Beam-beam effects

Lecture notes available at **https://xbuffat.web.cern.ch/beambeamCAS.pdf**

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Collective Effects and Impedances

CERN, Switzerland, Geneva

CERN Accelerator School – 20th November 2024

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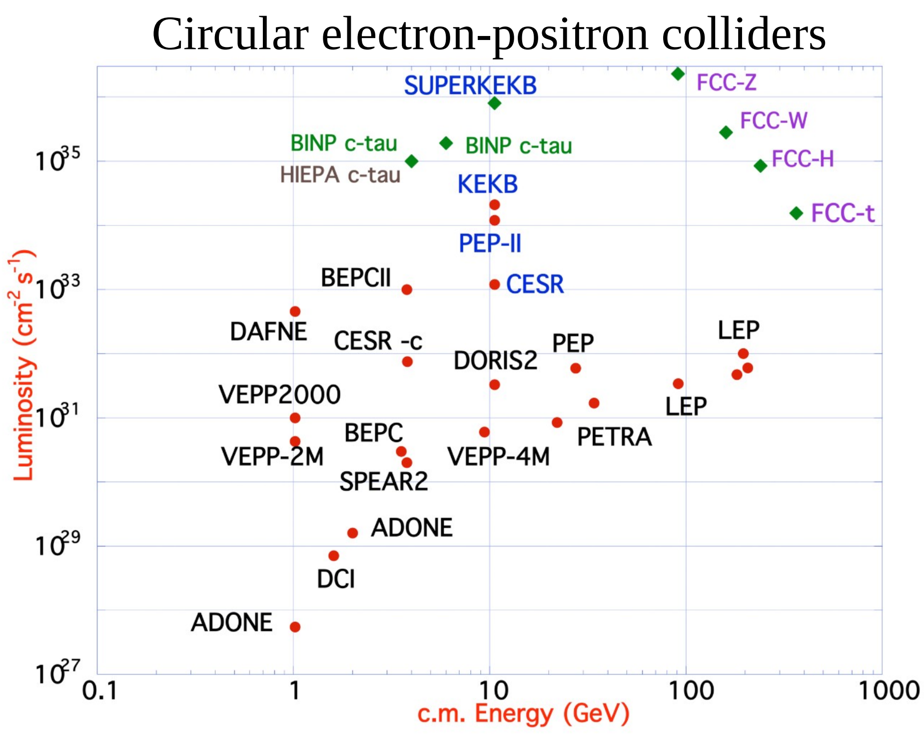
Collider pot-pourri

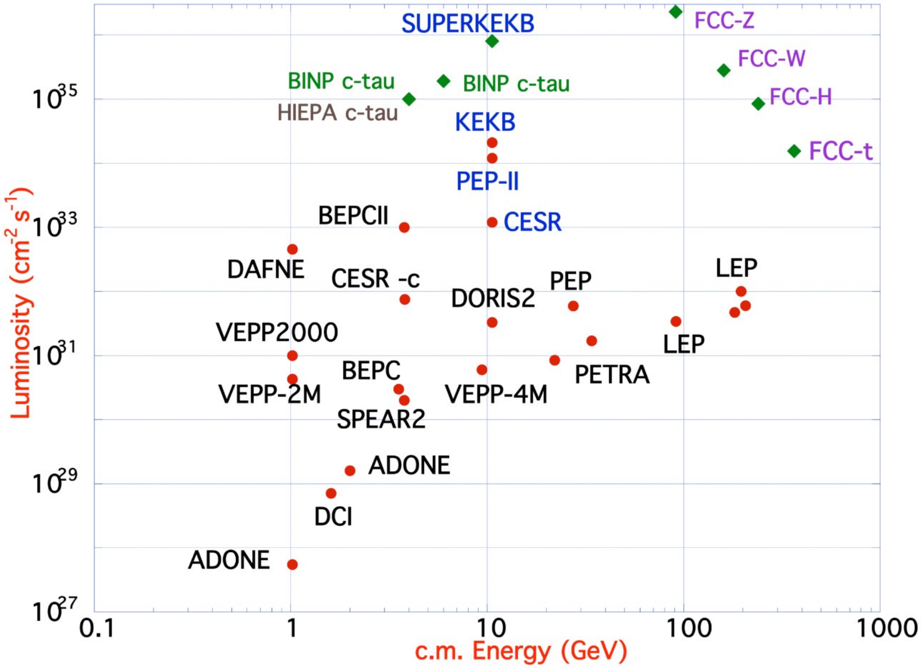
[Benedikt, SLC]



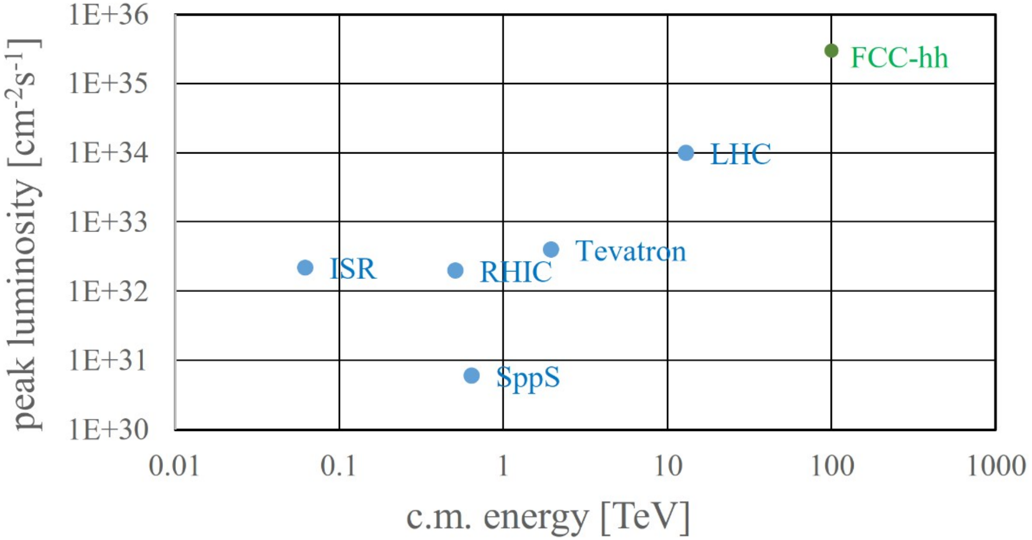
Collider pot-pourri

[Benedikt, SLC]

Circular electron-positron colliders



Circular hadron colliders



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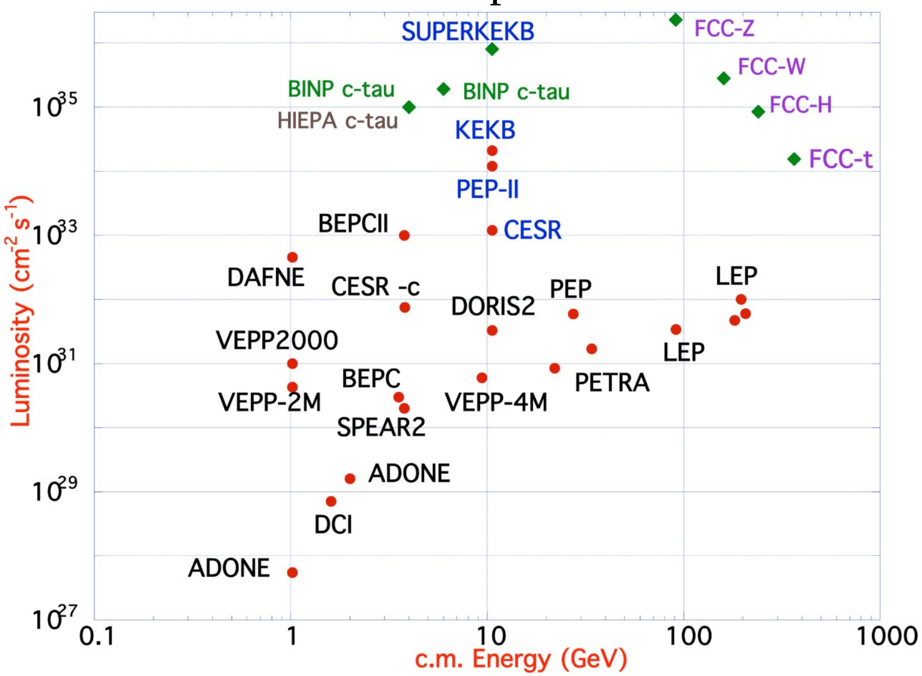

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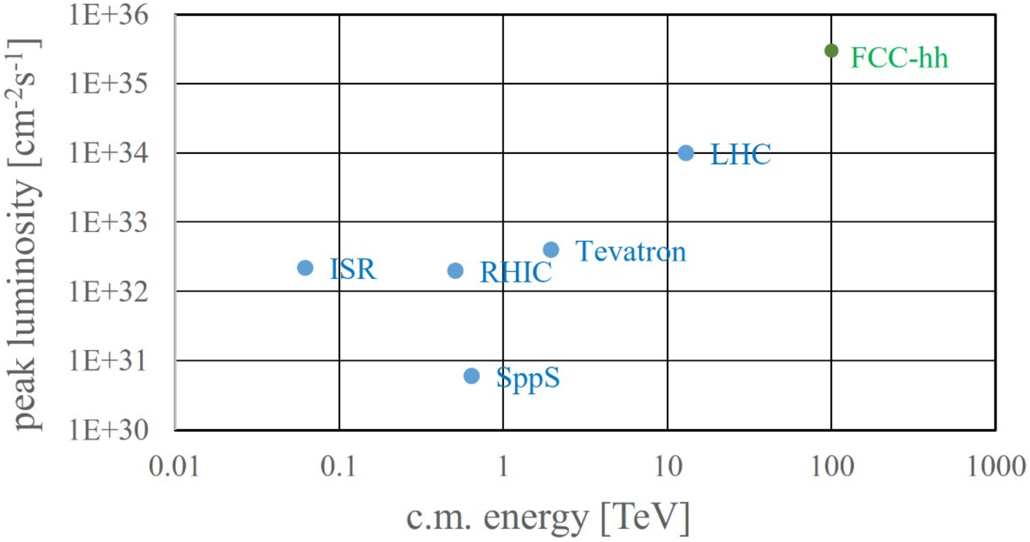

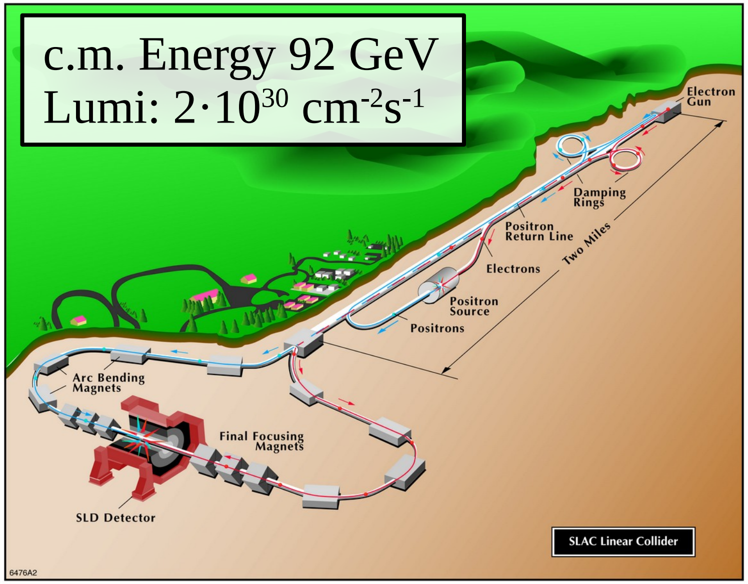

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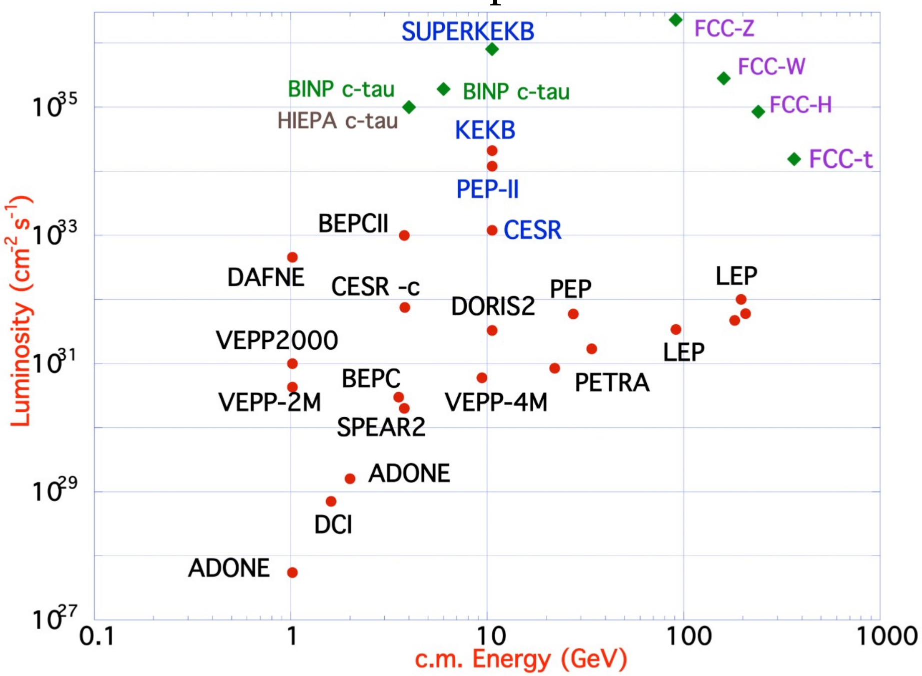

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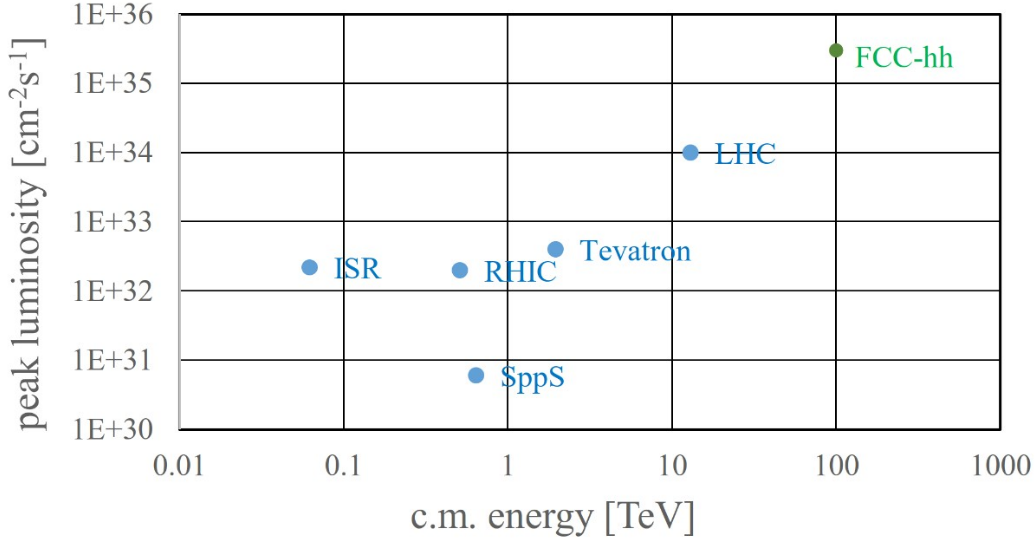
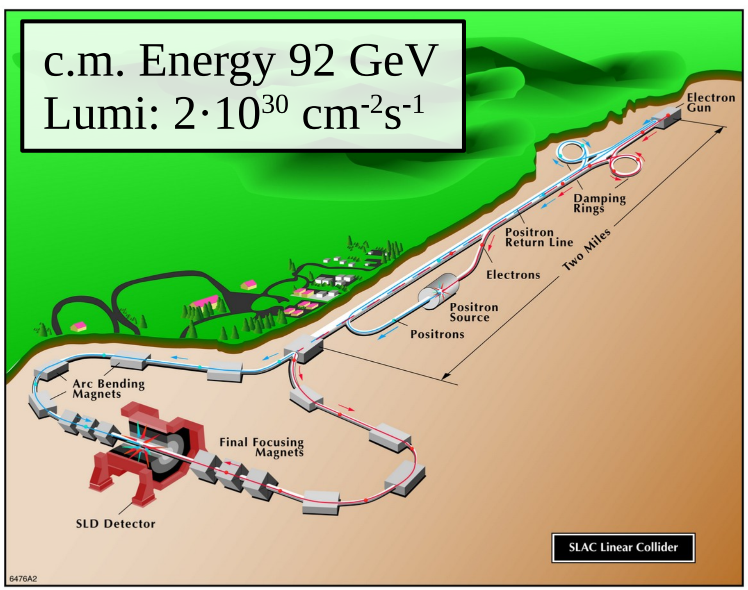
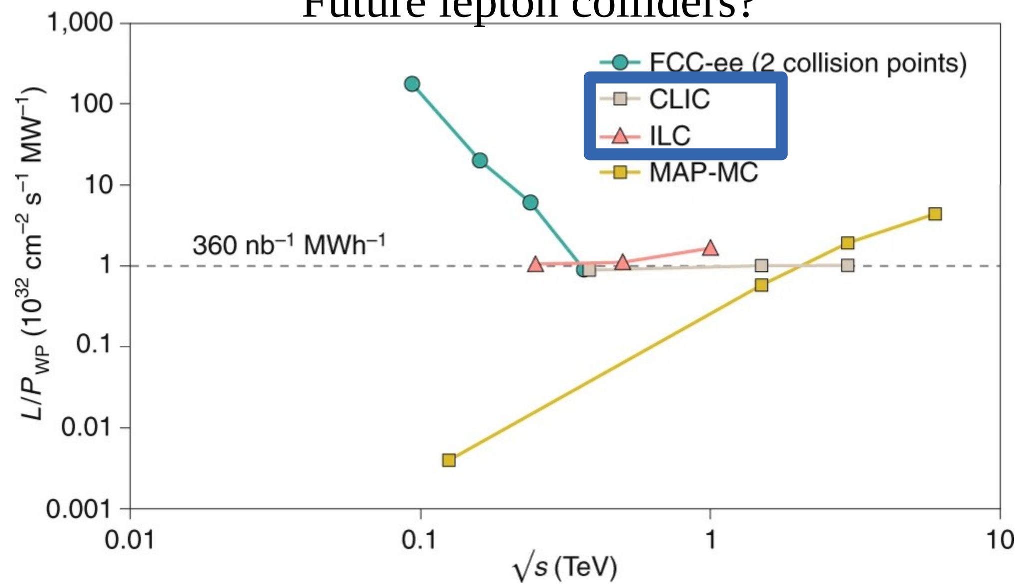
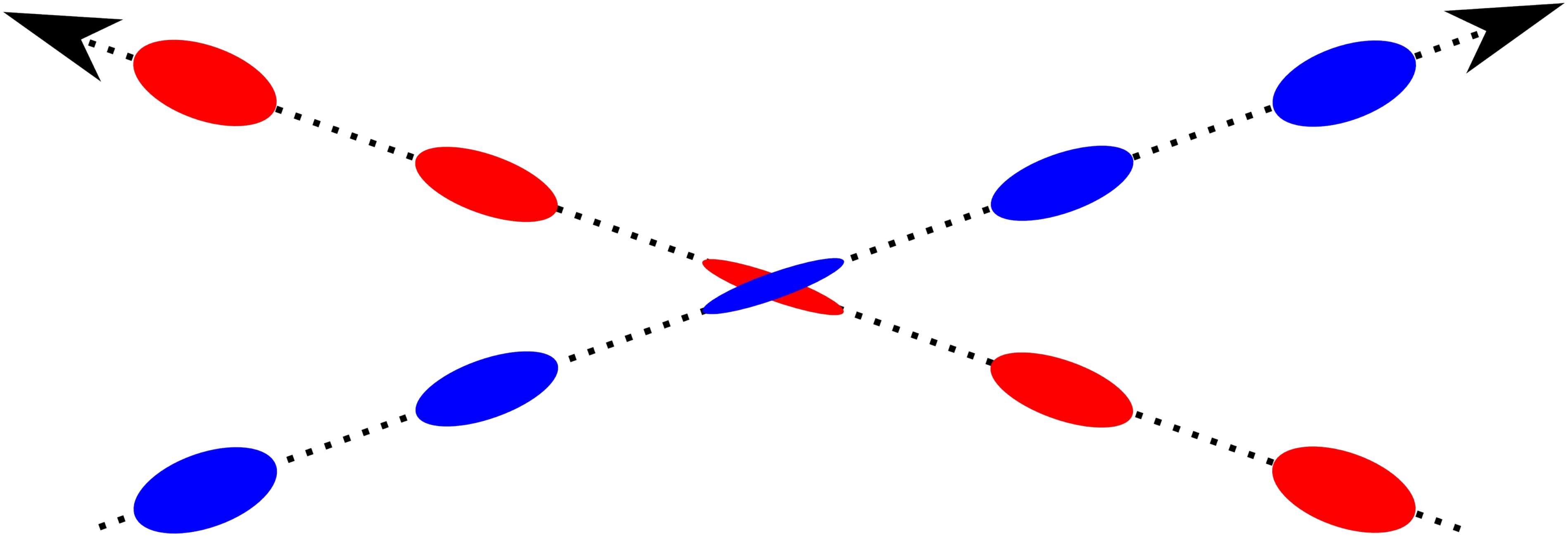




Future lepton colliders?

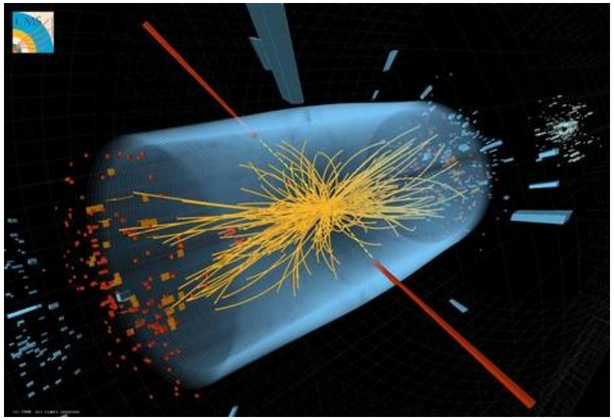


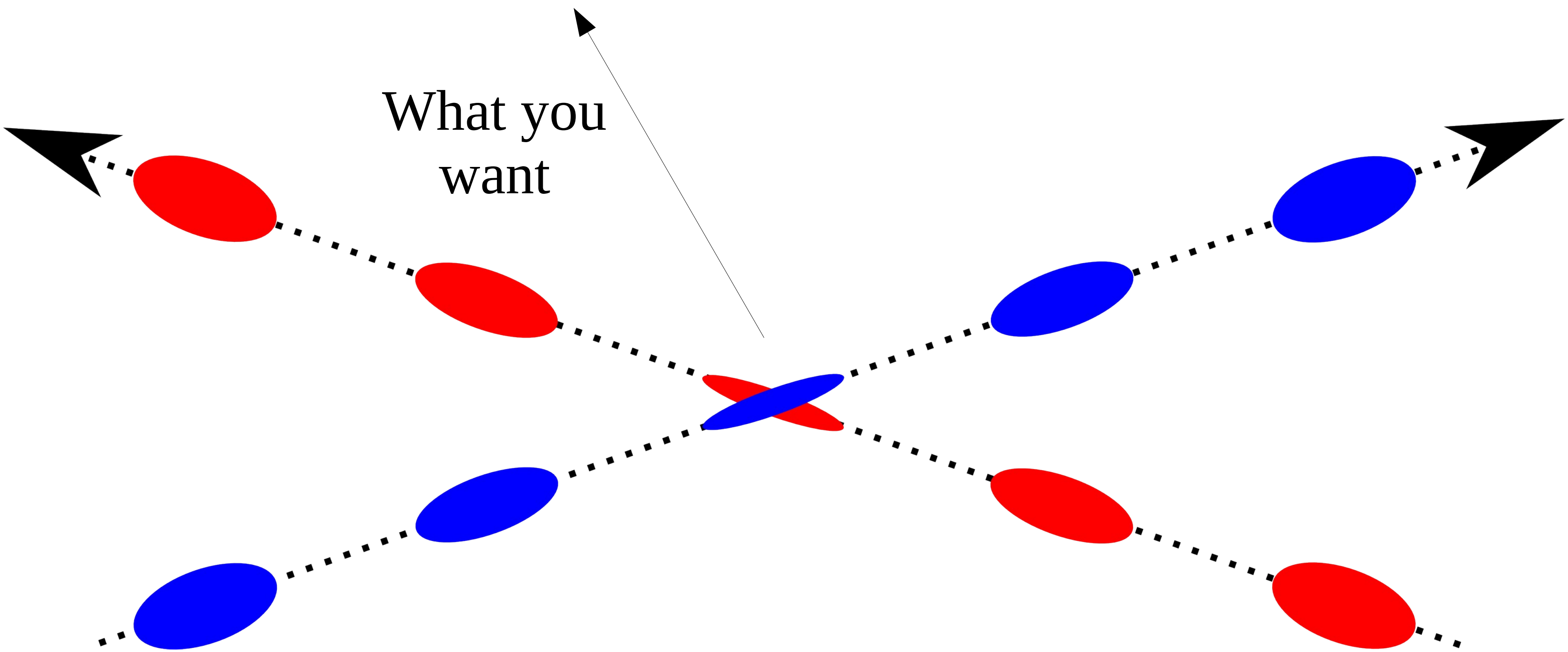
Beam-beam effects



Beam-beam effects

Candidate $H^0 \rightarrow \gamma\gamma$ event at CMS





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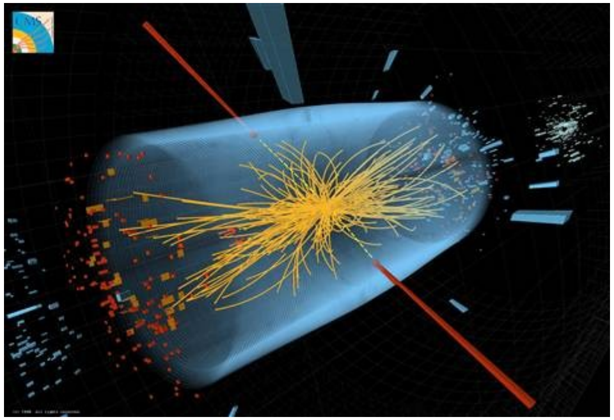
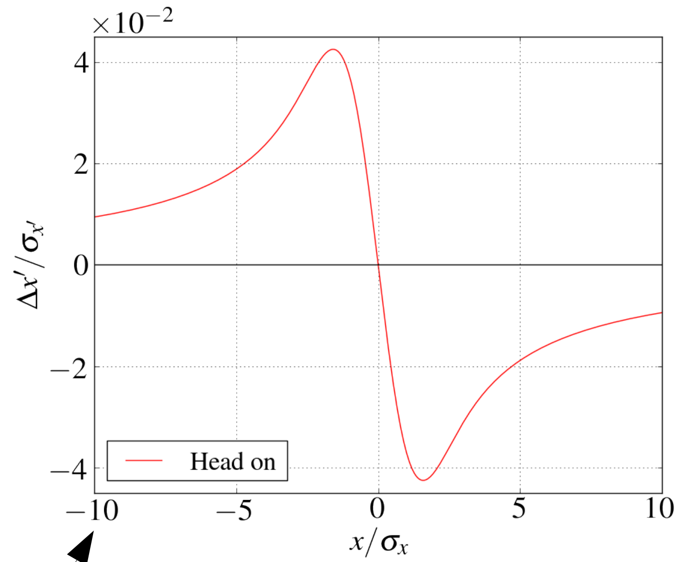
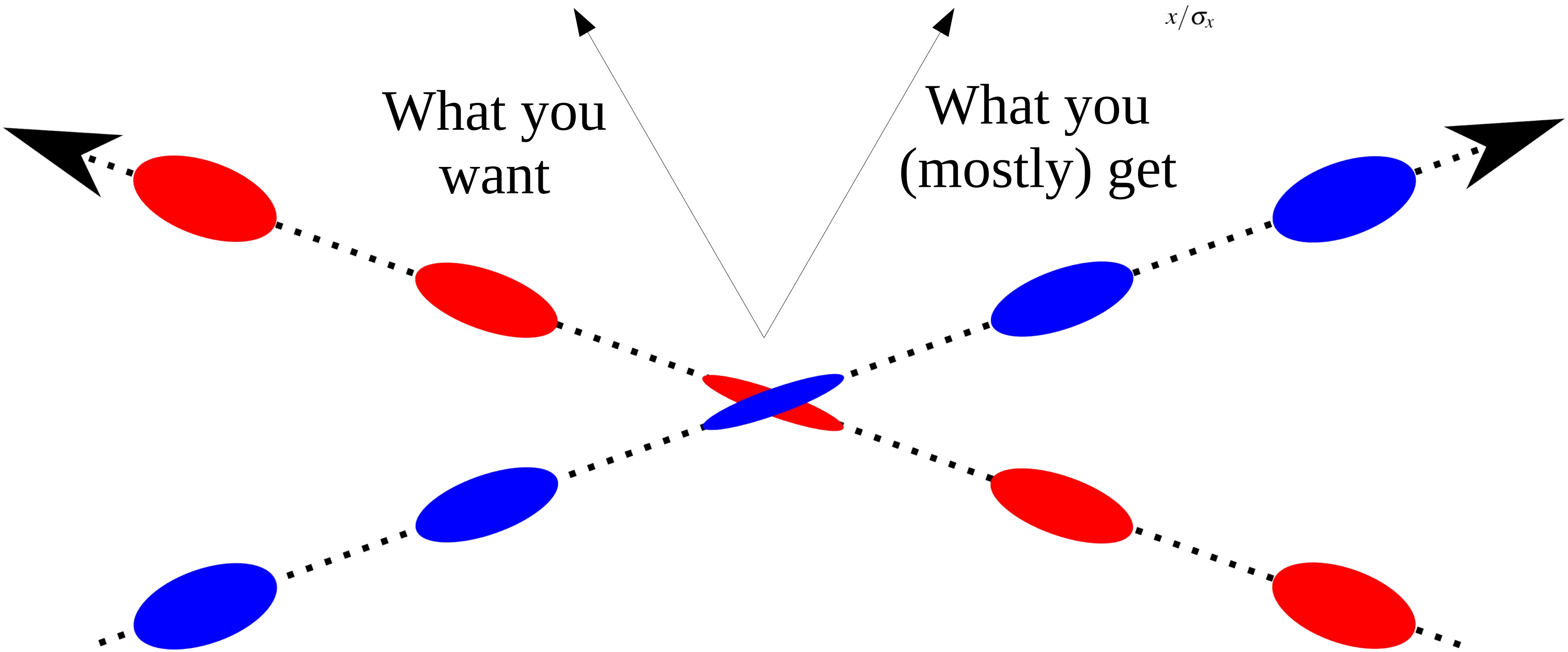




What you want

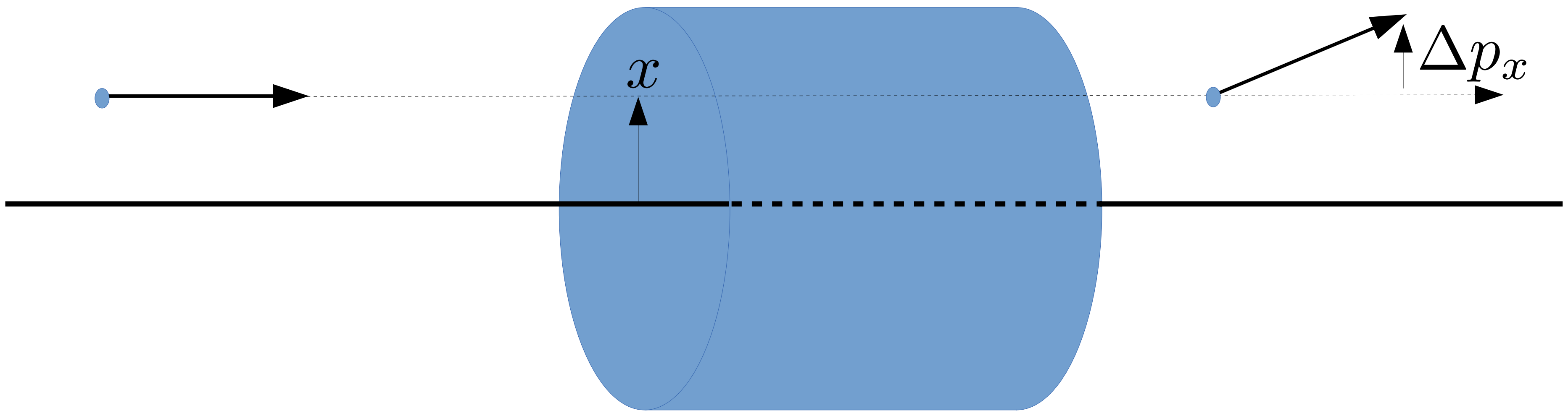
What you (mostly) get



Content

- The electromagnetic fields of colliding beams
- Dynamical effects
- Self-consistent solutions
- Non-linearities
- Beamstrahlung
- Summary

Kick from a uniformly charged cylinder



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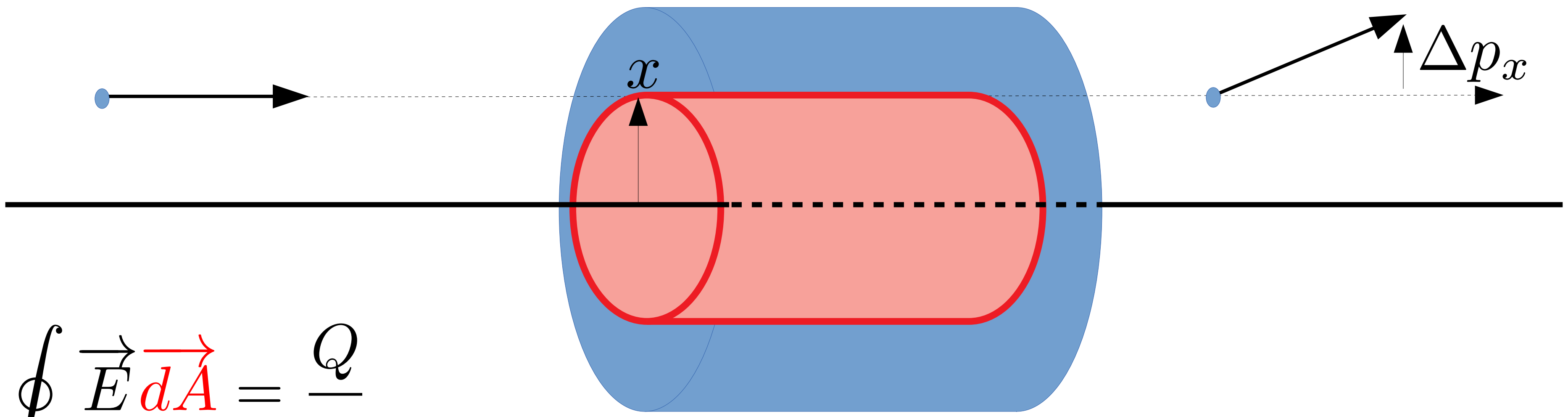

$$\oint \vec{E}\,\vec{dA} = \frac{Q}{\epsilon_0}$$

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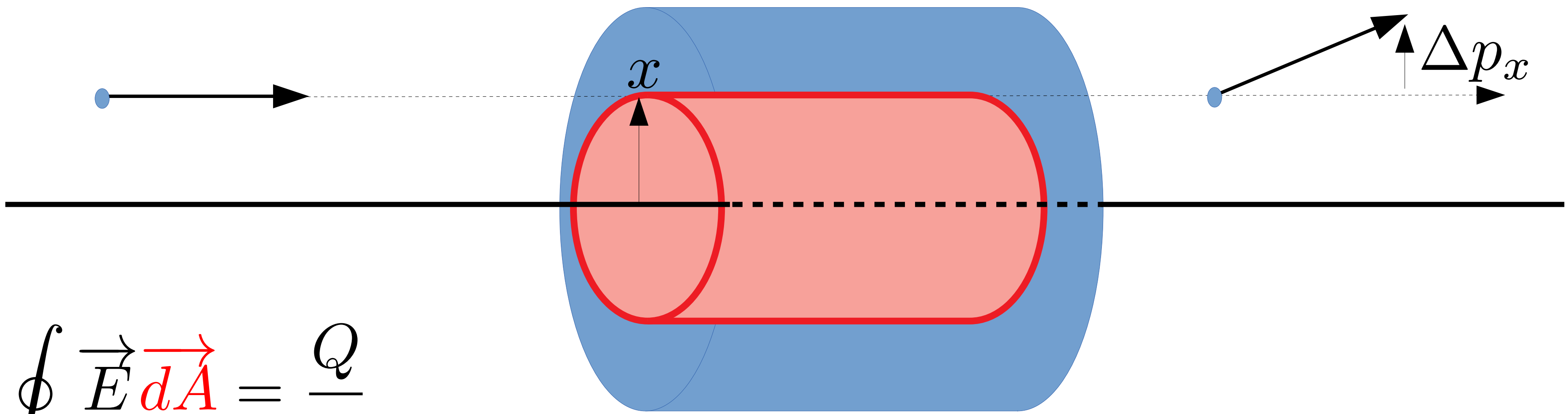

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long cylinder

Inside: $$E_x 2\pi x L = \frac{\pi x^2 L \rho}{\epsilon_0}$$

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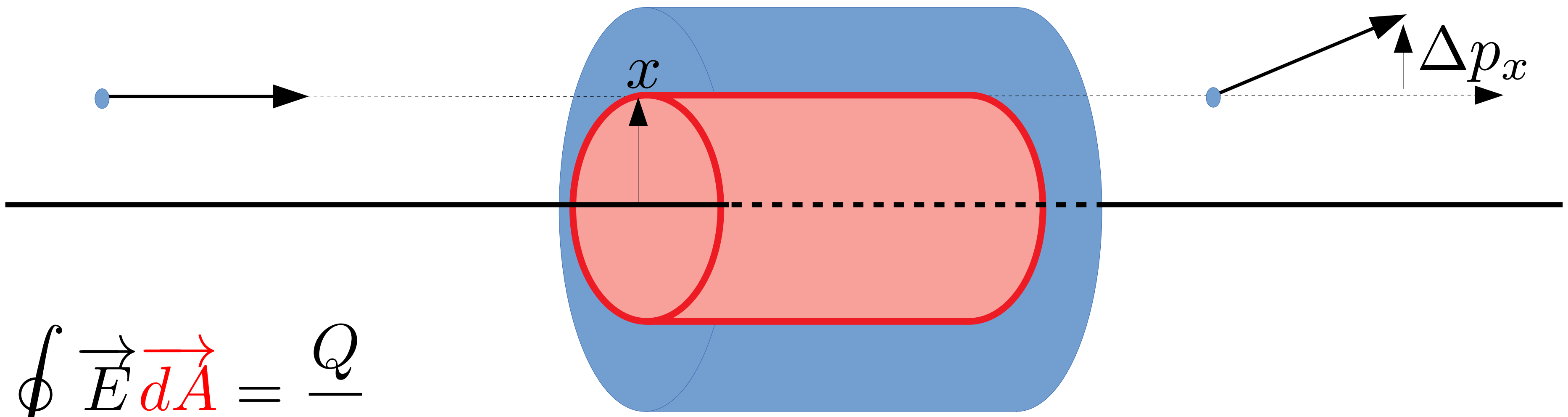

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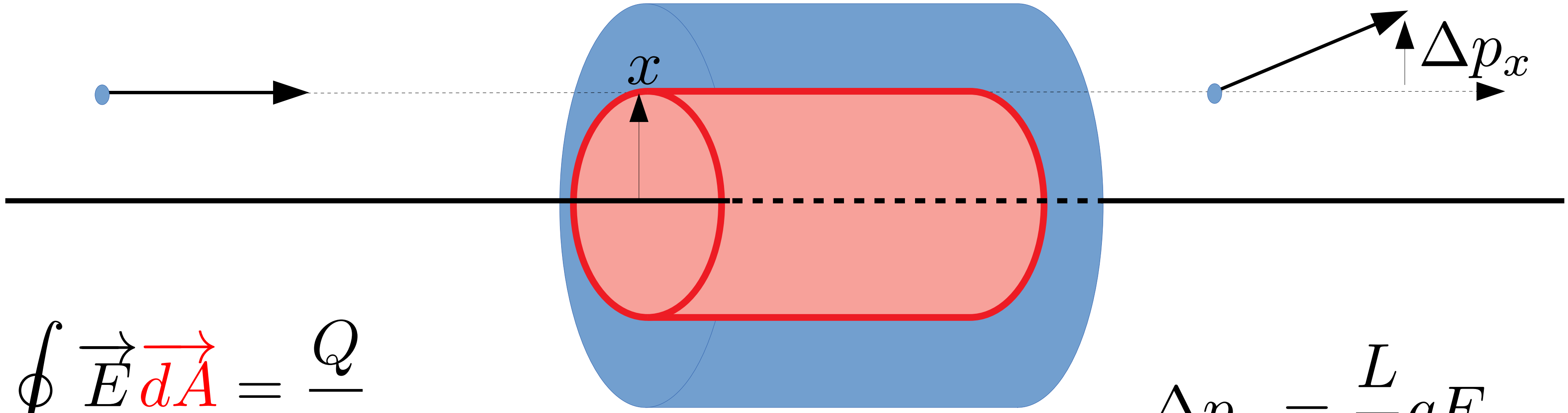

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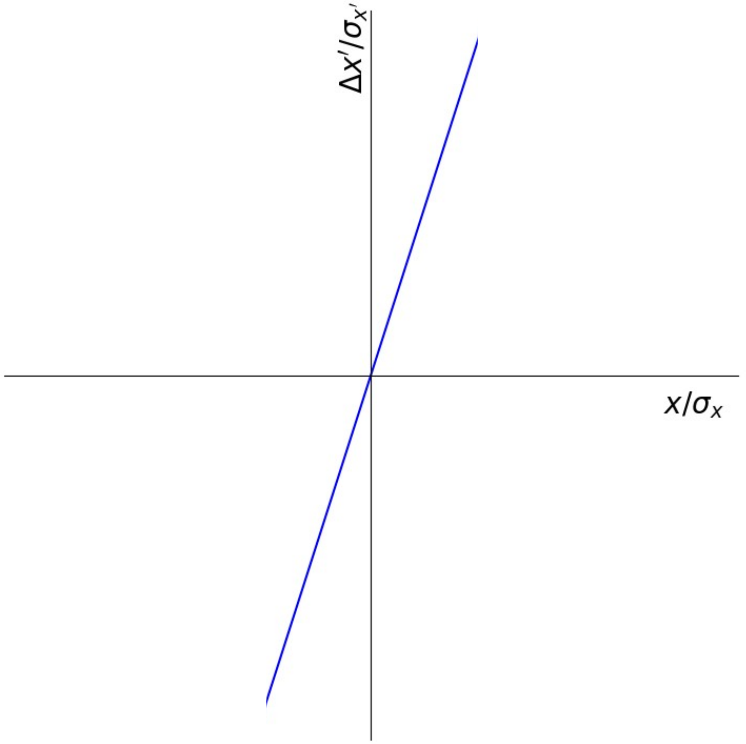
$$\Delta p_x = \frac{L}{c} q E_x$$



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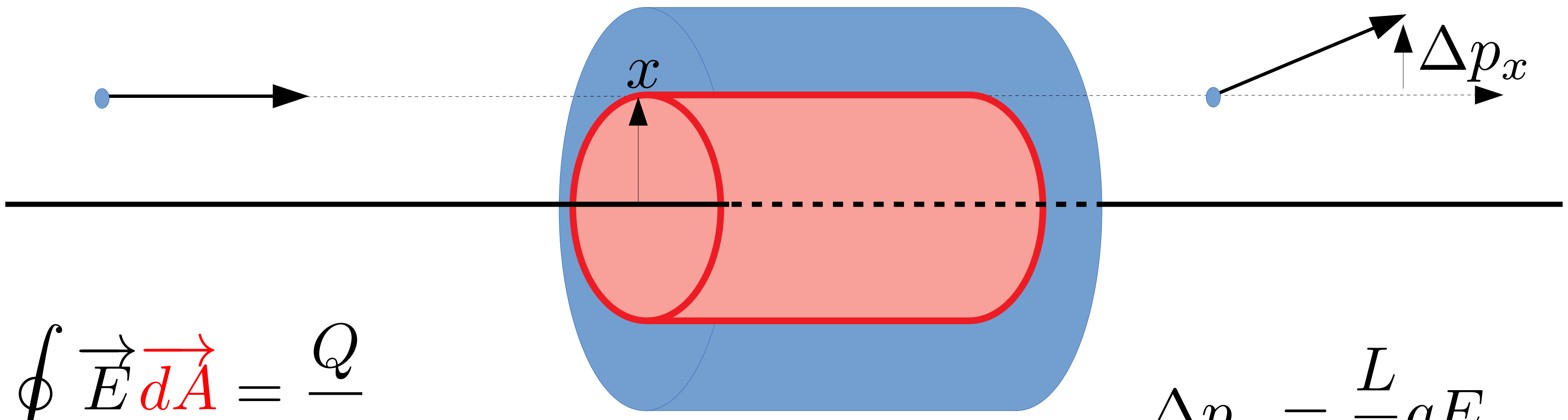


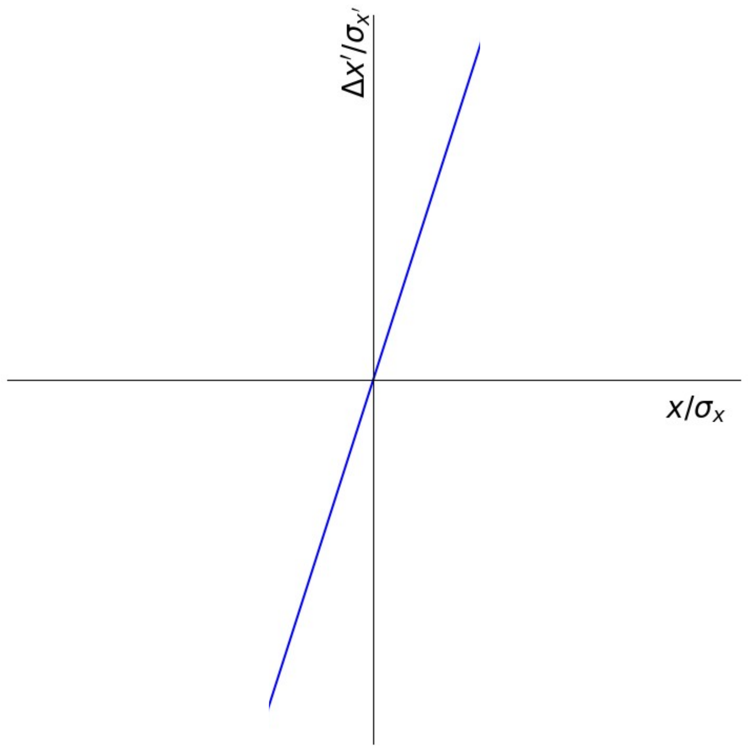
$$\oint \vec{E}\,\vec{dA} = \frac{Q}{\epsilon_0}$$

long cylinder

Inside: $$E_x 2\pi x L = \frac{\pi x^2 L \rho}{\epsilon_0} \qquad E_x = \frac{\rho}{2\epsilon_0} x$$

Outside: $$E_x 2\pi x L = \frac{Q_{tot}}{\epsilon_0}$$

$$\Delta p_x = \frac{L}{c} q E_x$$



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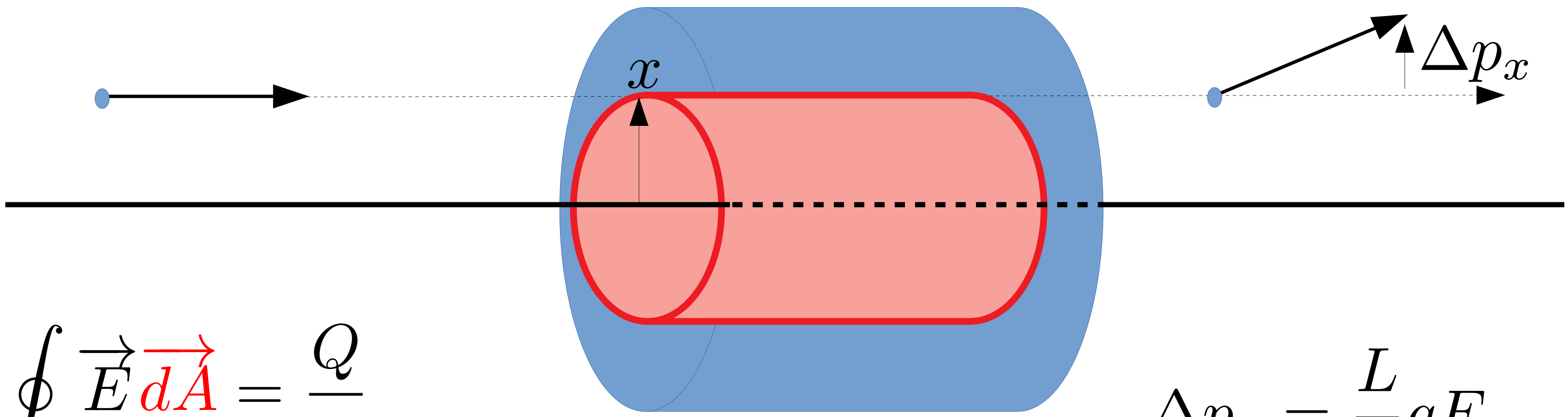

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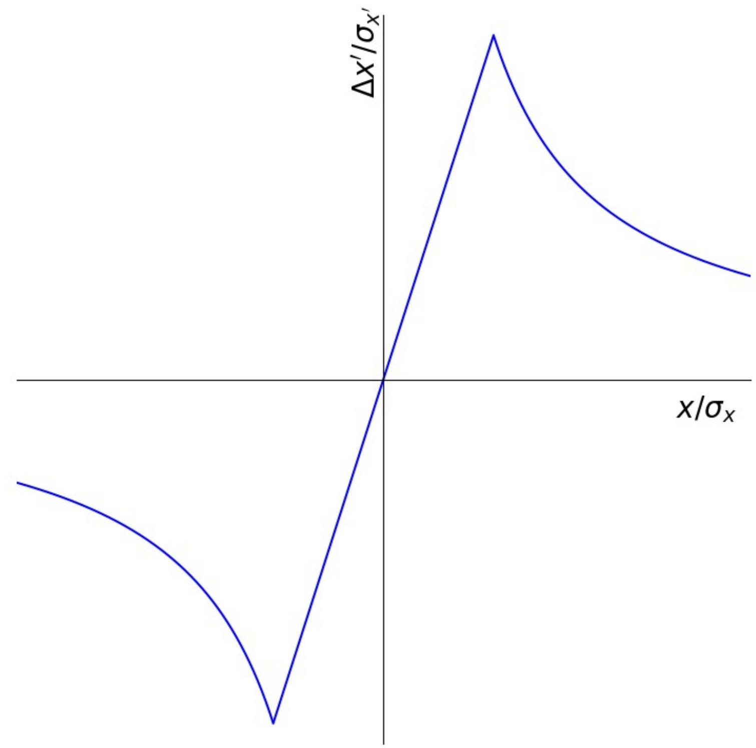
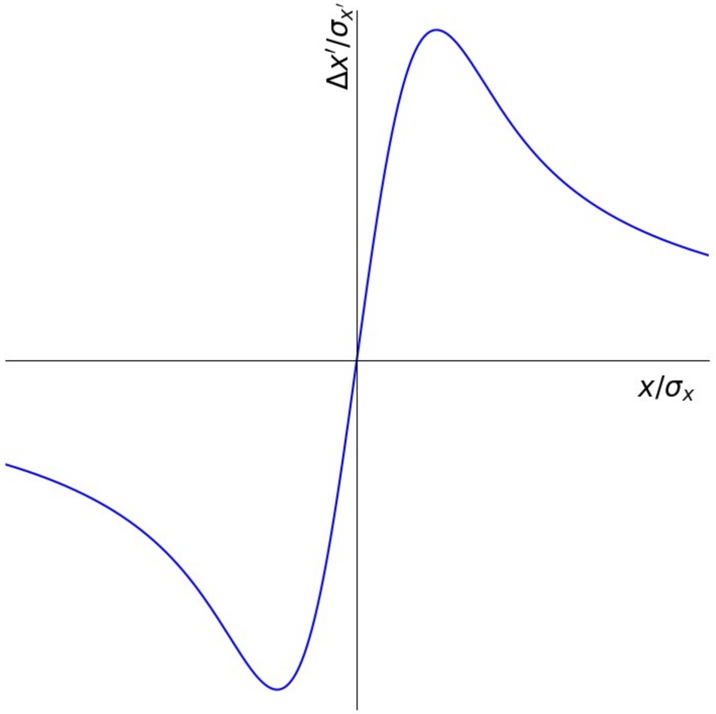


Kick from a round Gaussian beam

$$\Delta x' = \pm \frac{2Nr_0}{\gamma} \frac{x}{r^2} \left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)$$



Kick from a round Gaussian beam

- (i.e. focusing) for opposite charged beams
+ otherwise

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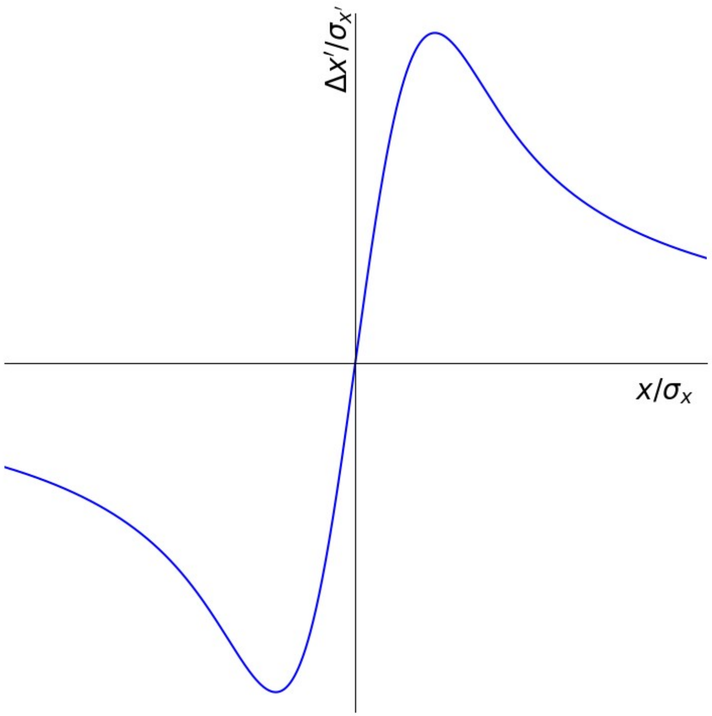

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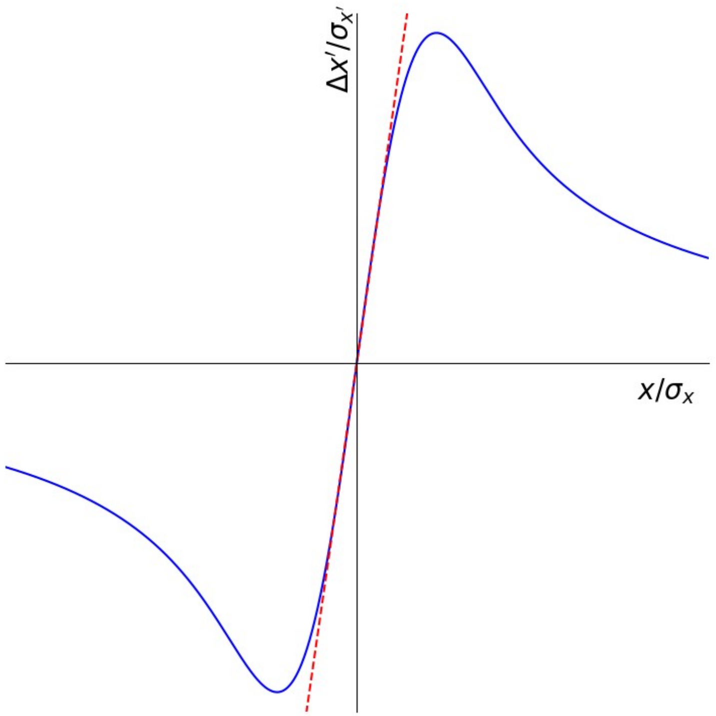

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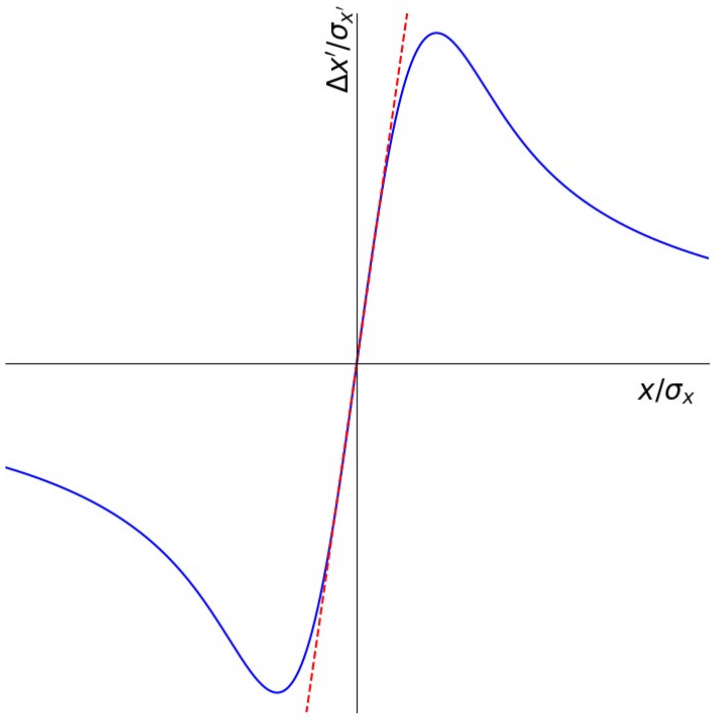
$$\cos(2\pi(Q_0 + \Delta Q_{BB})) = \cos(2\pi Q_0) - \frac{\beta_0^*}{2 f_{BB}} \sin(2\pi Q_0)$$



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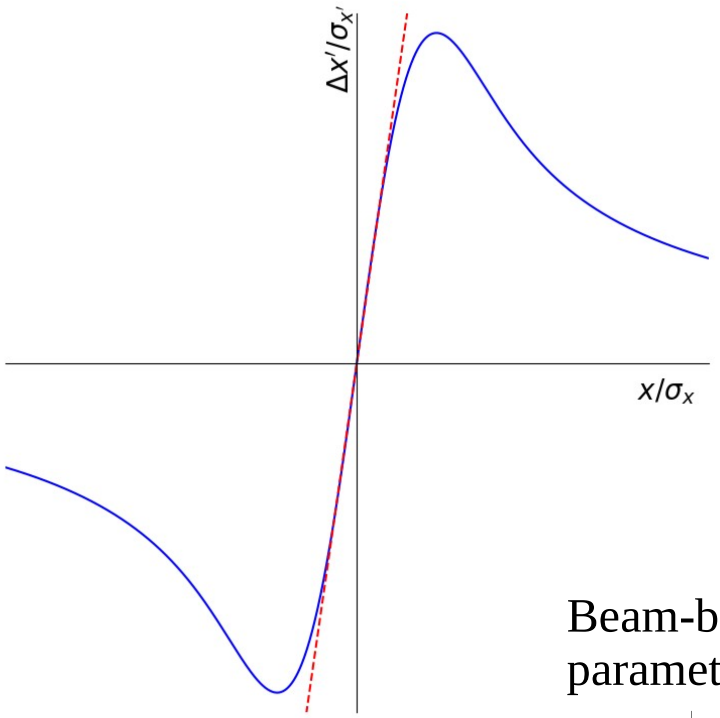

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- For small shifts and away from integer and half-integer resonances we have:

Beam-beam parameter

$$\Delta Q_{BB} \approx \mp \frac{Nr_0}{4\pi\epsilon_n} \equiv \xi$$

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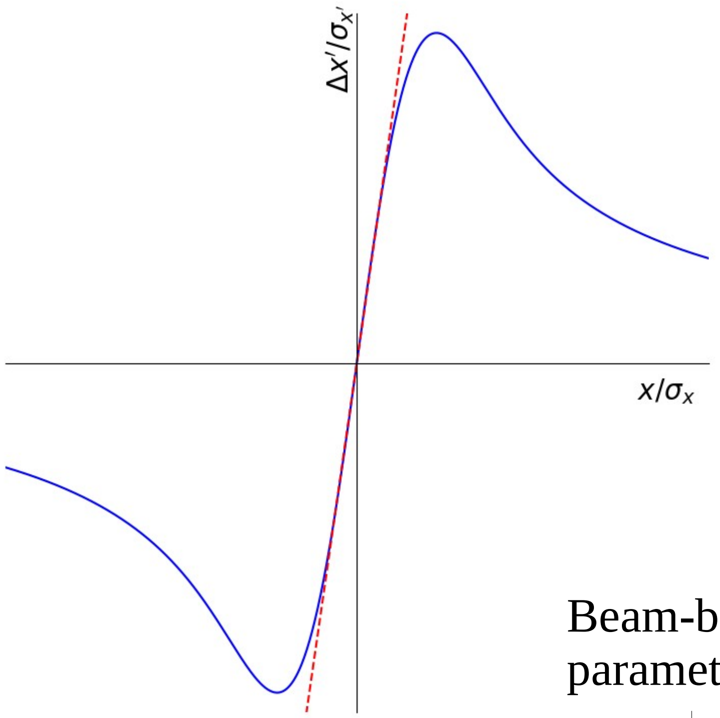

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- In these conditions the beam-beam tune shift is **independent of the beam energy** and of β^*

Kick from a Gaussian beam

[BassettiErskine]

- When the transverse beam sizes are not equal, the we get the Bassetti-Erskine formula:

$$\Delta y' + i\Delta x' = \frac{4Nr_0}{\gamma}\sqrt{\frac{\pi}{2(\sigma_x^2-\sigma_y^2)}}\left(w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2))}}\right) - e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}} w\left(\frac{\frac{\sigma_y}{\sigma_x}x + i\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)\right)$$

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Complex error function

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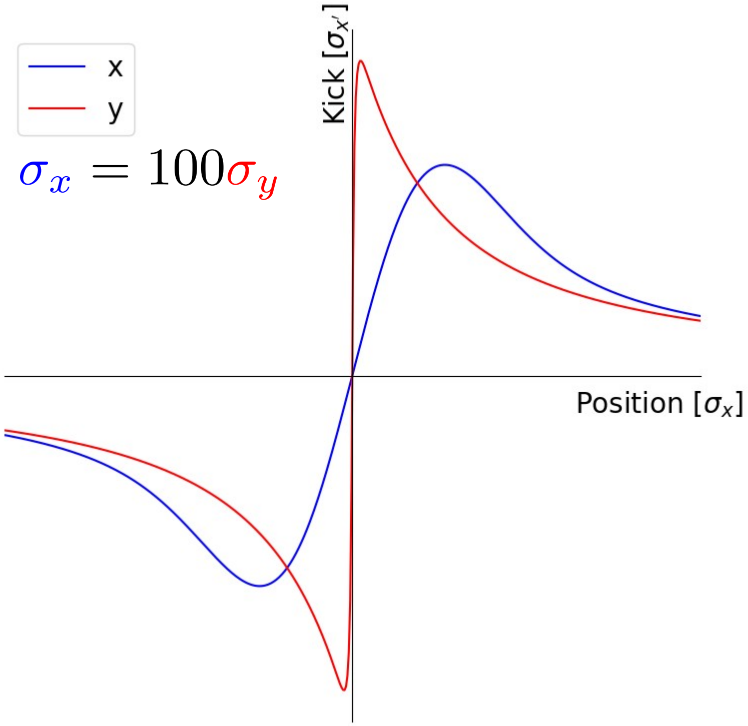

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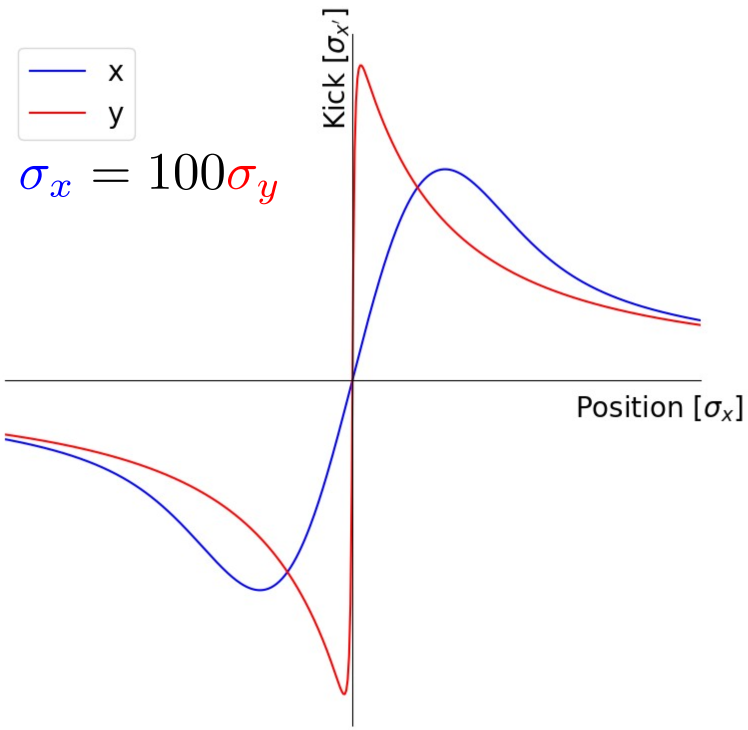

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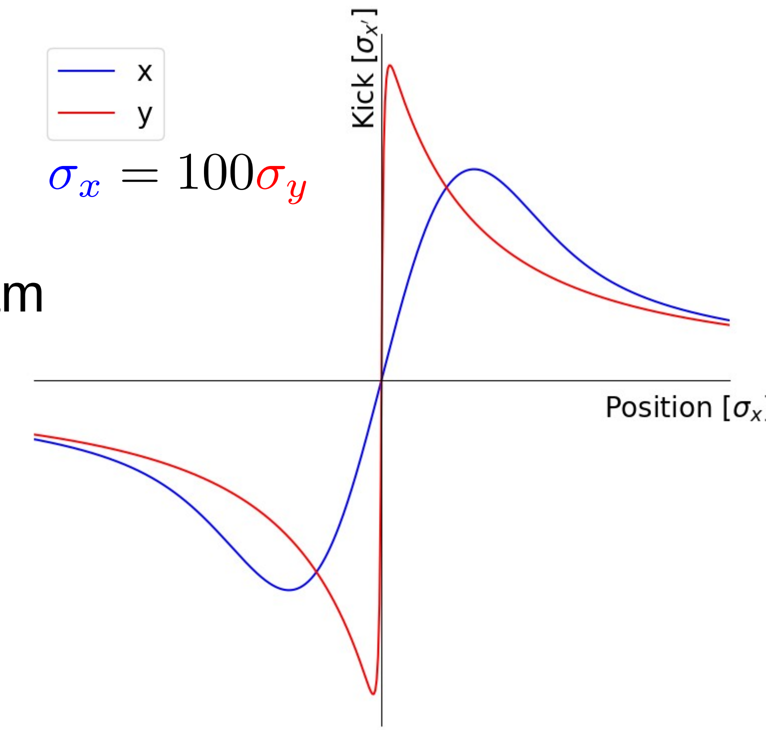

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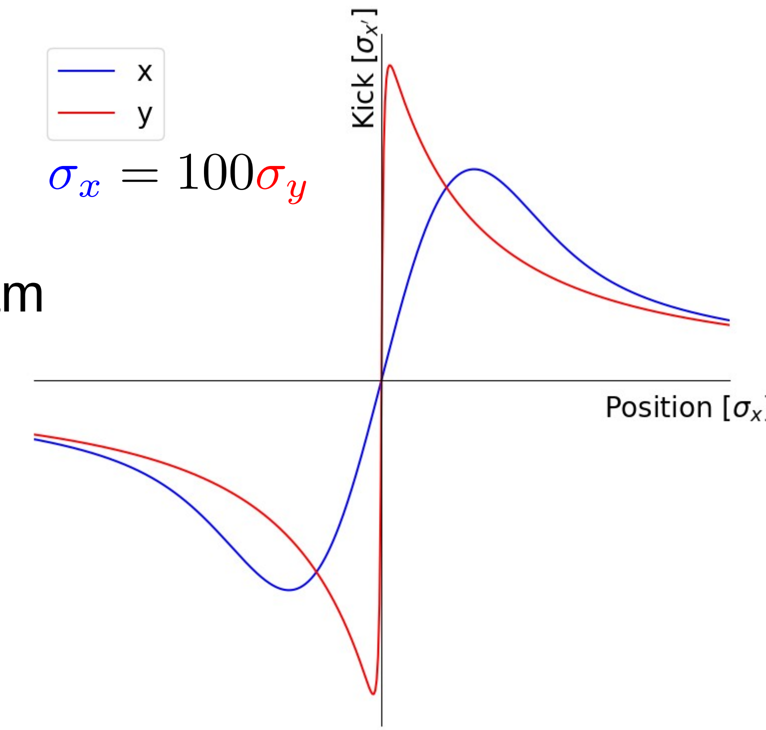


- When the beams are **not round** the beam-beam tune shift depends on the energy and the β*s

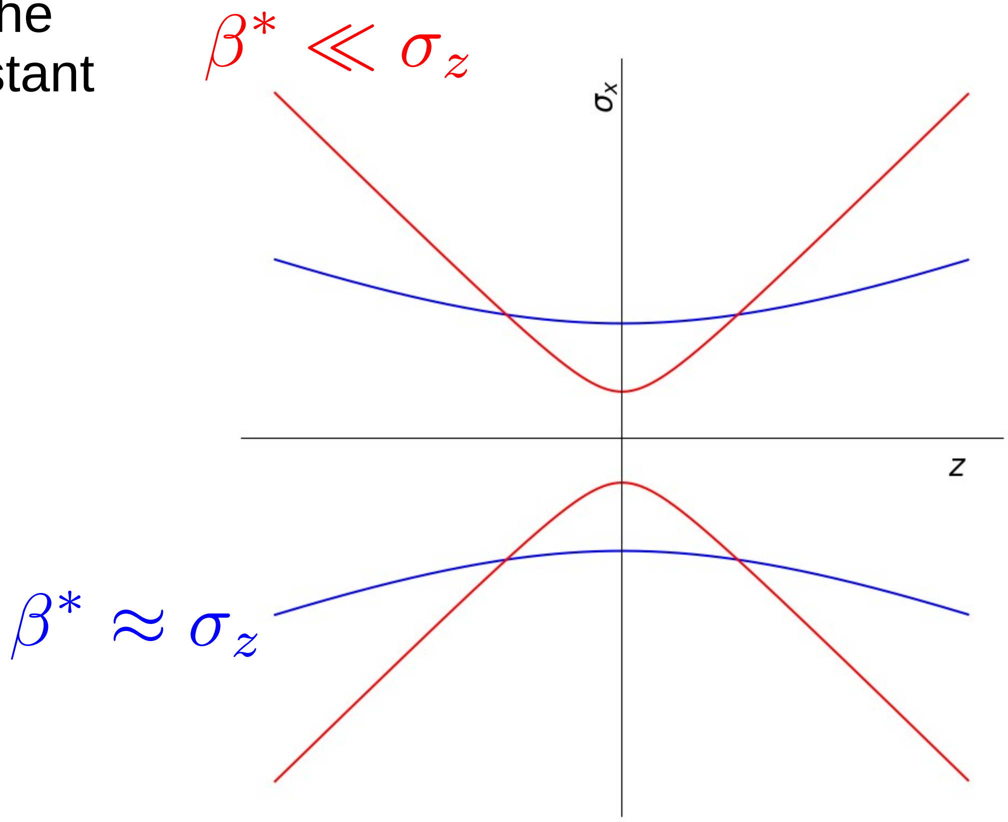
- For flat beams $\sigma_y \ll \sigma_x$

$$\xi_x = \frac{Nr_0}{2\pi\gamma\epsilon_x} \qquad \xi_{x,y} = \frac{Nr_0\beta^*_y}{2\pi\gamma\sigma_y\sigma_x}$$

Finite bunch length effects: Hourglass

- When the focusing at the IP is strong, the transverse beam size is no longer constant through the beam-beam interaction

  → Hourglass effect



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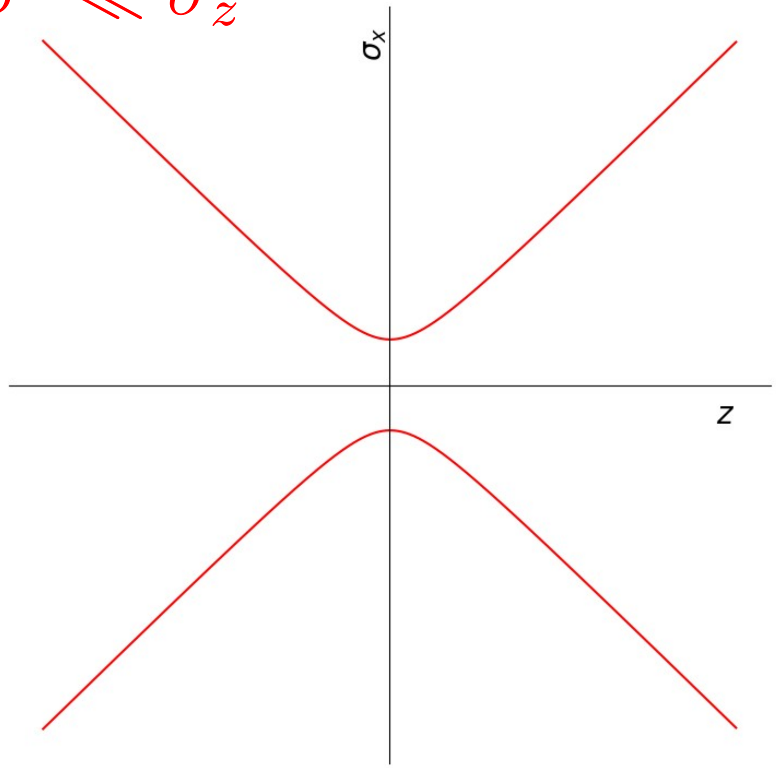
$\beta^* \ll \sigma_z$



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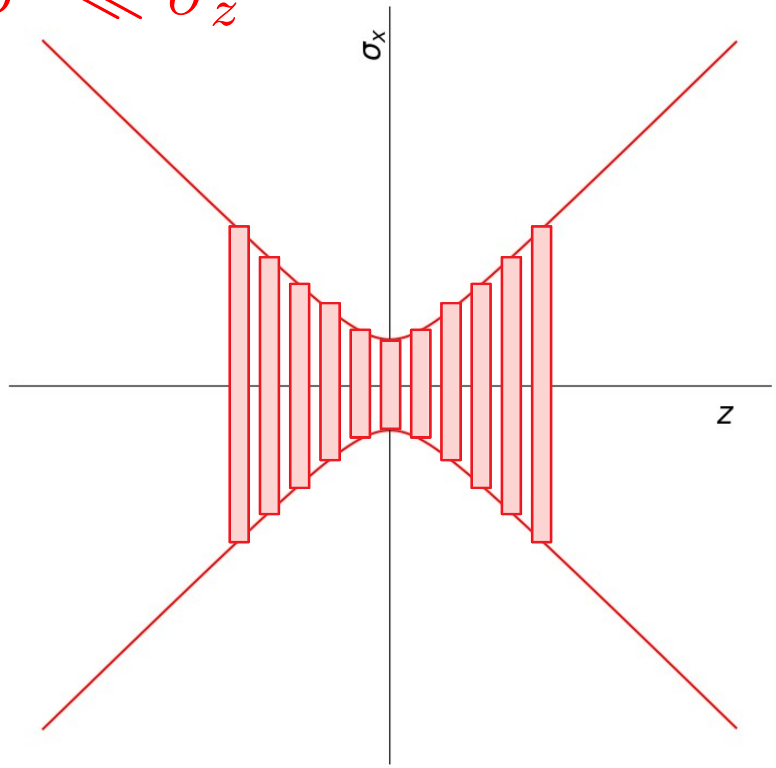

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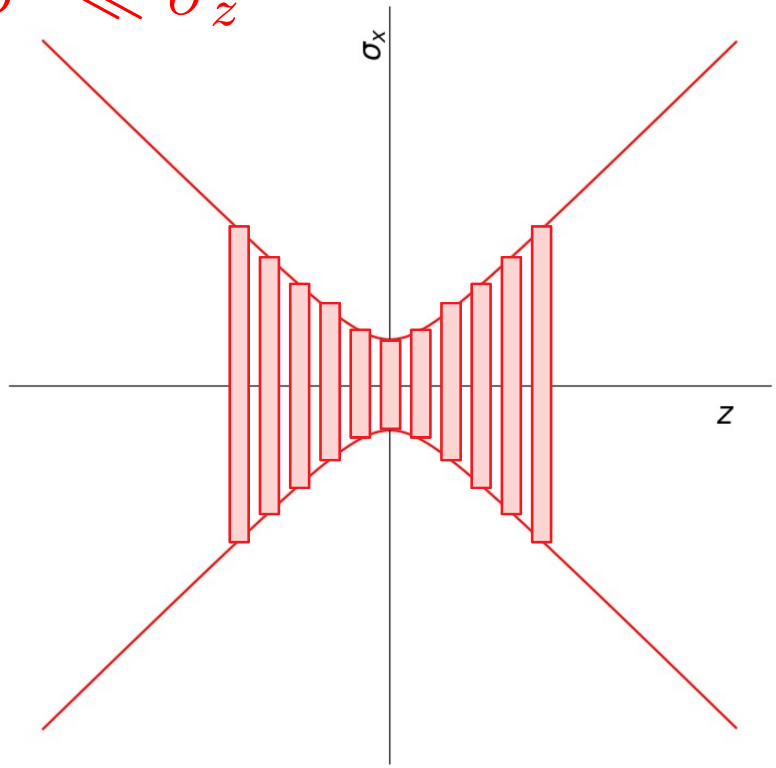

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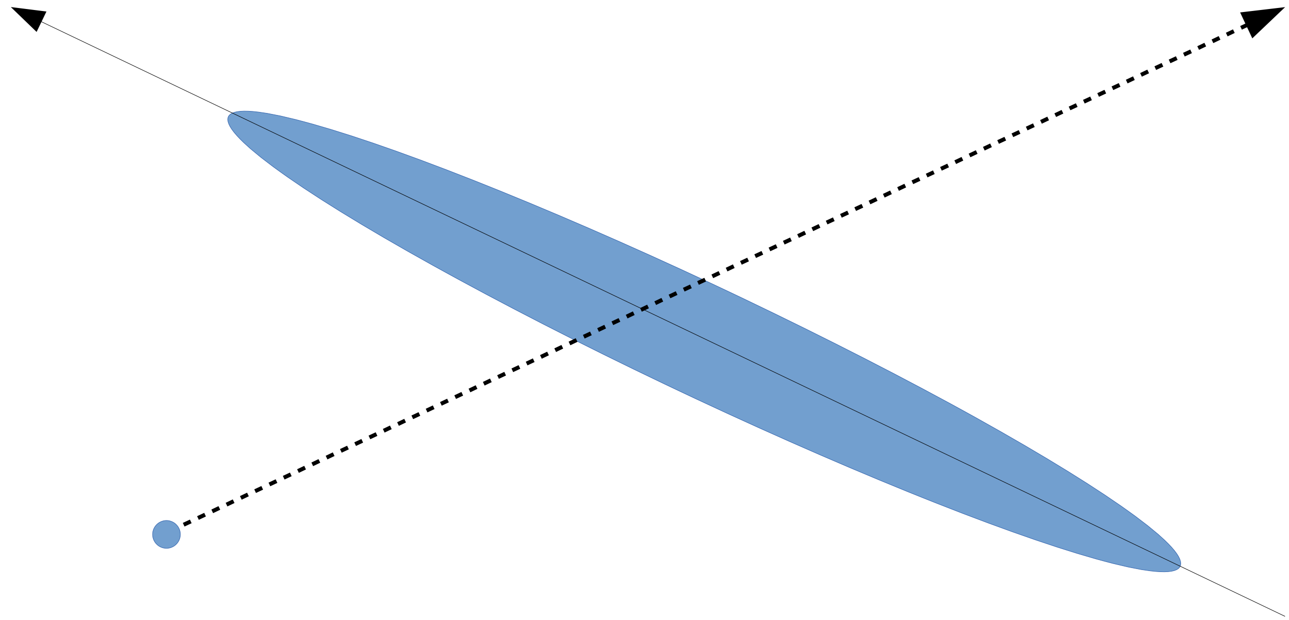
- Note: We assume that fields are purely transverse → ultra-relativistic approximation

Finite bunch length effects: Crossing angle [Hirata]

- When the beams collide with a crossing angle, the fields are no longer perpendicular to the propagation of the particle

  → Use a boosted frame that follows the transverse position of the particle



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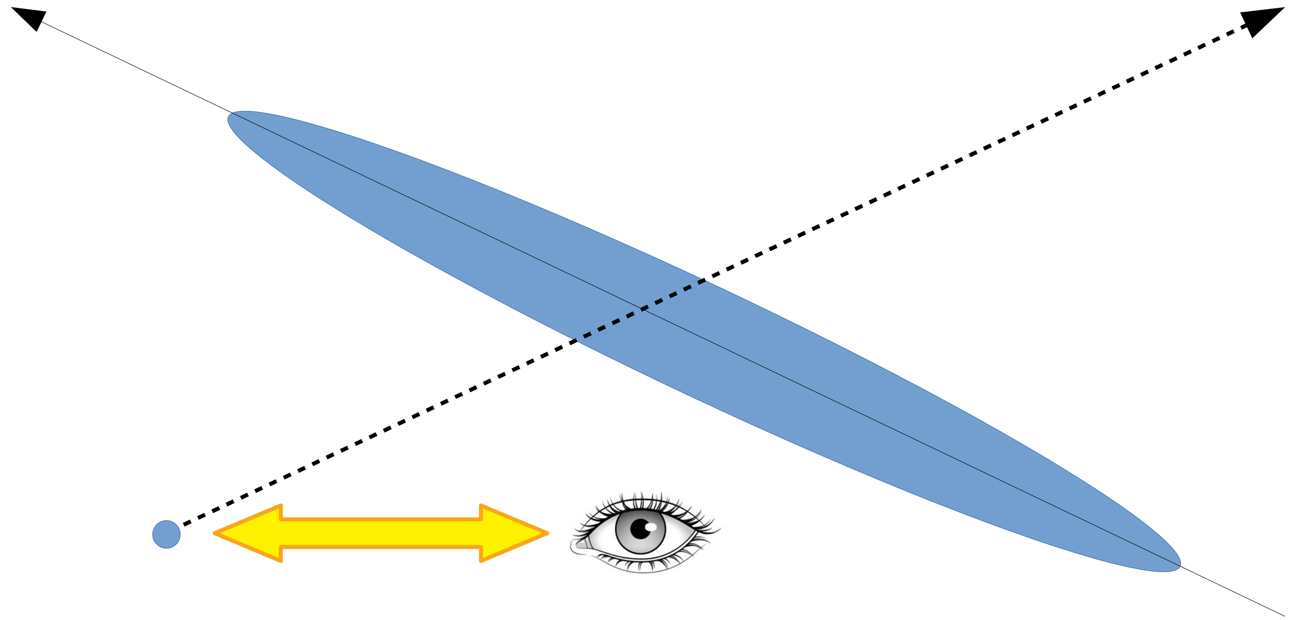

Finite bunch length effects: Crossing angle [Hirata]

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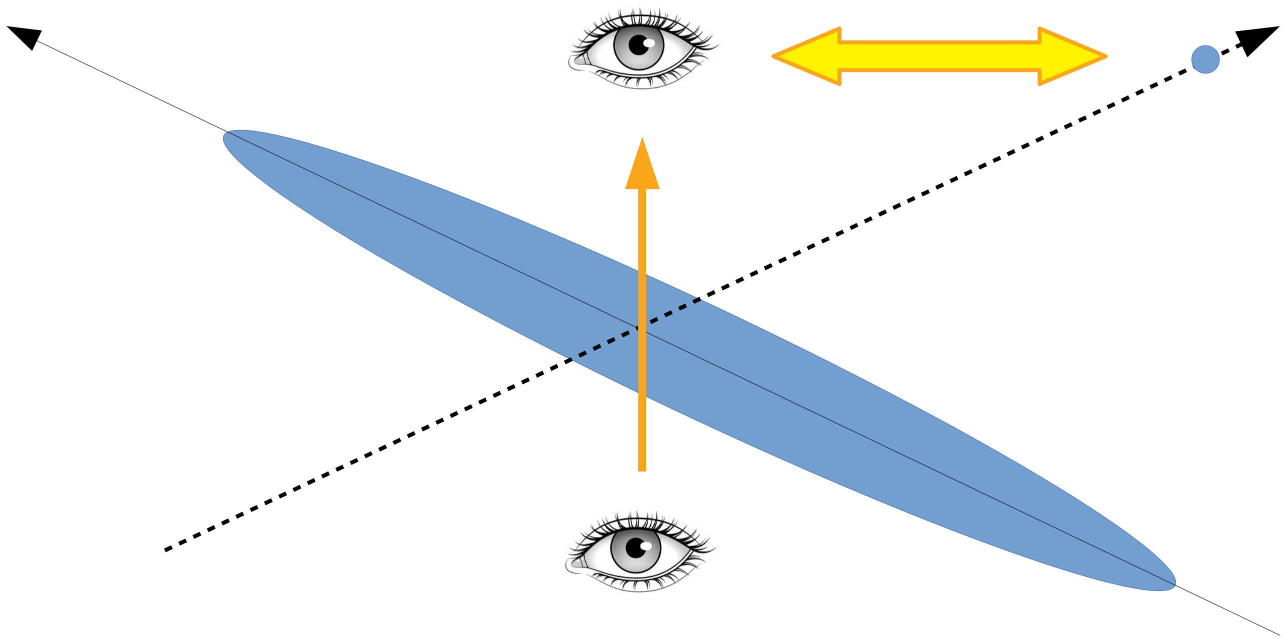

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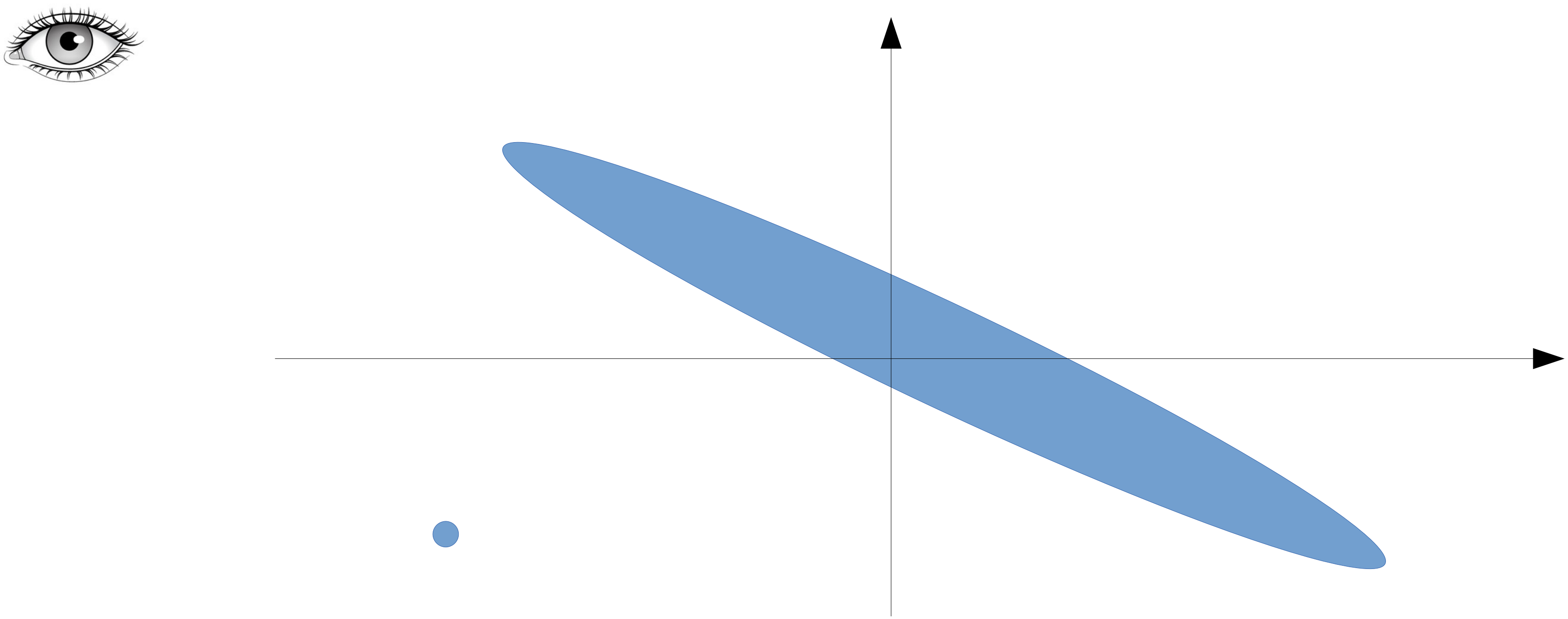

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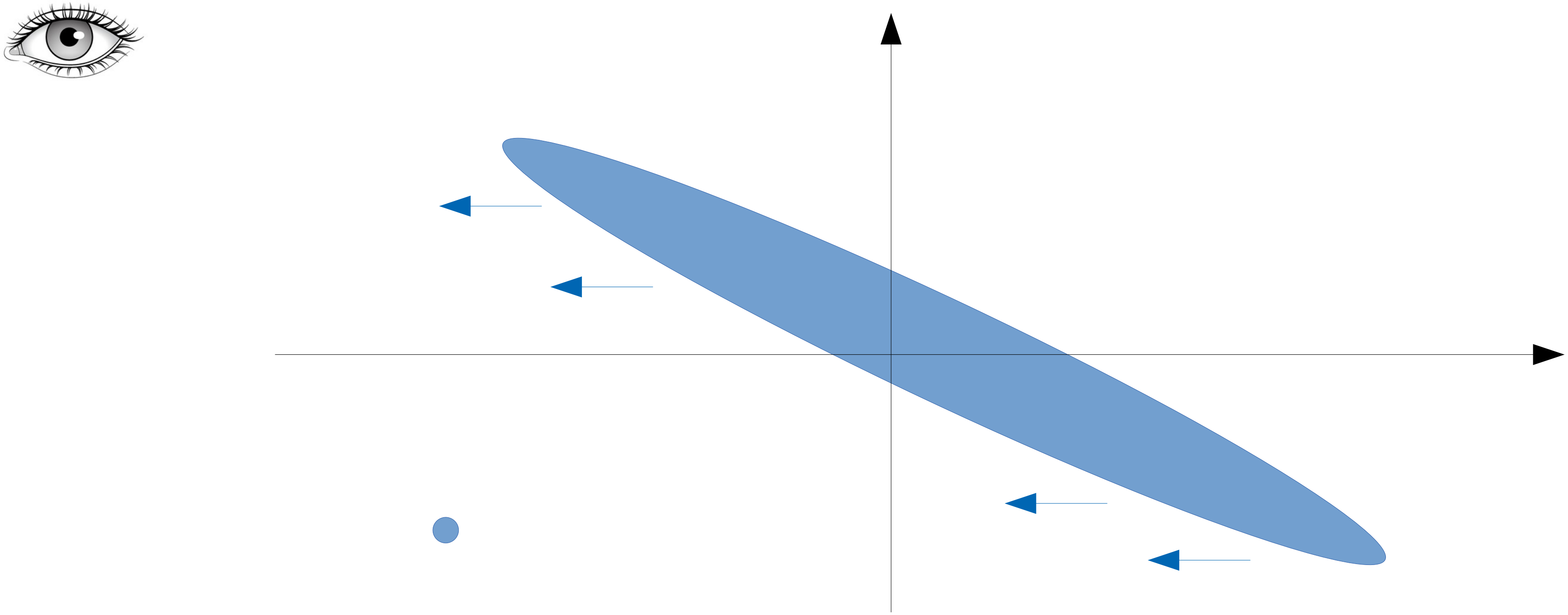

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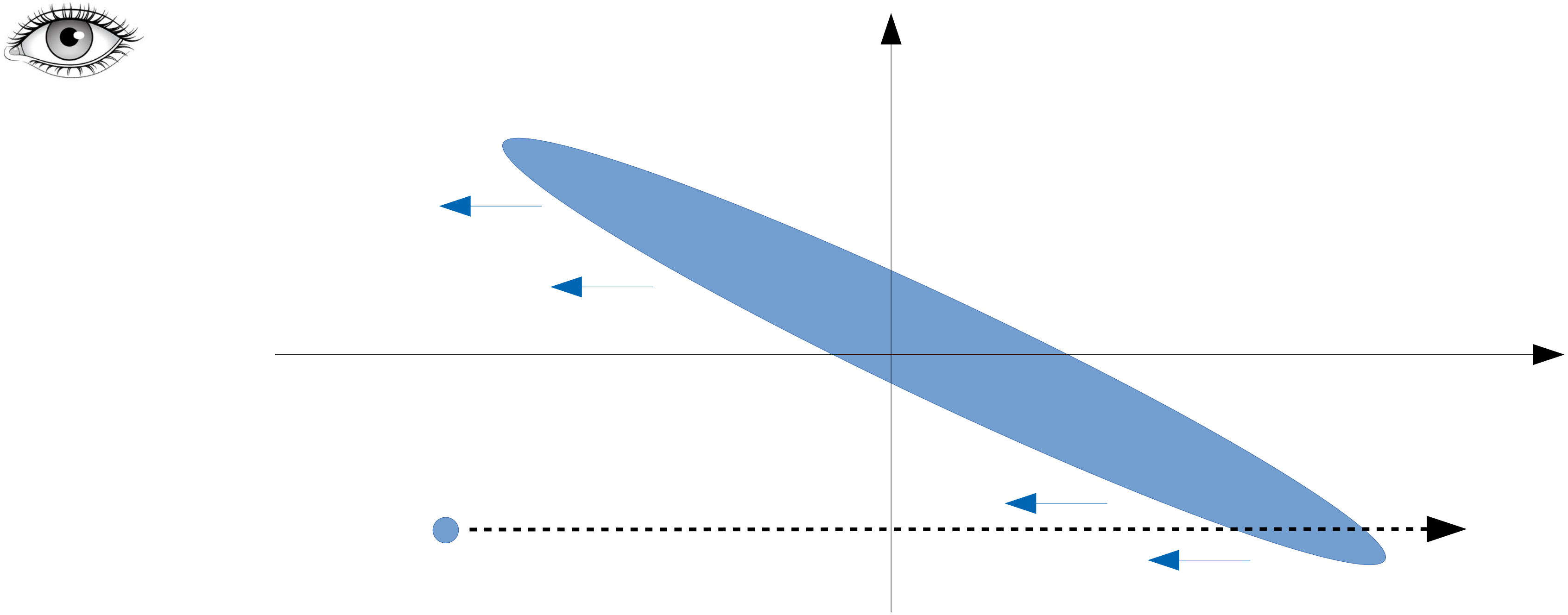

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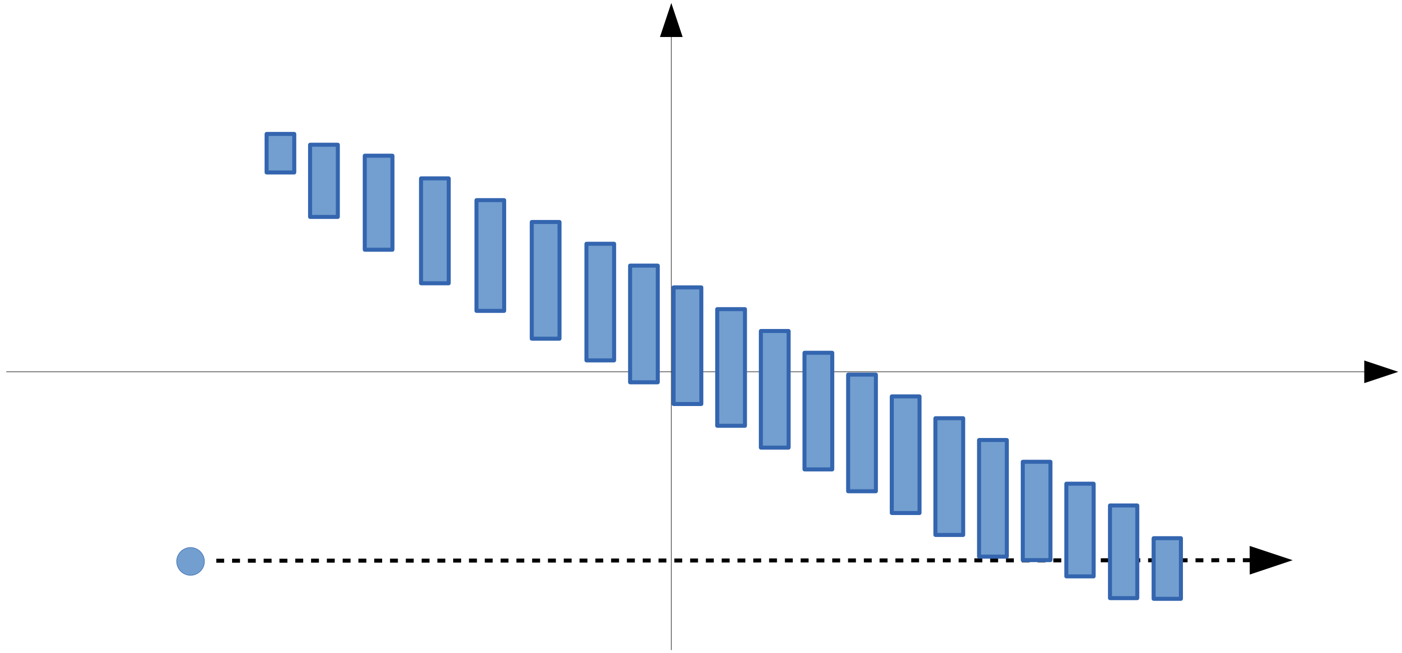


- In the boosted frame the collision can again be discretised in a set of beam-beam kicks with varying offset (and size if hourglass is strong)

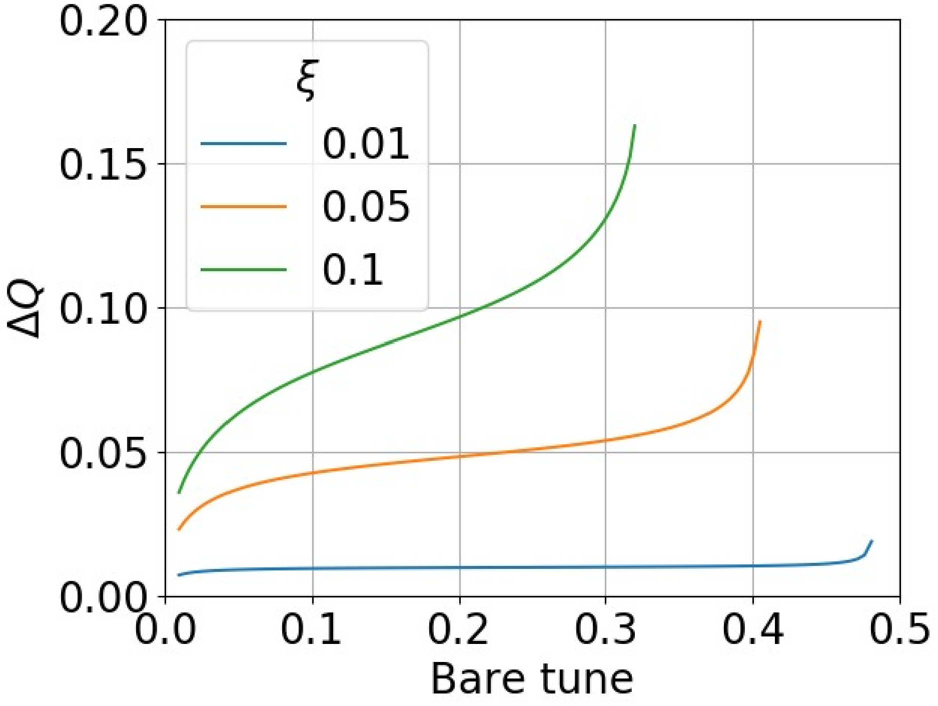
  → Bask to Bassetti-Erskine

Dynamic effects

[Chao]

- Taking into account only the linearised part of the beam-beam force, we can compute the new optics including beam-beam:

$$\cos(2\pi(Q_0 + \Delta Q_{BB})) = \cos(2\pi Q_0) - 2\pi\xi\sin(2\pi Q_0)$$



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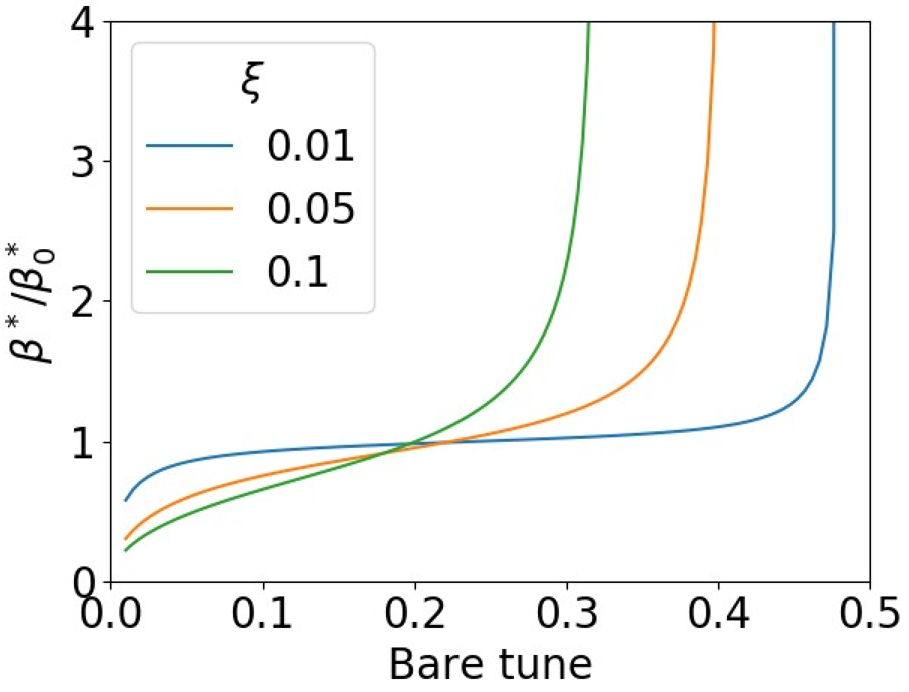

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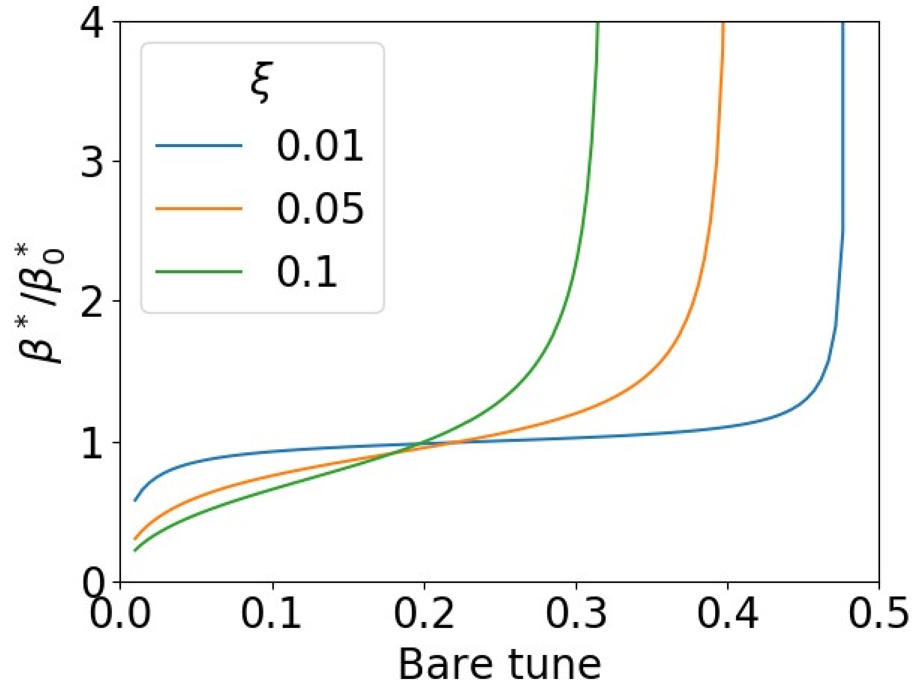


- In machines featuring strong synchrotron radiation, the change in optics leads to a change in equilibrium emittance:

$$\epsilon_0 = C_q \gamma_0^2 \frac{I_5}{j_x I_2} \qquad I_5 = \oint ds \frac{\gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_{px} + \beta_x \eta_{px}^2}{|\rho^3|}$$

Self-consistent solutions

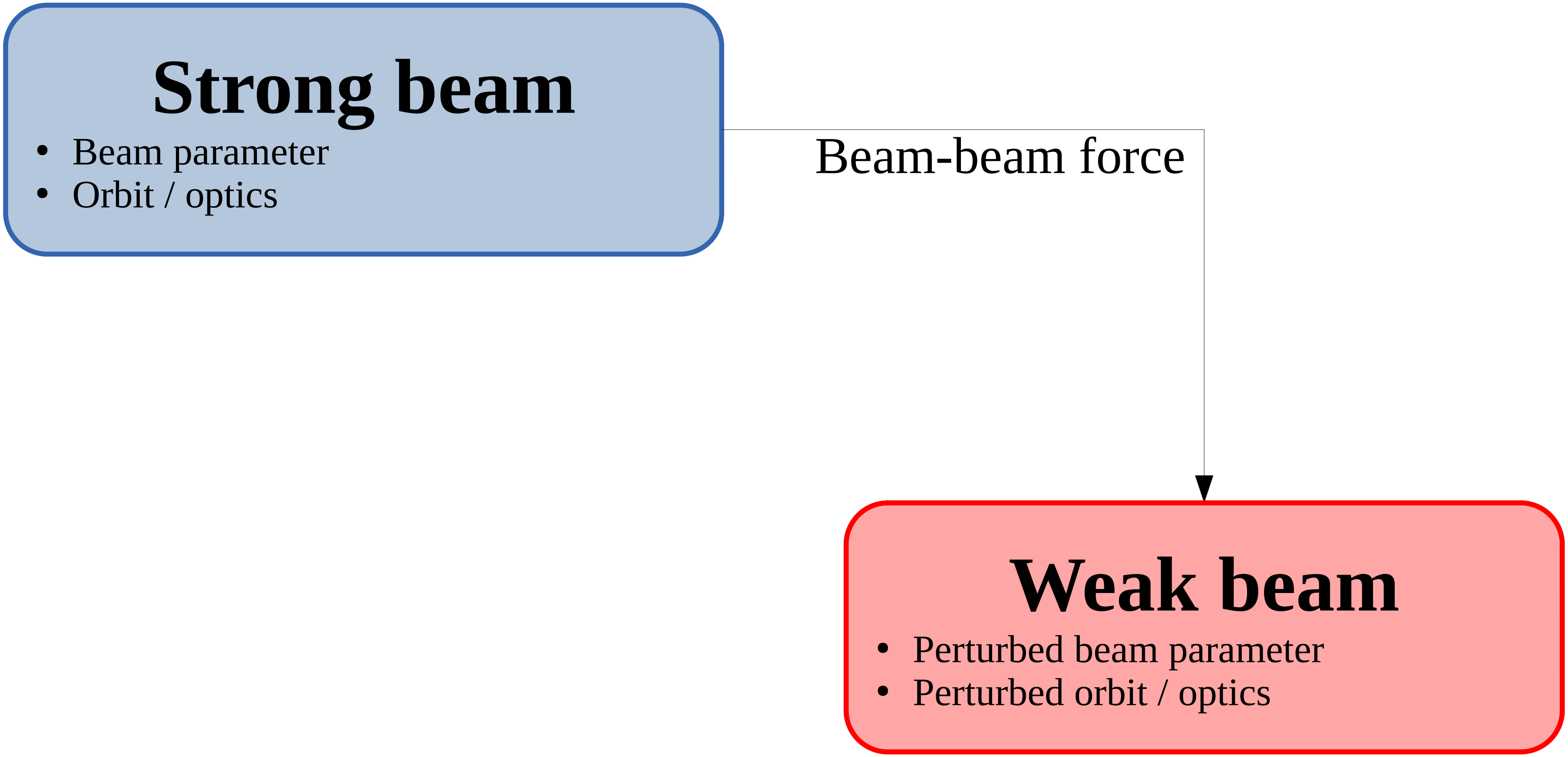
Strong beam

- Beam parameter
- Orbit / optics

Weak beam

- Beam parameter
- Orbit / Optics

Self-consistent solutions



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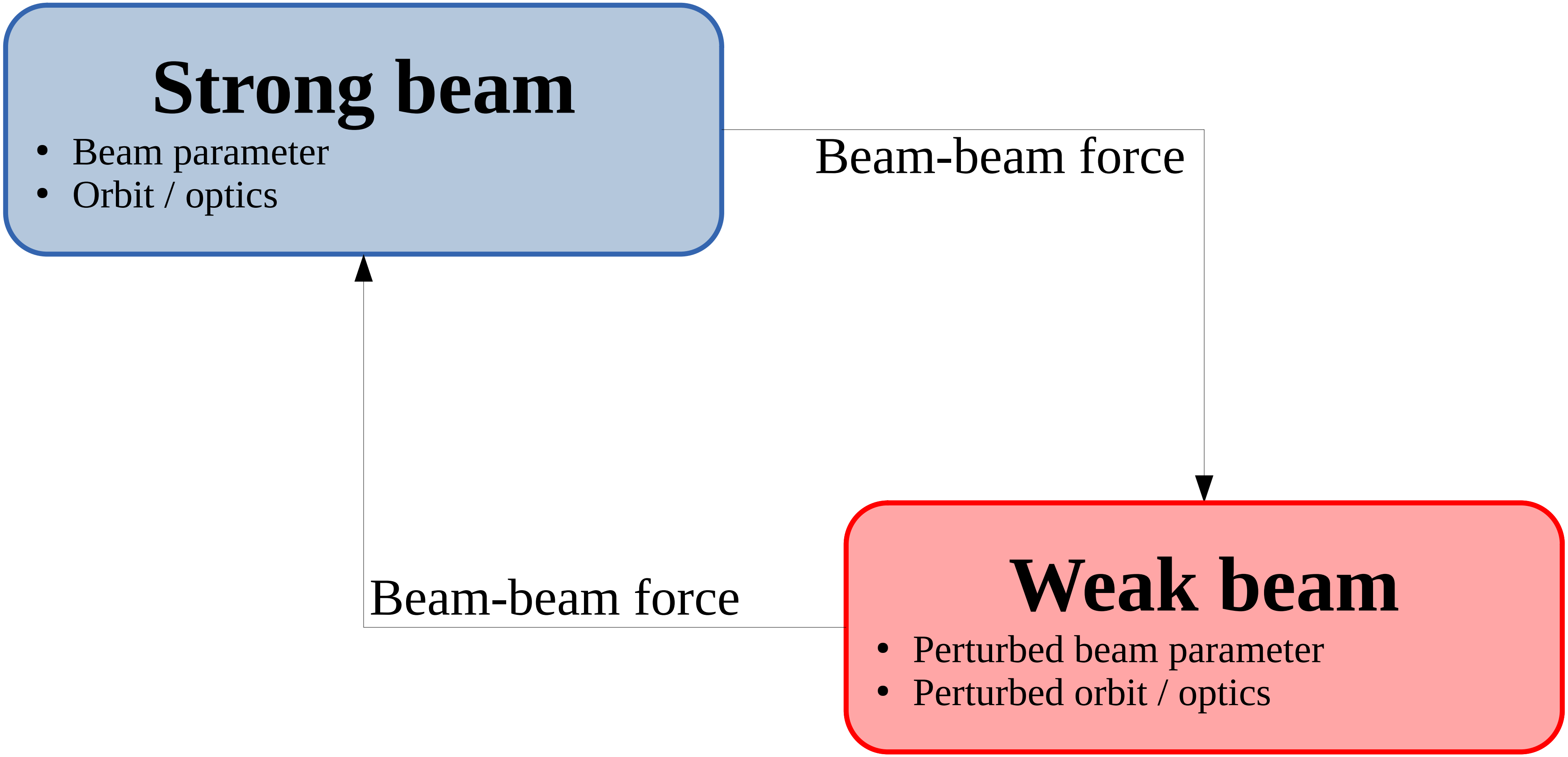

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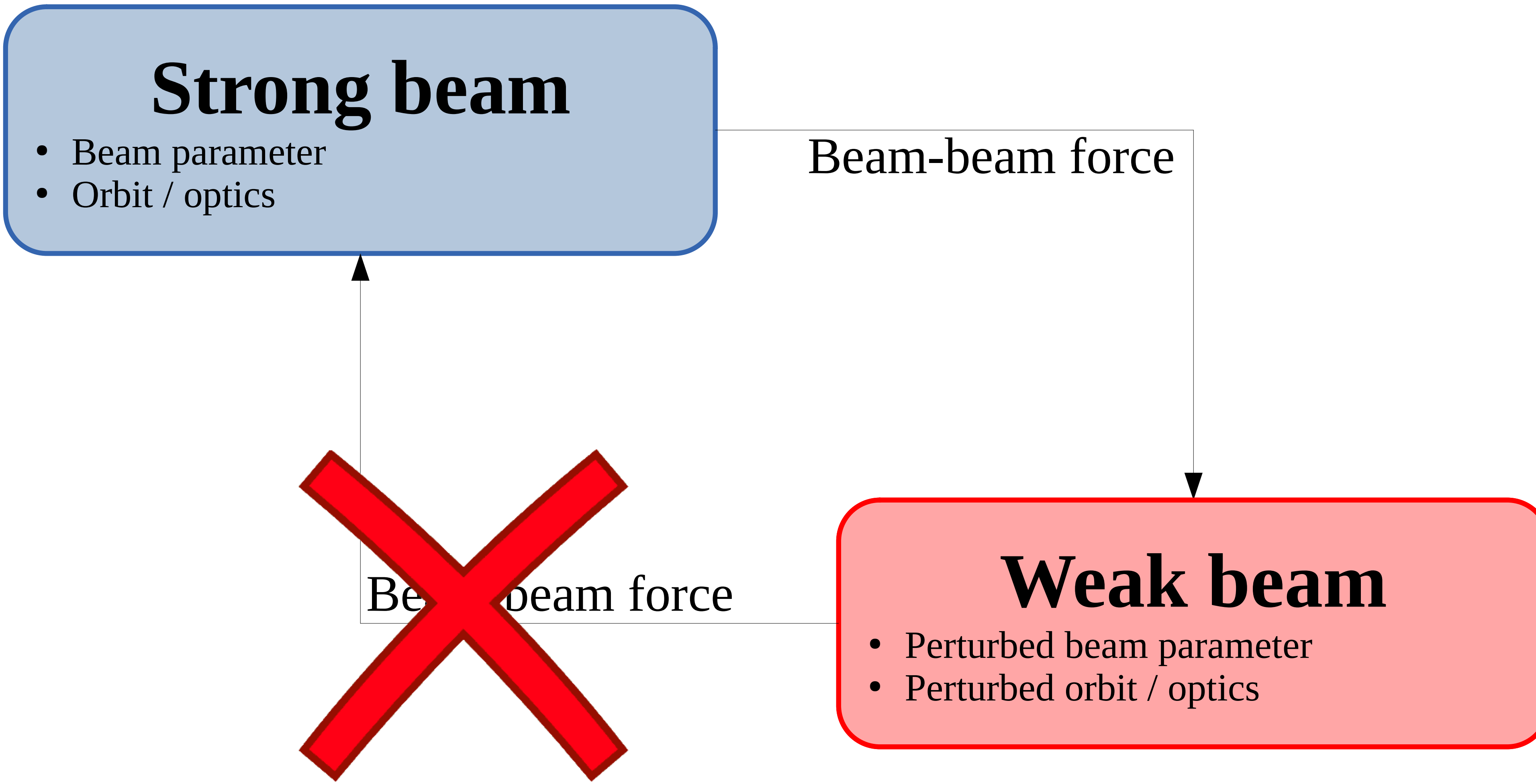
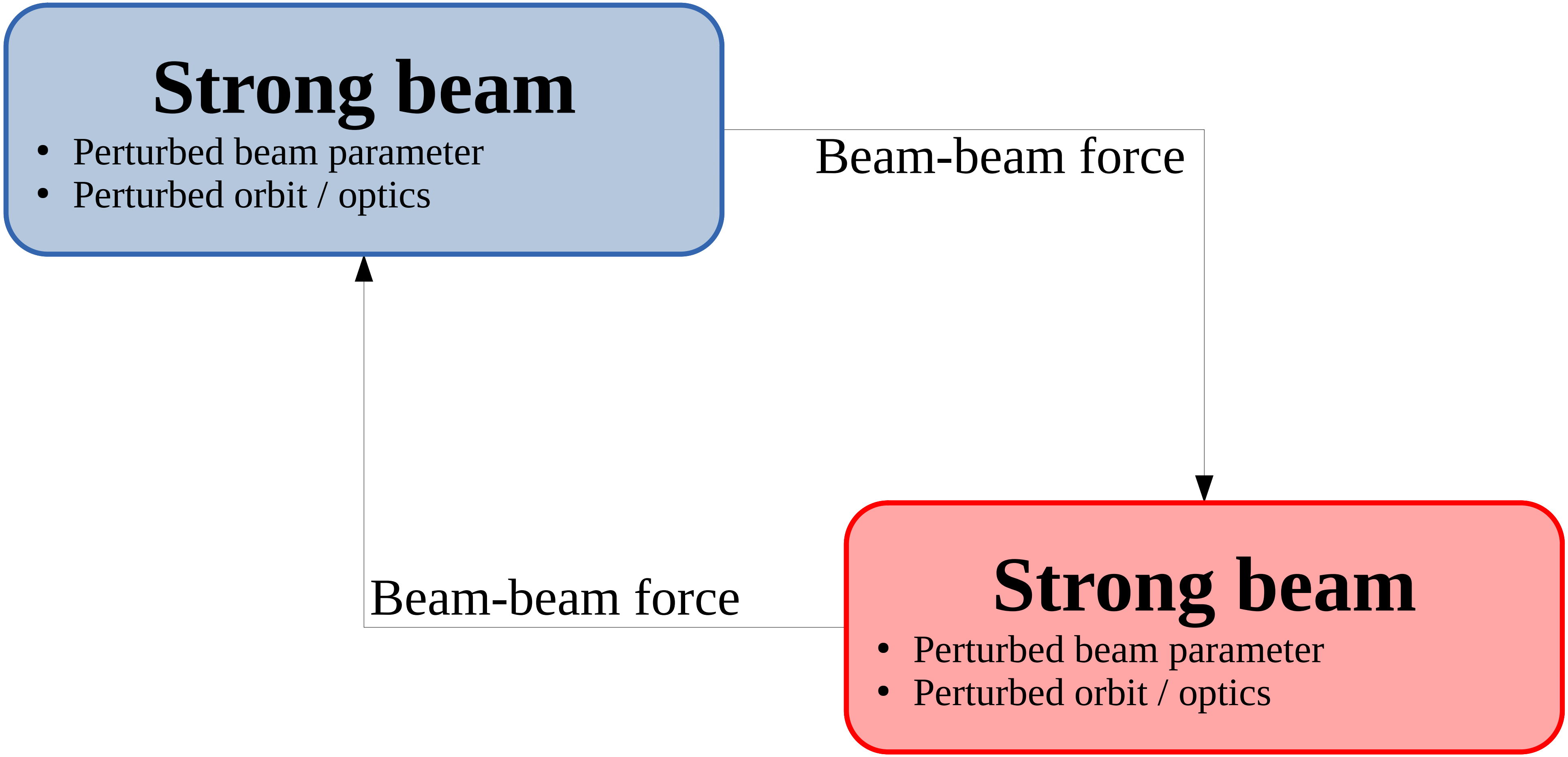


- When the impact of the weak beam on the strong beam is neglected, we talk about **weak-strong models**

Self-consistent solutions



- When the impact of the weak beam on the strong beam is neglected, we talk about **weak-strong models**
- If not, we talk about strong**-strong models**

→ Need self-consistent solutions

Flip-flop

[Chao, VEPP]

- The self-consistent dynamic β effect is obtained through a set of non-linear coupled equations:

$$\begin{cases} \left(\frac{\beta_0^*}{\beta_+^*}\right)^2 = 1 + 4\pi\xi\cot(2\pi Q_0)\frac{\beta_0^*}{\beta_-^*} - 4\pi^2\xi^2\left(\frac{\beta_0^*}{\beta_-^*}\right)^2 \\ \left(\frac{\beta_0^*}{\beta_-^*}\right)^2 = 1 + 4\pi\xi\cot(2\pi Q_0)\frac{\beta_0^*}{\beta_+^*} - 4\pi^2\xi^2\left(\frac{\beta_0^*}{\beta_+^*}\right)^2 \end{cases}$$

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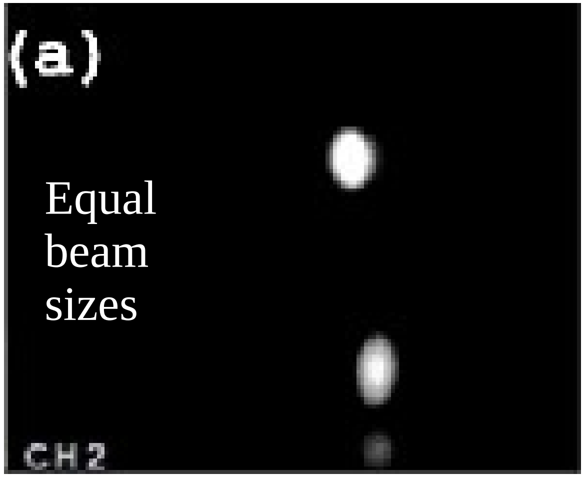

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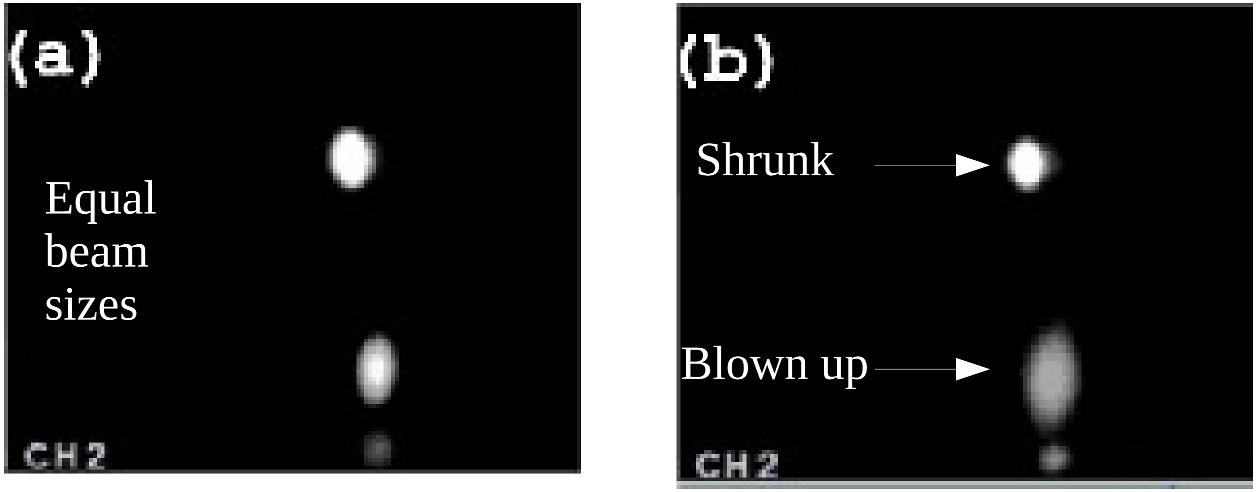

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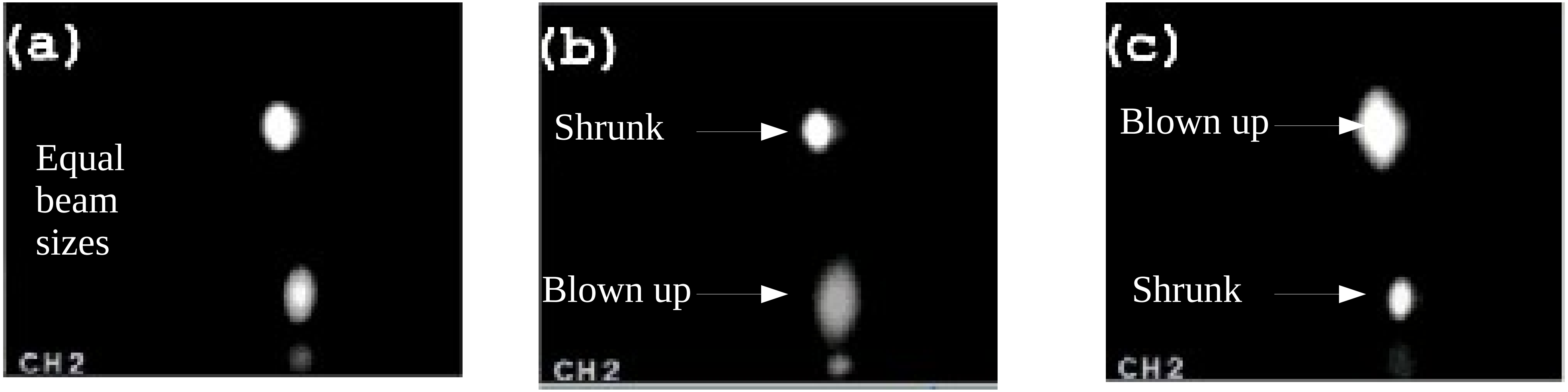
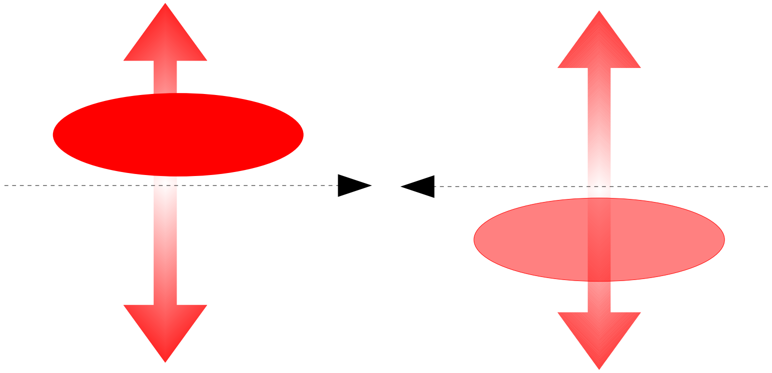


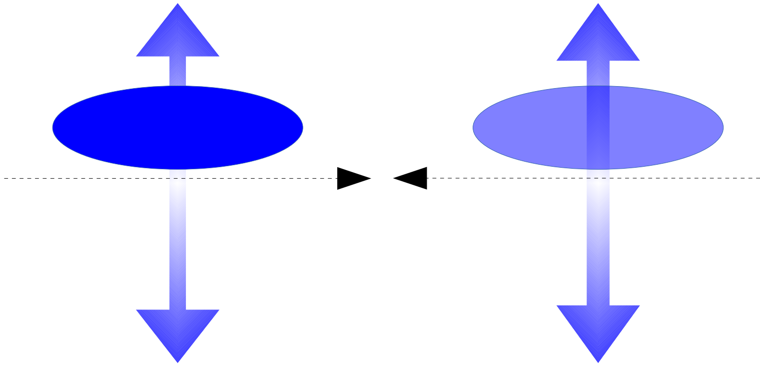
Coherent oscillations

[Pieloni]

Out of phase oscillations

In-phase oscillations





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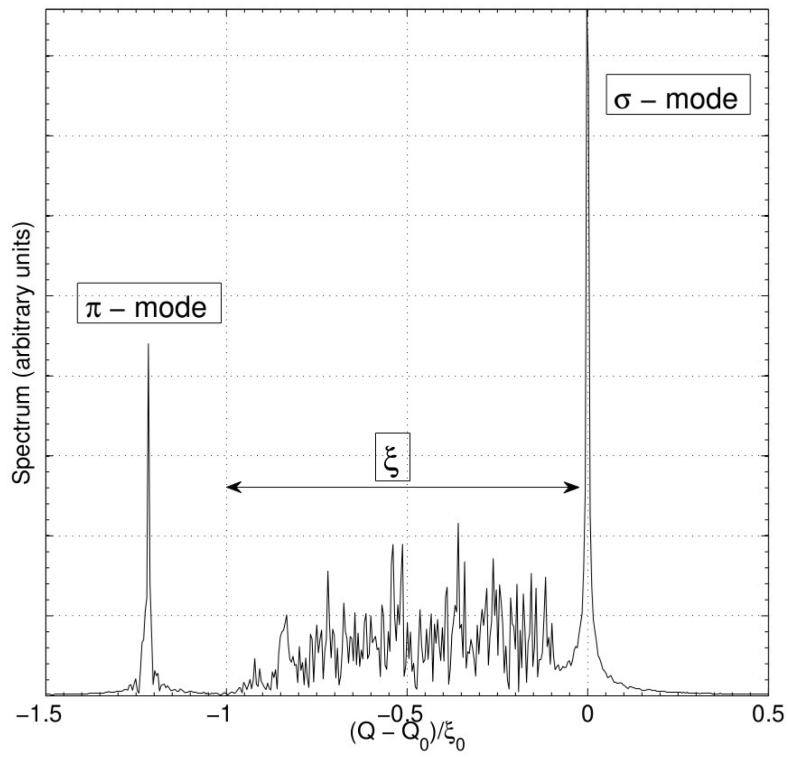

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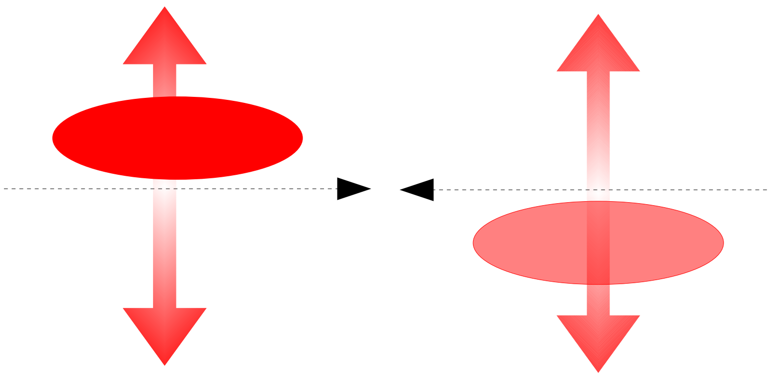

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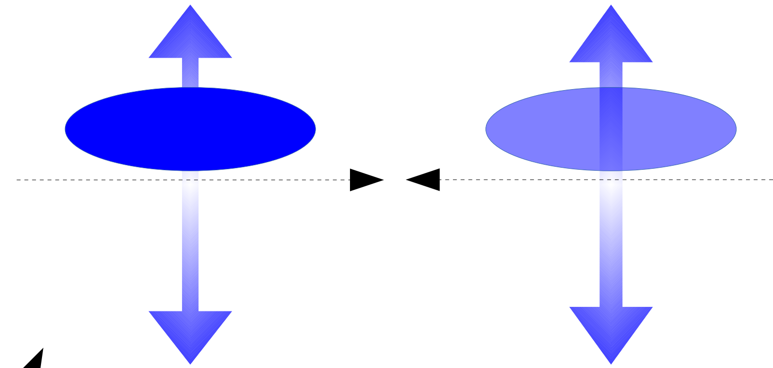

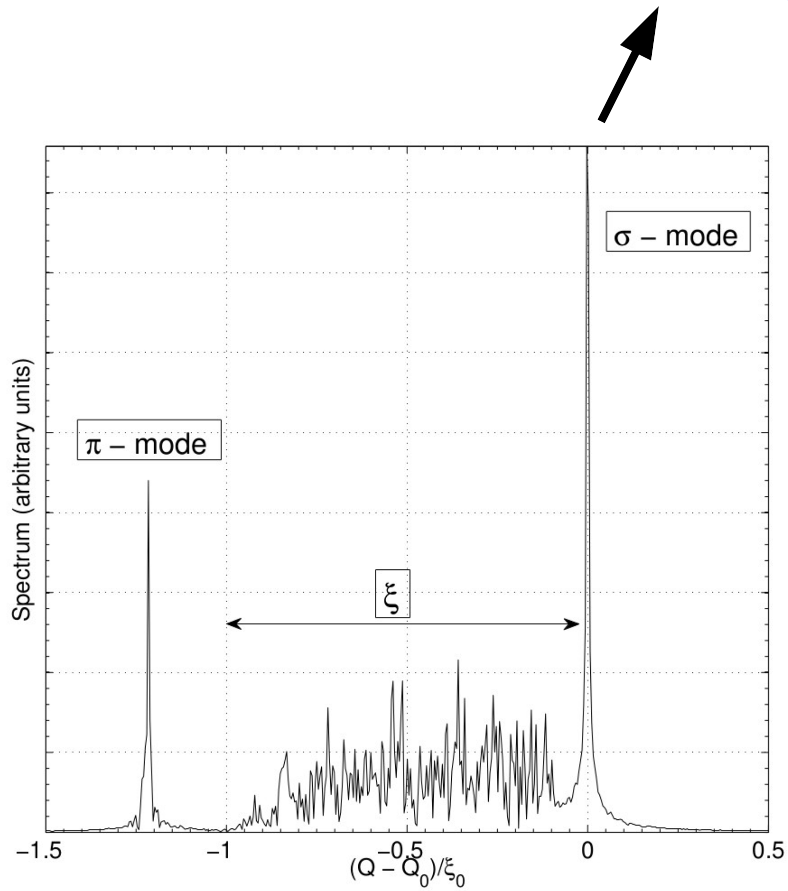

- The average beam-beam force is zero at each turn

$$\rightarrow Q_\sigma = Q_0$$

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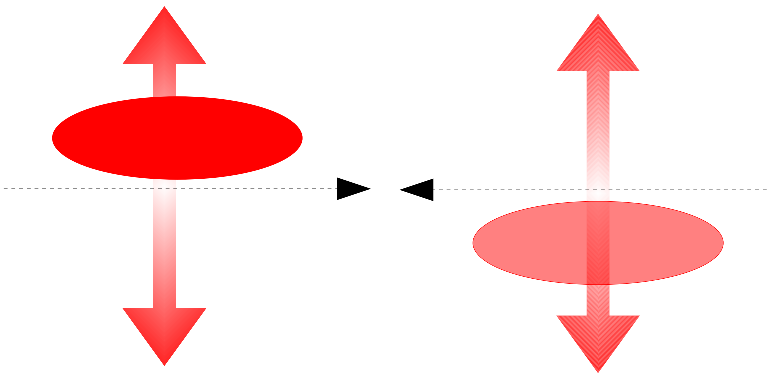
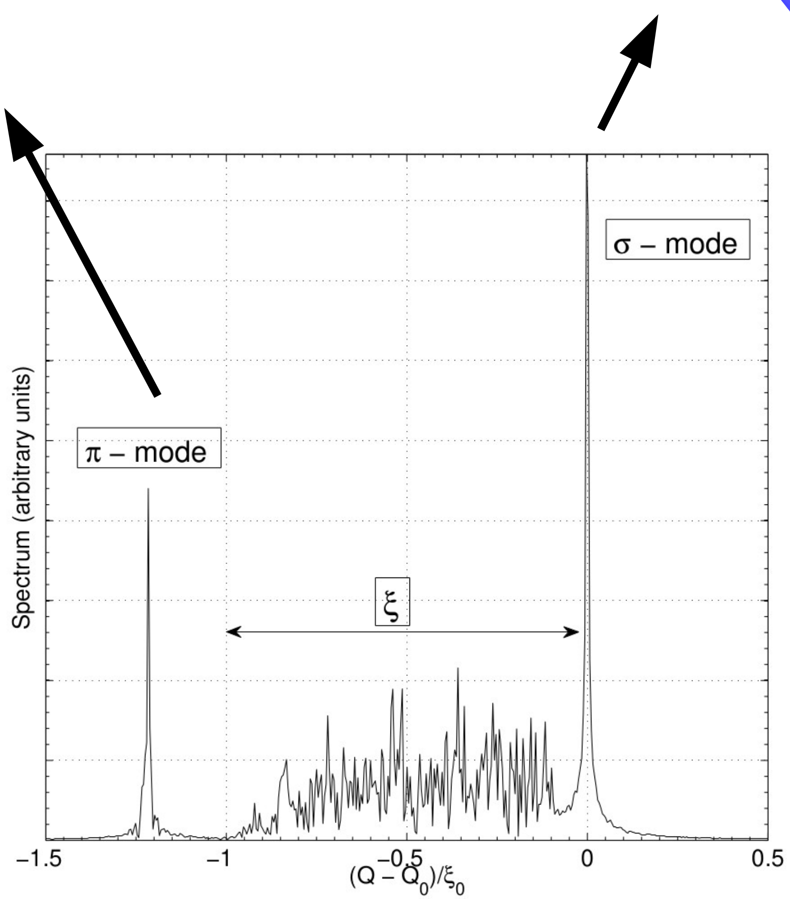


- The beams are offset at every turn

  → The frequency of the mode depends on the strength of the beam-beam force:

$$Q_\pi = Q_0 + Y \Delta Q_{BB}$$



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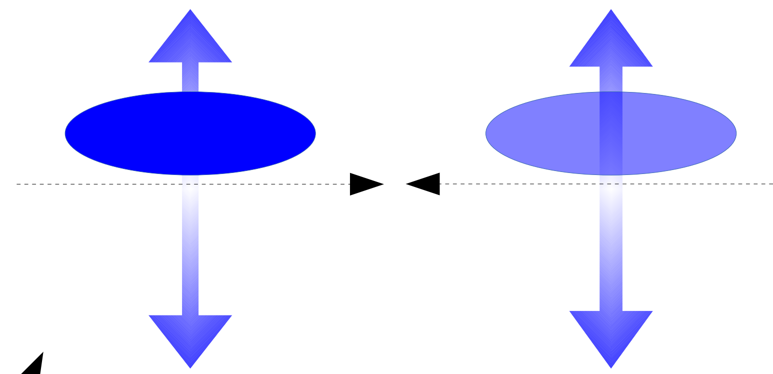

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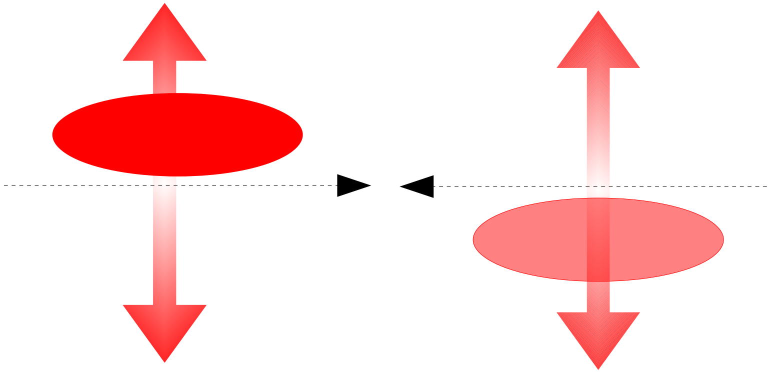

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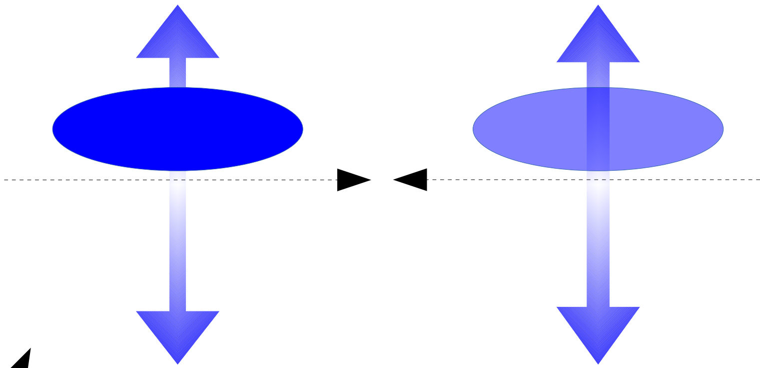

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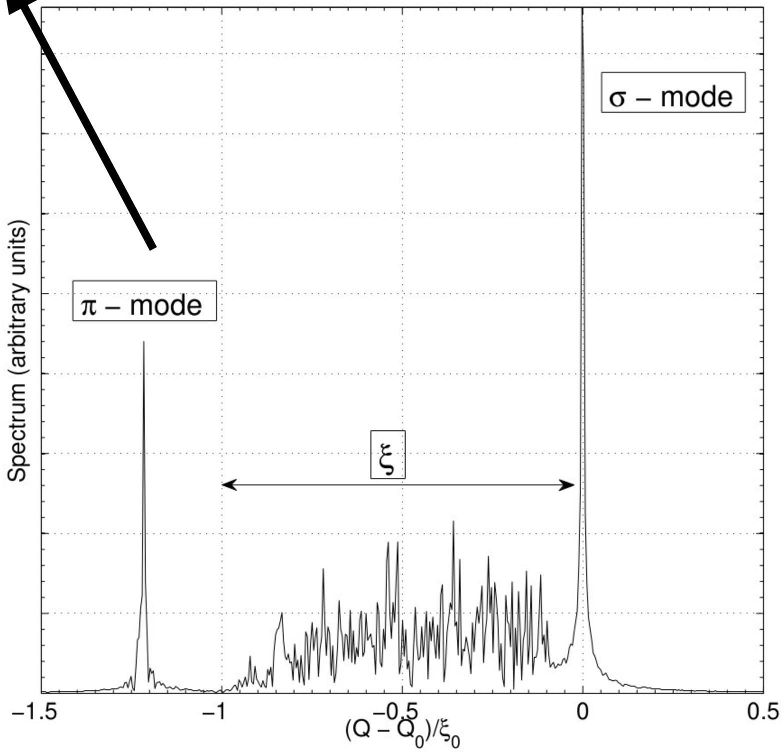
Yokoya factor (1.0 to ~1.3) due to the non-linearity of the force



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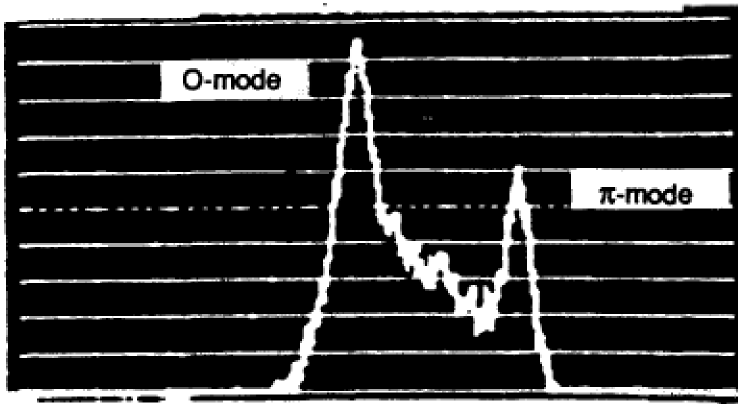
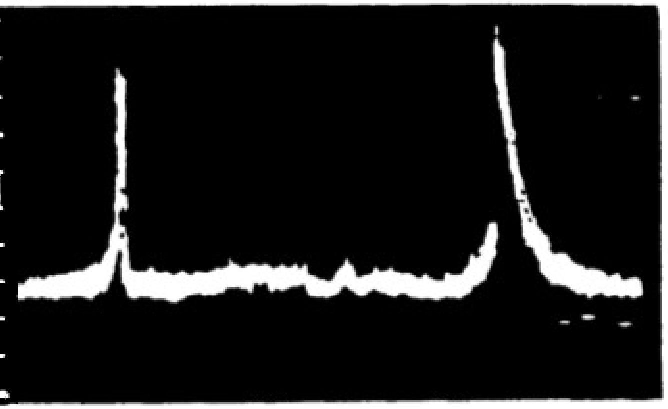
Coherent oscillation

TRISTAN

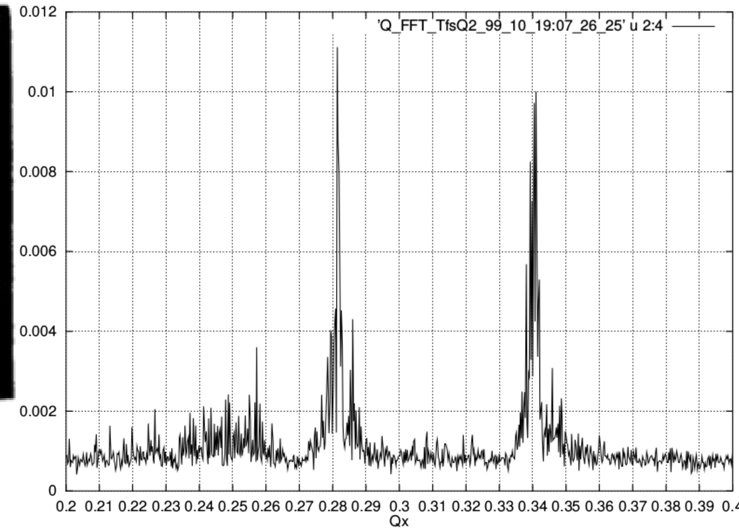


PETRA



Vertical eigenfrequencies of two colliding bunches.

LEP



Coherent oscillation

[TRISTAN, PETRA, RHIC, LHC]

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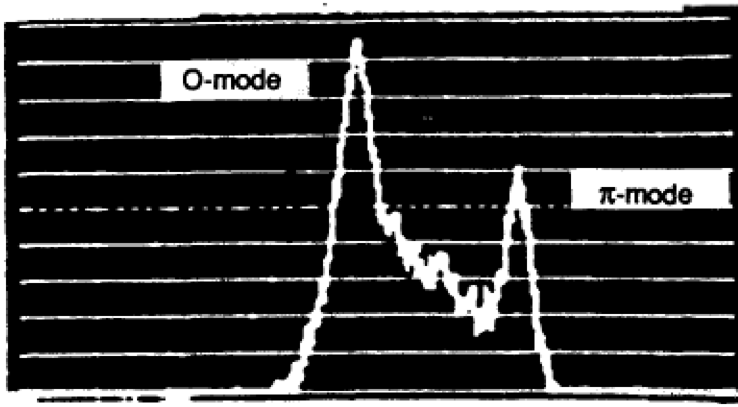

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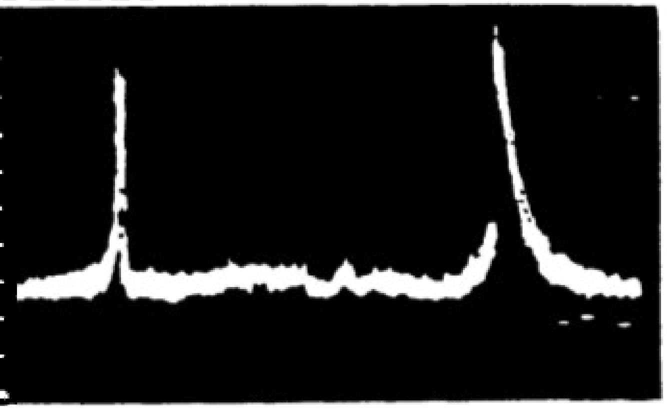

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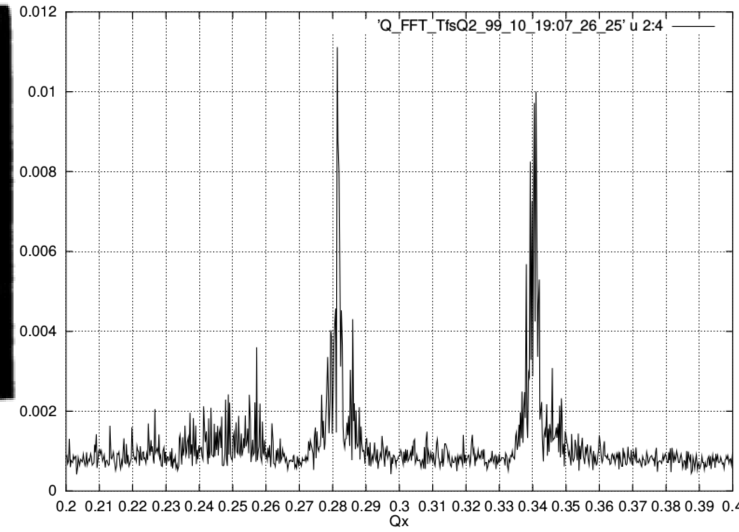
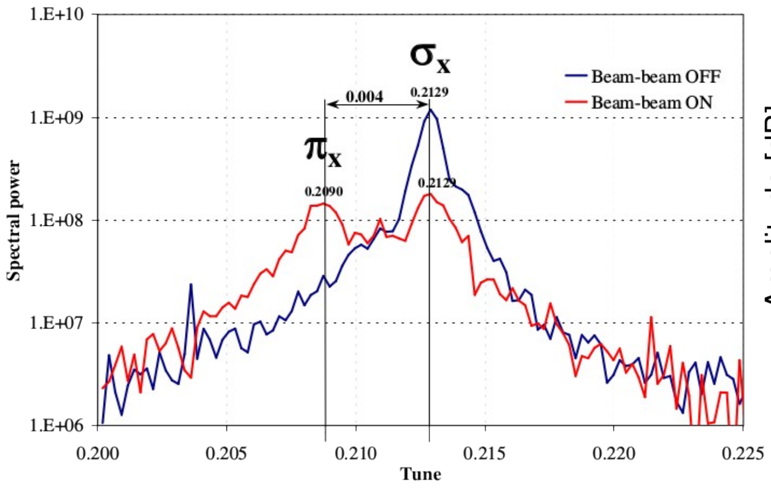
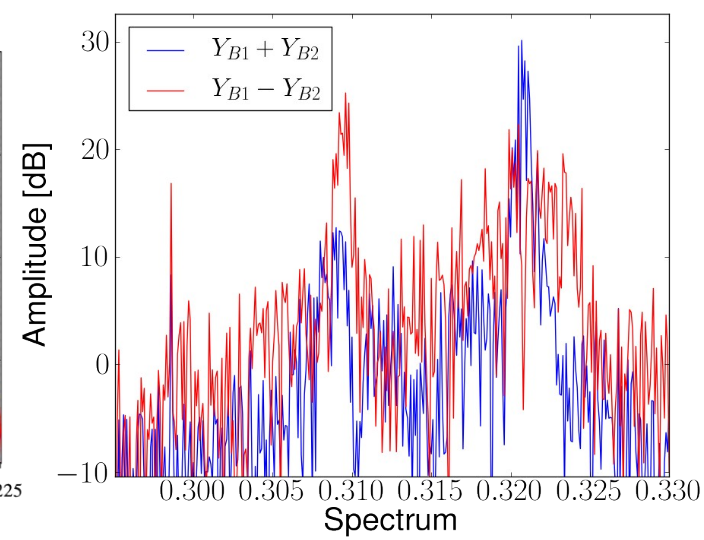


RHIC



LHC



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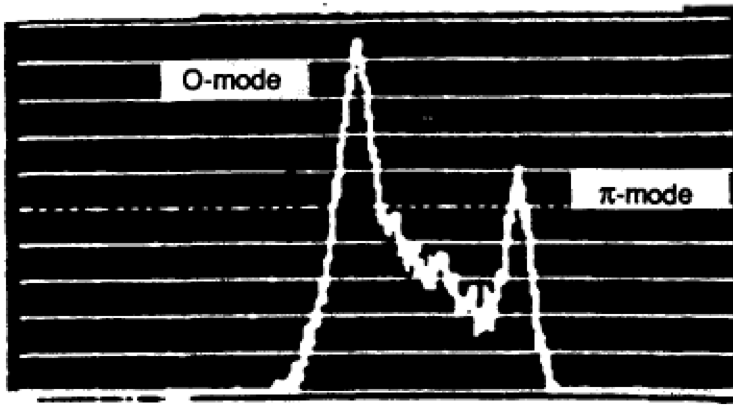

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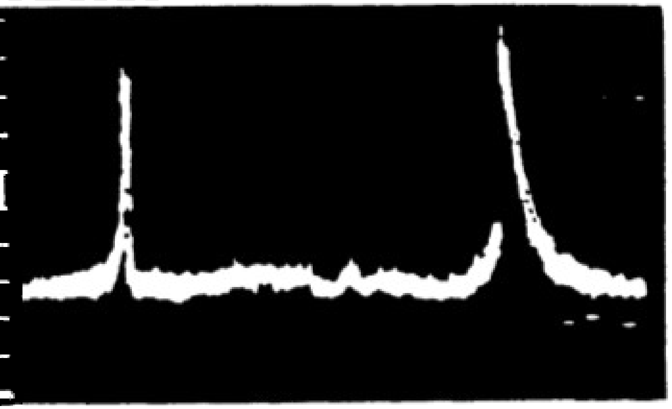

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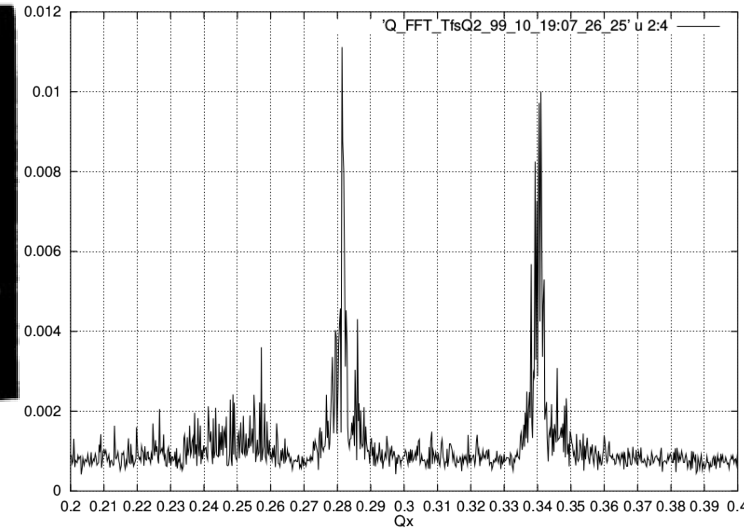

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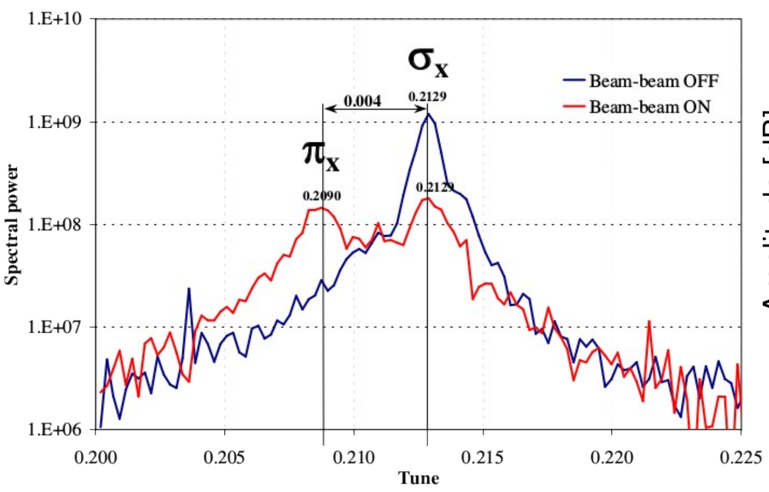

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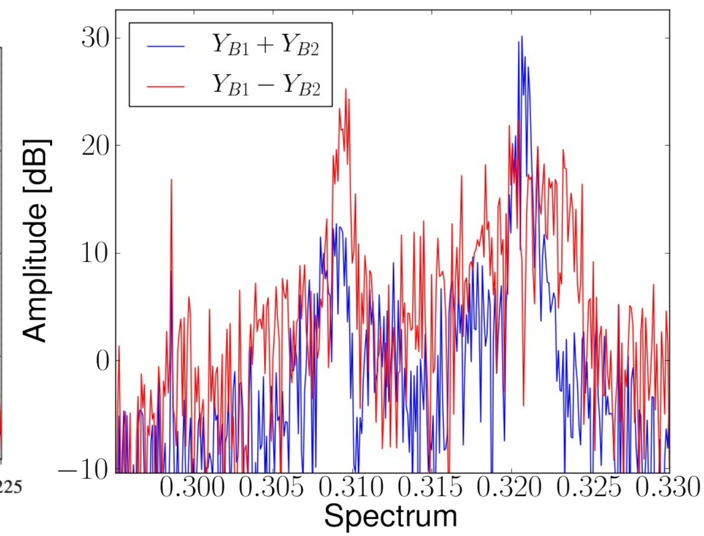


Sp$\bar{p}$S ?

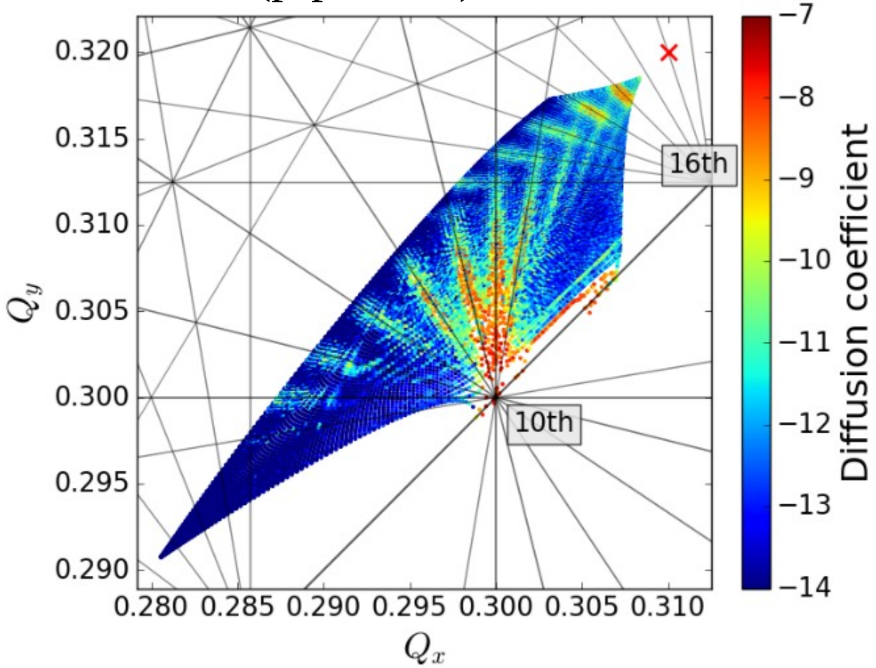
Tevatron ?

Non-linear effects

[Furuseth, Shatilov]

- Due to its non-linearity, the beam-beam force introduce a tune spread and drives resonances

LHC (p-p round)



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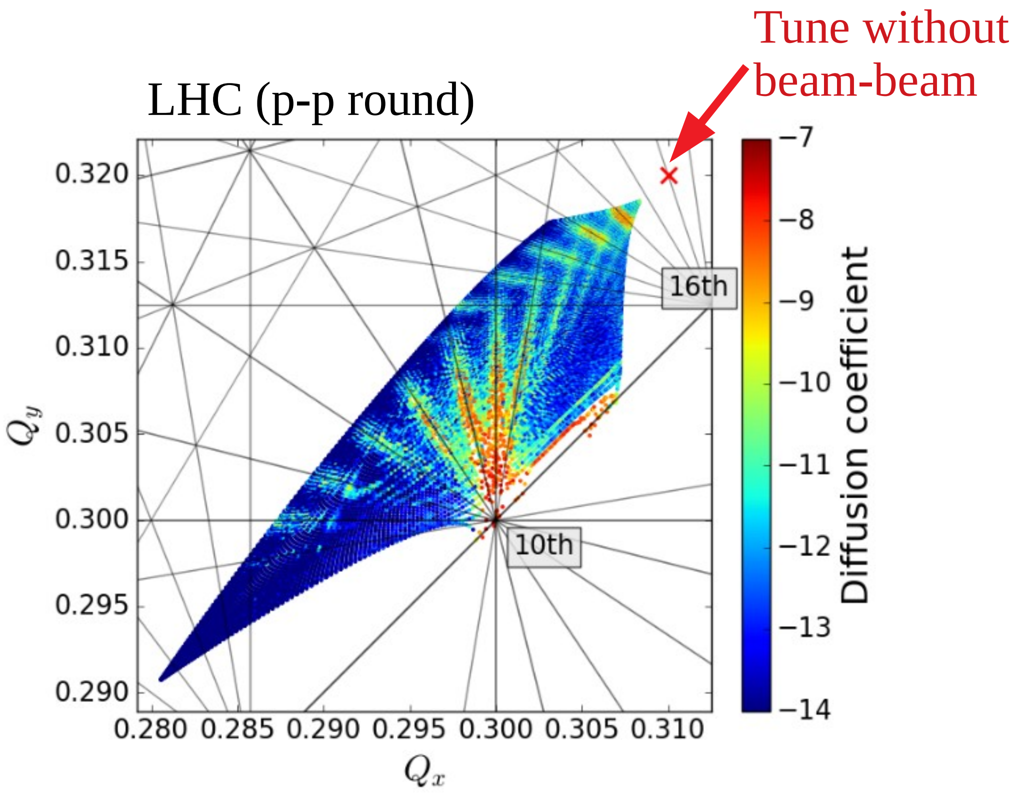

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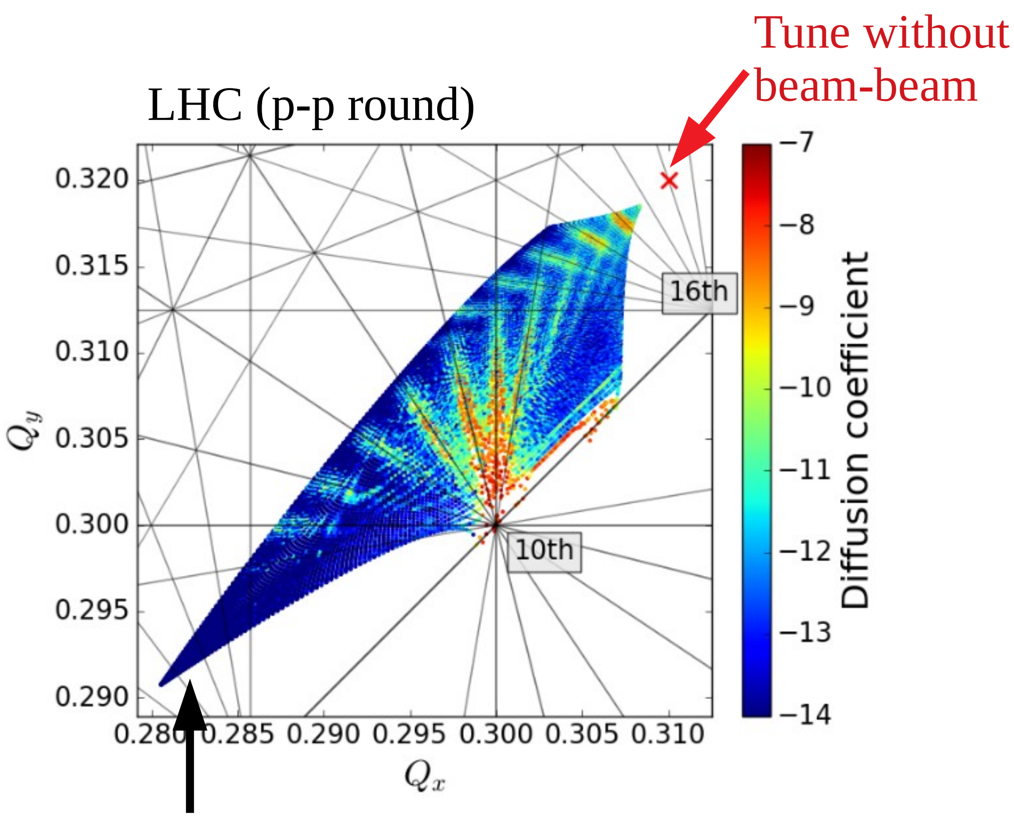

Particle oscillating with a small amplitude (~ Beam core)

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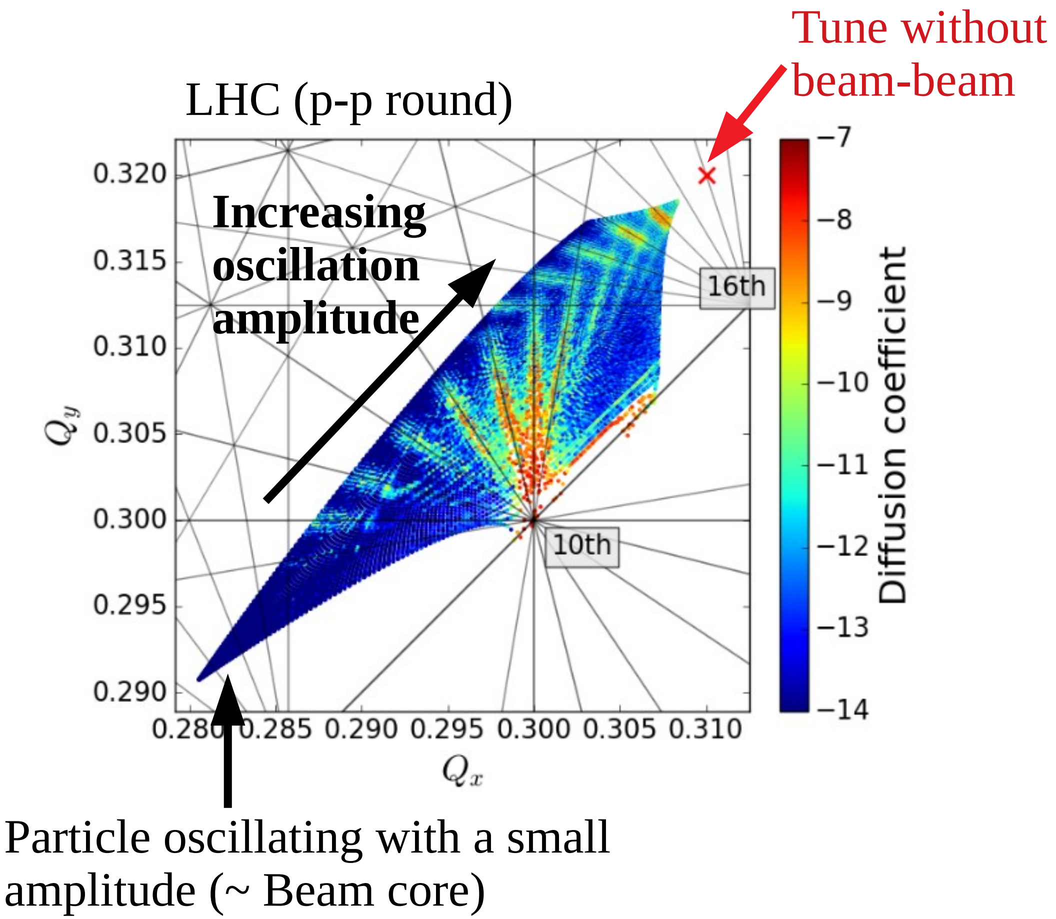

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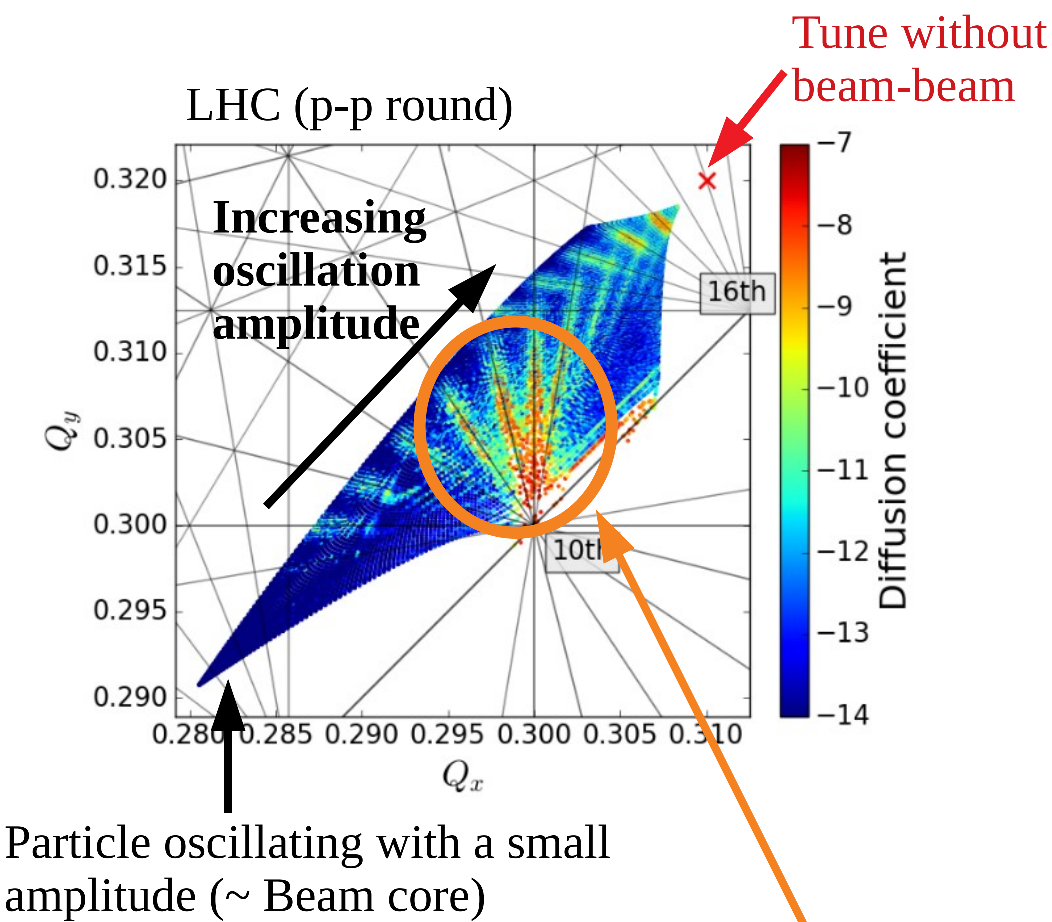

- Particle diffusion due to the non-linear force leads to emittance growth and/or beam losses

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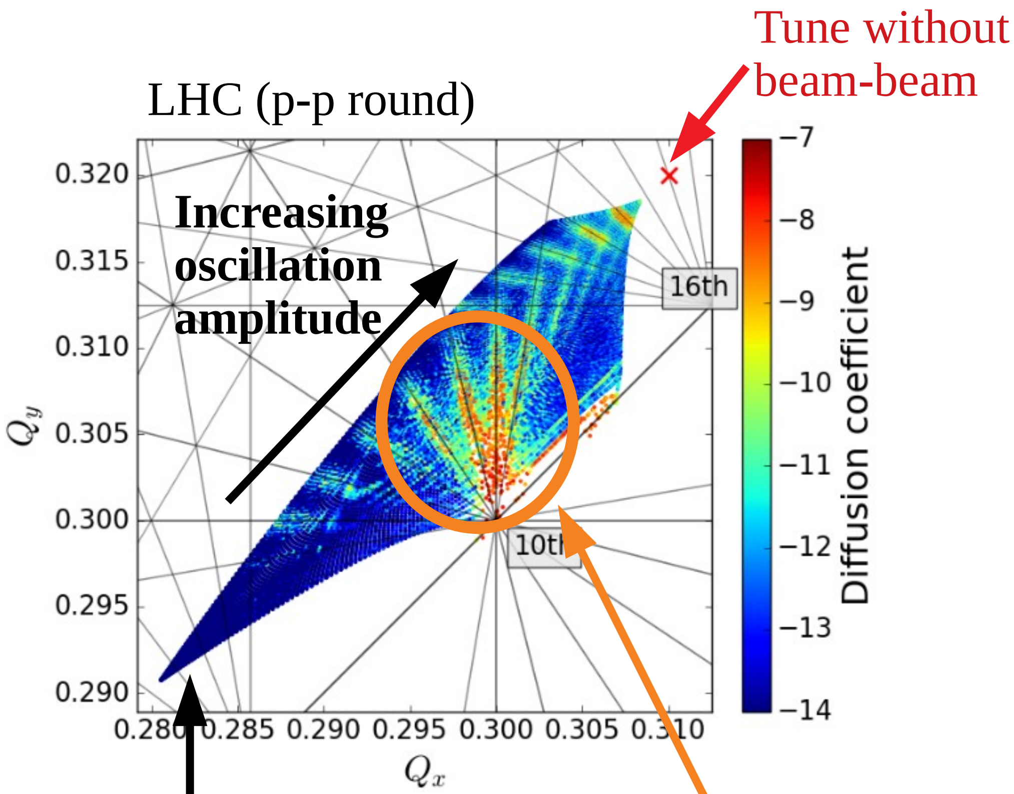

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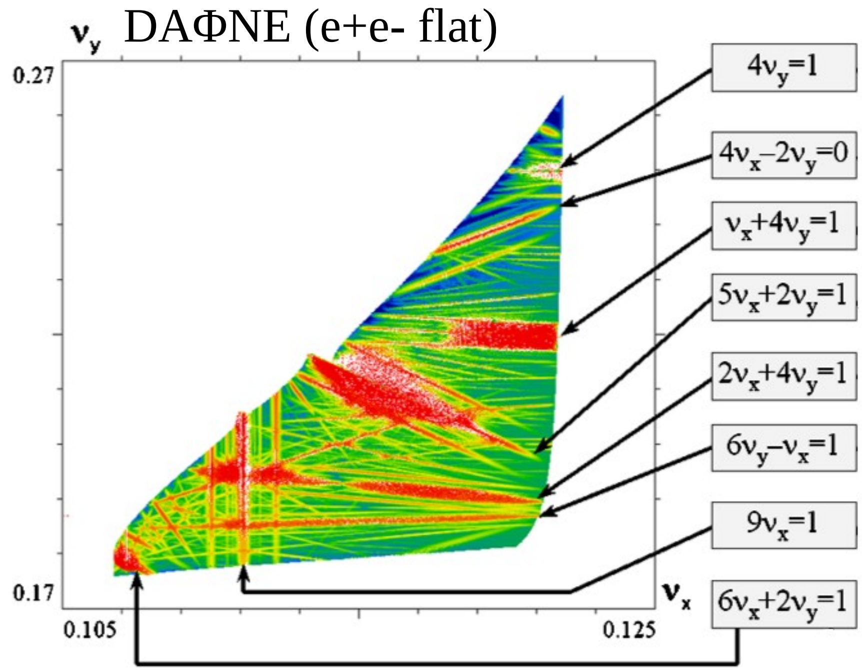

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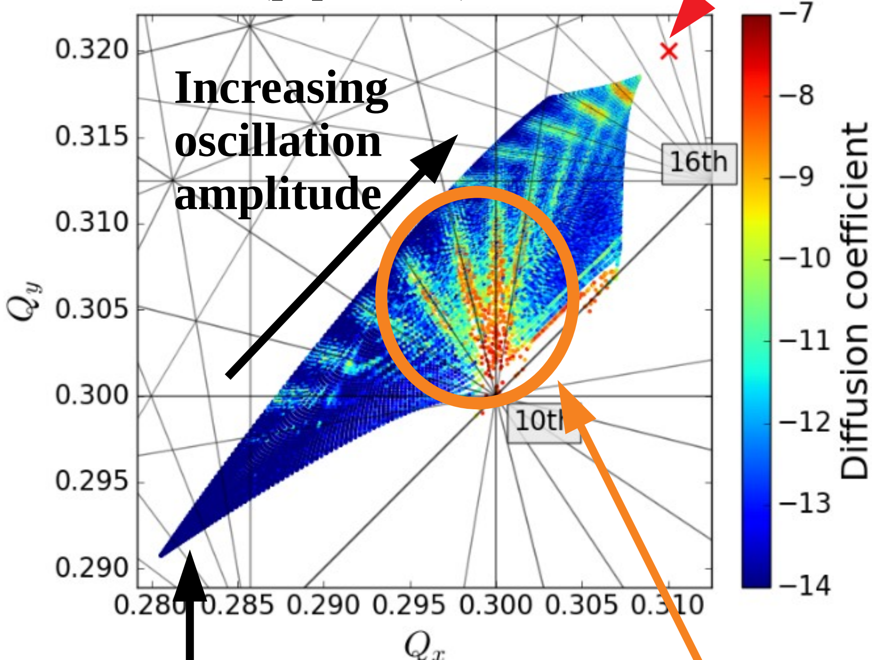
Tune without beam-beam

LHC (p-p round)



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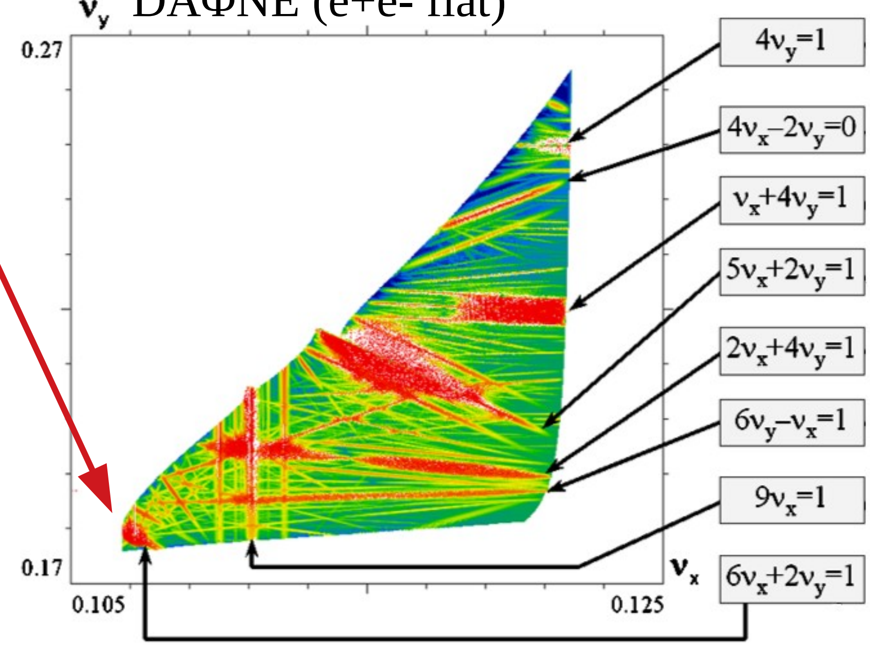
DAΦNE (e+e- flat)



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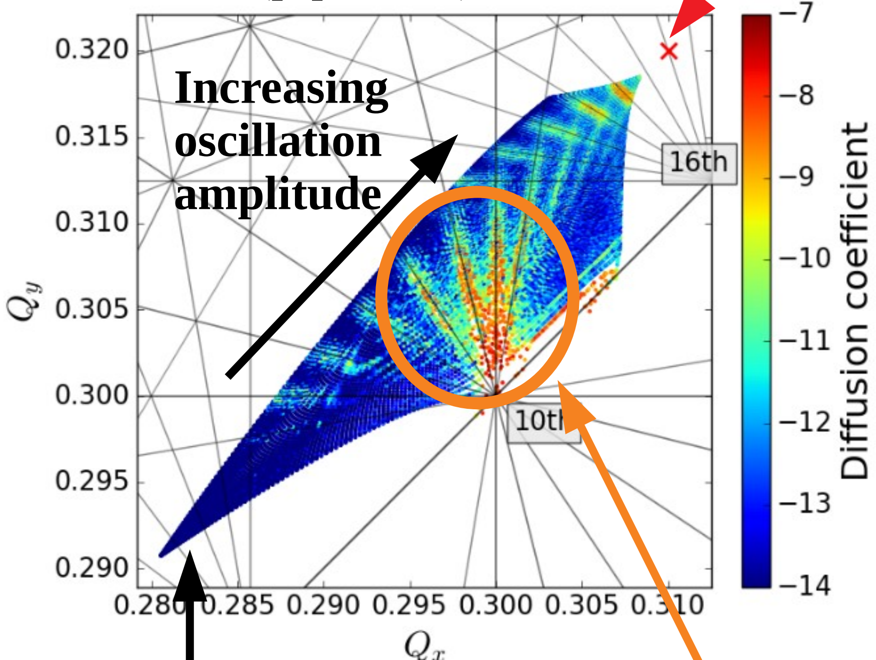

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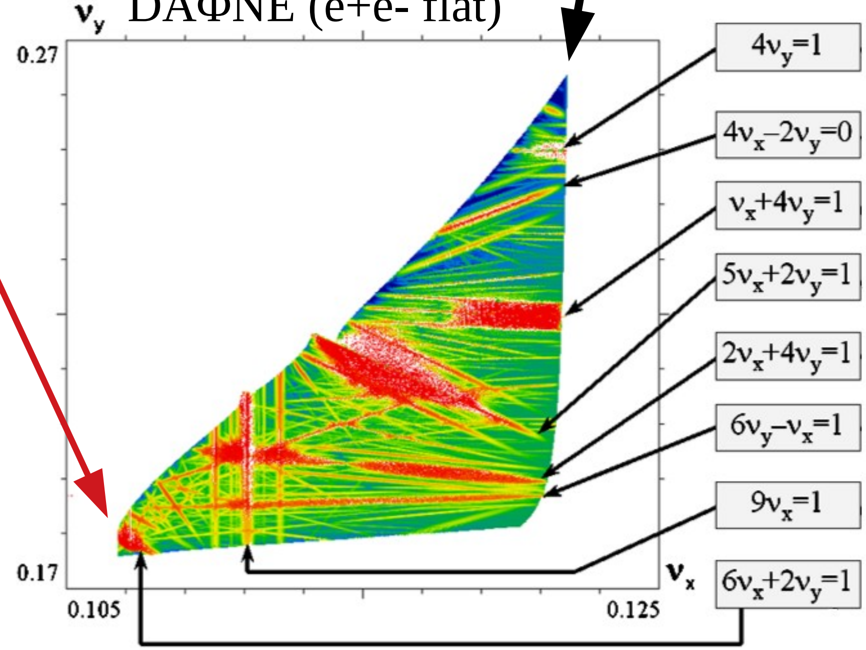

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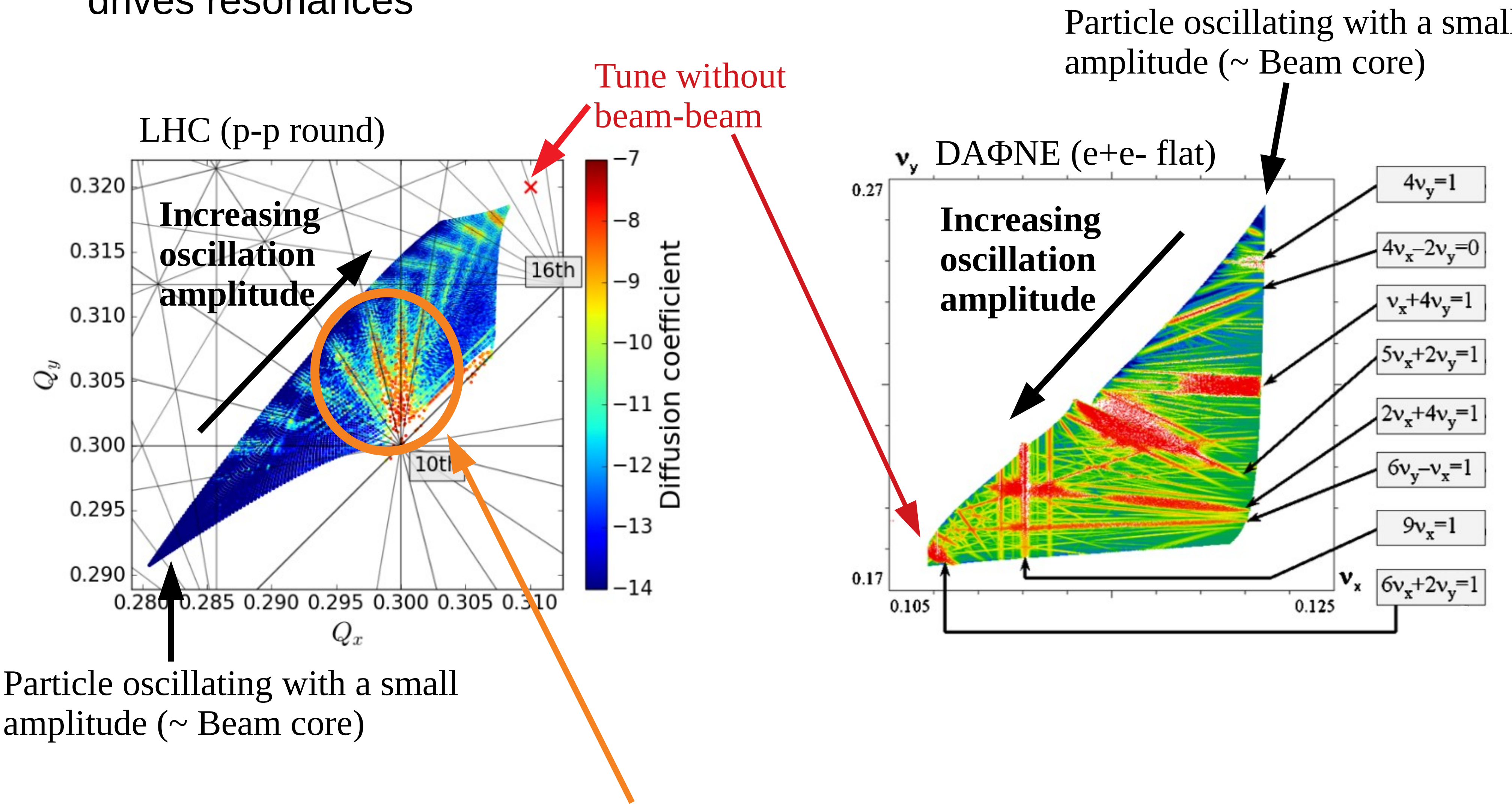

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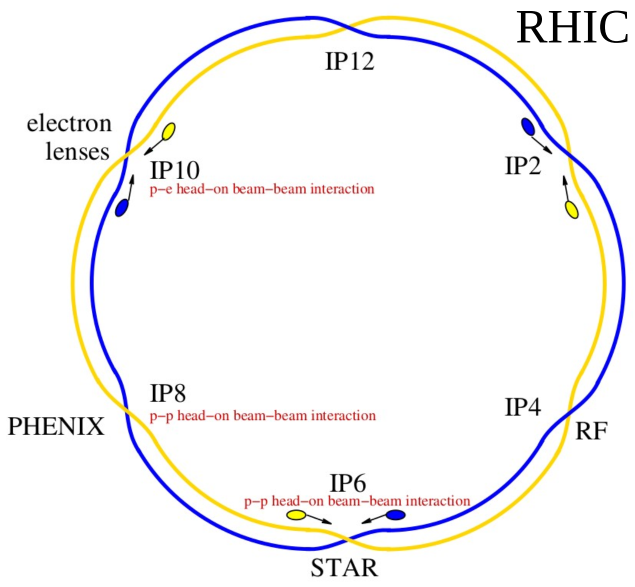
Compensation with an electron lens



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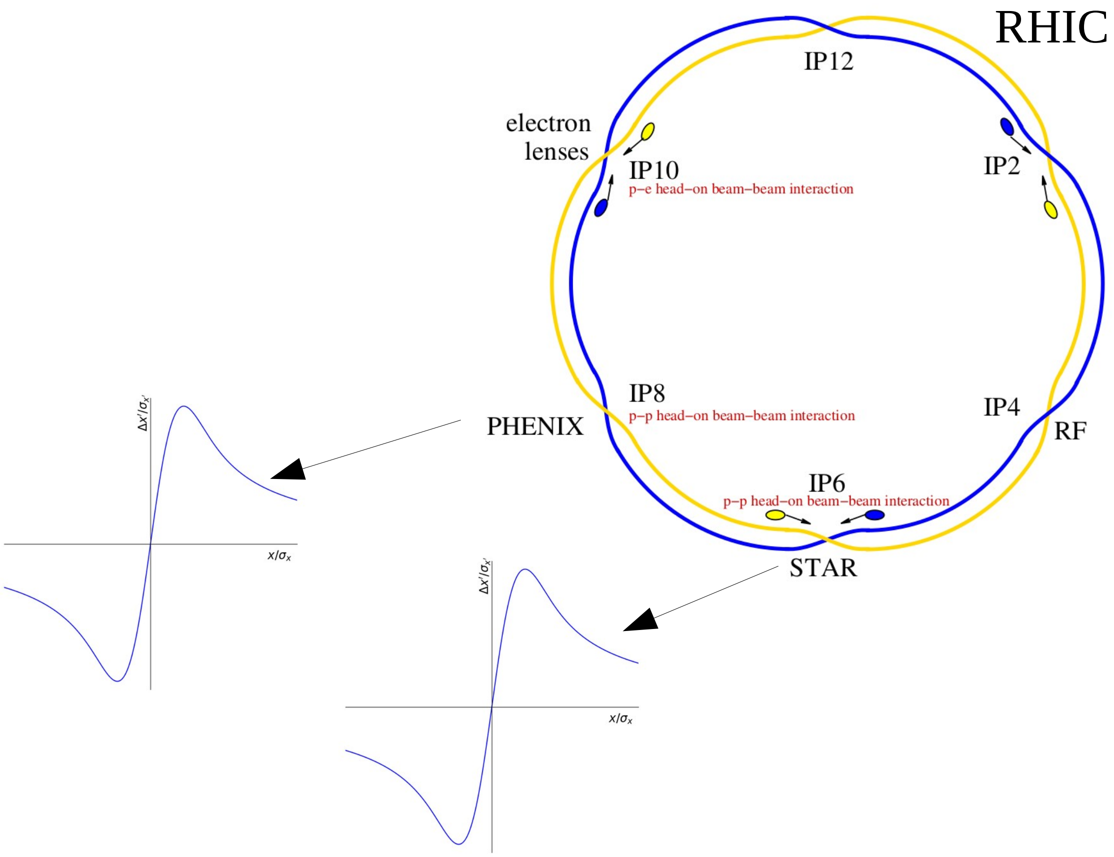

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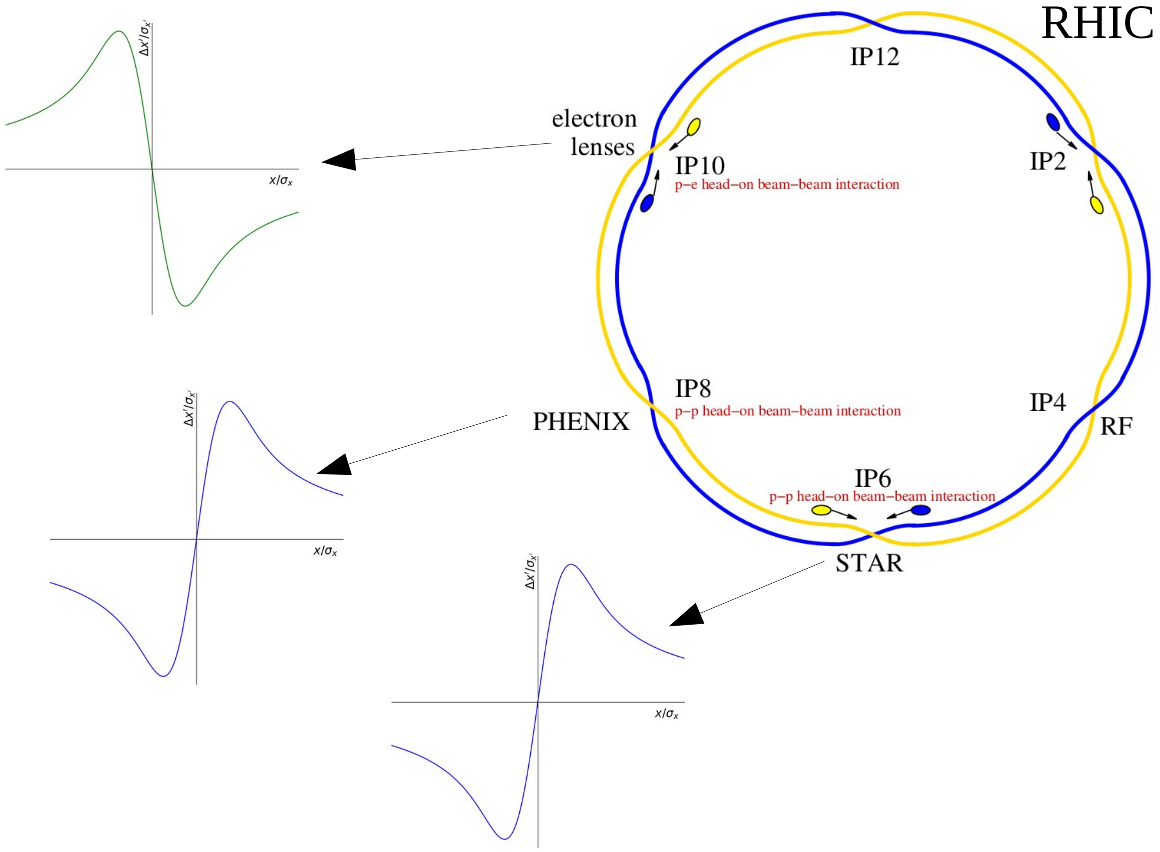

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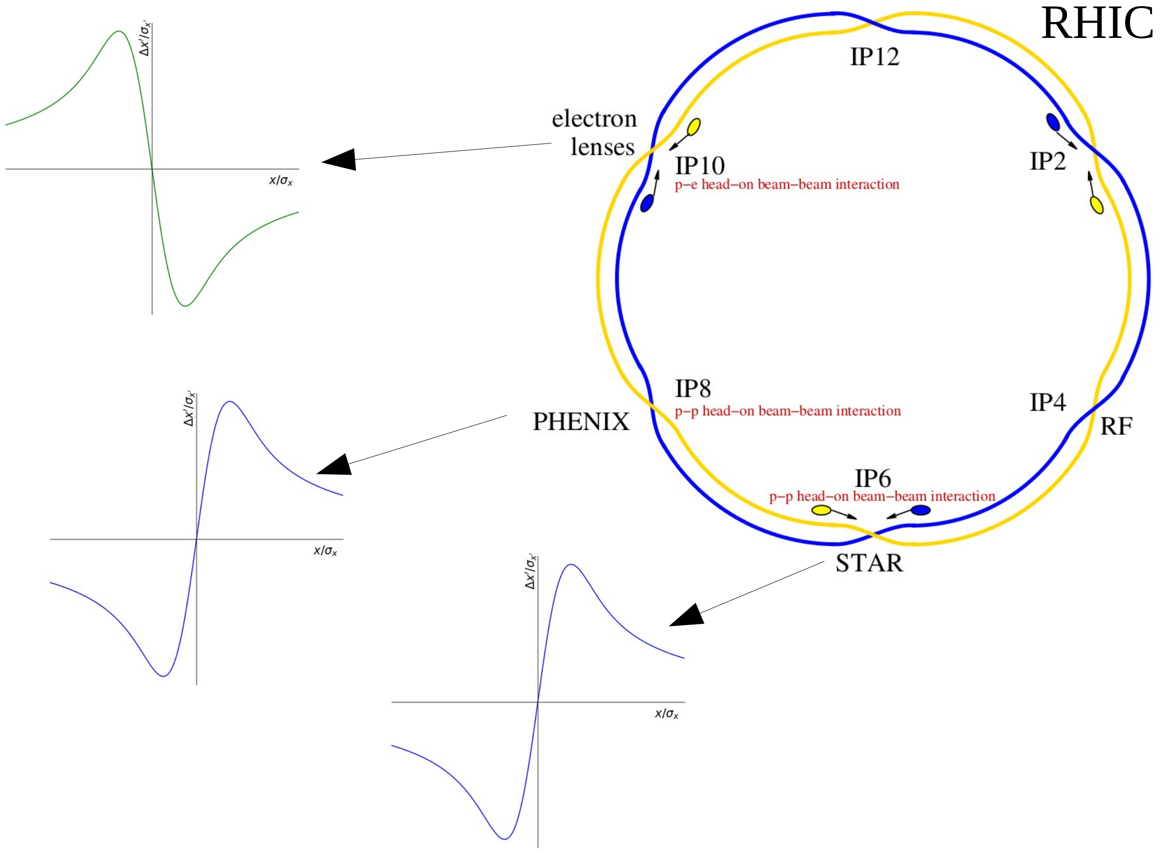

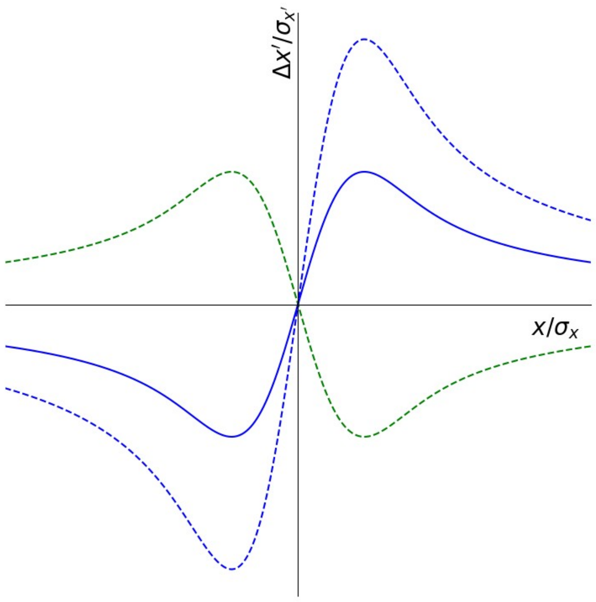

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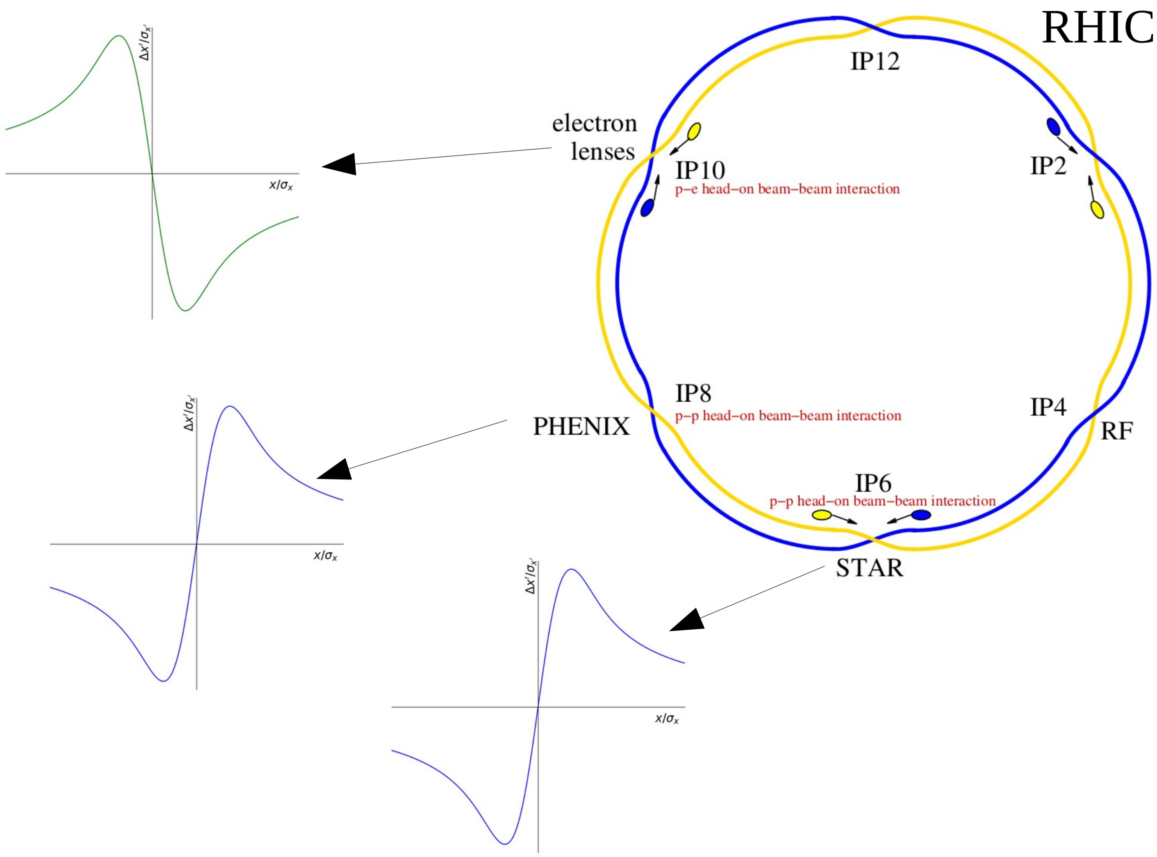

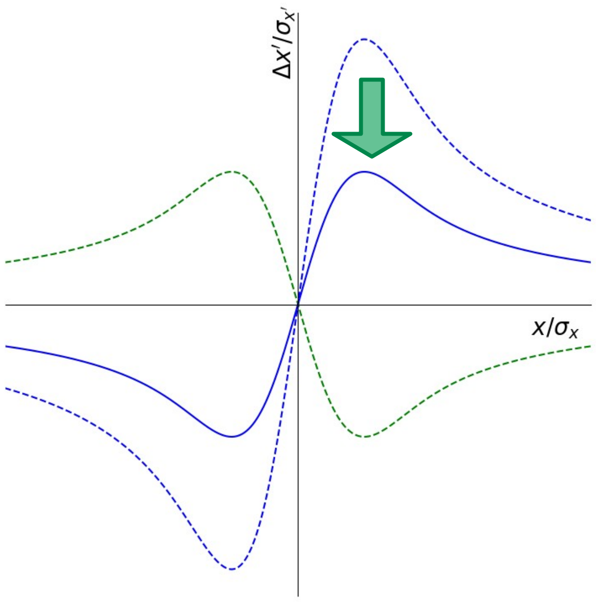

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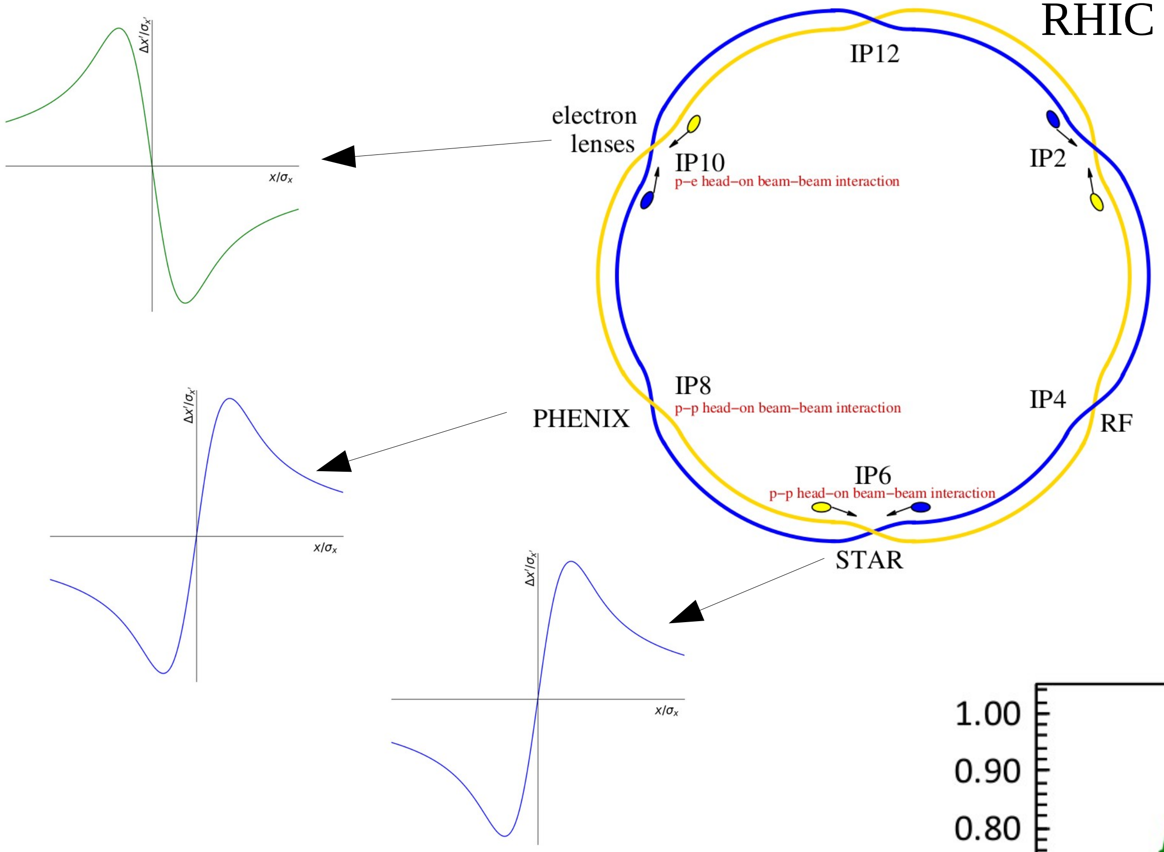

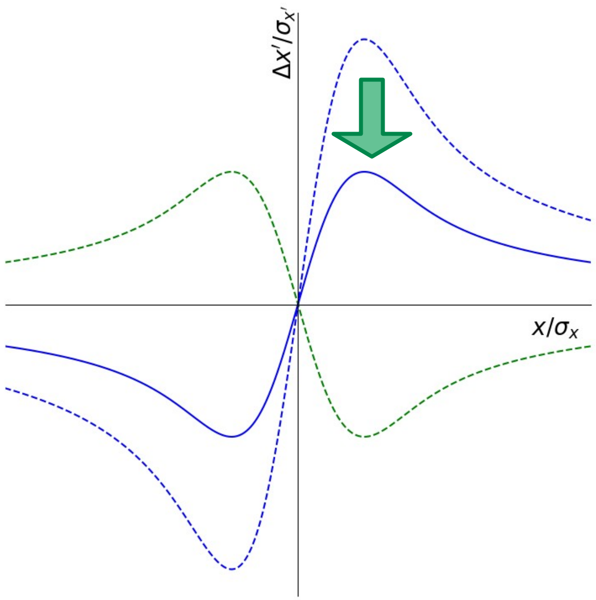

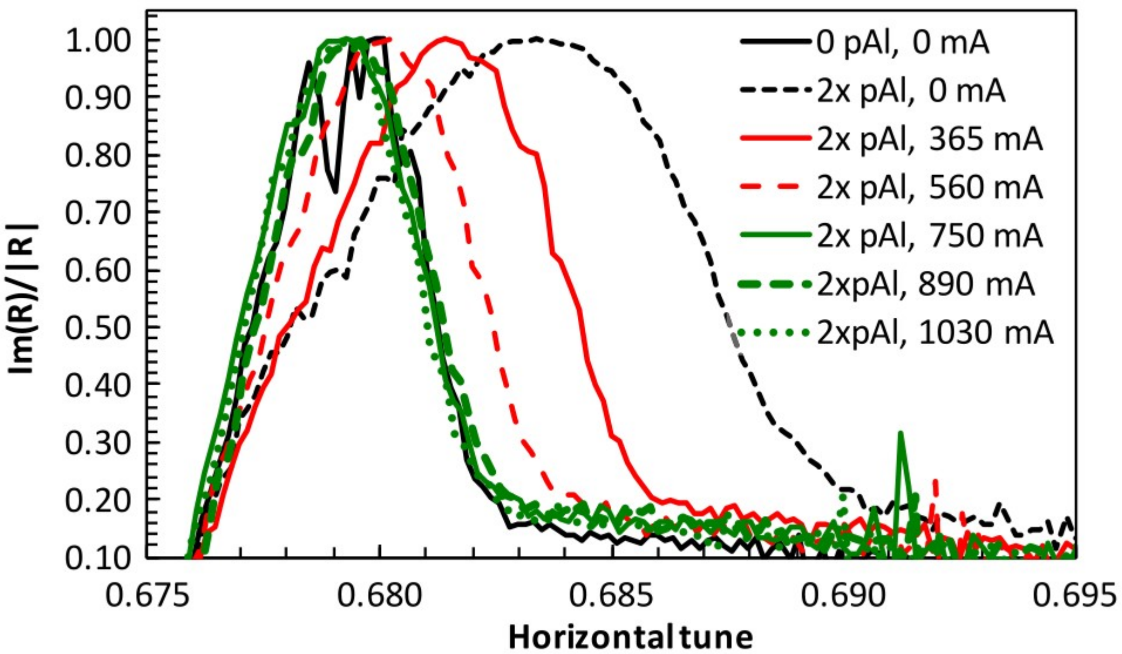

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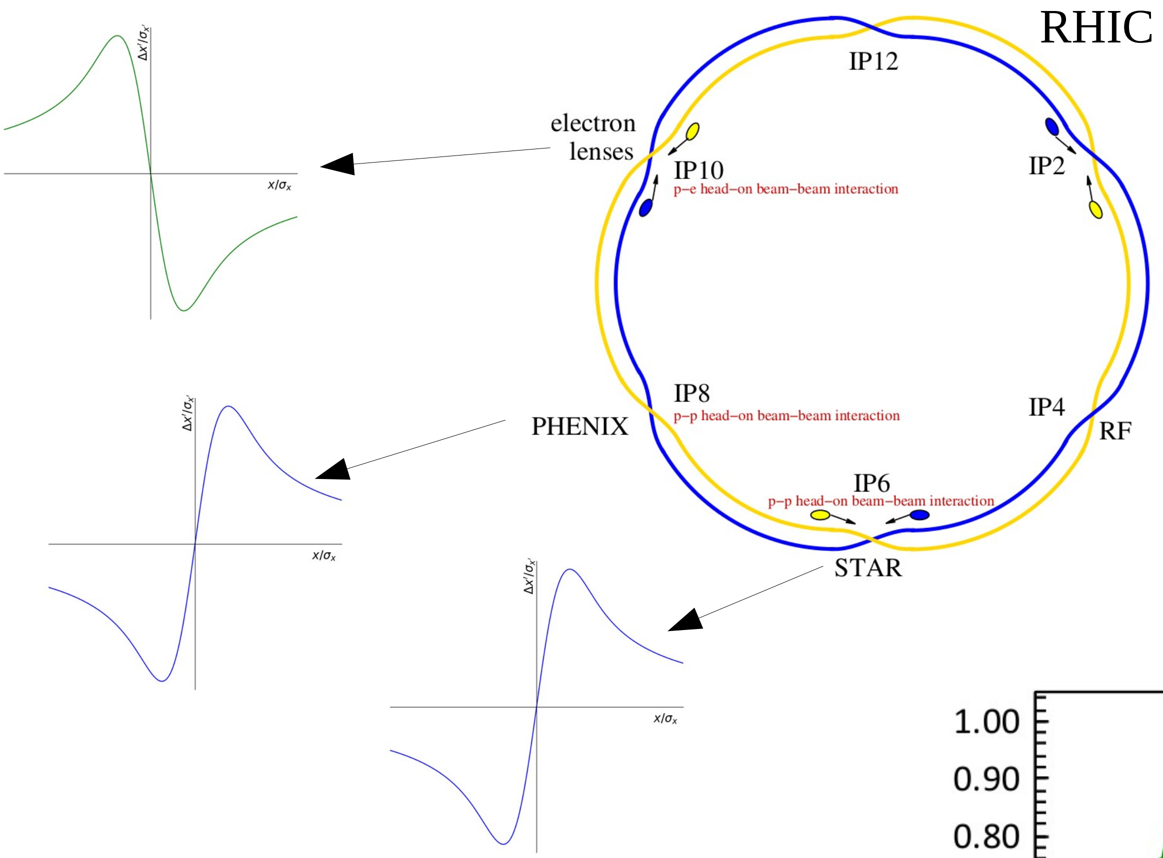

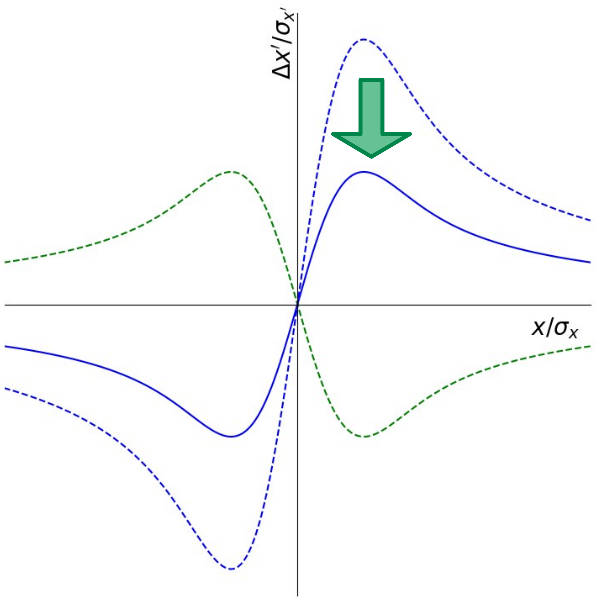

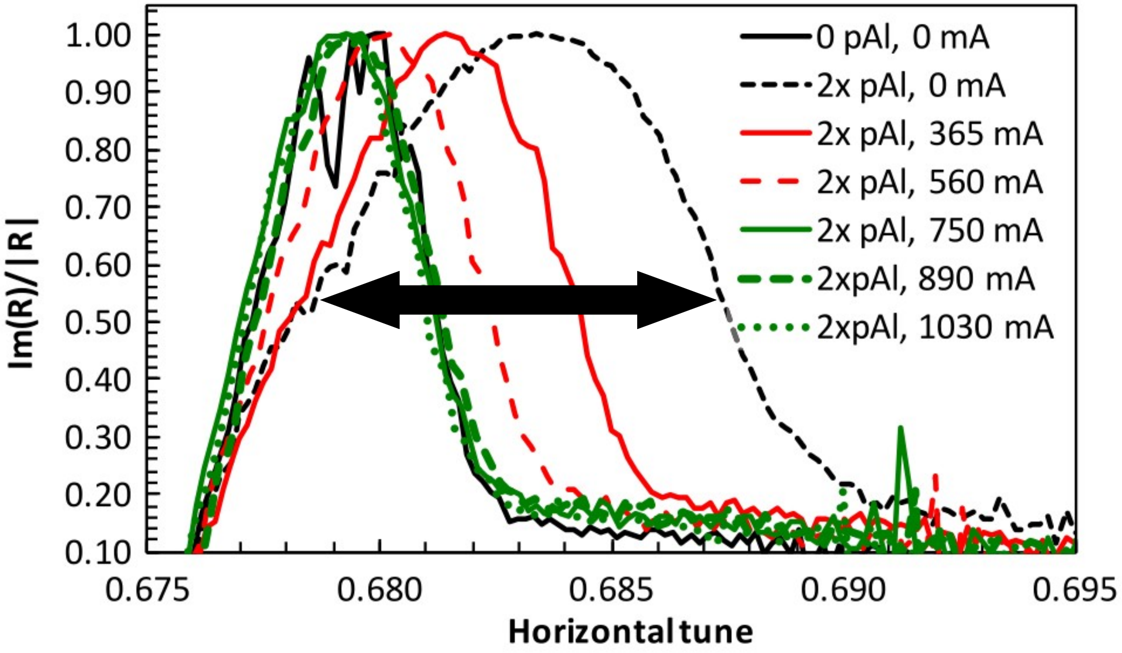

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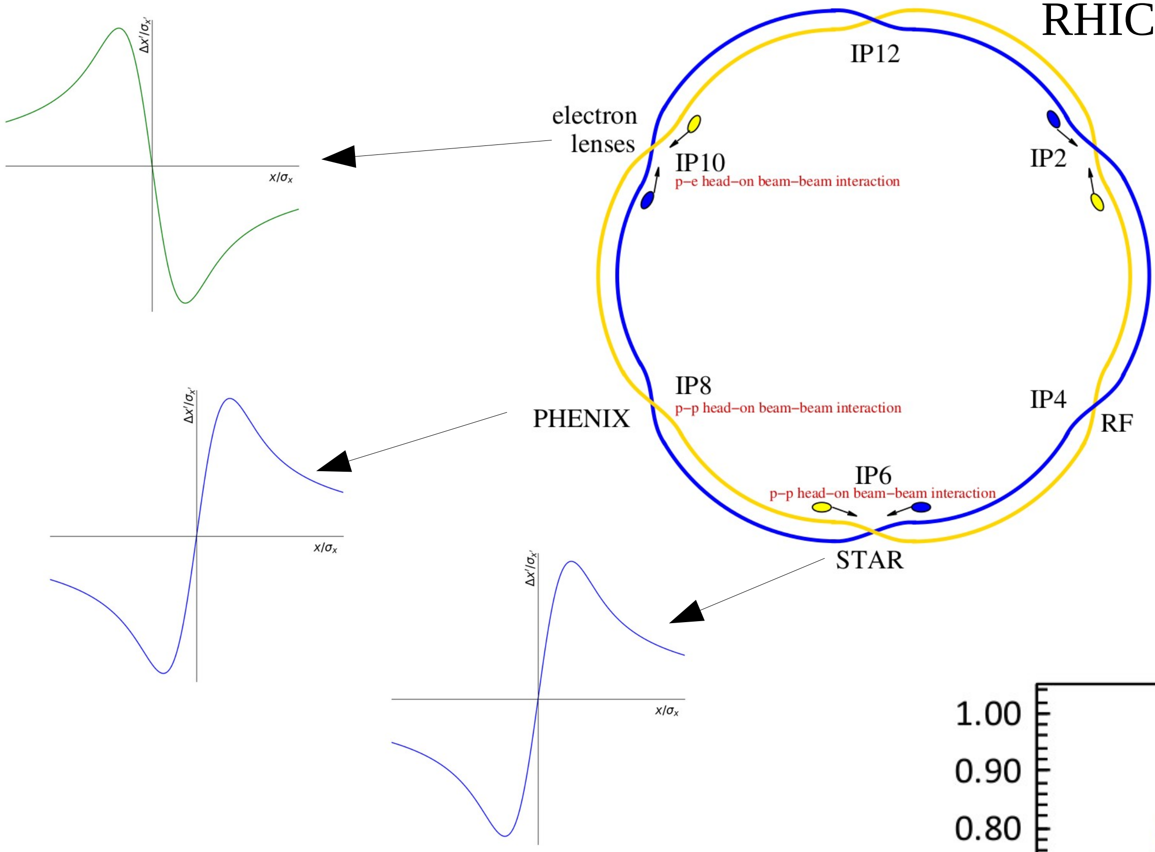
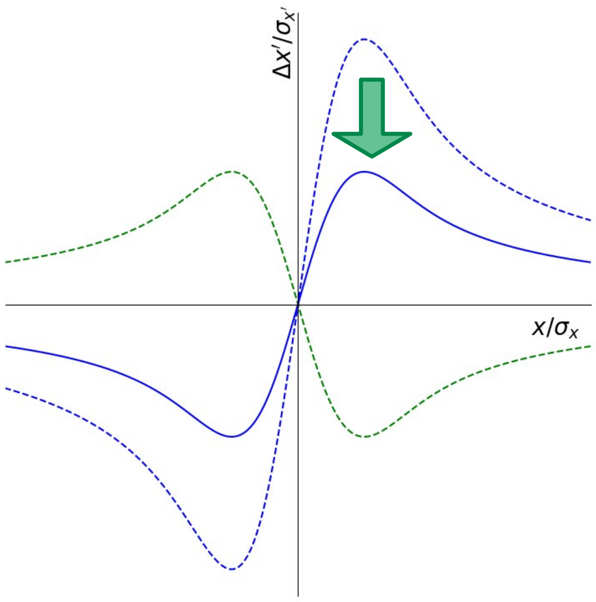
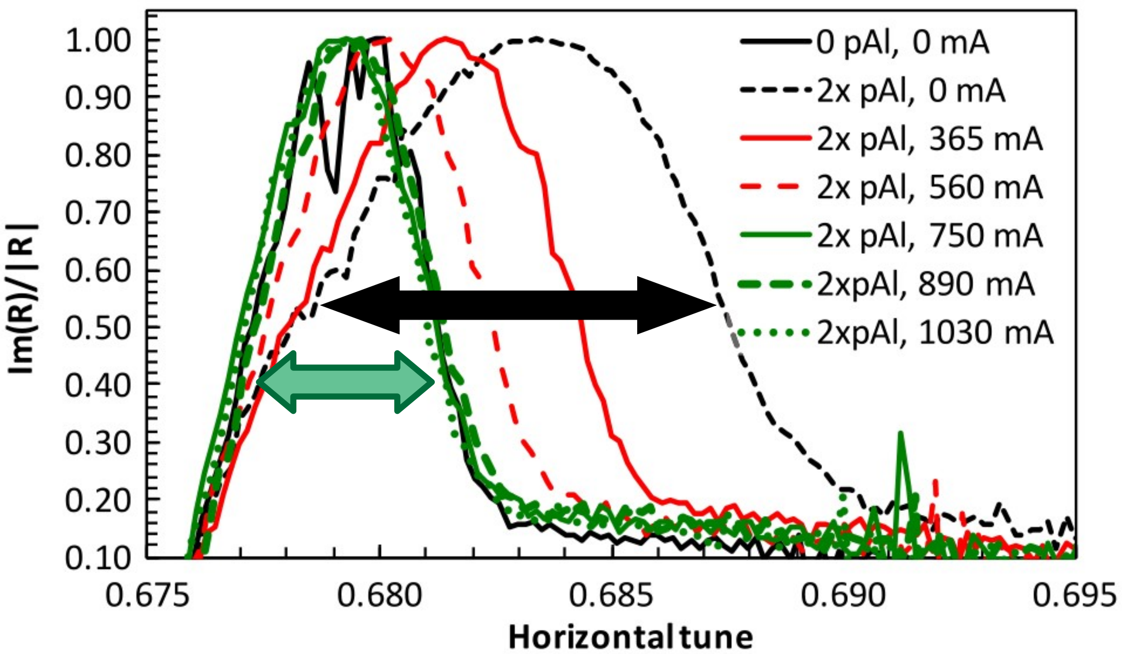




- Using an electron lens with a Gaussian profile on the tune spread is reduced

  → Improved beam quality preservation



Separation schemes

[Pretzel]

- Multibunch operation is key for most modern colliders

  → Increased luminosity without increasing the beam-beam force

$$\mathcal{L} = \frac{f_{rev} n_b N^2}{4\pi\sigma_x\sigma_y}$$

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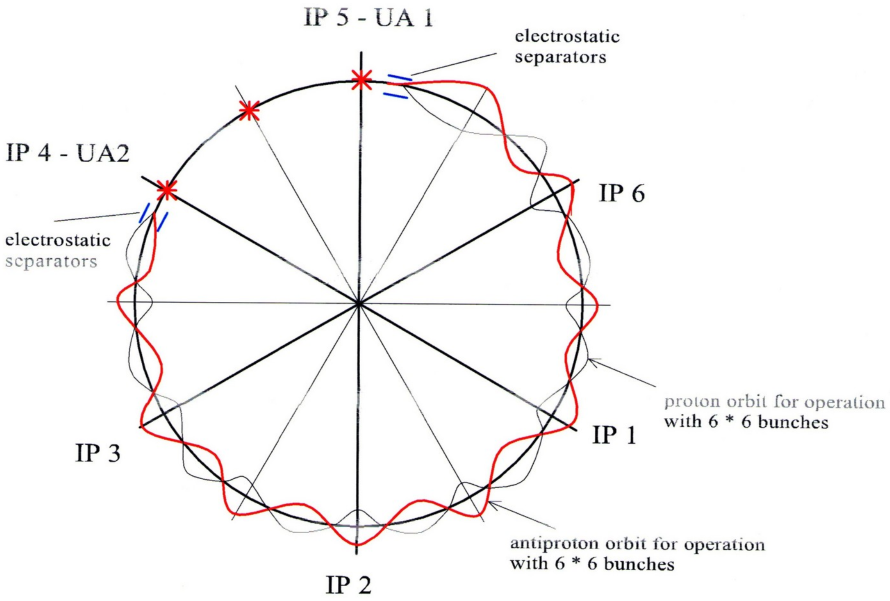
Two beams in one beam pipe
→ The pretzel scheme
(Sp$\bar{p}$S, CESR, LEP, Tevatron)



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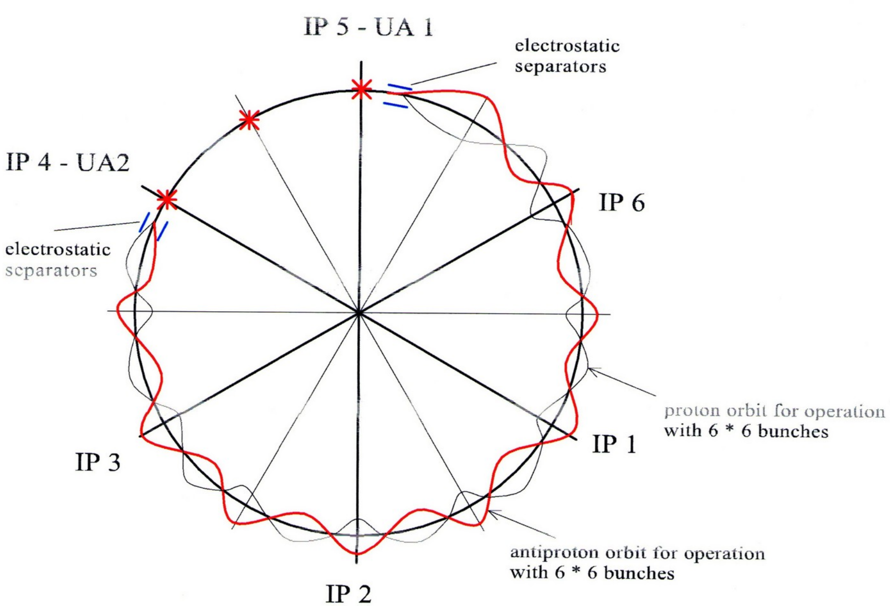
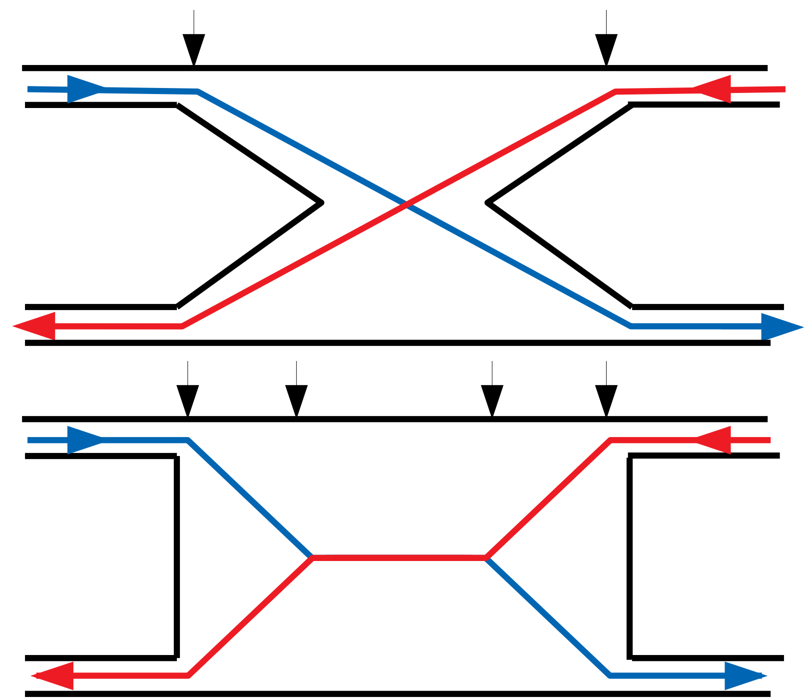


Beams in separate beam pipes
(DAΦNE, PEP-II, SuperKEKb, HERA, RHIC, LHC)



Long-range interactions

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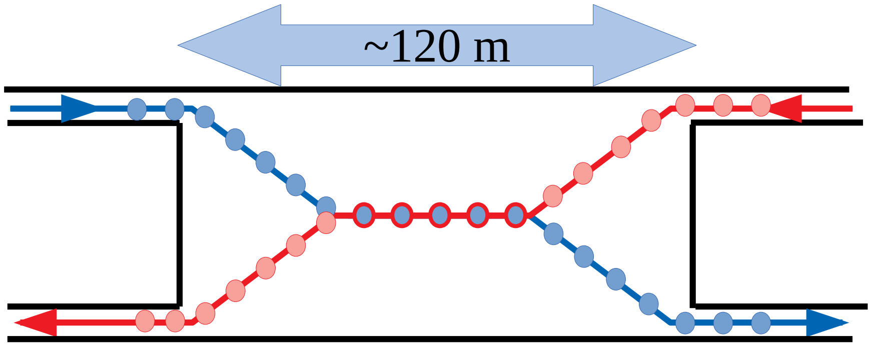

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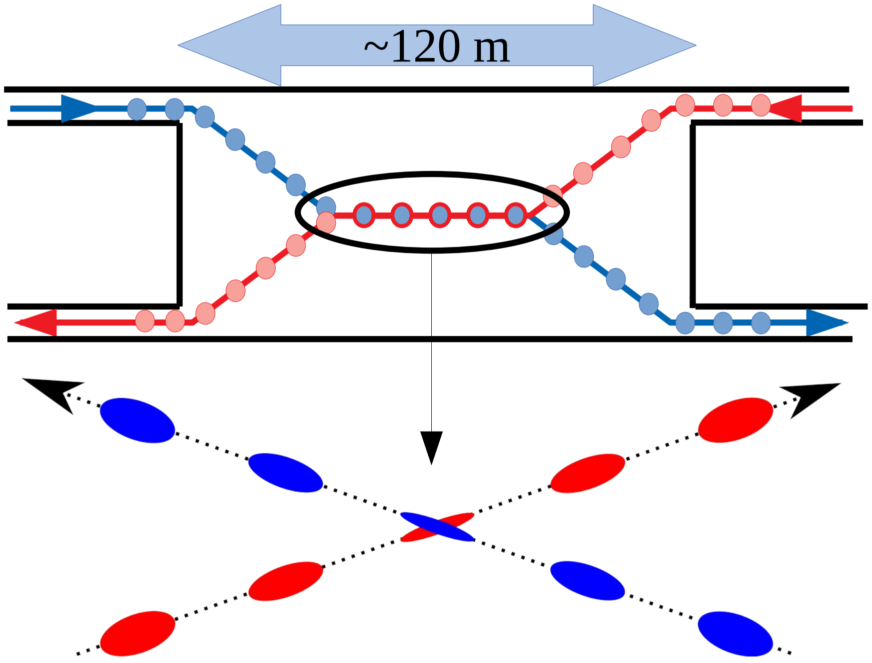

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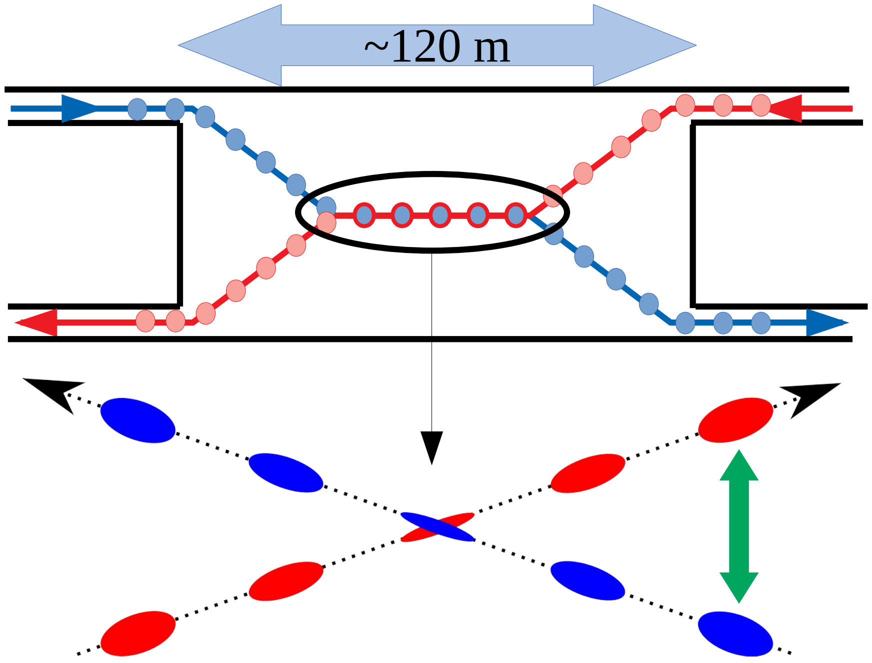

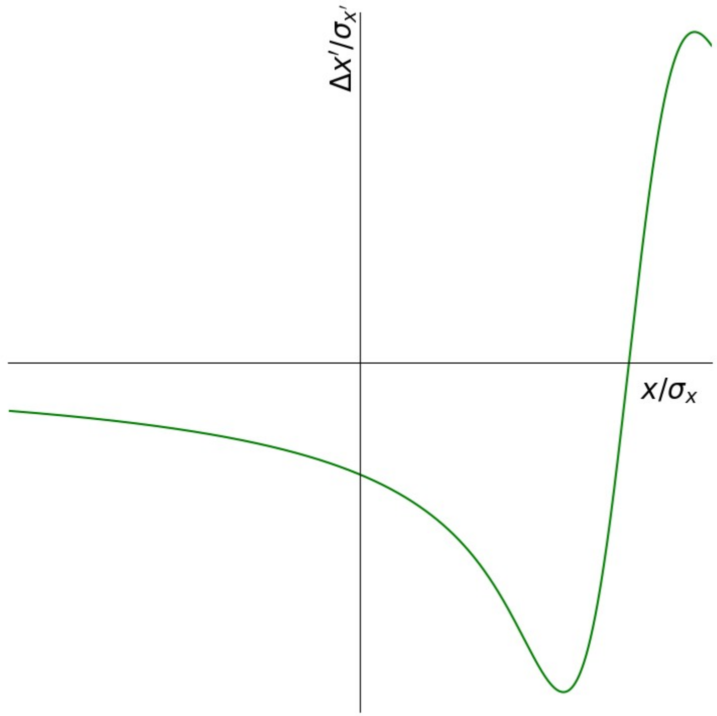

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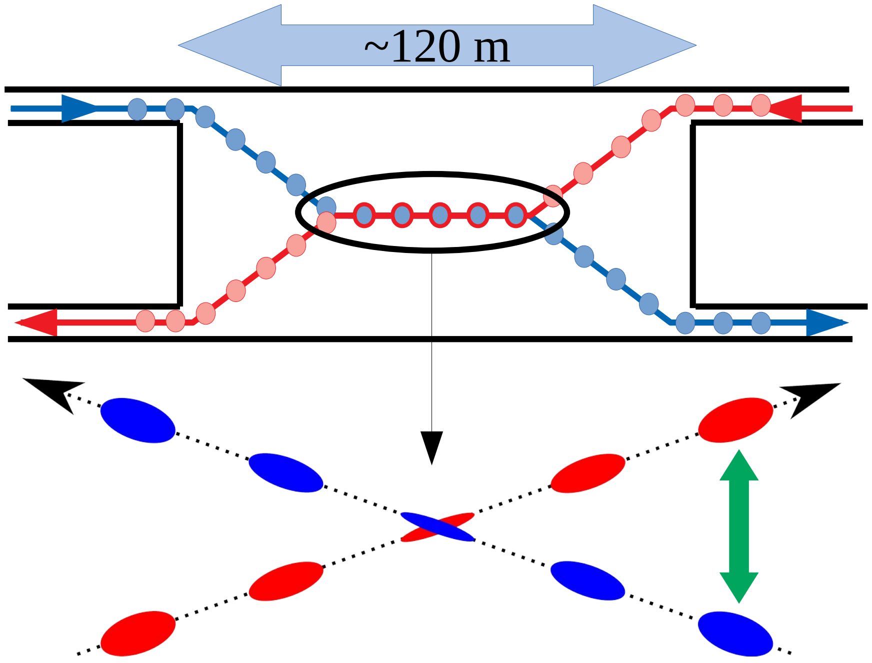
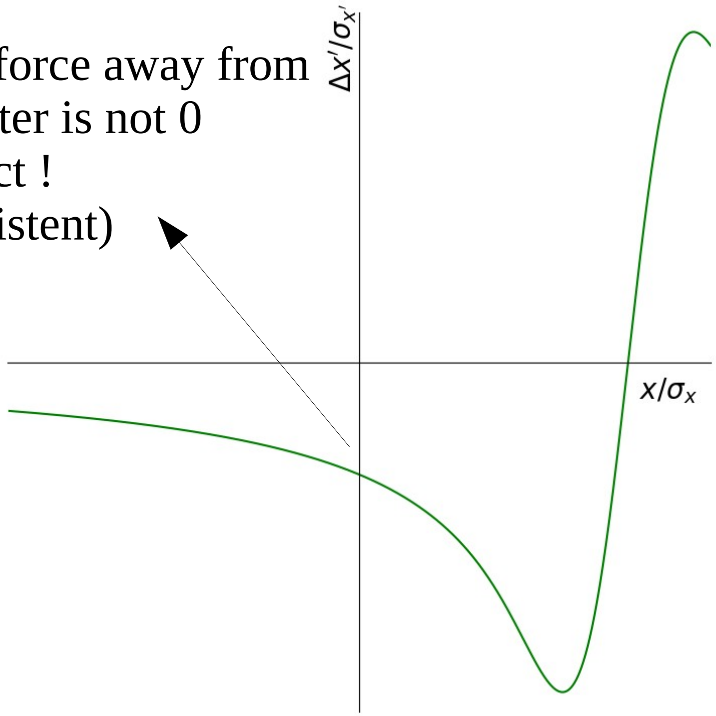


The average force away from the beam center is not 0
→ Orbit effect !
(self-consistent)



Long-range interactions

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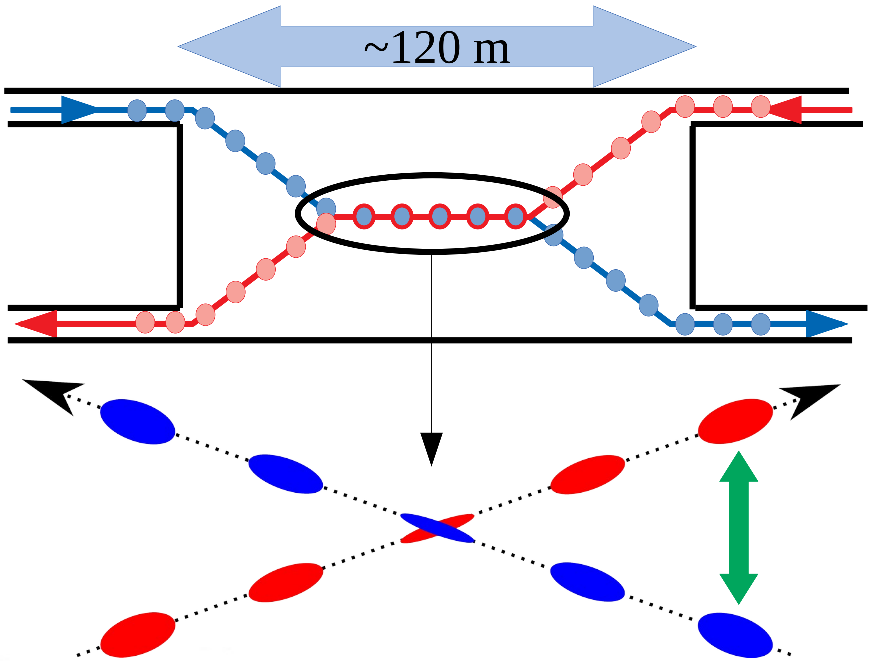

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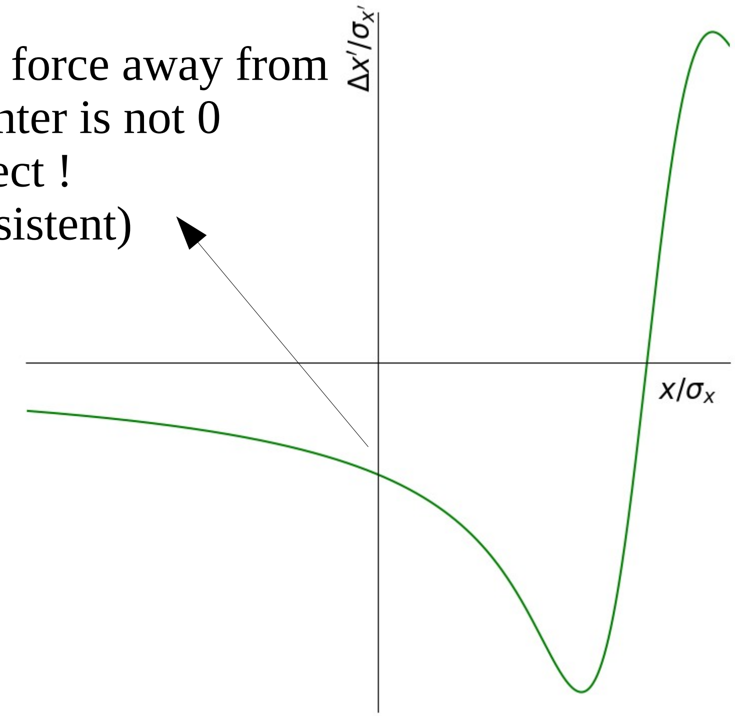

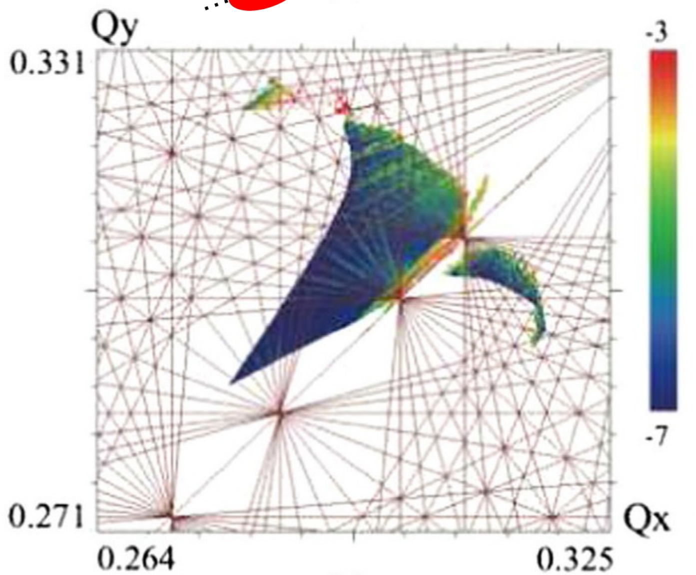

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→ Minimal crossing angle

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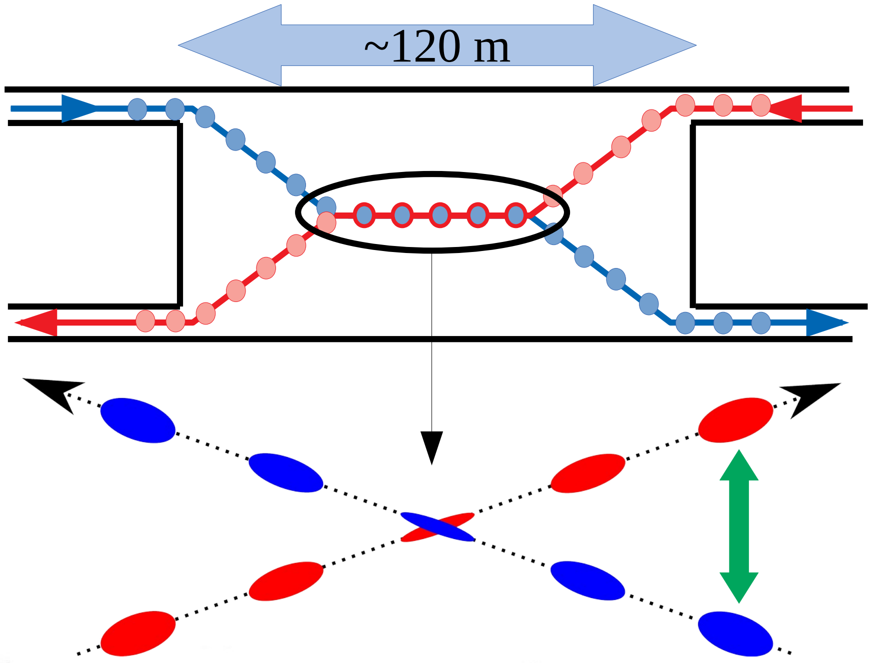

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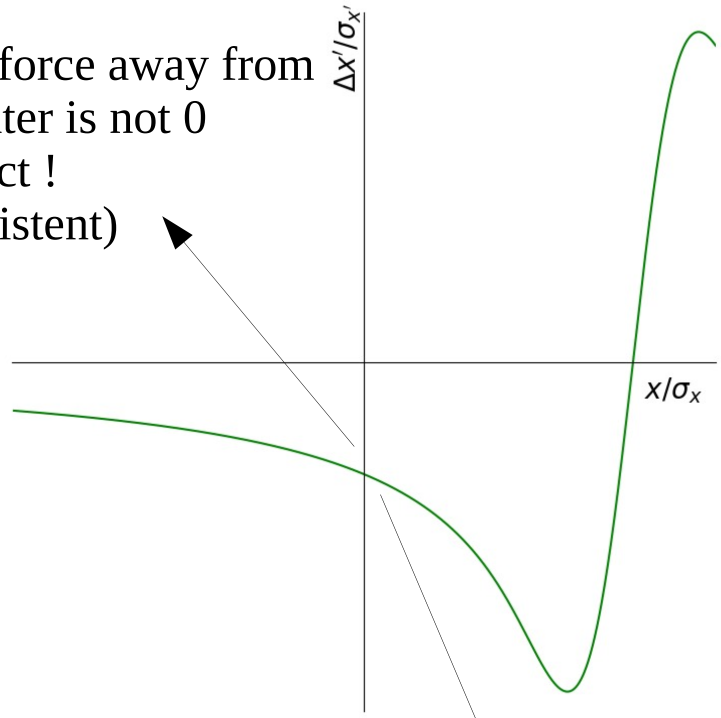

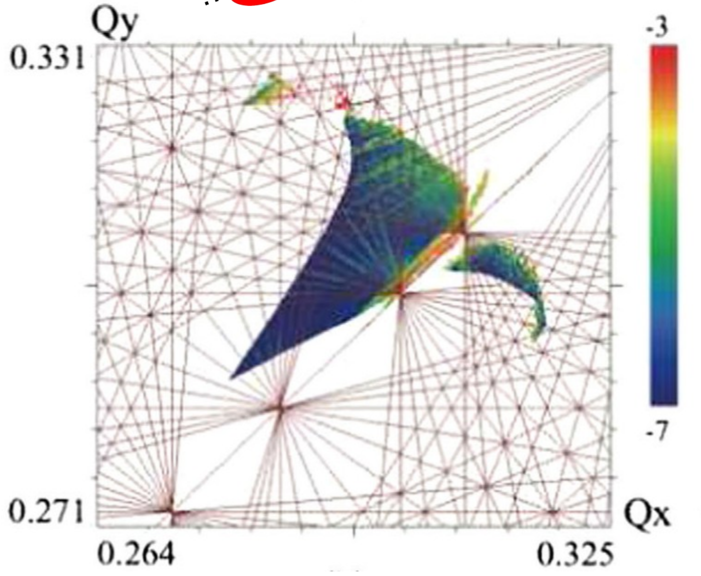

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Far from the beam center the force goes with 1/r
→ Comparable to the magnetic field of current carrying wire...
Compensation !

Crab cavity

[DQW]

- The crossing angle leads to a luminosity loss that can be mitigated with transverse deflecting cavities

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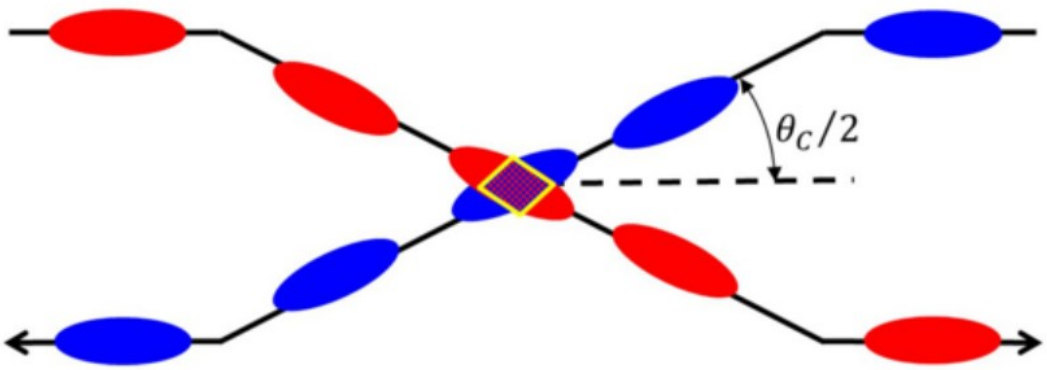

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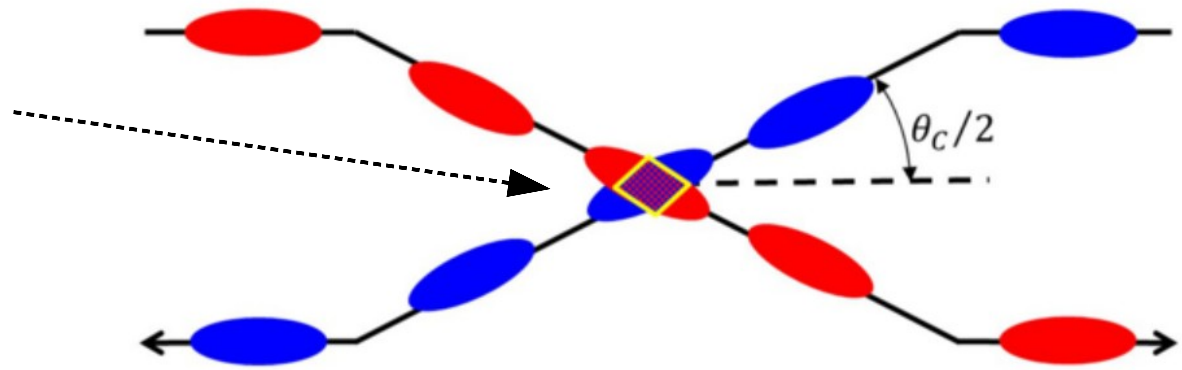

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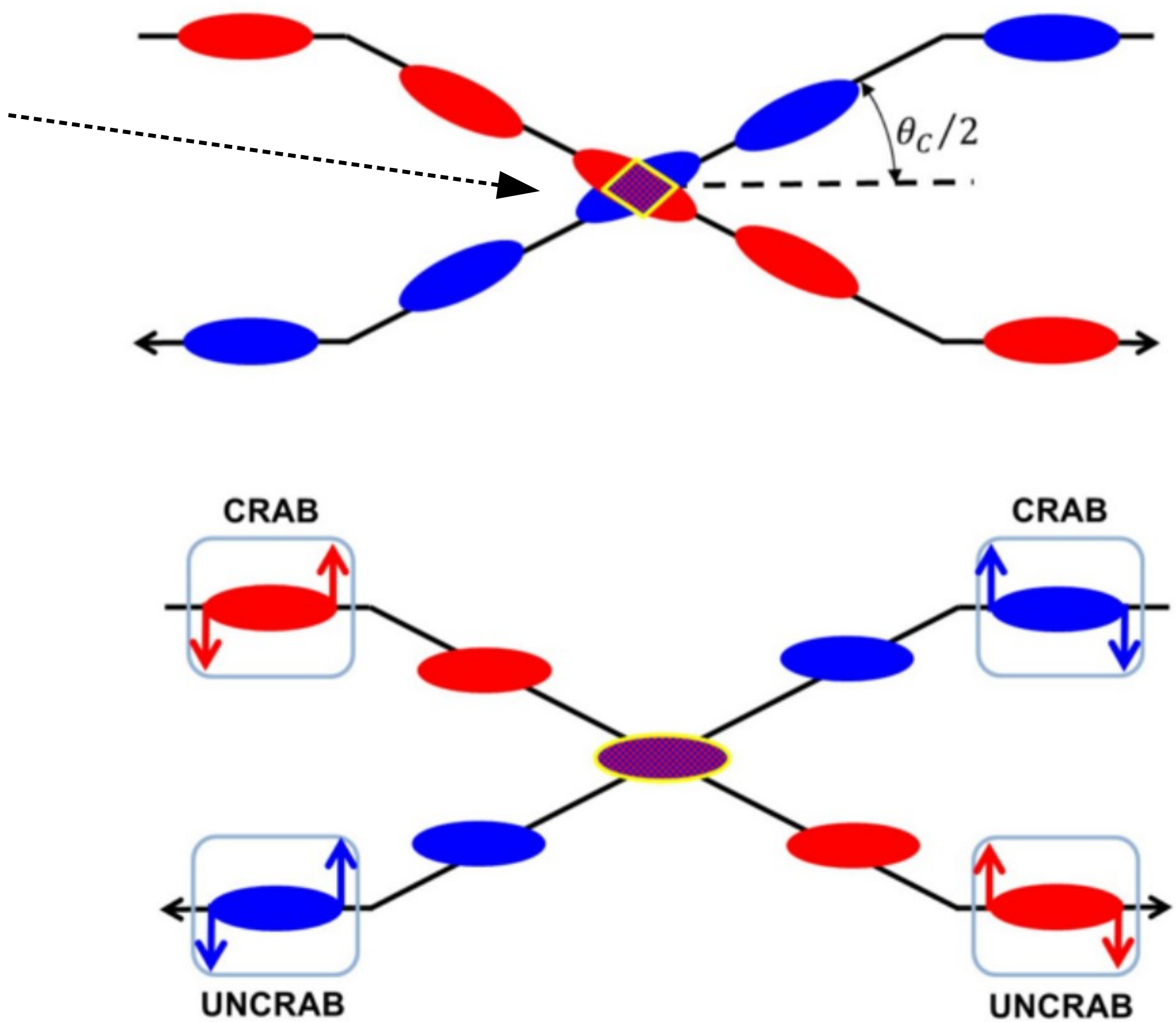

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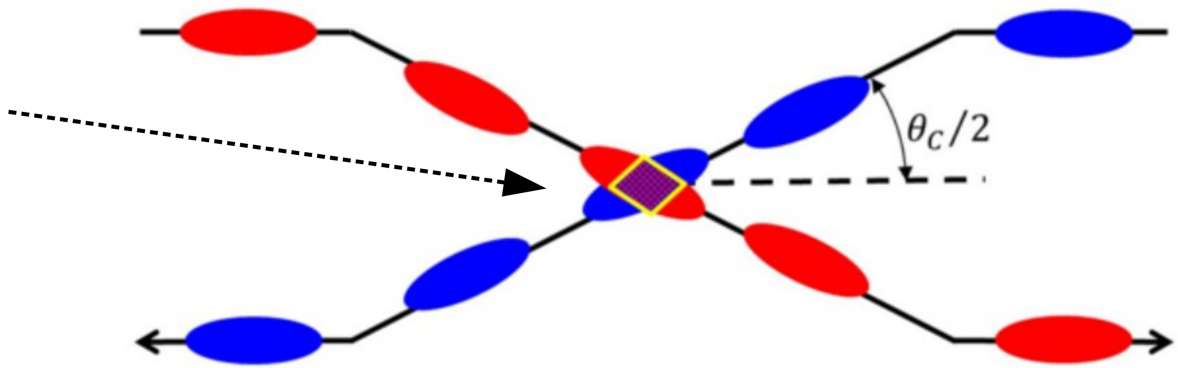
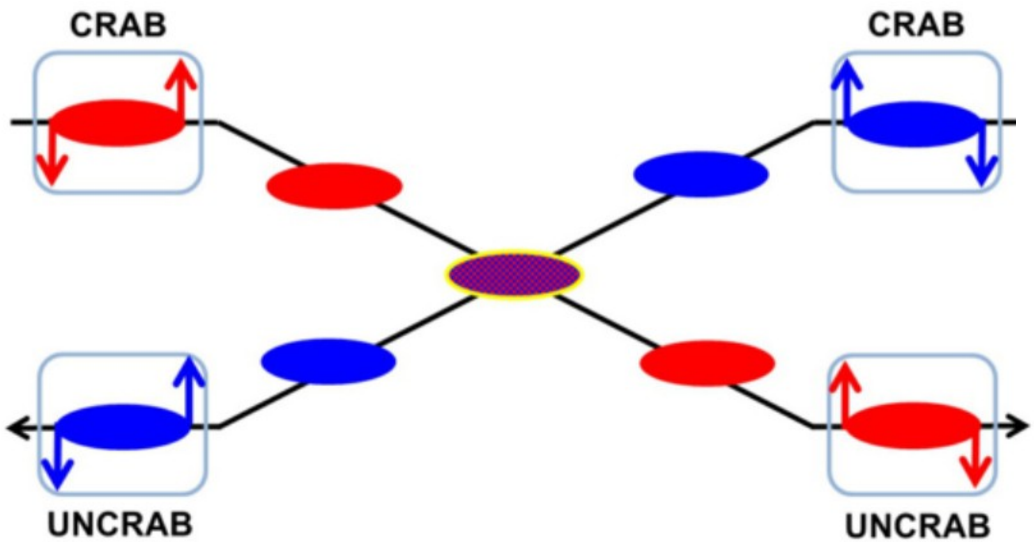
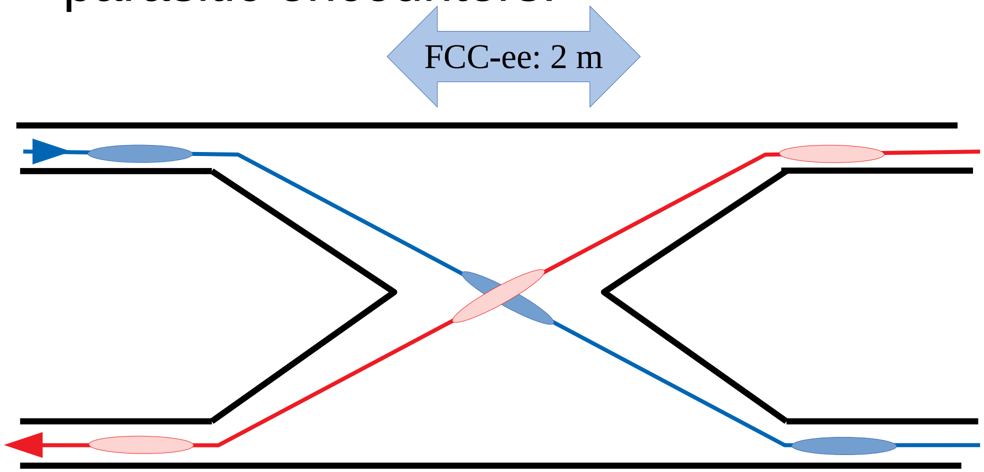




Large crossing angle and crab waist

[Raimondi, Shatilov]

- It is often needed to fully avoid parasitic encounters:



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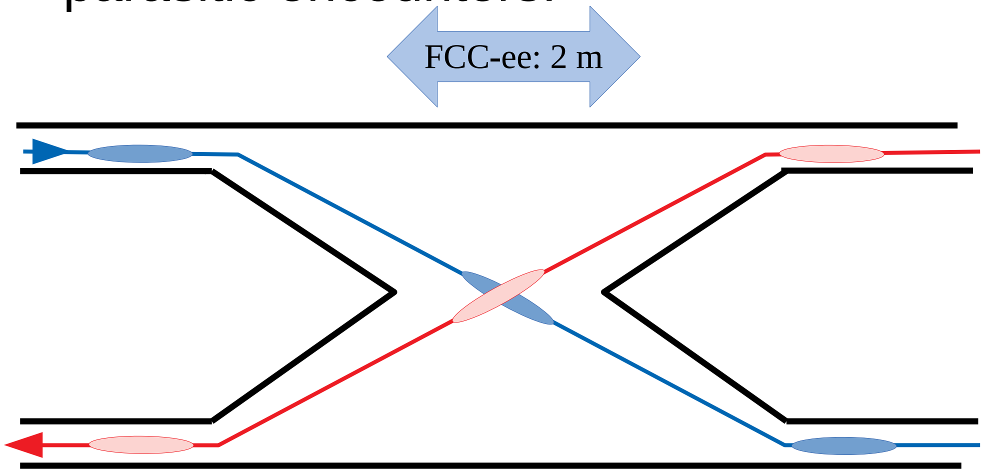

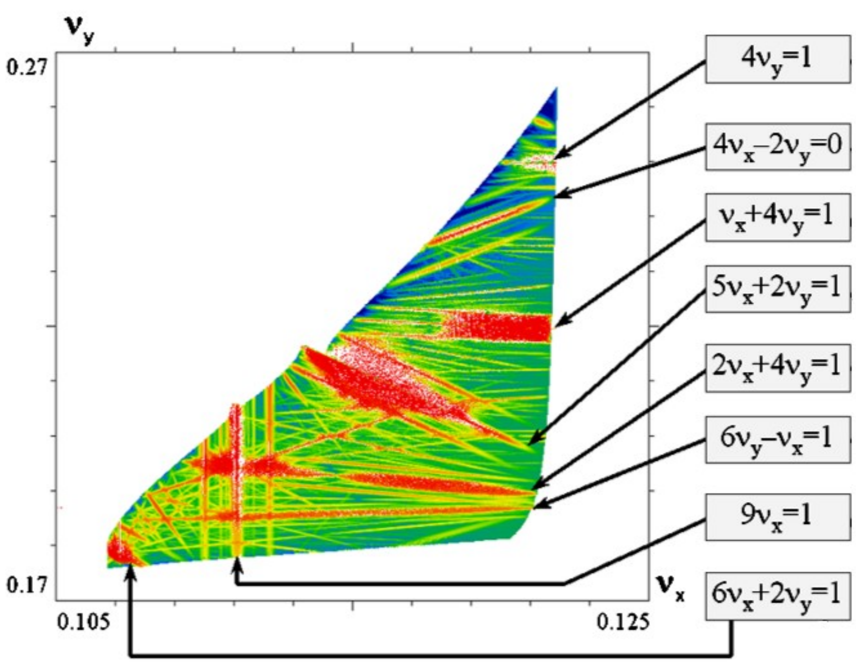

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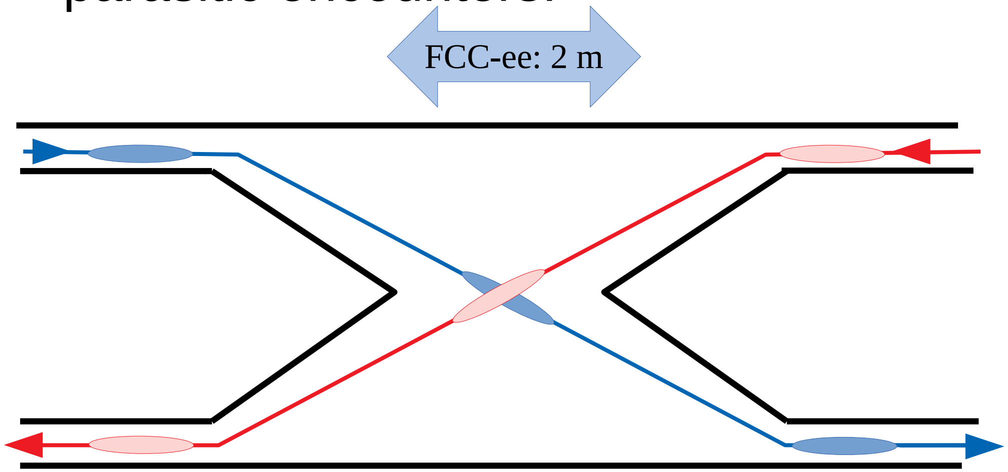
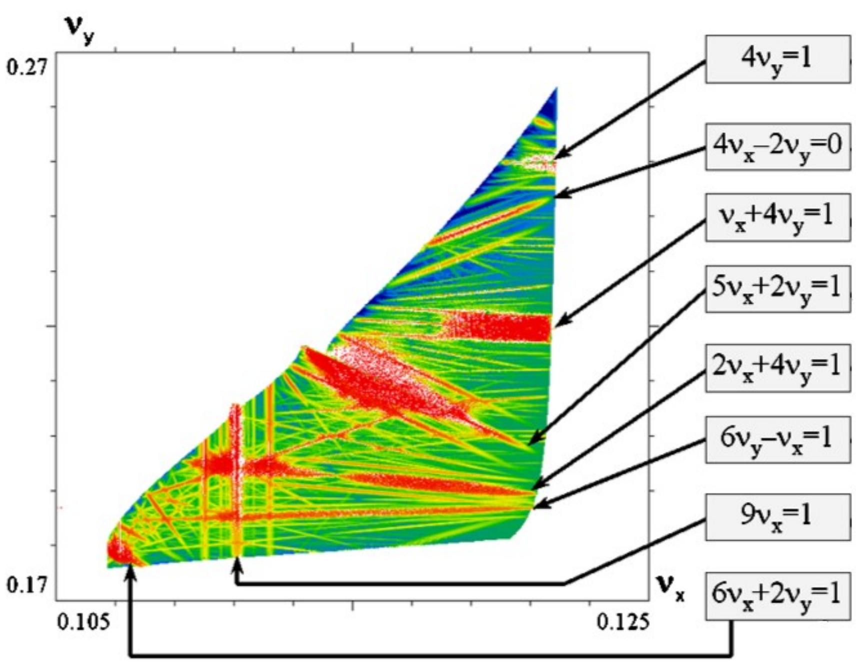
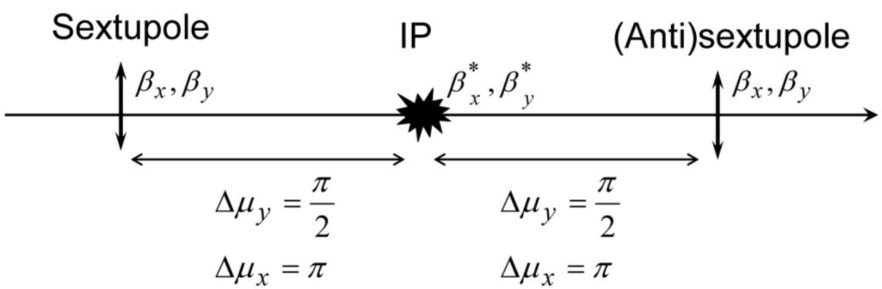




- So-called crab sextupoles can be used to improve the non-linear dynamics of the beam



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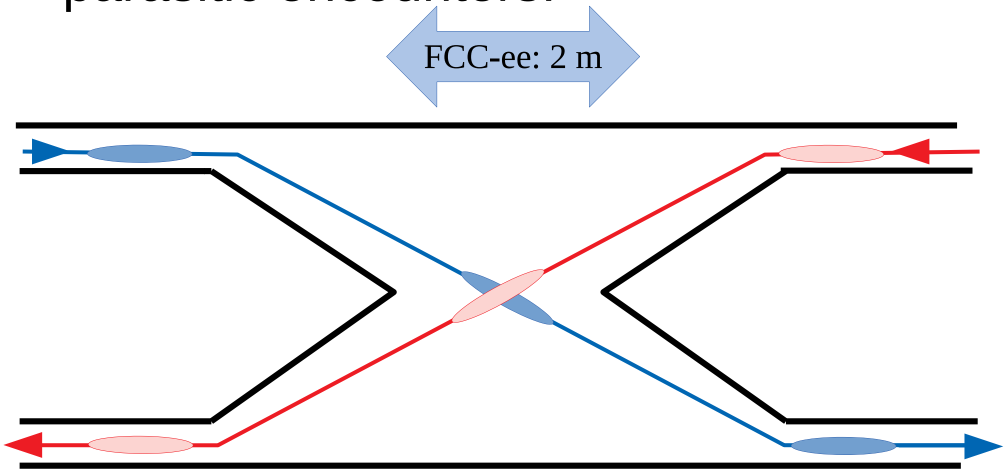

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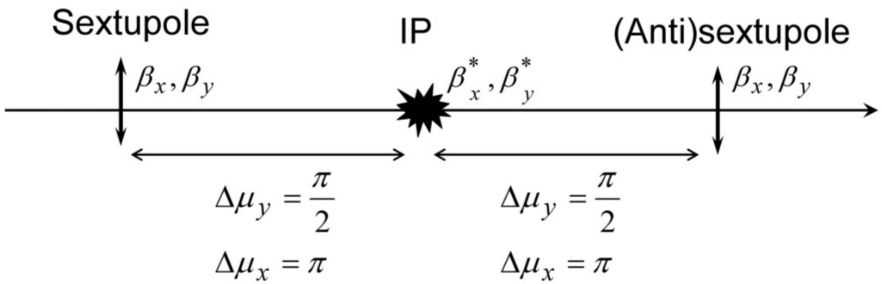

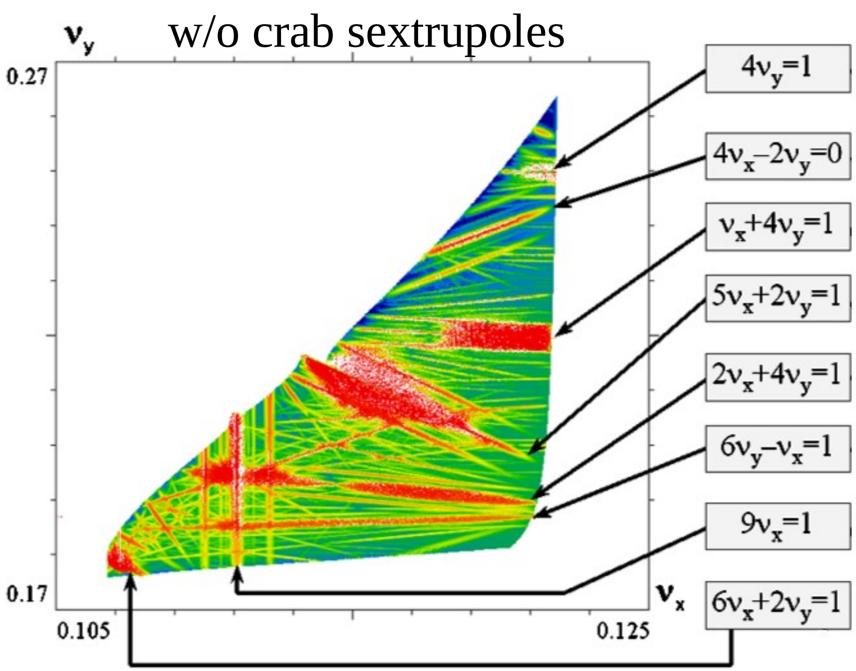

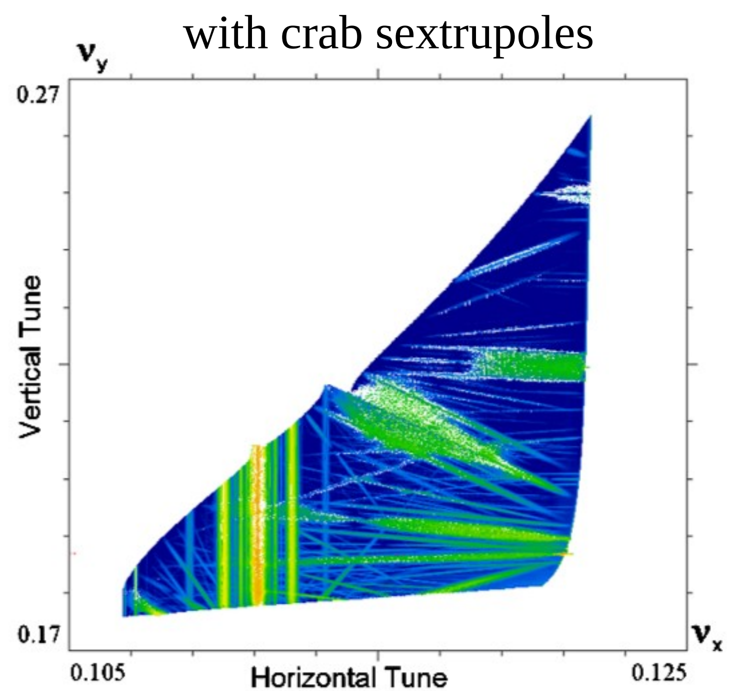

The crab confusion

$\neq$

Typical modern circular collider setups

Hadrons

- The head-on beam-beam parameter is limited by emittance preservation and beam losses

$$\xi \lessapprox 0.03 \longrightarrow \text{Electron lens compensation}$$

e^+e^-

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- Two pipes, many bunches and a small crossing angle $\theta \approx \mathcal{O}(100\ \mu\mathrm{rad})$
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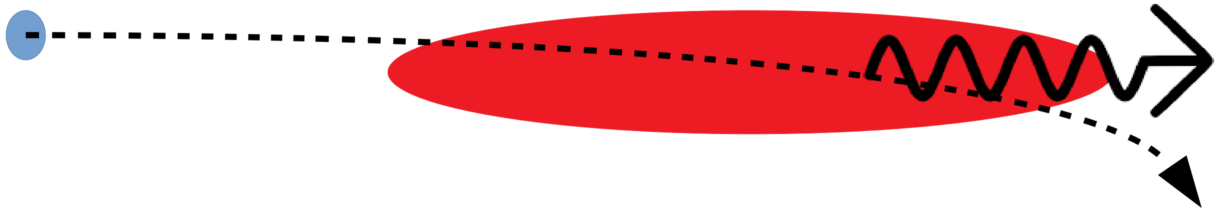
- In high energy electron-positron colliders the beam-beam force can be strong enough to generate high energy photons: Beamstrahlung



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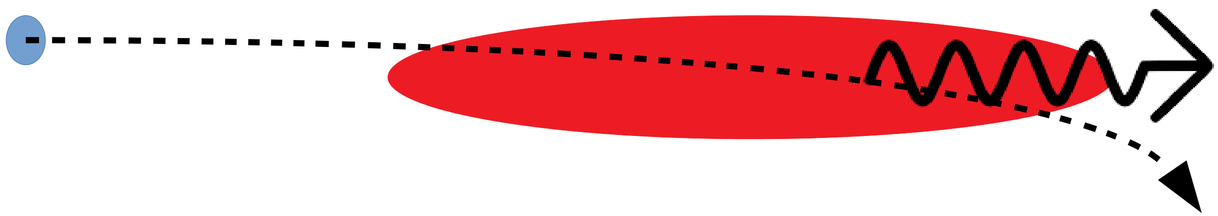

- We may apply the same formalism as for synchrotron radiation in a dipole, yet the bending radius is now defined by the non-linear beam-beam force

→ 'local' bending radius, for example:

$$n_\gamma \approx \frac{5}{2\sqrt{3}}\alpha\gamma \int ds \left\langle \frac{1}{\rho} \right\rangle_{x,y,z}$$

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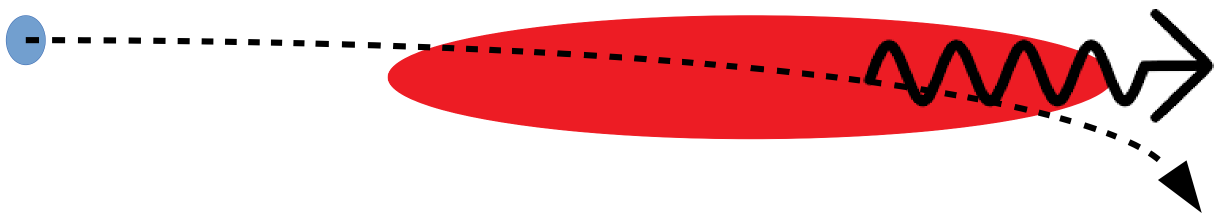

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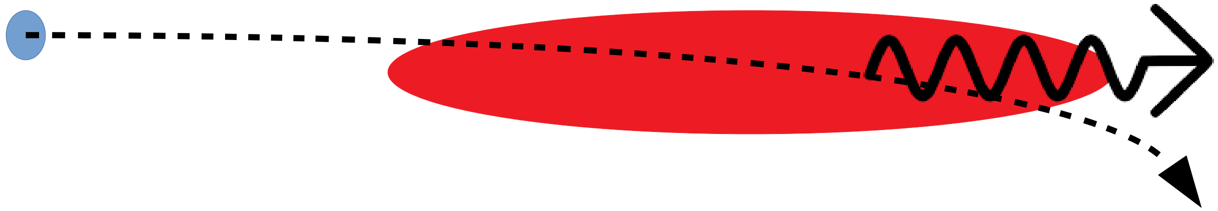

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  - Quantum excitation

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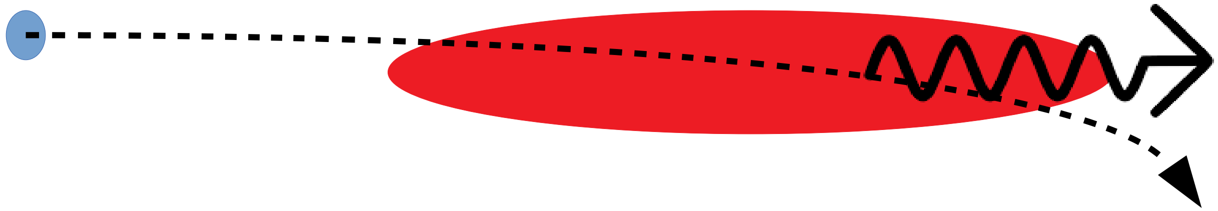
[BS]

- In high energy electron-positron colliders the beam-beam force can be strong enough to generate high energy photons: Beamstrahlung



- We may apply the same formalism as for synchrotron radiation in a dipole, yet the bending radius is now defined by the non-linear beam-beam force

  → 'local' bending radius, for example:

$$n_\gamma \approx \frac{5}{2\sqrt{3}} \alpha \gamma \int ds \left\langle \frac{1}{\rho} \right\rangle_{x,y,z}$$

- Similarly to synchotron radiation, beamstrahlung leads to
  - Energy loss (→ Touschek + Beamstrahlung + Bhabha scattering lifetime)
  - Emittance damping
  - Quantum excitation

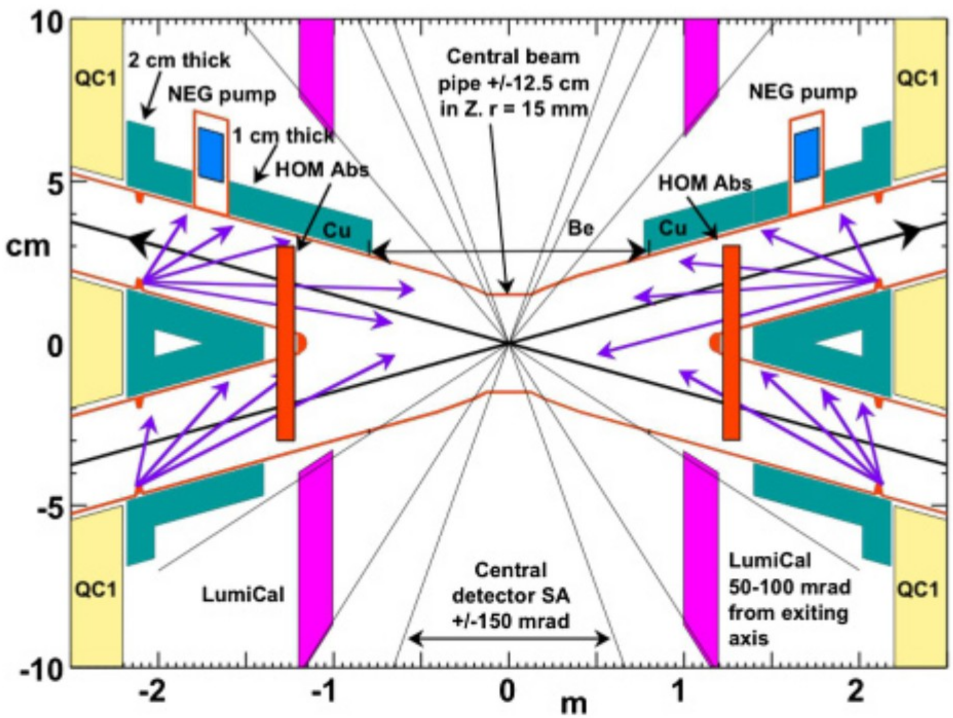
New equilibrium emittances
→ High momentum acceptance needed!

Beamstrahlung power

[FCCee, BSDump]

- For FCC-ee beamstrahlung generates hundreds of kW of photons propagating downstream of the IP

  → Need dedicated absorbers



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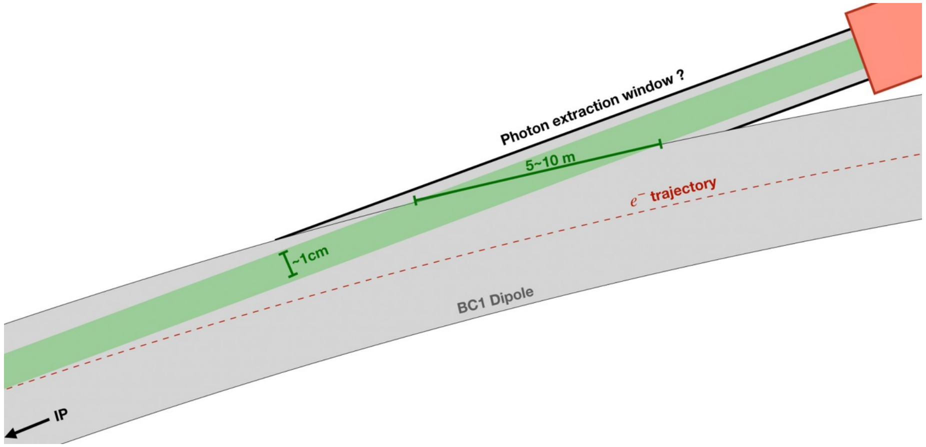
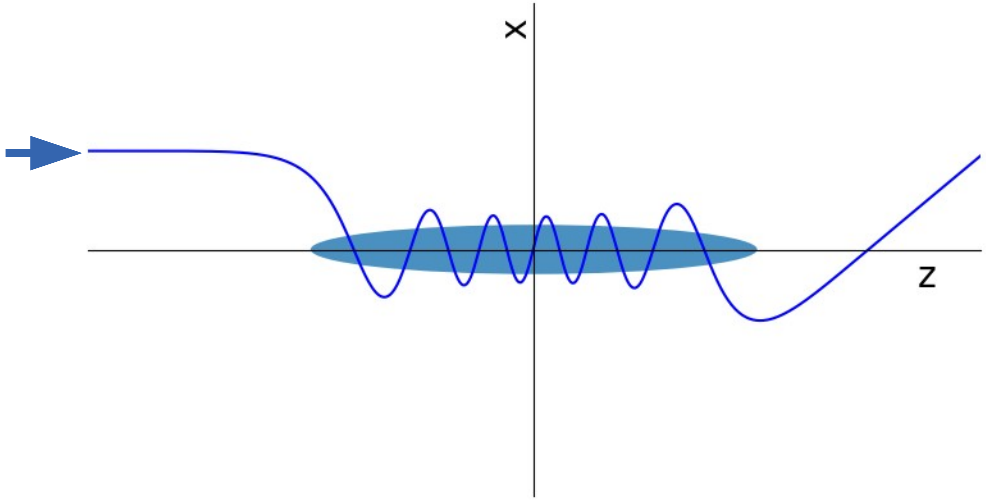


Beam-beam in linear colliders

[Schulte]

- As the beam quality does not have to be preserved after the collision, beam-beam forces can be much stronger in linear colliders



- The beam-beam force represents an additional focusing force at the IP which enhances the luminosity!
- The strength of the beam-beam force is rather characterised by the disruption parameter

$$D_{x,y} \equiv \frac{\sigma_z}{f_{x,y}} = \frac{2Nr_e\sigma_z}{\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

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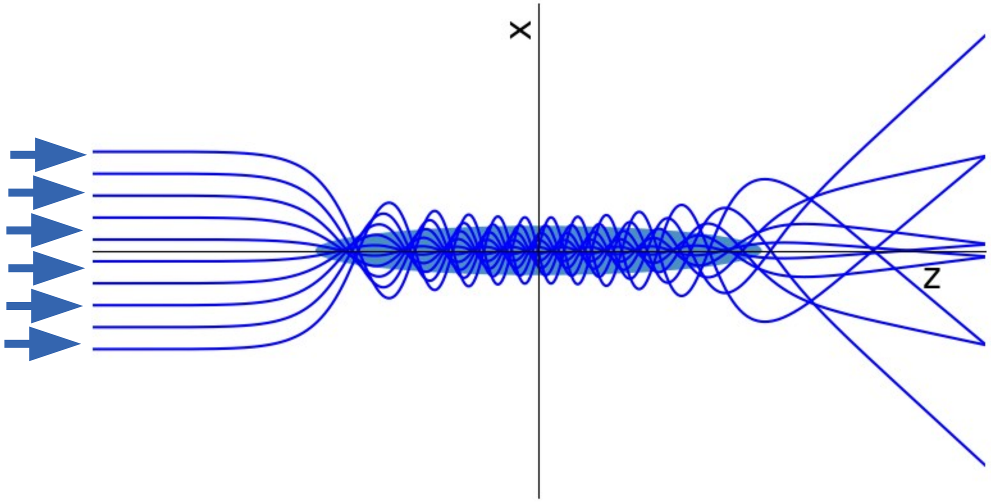

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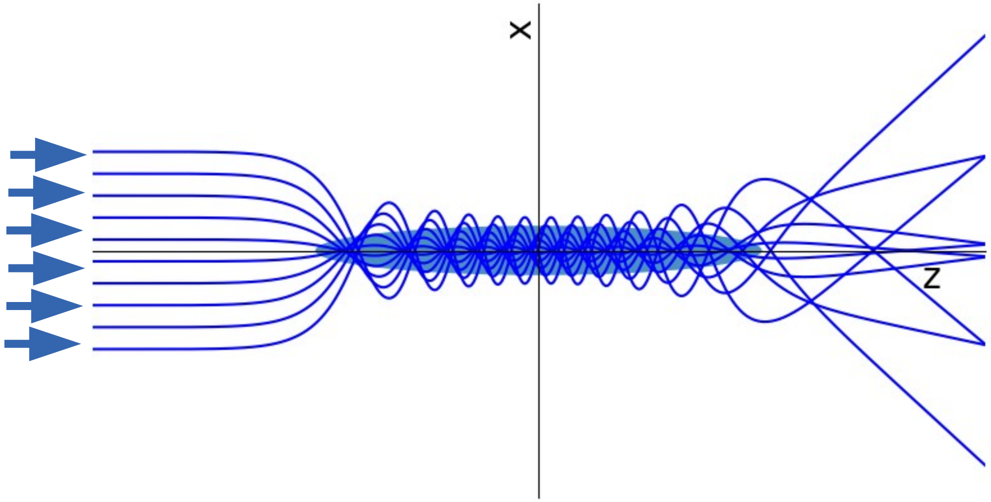

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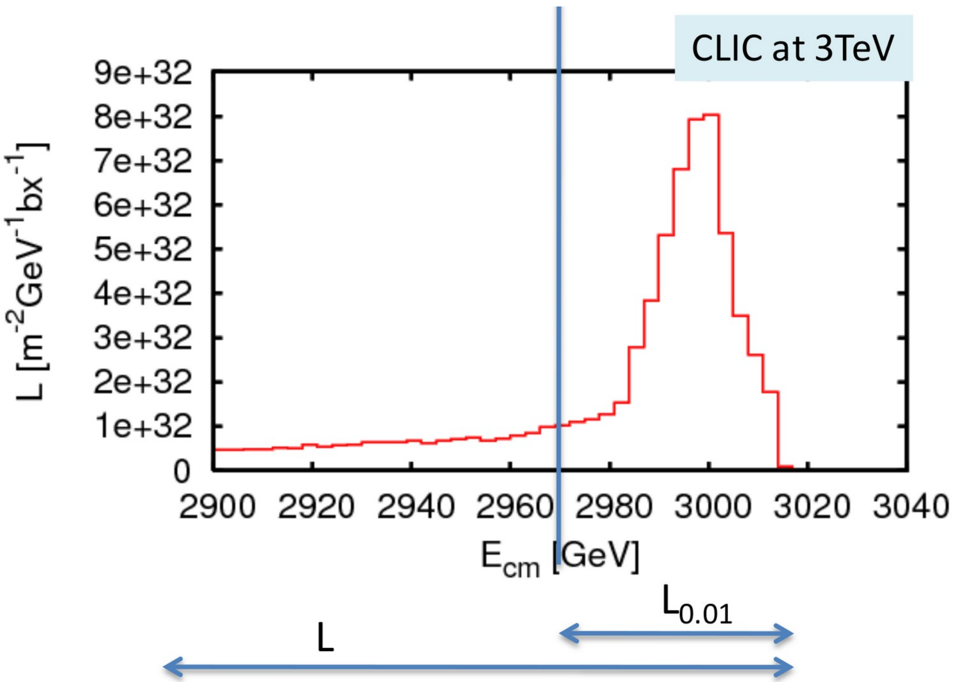

- As some particles lose energy to beamstrahlung during the collision, they may collide with a lower energy

→ Impact on luminosity spectrum

→ Need to maximise luminosity while minimising beamstrahlung

Summary

- The beam-beam force is obtained by subdividing the interaction of the two beams into a set of slice-particle interactions, where the Bassetti-Erkine formula applies (e.g. using a boosted frame)

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  - Tune shift, dynamic β, dynamic emittance, orbit effects, ...
- In the strong-strong regime, the modifications of the two beams have to be considered self-consistently
  - Beam-beam oscillation modes, flip-flop, ...
- The design of colliders is driven by the beam-beam effects in various ways

  → Maximisation of the luminosity minimising deterimental effects of the beam-beam interactions on the beam quality

  → Very different limits in hadron / e^+e^-, circular / linear colliders

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- *[Schulte] D. Schulte, Beam-beam effect in linear colliders, CERN Accelerator School on Intensity Limitations in Particle Beams , Geneva, 2015*

Related lecture notes

- *W. Herr and T. Pieloni, Beam-beam effects, CERN Accelerator School on Intensity Limitations in Particle Beams , Geneva, 2015*
- *X. Buffat, Coherent beam-beam effects, CERN Accelerator School on Intensity Limitations in Particle Beams , Geneva, 2015*
- *D. Schulte, Beam-beam effects in linear colliders, CERN Accelerator School on Intensity Limitations in Particle Beams , Geneva, 2015*
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