update on reheating after inflation and its gravitational wave probes

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abstract: The reheating stage remains a strongly model-dependent and theoretically poorly understood part of the inflationary paradigm. However, the possibility that it could one day be tested via its gravitational wave signatures, and that it may also be related to other relicts such as dark matter or baryon asymmetry, continues to motivate further studies. I review the general framework as well as some recent developments from this field, as inspired by this week's TH workshop on particle production in the early universe.

context [workshop]

Particle Production in the Early Universe

Sep 9 – 13, 2024
CERN
Europe/Zurich timezone

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Particle number changing processes in the early universe shaped the cosmos during several epochs of its evolution, including reheating after cosmic inflation, baryogenesis, and dark matter production. In many popular scenarios the semi-classical standard Boltzmann equations are insufficient for their quantitative description, e.g. due to the interplay between coherent oscillations and de-coherent scatterings, non-perturbative production, and thermal corrections to quasiparticle properties.

This workshop aims to bring together experts from around the world working on the development of methods for a quantitative description of nonequilibrium quantum processes, in particular those driving particle production. The focus will be on three methodological approaches, namely 1) first principles

This workshop aims to bring together experts from around the world working on the development of methods for a quantitative description of nonequilibrium quantum processes, in particular those driving particle production. The focus will be on three methodological approaches, namely 1) first principles QFT methods (thermal and non-thermal), 2) methods to treat non-perturbative production, 3) novel/ nonstandard mechanisms. The workshop is complementary to the rich menu of existing specialised meetings in the sense that it focuses on methodology and aims to bring together experts from different sub-communities using similar methods, fostering synergies and collaborations across fields.

Due to a limited number of places, we encourage early application (in particular before June 15) for this event. We also encourage the submission of abstracts for contributed talks.

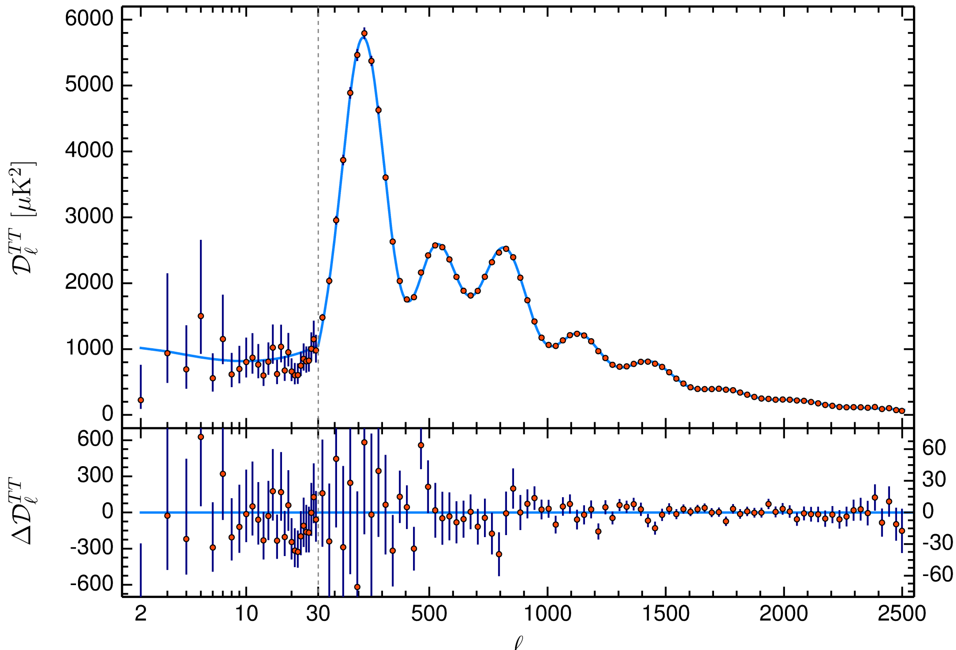
Organizers: Valerie Domcke (CERN), Marco Drewes (UC Louvain), Yohei Ema (University of Minnesota), Juraj Klaric (UvA, UniZg, Nikhef), Oleg Lebedev (University of Helsinki)

snapshot from program

		R1, CERN	12:35 - 14:00
Gravitational production of dark particles	*Andrew Long*	*4/3-006 - TH Conference Room, CERN*	14:00 - 14:20
On the anomalous gravitational fermion production in terms of level crossing	*Kohei Kamada*	*4/3-006 - TH Conference Room, CERN*	14:20 - 14:40
Adiabatic renormalization without infrared distortions in cosmological spacetimes	*Dr Francisco Torrenti*	*4/3-006 - TH Conference Room, CERN*	14:40 - 14:55
Unitarity, holomorphic cuts, and thermal effects in zero-temperature calculations	*Dr Peter Matak*	*4/3-006 - TH Conference Room, CERN*	14:55 - 15:10
Coffee		*4/2-011 - TH common room, CERN*	15:10 - 16:00
Upper Bound on Thermal Gravitational Wave Backgrounds from Hidden Sectors	*Yannis Georis*	*4/3-006 - TH Conference Room, CERN*	16:00 - 16:15
Gravitational wave background from vacuum and thermal fluctuations during axion-like inflation	*Philipp Klose*	*4/3-006 - TH Conference Room, CERN*	16:15 - 16:30
Neutrino Decoupling at NLO: Methods and Rates	*Miguel Escudero Abenza*	*4/3-006 - TH Conference Room, CERN*	16:30 - 16:45
Exploring General Vector Mediators in Inelastic Dark Matter Models	*Ana Luisa Foguel da Silva*	*4/3-006 - TH Conference Room, CERN*	16:45 - 17:00
CMB signature of non thermal dark matter production from self interacting dark sector	*Mr Sk Jeesun*		

introduction [inflation]

cmb observations show long-range correlations[1]



small momentum

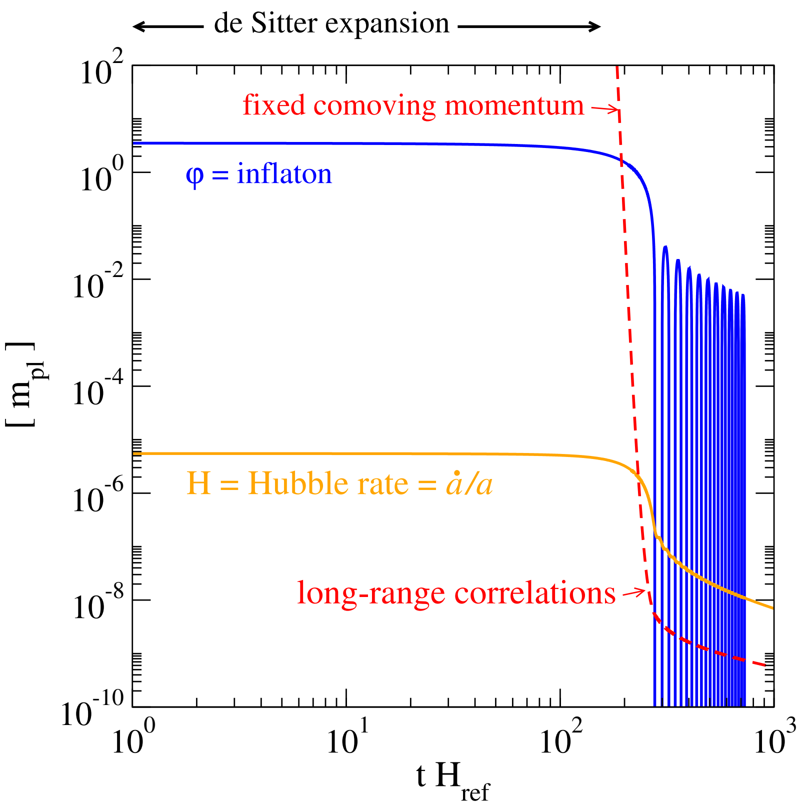
large separation

large momentum

small separation

[1] Y. Akrami *et al*, *Planck 2018 results. X. Constraints on inflation*, 1807.06211

a paradigm for generating correlated fluctuations: inflation



challenges

model building for inflation:

$\Rightarrow$ what is φ? what is its potential V? what sets its initial value?

model building for ending inflation ($\equiv$ reheating, to produce cmb):

$\Rightarrow$ want large couplings for transferring the energy in φ into the standard model, yet small couplings for sufficient "flatness" of V?[2]

methodology:

$\Rightarrow$ theoretical consistency: it is not easy to do non-equilibrium statistical quantum field theory in a general relativistic setting

[2] e.g. J. Yokoyama and A.D. Linde, *Is warm inflation possible?*, hep-ph/9809409

reheating [basic notions]

background equations ($\varphi = \bar\varphi + \delta\varphi$, $\nabla\bar\varphi = 0$, $\partial_T V \simeq 0$)[3]

$$\ddot{\bar\varphi} + 3H\dot{\bar\varphi} + \partial_\varphi V \simeq -\Upsilon\dot{\bar\varphi}\,,$$

$$\dot{e}_r + 3H\big(e_r + p_r\big) \simeq +\Upsilon\dot{\bar\varphi}^2$$

consistent with energy conservation $\dot{e} + 3H(e + p) = 0$, where $e = e_r + \underbrace{\dot{\bar\varphi}^2/2 + V}_{e_\varphi}$, $p = p_r + \underbrace{\dot{\bar\varphi}^2/2 - V}_{p_\varphi}$, $H^2 = 8\pi e/(3m_{\rm pl}^2)$

the friction Υ transfers energy from $\dot{\bar\varphi}$ to "radiation" (e_r, p_r)[4]

$\Rightarrow$ what is $\Upsilon(\bar\varphi, T, m_\varphi)$? how does the solution look like?

[3] e.g. A. Albrecht, P.J. Steinhardt, M.S. Turner and F. Wilczek, *Reheating an Inflationary Universe,* PRL 48 (1982) 1437

[4] on the level of $\delta\varphi$, there is also an opposite energy transfer from the plasma to the inflaton, via thermal fluctuations

approximate solution: "instantaneous reheating"

decoupled equations: patch a solution with $e = e_\varphi$ and $a = a_\varphi$, onto one with $e = e_r$ and $a = a_r$ at $t = t_{\text{match}}$, by requiring

$$\begin{cases} a_\varphi(t_{\text{match}}) = a_r(t_{\text{match}}) \\ e_\varphi(t_{\text{match}}) = e_r(t_{\text{match}}) \quad [\Rightarrow \dot{a}_\varphi(t_{\text{match}}) = \dot{a}_r(t_{\text{match}})] \end{cases}$$

but how to choose t_{match}?

number of relativistic particles

$$\Upsilon(\bar\varphi = 0, T_r, m_\varphi) \overset{!}{\simeq} H(\bar\varphi = 0, T_r, m_\varphi) \approx \frac{\sqrt{\underset{\downarrow}{g_r}}\, T_r^2}{m_{\text{pl}}}$$

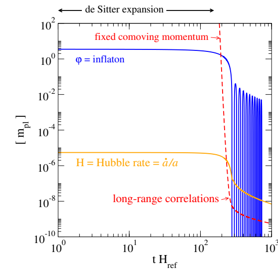
$\Rightarrow$ compute[5] Υ and solve for T_r

[5] often just the vacuum decay width at $T_r = 0$ is taken, but there are also more advanced computations, e.g. M. Drees and B. Najjari, *Energy spectrum of thermalizing high energy decay products in the early universe,* 2105.01935; K. Mukaida and M. Yamada, *Perturbative reheating and thermalization of pure Yang-Mills plasma,* 2402.14054

more accurate solution: "smooth reheating"

after inflation ends, $e_\varphi \gg e_r$ remains true for a while, and there is a long epoch of "matter domination",[6] during which

$$\ddot{\bar{\varphi}} + \underbrace{(3H + \Upsilon)}_{\text{non-linear}} \dot{\bar{\varphi}} + m_\varphi \bar{\varphi} \simeq 0$$



radiation can already exist, and Υ can be a non-trivial function of T

$\Rightarrow$ after determining Υ [hard], solve the evolution numerically [easy]

[6] the name originates from that $\langle \dot{\bar{\varphi}}^2 \rangle \approx m_\varphi^2 \langle \bar{\varphi}^2 \rangle$ across oscillations, implying $p_\varphi \approx 0$

another approach: “preheating”

if we look at the equations for perturbations around $\bar{\varphi}$ (typically for some spectator field $\delta\chi$), some momentum modes may be tachyonic, as if we had chosen a “wrong saddle point” $\bar{\varphi} \neq 0, \bar{\chi} = 0$

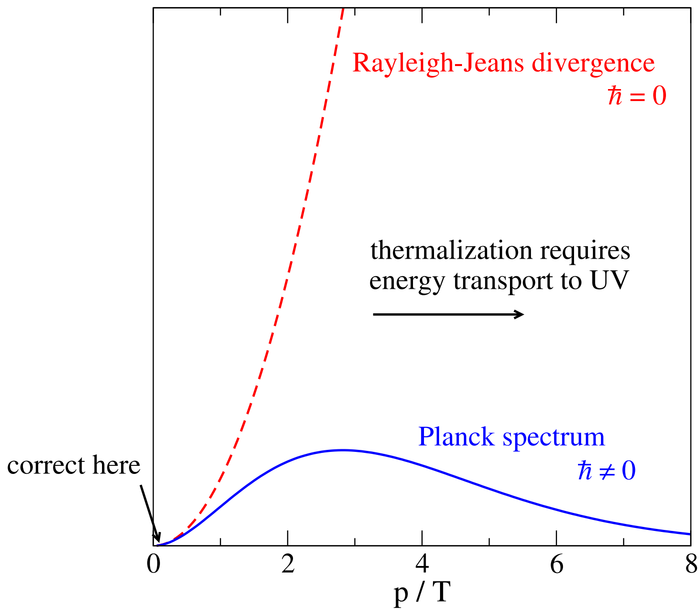
then we should solve the full non-linear classical equations of motion, to find a better saddle point $[\delta\mathcal{S}/\delta\{\varphi,\chi\} = 0]$

such dynamics could accelerate reheating[7]

7 e.g. L. Kofman, A.D. Linde and A.A. Starobinsky, *Towards the theory of reheating after inflation*, hep-ph/9704452; ...

preheating: challenge

solving the full non-linear classical equations cannot capture thermalization, since thermal momenta have occupancy of $\mathcal{O}(1)$ [8]



[8] in principle quantum corrections can be included approximately, see e.g. A. Tranberg and G. Ungersbäck, *Quantum tachyonic preheating, revisited*, 2312.08167, however they do not amount to a "small correction"

reheating [example of a pipeline]

remark (i): the inflaton φ does not need to thermalize

dynamics depends on the hierarchy between three scales:

H = hubble rate (from friedmann equations)

Υ = inflaton equilibration rate (needs to be computed[9])

Γ = radiation equilibration rate (needs to be computed[10])

cartoon of the reheating era (assuming $\Upsilon \ll \Gamma$):

$\Upsilon \ll \Gamma \ll H$: empty matter-dominated universe

$\Upsilon \ll H \ll \Gamma$: matter domination in the presence of radiation

$H \ll \Upsilon \ll \Gamma$: radiation-dominated universe ("hot big bang")

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[9] for formalism, see e.g. D. Bödeker and J. Nienaber, *Scalar field damping at high temperatures*, 2205.14166

[10] e.g. Y. Fu, J. Ghiglieri, S. Iqbal, A. Kurkela, *Thermalization of non-Abelian gauge theories at NLO*, 2110.01540

remark (ii): heat bath already during inflation?

suppose that $\Delta\dot{e}$ from inflaton compensates for hubble dilution, and look for a stationary solution

$$\overbrace{\dot{e}_r}^{\equiv 0} + 3H\bigl(e_r + p_r\bigr) \simeq \overbrace{\Upsilon\dot{\bar{\varphi}}^2}^{\equiv \Delta\dot{e}}$$

$$\overset{e_r+p_r = Ts_r}{\Longrightarrow} \quad \underbrace{3Ts_r}_{\text{strongly } T\text{-dependent}} \simeq \underbrace{\frac{\Upsilon}{H}\frac{(\partial_\varphi V)^2}{(3H+\Upsilon)^2}}_{\text{weakly } T\text{-dependent}}$$

a solution exists and represents a stable fixed point[11]

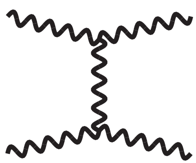
in a way this is related to the text-book temperature definition $T = \partial E/\partial S_{\text{boltzmann}}$, whence the equilibration rate Γ does not appear

[11] including the strong sphaleron rate (see below): K.V. Berghaus, P.W. Graham and D.E. Kaplan, *Minimal warm inflation*, 1910.07525; W. DeRocco, P.W. Graham and S. Kalia, *Warming up cold inflation*, 2107.07517

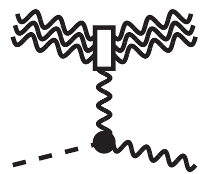
remark (iii): heating-up can be a self-amplifying process

if Υ scales with a positive power of T, release of energy into radiation increases Υ increases release of energy into radiation...

microphysically, this can originate via bose enhancement of bosonic decay products, but more importantly, via soft $2 \to 2$ scatterings off an already existing plasma



Γ



Υ

example of an interesting Υ: "sphaleron heating"

consider axion-like inflation with non-abelian gauge fields[12,13]

thanks to a shift symmetry, the potential remains "flat", even in the presence of thermal corrections

$$\mathcal{L} \supset \frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - V_0(\varphi) - \frac{\varphi\,\chi}{f_a}\,, \quad \chi \equiv \frac{\alpha\,\epsilon^{\mu\nu\rho\sigma}F^c_{\mu\nu}F^c_{\rho\sigma}}{16\pi}$$

normally $f_a \sim m_{\rm pl}$ but the confinement scale is $\Lambda_{\rm IR} \ll m_{\rm pl}$

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[12] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo Nambu-Goldstone bosons,* PRL 65 (1990) 3233; ...

[13] the abelian case also represents a much studied and very rich problem, see e.g. A. Caravano, E. Komatsu, K.D. Lozanov and J. Weller, *Lattice simulations of axion-U(1) inflation,* 2204.12874; R. von Eckardstein, M. Peloso, K. Schmitz, O. Sobol and L. Sorbo, *Axion inflation in the strong-backreaction regime: decay of the Anber-Sorbo solution,* 2309.04254; V. Domcke, Y. Ema and S. Sandner, *Perturbatively including inhomogeneities in axion inflation,* 2310.09186; P. Adshead, J.T. Giblin, R. Grutkoski and Z.J. Weiner, *Gauge preheating with full general relativity,* 2311.01504; ...

how to compute Υ? it depends on the frequency $\omega \sim i\partial_t$

through linear response theory,

$$\partial_\mu \left\{ \frac{\partial \mathcal{L}}{\partial_\mu \varphi} \right\} = \frac{\partial \mathcal{L}}{\partial \varphi} = -\left\langle \frac{\chi}{f_a} \right\rangle_\varphi = -(\delta m^2 + \Upsilon \partial_t + \ldots)\varphi + \mathcal{O}(\varphi^2) \, ,$$

Υ can be related[14,15] to the 2-point "spectral function" of χ,

$$\Upsilon(\omega) = \frac{1}{f_a^2} \frac{\rho_\chi(\omega)}{\omega}$$

this incorporates both vacuum decays $\varphi \to gg$ (for $\omega \gg \pi T$) [easy, up to NLO etc] and plasma scatterings $\varphi + X \to Y$ (for $\omega \ll \pi T$)

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14 L.D. McLerran, E. Mottola and M.E. Shaposhnikov, *Sphalerons and axion dynamics in high-T QCD*, PRD 43 (1991) 2027

15 ML and S. Procacci, *Minimal warm inflation with complete medium response*, 2102.09913

for $\omega \ll \pi T$, Υ can be estimated via classical simulations

generate gauge configurations at $t = 0$ with boltzmann weight

$$Z^{(\mathrm{cl})} = \int \mathcal{D}U_i \, \mathcal{D}\mathcal{E}_i \, \delta(G) \exp\Big\{ -\frac{1}{g^2 T a} \sum_{\mathbf{x}} \Big[\sum_{i,j} \mathrm{Tr}\big(\mathbb{1} - P_{ij}\big) + \sum_i \mathrm{Tr}\big(\mathcal{E}_i^2\big) \Big] \Big\}$$

$\uparrow$

[lattice spacing, not scale factor (raleigh-jeans!)]

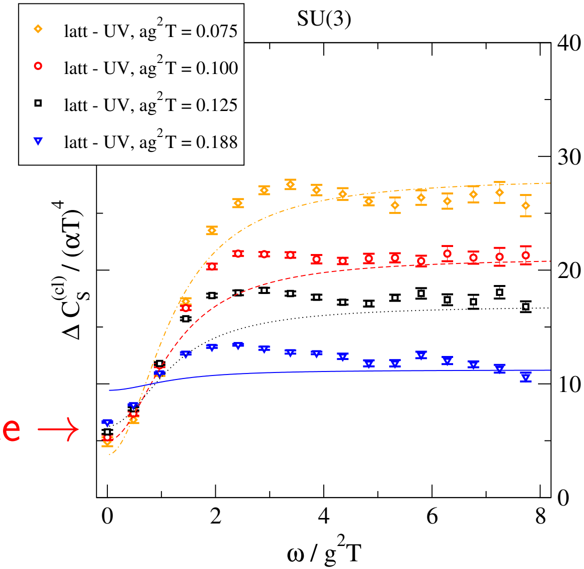
evolve fields to $t > 0$ with equations of motion

$$a \, \partial_t U_i(x) = i\mathcal{E}_i(x) U_i(x) \, ,$$

$$a \, \partial_t \mathcal{E}_i^b(x) = 2 \sum_{j \neq i} \mathrm{Im} \, \mathrm{Tr} \Big\{ T^b \Big[P_{ji}(x) + P_{-ji}(x) \Big] \Big\}$$

then measure the 2-point correlator of χ, and fourier-transform

the full frequency dependence[16]



sphaleron rate →

⇒ value can be quite different from the actual sphaleron rate

[16] ML, L. Niemi, S. Procacci, K. Rummukainen, *Shape of the hot topological charge density spectral function*, 2209.13804

the result can be transcribed into continuum

$$\Upsilon(\omega) \simeq \frac{\alpha^2(N_c^2-1)}{f_a^2}$$

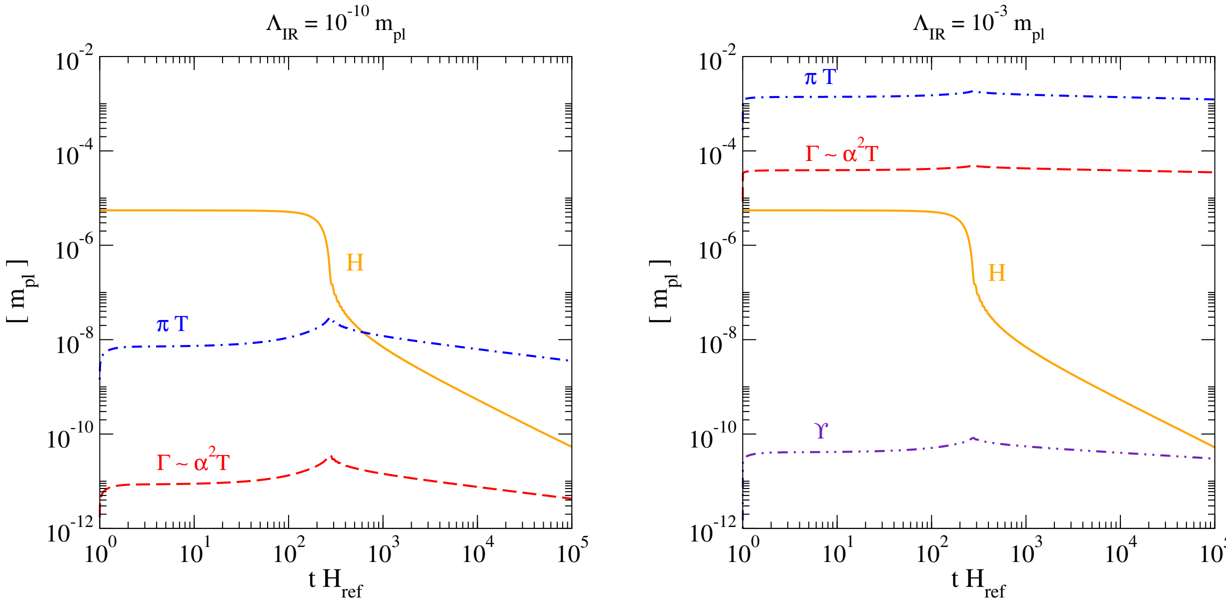
$$\times \Bigg\{ \underbrace{\kappa\,(\alpha N_c T)^3}_{\text{sphaleron rate}} \; \frac{1+\frac{\omega^2}{(c_{\rm IR}\alpha^2 N_c^2 T)^2}}{1+\frac{\omega^2}{(c_{\rm M}\alpha N_c T)^2}} + \underbrace{\Big[1+2\overset{\substack{\text{bose distribution}\\ \downarrow}}{n_{\rm B}}\Big(\frac{\omega}{2}\Big)\Big]\frac{\pi\omega^3}{(4\pi)^4}}_{\varphi\to gg} \Bigg\}$$

during reheating, $\omega \to m_\varphi$

this specific result is model-dependent,[17] but the general feature that there are different domains, $\pi T \ll m_\varphi$ and $\pi T \gg m_\varphi$, is universal

17 the influence of light fermions has been discussed e.g. in K.V. Berghaus, P.W. Graham, D.E. Kaplan, G.D. Moore and S. Rajendran, *Dark energy radiation,* 2012.10549; M. Drewes and S. Zell, *On sphaleron heating in the presence of fermions,* 2312.13739

this leads to smooth reheating, especially for large $\alpha(\Lambda_{\rm IR})$ [18]



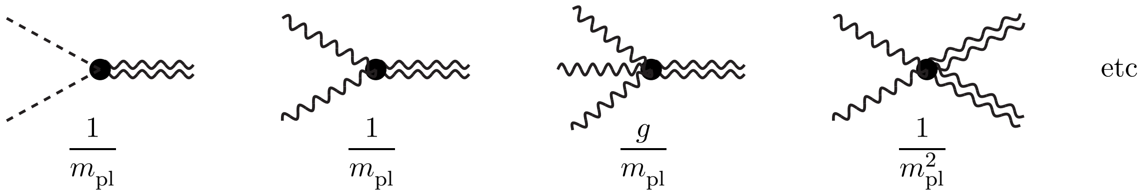
summary: with a fully determined Υ, reheating could be theoretically under fair control, even if numerical results are model-dependent

[18] H. Kolesova, ML, S. Procacci, *Maximal temperature of strongly coupled dark sectors*, 2303.17973

gravitational wave signatures [sketch]

overview

every particle species (including φ and g) couples to gravitational waves through the anisotropic part of its energy-momentum tensor[19]



similar processes as those that contribute to reheating also have a partial width to produce gravitational waves

this might be constrained by the fraction of relativistic energy density that gravitational waves carry (ΔN_{eff}), or future UHF experiments[20]

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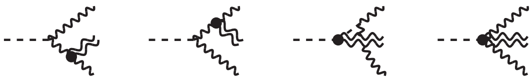
[19] e.g. S.Y. Choi, J.S. Shim and H.S. Song, *Factorization and polarization in linearized gravity*, hep-th/9411092

[20] N. Aggarwal *et al*, *Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies*, 2011.12414

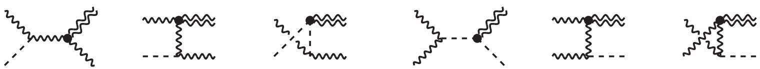
classes of processes that have been considered

IR

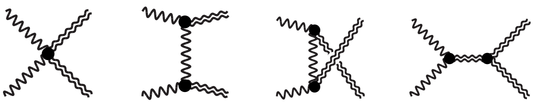
scalar-induced gravitational waves



“bremsstrahlung” ($1 \to 3$ decays)



$2 \to 2$ scatterings

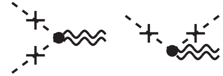


double-graviton production

UV

$\Rightarrow$ under which conditions are the different channels important?

scalar-induced gravitational waves



$\longleftarrow$ interactions not resolved

scalar $\equiv$ combination of $\delta\varphi$ and metric perturbations[21,22]

but wait... aren't these processes kinematically forbidden?

not if the field has a finite width "$\times$", whence it's not sharply on-shell

isn't the production rate then proportional to the width?

not if the energy is small compared with the width...

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[21] D. Baumann, P.J. Steinhardt, K. Takahashi and K. Ichiki, *Gravitational Wave Spectrum Induced by Primordial Scalar Perturbations*, hep-th/0703290; ...

[22] different language but perhaps related: S.Y. Khlebnikov and I.I. Tkachev, *Relic gravitational waves produced after preheating*, hep-ph/9701423; R. Easther and E.A. Lim, *Stochastic gravitational wave production after inflation*, astro-ph/0601617; J. Garcia-Bellido, D.G. Figueroa and A. Sastre, *Gravitational wave background from reheating after hybrid inflation*, 0707.0839; J.F. Dufaux, G. Felder, L. Kofman and O. Navros, *Gravity waves from tachyonic preheating after hybrid inflation*, 0812.2917; ...

example: thermalized inflaton with $k \gg H$ ( $\mathbf{k} = k\mathbf{e}_z$)[23]

$$T_{xy} \quad \supset \quad \partial_x \delta\varphi \, \partial_y \delta\varphi$$

$$\frac{\rho_{T_{xy}}(\omega, k)}{\omega} \quad \overset{\omega, k, \Upsilon \ll \pi T}{\approx} \quad \frac{\Upsilon}{T} \int_{\mathbf{p}} \frac{p_x^2 p_y^2 \, n_{\text{B}}(\epsilon_p)[1 + n_{\text{B}}(\epsilon_p)]}{\epsilon_p^2 \, [(v_z k + \omega)^2 + \Upsilon^2]}$$

↑ resummation

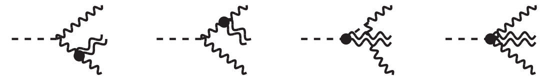
the result has the characteristic shape of a "transport peak":

high-energy regime $\omega, k \gg \Upsilon \Rightarrow$ suppression by Υ

hydrodynamic regime $\omega, k \ll \Upsilon \Rightarrow$ enhancement by $1/\Upsilon$

[23] P. Klose, ML, S. Procacci, *Gravitational wave background from non-Abelian reheating after axion-like inflation*, 2201.02317

bremsstrahlung ($1 \to 3$ **decays**)



for a heavy inflaton ($m_\varphi \gg \pi T$), oscillating in a matter-dominated epoch, this looks like the most obvious channel[24]

effectively we have here "resolved" the origin of the width "$\times$"

however, see above: the result is not reliable in the IR
(meaning $\omega < \max\{\Upsilon, H\}$ for φ, and $\omega < \max\{\Gamma, H\}$ for g)

[24] e.g. S. Kanemura and K. Kaneta, *Gravitational waves from particle decays during reheating,* 2310.12023; N. Bernal, S. Cléry, Y. Mambrini and Y. Xu, *Probing reheating with graviton bremsstrahlung,* 2311.12694; A. Tokareva, *Gravitational waves from inflaton decay and bremsstrahlung,* 2312.16691; G. Choi, W. Ke and K.A. Olive, *Minimal production of prompt gravitational waves during reheating,* 2402.04310; W. Hu, K. Nakayama, V. Takhistov and Y. Tang, *Gravitational Wave Probe of Planck-scale Physics After Inflation,* 2403.13882; B. Barman, N. Bernal, S. Cléry, Y. Mambrini, Y. Xu and Ó. Zapata, *Probing Reheating with Gravitational Waves from Graviton Bremsstrahlung,* 2405.09620; R. Inui, Y. Mikura and S. Yokoyama, *Gravitational waves from graviton Bremsstrahlung with kination phase,* 2408.10786; ...

$2 \to 2$ scatterings

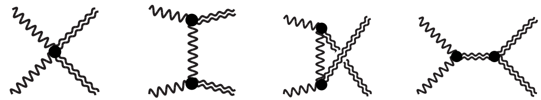
for $\pi T \gg m_\varphi$, and for standard model processes (---- → ∼∼∼), this is the most important channel, because not kinematically suppressed[25]

necessitates $T > 0$, so that a co-scatterer is available

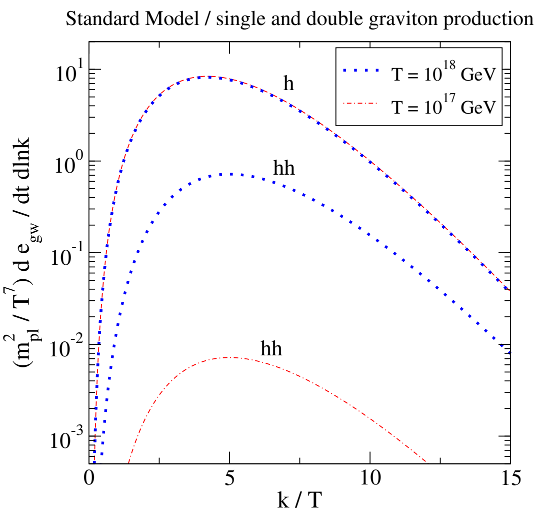
t-channel exchange can be IR divergent, and requires resummation, yielding then a logarithmically enhanced term ("leading log")

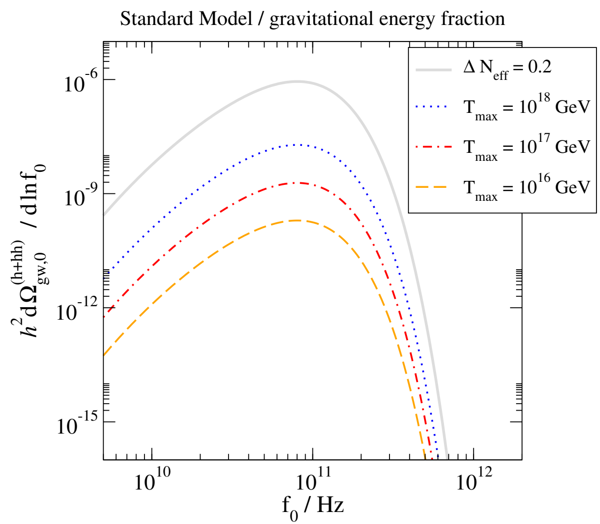
[25] J. Ghiglieri, ML, *Gravitational wave background from Standard Model physics: qualitative features,* 1504.02569; J. Ghiglieri, G. Jackson, ML, Y. Zhu, *Gravitational wave background from Standard Model physics: complete leading order,* 2004.11392; A. Ringwald, J. Schütte-Engel, C. Tamarit, *Gravitational waves as a big bang thermometer,* 2011.04731; P. Klose, ML, S. Procacci, *Gravitational wave background from non-Abelian reheating after axion-like inflation,* 2201.02317; L. Castells-Tiestos, J. Casalderrey-Solana, *Thermal emission of gravitational waves from weak to strong coupling,* 2202.05241; F. Muia, F. Quevedo, A. Schachner, G. Villa, *Testing BSM physics with gravitational waves,* 2303.01548; M. Drewes, Y. Georis, J. Klaric, P. Klose, *Upper Bound on Thermal Gravitational Wave Backgrounds from Hidden Sectors,* 2312.13855; Y. Xu, *Ultra-high Frequency Gravitational Waves from Scattering, Bremsstrahlung and Decay during Reheating*, 2407.03256; ...

double-graviton production



for testing validity of general relativity as an effective theory?[26]





26 J. Ghiglieri, J. Schütte-Engel, E. Speranza, *Freezing-In Gravitational Waves*, 2211.16513; J. Ghiglieri, ML, J. Schütte-Engel, E. Speranza, *Double-graviton production from Standard Model plasma*, 2401.08766

conclusions & outlook

traditionally cosmologists like scalar fields and weak couplings

as far as reheating goes, coupling the inflaton to a non-abelian sector could change the picture, because the latter can thermalize fast

after specifying the coupling between the inflaton and the gauge sector, Υ can be computed, and the smooth reheating dynamics solved

$\exists$ many sources of gravitational waves (some of them delicate)

as a bonus, non-abelian sectors may lead to phase transitions,[27] contribute to dark matter[28], affect baryon asymmetry[29], ...

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27 ...; P. Schwaller, *Gravitational Waves from a Dark Phase Transition*, 1504.07263; ...

28 ...; S. Biondini, H. Kolesova and S. Procacci, *Smooth reheating and dark matter via non-Abelian gauge theory*, 2406.10345; D. McKeen, R. Mizuta, D.E. Morrissey and M. Shamma, *Dark Matter from Dark Glueball Dominance*, 2406.18635

29 ...; A. Maleknejad, *$SU(2)_R$ and its axion in cosmology: A common origin for inflation, cold sterile neutrinos, and baryogenesis*, 2012.11516; ...