Applied Landau damping

Lecture notes available at **https://xbuffat.web.cern.ch/landaudampingCAS.pdf**

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CERN, Switzerland, Geneva

The CERN Accelerator School

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Recap

- Landau damping stems from the **interaction of single particles with waves**
  - A necessary condition for Landau damping is the a comparable velocity / frequency of the wave and the particles motion
- While collective forces such as wake fields or electron clouds tend to generate unstable modes of oscillation, Landau damping stabilises them **without emittance growth**
  - An external perturbation may also decay through a similar phenomenon, we rather talk about decoherence or filamentation. This mechanism leads to **emittance growth**
- Landau damping originates in the spread of oscillation frequencies of the particles in the beam
  - It is a **linear mechanism**, as in plasmas. However in accelerators the frequency spread often originates from **non-linear forces**

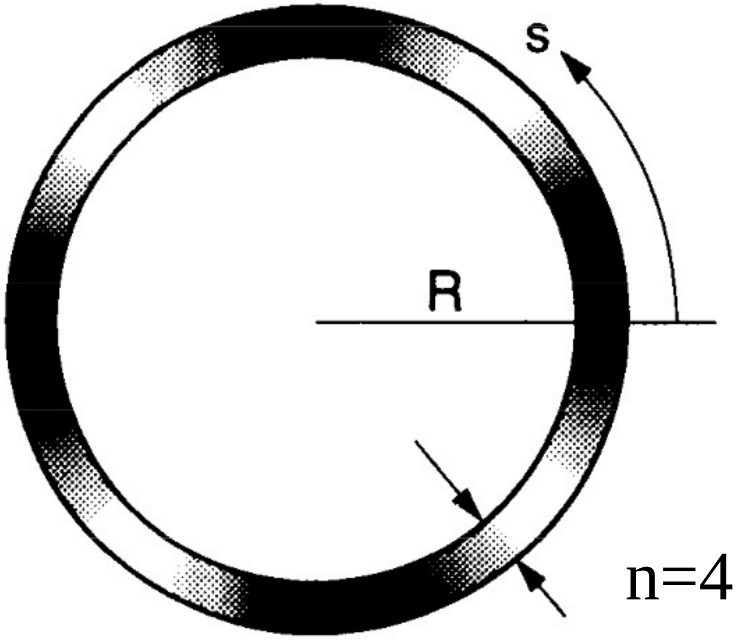
Content

- Part I (concept)
  - Wave – particle interaction
  - Van Kampen approach
  - Stability diagram and beam transfer function
- Part II (applications)
  - Longitudinal and transverse Landau damping in unbunched and bunched beams
  - Non-linear collective forces
  - Advanced Landau damping techniques

Longitudinal stability of unbunched beams

[Chao]

- In unbunched beam, density modulations may self-enhances under the influence an impedance (e.g. Negative mass instability)



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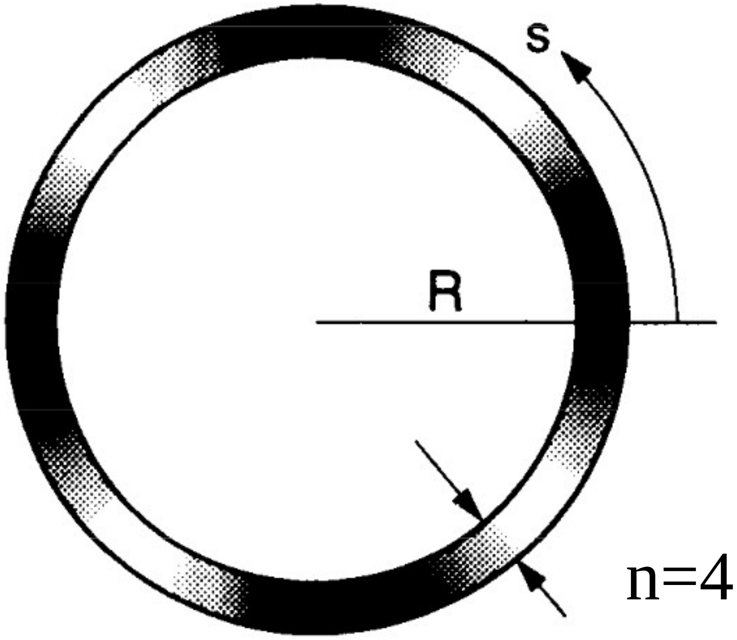
  - The dispersion relation takes a **special form**:



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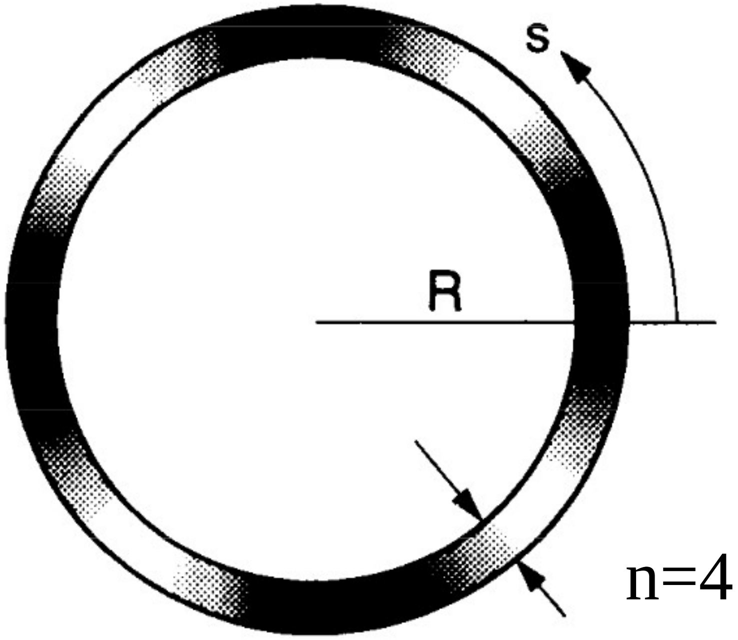

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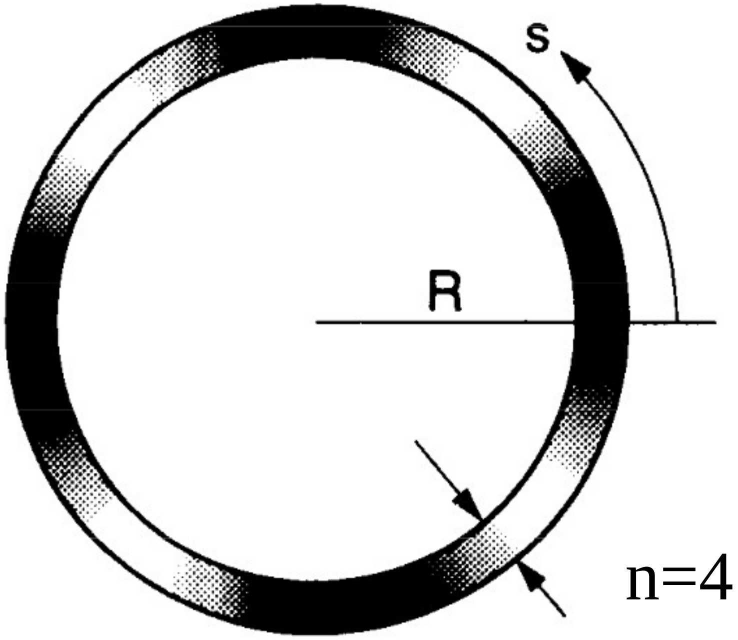

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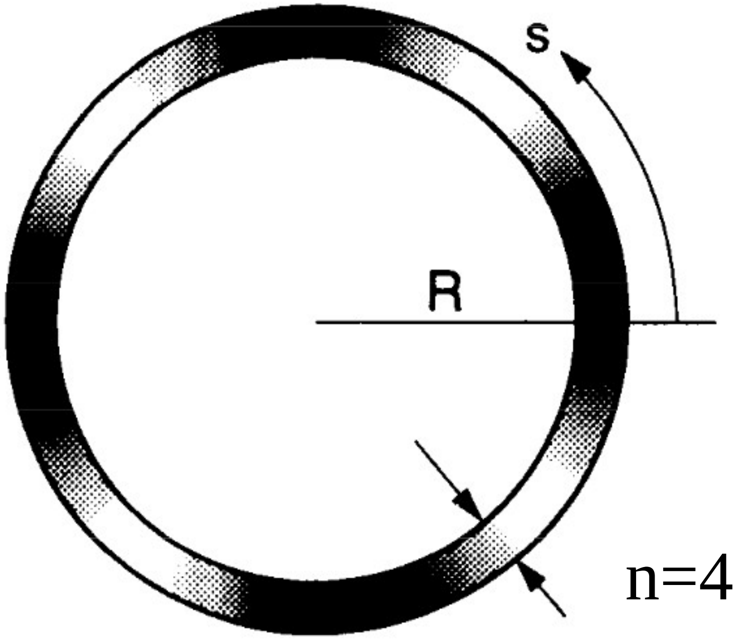

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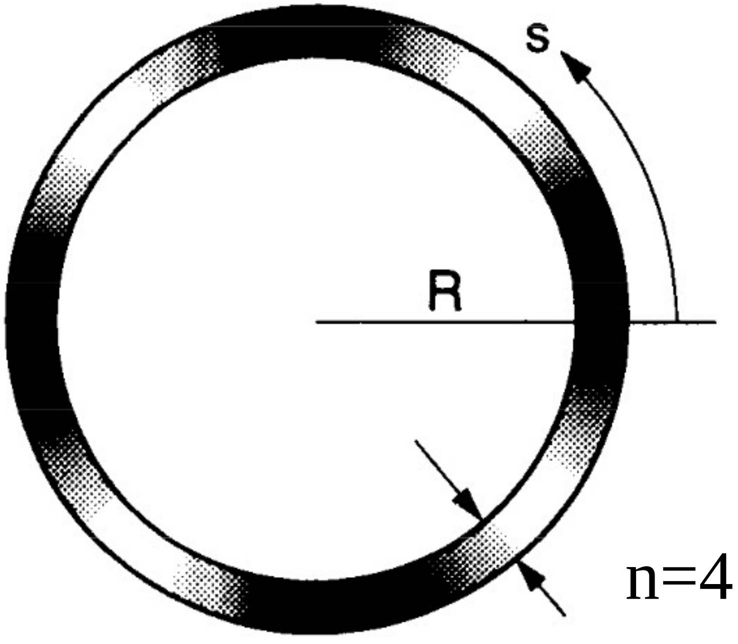

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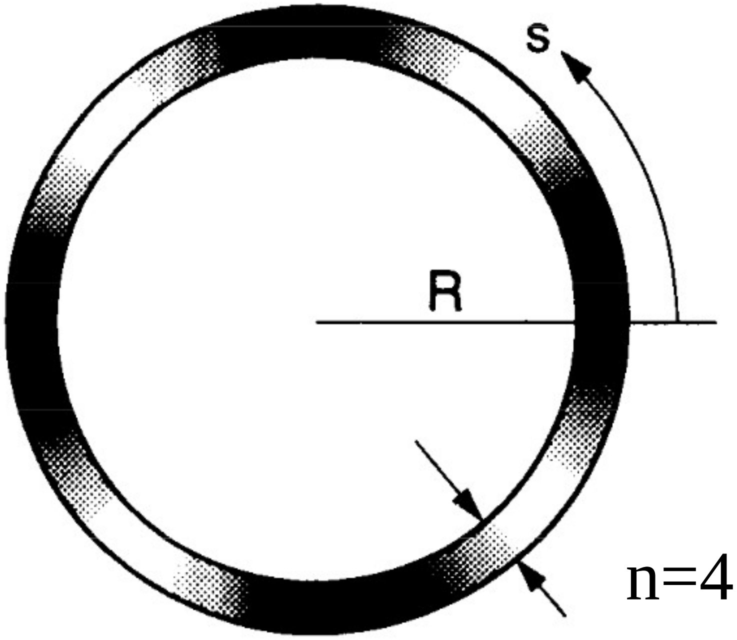

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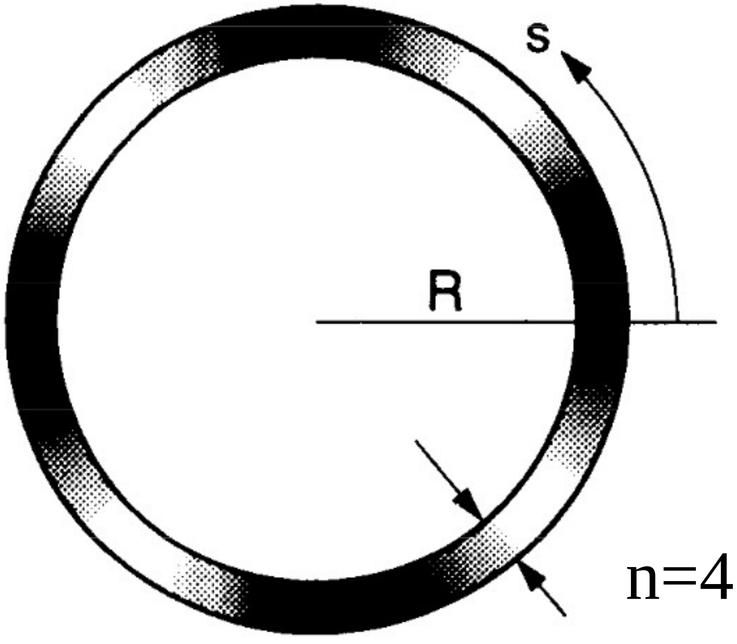

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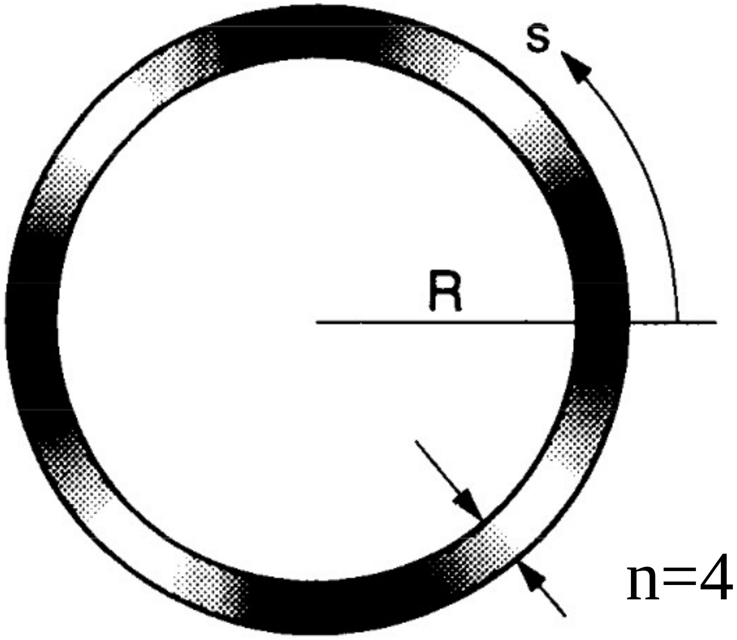

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Distribution of revolution frequencies

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Linked to the absence of focusing

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Stability diagram – Keil-Schnell criterion

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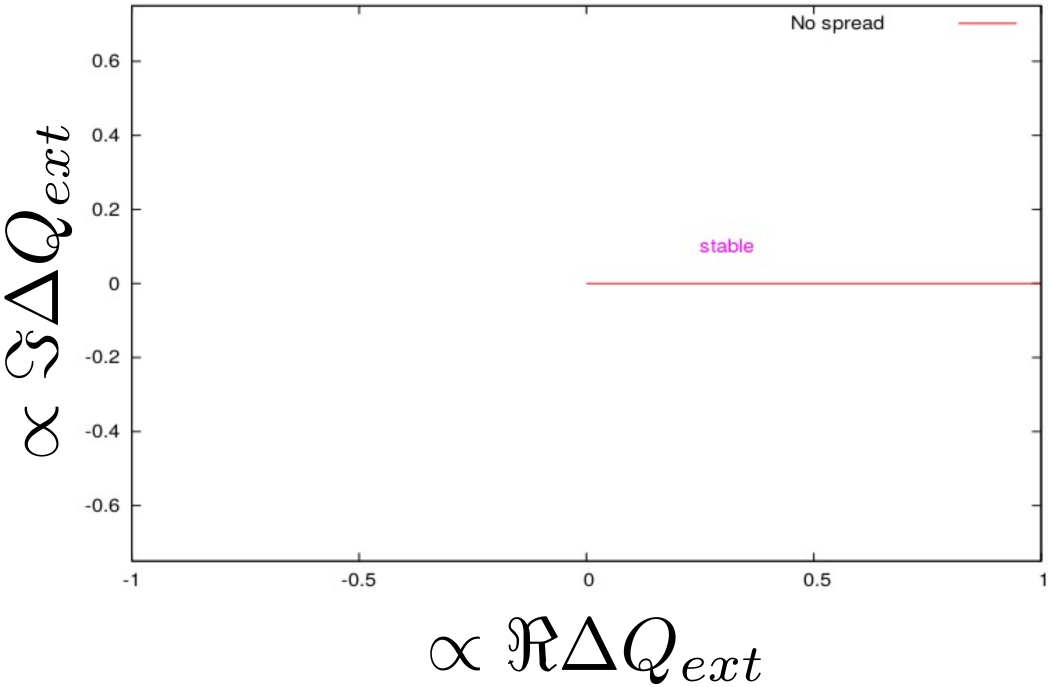

- The beam is always unstable without energy spread

Stability diagram – Keil-Schnell criterion

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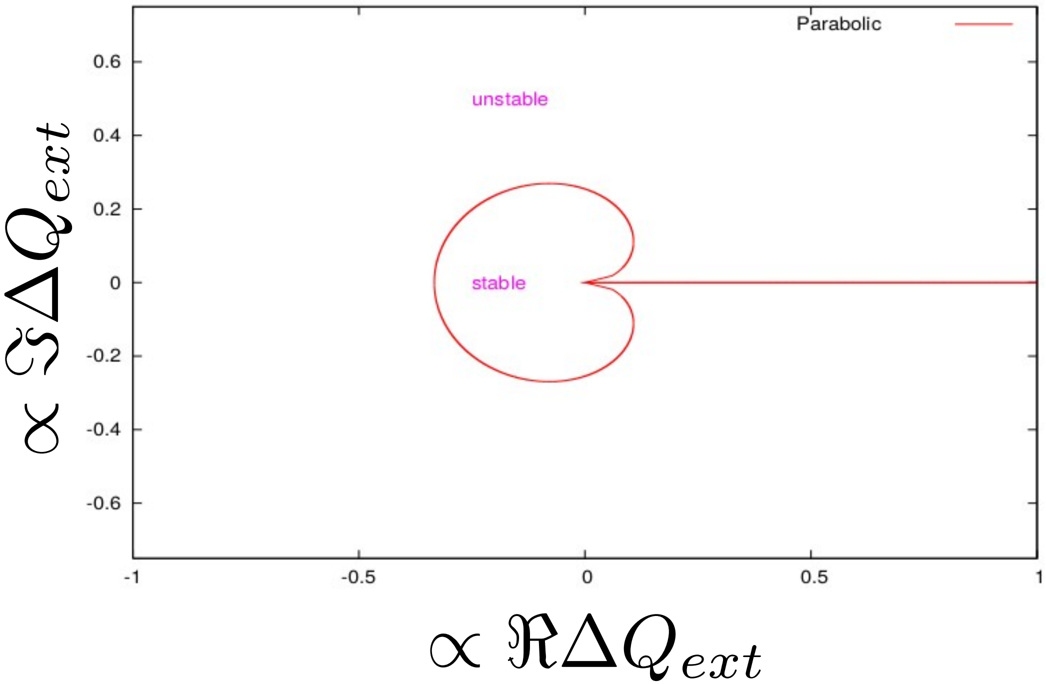

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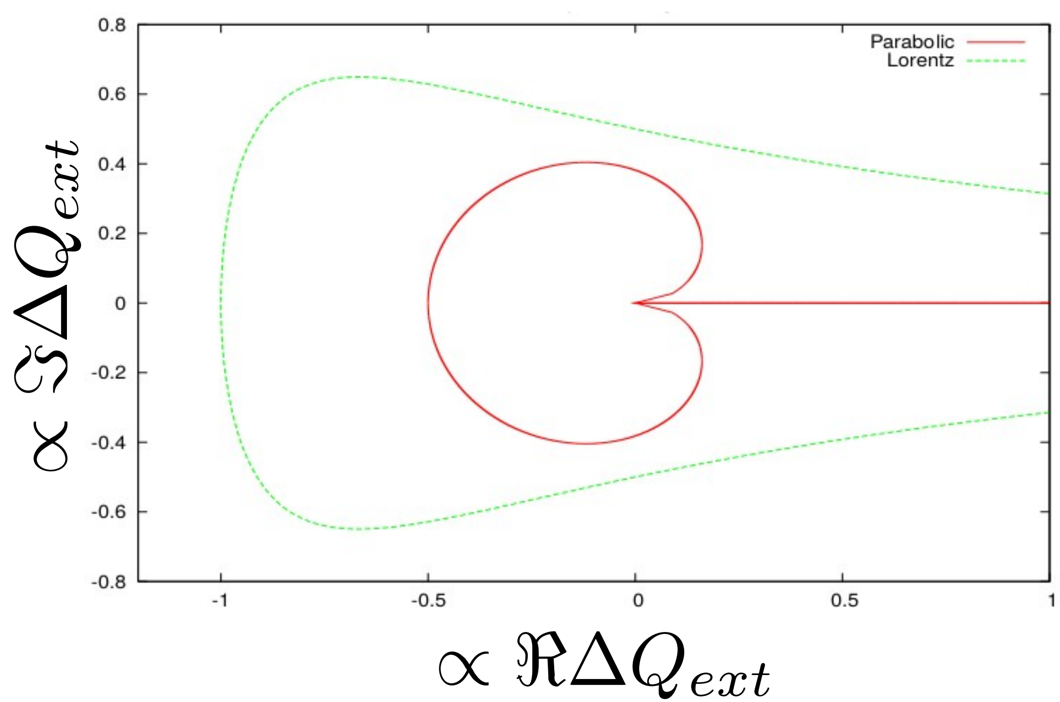

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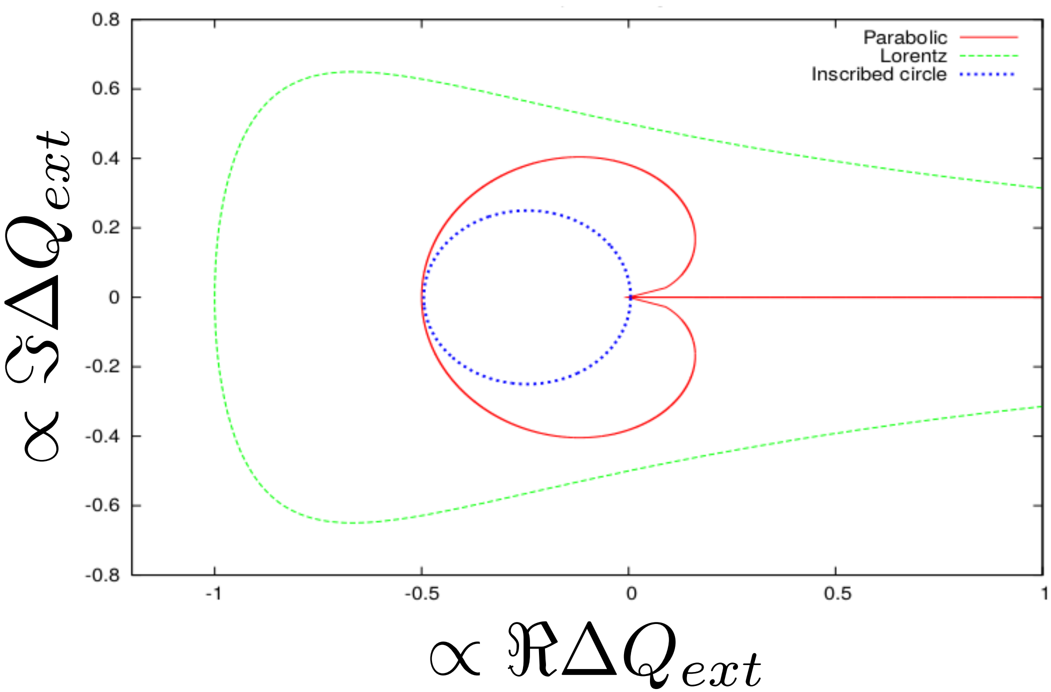

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- Usually the frequency distribution is poorly known, Keil-Schnell derived a conservative criterion based on the inscribed circle:

$$|\Delta\Omega_n| \lessapprox \frac{1}{4} n^2 \Delta\omega^2$$

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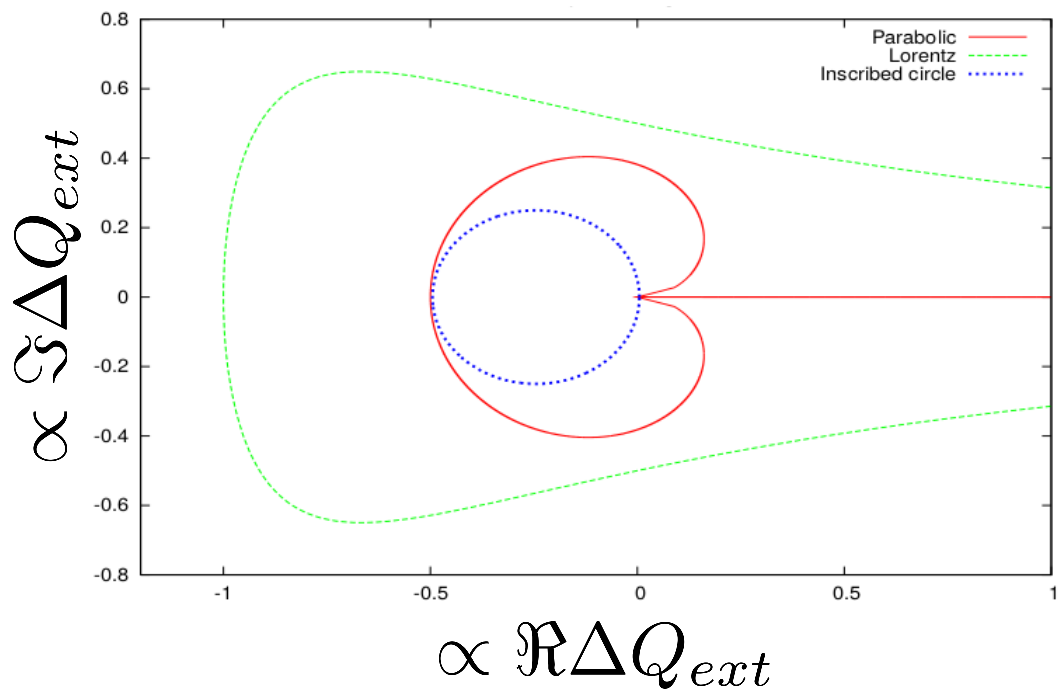

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- Revolution frequency spread:

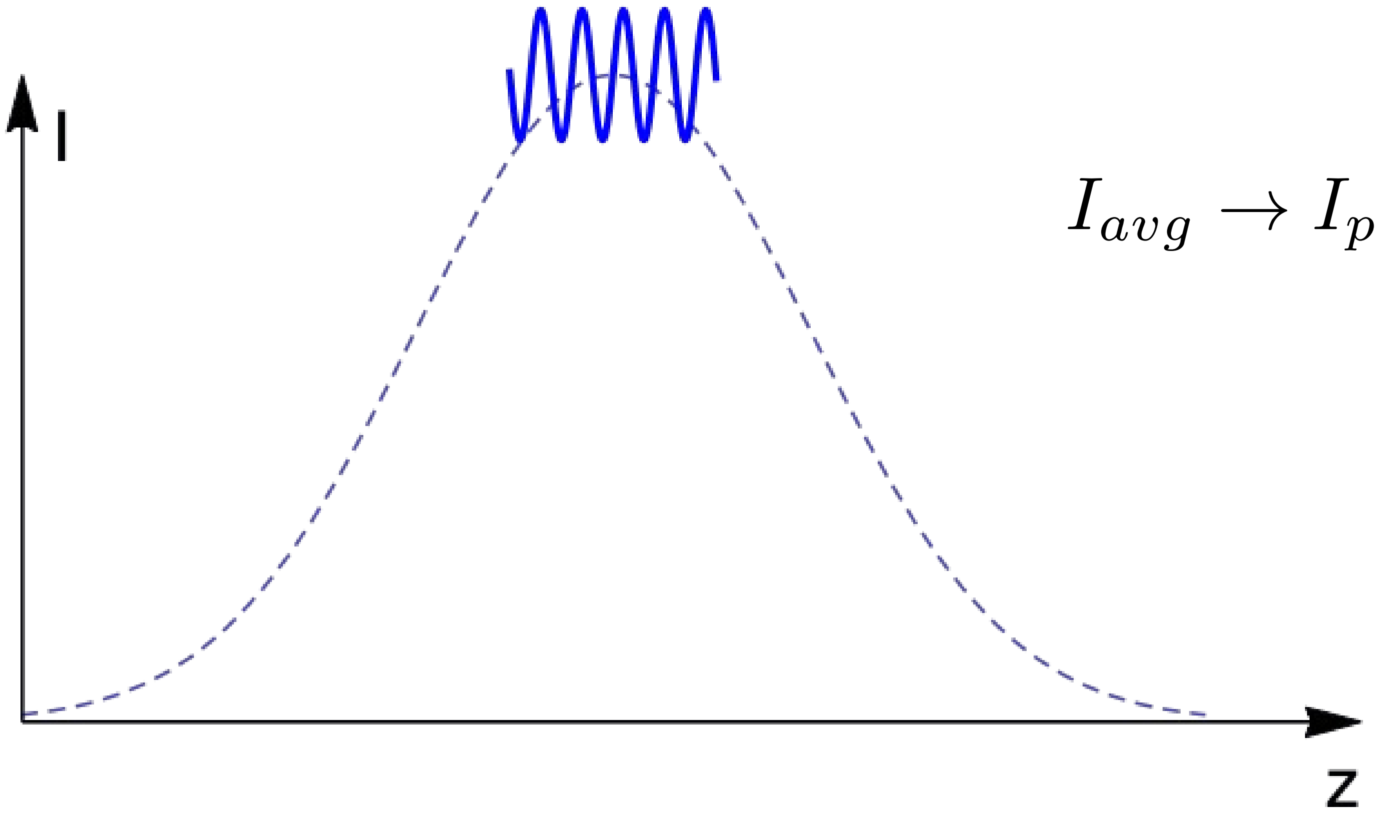
$$\Delta\omega \approx \omega_0 |\eta| \Delta\delta$$

Momentum spread

Microwave instability in bunched beams

- The KS criterion also provides a good indication of the requirement to stabilise the **microwave instability** in bunched beams

  → Keil-Schnell-Boussard criterion



$$I_{avg} \rightarrow I_{peak}$$

Transverse unbunched beams

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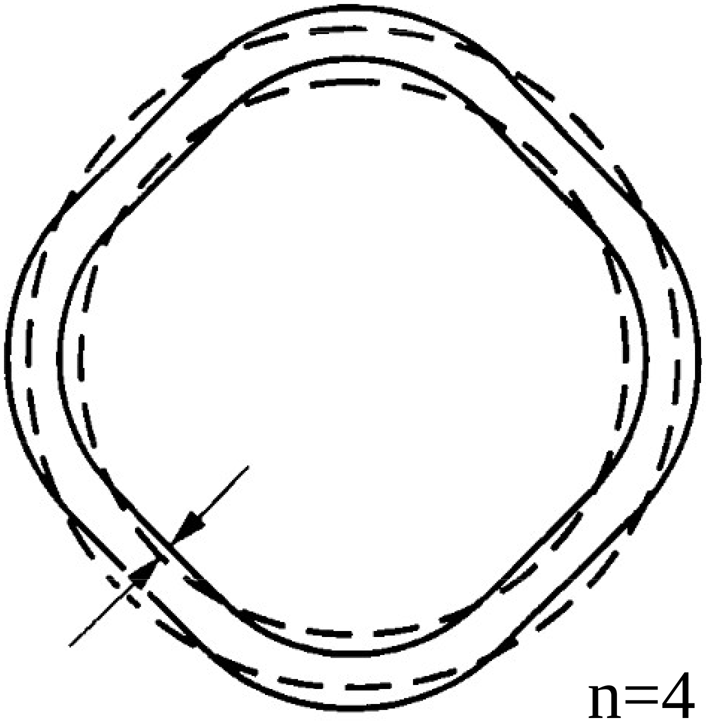
- Transverse oscillations of unbunched beams can also be driven unstable by the interaction with an impedance



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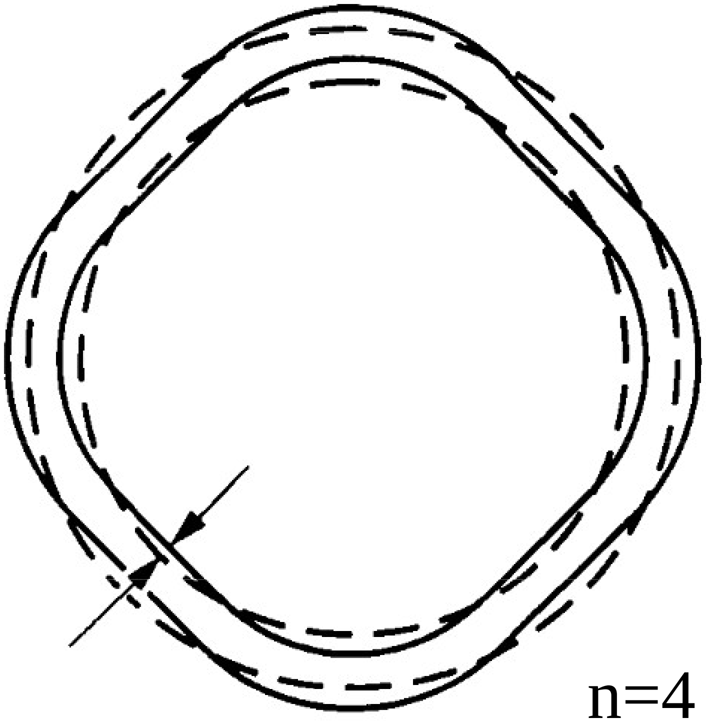

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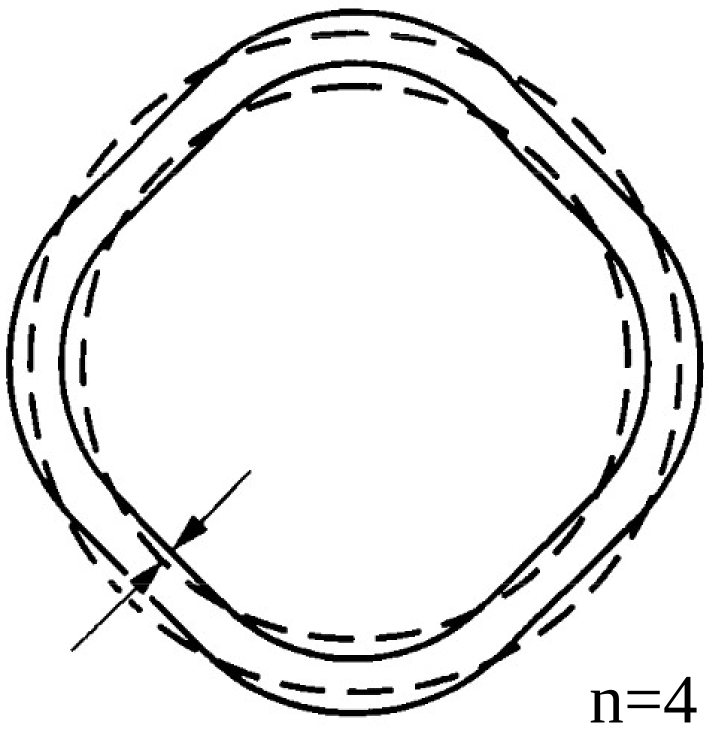

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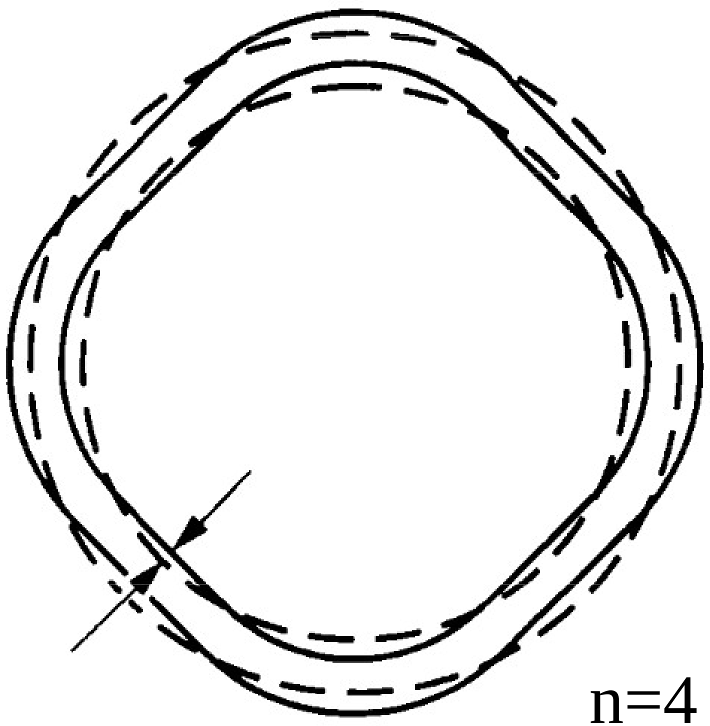

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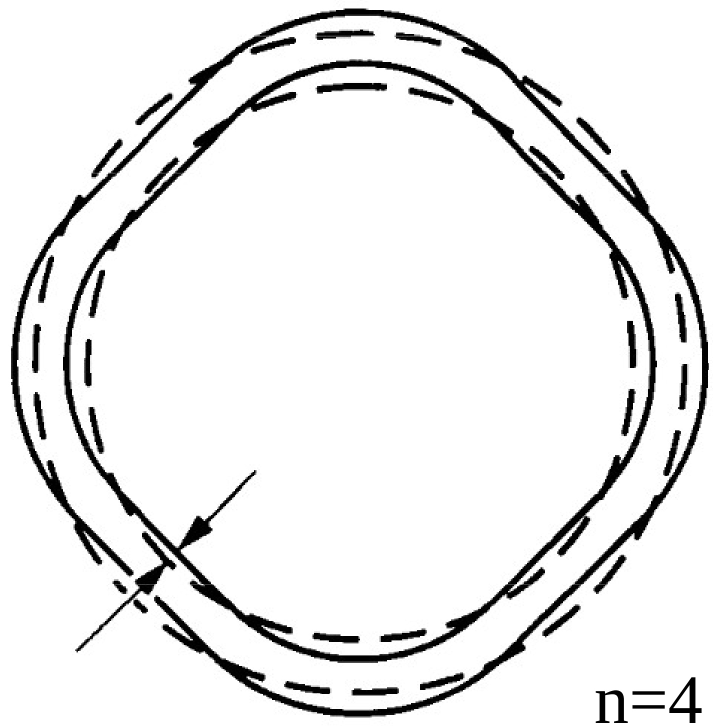

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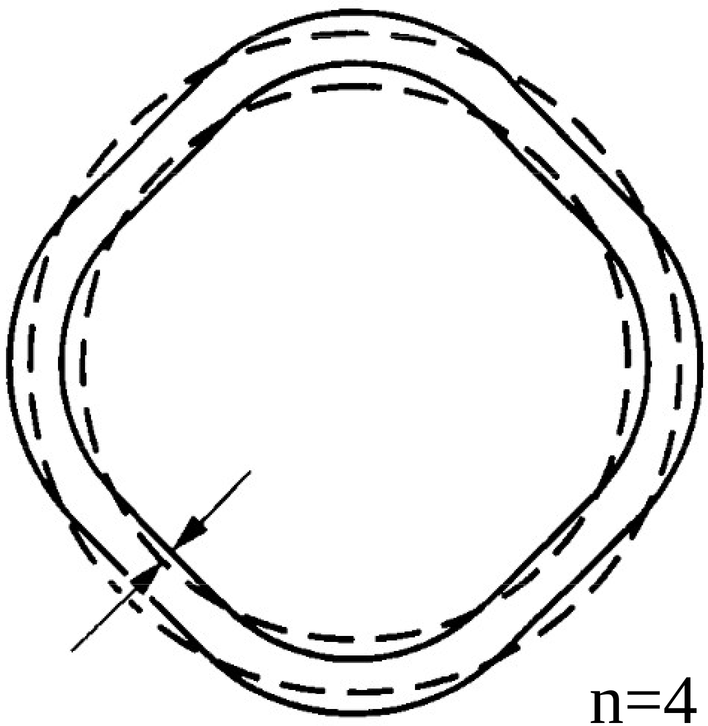

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- Simplified criterion: $|\Delta\Omega_n| \lessapprox \Delta\omega$

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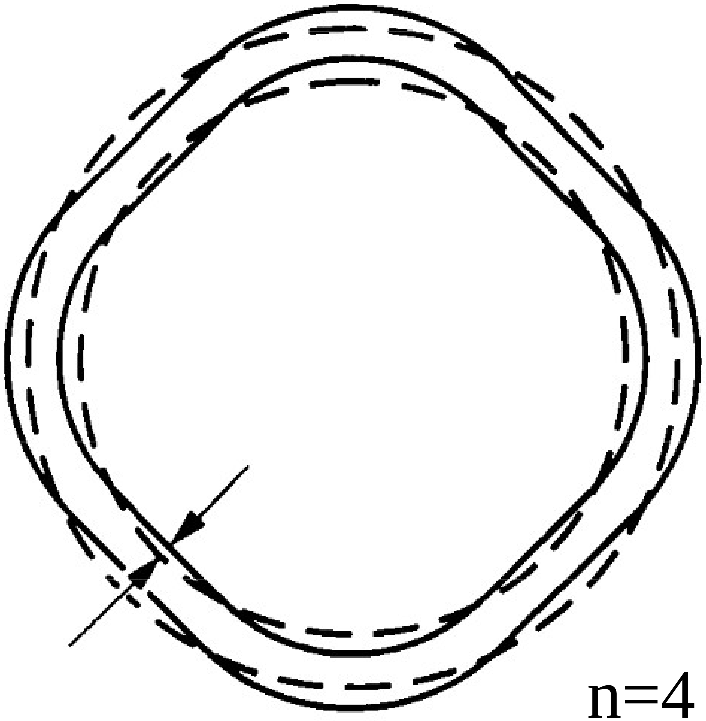

- Sources of transverse frequency spread:
  - Revolution frequency
  - Chromaticity (Q')

$$\Delta\omega = \omega_0|Q' - n\eta|\Delta\delta$$

Bunched beams

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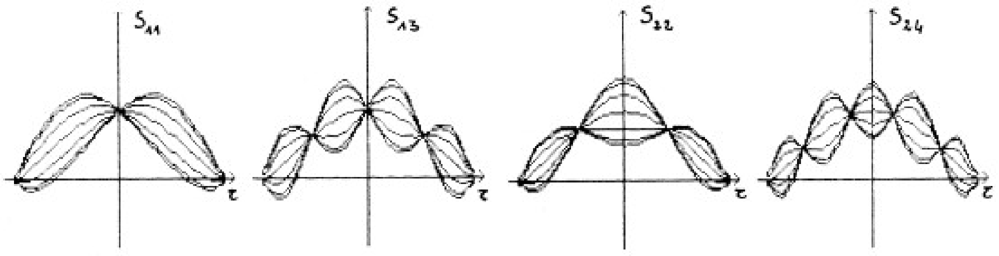

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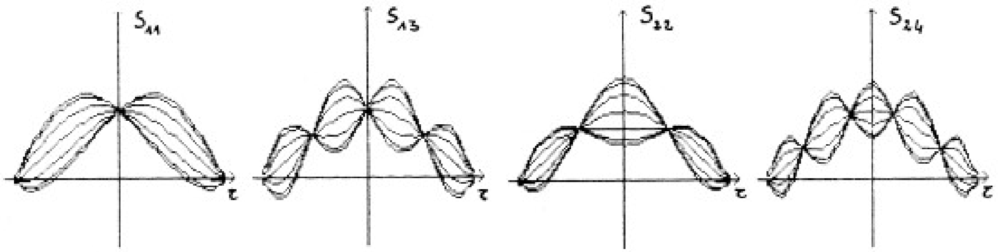

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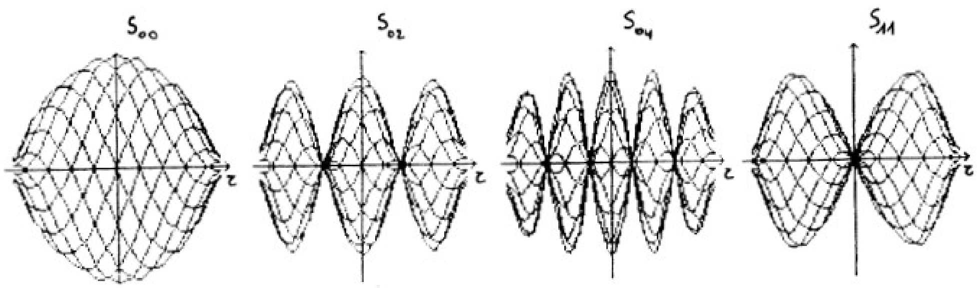

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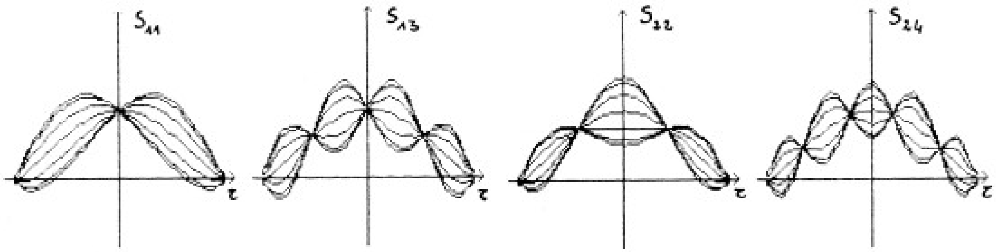
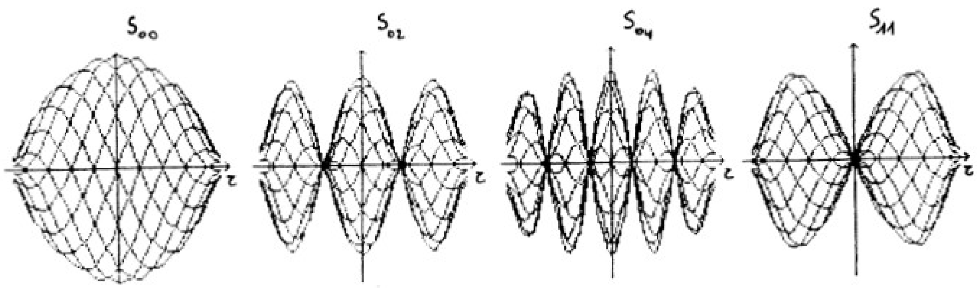


Mode frequency shift (e.g. from Sacharer formula) → $$|\Delta\Omega_n| \lessapprox \Delta\omega$$



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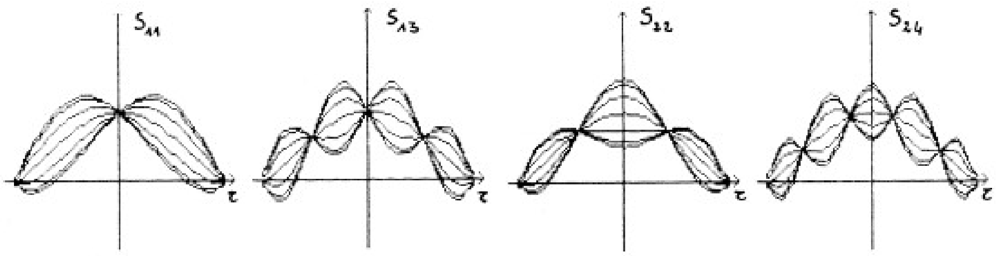

- Sources of longitudinal frequency spread:
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  - Non-linear forces

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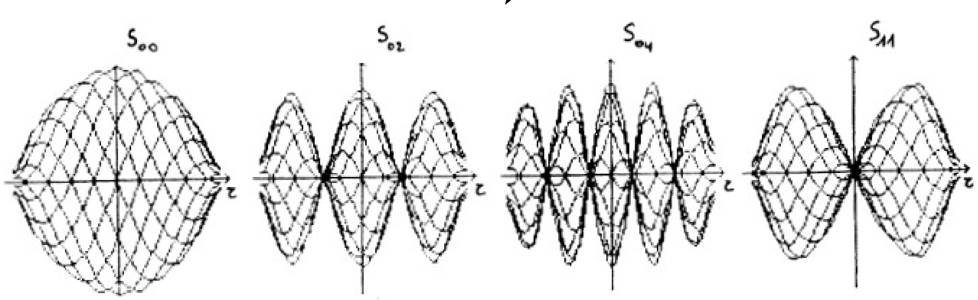

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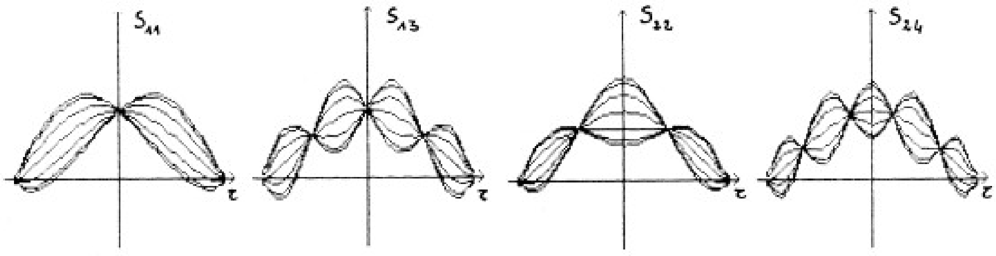

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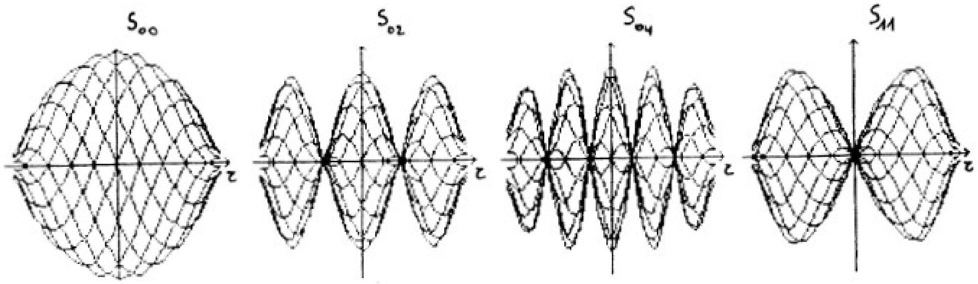
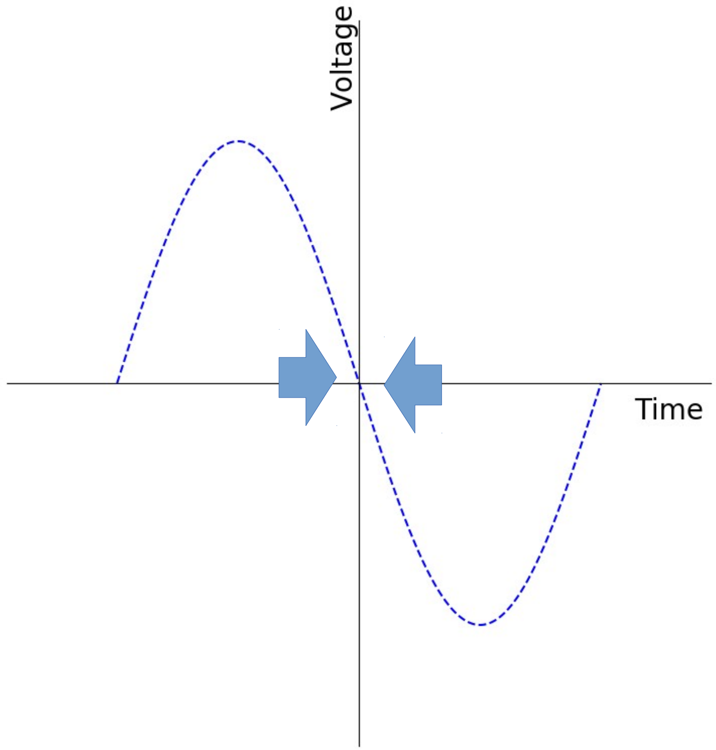


- Sources of transverse frequency spread:
  - Revolution frequency
  - Chromaticity
  - Non-linear forces

Longitudinal stability of bunched beams

- For bunched beams, the longitudinal focusing provokes oscillations around the fixed point with ω_s

  → As RF cavities function with sine wave, the focusing force is non-linear



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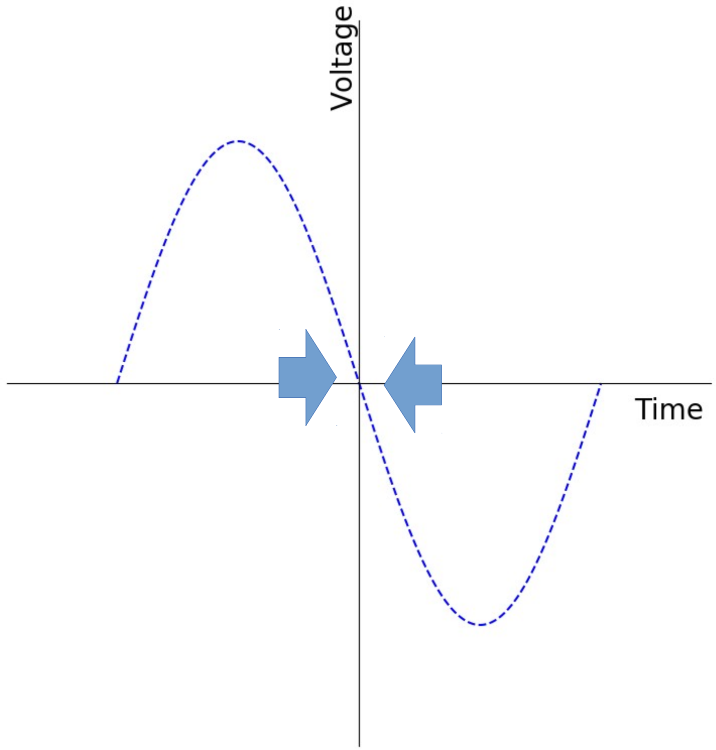
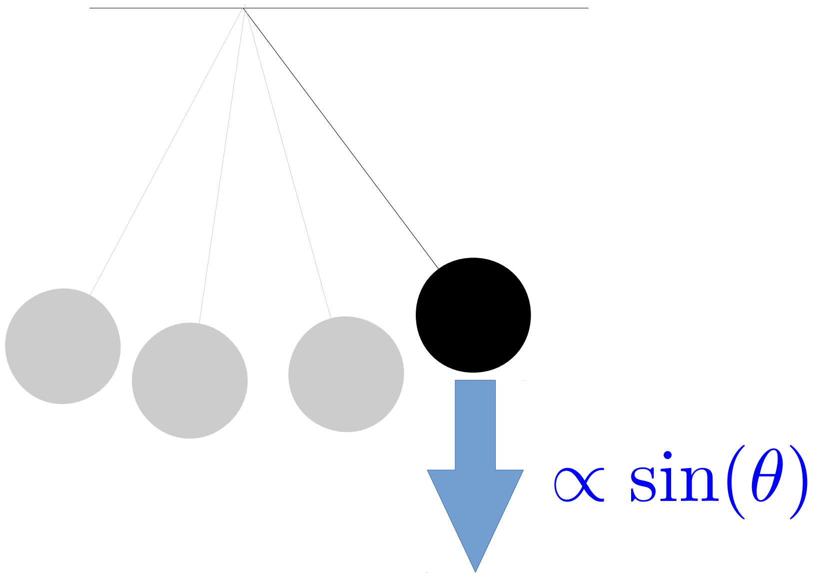
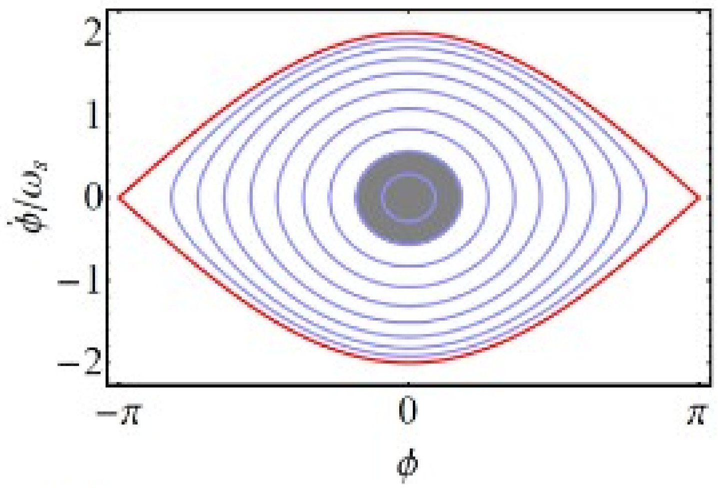




- The behaviour is identical to the pendulum **without** the small angle approximations

Longitudinal stability of bunched beams

[Damerau]



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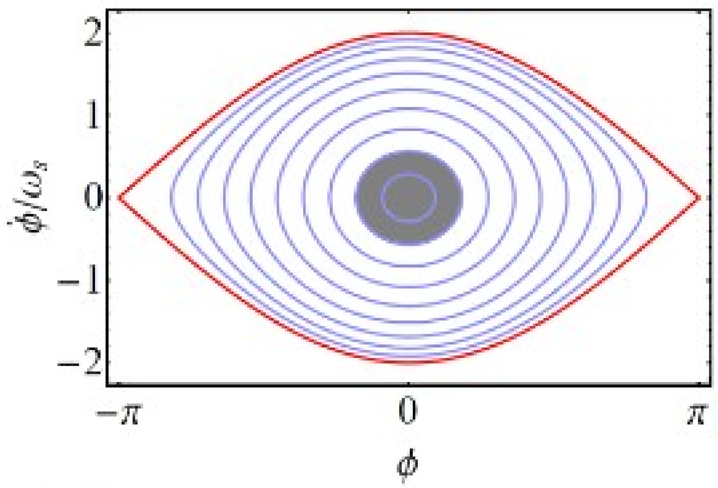
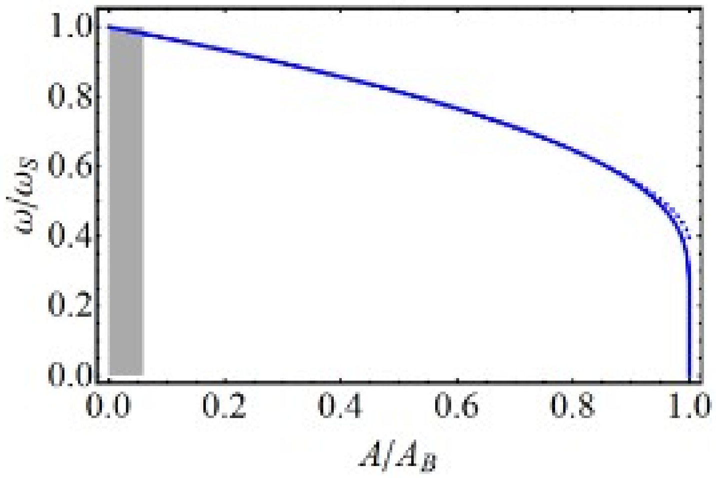
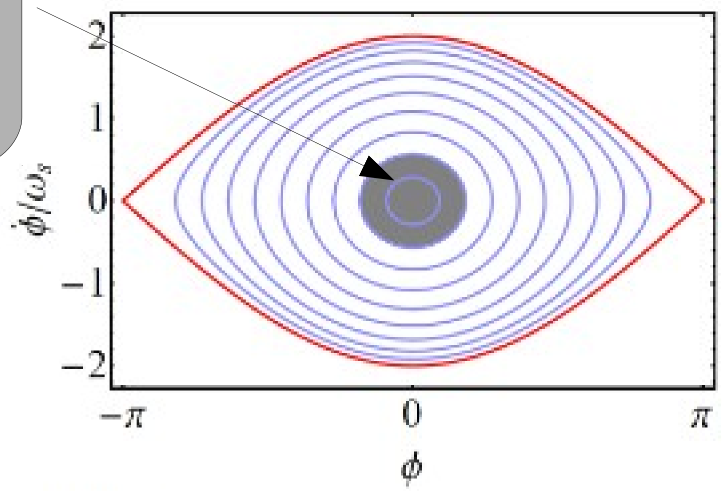


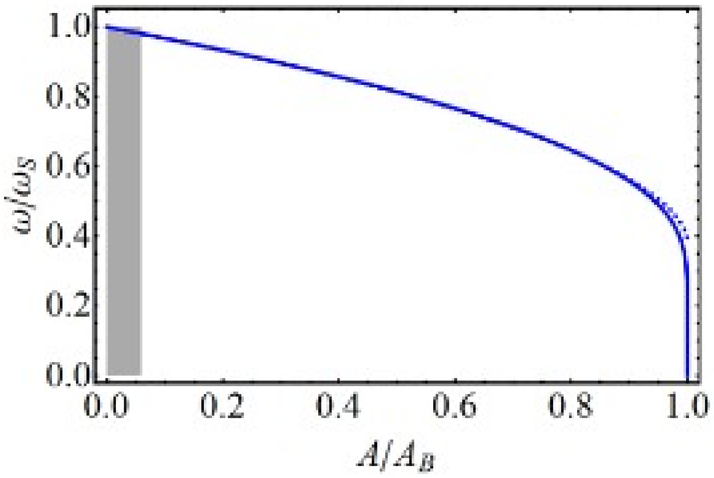


Longitudinal stability of bunched beams

[Damerau]

Small beam wrt to available bucket





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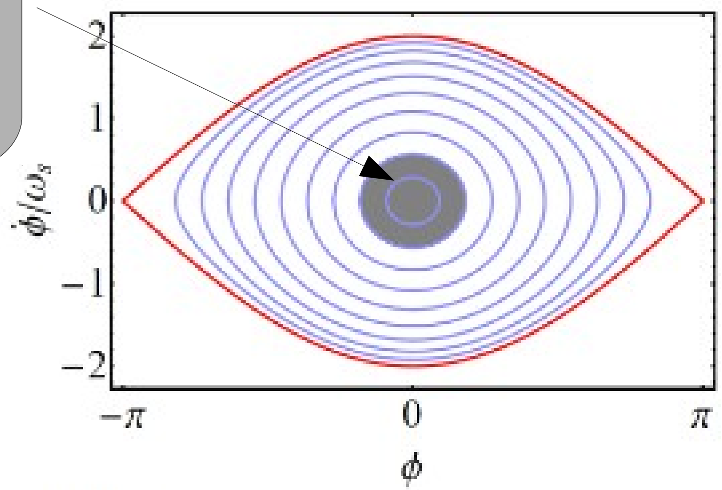
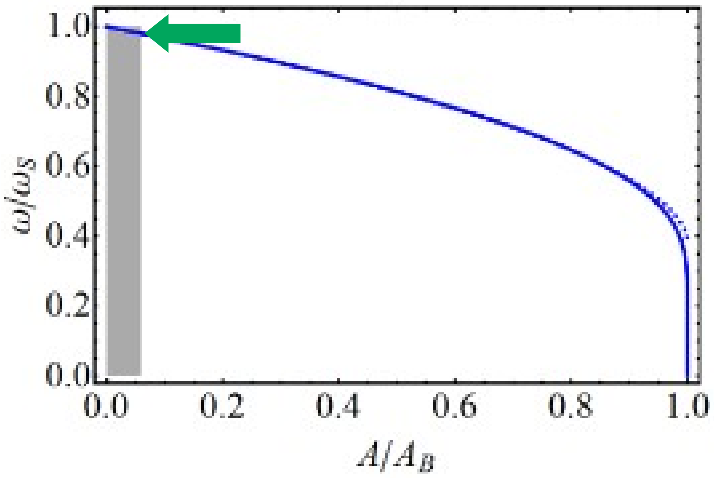
Small frequency spread





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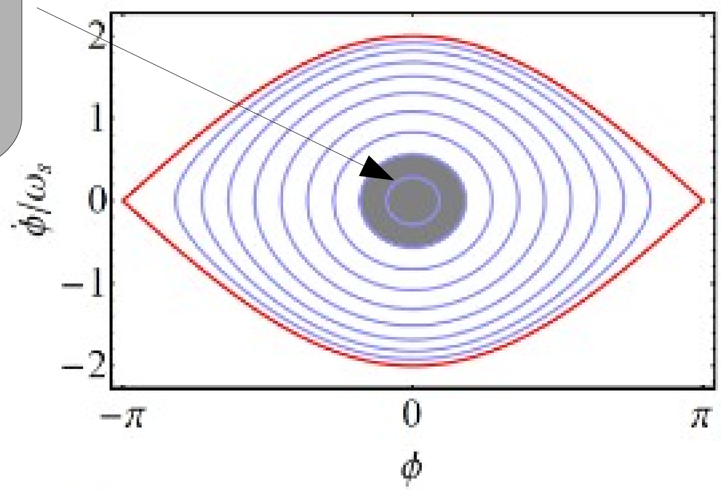
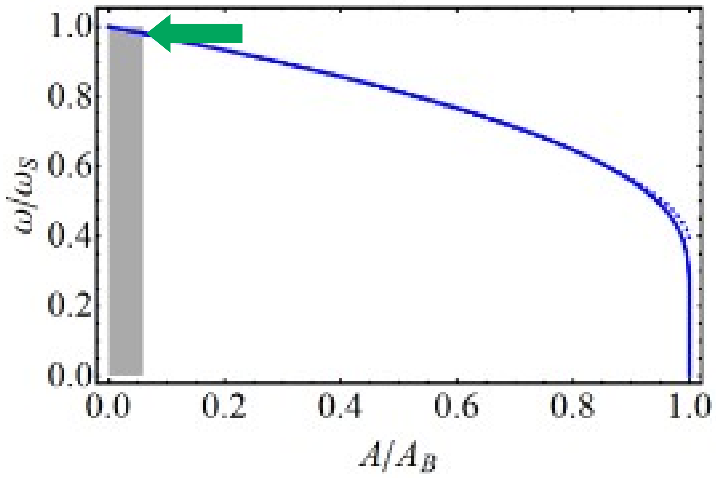


Small frequency spread



Weak Landau damping

Longitudinal stability of bunched beams

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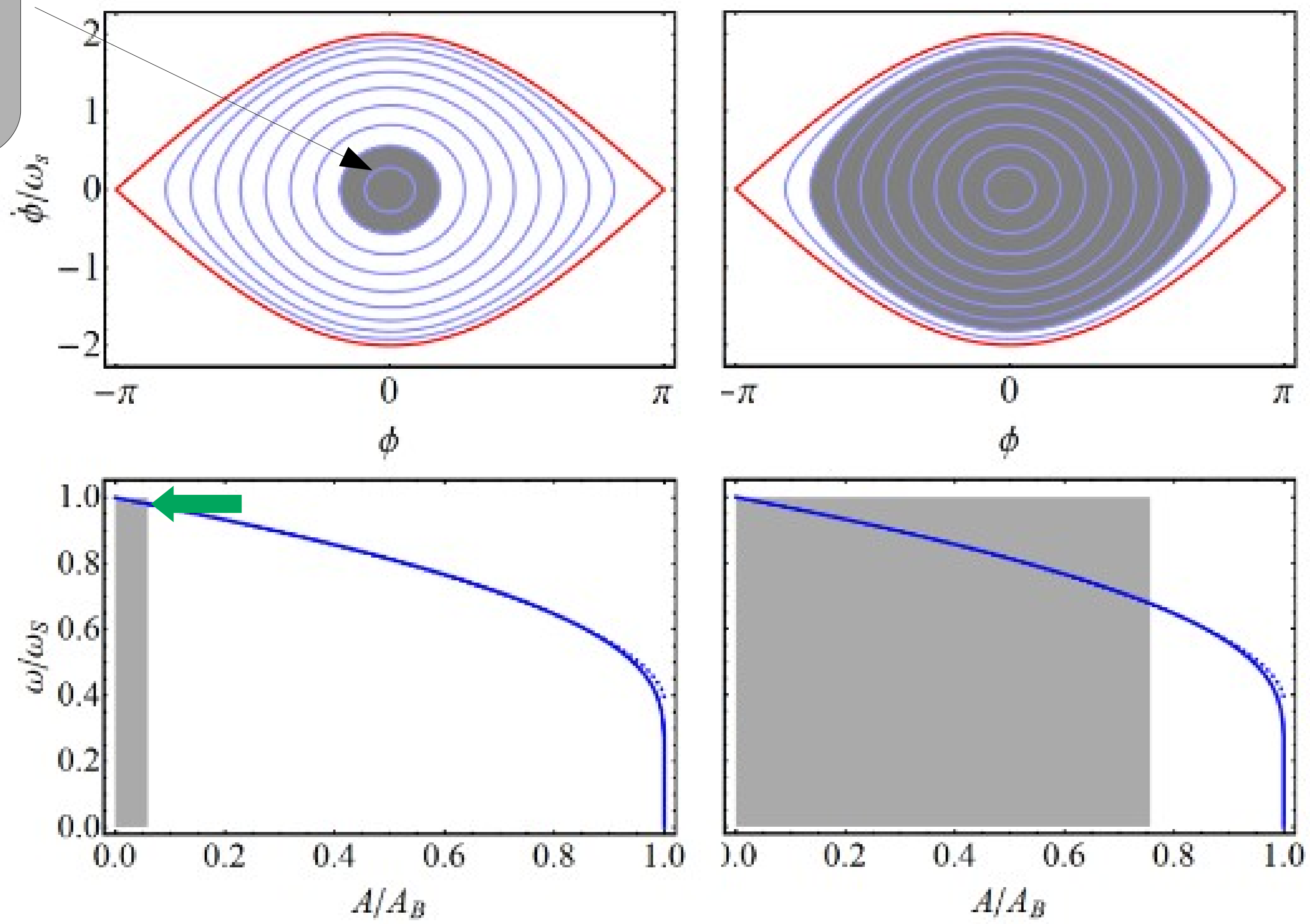
Small frequency spread

Weak Landau damping

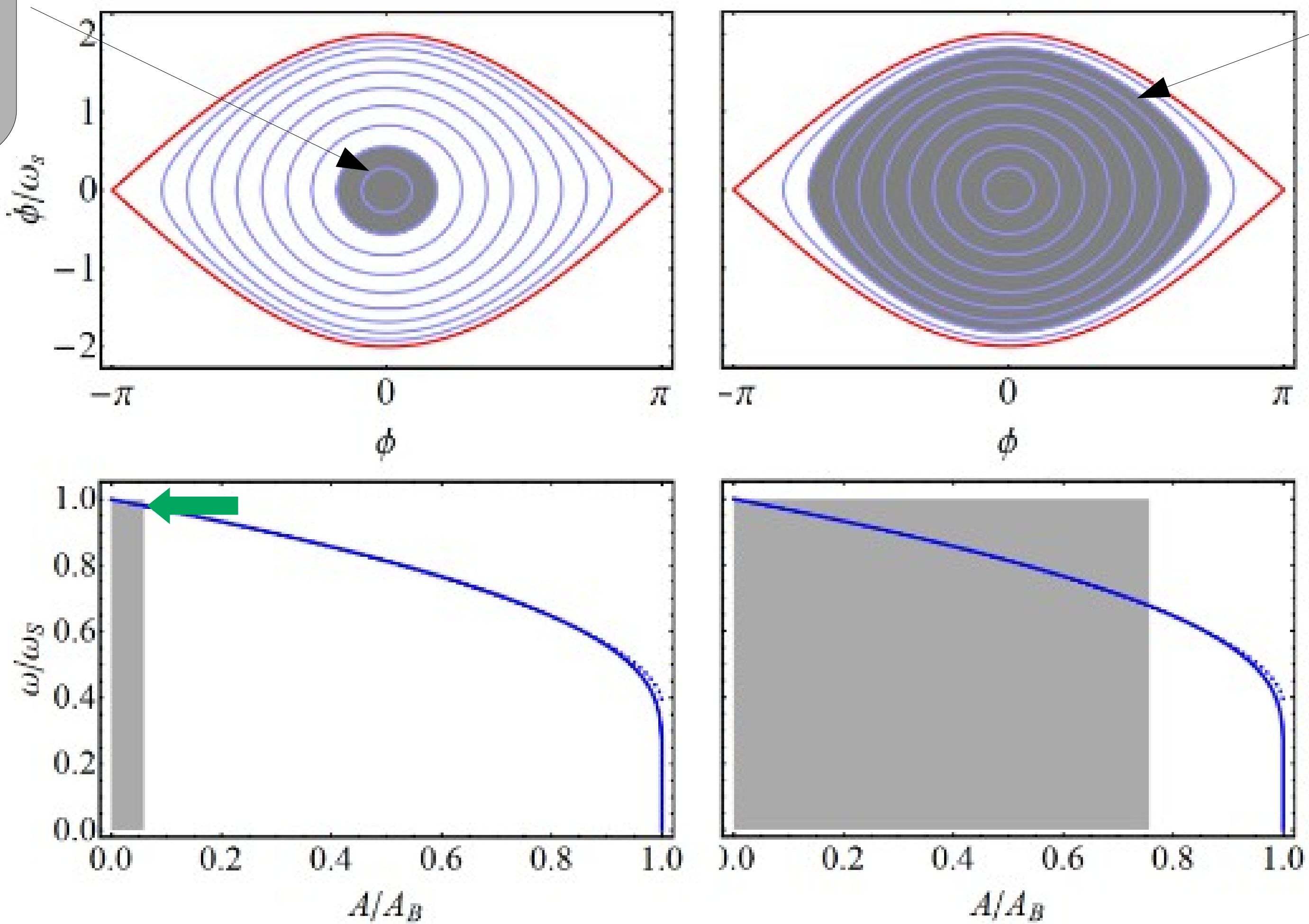


Longitudinal stability of bunched beams

[Damerau]

Small beam wrt to available bucket

Bucket is *filled*



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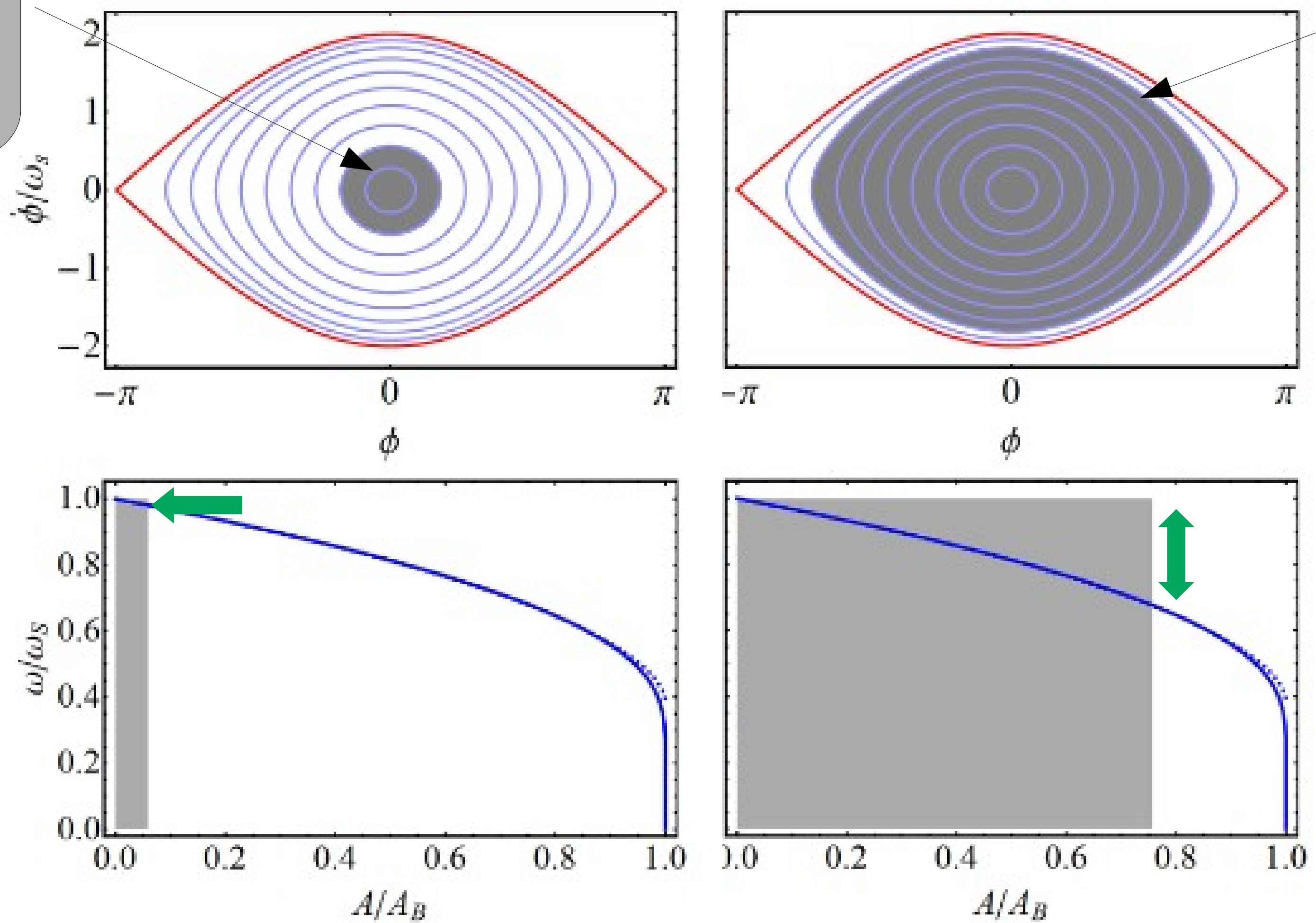

Bucket is *filled*

large frequency spread

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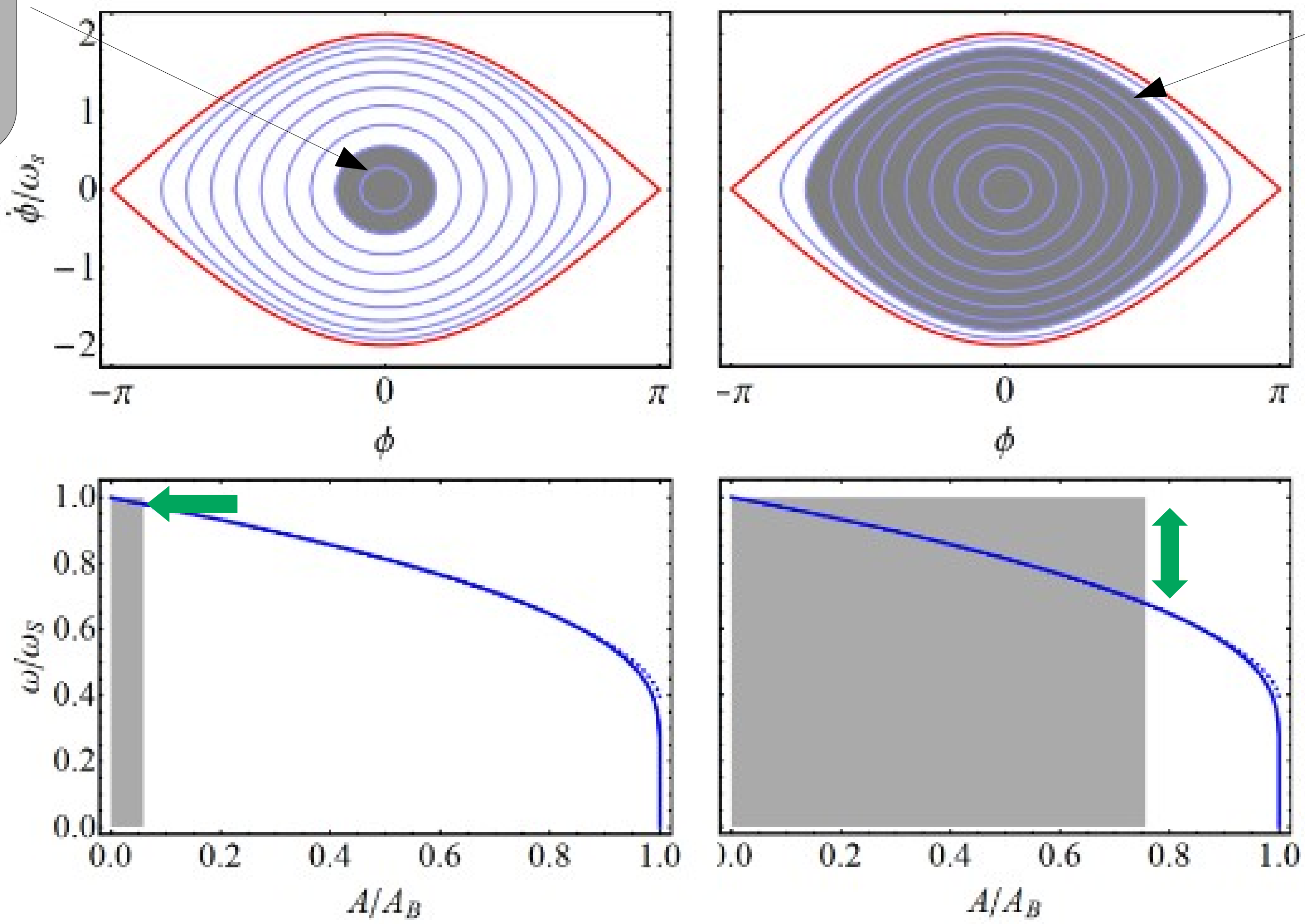

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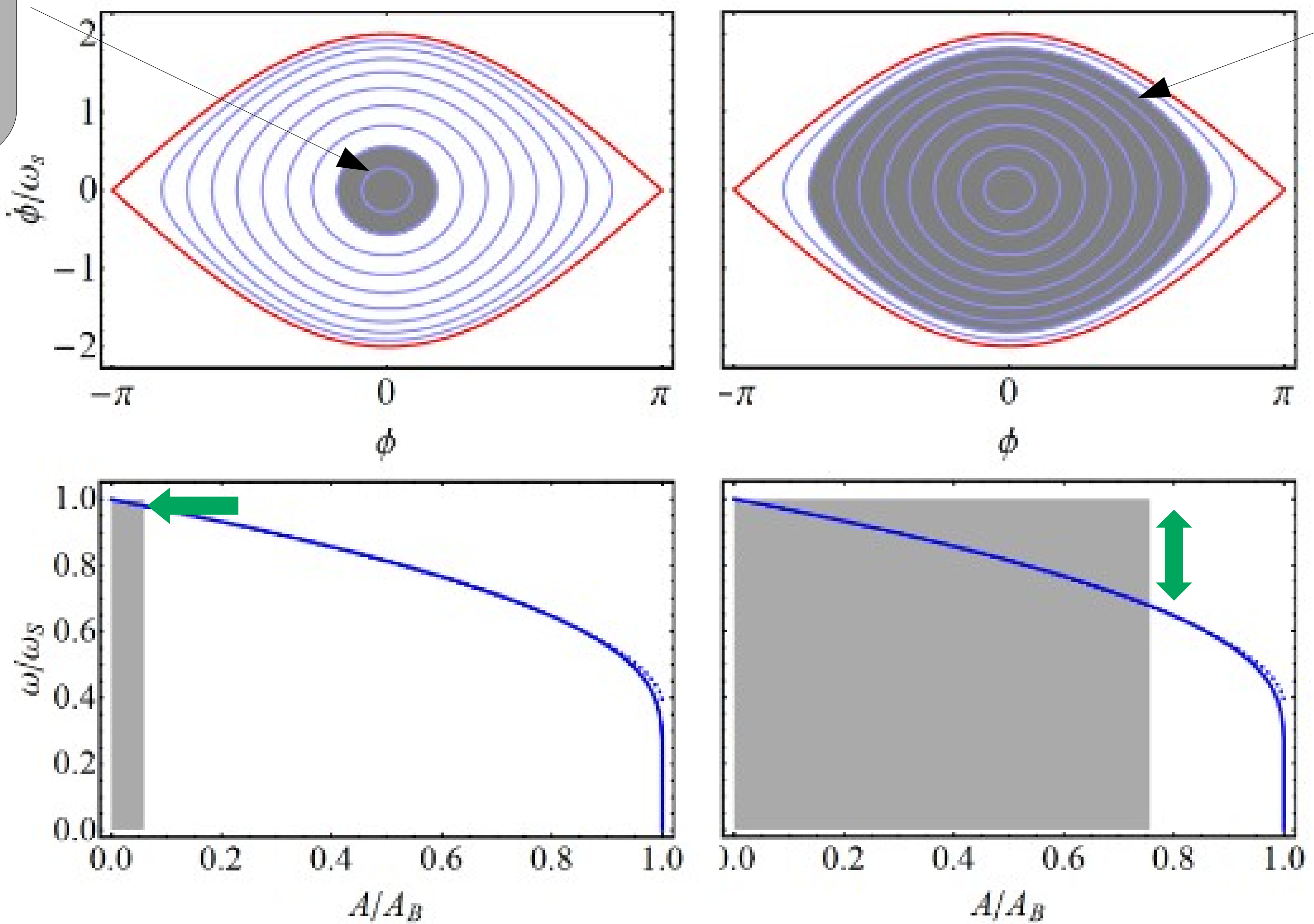


Small frequency spread

large frequency spread

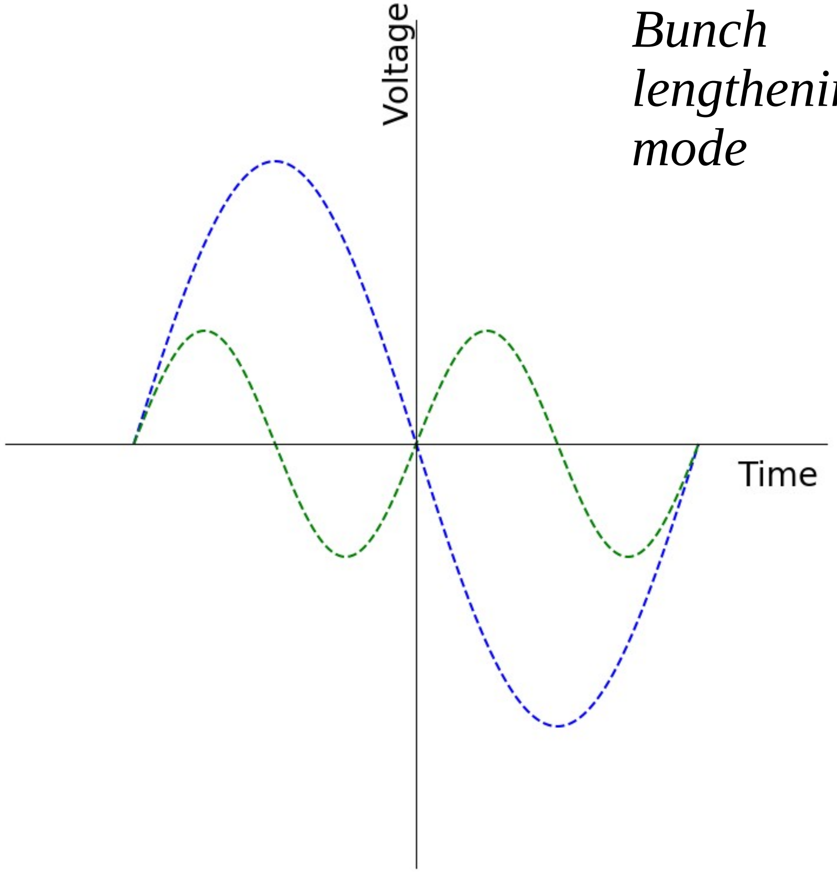
Weak Landau damping

Strong Landau damping

- **Filling the available bucket** is key to maintain Laudau damping in the longitudinal plane

Double harmonic RF

- With a second harmonic RF (featuring a lower voltage) the total voltage becomes more non-linear



Bunch lengthening mode

Double harmonic RF

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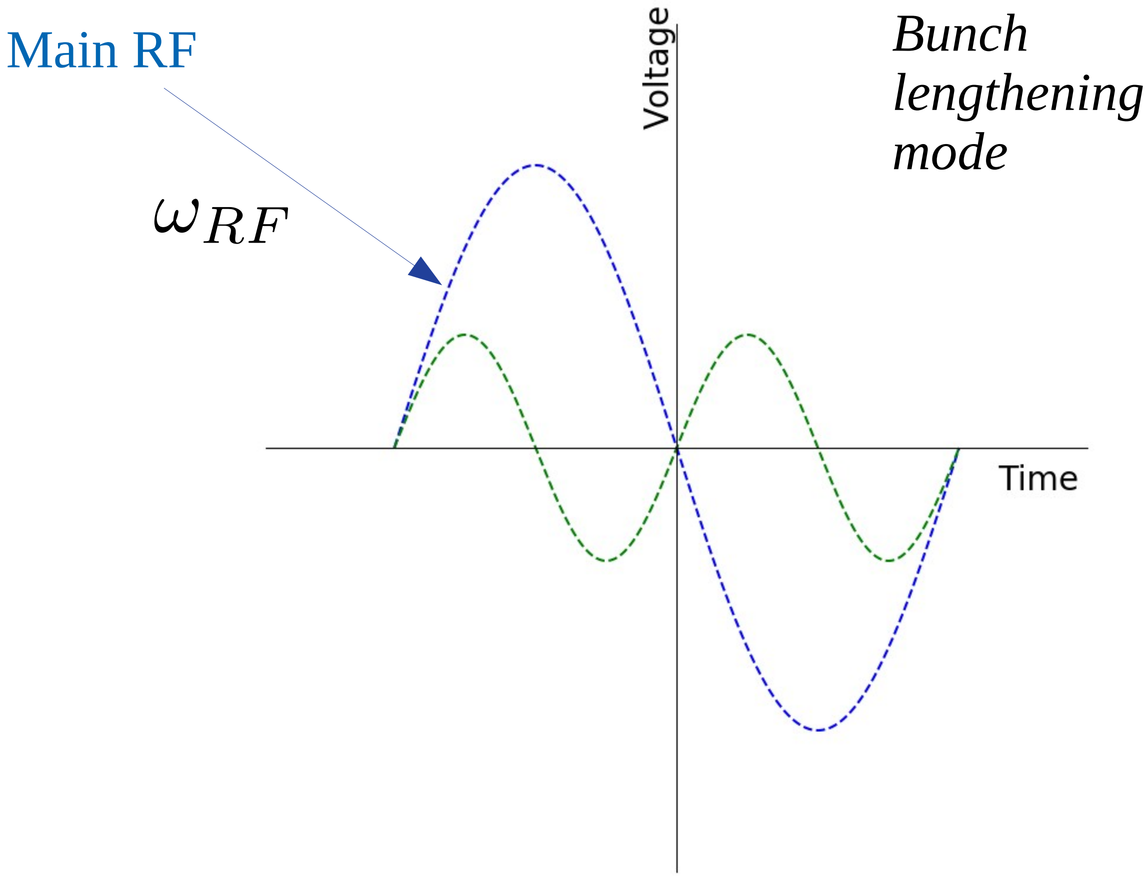

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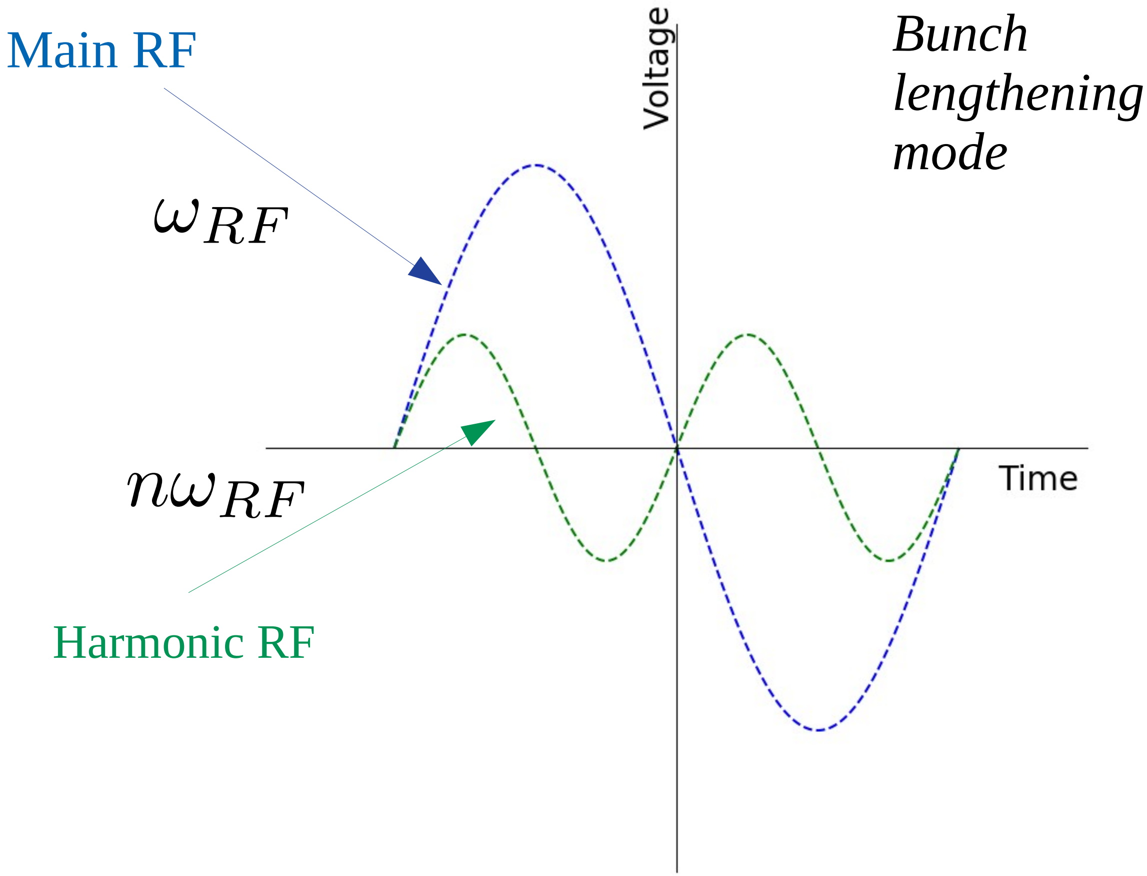

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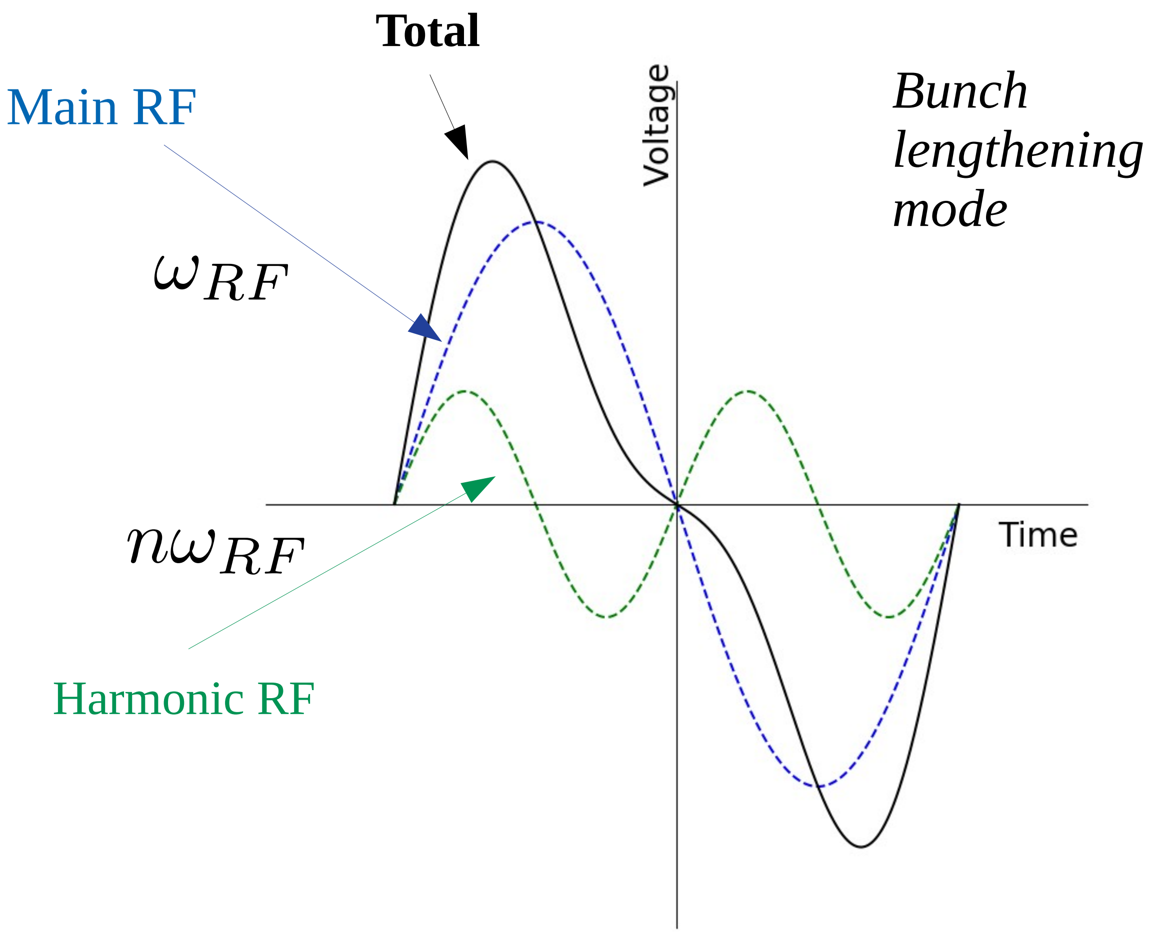

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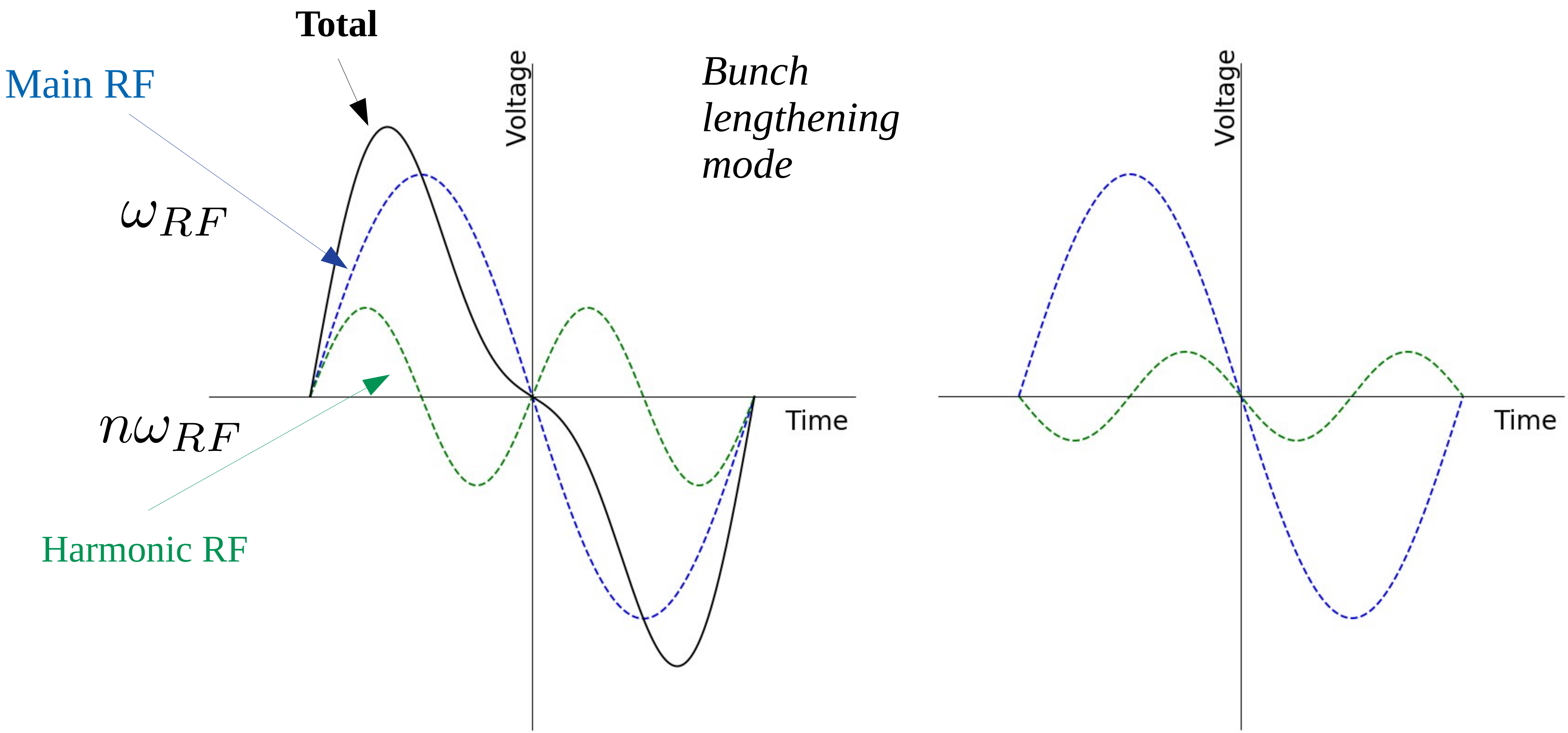

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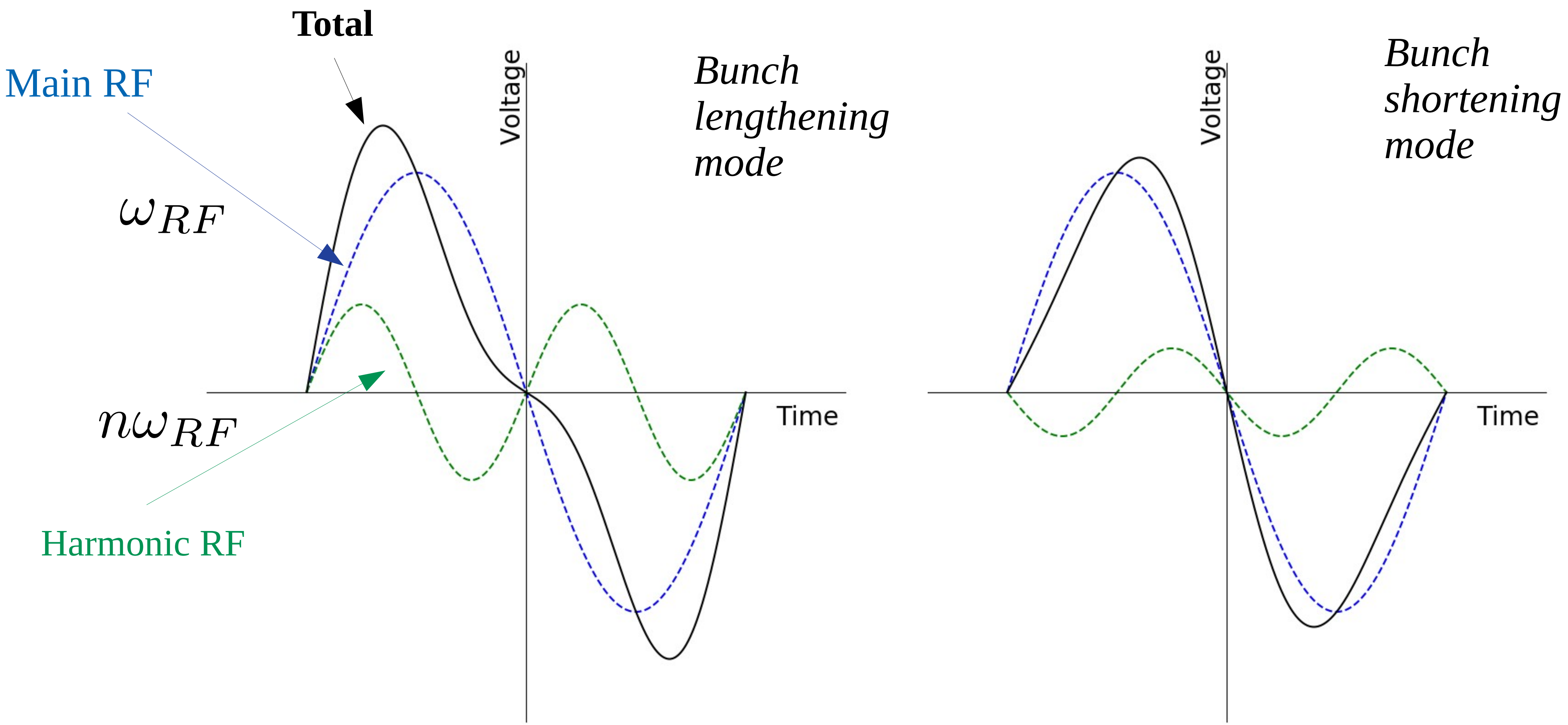

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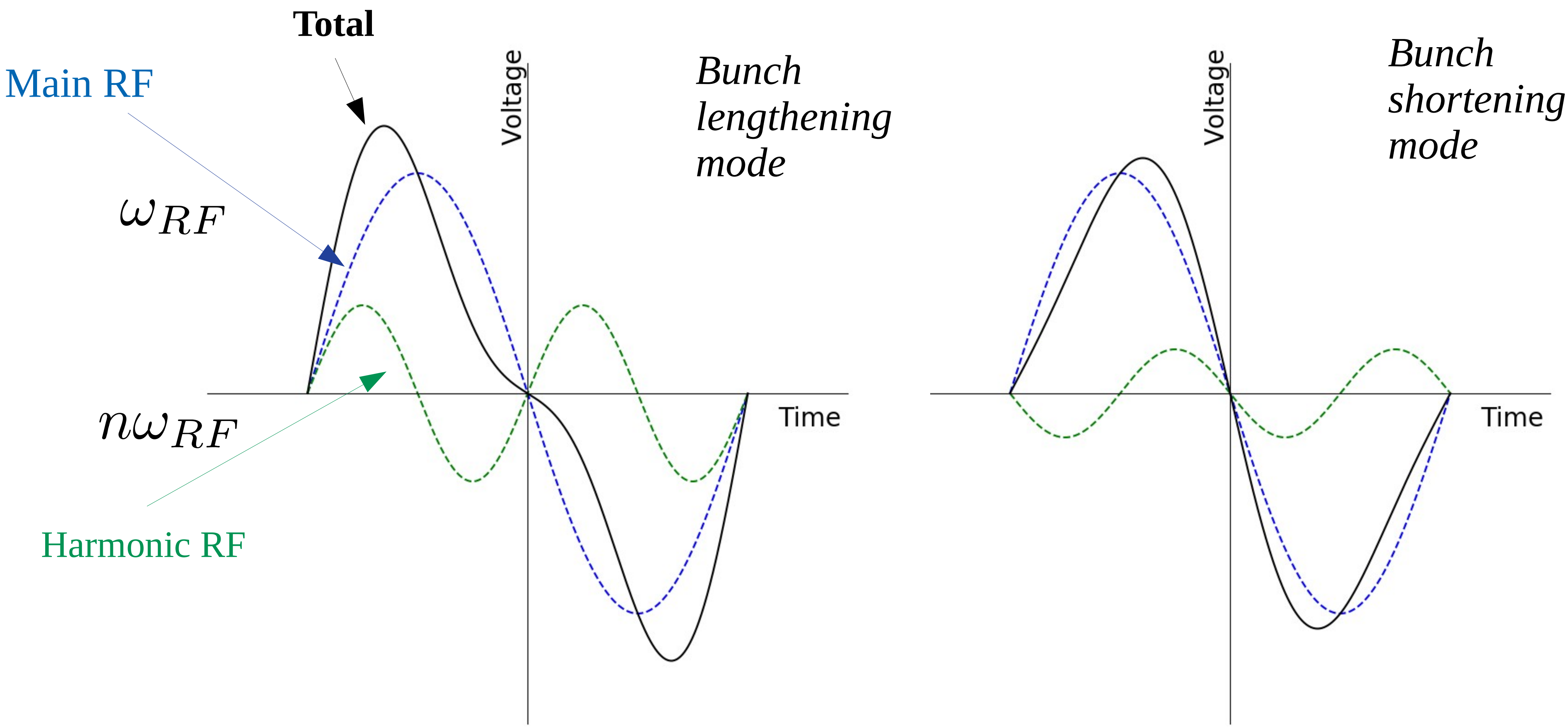


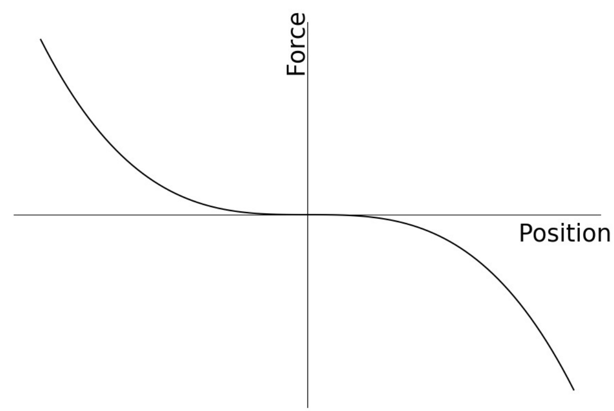
- The tune spread can be enhanced (or reduced) depending on the relative phase and voltage of the two RF systems

  → Improve / deteriorate Landau damping

Transverse stability

- In high energy machines, the frequency spread linked to revolution frequency and the chromaticity is usually small
- Chromatic sextupole magnets are non-linear, yet to first order they don’t contribute to the transverse tune spread

→ Dedicated octupole magnets (aka Landau octupoles)



$$\omega(J) = 2\pi(Q_0 + aJ)$$

Practical aspects

Optics

Slippage factor

chromaticity correction

Practical aspects

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chromaticity correction

RF cavities

Frequency, voltage,
harmonic systems

Practical aspects

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Magnets
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Impedance
Beam pipe dimensions,
Beam equipment designs
(e.g. instrumentation, collimator,
Vacuum valves),
Material choices, transistions

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Operation
Adiabatic damping during energy ramp
(RF voltage functions, longitudinal blowup)

Fuego’s theoretical catch

$$\frac{-1}{\Delta\Omega_n} = \int dJ \frac{J\frac{df_0}{dJ}}{\Omega - \omega(J)}$$

?

$$\frac{-1}{\Delta\Omega_n} = \int d\omega \frac{\rho(\omega)}{\Omega - \omega}$$

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$\approx$

- In plasmas, only the density of velocity matters

  → A treatment based on the frequency distribution remains a **reasonable approximation** for many applications in accelerator

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- In plasmas, only the density of velocity matters

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- When the frequency spread arise from non-linear forces, the treatment is slighty different

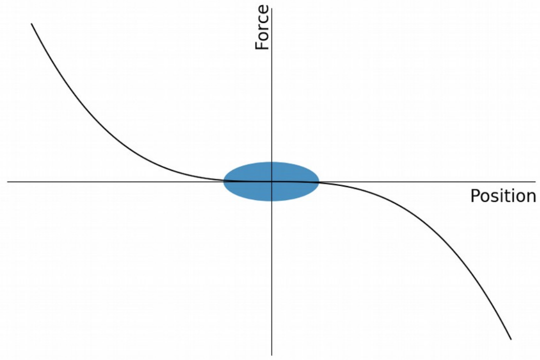
$$\omega(J) = \frac{\partial H}{\partial J}$$

Non-linear collective forces

- Some collective forces are non-linear, they have an impact on Landau damping
  - Due to their dynamic nature, they lead to different behaviours

    → Different dispersion relations

External forces



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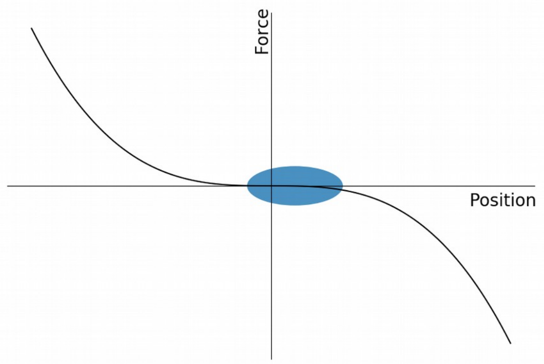

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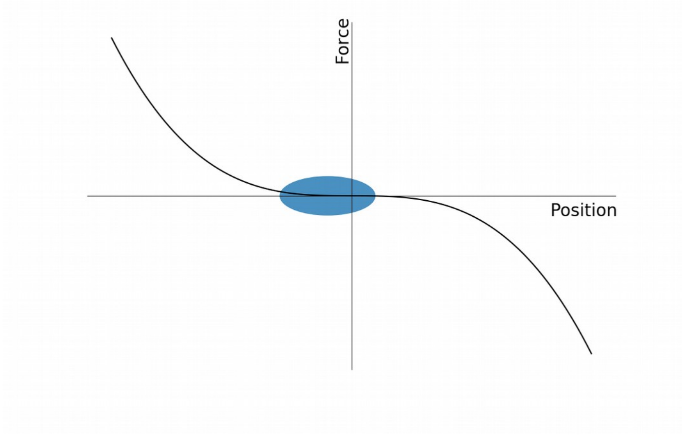

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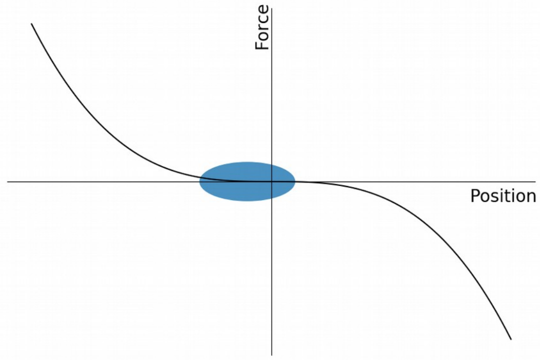
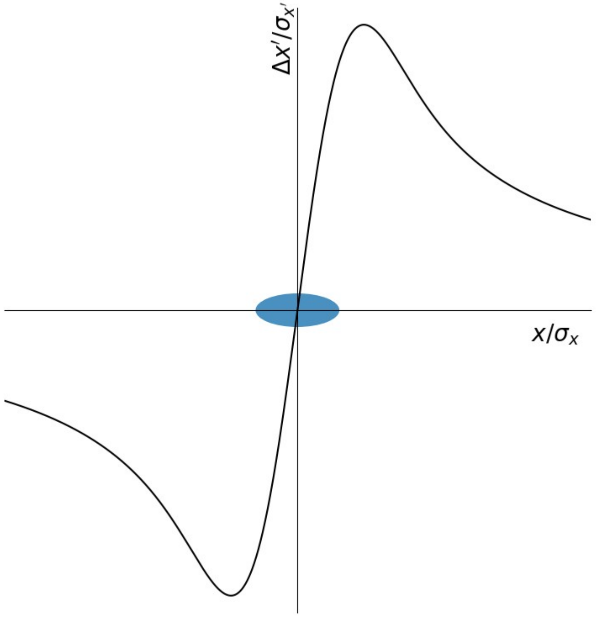


Space-charge



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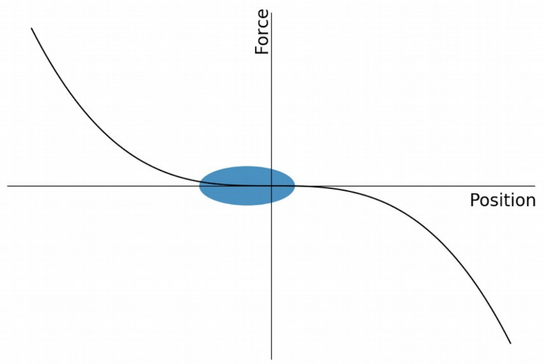

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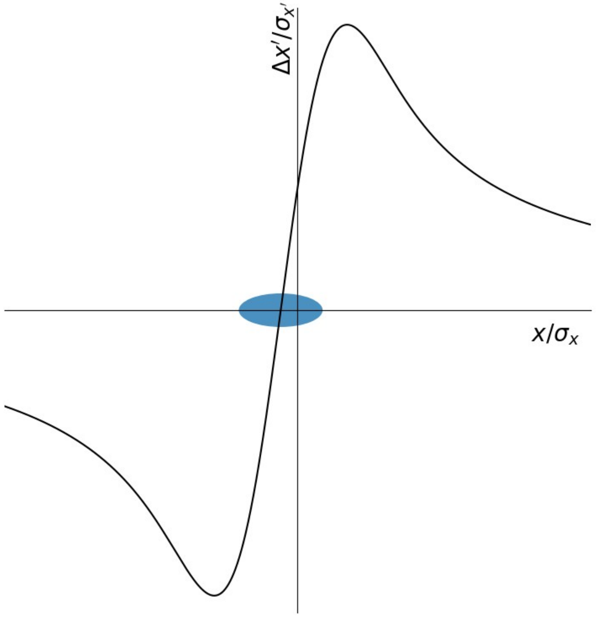

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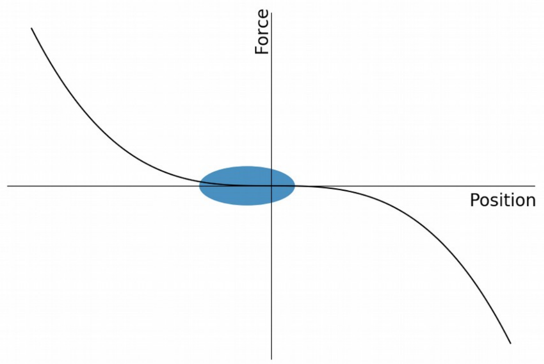

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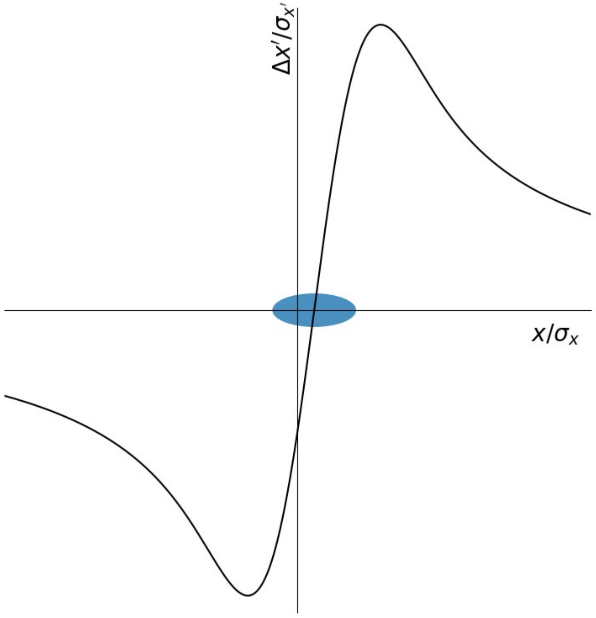

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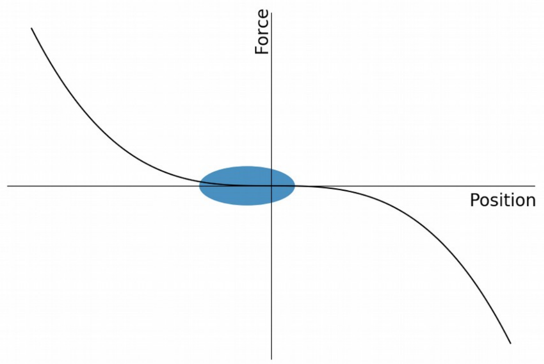

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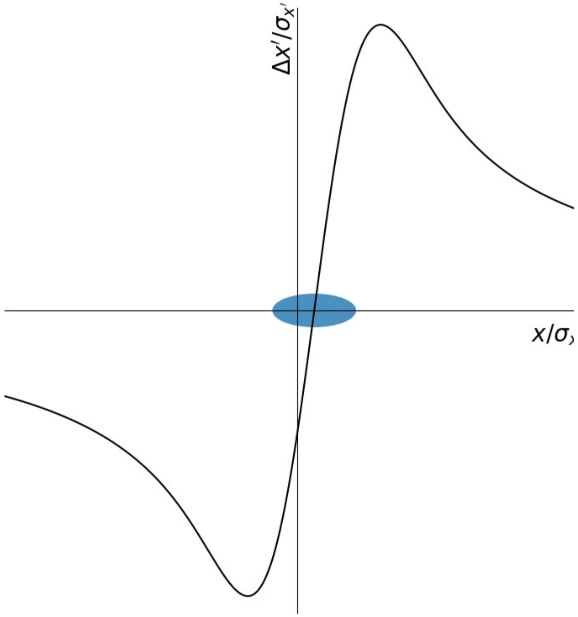
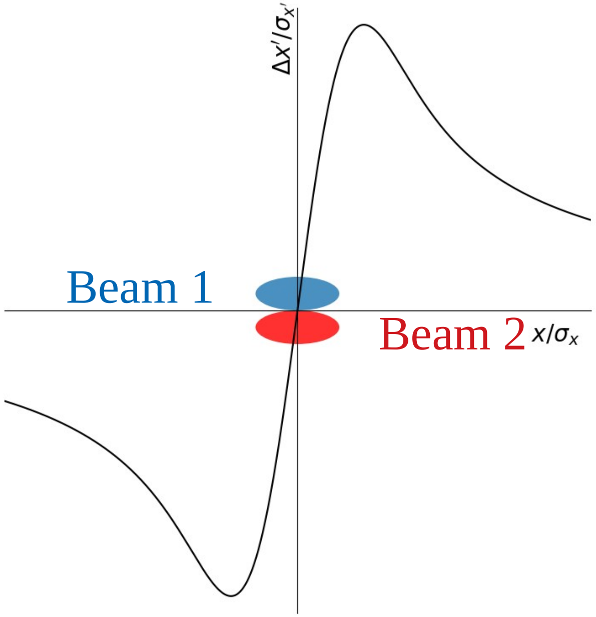


Beam-beam
σ-mode



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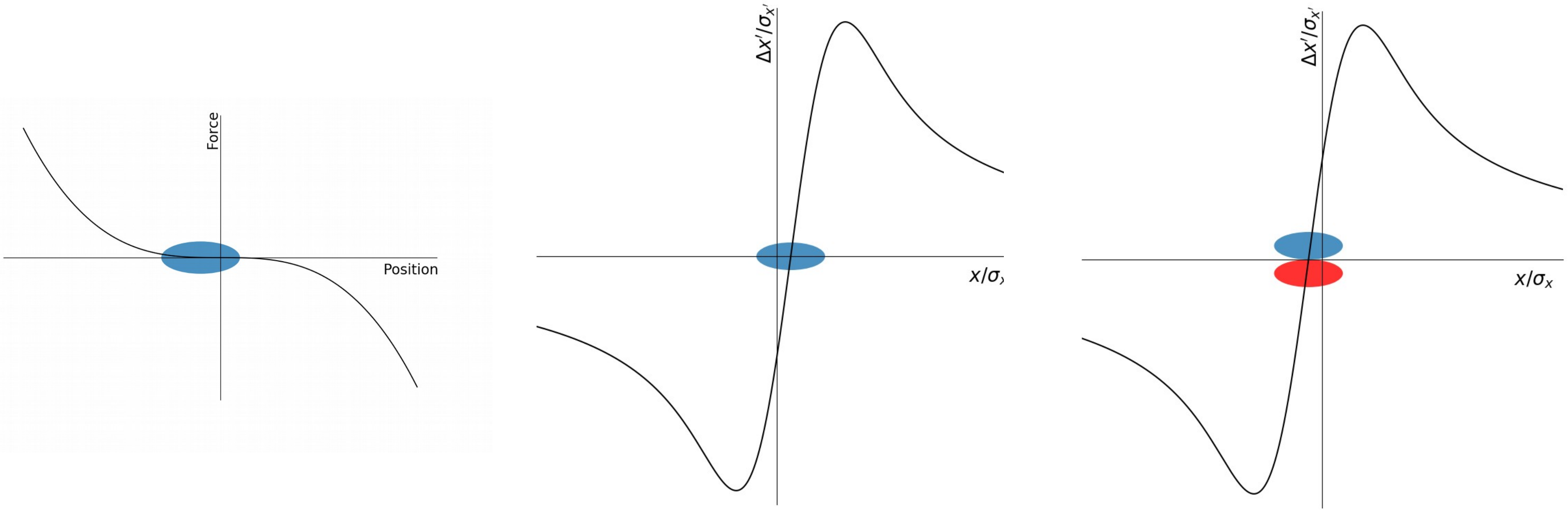

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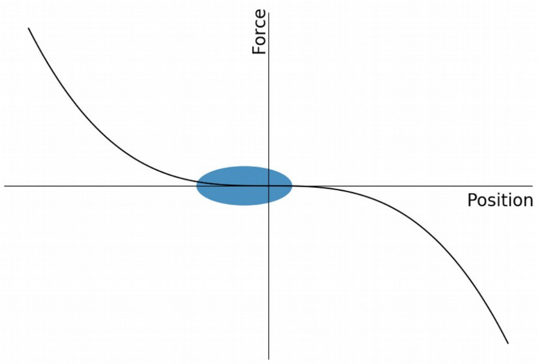

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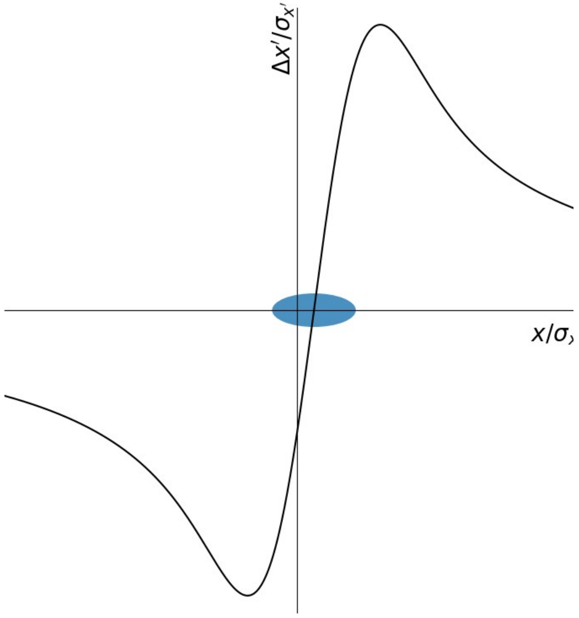

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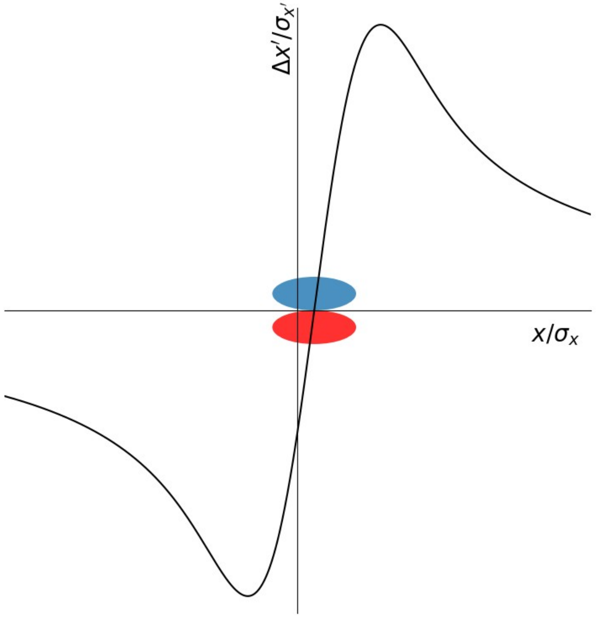

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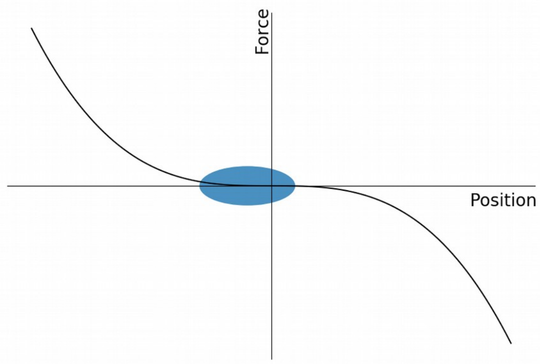

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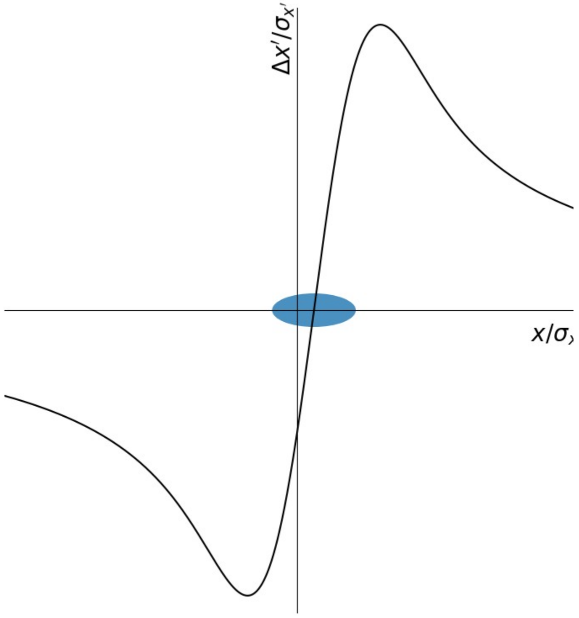
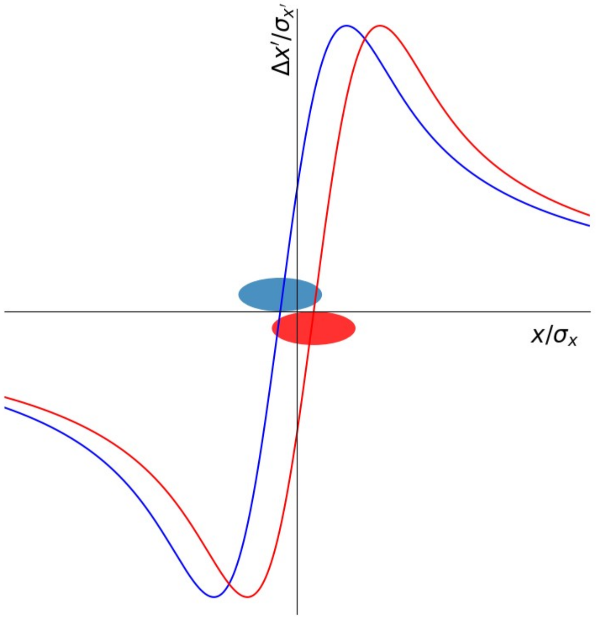


Beam-beam
π-mode



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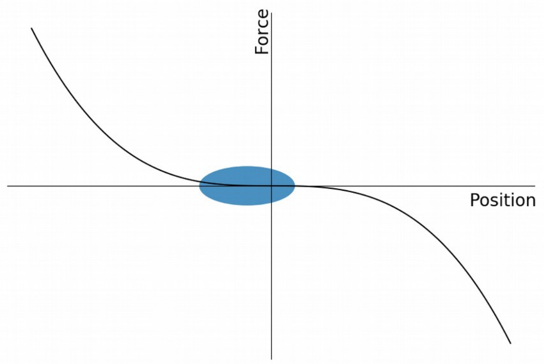

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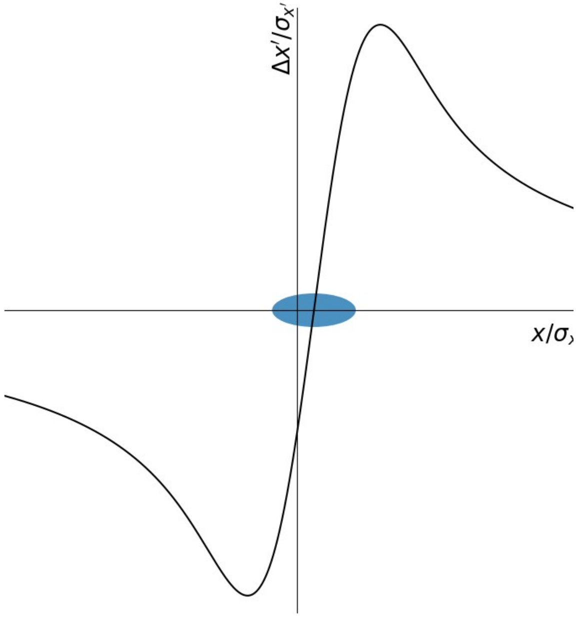

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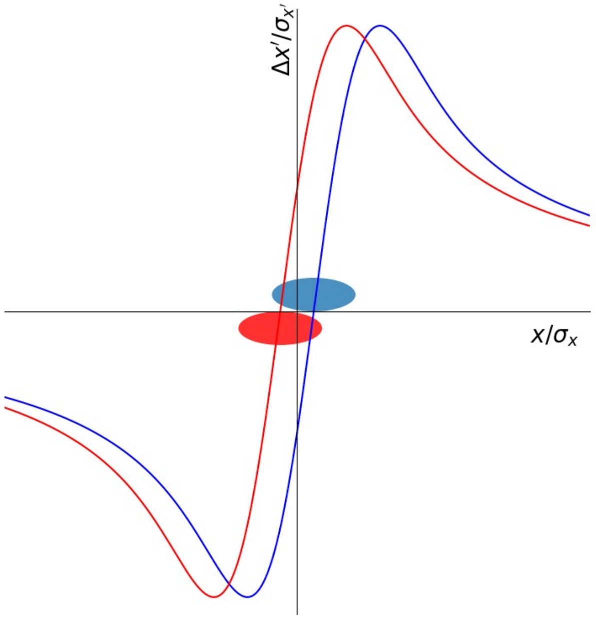

Stability of the rigid bunch mode with space-charge [Metral, Kornilov]

$$\int dJ_x dJ_y \frac{J_x \frac{\partial f_0}{\partial J_x} \left(\Delta Q_n^x - \Delta Q_{SC}^x(J_x, J_y)\right)}{Q^x - Q_0^x - \Delta Q^x(J_x, J_y) - \Delta Q_{SC}^x(J_x, J_y) - nQ_s} = -1$$

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Tune shift of mode n

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Tune of the mode including Landau damping

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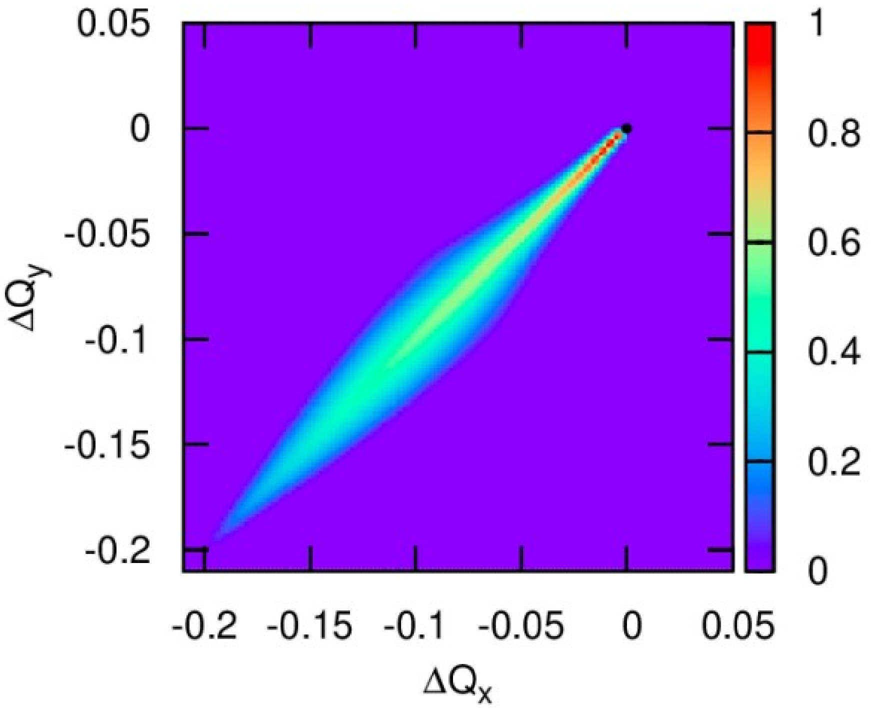

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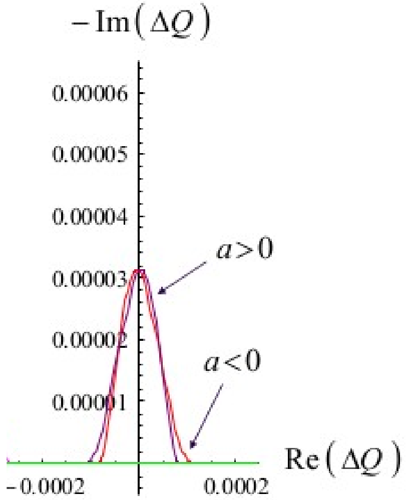

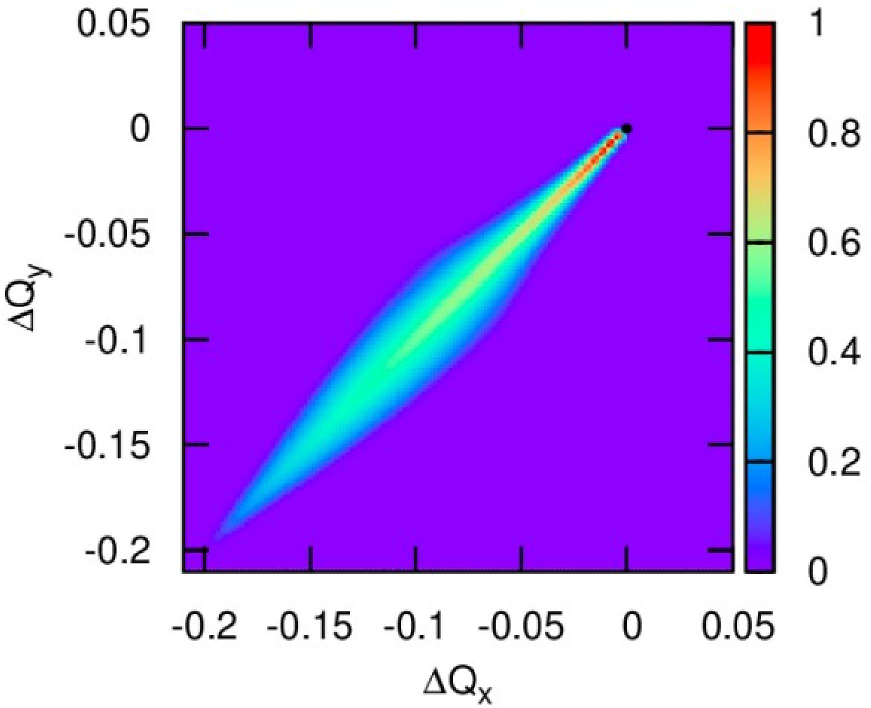

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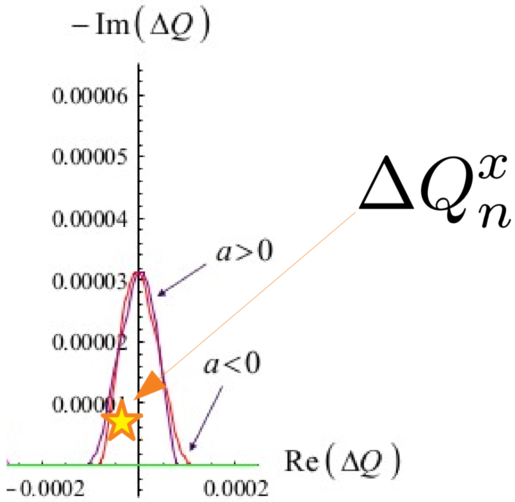

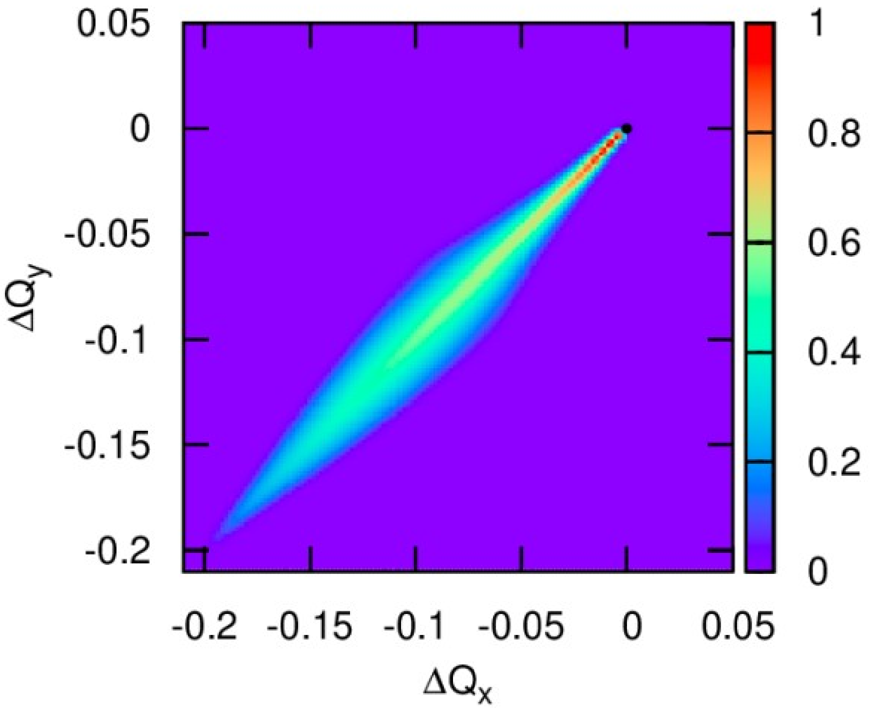

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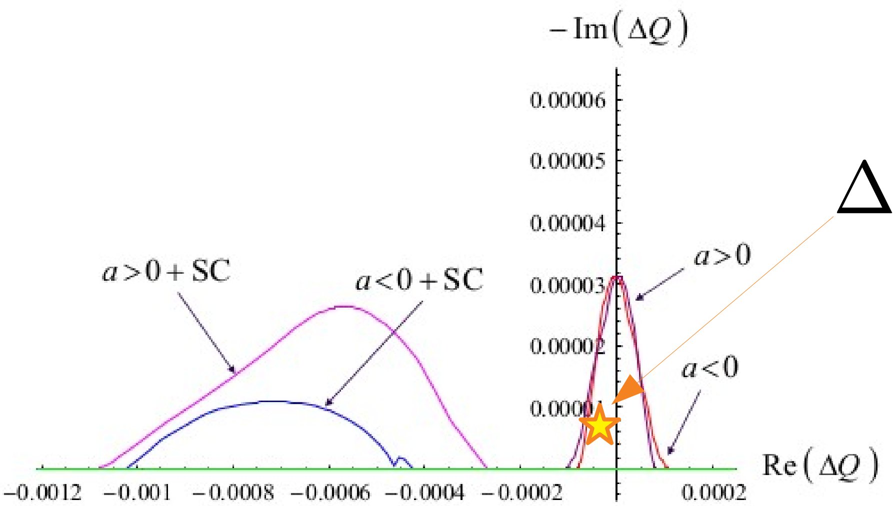
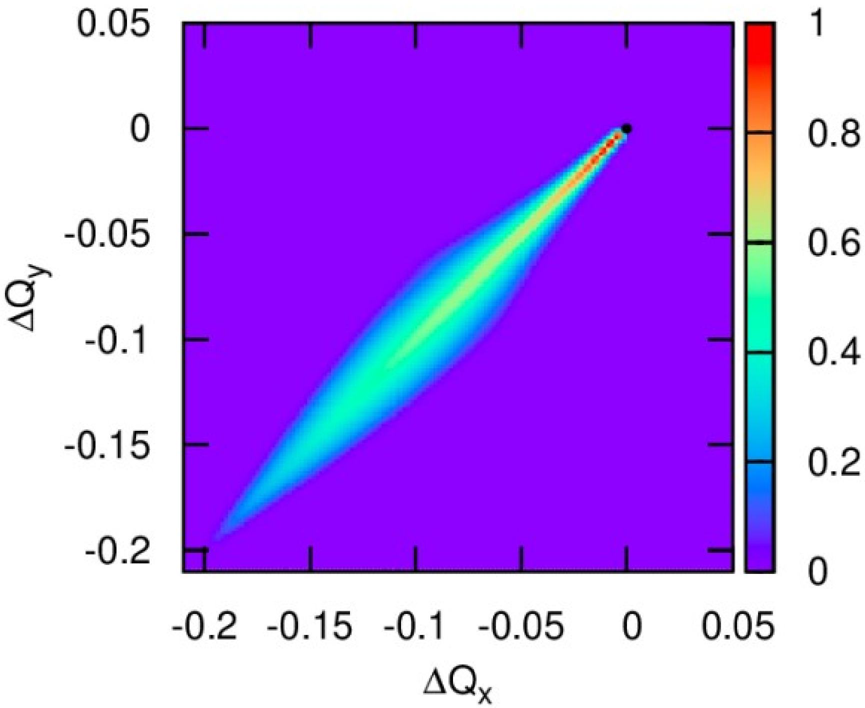


ΔQ_n^x



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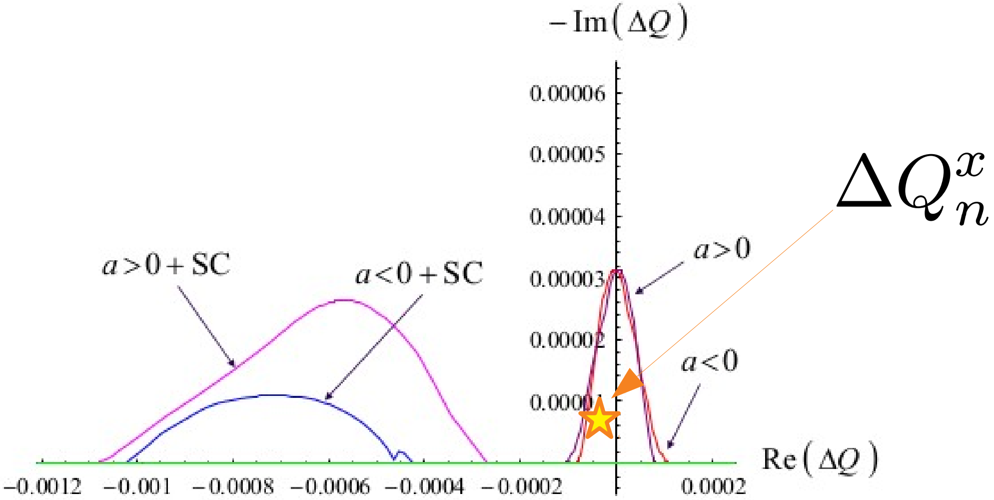
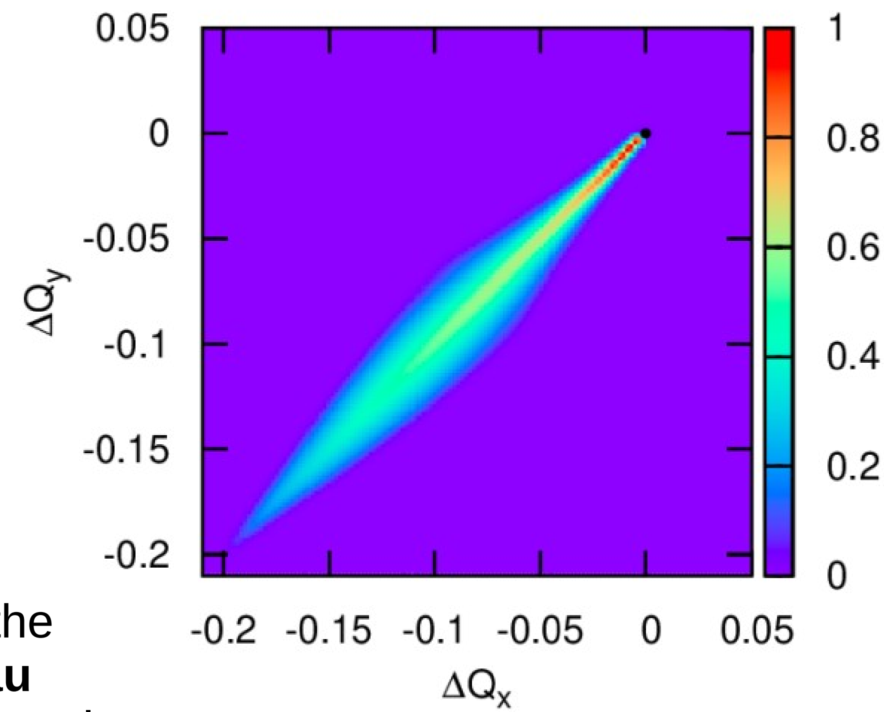
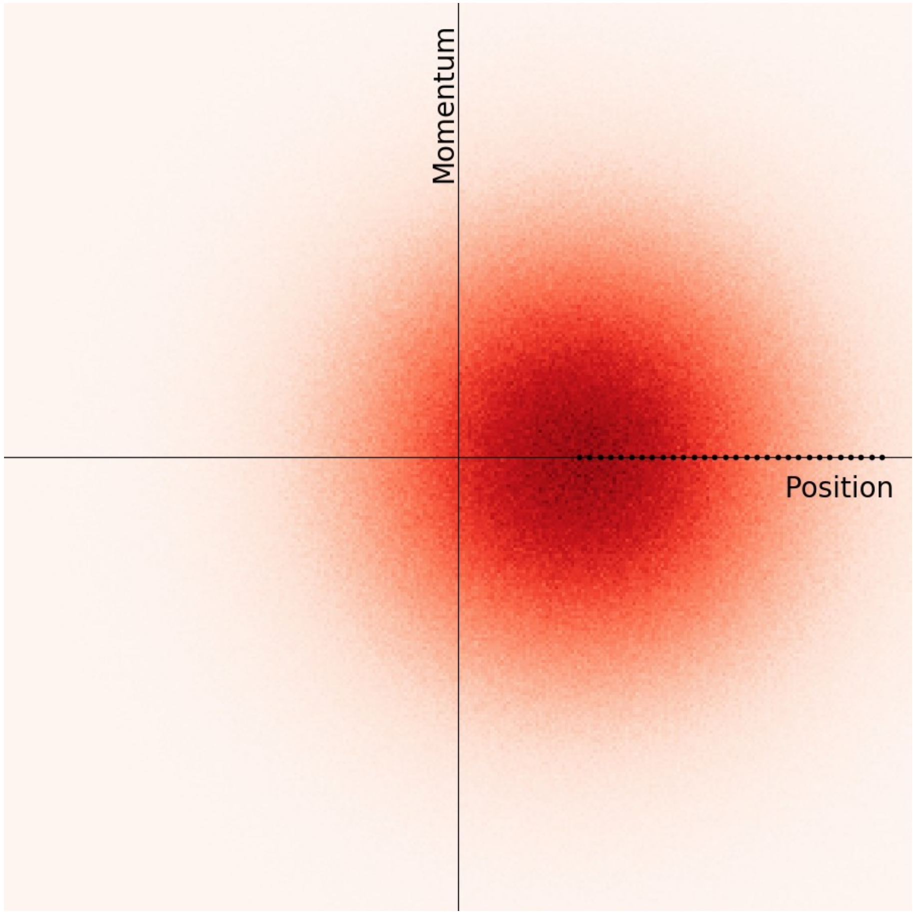




- By shifting the so-called incoherent spectrum from the coherent modes, space-charge can **remove Landau damping** for modes otherwise stabilised e.g. by octupoles

Decoherence of the rigid bunch mode with space-charge

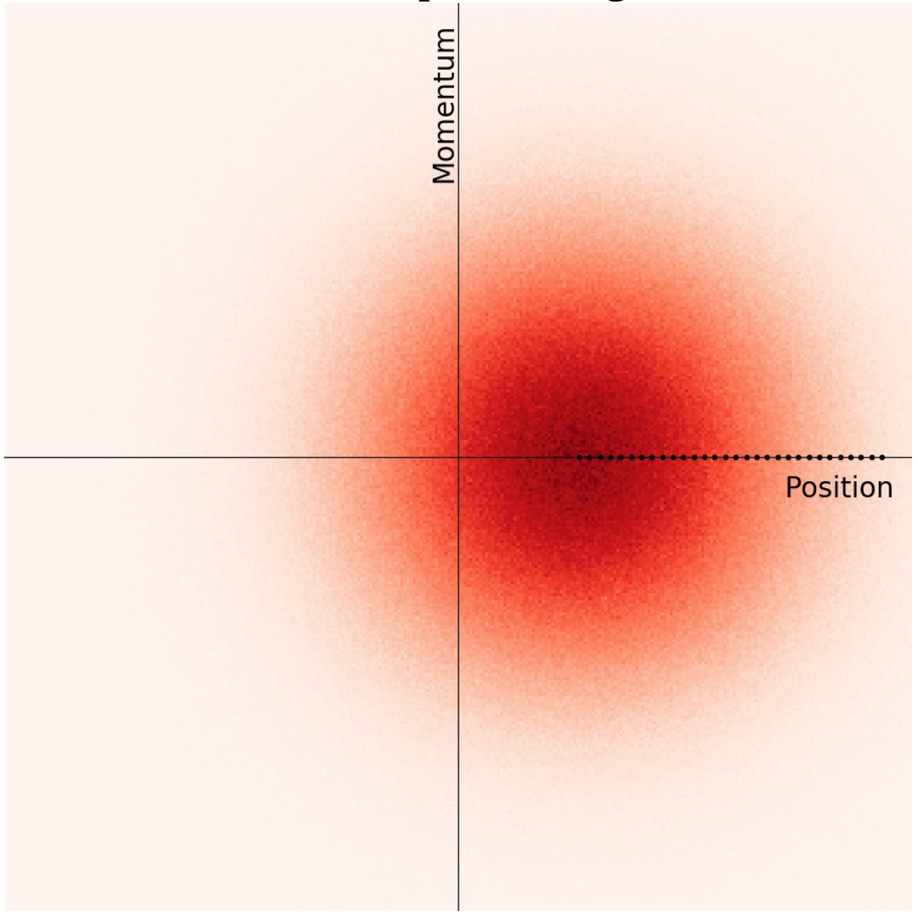
Without space-charge



Turn

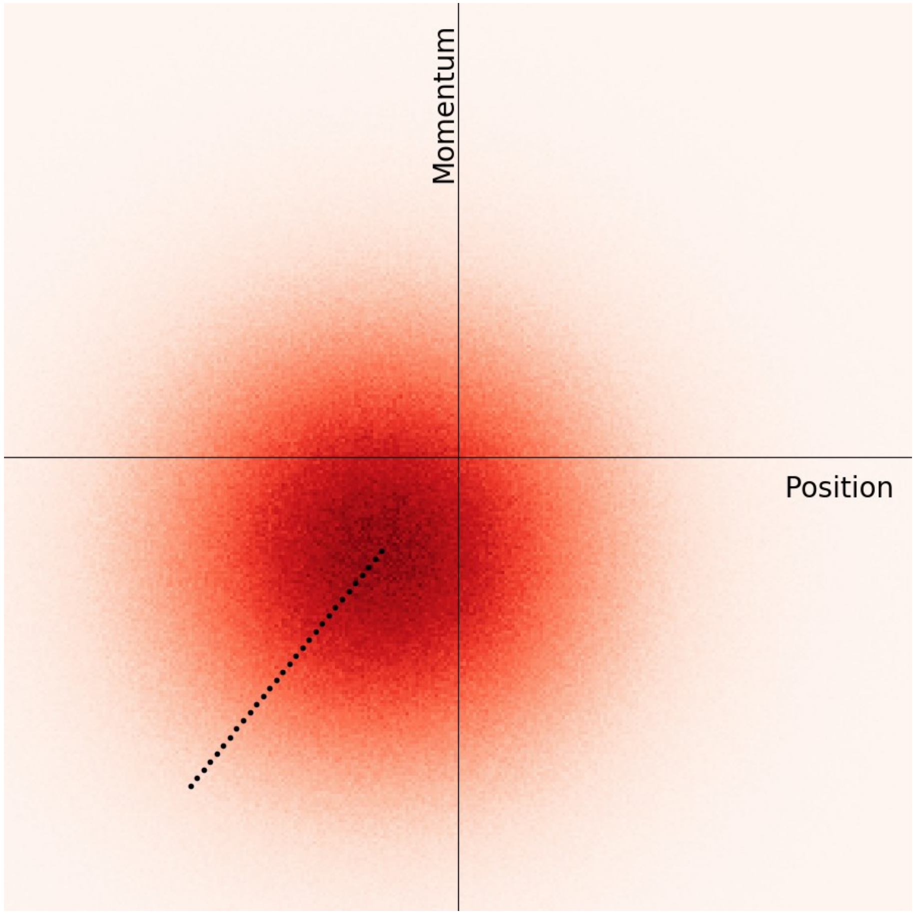
0

With space-charge



Decoherence of the rigid bunch mode with space-charge

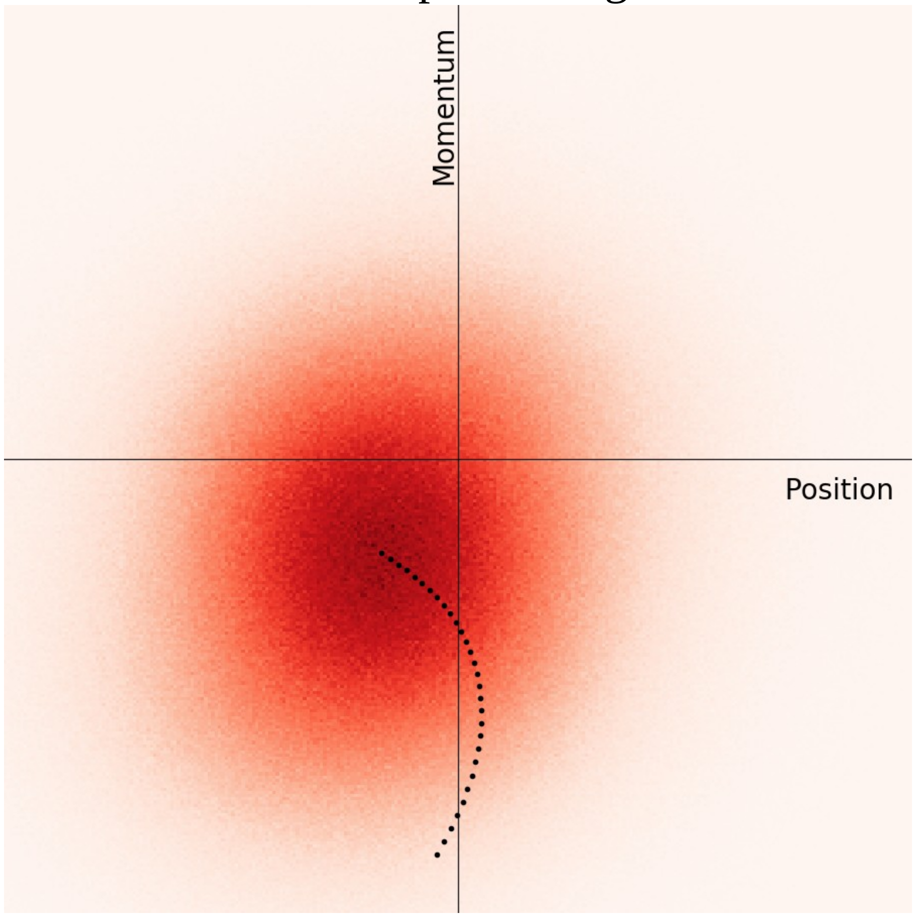
Without space-charge



Turn

14

With space-charge



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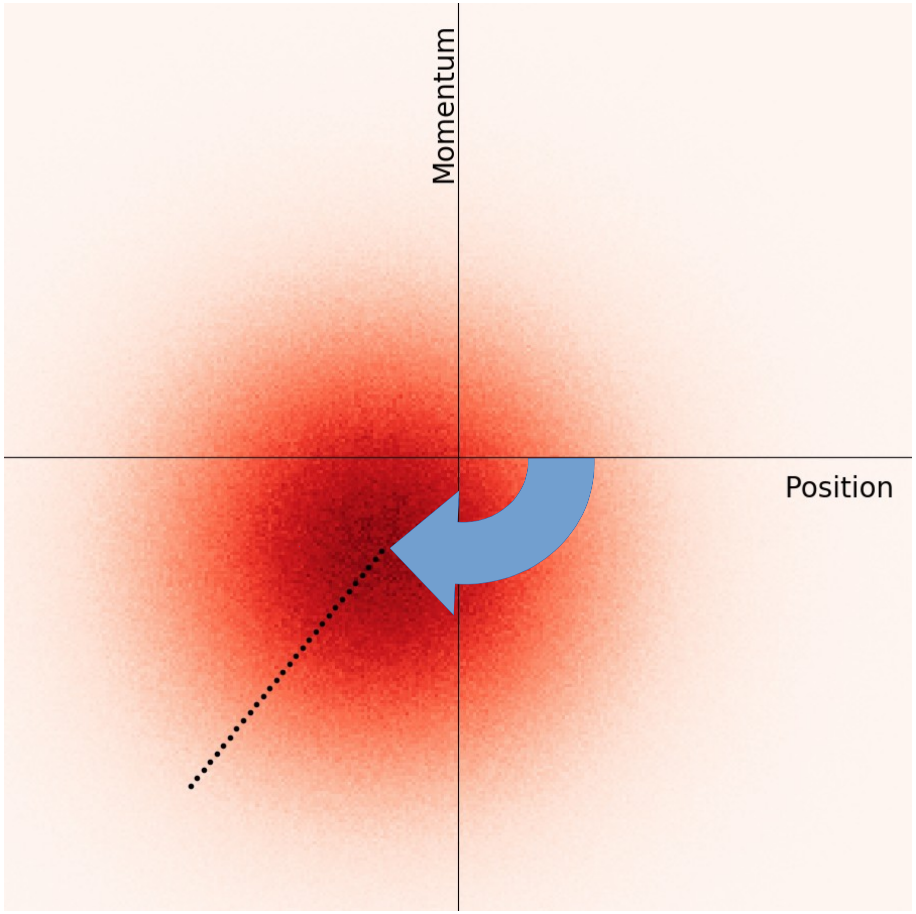
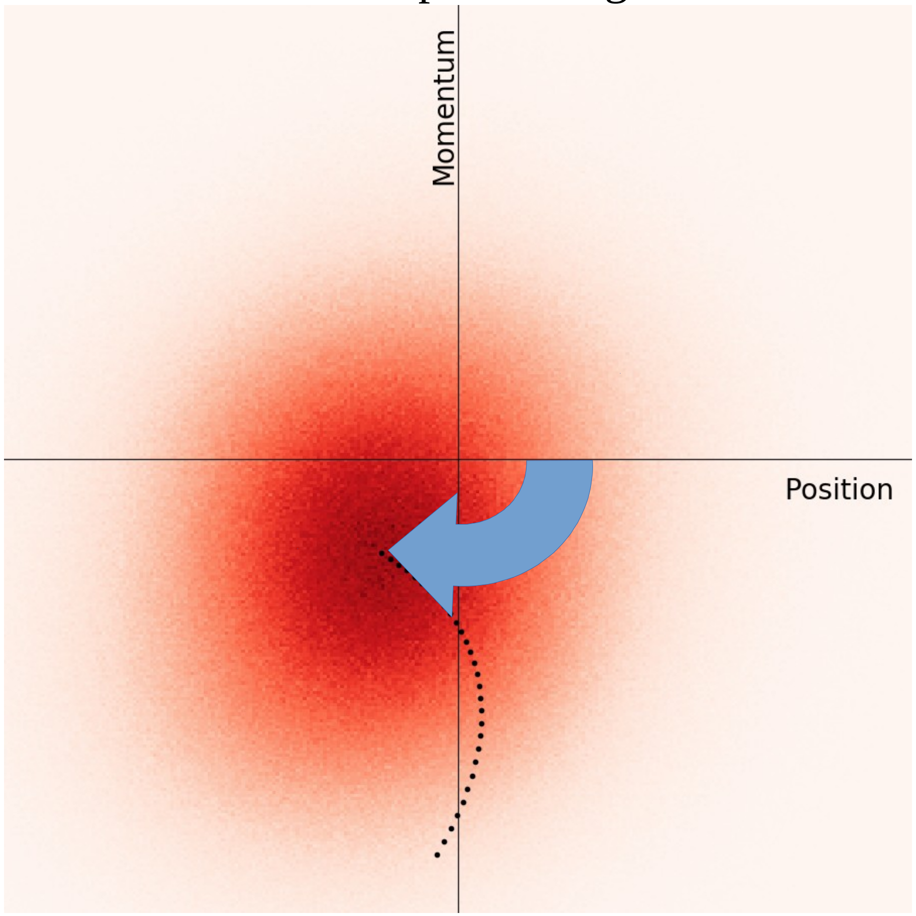


Turn

14

With space-charge



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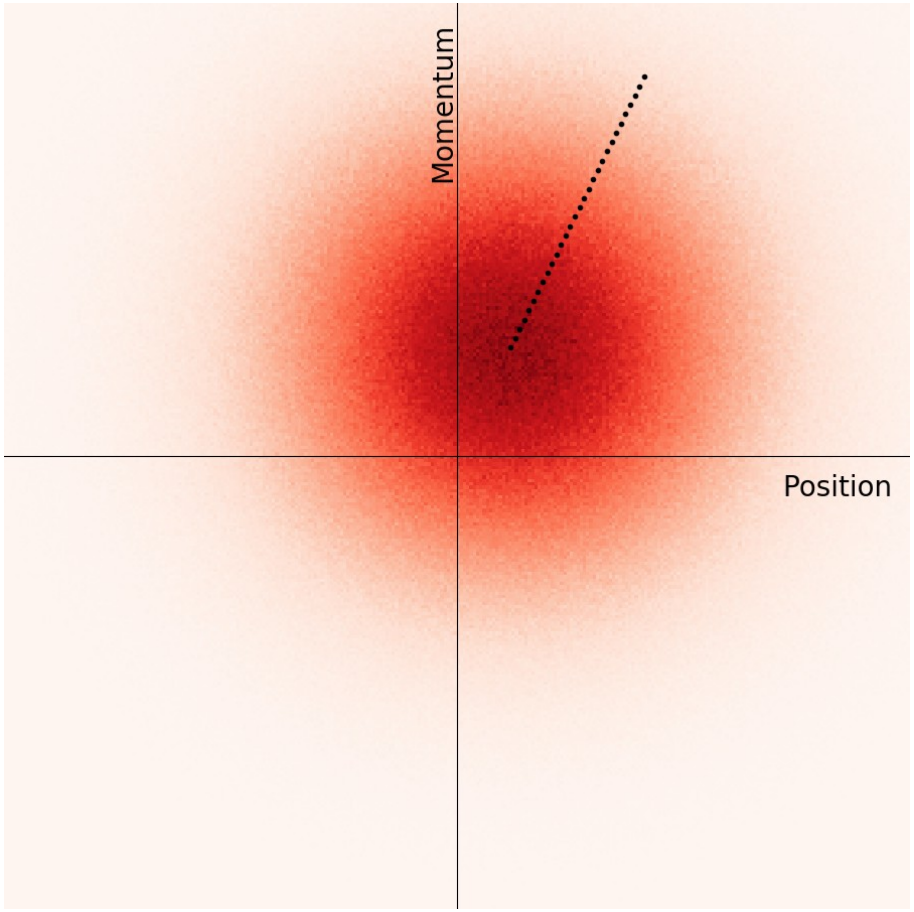
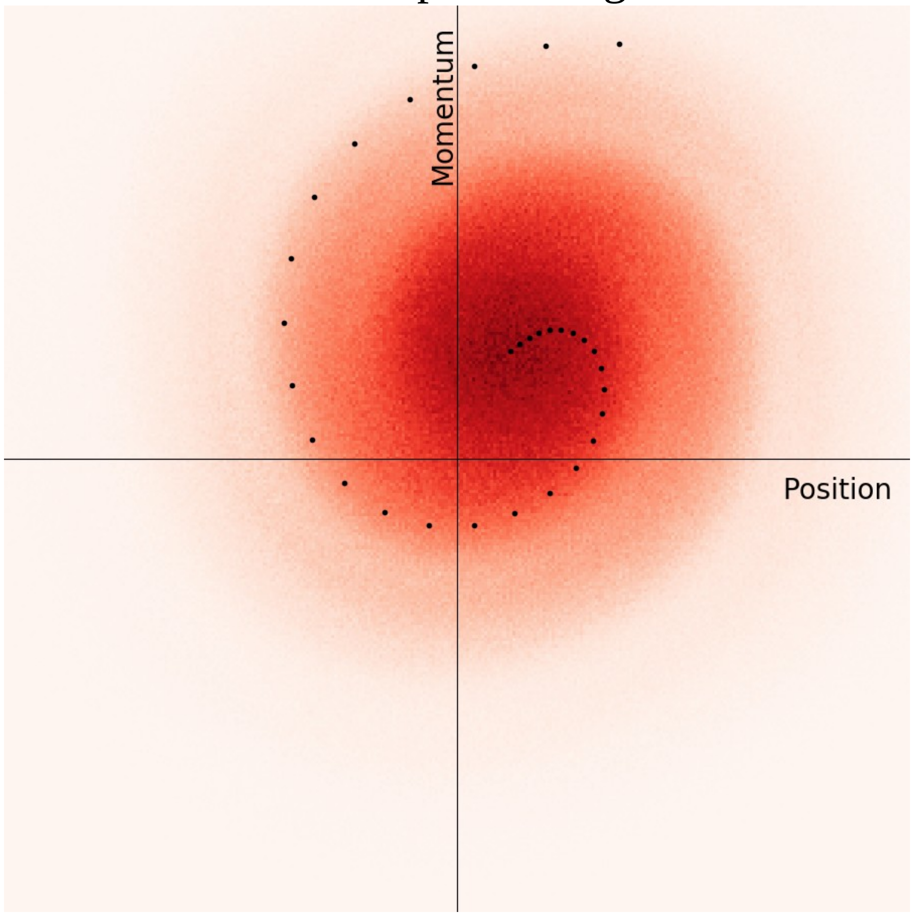


Turn

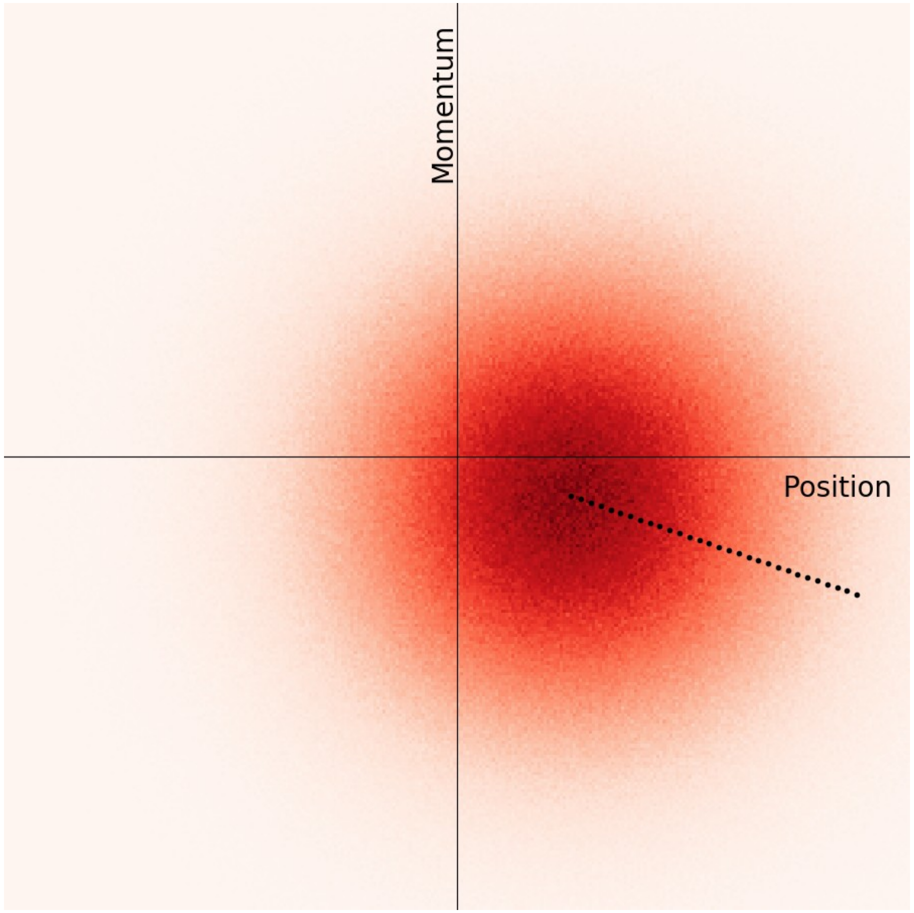
99

With space-charge



Decoherence of the rigid bunch mode with space-charge

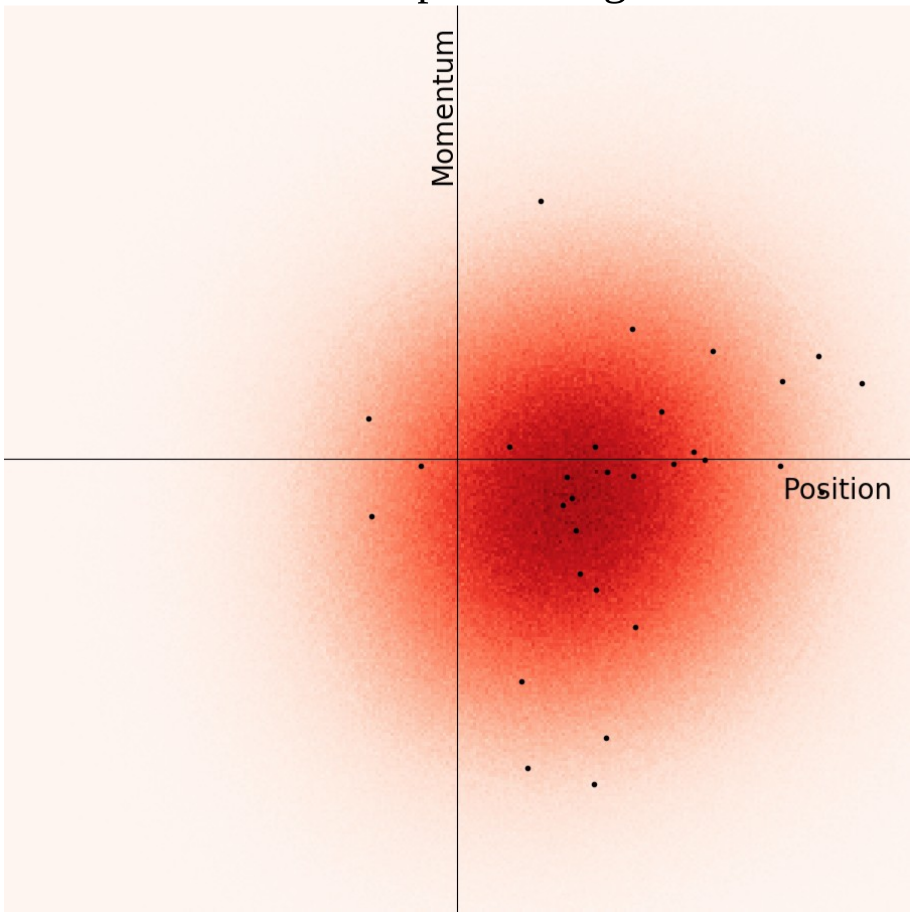
Without space-charge



Turn

6000

With space-charge



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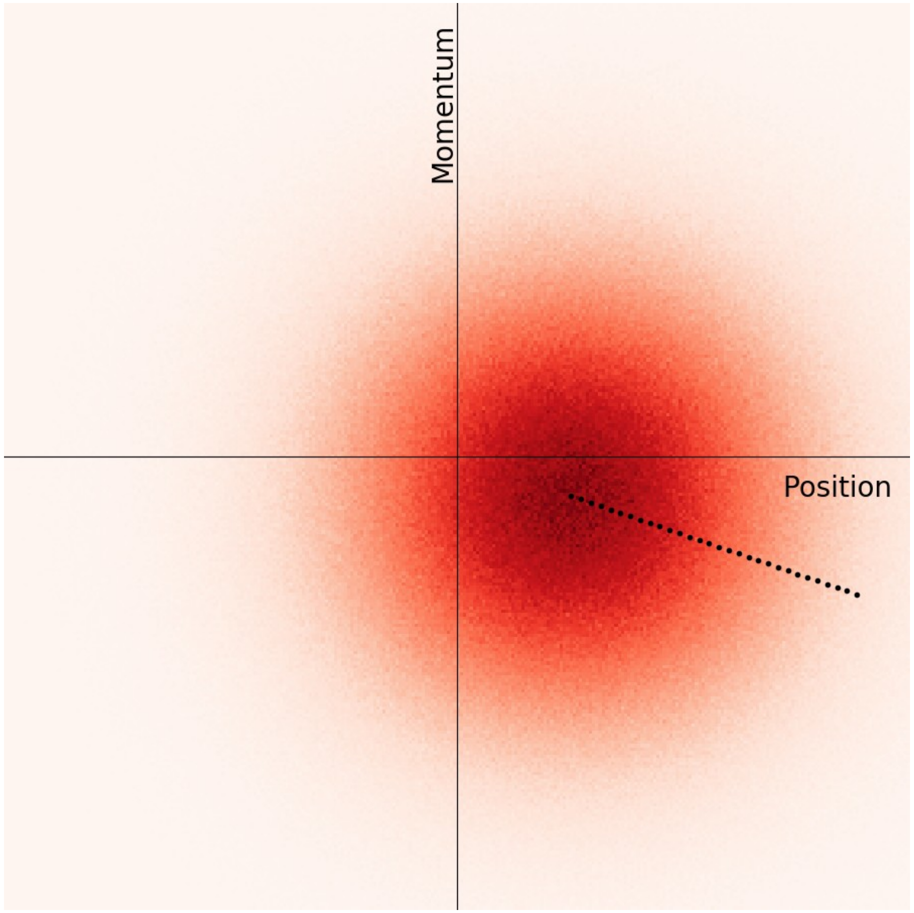
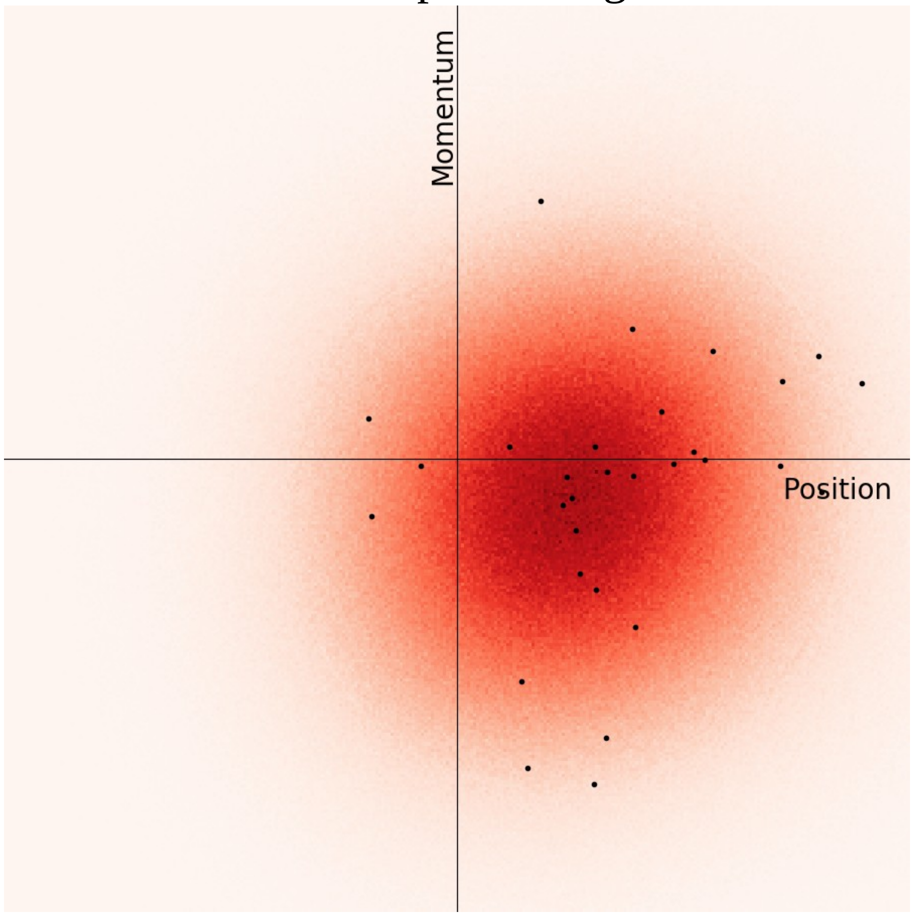
Without space-charge



Turn

6000

With space-charge



- The motion of the centroid is not affected by space-charge

  → Coherent mode

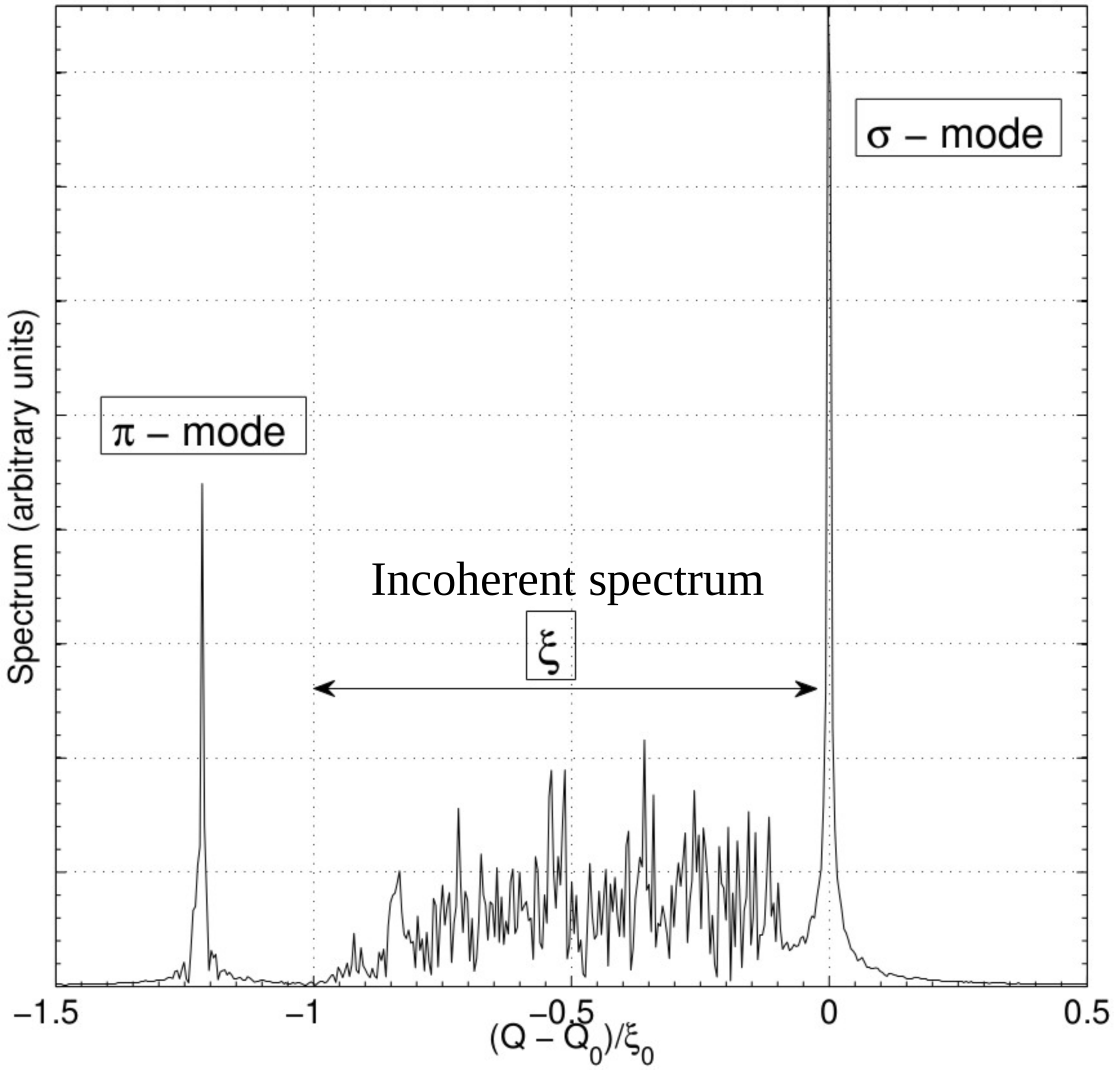
- The motion of single particles around the centroid is affected

  → Incoherent tune spread

Stability of the rigid bunch mode with beam-beam [Pieloni]

- A similar effect occurs with the coherent modes generated by beam-beam interactions

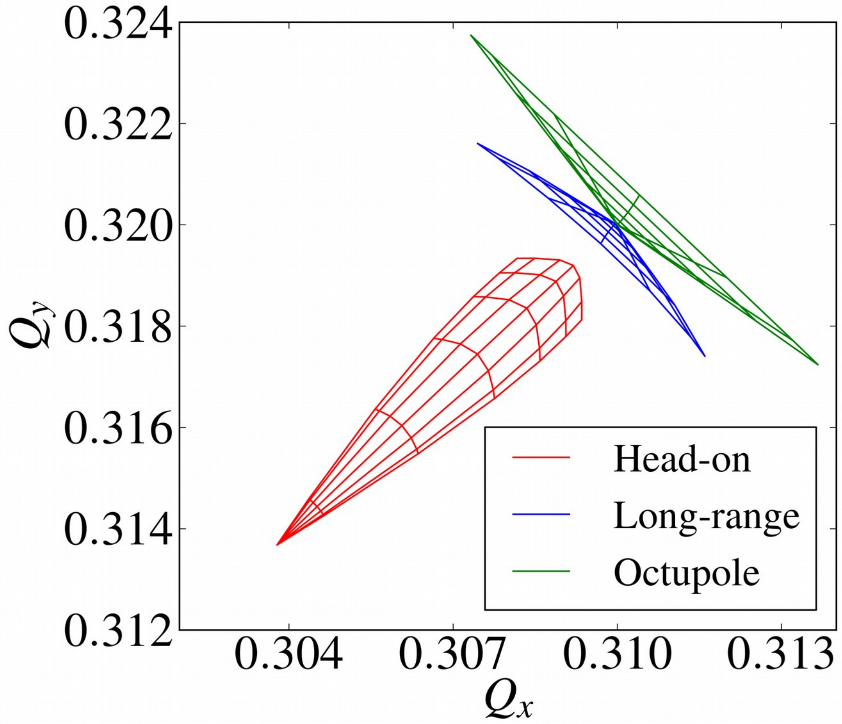
  → They are outside of the incoherent spectrum, Landau damping is lost



Stability diagrams with beam-beam

[Buffat, Chao2]

- If the coherent modes are suppressed (e.g. with an active feedback), the remaining tune spread can be beneficial for other modes



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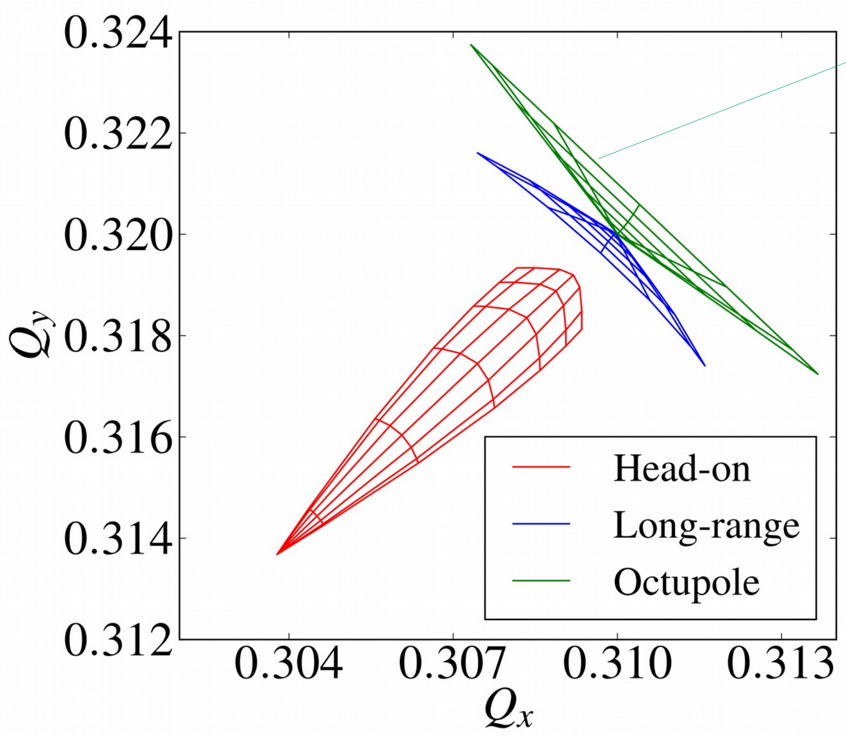

$$\omega(J) = 2\pi(Q_0 + aJ)$$

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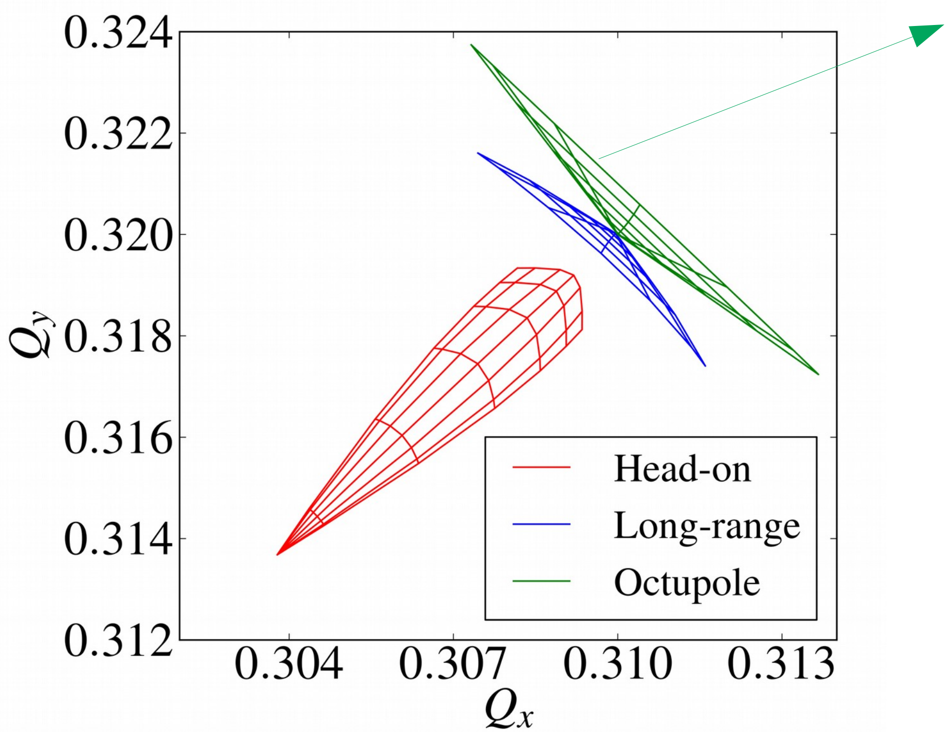

$$\omega(J) = 2\pi(Q_0 + aJ)$$

$$\frac{-1}{\Delta\Omega_n} = \int dJ \frac{J \frac{df_0}{dJ}}{\Omega - \omega(J)}$$

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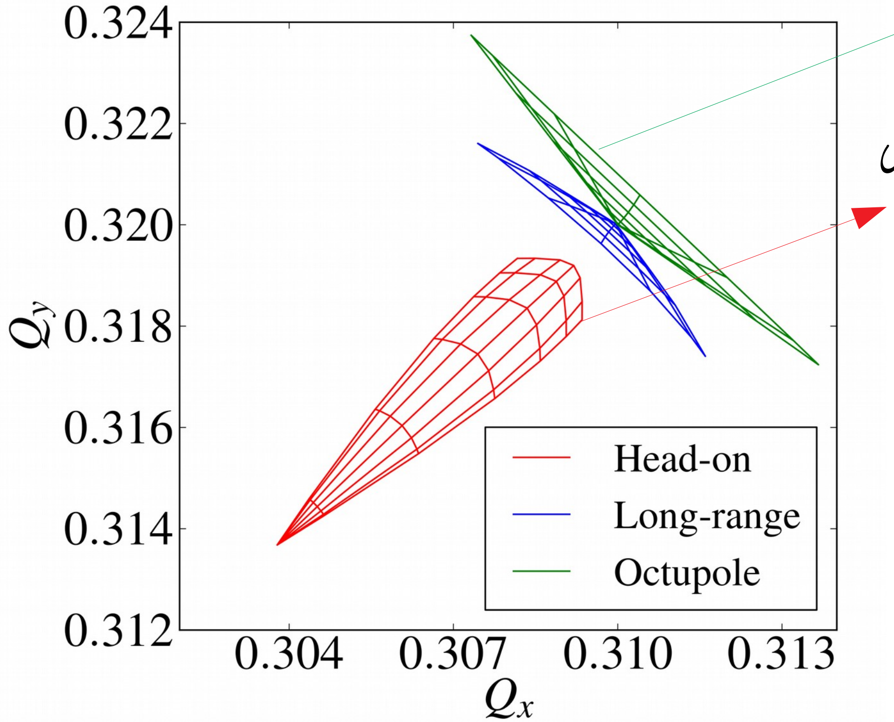

$$\omega(J) = 2\pi(Q_0 + aJ)$$

$$\begin{aligned}\omega(J) &= 2\pi Q_0 \\ &- \frac{N r_0}{\gamma J}\left(1 - e^{-\frac{J\beta}{2\sigma^2}} I_0\left(\frac{J\beta}{2\sigma^2}\right)\right)\end{aligned}$$

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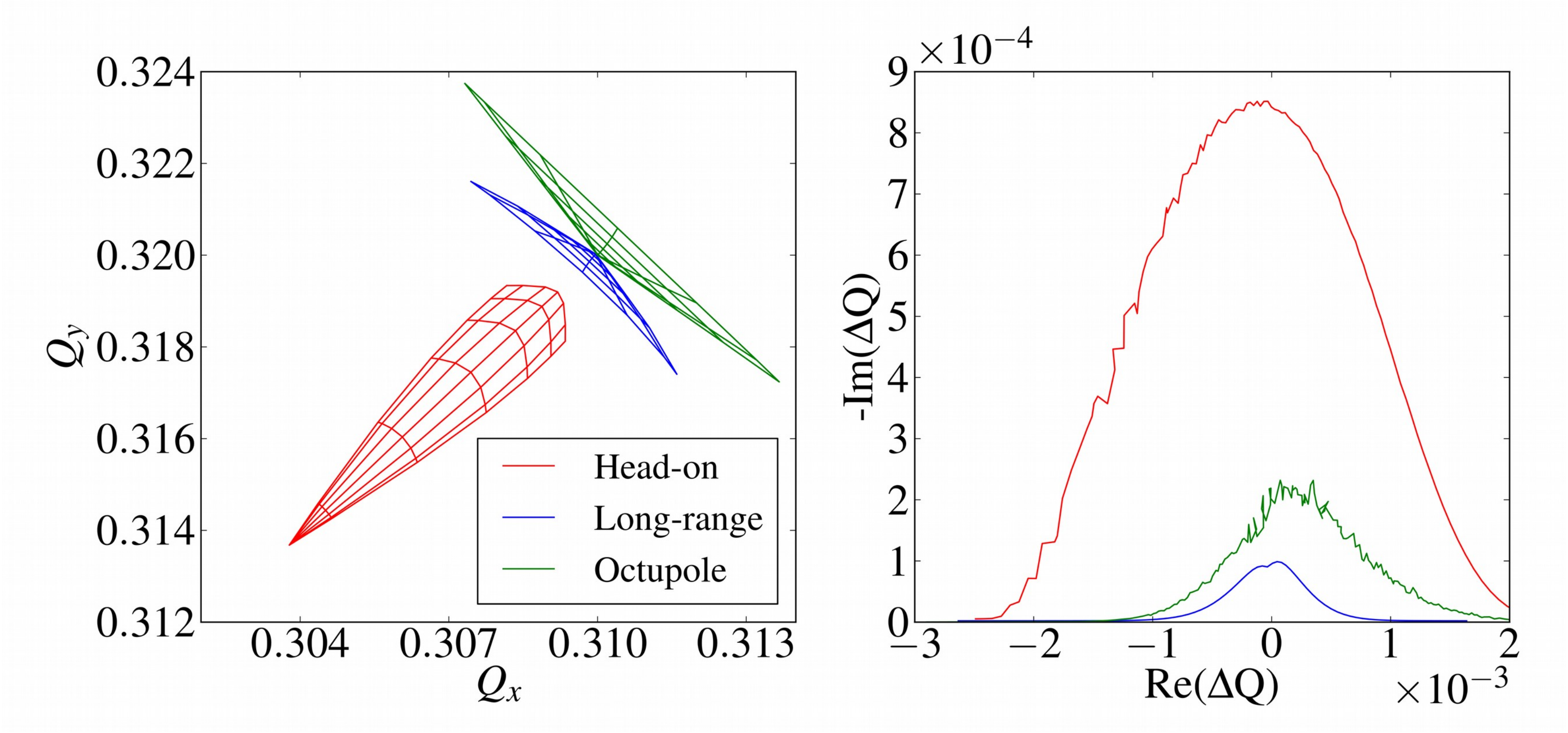

- Due to its different dependence on the action, the amplitude detuning due to head-on beam-beam interactions is more efficient at producing Landau damping than octupoles!

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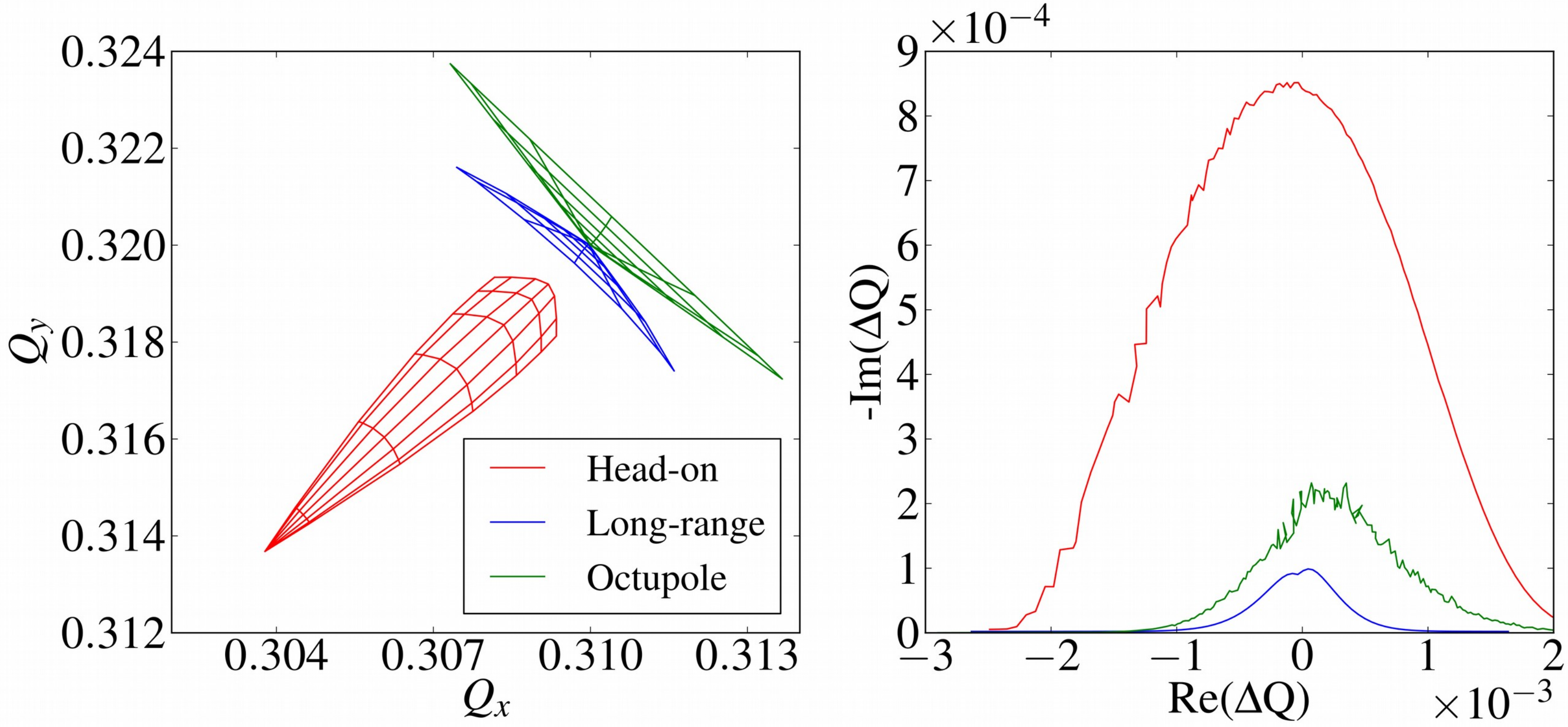

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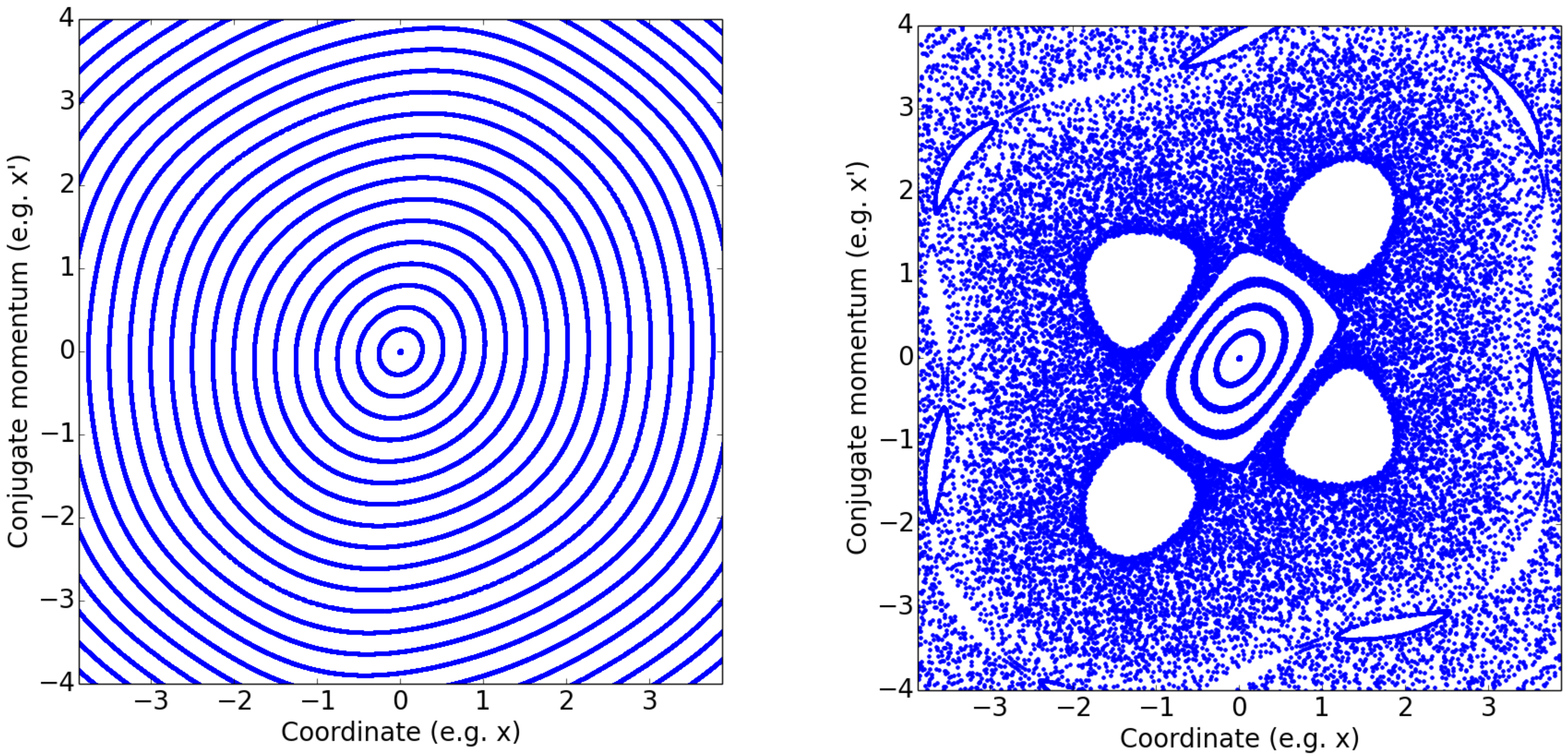
→ Maybe we should be inspired ?

The issue with non-linear forces

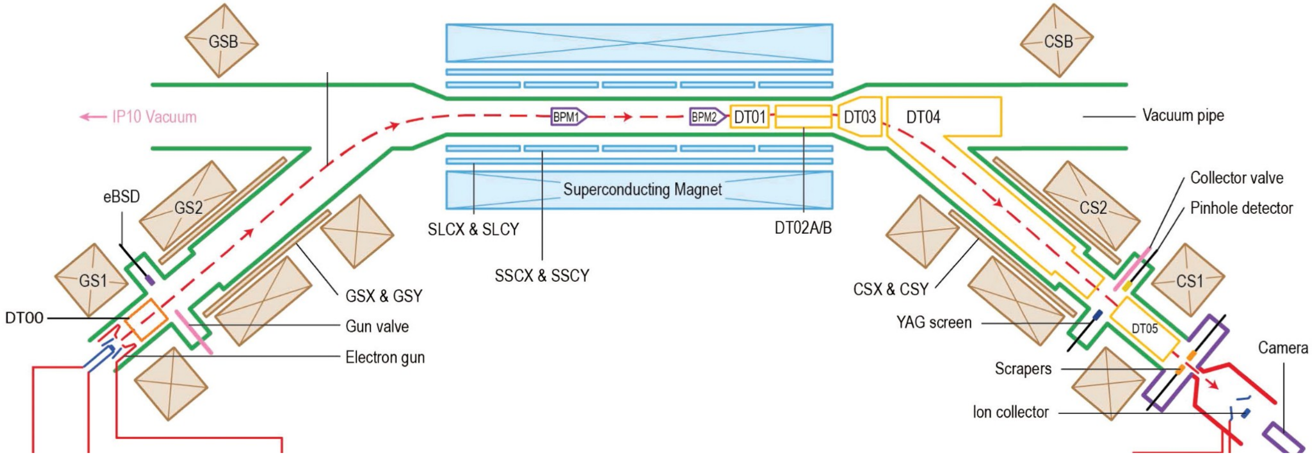


- Along with the tune spread required for Landau damping, non-linearities come with detrimental effect for the single particle trajectories:
  - Resonances, chaotic motion and eventually beam quality degradation (particles losses, emittance growth)

→ The **amount of Landau** damping that can be obtained with octupoles is **limited** by their impact on beam losses

Electron lens

[RHIC, elens]



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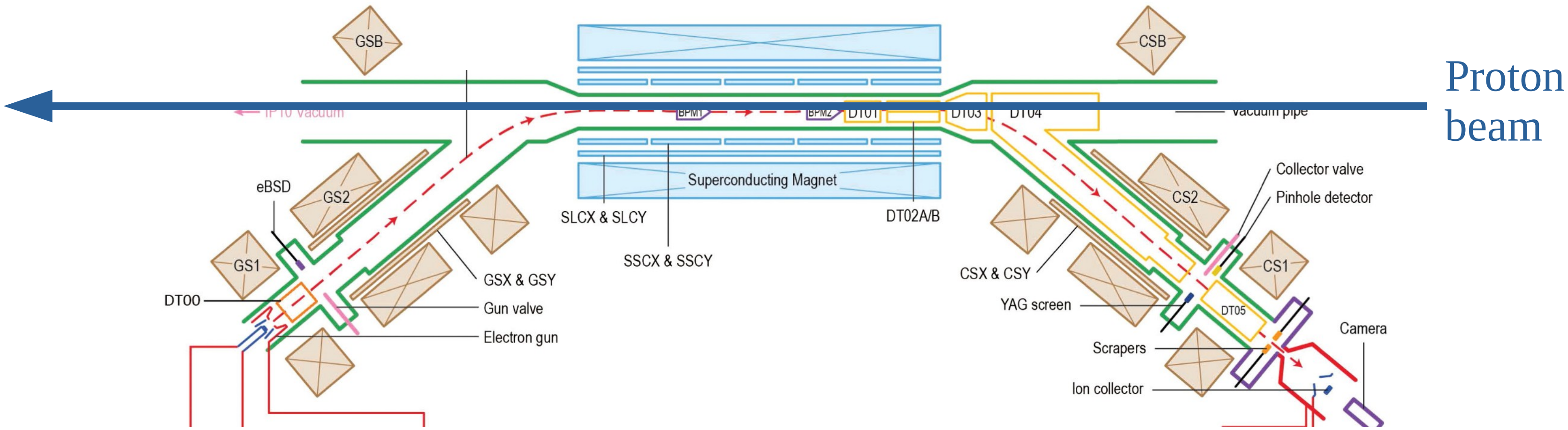

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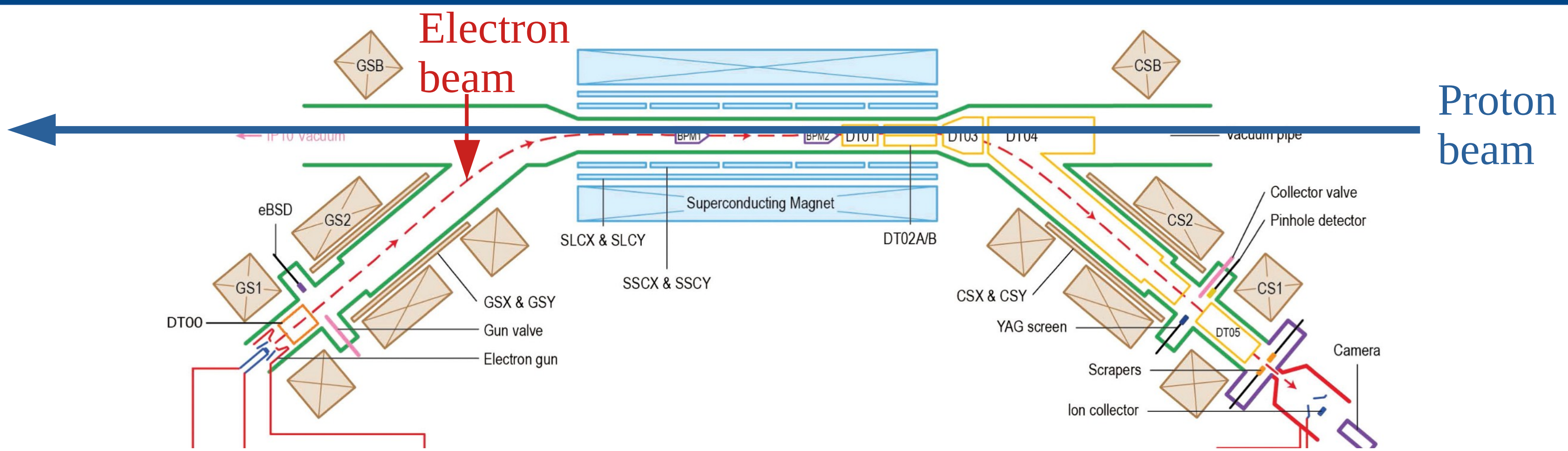

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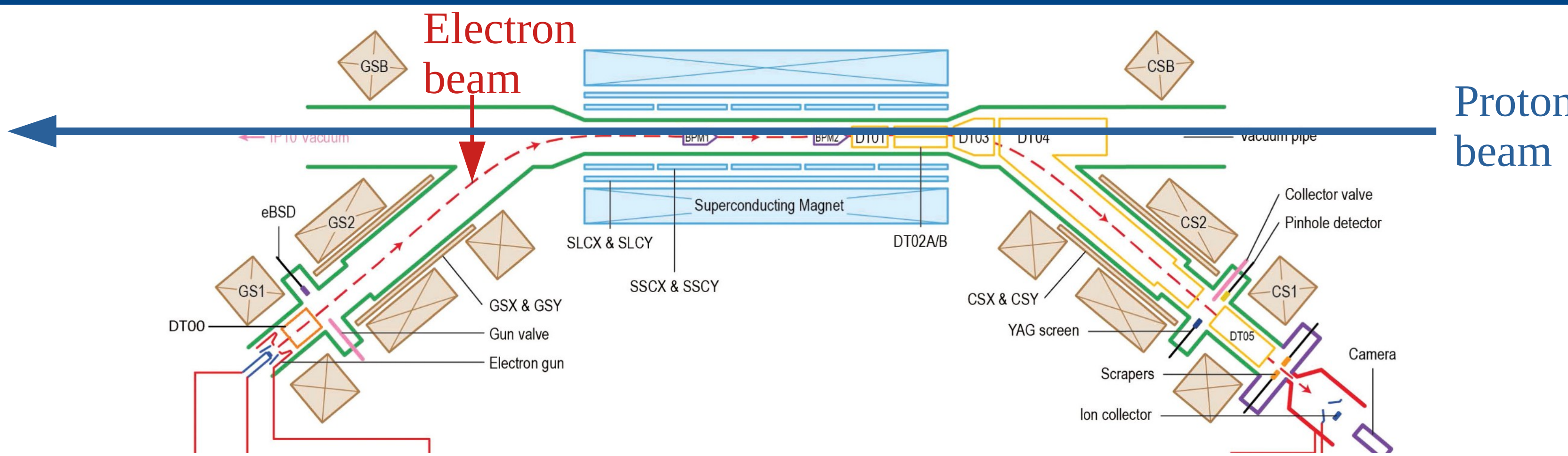

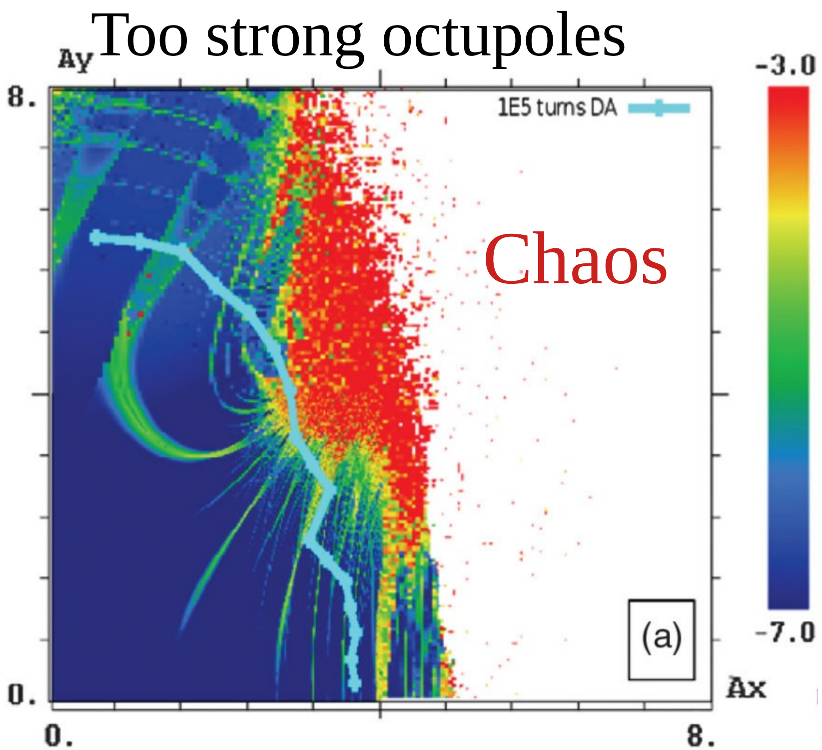

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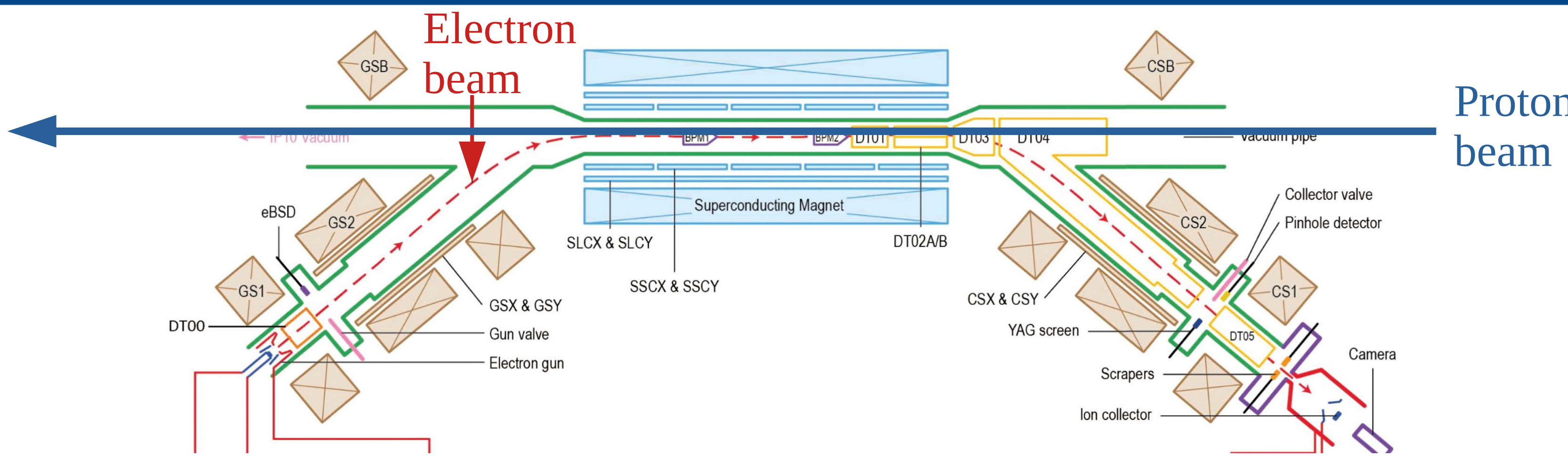

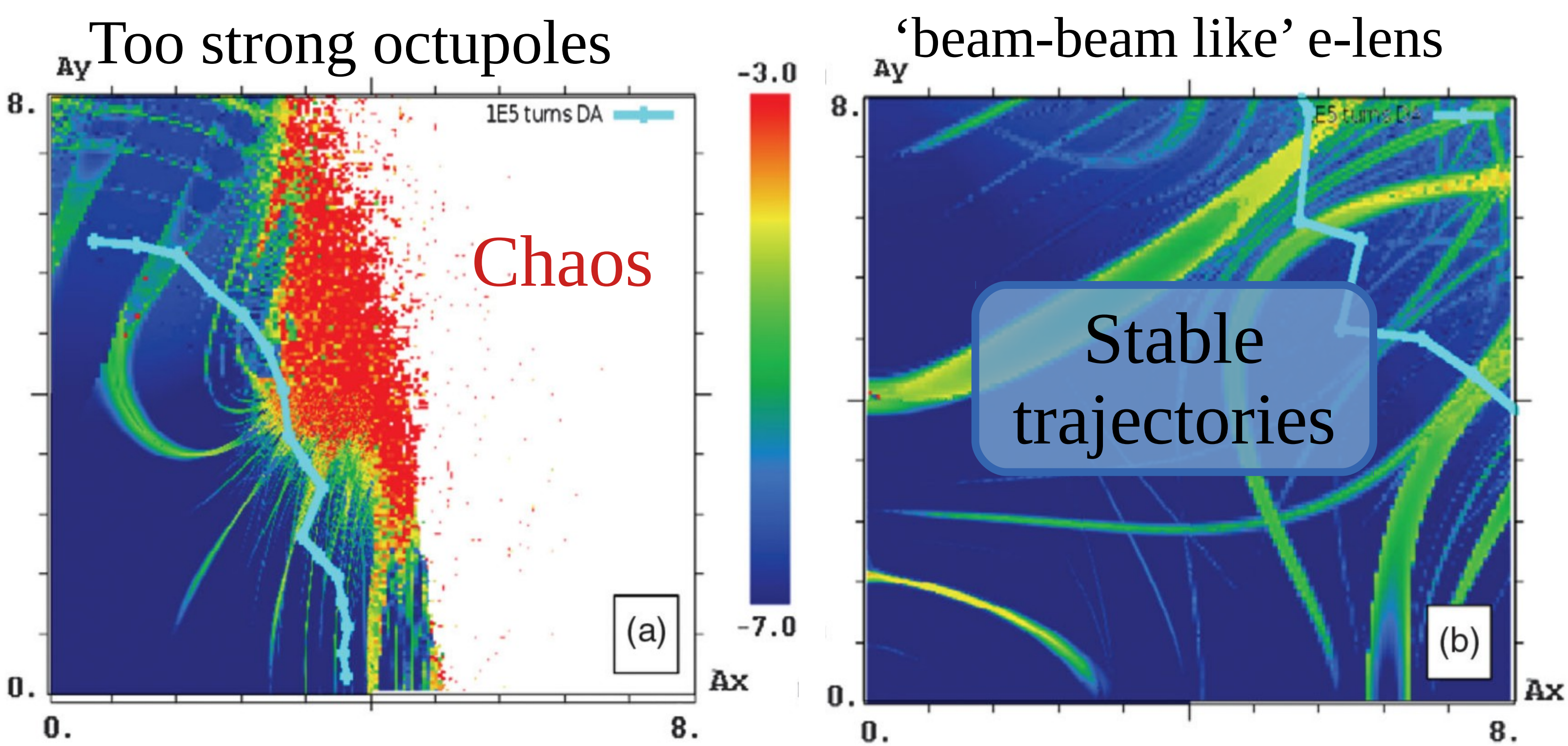

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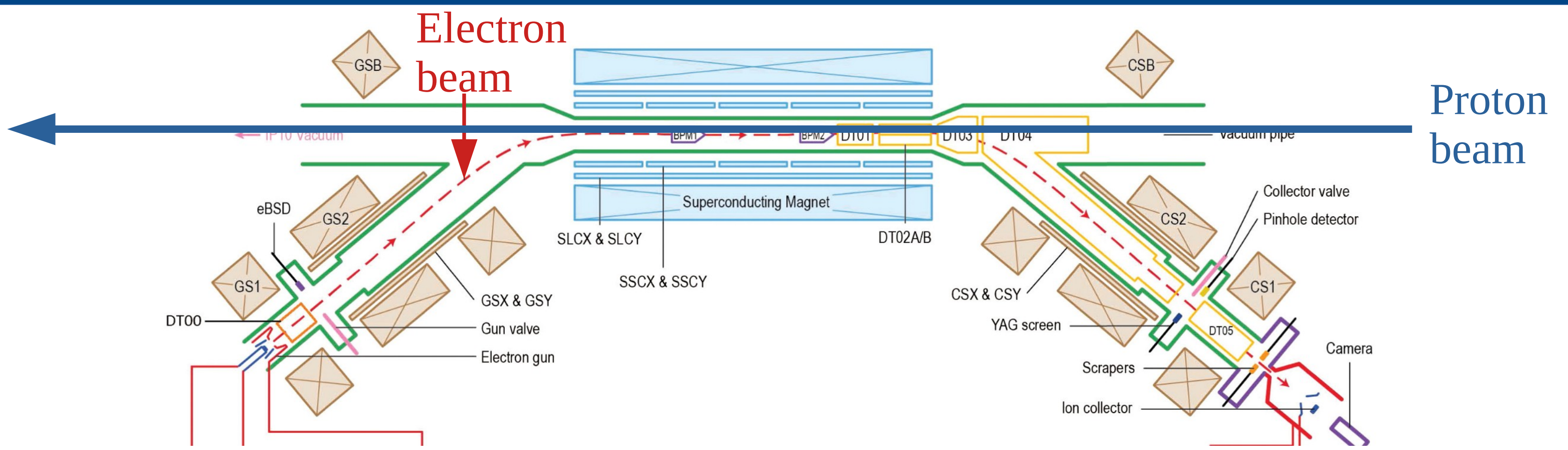
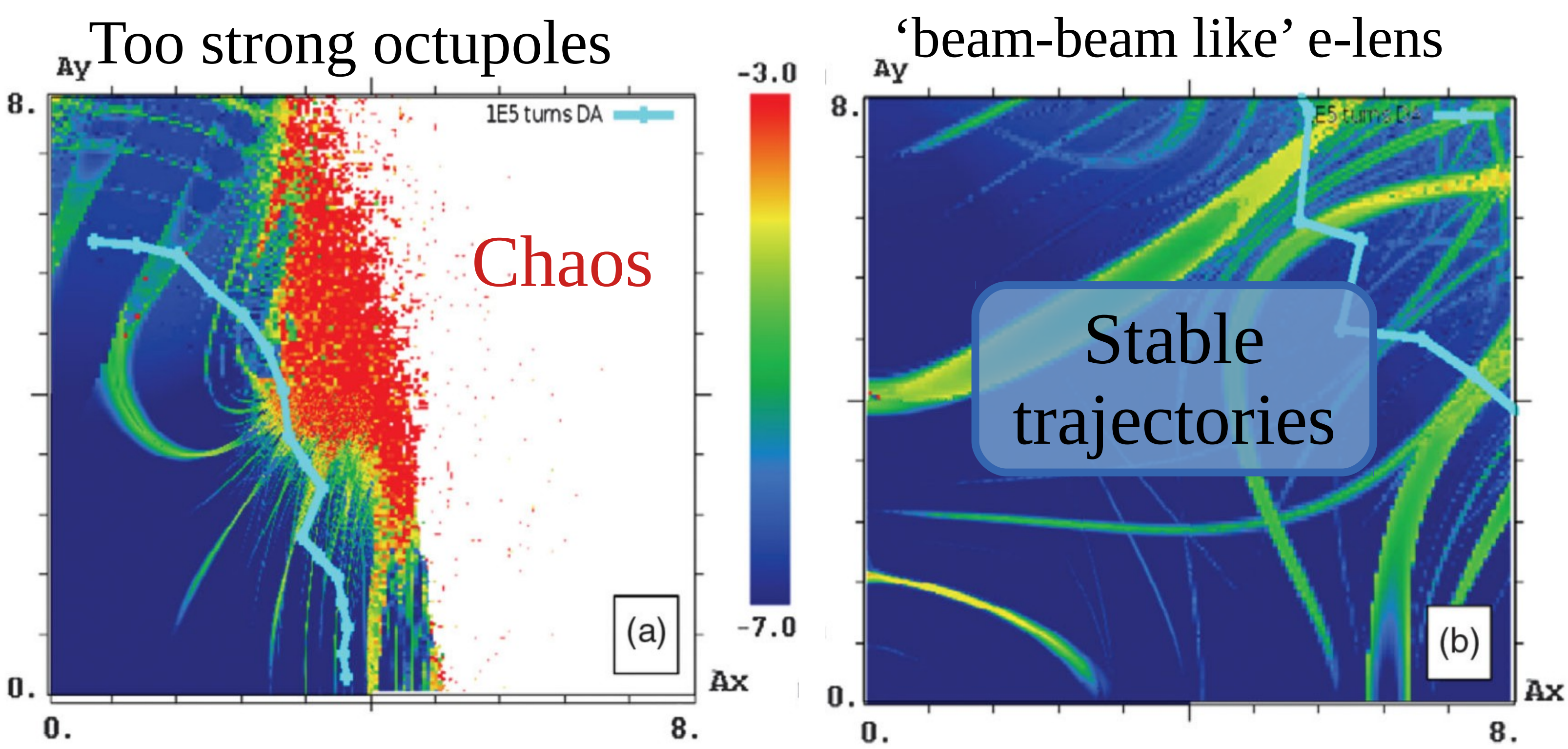


The gun design allows for various electron beam shapes



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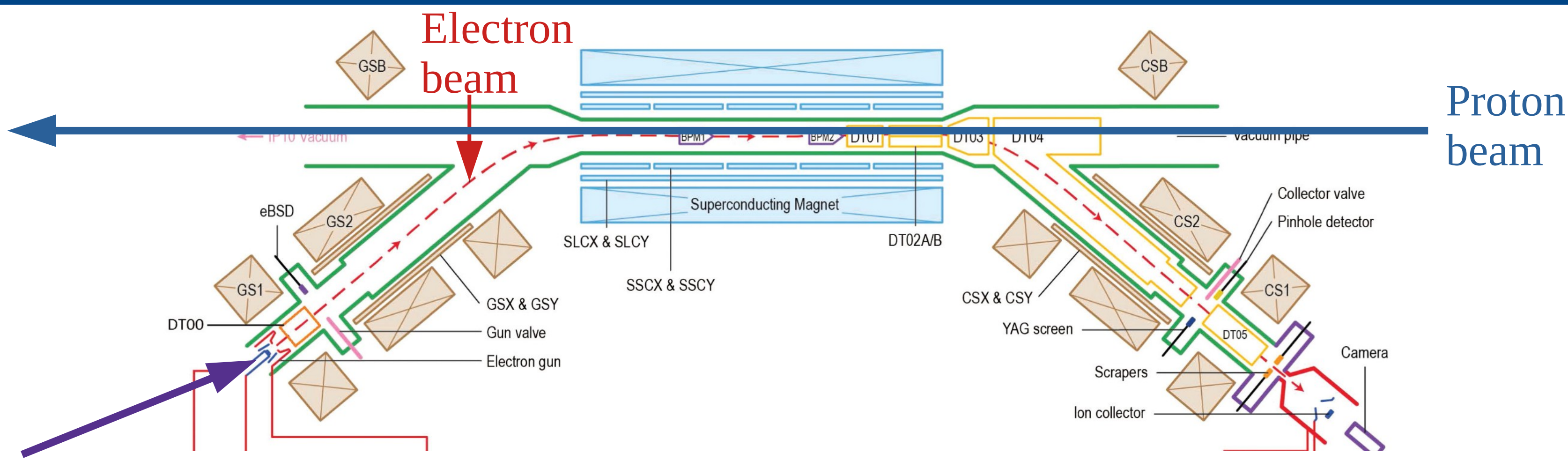
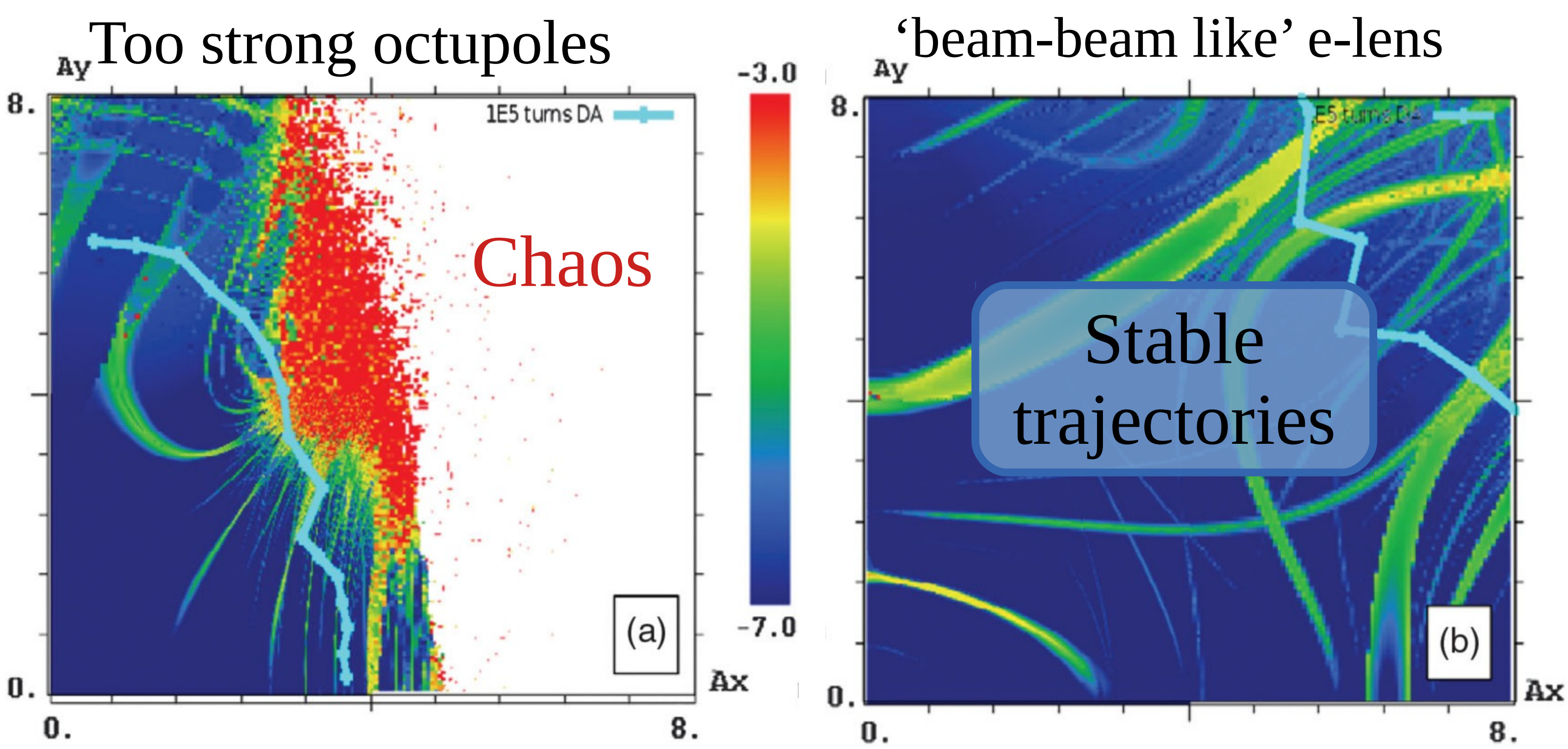


The gun design allows for various electron beam shapes
→ Optimise the force to maximise Landau damping with least impact on the beam quality

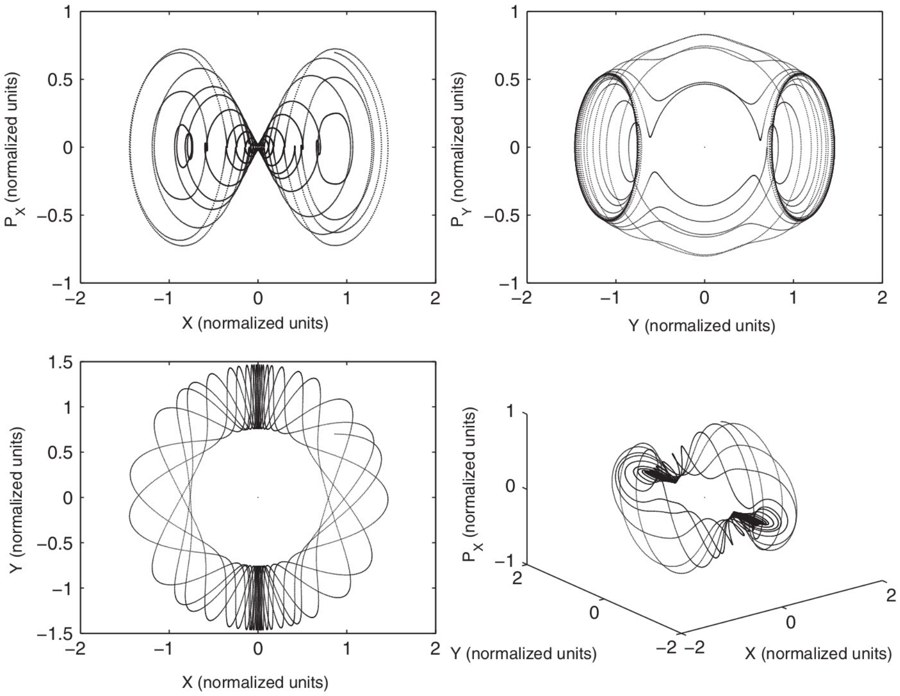


Non-linear integrable optics

[NLIO,IOTA, McMillan]

- It is possible to introduce ‘good’ non-linearities that generate a tune spread yet maintaining some invariants of motion

  → Possibly strong Landau damping without deterioration of the beam quality



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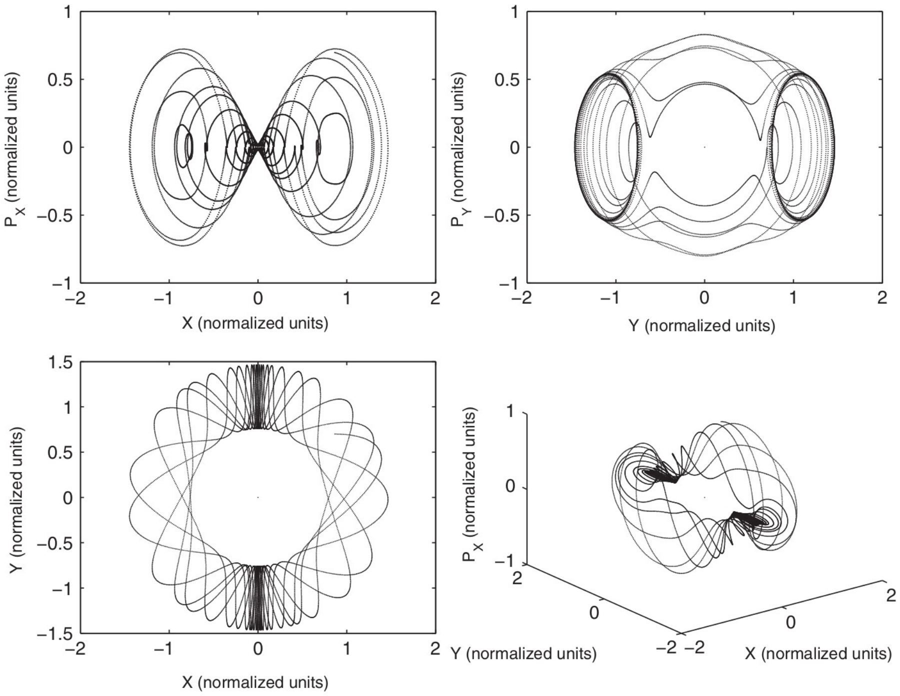

A series of independently powered octupoles to generate a non-linear integrable optics at IOTA

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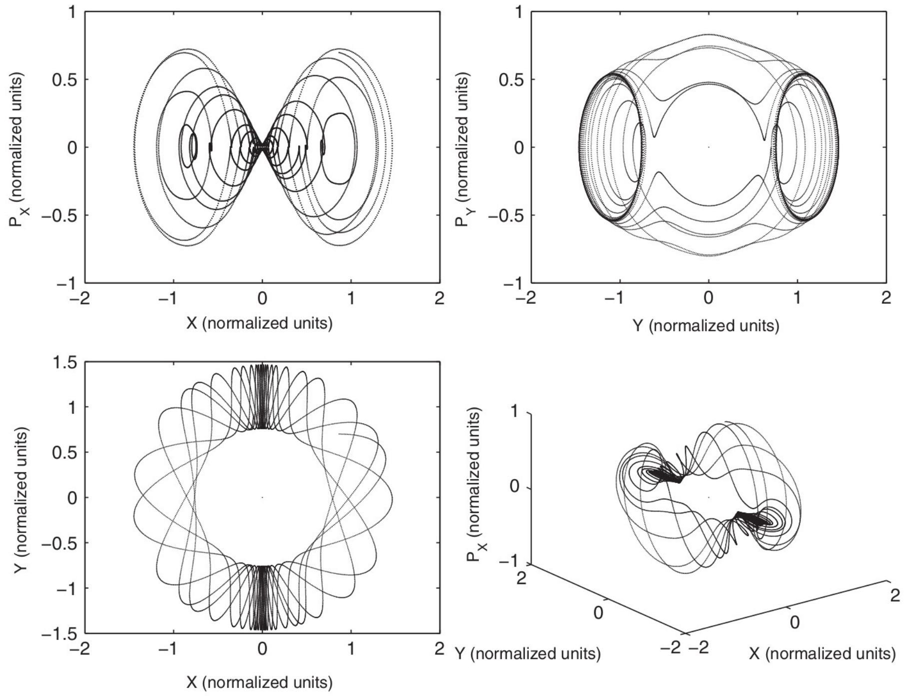

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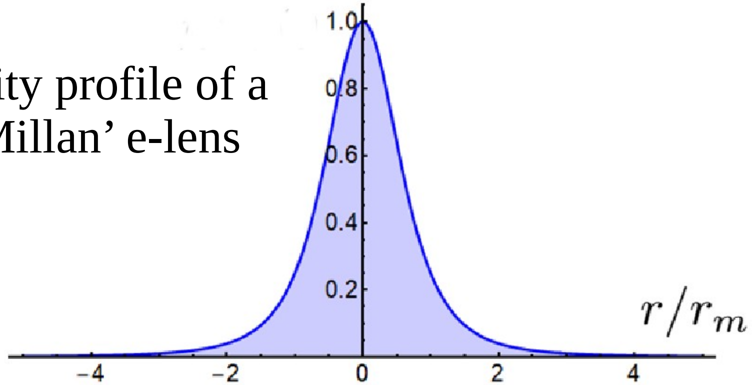
Density profile of a 'McMillan' e-lens



Transverse detuning with longitudinal amplitude

[Schenk. RFQ]

$$\frac{\int dr r f_0(r) \left| H_l^k(r) \right|^2}{\Delta\Omega_{ext}^{l,k}} = \int dr \frac{r f_0(r) \left| H_l^k(r) \right|^2}{\Omega^{l,k} - \omega(r) - l\omega_s}$$

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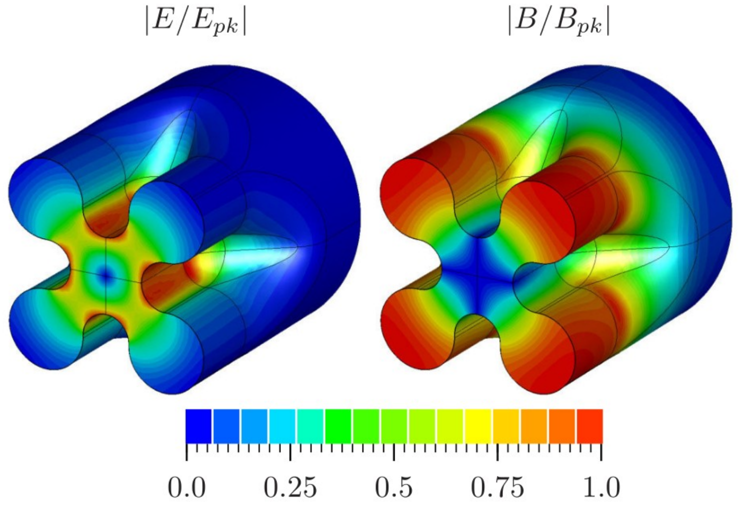
Longitudinal oscillation amplitude

- Transverse detuning with longitudinal amplitude can be achieved with
  - Dedicated optics (non-linear chromaticity)
  - RF quadrupoles



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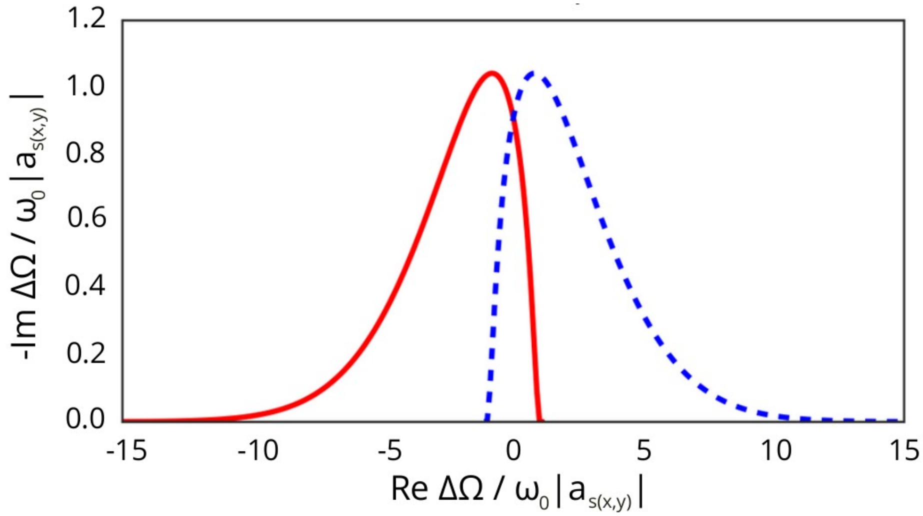

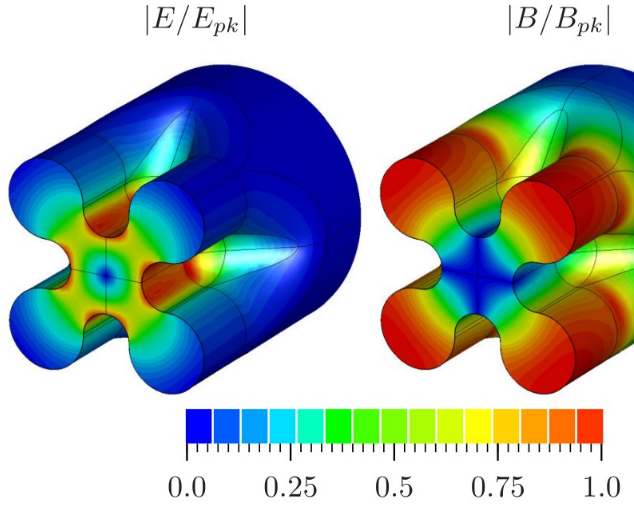

Summary

- In some cases Landau damping arise naturally in accelerators
  - Momentum spread
  - Chromatic spread
  - Non-linearity of the longitudinal focusing (RF wave)
  - Non-linearity of collective forces (Space-charge, beam-beam)

    → **Watch out !** Due to their dynamic nature, the collective forces can lead to loss of Landau damping, by shifting the coherent mode frequencies w.r.t. the incoherent spectrum

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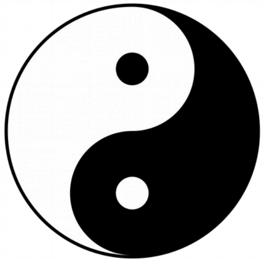
- Several aspect of accelerator design are driven by the need for Landau damping (Beam parameters, optics, operation, ...)
- Landau damping is **beneficial** to maintain the beam quality, however the means to generate Landau damping can have a bad impact on the trajectories of single particles, leading to a **deterioration** of the beam quality



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