2024 CAS course on “RF for Accelerators”

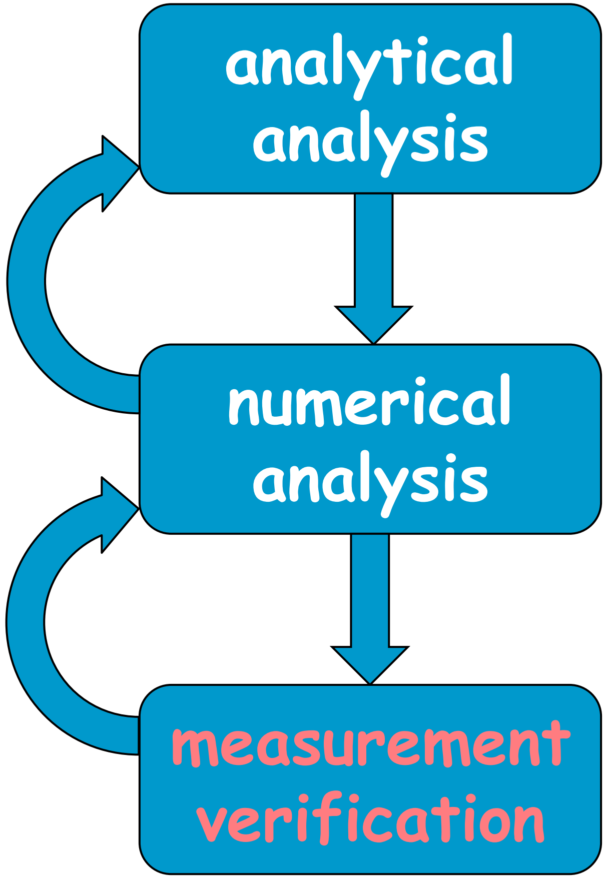
RF Measurements

– *Computer Lab Intro* –

Manfred Wendt – CERN

Design Workflow

- **Limited, simple geometries for analytical solution for electromagnetic field problems**
- **Benchmark numerical EM codes to known analytical solutions for a similar, simple geometry**
  - **here: perfect cylinder as resonant cavity with ideal materials**
    - Perfect electric conductor (PEC)
- **Benchmark the final, optimized geometry with different numerical codes (if possible)**
- **Manufacturing a prototype and compare RF measurement with numerical results**



Software, Numerical Codes for RF Engineering

- **Electromagnetic field problems**
  - **Dassault CST Studio Suite – commercial**
  - **Ansys HFSS – commercial**
  - **GdfidL (“Gitter drüber, fertig ist die Laube”) – commercial**
  - **ACE3P – SLAC development**
  - **Open source, e.g., Meep, grpMax, openEMS, FEniCS, Elmer FEM, FreeFEM, Bempp,… (still too many hurdles for practical use)**
- **Circuit level simulations**
  - **Keysight Pathwave ADS (Advance Design System) – commercial**
  - **AWR Microwave Office – commercial**
  - **Ansoft Designer – commercial**
  - **Various commercial SPICE versions, e.g., Pspice, HSPICE, PrimeSim SPICE,…**
  - **Open source / freeware: Qucs, Qucs-S, QucsStudio,**
    various free SPICE versions, e.g., Berkeley SPICE, Ltspice Ngspice
- **Other RF software**
  - **Too many to list. Here we will use:**
    ***Dellsperger Smith*, a software to introduce the *Smith* chart**

Cylindrical Resonator Equations (1)

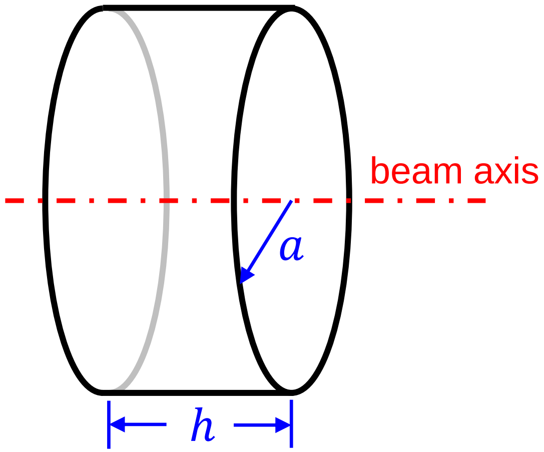
- **Eigenfrequencies**
  - **For the TE_{nml}-modes:**

$$f_{TEnml} = \frac{c}{2\pi}\sqrt{\left(\frac{p'_{nm}}{a}\right)^2 + \left(\frac{\pi l}{h}\right)^2} \quad [Hz]$$

  - **For the TM_{nml}-modes:**

$$f_{TMnml} = \frac{c}{2\pi}\sqrt{\left(\frac{p_{nm}}{a}\right)^2 + \left(\frac{\pi l}{h}\right)^2} \quad [Hz]$$

    - p_{nm} and p'_{nm} are the zeros of the Bessel function of 1st kind J_0, respectively the zeros of the derivative of the Bessel function of 1st kind J'_0
    - $c \cong 2.998 \times 10^8\ m/s$ speed of light

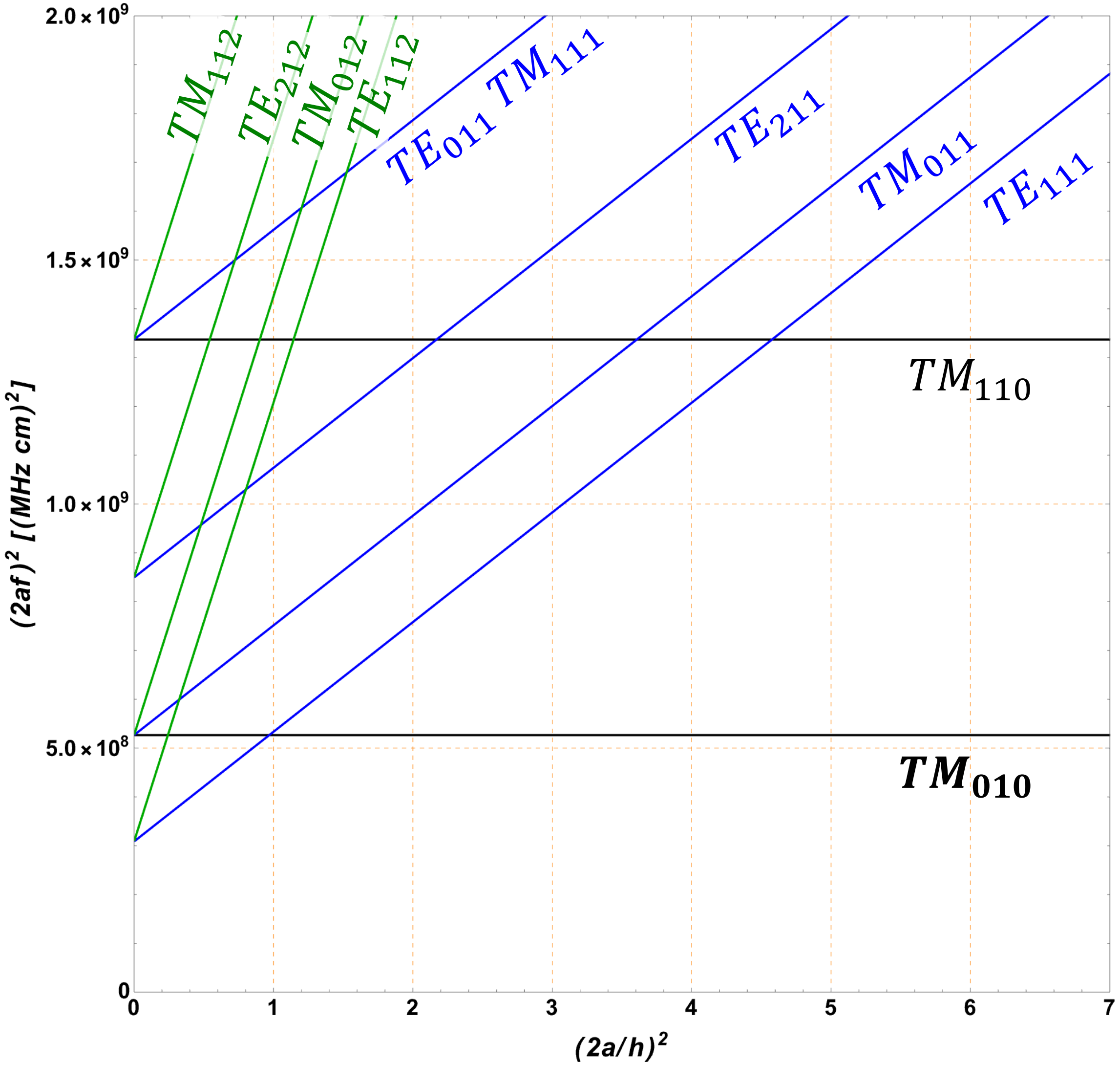


- **“Ideal” cylindrical resonator**
  - **No beam ports**
  - **Diameter: $2a$**
  - **Height: h**
  - **Vacuum: $\varepsilon_r = 1$**

n	p'_{n1}	p'_{n2}	p'_{n3}
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

n	p_{n1}	p_{n2}	p_{n3}
0	**2.405**	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

Cylindrical Resonator Equations (2)



- **Analytical equations for the TM_{010} accelerating mode**
  - **TM_{010}-mode related wavelength:**

$$\lambda_{TM010} = \frac{c}{f_{TM010}} = \frac{2\pi}{p_{01}} a \cong 2.61274\, a \;\; [m]$$

$$a = \frac{p_{01}}{2\pi} \lambda_{TM010} \cong 0.38274\, \lambda_{TM010} \;\; [m]$$

  - **TM_{010}-mode resonance frequency:**

$$f_{TM010} = \frac{c}{\lambda_{TM010}} \;\; [Hz]$$

Mode chart for cylindrical resonators

Cylindrical Resonator Equations (3)

- **Quality factor – unloaded Q_0 (Q-factor or Q-value)**

$$\boldsymbol{Q = \frac{a}{\delta}\left[1 + \frac{a}{h}\right]^{-1}}$$

  - **with the skin-depth:**

$$\delta = \sqrt{\frac{2}{\omega_{TM010}\sigma\mu}} \quad [m]$$

    - with:

$$\omega_{TM010} = 2\pi f_{TM010}$$

$$\mu = \mu_r \mu_0 \cong 1 \times 4\pi \cdot 10^{-7}\ H/m$$

(non-magnetic media)

$$\sigma\ [S/m]$$

(conductivity of the material of the cavity walls)

- **“Geometry factor” R/Q (R-over-Q)**

$$\boldsymbol{R/Q = \frac{4\,\eta_0}{\pi p_{01}^3 J_1^2(p_{01})} \frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a} \quad [\Omega]}$$

$$R/Q \cong 128\ \Omega \frac{\sin^2\left(\frac{p_{01}}{2}\frac{h}{a}\right)}{h/a} [\Omega]$$

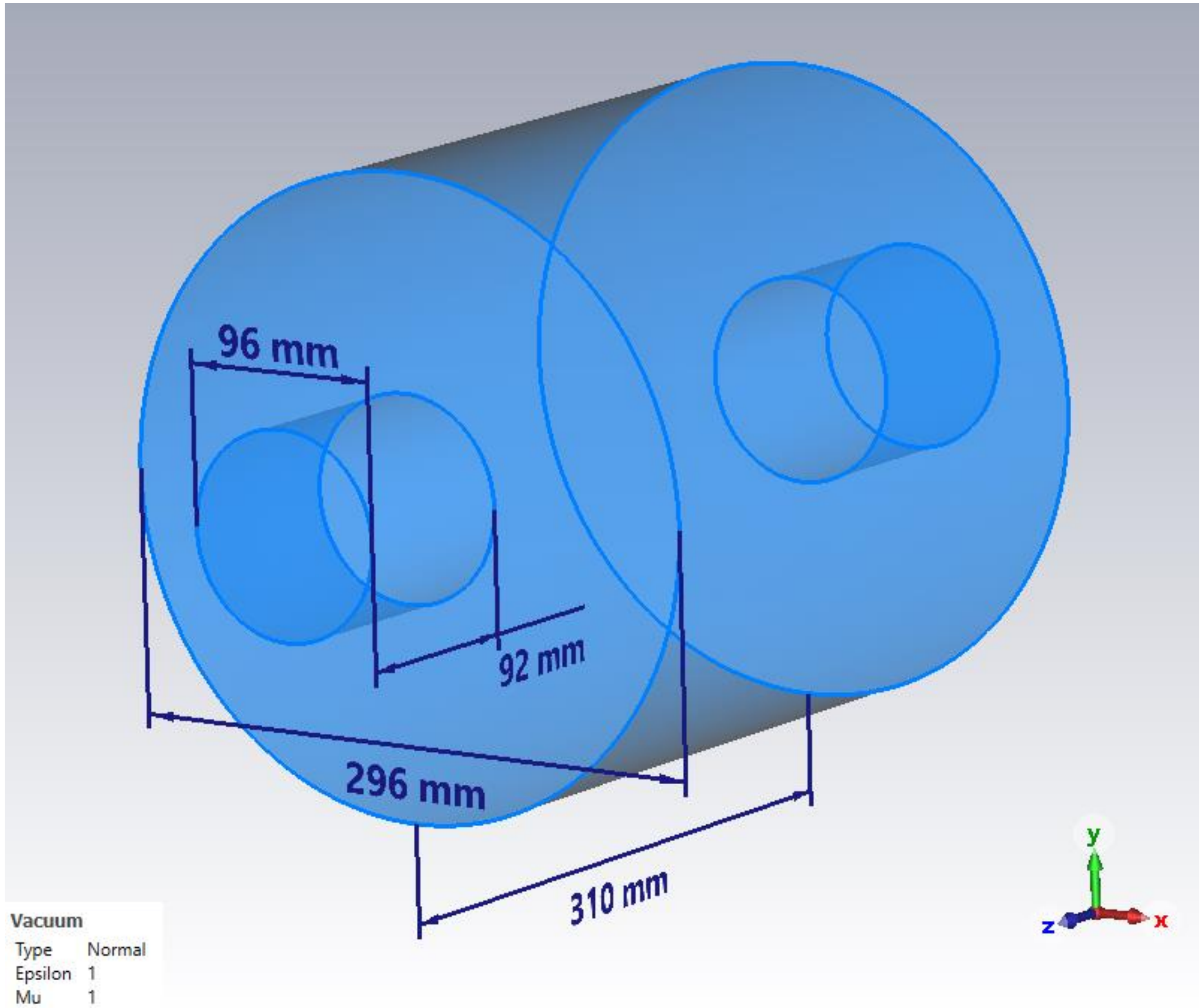
$$\approx 185\ \Omega\ h/a\ [\Omega] \quad \text{for: } \frac{p_{01}}{2}\frac{h}{a} \ll 1$$

  - **With:** $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cong 120\pi\ [\Omega] \cong 377\ [\Omega]$

$$p_{01} = 2.40483$$

$$J_1(p_{01}) = 0.519147$$

Cylindrical "Pill-box" Cavity



- **Properties**
  - **cavity radius** $a = 148\ mm$
  - **cavity height** $h = 310\ mm$
  - **material conductivity**
    stainless steel $\sigma = 1.33 \times 10^6\ S/m$
  - **beam-port radius** $a_{port} = 48\ mm$
  - **Beam-port length** $h_{port} = 92\ mm$
- **Results analytical (no beam-ports):**
  - **Eigen-mode frequencies**
    - Depend on the object size

      $f_{TE111} = 766\ MHz$

      $f_{TM010} = 776\ MHz$
  - **Q-factor (unloaded)**
    - Depend on the wall material conductivity

      $Q_0(TM010) = 6322$
  - **Characteristic impedance of the eigen-mode**
    - Depends on the shape of the object

      $R/Q(TM010) = 10.4\ \Omega$