Technische Universität München
T31 SFB 1258 Projekt D04
„Neutrinos and Dark Matter in Astro- and Particle Physics“

Excited bound states and their role in Dark Matter production

Tobias Binder

10th September 2024

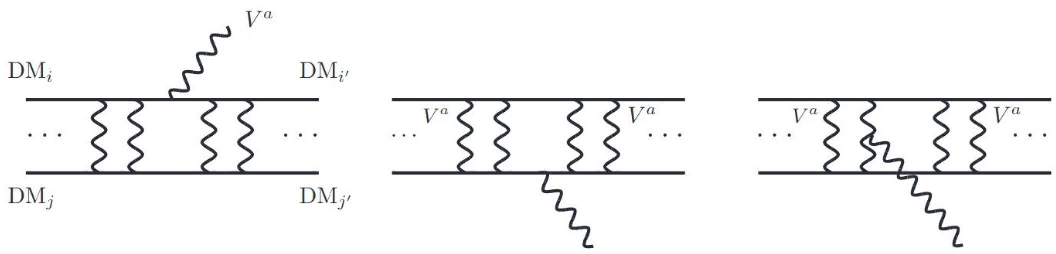
Based on:
arXiv:2308.01336

In collaboration with:
Mathias Garny, Jan Heisig, Stefan Lederer and Kai Urban

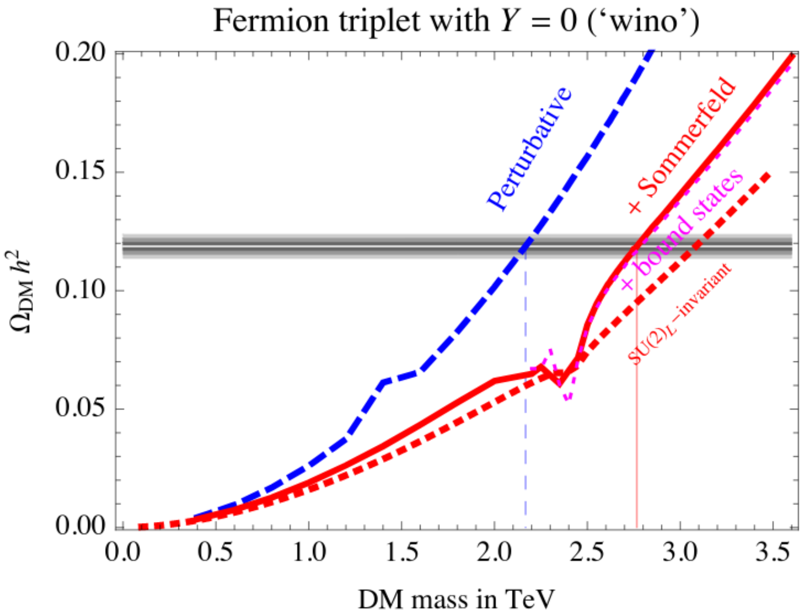
Particle production in the early Universe, CERN

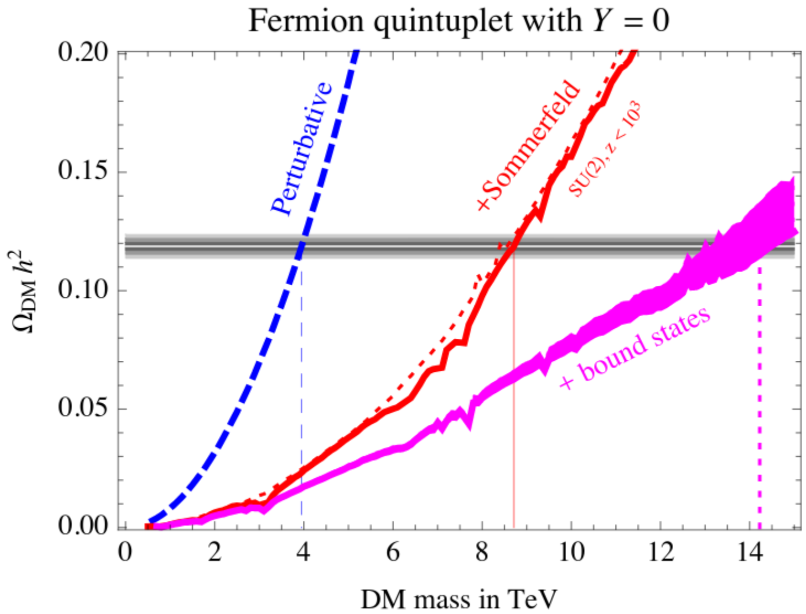
Minimal Dark Matter examples

[Mitridate *et al.* 17]



(See also
[Hisano et al. 03,05,06,
Harz & Petraki 18])





Classification of bound-state formation

Leading multipole: $\langle \psi_{nl} | \, r^X \, | \psi_{\mathbf{p}} \rangle$

- **Monopole (X=0)** [Oncala & Petraki 19,21]
- **Dipole (X=1):**
  - Wino, Minimal DM, Colored co-annihilation [Ellis et al. 15, Mitridate et al. 17, Harz & Petraki 18, ...]
  - Dark U(1), SU(N) [Harling et al. 14, ..., Asadi 21, Biondini et al. 23]
- **Quadrupole (X=2)** [Wise et al. 14,16, Petraki et al. 15, Biondini 21,22]

This talk: highly excited bound states in perturbative, unbroken gauge theories (dipole).

Bound-state formation in U(1) gauge theory

$$\mathcal{S}(\chi\bar{\chi}) \to \mathcal{B}(\chi\bar{\chi})_{nl} + \gamma$$

$$(\sigma v)_{nl} = \tfrac{4\alpha}{3}\Delta E^3 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_{\mathbf{p}}\rangle|^2$$

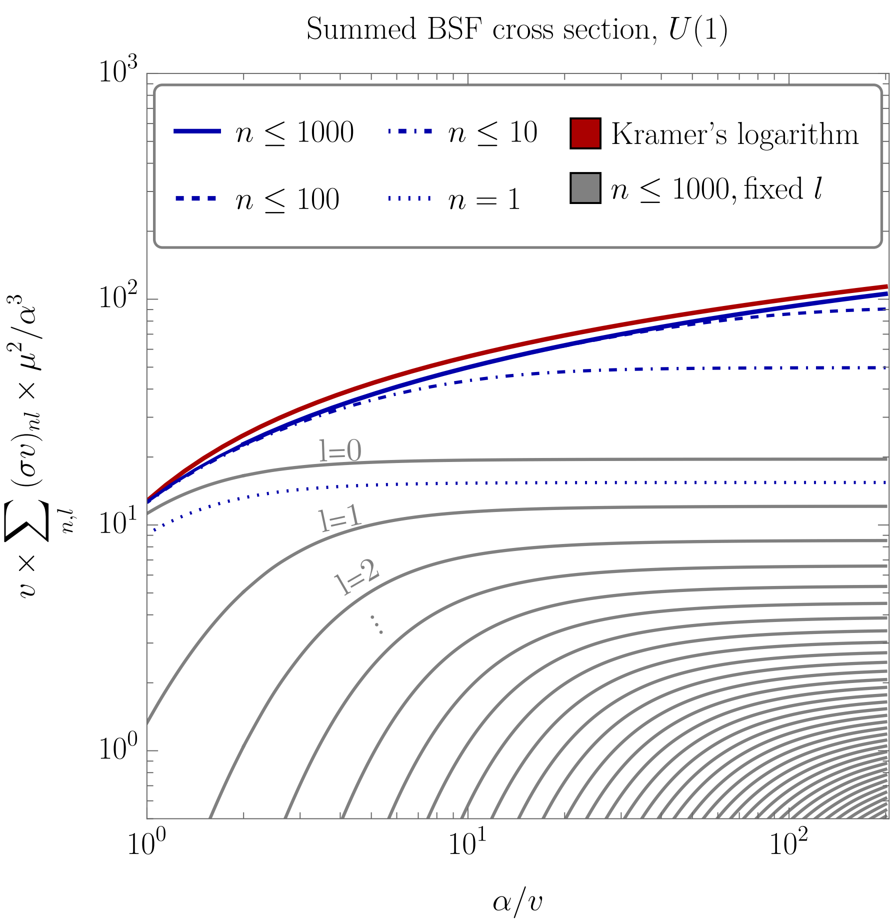
(e.g. hydrogen, (dark) positronium, complex scalars)

- Up to half a million bound states:

  all $\mathrm{n} \leq 1000, l \leq n-1.$

- Confirm Kramer's logarithm within expected error as a check:

$$\sum_{n,\ell}(\sigma v)_{n\ell} \simeq \frac{32\pi}{3\sqrt{3}}\frac{\alpha^2}{\mu^2}\frac{\alpha}{v}\left[\log(\alpha/v)+\gamma_E\right], \text{ for } v \ll \alpha.$$



Bound-state formation in SU(3) gauge theory

$3 \otimes \bar{3} = 1 \oplus 8$

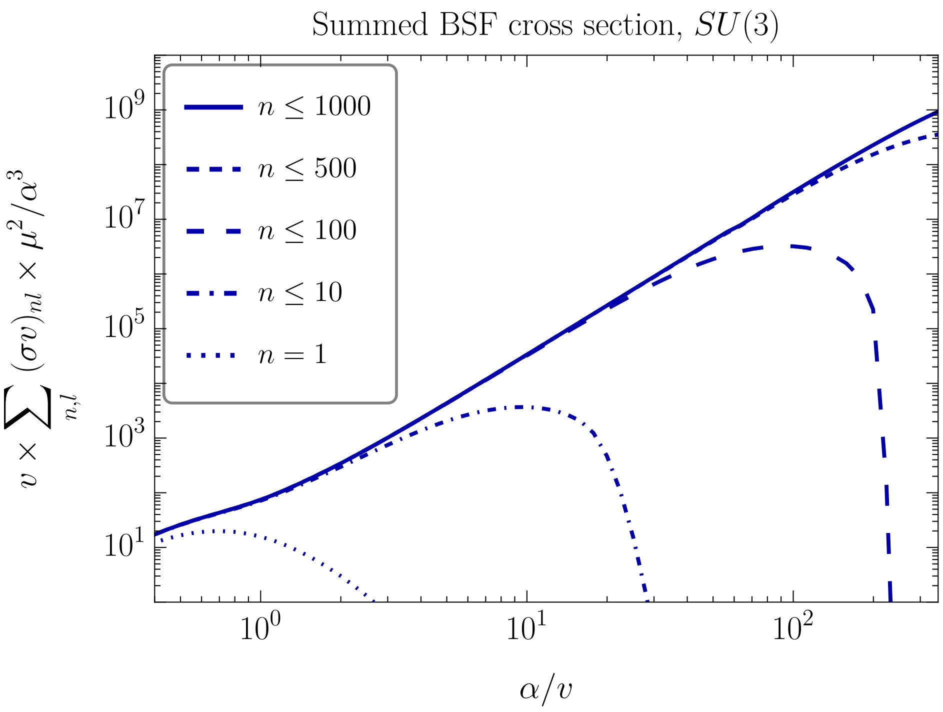
$\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$

$(\sigma v)_{nl} = \frac{C_F}{N_c^2}\frac{4\alpha}{3}\Delta E^3 |\langle \psi^1_{nl}|\, \mathbf{r}\, |\psi^8_{\mathbf{p}}\rangle|^2$

(e.g. quarkonium, squark)

- Assume constant coupling
- Low velocity scaling much stronger:

  $\sum_{nl}(\sigma v)_{nl} \propto v^{-4}$ for $v \ll \alpha$
- Raises concerns about partial wave-unitarity violation



Bound-state formation in SU(3) gauge theory

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$$

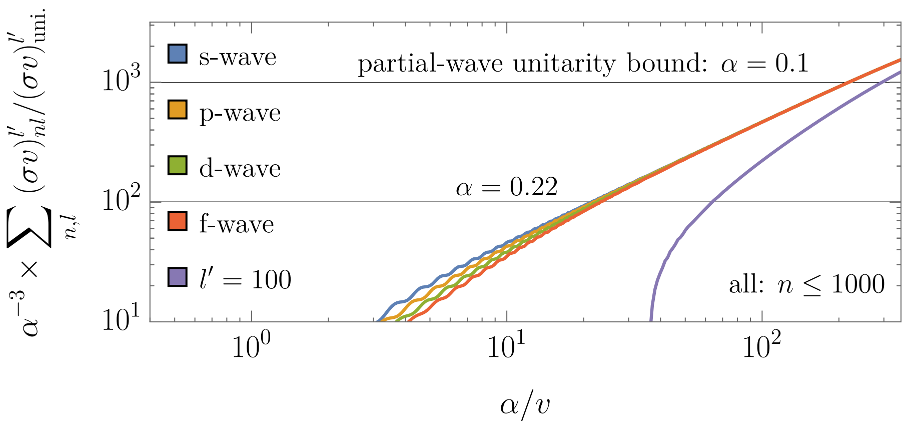
$$(\sigma v)_{nl} = \frac{C_F}{N_c^2}\frac{4\alpha}{3}\Delta E^3 |\langle\psi^1_{nl}|\, \mathbf{r}\, |\psi^8_{\mathbf{p}}\rangle|^2$$

(e.g. quarkonium, squark)

- Unitarity condition:

$$\sum_{nl}(\sigma v)^{l'}_{nl} \leq (\sigma v)^{l'}_{\text{uni.}} = \frac{\pi(2l'+1)}{\mu^2 v}$$

- Observe partial-wave unitarity violation in the perturbative regime

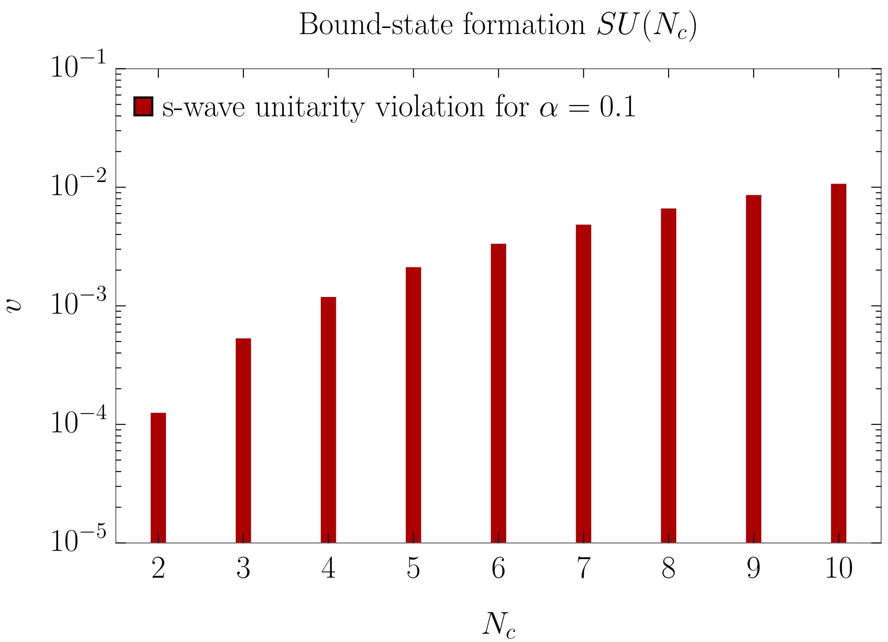


Partial wave unitarity violation in SU(N)

- We observe partial wave unitarity violation in SU(N) gauge theories for perturbatively small couplings

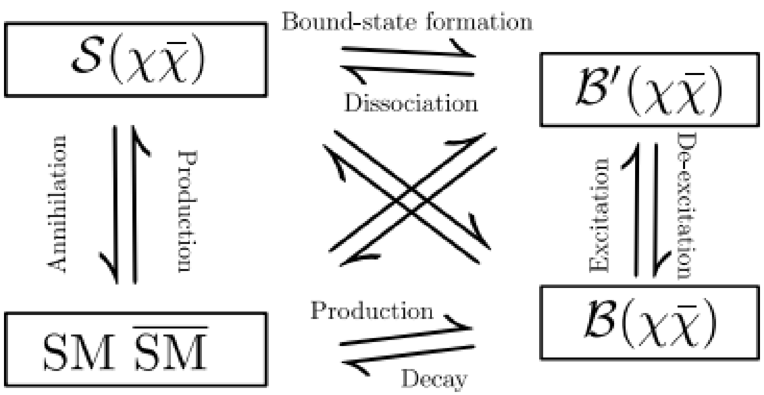
- More generally: if the initial state is less attractive than the final state, partial wave unitarity will be violated at a finite velocity

- Mechanism behind unitarization unknown



In the following, focussing on the regime consistent with perturbativity and unitarity

Effective cross section



Quasi steady-state

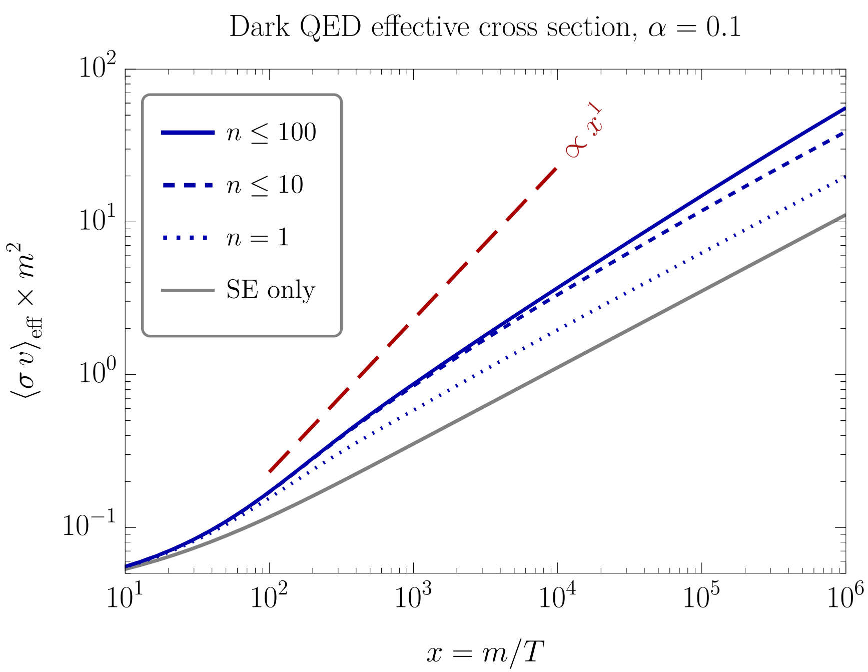
$$\dot{n} + 3Hn = -\langle \sigma v \rangle_{\text{eff}} (n^2 - n_{\text{eq}}^2)$$

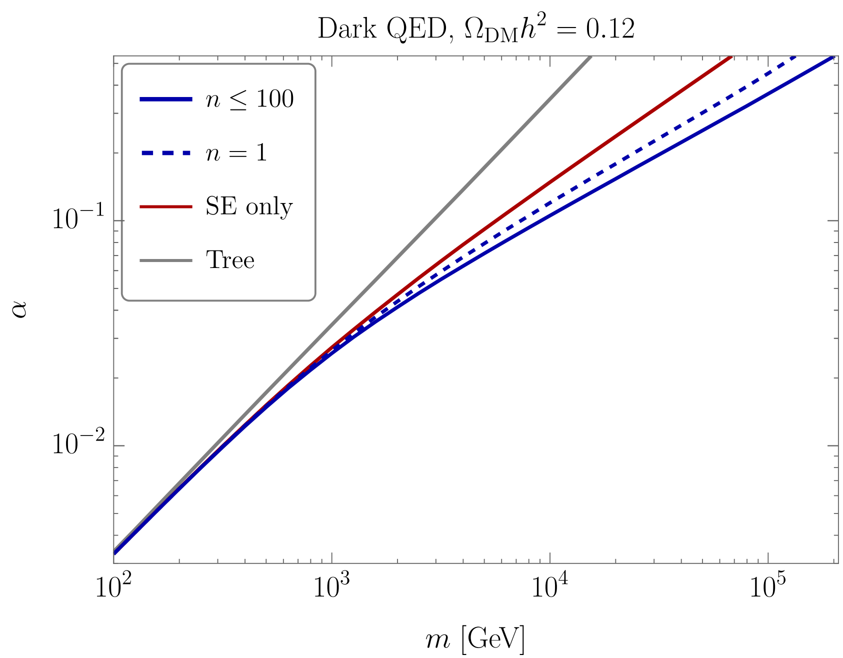
$$\langle \sigma v \rangle_{\text{eff}} = \langle \sigma v \rangle_{\text{ann}} + \sum_{\mathcal{B}} \langle \sigma v \rangle_{\mathcal{B}} R_{\mathcal{B}}$$

[Ellis et al. 15, Mitridate et al. 17,
Binder et al. 21, Garny & Heisig 21]

- Effective cross section encodes complex interplay between annihilation, bound-state formation, excitation, bound-state decay and reverse processes

Effective cross section: Dark QED sector

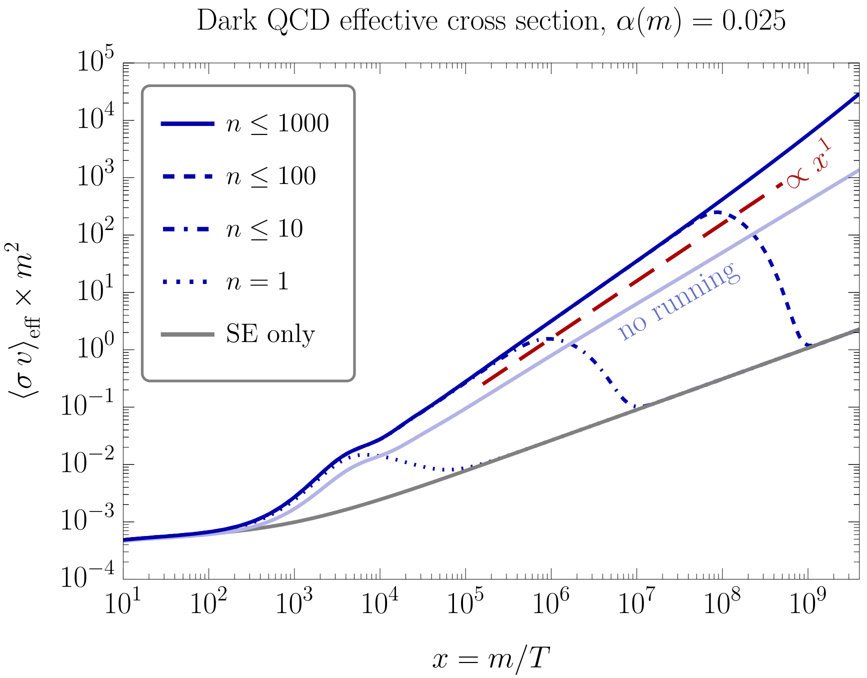




- Includes about 5000 bound states and all possible dipole transitions (~10^6).
- Dark QED indeed freezes out.
- Upper bound on DM mass consistent with perturbative unitarity is 0.2 PeV.

Effective cross section: Dark QCD sector

- s-wave bound states only, dominant decay mode
- Running coupling effects lead to „eternal depletion“ in the perturbative regime, i.e. no freeze-out
- Slope increases with N in SU(N)

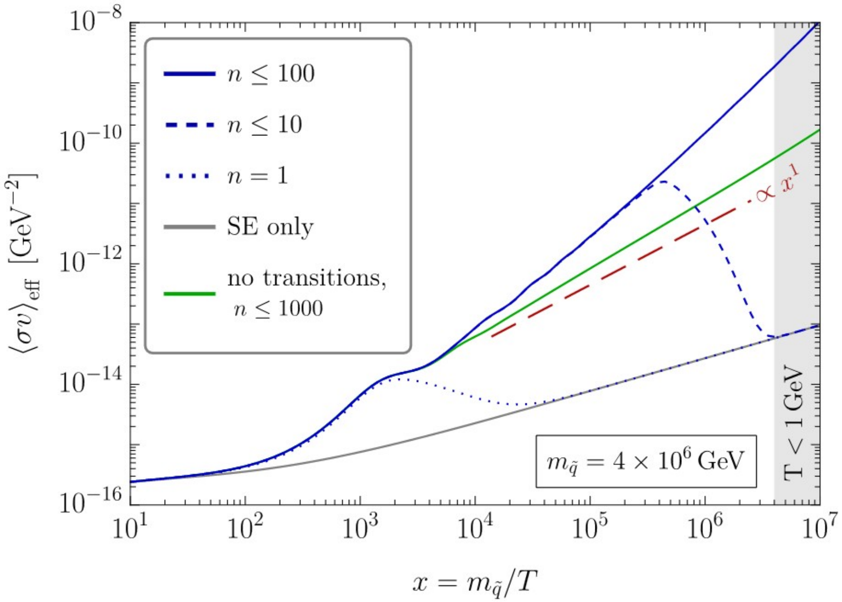


SM SU(3) and U(1) charged mediator model

„t-channel“ simplified toy model:

$$\mathcal{L} \supset \lambda_\chi \tilde{q} \bar{q}_R \chi + h.c.$$

- $\tilde{q}$: scalar mediator, carries SM electric and color charge
- q_R: right handed SM quark
- χ : Majorana Fermion Dark Matter

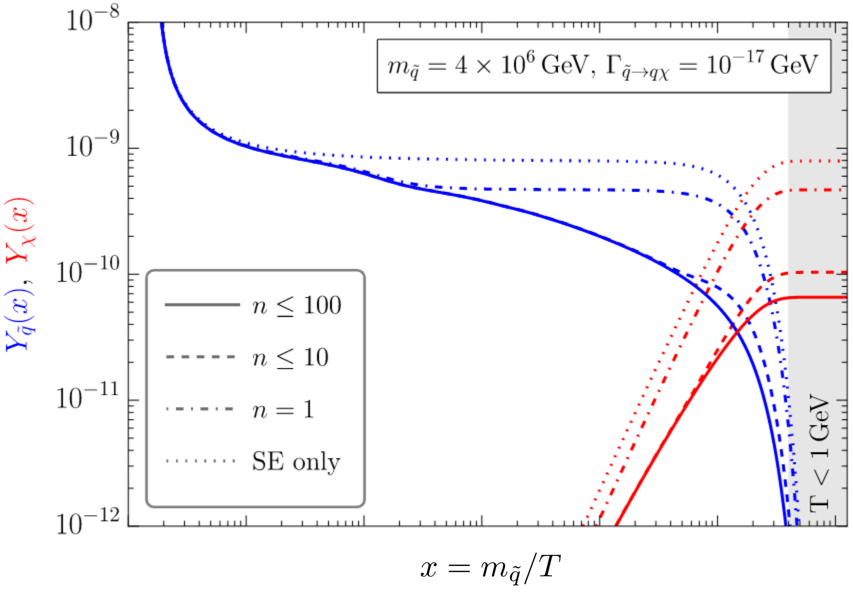


Possible DM production scenarios:

$$\begin{aligned} \Gamma_{\rm conv}^{\chi\to\tilde{q}} &\gg H(m_{\tilde{q}}) && \text{coannihilation}\,, \\ \Gamma_{\rm conv}^{\chi\to\tilde{q}} &\sim H(m_{\tilde{q}}) && \text{conversion-driven}\,, \\ \Gamma_{\rm conv}^{\chi\to\tilde{q}} &\ll H(m_{\tilde{q}}) && \text{superWIMP/freeze-in}\,. \end{aligned}$$

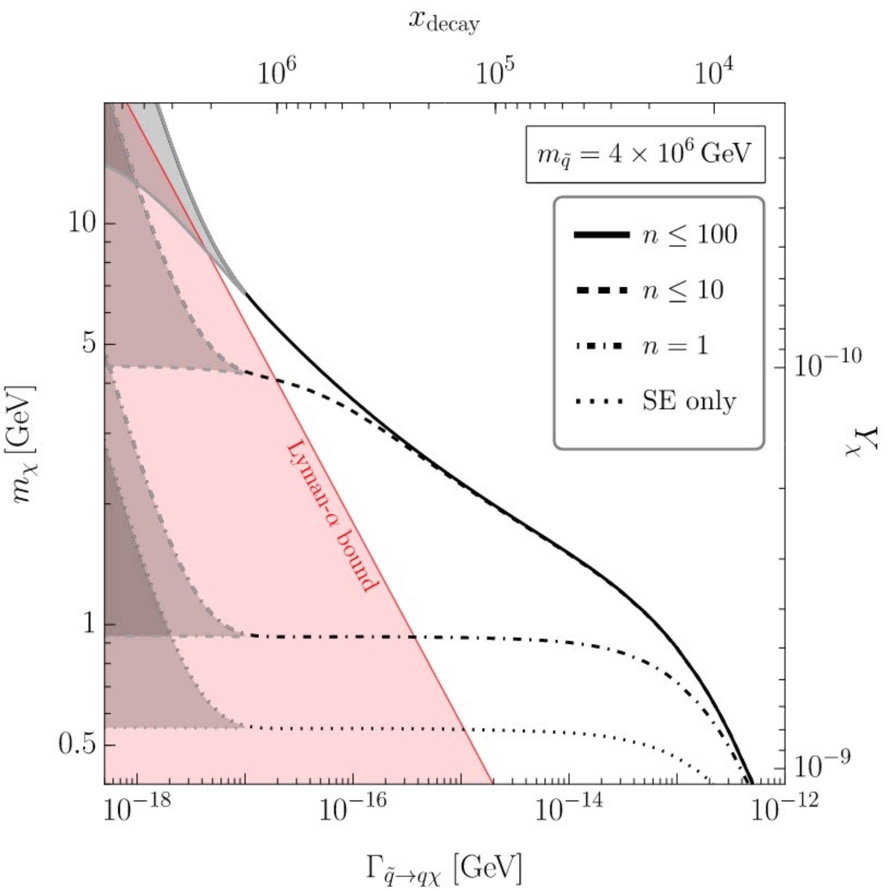
SuperWIMP regime

- superWIMP mechanism: late decay of mediator into DM, final DM yield independent of actual size of the conversion rate.
- Continous depletion of mediator yield from bound state effects.
- → introduces a *dependence* of the DM yield on the conversion rate as a novel feature.



Constraints

- DM produced relativistically from heavy mediator decay
- DM can be „too hot“, i.e., substructure can be erased by free-streaming effect
- Substructure probed by Ly-alpha observations
- Bound state effects open up parameter space
- Corrections to the DM mass up to an order of magnitude



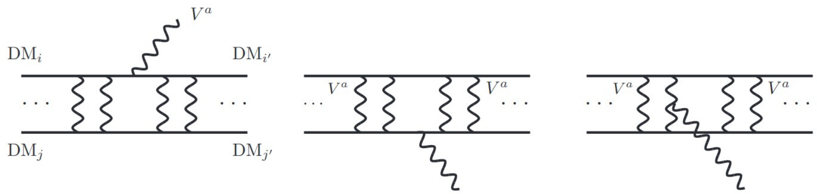
Summary & Conclusion

- Highly excited bound states can play an important role for predicting the DM relic abundance precisely.
- Can lead to „eternal freeze-out“ in unbroken non-abelian gauge theories
- SuperWIMP regime:

  - *novel feature*: bound state effects can introduce a dependence of the DM yield on the mediator lifetime

  - *DM mass corrections*: by up to an order of magnitude

- unitarization of bound-state formation in unbroken non-Abelian gauge theories within the regime of perturbatively small couplings (?)



Colored co-annihilation examples

- **I.e., co-annihilating partner charged under SM SU(3)**
- **Longe-range effects impact** $(\Delta m_\chi, m_\chi)$**- plane**
  - Squark (scalar triplet)
  - Gluino (fermion octet)

[Ellis *et al.* 15, Liew & Luo 16, Mitridate *et al.* 17]

[Harz & Petraki 18,19]

- **+ Higgs**
  - Additional attractive contribution
  - (squark) octet can be bounded

[Gross *et al.* 18, Fukuda & Luo & Shirai 18]

- **Non-perturbative regime**
  (for mass splitting below confinging scale)

DM production scenarios

„t-channel“ simplified model:

$$\mathcal{L} \supset \lambda_\chi \tilde{q}\bar{q}_R\chi + h.c.$$

DM production can be classified into:

$$\begin{aligned}
\Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\gg H(m_{\tilde{q}}) & \text{coannihilation}\,,\\
\Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\sim H(m_{\tilde{q}}) & \text{conversion-driven}\,,\\
\Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\ll H(m_{\tilde{q}}) & \text{superWIMP/freeze-in}\,.
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}Y_{\tilde{q}}}{\mathrm{d}x} &= \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\Bigg[\frac{1}{2}\langle\sigma_{\tilde{q}\tilde{q}^\dagger}v\rangle_{\text{eff}}\left(Y_{\tilde{q}}^2 - Y_{\tilde{q}}^{\text{eq}\,2}\right) \qquad (19)\\
&+\langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\text{eq}}Y_{\tilde{q}}^{\text{eq}}\right) + \frac{\Gamma_{\text{conv}}^{\tilde{q}\to\chi}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\text{eq}}}{Y_\chi^{\text{eq}}}\right)\Bigg]\,,\\
\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} &= \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\Bigg[\langle\sigma_{\chi\chi}v\rangle\left(Y_\chi^2 - Y_\chi^{\text{eq}\,2}\right) \qquad (20)\\
&+\langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\text{eq}}Y_{\tilde{q}}^{\text{eq}}\right) - \frac{\Gamma_{\text{conv}}^{\tilde{q}\to\chi}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\text{eq}}}{Y_\chi^{\text{eq}}}\right)\Bigg]\,,
\end{aligned}$$

pNREFT

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

$$\mathcal{L} \xrightarrow{m} \mathcal{L}^{\mathrm{NR}} \xrightarrow{\alpha m} \mathcal{L}^{\mathrm{pNR}}$$

Non-relativistic effective field theory for the ultra-soft scale $\alpha^2 m$

- *potential* non-relativistic (pNR) QED:

$$\mathcal{L}^{\mathrm{pNRQED}} \supset \int \mathrm{d}^3 r\; S^\dagger(\mathbf{x},\mathbf{r},t)\left[i\partial_t + \frac{\nabla^2_{\mathbf{x}}}{4m} + \frac{\nabla^2_{\mathbf{r}}}{m} - V(r) + i2\frac{\pi\alpha^2}{m^2}\delta^3(\mathbf{r}) + \mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t)\right] S(\mathbf{x},\mathbf{r},t)$$

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$

$$(\sigma v)_{nl} = \frac{4\alpha}{3}|\langle\psi_{nl}|\,\mathbf{r}\,|\psi_v\rangle|^2 \Delta E^3$$

pNREFT

[Pineda & Soto 1997, Beneke 98,99, Brambilla et al. 2000, 2005]

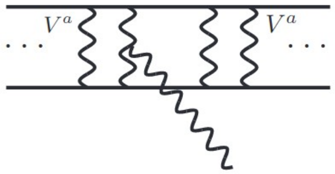
$$\mathcal{L} \xrightarrow{m} \mathcal{L}^{\mathrm{NR}} \xrightarrow{\alpha m} \mathcal{L}^{\mathrm{pNR}}$$

Non-relativistic effective field theory for the ultra-soft scale $\alpha^2 m$

- *potential* non-relativistic (pNR) SU(N) in the weakly coupled regime:

$$\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots$$

$$\mathcal{L}_{\mathrm{pNREFT}} \supset \int \mathrm{d}^3 r \ \mathrm{Tr} \Big[\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger (iD_0 - H_{\mathrm{adj}})\mathrm{Adj} - V_A(\mathrm{Adj}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) - \frac{V_B}{2} \mathrm{Adj}^\dagger \{\boldsymbol{r} \cdot g\boldsymbol{E}, \mathrm{Adj}\} + \cdots \Big] .$$



Included, e.g. [Binder et al. 2021]

e.g. quarkonium, squark:

$3 \otimes \bar{3} = 1 \oplus 8$ $\quad\quad$ $\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$ $\quad\quad$ $(\sigma v)_{nl} = \frac{C_F}{N_c^2} \frac{4\alpha}{3} \Delta E^3 |\langle \psi^1_{nl} | \mathbf{r} | \psi^8_{\mathbf{p}} \rangle|^2$

Positronium example

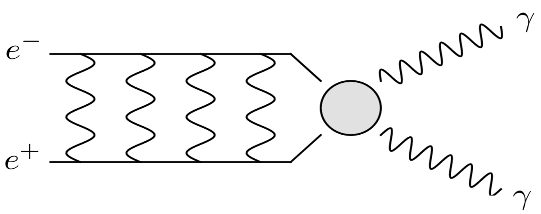
Bound-state decay and Sommerfeld enhancement:

$$\Gamma_n = (\sigma v)_0 \times |\psi_n(r=0)|^2$$

Pirenne & Wheeler 1946

$$(\sigma v) = (\sigma v)_0 \times |\psi_v(r=0)|^2$$
$$\propto (\sigma v)_0 \, (\alpha/v) \, , \text{ for } v \lesssim \alpha.$$

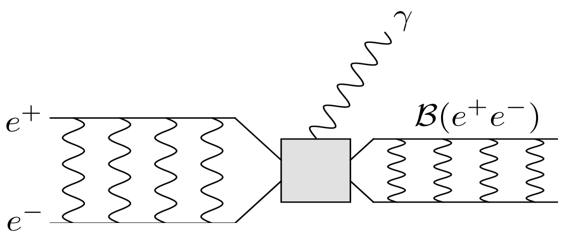
Sakharov 1948 (Sommerfeld 1931)



Bound-state formation (recombination):

$$(\sigma v)_{nl} = \frac{4\alpha}{3} | \langle \psi_{nl} | \, \mathbf{r} \, | \psi_v \rangle \, |^2 \Delta E^3$$
$$\sim 3 \times \text{annihilation}, \text{ for } v \lesssim \alpha.$$

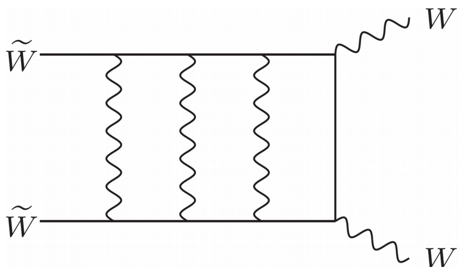
(and n=1,l=0.)



(originates from the Electric Dipole Operator „gr.E“, see e.g. Landau&Lifshitz)

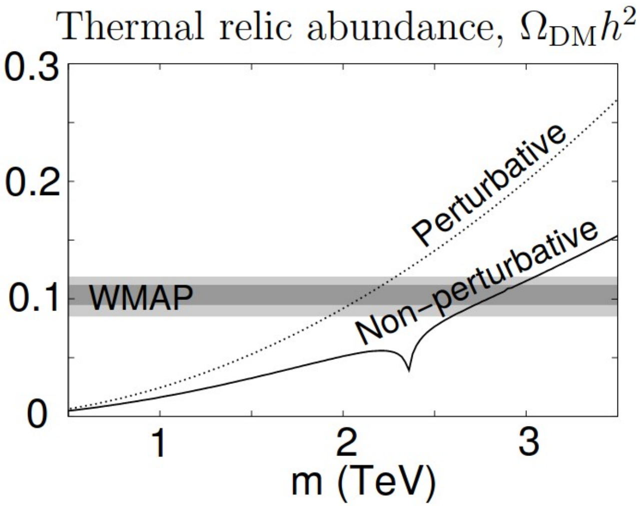
Wino Dark Matter example

$$\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{W}^0 \\ \widetilde{W}^- \end{pmatrix}$$



- Majorana Fermion, SU(2) Triplet, zero Hypercharge („most minimal WIMP“)
- Sommerfeld-enhanced annihilation allows for heavier Wino masses
- ID signal mass sensitive, see e.g.

  [Rinchiuso, Slatyer et al. 20]



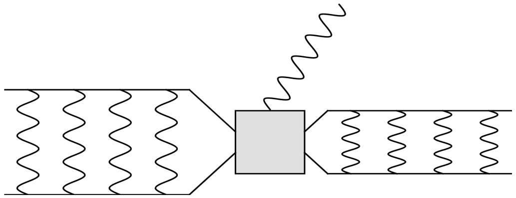
[Hisano et al. 03,05,06]

General dipole transition matrix elements

- „gr.E“ leads to matrix elements of the form:

$$\langle \psi_f | \, \mathbf{r} \, | \psi_i \rangle = \int \mathrm{d}^3 r \; \psi_f^\star(\mathbf{r}) \; \mathbf{r} \; \psi_i(\mathbf{r}).$$

$$\mathrm{V}_{i/f} = -\frac{\alpha_{i/f}^{\mathrm{eff}}}{r}$$



- E.g.: (chromo-) electric dipole transitions of pairs in unbroken U(1) and SU(N) gauge theories

- Analytic result in terms of recurrence relations* allows for efficient and numerically stable evaluation.

- Tested against know results for low excitations

*) in QED limit consistent with
[W. Gordon, Zur Berechnung der Matrizen beim Wasserstoffatom, Annalen der Physik 394 (1929)]