Gravitational production of dark particles: quantum interference fringes

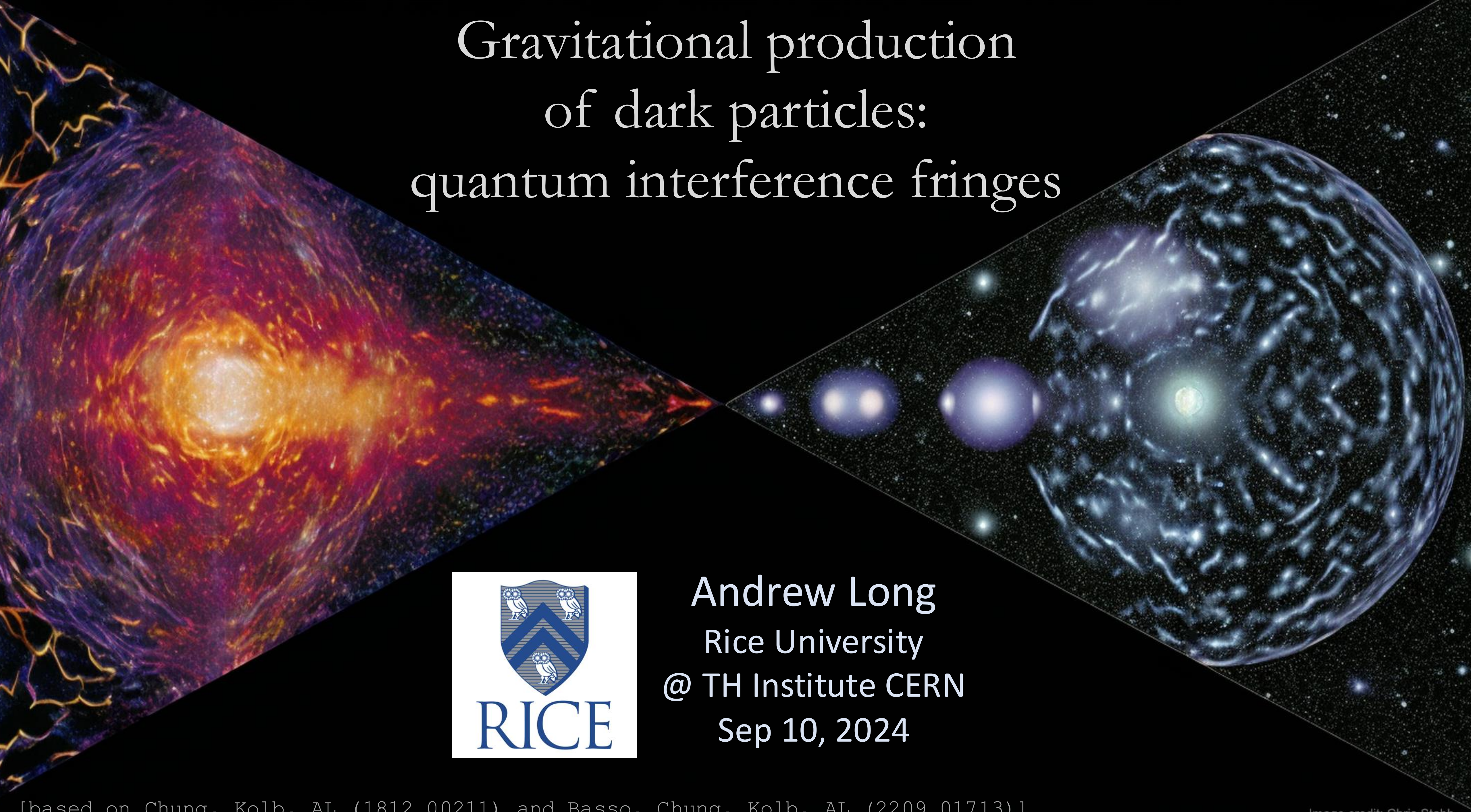


Image credit: Chris Stabb

Andrew Long
Rice University

@ TH Institute CERN
Sep 10, 2024

[based on Chung, Kolb, AL (1812.00211) and Basso, Chung, Kolb, AL (2209.01713)]

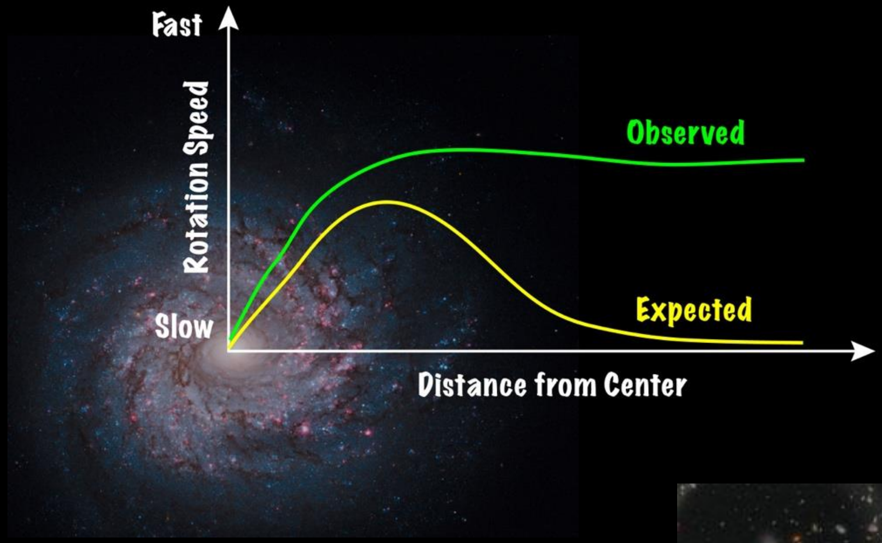
motivation

making dark matter from gravity

dark matter pulls on things

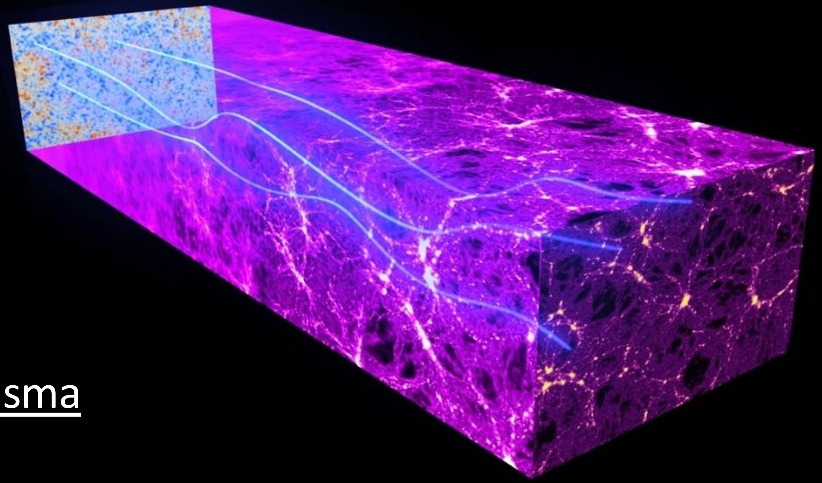
Dark matter pulls on stars in galaxies

(galactic rotation curves)



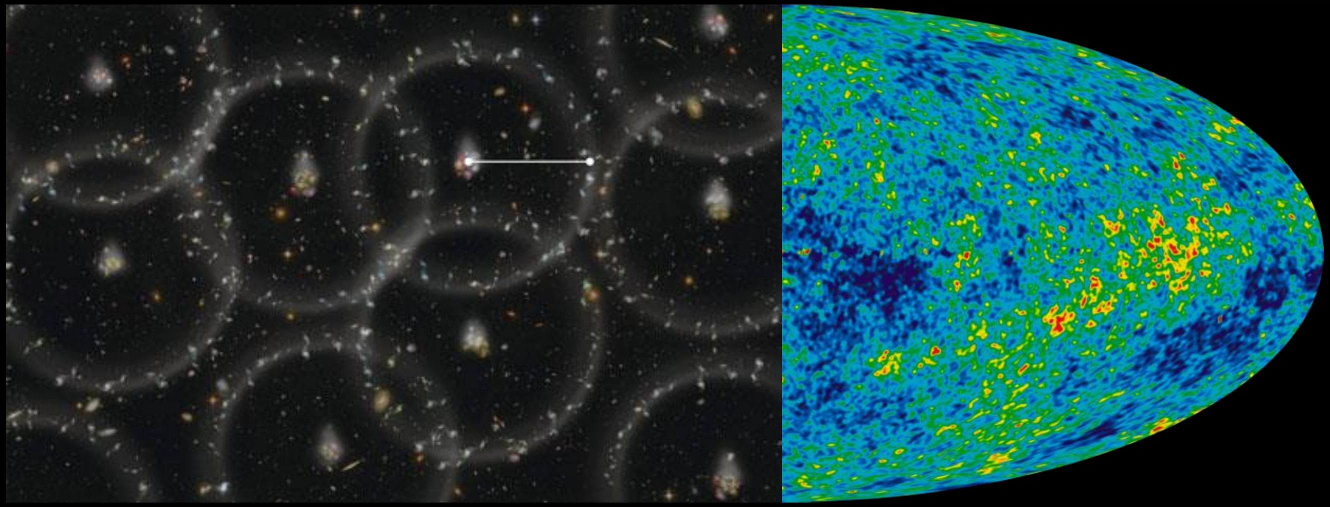
Dark matter pulls on light

(gravitational lensing)



Dark matter pulled on e^-p^+ plasma

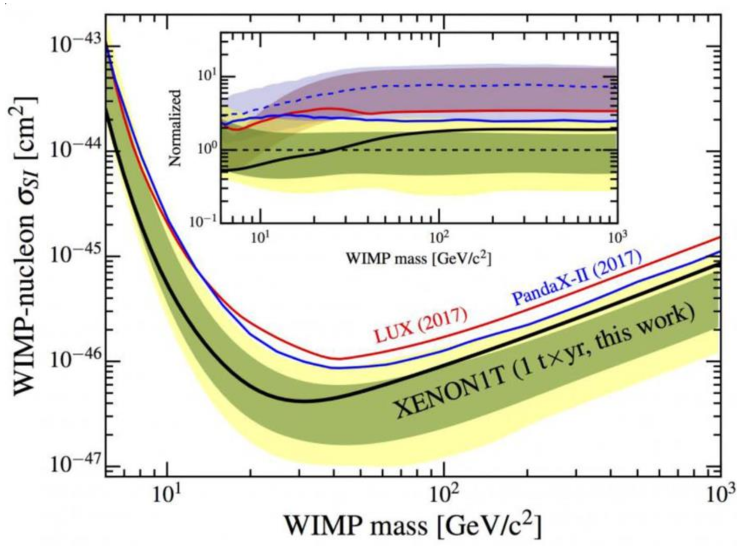
(CMB & large scale structure)



no evidence (yet) of dark matter bumping into things

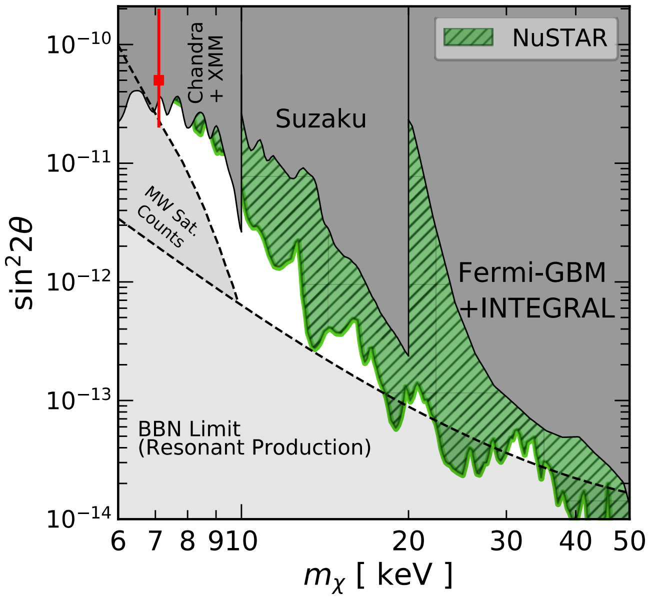
No dark matter bumping into things

(direct detection; 1805.12562)



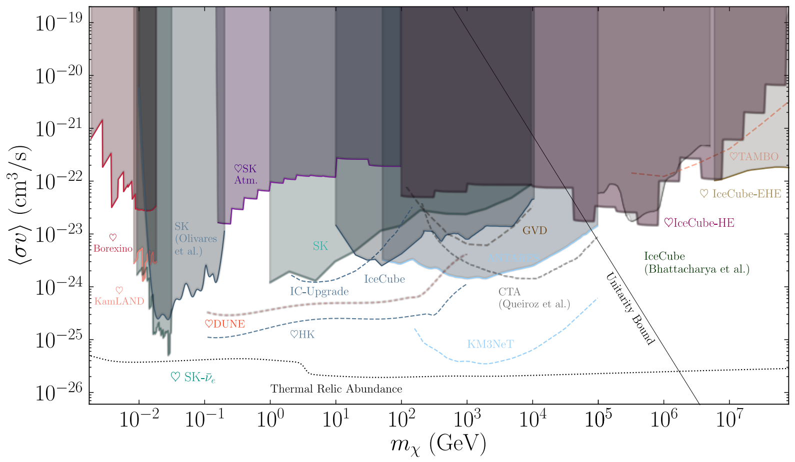
No dark matter decaying into things

(X-ray emission; 1908.09037)



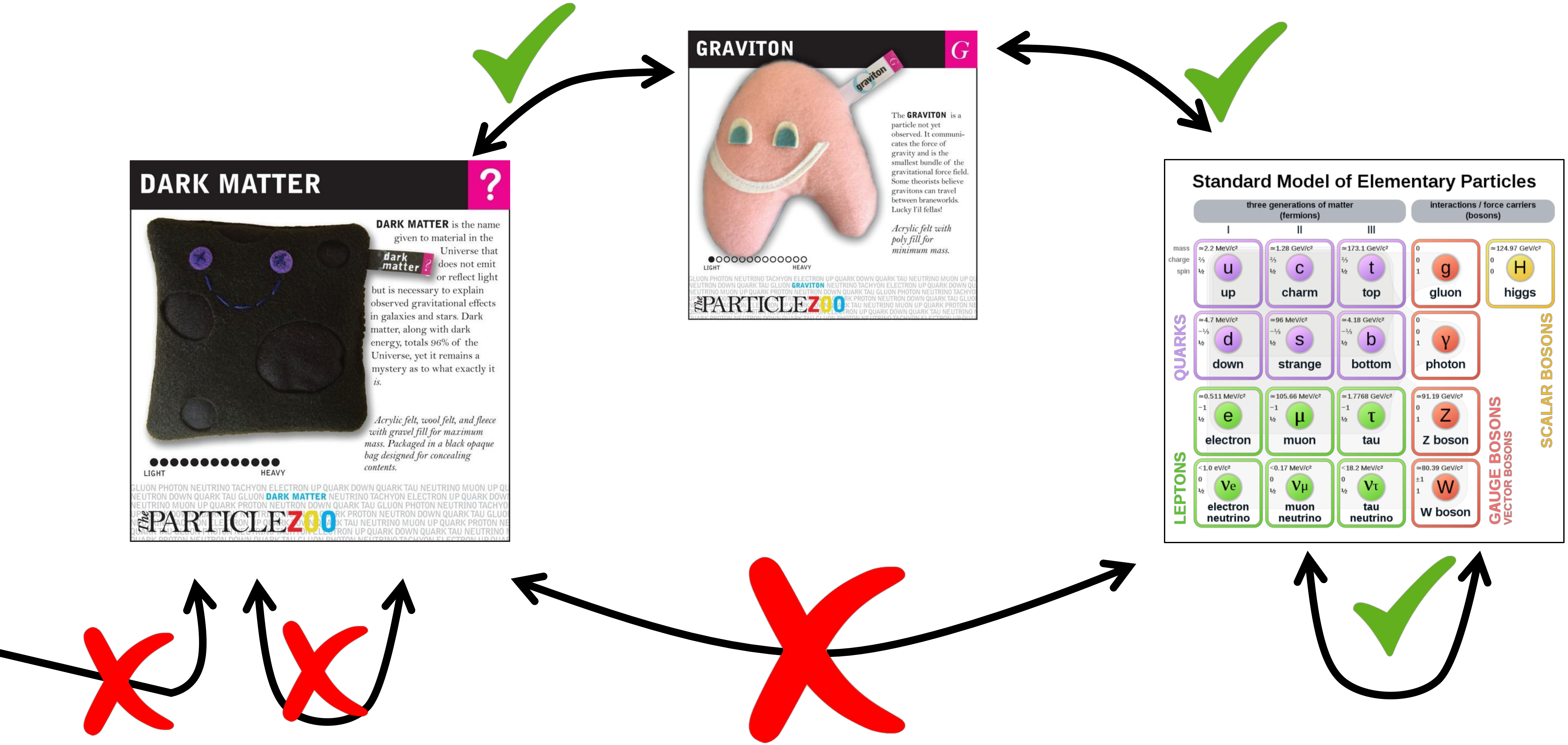
No dark matter bumping into itself

(annihilation to ν's; 1912.09486)



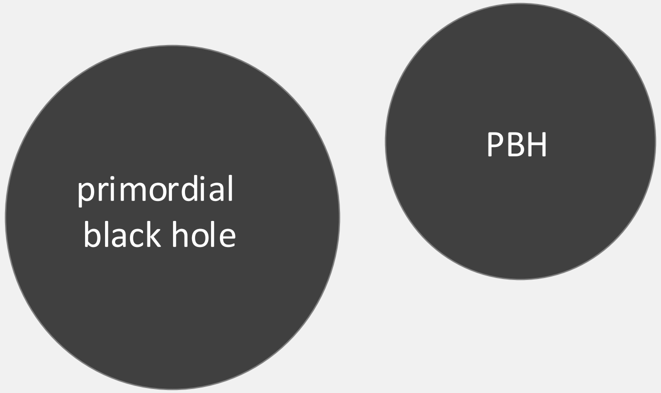
(notwithstanding hints of new physics, there's no overwhelming evidence)

the hypothesis:



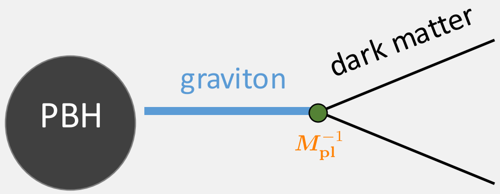
Several ways to create dark matter from gravity

the DM is a collection
of primordial black holes



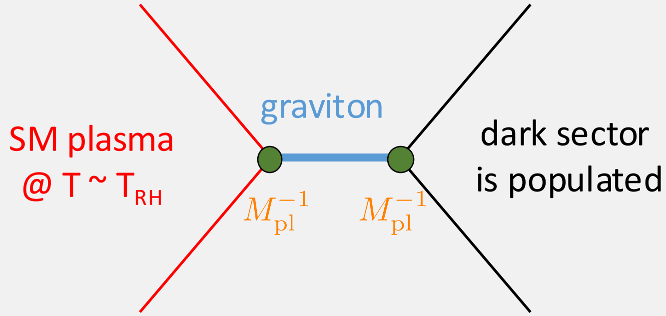
[many authors, esp. after LIGO BBH merger (2016)]

the DM is produced from
PBH evaporation



[Fujita++ (2014)], [Lennon++ (2017], [Morrison, Profumo, Yu (2018)], [Hooper, Krnjaic, McDermott (2019)]

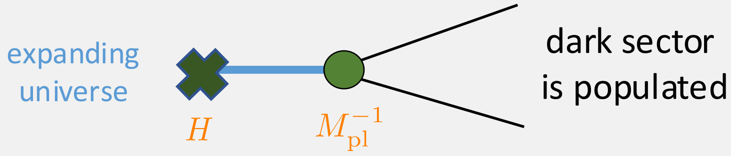
the DM is a produced from
thermal freeze-in



[Garny, Sandora, & Sloth (2015)], [Garcia, Kaneta, Mambrini, Olver, Verner (2021)], [Clery, Mambrini, Olive, Sherkin, Verner (2022)]

this talk:
cosmological
gravitational particle
production (CGPP)

the DM is produced from
cosmological expansion
during (or at the end) of inflation



[Kuzmin & Tkachev (1999)] , [Chung, Kolb, Riotto (1999)] , [review by Kolb & AL (2312.09042)]

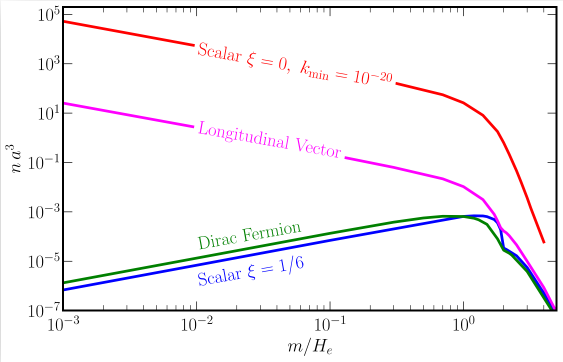
CGPP for dark matter – lots of studies!

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \, \mathscr{L}$$

spin-0 (scalar field)

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \xi \varphi^2 R$$

Chung, Kolb, & Riotto (1998)
Kuzmin & Tkachev (1998)
Herring, Boyanovsky, & Zentner (2020)
Ling & AL (2101.11621)
Lebedev, Solomko, & Yoon (2022)
Brandenberger, Kamali, & Ramos (2023)
Garcia, Pierre, & Verner (2023)



spin-1/2 (spinor field)

$$\mathscr{L} = \frac{i}{2} \bar{\Psi} \underline{\gamma}^\mu (\nabla_\mu \Psi) - \frac{1}{2} m \bar{\Psi} \Psi + \mathrm{h.c.}$$

Kuzmin & Tkachev (1998)
Chung, Everett, Yoo, & Zhou (2011)
Hashiba, Ling, & AL (2206.14204)
Lebedev++ (2023)

spin-1 (vector field)

Dimopoulos (2006) – not for DM; Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); Kolb & AL (2009.03828)

$$\mathscr{L} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_1 R g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_2 R^{\mu\nu} A_\mu A_\nu$$

spin-3/2 (vector-spinor field)

Kallosh, Kofman, Linde, & Van Proeyen (1999); Giudice, Riotto, & Tkachev (1999); Lemoine (1999); Kolb, AL, & McDonough (2102.10113); Kaneta, Ke, Mambrini, Olive, Verner (2023)

$$\mathscr{L} = \frac{i}{4} \bar{\Psi}_\mu (\underline{\gamma}^\mu \underline{\gamma}^\rho \underline{\gamma}^\sigma - \underline{\gamma}^\sigma \underline{\gamma}^\rho \underline{\gamma}^\mu)(\nabla_\rho \Psi_\sigma) + \frac{1}{2} m \bar{\Psi}_\mu \underline{\gamma}^\mu \underline{\gamma}^\sigma \Psi_\sigma + \mathrm{h.c.}$$

spin-2 (tensor field)

$$\mathscr{L} = \tfrac{1}{2} \nabla h_{\mu\nu} \nabla h^{\mu\nu} - \tfrac{1}{2} m^2 h_{\mu\nu} h^{\mu\nu} + \cdots$$

Alexander, Jenks, & McDonough (2020)
Kolb, Ling, AL, & Rosen (2302.04390)

larger reps (Kalb-Ramond)

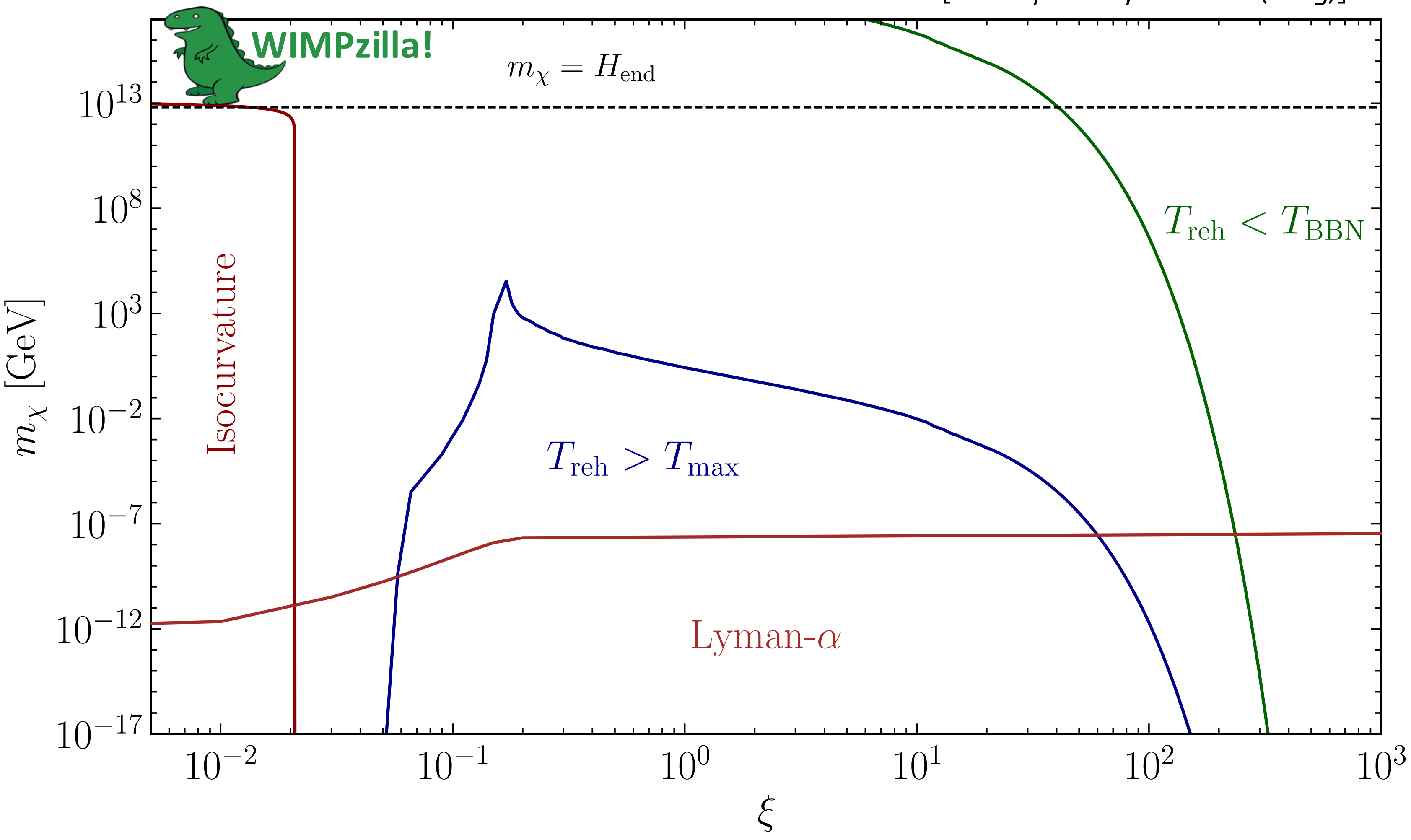
Capanelli, Jenks, Kolb, McDonough (2023)

Highlight: results for CGPP of spin-0 DM

$$\mathscr{L} \supset -\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$$

see also: [Chung, Kolb, Riotto, & Senatore (2005)], [Ling & AL (2020)], [Kolb, AL, McDonough, & Payeur (2022)], [Lebedev, Solomko, & Yoon (2022)]

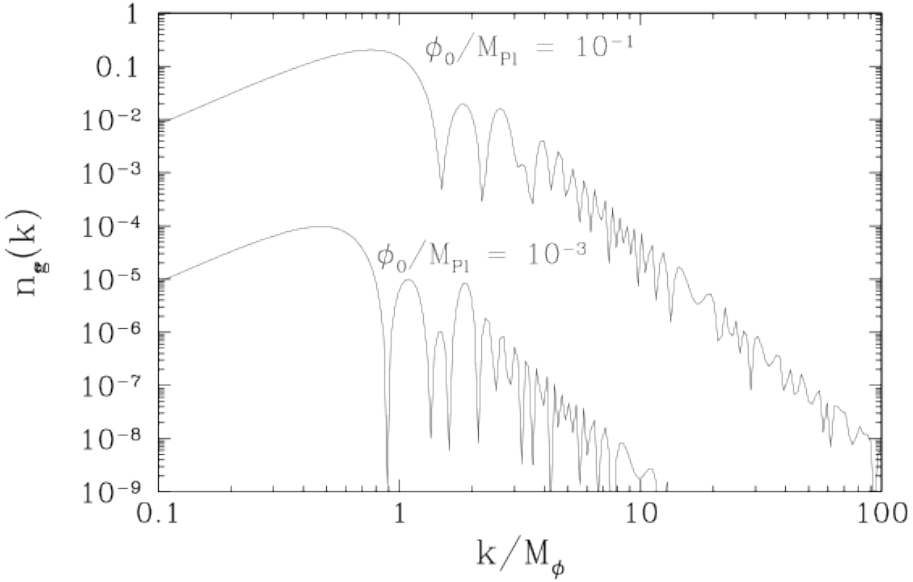
[Garcia, Pierre, & Verner (2023)]



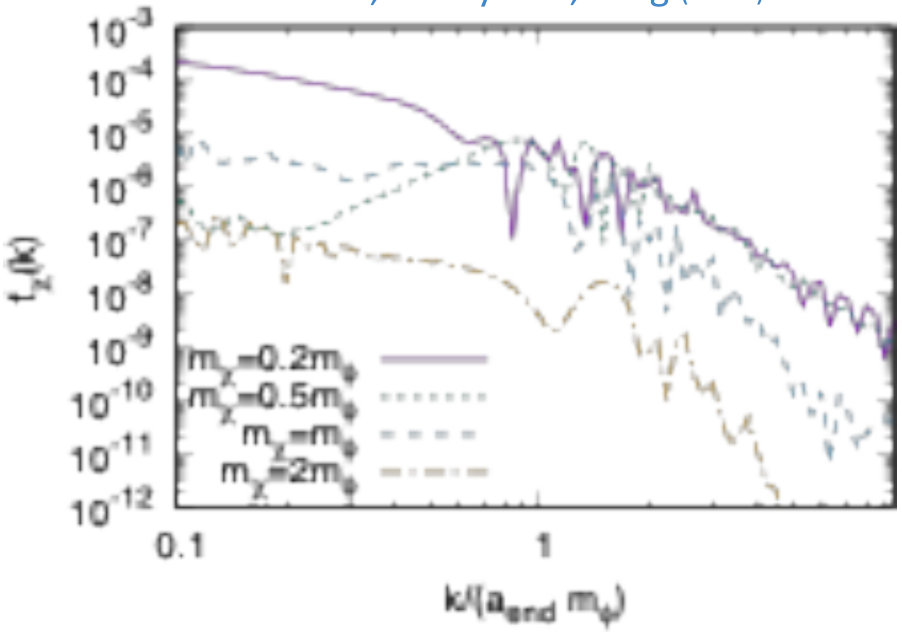
Today's topic: what's up with these wiggles?

spectra of dark particles arising from CGPP

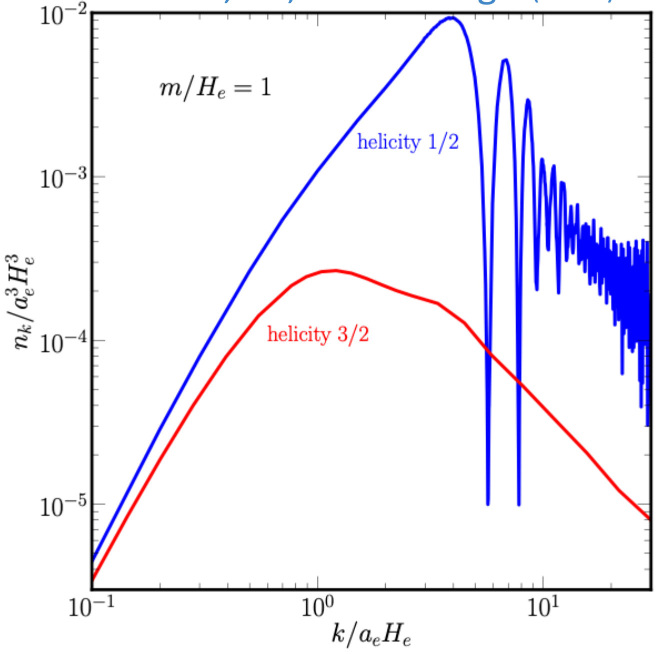
Giudice, Riotto, Tkachev (1999)



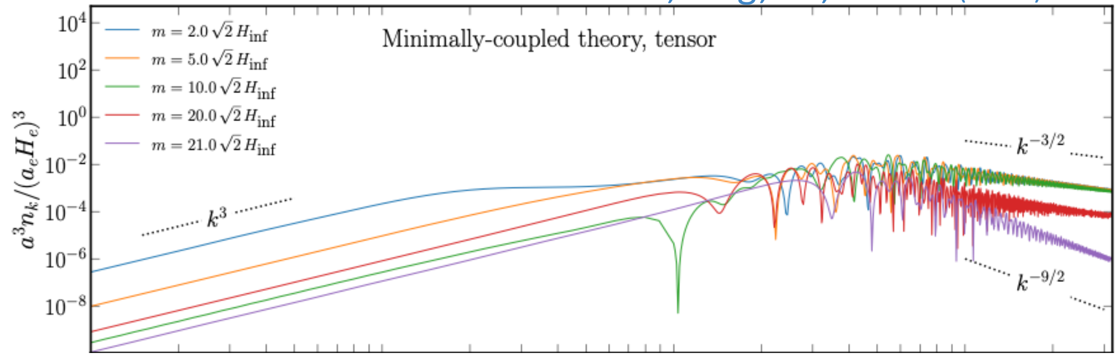
Ema, Nakayama, Tang (2018)



Kolb, AL, McDonough (2021)



Kolb, Ling, AL, Rosen (2023)



CGPP calculation
in the Bogolubov formalism

CGPP in a nutshell

conformal couplin
to gravity

QFT in curved spacetime: $$S = \int d^4x \sqrt{-g} \left[\tfrac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \tfrac{1}{2} m_\chi^2 \chi^2 + \tfrac{1}{2} \xi \chi^2 R \right] \quad (\xi = 1/6)$$

inflation + reheating: FRW: $$(\mathrm{d}s)^2 = a(\eta)^2 \left[(\mathrm{d}\eta)^2 - |\mathrm{d}\boldsymbol{x}|^2 \right]$$

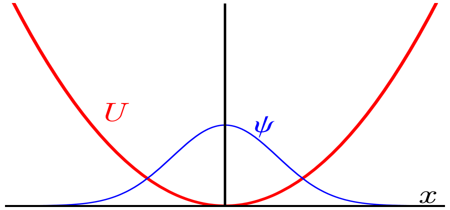
Fourier decomposition: $$\chi(\eta, \boldsymbol{x}) = \frac{1}{a(\eta)} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \, a_{\boldsymbol{k}} \, \chi_k(\eta) \, e^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \mathrm{c.c.}$$

equations of motion

$$\chi_k''(\eta) + \omega_k^2(\eta) \, \chi_k(\eta) = 0$$

$$\omega_k^2(\eta) = k^2 + a(\eta)^2 m_\chi^2$$

a harmonic oscillator with time-dependent frequency



Ansatz: leading-order WKB approx.

$$\chi_k(\eta) = \tilde{\alpha}_k(\eta) \, \chi_k^{(+)}(\eta) + \tilde{\beta}_k(\eta) \, \chi_k^{(-)}(\eta)$$

$$\chi_k^{(\pm)}(\eta) = \frac{\exp[\mp \mathrm{i} \int^\eta \mathrm{d}\eta' \omega_k(\eta')]}{\sqrt{2\omega_k(\eta)}} \qquad |\tilde{\alpha}_k|^2 - |\tilde{\beta}_k|^2 = 1$$

new mode functions

$$\partial_\eta \tilde{\alpha}_k = \frac{\omega_k'}{2\omega_k} \, \tilde{\beta}_k \, \mathrm{e}^{2\mathrm{i} \int^\eta \mathrm{d}\eta' \omega_k(\eta')}$$

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comoving number density of CGPP:

$$a^3 n_k = \frac{k^3}{2\pi^2} \left| \tilde{\beta}_k(\infty) \right|^2$$

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conformal coupling to gravity

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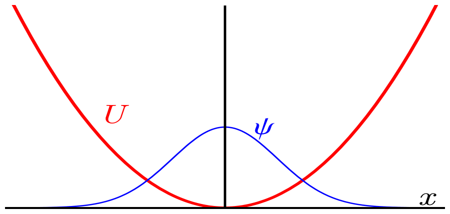

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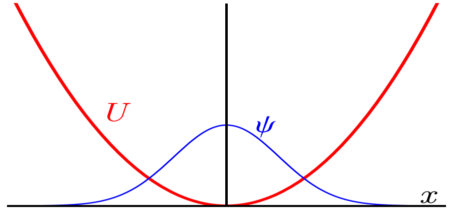

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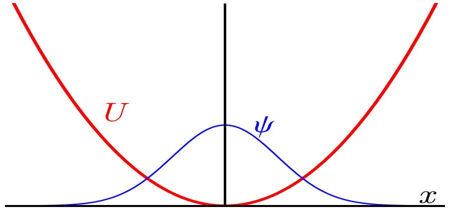

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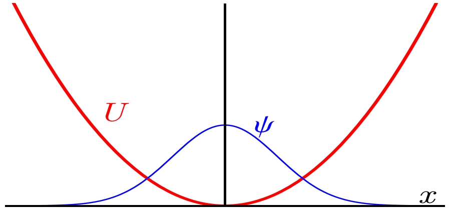

CGPP is inefficient

$$\tilde{\alpha}_k(\eta) \approx 1$$

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Resonant contributions

Bogolubov coefficients:

$$\beta_k \approx \int_{-\infty}^{\infty} \mathrm{d}t\, \Gamma_k(t)\, \mathrm{e}^{-2\mathrm{i}\int^t \mathrm{d}t' \sqrt{k^2/a(t')^2 + m_\chi^2}}$$

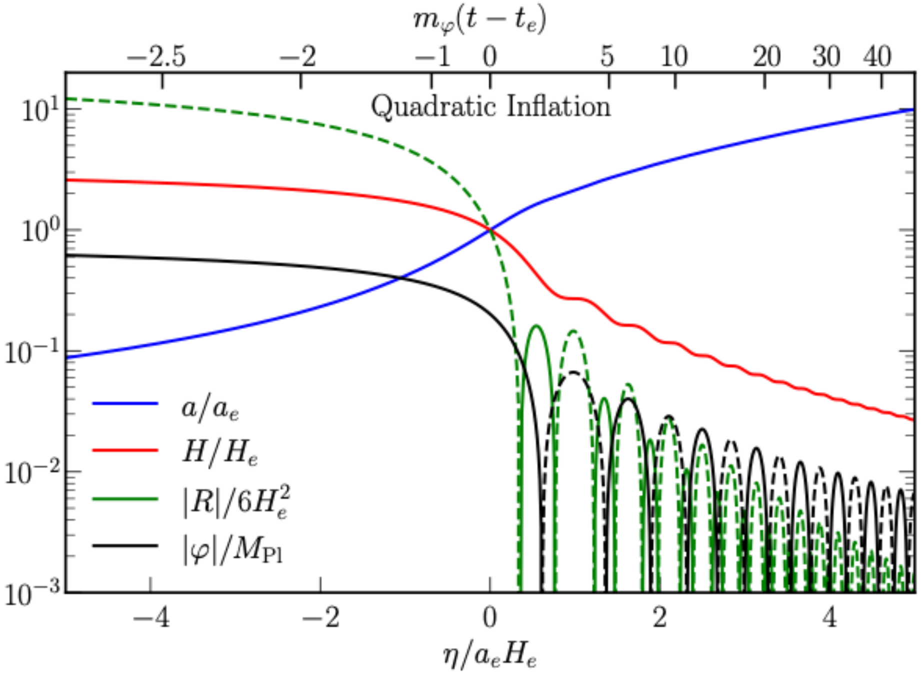
where

$$\Gamma_k(t) = \frac{m_\chi^2\, H(t)/2}{k^2/a(t)^2 + m_\chi^2}$$

this integral resembles a Fourier transform

it selects out oscillatory components when the inflaton oscillates

Background evolution (example):



Decompose into slow- and fast-varying pieces:

$$\Gamma_k(t) \sim \sum_{n=-\infty}^{+\infty} \Gamma_{\rm slow}^{(n)}(t)\, \mathrm{e}^{\mathrm{i}nm_\phi t}$$

resonant contributions ←→ scattering channels

$$\beta_k = \sum_{n=1}^{\infty} \beta_k^{(n\phi\to 2\chi)}$$

resonance condition ←→ energy conservation

$$nm_\phi \approx 2\sqrt{k^2/a(\bar{t}_k^{(n)})^2 + m_\chi^2}$$

Analytic expressions & power-law behavior

Evaluate resonant contributions using stationary phase approximation:

$$\beta_k^{(n\phi\to 2\chi)} = \mathcal{A}_k^{(n\to 2)}\, e^{i\Phi_k^{(n\to 2)}}$$

$$\Delta\Phi_k^{(n\to 2)} = \Phi_k^{(n\to 2)} - \Phi_{k,\text{leading}}^{(n\to 2)}$$

$$\kappa_n = \frac{2k/\underline{a}}{\sqrt{(nm_\phi)^2 - (2m_\chi)^2}}$$

$$\mathcal{A}_k^{(1\to2)} = -\kappa_1^{-15/4}\, 3\alpha_3 \sqrt{\frac{-\frac{i}{2}\pi}{\frac14 - r_\chi^2}}\, r_\chi^2 \left(1 + \mathcal{O}(\kappa_1^{-3})\right) , \tag{4.3a}$$

$$\mathcal{A}_k^{(2\to2)} = \kappa_2^{-9/4} \frac{3}{16} \sqrt{\frac{-i\pi}{1 - r_\chi^2}}\, r_\chi^2 \left(1 + \frac{x_0 + x_1 r_\chi^2 + x_2 r_\chi^4 - 416 r_\chi^6 + 384 r_\chi^8}{1024(1-r_\chi^2)^2}\kappa_2^{-3} + \mathcal{O}(\kappa_2^{-6})\right) , \tag{4.3b}$$

$$\mathcal{A}_k^{(3\to2)} = \kappa_3^{-15/4} \frac{\alpha_3}{9} \sqrt{\frac{-\frac32 i\pi}{\frac94 - r_\chi^2}}\, r_\chi^2 \left(1 + \mathcal{O}(\kappa_3^{-3})\right) , \tag{4.3c}$$

$$\mathcal{A}_k^{(4\to2)} = \kappa_4^{-21/4} \frac{3\left(-21 + 68\alpha_3^2 + 24\alpha_4 + 12 r_\chi^2\right)}{4096} \sqrt{\frac{-2i\pi}{4 - r_\chi^2}}\, r_\chi^2 \left(1 + \mathcal{O}(\kappa_4^{-3})\right) , \tag{4.3d}$$

$$\Delta\Phi_k^{(1\to2)} = \kappa_1^{-3/2} \left(\frac{y_0^{(1)} + y_1^{(1)} r_\chi^2 - 1280 r_\chi^4}{480\left(1 - 4r_\chi^2\right)} + z^{(1)} + \mathcal{O}(\kappa_1^{-3}) \right) , \tag{4.4a}$$

$$\Delta\Phi_k^{(2\to2)} = \kappa_2^{-3/2} \left(\frac{y_0^{(2)} + y_1^{(2)} r_\chi^2 - 80 r_\chi^4}{960\left(1 - r_\chi^2\right)} + z^{(2)} + \mathcal{O}(\kappa_2^{-3}) \right) , \tag{4.4b}$$

$$\Delta\Phi_k^{(3\to2)} = \kappa_3^{-3/2} \left(\frac{y_0^{(3)} + y_1^{(3)} r_\chi^2 - 1280 r_\chi^4}{12960\left(9 - 4r_\chi^2\right)} + z^{(3)} + \mathcal{O}(\kappa_3^{-3}) \right) , \tag{4.4c}$$

$$\Delta\Phi_k^{(4\to2)} = \kappa_4^{-3/2} \left(\frac{y_0^{(4)} + y_1^{(4)} r_\chi^2 + y_2^{(4)} r_\chi^4 + 2588 r_\chi^6}{960\left(4 - r_\chi^2\right)\left(-21 + 68\alpha_3^2 + 24\alpha_4 + 12 r_\chi^2\right)} + z^{(4)} + \mathcal{O}(\kappa_4^{-3}) \right) , \tag{4.4d}$$

[see Basso, Chung, Kolb, AL (2209.01713) for additional details]

Analytic expressions & power-law behavior

Evaluate resonant contributions using stationary phase approximation:

$$\beta_k^{(n\phi\to2\chi)} = \mathcal{A}_k^{(n\to2)}\, e^{i\Phi_k^{(n\to2)}}$$

$$\Delta\Phi_k^{(n\to2)} = \Phi_k^{(n\to2)} - \Phi_{k,\mathrm{leading}}^{(n\to2)}$$

$$\kappa_n = \frac{2k/a}{\sqrt{(nm_\phi)^2 - (2m_\chi)^2}}$$

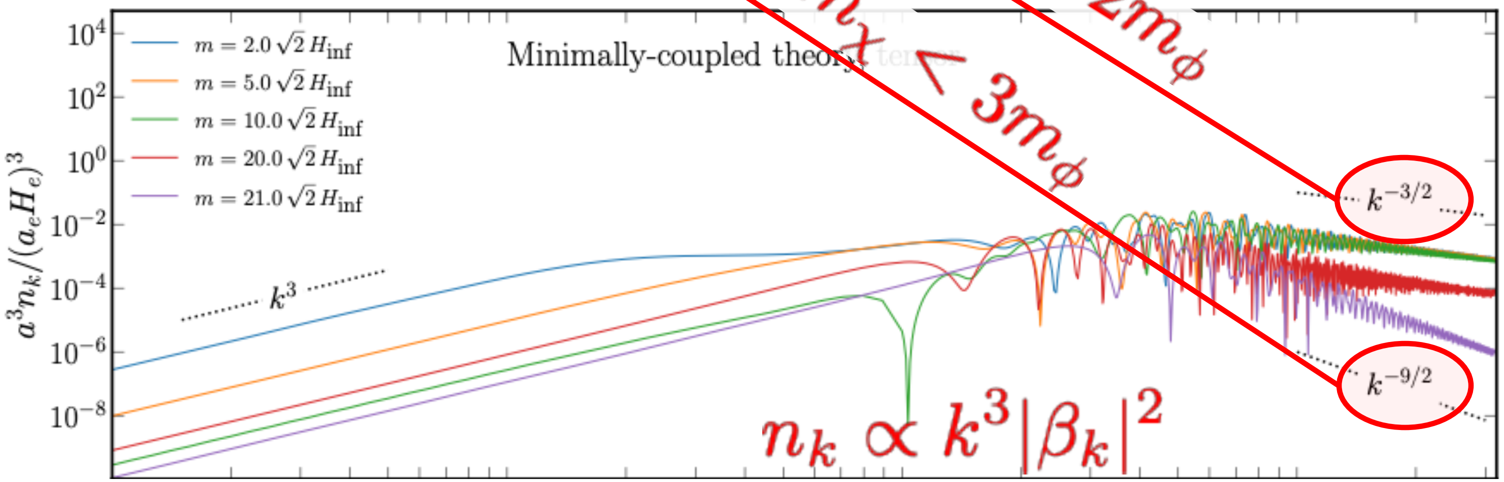
$$\mathcal{A}_k^{(1\to2)} = -\kappa_1^{-15/4}\, 3\alpha_3 \sqrt{\frac{-\frac{i}{2}\pi}{\frac{1}{4} - r_\chi^2}}\, r_\chi^2 \left(1 + \mathcal{O}(\kappa_1^{-3})\right) , \tag{4.3a}$$

$$\mathcal{A}_k^{(2\to2)} = \kappa_2^{-9/4}\, \frac{3}{16} \sqrt{\frac{-i\pi}{1 - r_\chi^2}}\, r_\chi^2 \left(1 + \frac{x_0 + x_1 r_\chi^2 + x_2 r_\chi^4 - 416 r_\chi^6 + 384 r_\chi^8}{1024(1-r_\chi^2)^2}\kappa_2^{-3} + \mathcal{O}(\kappa_2^{-6})\right) , \tag{4.3b}$$

$$\mathcal{A}_k^{(3\to2)} = \kappa_3^{-15/4}\, \frac{\alpha_3}{9} \sqrt{\frac{-\frac{3}{2}i\pi}{\frac{9}{4} - r_\chi^2}}\, r_\chi^2 \left(1 + \mathcal{O}(\kappa_3^{-3})\right) \tag{4.3c}$$

$$\mathcal{A}_k^{(4\to2)} = \kappa_4^{-21/4}\, \frac{3\left(-21 + 68\alpha_3 \ldots 24\alpha_4 + 12 r_\chi^2\right)}{4096} \sqrt{\ldots [illegible]}\, r_\chi^2 \left(1 + \mathcal{O}(\kappa_4^{-3})\right) , \tag{4.3d}$$

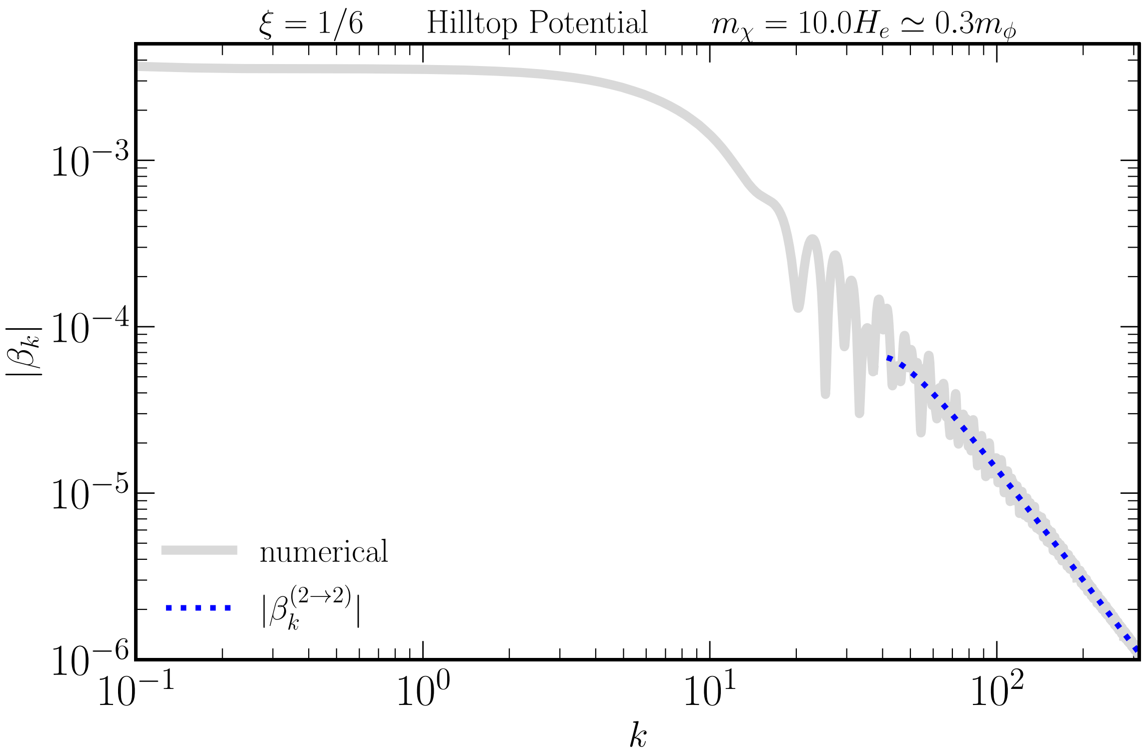
power-law scaling agrees with earlier numerical work:

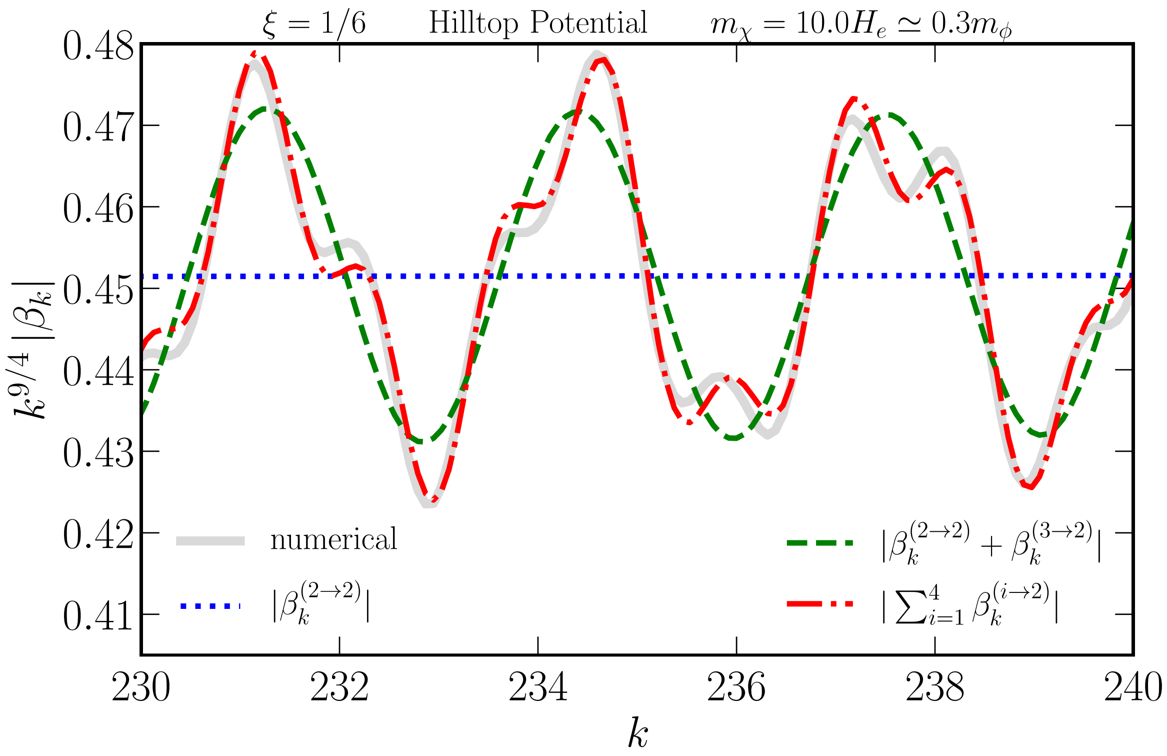


[see Basso, Chung, Kolb, AL (2209.01713) for additional details]

Numerical validation

Compare two approaches: direct numerical integration & stationary phase approximation.

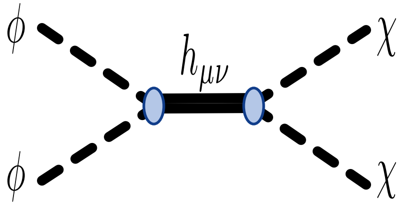




The oscillatory behavior (with wavenumber k) in the spectrum (Bogolubov coefficient β_k) is captured by interference between resonant contributions to the Bogolubov integral, which are associated with different inflaton annihilation channels $n\phi \to 2\chi$

Scattering description

Ema, Nakayama, & Tang (2018):
CGPP arising from inflaton oscillations corresponds to inflaton annihilations



$$\sigma_{\phi\phi\to\chi\chi} \sim m_\chi^4 / M_{\rm pl}^4 m_\phi^2$$

$$\Gamma_{\phi\phi\to\chi\chi} \simeq \frac{C}{16\pi} \frac{\Phi^2}{M_{\rm pl}^4} \frac{m_\chi^4}{m_\phi}$$

$$\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3/2}$$
$$\phi\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-9/2}$$
$$n\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3(2n-3)/2}$$

Chung, Kolb, & AL (2018):
We arrive at the same power-law scaling relations from the Bogolubov formalism

$$\mathcal{A}_k^{(2\to2)} \propto k^{-9/4} \quad \Rightarrow \quad n_k \propto k^{-3/2}$$
$$\mathcal{A}_k^{(3\to2)} \propto k^{-15/4} \quad \Rightarrow \quad n_k \propto k^{-9/2}$$
$$\mathcal{A}_k^{(3\to2)} \propto k^{-21/4} \quad \Rightarrow \quad n_k \propto k^{-15/2}$$

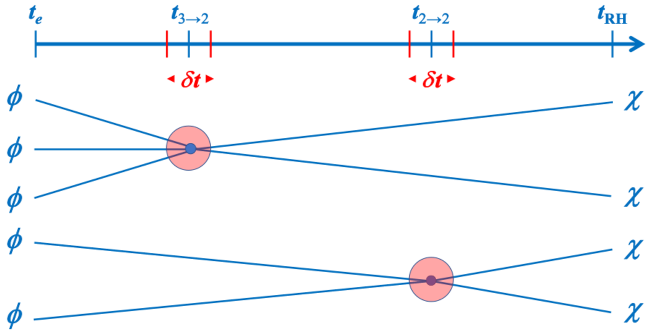
Basso, Chung, Kolb, & AL (2022):
We study interference effects using the Bogolubov formalism. How does the interference correspond to a scattering?

Naively seems like interference is impossible:

- initial states are different (e.g., 2ϕ vs 3ϕ)
- final states are different ($E_\chi = m_\phi$ vs $3m_\phi/2$)

These issues are resolved because:

- the initial inflaton coherent state is a state of indefinite particle number
- Early 3-to-2 scatterings interfere with late 2-to-2 scatterings -- energy lost through redshift



Summary

Context: Cosmological gravitational particle production (CGPP) arises when quantum fields `feel' the homogeneous expansion of the universe during inflation or at the end of inflation.

CGPP provides a simple explanation for the origin of dark matter (across wide mass & spin), and it leads to an unavoidable production of any (non-conformal) hidden-sector particles.

Question: What's the origin of wiggles seen in (some) spectra of particles arising from CGPP?

Answer: Interference of resonant contributions to the Bogolubov integral, which can also be interpreted as quantum interference between different annihilation channels: $n\phi \rightarrow 2\chi$.

Points for discussion:

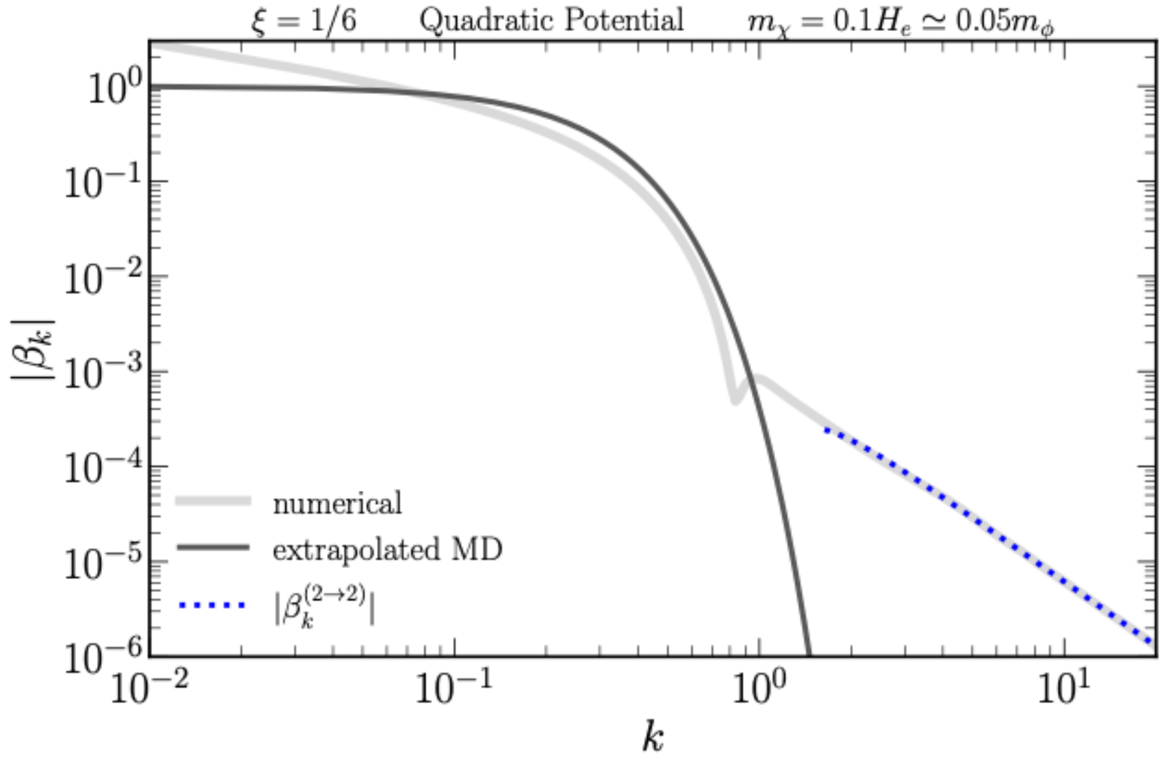
- The interference fringes don't impact the total abundance of CGPP appreciably.
- So, what are possible observable signatures of the interference fringes?
- More work needed to establish a rigorous scattering description of CGPP interference.
- We focused on a conformally coupled scalar field – how different for fields with spin?
- We focused on CGPP – is there also an impact on preheating due to a non-grav coupling?

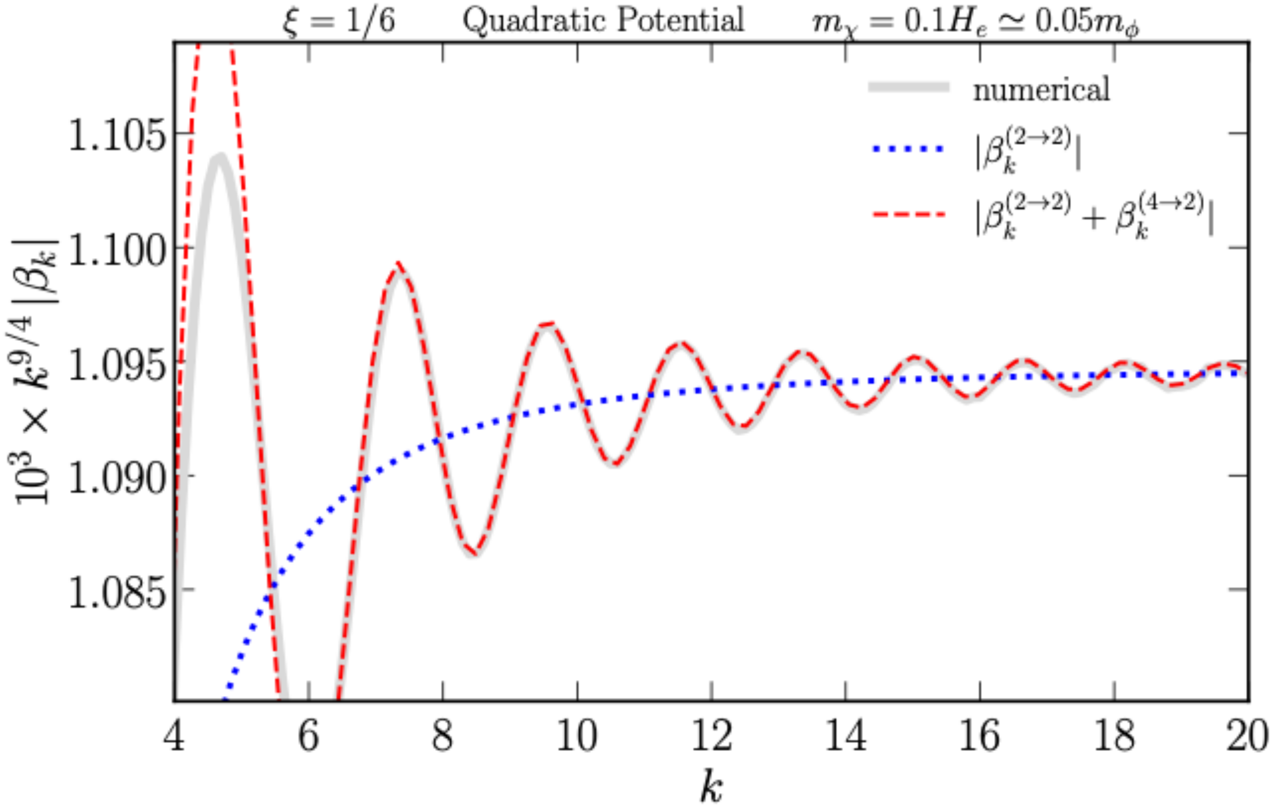
backup slides

numerical vs analytical
for a minimally-coupled scalar

Numerical validation – quadratic inflaton potential

For a quadratic inflaton potential, the hierarchy between H_e and m_ϕ is smaller (than for hilltop)





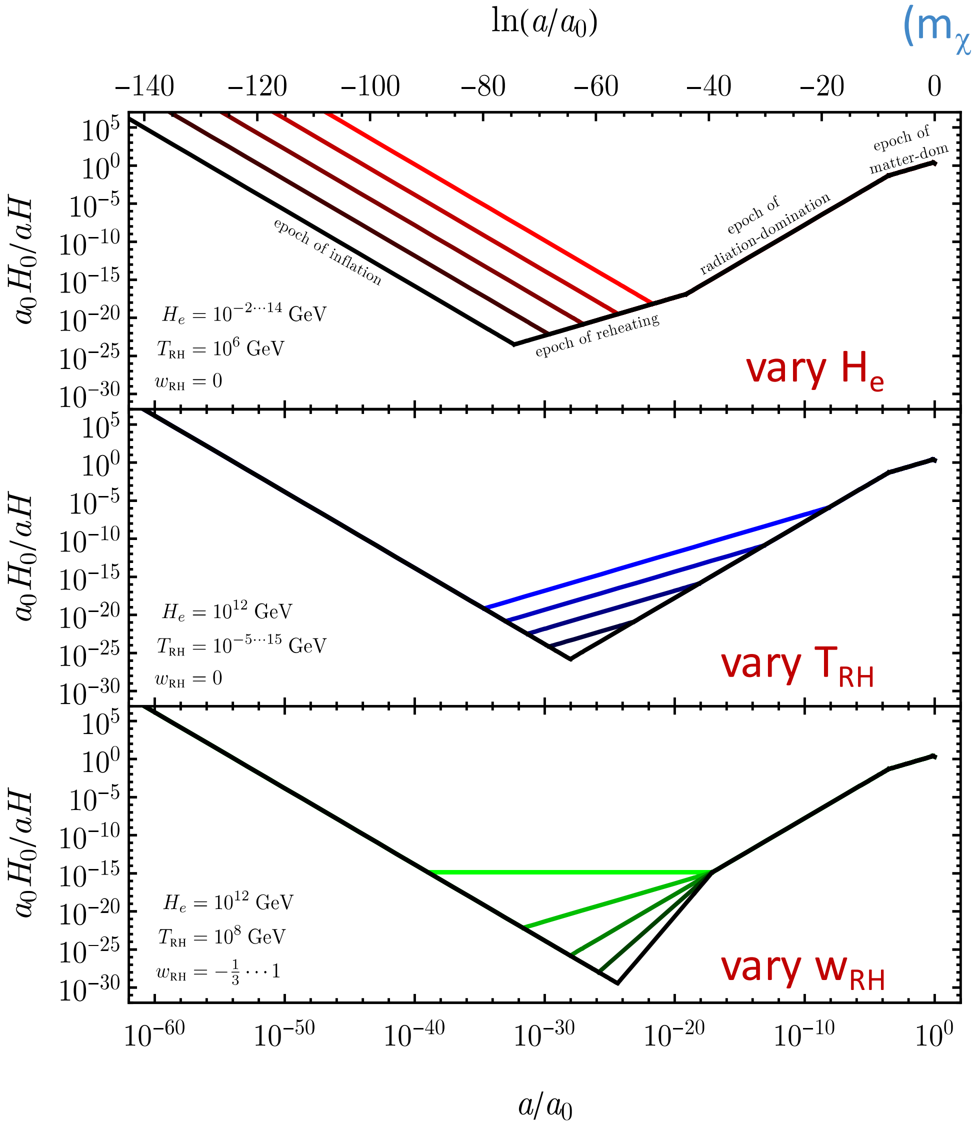
The analytical approximation converges more quickly – already 2→2 and 4→2 contributions give an excellent fit to the direct numerical calculation

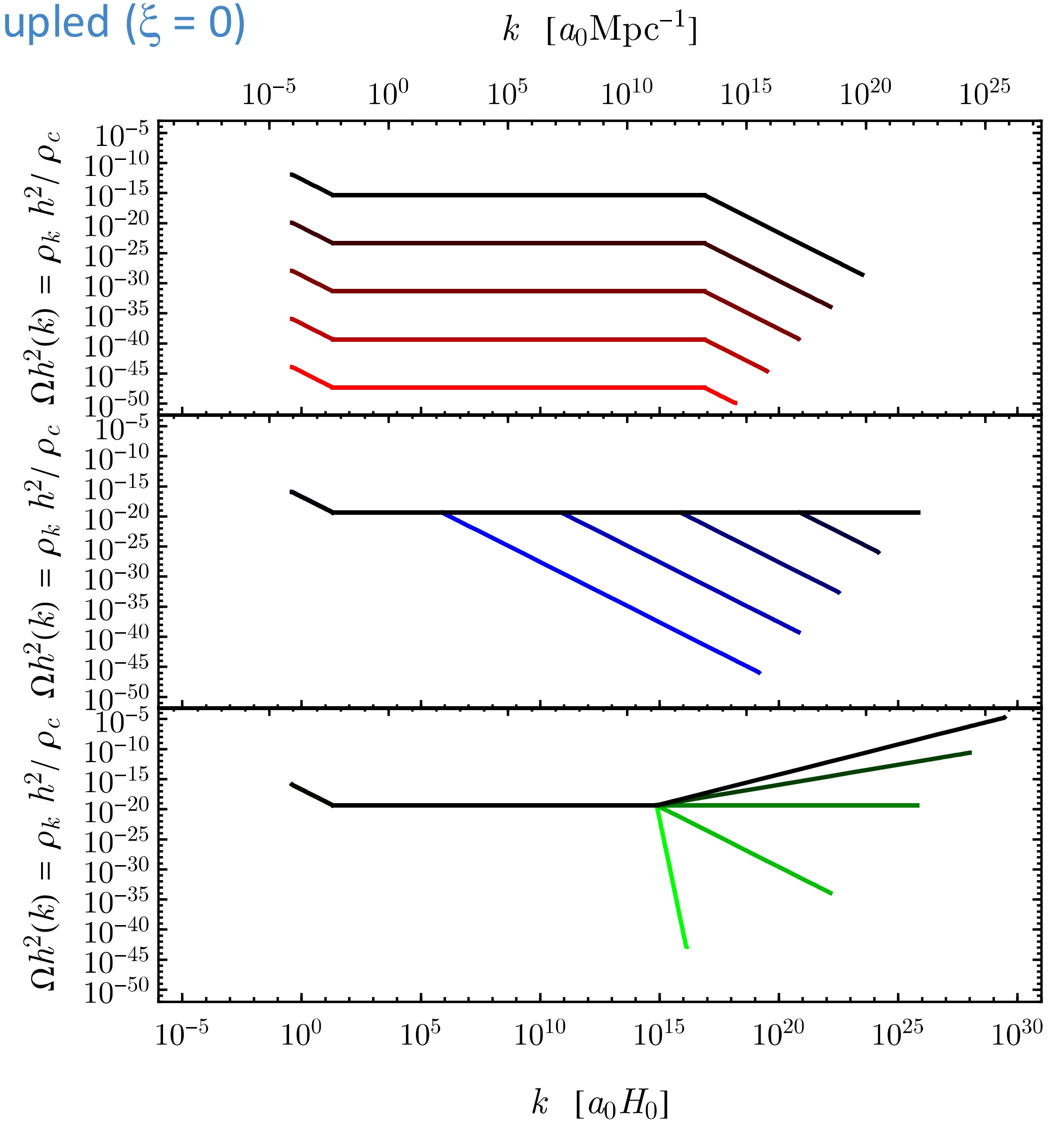
modeling reheating

how it impacts the spectrum

Effect of reheating epoch

For illustration, consider a scalar field that is light ($m_\chi < H_{inf}$) and minimally-coupled ($\xi = 0$)





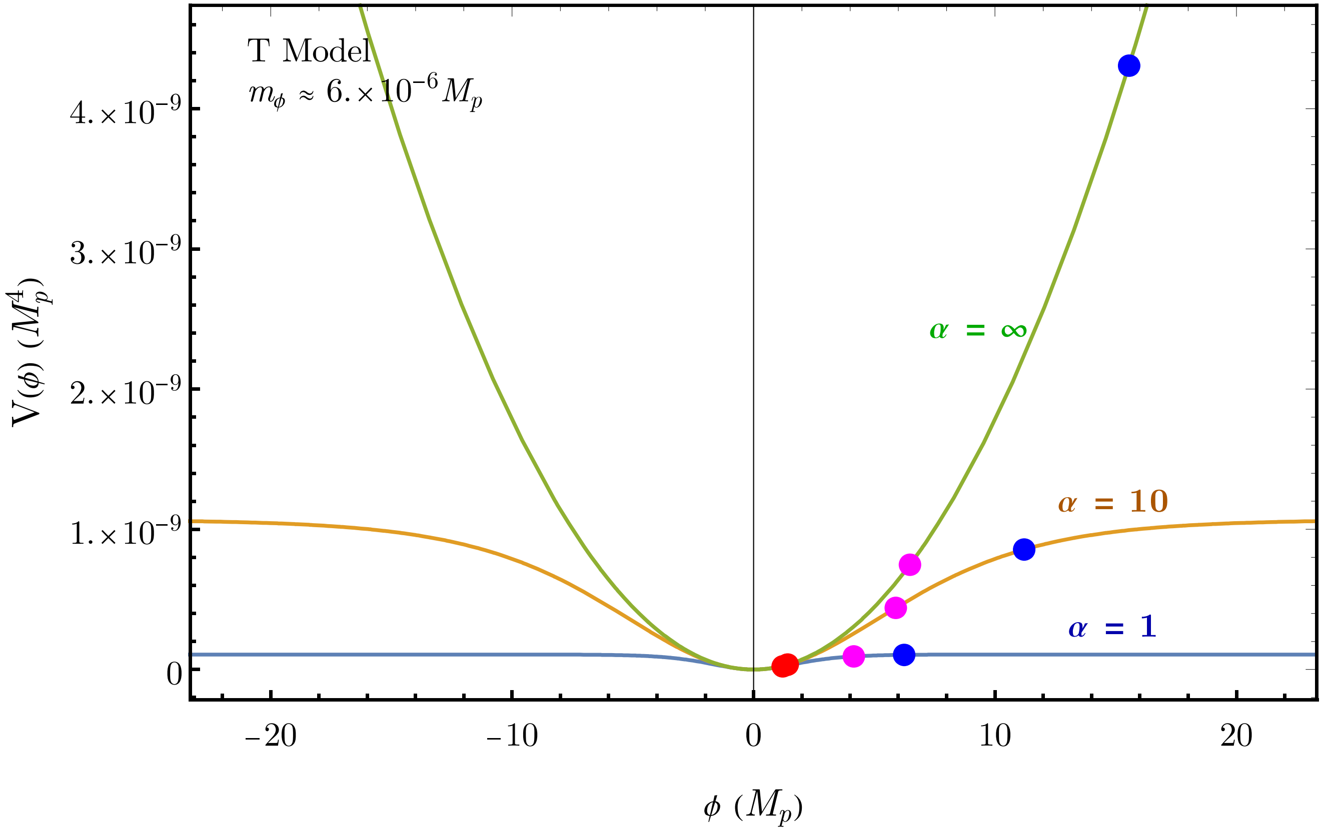
phenomenological considerations
in a concrete model

Example: alpha attractor

[Ling & AL (2101.11621)]

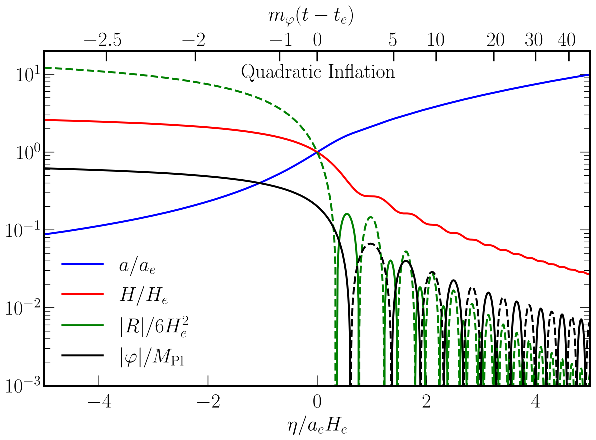
T-model
alpha attractor

$$V_T(\phi) = \alpha \mu^2 M_p^2 \tanh^2 \frac{\phi}{\sqrt{6\alpha} M_p}$$



$$\Rightarrow \begin{cases} \phi(t) \\ a(t) \end{cases}$$

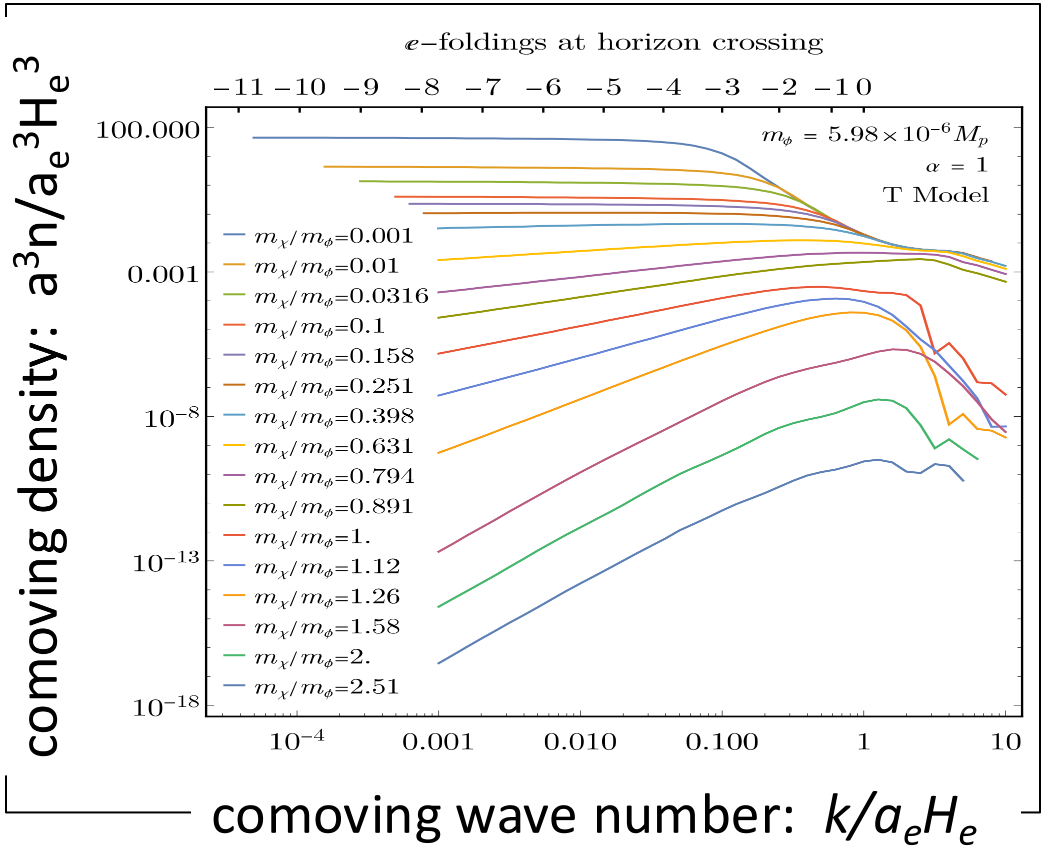
FRW background



Numerical results

[Ling & AL (2101.11621)]

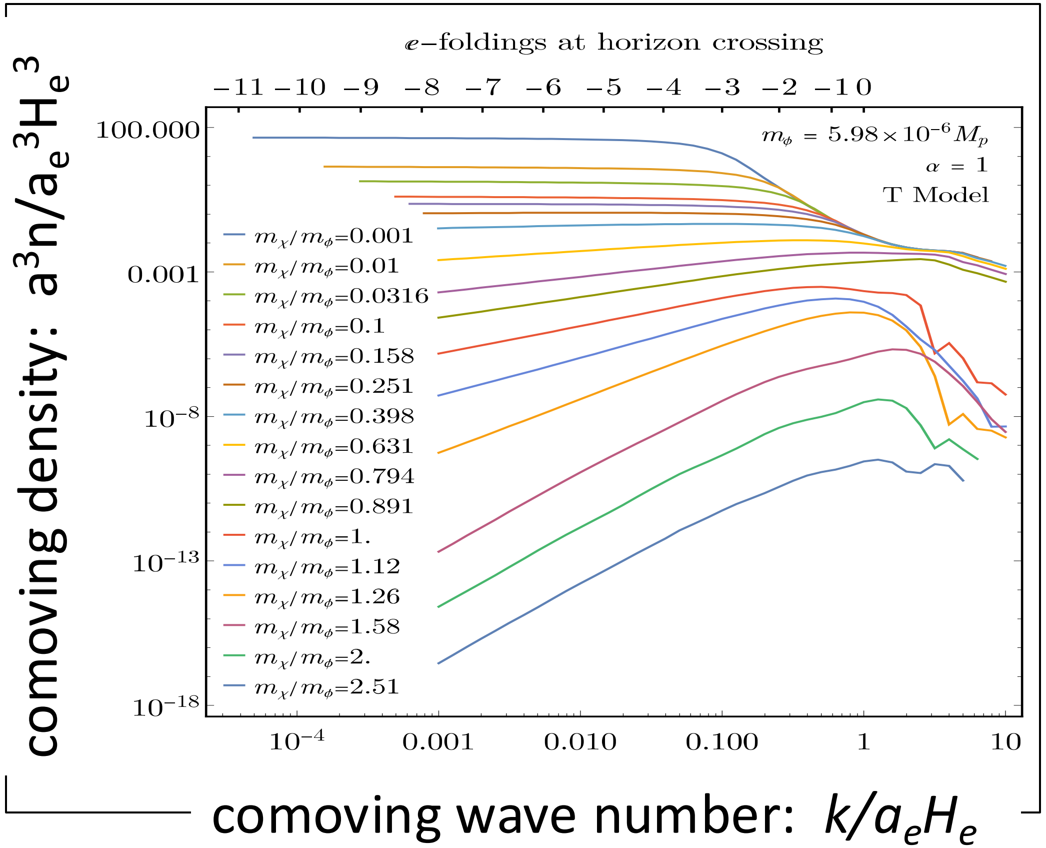
comoving number density



Numerical results

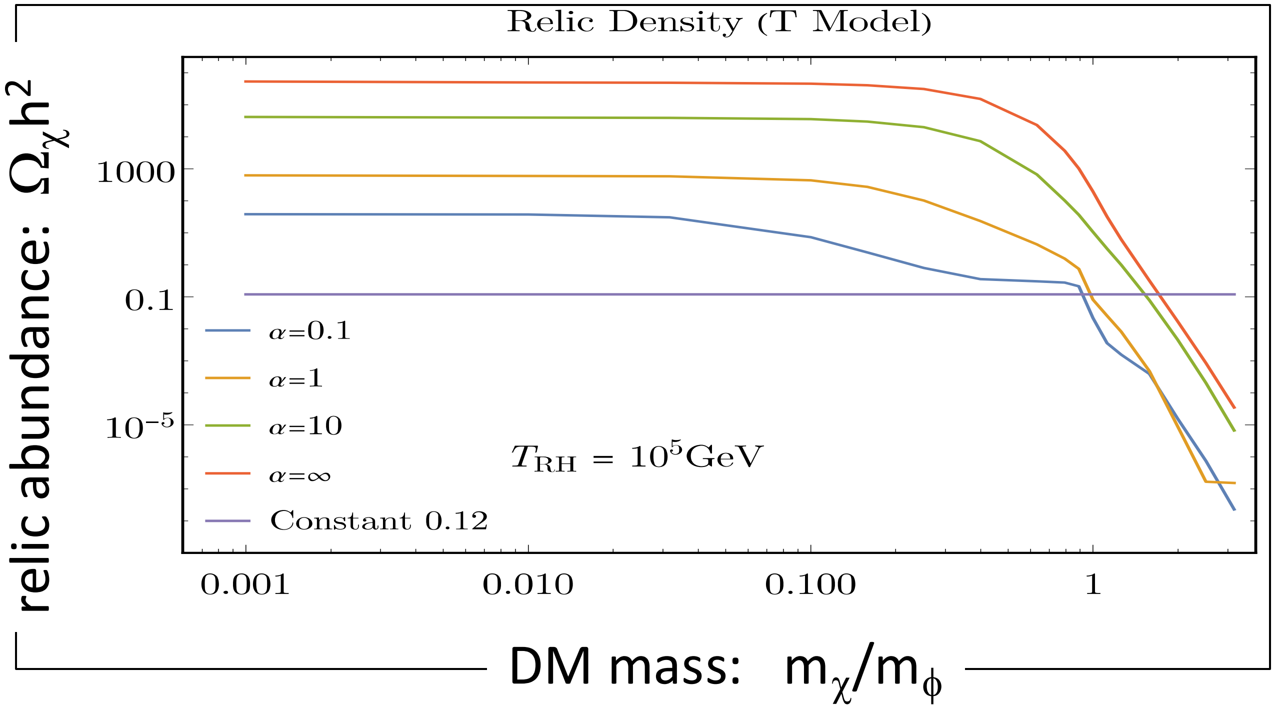
[Ling & AL (2101.11621)]

comoving number density



$$\Omega_\chi h^2 \simeq (0.114)\left(\frac{m_\chi}{10^{10}\text{ GeV}}\right)\left(\frac{H_e}{10^{10}\text{ GeV}}\right)\left(\frac{T_{\rm RH}}{10^{8}\text{ GeV}}\right)\left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$

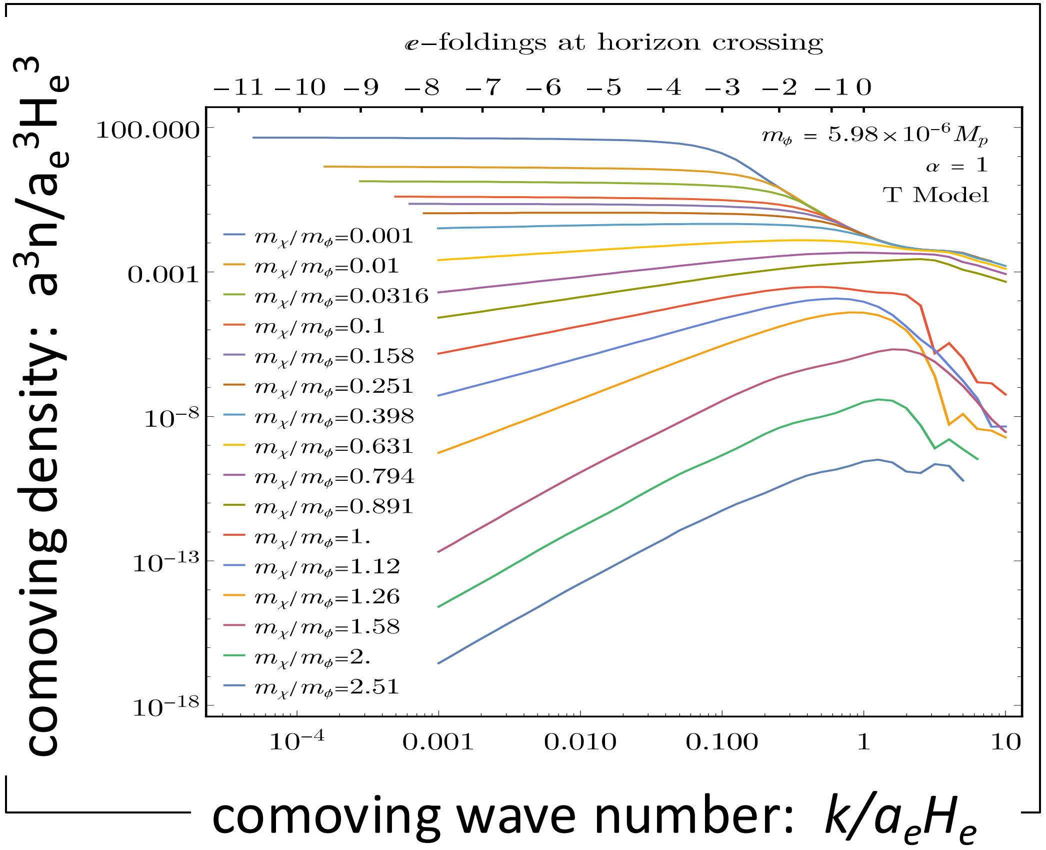
relic abundance



Numerical results

[Ling & AL (2101.11621)]

comoving number density



$$k = a_e H_e \qquad \text{(Hubble-scale modes at the end of inflation)}$$

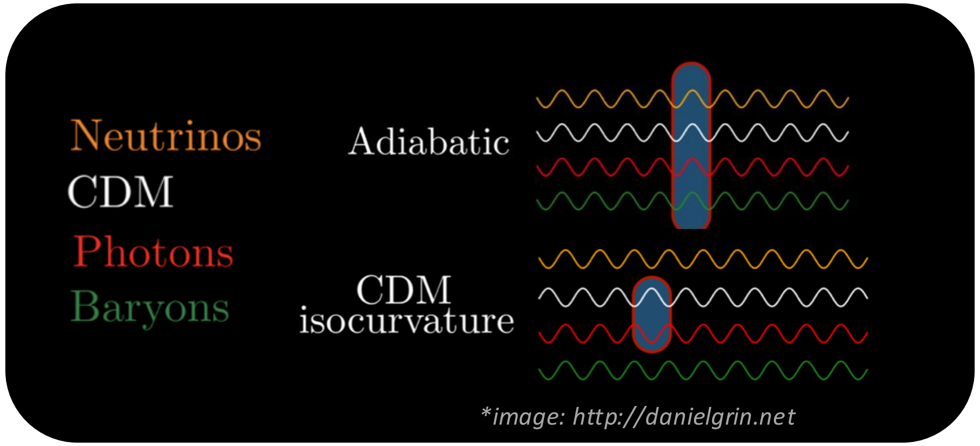
$$\lambda_{\text{phys,e}} = 2\pi a_e/k \simeq \left(10^{-29}\ \text{meters}\right)\left(\frac{H_e}{10^{14}\ \text{GeV}}\right)^{-1}$$

$$\lambda_{\text{phys,0}} = 2\pi a_0/k \simeq \left(100\ \text{meters}\right)\left(\frac{H_e}{10^{14}\ \text{GeV}}\right)^{-1/3}\left(\frac{T_{\text{RH}}}{10^{9}\ \text{GeV}}\right)^{-1/3}$$

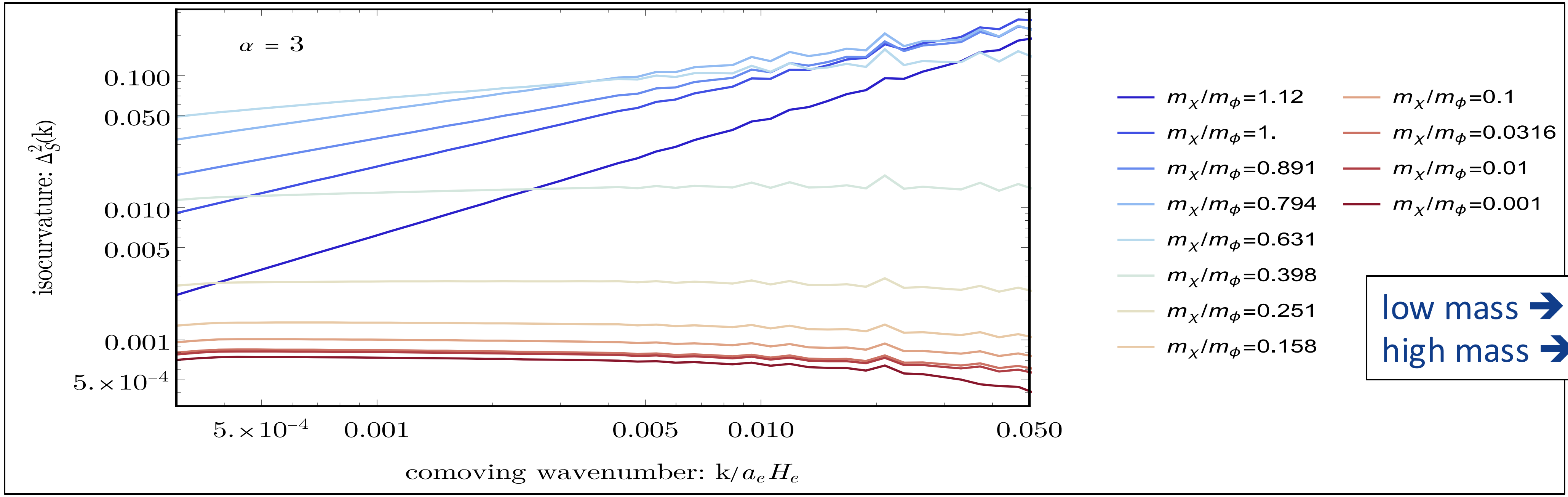
$$p_{\text{phys,0}} = \hbar k/a_0 \simeq \left(5\times10^{-18}\ \text{GeV}\right)\left(\frac{H_e}{10^{14}\ \text{GeV}}\right)^{1/3}\left(\frac{T_{\text{RH}}}{10^{9}\ \text{GeV}}\right)^{1/3}$$

CMB isocurvature

[Ling & AL (2101.11621)]



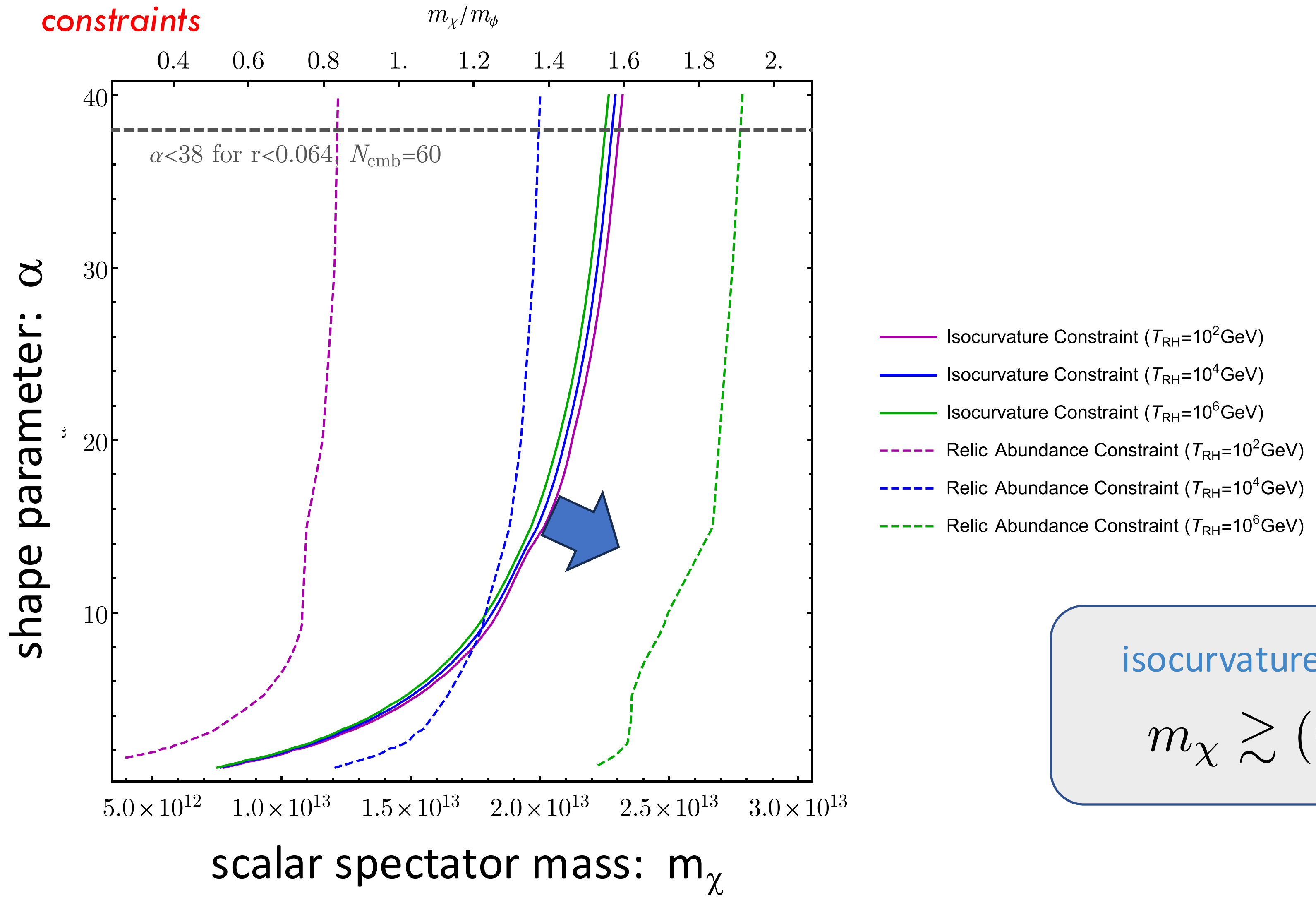
$$\Delta^2_{\mathcal{S}}(k_{\rm cmb}) < 7.3 \times 10^{-11}$$
$$k_{\rm cmb} = 0.002\ {\rm Mpc}^{-1} a_0$$
$$k_{\rm cmb}/a_e H_e \approx e^{-50} \simeq 2 \times 10^{-22}$$



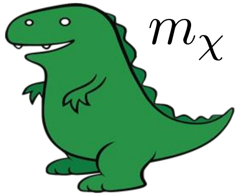
low mass ➔ red tilt
high mass ➔ blue tilt

Parameter space

[Ling & AL (2101.11621)]



WIMPzilla!



$m_\chi \gtrsim m_\phi$

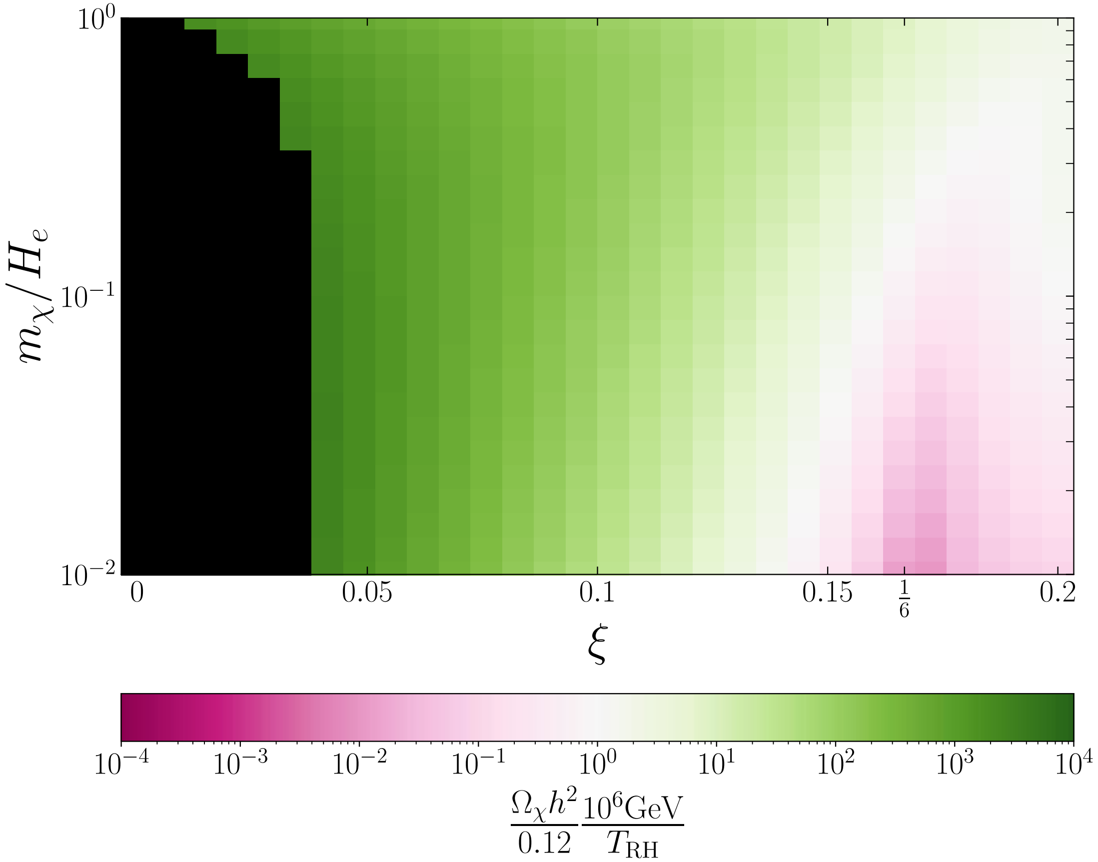
isocurvature avoidance:

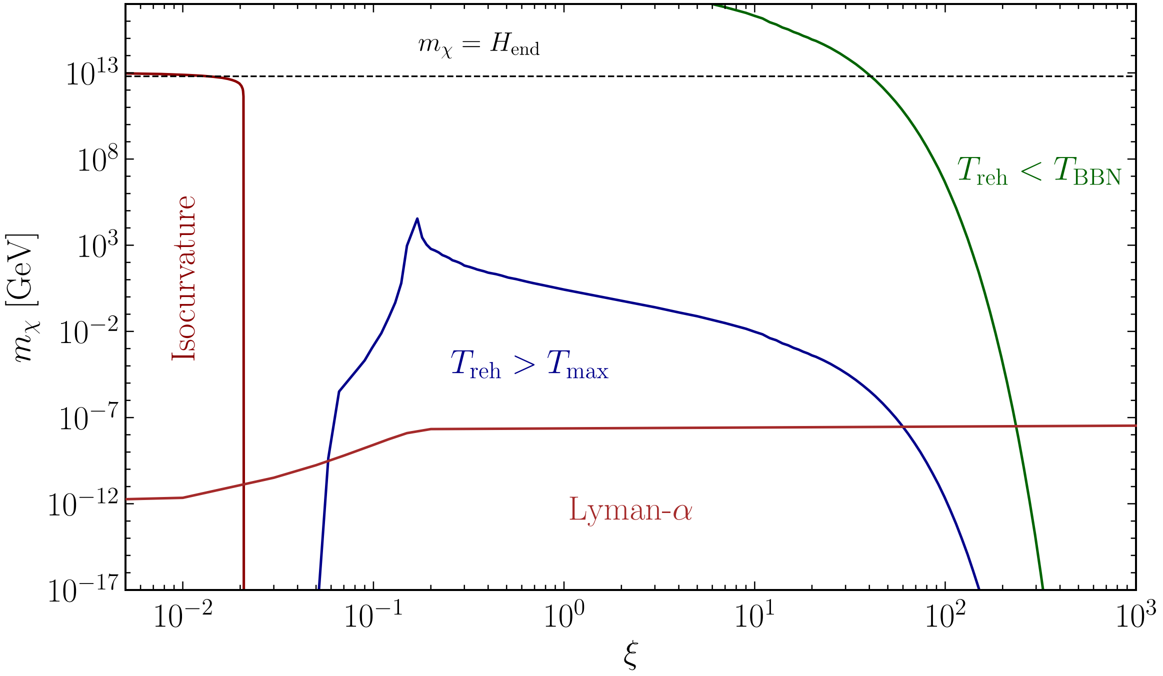
$$m_\chi \gtrsim (0.8 - 1.6)\, m_\phi$$

going non-minimal

[Kolb, AL, McDonough, & Payeur (2022)], [Garcia, Pierre, & Verner (2023)]
see also: [Markkanen, Rajantie, & Tenkanen (2018); Tenkanen (2019)]

$$\mathscr{L} \supset -\tfrac{1}{2} m_\chi^2 \chi^2 + \tfrac{1}{2}\xi R \chi^2$$





isocurvature constraints on ultra-light scalar GPP
can be avoided by introducing
a "small" non-minimal coupling to gravity