Real dynamics of thermal false vacuum decay

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Particle Production in the Early Universe

CERN

Sep 10, 2024

Motivation

- **First order phase transitions in the early universe**

  GW signal…

- **EW baryogenesis**

  Dynamics of the expanding bubble…

- **Higgs vacuum metastability**

  Wall is coming?..

- **Experimental tests of nucleation theory**

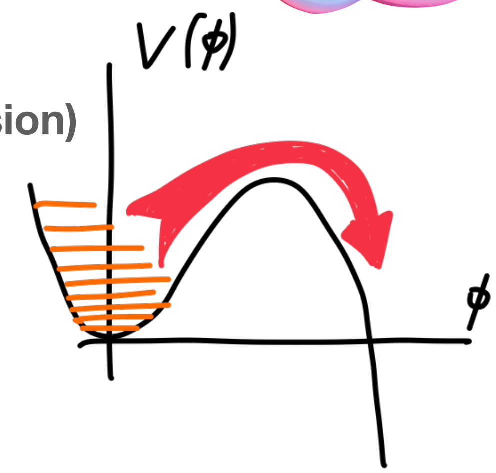
  A. Zenesini et al. **False vacuum decay via bubble formation in ferromagnetic superfluids,** Nature Physics 20, 558–563 (2024)

Methods

Consider vacuum decay at finite temperature via **classical thermal** jumps of the field over the barrier.

i.e. at temperatures high (classical regime) but not too high (exponential — Boltzmann — suppression)

The decay happens through the formation of special thermodynamic fluctuation: **critical bubble.**



Euclidean (equilibrium) approach

- Gibbs 1875 — first discussion of the critical bubble, its energy in the thin-wall approximation
- Wigner 1937 — Transition State Method for chemical reactions: saddle point, negative mode, zero modes
- Langer 1969 — Classical-statistical theory of metastability: many d.o.f. + external heat bath
- Affleck 1980 — Quantum-statistical theory of metastability: 1 d.o.f., no external heat bath
- Linde 1982 — Decay of false vacuum at finite temperature: field theory, different regimes

Growth rate of the critical bubble's unstable mode

Free energy around the false vacuum

Decay probability per unit time per unit volume

$$\Gamma_E = \frac{\omega_-}{\pi T}\frac{\mathrm{Im}F}{\mathscr{V}}$$

Volume

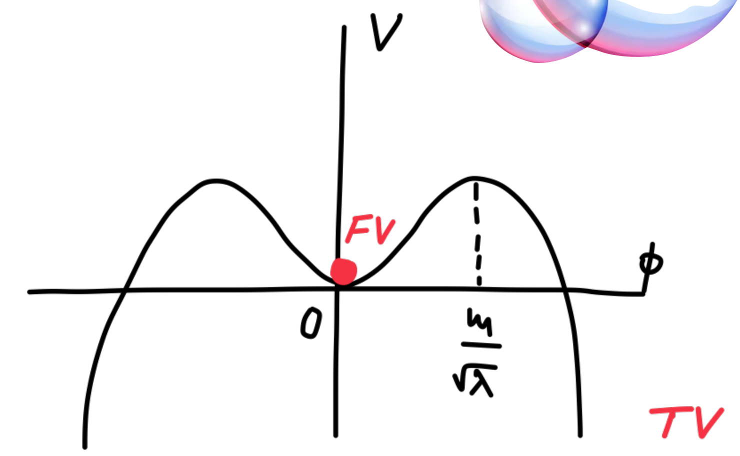
Temperature

To test the predictions of the Euclidean approach and to study dynamical properties of the phase transition we use

Real-time simulations

- Grigoriev, Rubakov, Shaposhnikov — Sphaleron transitions, Hamiltonian dynamics
- Alford, Feldman, Gleiser — Vacuum decay, Langevin dynamics
- Gould, Moore, Rummukainen — Vacuum decay, ``multi-canonical sampling” + real-time evolution

Simplest Setup



Scalar field theory in 1+1 dimensions: $S = \int \mathrm{d}t\,\mathrm{d}x \left(-\frac{(\partial_\mu \phi)^2}{2} - \frac{m^2\phi^2}{2} + \frac{\lambda\phi^4}{4} \right)$

- **Euclidean theory predicts:** $E_b = \frac{4m^3}{3\lambda}$, $\Gamma_E = \frac{6m^2}{\pi}\sqrt{\frac{E_b}{2\pi T}}\,\mathrm{e}^{-E_b/T}$

  barrier (critical bubble) energy

- **We want to measure the decay rate (among other things) in "first-principle" classical lattice simulations**

We prepare a suite of simulations with the initial thermal Rayleigh-Jeans spectrum:

Fourier modes of the field and momentum

$$\langle |\tilde{\phi}_j|^2 \rangle = T/\Omega_j^2\,, \quad \langle |\tilde{\pi}_j|^2 \rangle = T$$

$$\Omega_j^2 = 2(1-\cos k_j a)/a^2 + m_{th}^2\,, \quad k_j = 2\pi j/L$$

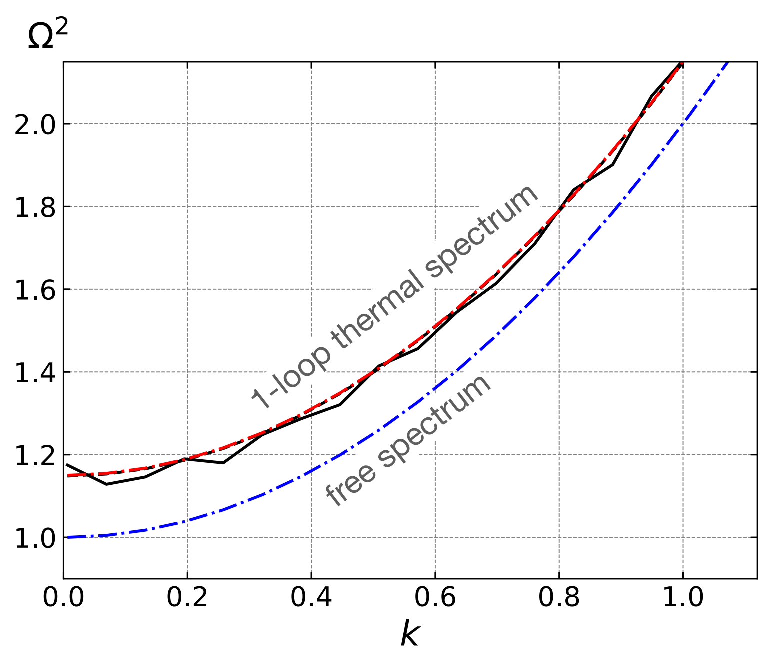
lattice spacing

box size

$$m_{th}^2 = m^2 - \boxed{\frac{3\lambda T}{2m}}$$

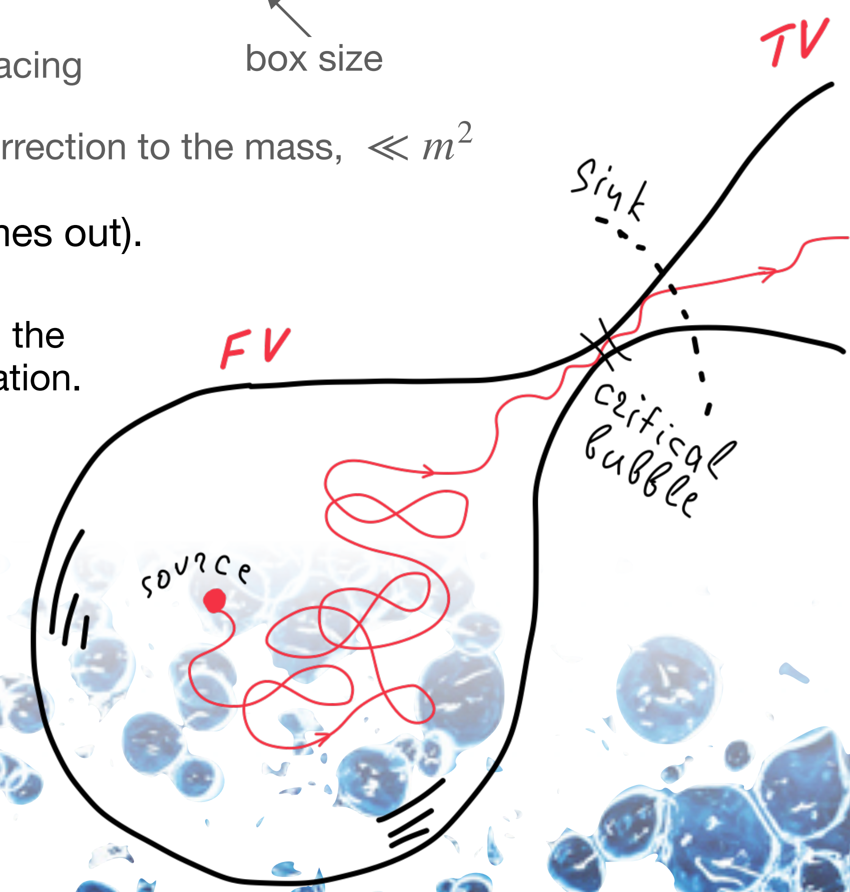
thermal correction to the mass, $\ll m^2$

…and evolve them until they decay (or simulation times out).



Measurement of the dispersion relation in the theory with the stable potential

We checked that this is an equilibrium state by evolving the theory with the stable potential using the Langevin equation.



What does it mean “decay rate”?

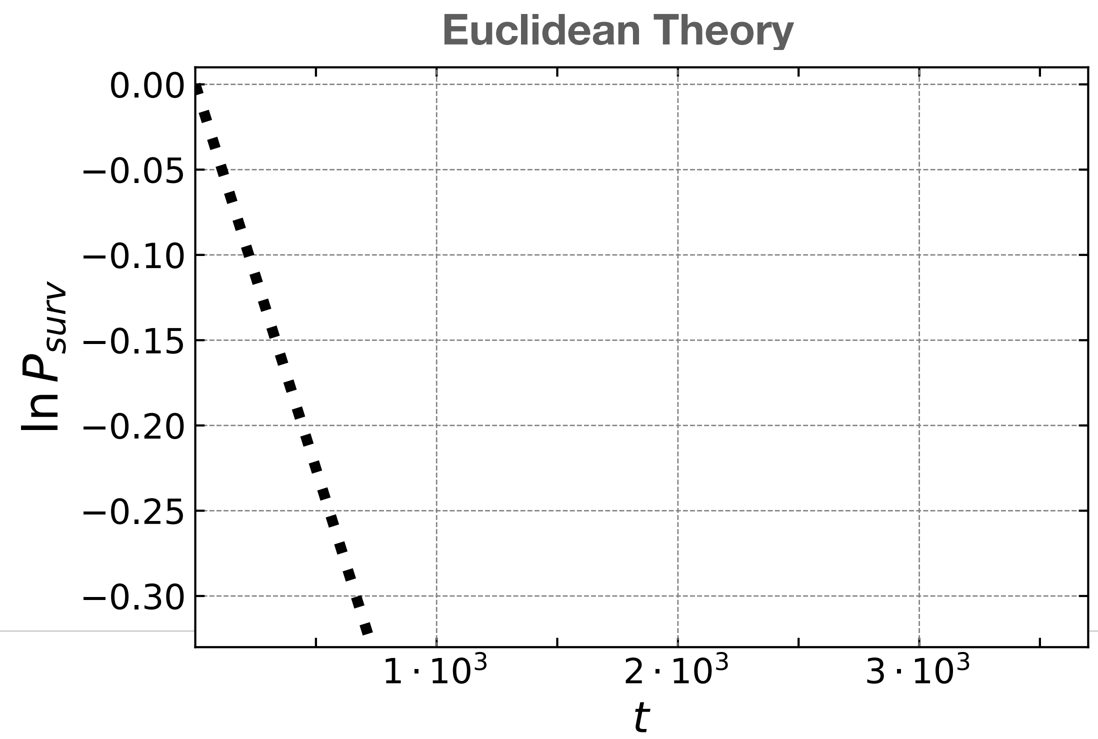
- Introduce **survival probability** $P_{surv}(t)$

For decays obeying the exponential distribution, it follows the law:

(we exclude early-time transients)

$$\ln P_{surv}(t) = \text{const} - \Gamma L \cdot t$$

This is decay rate



First surprise

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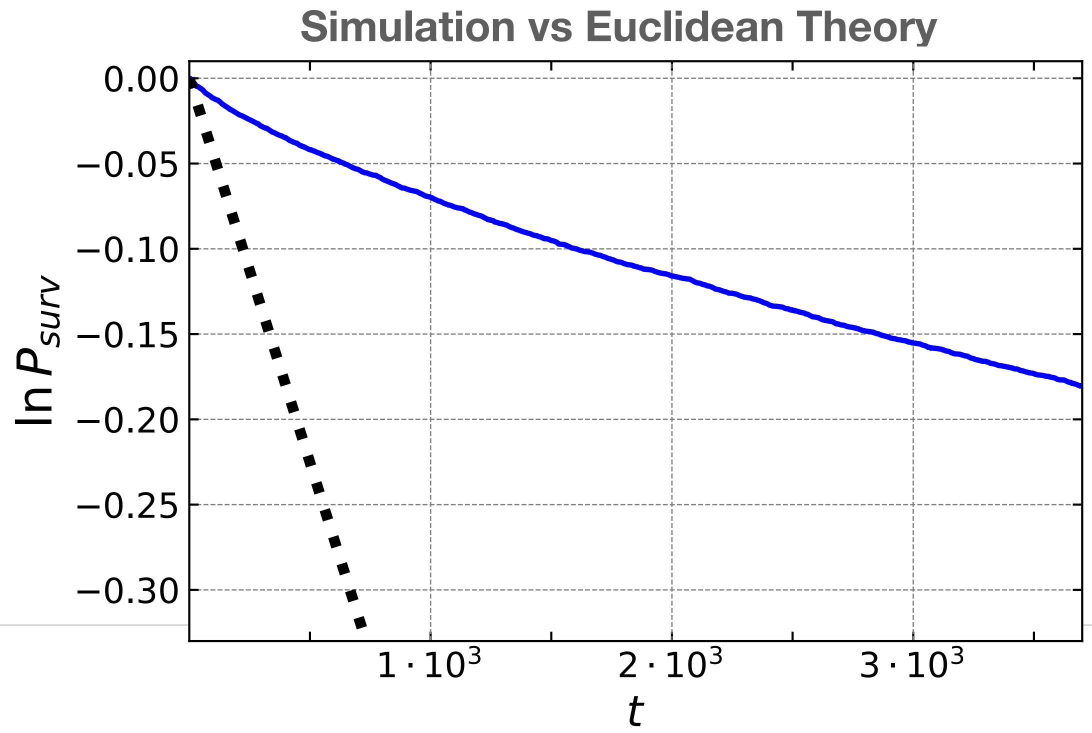
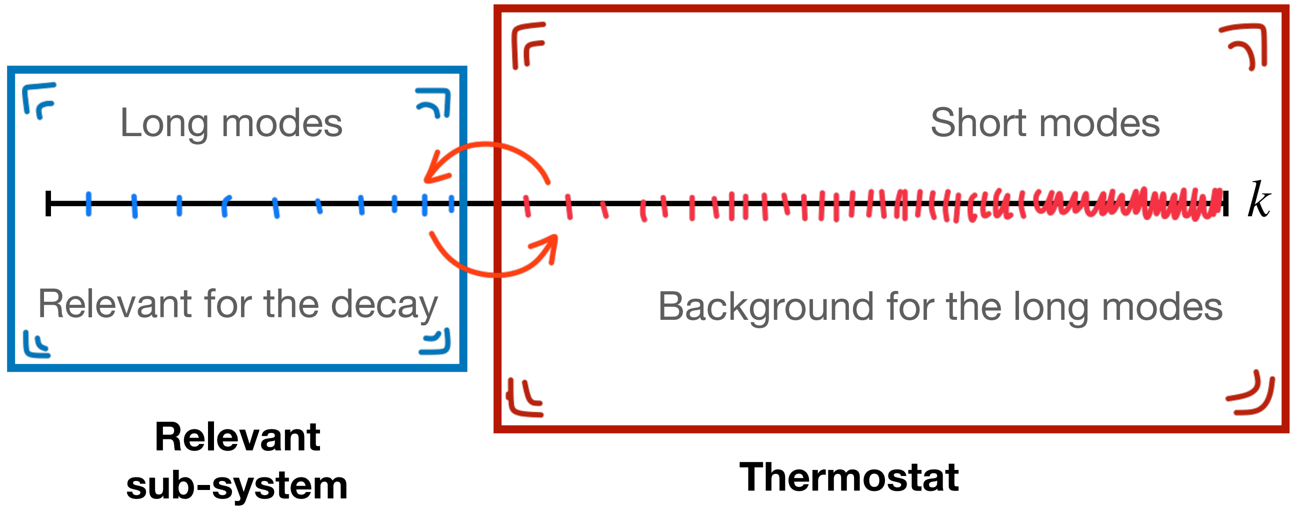


- Decay rate found in simulations is smaller than the Euclidean prediction
- It is, moreover, time-dependent, getting even smaller with time

What does it mean “thermal”?

- For the **Hamiltonian** evolution, it means the following:



But thermalisation in the theory is very **inefficient**: for modes with $\omega \sim m \sim$ (bubble size)$^{-1}$, the thermalisation time is

$$t_{th} \sim \frac{(2\pi)^3}{\tilde{T}^4}, \qquad \tilde{T} = \frac{\lambda T}{m^3} \ll 1$$

(due to $2 \to 4$ and $3 \to 3$ scattering processes)

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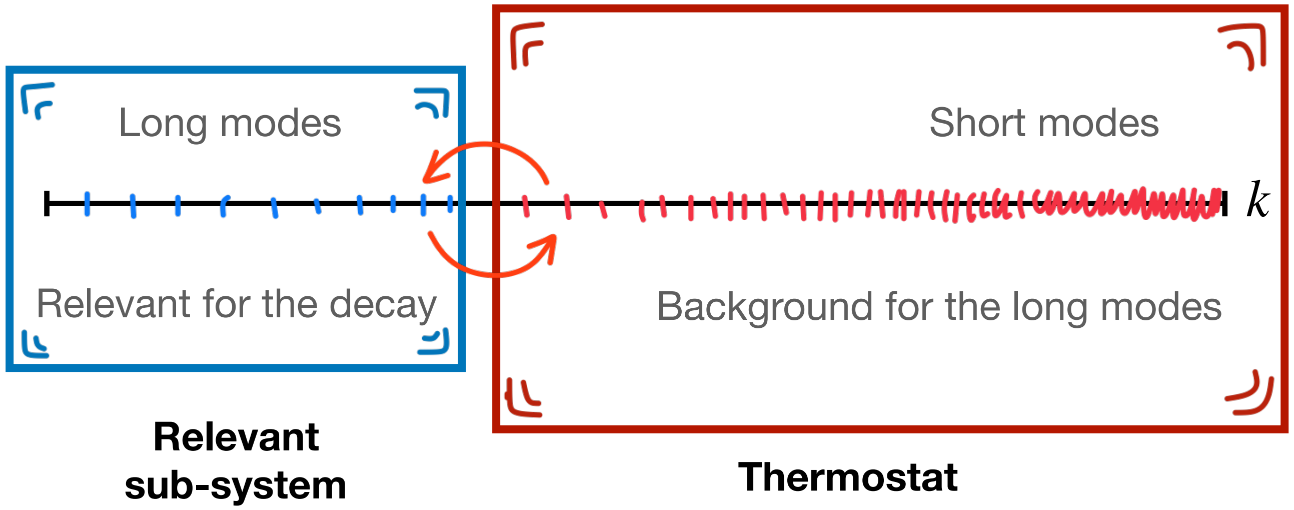

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(due to $2 \to 4$ and $3 \to 3$ scattering processes)

- Compare this with the decay time: $t_{dec} \sim (\Gamma L)^{-1}$

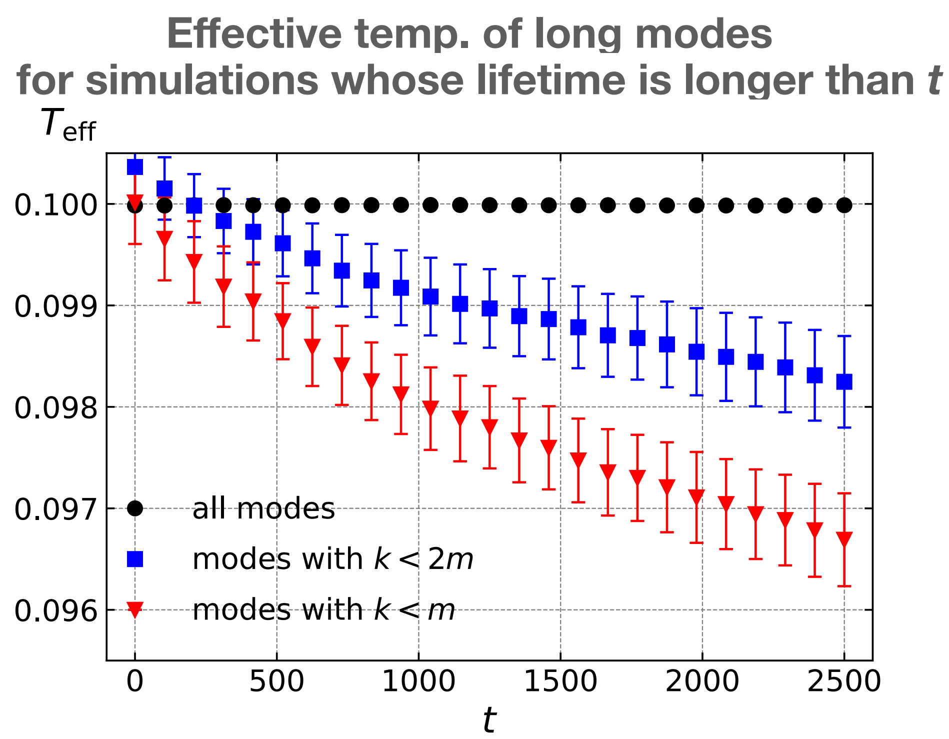
In our simulations it happens that $t_{th} > t_{dec}$ (hardly relevant for cosmology, but can be relevant for experiments)

This leads to the interesting effect.

Classical Zeno effect

- Because of inefficient thermalisation, the initial power contained in the long modes is preserved during the simulation.
- The configuration which, due to a statistical fluctuation, has a higher initial long-mode power decays faster. The one with lower power lives longer.

→ Statistical properties of the ensemble change with time: long modes cool down.



- Effective temp. of long modes drops by a few per cent during the run: enough to visibly suppress the decays.

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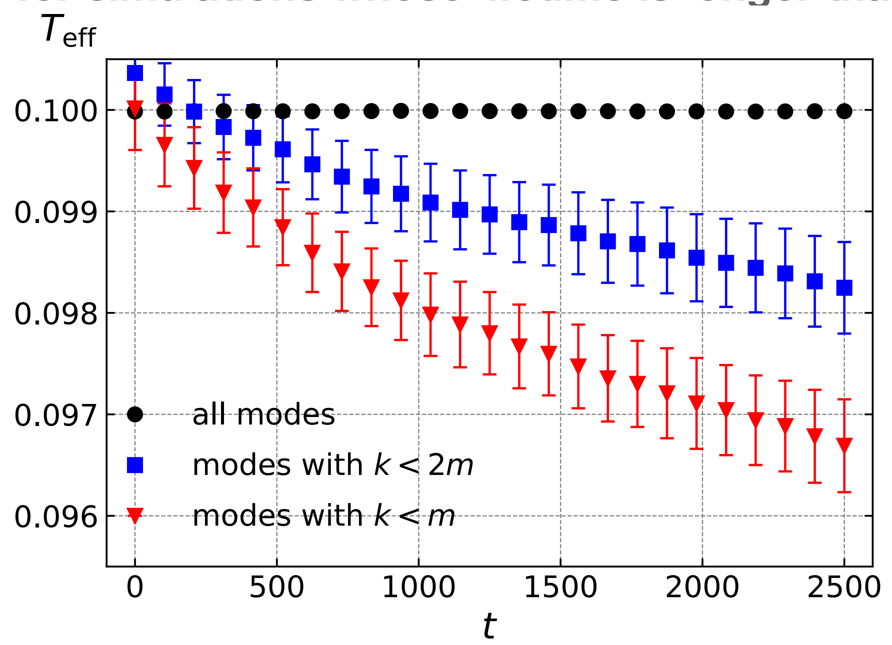
Effective temp. of long modes for simulations whose lifetime is longer than *t*



- Effective temp. of long modes drops by a few per cent during the run: enough to visibly suppress the decays.

Decay is a non-Markovian process (in this regime).

The longer we observe the system, the less chance it has to decay in the future: classical Zeno effect.

To find the unbiased rate, we extrapolate the slope of the survival probability curve to zero.

Second surprise

- We measure the (unbiased) decay rate at different temps. and fit with the formula (recall that $\Gamma_E = A_E \exp(-E_b/T)$)

$$\ln \Gamma(T) = -\frac{1}{2} \ln T + \ln A - \frac{B}{T}$$

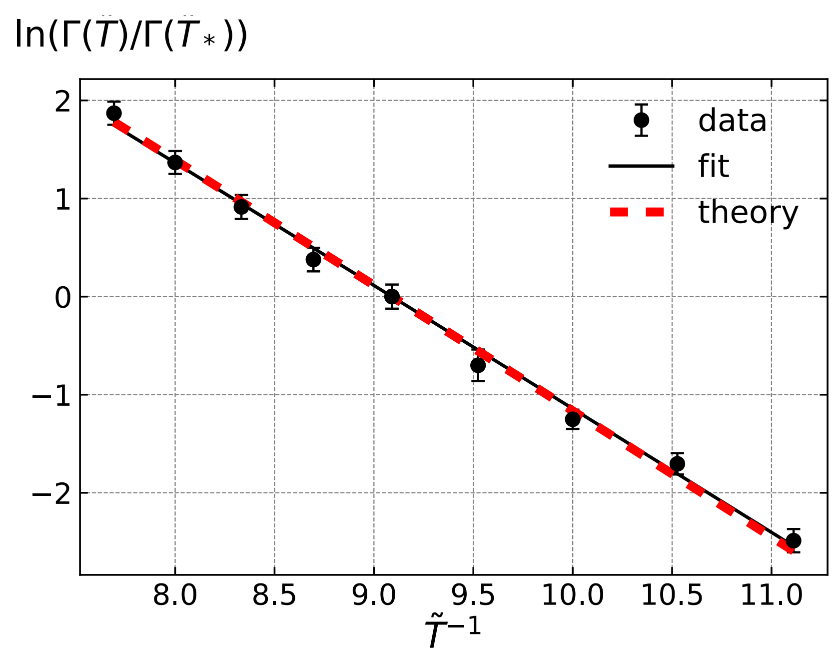
critical bubble energy

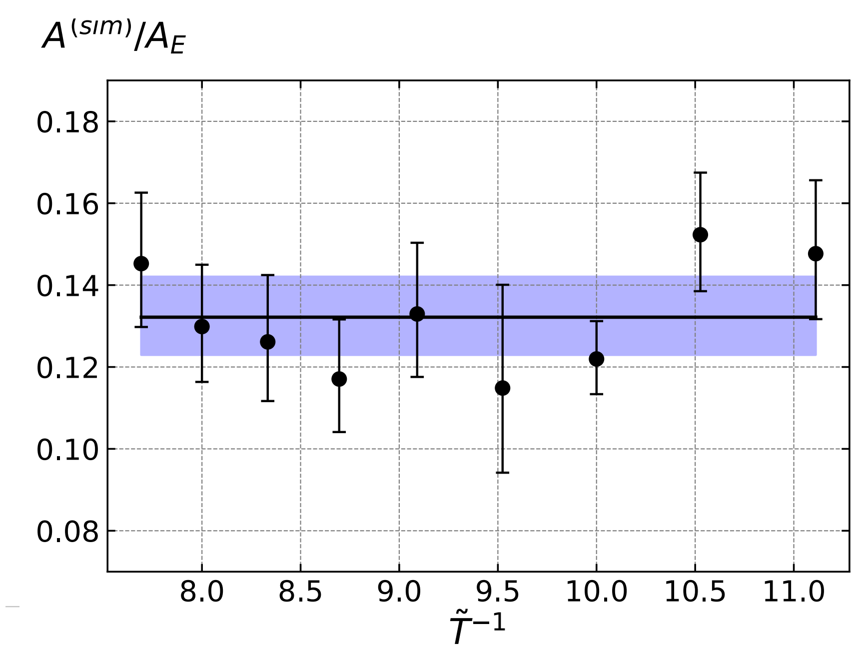
prefactor (with the zero mode excluded)

from the zero mode in the prefactor

One can measure *A* and *B* separately, using the ratio $\Gamma(T)/\Gamma(T_*)$ to find *B*, with some reference temp. T_*

Or one can make the 2-parameter fit, the result is the same (within the errorbars).





- Critical bubble energy agrees with the Euclidean theory (<2% error bar)
- The measured prefactor is smaller by a factor ~8.

? Something wrong with thermalisation again? Violation of thermal equilibrium near the critical bubble?

More evidence: Langevin evolution

We can reduce artificially the thermalisation time by coupling the system to an external heat bath.

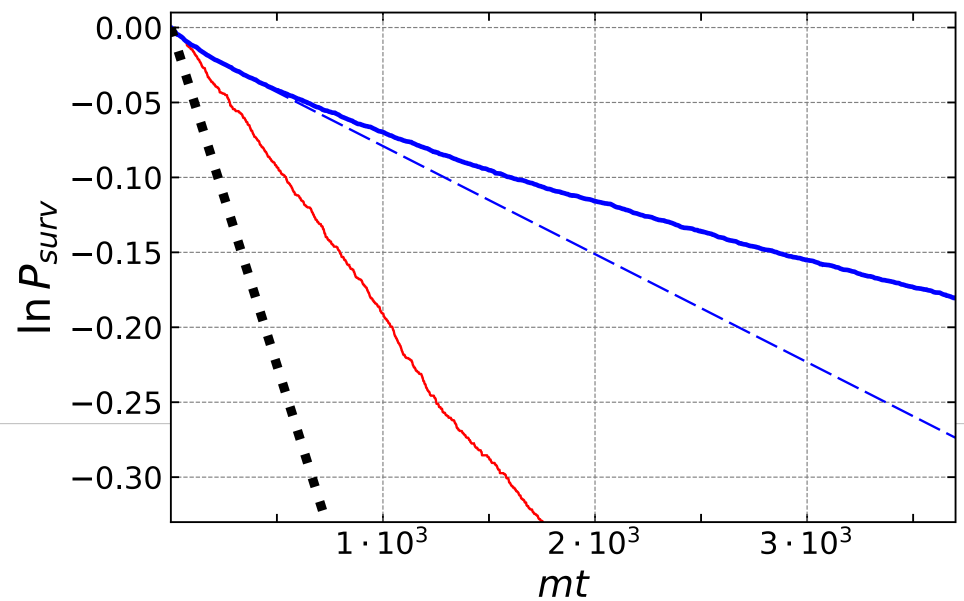
$$\begin{cases} \ddot{\phi} + \eta\dot{\phi} - \phi'' + m^2\phi - \lambda\phi^3 = \xi \\ \langle \xi(t,x)\rangle = 0\ , \ \ \langle \xi(t,x)\xi(t',x')\rangle = 2\eta T\delta(t-t')\delta(x-x') \end{cases}$$

$$\longrightarrow \quad t_{th} \sim \eta^{-1}$$

Noise and dissipation change the dynamics of vacuum decay.

They don't change the critical bubble.

Simulation at zero vs non-zero noise ($\eta = 10^{-2}m$)

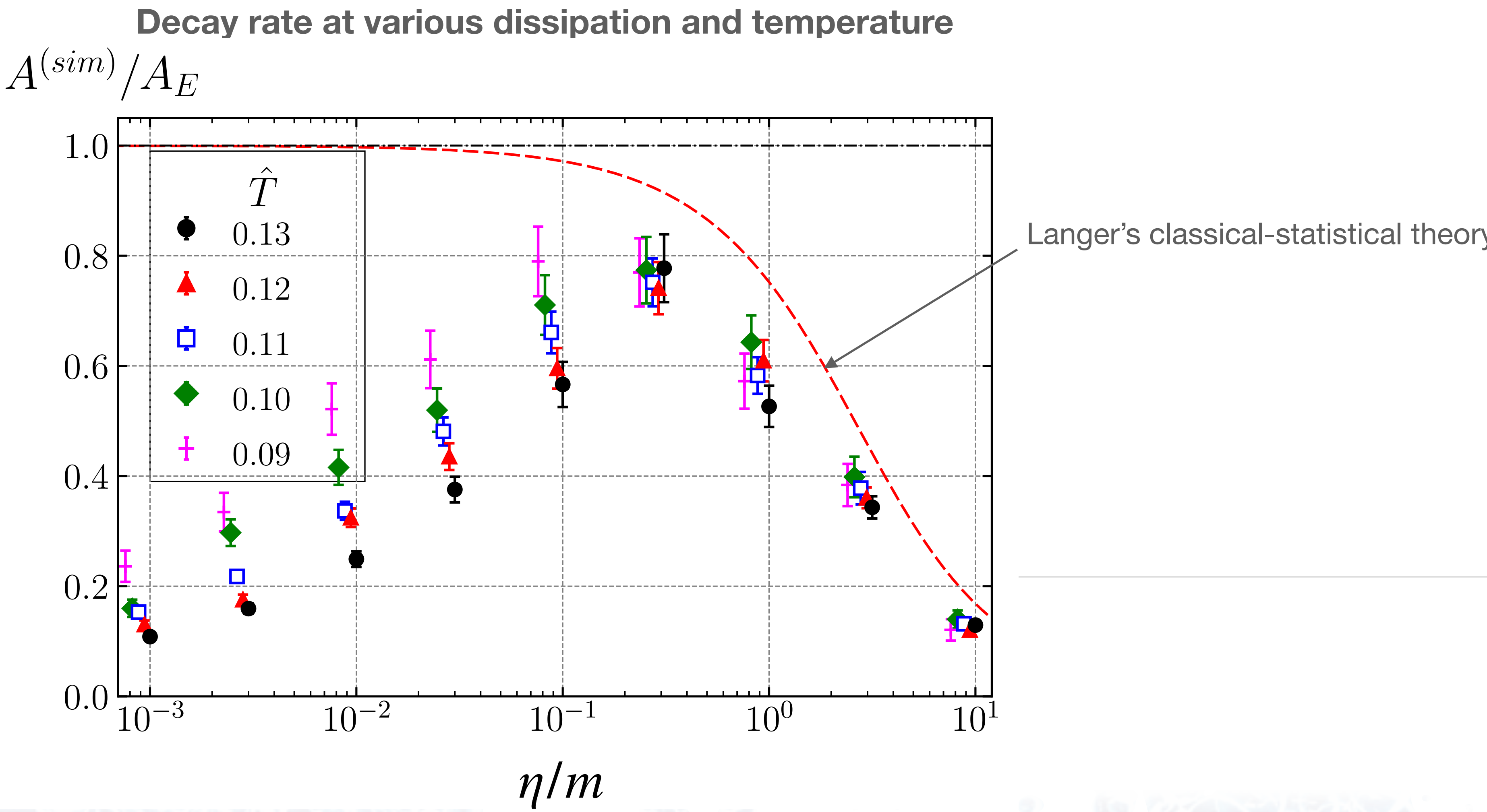


- No Zeno effect as long as $\eta \gtrsim \Gamma L$
- Decay rate increases, but still below the Euclidean bound

Langevin dynamics: decay rate

We observe the following behavior:

- As dissipation increases, Γ increases as well. It reaches maximum at $\eta \simeq 3 \cdot 10^{-1} m$, then starts decreasing due to over-damping.
- Γ tends to increase when T goes down.



Violation of equilibrium condition

In Physical Chemistry, the analog of Euclidean Theory is Transition State Theory (TST).

Hanggi, Talkner, Borkovec, Rev.Mod.Phys. 62 (1990)

TST deals with particles (one or few d.o.f.) in the external heat bath, $\eta > 0$.

It is known that TST is violated if there is no equilibrium around the barrier.
The following condition must be satisfied:

$$\eta \gg \frac{\omega_- T}{E_b}$$

We can generalise this condition to **Langevin** dynamics of field theory.

This is done by careful examination of Langer's work.

For the **Hamiltonian** dynamics of field theory, we suggest the following condition:

All our current and future results are consistent with it.

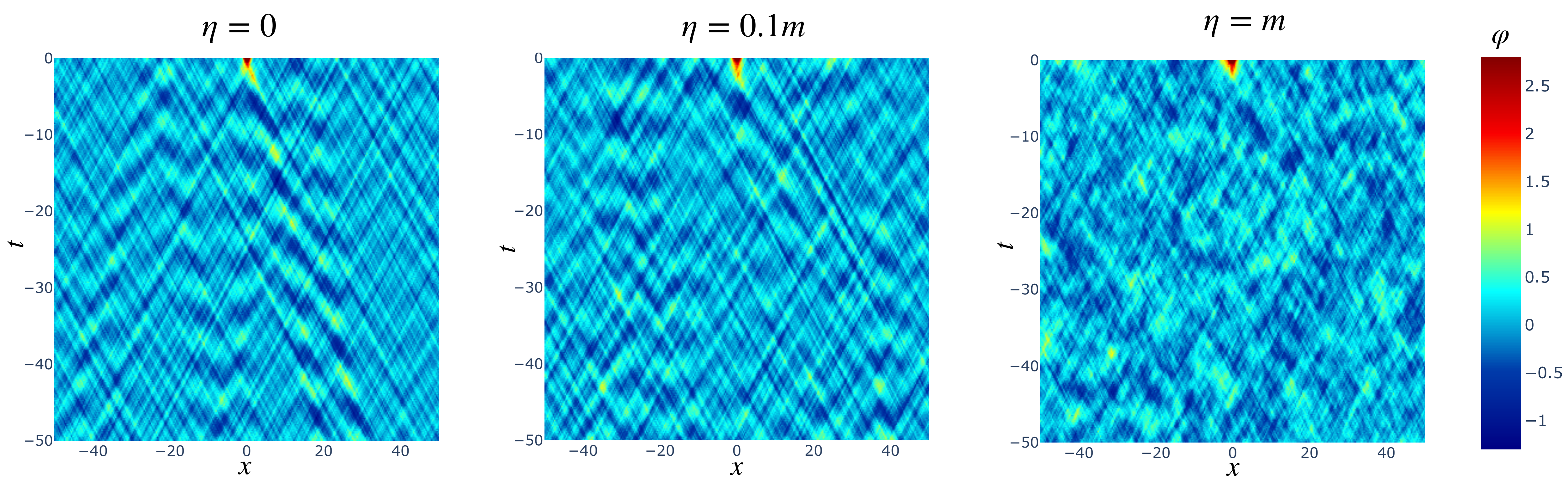
Effective free energy of the critical bubble

$$t_{th} \lesssim \frac{\mathcal{F}_b}{\omega_- T}$$

- It is generally violated for weakly-coupled theories with one coupling (one field)
- In theories with many fields, it must be examined on a case-by-case basis.

Dynamics of vacuum decay

When equilibrium is violated, interesting features appear in the field evolution prior to the decay.



At small dissipation, we observe a population of nonlinear waves with $\omega < m$ – **oscillons**.

They disappear when $\eta > 0.1m$ and the system evolves due to the stochastic terms.

Dynamics of vacuum decay

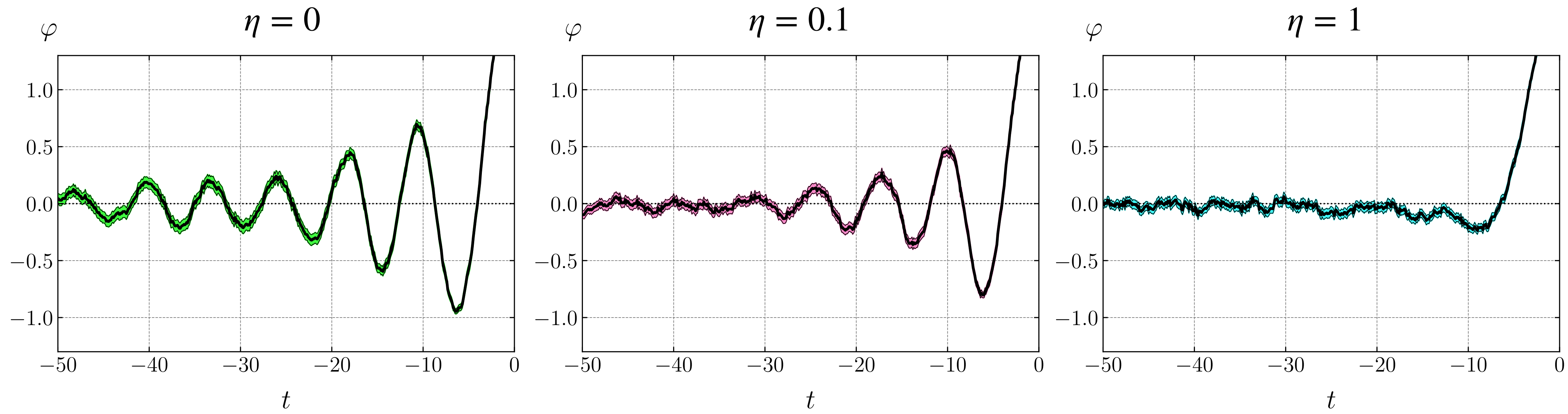
In the Hamiltonian dynamics, every critical bubble is preceded by an oscillon.

Johnson, Pîrvu, Sibiryakov, 2312.13364

We can track its trajectory.

Thanks to Dalila's smart numerical routine.

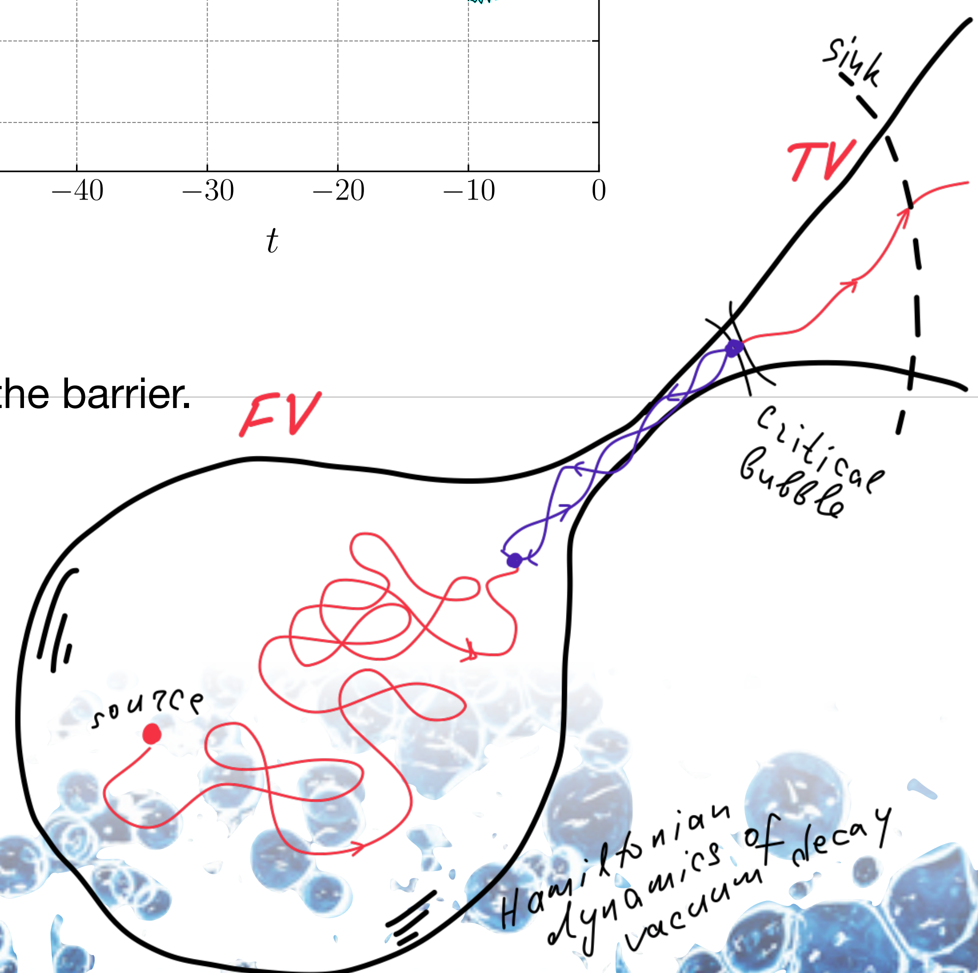
Stacking many oscillons together, we get the average **oscillonic precursor** to the critical bubble:



In our system, the presence of oscillons indicate violation of thermal equilibrium near the barrier.

Thus, they are correlated with the diminishing decay rate.

But how deep is this correlation?..



Discussion

- We also measured the critical bubble profile. It agrees with the Euclidean prediction. 2408.06411

  No classical thermal corrections to the critical bubble in 1+1.

- We have repeated the measurements in theories of many scalar fields: 24xx.xxxxx
  one decaying ϕ and N spectators — explicit heat bath — generating the effective (thermal) potential for ϕ.

  Varying the couplings, we managed to get fast thermalisation and recover the Euclidean decay rate.

  The critical bubble shape agrees with the thermal effective potential for ϕ (i.e. the Euclidean calculation) even if there is no fast thermalisation.

- We are now studying properties of the oscillonic precursors…

  Do they correlate in space? Can we tune them to delay the phase transition?
  Vacuum decay due to collisions of oscillons?

- Our results are not directly applicable to sphaleron transitions (or production&collisions of kinks)

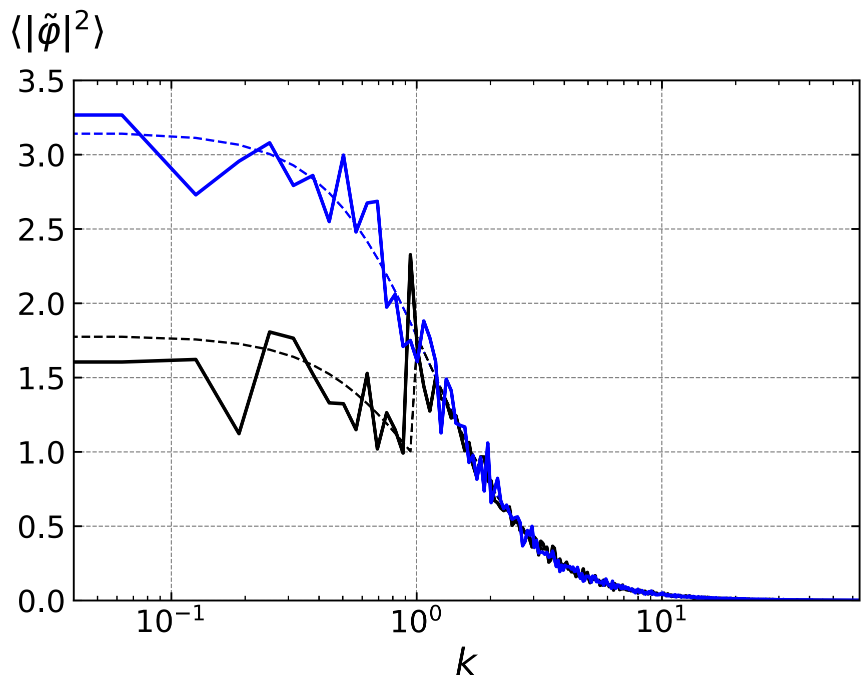
  But they suggest that there might be some interesting non-equilibrium dynamics associated with them.

- We need to compute "analytically" the prefactor in the decay rate for the Hamiltonian dynamics

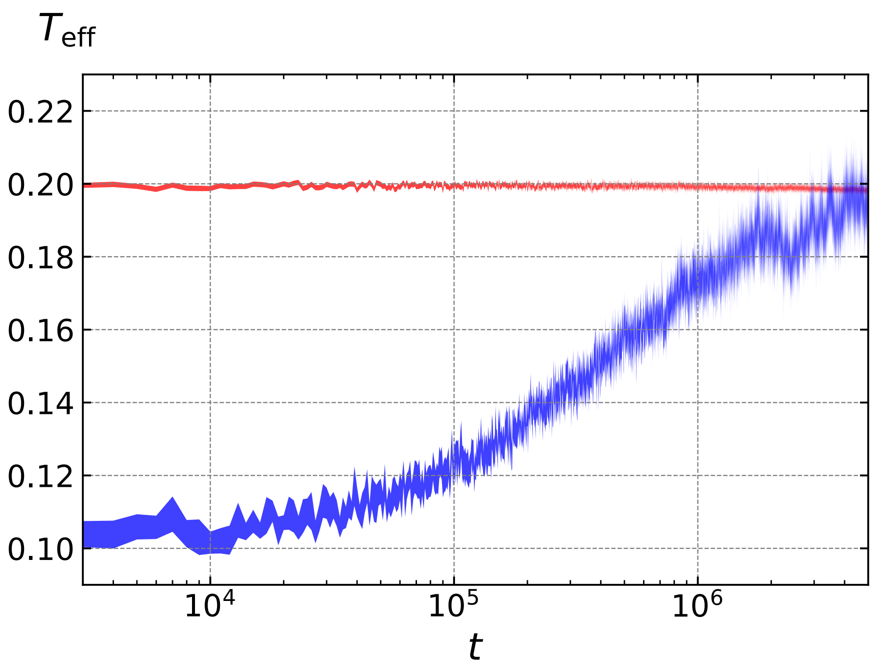
backup slides

Thermalisation time

We perform the numerical experiment estimating the thermalisation time of long modes in the Hamiltonian system.



Initial and final spectra

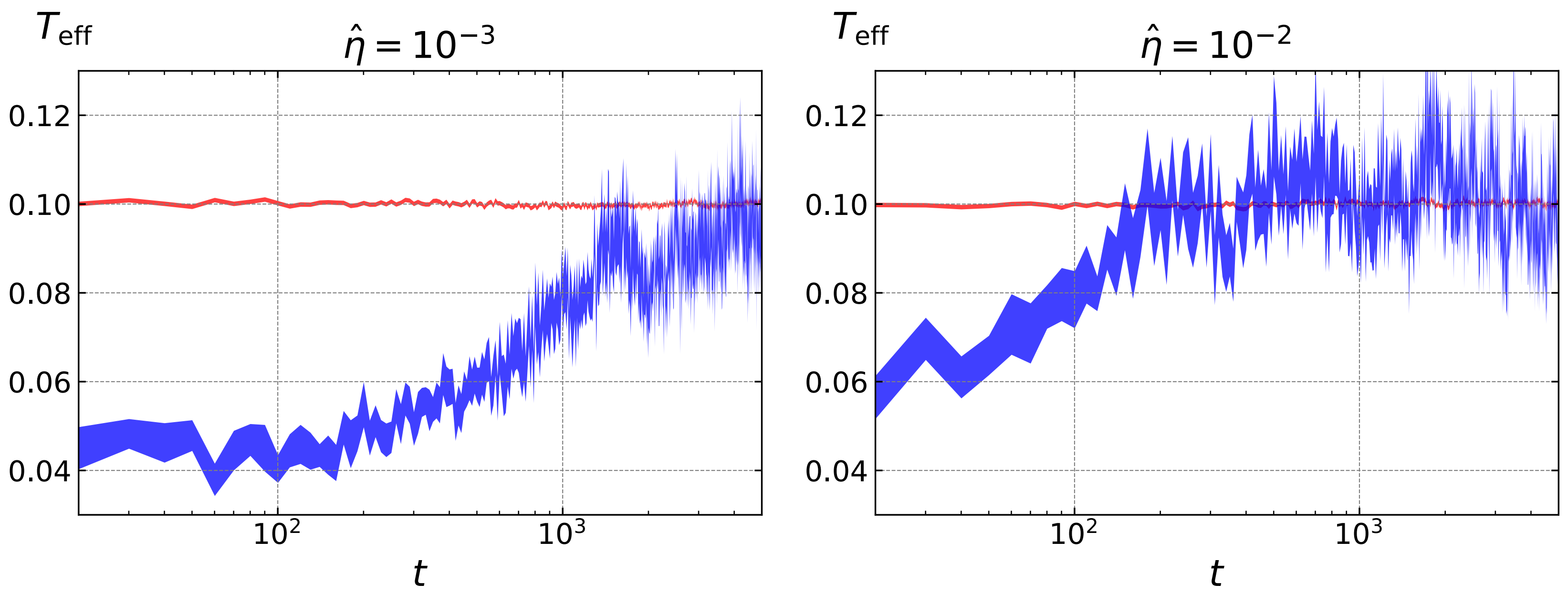


Effective temp. of long modes ($k < m$, blue) vs temp. of all modes (red)

The result agrees with the theoretical estimate $t_{th} \sim \dfrac{(2\pi)^3}{\tilde{T}^4}$

Thermalisation with external heat bath

We perform the numerical experiment estimating the thermalisation time of long modes with the Langevin evolution.



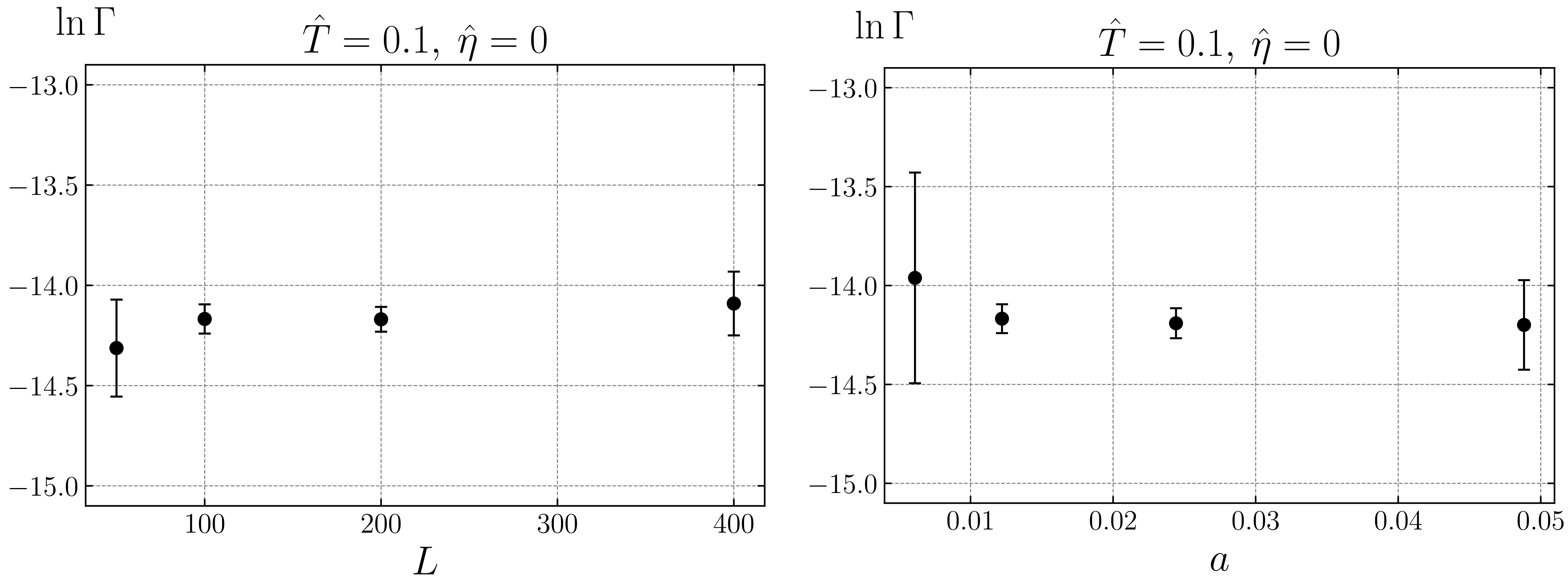
Effective temperature of long modes ($k < m$, blue) and the temperature of the ensemble ($k > m$, red)

The result agrees with the estimate $t_{th} \sim \eta^{-1}$.

Box size and lattice spacing

In simulations, we take $L = 100$ and $a \simeq 0.01$ (in units of mass).

The plots below demonstrate insensitivity of the decay rate to L and a.



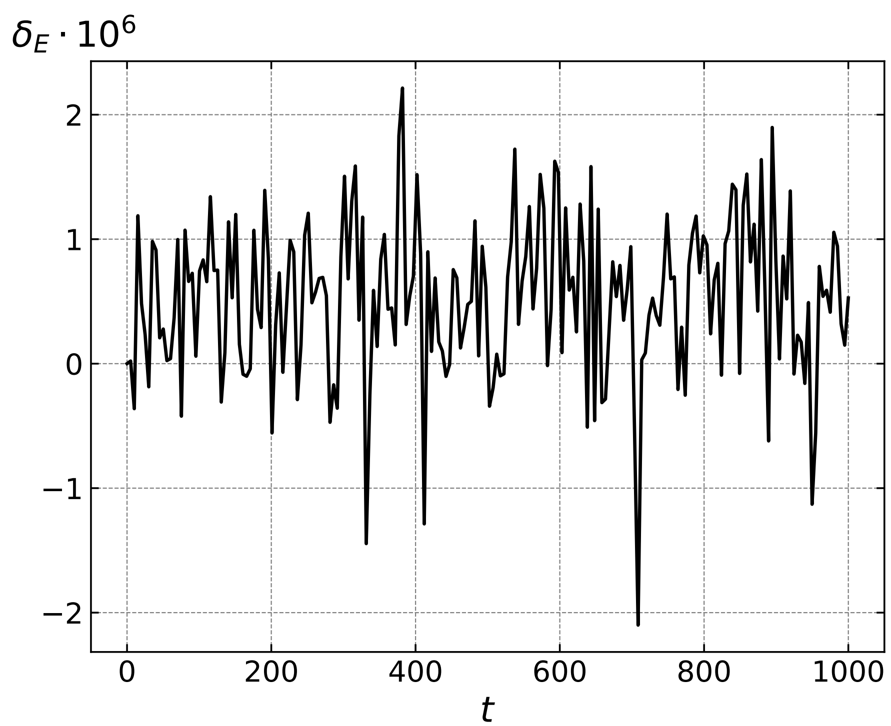
We use this to put the upper bound on the systematic error of the decay rate measurement.

Accuracy of numerical scheme

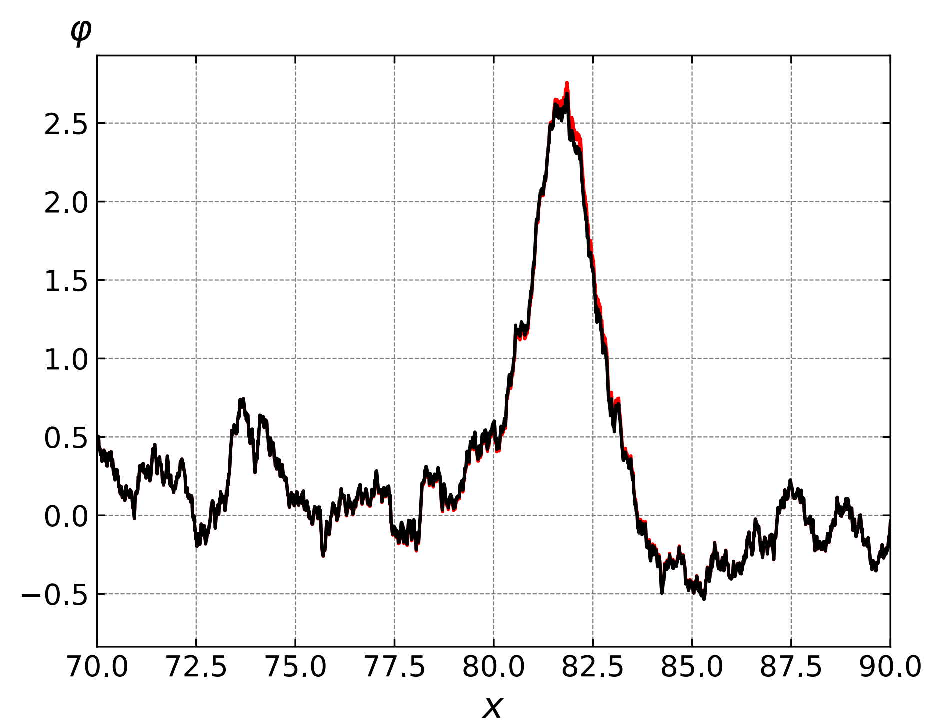
Hamiltonian dynamics

We use the 4th order pseudo-spectral, operator-splitting scheme.

The plots below show that it is enough to take $h/a \simeq 0.8$ to achieve the relative energy non-conservation $\lesssim 10^{-6}$.



Relative energy variation



Two decaying configurations evolved from the same initial state, with $h/a = 0.4, 0.8$.

Accuracy of numerical scheme

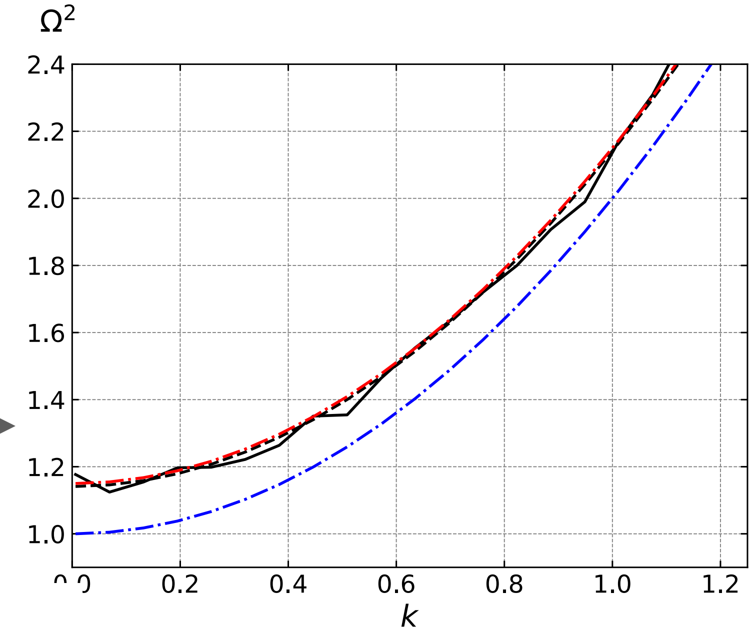
Langevin dynamics

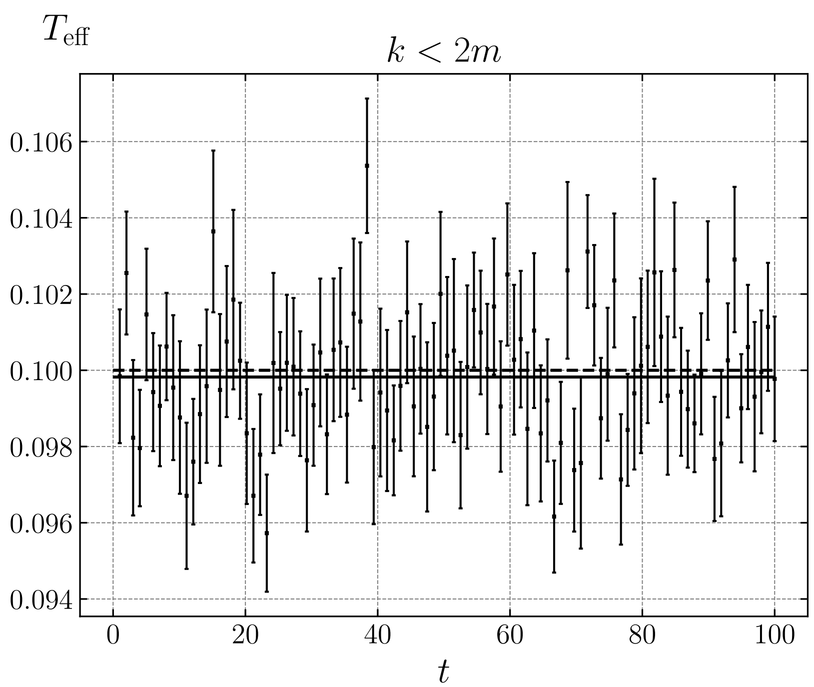
We use the 3rd strong order pseudo-spectral, operator-splitting scheme.

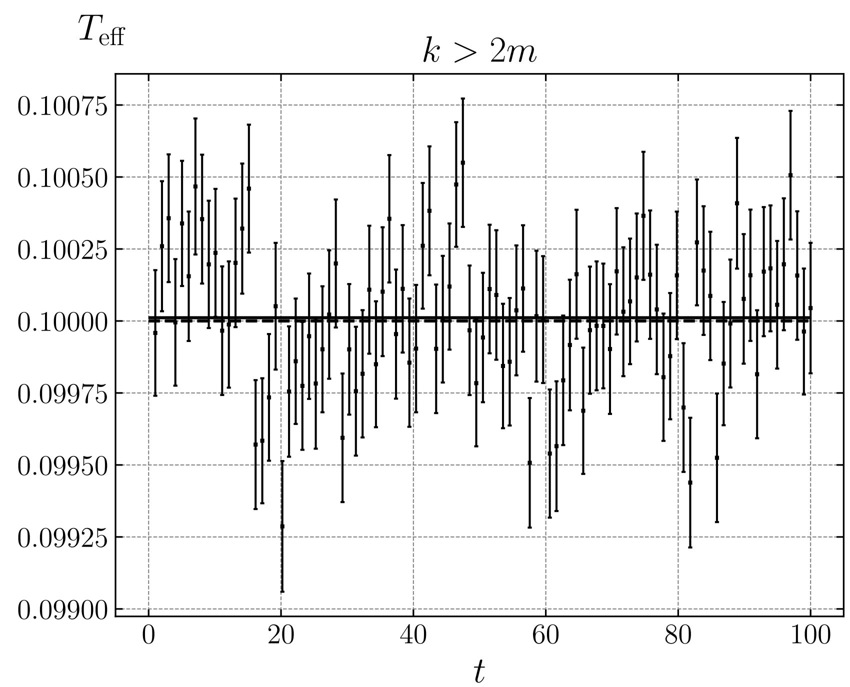
We took it from [Telatovich, Li, 1706.04237] but corrected their mistake.

The timestep is $h/a \simeq 0.25$ at $\eta \lesssim 1$ and $h/a \simeq 0.1$ at $\eta > 1$.

Dispersion relation measured in simulations (black), compared with the free (blue) and thermally-corrected (red) ones.







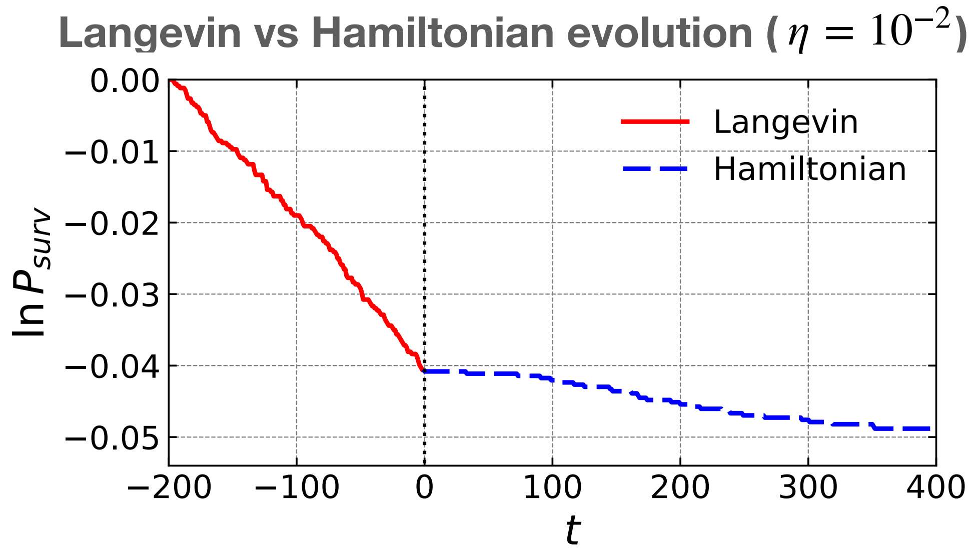
Effective temp. of long and short modes measured during the simulation.

Langevin vs Hamiltonian evolution

Let's make the following numerical experiment.

- Evolve the ensemble with non-zero η for $t \gg \eta^{-1}$ so that all surviving configurations reach equilibrium with the heat bath.

- Decouple the ensemble from the heat bath by setting $\eta = 0$.



The decay rate changes abruptly to the one that we got before for the Hamiltonian evolution.

Thus, the deviation of the rate from equilibrium is really due to the field dynamics near the barrier.

More observables

- **Shape of the critical bubble** $\phi_b(x)$

  Should we compute the bubble using the bare potential or an effective potential?

  If effective, which fields to include and when?

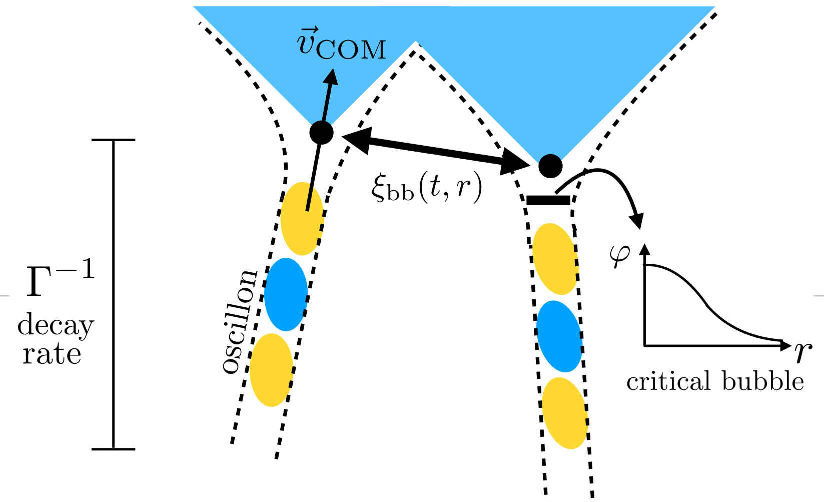
- **Dynamics of bubble nucleation**

  Euclidean theory tells us little about how the critical bubble actually forms out of thermal fluctuations.

  This dynamics is quite interesting: bubble velocities, oscillonic precursors…

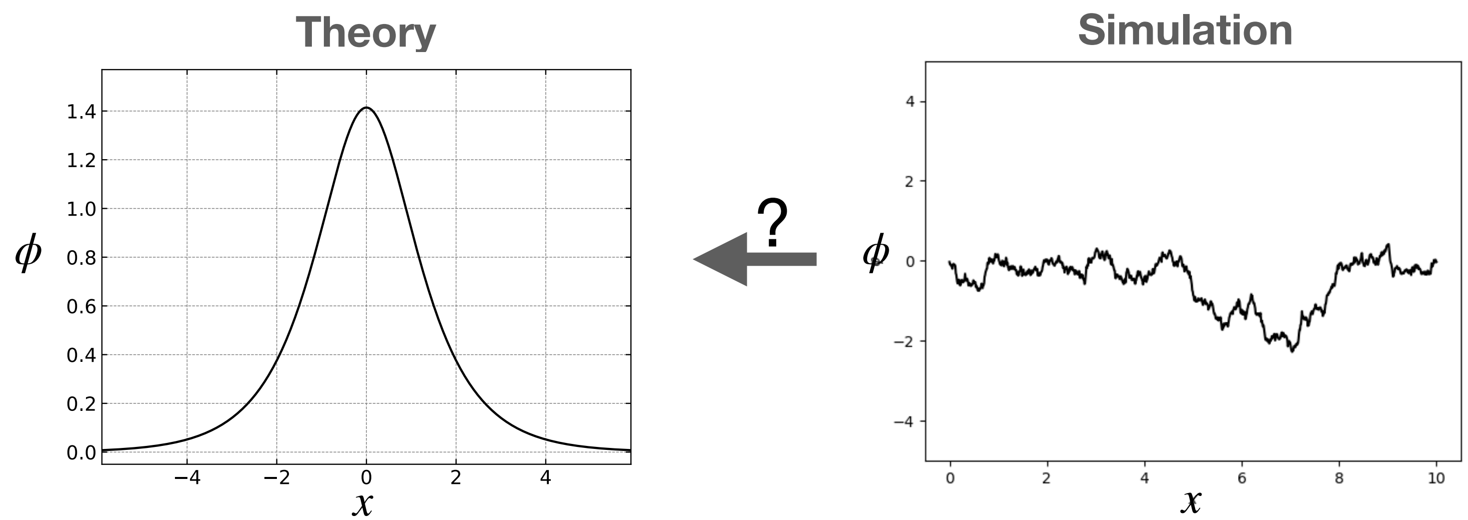
  Gleiser, Kolb… hep-ph/0409179, 0708.3844

  Johnson, Pîrvu, Sibiryakov, 2312.13364



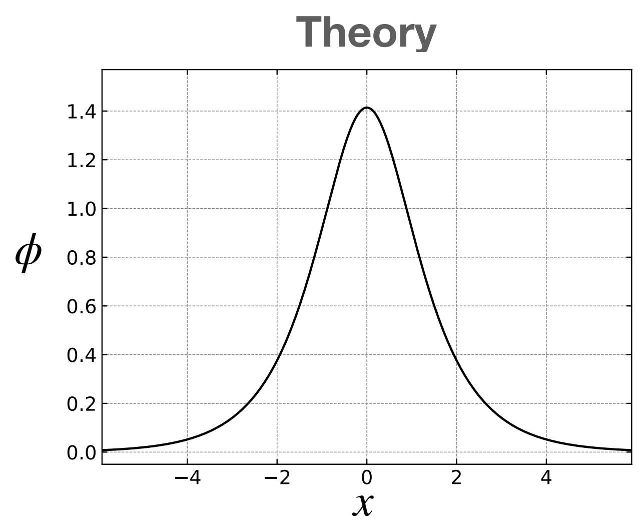
from 2312.13364

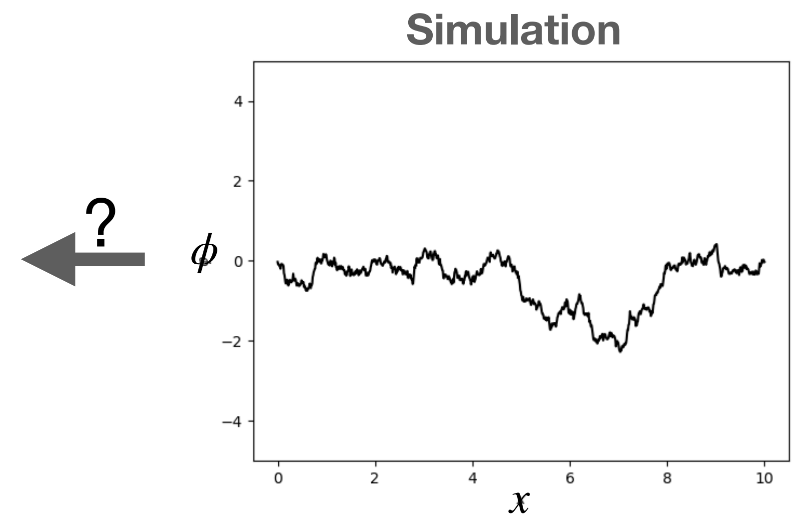
Critical bubble profile



Take many simulations, **synchronise** them in space and time, produce the average.

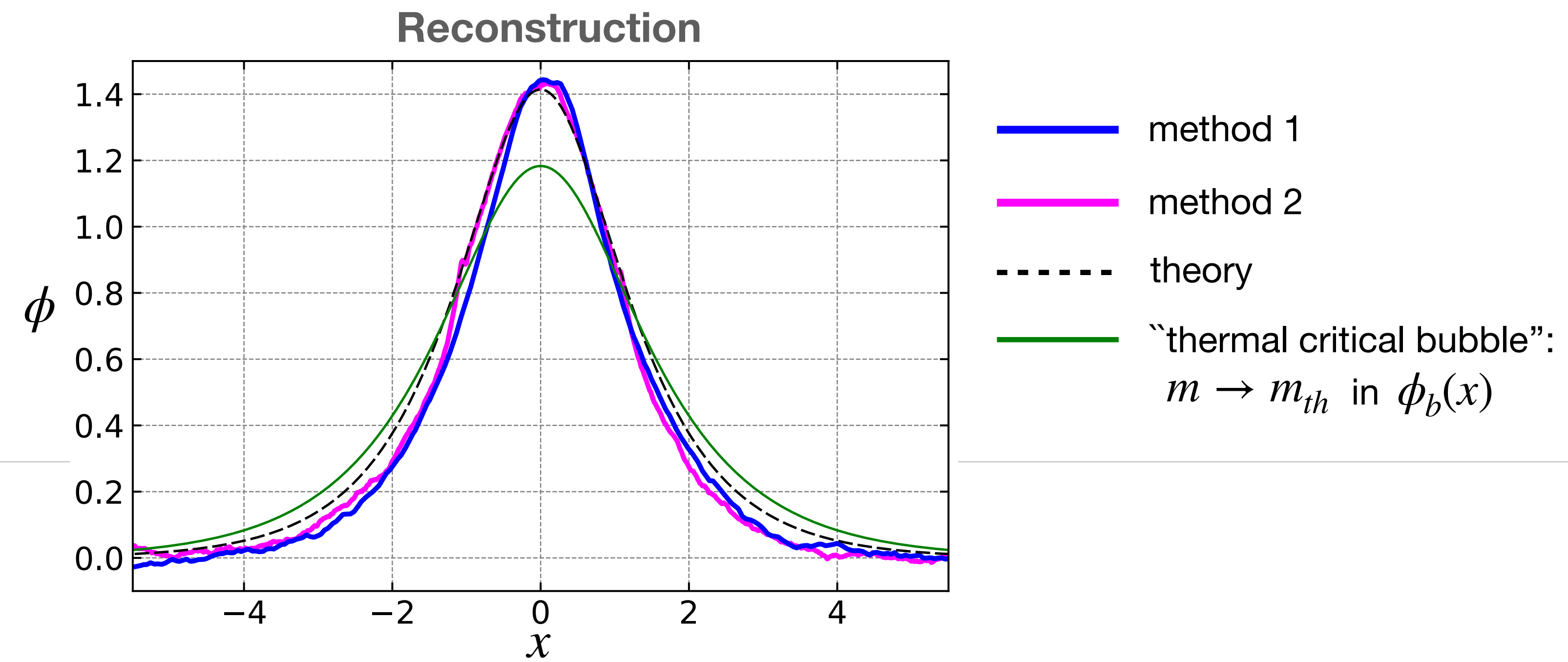
Critical bubble profile





Take many simulations, **synchronise** them in space and time, produce the average, pinpoint the critical bubble.

We employ two different reconstruction routines. They agree with each other and with the Euclidean prediction.



No surprise here: the critical bubble is determined by the bare potential; fluctuations contribute to the prefactor.

Things can be different with many fields!