Landau damping

Lecture notes available at **https://xbuffat.web.cern.ch/landaudampingCAS.pdf**

The CERN Accelerator School

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Beams Department – Accelerator and Beam Physics
Collective Effects and Impedances

CERN, Switzerland, Geneva

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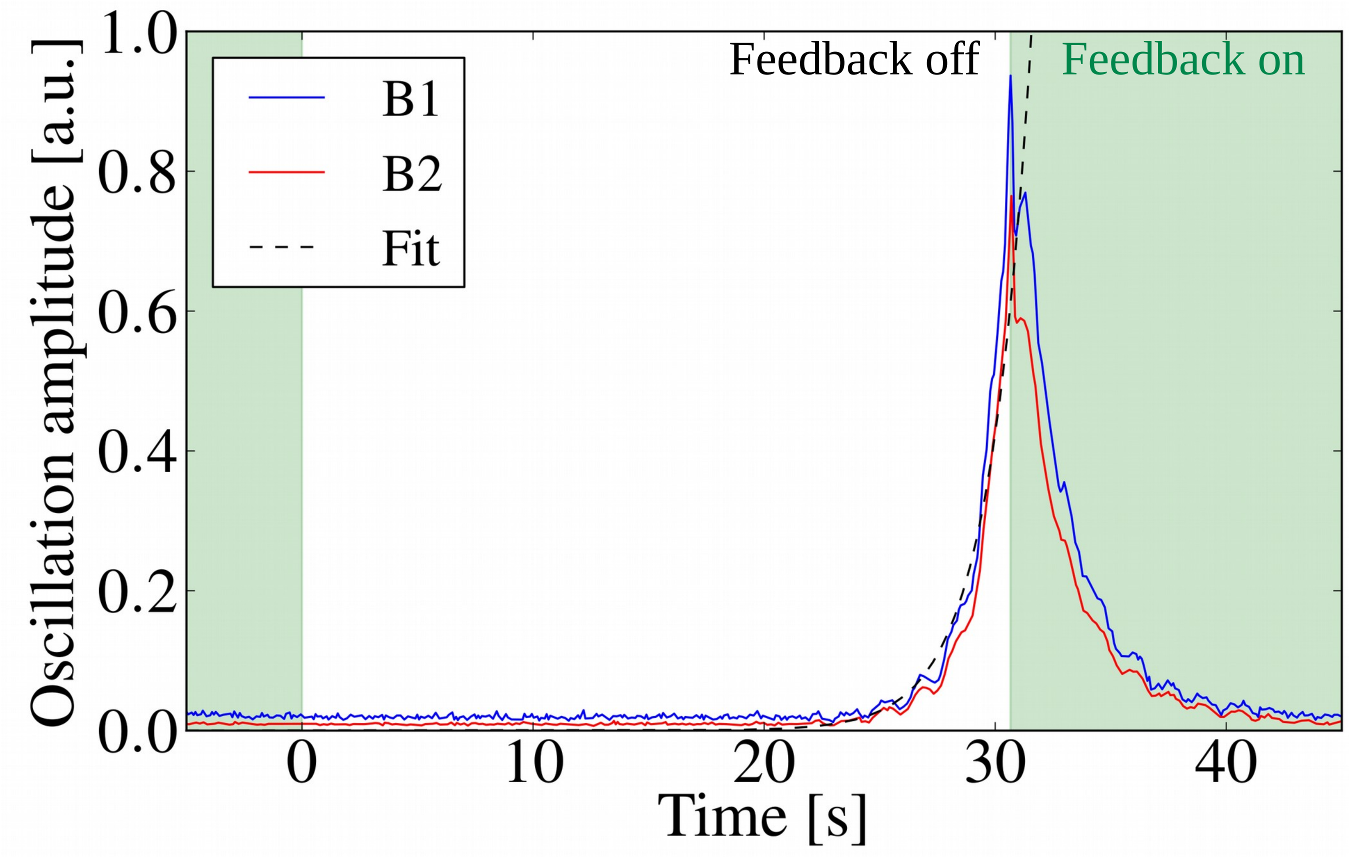
Beam instabilities

- Beams tend to self-destruct via self-amplified oscillations



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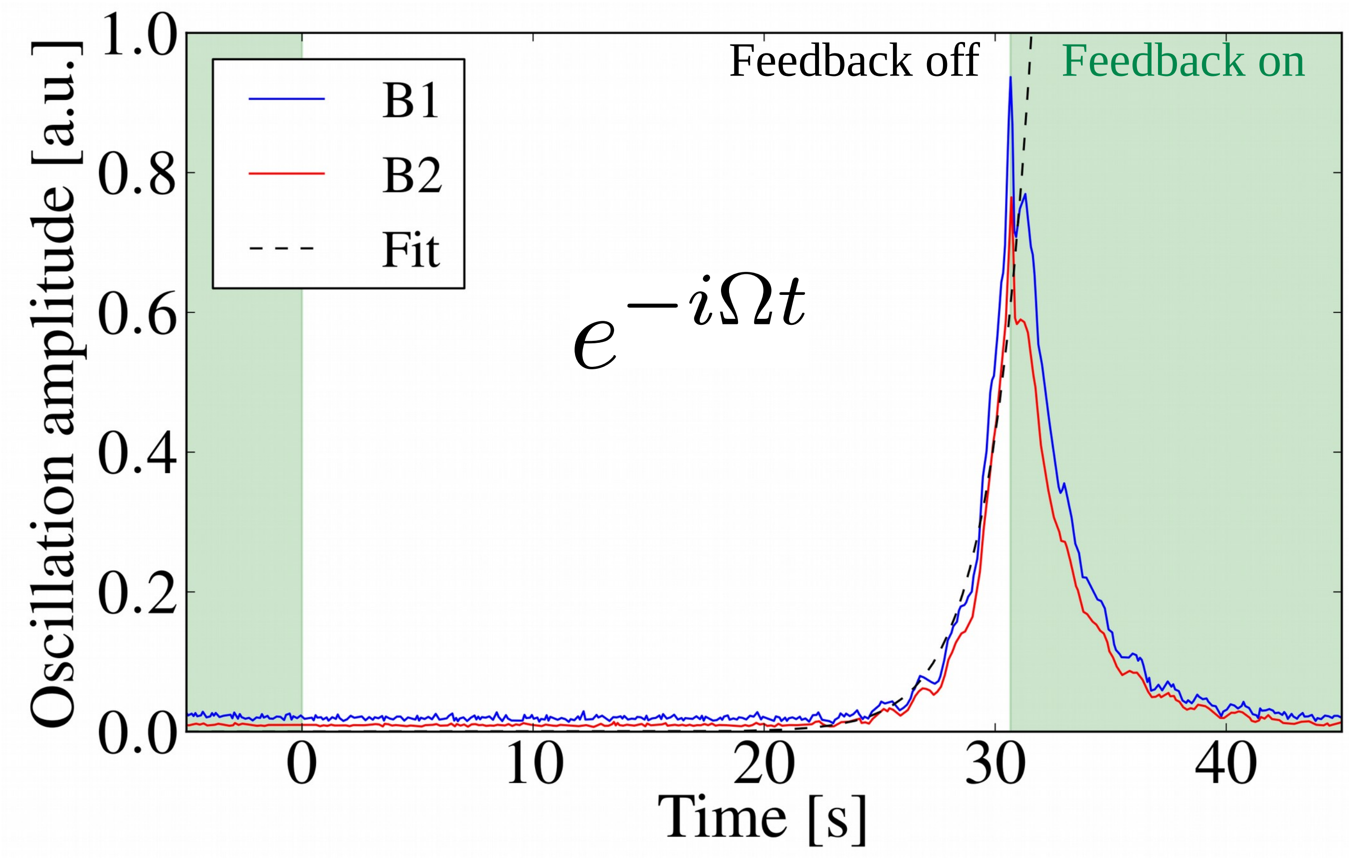

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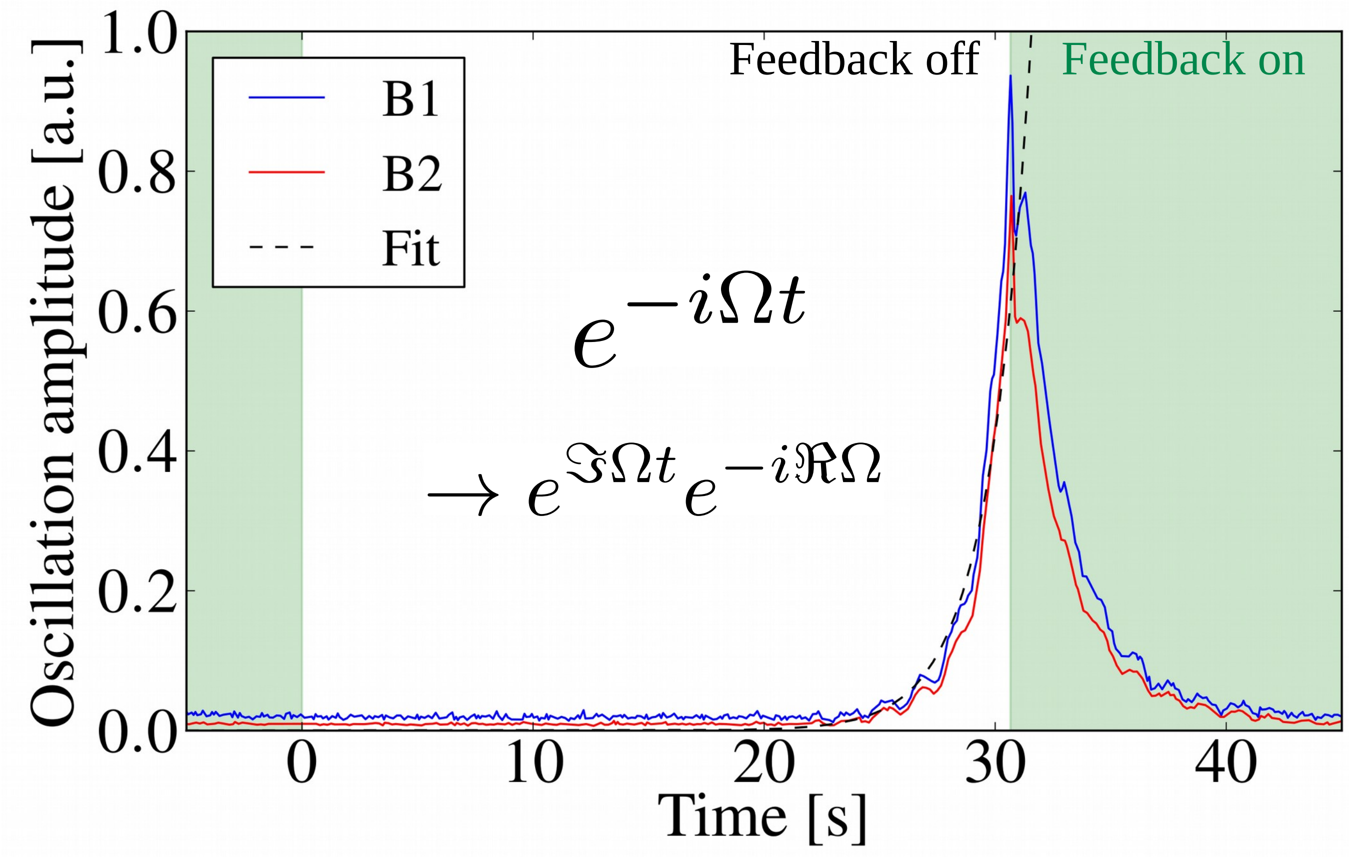

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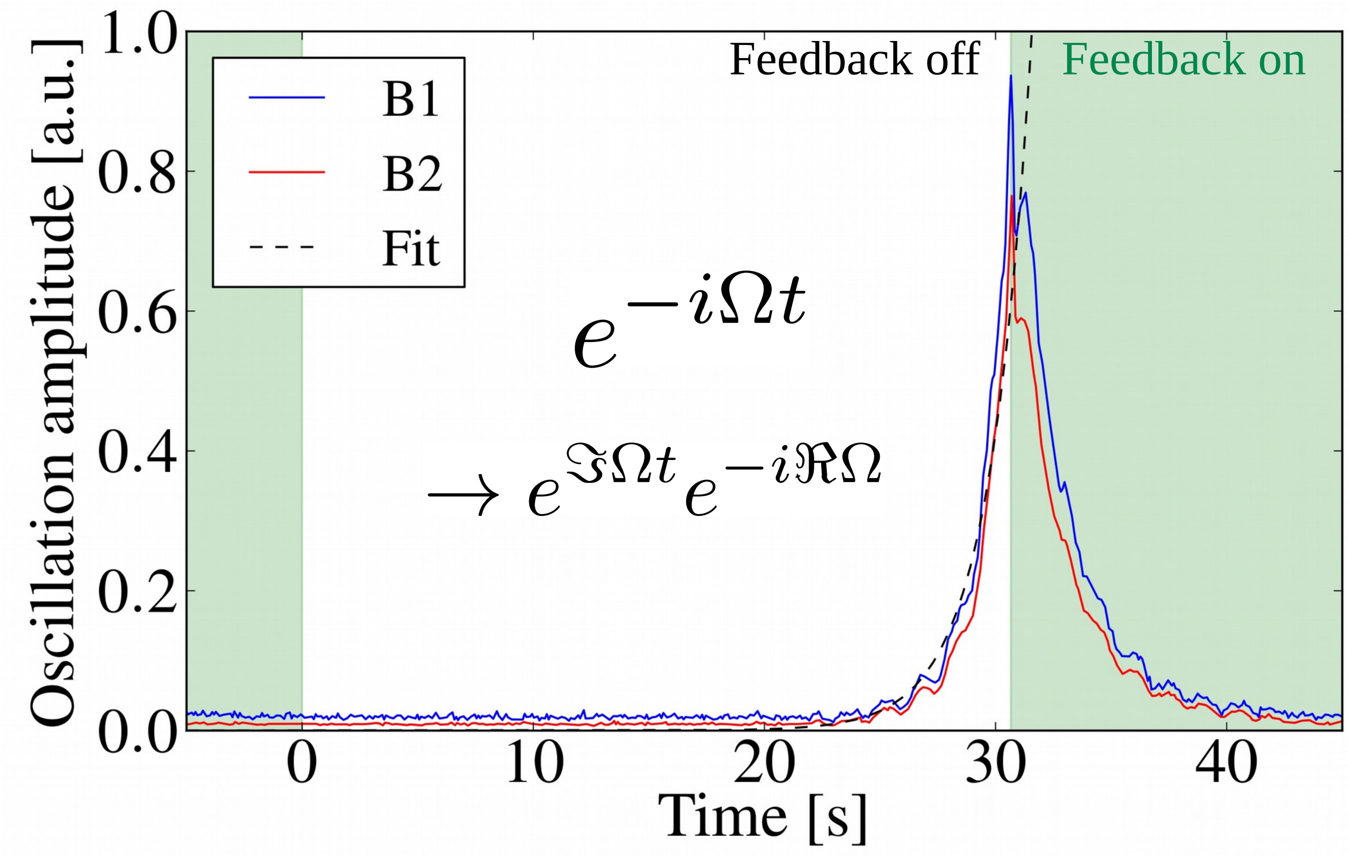
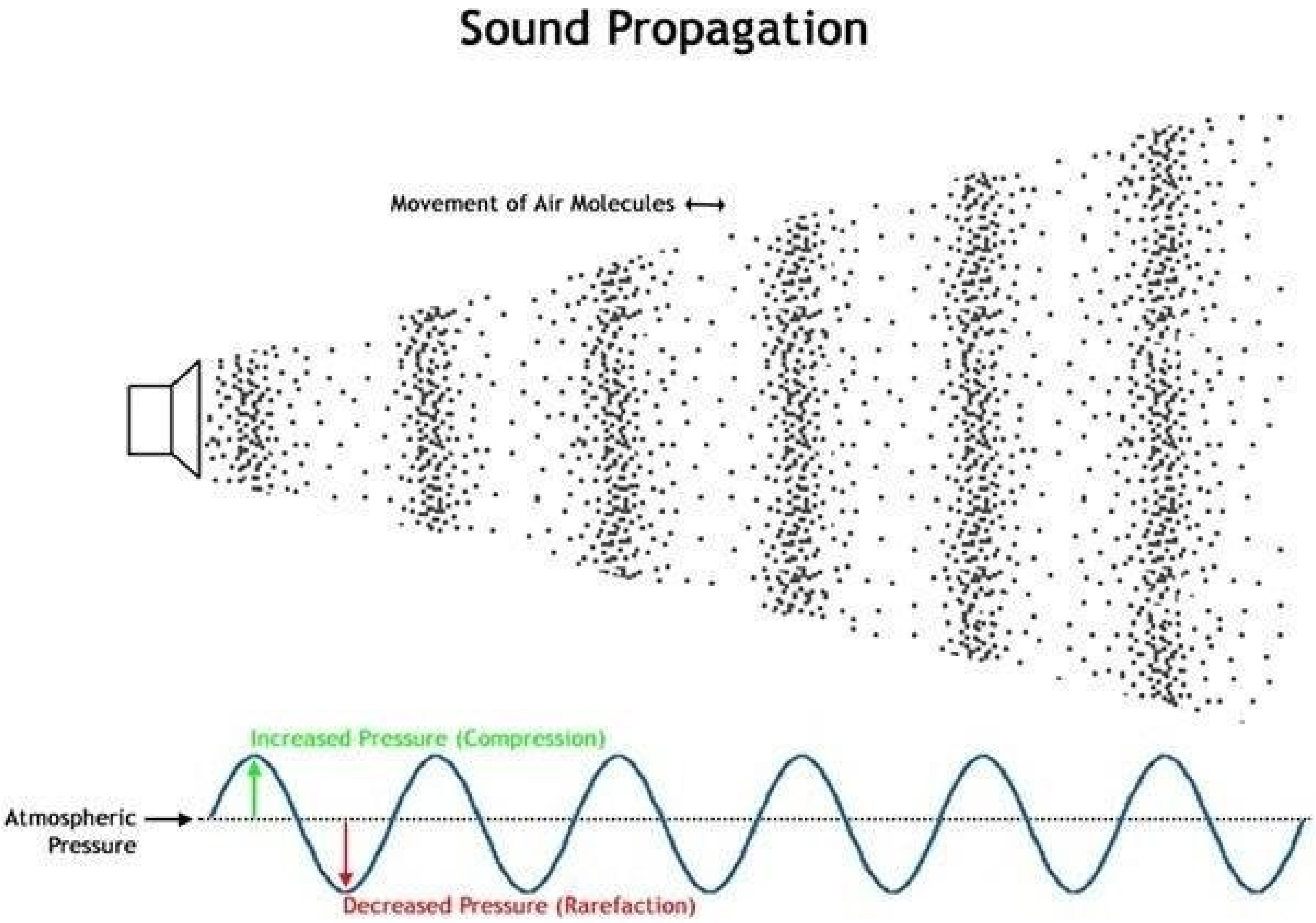


→ Landau damping is (almost) **always needed** to obtain good quality beams

Content

- Part I (concept)
  - Wave – particle interaction
  - Decoherence
  - Landau damping using Van Kampen approach
  - Stability diagram and beam transfer function
- Part II (applications)
  - Longitudinal and transverse Landau damping in unbunched and bunched beams
  - Non-linear collective forces
  - Advanced Landau damping techniques

Single particle motion ≠ collective motion



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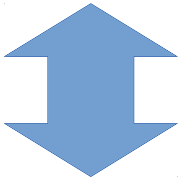
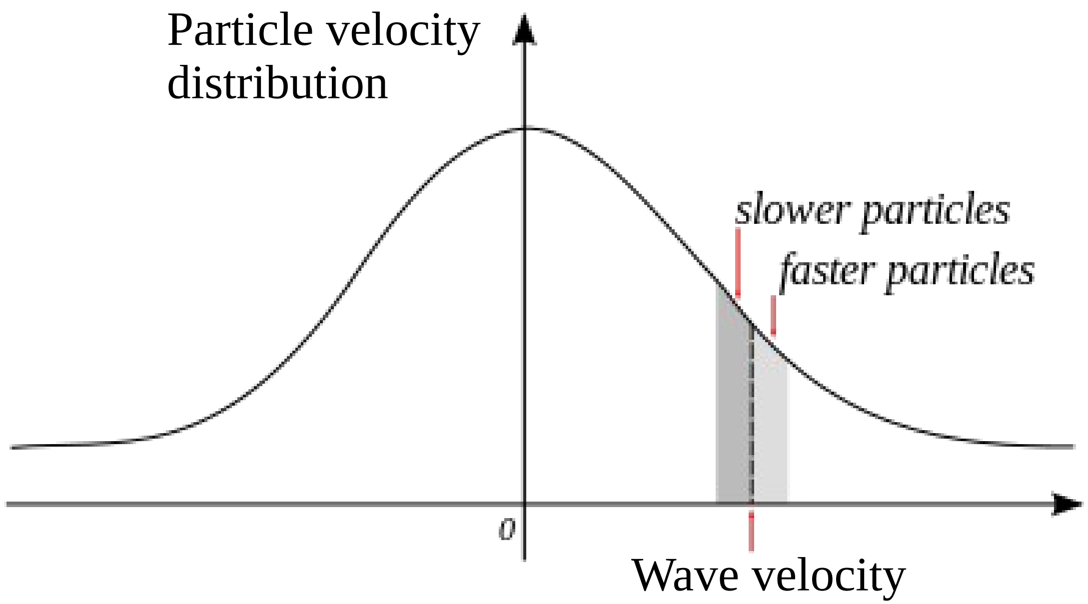


- **Particles** can exchange energy with a wave when they have **a similar velocity**

Damping of collective motion

[WikiLandau, WikiTwoStream]



Damping of collective motion

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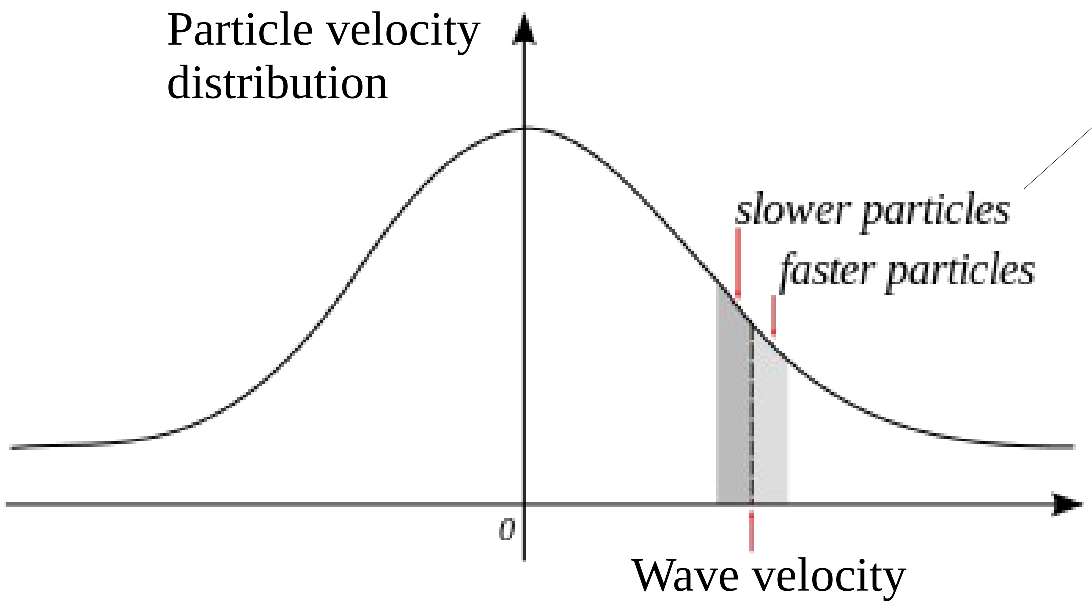
Gain energy from the wave



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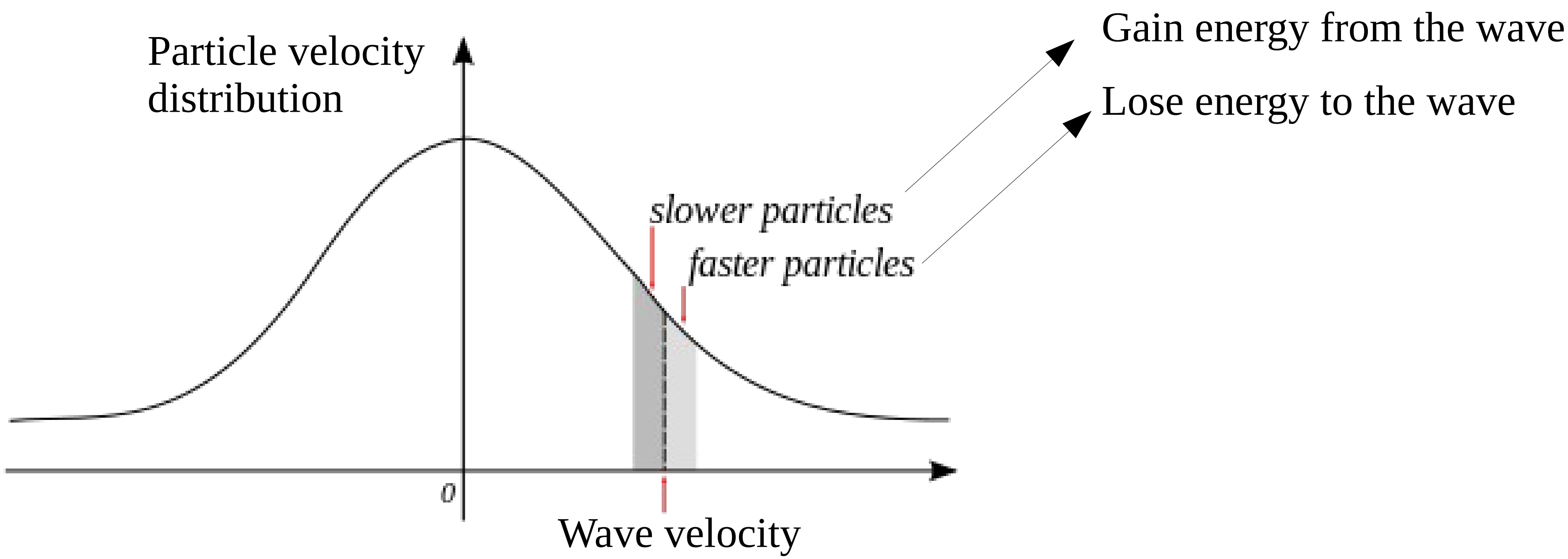

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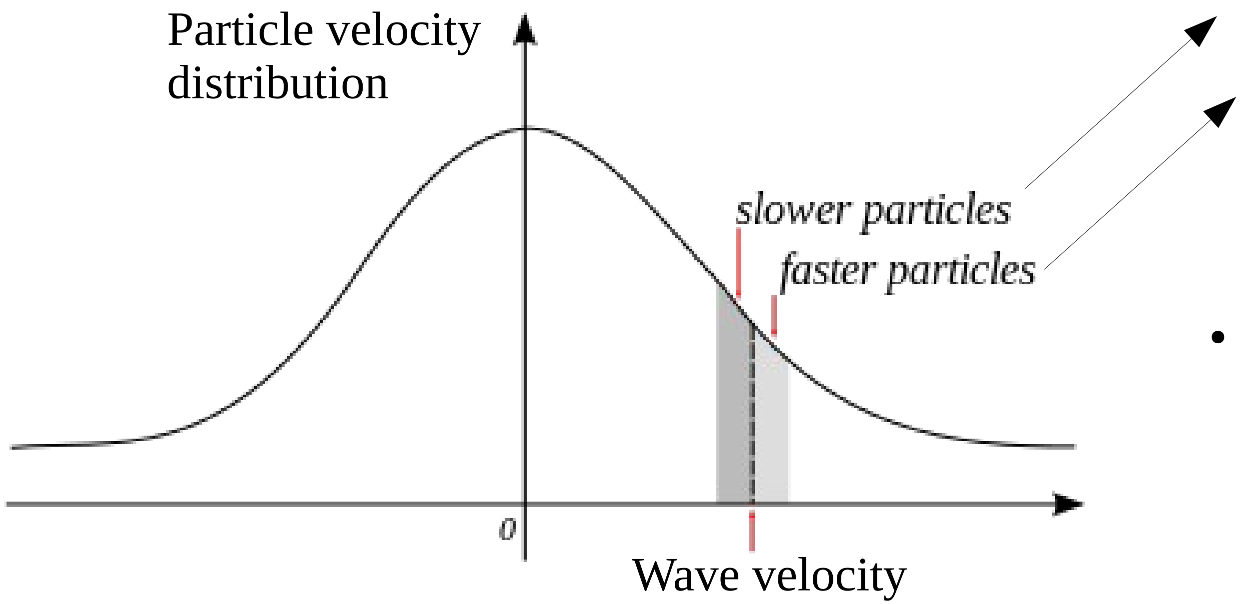

- The wave is damped when it loses more energy than it gains from particles → **Landau damping**

Damping of collective motion

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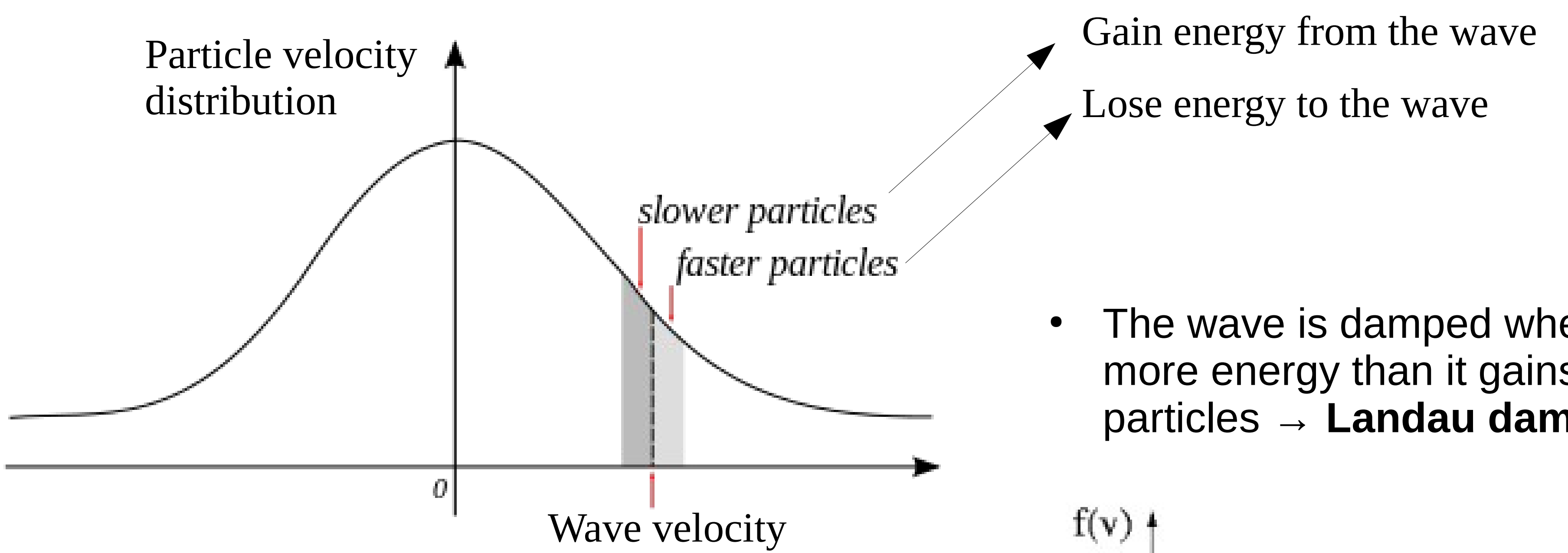

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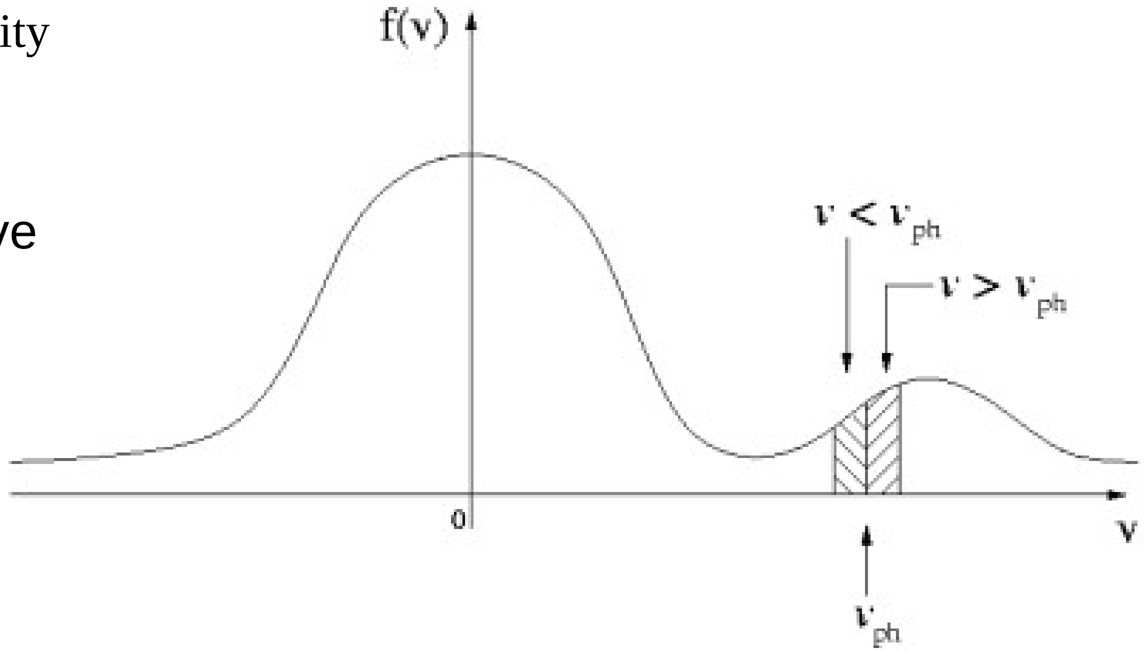

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Damping of collective motion

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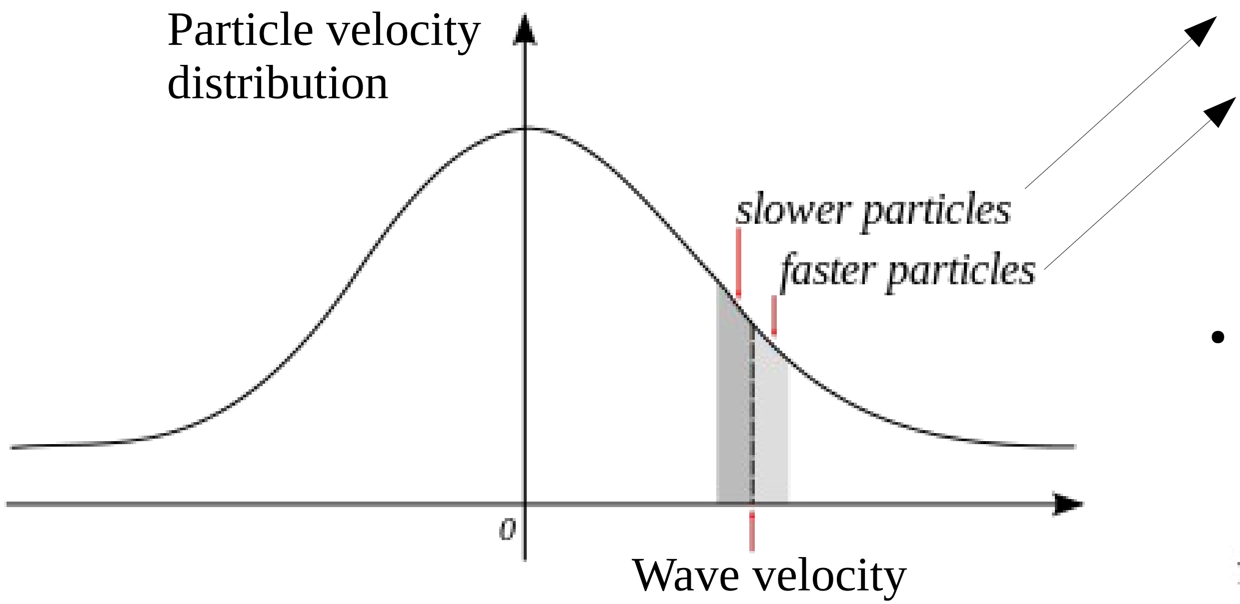

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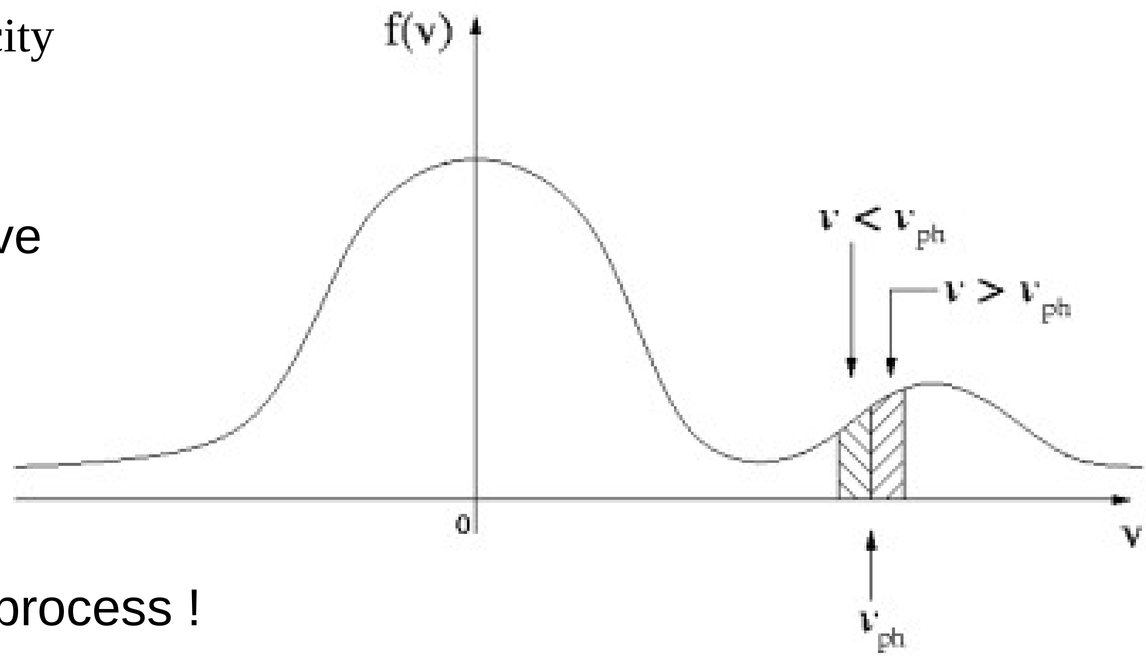

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- Landau damping is **collisionless** process !

  The interaction between the particles and the wave occures only via the collective force (e.g. electromagnetic fields)

- **Landau damping prevents instabilities to happens**

Damping of collective motion
A little subtlety for accelerators

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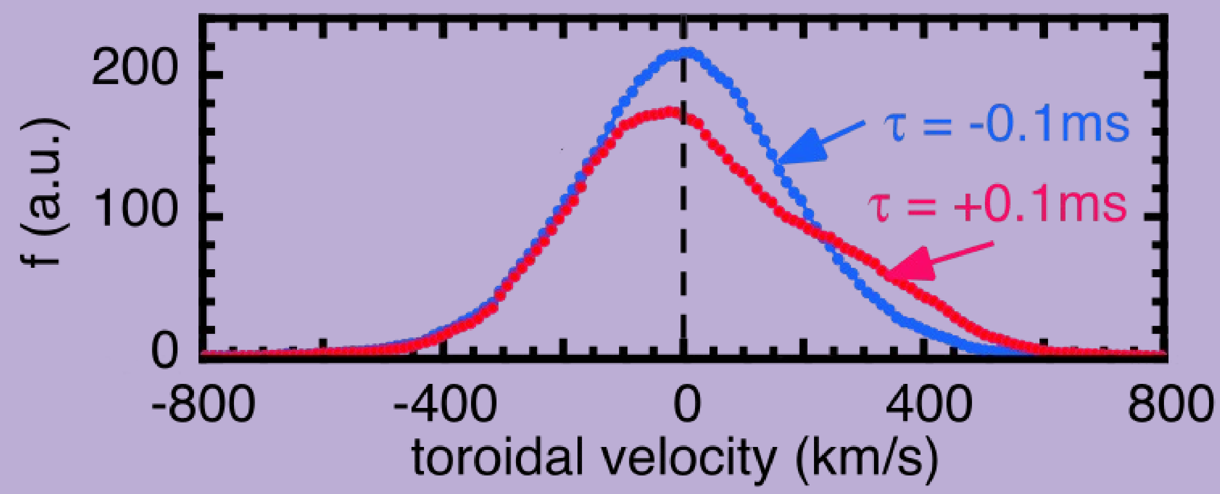

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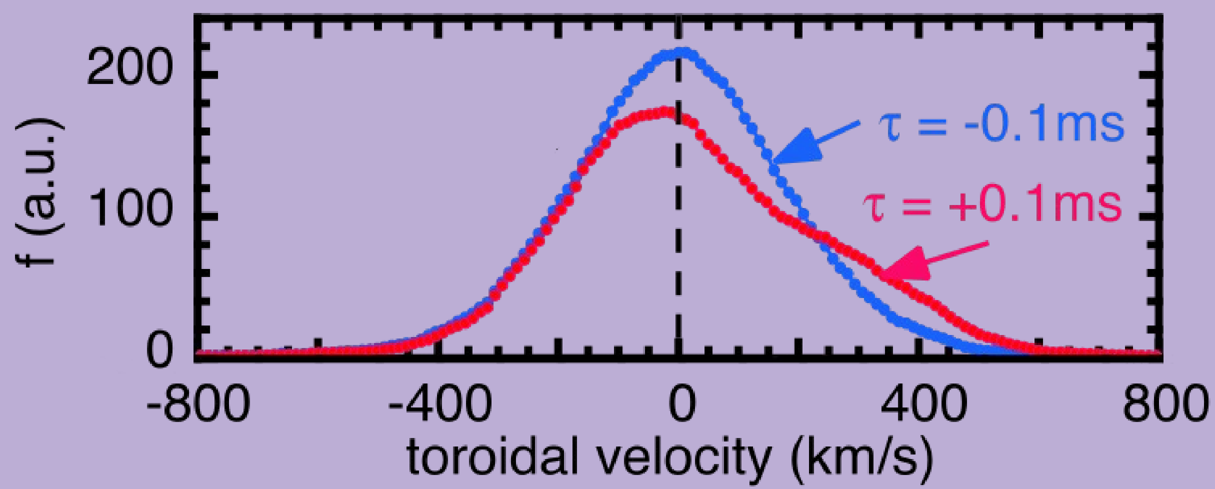
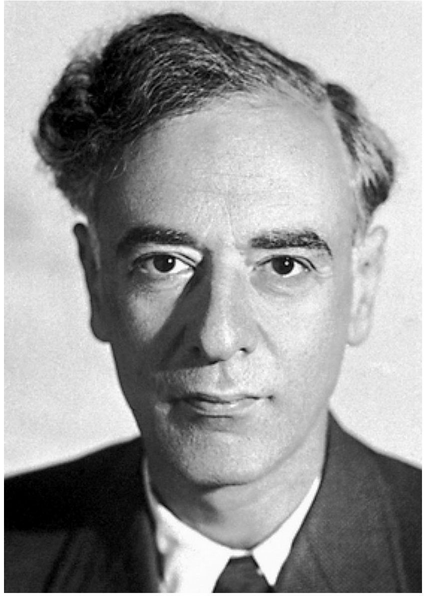


- In accelerators we refer to this effect as **decoherence** or filamentation

→ The main difference with Landau damping is the corresponding **emittance growth**

Landau damping in practice

[WikiLevLandau, WikiAndromeda, LIGO, ITER,LHC, QGP, Firefly]



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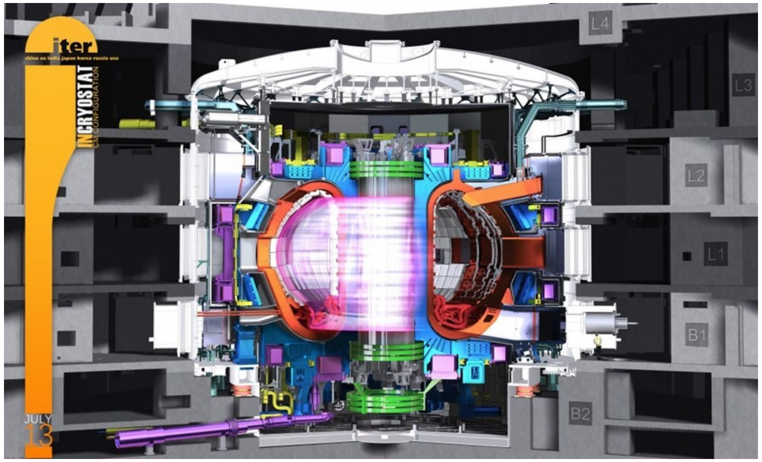

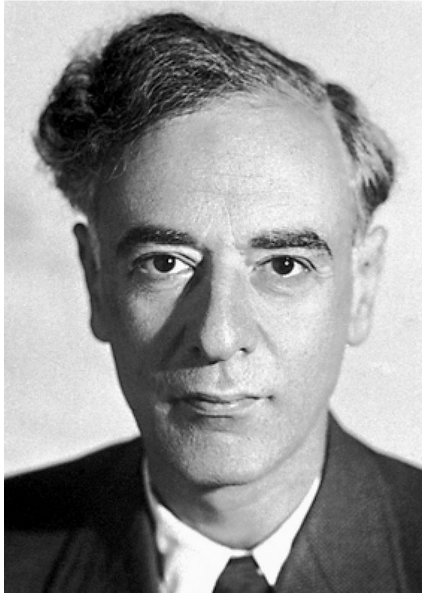

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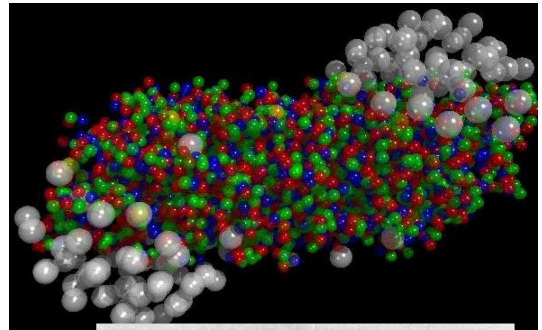

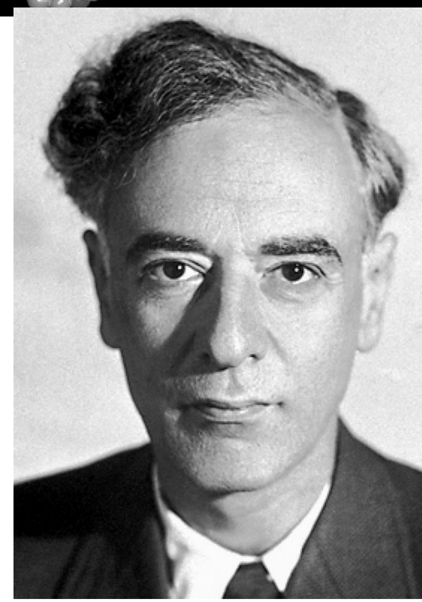

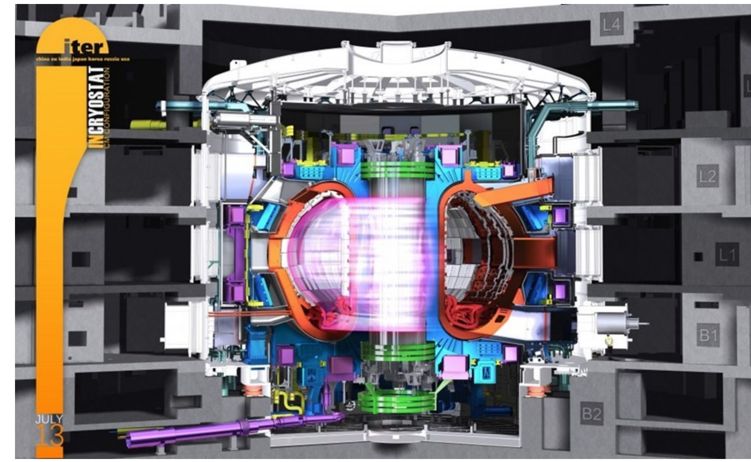

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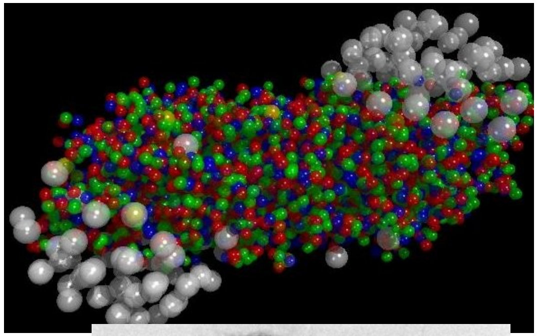

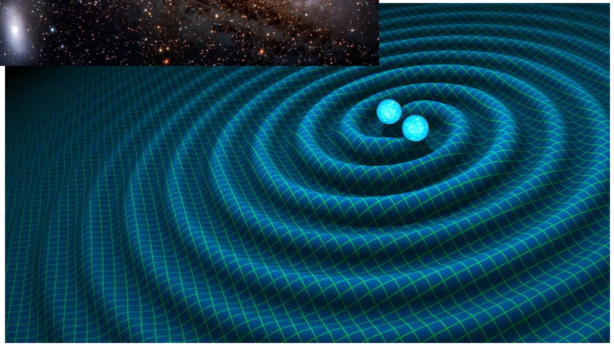

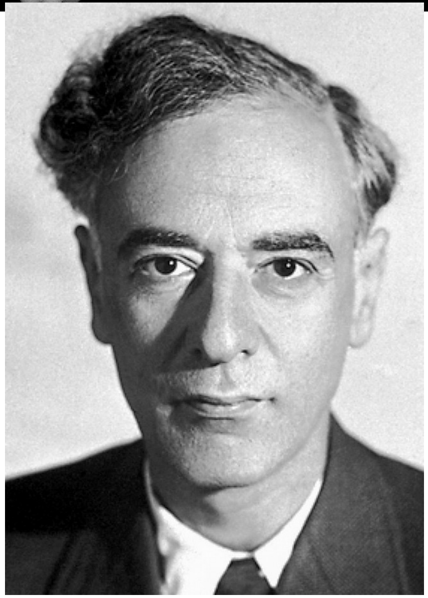

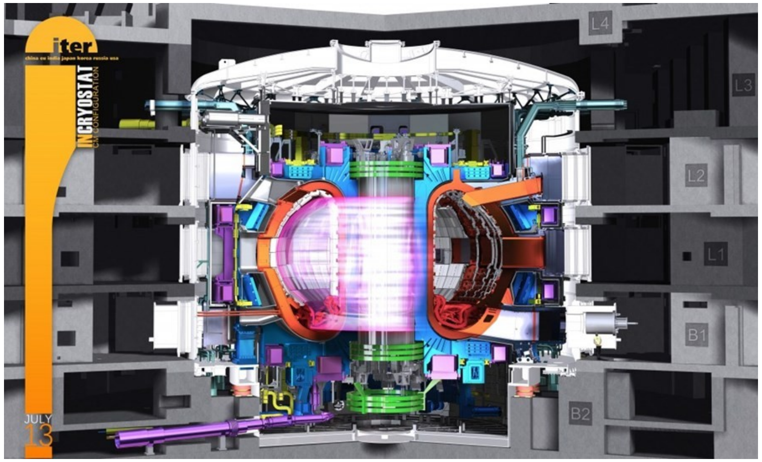

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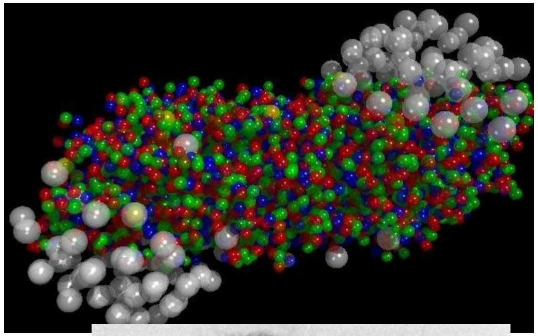

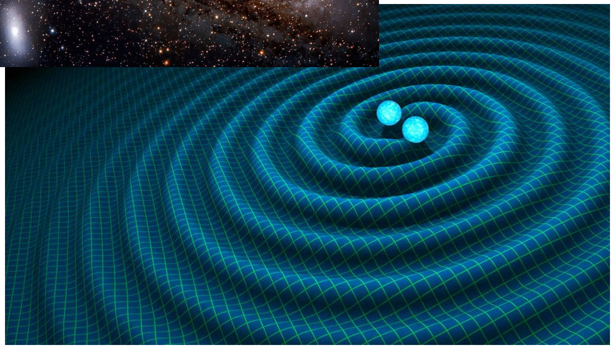

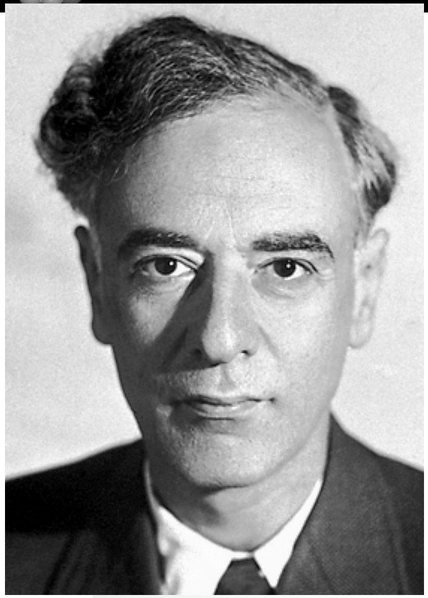

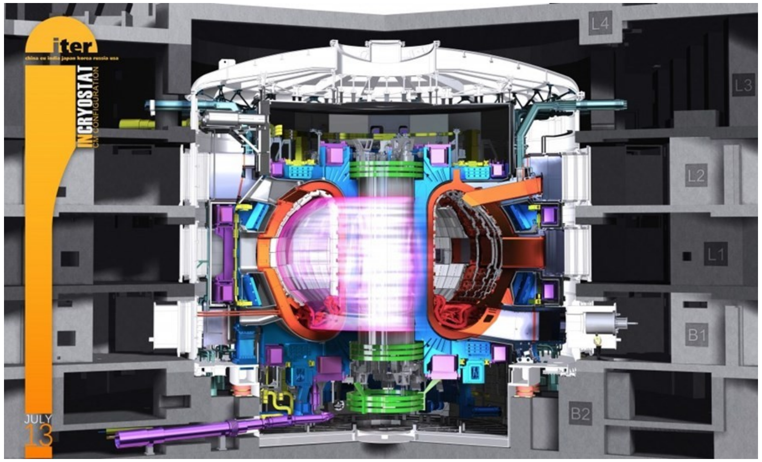

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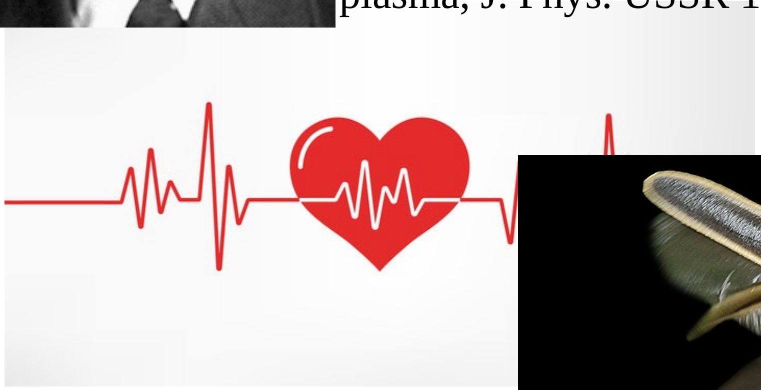

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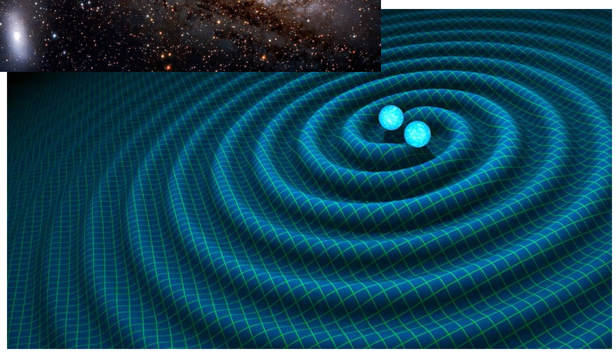

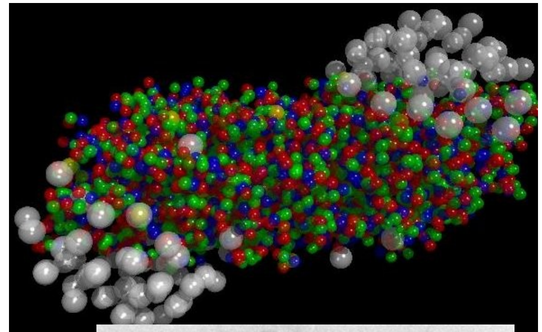

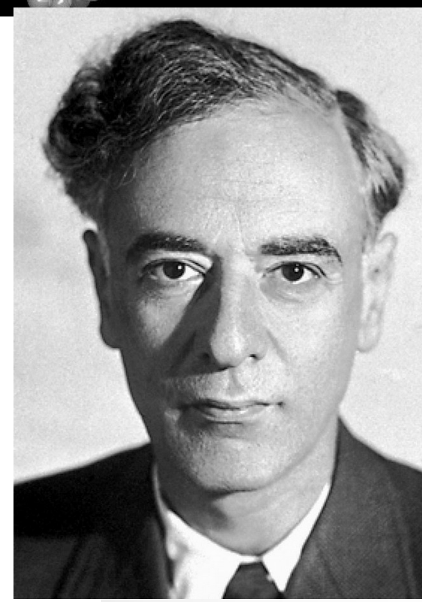

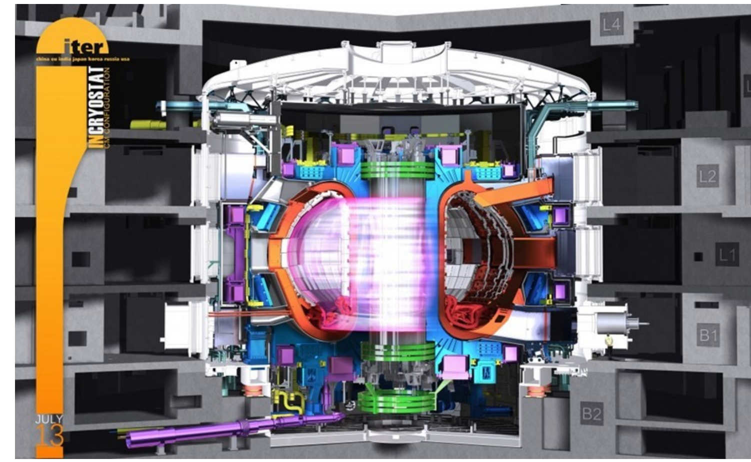

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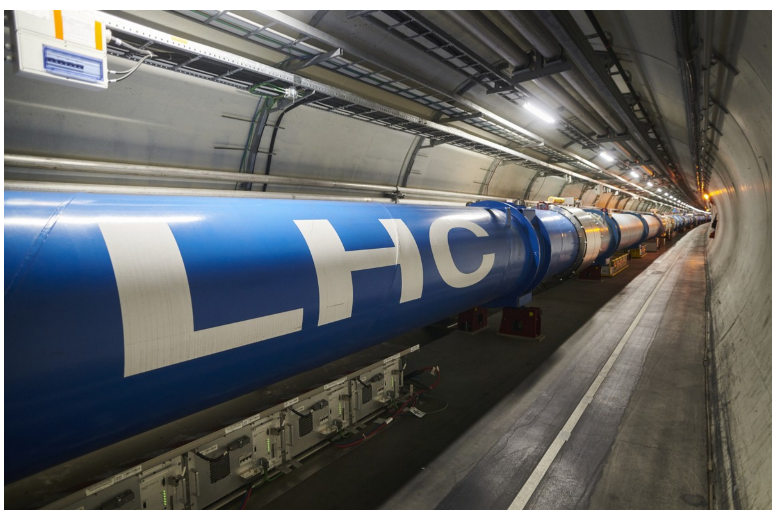
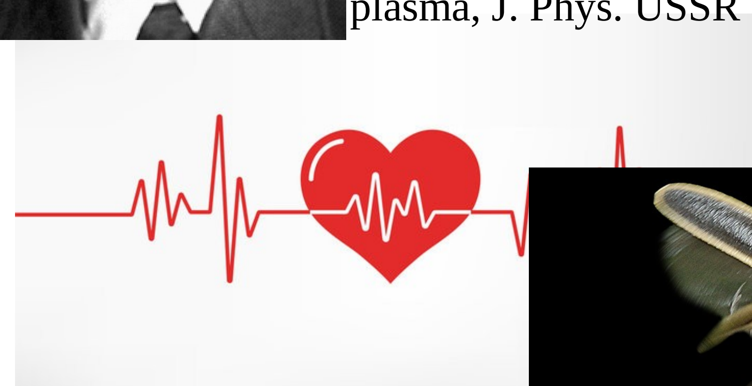
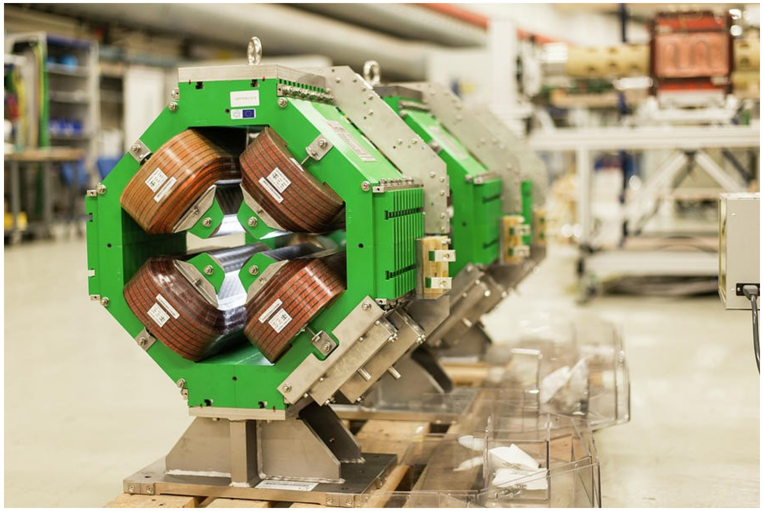


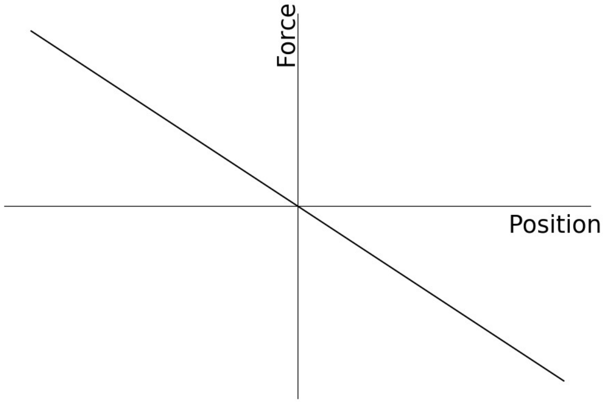


Decoherence in beams

[Sextupole, Octupole]

- The velocity spread is usually small in particle beams → an analogous effect occurs thanks to the **tune spread**





Linear force
→ Fixed oscillation frequency

$$\omega = \omega_0 = 2\pi Q_0$$

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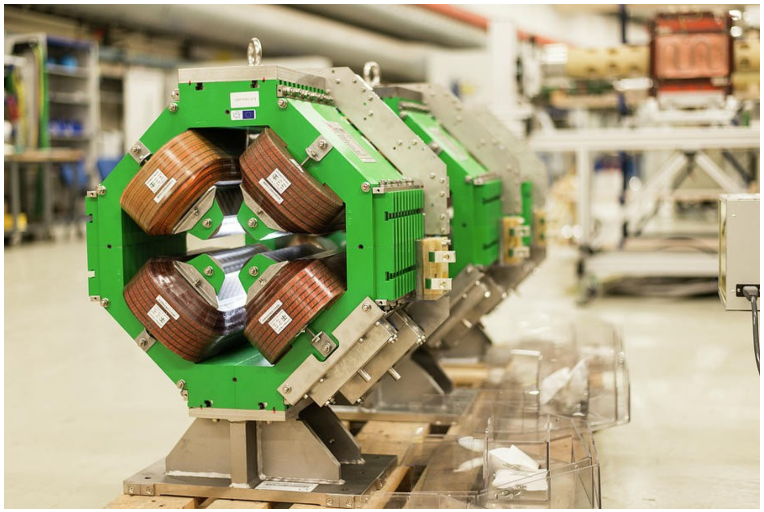

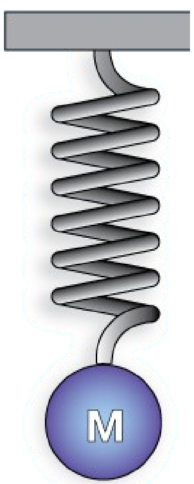

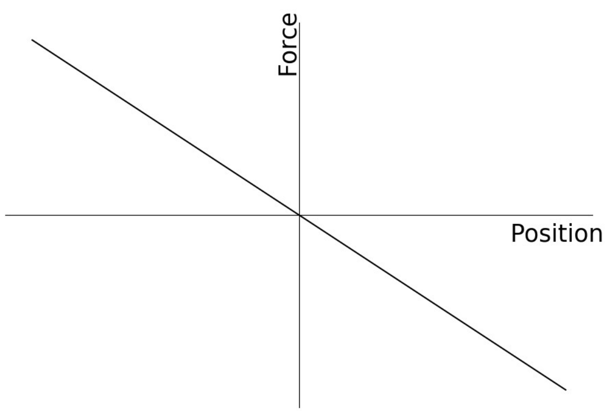

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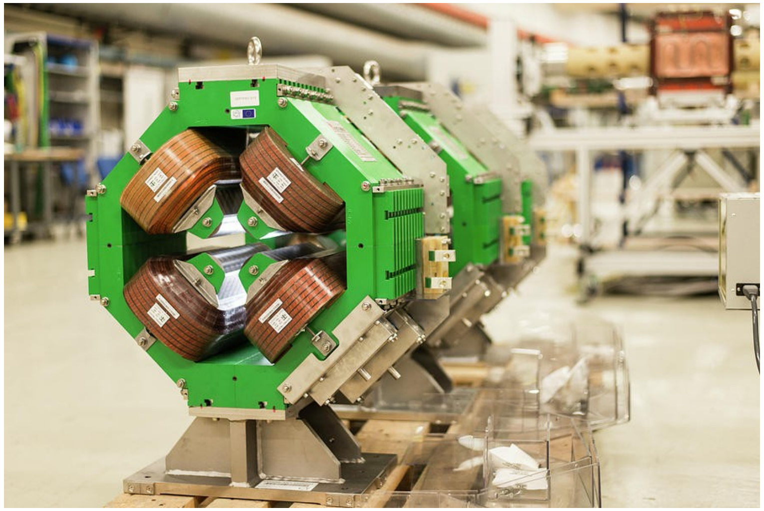

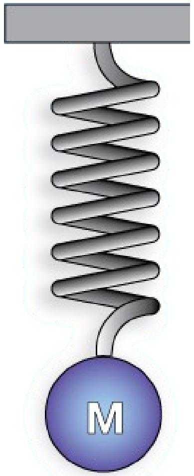

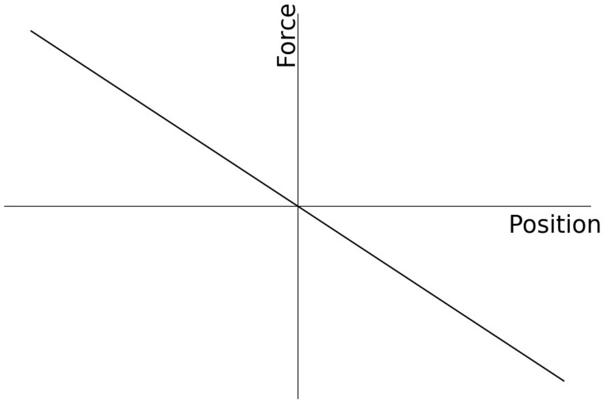

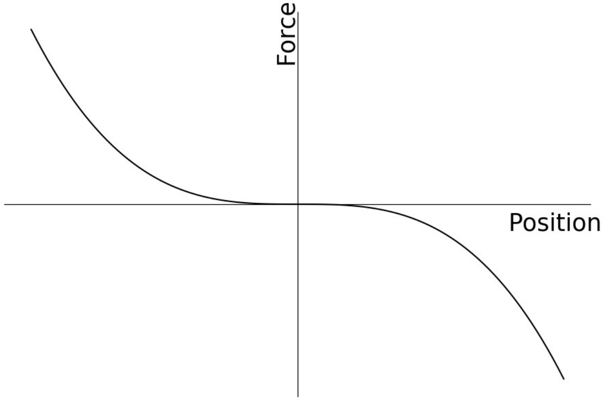

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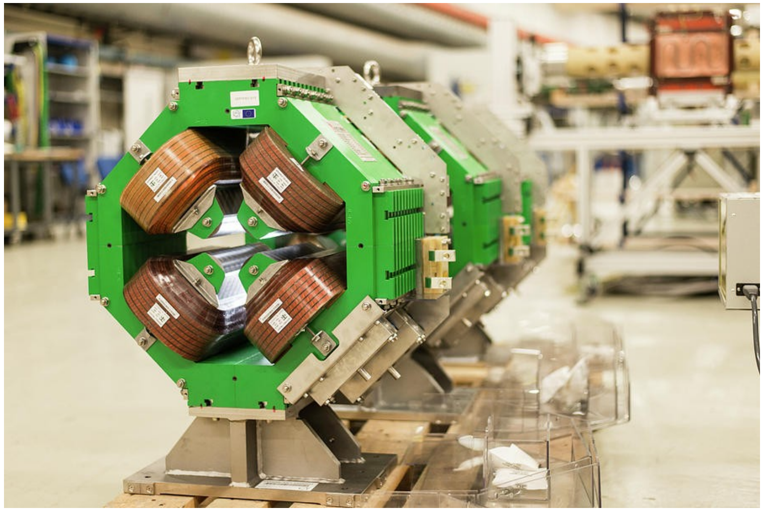

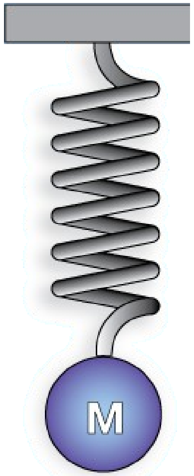

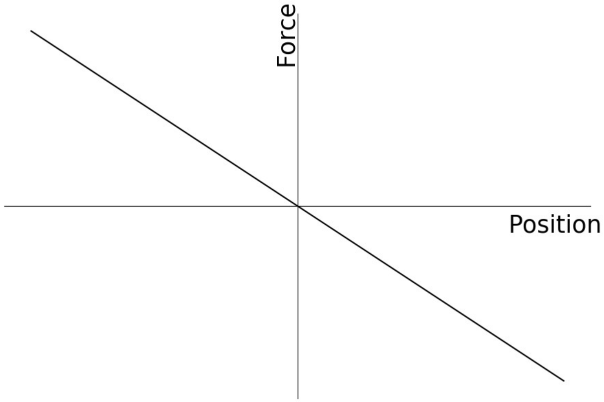

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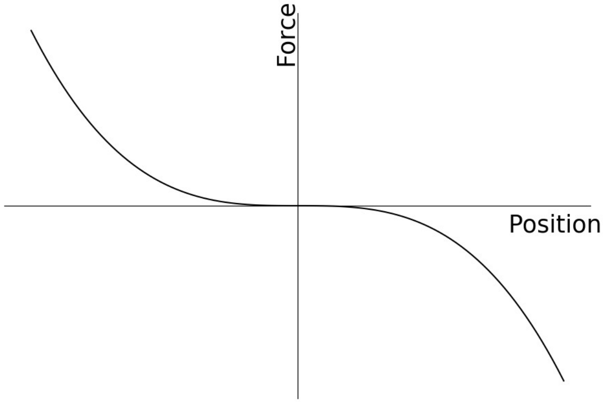
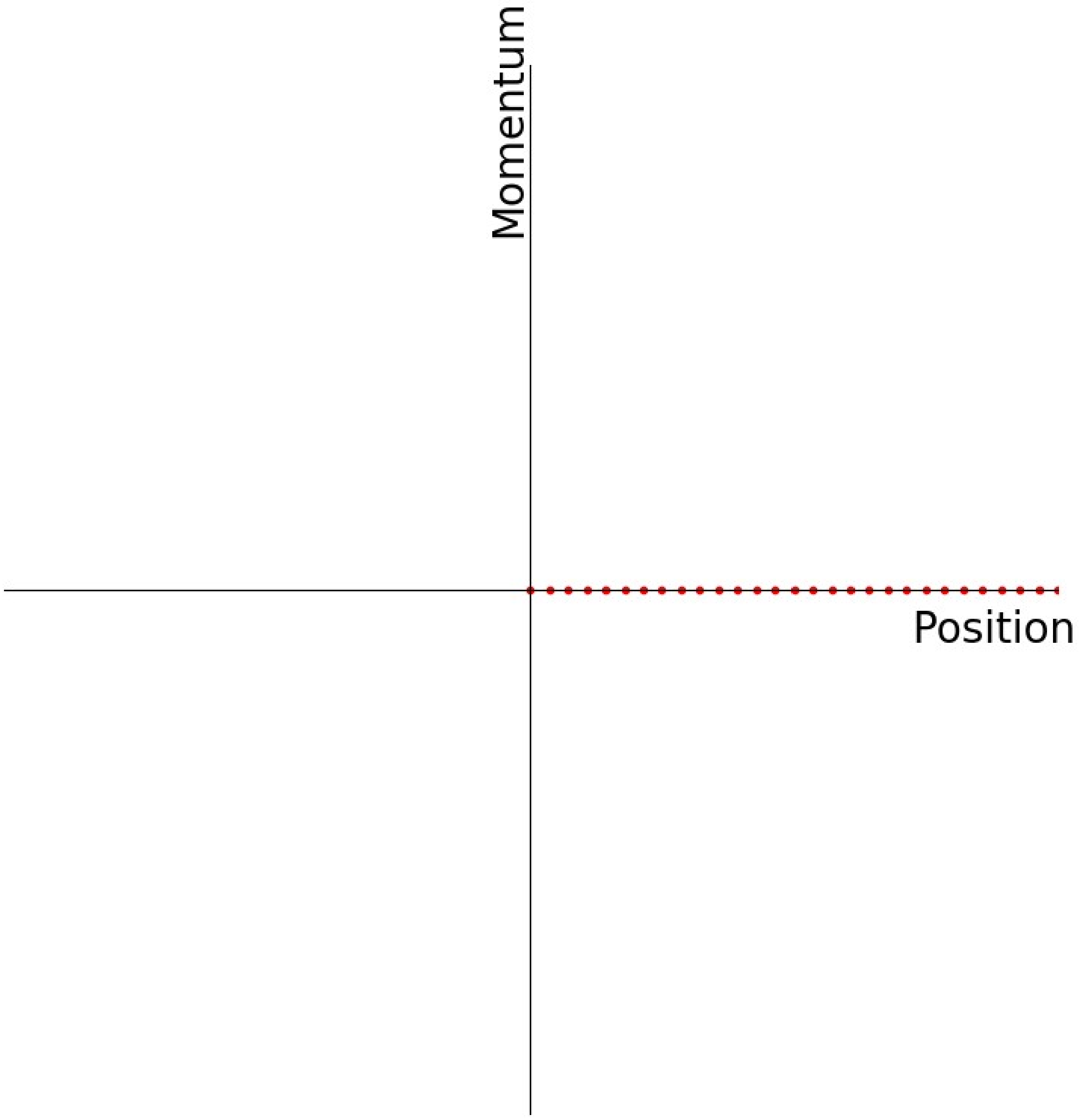


Non linear force
→ Amplitude dependent frequency / **detuning**

$$\omega(J) = 2\pi(Q_0 + aJ)$$

Decoherence



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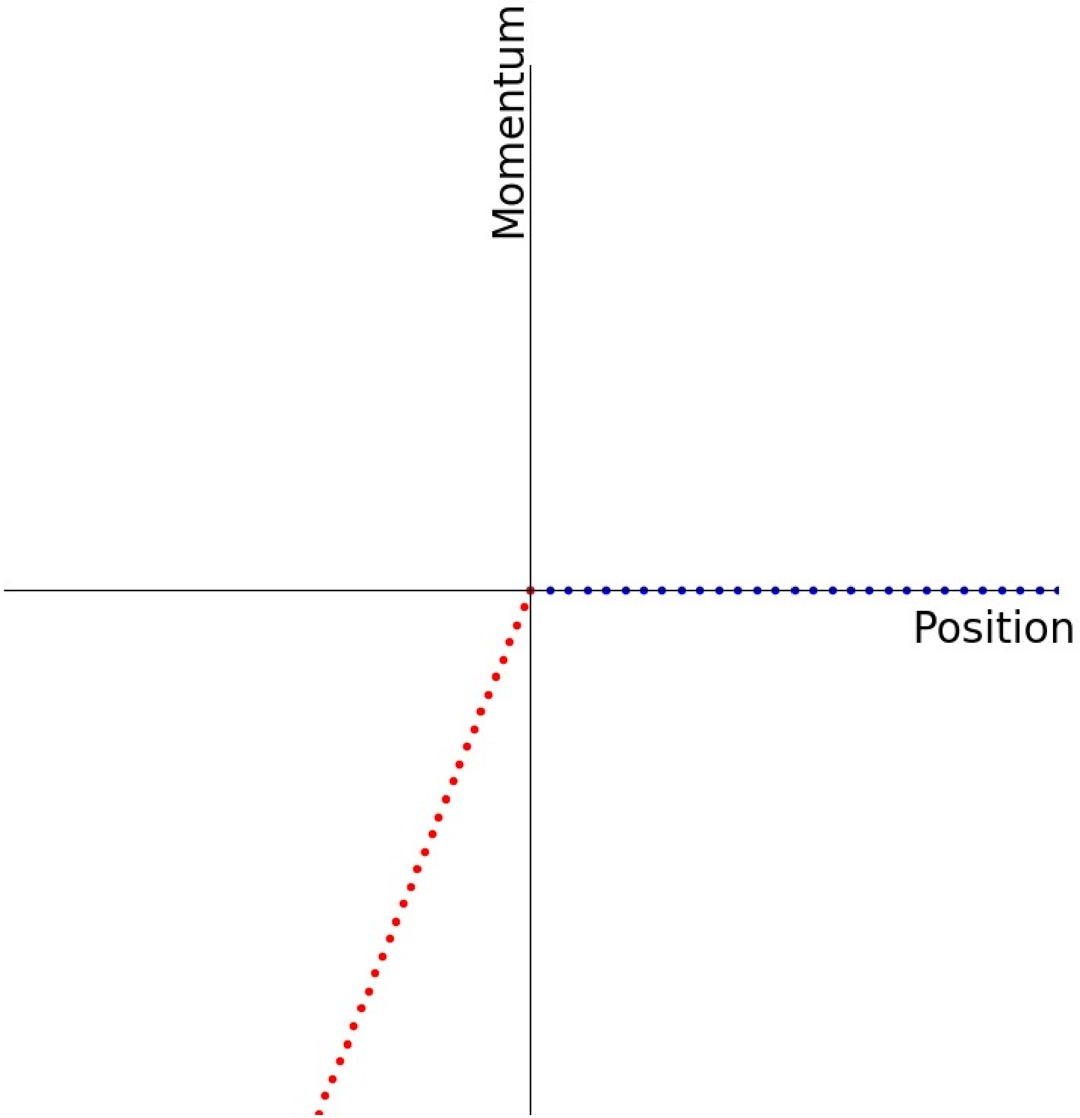

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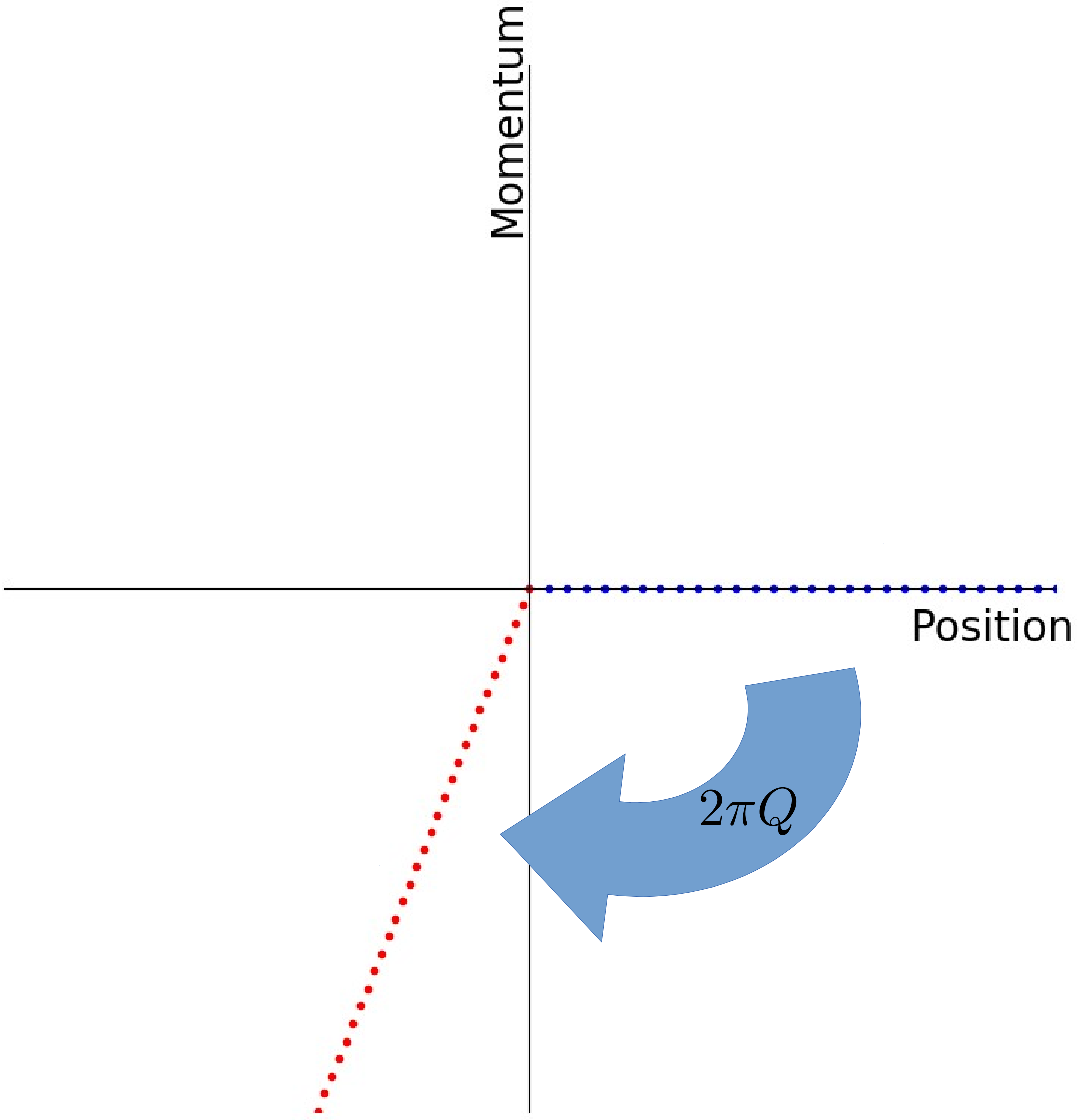

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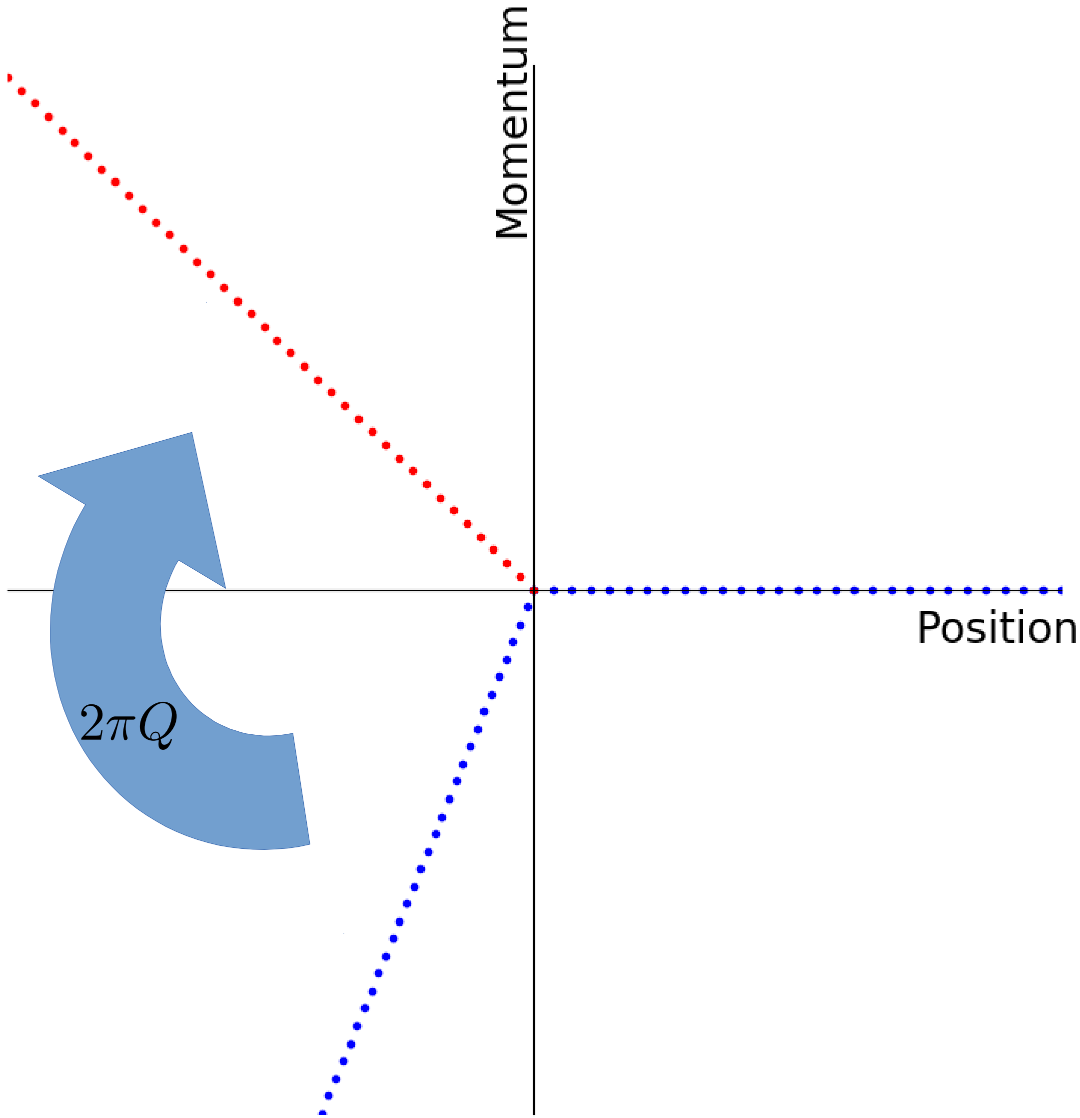

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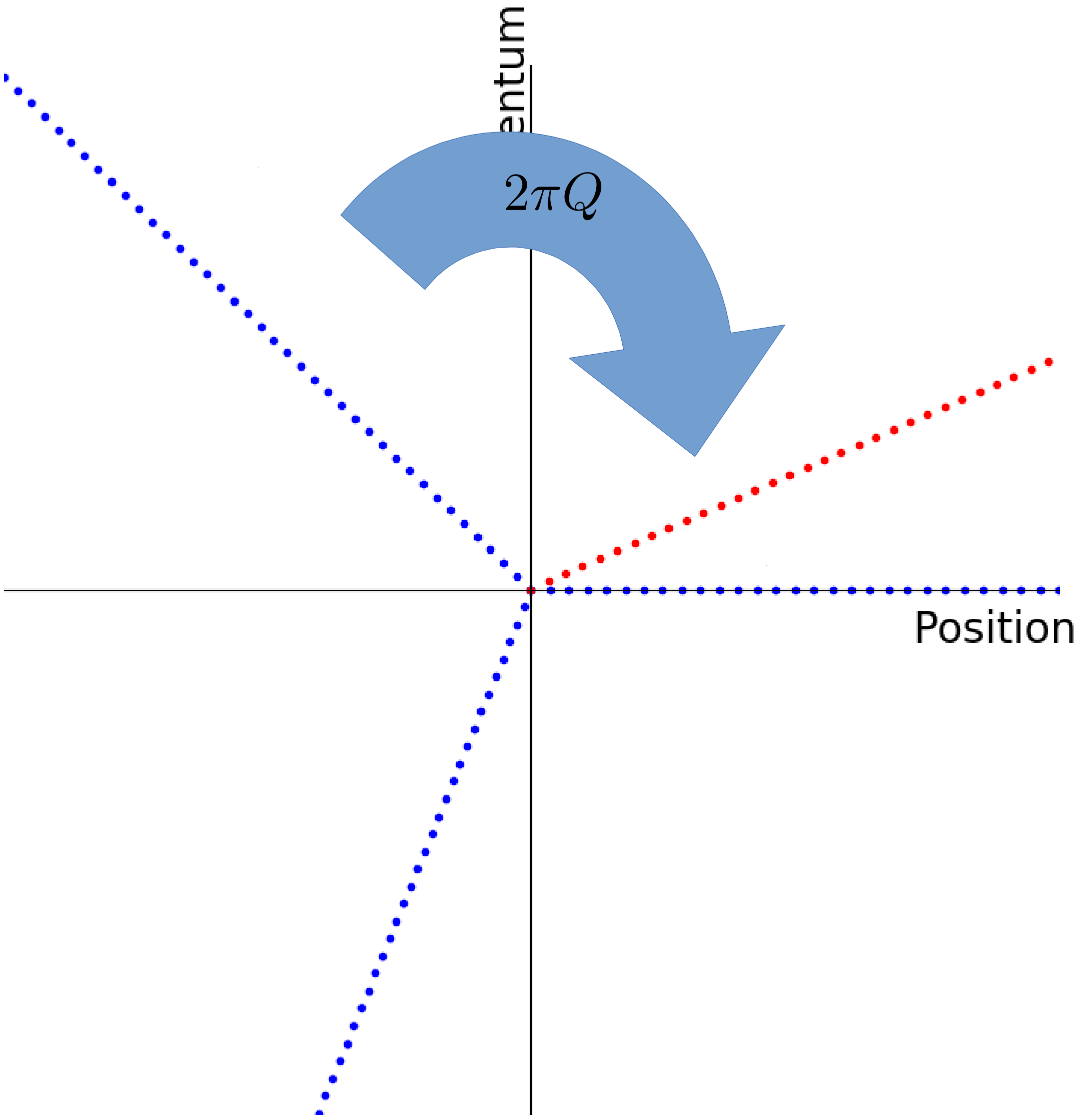

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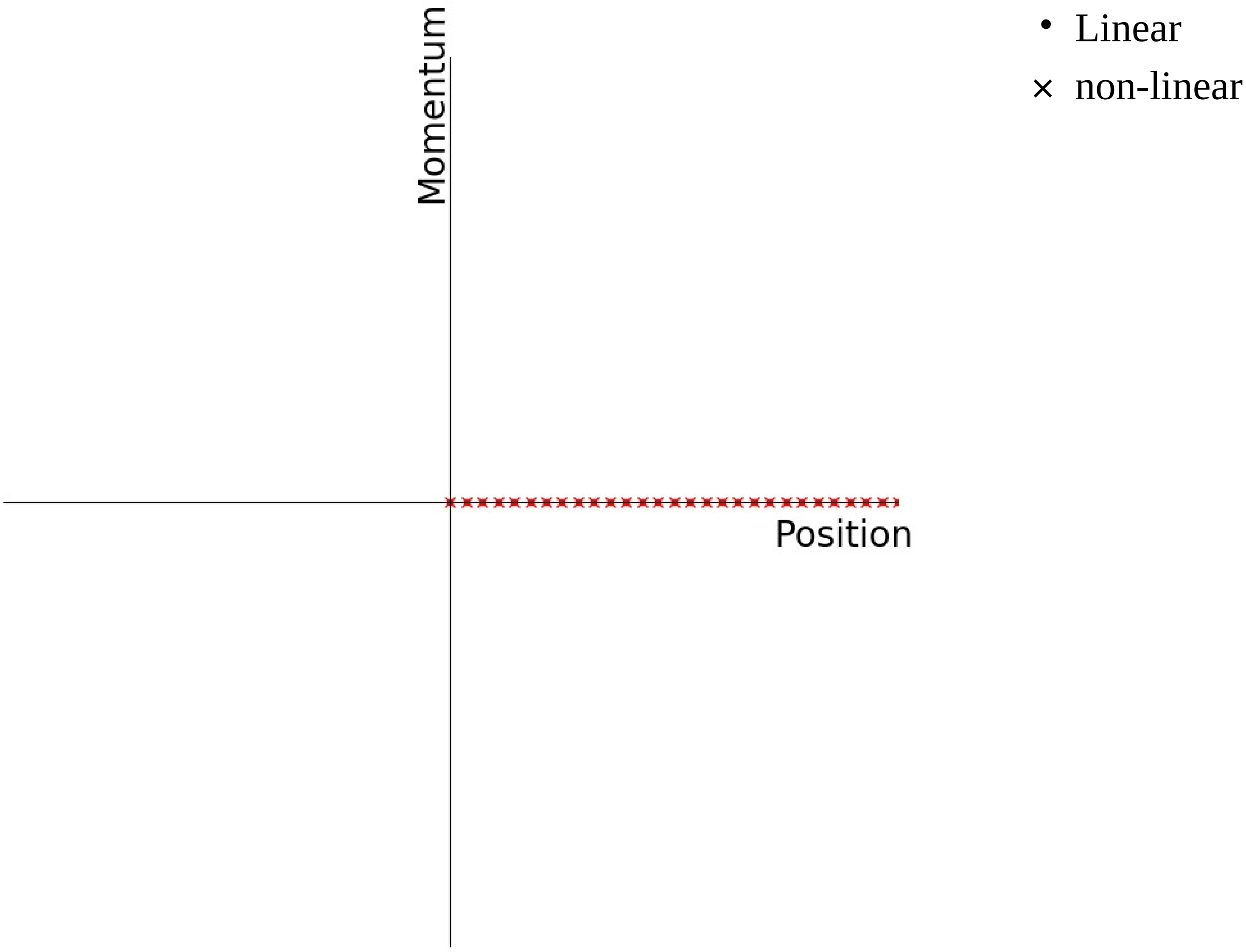

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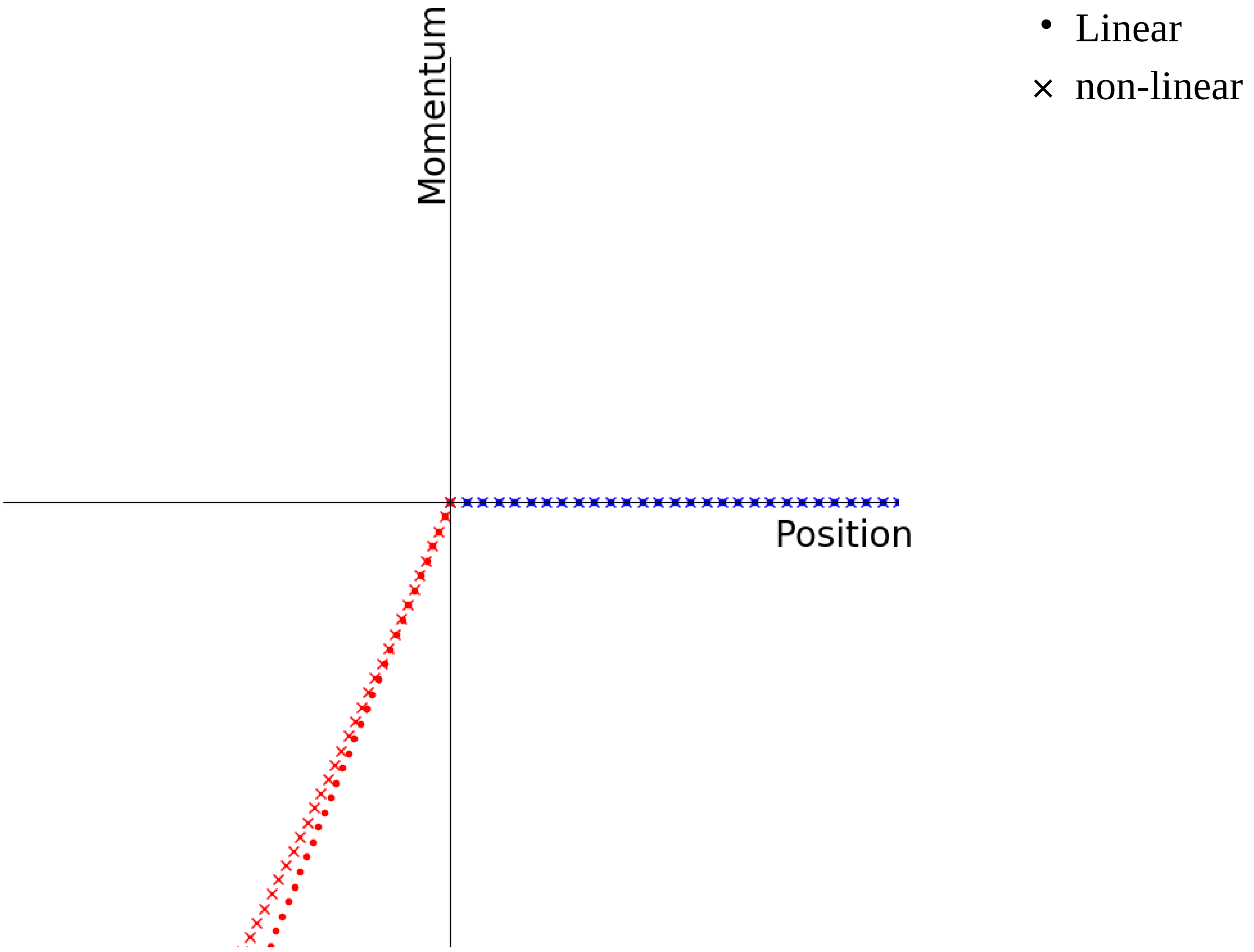

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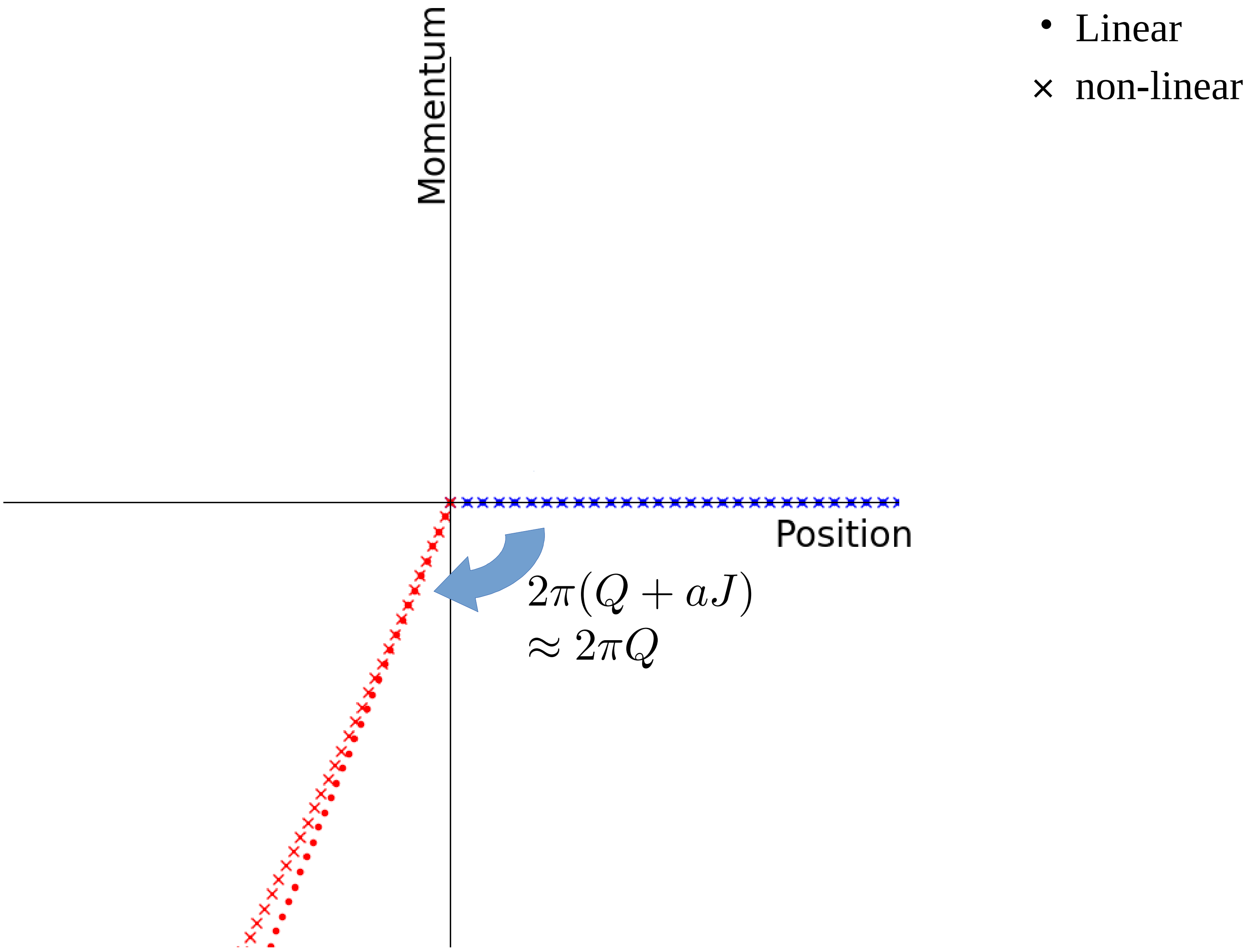

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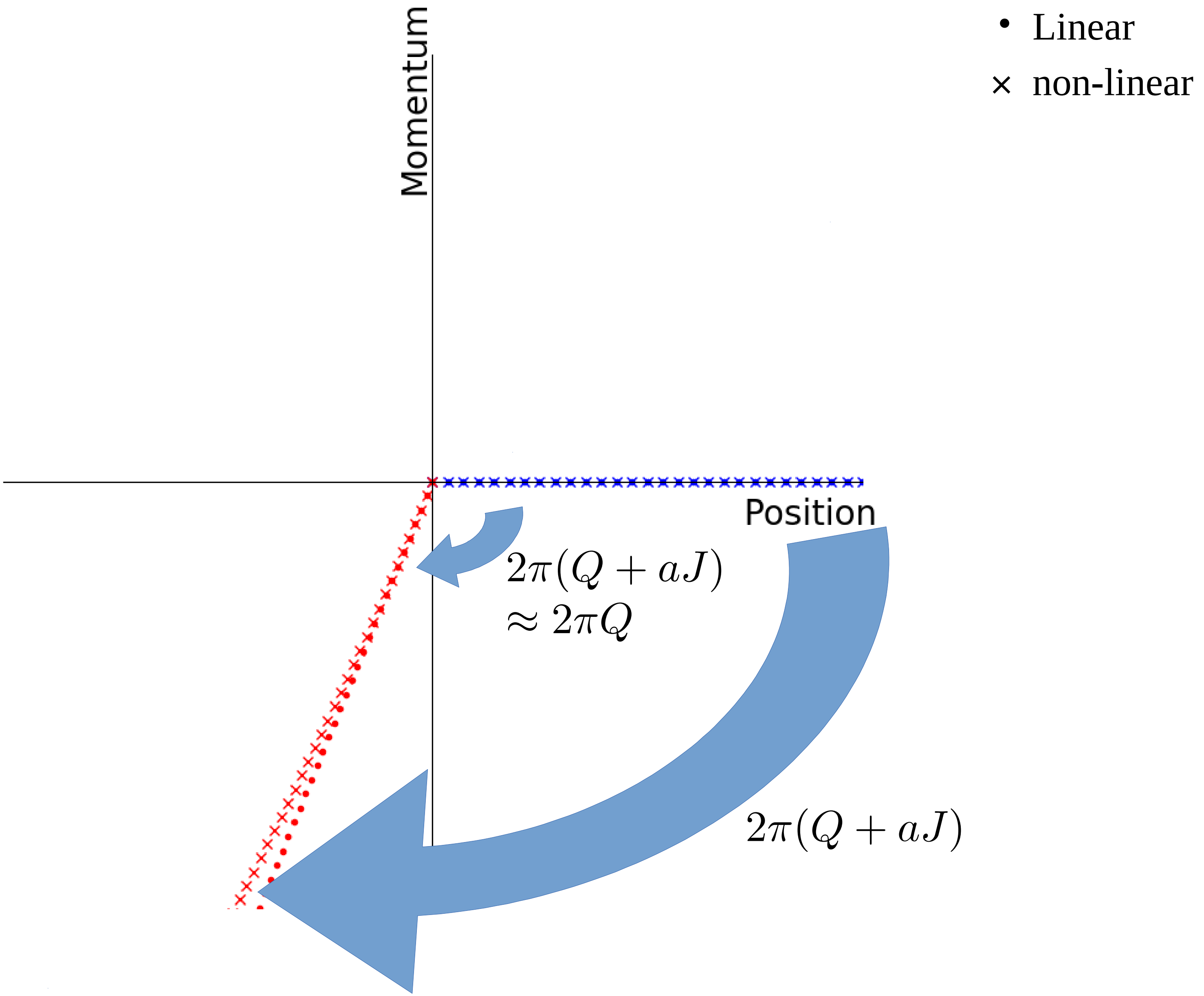

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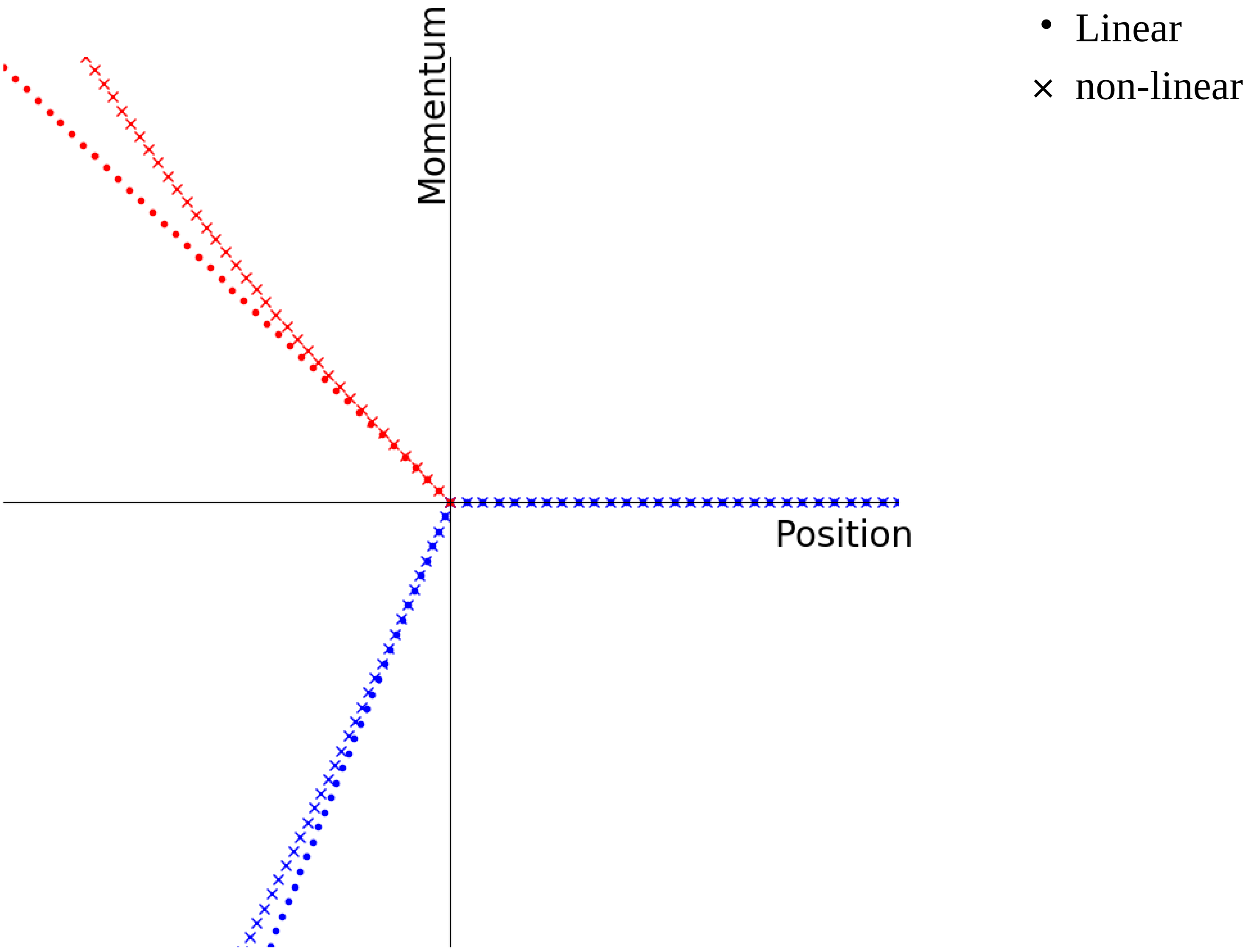

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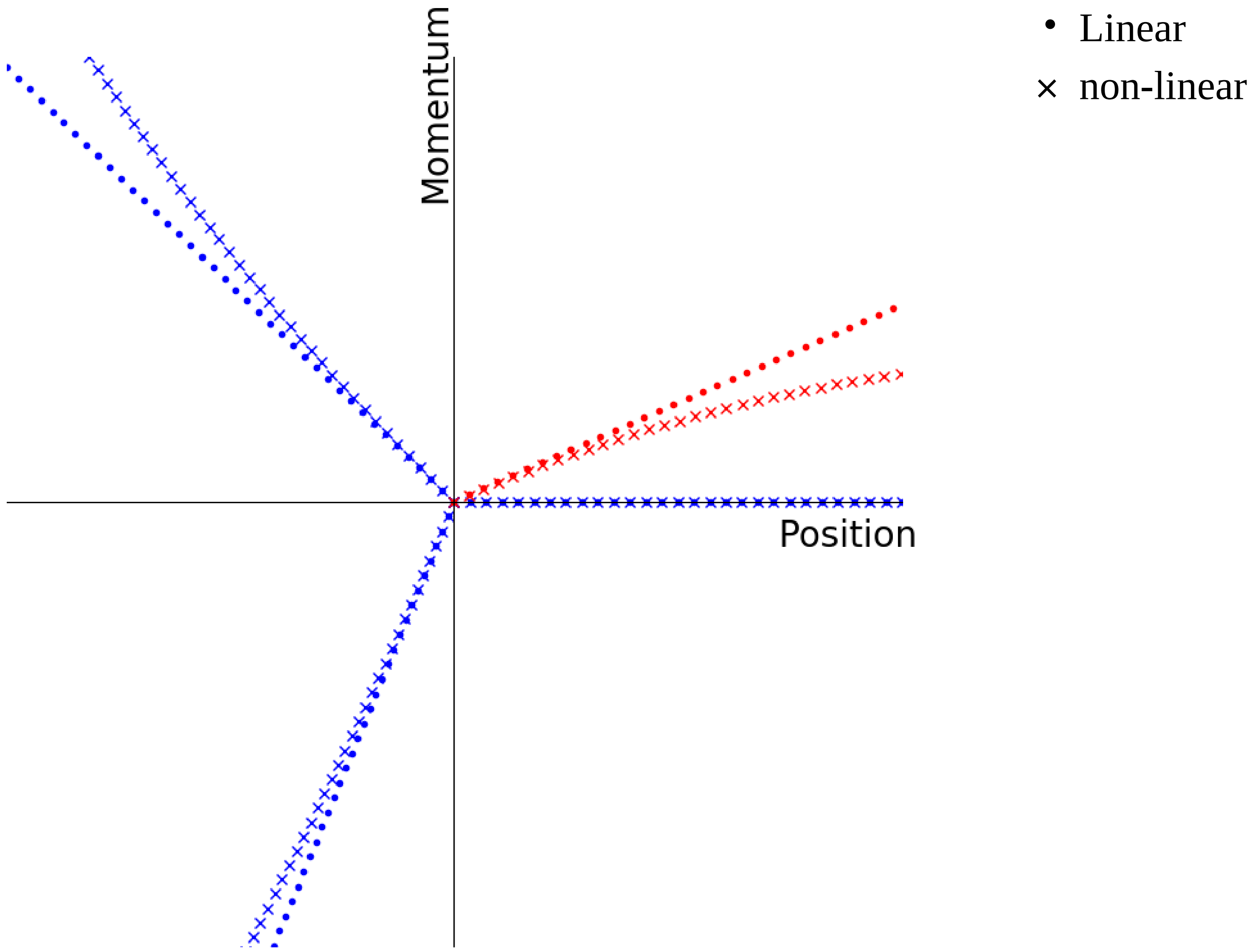
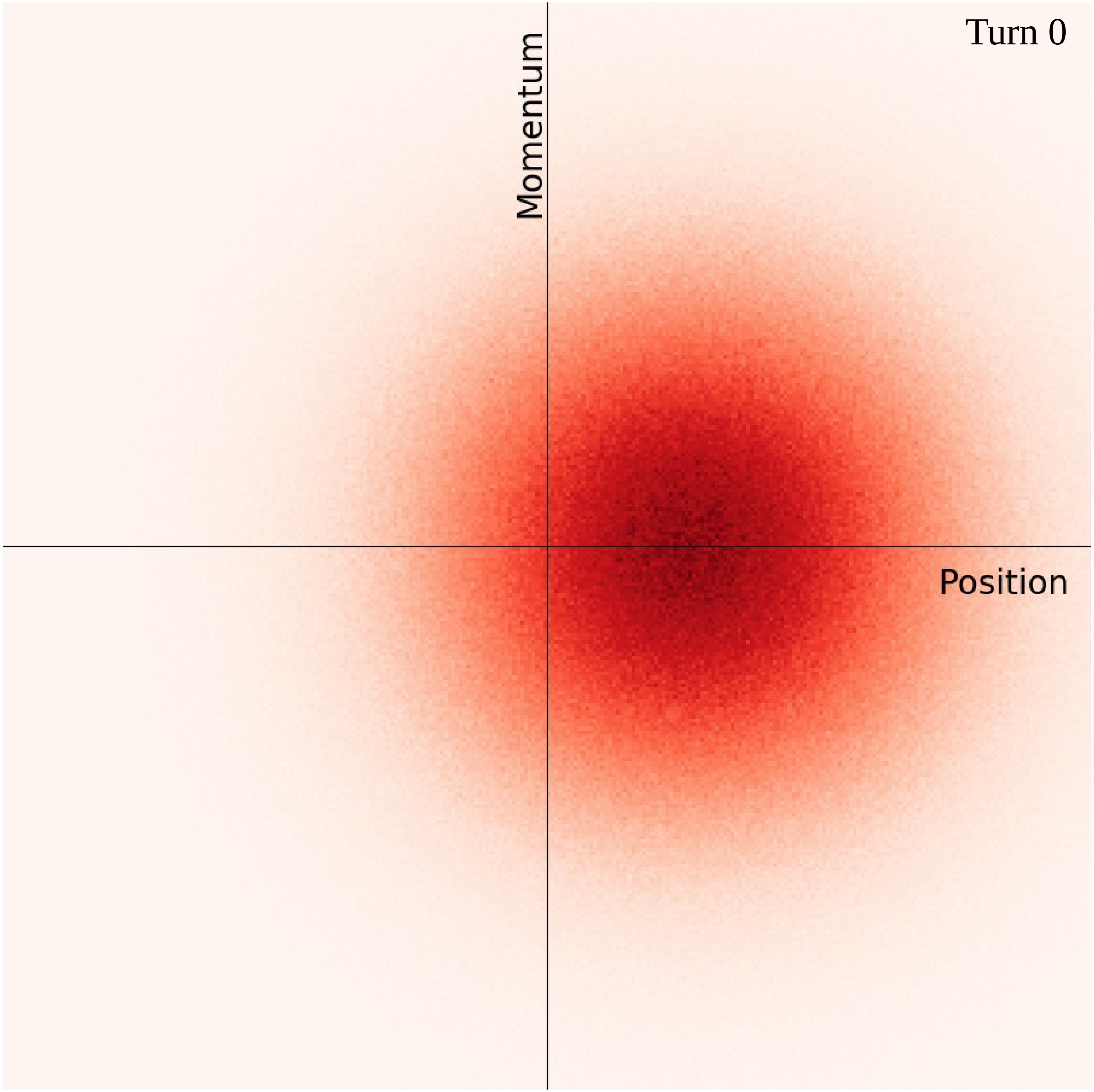


Perturbation without decoherence



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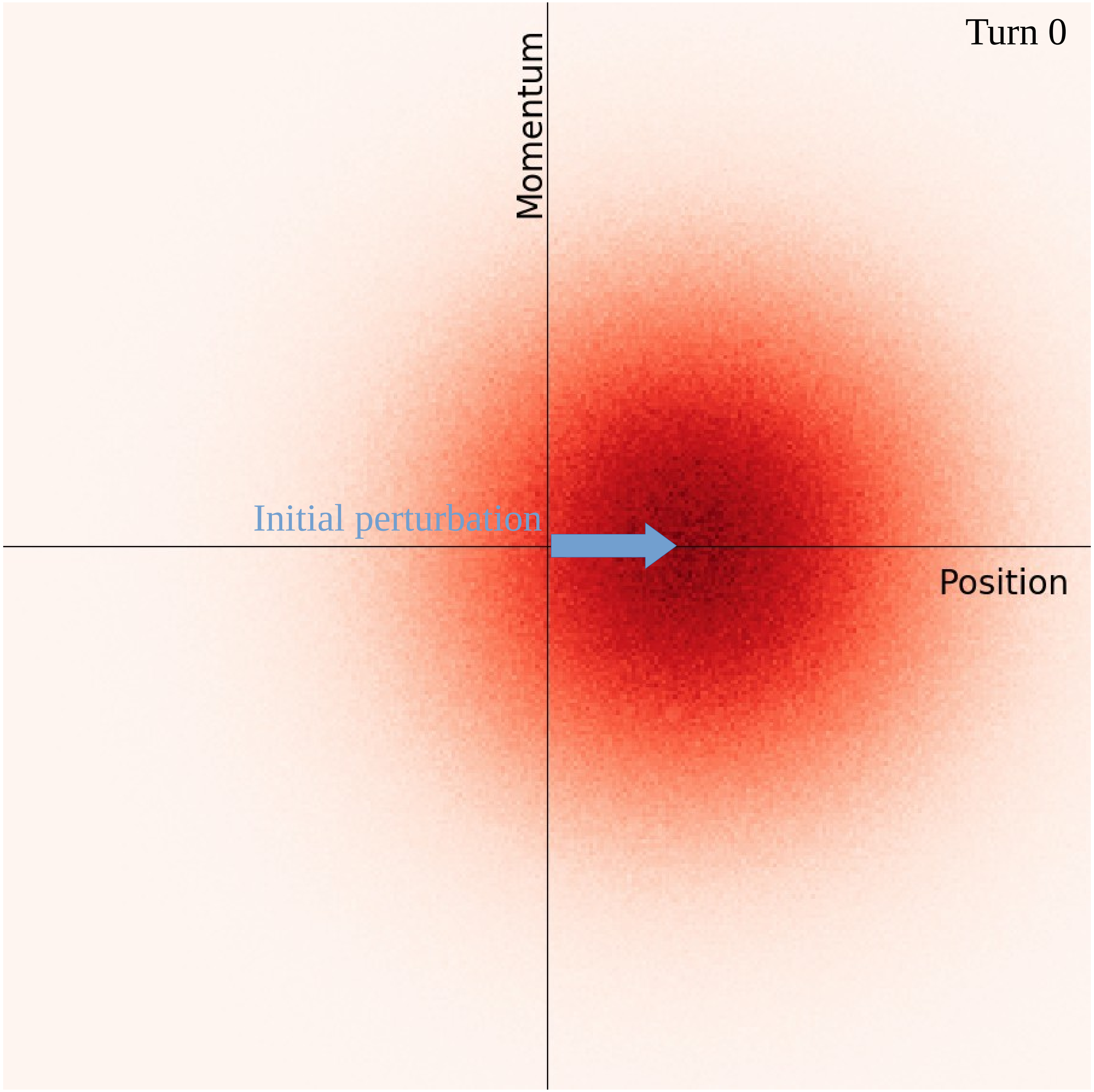

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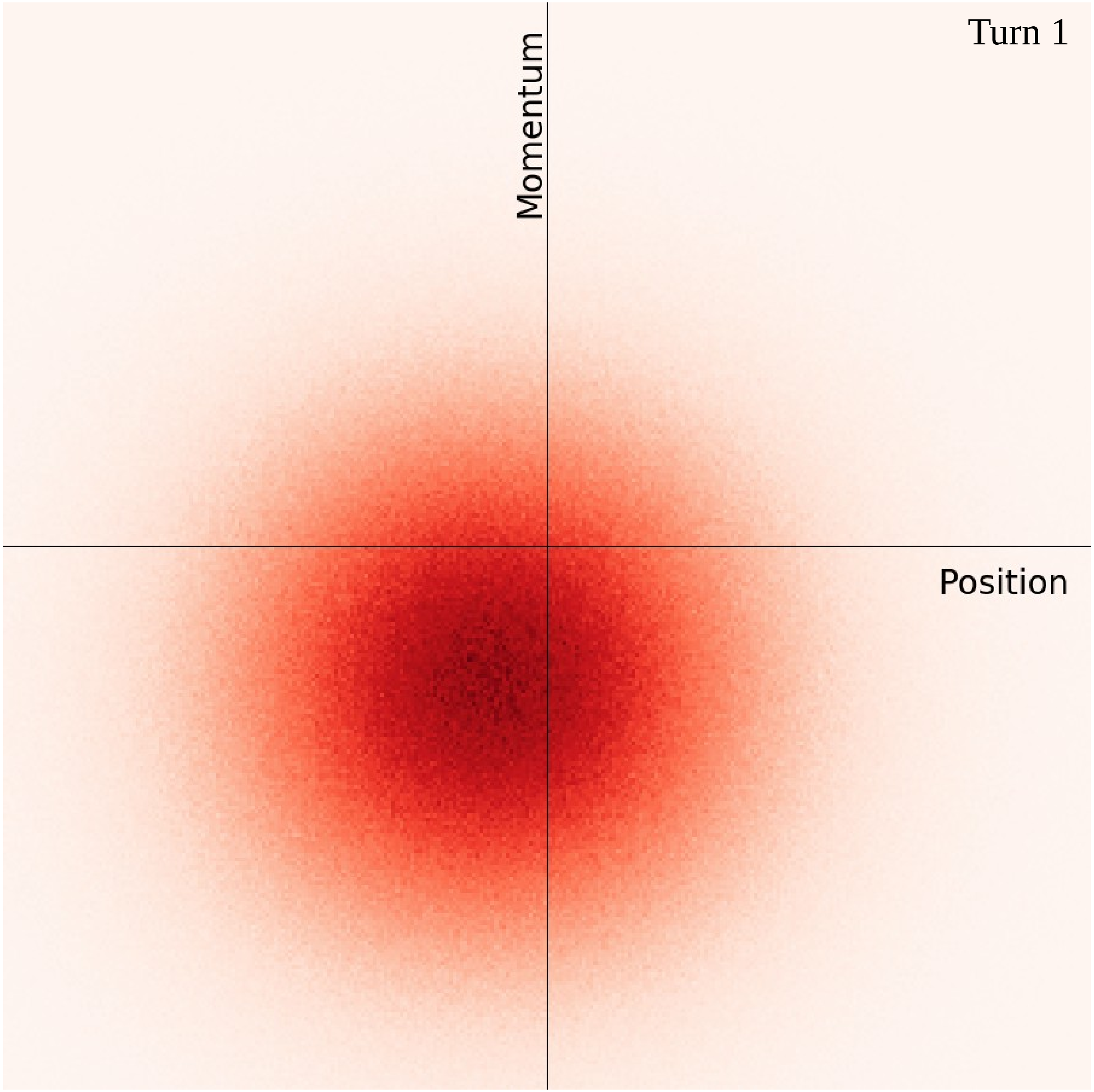

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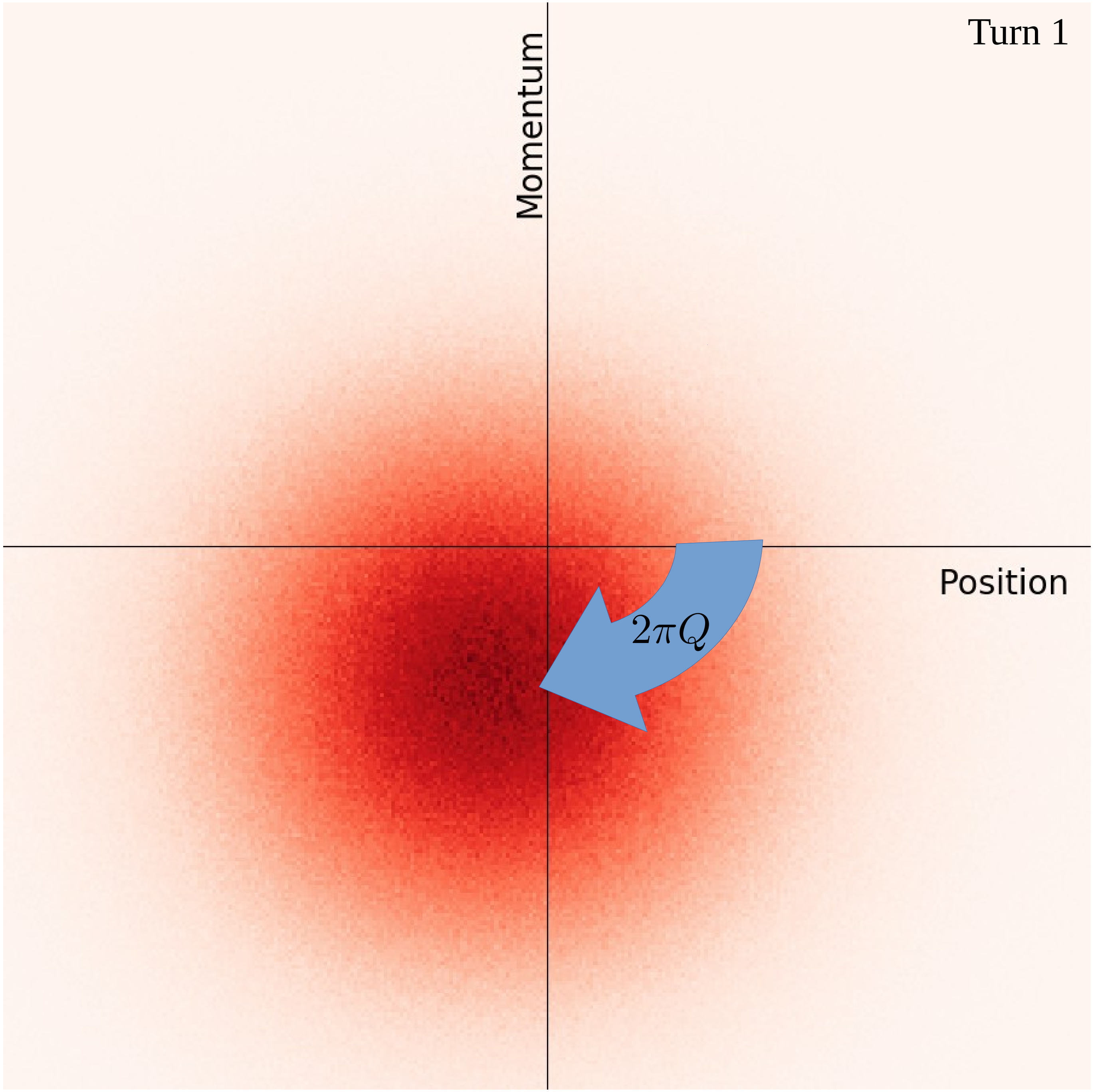

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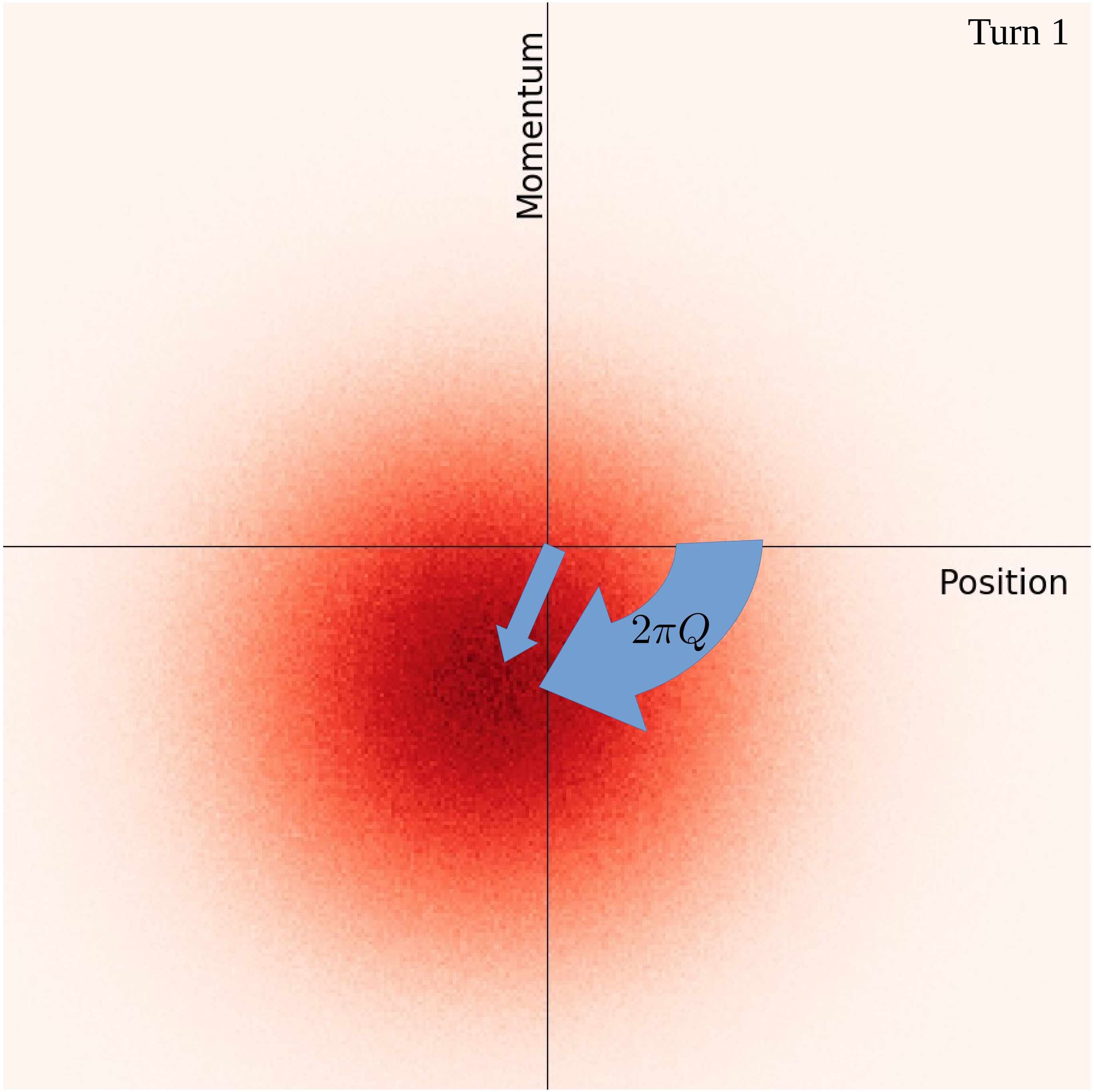

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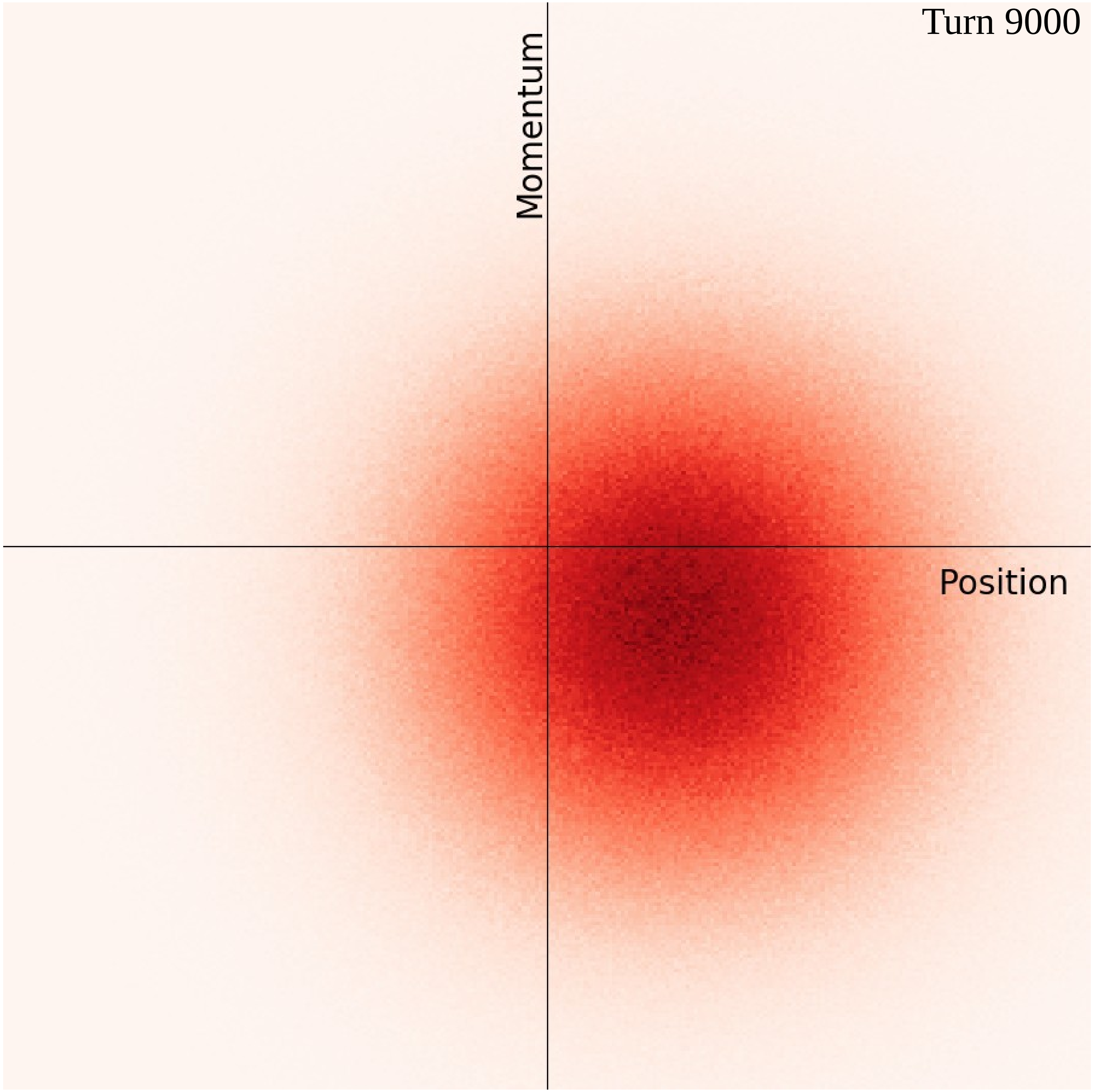

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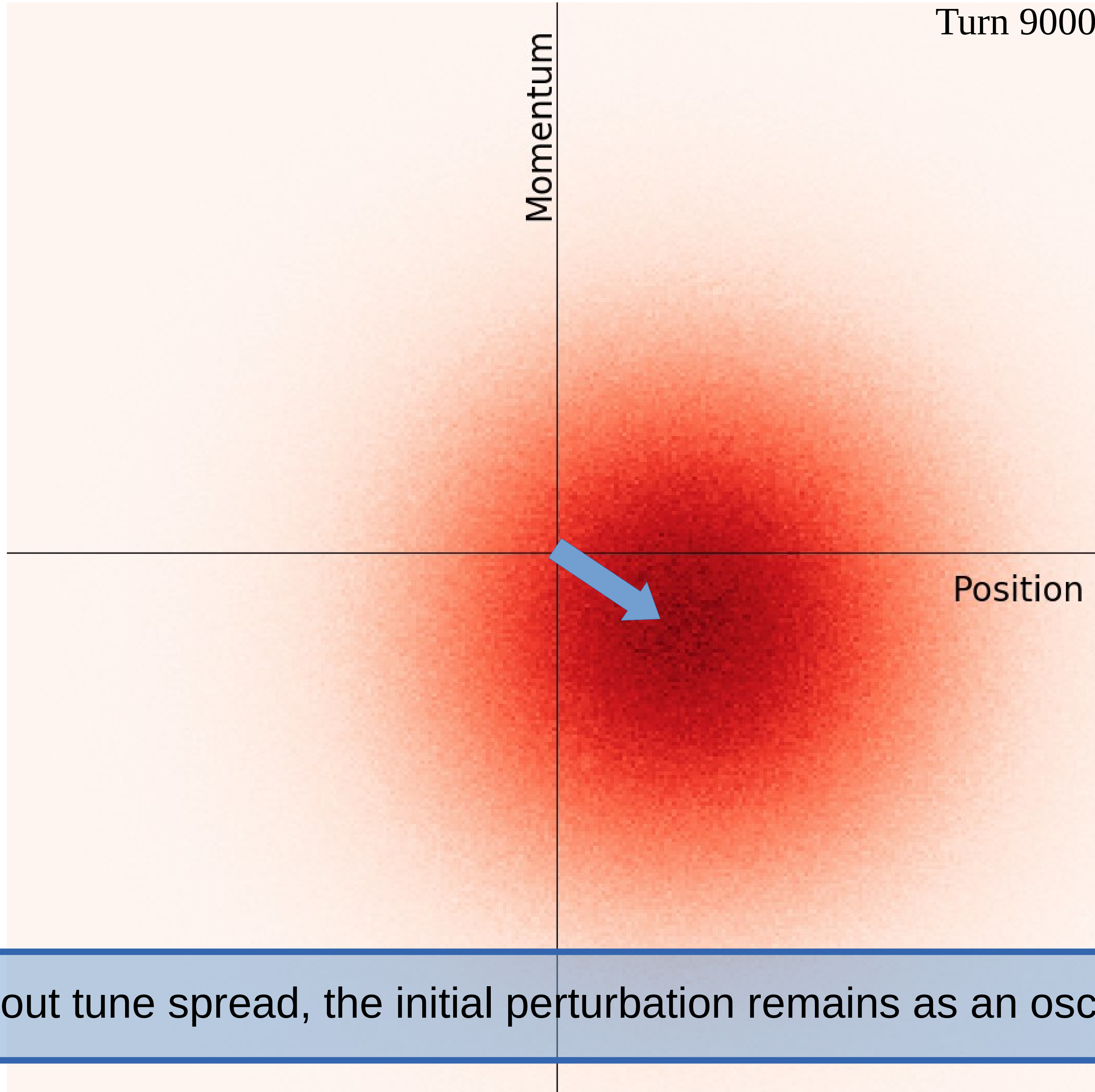
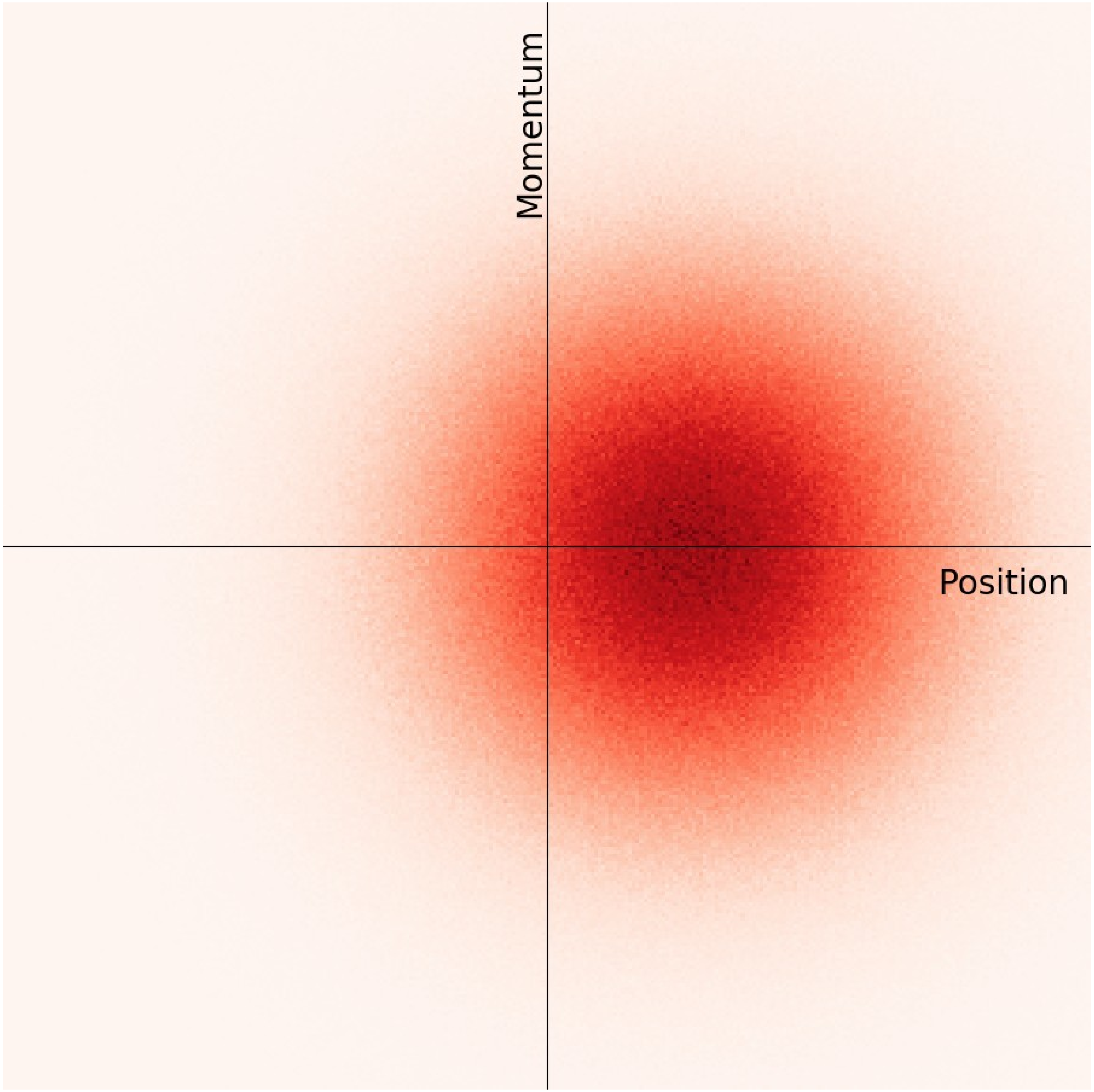


- Without tune spread, the initial perturbation remains as an oscillation

Decoherence



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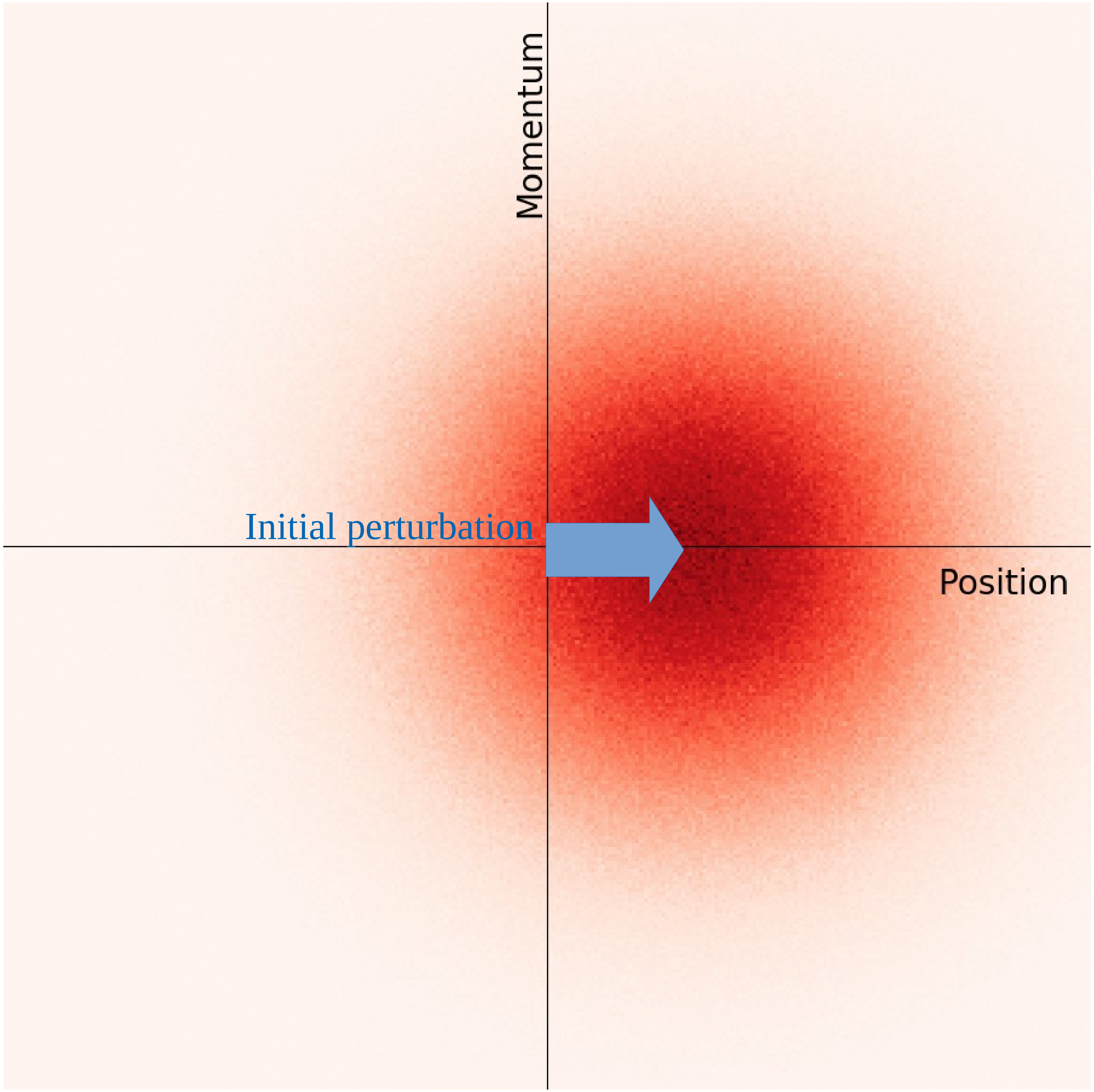

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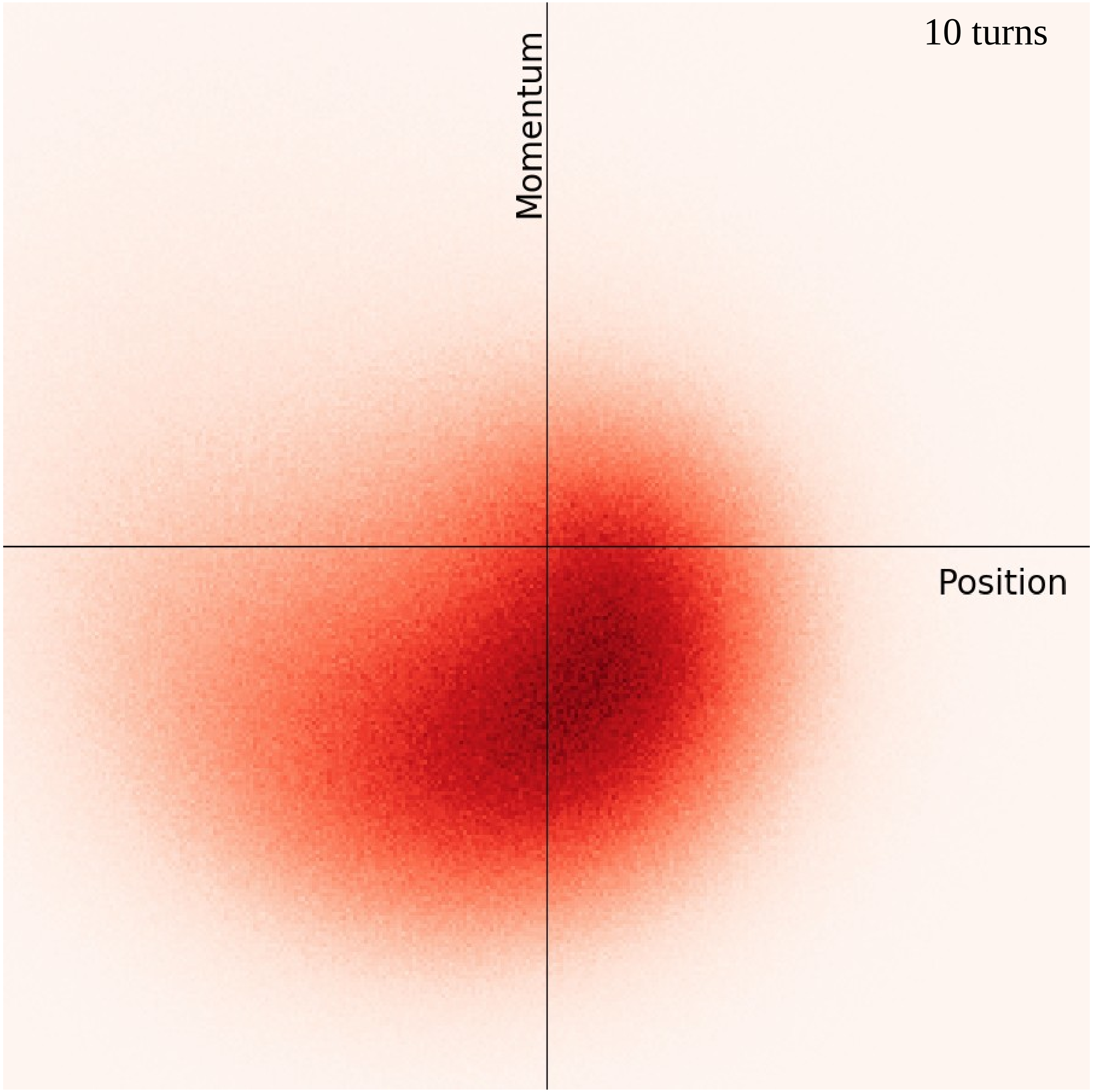

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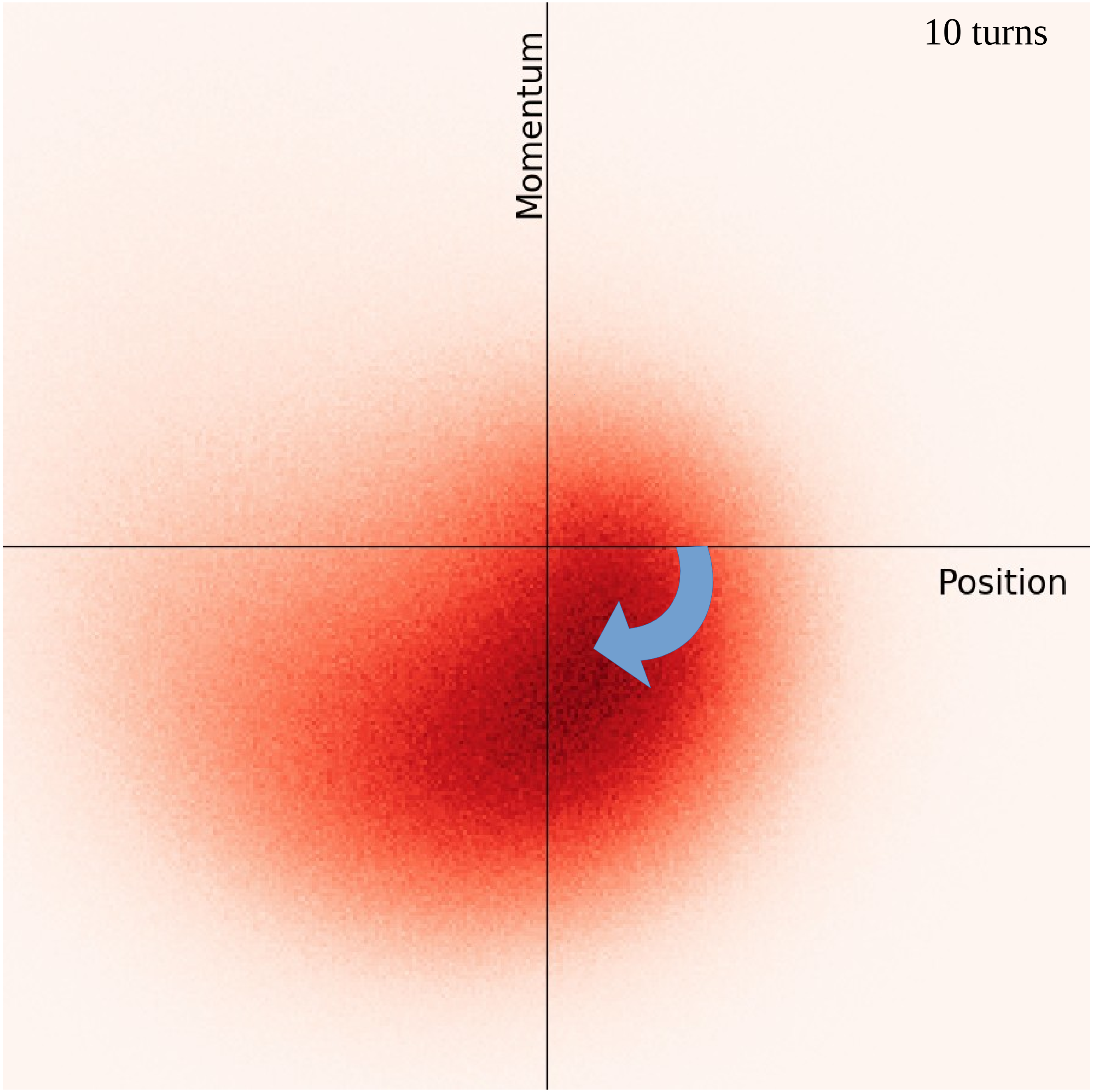

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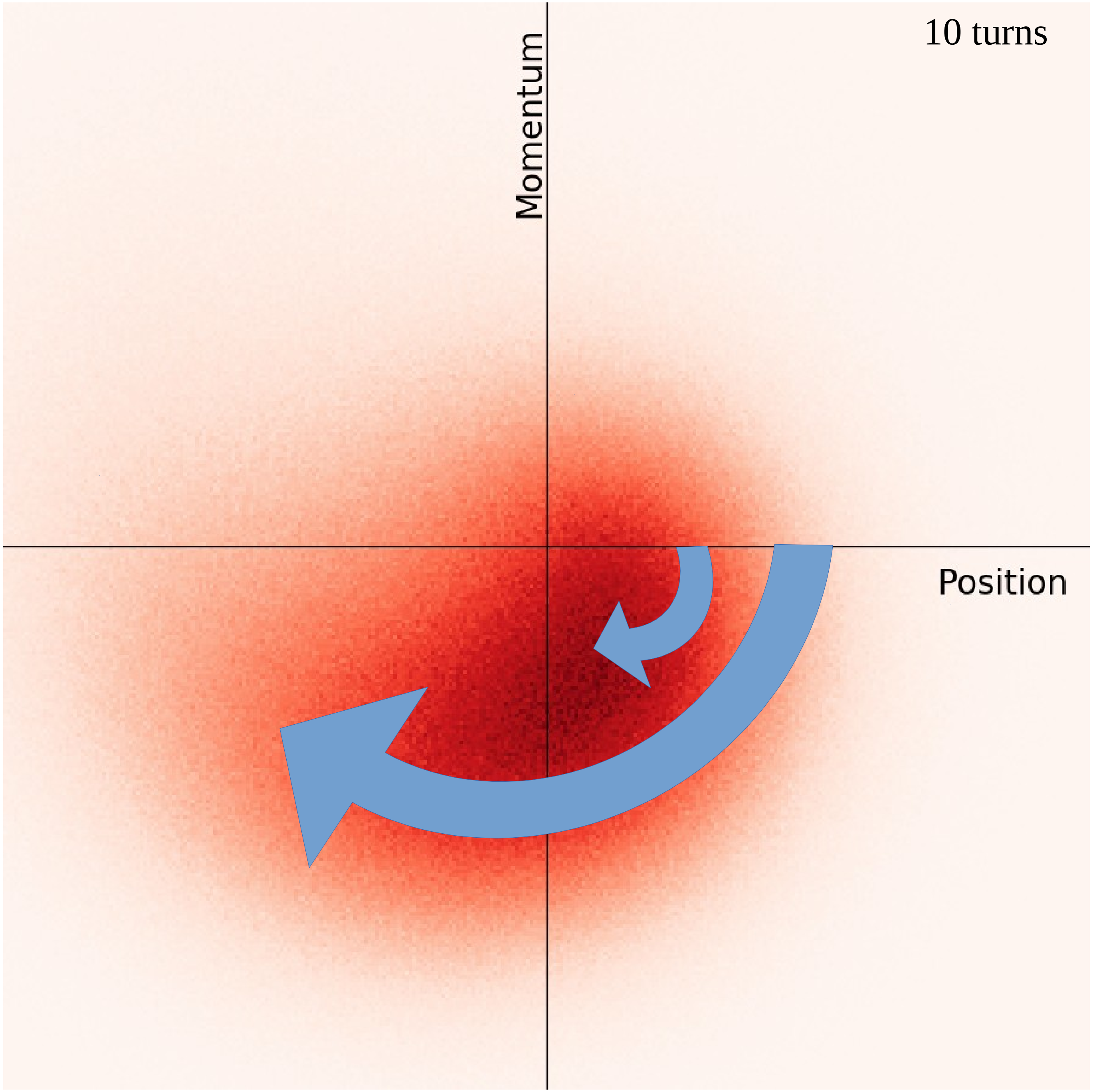

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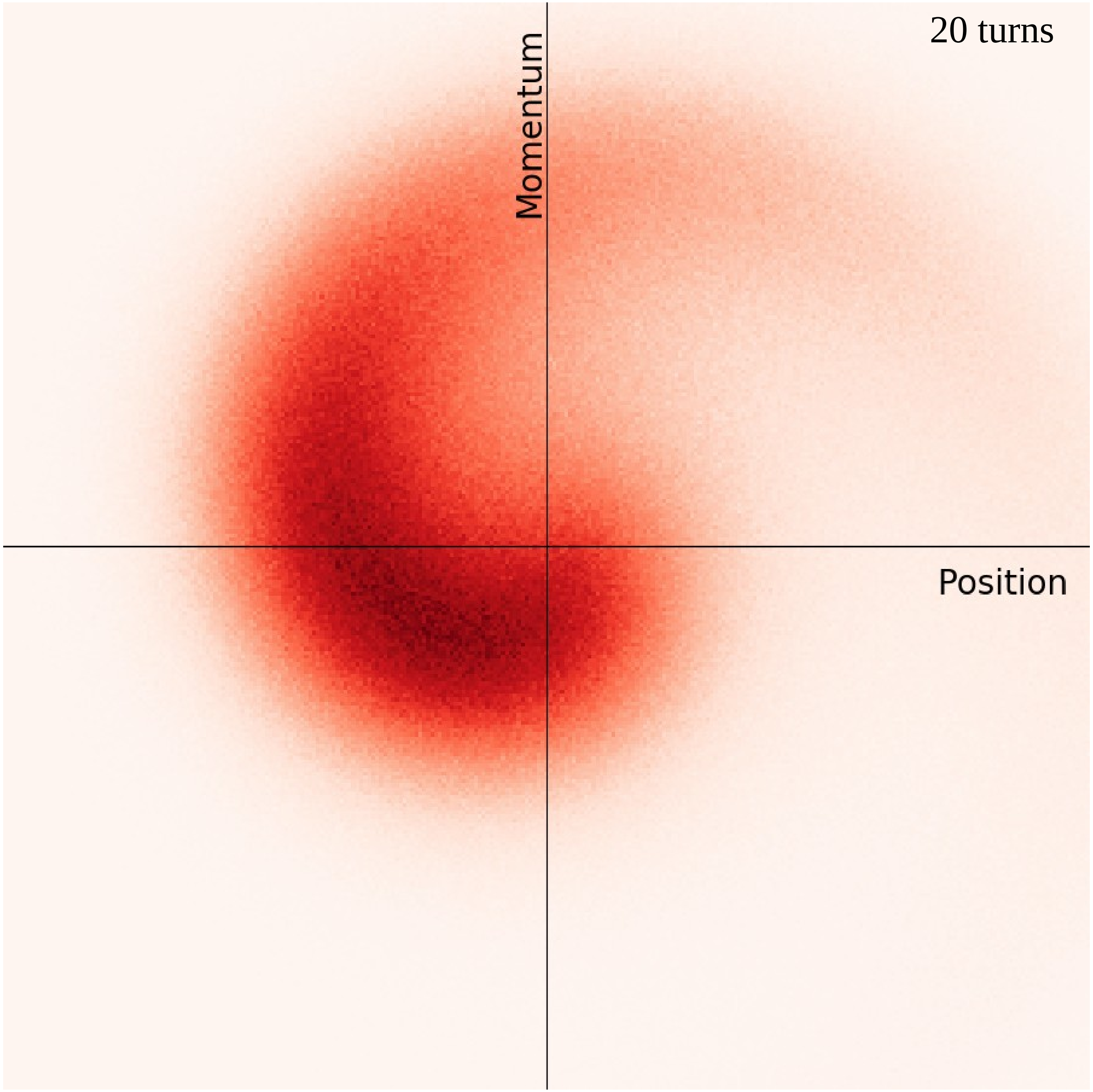

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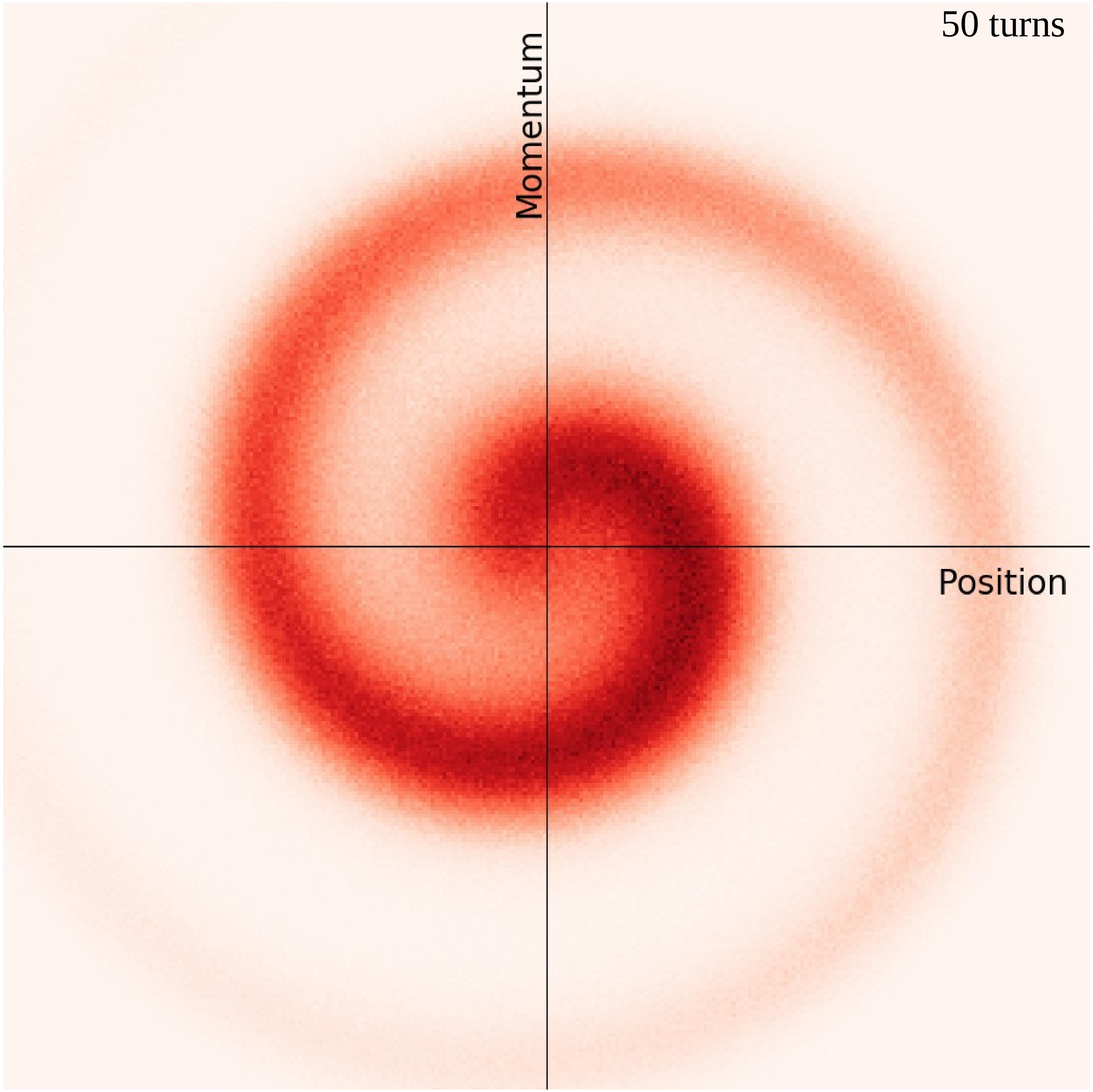

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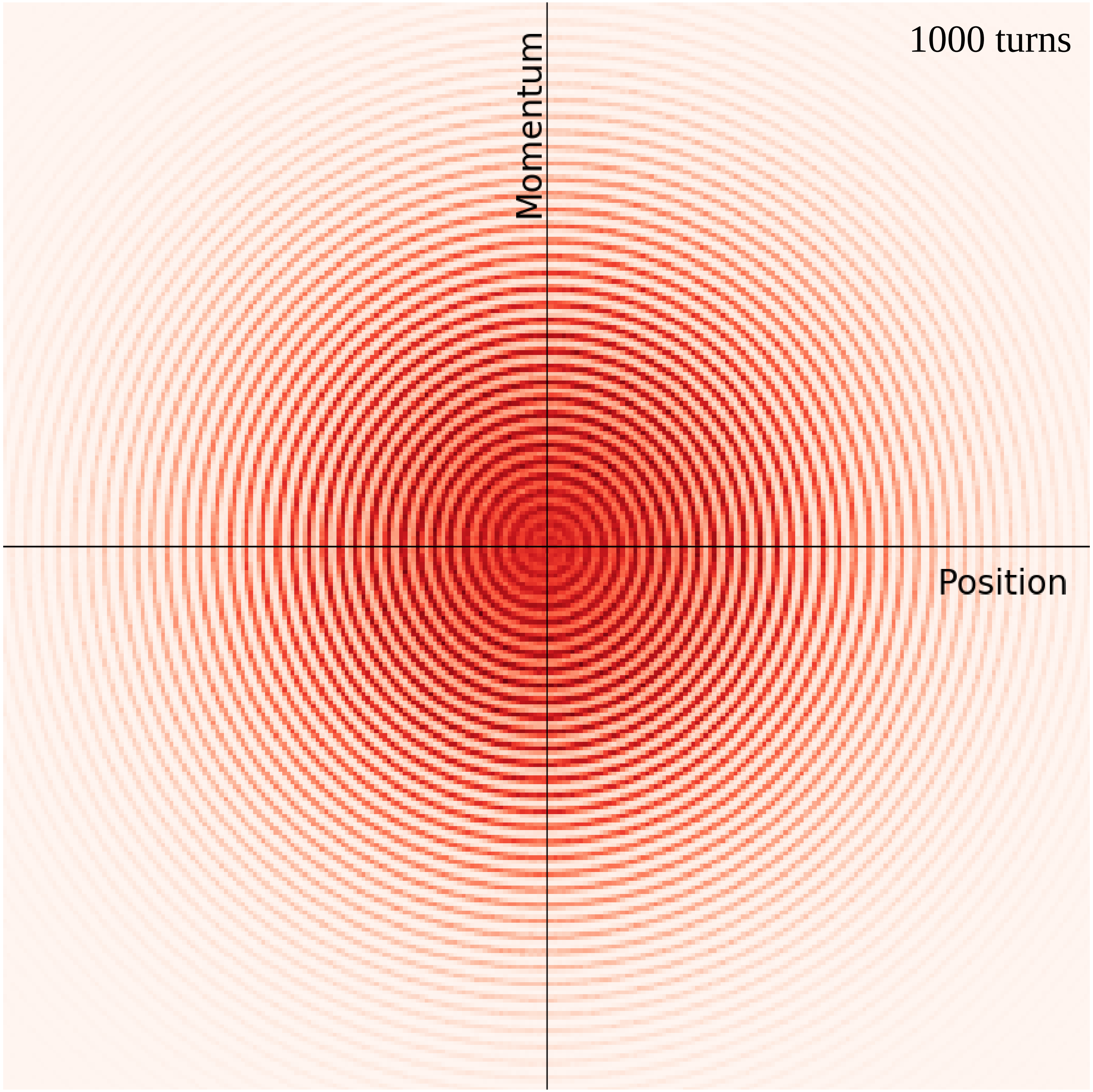

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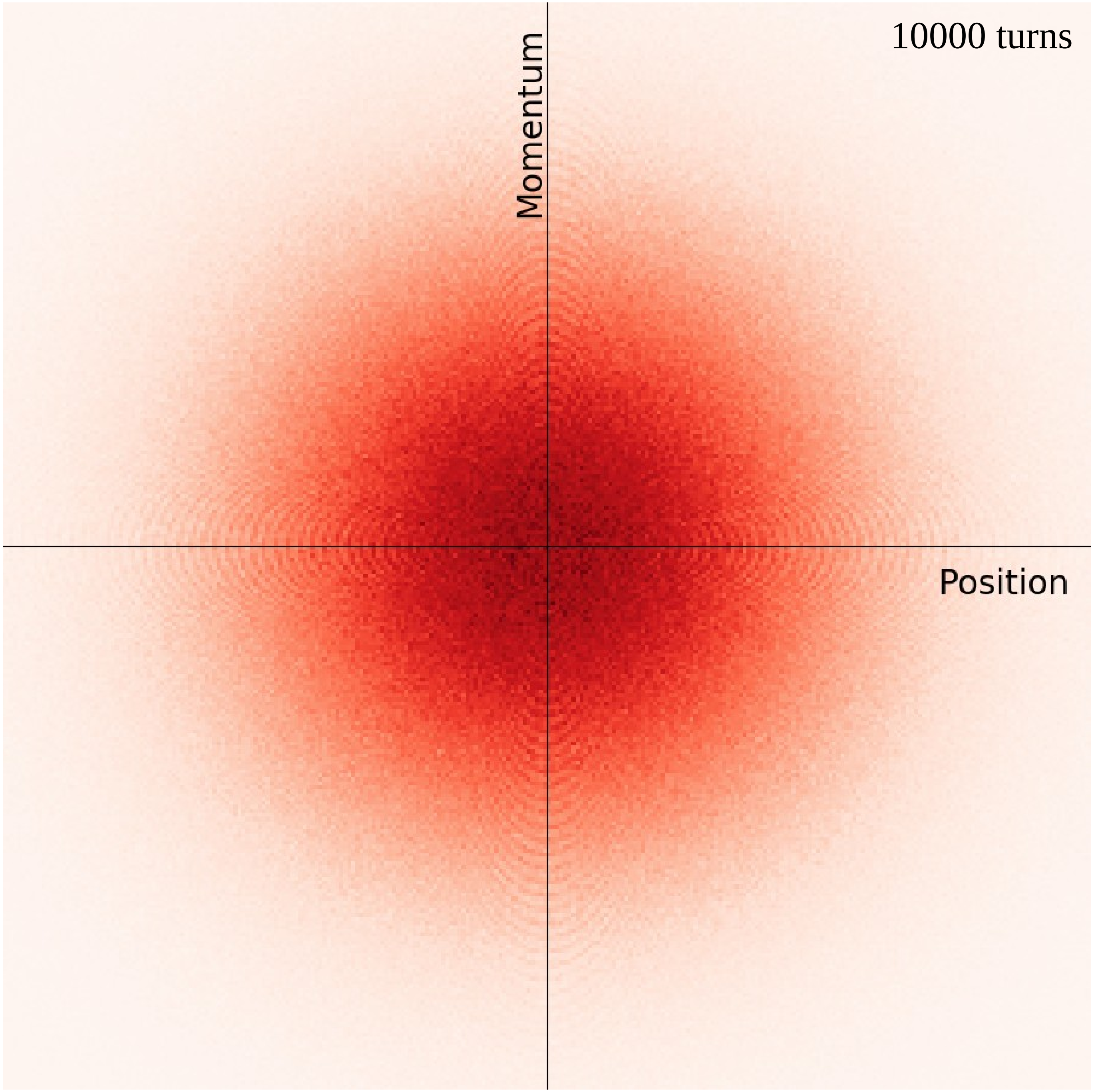

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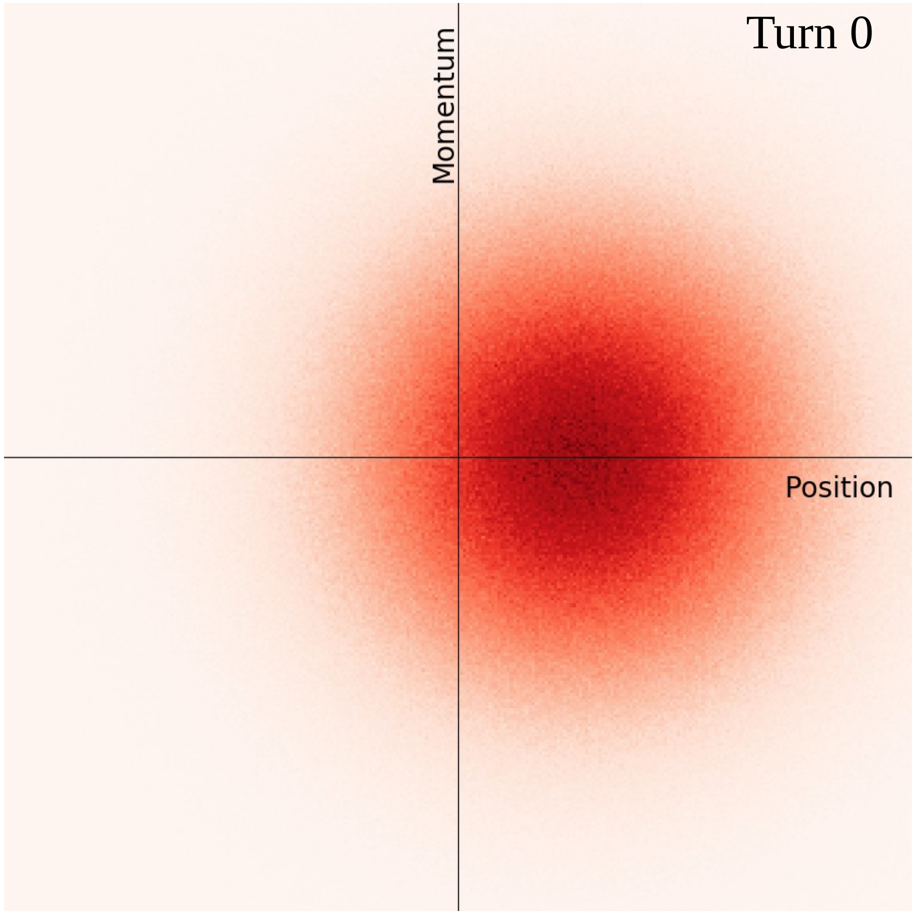
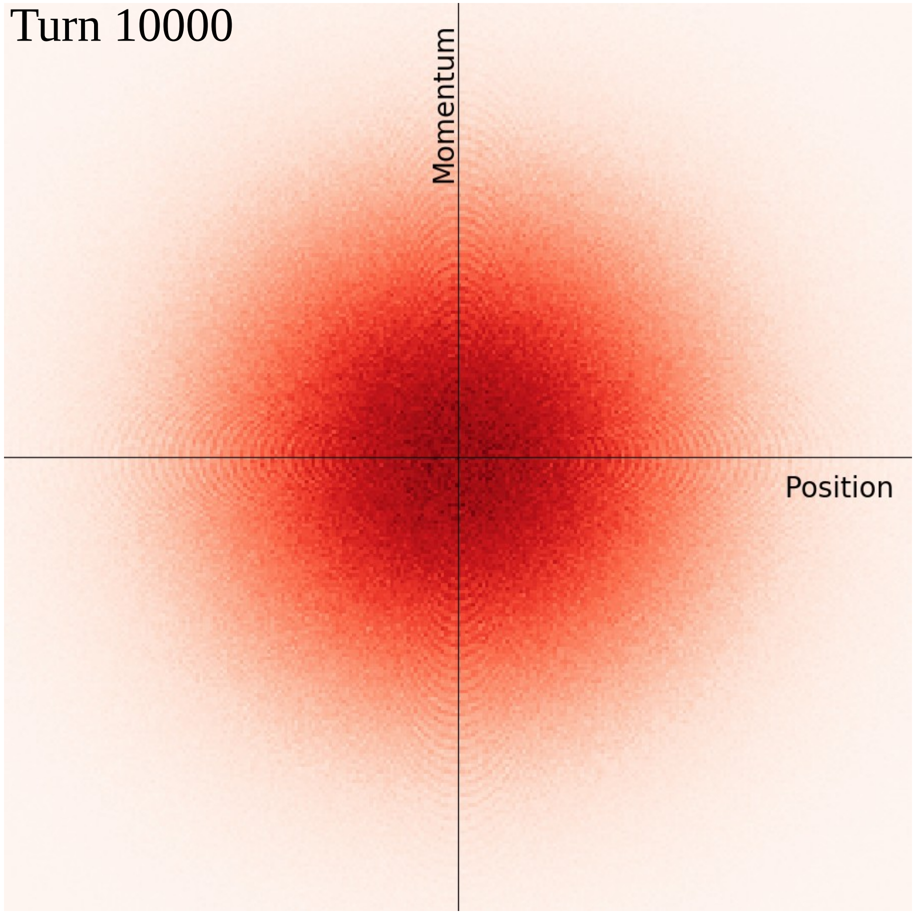
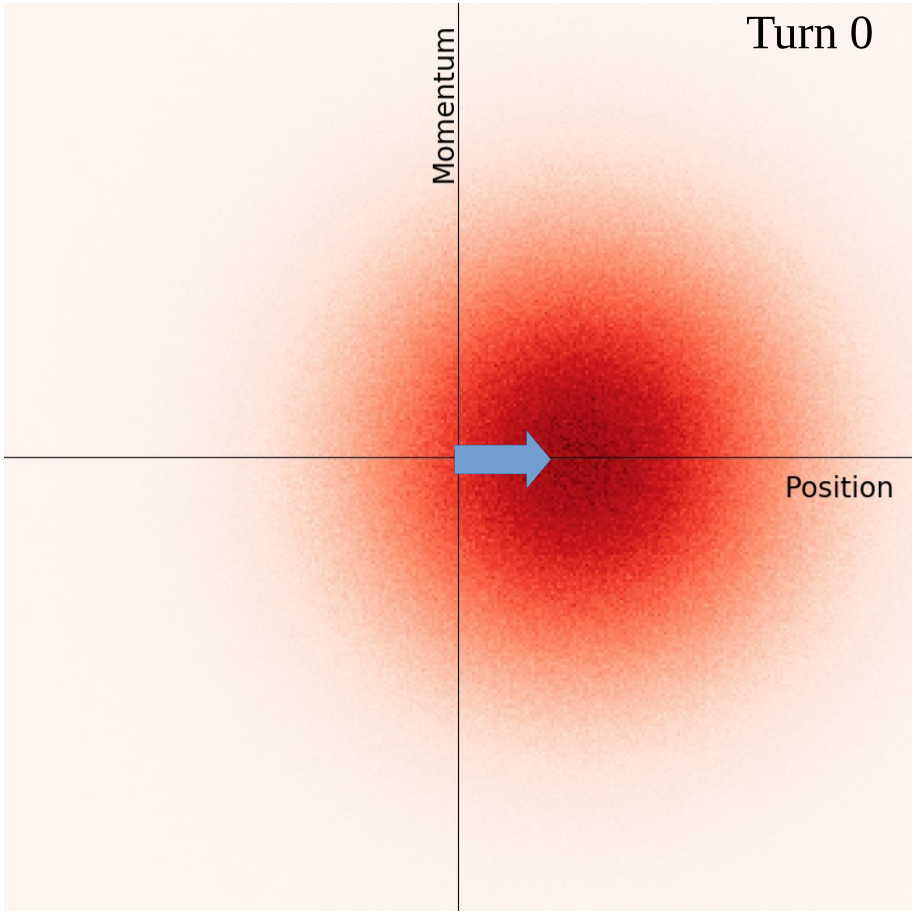
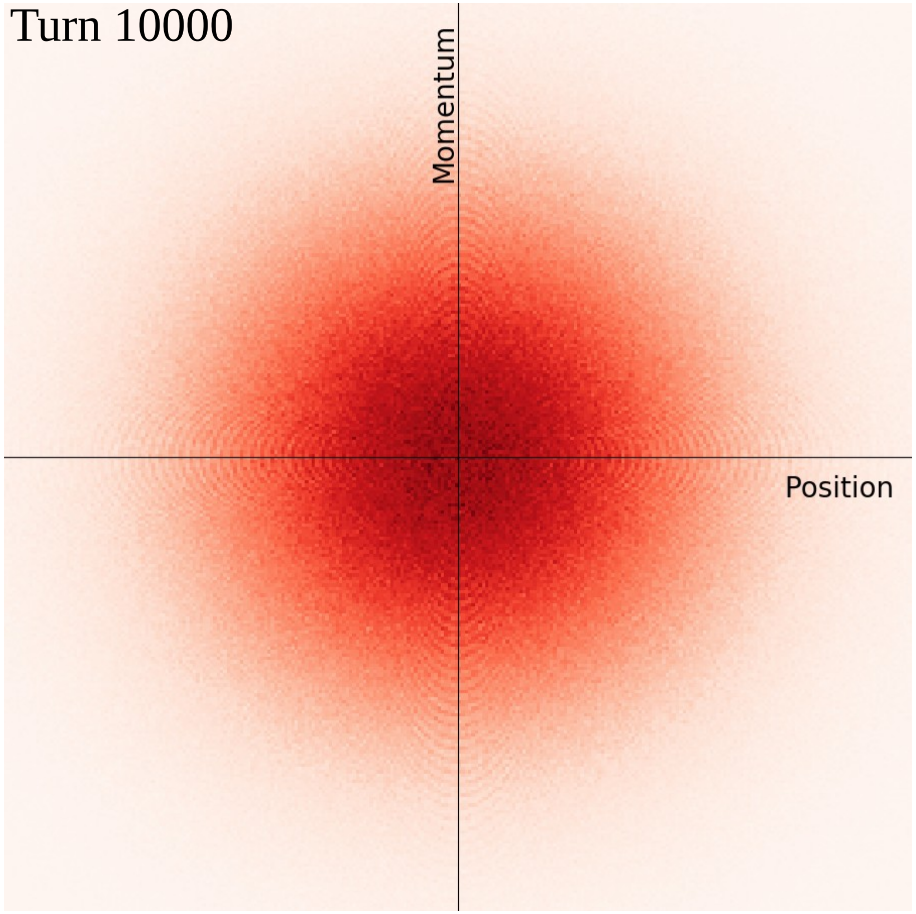




- Due to the tune spread, the initial perturbation is damped at the expense of a change of distribution → **emittance growth**

Decoherence





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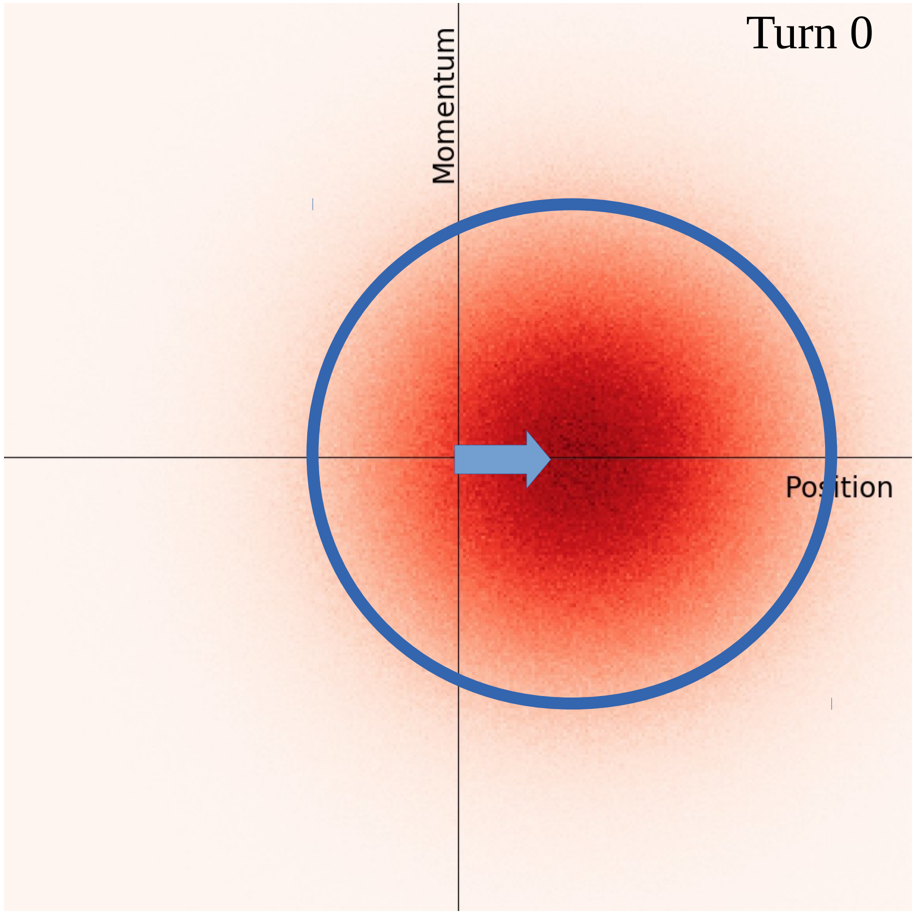

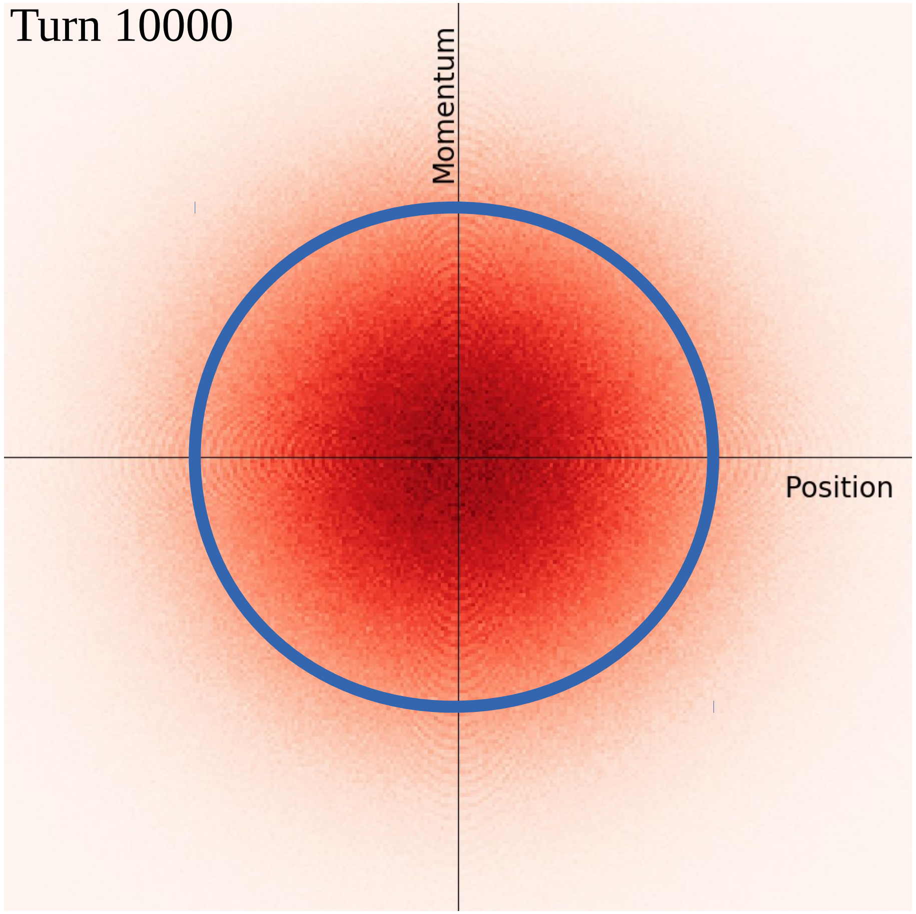

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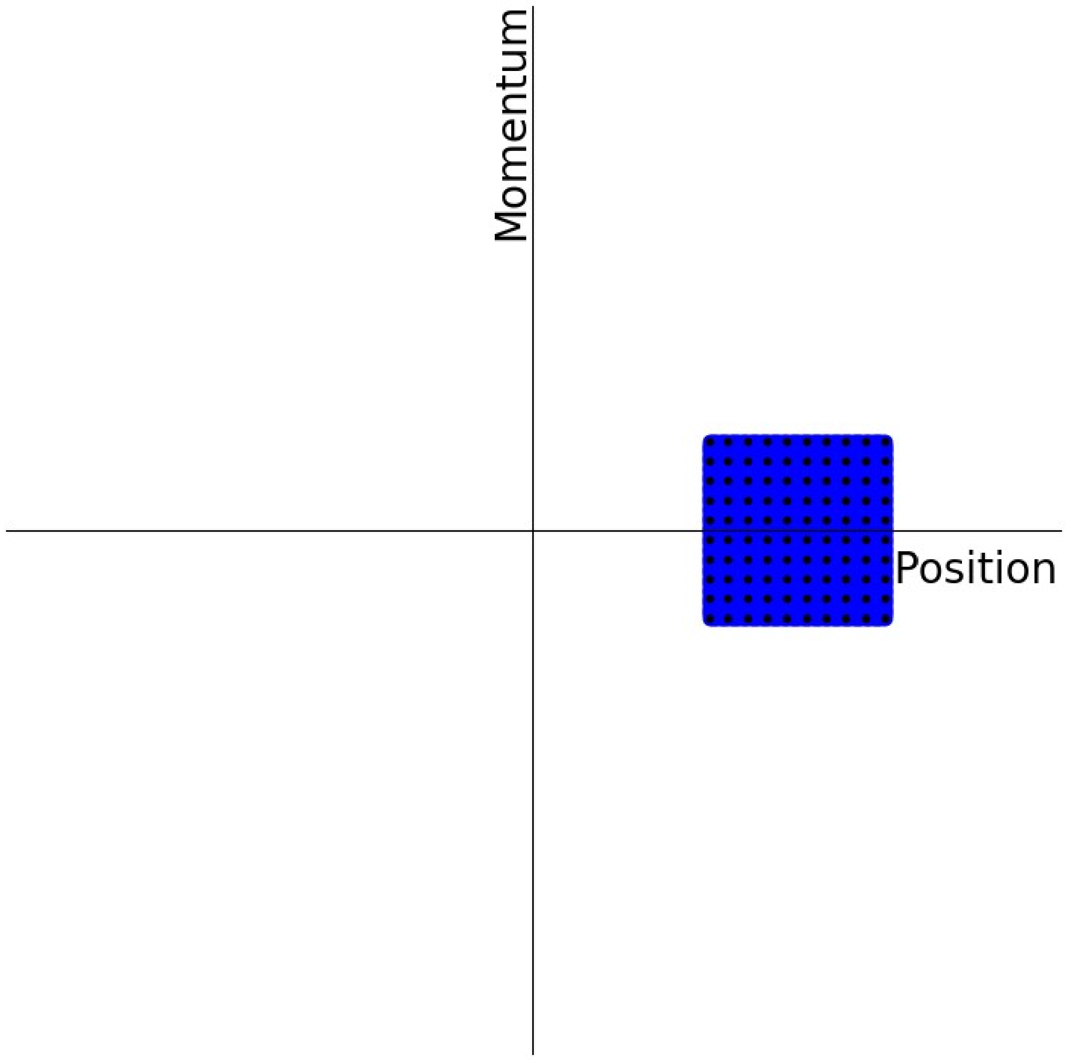
Liouville theorem for Hamiltonian systems



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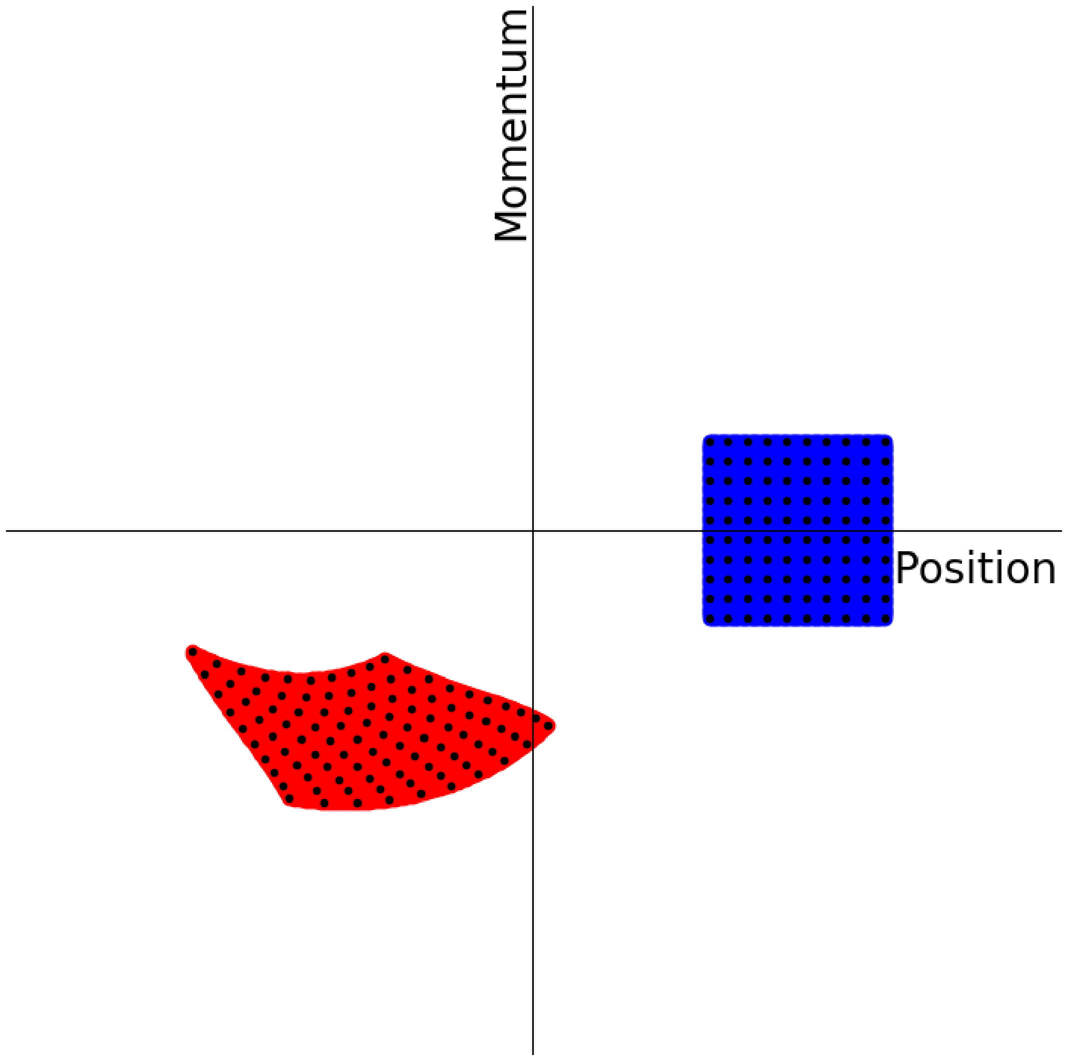

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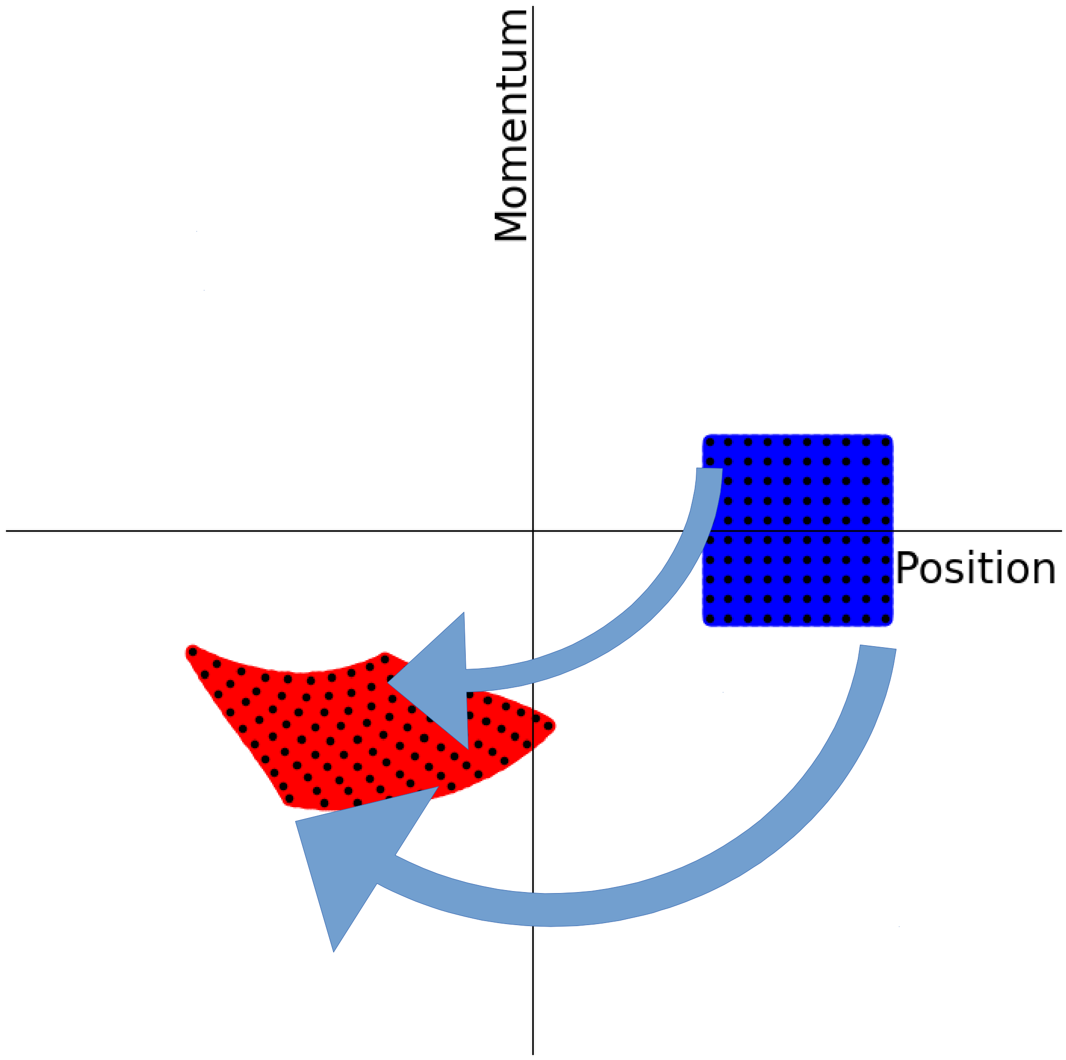

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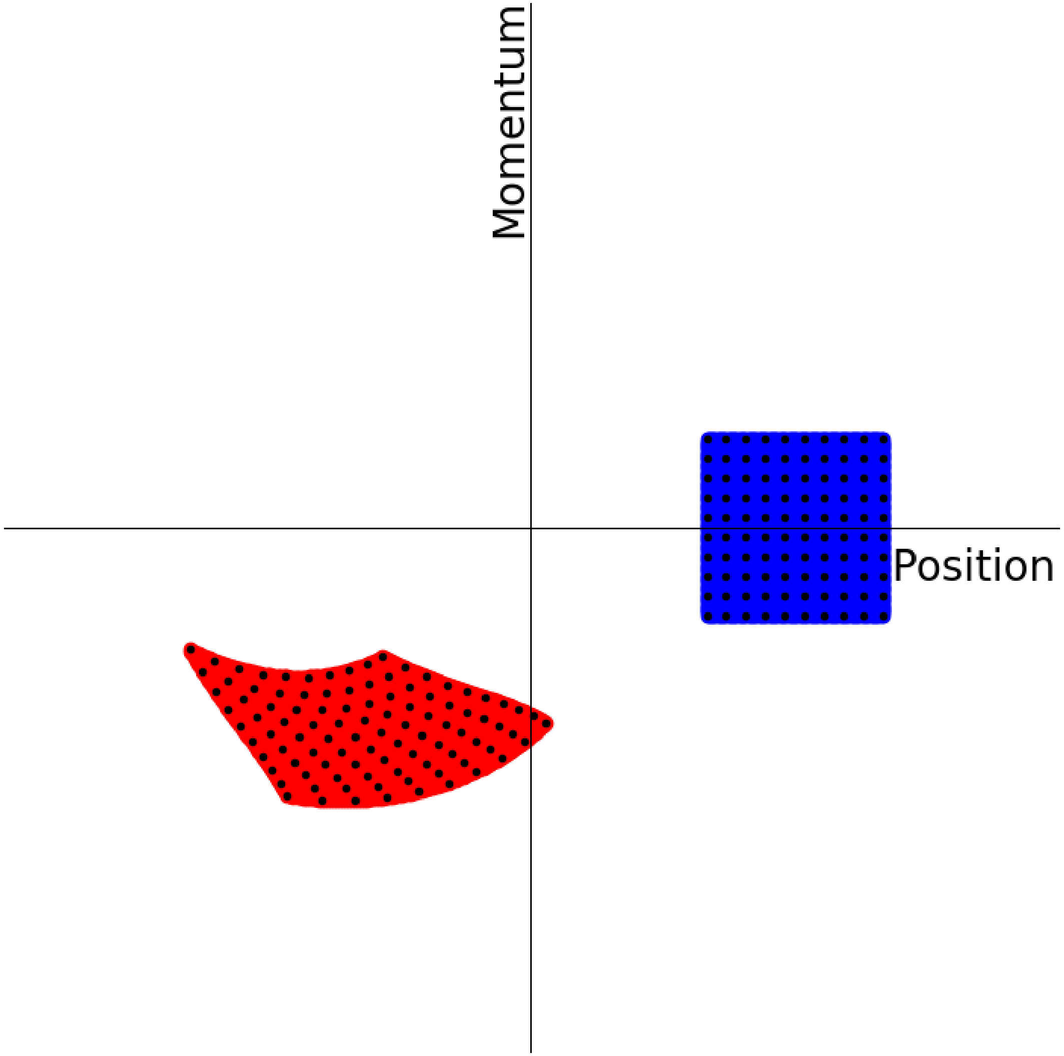

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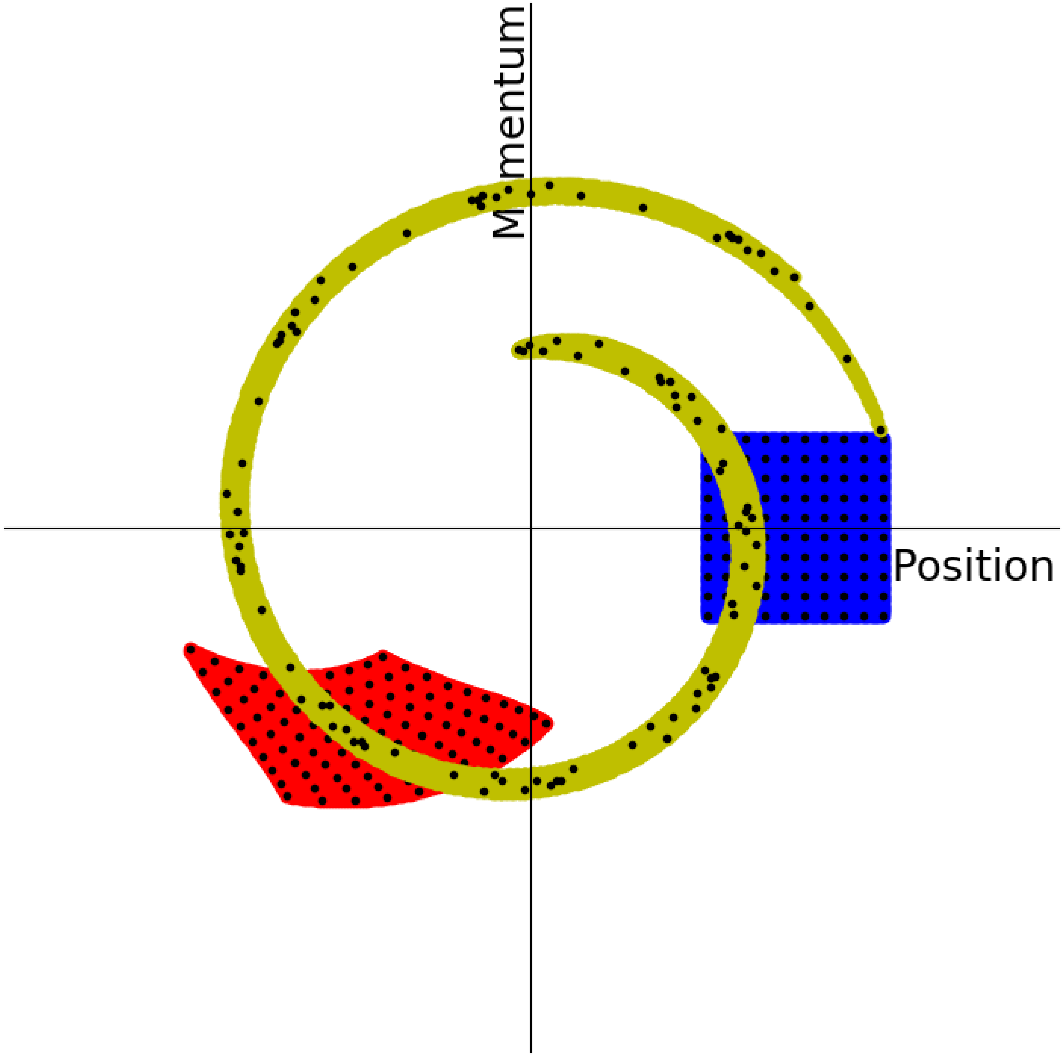

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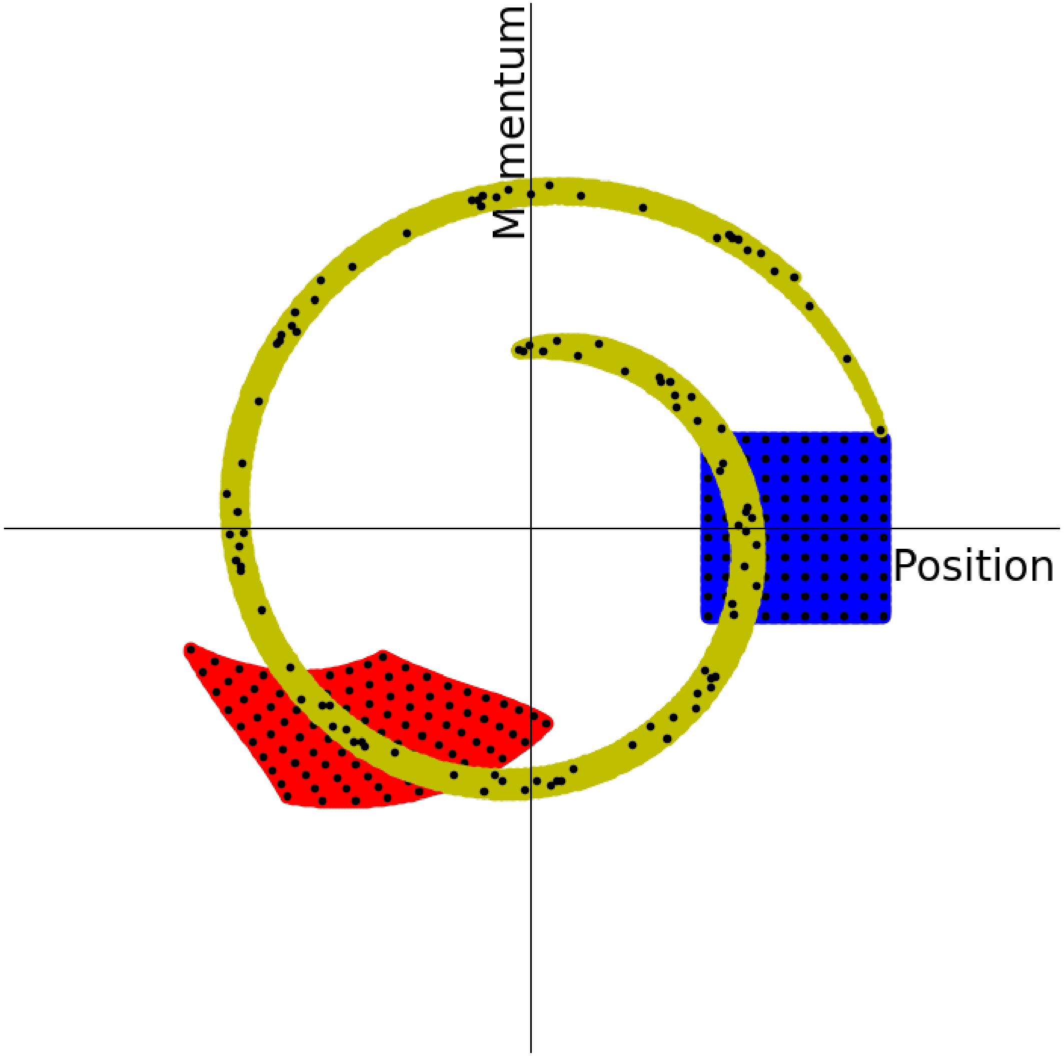

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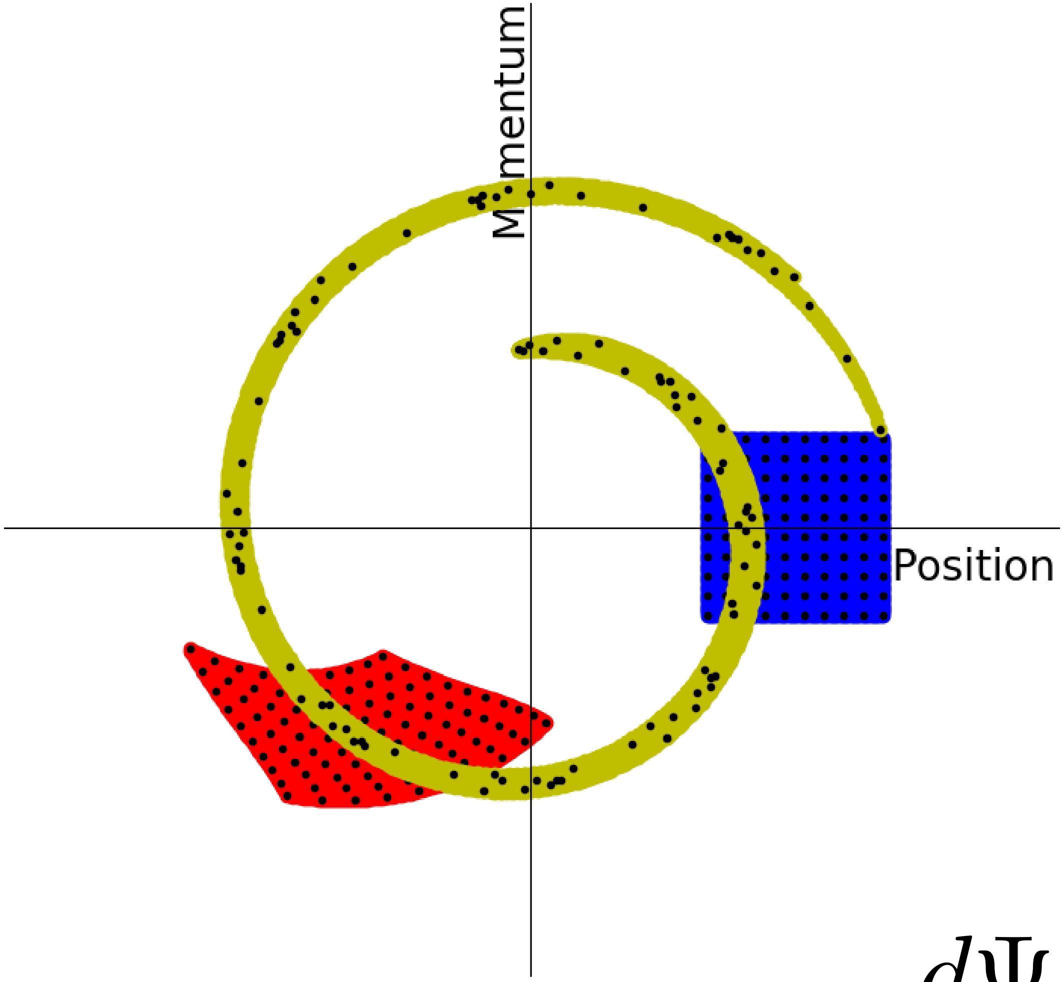
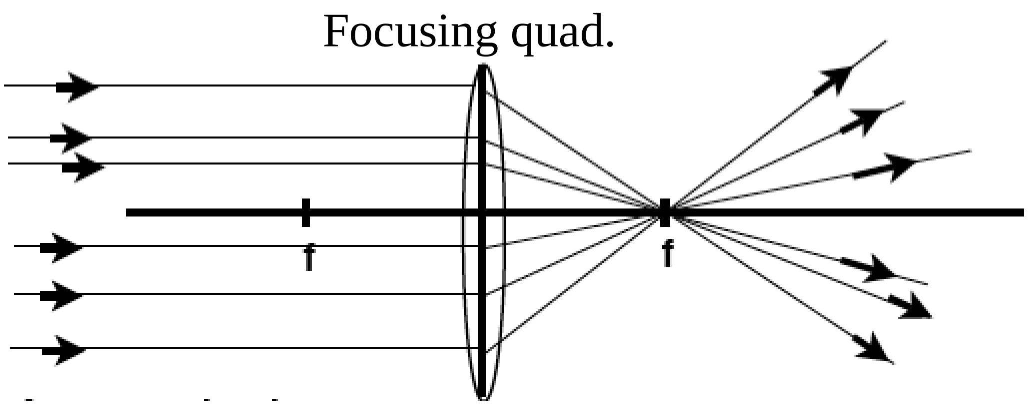


- Even with distorted trajectories,the phase-space density is preserved:

$$\frac{d\Psi}{dt} = 0$$

Liouville theorem: A simple illustration



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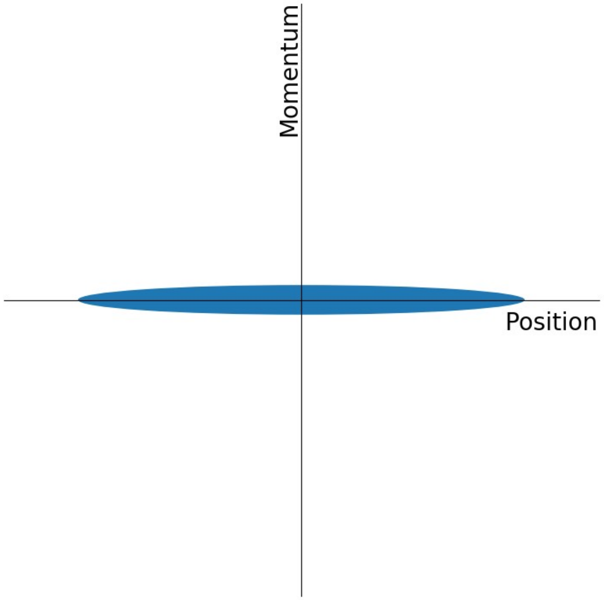

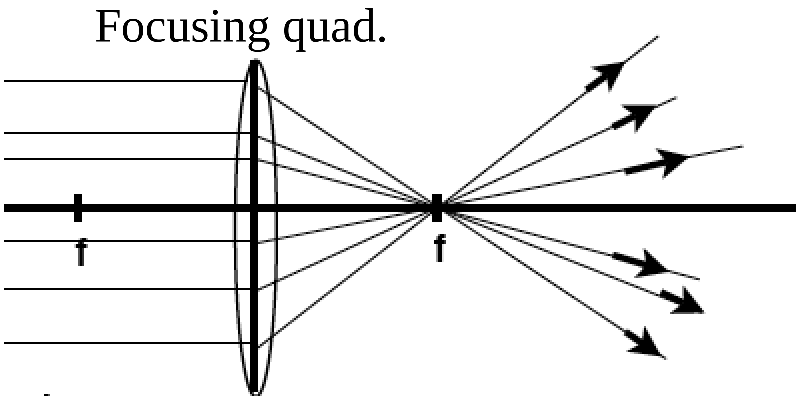

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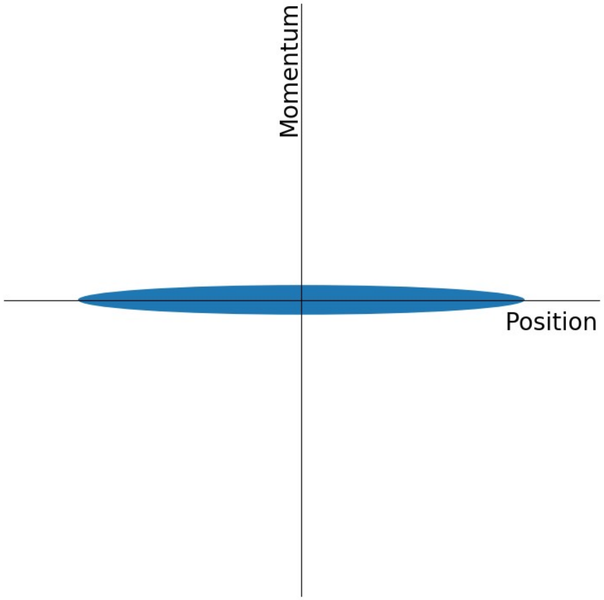

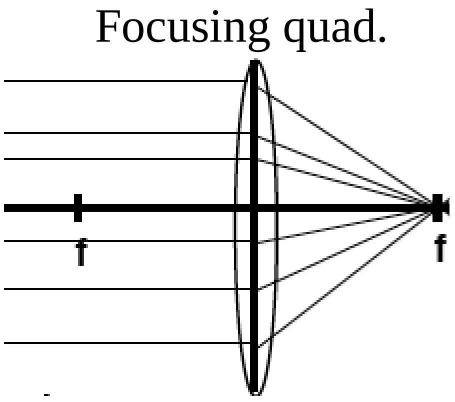

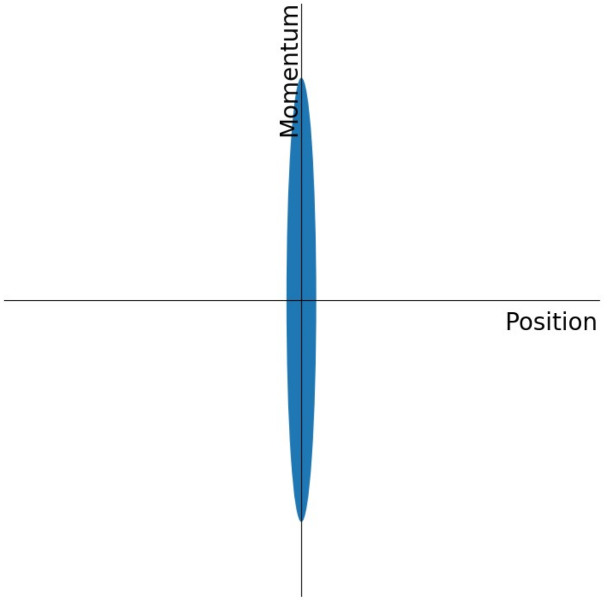

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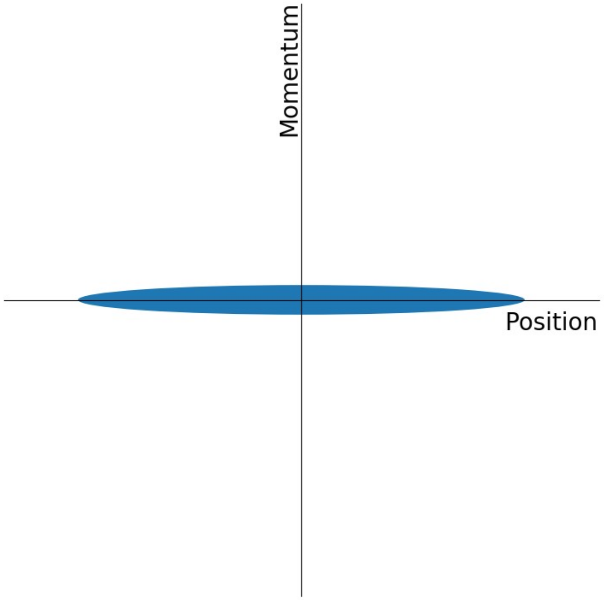

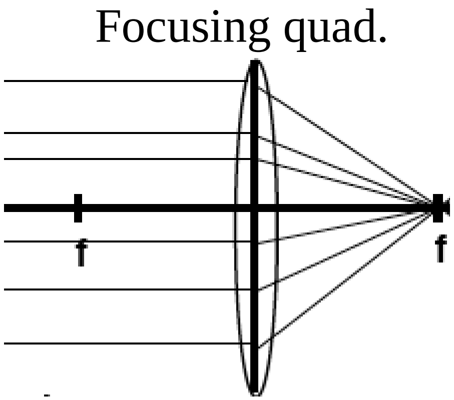

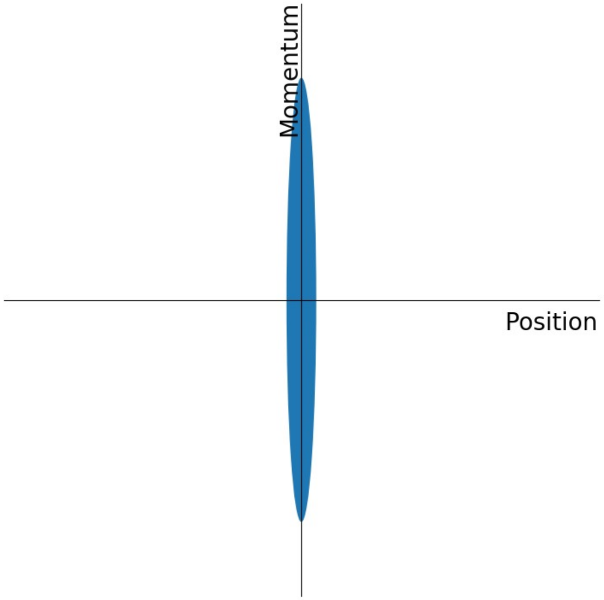

- The conservation of the emittance is a consequence of Liouville theorem

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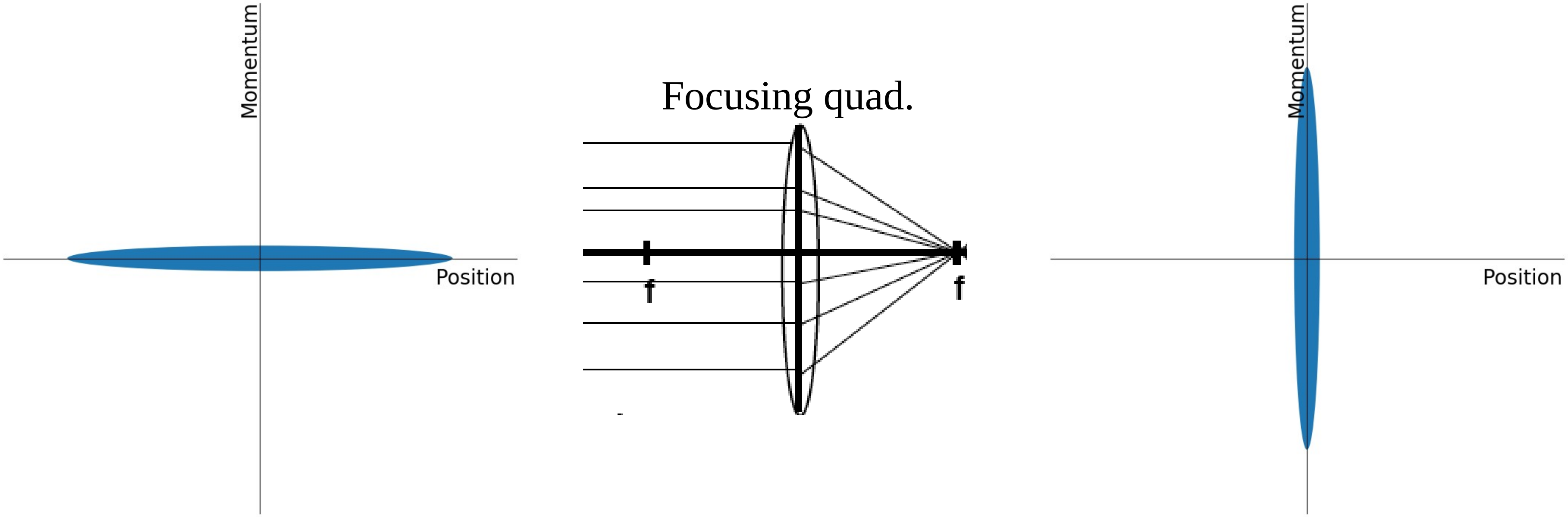

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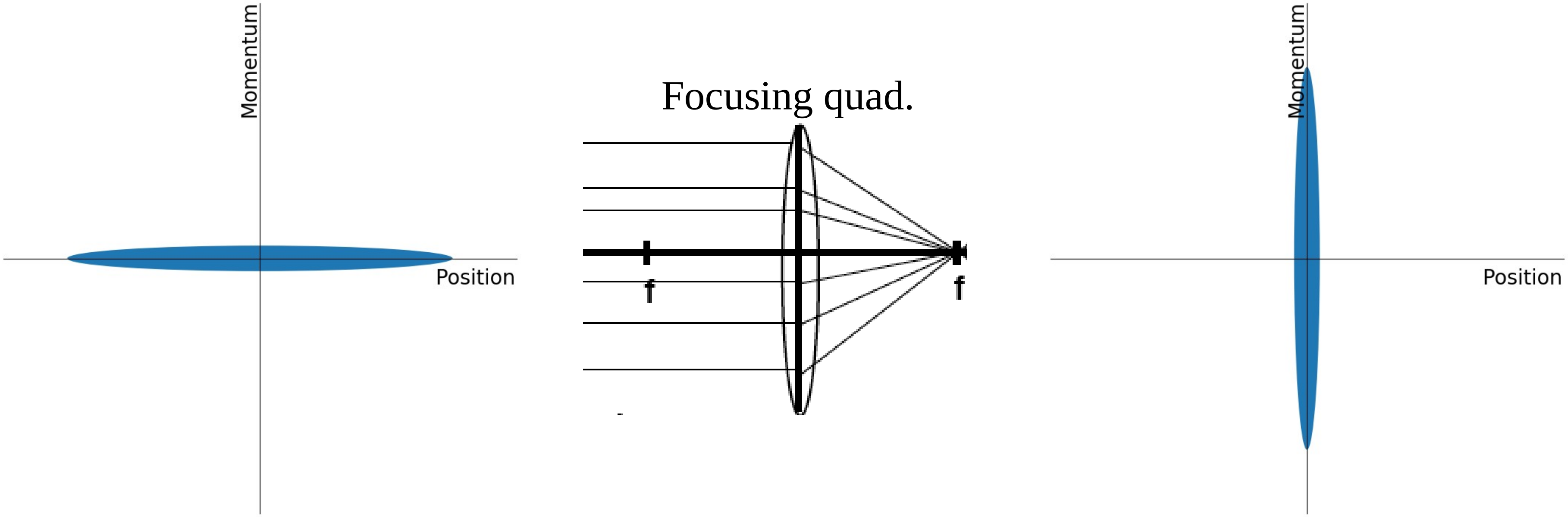

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    → Non-conservative forces such as intrabeam scattering or the emission of synchrotron radiation cannot be described with Hamilton's equation: Liouville theorem does not apply

Vlasov equation for particle beams

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Degree of freedom

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[Ruggiero]

- Let's consider a single degree of freedom and use action angle variables:

$$\begin{aligned} x &= \sqrt{2J}\cos(\theta) \\ p_x &= \sqrt{2J}\sin(\theta) \end{aligned}$$

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Example of solution: Exponential distribution in action (Gaussian in x, p_x) :

$$\Psi_0 = \frac{1}{2\pi\epsilon}e^{-\frac{J}{\epsilon}}$$

Perturbation theory with an external force

[Ruggiero]

- Let's consider a first order perturbation of the distribution: $\Psi = \Psi_0 + \Psi_1$

Perturbation theory with an external force

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- Let's consider a first order perturbation of the distribution: $\Psi = \Psi_0 + \Psi_1$

- And a first order perturbation of the Hamiltonian by an external force:

Perturbation theory with an external force

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- And a first order perturbation of the Hamiltonian by an external force: $H = H_0 + H_1$

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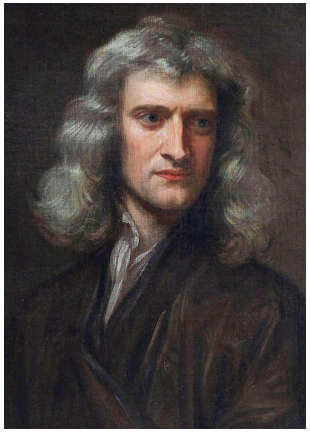

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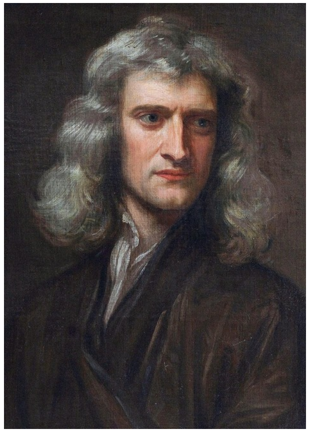

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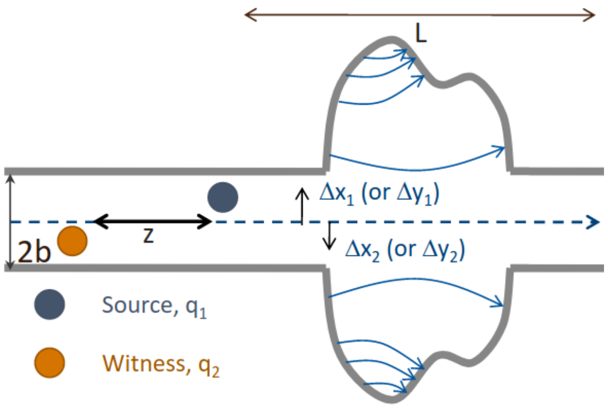
The collective force

[Wake, Pinch, Ruggiero]



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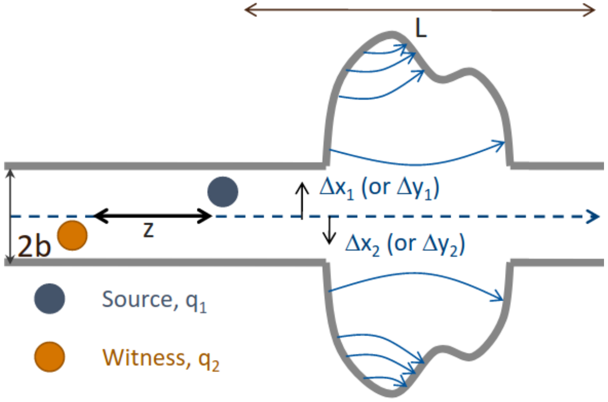

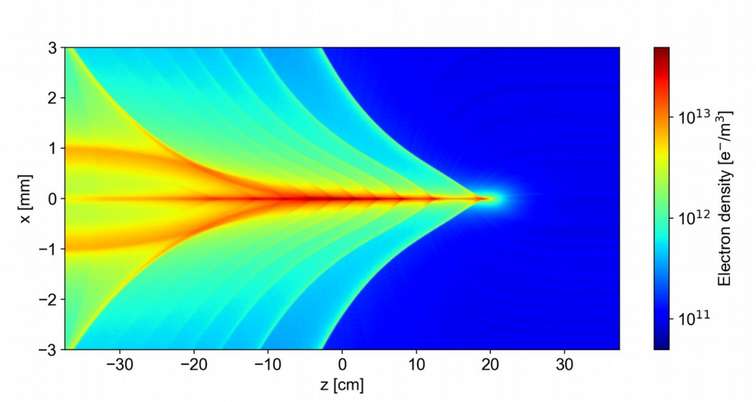

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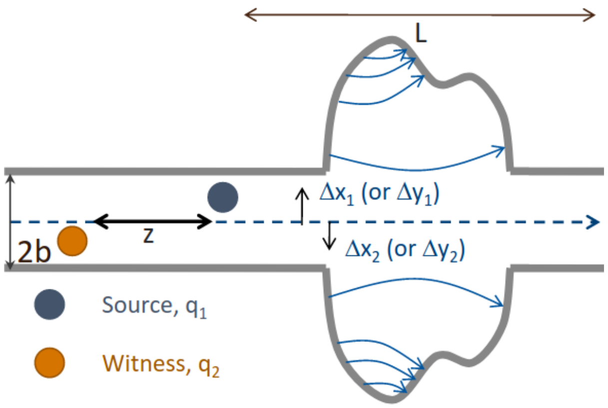

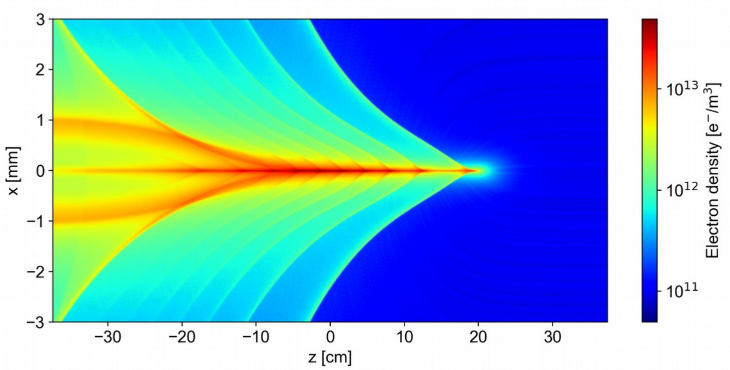

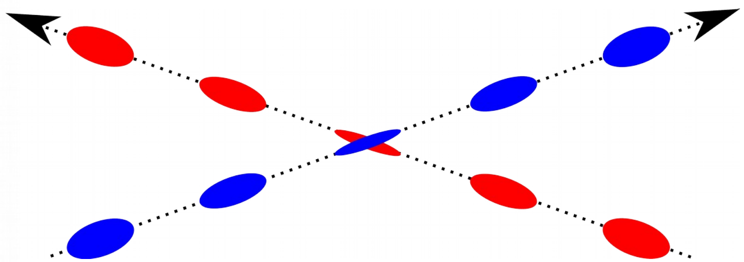

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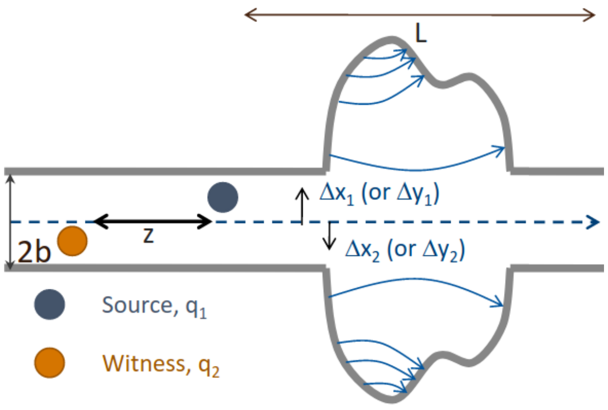

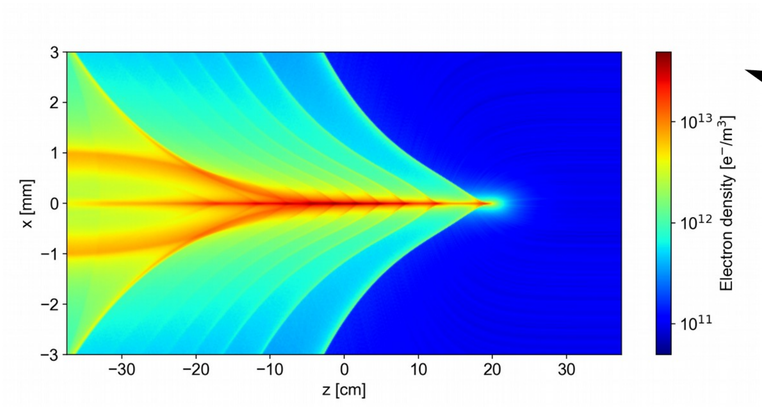

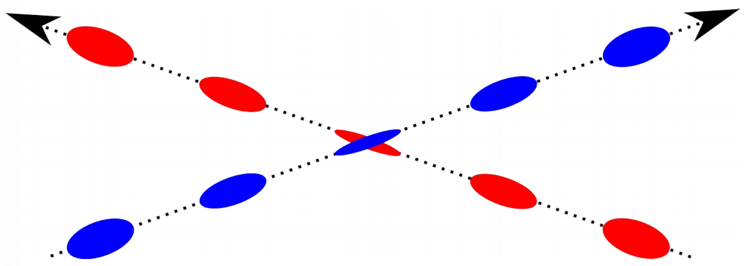

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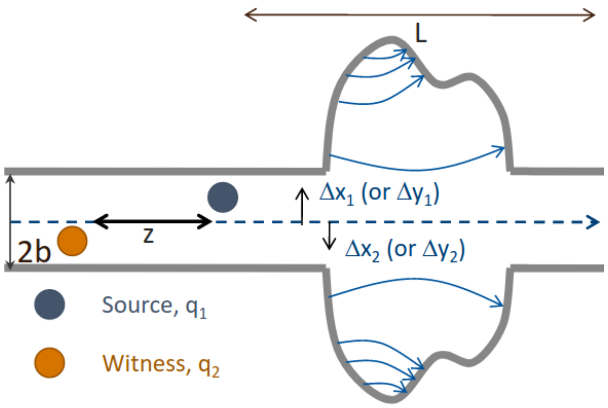

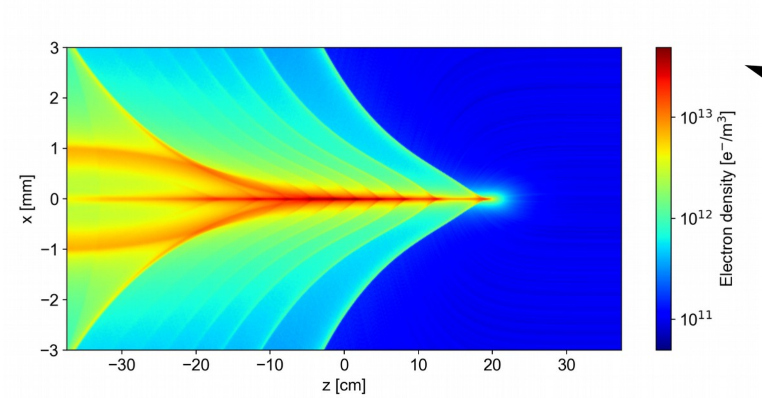

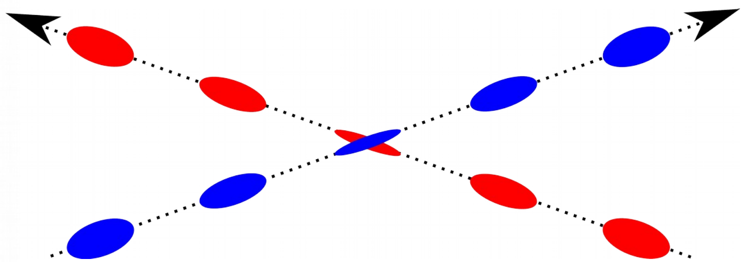

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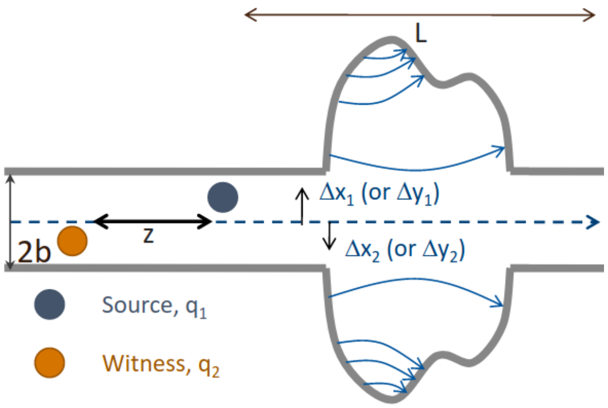

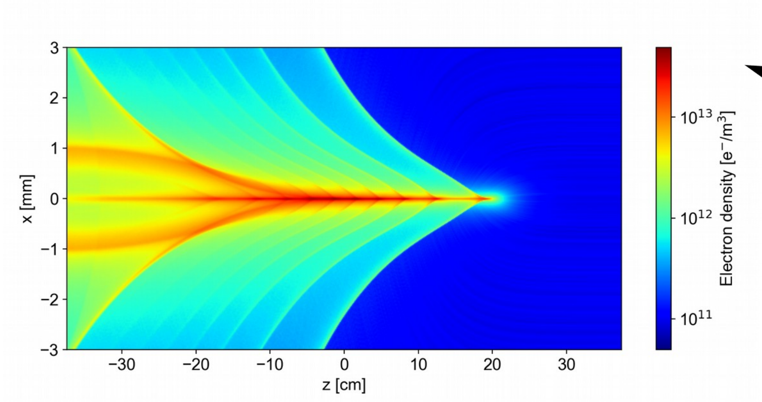

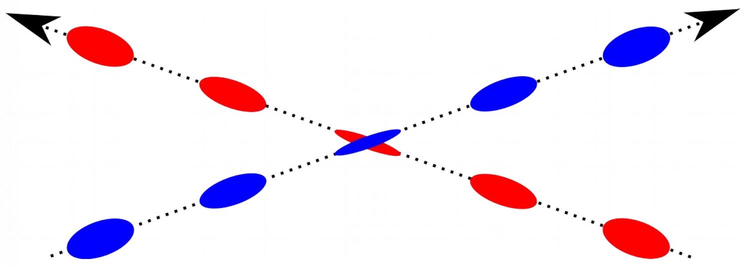

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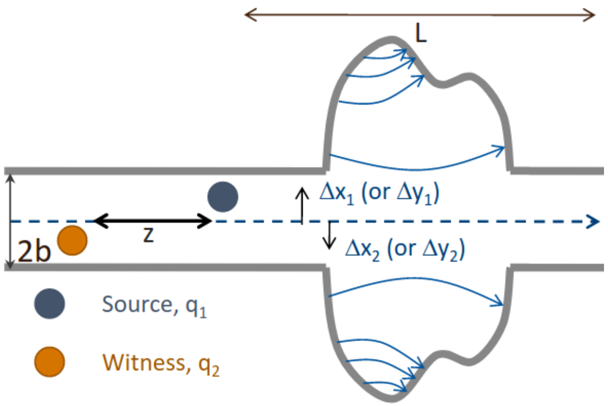

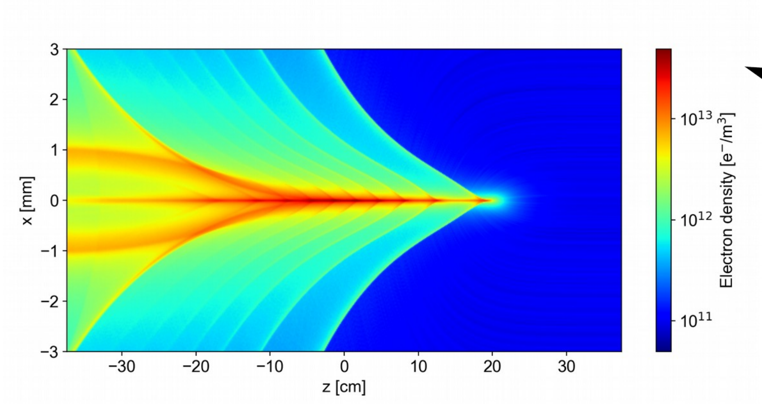

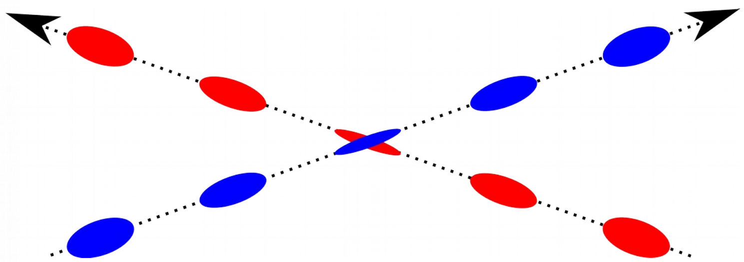

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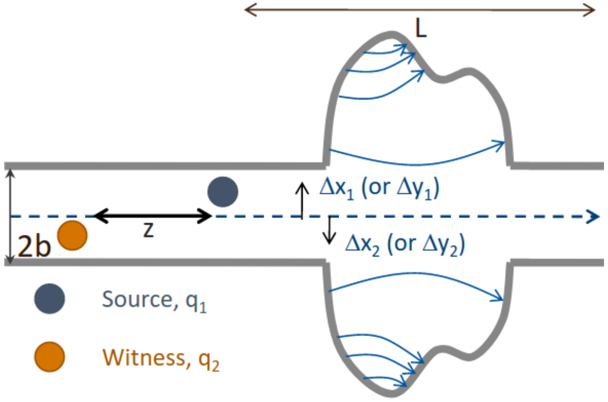

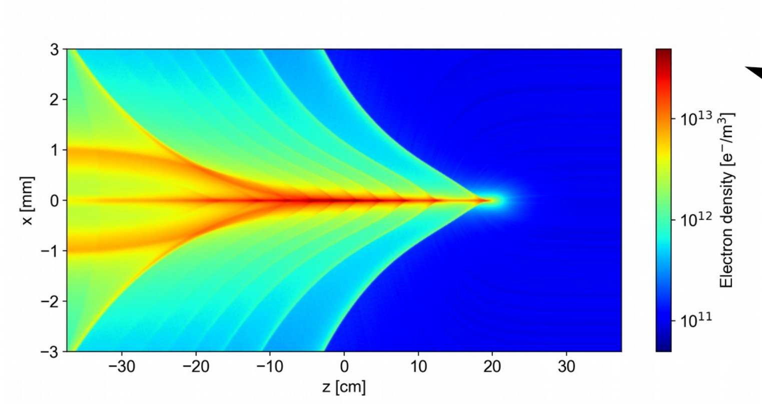

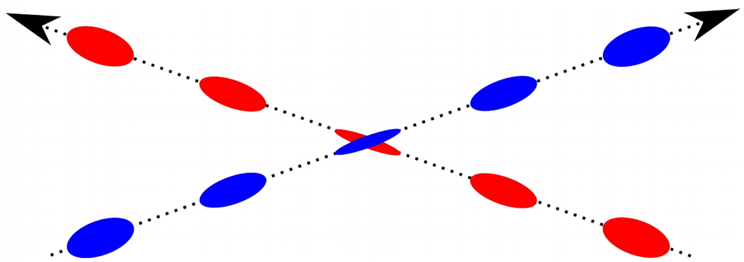

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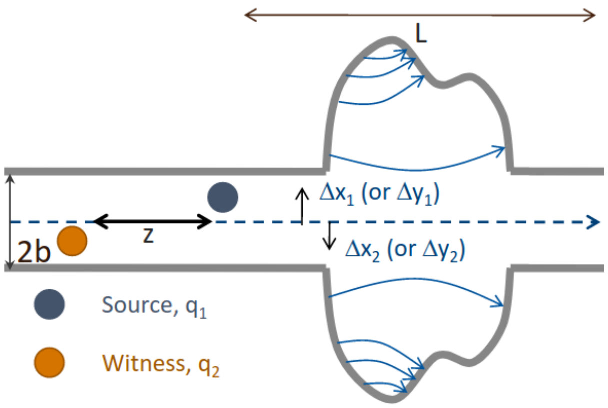

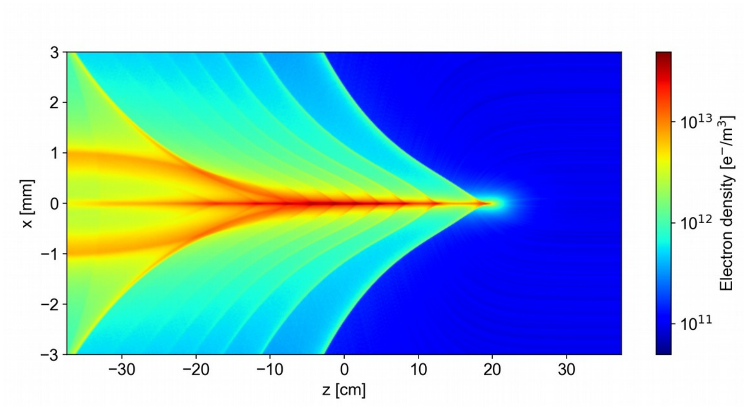

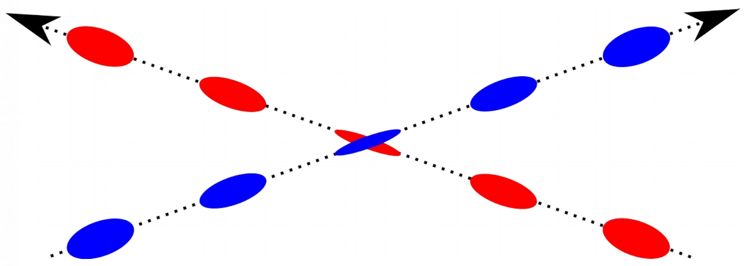

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Coherent mode

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- The dispersion relation links the coherent mode frequency with the frequency shift due to the collective force via the tune spread

$$\frac{-1}{\Delta\Omega_{ext}} = \int dJ\frac{J\frac{df_0}{dJ}}{\Omega_c - \omega(J)}$$

The stability diagram

[Ruggiero]

$$f_0 = \frac{1}{\epsilon} e^{-\frac{J}{\epsilon}} \qquad \omega(J) = \omega_0 \left(Q_0 + aJ \right)$$

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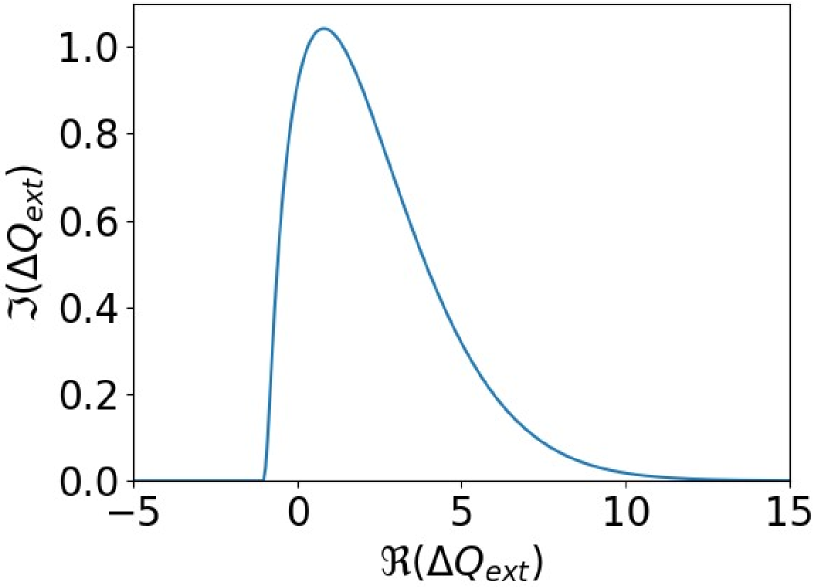

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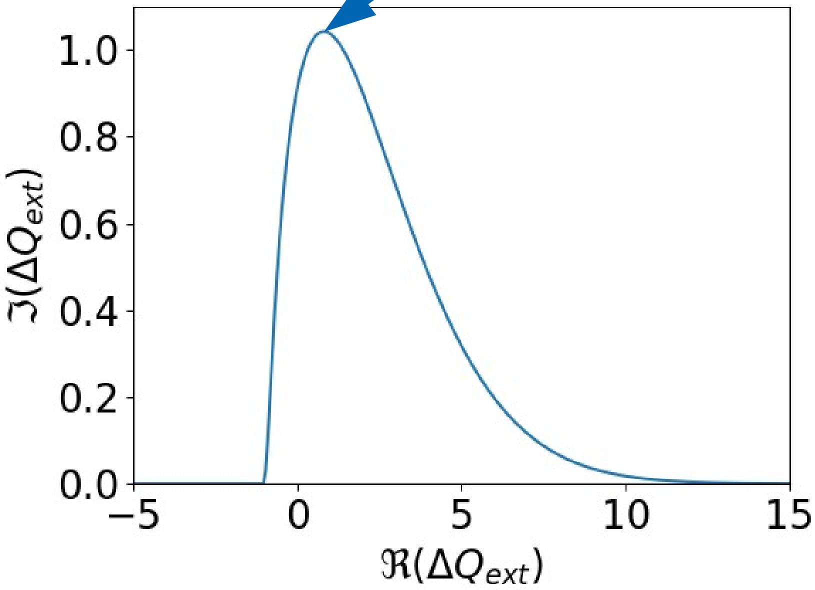

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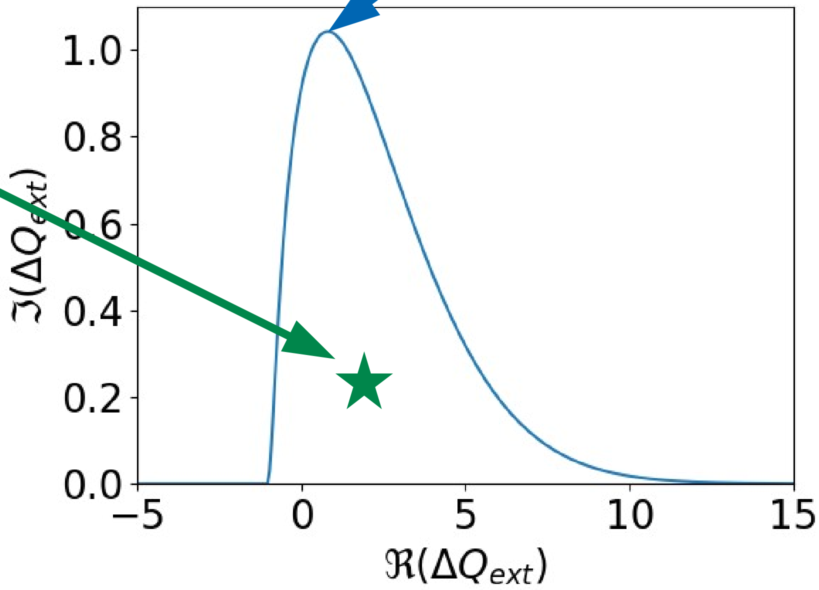
The beam is stable



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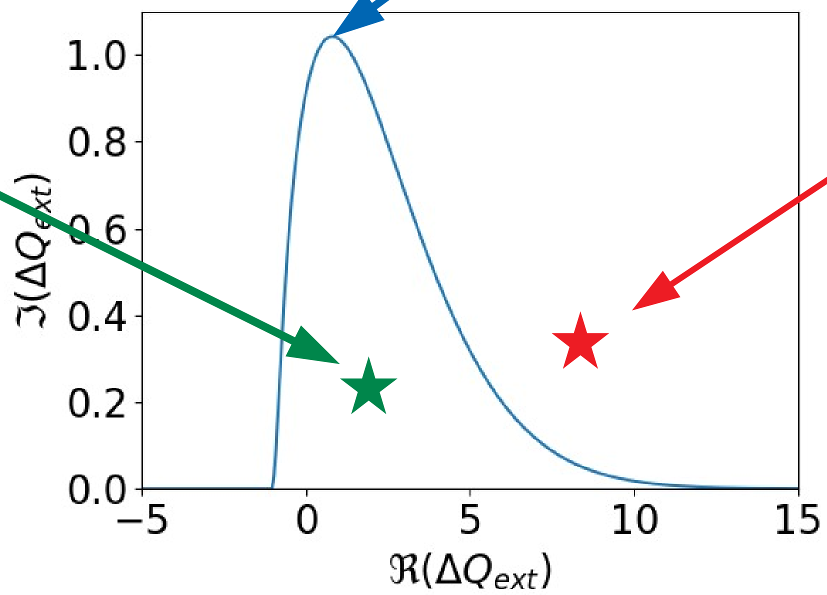

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[Ruggiero]

$$f_0 = \frac{1}{\epsilon} e^{-\frac{J}{\epsilon}} \qquad \omega(J) = \omega_0 \left(Q_0 + aJ\right)$$

- Using a Gaussian distribution and linear detuning, we can write the dispersion relation:

$$\frac{-1}{\Delta Q_{ext}} = 1 - q e^q E_1(q) \quad q \equiv \frac{Q_c - Q_0}{a\epsilon}$$

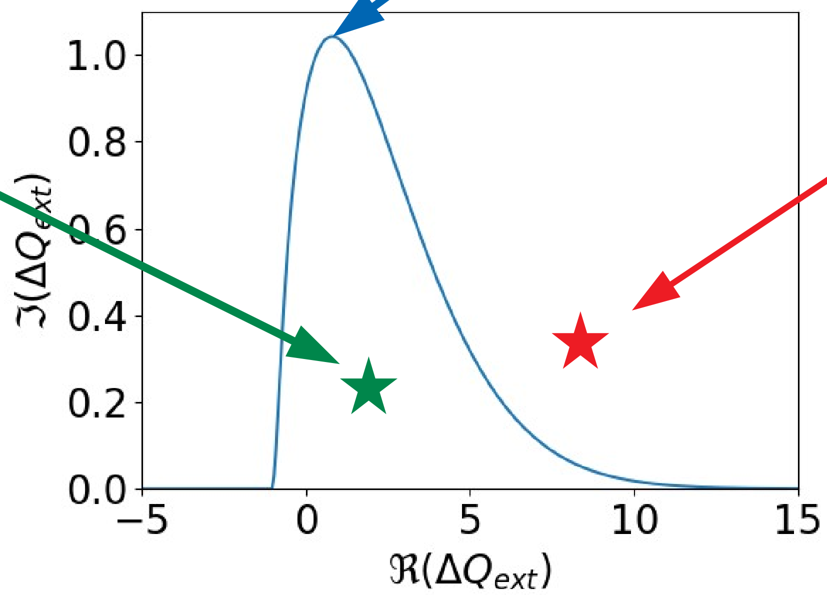
$$\Omega_c = \omega_0 Q_c$$
$$\Psi_c = g_c(J) e^{i(\theta - \Omega_c t)}$$

$$\Im Q_c \to 0$$

If the tune shift due to the collective force is **inside** of the stability diagram

$$\Im Q_c < 0$$

The beam is stable



If the tune shift due to the collective force is **outside** of the stability diagram

$$\Im Q_c > 0$$

The beam is unstable

- The **stability diagram is a very common way of representing Landau damping** when the impact of the collective force can be represented by a complex tune shift

Forced beam oscillations

[Ruggiero]

$$F_{ext} = A_{ext} e^{-i\Omega t}$$

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$$\frac{\partial \Psi_1}{\partial t} + \omega(J) \frac{\partial \Psi_1}{\partial \theta} - \sqrt{2J} \sin(\theta) A_{ext} e^{i\Omega t} \frac{\partial \Psi_0}{\partial J} = 0$$

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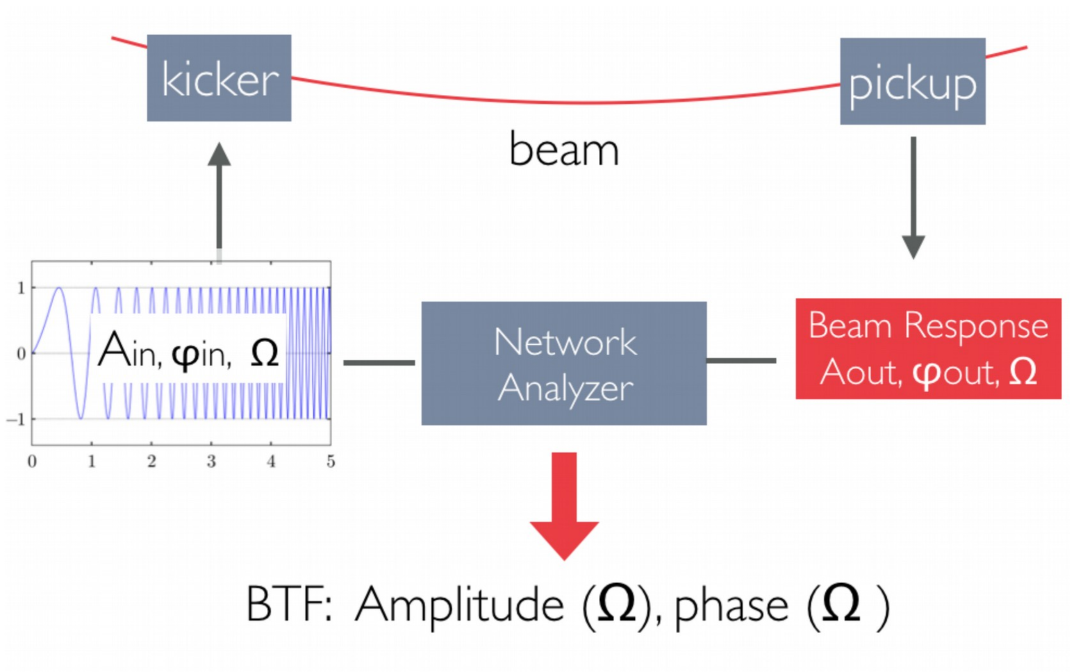
- The beam oscillation amplitude normalised to the excitation amplitude is called the **beam transfer function** → A measurable quantity that directly relates to the **stability diagram**

Beam transfer function measurement

[Tambasco]



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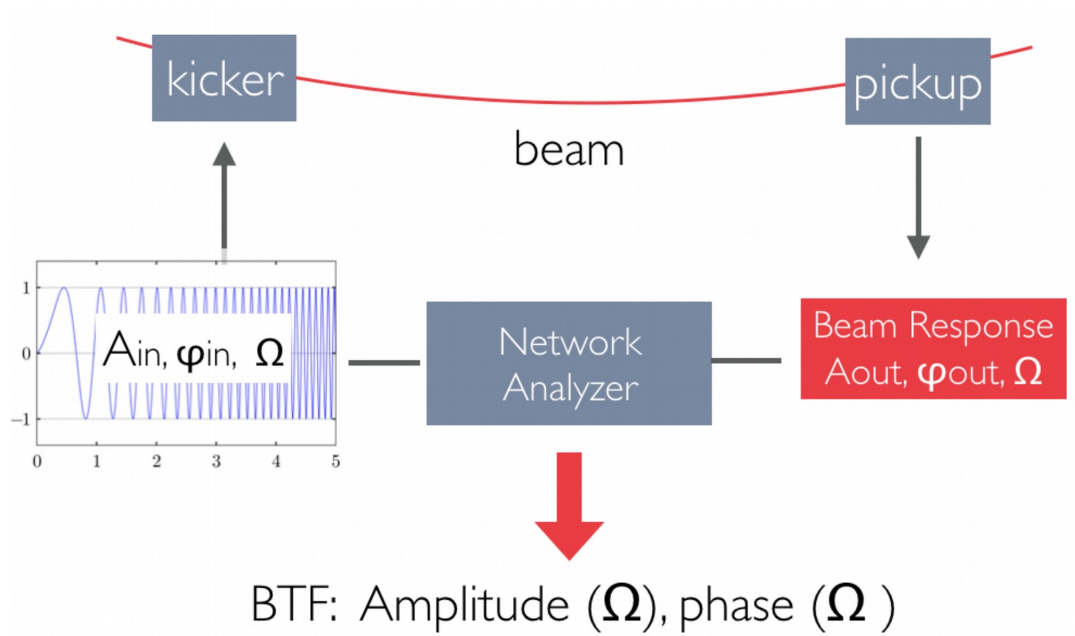
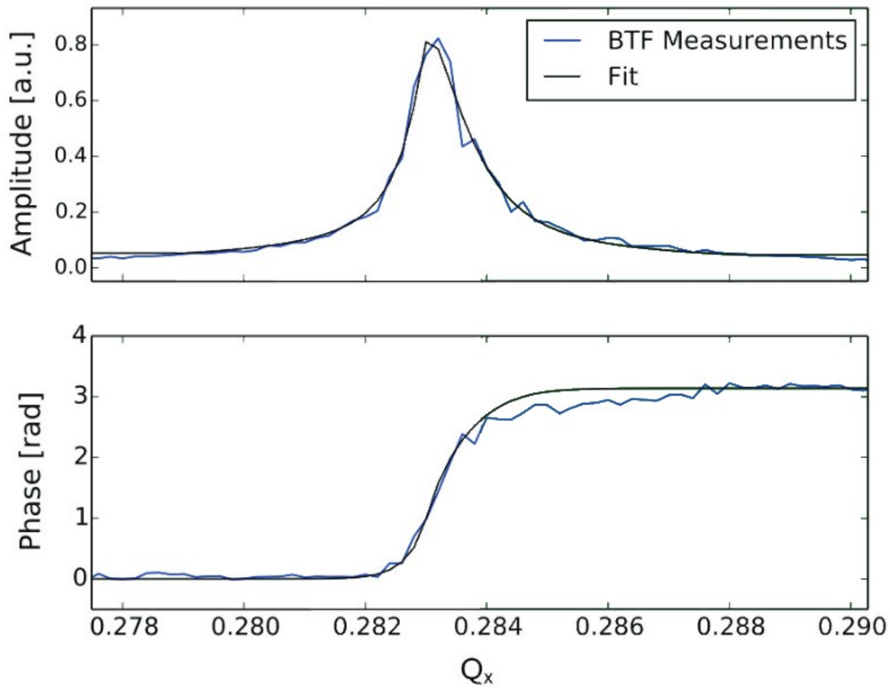


Measurement at LHC



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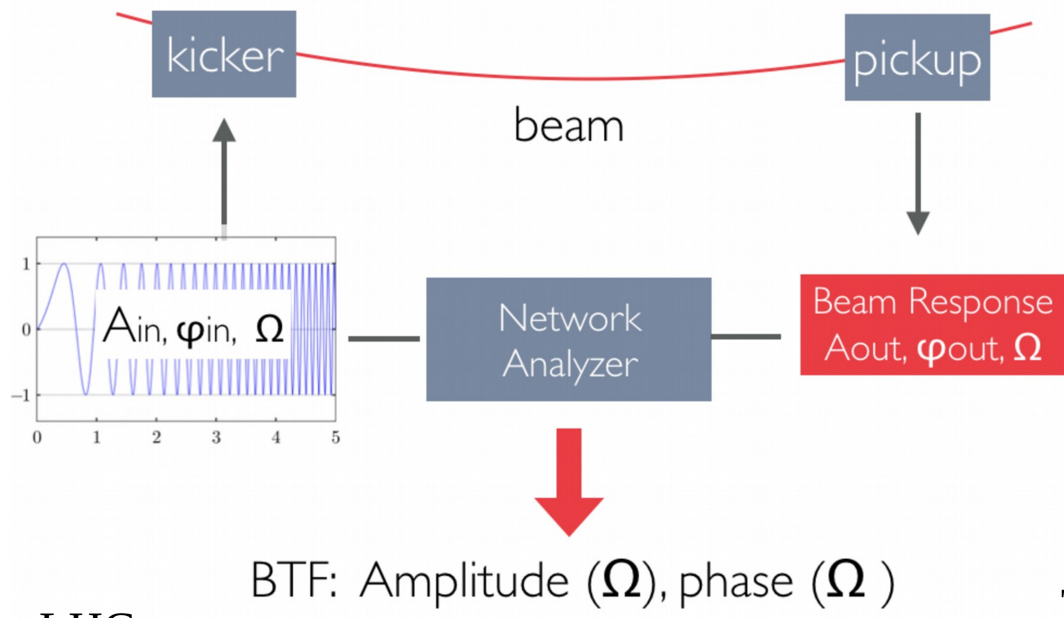

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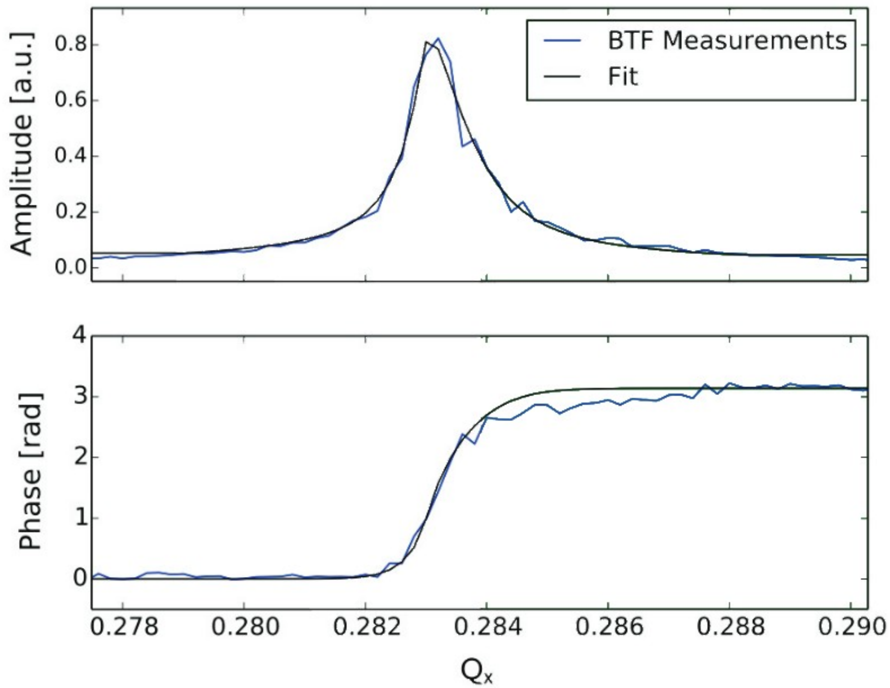
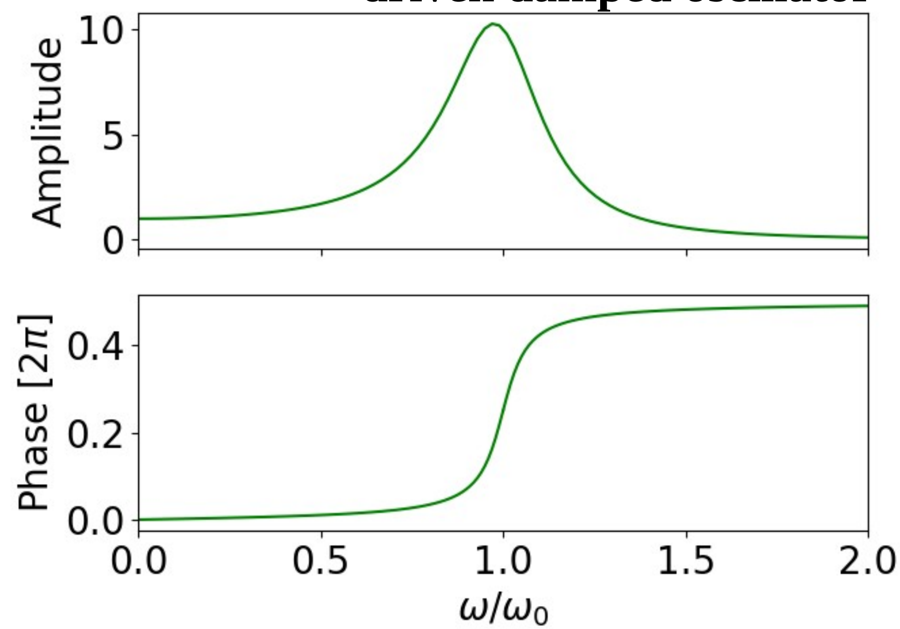


Transfer function of the **driven damped oscillator**



Beam transfer function and stability diagram [Tambasco]

$$\frac{\langle x \rangle}{A_{ext}} = \int dJ \frac{J \frac{df_0}{dJ}}{\Omega - \omega(J)}$$

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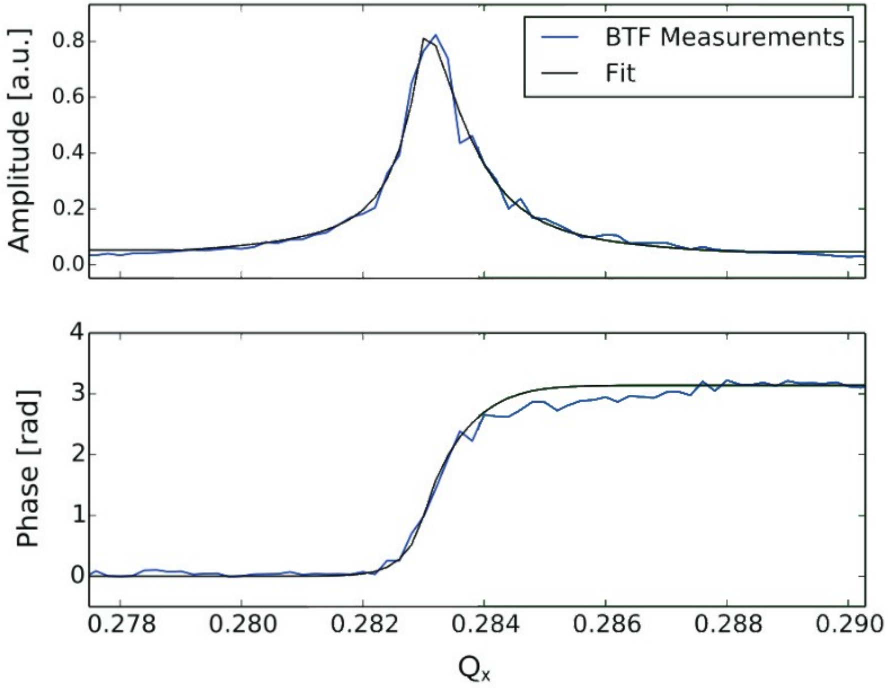

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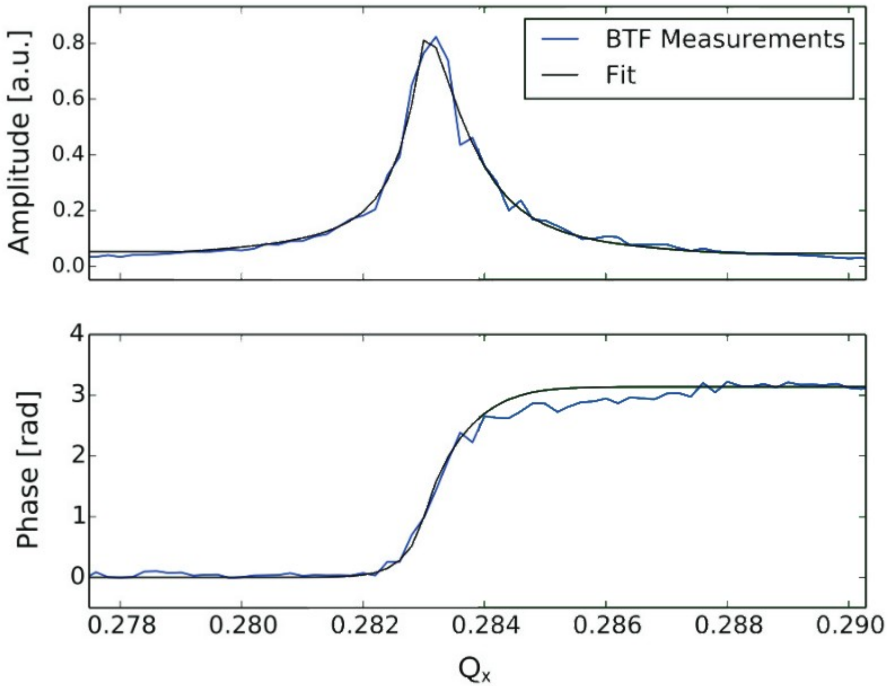
Measurement at LHC



Beam transfer function and stability diagram [Tambasco]

$$\frac{\langle x \rangle}{A_{ext}} = \int dJ \frac{J \frac{df_0}{dJ}}{\Omega - \omega(J)} \quad \Rightarrow \quad \frac{-1}{\Delta\Omega_{ext}} = \int dJ \frac{J \frac{df_0}{dJ}}{\Omega_c - \omega(J)}$$

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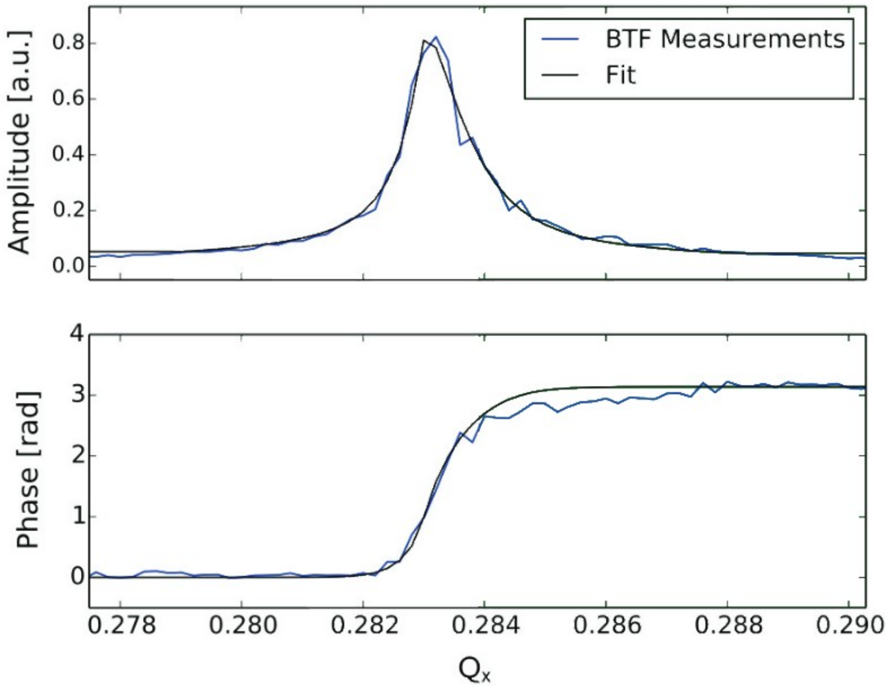

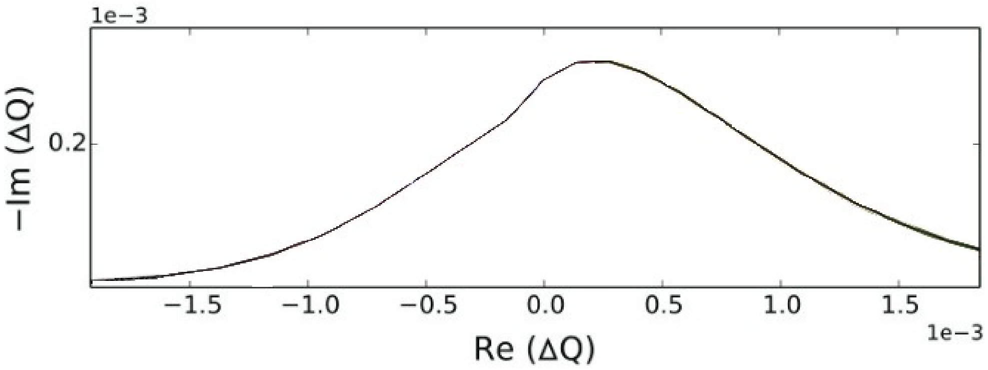

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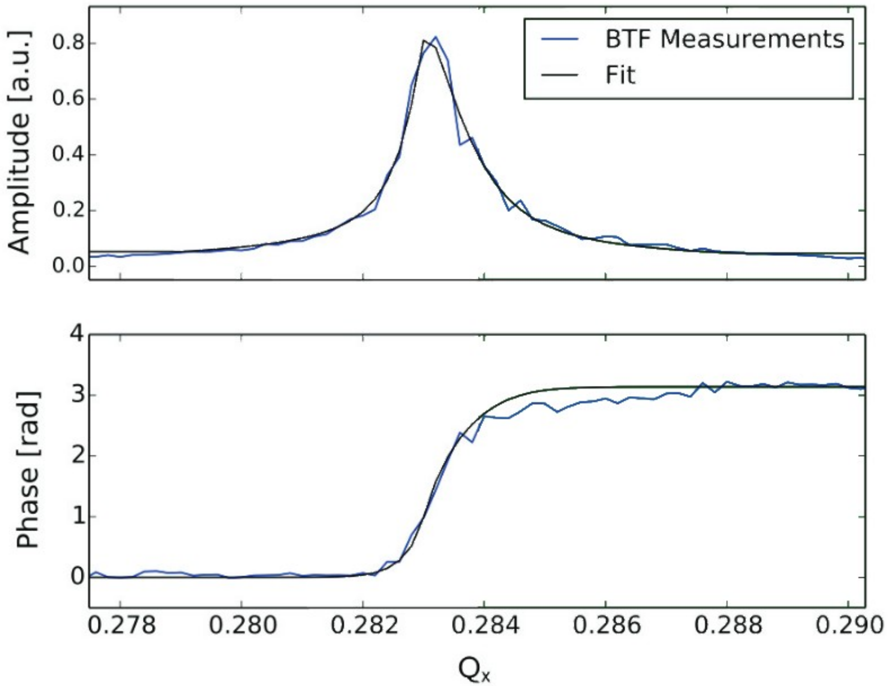

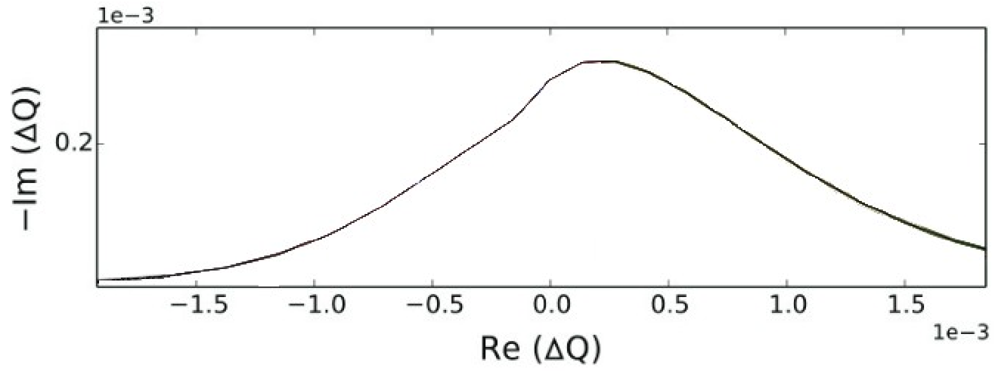

- The BTF is an interesting way to **quantify experimentally** Landau damping

Recap

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- Landau damping stems from the **interaction of single particles with waves**
  - A necessary condition for Landau damping is the a comparable velocity / frequency of the wave and the particles motion

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- While collective forces such as wake fields or electron clouds tend to generate unstable modes of oscillation, Landau damping stabilises them **without emittance growth**
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Recap

- Landau damping stems from the **interaction of single particles with waves**
  - A necessary condition for Landau damping is the a comparable velocity / frequency of the wave and the particles motion
- While collective forces such as wake fields or electron clouds tend to generate unstable modes of oscillation, Landau damping stabilises them **without emittance growth**
  - An external perturbation may also decay through a similar phenomenon, we rather talk about decoherence or filamentation. This mechanism leads to **emittance growth**
- Landau damping originates in the spread of oscillation frequencies of the particles in the beam
  - It is a **linear mechanism**, as in plasmas. However in accelerators the frequency spread often originates from **non-linear forces**

“Now what ?”

– Fuego,a down-to-earth rabbit

- Ok, in the second part we’ll address practical applications...

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