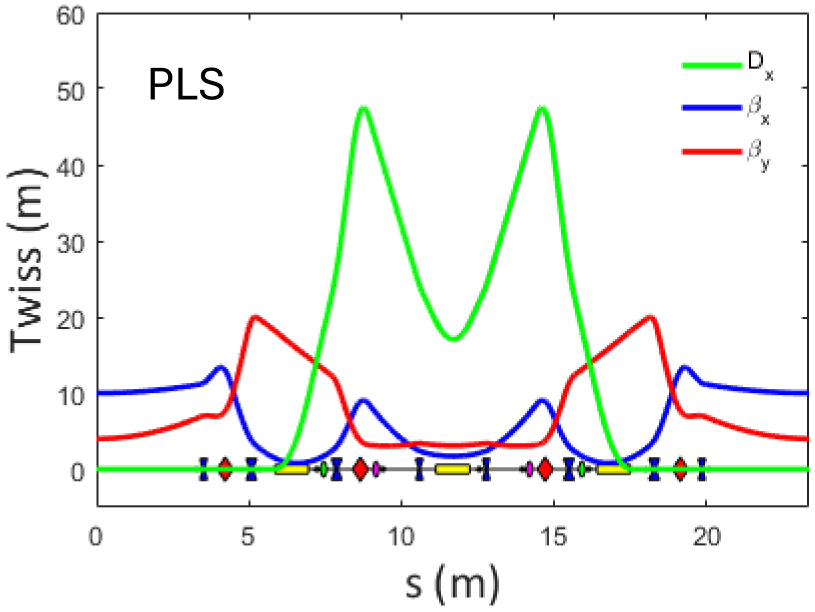
ALS (USA)

SLS (Switzerland)

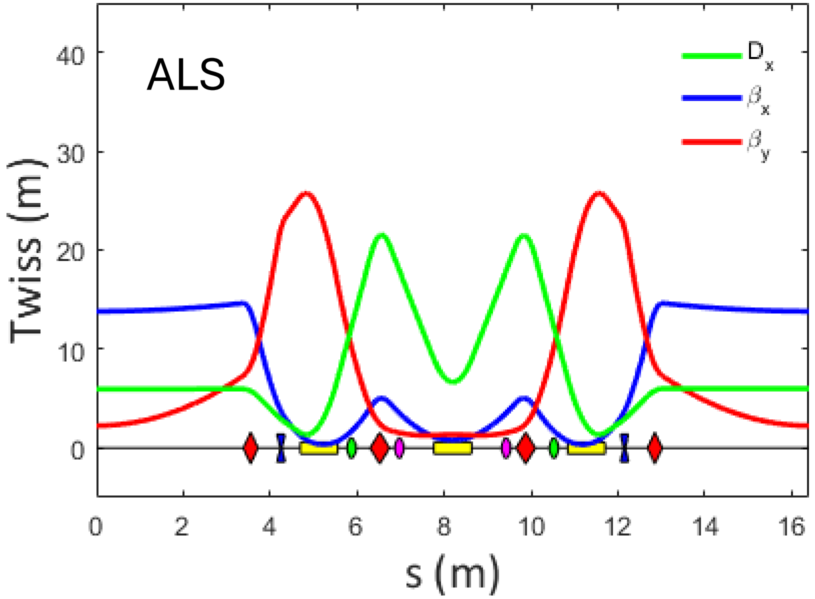
PLS (Korea)

TLS (Taiwan)

…



This design consists of three dipoles per cell. In this case, the outer dipoles are tuned produce zero dispersion in the straights (achromat), and the optics in the central dipole have a waist in the middle (TME).



As with DBA lattices, the TBA lattice can be detuned for low emittance. Example for Swiss Light Source (12 TBA cells, 2.4 GeV, 288 m):

Zero-dispersion mode: ε_0 = 4.8 nm.rad

Low emittance mode: ε_0 = 3.9 nm.rad (20% lower)

Emittance reduction: transverse gradient bends

Up to now we have made the assumption that the horizontal damping partition number $j_x \approx 1$. An alternative way to reduce the emittance is by transferring part of the damping from the longitudinal plane into the horizontal plane (i.e. by increasing j_x). This can be achieved by adding a transverse gradient to the bending magnets.

$$I_4 = \oint \frac{\eta_x(s)}{\rho(s)}\left(\frac{1}{\rho^2(s)} + 2k_1(s)\right) ds \tag{31}$$

For stable motion in both planes:

$$j_x, j_z > 0 \qquad -2 < \frac{I_4}{I_2} < 1$$

Damping partition numbers:

$$j_x = 1 - \frac{I_4}{I_2} \qquad j_z = 2 + \frac{I_4}{I_2}$$

Emittance lowered by increasing j_x:

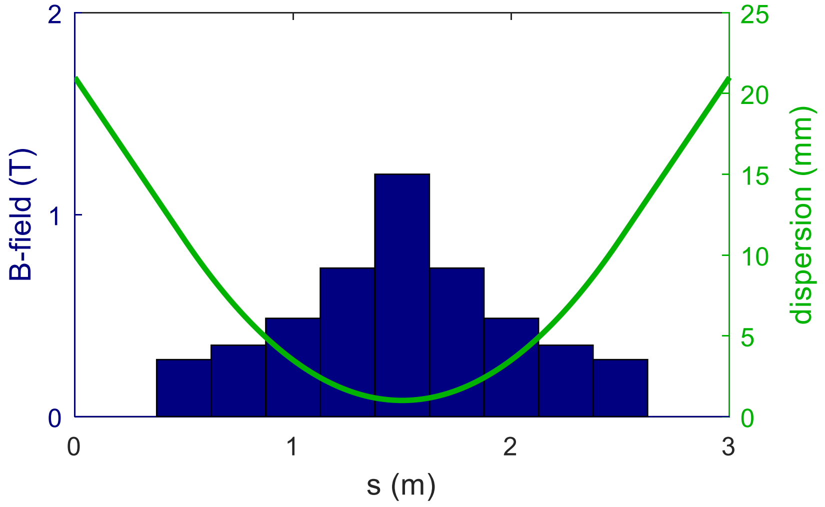
$$\epsilon_x = C_q \gamma^2 \frac{I_5}{j_x I_2}$$

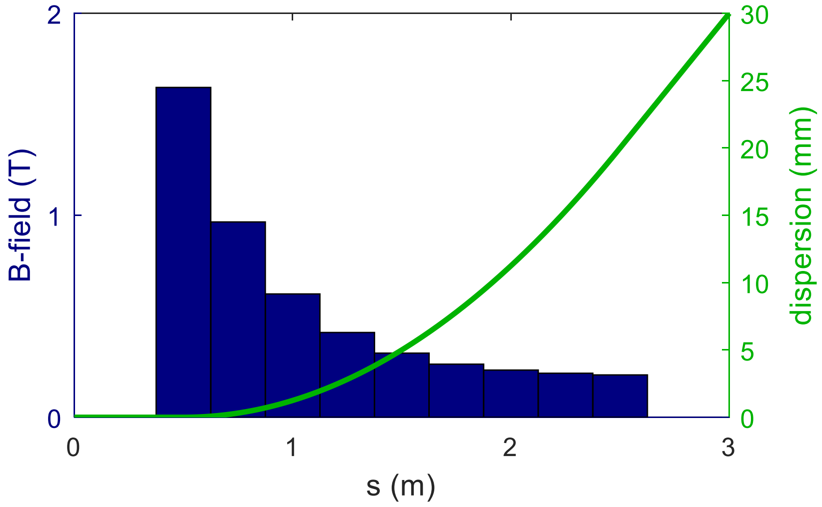
Energy spread increases:

$$\sigma_E^2 = C_q \gamma^2 \frac{I_3}{j_z I_2}$$

Emittance reduction: longitudinally-varying bends

Another way to reduce the emittance is by varying the B-field within the dipole, such that the deflection is maximised at the location where the dispersion is smallest





Emittance lowered by minimising I_5:

$$\varepsilon_x = C_q \gamma^2 \frac{I_5}{J_x I_2} \qquad I_5 = \oint \frac{\mathcal{H}_x(s)}{\rho^3(s)} ds \qquad \mathcal{H}_x(s) = \gamma_x D_x^2 + 2\alpha_x D_x D_x' + \beta_x D_x'^2$$

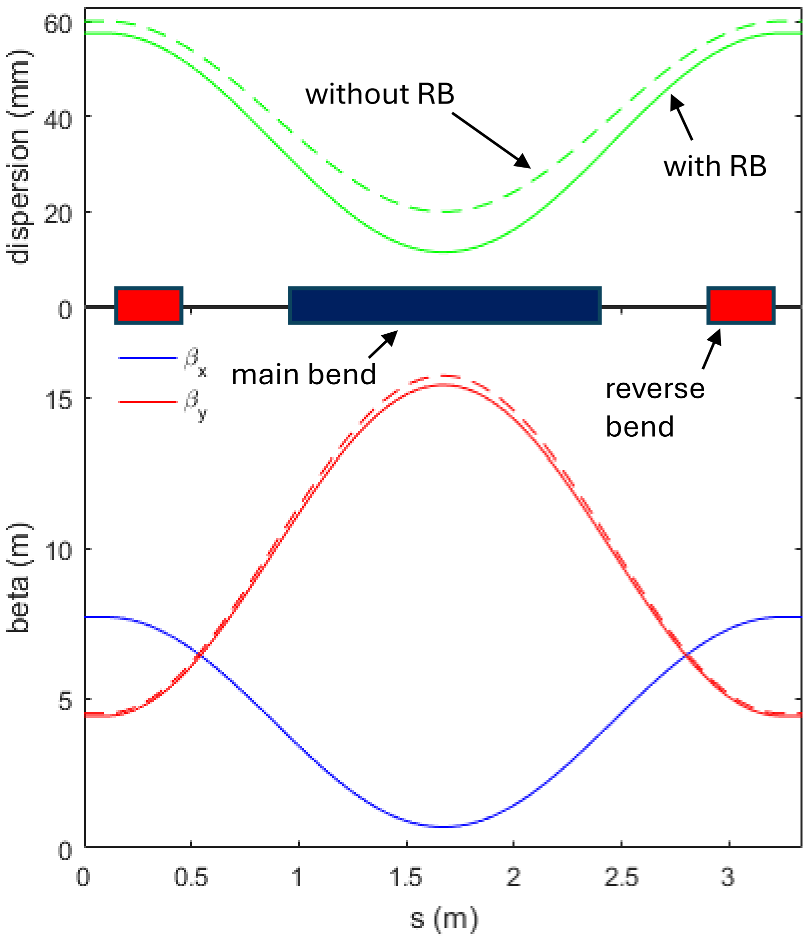
- Total bend angle is kept constant
- Can get below TME of uniform dipole
- Have the benefit of producing hard x-rays where B-field is large

Emittance reduction: reverse bends

Rather than having all bending magnets deflect the beam in the same direction, some storage ring designs include weak dipoles of the opposite polarity. These provide an additional handle with which to control the dispersion function.

- Beta-functions largely unchanged
- Disentangle dispersion from beta-function, allowing the TME to be reached with moderate phase advance
- Located at large $\mathcal{H}_x(s)$, so also contribute to ε_x and σ_E
- Lead to very small or even negative momentum compaction factor

This technique is being exploited by many facility upgrades (e.g. SLS-2, APS-U, Diamond-II, amongst others)



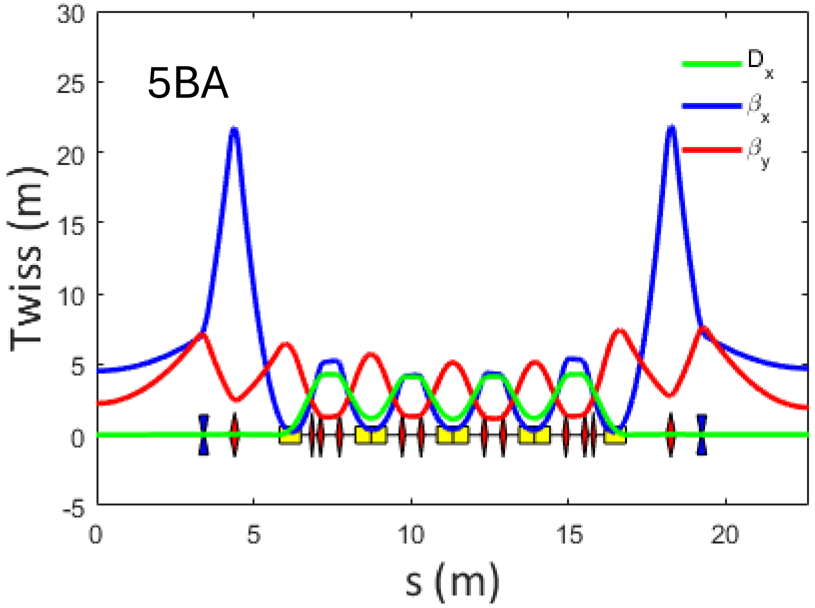
Multi-bend achromat (MBA) lattices

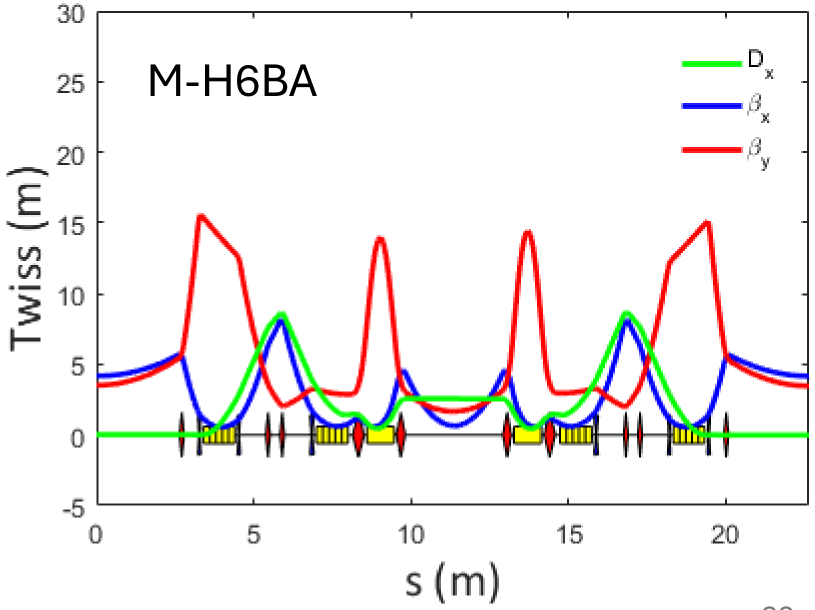
The latest generation of electron storage rings are diverse in the way they make use of the scaling of the emittance with the number of bending magnets (i.e. with bend angle):

- Transverse gradient, longitudinally-varying and reverse bends are used to reach the lowest emittance.

- Hybrid lattices provide locations where the dispersion is allowed to grow. This gives efficient correction of the chromaticity as the sextupoles can be placed where dispersion is large.

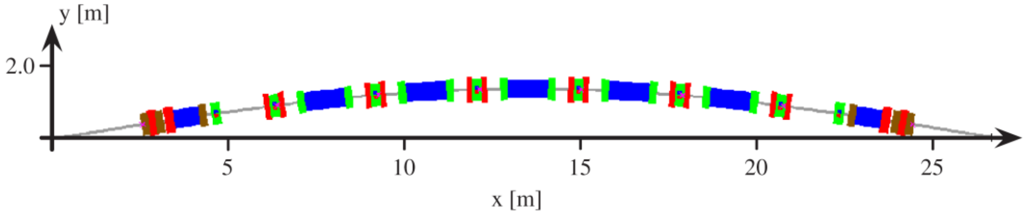
- Arc sections can be split to provide additional space for insertion device beamlines.

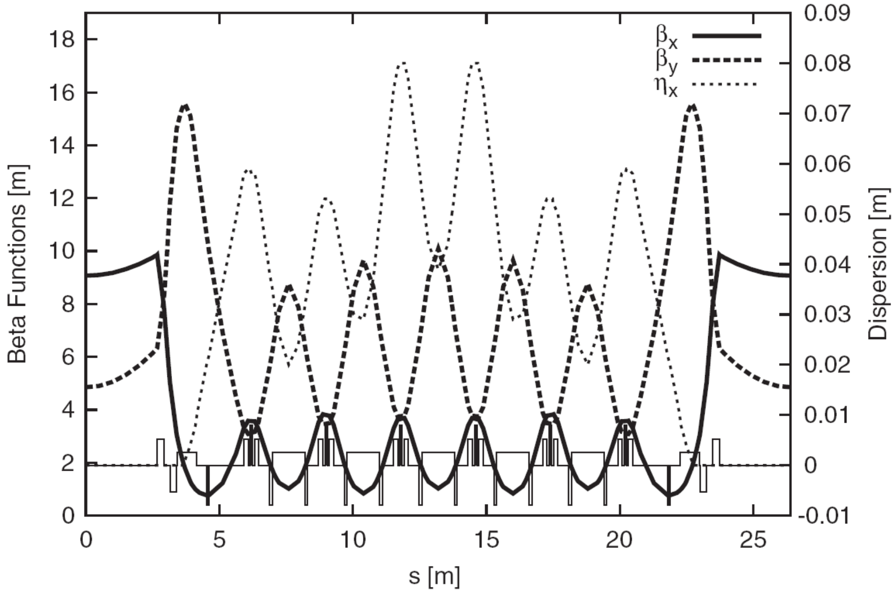




Multi-bend achromat (MBA) lattices

The MAX-IV storage ring in Sweden was the first to be built using an MBA lattice. There are seven bends per arc, with a vertically-focussing gradient in the bending magnets (increasing j_x to lower the emittance).





Parameter	Value
Beam Energy	3 GeV
Circumference	528 m
Number of Cells	20
Horizontal Emittance (no IDs)	326 pm
Horizontal Emittance (with IDs)	263 pm

S.C. Leeman et al, "Beam dynamics and expected performance of Sweden's new storage ring light source: MAX IV", PRST-AB 12, 120701, (2009)

Summary (1)

The natural emittance in a storage ring is determined by the balance between the radiation damping (given by I_2) and the quantum excitation (given by I_5)

The quantum excitation depends on the lattice functions. Different styles of lattice can be used, depending on the emittance specification for the storage ring.

In general, for small bending angle θ the natural emittance can be written as:

$$\varepsilon_0 \approx F C_q \gamma^2 \theta^3 \quad (32)$$

Where θ is the bending angle of a single dipole and the numerical factor F is determined by the lattice style.

Summary (2)

$$\varepsilon_0 \approx F C_q \gamma^2 \theta^3$$

Lattice style	F
90° FODO	$2\sqrt{2}$
137° FODO	1.2
Double bend achromat (DBA)	$\frac{1}{4\sqrt{15}}$
Multi bend achromat (MBA)	$\frac{1}{12\sqrt{15}}\left(\frac{M+1}{M-1}\right)$
Theoretical minimum emittance (TME)	$\frac{1}{12\sqrt{15}}$

Summary (3)

Achromats have been popular choices for storage ring lattices in third and fourth generation synchrotron light sources for two reasons:

- They provide lower natural emittance than FODO lattices
- They provide zero-dispersion locations appropriate for insertion devices (wigglers and undulators)

Increasing the number of bends in an achromat cell (multi bend achromats) and detuning an achromat (to allow some dispersion in the straights) can help to achieve a lower emittance

Third-generation light sources were typically built using double bend achromats (such as ESRF, APS, Spring-8, Diamond, Soleil, ...) or triple bend achromats (such as ALS, SLS).

Many of these are being upgraded with fourth-generation MBA lattices to benefit from the lower emittance electron beams and hence higher-brightness photon beams.

Summary (4)

The opening angle of the synchrotron radiation places a lower limit (typically a fraction of a picometer) on the vertical emittance.

In practice the vertical emittance is dominated by alignment and tuning errors (betatron coupling and vertical dispersion).

Natural emittances of a few nanometres are typical for third generation and around a hundred picometres are typical for fourth generation light sources.

Storage rings for light sources often operate with vertical emittances of around 0.1-1% of the horizontal (natural) emittance: this requires careful tuning and correction of alignment errors.

References

These lectures are based on ones prepared by A. Wolski and given at the CERN Advanced Accelerator School in November 2022, Sevrier, France:

[1] https://indico.cern.ch/event/1126689/

Other useful references on this topic include:

[2] S.Y. Lee, “Accelerator Physics”, World Scientific Publishing, (1999)

[3] M. Borland et al., “Lattice design challenges for fourth-generation storage-ring light sources”, J. Synch. Rad. 21, 912-936, (2014)

[4] T. Raubenheimer, “The Generation and acceleration of low emittance flat beams for future linear colliders”, SLAC Report 387, p.19, (1991)

[5] A. Streun, “The anti-bend cell for ultralow emittance storage ring lattices”, Nuclear Inst. Meth. In Phys. Res. A, 737, p. 148-154, (2014)

[6] P. Raimondi, S. Liuzzo, “Towards a diffraction limited light source”, Phys. Rev. Accel. Beams, 26, 021601, (2023)

[7] B. Riemann, A Streun, “Low emittance lattice design from first principles: reverse bending and longitudinal gradient bends”, Phys. Rev. Accel. Beams, 22, 021601, (2019)