The Open Effective Field Theory of Inflation

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What we know about the very early universe

We know that primordial perturbations are characterised by:

- Approximate scale invariance.
- Low tensor-to-scalar ratio.

Observational constraints from:

- CMB power spectrum.
- Matter power spectrum.

The theoretical description of the very early universe is still at large.

Number of models $\gg$ Measurements

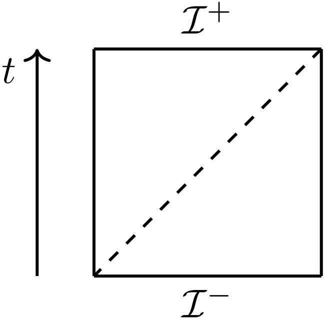
Effective Field Theories

What observations are compatible with IR symmetries?

How big is the EFT that we need to parametrise our ignorance?

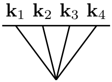
Fields in the very early universe

Primordial spacetime was well approximated by the de Sitter spacetime.



Primordial perturbations are correlators of ζ and γ_{ij} on $\mathcal{I}^+$. An expanding spacetime leads to particle production:

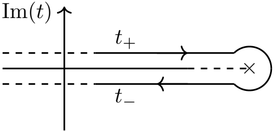
$$\langle \hat{\zeta}_{k_1} \hat{\zeta}_{k_2} \hat{\zeta}_{k_3} \hat{\zeta}_{k_4} \rangle =$$



- ζ and γ_{ij} are described by massless particles of spin 0 and 2.
- Heavier particles do not leave an imprint on the reheating surface. Particle production is Boltzmann suppressed.

In-In formalism

Primordial perturbations can be computed using the in-in formalism:



$$, \ \langle \hat{\mathcal{O}}(t_0) \rangle = \langle \bar{T} \left\{ e^{i \int_{t_+} \hat{\mathcal{H}}_{\text{int}}} \right\} \hat{\mathcal{O}}(t_0) \, T \left\{ e^{-i \int_{t_-} \hat{\mathcal{H}}_{\text{int}}} \right\} \rangle$$

This can be rewritten as:

$$\langle \hat{\zeta}_{\mathbf{k}_1}(t_0) \hat{\zeta}_{\mathbf{k}_2}(t_0) \hat{\zeta}_{\mathbf{k}_3}(t_0) \rangle = \int \mathcal{D}\zeta_{\mathbf{k}}(t_0) \ \zeta_{\mathbf{k}_1}(t_0) \zeta_{\mathbf{k}_2}(t_0) \zeta_{\mathbf{k}_3}(t_0) \ e^{iF[\zeta_{\mathbf{k}}(t_0)]}$$

where $F[\zeta_{\mathbf{k}}(t_0)]$ is the influence functional:

$$e^{iF[\zeta_{\mathbf{k}}(t_0)]} = \int_{\text{BD}}^{\zeta_{\mathbf{k}}(t_0)} \mathcal{D}\zeta^{+}_{\mathbf{k}}(t) \int_{\text{BD}}^{\zeta_{\mathbf{k}}(t_0)} \mathcal{D}\zeta^{-}_{\mathbf{k}}(t) \ e^{iS[\zeta^{+}(t)] - iS[\zeta^{-}(t)]}$$

Effective Field Theory of Inflation

Based on the symmetries of quasi de Sitter and the field content (ζ and γ_{ij}), an EFT description is built. Focusing on the scalar fluctuations $\zeta = -H\pi$:

$$\mathcal{L}(\pi) = \frac{f_\pi^4}{c_s^3}\left[\frac{\dot{\pi}^2}{2} - \frac{c_s^2(\partial_i\pi)^2}{2a^2(t)} + C_{\dot{\pi}^3}\dot{\pi}^3 + C_{\dot{\pi}(\partial_i\pi)^2}\frac{\dot{\pi}(\partial_i\pi)^2}{a^2(t)} + ...\right]$$

It is an EFT of broken time translations. Non-linearly realised symmetries constrain the Wilson coefficients:

$$C_{\dot{\pi}^3} = \frac{1}{2}(1-c_s^2)(1+\frac{2\tilde{c}_3}{3c_s^2})\ ,\ C_{\dot{\pi}(\partial_i\pi)^2} = -\frac{1}{2}(1-c_s^2)$$

One can impose constraints on the speed of sound and the Wilson coefficients:

$$c_s > 0.021\ ,\ \Gamma(c_s, \tilde{c}_3) < \frac{32\pi}{H^4}f_\pi^4$$

Summary

We do not have a complete picture of the very early universe:

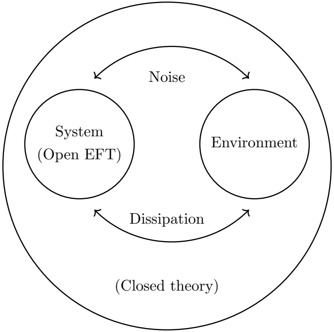
- We can focus on descriptions that respect scale invariance and low tensor-to-scalar ratios.
- We approximate the very early universe by de Sitter spacetime.
- We use the in-in formalism.
- (Closed) Bottom-up approach: EFT of Inflation, described by an action.

Open Quantum Systems I

In cold single field inflation, the closed system approach is a good approximation thanks to the Boltzmann suppression of the heavier fields.

Quantum production → Boltzmann suppression → Good closed EFT

In other models of the very early universe, the heavy sector can be very populated. Therefore the closed EFT description is not enough.



- Dissipation: Energy from the system flows into the environment.
- Noise: Fluctuations of the environment generate excitations of the system.

Closed EFT → Action $S[\zeta(t)]$

Open EFT → Influence functional $F[\zeta(t_0)]$

Open Quantum Systems II

The influence functional depends on π^+ and π^-:

$$e^{iF[\pi_{\mathbf{k}}(t_0)]} = \int_{\mathrm{BD}}^{\pi_{\mathbf{k}}(t_0)} \mathcal{D}\pi_{\mathbf{k}}^{+}(t) \int_{\mathrm{BD}}^{\pi_{\mathbf{k}}(t_0)} \mathcal{D}\pi_{\mathbf{k}}^{-}(t)\ e^{i\mathcal{I}[\pi^+(t),\pi^-(t)]}$$

$$\langle \hat{\pi}_{\mathbf{k}_1}(t_0)\hat{\pi}_{\mathbf{k}_2}(t_0)\hat{\pi}_{\mathbf{k}_3}(t_0)\rangle = \int \mathcal{D}\pi_{\mathbf{k}}(t_0)\ \pi_{\mathbf{k}_1}(t_0)\pi_{\mathbf{k}_2}(t_0)\pi_{\mathbf{k}_3}(t_0)\ e^{iF[\pi_{\mathbf{k}}(t_0)]}$$

The more convenient basis is:

$$\pi_r = \frac{1}{2}(\pi^+ + \pi^-)\ ,\ \pi_a = (\pi^+ - \pi^-)$$

It satisfies three conditions from UV unitarity:

$$\mathcal{I}[\pi_r, \pi_a = 0] = 0\ ,\ (\mathcal{I}[\pi_r, -\pi_a])^* = \mathcal{I}[\pi_r, \pi_a]\ ,\ \mathrm{Im}\,(\mathcal{I}[\pi_r, \pi_a]) \geq 0$$

We also impose scale invariance and non-linearly realised symmetries.

Quadratic theory

The conditions on $\mathcal{I}$ leave the quadratic order to be:

$$\mathcal{I}^{(2)} = \dot{\pi}_r \dot{\pi}_a - \frac{c_s^2 \left(\partial_i \pi_r \partial^i \pi_a\right)}{a^2(t)} - \underbrace{2\gamma_1 \dot{\pi}_r \pi_a}_{\text{dissipation}} + i \underbrace{\left[\beta_1 \pi_a^2 - \beta_2 \dot{\pi}_a^2 + \frac{\beta_3 \left(\partial_i \pi_a\right)^2}{a^2(t)}\right]}_{\text{diffusion}}$$

We can write a Langevin equation from this effective functional:

$$\ddot{\pi} + (3H + \gamma_1)\dot{\pi} - c_s^2 \partial_i^2 \pi = \xi + \dots \,, \ \pi = \pi_H(t) + \int G_R(t, t')\xi(t')$$

where the stocastic noise ξ is gaussian:

$$\langle \xi_{\mathbf{k}}(t_1) \xi_{\mathbf{k}'}(t_2) \rangle = \beta_1 (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \delta(t_1 - t_2)$$

π_H decays and the sourced term generates the power spectrum.

Power spectrum

The sourced power spectrum is given by the integral:

$$P_k(t) = \beta_1 \int dt' \left(G_R(t,t')\right)^2$$

We recover a scale invariant power spectrum:

$$\Delta_\zeta^2(k) = \frac{2\beta_1}{H^2}\frac{H^4}{f_\pi^4}\frac{2^{\frac{\gamma}{H}}\Gamma\left(\frac{\gamma}{2H}+\frac{1}{2}\right)\Gamma\left(\frac{\gamma}{2H}+\frac{3}{2}\right)^2}{\Gamma\left(\frac{\gamma}{2H}+1\right)\Gamma\left(\frac{\gamma}{H}+\frac{5}{2}\right)}.$$

We recover the results from warm inflation taking $\gamma \gg H$ and $\beta_1 \propto \gamma T$:

$$\Delta_\zeta^2 \propto \frac{T}{H}\frac{H^4}{f_\pi^4}\sqrt{\frac{\gamma}{H}}$$

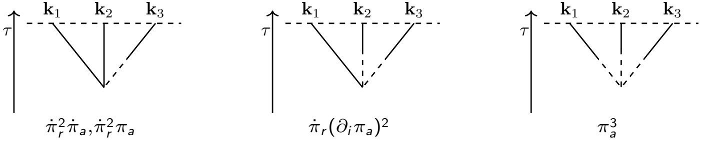
To study the bispectrum signal we go to next order in the expansion of $\mathcal{I}[\pi_r, \pi_a]$.

Feynman rules

The vertices are constrained by the IR symmetries:

$$\mathcal{I}^{(3)} \supset \dot{\pi}_r^2 \dot{\pi}_a \ , \ \dot{\pi}_r^2 \pi_a \ , \ \dot{\pi}_r (\partial_i \pi_a)^2 \ , \ \pi_a^3 ...$$

There are three types of diagrams:

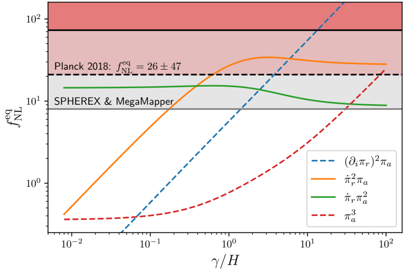


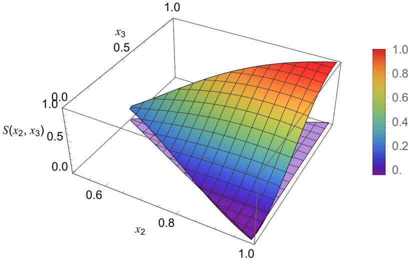
There are two kind of lines:

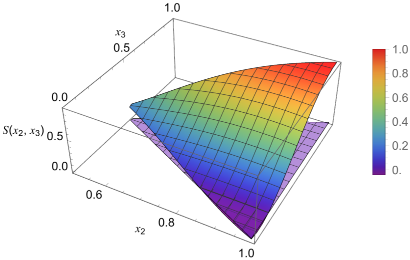
- **Dashed**: Propagation of the system fluctuations sourced by noise: $G^R_{\mathbf{k}}(t_0, t) = \langle \hat{\pi}_r(t_0) \hat{\pi}_q(t) \rangle$.
- **Dressed**: Propagation of the system fluctuations dressed by the noise: $G^K_{\mathbf{k}}(t_0, t) = \langle \{ \hat{\pi}_r(t_0), \hat{\pi}_r(t) \} \rangle$.

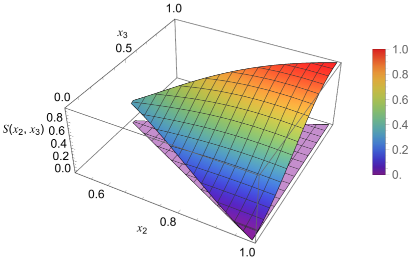
Bispectrum I

Equilateral shapes are dominant at large dissipation



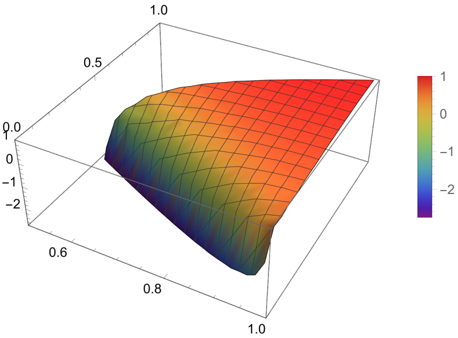






Bispectrum II

Folded singularities are enhanced at small dissipation:



This can be seen from subhorizon production:

$$\text{subhorizon} \sim \text{flat space}$$

This imposes that perturbations are generated in the folded configuration:

$$\underbrace{\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0}_{\text{Momentum}}\,, \quad \underbrace{k_{\text{max}} = \sum_{b \neq \text{max}} k_b .}_{\text{Energy}}$$

Large dissipation would dampen them, but at $\gamma \ll H$ we can expect them to reach the boundary.

Conclusion & Outlook

The Open EFT approach allows us to study the very early universe when a local closed EFT is not enough. This is a bottom-up approach.

- We have built a quadratic effective influence functional for inflation that reproduces a scale invariant power spectrum and warm inflation results.
- We have built a set of Feynman rules that can reproduce the results found in the literature.

On the phenomenological side of things:

- $\gamma \ll H$: folded shapes become important due to subhorizon production.
- $\gamma \gg H$: equilateral shapes are dominant.

In the future:

- Observational and theoretical constrains of the Wilson coefficients.
- Extension to tensor modes.
- Dissipative Inflation Bootstrap?