Adiabatic renormalization without infrared distortions in cosmological spacetimes

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Based on:
Ferreiro & FT, *PLB* 840 (2023) [**2212.01078**]
Ferreiro, Monin & FT, *PRD* 109 (2024) [**2311.08986**]

Particle Production in the Early Universe **- CERN - 10th September 2024**

Scalar field in curved spacetime

- Scalar field + FLRW background + non-minimal coupling to curvature: $[ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2)]$

$$\boxed{\phi'' + 2\frac{a'}{a}\phi' - \nabla^2\phi + m^2\phi + \xi R(\tau)\phi = 0} \qquad R(\tau) = 6\frac{a''}{a^3}$$

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Coincident spacetime points: $(x = x')$

$$\langle\phi^2\rangle = \int d\log k\,\Delta_{\phi^2}(k)\,; \qquad \Delta_{\phi^2}(k) \equiv \frac{k^3|\chi_k|^2}{2\pi^2a^2}$$

Power spectrum

Example: de Sitter spacetime

EXAMPLE:	de Sitter $a(\tau) = -(H\tau)^{-1}$	+	light field $m \ll H$	+	Bunch- Davies vacuum $\chi_k \propto \sqrt{\tau} H^{(1)}_{\nu[m]}(-k\tau)$

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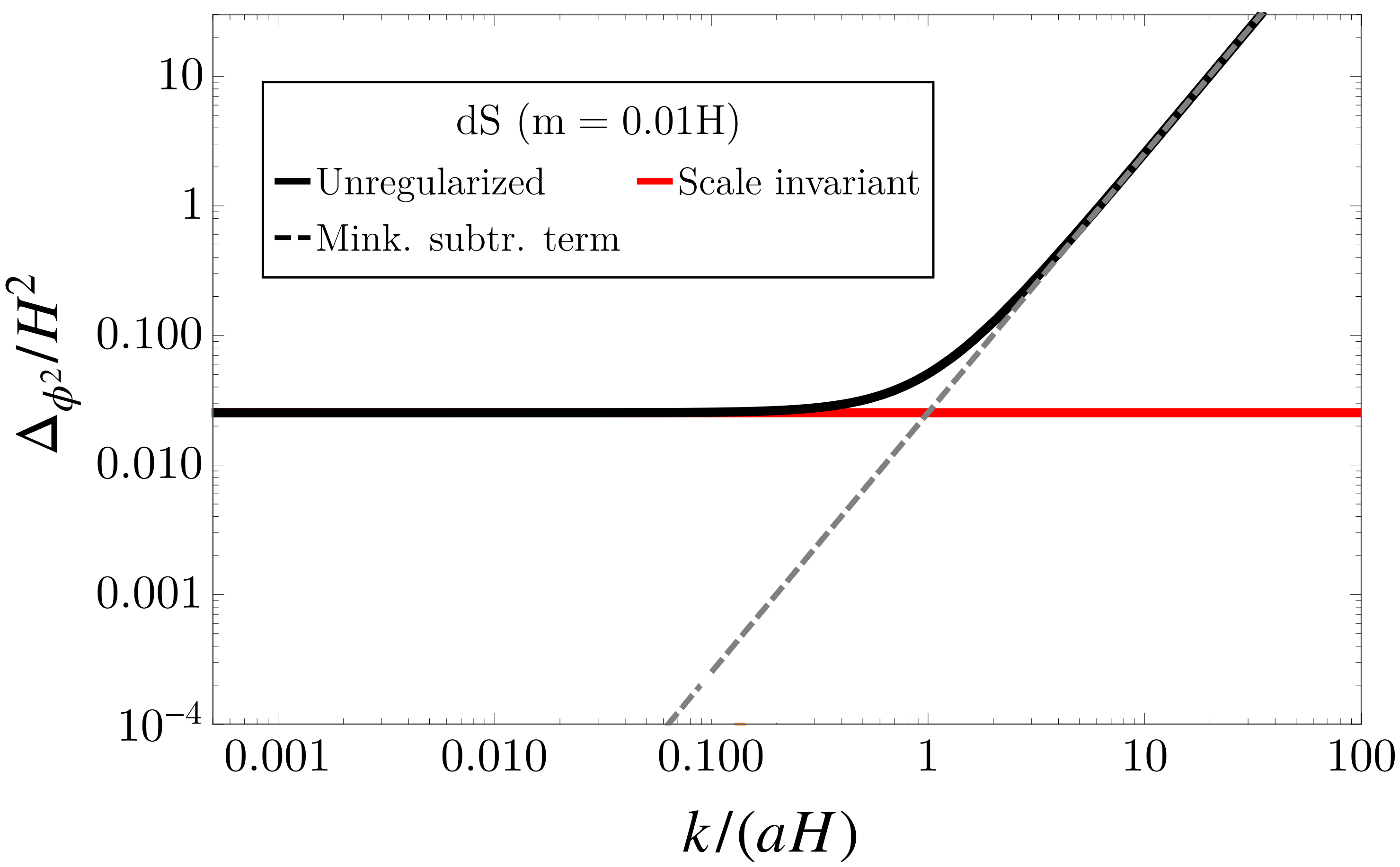

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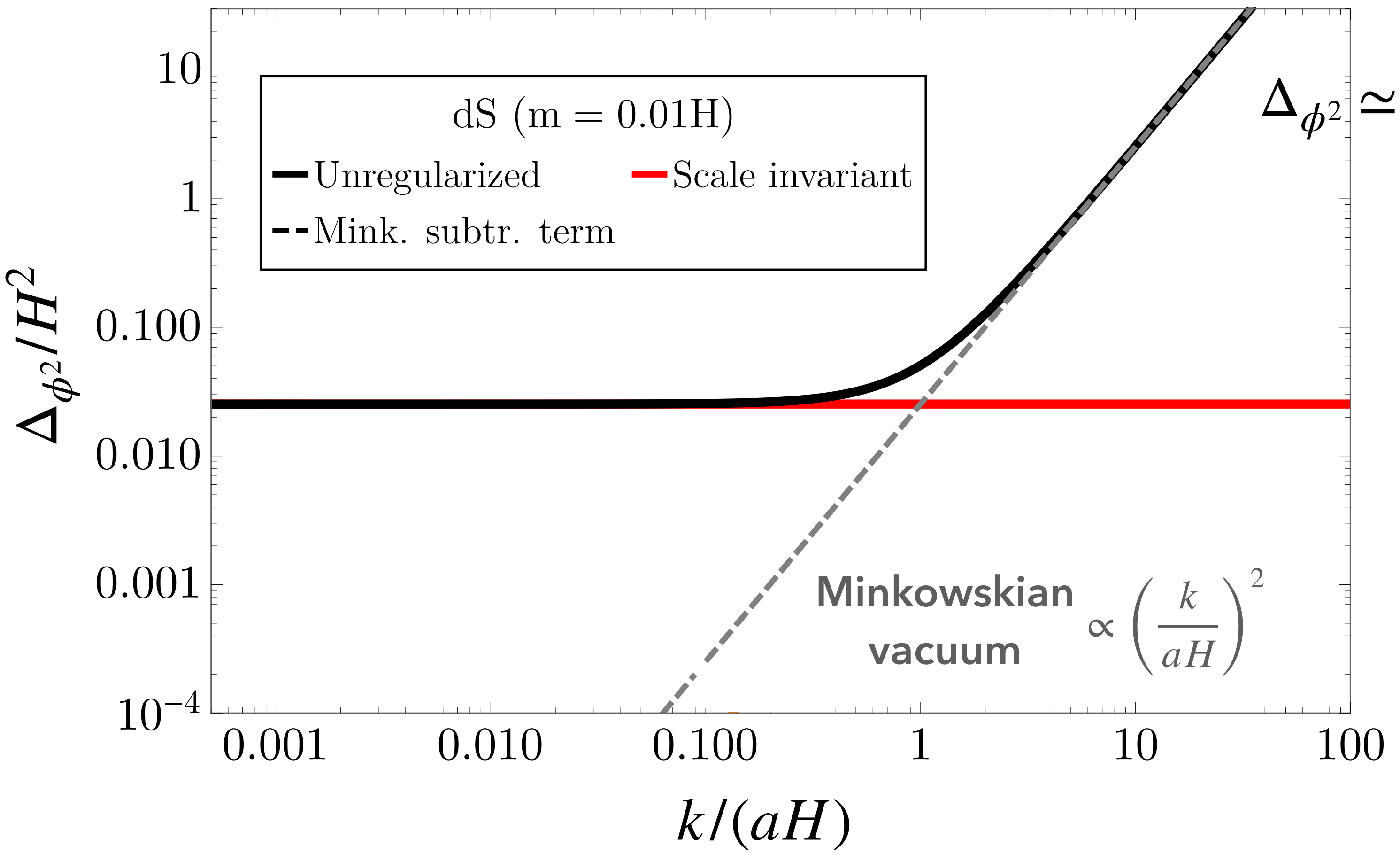

$$\Delta_{\phi^2} \simeq \frac{H^2}{4\pi^2}\left[1 + \left(\frac{k}{aH}\right)^2\right]$$

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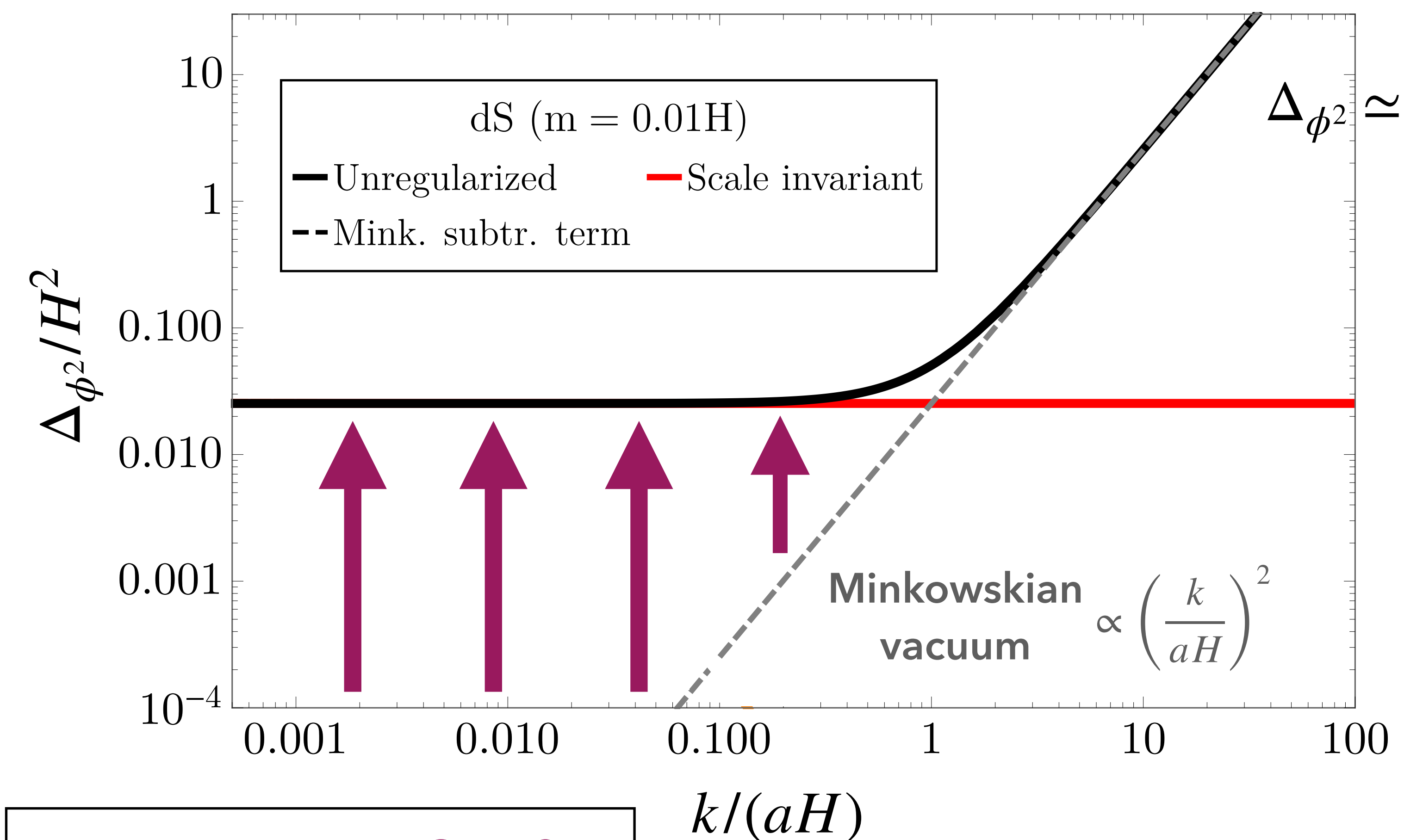

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AMPLIFIED MODES
at super-horizon scales
$k \lesssim aH$

Expectation values

$[h_k \equiv \chi_k / a]$

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- **STRESS-ENERGY TENSOR:** $\langle T_{\mu\nu} \rangle = -g_{\mu\nu} \langle p \rangle + (\langle p \rangle + \langle \rho \rangle) u_\mu u_\nu$

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$\langle \phi^2 \rangle$ contains **quadratic** and **logarithmic** UV divergences

$\langle T_{\mu\nu} \rangle$ contains **quartic**, **quadratic** and **logarithmic** UV divergences

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subtraction terms

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- Equivalent to deWitt-Schwinger expansion in general curved backgrounds.

[e.g. del Rio & Navarro-Salas (2014)]

Infrared distortions

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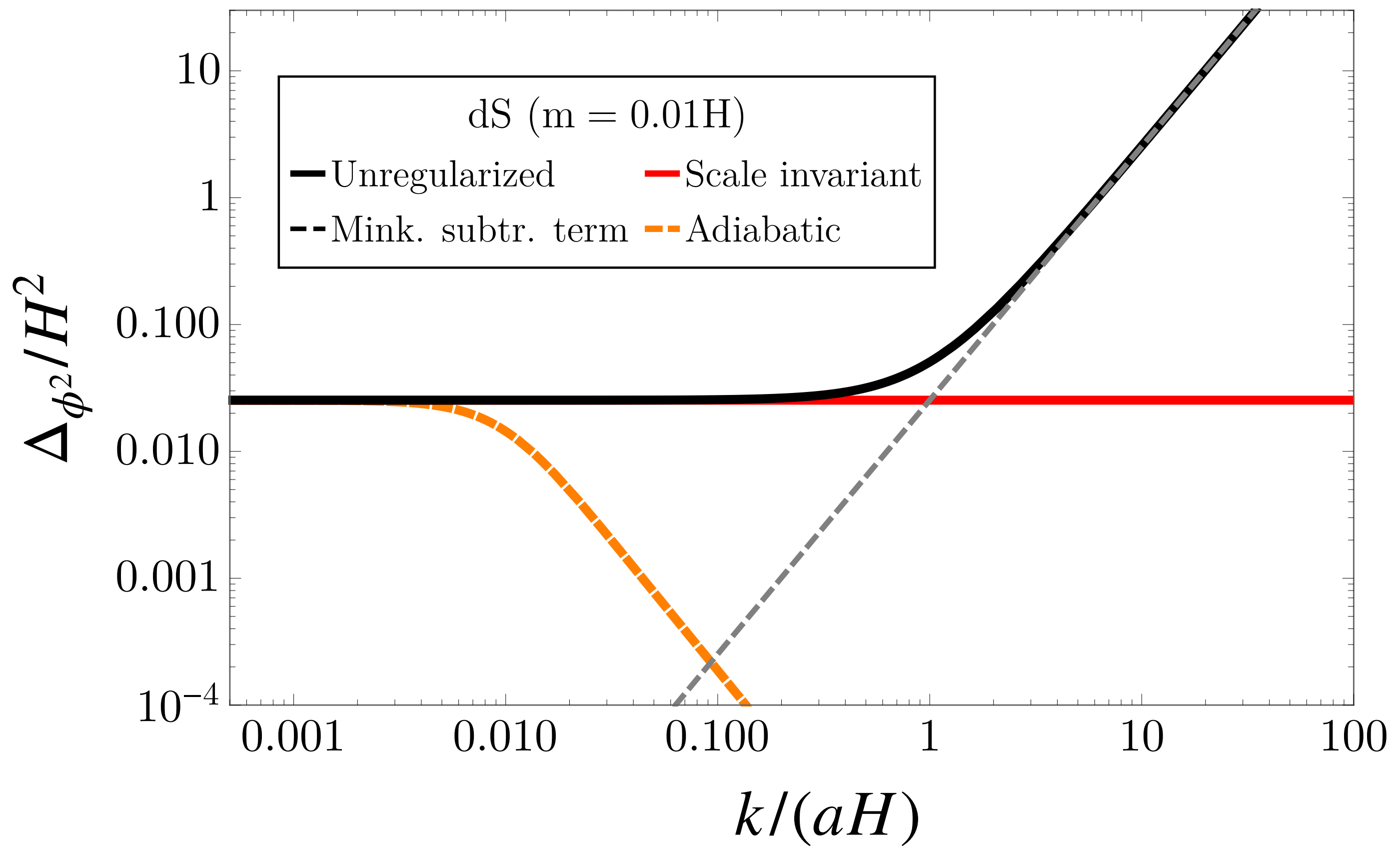

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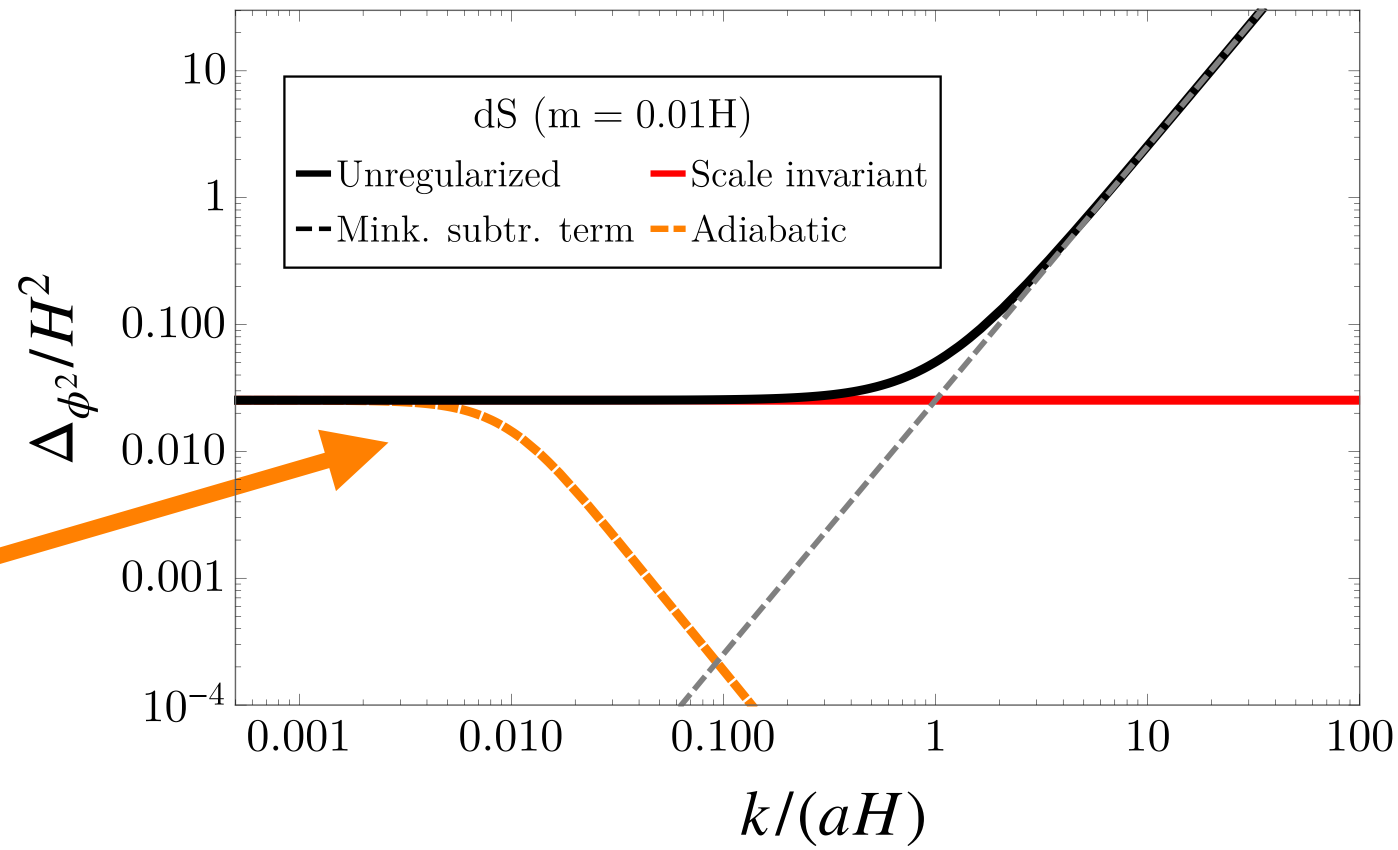

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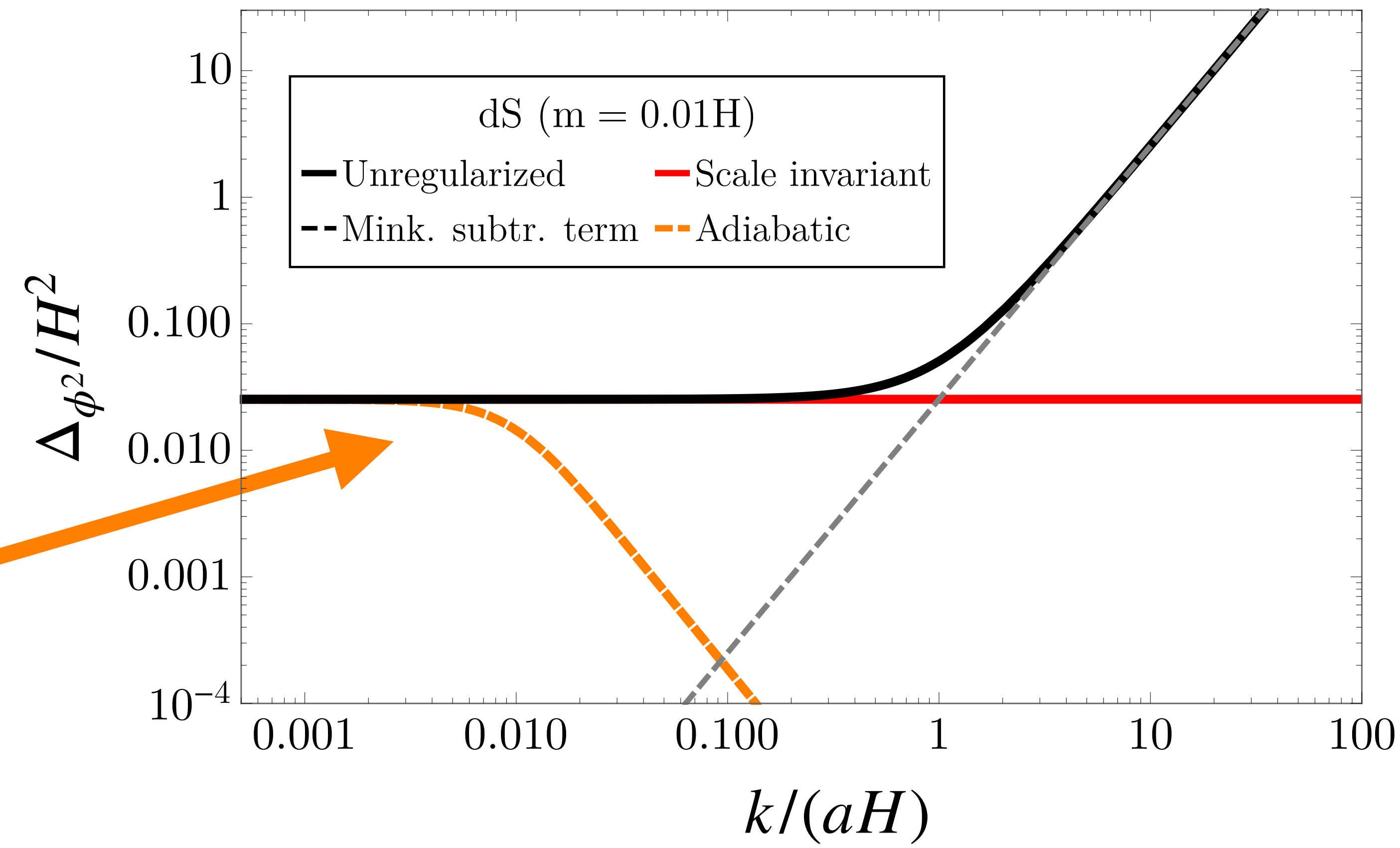

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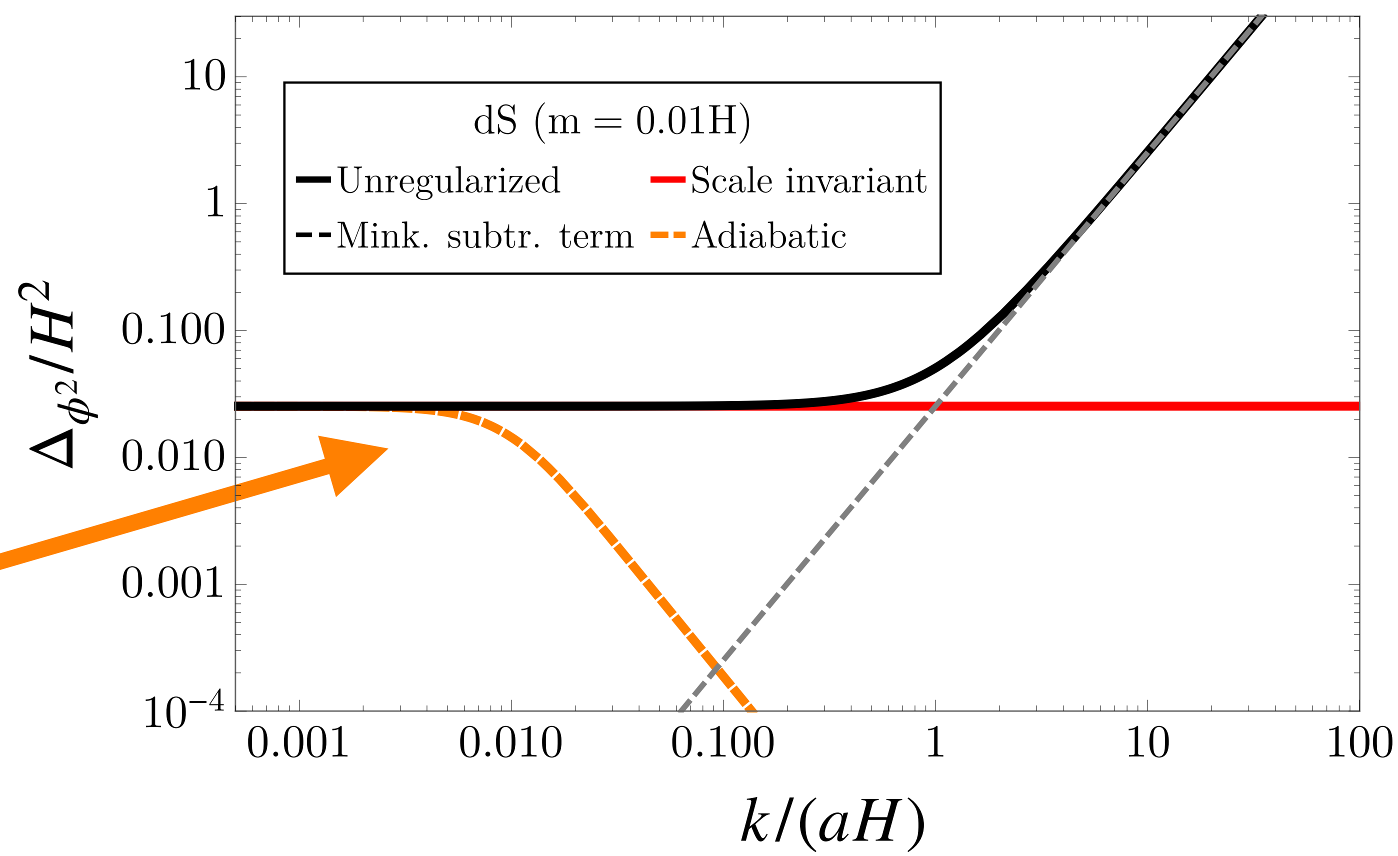

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- Implications?

Agullo, Navarro-Salas, Olmo, Parker (2008,09).
Finelly, Marozzi, Vacca, Venturi (2007)
Durrer, Marozzi, Rinaldi (2009)
Urakawa, Starobinsky (2009)
Bastero-Gil, Berera, Mahajan, Rangarajan (2013)
Markkanen (2015)
Animali, Conzinu, Marozzi (2022)
Corba, Sorbo (2022)
Pla, Stefanek (2024)

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- Two different regularization methods compatible with **locality** and **covariance** can differ by a **finite amount of geometrical terms** *[Wald 1995]*:

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- We develop a new regularization method that solves both problems:

Physical scale adiabatic regularization (PSAR)

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Physical scale adiabatic regularization

PSAR method for the **two-point function**

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M : arbitrary mass scale

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➤ Higher orders are obtained by solving the mode equation iteratively:

$$\chi_k'' + \left(\widetilde{\omega}^2 \underbrace{- M^2 a^2 + m^2 a^2 + \left(\xi - \frac{1}{6}\right) a^2 R}_{\text{2nd order}} \right) \chi_k = 0$$

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2nd order: $-M^2a^2 + m^2a^2 + \left(\xi - \frac{1}{6}\right)a^2R$

- Subtraction terms are obtained by expanding up to **second order**:

$$\mathcal{S}_{\phi^2} = \frac{1}{2a^2\widetilde{\omega}} + \frac{(M^2 - m^2)}{4\widetilde{\omega}^3} - \frac{(\xi - \frac{1}{6})R}{4\widetilde{\omega}^3} - \frac{3}{16}\frac{\widetilde{\omega}'^2}{a^2\widetilde{\omega}^5} + \frac{\widetilde{\omega}''}{8a^2\widetilde{\omega}^4}$$

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$$\mathcal{S}_{\phi^2} \equiv \frac{1}{2a^2\sqrt{k^2 + m^2a^2}} - \frac{(\xi - \frac{1}{6})R}{4(k^2 + M^2a^2)^{3/2}}$$

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We set $M = m$ so that $\langle : \phi^2 : \rangle = 0$ in Minkowski spacetime

finite contribution $\propto R$

$$\mathcal{S}_{\phi^2} \equiv \frac{1}{2a^2\sqrt{k^2 + m^2 a^2}} - \frac{(\xi - \frac{1}{6})R}{4(k^2 + M^2 a^2)^{3/2}}$$

M : arbitrary mass scale

- The difference between prescriptions can be written as a **sum of geometric terms:**

$$\langle : \overline{\phi^2(x)} : \rangle - \langle : \phi^2(x) : \rangle = \frac{\xi - \frac{1}{6}}{16\pi^2} \log\left(\frac{m^2}{M^2}\right) R + \frac{R}{288\pi^2}$$

PSAR method — *Adiabatic method*

Example: de Sitter spacetime

Regularized power spectrum with the PSAR method

$$:\Delta_{\phi^2}: = \frac{H^2 x^3}{8\pi}\left(|H_\nu^{(1)}(x)|^2 - \frac{2\pi^{-1}}{\sqrt{m_H^2 + x^2}} - \frac{2\pi^{-1}}{(M_H^2 + x^2)^{\frac{3}{2}}} \right)$$

$$x \equiv k/(aH)$$

$$M_H \equiv \frac{M}{H} \qquad m_H \equiv \frac{m}{H}$$

$$\nu \equiv \sqrt{\frac{9}{4} - m_H^2}$$

Example: de Sitter spacetime

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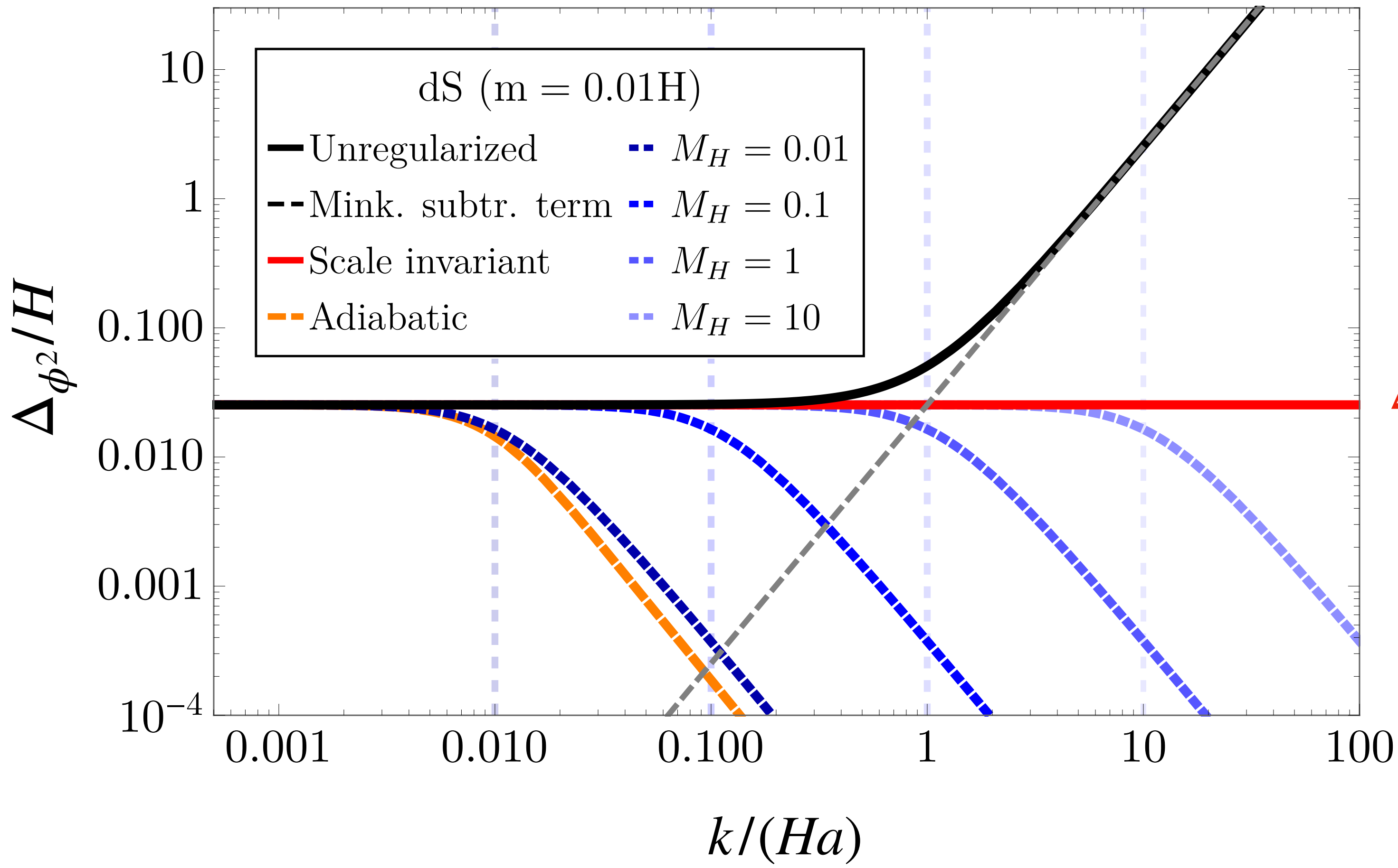

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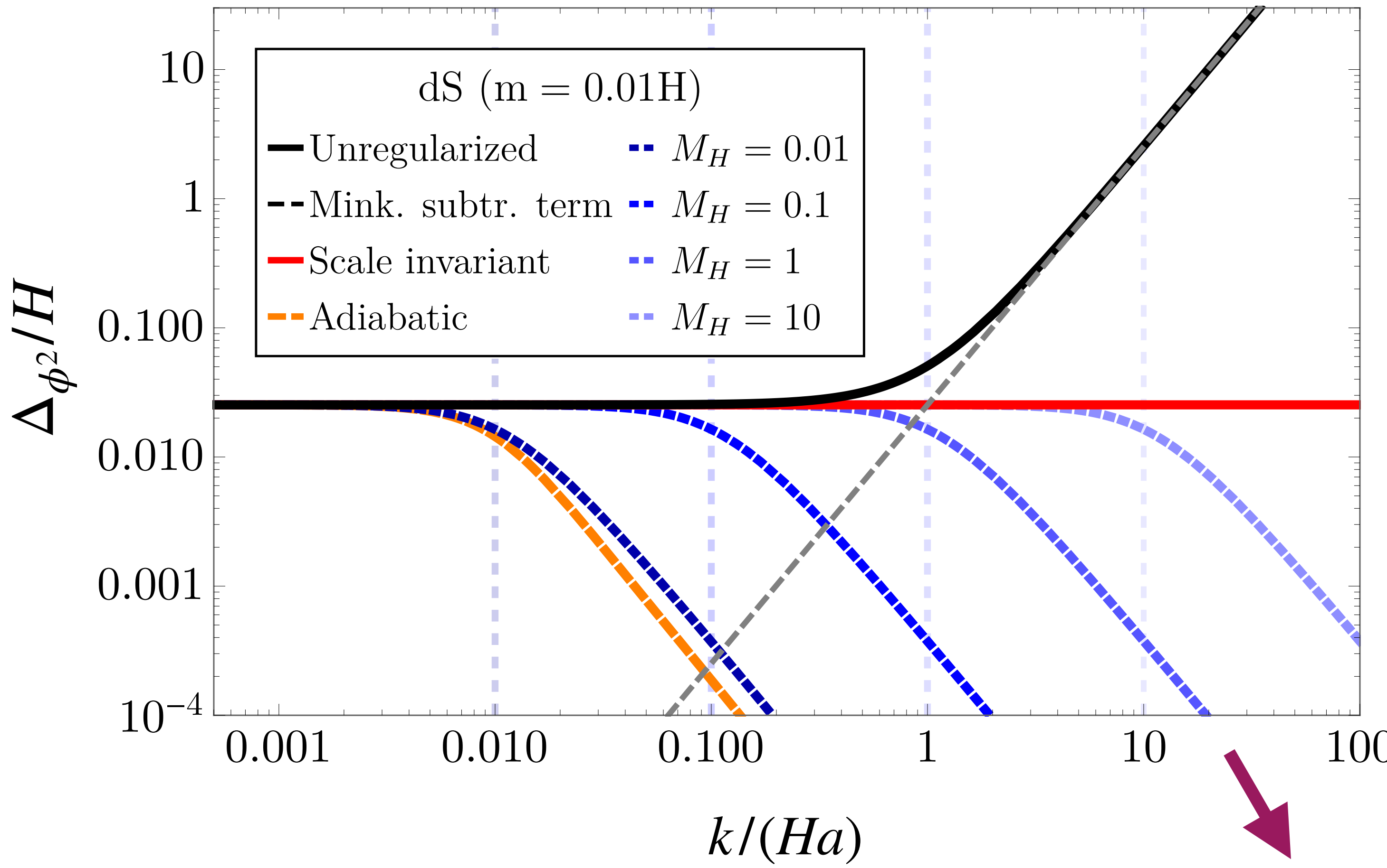

$$\Delta_{\phi^2}^{(0s)} \simeq \frac{H^2}{4\pi^2}$$

setting $M = H$ removes the UV tail while preserving the infrared amplified part of the spectrum

PSAR method for the stress-energy tensor

PSAR method for the **regularized stress-energy tensor**

We expand up to **fourth adiabatic order** and modify the terms so that:

1. $\langle :T_{ab}: \rangle = 0$ in Minkowski spacetime
2. $\nabla^a \langle :T_{ab}: \rangle = 0$ (conserved tensor)
3. No distortions for $k \lesssim k_+$, with k_+ the maximum amplified momentum.

Ferreiro, Monin & F.T, PRD (2024) [2311.08986]

PSAR method for the stress-energy tensor

Subtraction terms depend on two arbitrary mass scales: M_2 and M_4

PSAR method for the stress-energy tensor

Subtraction terms depend on two arbitrary mass scales: M_2 and M_4

- **Energy density:**

$$\mathcal{S}_\rho = \frac{\omega}{2a^4} + \left(\xi - \frac{1}{6}\right)\left(\frac{9M_2^2 m^2 a'^2}{4a^2\omega_2^5} - \frac{3m^2 a'^2}{4a^4\omega_2^3} - \frac{9M_2^4 a'^2}{4a^2\omega_2^5} - \frac{3M_2^2 a'^2}{4a^4\omega_2^3} - \frac{3a'^2}{2a^6\omega_2}\right)$$
$$+\left(\xi - \frac{1}{6}\right)^2\left(\frac{27M_4^2 a'^2 a''}{2a^5\omega_4^5} + \frac{9a'^2 a''}{a^7\omega_4^3} - \frac{9a^{(3)}a'}{2a^6\omega_4^3} + \frac{9a''^2}{4a^6\omega_4^3}\right)$$

$$\omega \equiv \sqrt{k^2 + m^2a^2}$$
$$\omega_2 \equiv \sqrt{k^2 + {M_2}^2a^2}$$
$$\omega_4 \equiv \sqrt{k^2 + {M_4}^2a^2}$$

PSAR method for the stress-energy tensor

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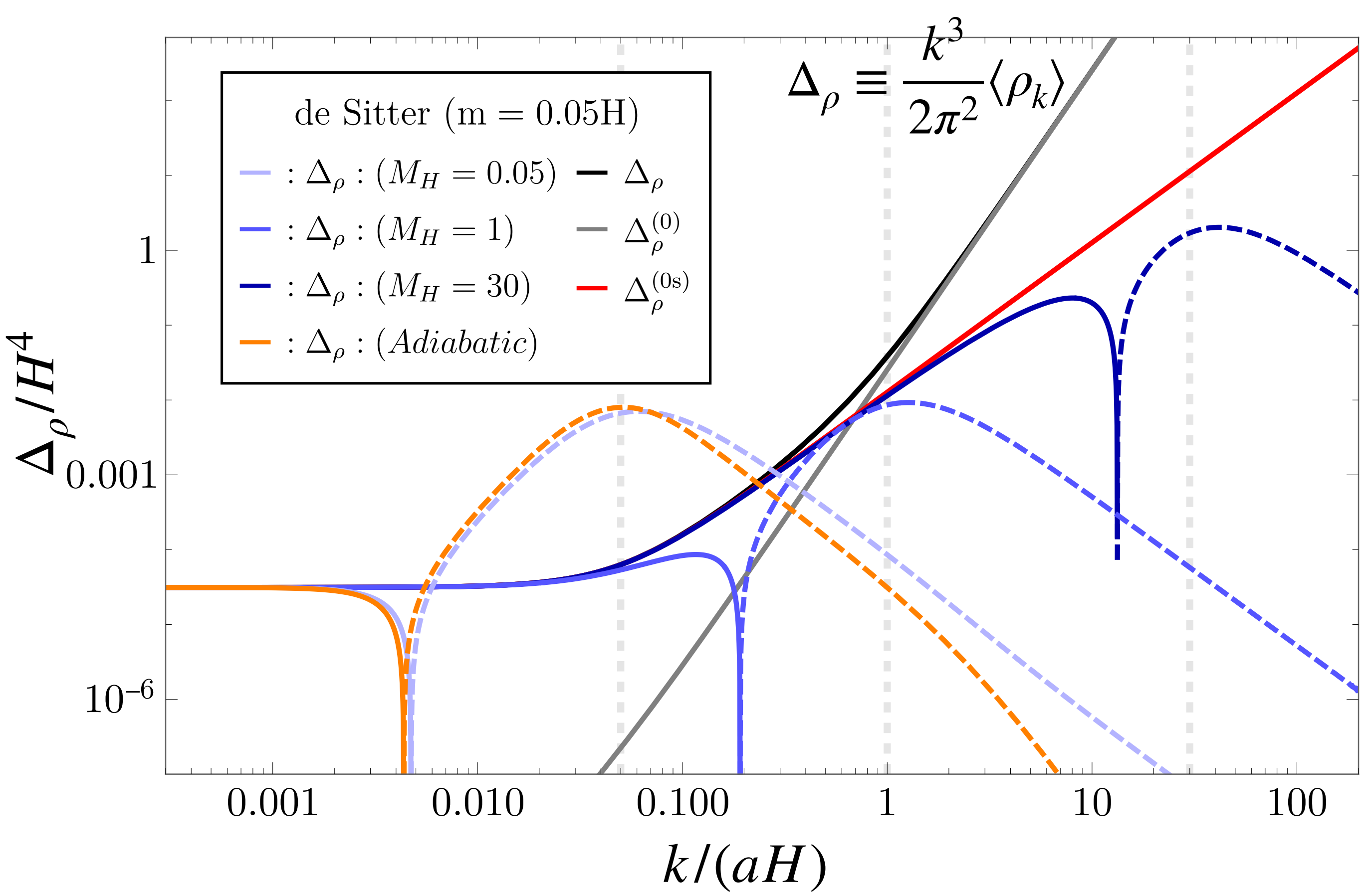
$$\omega \equiv \sqrt{k^2 + m^2 a^2}$$
$$\omega_2 \equiv \sqrt{k^2 + M_2{}^2 a^2}$$
$$\omega_4 \equiv \sqrt{k^2 + M_4{}^2 a^2}$$

- **Pressure density:**

$$\mathcal{S}_p = -\frac{m^2}{6a^2\omega} + \frac{\omega}{6a^4} + \left(\xi - \frac{1}{6}\right)\left(\frac{3M_2^4 a''}{2a\omega_2^5} + \frac{M_2^2 a''}{2a^3\omega_2{}^3} + \frac{a''}{a^5\omega_2} - \frac{15M_2^6 a'^2}{4\omega_2^7} - \frac{3M_2^2 a'^2}{4a^4\omega_2^3} - \frac{3a'^2}{2a^6\omega_2}\right.$$
$$\left. - \frac{3m^2 M_2^2 a''}{2a\omega_2^5} + \frac{m^2 a''}{2a^3\omega_2^3} + \frac{15m^2 M_2^4 a'^2}{4\omega_2^7} - \frac{3m^2 M_2^2 a'^2}{2a^2\omega_2^5} - \frac{m^2 a'^2}{4a^4\omega_2^3}\right)$$
$$+\left(\xi - \frac{1}{6}\right)^2\left(\frac{3a^{(4)}}{2a^5\omega_4^3} - \frac{27M_4^2 a''^2}{4a^4\omega_4^5} - \frac{15a''^2}{4a^6\omega_4^3} - \frac{9M_4^2 a^{(3)}a'}{a^4\omega_4^5} - \frac{15a^{(3)}a'}{2a^6\omega_4^3} + \frac{45M_4^4 a'^2 a''}{2a^3\omega_4^7} + \frac{18M_4^2 a'^2 a''}{a^5\omega_4^5} + \frac{12a'^2 a''}{a^7\omega_4^3}\right)$$

Example: de Sitter spacetime

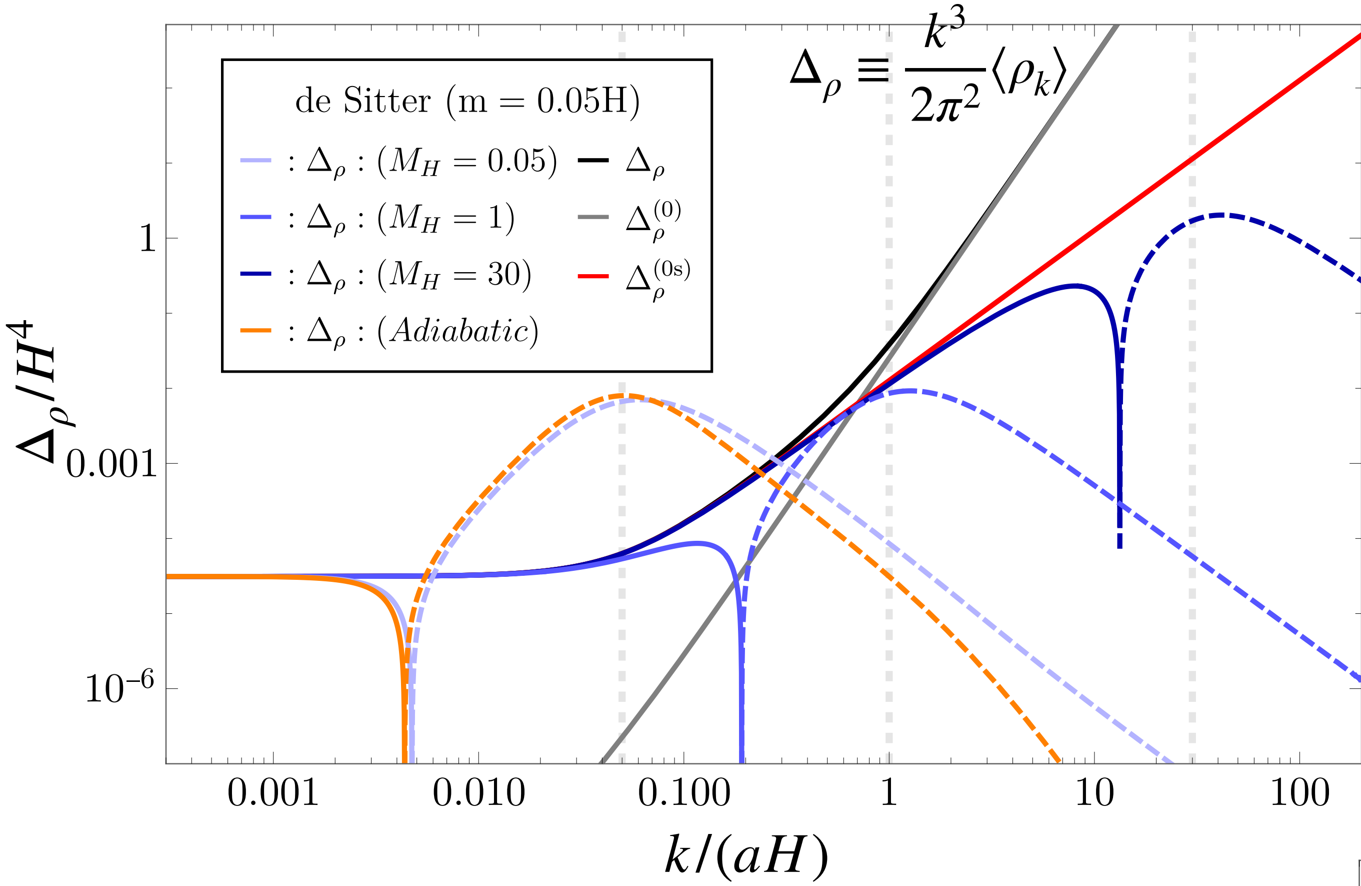


$$\Delta_\rho^{(0s)} \equiv \frac{k^3}{2\pi^2}\left(\langle \rho_k \rangle - \mathcal{S}_\rho^{(0)}\right)$$

ENERGY DENSITY

$$: \Delta_\rho :\equiv \frac{k^3}{2\pi^2}\left(\langle \rho_k \rangle - \mathcal{S}_\rho\right)$$

Example: de Sitter spacetime



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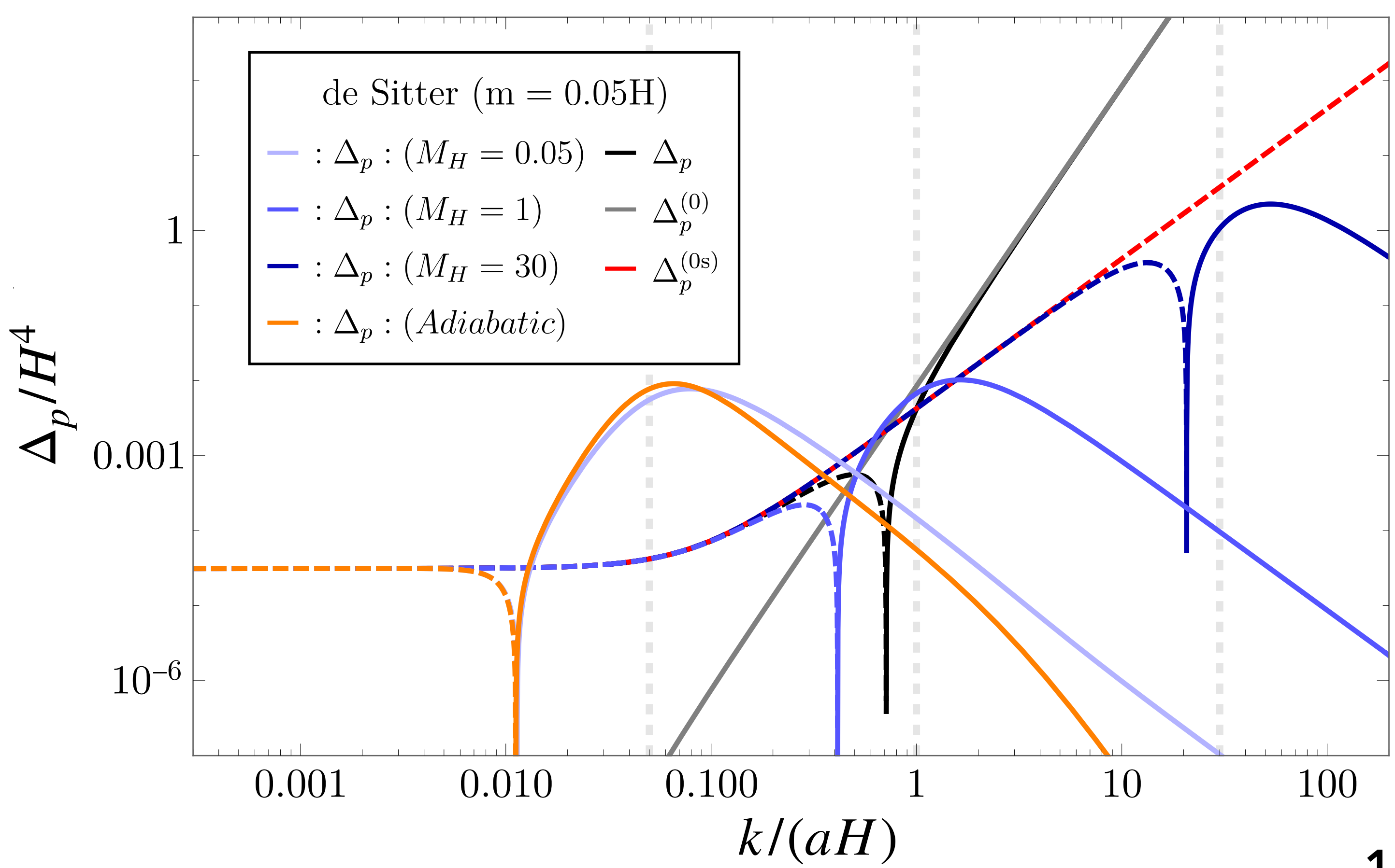


Conclusions

➤ **Other relevant work (see 2311.08986 for details!):**

- **Numerical implementation** of our regularization method and application to **geometric reheating.**
- Interpretation of our proposed subtraction terms as **renormalization of coupling constants** in the Einstein equations.

Conclusions

- **Other relevant work (see 2311.08986 for details!):**
  - **Numerical implementation** of our regularization method and application to **geometric reheating.**
  - Interpretation of our proposed subtraction terms as **renormalization of coupling constants** in the Einstein equations.

- **Possible extensions:**
  - PSAR method for **fermions?** (see *Landete, Navarro-Salas & F.T. PRD (2014)*)
  - Include interactions to **homogeneous time-dependent fields**? (e.g. preheating, Schwinger effect, etc).

Thank you!

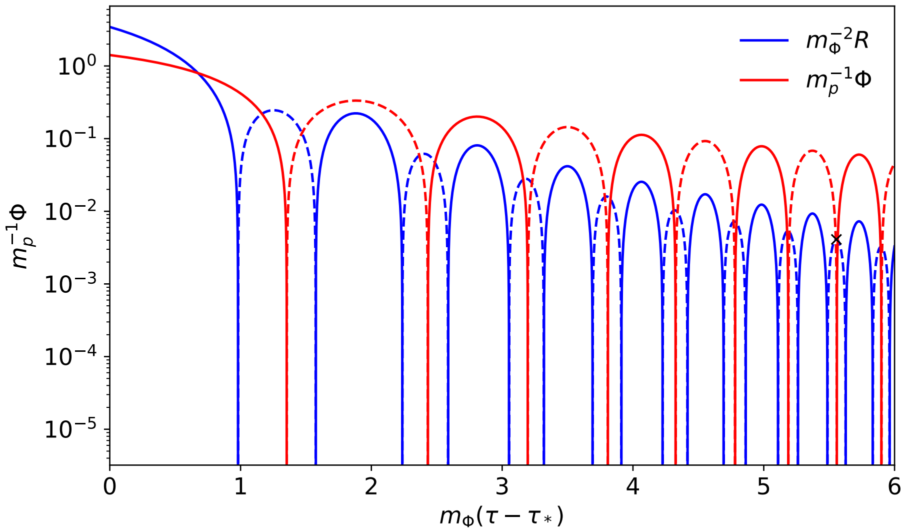
Backup slides

Example: geometric reheating

➤ The Ricci scalar oscillates after inflation between positive and negative values:

$$V(\Phi) = \frac{1}{2} m_\Phi^2 \Phi^2$$

Inflaton potential



Example: geometric reheating