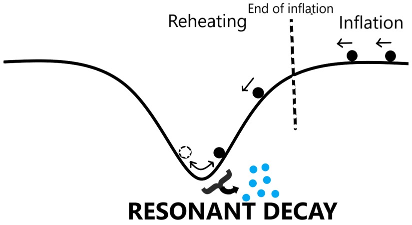
Imperial College London

10 September 2024
at Particle Production in the Early Universe, CERN

Non-Gaussianity from preheating with scale dependence

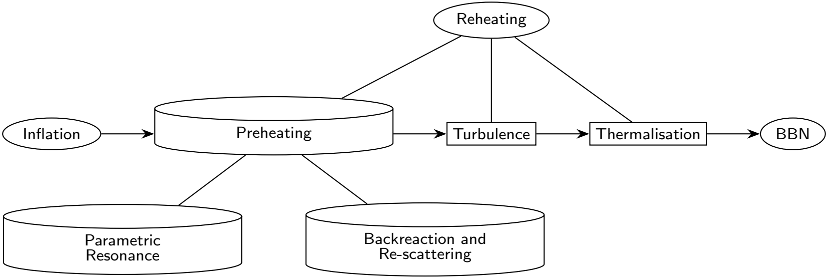


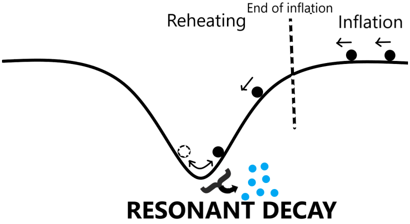
Pulkit S. Ghoderao
with Arttu Rajantie

Based on JCAP05 (2024) 106 (arXiv:2311.02173)

Preheating

What is it?





$$V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

Preheating

Why study it?

- Inflationary observables (tensor-to-scalar ratio, non-Gaussianity) are proportional to slow-roll parameters which are generic across models
- To distinguish between different models look for non-Gaussianity generated from preheating instead
- Found to have large detectable non-Gaussianity in massless preheating model *Chambers and Rajantie,2008 Bond et al.,2009*
- Integral part of our universe's evolution

Non-Gaussianity

The observable we wish to calculate

Curvature perturbation

$$\zeta = \delta N \equiv N - \overline{N}, \text{ where } N \equiv \ln\left(\frac{a(\rho_{\mathrm{ref}})}{a_{\mathrm{ini}}}\right)$$

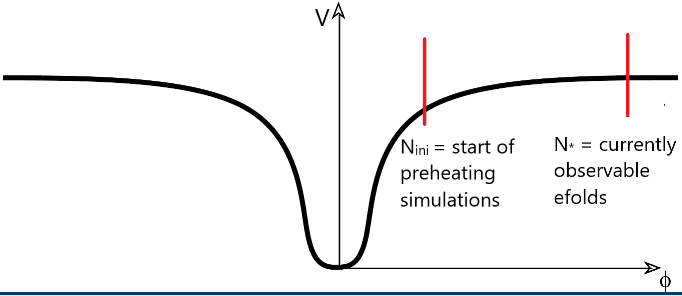
Non-Gaussianity parameter

$$f_{\mathrm{NL}} \sim -\frac{5}{6}\frac{\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle}{(\langle\zeta(k_1)\zeta(k_2)\rangle + \mathrm{perms})^2}$$

Scales

Separate Universe picture

$k < a_* H_*$: Larger than observable universe
$a_{\text{ini}} H_{\text{ini}} < k < a_* H_*$: Constitute separate universes
$k > a_{\text{ini}} H_{\text{ini}}$: Shorter than a single separate universe



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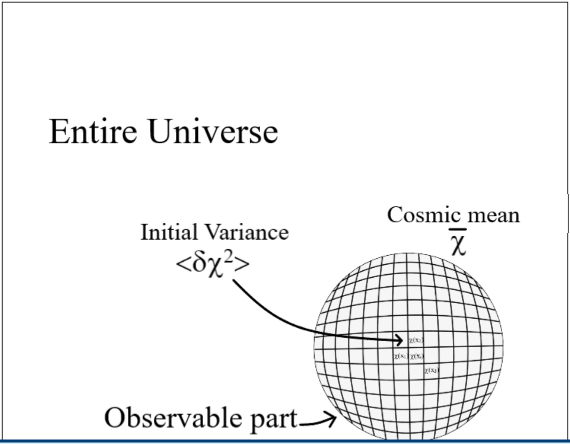

Populating the separate universes

- Spectator has quantum fluctuations at initial time slice

$$\text{Variance } \Sigma = \int \mathcal{P}_\chi(k) \frac{dk}{k}$$

Calculable from evolution during inflationary stage

$$Eg.\ \Sigma = \frac{H^2}{4\pi^2}\mathcal{N} \quad \text{(for scale-invariant power spectrum)}$$

- $\zeta(x) = \delta N(\chi(x))$
- Populate the Hubble volumes with Gaussian pdf

$$p_G(\chi) = \frac{1}{\sqrt{2\pi\langle\delta\chi^2\rangle}} \exp\left(-\frac{(\chi - \overline{\chi})^2}{2\langle\delta\chi^2\rangle}\right) \qquad (1)$$

Scale dependence during inflation I

Initial Conditions

Mode equation: $$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k + m_\chi^2(\phi)\chi_k = 0 \qquad (2)$$

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1) t < horizon crossing 2) horizon crossing < t < initial slice
Cosmic Variance

$$\langle \overline{\chi}^2 \rangle = \int_{\mathcal{N}_*}^{\infty} \underbrace{\frac{H(\mathcal{N}_k)^2}{4\pi^2}}_{\text{at horizon crossing}} \overbrace{e^{-3F(\mathcal{N}_k,\mathcal{N}_{\text{ini}})}}^{\text{over-damped envelope}} \underbrace{\left(\frac{1}{H(\mathcal{N}_k)}\frac{dH(\mathcal{N}_k)}{d\mathcal{N}_k} - 1\right)}_{dk/k} d\mathcal{N}_k \qquad (3)$$

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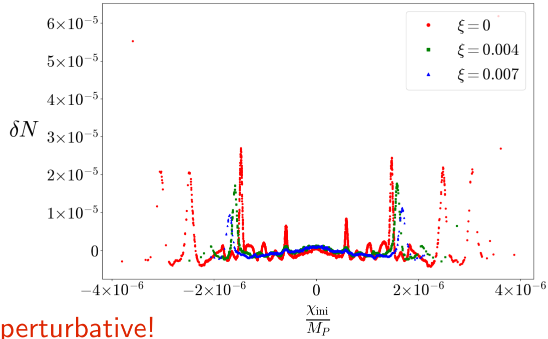
Initial Variance

$$\langle \delta\chi^2 \rangle = \int_{\mathcal{N}_{\text{ini}}}^{\mathcal{N}_*} \frac{H(\mathcal{N}_k)^2}{4\pi^2} e^{-3F(\mathcal{N}_k,\mathcal{N}_{\text{ini}})} \left(\frac{1}{H(\mathcal{N}_k)}\frac{dH(\mathcal{N}_k)}{d\mathcal{N}_k} - 1\right) d\mathcal{N}_k \qquad (4)$$

Lattice simulations

Full non-linear simulations in real space

- Divide observable universe into $\sim$ 10,000 separate universes
- Run a lattice simulation with different χ_{ini} value in each separate universe
- Plot how much each universe expands by (δN) as a function of χ_{ini}



Non-perturbative!

Non-perturbative delta N formalism

Going from $N(\chi)$ to $f_{\rm NL}$

$$\langle \zeta_\chi(\vec{x}_1)\cdots\zeta_\chi(\vec{x}_n)\rangle = \int d\chi_1\cdots d\chi_n \delta N(\chi_1)\cdots\delta N(\chi_n) p(\chi_1,\ldots,\chi_n)$$

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$$p = p_G(\chi_1, ..., \chi_n) = \frac{1}{(2\pi)^{n/2}\sqrt{\det \Sigma_{ij}}} \exp\left(-\frac{1}{2}(\chi_i - \overline{\chi}_i)\Sigma^{-1}_{ij}(\chi_j - \overline{\chi}_j)\right)$$

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Expand in terms of correlator Σ_{ij}

$$\langle \zeta_\chi(\vec{x}_1)\zeta_\chi(\vec{x}_2) \rangle = \tilde{N}_\chi^2 \Sigma_{12} + \frac{1}{2}\tilde{N}_{\chi\chi}^2 \Sigma_{12}^2 + \frac{1}{4}\tilde{N}_{\chi\chi\chi}^2 \Sigma_{12}^3 + \text{Order}(\Sigma^4), \quad (5)$$

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where non-perturbative coefficients are:

$$\tilde{N}_\chi = \frac{1}{\Sigma}\int d\chi\, p_G(\chi)\,\delta\chi\,\delta N(\chi),\;\; \tilde{N}_{\chi\chi} = \frac{1}{\Sigma^2}\int d\chi\, p_G(\chi)\,\delta\chi^2\,\delta N(\chi),\;\; etc.$$

Similarly the three point curvature correlator

Similarly the three point curvature correlator

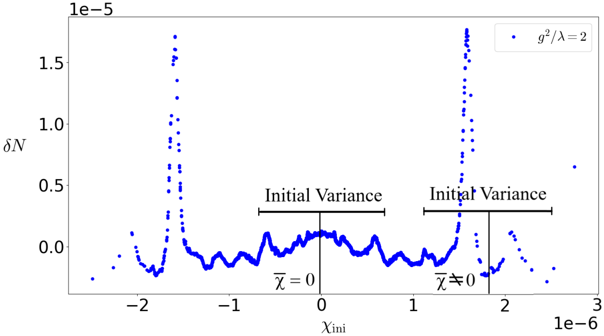
$$\langle \zeta_\chi(\vec{x}_1)\zeta_\chi(\vec{x}_2)\zeta_\chi(\vec{x}_3)\rangle =$$
$$\tilde{N}_\chi \tilde{N}_{\chi\chi} \tilde{N}_\chi (\Sigma_{12}\Sigma_{23} + \text{perms}) +$$
$$\frac{1}{2}\tilde{N}_{\chi\chi}\tilde{N}_{\chi\chi\chi}\tilde{N}_\chi(\Sigma_{12}^2\Sigma_{23} + \text{perms}) + \tilde{N}_{\chi\chi}\tilde{N}_{\chi\chi}\tilde{N}_{\chi\chi}(\Sigma_{12}\Sigma_{23}\Sigma_{31}) + \text{Order}(\Sigma^4), \tag{6}$$

At leading order *Imrith, Mulryne and Rajantie,2019*

$$f_{\rm NL} = -\frac{5}{6}\tilde{N}_\chi^2 \tilde{N}_{\chi\chi}\frac{\mathcal{P}_\chi(k)^2}{\mathcal{P}_\zeta(k)^2}$$

Provided mean is non-zero (not true for all inflationary models)

- If mean is zero and
- potential is symmetric (as happens for most inflationary models)



Odd coefficients: $\tilde{N}_\chi = \frac{1}{\Sigma}\int d\chi\, p_G(\chi)\delta\chi\delta N(\chi) \to 0$, $\tilde{N}_{\chi\chi\chi} \to 0$

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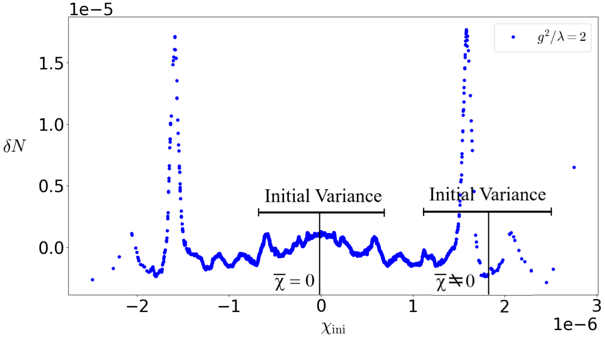

Odd coefficients: $\tilde{N}_\chi = \frac{1}{\Sigma} \int d\chi\, p_G(\chi)\delta\chi\delta N(\chi) \to 0$, $\tilde{N}_{\chi\chi\chi} \to 0$

Go to higher order in expansion

$$f_{\rm NL} = -\frac{5}{6}\frac{\tilde{N}^3_{\chi\chi}}{P_\zeta(k)^2}\int \frac{d^3q}{(2\pi)^3}\,\Sigma(q)\Sigma(|\vec{q}-\vec{k}_1|)\Sigma(|\vec{q}+\vec{k}_3|) \quad (7)$$

Scale dependence during inflation II

In calculating f_{NL}

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In calculating $f_{\rm NL}$

Scale dependence not included *Boubekeur and Lyth,2006*:

$$\Sigma(k) = \frac{2\pi^2 \mathcal{P}_\chi}{k^3}, \quad \mathcal{P}_\chi = \frac{H_*^2}{4\pi^2},$$

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giving

$$f_{\rm NL} \approx -\frac{5}{6} \frac{\tilde{N}_{\chi\chi}^3 \mathcal{P}_\chi^3}{\mathcal{P}_*^2} \int_{L^{-1}}^{k_*} \frac{dq}{q} = -\frac{5}{6} \frac{\tilde{N}_{\chi\chi}^3 \mathcal{P}_\chi^3}{\mathcal{P}_*^2} \ln\left(k_* L\right), \tag{8}$$

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$$f_{\mathrm{NL}} \approx -\frac{5}{6}\frac{\tilde{N}_{\chi\chi}^3 \mathcal{A}_\chi^3 k_*^{2n_\chi}}{\mathcal{P}_*^2}\int_0^{k_*}\frac{dq}{q^{1-n_\chi}} = -\frac{5}{6}\frac{\tilde{N}_{\chi\chi}^3 \mathcal{A}_\chi^3}{n_\chi \mathcal{P}_*^2} k_*^{3n_\chi} \qquad (9)$$

Our preheating model

Motivated by non-minimal coupling to gravity

- $\xi\phi^2 R$ term in action $\rightarrow$ makes it observationally viable
- Assuming $\xi << 1$
- In Einstein frame,

$$V(\phi,\chi) = \frac{\lambda}{4}\left(\frac{M_P}{\sqrt{\xi}}\tanh(\frac{\sqrt{\xi}}{M_P}\phi)\right)^4 + \frac{g^2}{2}\chi^2\left(\frac{M_P}{\sqrt{\xi}}\tanh(\frac{\sqrt{\xi}}{M_P}\phi)\right)^2 \tag{10}$$

Application of our formalism to model

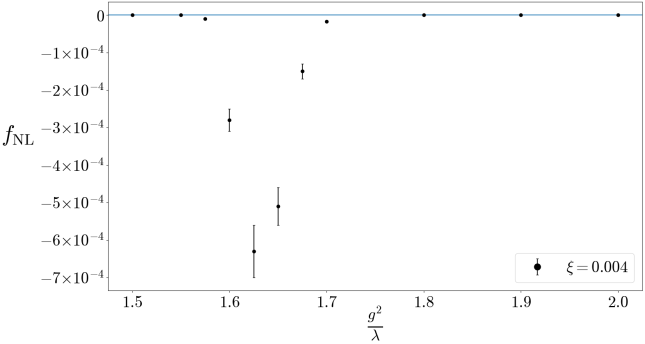
- Cosmic variance is negligible compared to initial variance,

$$\langle\overline{\chi}^2\rangle << \langle\delta\chi^2\rangle$$

  Eg. For $g^2/\lambda = 2$ and $\xi = 0.004$,
  $\langle\overline{\chi}^2\rangle = 2.205 \times 10^{-15} M_P^2$, $\langle\delta\chi^2\rangle = 4.176 \times 10^{-13} M_P^2$

- f_{NL} is largest for the lowest possible ξ allowed by tensor-to-scalar observations $\xi > 0.004$.

f_{NL} vs g^2/λ plot



- For $g^2/\lambda = 2$ and $\xi = 0.004$, scale invariant approximation gives $f_{\mathrm{NL}} \sim$ Order(1). But including scale dependence gives $f_{\mathrm{NL}} \sim$ Order(10^{-9}) Quite significant change!
- Scanning over g^2/λ, f_{NL} is largest around $g^2/\lambda = 1.625$ given by $f_{\mathrm{NL}} = -(6.3 \pm 0.7) \times 10^{-4}$ Actual numeric value! that is also significantly greater from the above $g^2/\lambda = 2$ value.

Outlook

- Current observational bound on $f_{\mathrm{NL}}^{\mathrm{local}} = -0.9 \pm 5.1$ *Akrami et al.,2020*
- However NASA's SPHEREx mission set to launch next year will bring error to $< 1 \implies$ Both detection and non-detection will rule out models of inflation based on preheating
- Exact numerical values obtained through the formalism can distinguish between inflationary models
- The model we considered did not yield sufficiently high f_{NL} to be detectable in the near future
- Our method can be readily applied to any other inflation and reheating model perhaps yielding high, detectable non-Gaussianity

Recipe

Arbitrary inflation theory $\rightarrow$ Formalism $\rightarrow$ f_{NL} $\leftarrow$ Observations

1. Calculate cosmic variance and initial variance from inflation (Negligible cosmic variance forces mean to be zero)
2. Draw a bunch of initial χ values ($\sim 10,000$) from Gaussian distribution with appropriate mean and calculated initial variance
3. Perform a separate lattice simulation for each of the χ values and collate $N(\chi)$
4. If cosmic variance is non-zero, use the leading order f_{NL} formula. If cosmic variance is zero use the next order f_{NL} formula
5. Compare with observed f_{NL} to keep or rule out inflationary model

Thank you for your attention!

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Extra slides

Assuming fixed H_0, $\Sigma(k) = A/k^{3-n_s}$ where,

$$A = \frac{H_0^{2-n_s}}{2} \exp\left(3\left(N_{\text{crit}} + \frac{\sqrt{32\xi+1}-1}{48\xi}\sqrt{9-48\frac{g^2}{\lambda}\sqrt{\frac{\lambda}{12}}\frac{M_P}{H_0}} - \frac{2\frac{g^2}{\lambda}}{\sqrt{9-48\frac{g^2}{\lambda}\xi}}\tanh^{-1}\left(\sqrt{\frac{3-16\frac{g^2}{\lambda}\sqrt{\frac{\lambda}{12}}\frac{M_P}{H_0}}{3-16\frac{g^2}{\lambda}\xi}}\right)\right.\right. \tag{11}$$

$$n_s = 3 - \sqrt{9-48\frac{g^2}{\lambda}\xi\left(\frac{16\xi N_{\text{obs}}+\sqrt{32\xi+1}+1}{16\xi N_{\text{obs}}+\sqrt{32\xi+1}-1}\right)} \tag{12}$$

Parameter constraints from Planck 2018 observations

- Inflaton power spectrum, $\mathcal{P}_* = 2.1 \times 10^{-9} \implies \lambda$ is completely fixed by a particular choice of ξ irrespective of g^2/λ
- Tensor to scalar ratio, $r < 0.1 \implies \xi > 0.004$ irrespective of g^2/λ
- $k_{\rm phys} = 0.05 \mathrm{MPc}^{-1}$
  $k_* = (a_0/a_{\rm end}) k_{\rm phys}$
  Assuming instantaneous reheating Dai, Kamionkowski and Wang 2014,

$$\ln\left(\frac{a_0}{a_{\rm end}}\right) = \frac{1}{4}\ln\left(\frac{30}{g_{\rm eff}\pi^2}\right) + \frac{1}{3}\ln\left(\frac{11}{43}g_{\rm eff}\right) + \ln\left(\frac{\rho_{\rm end}^{1/4}}{T_0}\right) \approx 65, \tag{13}$$

with $g_{\rm eff} \sim 100,\ T_0 = 2.725K$ and

$$\rho_{\rm end} = \frac{3\lambda}{8}\left(\frac{M_P}{\sqrt{\xi}} \tanh\left(\frac{\sqrt{\xi}}{M_P}\phi_{\rm end}\right)\right)^4. \tag{14}$$

Therefore $k_* \sim 2.225 \times 10^{-30} M_P$.

HLattice

Program written in FORTRAN language. Simulates scalar fields and gravity during inflation and reheating.

- Variable evolved: $\beta_{ij} = \ln(g_{ij})$, where 3×3 metric $g_{ij} = a(t)^2(\delta_{ij} + h_{ij})$ is in synchronous gauge
- Scale factor at each step: $a(t) = \frac{1}{L^3}\left(\int \sqrt{g} d^3x\right)^{1/3}$
- Spatial gradients using a specified discretisation scheme
- Symplectic sixth order integrator with fourth order Runge-Kutta integrator to obtain β_{ij} at each time step

Preheating

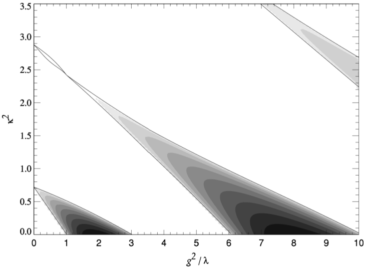
Perturbative introduction *Greene et al.,1997*

flat FRW metric: $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$; $H = \dot{a}/a$

- Divide into background and fluctuation
  $\phi = \overline{\phi} + \delta\phi, \chi = \overline{\chi} + \delta\chi$
- Equation of motion for inflaton background
  $\ddot{\overline{\phi}} + 3H\dot{\overline{\phi}} + \lambda\overline{\phi}^3 = 0 \leftarrow (dt = a d\tau, \phi = \varphi/a) \rightarrow \overline{\varphi}'' + \lambda\overline{\varphi}^3 = 0$
  $$\overline{\varphi} = \mathrm{cn}(\tau, 1/\sqrt{2})$$
- Quantise fluctuations:
  $$\delta\chi \sim \int d^3k\ (\hat{a}_k\chi_k + \hat{a}_k^\dagger\chi_k^*)$$
- First order fluctuations:
  $$\delta\chi_k'' + \left(\kappa^2 + \frac{g^2}{\lambda}\overline{\varphi}\right)\delta\chi_k = 0$$

- Second order differential equation with oscillating term $\Longrightarrow$ Floquet theory

$$\delta\chi_k \sim e^{\mu(g^2/\lambda,\kappa)\tau}$$



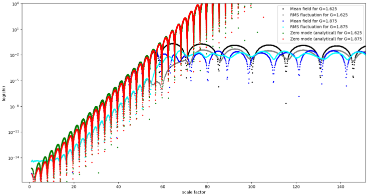
(Fig. credit: *Greene et al.,1997*)

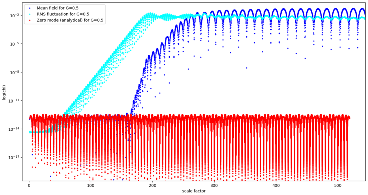
Parametric Resonance

Preheating

Full non-linear simulations in real space

Smallness of coupling constant $\lambda \to$ Growth of quantum modes $\to$ Large occupation numbers $\to$ Transition to semi-classical behaviour *Polarski and Starobinsky,1996* $\to$ Classical simulations with random Gaussian initial conditions *Khlebnikov and Tkachev,1996*



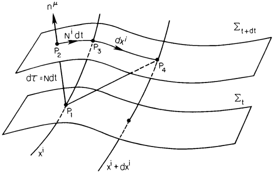


Evolution of field χ in and out of resonance with parameter g^2/λ

Separate Universe Approximation

$a(t,x) = a(t)$ at different x

- Slices and threads: ADM formalism $3+1$ split



(Image credit: *Kolb and Turner,1990*)

$$ds^2 = -\mathcal{N}^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- Causality $\implies$ Separate Hubble volumes
- Zeroeth order expansion of ADM equations in k/aH
  $\implies$ each Hubble volume is separately FRW

delta N formalism

- local vs background scale factor

$$\tilde{a}(t,x) = a(t)e^{\psi(t,x)} \tag{15}$$

- local number of efolds

$$\begin{aligned}\tilde{N}(t_1 \to t_2; x) &= \int_{t_1}^{t_2} \mathcal{N} dt\ \tilde{H} = \int_{t_1}^{t_2} \mathcal{N} dt \frac{1}{\tilde{a}} \frac{d\tilde{a}}{\mathcal{N} dt} \\ &= \int_{t_1}^{t_2} dt \left(\frac{\dot{a}}{a} + \dot{\psi} \right) = \ln\left(\frac{a(t_2)}{a(t_1)} \right) + \psi(t_2, x) - \psi(t_1, x)\end{aligned} \tag{16}$$

- delta N formula

$$\delta N(t_1 \to t_2, x) = \psi(t_2) - \psi(t_1) \tag{17}$$