Transverse Beam Emittance Diagnostics

CAS - Advanced Accelerator Physics

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Spa (Belgium), 11.-22.11.2024

Accelerator Key Parameters

Light Source vs. Collider

light source: spectral brilliance

- measure for phase space density of photon flux

$$B = \frac{\text{Number of photons}}{[\text{sec}][\text{mm}^2][\text{mrad}^2][0.1\%\,\text{bandwidth}]}$$

- user requirement: high brightness
  → lot of monochromatic photons on sample
- connection to machine parameters

$$B \propto \frac{N_\gamma}{\sigma_x \sigma_{x'} \sigma_z \sigma_{z'}} \propto \frac{I}{\varepsilon_x \varepsilon_z}$$

requirement: high quality accelerator

- high beam current → instabilities, high heat load…
- small transverse emittance
  → generate small emittance (lattice design)
  → preserve emittance (instabilities → feedback)
  → measure small emittance

collider: luminosity

- measure for the collider performance

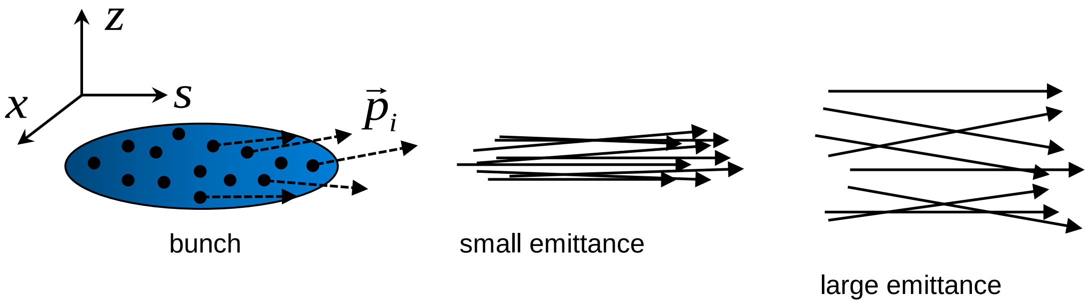
$$\dot{N} = L \cdot \sigma$$

relativistic invariant proportionality factor between cross section σ (property of interaction) and number of interactions per second

- user requirement: high luminosity
  → lot of interactions in reaction channel
- connection to machine parameters

$$L \propto \frac{I_1 \cdot I_2}{\varepsilon}$$

for two identical beams with emittances $\varepsilon_x = \varepsilon_z = \varepsilon$



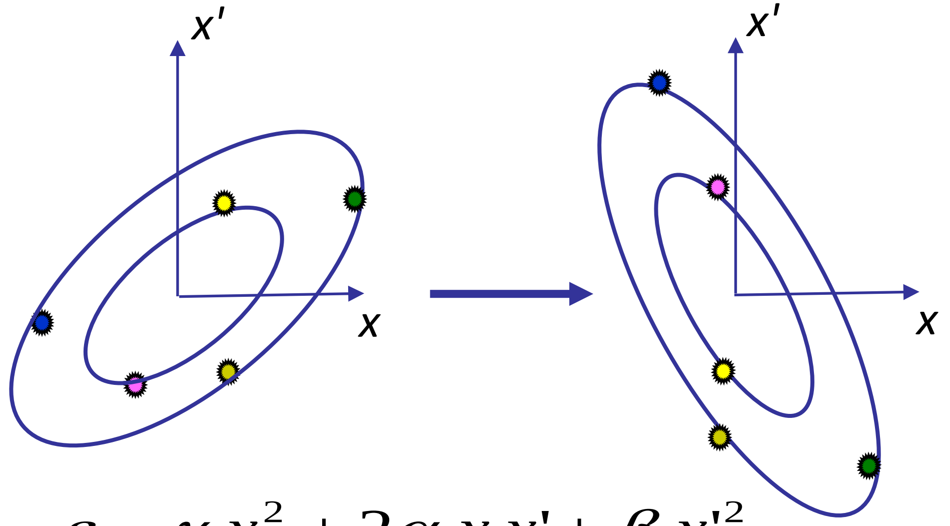
Transverse Emittance

Projection of Trace Space Volume

trace space

- space defined by position ***x*** & divergence ***x'***
- separate horizontal, vertical (and longitudinal) plane

- under linear forces (dipoles, quadrupoles)
  - any particle moves on an ellipse in trace space *(x,x')*
  - ellipse rotates in magnets and shears along drifts
    - → but area is preserved: **emittance**



$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

(*α, β, γ, ε*: Courant-Snyder or Twiss parameters)

transformation along accelerator

- knowledge of the magnet structure (beam optics) → transformation from initial (i) to final (f) location
  - single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \underbrace{\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}}_{R} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

  - transformation of optical functions

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} R_{11}{}^2 & -2R_{11}R_{12} & R_{12}{}^2 \\ -R_{11}R_{21} & 1+R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}{}^2 & -2R_{21}R_{22} & R_{22}{}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$

Transverse Emittance Ellipse

Propagation along Accelerator

$$\alpha(s) = -\frac{\beta'(s)}{2}$$

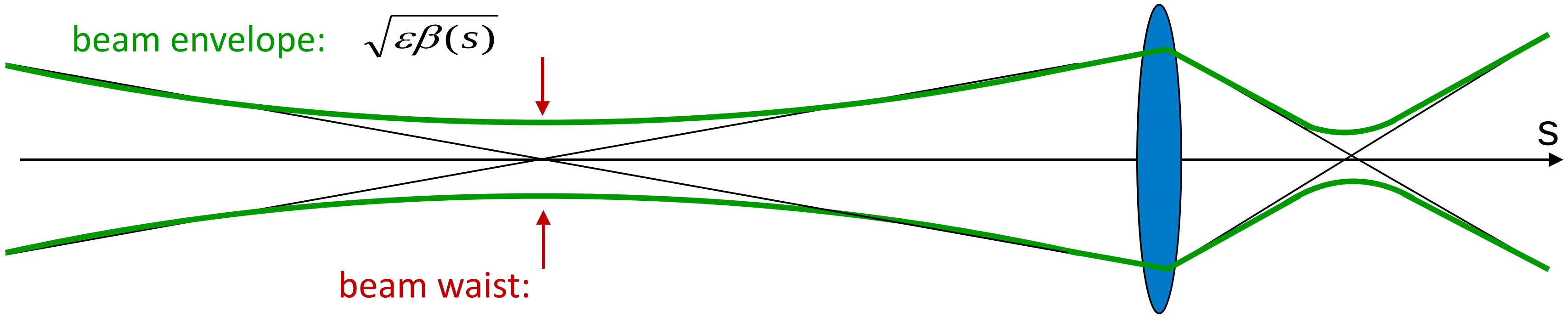
$$\gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$$

$$x(s) = \sqrt{\varepsilon\beta(s)} \cdot \cos[\Psi(s)+\Phi]$$

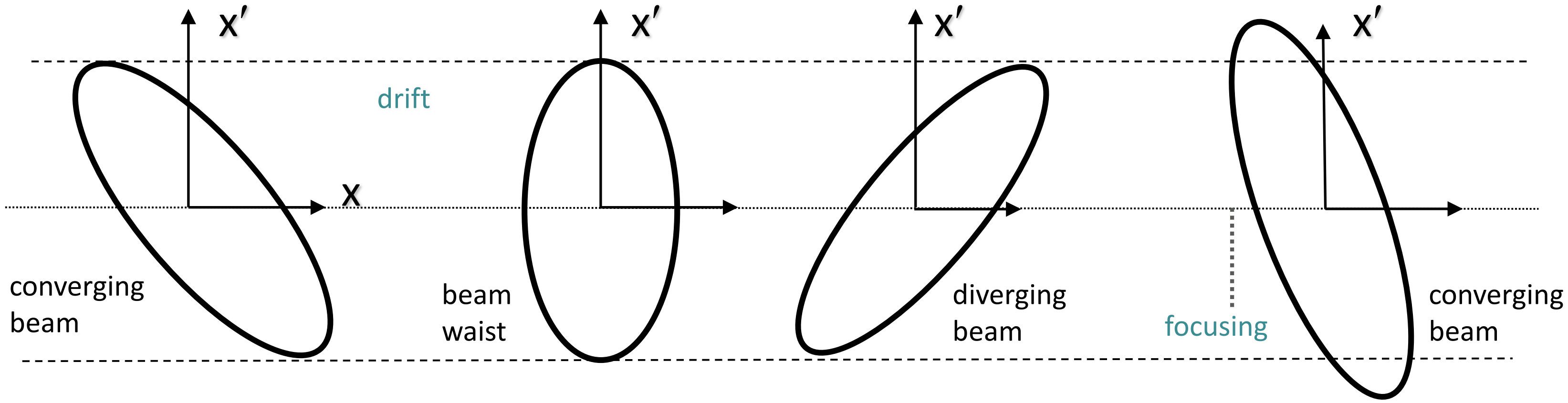
change of ellipse shape and orientation

- area is preserved

$$\varepsilon = \gamma(s)\,x(s)^2 + 2\alpha(s)\,x(s)\,x'(s) + \beta(s)\,x'(s)^2$$

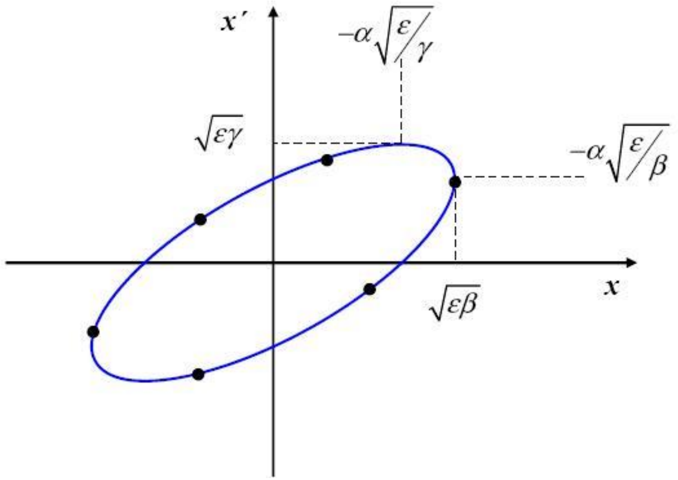


→ minimum in envelope → minimum in β → β′ = 0 → α = 0



Emittance and Beam Matrix

Emittance Representations



beam matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11} \cdot \Sigma_{22} - \Sigma_{12}^2}$$

- transformation of beam matrix

$$\Sigma(s_1) = \mathrm{R}\Sigma(s_0)\mathrm{R}^T \quad \text{with} \quad R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

via Twiss (Courant-Snyder) parameters

$$\varepsilon = \gamma\, x^2 + 2\alpha\, x\, x' + \beta\, x'^2$$

statistical definition

P.M. Lapostolle, IEEE Trans. Nucl. Sci. NS-18, No.3 (1971) 1101

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

with 2nd moments of beam distribution ρ(x)

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{\infty} \mathrm{d}x\, x^2 \cdot \rho(x)}{\int_{-\infty}^{\infty} \mathrm{d}x\, \rho(x)}$$

- ε_{rms} is measure of spread in trace space
- root-mean-square (rms) of distribution

$$\sigma_x = \langle x^2 \rangle^{1/2}$$

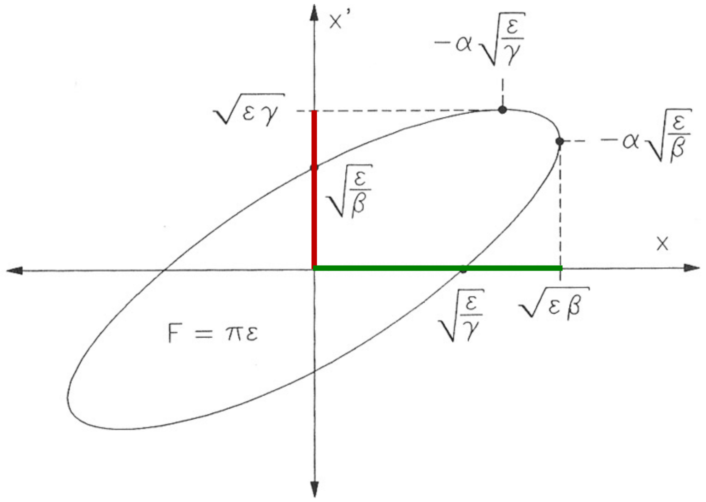
- ε_{rms} is useful definition for non-linear beams
  - usually restriction to certain range
    → c.f. 90% of particles instead of [-∞,+∞]

Emittance Measurement

Principle

measurement of projected area of transverse trace space volume

- not directly accessible for beam diagnostics



- accessible quantities
  - beam size $\sqrt{\Sigma_{11}} = \sqrt{\langle x^2 \rangle} = \sqrt{\varepsilon\,\beta}$
  - beam divergence $\sqrt{\Sigma_{22}} = \sqrt{\langle x'^2 \rangle} = \sqrt{\varepsilon\,\gamma}$
    - divergence measurements seldom in use
      → restriction to profile measurements

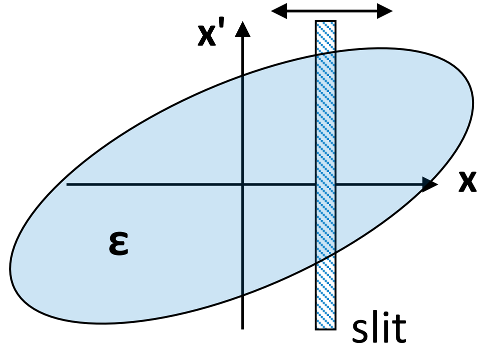
- measurement schemes
  - mapping of trace space
    → restrict to (infenitesimal) element in space coordinate, convert angles *x'* in position *x*
  - beam matrix based measurements
    → determination of beam matrix elements

$$\varepsilon = \sqrt{\det\Sigma} = \sqrt{\Sigma_{11}\cdot\Sigma_{22} - \Sigma_{12}^2}$$

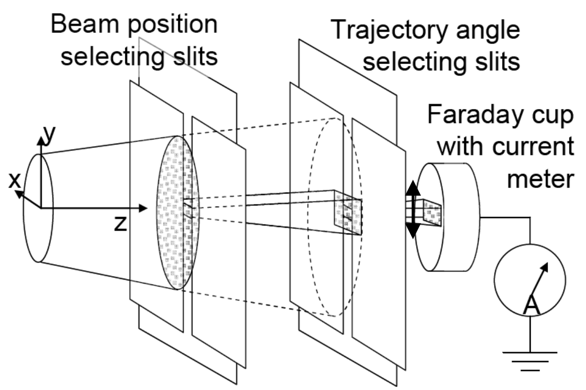
Trace Space Mapping

Principle

slit scan method



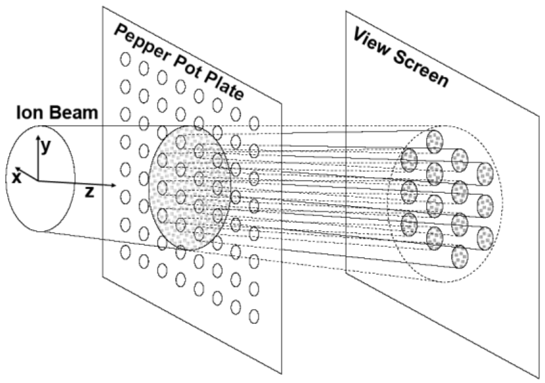
- low energy beams often space charge limited → cutting out small beamlet
- slit produces vertical slice in transverse phase space
- measure intensity as function of x' → propagate beamlet along drift space
- moving of slit → scan of phase space ($\mathbf{N_x}$ x $\mathbf{N_{x'}}$ measurements)

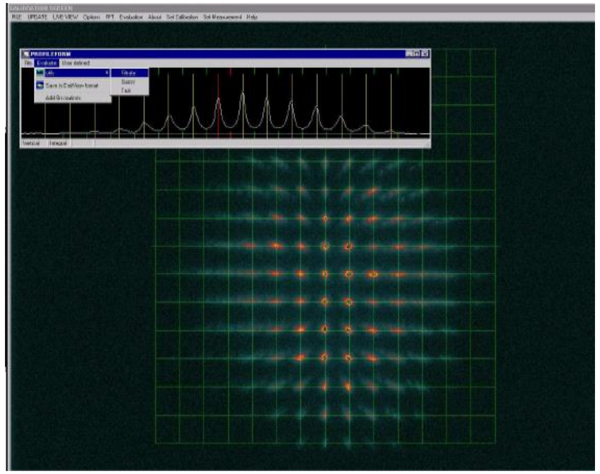


M.P.Stockli, Proc. BIW 2006, p.25

- monitor with x' resolution instead of scan
  - SEM, profile grid,...
    → $\mathbf{N_x}$ measurements

- 2-dimensional extension: Pepper pot
  → **1** measurement
  → $\mathbf{N_x}$ x $\mathbf{N_{x'}}$ holes





P.Forck, *Lecture Notes on Beam Instrumentation and Diagnostics*, JUAS 2006

Beam Matrix based Measurements

Principle

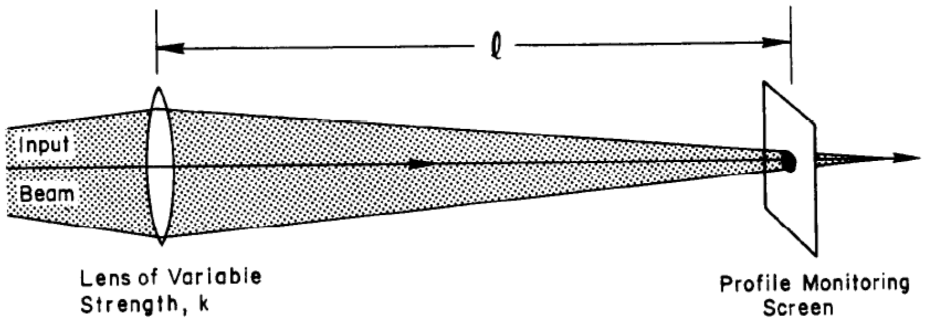
task: determination of 3 beam matrix elements Σ_{11}, Σ_{22}, $\Sigma_{12} = \Sigma_{21}$

- remember: beam matrix Σ(s) depends on location → determination at same location
- how to measure element Σ_{12} ???

idea: exploit transformation properties of beam matrix

- instead of beam matrix measurement at one accelerator location → (minimum) 3 **profile** measurements under different conditions

- quadrupole scan
  - change of matrix elements R via **change of beam optics**
  - sequential measurement with **one** monitor using **different** quadrupole settings



- multi-screen method
  - change of matrix elements R via **change of monitor positions**
  - measurement with **several** monitors using **one** optics setting

Beam Matrix based Measurements

Access to Matrix Elements

profile monitor

- measurement of $\sigma = \sqrt{\Sigma_{11}}$
- other matrix elements can be inferred from beam profiles taken under various transport conditions

  → knowledge of transport matrix M required: $\Sigma^b = R \cdot \Sigma^a \cdot R^T$ with $R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$

measurement of at least 3 profiles for 3 matrix elements

$$\Sigma_{11}^a$$

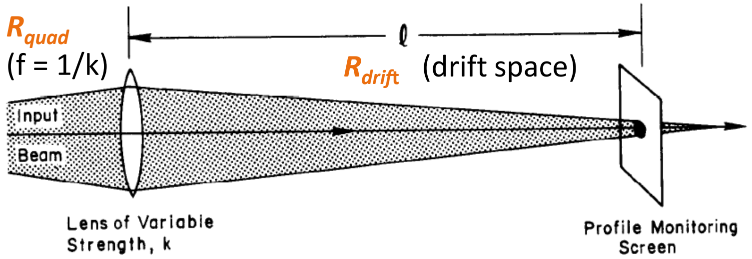
$$\Sigma_{11}^b = R_{11}^2 \cdot \Sigma_{11}^a + 2R_{11}R_{12} \cdot \Sigma_{12}^a + R_{12}^2 \cdot \Sigma_{22}^a$$

$$\Sigma_{11}^c = \overline{R}_{11}^2 \cdot \Sigma_{11}^a + 2\overline{R}_{11}\overline{R}_{12} \cdot \Sigma_{12}^a + \overline{R}_{12}^2 \cdot \Sigma_{22}^a$$

- **measurement :** profiles $\sigma^{a,b,c} = \sqrt{\Sigma_{11}^{a,b,c}}$
- **known:** transport optics $R, \overline{R}$
- **deduced:** matrix elements $\Sigma_{11}^a, \Sigma_{12}^a, \Sigma_{22}^a$

→ more than 3 profile measurements favourable, data subjected to least-square analysis

quadrupole scan



quadrupole transfer matrix: $R_{quad} = \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$

drift space transfer matrix $R_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$

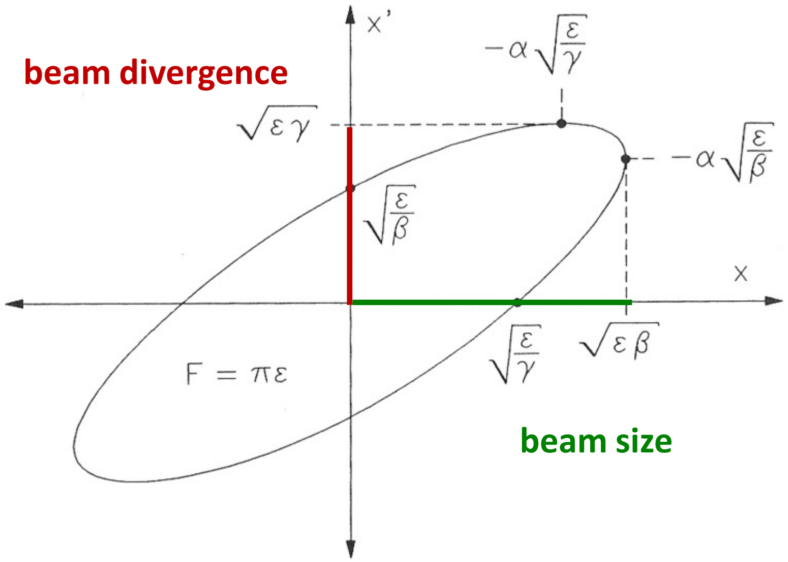
$$\boldsymbol{R = R_{quad} R_{drift}}$$

Circular Accelerators

Emittance Diagnostics in circular Accelerators

periodicity with circumference L

- one-turn transport matrix: $R(s+L) = R(s)$
- Courant-Snyder / Twiss parameters $\alpha(s), \beta(s), \gamma(s)$ uniquely defined at each location in ring
- measurement at one location in ring sufficient to determine ε
  - → measured quantity: beam profile / angular distribution



classification

- imaging
  - → beam size
- interference
  - → beam size
- projection
  - → beam divergence

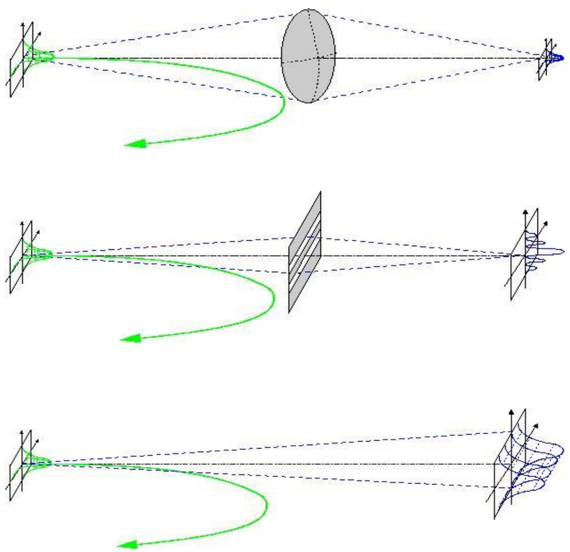
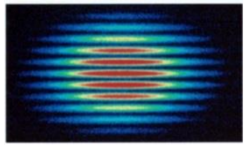
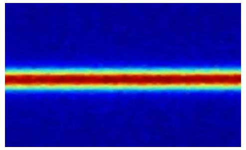


image size



interference pattern

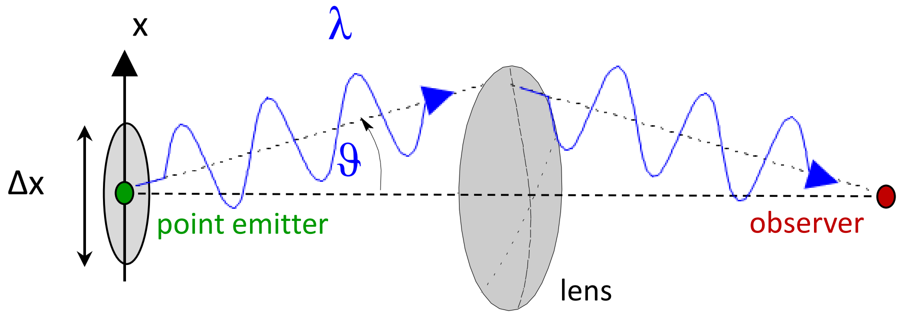


angular distribution

Imaging of small Beam Sizes

Fundamental Resolution Limit

point observer detecting photons from point emitter



$$\Delta x$$

$$\Delta p_x = 2\hbar k \cdot \sin\vartheta \approx 2 \cdot \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} \cdot \sin\vartheta$$

NA = sinϑ: numerical aperture

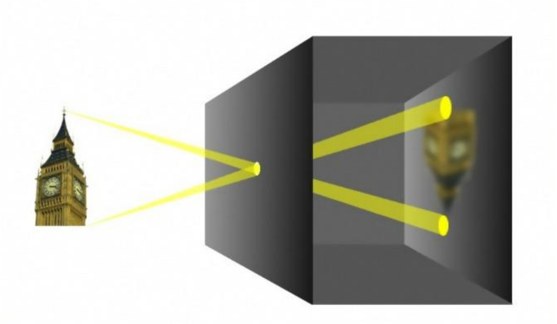
uncertainty principle: $\Delta x \cdot \Delta p_x \approx h \quad \Rightarrow \quad \Delta x \approx \frac{\lambda}{2\sin\vartheta}$

➡ high resolution: (i) **small λ** (ii) **high NA**

X-ray imaging

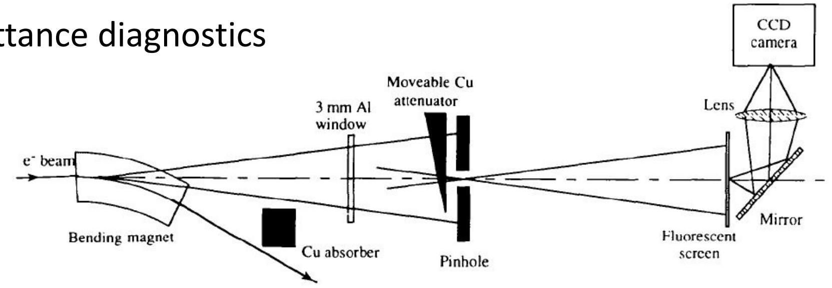
(example PETRA III: E = 6 GeV, λ = 500 nm ⇨ $\Delta\sigma_{res} \approx 140$ µm $\sigma_{beam,v} \approx 12$ µm)

- simple X-ray imaging system: pinhole camera
- Camera Obscura - description of phenomenon already by Aristoteles (384-322 b.C.) in „Problemata“
  - principle



  - realization for emittance diagnostics

P.Elleaume *et al.*, J.Synchrotron Rad. 2 (1995) , 209



Interference and Coherence

Recall

coherence in classical optics

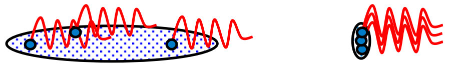
- ability of interference of light
  - i.e. fix phase relation between wave trains
- contrast of interference pattern
  - measure for coherence

low degree of coherence

high degree of coherence

- spatial coherence
  - application for transverse beam diagnostics

- temporal coherence
  - application for longitudinal beam diagnostics
    → bunch length measurements, Coherent Radiation Diagnostics



long bunch ($\lambda<\sigma_t$)　　short bunch ($\lambda>\sigma_t$)

synchrotron radiation interferometer

T. Mitsuhashi , Proc. Joint US-CERN-Japan-Russia School of Particle Accelerators, Montreux, 11-20 May 1998 (World Scientific), pp. 399-427.

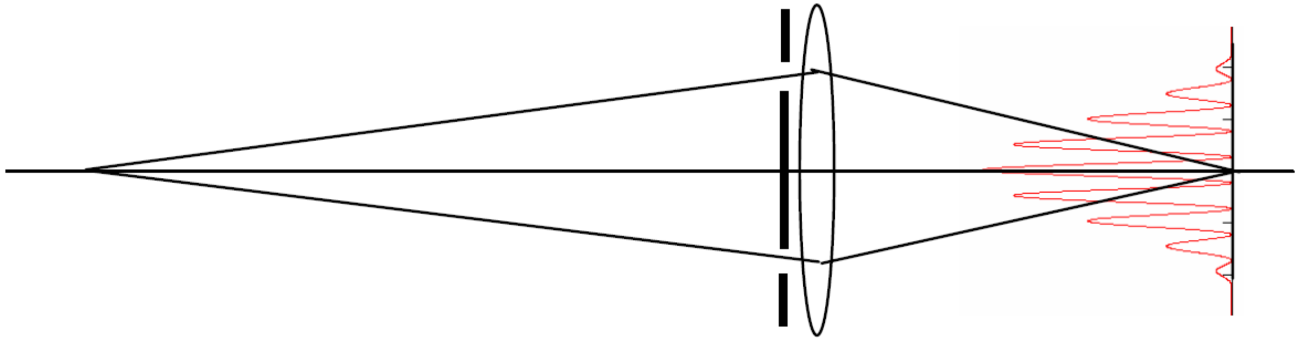
- principle borrowed from astronomy → Michelson's stellar interferometry
- fundamental resolution limit → uncertainty principle
  - interferometric measurement requires precise phase determination ($\Delta\Phi$ small)
  - fluctuation in amplitude (in number of photons Δn) large, i.e. sufficient intensity required

$$\Delta n \cdot \Delta\Phi \geq 1$$

Synchrotron Radiation Interferometer

Principle

point source



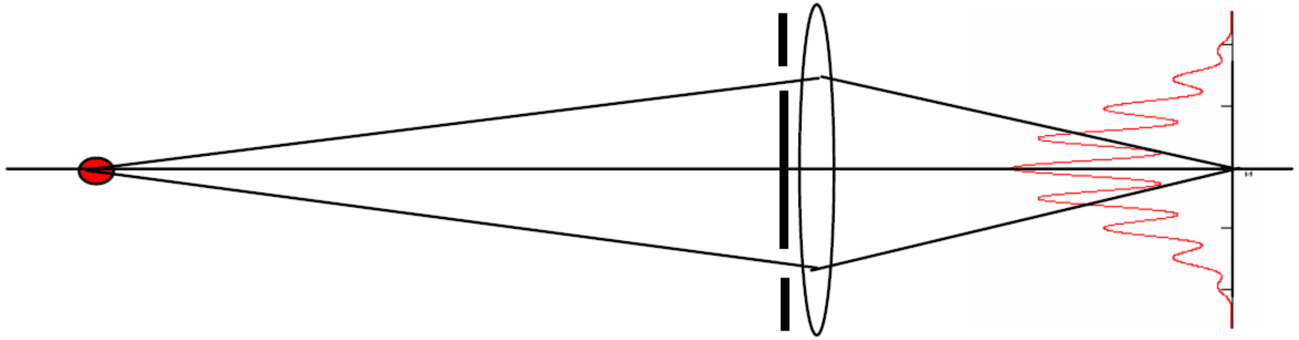
$V = 1$

contrast quantification

visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

extended source



$V < 1$

➡ **visibility contains information about source size**

Mathematical Formulation

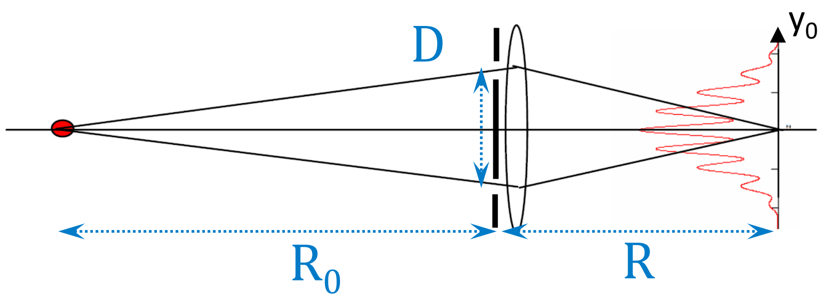
Degree of Coherence and Beam Size

1st order degree of spatial coherence

$$\gamma(D) = \frac{\langle \vec{E}_1 \cdot \vec{E}_2^* \rangle}{\sqrt{|\vec{E}_1|^2 \cdot |\vec{E}_2|^2}}$$

γ: normalized complex correlation function

with $|\gamma| = V$ (visibility)



- intensity distribution of spatial partial coherent source

$$I = \left\langle \left| \vec{E}_1 + \vec{E}_2 \right|^2 \right\rangle = I_{inc} \left(1 + |\gamma| \cdot \cos\varphi'\right) \qquad I_{inc} = I_1 + I_2 \text{ und } I_1 \approx I_2$$

→ taking into account interference at single slit:

$$I(y_0) = I_{inc} \left[\operatorname{sinc}\left(\frac{2\pi\, a}{\lambda\, R} y_0 \right) \right]^2 \left[1 + |\gamma| \cdot \cos\left(\frac{2\pi\, D}{\lambda\, R} y_0 + \varphi \right) \right]$$

van Cittert-Zernike theorem (far field)

- relation between degree of coherence and intensity distribution in source plane

  → Fourier transform

$$\gamma(D) = \int dy \; f(y) \cdot \exp\{-i\, 2\pi\, \nu_y \cdot y\}$$

with $\nu_y = \dfrac{D}{\lambda\, R_0}$

spatial frequency

(in line pairs / mm)

- spatial equivalent to Wiener-Khinchine theorem

(autocorrelation spectroscopy)

- Gaussian beam distribution $f(y)$

  → analytical solution of Fourier transform

$$\sigma_y = \frac{\lambda R_0}{\pi D} \sqrt{\frac{1}{2} \ln \frac{1}{|\gamma(D)|}}$$

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