Phenomenology of Inflaton Fragmentation during Reheating

Marcos A. G. García

+ M. Pierre (DESY), A. Pereyra, F. Barreto (IFUNAM), K. Olive (Minnesota), Y. Mambrini, M. Gross, J.-H. Yoon (IJCLab)

2306.08038, 2308.16231, 2403.04848, 2404.16932

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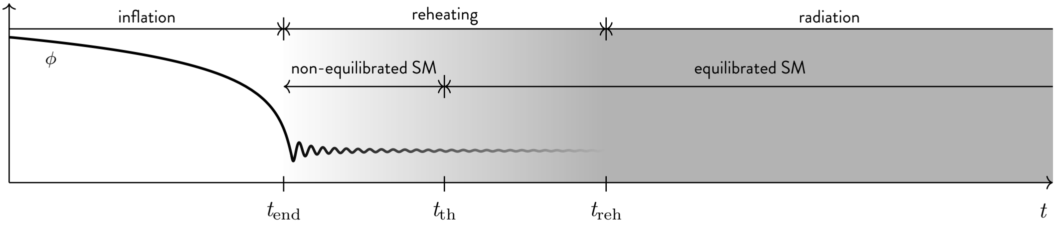
Inflation and reheating



Inflation: a slowly rolling scalar field

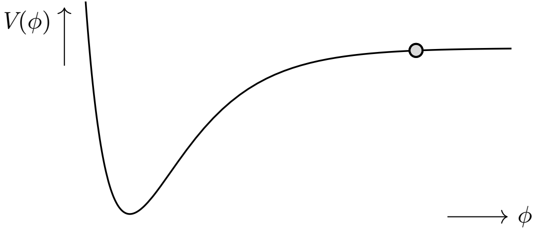
$$V(\phi) = \frac{3}{4} m_\phi^2 M_P^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^2 \quad \text{(Starobinsky)}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\phi}{3M_P^2}\right)^{1/2}$$

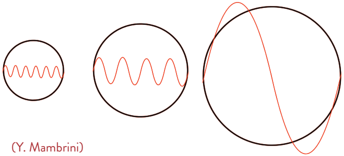
$$\frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho_\phi$$

$$\frac{1}{2}\dot{\phi}^2 - V(\phi) = p_\phi$$

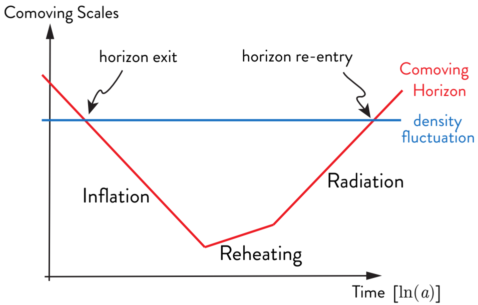


Primordial fluctuations

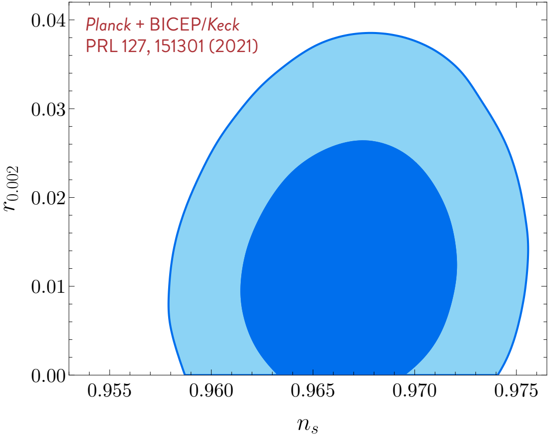
Quantum fluctuations in ϕ, g are stretched by the expansion



(Y. Mambrini)



Reheating represents the main theoretical uncertainty for a given inflation model



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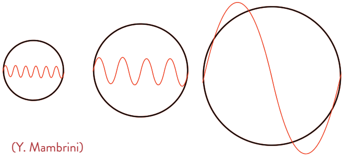

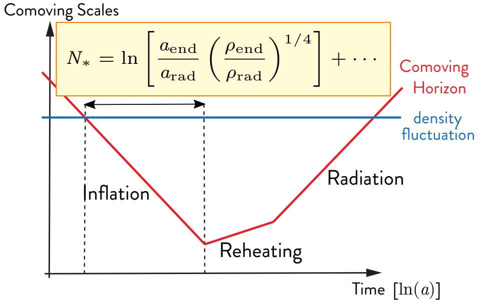

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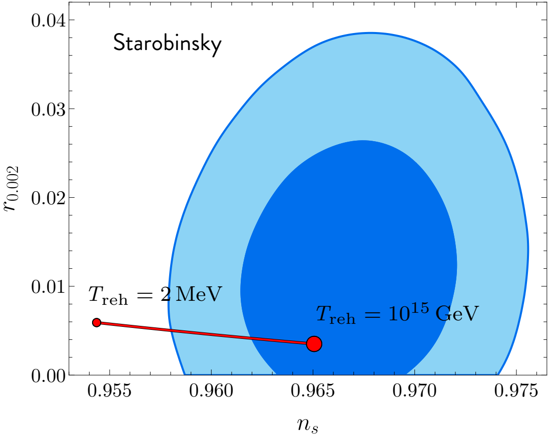
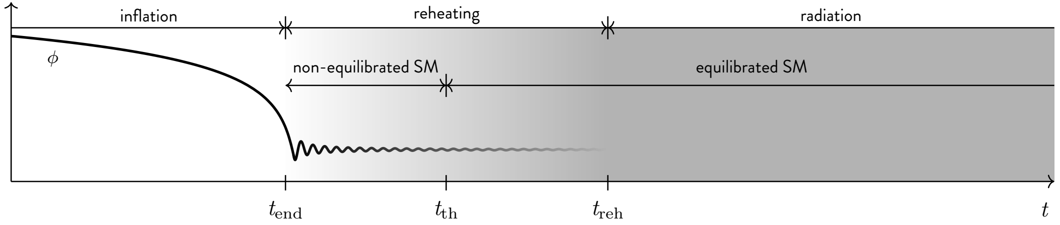
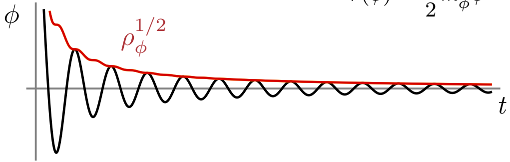


Coherent oscillations



The end of inflation: $\ddot{a} = 0 \quad \Leftrightarrow \quad w = -1/3 \quad \Leftrightarrow \quad \dot{\phi}^2 = V(\phi)$

$$V(\phi) \approx \frac{1}{2} m_\phi^2 \phi^2$$



quadratic minimum ≈ matter

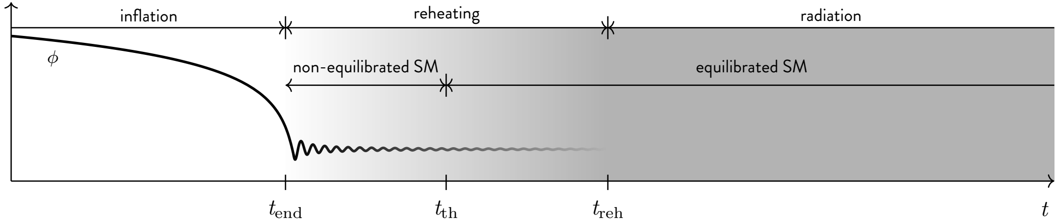
$$\phi(t) \simeq \phi_0(t)\mathcal{P}(t) = \phi_0(t)\cos(m_\phi t)$$

$$\langle \dot{\phi}^2 \rangle \simeq \langle \phi V'(\phi) \rangle = 2\langle V(\phi) \rangle$$

$$\rho_\phi \simeq 2\langle V(\phi) \rangle = V(\phi_0)$$

$$p_\phi \simeq 0$$

Coherent oscillations



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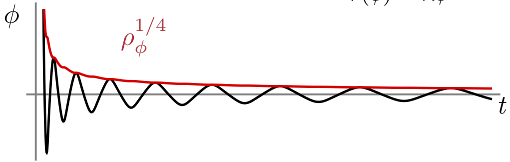
$V(\phi) \approx \lambda \phi^4$



quartic minimum ≈ radiation

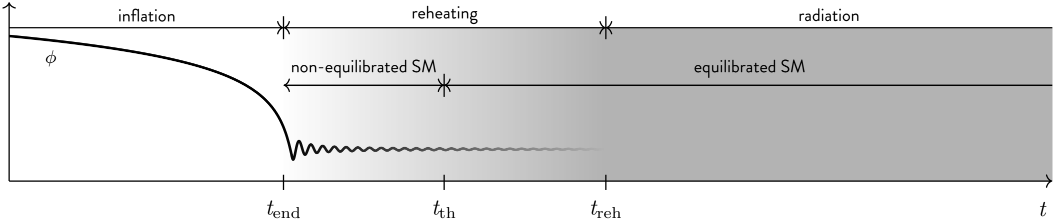
$$\phi(t) \simeq \phi_0(t)\mathcal{P}(t) = \phi_0(t)\sum_n \mathcal{P}_n e^{-in\omega_\phi t}$$

$$\langle \dot{\phi}^2 \rangle \simeq \langle \phi V'(\phi) \rangle = 4 \langle V(\phi) \rangle$$

$$\rho_\phi \simeq 3\langle V(\phi) \rangle = V(\phi_0)$$

$$p_\phi \simeq \rho_\phi / 3$$

Coherent oscillations



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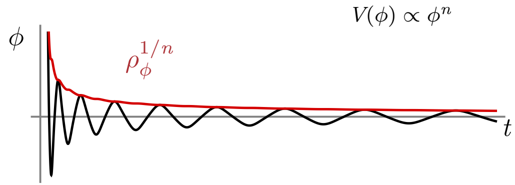


$$\phi(t) \simeq \phi_0(t)\mathcal{P}(t) = \phi_0(t)\sum_n \mathcal{P}_n e^{-in\omega_\phi t}$$

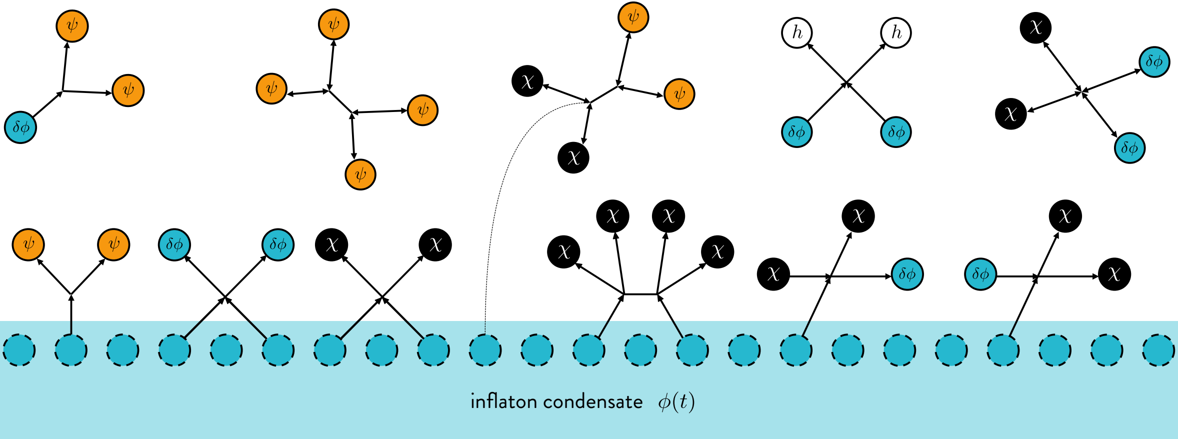
$$\langle \dot{\phi}^2 \rangle \simeq \langle \phi V'(\phi) \rangle$$

$$p_\phi \simeq \frac{n-2}{n+2}\rho_\phi \equiv w_\phi \rho_\phi$$

Particle production during reheating

self-interactions · with SM · with DM

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \lambda M_P^4\left(\frac{\phi}{M_P}\right)^n - y\phi\bar{\psi}\psi - \frac{1}{2}\sigma\phi^2\chi^2 + \cdots$$



1. Inflation 2. Reheating 3. Dark preheating 4. Self-resonance

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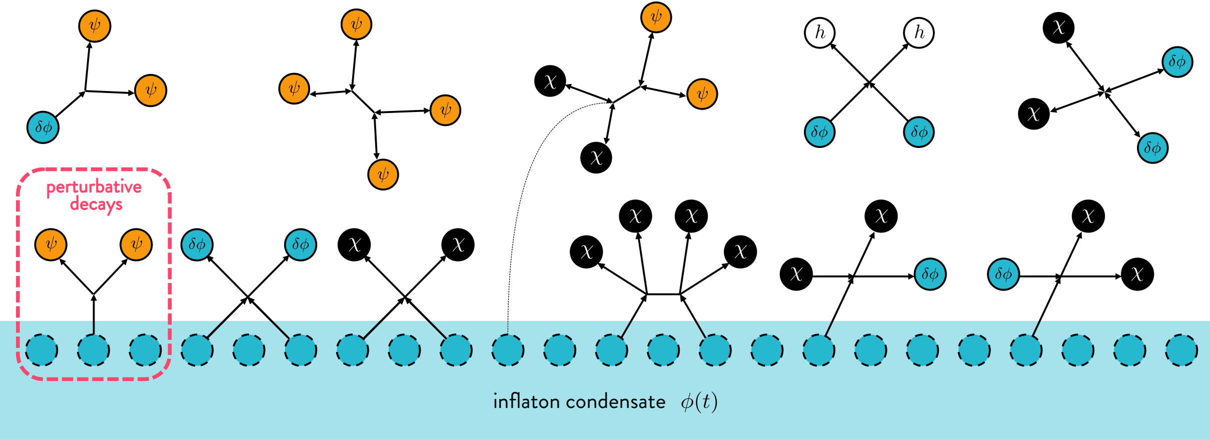


The perturbative picture

Phenomenological approach: decay of ϕ quanta $\longleftrightarrow$ friction (dissipation) in $\phi(t)$

$$\ddot{\phi} + (3H + \Gamma_\phi)\dot{\phi} + V'(\phi) \;=\; 0$$

$$\Downarrow$$

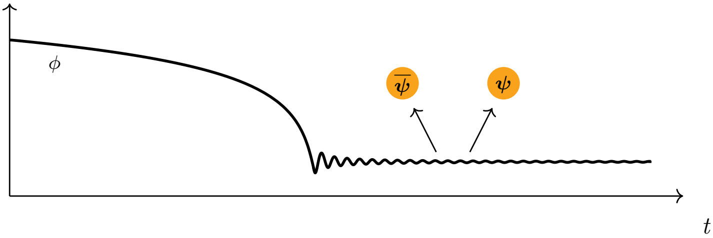
$$\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi \;=\; -\Gamma_\phi(1+w_\phi)\rho_\phi$$

Reheating as the exchange of energy between two ideal fluids

$$T^{\mu\nu} \;=\; T^{\mu\nu}_\phi + T^{\mu\nu}_R \;=\; \rho_\phi\,\mathrm{diag}(1, w_\phi, w_\phi, w_\phi) \;+\; \rho_R\,\mathrm{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})$$

$$\nabla_\mu T^{\mu\nu} \;=\; 0 \qquad \Rightarrow \qquad \dot{\rho}_R + 4H\rho_R \;=\; \Gamma_\phi(1+w_\phi)\rho_\phi$$

Decay of the condensate



Oscillation modes as quasi-particles

$$\phi(t)|0\rangle = \phi_0(t)\sum_n \mathcal{P}_n e^{-iK_n\cdot x}|0\rangle$$

$$K_n \equiv (n\omega_\phi, \mathbf{0})$$

$$\omega_\phi \propto m_\phi(t) \equiv \sqrt{V''(\phi_0(t))}$$

$$f_\phi(K) = (2\pi)^3 n_\phi(t)\delta^{(3)}(\boldsymbol{K})$$

$$\frac{\partial f_\psi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\psi}{\partial|\boldsymbol{P}|} = \frac{1}{P^0}\sum_{n=1}^{\infty}\int\frac{d^3\boldsymbol{K}}{(2\pi)^3 n_\phi}\frac{d^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(K_n - P - P')|\overline{\mathcal{M}_n}|^2 f_\phi(K)$$

$$\Downarrow$$

$$\dot{\rho}_\psi + 4H\rho_\psi = \sum_{n=1}^{\infty}\int\frac{d^3\boldsymbol{P}}{(2\pi)^3}\frac{d^3\boldsymbol{K}}{(2\pi)^3 n_\phi}\frac{d^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(K_n - P - P')|\overline{\mathcal{M}_n}|^2 f_\phi(K)$$

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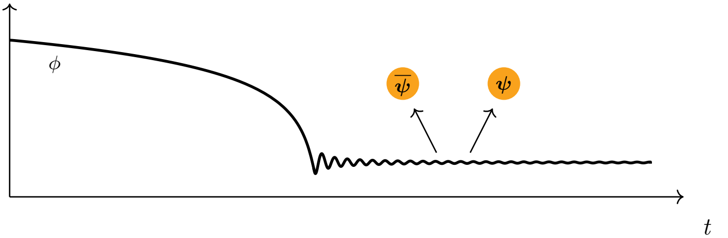

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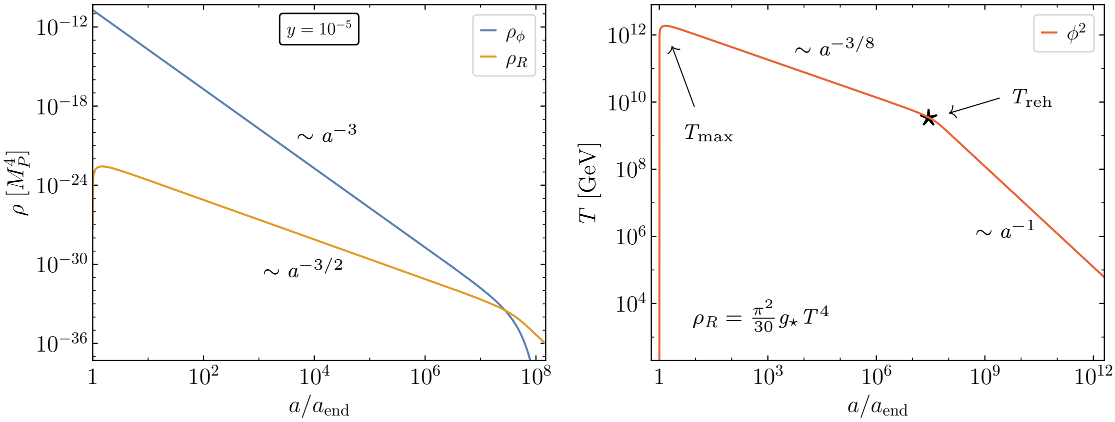
$$\Gamma_\phi = \frac{1}{8\pi(1+w_\phi)\rho_\phi}\sum_{n=1}^{\infty}\langle|\overline{\mathcal{M}_n}|^2 n\omega_\phi\beta_n\rangle\,, \quad \beta_n = \sqrt{\left(1-\frac{(m_1+m_2)^2}{n^2\omega_\phi^2}\right)\left(1-\frac{(m_1-m_2)^2}{n^2\omega_\phi^2}\right)}$$

MG, K. Kaneta, Y. Mambrini, K. Olive, JCAP 04 (2021), 012

Perturbative reheating

For $V(\phi) \propto \phi^2$

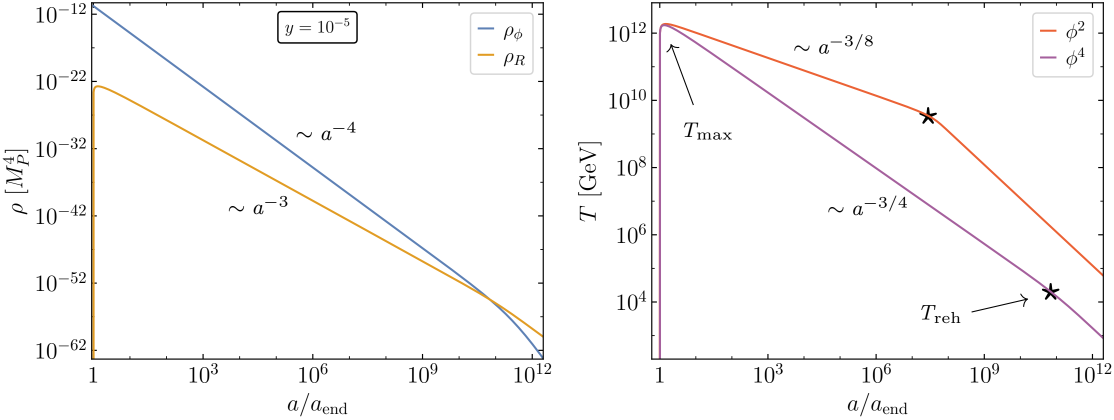
$$\Gamma_\phi = \frac{y^2}{8\pi} m_\phi = \text{const.}\,, \qquad T_{\rm reh} = y \left(\frac{9 m_\phi^2 M_P^2}{40 \pi^4 g_{\rm reh}} \right)^{1/4}$$



Perturbative reheating

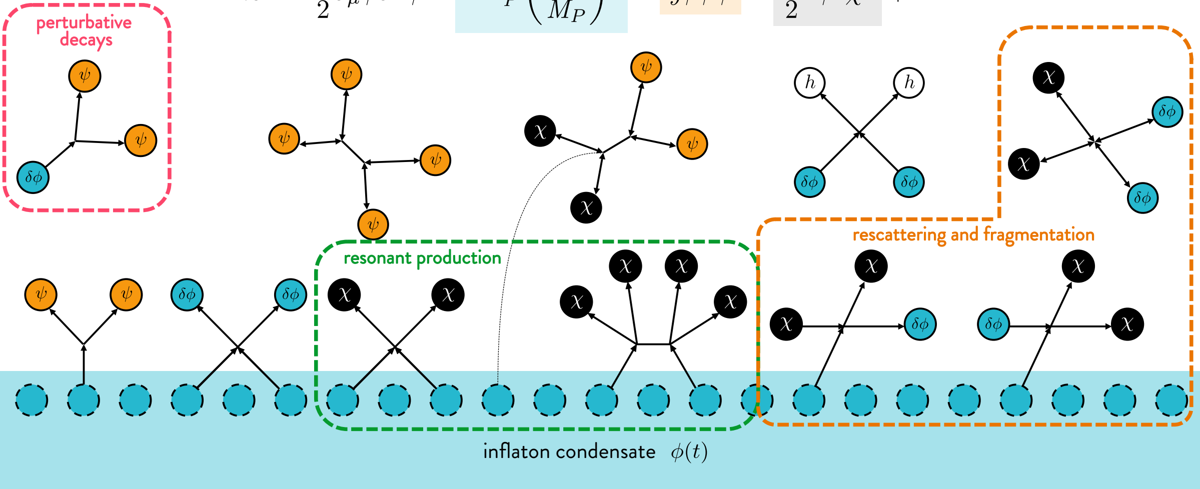
For $V(\phi) \propto \phi^4$

$$\Gamma_\phi = \alpha^2 \frac{y^2}{8\pi} m_\phi(t) \propto a^{-1}, \qquad T_{\rm reh} = \left(\frac{30\lambda}{\pi^6 g_{\rm reh}}\right)^{1/4} \alpha^2 y^2 M_P, \qquad \alpha \simeq 0.71$$



Fragmentation from dark preheating

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \lambda M_P^4\left(\frac{\phi}{M_P}\right)^2 - y\phi\bar{\psi}\psi - \frac{1}{2}\sigma\phi^2\chi^2 + \cdots$$



Resonant particle production

Bosonic effects can exponentially enhance the rate at which particles are produced from the oscillating inflaton

The dark matter field $\chi(t, \mathbf{x})$ satisfies the Heisenberg equation of motion

$$\left(\frac{d^2}{dt^2} - \frac{\nabla^2}{a^2} + 3H\frac{d}{dt} + m_\chi^2 + \sigma\phi^2 \right) \chi \;=\; 0$$

In conformal time, $d\tau \;=\; dt/a$, introducing

$$X(\tau, \mathbf{x}) \;=\; a\,\chi(\tau, \mathbf{x}) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\, e^{-i\boldsymbol{k}\cdot\mathbf{x}} \left[X_k(\tau)\hat{A}_{\boldsymbol{k}} + X_k^*(\tau)\hat{A}^\dagger_{-\boldsymbol{k}} \right]$$

the equation of motion is

$$X_k'' + \underbrace{\left[k^2 - \frac{a''}{a} + a^2 m_\chi^2 + \sigma\, a^2 \phi^2 \right]}_{\omega_k^2} X_k \;=\; 0$$

with Bunch-Davies initial condition $\quad X_k(\tau_0) \;=\; \dfrac{e^{-i\omega_k \tau}}{\sqrt{2\omega_k}}$

Resonant particle production

For $V(\phi) = \frac{1}{2} m_\phi^2 \phi^2$, after a few oscillations, **resonant** growth inside *Floquet* bands

$$\frac{d^2 x_q}{dz^2} + (A_q - 2\kappa \cos 2z) x_q = 0$$

where

$$x_q(t, \mathbf{x}) = a(t)^{1/2} X_q(t, \mathbf{x}) \,, \qquad \kappa = \frac{1}{8} \left(\frac{\sigma}{\lambda}\right) \left(\frac{\phi_0}{M_P}\right)^2 \,,$$

$$z = m_\phi t + \frac{\pi}{2} \,, \qquad A_q = q^2 \left(\frac{a_{\rm end}}{a}\right)^2 + 2\kappa \,,$$

with $q = \dfrac{k}{m_\phi a_{\rm end}}$. Floquet's theorem guarantees a solution of the form

$$x_q(z) = e^{\mu_q z} g_1(z) + e^{-\mu_q z} g_2(z)$$

Y. Shtanov, J. Traschen and R. Brandenberger, PRD 51 (1995) 5438; L. Kofman, A. Linde, A. Starobinsky, PRD 56 (1997) 3258

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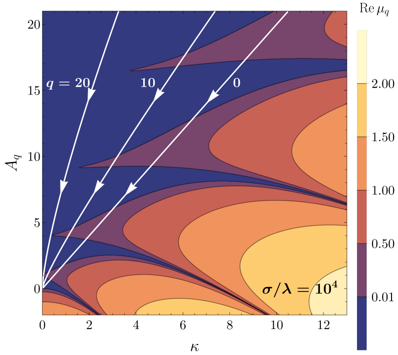
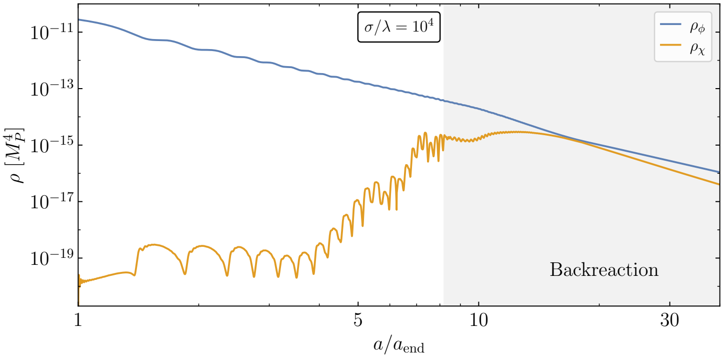




Backreaction and fragmentation

The explosive production of particles can disrupt the homogeneous inflaton condensate by re-scatterings, leading to the **fragmentation** of the condensate. The full system

$$\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2 \phi}{a^2} + \partial_\phi V(\phi,\chi) &= 0\,, \\
\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2 \chi}{a^2} + \partial_\chi V(\phi,\chi) &= 0\,, \\
\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2a^2}(\nabla\chi)^2 + V(\phi,\chi) &= \rho_\phi + \rho_\chi \;=\; 3H^2 M_P^2
\end{aligned}$$

can be solved by finite-difference techniques on a spatial lattice

D. Figueroa et al., Comput. Phys. Commun. 283, 108586 (2023)

CosmoLattice

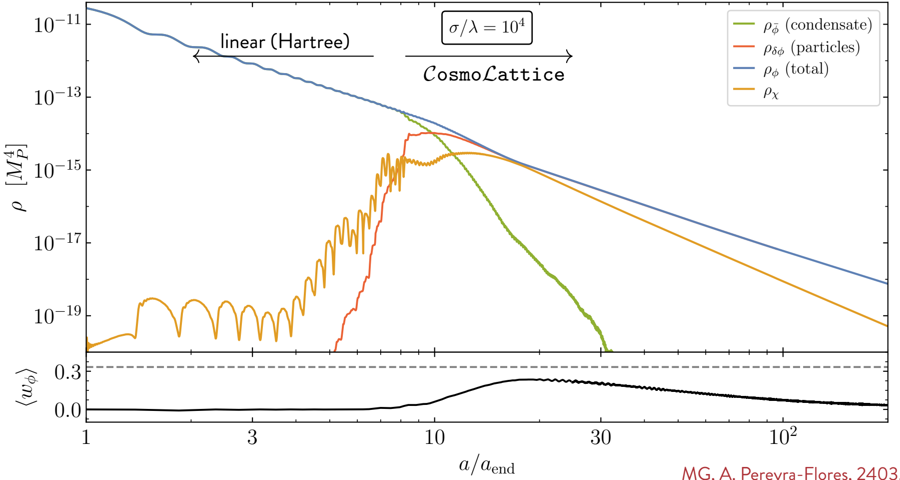
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

Backreaction and fragmentation

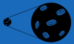
$$\rho_{\bar\phi} = \frac{1}{2}\dot{\bar\phi}^2 + V(\bar\phi)$$

$$\rho_{\delta\phi} = \rho_\phi - \rho_{\bar\phi} = \frac{1}{(2\pi)^3 a^4}\int d^3\boldsymbol{k}\,\omega_k f_{\delta\phi}(k)$$

$$\rho_\phi = \overline{\frac{1}{2}\dot\phi^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)}$$



MG, A. Pereyra-Flores, 2403.04848



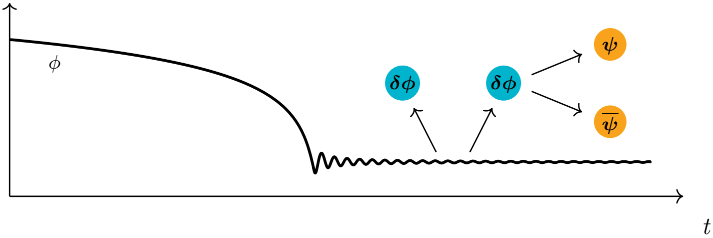
1. Inflation

2. Reheating

3. Dark preheating

4. Self-resonance

Decay of the fragmented inflaton



After fragmentation, the free inflaton quanta dominate the decays

$$\dot{\rho}_\psi + 4H\rho_\psi = \int \frac{d^3\boldsymbol{P}}{(2\pi)^3}\frac{d^3\boldsymbol{K}}{(2\pi)^3 2K^0}\frac{d^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(K-P-P')|\overline{\mathcal{M}}|^2 f_{\delta\phi}(K)$$

$$= \Gamma_{\delta\phi} m_\phi n_{\delta\phi} \approx \Gamma_{\delta\phi}\left(\frac{m_\phi}{\bar{E}_\phi}\right)\rho_{\delta\phi}$$

$$\Gamma_{\delta\phi} = \frac{|\mathcal{M}_{\delta\phi\to\bar{\psi}\psi}|^2}{16\pi m_\phi(t)}\sqrt{1-\frac{4m_\psi^2}{m_\phi(t)^2}}$$

MG, M. Pierre, JCAP 11 (2023) 004; MG et al., JCAP 12 (2023) 028

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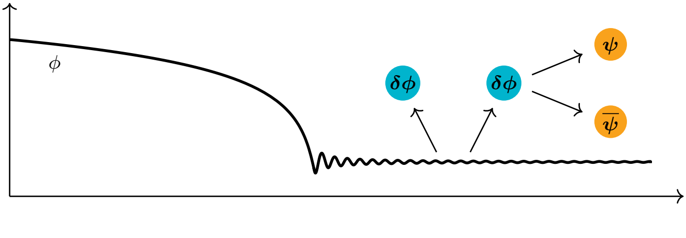

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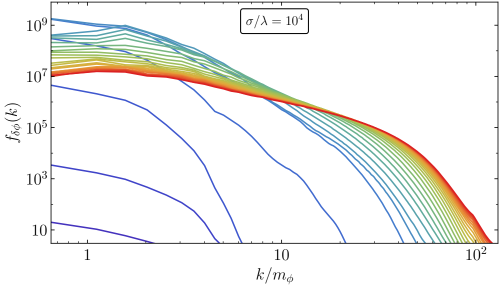
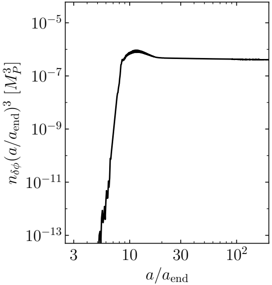
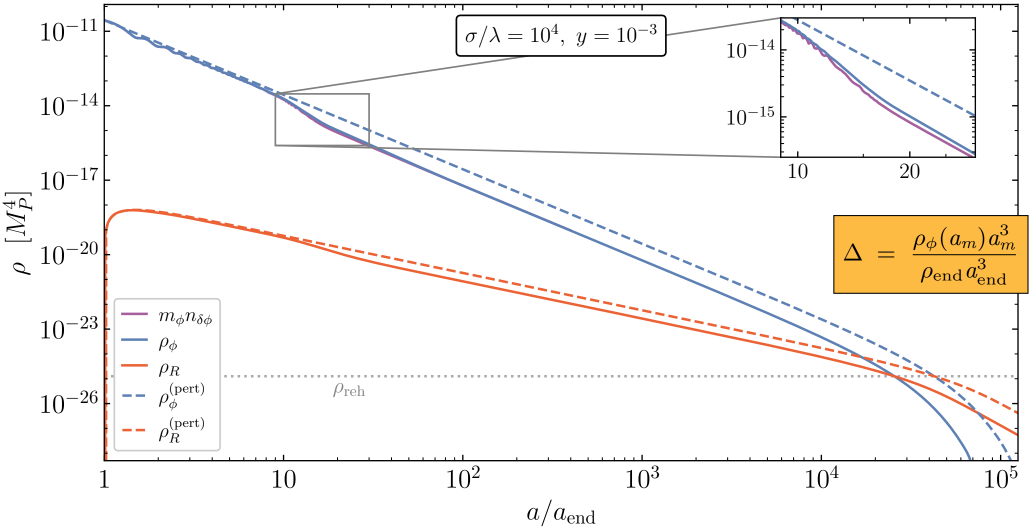




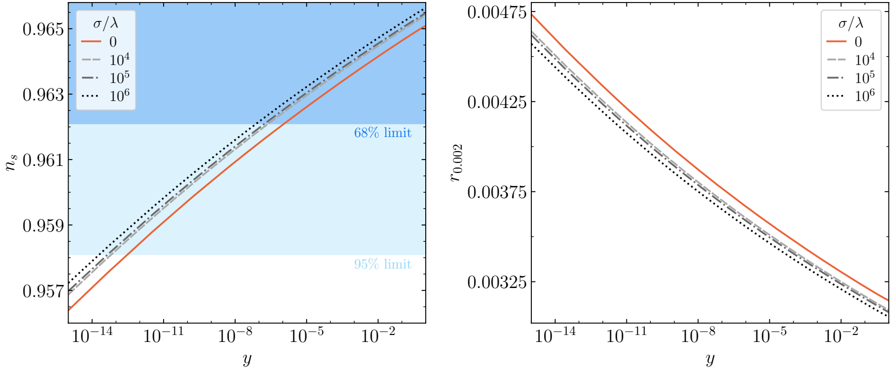
ϕ-χ **interactions** excite the inflaton UV modes (inflaton particles)

After backreaction the number of inflatons in a comoving volume is conserved, save for decays

Reheating after fragmentation



CMB observables (Starobinsky)



$$N_* - N_*^{(\text{pert})} \simeq -\frac{1}{3}\ln\Delta \simeq \begin{cases} 0.51\,, & \sigma/\lambda = 10^4\,, \\ 0.63\,, & \sigma/\lambda = 10^5\,, \\ 0.90\,, & \sigma/\lambda = 10^6\,, \end{cases} \quad \Rightarrow \quad \begin{aligned} n_s &\simeq n_s^{(\text{pert})} - \frac{2\ln\Delta}{3(N_* + 2.5)^2} \\ r &\simeq r^{(\text{pert})} + \frac{8\ln\Delta}{(N_* + 2.5)^3} \end{aligned}$$

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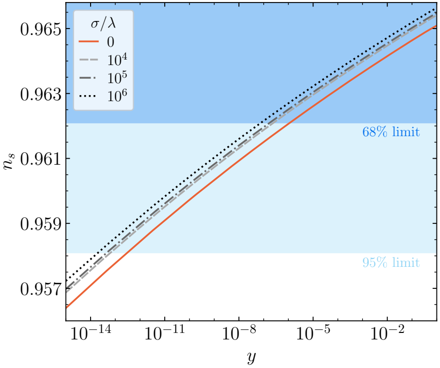

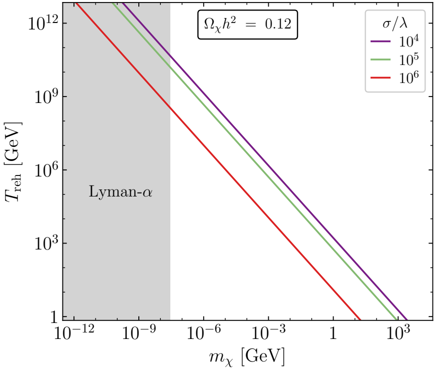

σ/λ	Lyman-α				CMB 68%	CMB 95%
0			$T_{\rm reh}$	$\gtrsim$	6.3×10^8 GeV	(203 GeV)
10^4	4.5×10^{10} GeV	$\gtrsim$	$T_{\rm reh}$	$\gtrsim$	1.5×10^8 GeV	(42 GeV)
10^5	1.5×10^{10} GeV	$\gtrsim$	$T_{\rm reh}$	$\gtrsim$	1.0×10^8 GeV	(29 GeV)
10^6	3.4×10^8 GeV	$\gtrsim$	$T_{\rm reh}$	$\gtrsim$	4.9×10^7 GeV	(12 GeV)

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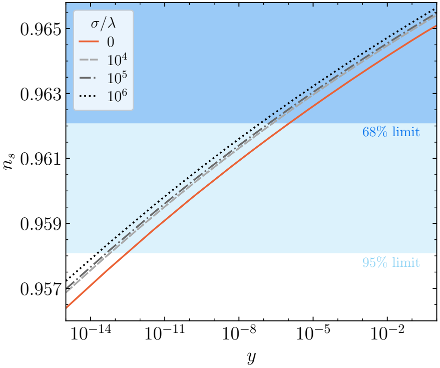
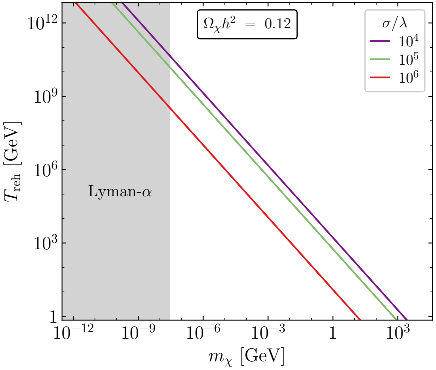
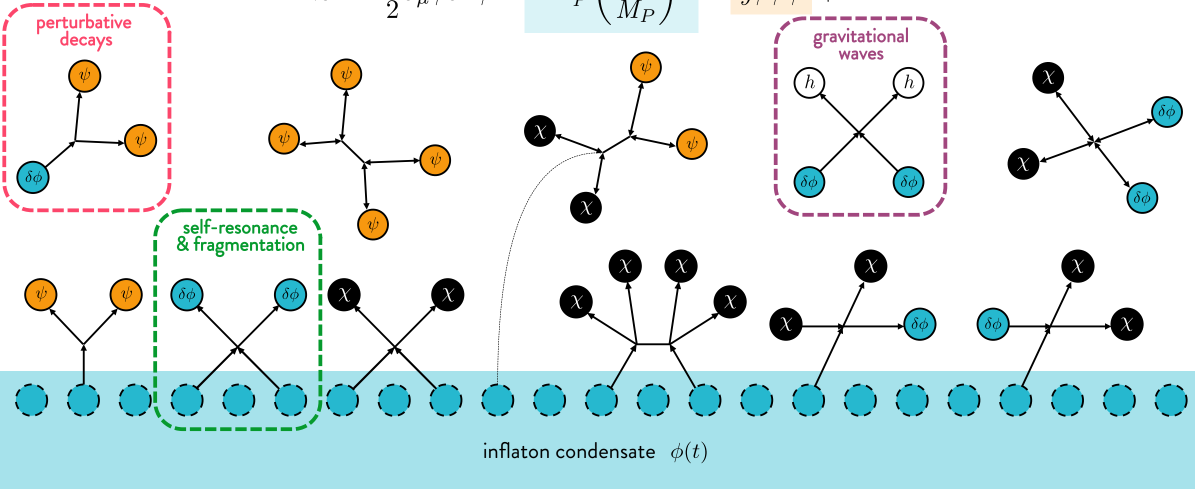




σ/λ	Lyman-α				CMB 68%	CMB 95%
10^4	30 eV	$\lesssim$	m_χ	$\lesssim$	9 keV	(40 GeV)
10^5	30 eV	$\lesssim$	m_χ	$\lesssim$	5 keV	(20 GeV)
10^6	30 eV	$\lesssim$	m_χ	$\lesssim$	0.2 keV	(1 GeV)

Fragmentation from self-interactions

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \lambda M_P^4 \left(\frac{\phi}{M_P}\right)^4 - y\phi\bar{\psi}\psi + \cdots$$



Parametric self-resonance

Even in the absence of couplings to other fields, the oscillations of ϕ may not survive forever

The inhomogeneous fluctuation $\delta\phi(t, \boldsymbol{x})$ satisfies, at the linear level,

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{\nabla^2 \delta\phi}{a^2} + V''(\phi(t))\,\delta\phi = 0$$

In conformal time, $d\tau = dt/a$, introducing

$$\Phi(\tau, \boldsymbol{x}) = a\,\delta\phi(\tau, \boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \left[\Phi_k(\tau)\hat{a}_{\boldsymbol{k}} + \Phi_k^*(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}\right]$$

the equation of motion is

$$\Phi_k'' + \underbrace{\left[k^2 - \frac{a''}{a} + V''(\phi)a^2\right]}_{\omega_k^2}\Phi_k = 0$$

with Bunch-Davies initial condition $\Phi_k(\tau_0) = \dfrac{e^{-i\omega_k \tau}}{\sqrt{2\omega_k}}$

Quartic self-resonance

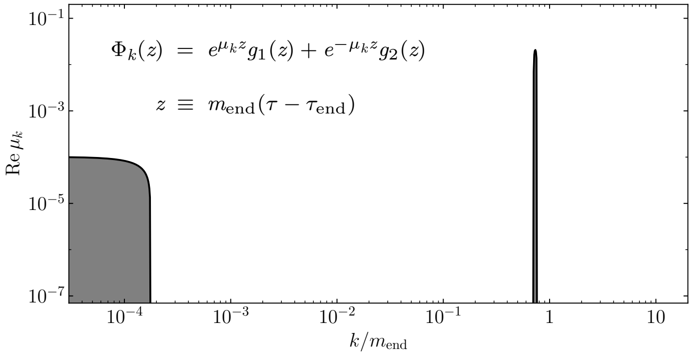
For $V(\phi) = \lambda\phi^4$, after a few oscillations,

$$\Phi_k'' + m_{\rm end}^2 \underbrace{\left[k^2 + {\rm sn}^2\left(\frac{m_{\rm end}}{\sqrt{6}}\Delta\tau, -1\right)\right]}_{\text{parametric resonance}} \Phi_k = 0$$

Resonant growth inside Floquet bands

Occupation number (PSD):

$$f_{\delta\phi}(k,t) = \frac{1}{2\omega_k}\left|\omega_k\Phi_k - i\Phi_k'\right|^2$$



Quartic fragmentation

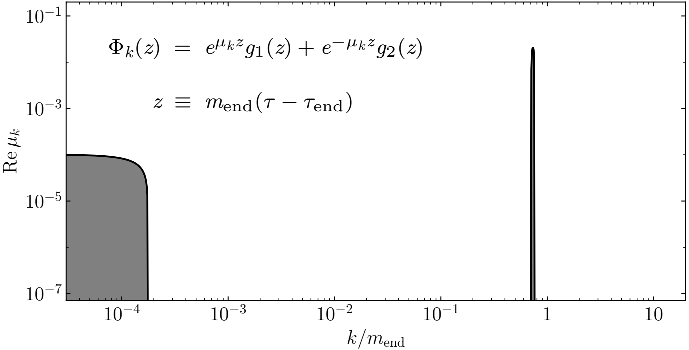
For $V(\phi) = \lambda\phi^4$, after a few oscillations,

$$\Phi_k'' + m_{\rm end}^2\left[\kappa^2 + 3\,\mathrm{sn}^2\left(\frac{m_{\rm end}}{\ldots}\Delta\tau, -\frac{1}{2}\right)\right]\Phi = 0 \quad\longrightarrow\quad \ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + \partial_\phi V = 0$$

Resonant growth inside *Floquet* bands

Rapid onset of nonlinearity, $\delta\phi \sim \phi$

Use ($\mathcal{C}$osmo)$\mathcal{L}$attice codes

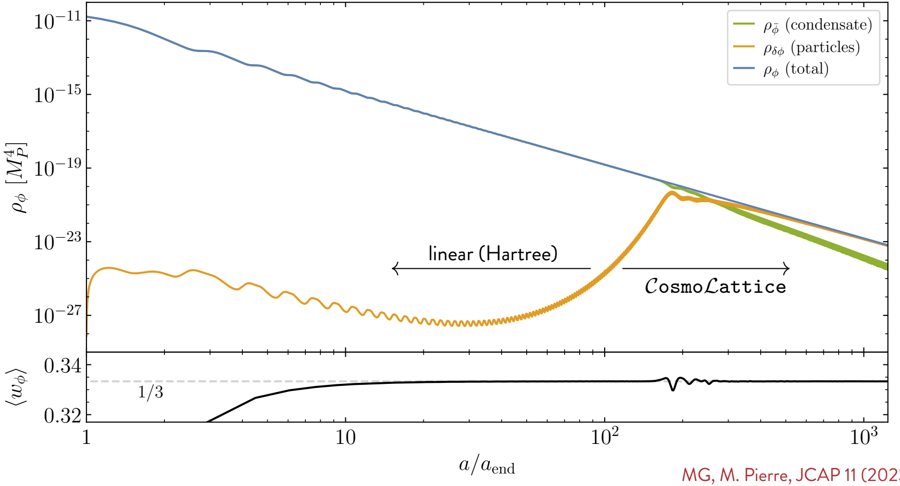


Quartic fragmentation

$$\rho_{\bar\phi} = \frac{1}{2}\dot{\bar\phi}^2 + V(\bar\phi)$$

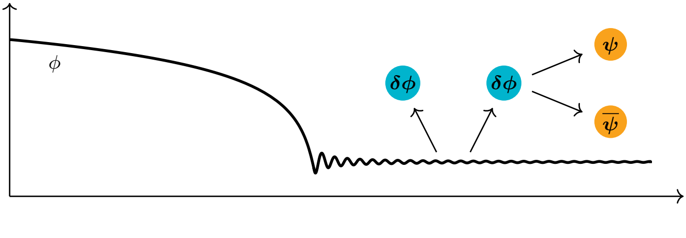
$$\rho_{\delta\phi} = \rho_\phi - \rho_{\bar\phi} = \frac{1}{(2\pi)^3 a^4}\int d^3\mathbf{k}\,\omega_k f_{\delta\phi}(k)$$

$$\rho_\phi = \overline{\frac{1}{2}\dot\phi^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)}$$

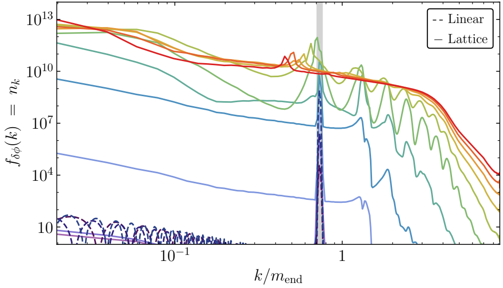


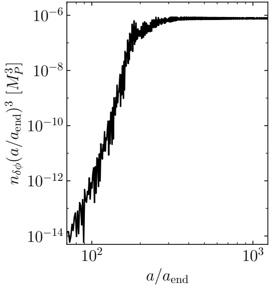
MG, M. Pierre, JCAP 11 (2023) 004

Decay of the fragmented inflaton



After fragmentation, the free inflaton quanta dominate the decays





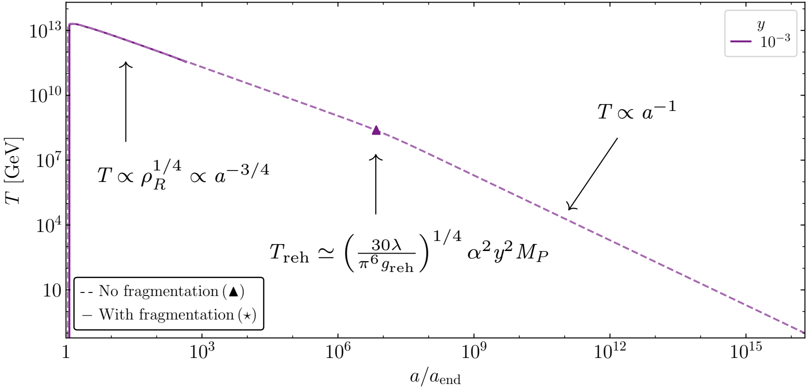
Self-interactions excite the inflaton UV modes (inflaton particles)

After backreaction the number of inflatons in a comoving volume is conserved, save for decays

Reheating temperatures, self-resonance (ignoring fragmentation)

$$\dot{\rho}_\phi + 4H\rho_\phi = -\frac{y^2}{8\pi}\left(\frac{4}{3}\alpha^2 m_\phi \rho_\phi\right) \qquad \rho_\phi \simeq \rho_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^4$$

$$\dot{\rho}_R + 4H\rho_R = \frac{y^2}{8\pi}\left(\frac{4}{3}\alpha^2 m_\phi \rho_\phi\right) \quad \Rightarrow \quad \rho_R \simeq \frac{\alpha^2}{\pi} y^2 \lambda^{1/4} \rho_{\rm end}^{3/4} M_P \left(\frac{a_{\rm end}}{a}\right)^3$$

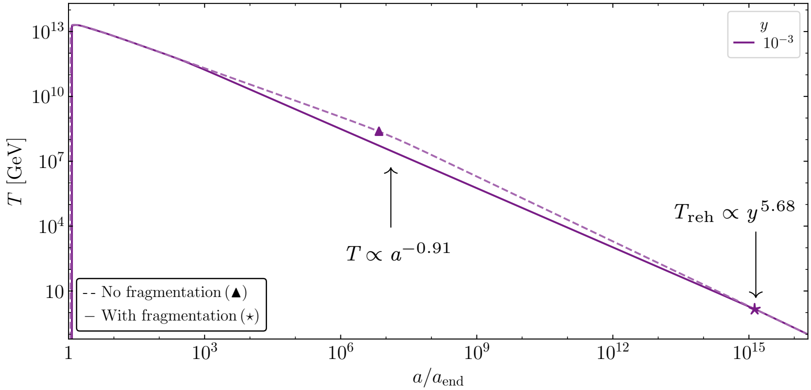


Reheating temperatures, self-resonance

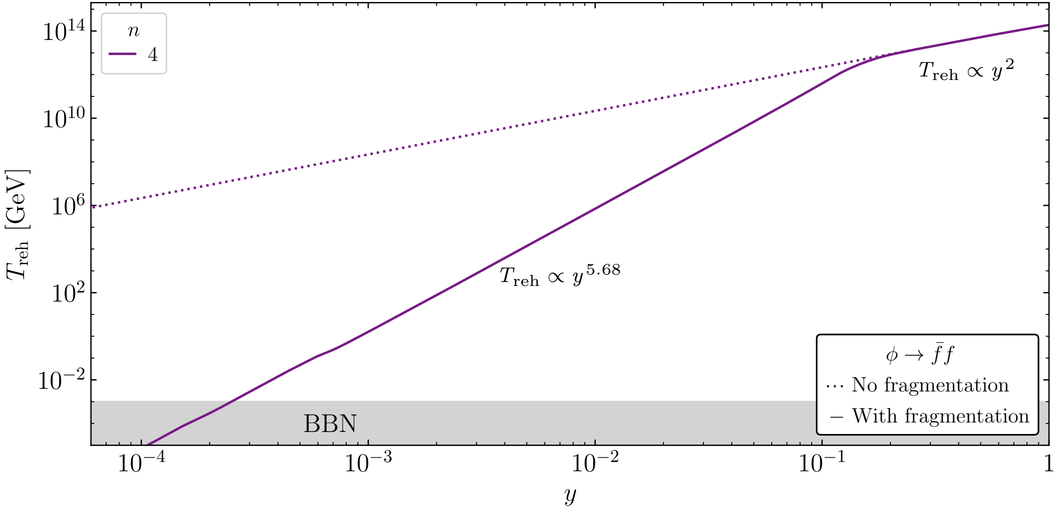
$$\dot{\rho}_\phi + 4H\rho_\phi = -\frac{y^2}{8\pi} m_\phi^2 n_{\delta\phi}$$
$$\dot{\rho}_R + 4H\rho_R = \frac{y^2}{8\pi} m_\phi^2 n_{\delta\phi}$$

$$\Rightarrow$$

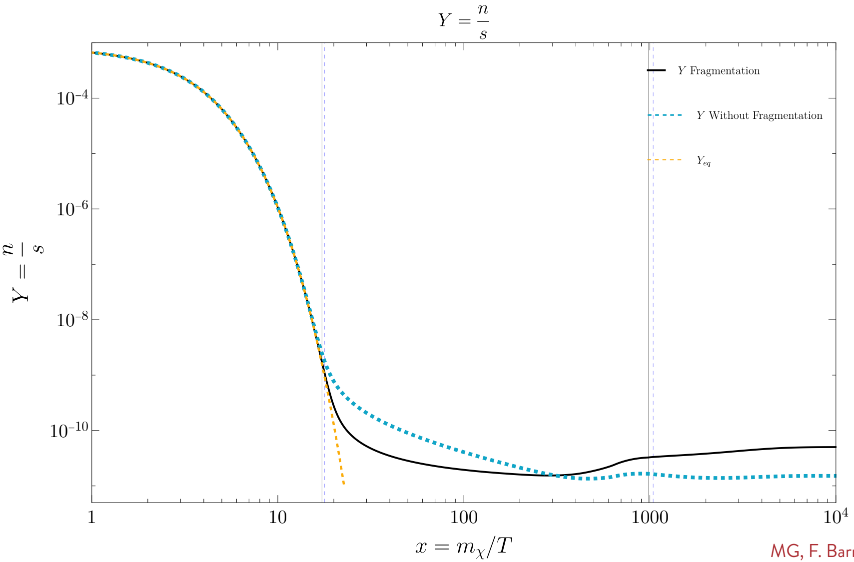
$$\rho_\phi \simeq \rho_{\rm end}\left(\frac{a_{\rm end}}{a}\right)^4$$
$$\rho_R \simeq (10^{-14})\,\frac{y^2}{8\pi}\left(\frac{M_P^4}{\rho_{\rm end}}\right)^{1/2} M_P^4 \left(\frac{a}{a_{\rm end}}\right)^{-3.65}$$



Reheating temperatures, self-resonance

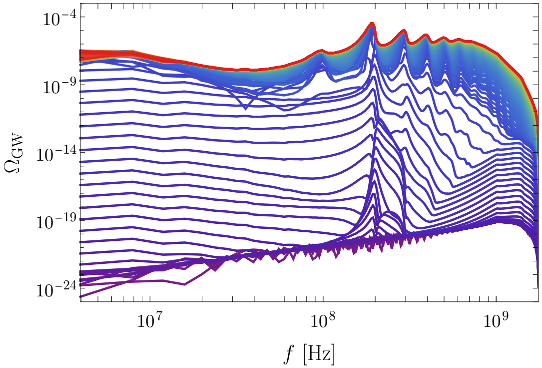


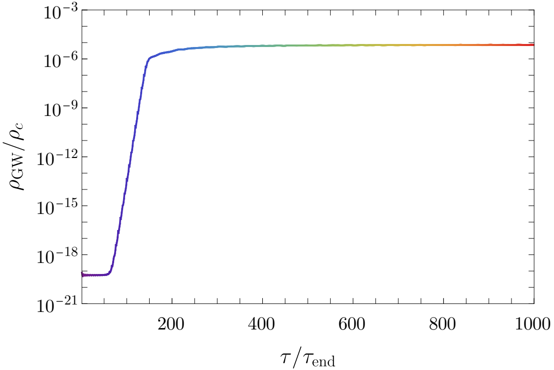
Dark matter freeze-out (after fragmentation)



MG, F. Barreto, in preparation

Induced gravitational waves

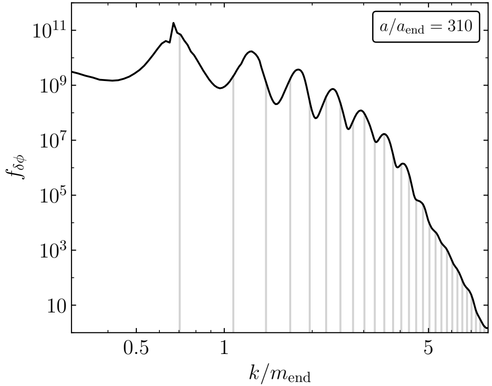


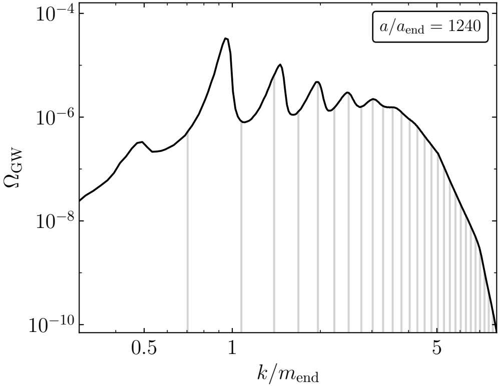


$$\Omega_{\mathsf{GW}}(k) = \frac{1}{\rho_c}\frac{d\rho_{\mathsf{GW}}}{d\log k} \qquad \rho_{\mathsf{GW}} = \frac{M_P^2}{4a^4}\frac{1}{V}\int d^3\boldsymbol{k}\,\bar{h}'_{ij}(\tau,\boldsymbol{k})\,\bar{h}'^{*}_{ij}(\tau,\boldsymbol{k})$$

$$f \simeq 1.46\times 10^{8}\left(\frac{k}{0.7\,m_{\rm end}}\right)\,\mathrm{Hz}$$

Peak structure



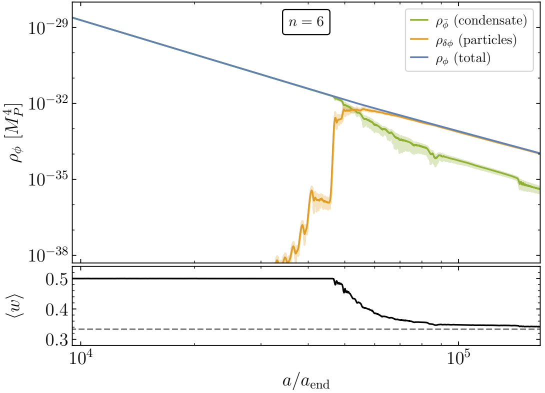


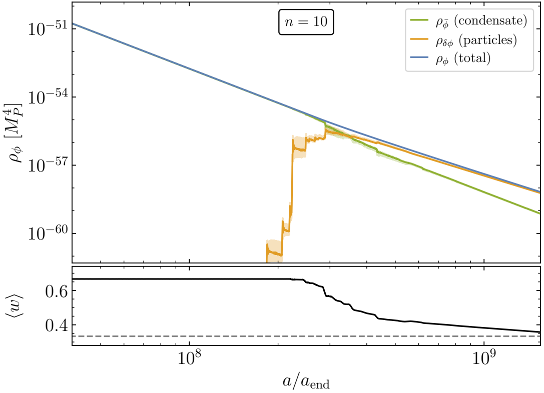
The Boltzmann approximation reveals the peak structure

$$f_{\delta\phi}(k,t) \simeq \frac{\pi}{c^2}\left(\frac{m_{\rm end}}{H_{\rm end}}\right)\left(\frac{a(t)}{a_{\rm end}}-1\right)\sum_{n=1}^{\infty}\frac{|\hat{\mathcal{P}}_n|^2}{n^2\beta_n}\delta\left(\frac{k}{m_{\rm end}}-\frac{1}{2}nc\beta_n\right)$$

Self-resonance: $V(\phi) \propto \phi^n, n > 4$

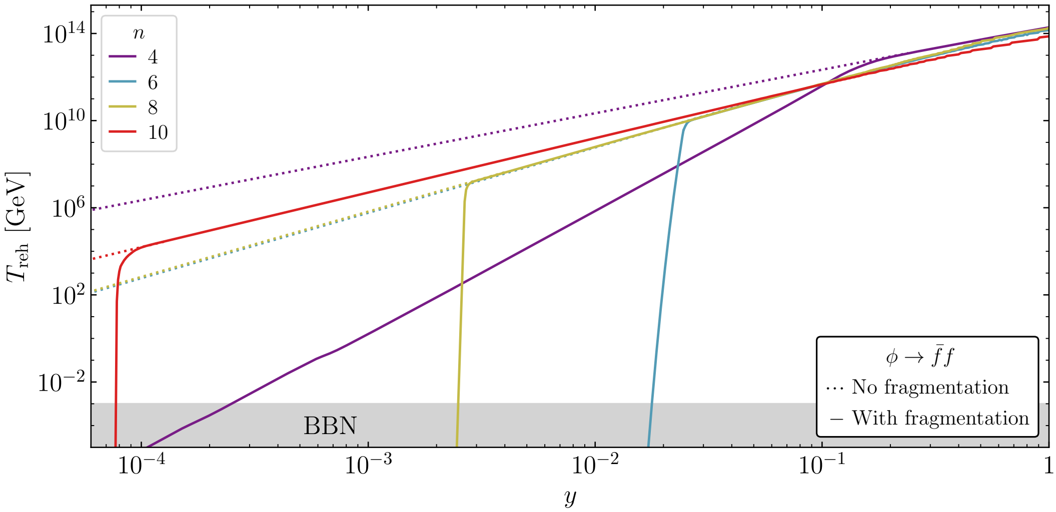
Fragmentation leads to radiation domination, $w = \dfrac{n-2}{n+2} \longrightarrow \dfrac{1}{3}$ (not the same as reheating)



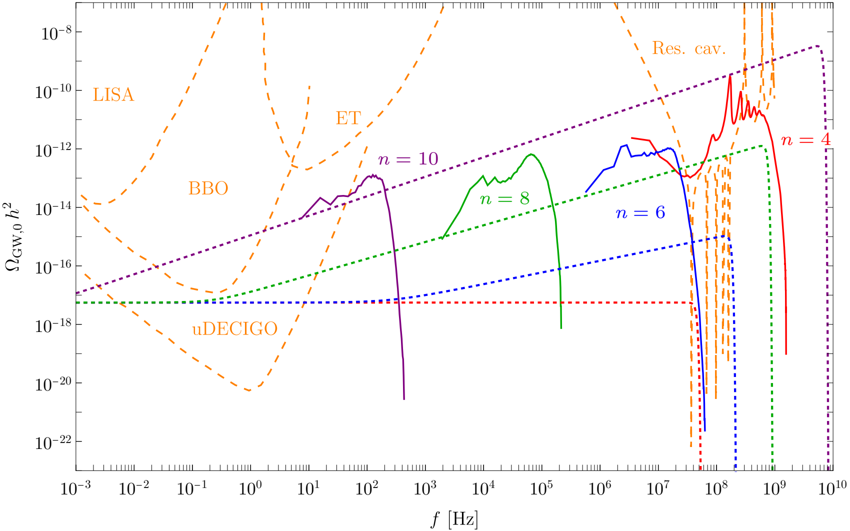


K. Lozanov, M. Amin, PRL 119, 061301 (2017); MG et al., JCAP 12, 028 (2023)

Reheating temperatures, self-resonance

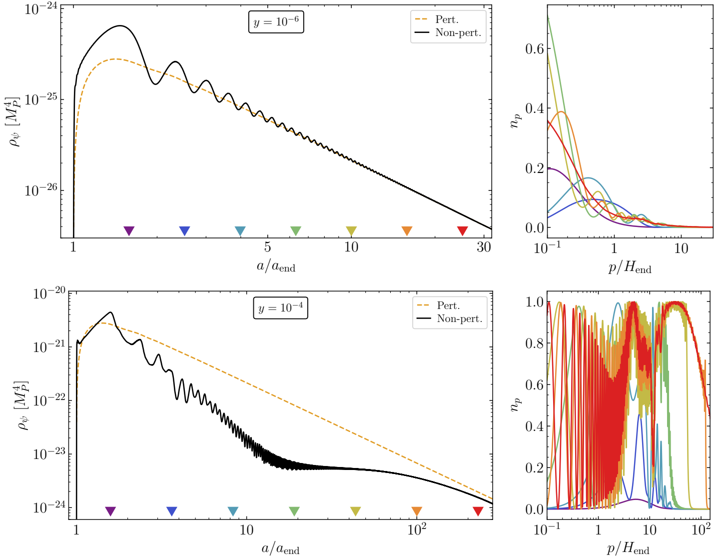


Induced gravitational waves



MG, M. Pierre, JCAP 12 (2023) 028; MG, M. Pierre, 2404.16932

Other non-perturbative effects



Fragmentation calls for $y > 10^{-4}$ in flat potentials, but ...

... in quadratic potentials, $y > 10^{-5}$ is the realm of fermion preheating

The backreaction regime is difficult to explore numerically

MG et al., JCAP 03 (2022) 016

Final thoughts

Summary

- Combined lattice + Boltzmann approach allows a description of post-fragmentation reheating
- Quantifiable impact on observables
- More to do to include fluctuation + dissipation
- (Bosonic) preheating calls for (fermionic) preheating

Conclusion

There's still a lot to do!

Thank You!