

# Upper Bound on Thermal Gravitational Wave Backgrounds from Hidden Sectors

---

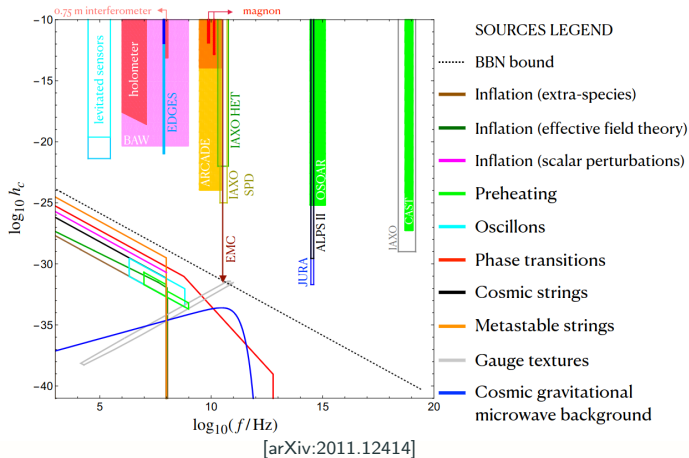
Yannis Georis

based on work in collaboration with M. Drewes, J. Klarić and P. Klose  
[arXiv:2312.13855]

CERN TH Institute *Particle Production in the Early Universe*  
September 10, 2024



# Ultra-High-Frequency Gravitational Waves



- No known astrophysical background at frequencies above  $10^4$  Hz  
 → Powerful probe of cosmological backgrounds
- Many different sources: inflation, preheating, topological defects, ...

# Gravitational Waves from thermal fluctuations

- Even in **equilibrium**, thermal plasma emit gravitational waves from **microscopic processes**

$$\frac{d\dot{e}_{\text{gw}}}{d \ln k} + 4H \frac{de_{\text{gw}}}{d \ln k} = 16\pi^2 \left( \frac{k}{2\pi a} \right)^3 \frac{\Pi(k/a)}{m_{\text{pl}}^2} \sim \langle T^{\mu\nu} T^{\rho\sigma} \rangle$$

- GW production rate** governed by the self-energy

$$\mathcal{L} \supset \frac{1}{2} \frac{\sqrt{8\pi}}{m_{\text{pl}}} h_{\mu\nu} T^{\mu\nu}$$

- Assuming standard cosmic history

$$h^2 \Omega_{\text{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left( \frac{f}{\text{Hz}} \right)^3 \times \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT'}{m_{\text{pl}}} \frac{\Pi(2\pi f a_0 / a')}{8 T'^4} .$$

- Contribution is small but **unavoidable** ! Act as cosmic **GW floor**.

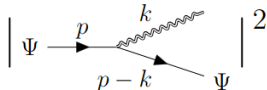
# Gravitational Waves from thermal fluctuations

- Even in **equilibrium**, thermal plasma emit gravitational waves from **microscopic processes**

$$\frac{d\dot{e}_{\text{gw}}}{d \ln k} + 4H \frac{de_{\text{gw}}}{d \ln k} = 16\pi^2 \left( \frac{k}{2\pi a} \right)^3 \frac{\Pi(k/a)}{m_{\text{pl}}^2} \sim \langle T^{\mu\nu} T^{\rho\sigma} \rangle$$

- GW production rate** governed by the self-energy

$$\mathcal{L} \supset \frac{1}{2} \frac{\sqrt{8\pi}}{m_{\text{pl}}} h$$



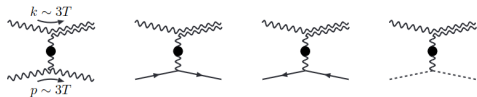
- Assuming standard cosmic history

$$h^2 \Omega_{\text{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left( \frac{f}{\text{Hz}} \right)^3 \times \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT'}{m_{\text{pl}}} \frac{\Pi(2\pi f a_0 / a')}{8 T'^4}.$$

- Contribution is small but **unavoidable** ! Act as cosmic **GW floor**.

## Boltzmann vs hydrodynamic regime

- For hard graviton momentum/frequency  $k \sim \pi T$ ,  $\Pi$  dominated by particle scatterings (**Boltzmann regime**) e.g.

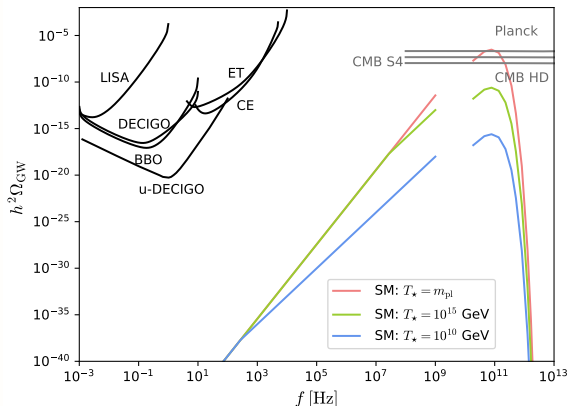


→ Computed at LO for the SM by [Ghiglieri/Jackson/Laine/Zhu, 2004.11392]

- For soft momentum  $k \ll T$ , long-range hydrodynamic fluctuations dominate (**hydrodynamic regime**)

→ Estimated for the SM by [Ghiglieri/Laine, 1504.02569]

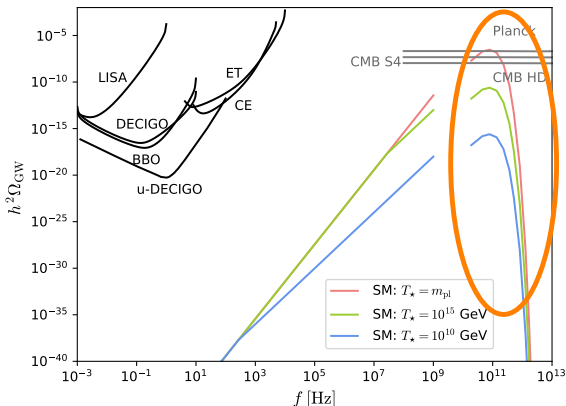
# Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

- GW spectrum peaks for  $f \sim 10^{11}$  Hz (Model independent:  $k \sim \pi T$ )

# Standard Model GW background

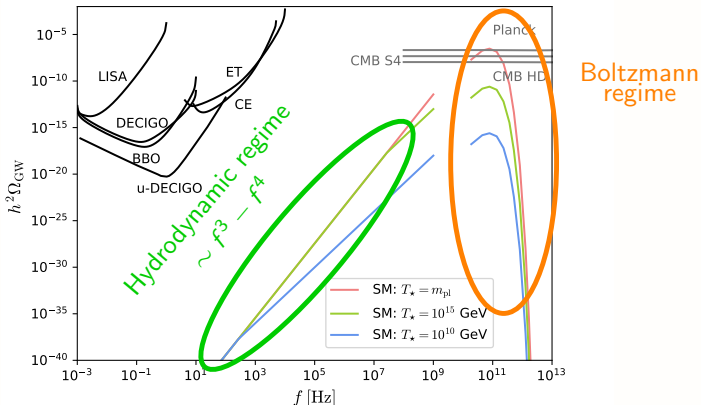


Boltzmann  
regime

[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

- GW spectrum peaks for  $f \sim 10^{11}$  Hz (Model independent:  $k \sim \pi T$ )

# Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

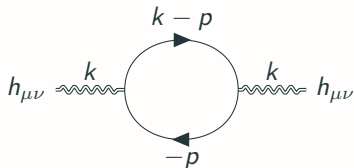
- GW spectrum peaks for  $f \sim 10^{11}$  Hz (Model independent:  $k \sim \pi T$ )



# Gravitational Waves from hidden sectors

- Stress-energy tensor given at leading order by

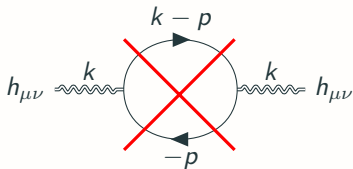
$$T_{ij}(x) \supset \frac{c_X}{4} \bar{\psi}_x i D_{ij} \psi_x, \quad i D_{ij} = \gamma_i i \partial_j + \gamma_j i \partial_i$$



# Gravitational Waves from hidden sectors

- Stress-energy tensor given at leading order by

$$T_{ij}(x) \supset \frac{c_X}{4} \bar{\psi}_x iD_{ij}\psi_x, \quad iD_{ij} = \gamma_i i\partial_j + \gamma_j i\partial_i$$

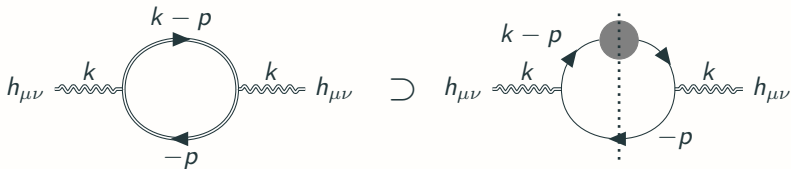


- Tree level contributions vanishes for kinematical reasons: Need to resum (in the hydrodynamic regime) !

# Gravitational Waves from hidden sectors

- Stress-energy tensor given at leading order by

$$T_{ij}(x) \supset \frac{c_X}{4} \bar{\psi}_x iD_{ij}\psi_x, \quad iD_{ij} = \gamma_i i\partial_j + \gamma_j i\partial_i$$



- In hydrodynamic regime, enhanced for feebly interacting particles

Plasma shear viscosity  $\Pi(k) \sim 8T\eta \sim \frac{T^4}{\Upsilon}$  Particles' width

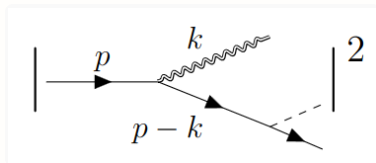
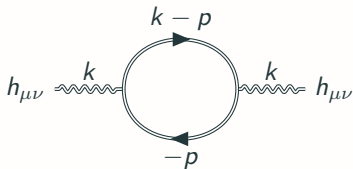
→ Can dominate SM contribution !

*Is it a good way to probe hidden sectors ?*

# Gravitational Waves from hidden sectors

- Stress-energy tensor given at leading order by

$$T_{ij}(x) \supset \frac{c_X}{4} \bar{\psi}_x iD_{ij}\psi_x, \quad iD_{ij} = \gamma_i i\partial_j + \gamma_j i\partial_i$$



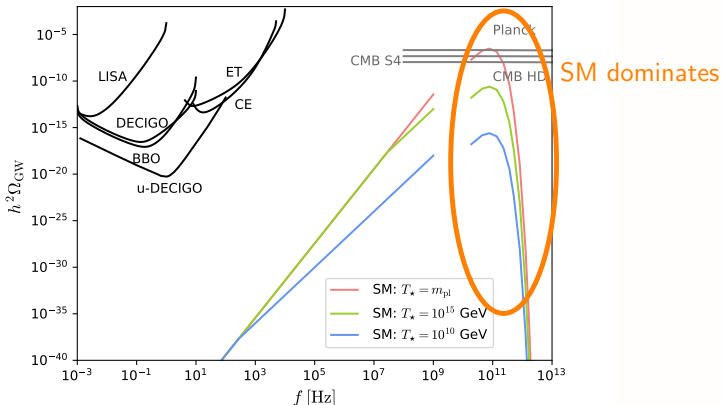
- In hydrodynamic regime, enhanced for feebly interacting particles

Plasma shear viscosity  $\Pi(k) \sim 8T\eta \sim \frac{T^4}{\Upsilon}$  Particles' width

→ Can dominate SM contribution !

*Is it a good way to probe hidden sectors ?*

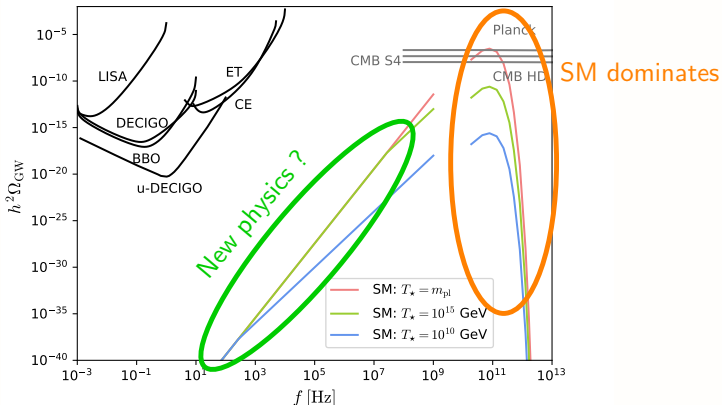
# Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

- GW spectrum peaks for  $f \sim 10^{11}$  Hz (Model independent:  $k \sim \pi T$ )

# Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

- GW spectrum peaks for  $f \sim 10^{11}$  Hz (Model independent:  $k \sim \pi T$ )

# GW production rate

- Fermionic production rate in real time (in-in) formalism

$$\Pi(k) = -\frac{c_X^2}{8} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \int \frac{dp_0}{2\pi} \mathbb{L}^{ij;kl} p_i p_k \text{Tr}[\gamma_j iS_p^> \gamma_l iS_{p-k}^< + \gamma_j iS_p^< \gamma_l iS_{p-k}^>]$$

Traceless-transverse projector  $\mathbb{L}^{ij;kl}$

Derivative coupling  $p_i p_k$

$\sim (\not{p} + m) \frac{\Gamma_p}{\Omega_p^2 + \Gamma_p^2} (1 - f)$

- After some algebra,

$$\Pi(k) \stackrel{m \ll T}{\simeq} g_X \frac{16\pi^2}{225} T^5 \frac{5\Upsilon_{av}}{k^2 + 10\Upsilon_{av}^2}, \quad g_X = \begin{cases} 1 & \text{Spin 0} \\ 2 \cdot c_X & \text{Spin } \frac{1}{2} \end{cases}$$

- Hydrodynamic regime:

$$k < \sqrt{10}\Upsilon_{av}, \Pi(k) \sim \frac{1}{\Upsilon_{av}},$$

- Boltzmann regime:

$$k > \sqrt{10}\Upsilon_{av}, \Pi(k) \sim \Upsilon_{av}$$

- For renormalisable interactions  $\Upsilon_{av} = yT$

$$k < \sqrt{10}\Upsilon_{av} \longleftrightarrow f < f_c = y \cdot 6 \cdot 10^{10} \text{Hz}$$

# Hubble suppression

- GW evolution equation

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} = \frac{16\pi T_{ij}}{a^2 m_{\text{Pl}}^2}$$

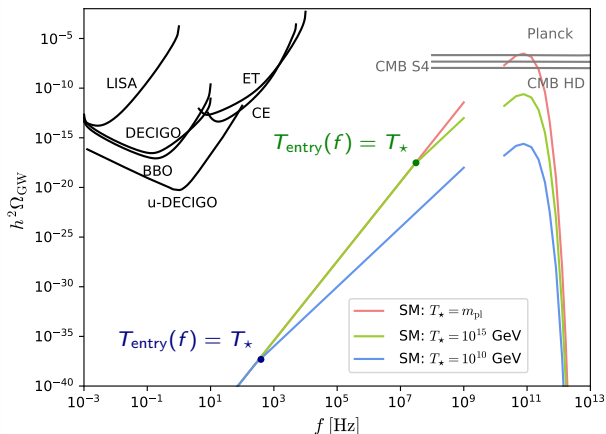
- 2 regimes
  1. Super-Horizon ( $k < H$ ) modes are static
  2. Sub-Horizon ( $k > H$ ) modes
- GW production is delayed until  $k = \frac{2\pi f a_0}{a} > H = T^2/M_0$
- In terms of temperature, production is delayed until

$$T < T_{\text{entry}}(f) \approx 4 \cdot 10^7 \text{ GeV} \frac{f}{\text{Hz}} \quad \text{frequency dependent !}$$

→ SM contribution behaves as  $f^4$  at low frequencies, not  $f^3$  !



# Standard Model GW background



[See Ghiglieri/Jackson/Laine/Zhu '15,'20, Ringwald/Schütte-Engel/Tamarit '20]

## Upper bound on GW emission

- Production rate in the relativistic regime

$$\Pi(k) \stackrel{m \ll T}{\simeq} g_X \frac{16\pi^2}{225} T^5 \frac{5\Upsilon_{\text{av}}}{k^2 + 10\Upsilon_{\text{av}}^2}$$

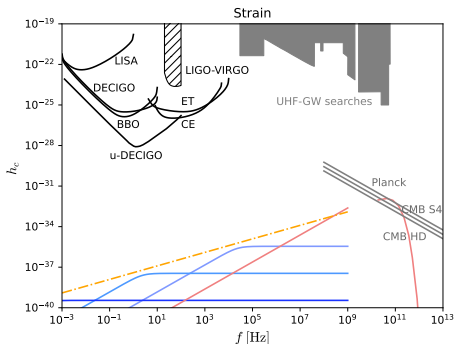
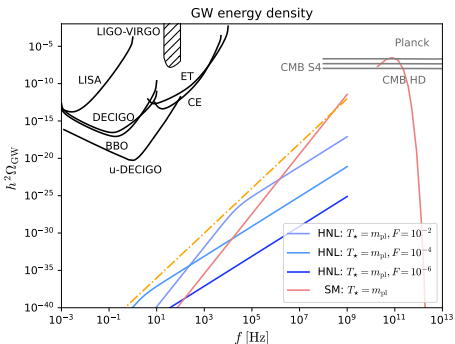
→ Maximised for a width  $\Upsilon_{\text{av}} = k/\sqrt{10}$  ( $\Upsilon_{\text{av}} = k/2$  in the non-relativistic case)

- Leads to the **model-independent upper bound**

$$h^2 \Omega_{\text{gw}}(f) < 4.9 \cdot 10^{-40} \times g_X \left( \frac{f}{\text{Hz}} \right)^3$$

- Tradeoff:** Larger enhancement for smaller  $\Upsilon$  but arises for smaller frequencies
- Does not apply in case of
  1. Out-of-equilibrium dynamics
  2. Hidden sector hotter than SM
  3. Beyond radiation domination

# Illustration: Right-handed neutrinos



[Drewes/YG/Klarić/Klose, 2312.13855]

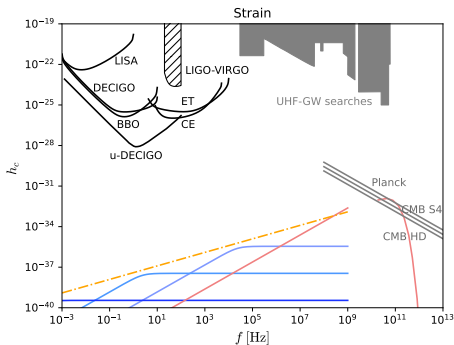
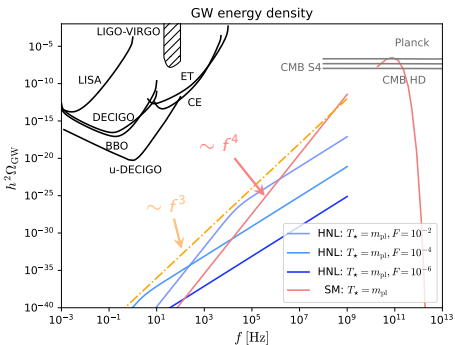
- Coupled to SM through Yukawa coupling

$$\mathcal{L} \supset F\psi(\tilde{\phi}^\dagger l) + \text{h.c.}$$

- Right-handed neutrino width

$$\Upsilon_{\text{av}} \simeq 0.2 \frac{F^2 T}{16\pi},$$

# Illustration: Right-handed neutrinos



[Drewes/YG/Klarić/Klose, 2312.13855]

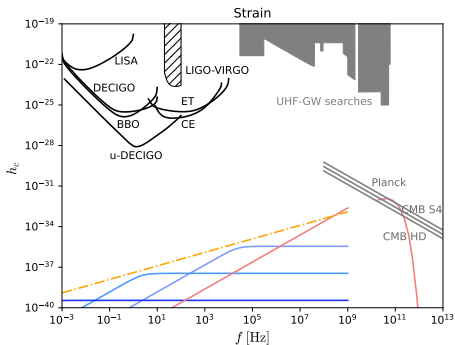
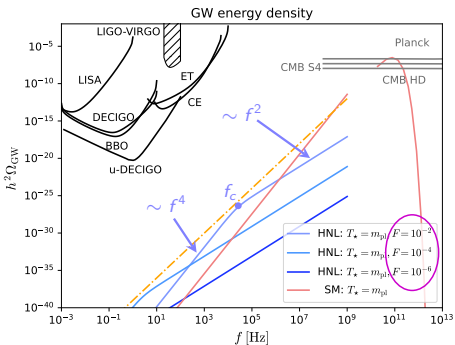
- Coupled to SM through Yukawa coupling

$$\mathcal{L} \supset F\psi(\tilde{\phi}^\dagger l) + \text{h.c.}$$

- Right-handed neutrino width

$$\Upsilon_{\text{av}} \simeq 0.2 \frac{F^2 T}{16\pi},$$

# Illustration: Right-handed neutrinos



[Drewes/YG/Klarić/Klose, 2312.13855]

- Coupled to SM through Yukawa coupling

$$\mathcal{L} \supset F\psi(\tilde{\phi}^\dagger l) + \text{h.c.}$$

- Right-handed neutrino width

$$\Upsilon_{\text{av}} \simeq 0.2 \frac{F^2 T}{16\pi},$$

## Summary and outlook

- Ultra-High-Frequency GWs are powerful probes of new physics because of lack of astrophysical background
- Can be produced by plasma in thermal equilibrium  
→ Constitutes an irreducible background for every theories
- Background can be enhanced for feebly interacting particles
- Upper bound on such background is very restrictive
- Inclusion of hydrodynamic and Hubble suppression crucial for accurate estimate of GW emission
- Formalism can also be applied to non-equilibrium situations

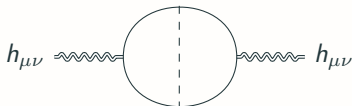
*Thanks for your attention!*

## Backup slides

---

# Circumventing our upper bound

- Non-Standard Cosmic History, e.g.
  1. Entropy transfer between SM and hidden sector
  2. Large number of degrees of freedom in the hidden sector
- Contributions from vertex-type diagrams e.g.



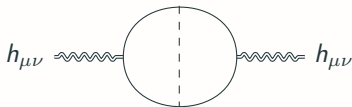
→ Only plays a role at high frequencies  $f \gg f_c$  !

- Out-of-equilibrium dynamics, e.g.
  1. Freeze-in
  2. Peaked distribution function (not only dominated by on-shell region)
  3. Scales other than  $T, m$  e.g. condensate oscillation during reheating



# Circumventing our upper bound

- Non-Standard Cosmic History, e.g.
  1. Entropy transfer between SM and hidden sector
  2. Large number of degrees of freedom in the hidden sector
- Contributions from vertex-type diagrams e.g.



→ Only plays a role at high frequencies  $f \gg f_c$  !

- Out-of-equilibrium dynamics, e.g.
  1. Freeze-in Underabundant if thermal production !
  2. Peaked distribution function (not only dominated by on-shell region)
  3. Scales other than  $T, m$  e.g. condensate oscillation during reheating

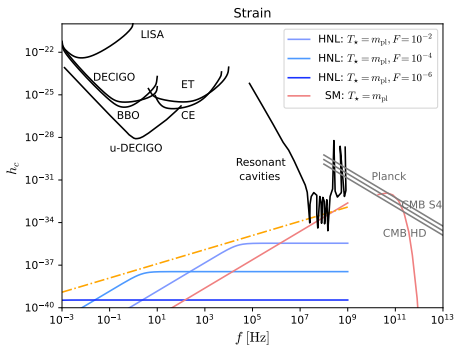
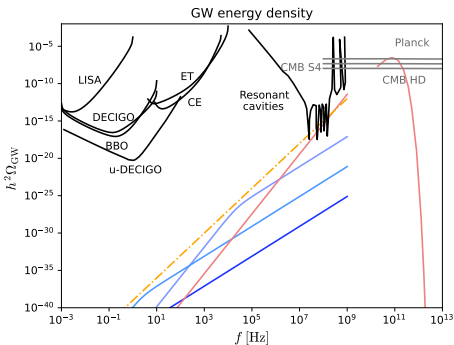
# $N_{\text{eff}}$ as Big Bang thermometer

	SM	$\nu$ MSM	SMASH	MSSM
$T_{\text{max}} [\text{GeV}] <$	$(1.2-5.1) \times 10^{19}$	$(1.3-5.4) \times 10^{19}$	$(1.4-6.0(1)) \times 10^{19}$	$(2.3-9.4) \times 10^{19}$
$T_{\text{max}}^{\Delta N_{\text{eff}}=0.001} [\text{GeV}] <$	$2.3 \times 10^{17}$	$2.4 \times 10^{17}$	$2.7 \times 10^{17}$	$4.39 \times 10^{17}$

[Ringwald/Schütte-Engel/Tamarit '20]

- Can (in theory) probe the maximal temperature of the SM plasma by measuring  $N_{\text{eff}}$

# Resonant cavity searches

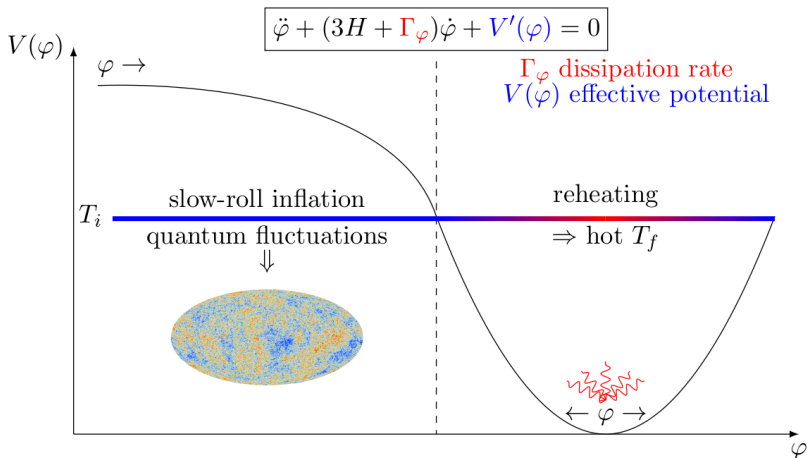


[Drewes/YG/Klarić/Klose, 2312.13855]

Resonant cavity searches [Herman/Lehoucq/Füzfa, '22] can potentially test these models but rely on unknown technology !

# Beyond SM radiation domination: Inflation and reheating

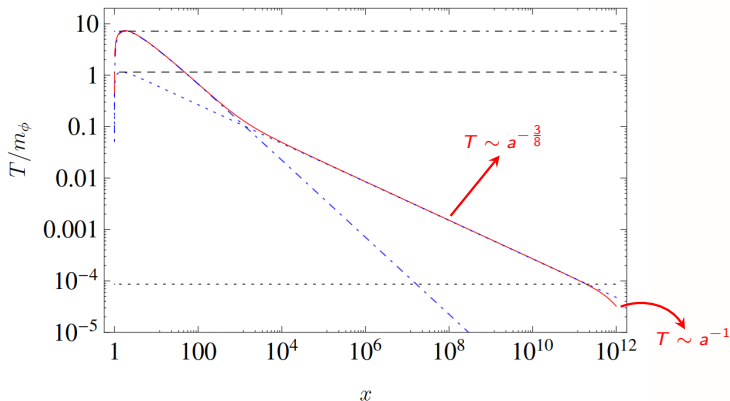
- Many processes predict deviations from SM radiation domination



[credit: Gilles Buldgen]

# Beyond SM radiation domination: Reheating

- Many processes predict deviations from SM radiation domination
- *E.g.* inflaton decay during reheating slows down redshifting



[M. Drewes, 1406.6243]

## GWs beyond SM radiation domination

- Can we adapt our formula to GW production if
  1. SM do not dominate energy budget ?
  2. entropy exchange between SM and hidden sector ?
- If production is still thermal:

$$h^2 \Omega_{\text{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left( \frac{f}{\text{Hz}} \right)^3 \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT'}{m_{\text{Pl}}} \frac{\Pi\left(\frac{2\pi f a_0}{a}\right)}{8 T'^4} \left| \frac{d \ln a}{d \ln T'} \right| \left( \frac{T'}{\bar{T}'} \right) \left( \frac{\rho_{\text{SM}}}{\rho_{\text{tot}}} \right)^{\frac{1}{2}}$$

# GWs beyond SM radiation domination

- Can we adapt our formula to GW production if
  1. SM do not dominate energy budget ?
  2. entropy exchange between SM and hidden sector ?
- If production is still thermal:

$$h^2 \Omega_{\text{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left( \frac{f}{\text{Hz}} \right)^3 \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT'}{m_{\text{Pl}}} \frac{\Pi\left(\frac{2\pi f a_0}{a}\right)}{8 T'^4} \left| \frac{d \ln a}{d \ln T'} \right| \left( \frac{T'}{\bar{T}} \right) \left( \frac{\rho_{\text{SM}}}{\rho_{\text{tot}}} \right)^{\frac{1}{2}}$$

Radiation domination

Non-standard cosmic history

# GWs beyond SM radiation domination

- Can we adapt our formula to GW production if
  1. SM do not dominate energy budget ?
  2. entropy exchange between SM and hidden sector ?
- If production is still thermal:

$$h^2 \Omega_{\text{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left( \frac{f}{\text{Hz}} \right)^3 \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT'}{m_{\text{Pl}}} \frac{\Pi\left(\frac{2\pi f a_0}{a}\right)}{8 T'^4} \left| \frac{d \ln a}{d \ln T'} \right| \left( \frac{T'}{\bar{T}} \right) \left( \frac{\rho_{\text{SM}}}{\rho_{\text{tot}}} \right)^{\frac{1}{2}}$$

Radiation domination Non-standard cosmic history

- Many works on graviton bremsstrahlung during reheating: expect hydrodynamic and Hubble suppression effect to be relevant at low frequencies !
- During inflation,  $\Gamma \ll H < k$ 
  - No hydrodynamic suppression expected



# GWs beyond SM radiation domination

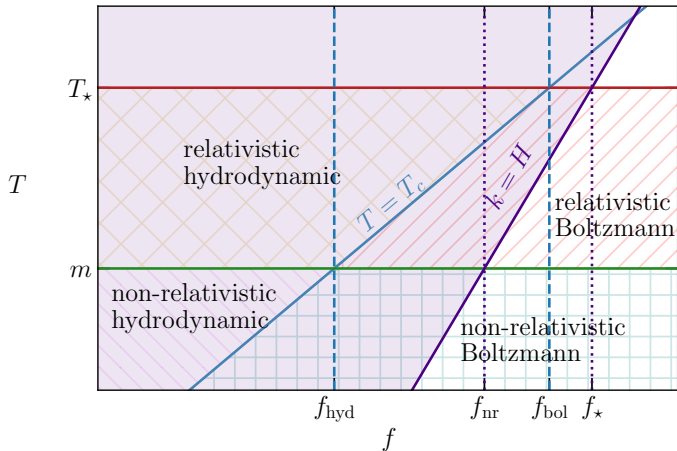
- Can we adapt our formula to GW production if
  1. SM do not dominate energy budget ?
  2. entropy exchange between SM and hidden sector ?
- If production is still thermal:

$$h^2 \Omega_{\text{gw}}(f) \approx 2.02 \cdot 10^{-38} \times \left( \frac{f}{\text{Hz}} \right)^3 \int_{T_{\text{min}}}^{T_{\text{max}}} \frac{dT'}{m_{\text{Pl}}} \frac{\Pi\left(\frac{2\pi f a_0}{a}\right)}{8 T'^4} \left| \frac{d \ln a}{d \ln T'} \right| \left( \frac{T'}{\bar{T}} \right) \left( \frac{\rho_{\text{SM}}}{\rho_{\text{tot}}} \right)^{\frac{1}{2}}$$

Radiation domination Non-standard cosmic history

- Many works on graviton bremsstrahlung during **reheating**: expect **hydrodynamic** and **Hubble suppression** effect to be **relevant** at low frequencies !
- During inflation,  $\Gamma \ll H < k$   
→ No hydrodynamic suppression expected

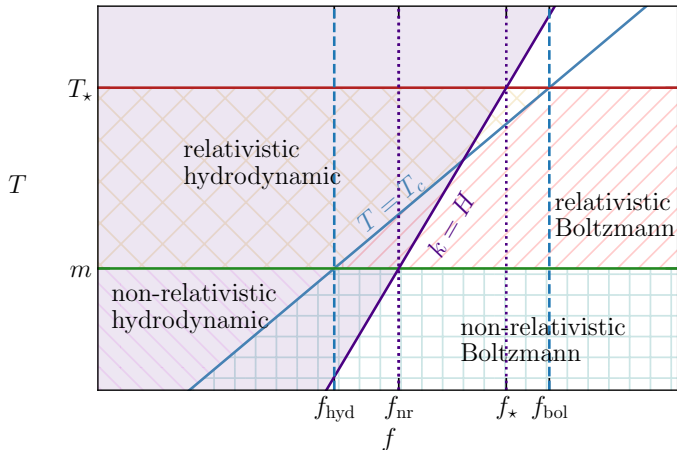
# Production regimes



[Drewes/YG/Klarić/Klose, 2312.13855]

- Different possible scenarios depending on Hubble vs hydrodynamics scale

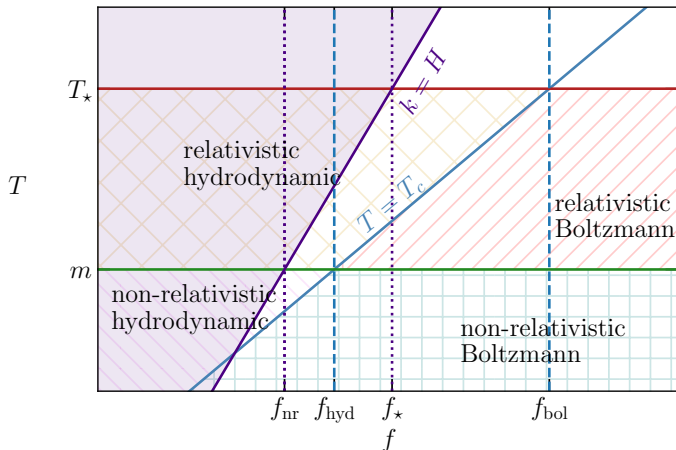
# Production regimes



[Drewes/YG/Klarić/Klose, 2312.13855]

- Different possible scenarios depending on Hubble vs hydrodynamics scale

# Production regimes



[Drewes/YG/Klarić/Klose, 2312.13855]

- Different possible scenarios depending on Hubble vs hydrodynamics scale

# Case 1

$$h^2 \Omega_{\text{gw}}^{T>m}(f) \stackrel{f_x > f_{\text{bol}}}{\simeq} g_X \frac{1.6 \cdot 10^{-40}}{2(d-4)+1} \left( \frac{f}{\text{Hz}} \right)^2 \frac{f_{\text{bol}}}{\text{Hz}} \begin{cases} 0 & \text{for } f < f_{\text{nr}}, \\ \left( \frac{f}{f_\star} \right)^{2(d-4)} & \text{for } f_{\text{nr}} < f < f_\star, \\ \frac{f_\star}{f} & \text{for } f_\star < f. \end{cases}$$

## Case 2

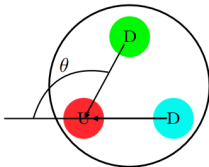
$$h^2 \Omega_{\text{gw}}^{T>m}(f) \stackrel{f_* \gg f_{\text{bol}}}{\simeq} g_X \frac{1.6 \cdot 10^{-40}}{2(d-4)+1} \left( \frac{f}{\text{Hz}} \right)^2 \frac{f_{\text{bol}}}{\text{Hz}} \begin{cases} 0 & \text{for } f < f_{\text{nr}}, \\ \left( \frac{f}{f_*} \right)^{2(d-4)} & \text{for } f_{\text{nr}} < f < f_X, \\ \beta \frac{f_X}{f_{\text{bol}}} \left( \frac{f}{f_X} \right)^{\frac{1}{2(d-4)}} & \text{for } f_X < f < f_{\text{bol}}, \\ \frac{f_*}{f} & \text{for } f_{\text{bol}} < f. \end{cases}$$

## Case 3

$$h^2 \Omega_{\text{gw}}^{T>m}(f) \underset{f_{\text{hyd}} \gg f_{\text{nr}}}{\overset{f_{\text{bol}} \gg f_{\star}}{\approx}} g_X \frac{1.6 \cdot 10^{-40}}{2(d-4)+1} \left( \frac{f}{\text{Hz}} \right)^2 \frac{f_{\text{bol}}}{\text{Hz}} \begin{cases} 0 & \text{for } f < f_{\text{hyd}}, \\ \beta \frac{f_X}{f_{\text{bol}}} \left( \frac{f}{f_X} \right)^{\frac{1}{2(d-4)}} & \text{for } f_{\text{hyd}} < f < f_{\text{bol}}, \\ \frac{f_{\star}}{f} & \text{for } f_{\text{bol}} < f. \end{cases}$$

## Example 2: Axion-like particles

- Neutron EDM  $|d_n| \lesssim 10^{-26} e \cdot \text{cm}$



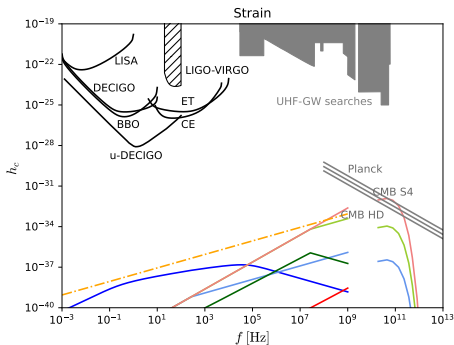
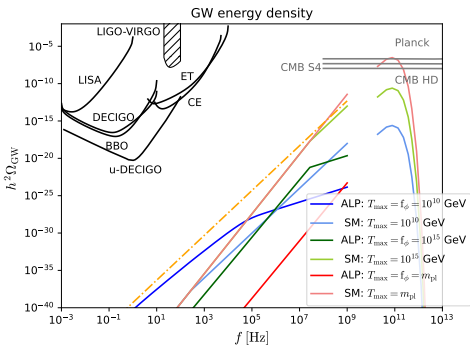
- QCD axion initially introduced as solution to the strong CP-problem

$$\mathcal{L} \supset \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2) - \frac{\alpha}{16\pi f_\phi} \phi \tilde{G}_{\mu\nu} G^{\mu\nu}$$

- In general, axion-like particles are good candidates to
  - form Dark Matter
  - drive inflation



## Example 2: Axion-like particles



[Drewes/YG/Klarić/Klose, 2312.13855]

- Thermal width

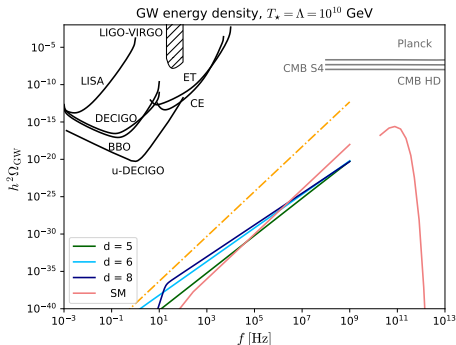
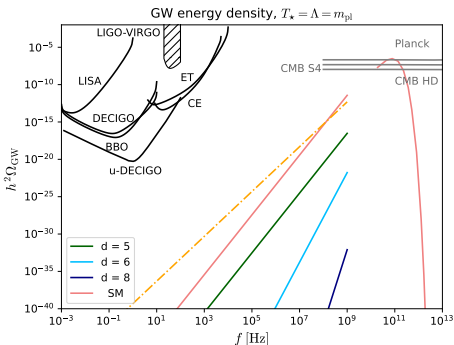
$$\Upsilon_{\text{av}} \stackrel{m \ll T}{=} \kappa n_c^3 (n_c^2 - 1) \frac{\alpha^5 T^3}{f_{\phi}^2}, \quad \kappa \approx 1.5, \quad \frac{1}{\alpha} \approx \frac{22 n_c}{12\pi} \ln \left( \frac{2\pi T}{\Lambda_{\text{IR}}} \right)$$

## Frequency dependence of the GW spectrum $T^* = m_{\text{pl}}$

$$h^2 \Omega_{\text{gw}}^{T^* > m}(f) \stackrel{f_x > f_{\text{bol}}}{\simeq} g_X \frac{1.6 \cdot 10^{-40}}{2(d-4)+1} \left( \frac{f}{\text{Hz}} \right)^2 \frac{f_{\text{bol}}}{\text{Hz}} \begin{cases} 0 & \text{for } f < f_{\text{nr}}, \\ \left( \frac{f}{f_*} \right)^{2(d-4)} & \text{for } f_{\text{nr}} < f < f_*, \\ \frac{f_*}{f} & \text{for } f_* < f. \end{cases}$$

- Such frequency scaling depends on the ratios between  $T_*$ ,  $\Lambda$ ,  $m$ , ...  
→ Can extract information on the particle's properties from the scaling !

## Example 3: Higher dimensional operators



[Drewes/YG/Klarić/Klose, 2312.13855]

- Assuming generically that the width scales as

$$\Upsilon_{\text{av}} \simeq y T \left( \frac{T}{\Lambda} \right)^{2(d-4)} \begin{cases} 1 & T \gg m \\ \left( \frac{m}{T} \right)^n & T \lesssim m \end{cases}, \quad n \leq 1 + 2(d-4)$$

- Unavoidable contribution to the width (at least) at  $d = 8$  from graviton exchanges

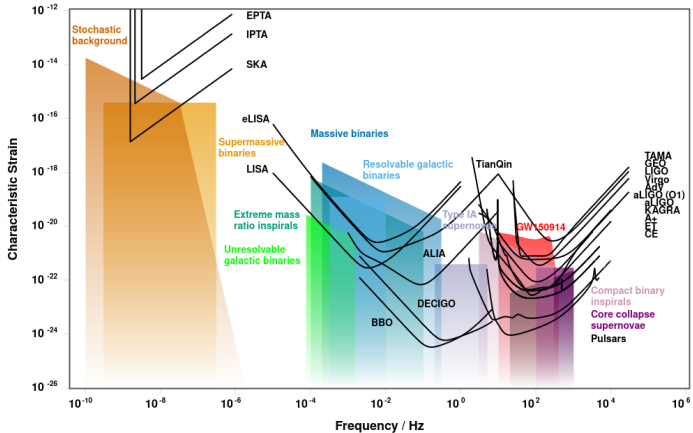
# Frequency dependence of the GW spectrum $T^* = 10^{10}$ GeV

$$h^2 \Omega_{\text{gw}}^{T > m}(f) \stackrel{f_* \gg f_{\text{bol}}}{\simeq} g_X \frac{1.6 \cdot 10^{-40}}{2(d-4)+1} \left( \frac{f}{\text{Hz}} \right)^2 \frac{f_{\text{bol}}}{\text{Hz}} \begin{cases} 0 & \text{for } f < f_{\text{nr}}, \\ \left( \frac{f}{f_*} \right)^{2(d-4)} & \text{for } f_{\text{nr}} < f < f_X, \\ \beta \frac{f_X}{f_{\text{bol}}} \left( \frac{f}{f_X} \right)^{\frac{1}{2(d-4)}} & \text{for } f_X < f < f_{\text{bol}}, \\ \frac{f_*}{f} & \text{for } f_{\text{bol}} < f. \end{cases}$$

- Such frequency scaling depends on the ratios between  $T_*$ ,  $\Lambda$ ,  $m$ , ...  
 → Can extract information on the particle's properties from the scaling !

# Gravitational Waves landscape

CMB B-modes



Ultra-high-frequency GW detectors

[Moore/Cole/Berry, 1408.0740]

- Combination of ground- and space-based interferometers + PTAs will  $\approx$  cover the frequency band  $[10^{-9}, 10^4]$  Hz