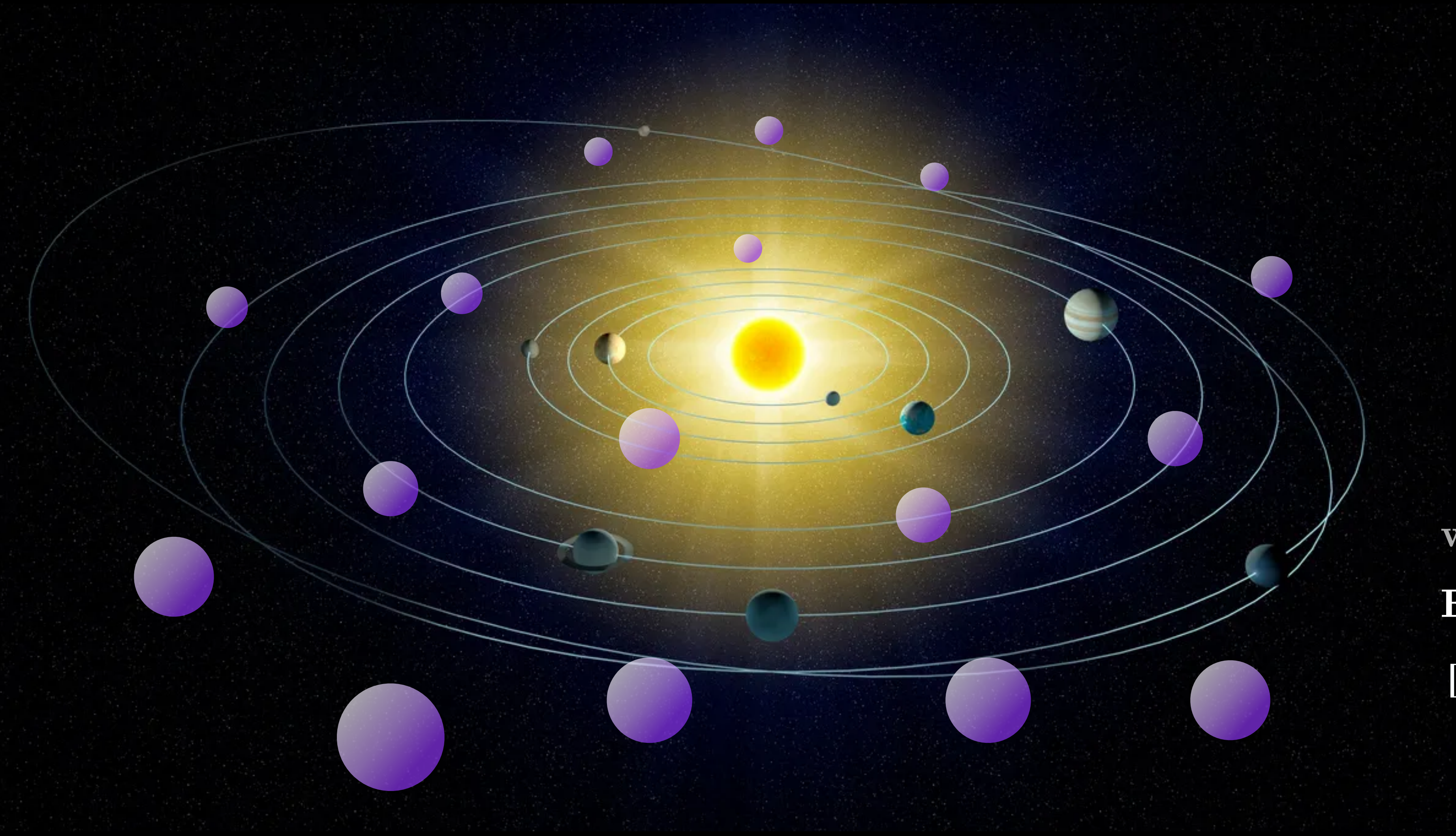
Axion Dark Matter: Production and Clustering

Marco Gorghetto



with

E. Hardy and G. Villadoro

[2405.19389, 2007.04990]

Outline

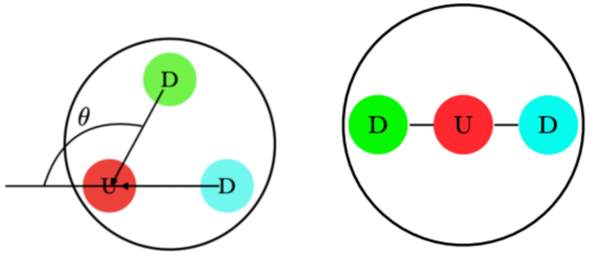
- Axions and post-inflationary scenario

- Structure formation and axion stars

QCD axion:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial a)^2 + \frac{a}{\boxed{f_a}}\frac{\alpha_s}{8\pi} G\tilde{G} + \dots \qquad \Rightarrow \qquad m = \frac{\chi_{\rm top}^{1/2}}{f_a} \simeq 0.57\,{\rm meV}\left(\frac{10^{10}\ {\rm GeV}}{f_a}\right)$$

- Dynamically explains no neutron EdM

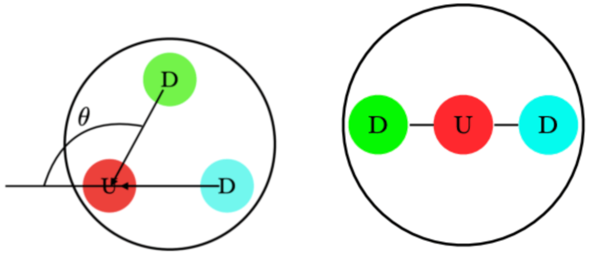


[picture from A. Hook]

QCD axion:

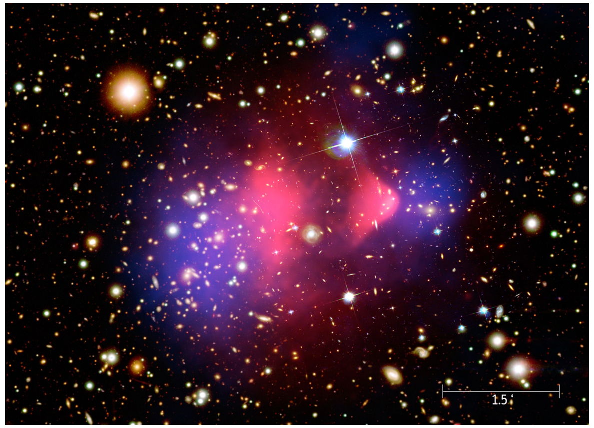
$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial a)^2 + \frac{a}{\boxed{f_a}}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots \qquad \Rightarrow \qquad m = \frac{\chi_{\rm top}^{1/2}}{f_a} \simeq 0.57\,{\rm meV}\left(\frac{10^{10}\ {\rm GeV}}{f_a}\right)$$

- Dynamically explains no neutron EdM

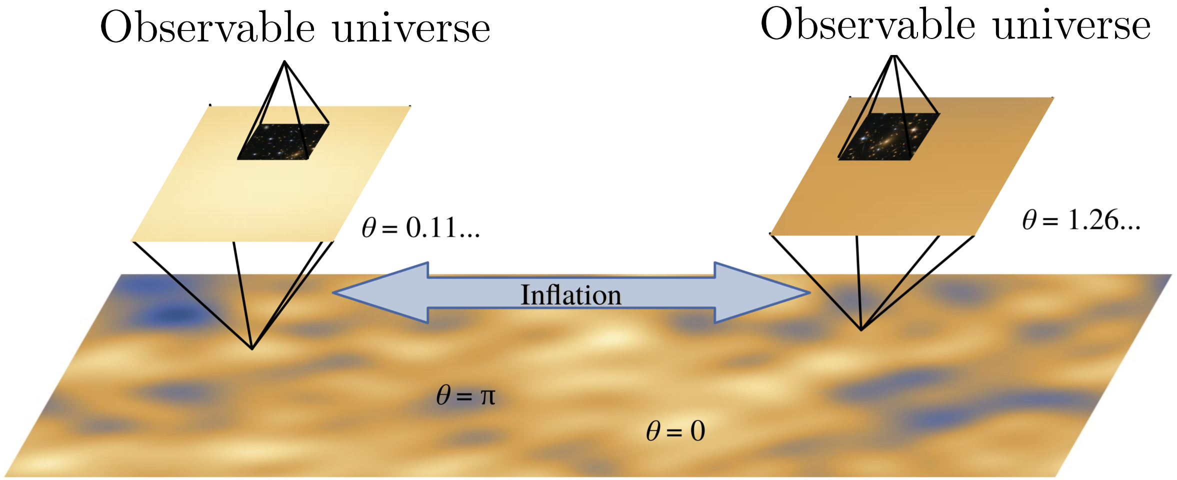


[picture from A. Hook]

- Contributes to all/part of the dark matter



Pre-inflationary



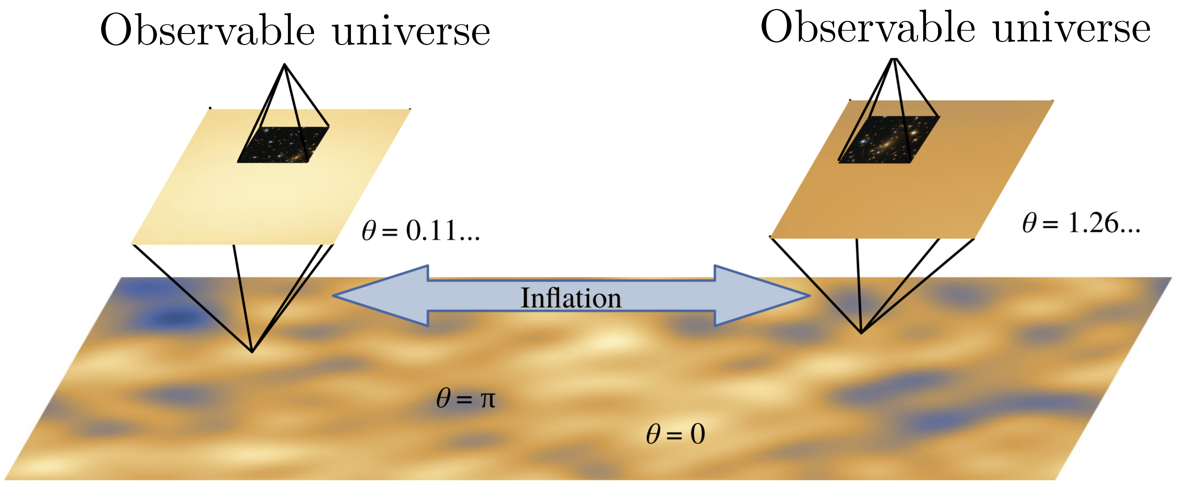
$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

$$\Omega_a \simeq \theta_0^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{1.2} \Omega_{\mathrm{DM}}$$

misalignment

Post-inflationary

Pre-inflationary



$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

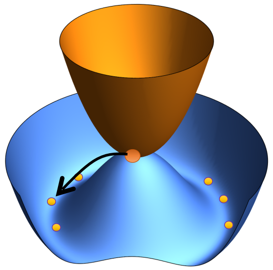
$$\Omega_a \simeq \theta_0^2 \left(\frac{f_a}{10^{12}\,\text{GeV}} \right)^{1.2} \Omega_{\text{DM}}$$

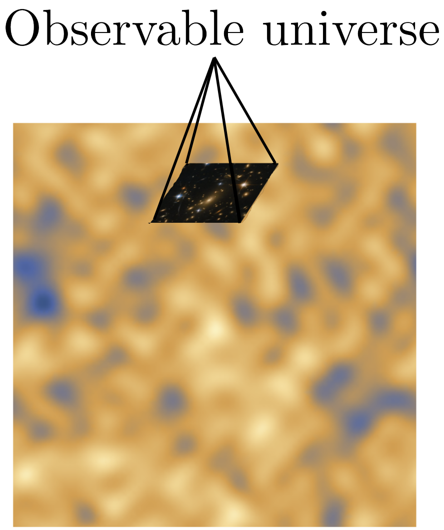
misalignment

Post-inflationary

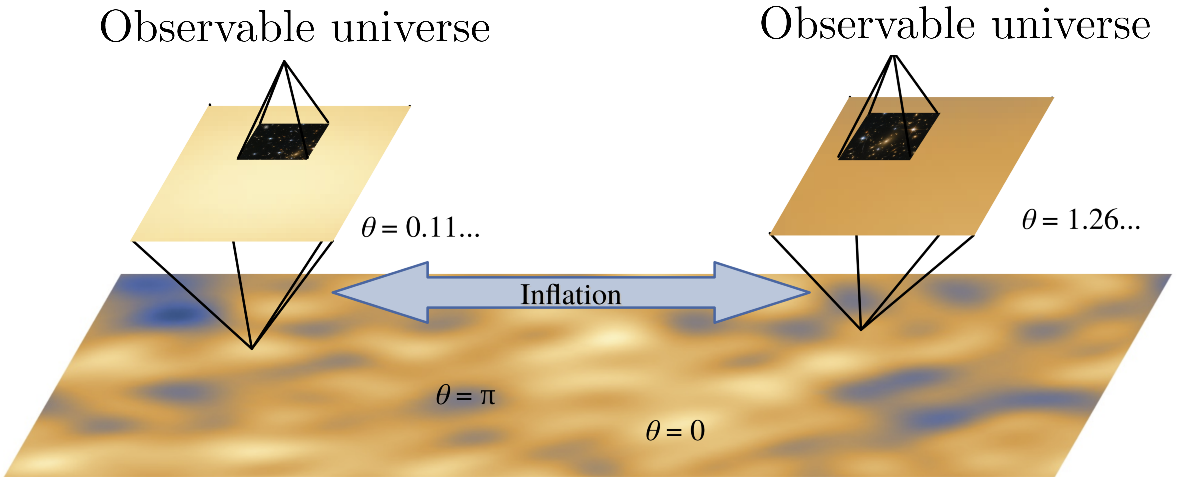
$T \gtrsim f_a$

$T \lesssim f_a$





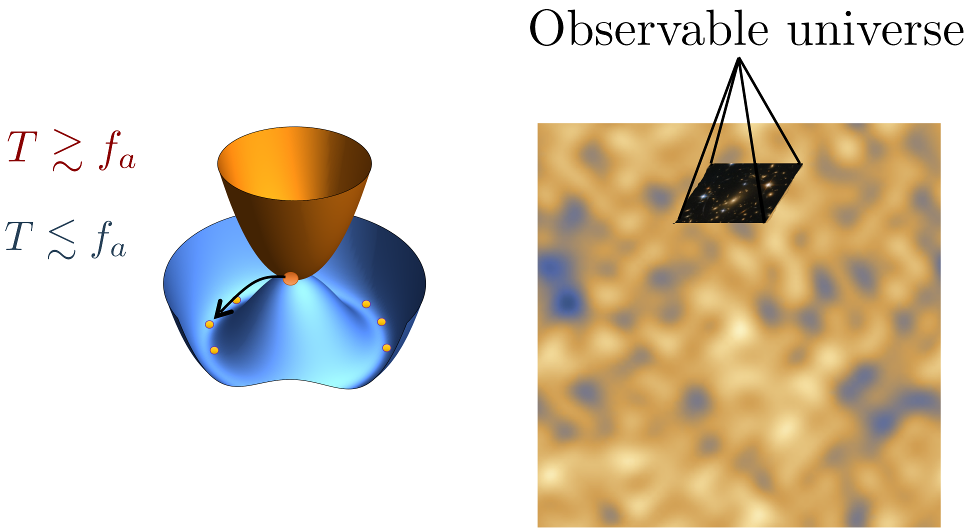
Pre-inflationary

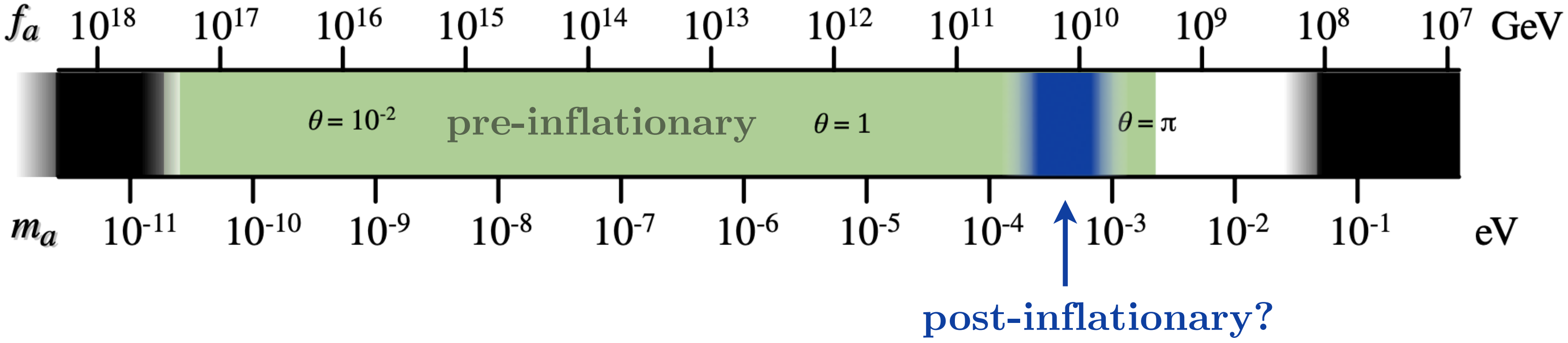


$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi] \qquad \Omega_a \simeq \theta_0^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{1.2} \Omega_{\mathrm{DM}}$$

misalignment

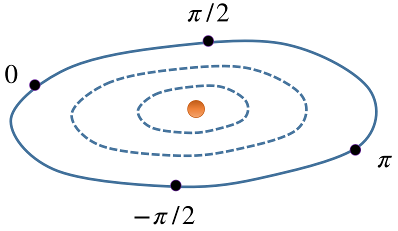
Post-inflationary





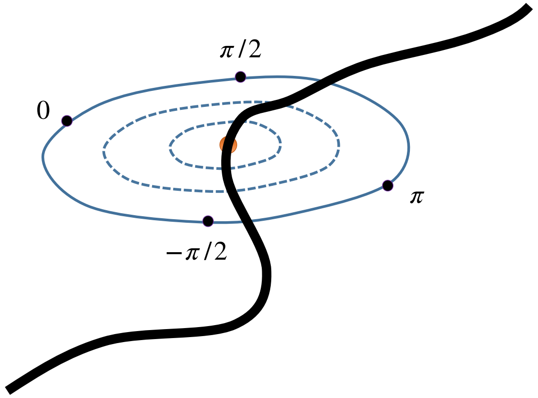
Kibble mechanism $\Longrightarrow$ **Axion strings**

@ $T \simeq f_a$ (or $H \simeq f_a$)



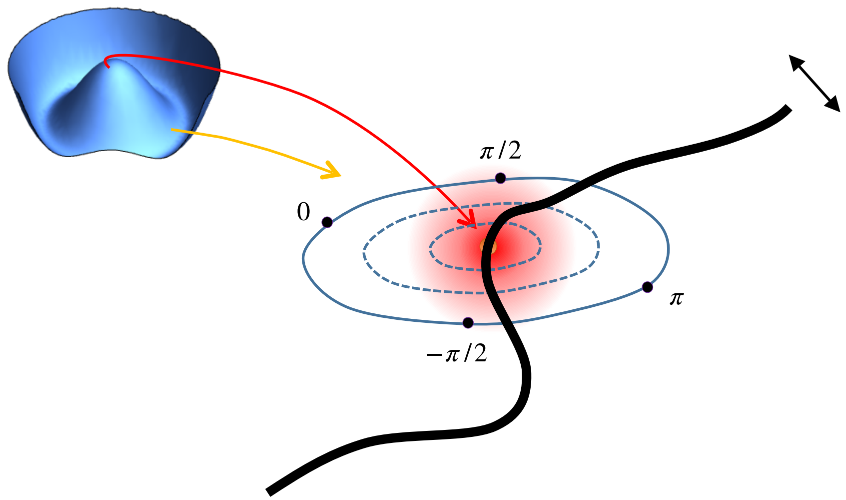
@ $T \simeq f_a$ (or $H \simeq f_a$)

Kibble mechanism $\Longrightarrow$ **Axion strings**



@ $T \simeq f_a$ (or $H \simeq f_a$)

Kibble mechanism $\Longrightarrow$ **Axion strings**

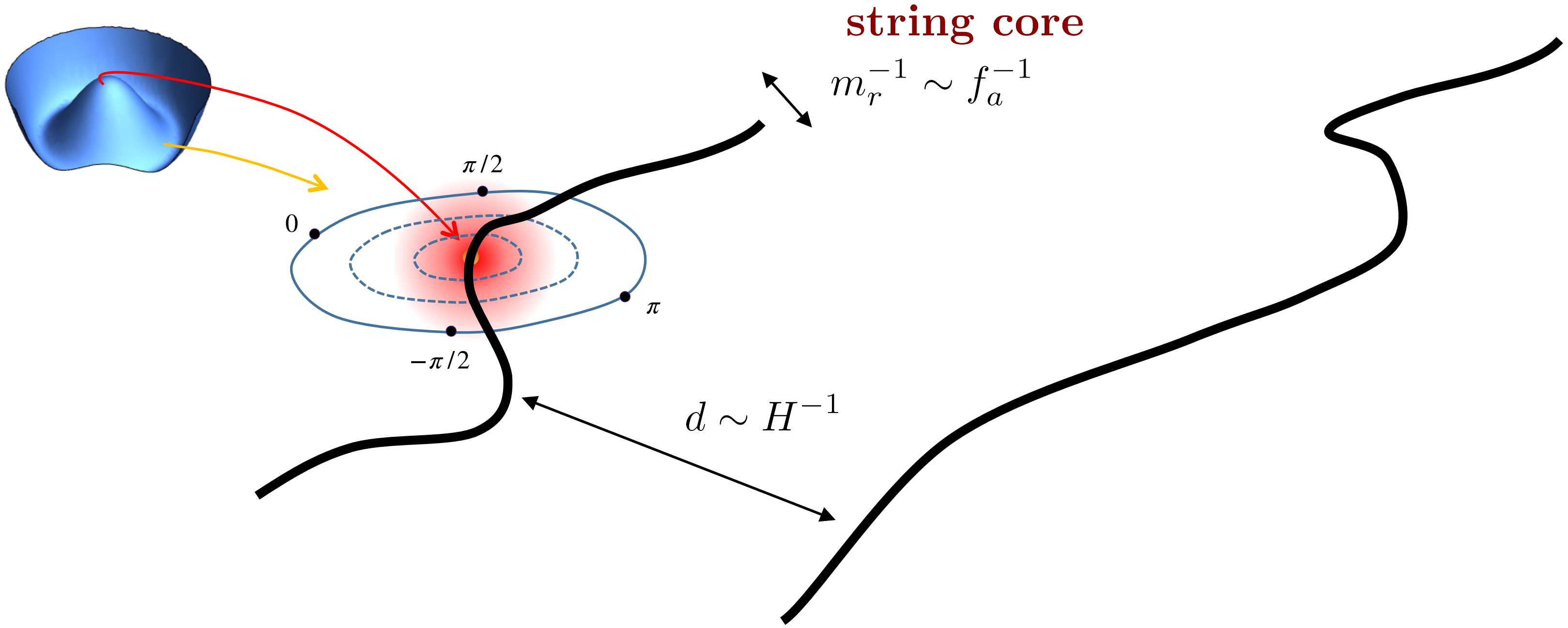


string core

$m_r^{-1} \sim f_a^{-1}$

@ $T \simeq f_a$ (or $H \simeq f_a$)

Kibble mechanism $\Longrightarrow$ **Axion strings**



string tension

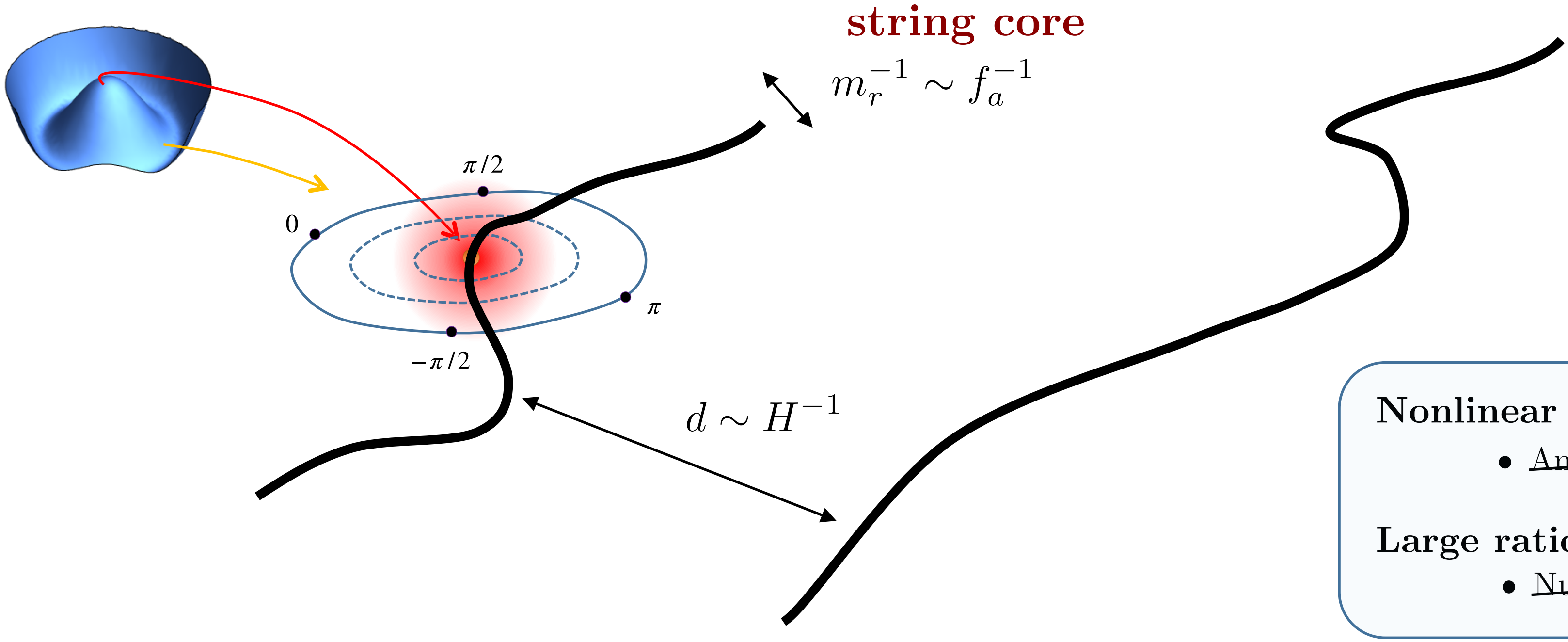
$$\mu = \frac{E}{L} \sim \underbrace{\boxed{\pi f_a^2}}_{\text{core}} \underbrace{\boxed{\log \frac{d}{m_r^{-1}}}}_{\text{axion gradient}} \sim \pi f_a^2 \log \frac{m_r}{\underset{\searrow\, T^2/M_p}{H}}$$

↑

grows logarithmically in time

@ $T \simeq f_a$ (or $H \simeq f_a$)

Kibble mechanism $\Longrightarrow$ **Axion strings**



Nonlinear dynamics: ☹
- ~~Analytical approach~~

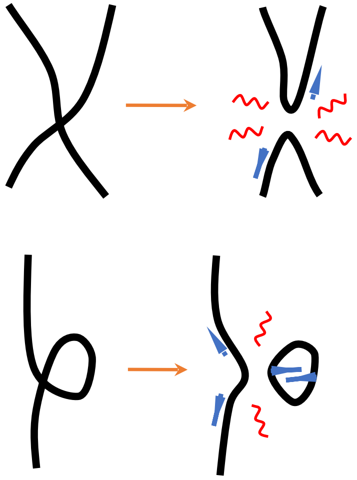
Large ratio of scales: ☹
- ~~Numerical approach~~

string tension

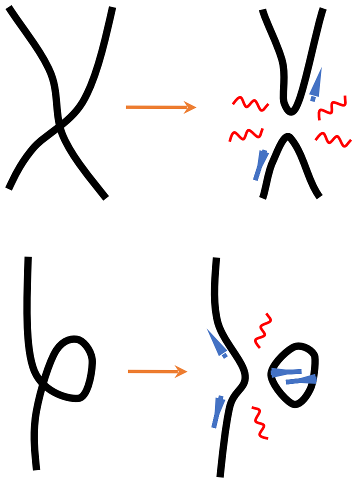
$$\mu = \frac{E}{L} \sim \underbrace{\boxed{\pi f_a^2}}_{\text{core}} \underbrace{\boxed{\log \frac{d}{m_r^{-1}}}}_{\text{axion gradient}} \sim \pi f_a^2 \log \frac{m_r}{\underbrace{H}_{T^2/M_p}}$$

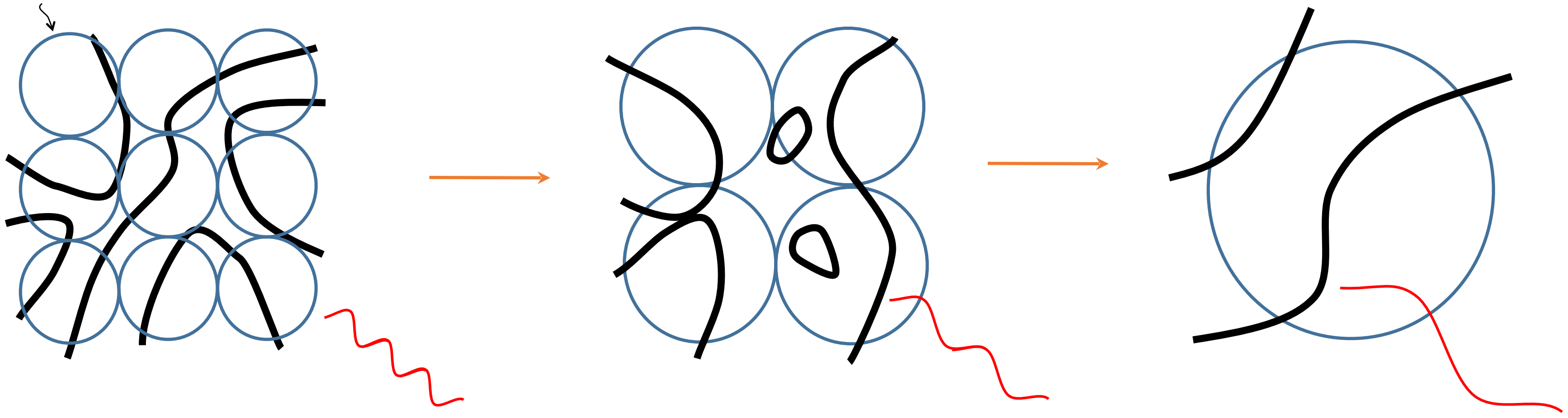
⬆ grows logarithmically in time

The Scaling Regime



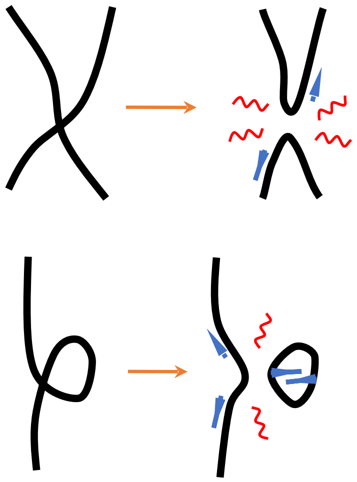
The Scaling Regime

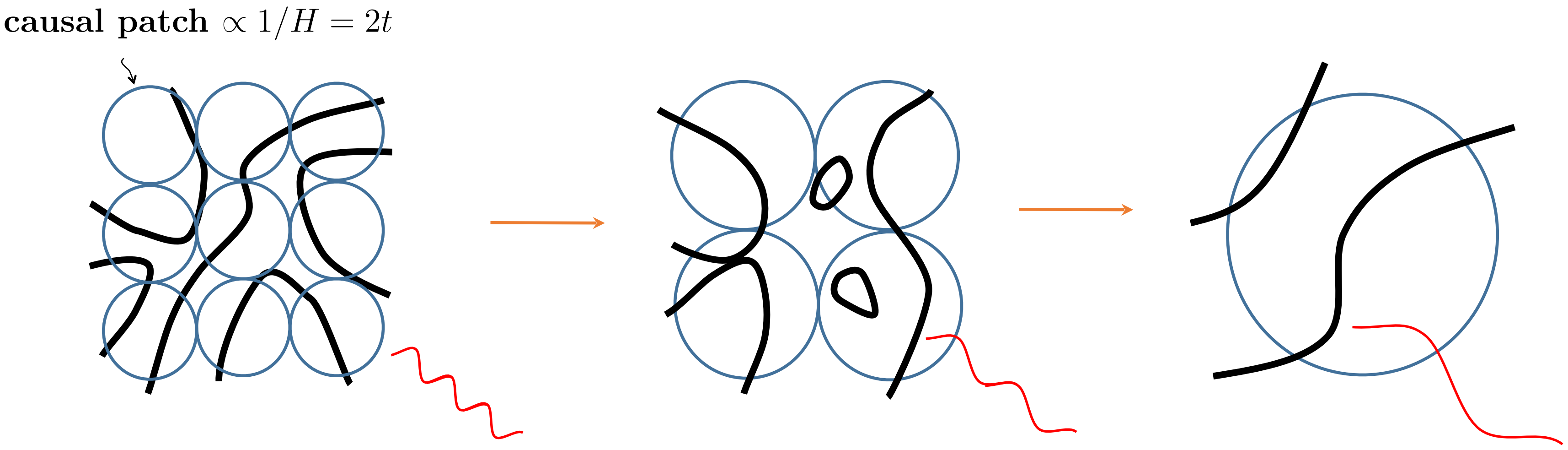




causal patch $\propto 1/H = 2t$

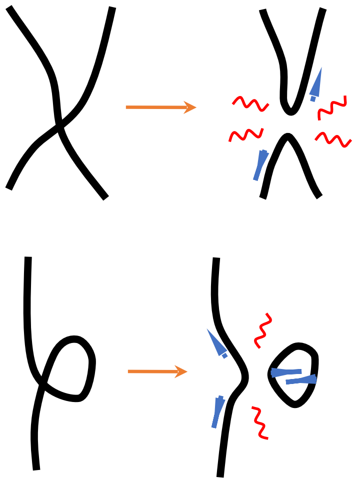
The Scaling Regime



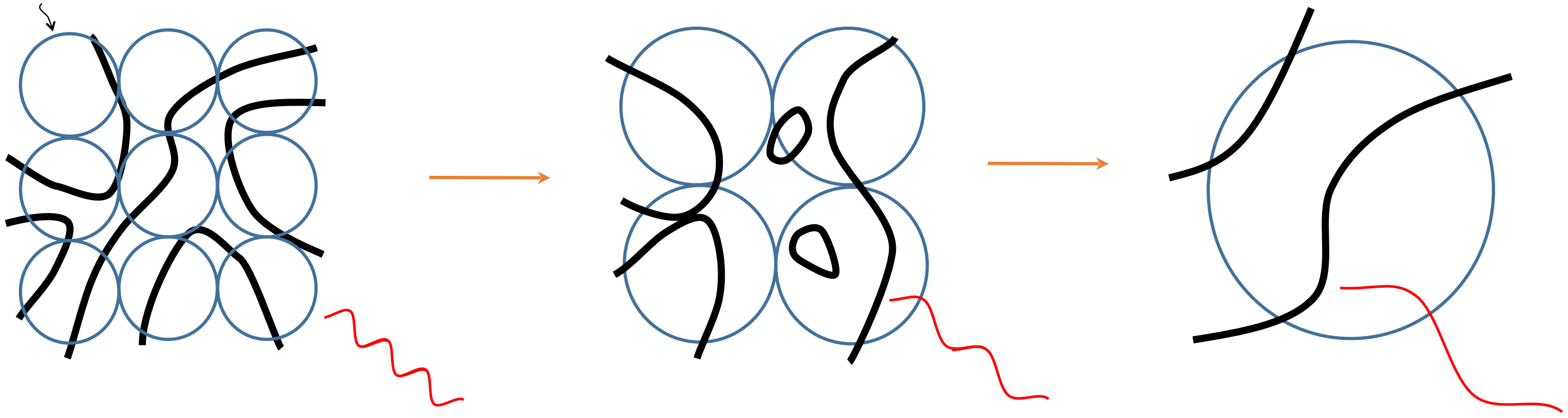


rate of energy loss: Γ

The Scaling Regime



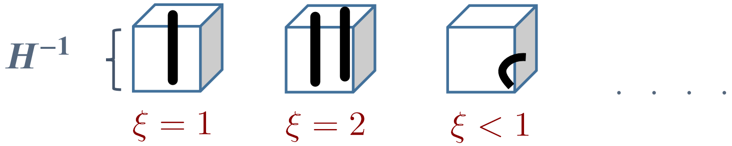
causal patch $\propto 1/H = 2t$

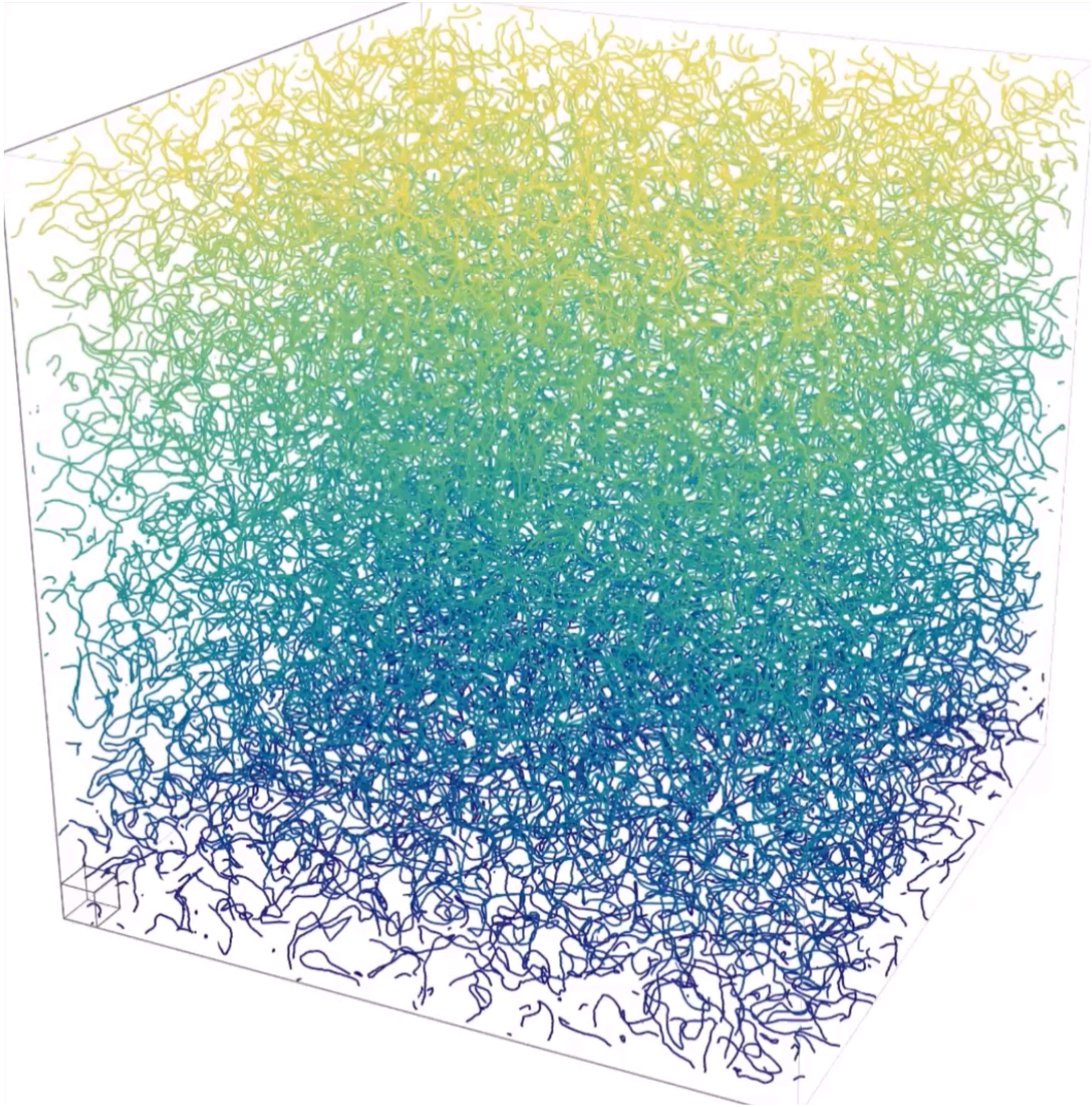


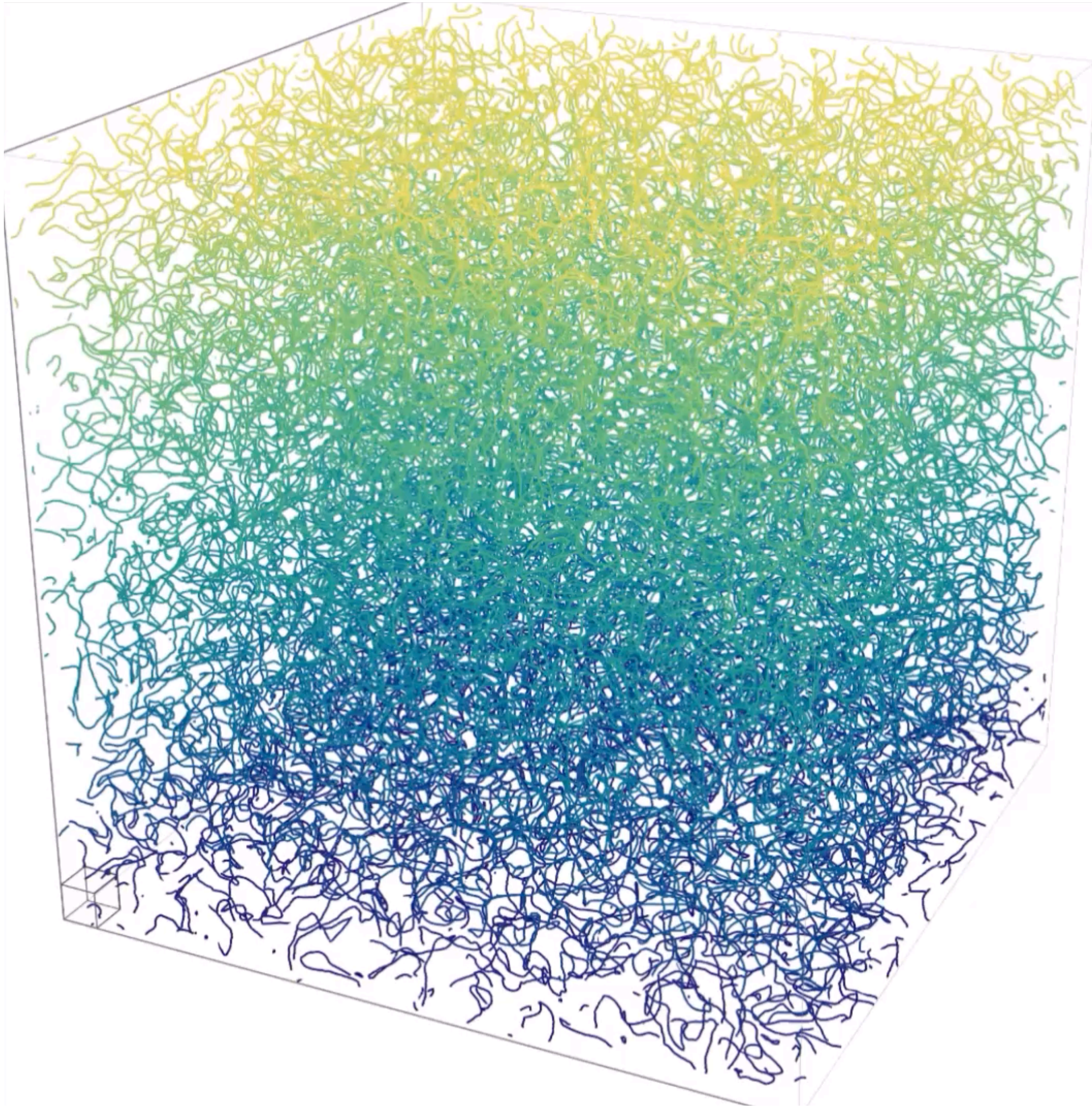
rate of energy loss:

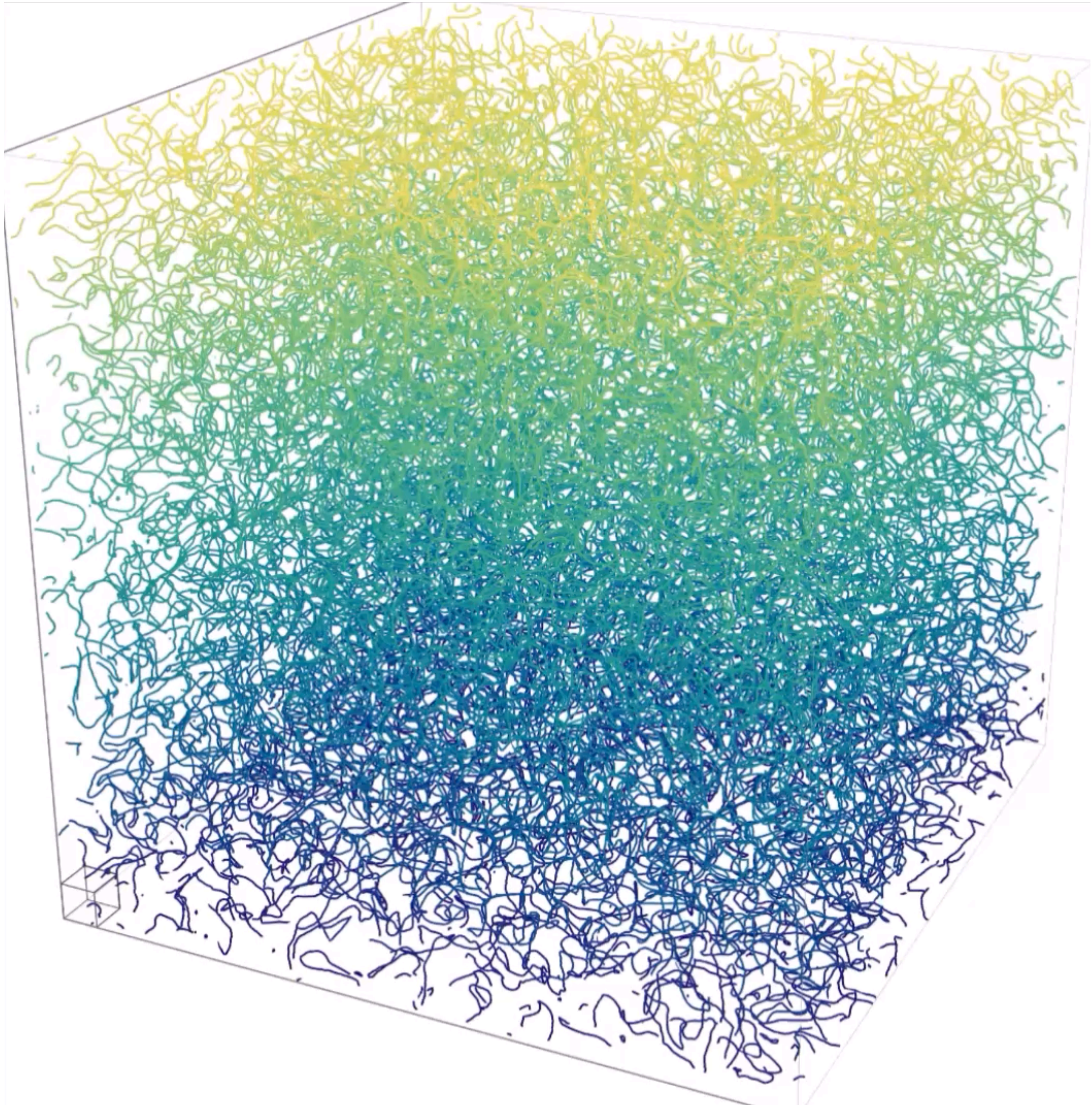
$$\Gamma \simeq \frac{\boxed{\xi}\mu}{t^3}$$

number of strings
per Hubble patch



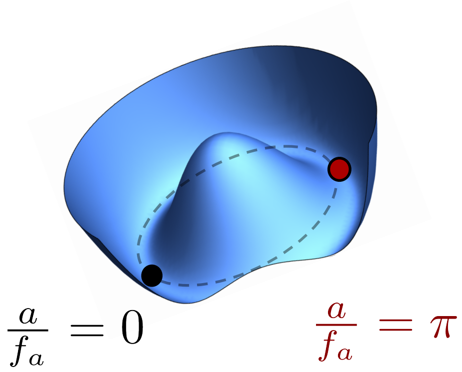




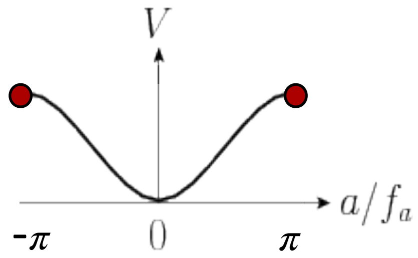


Domain Walls

@ $T \simeq 1\,\mathrm{GeV} \quad (\; m = H \equiv H_\star \;)$

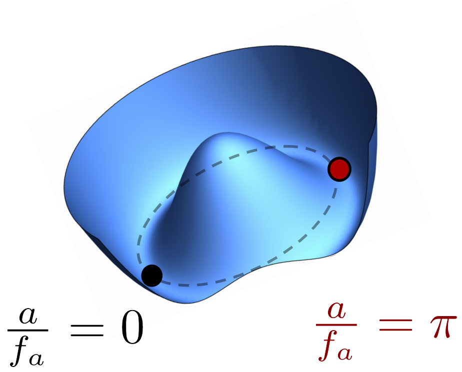


Axion potential from QCD:

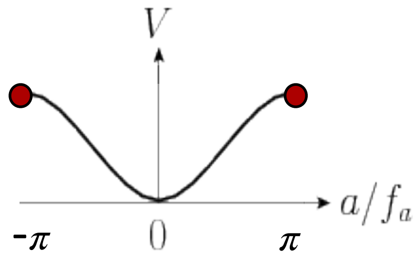


Domain Walls

@ $T \simeq 1\,\mathrm{GeV}$ $\quad (\, m = H \equiv H_\star \,)$

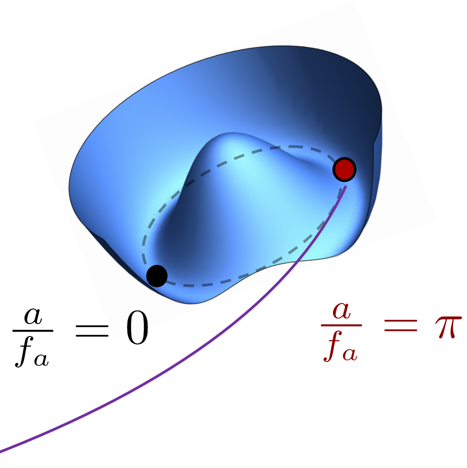


Axion potential from QCD:

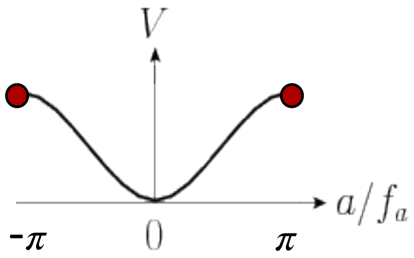


Domain Walls

$@\, T \simeq 1\,\mathrm{GeV} \quad (\ m = H \equiv H_\star\ )$

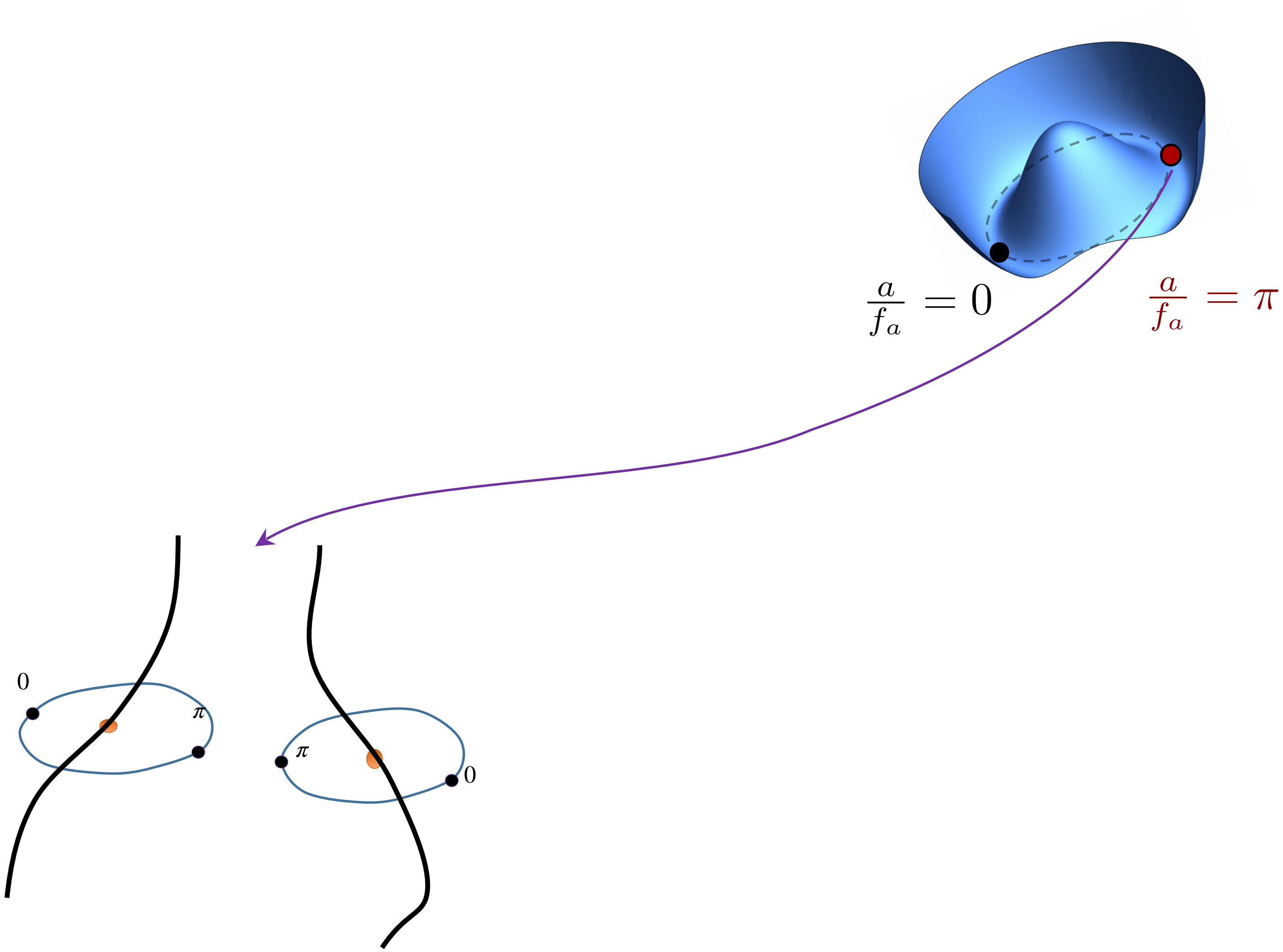


Axion potential from QCD:

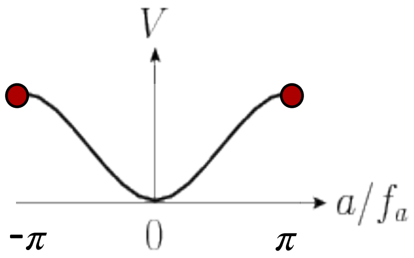


Domain Walls

$@\, T \simeq 1\,\mathrm{GeV} \quad (\ m = H \equiv H_\star\ )$

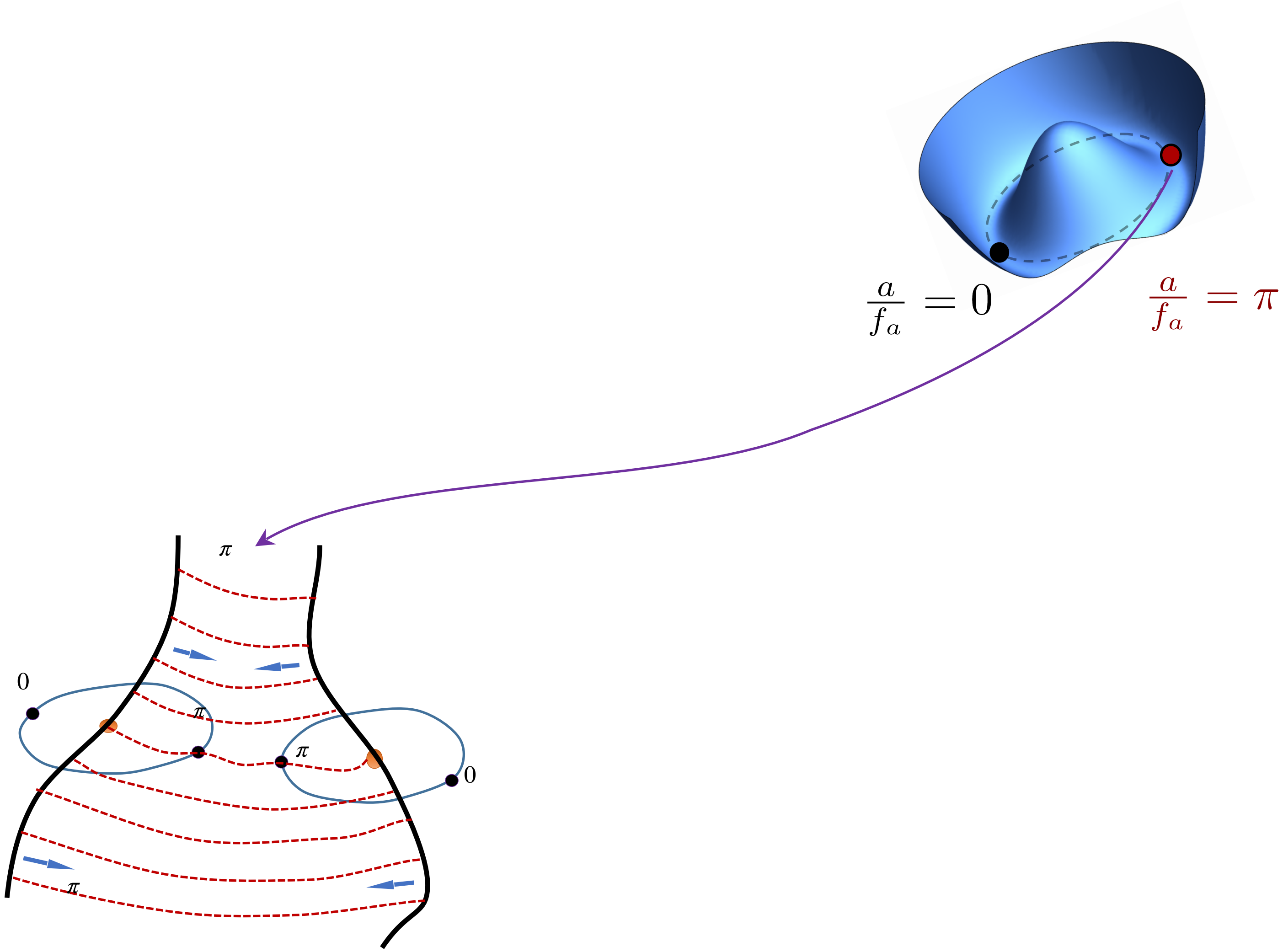


Axion potential from QCD:

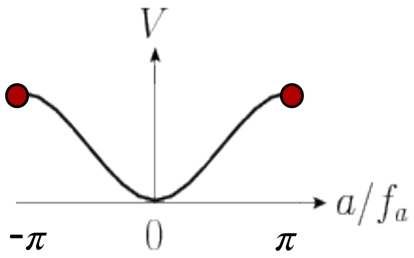


Domain Walls

@ $T \simeq 1\,\mathrm{GeV} \quad (\ m = H \equiv H_\star\ )$

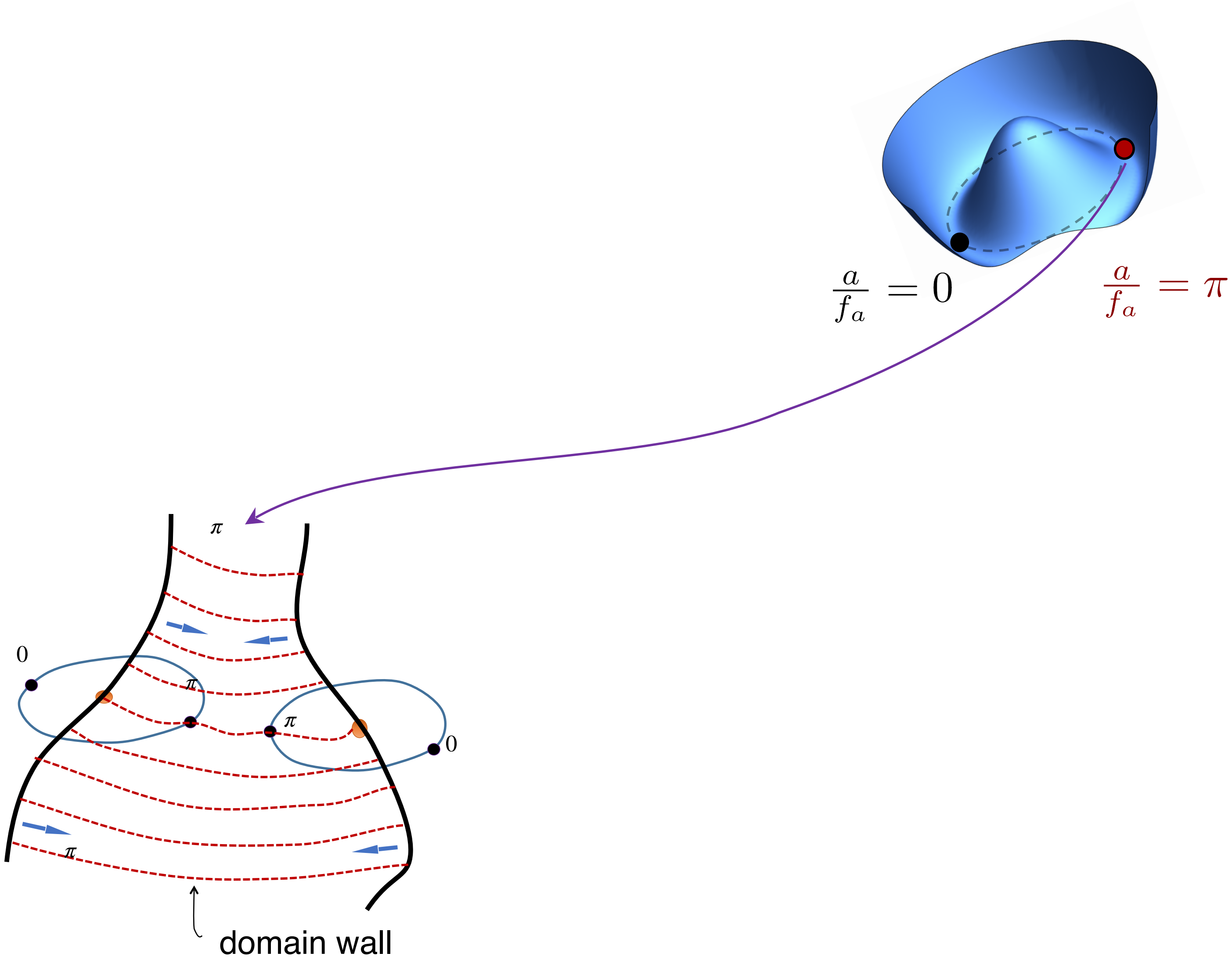


Axion potential from QCD:

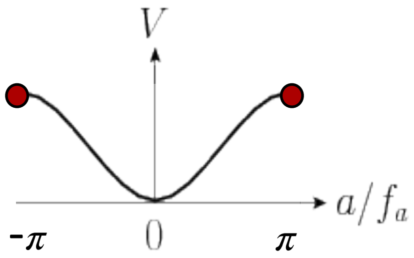


Domain Walls

$@\ T \simeq 1\,\mathrm{GeV} \quad (\ m = H \equiv H_\star\ )$



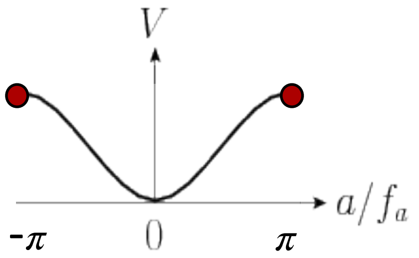
Axion potential from QCD:

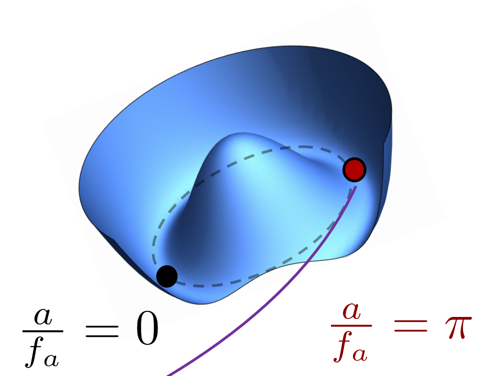


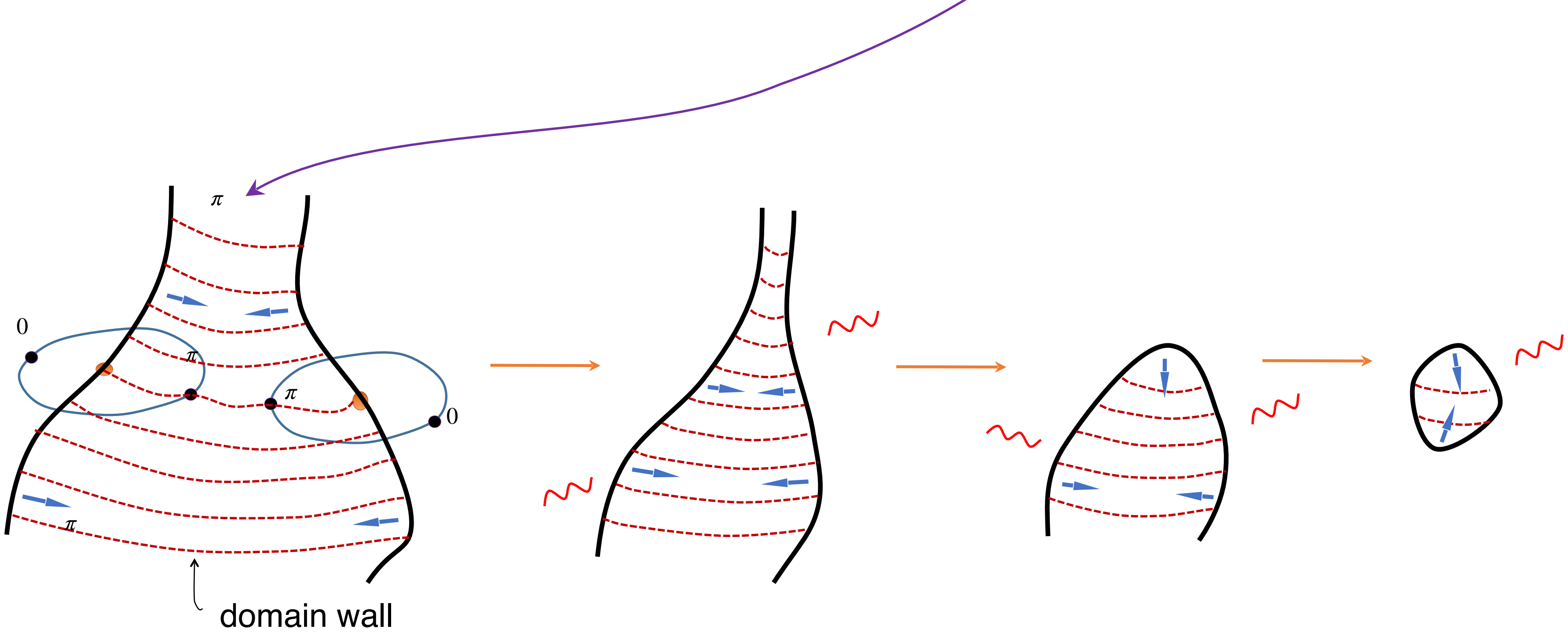
Domain Walls

@ $T \simeq 1\,\text{GeV} \quad (\ m = H \equiv H_\star\ )$

Axion potential from QCD:

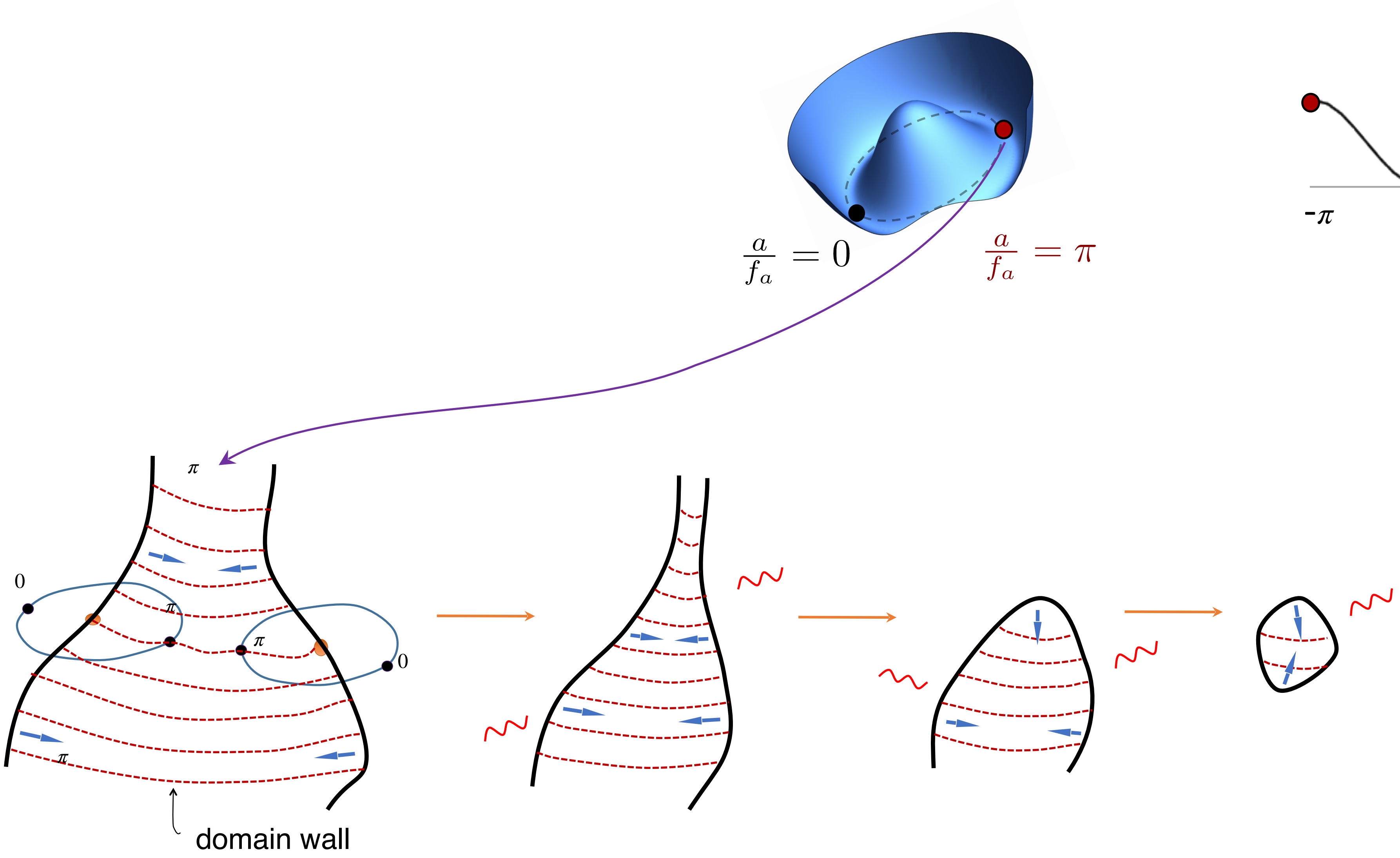




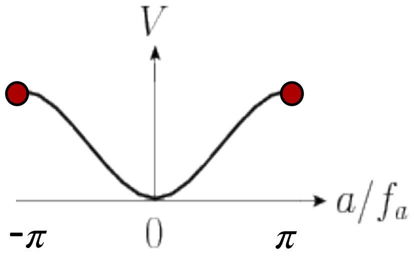


Domain Walls

@ $T \simeq 1\,\mathrm{GeV} \quad (\ m = H \equiv H_\star\ )$

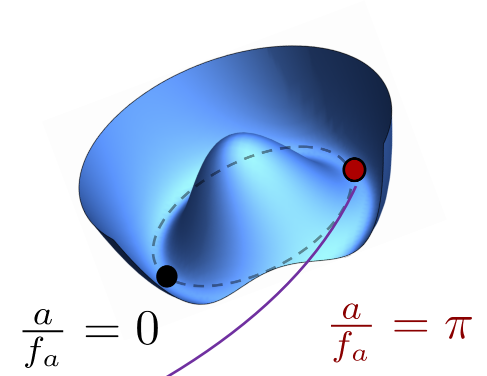


Axion potential from QCD:

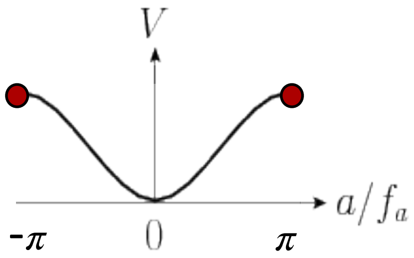


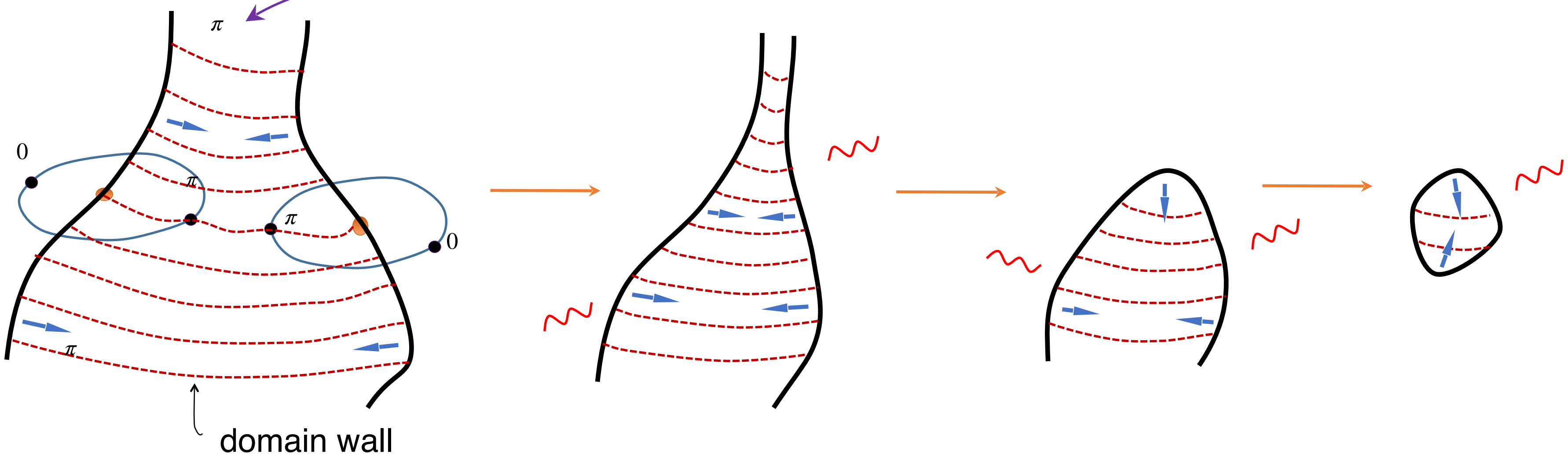
Domain Walls

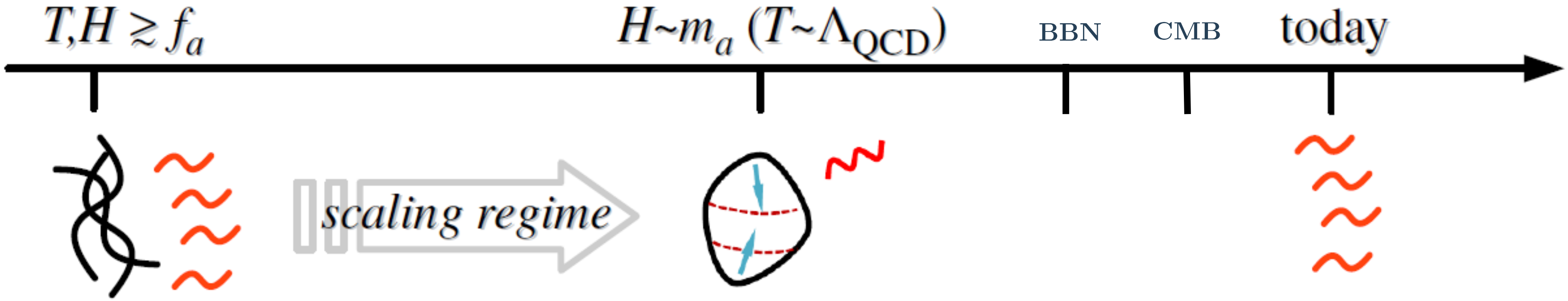
@ $T \simeq 1\,\mathrm{GeV} \quad (\, m = H \equiv H_\star \,)$



Axion potential from QCD:





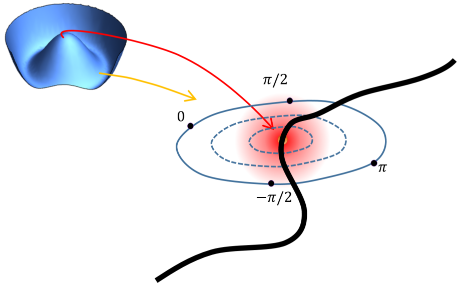


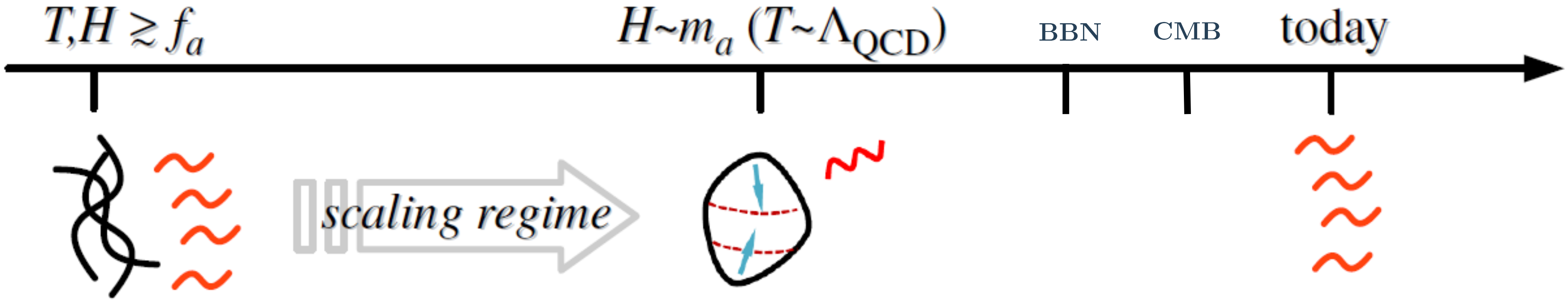
$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$

strings form

relic axions
and gravitational waves

strings





$T,H \gtrsim f_a$

$H \sim m_a\ (T \sim \Lambda_{QCD})$

BBN

CMB

today

scaling regime

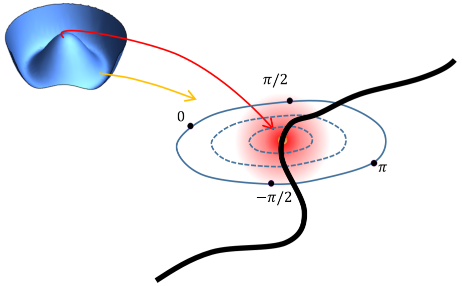
$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70}$

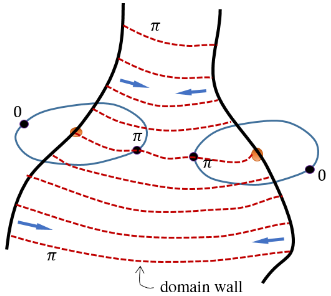
strings form

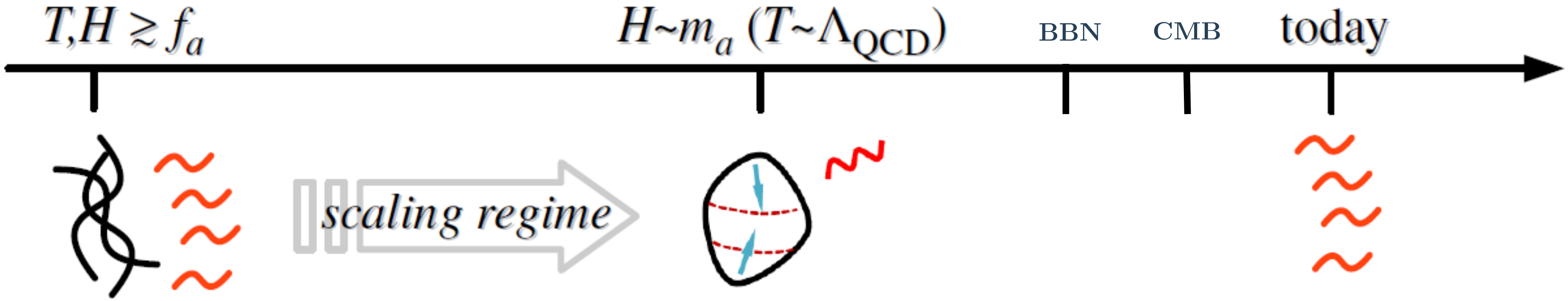
domain walls form and annihilate

relic axions and gravitational waves

strings **domain walls**







$T,H \gtrsim f_a$ $H\sim m_a$ $(T\sim\Lambda_{\mathrm{QCD}})$ BBN CMB today

scaling regime

$\log(m_r/H)$ $\sim \mathbf{1 \div 15}$ $\sim \mathbf{70}$

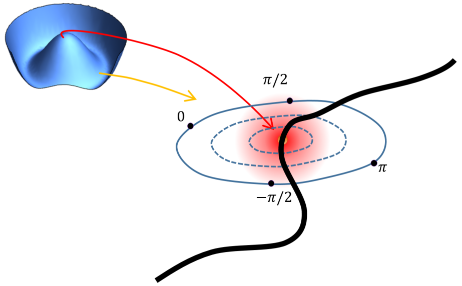
strings form

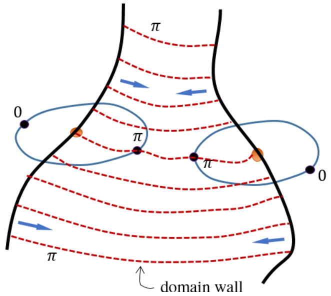
domain walls form and annihilate

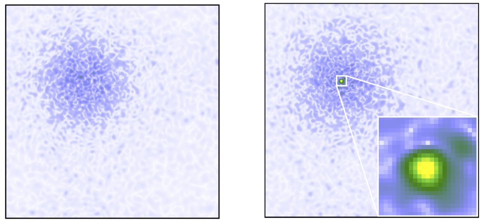
relic axions and gravitational waves

strings **domain walls**

axion stars
miniclusters

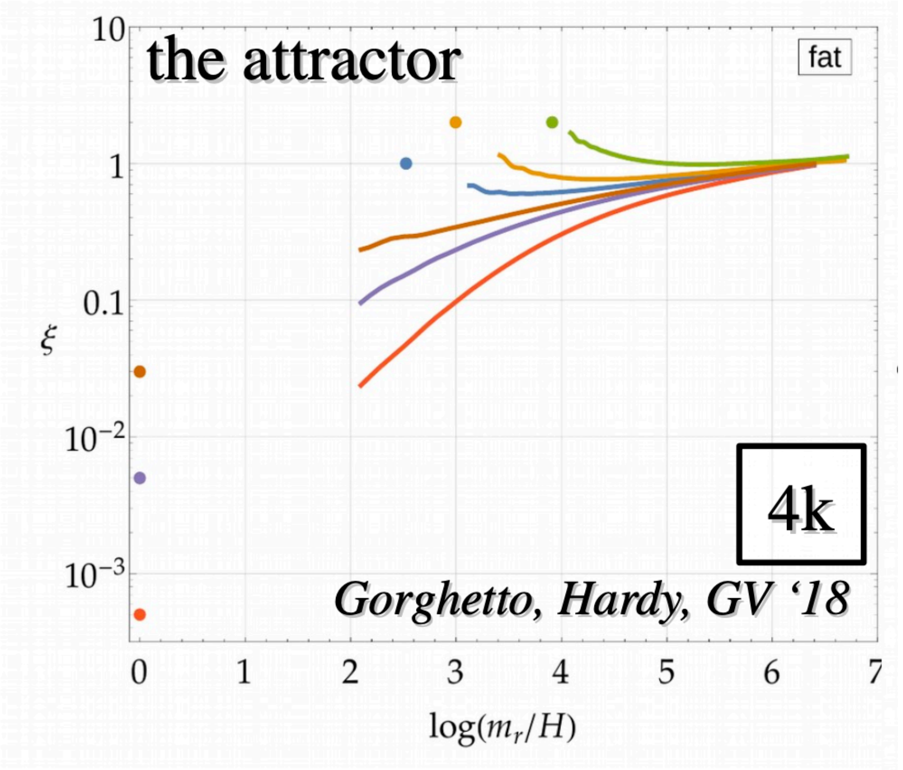




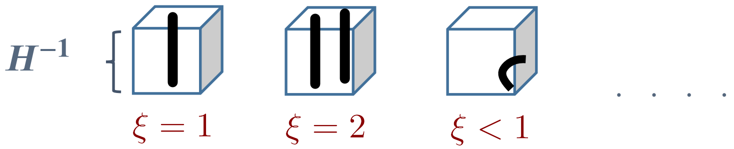


[from 1804.05857]

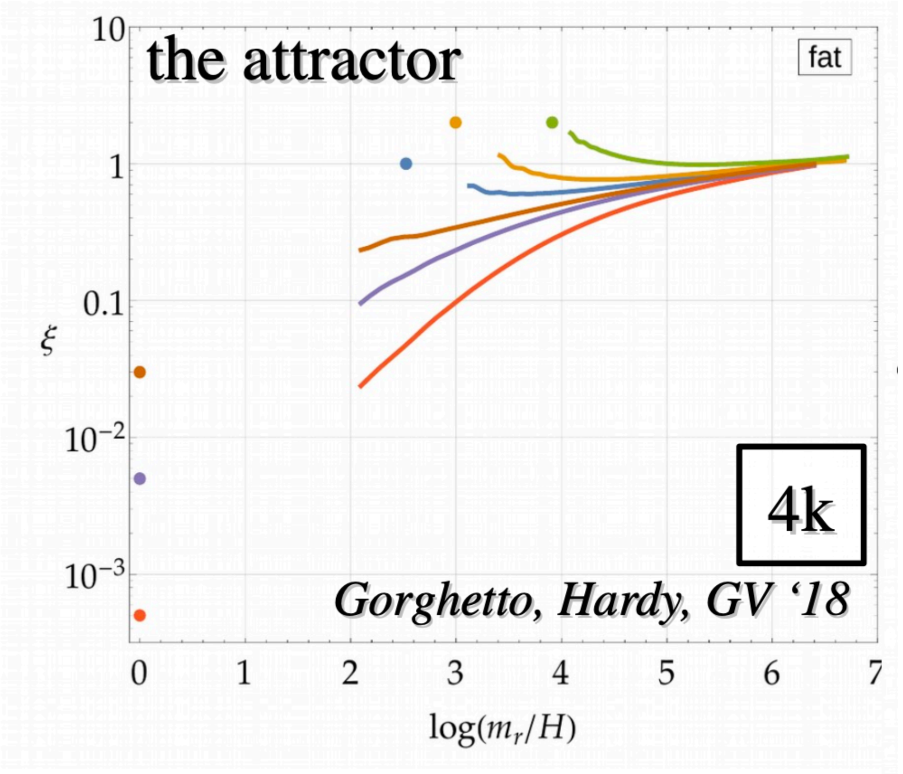
The Scaling Regime


the attractor

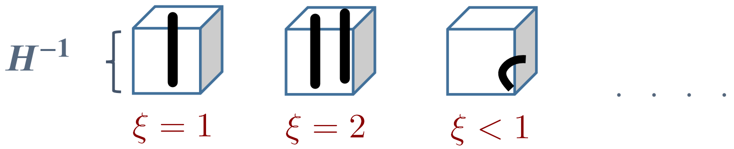
Gorghetto, Hardy, GV ‘18



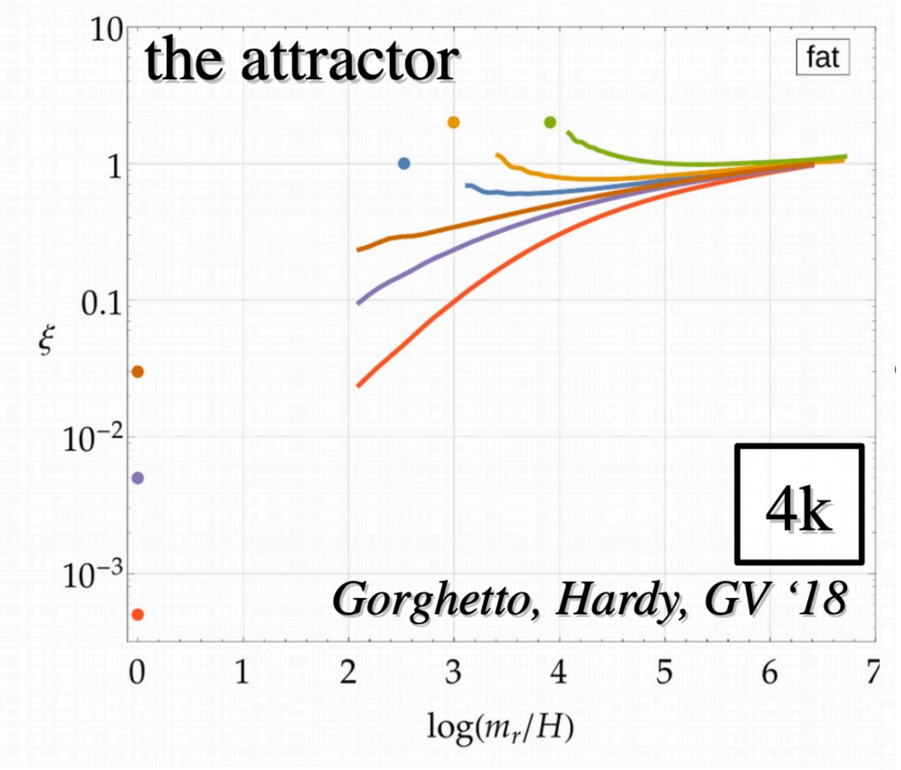
The Scaling Regime

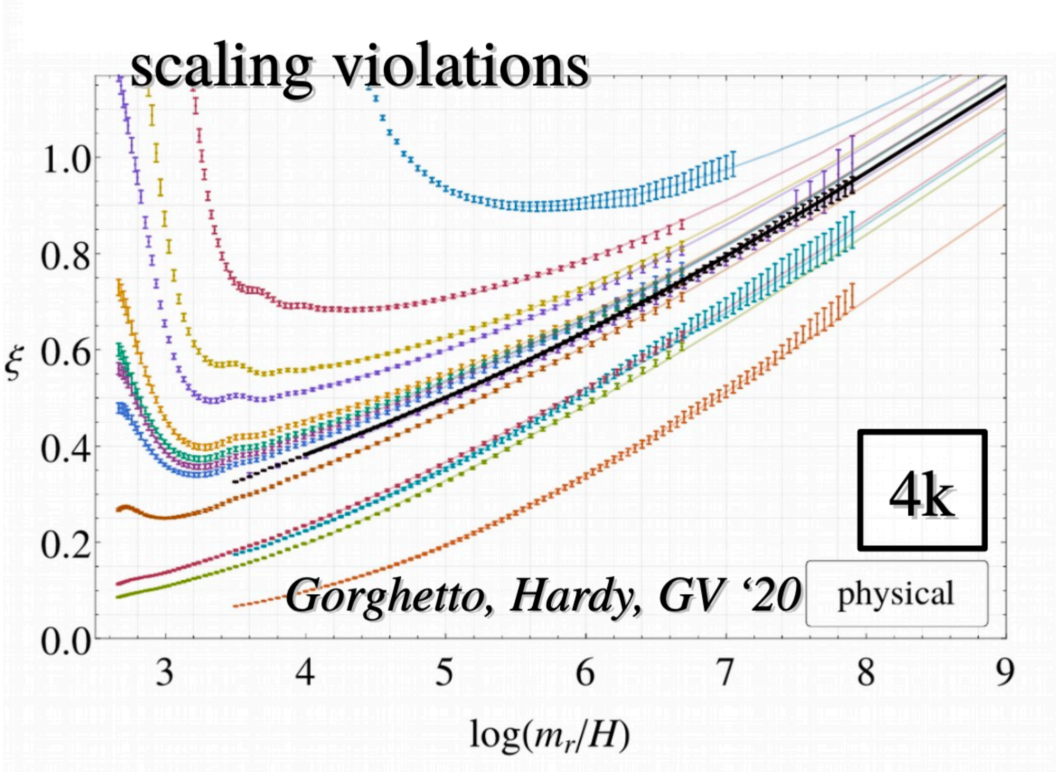


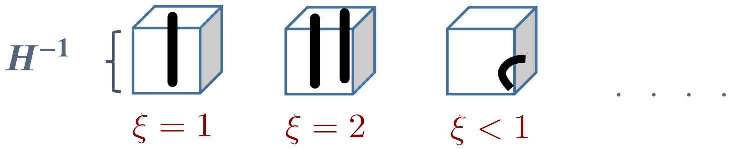
Gorghetto, Hardy, GV '18



The Scaling Regime



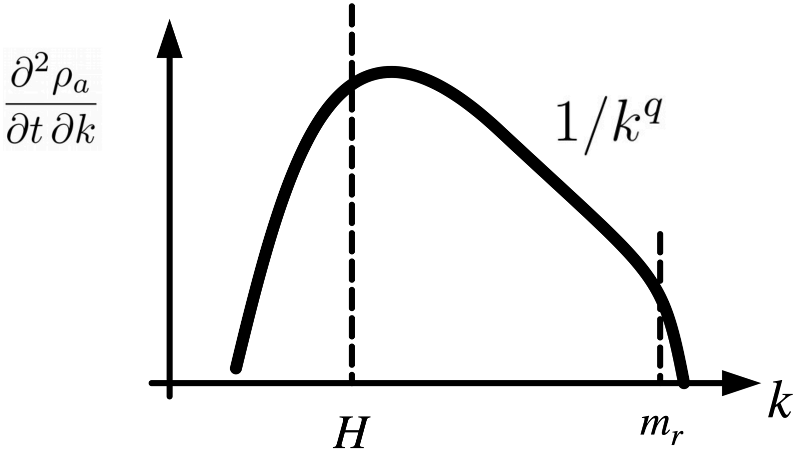




$$\xi \to \frac{\log(m_r/H)}{4 \div 5}$$

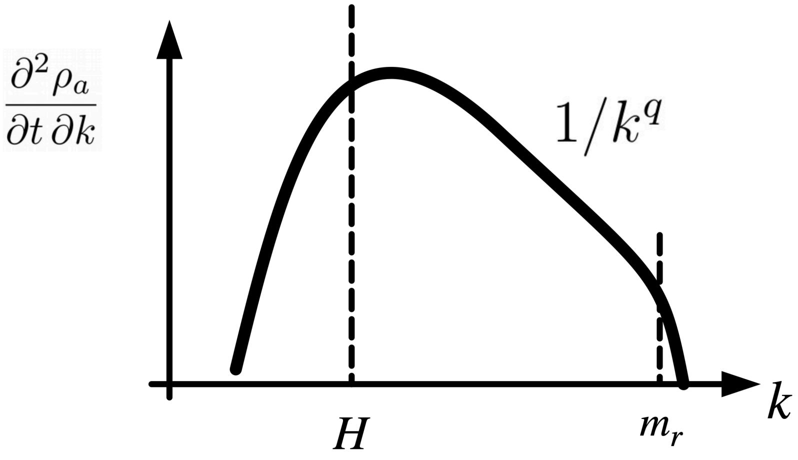
$$\boxed{\xrightarrow{\log \to 70} 15(2)}$$

The Spectrum



$$n \sim \frac{\rho}{\langle k \rangle}$$

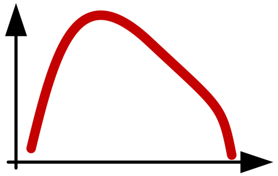
The Spectrum

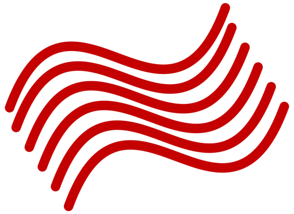


$$n \sim \frac{\rho}{\langle k \rangle}$$

IR

$q > 1$



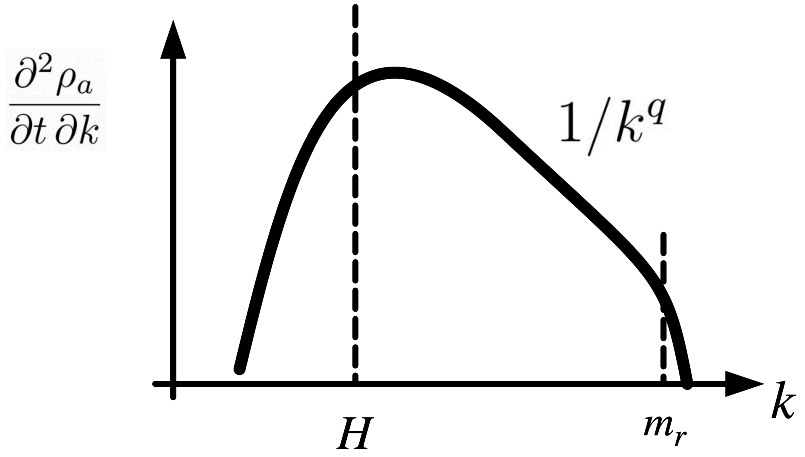


Davies, Shellard, ...

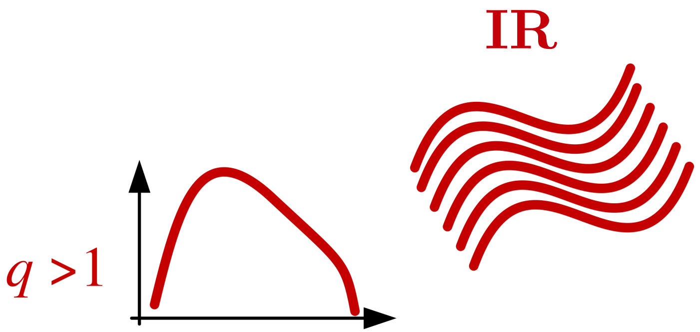
$$n \sim \frac{\rho}{H} \sim \xi \log f^2 H \sim \boxed{\xi \log} n^{mis}$$

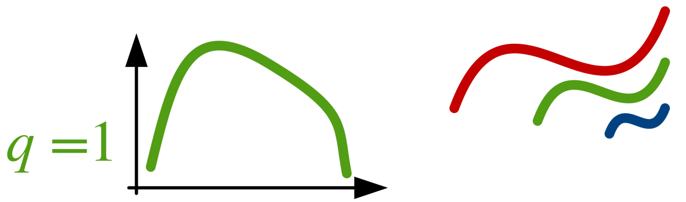
$\sim 10^3$

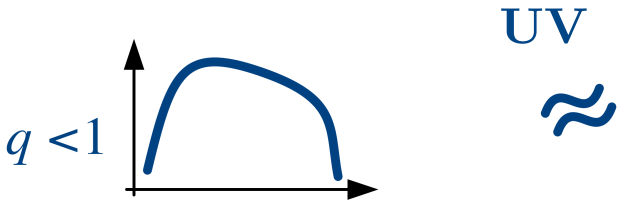
The Spectrum



$$n \sim \frac{\rho}{\langle k \rangle}$$







Davies, Shellard, ...

$$n \sim \frac{\rho}{H} \sim \xi \log f^2 H \sim \boxed{\xi \log} n^{mis}$$

$\sim 10^3$

Sikivie, ...

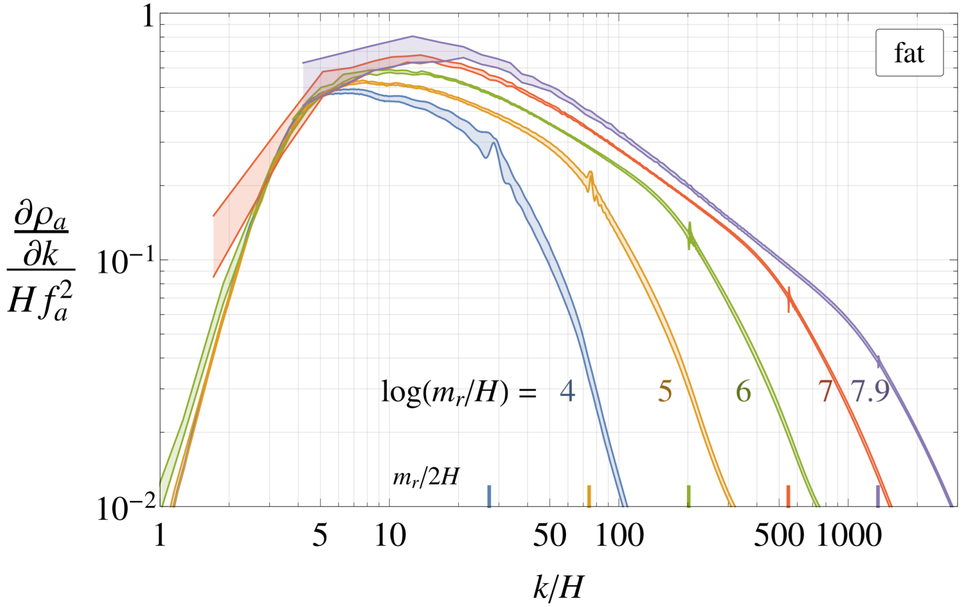
$$n \sim \frac{\rho}{H \log} \sim \xi f^2 H \sim \boxed{\xi} n^{mis}$$

$$n \sim \frac{\rho}{H} \left(\frac{H}{m_r}\right)^{1-q} \sim n^{mis} \boxed{\left(\frac{H}{m_r}\right)^{1-q}}$$

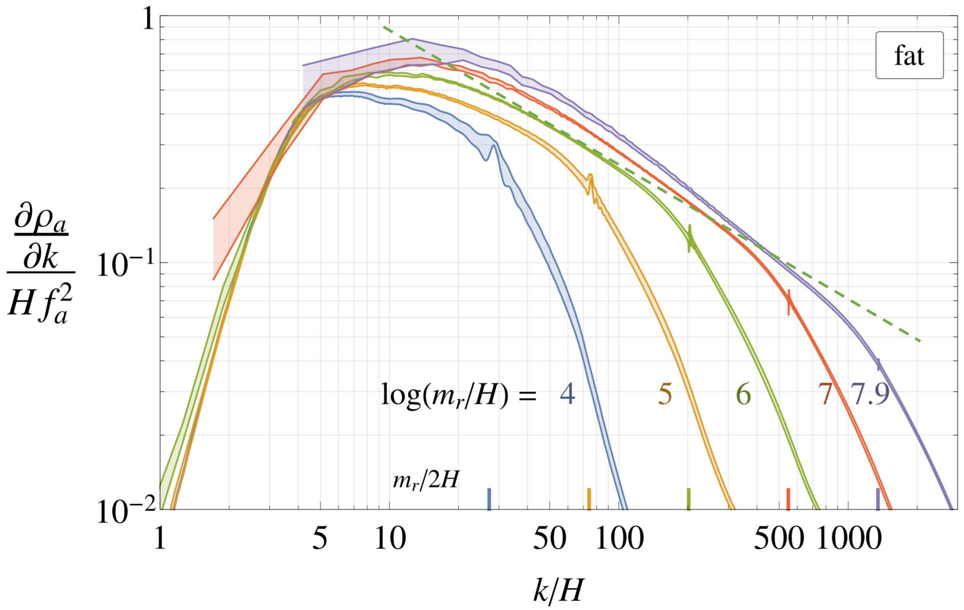
$\ll 1$

The Spectral Index

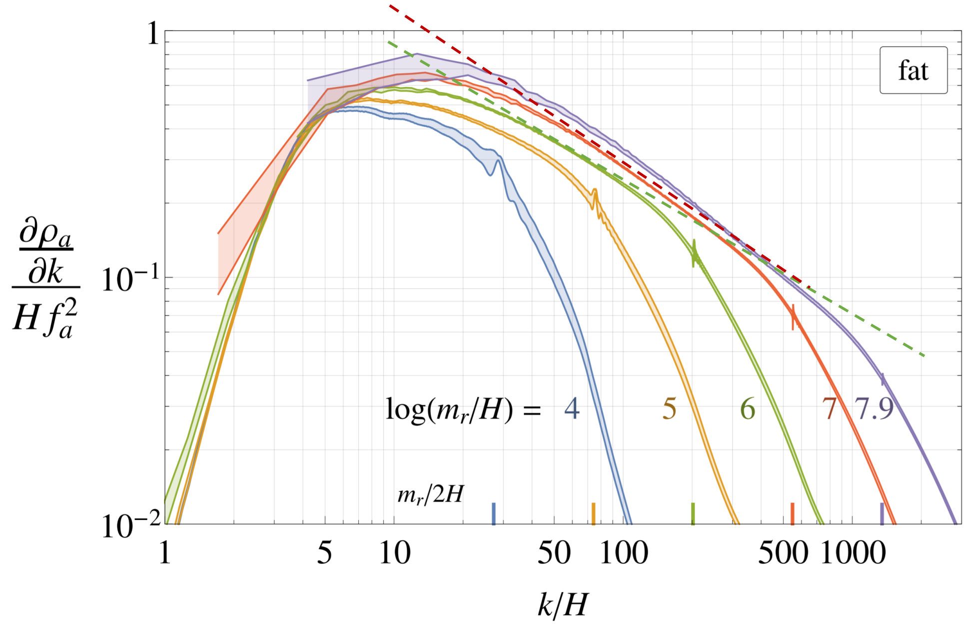
The Spectral Index



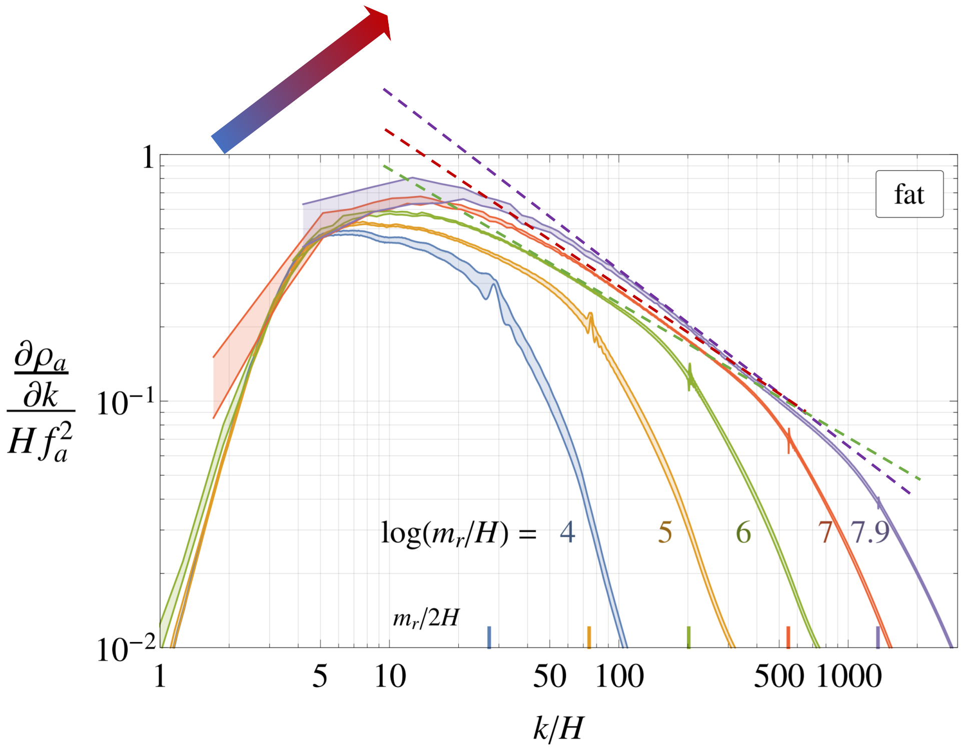
The Spectral Index



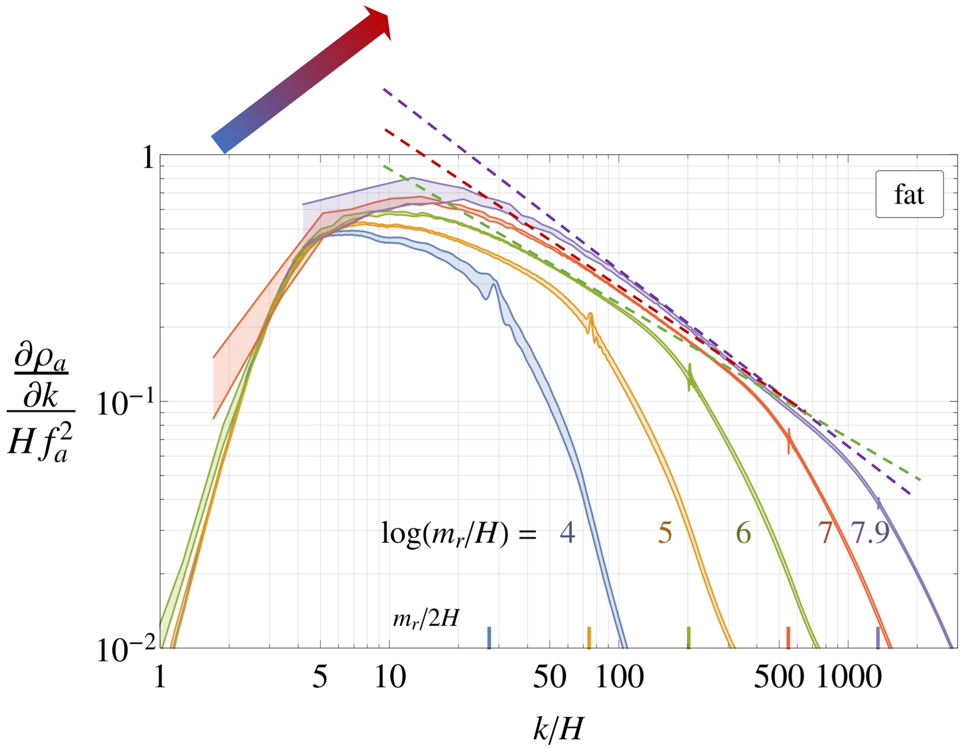
The Spectral Index



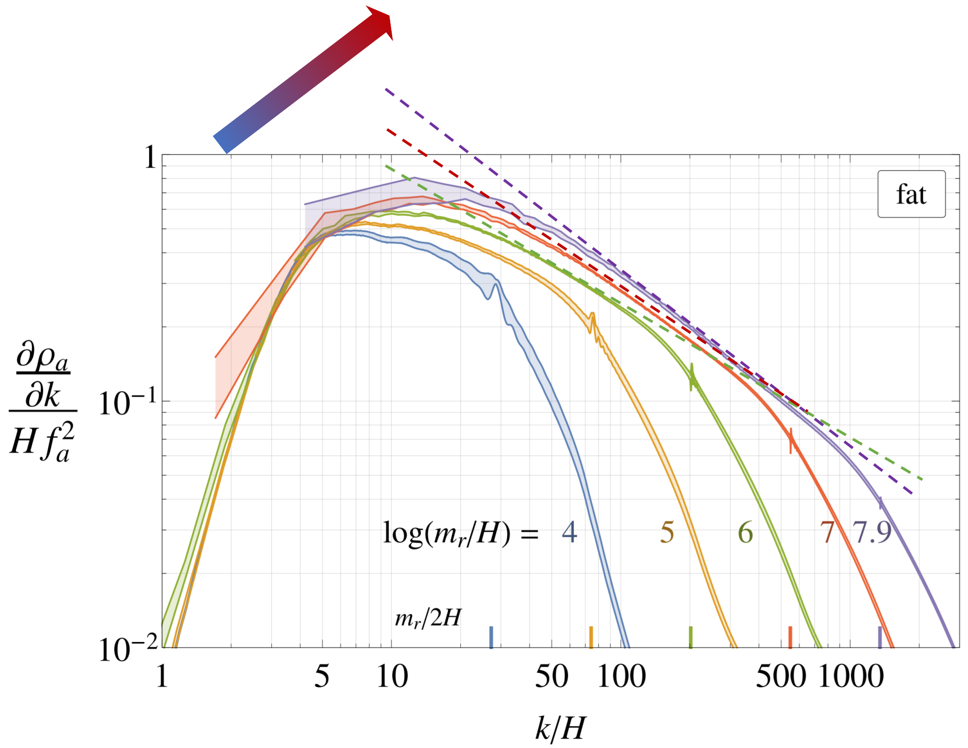
The Spectral Index

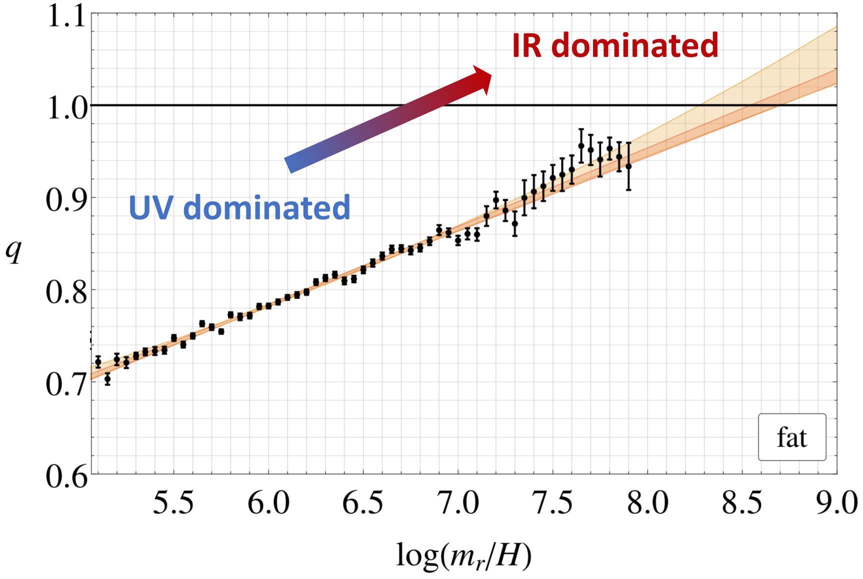


The Spectral Index

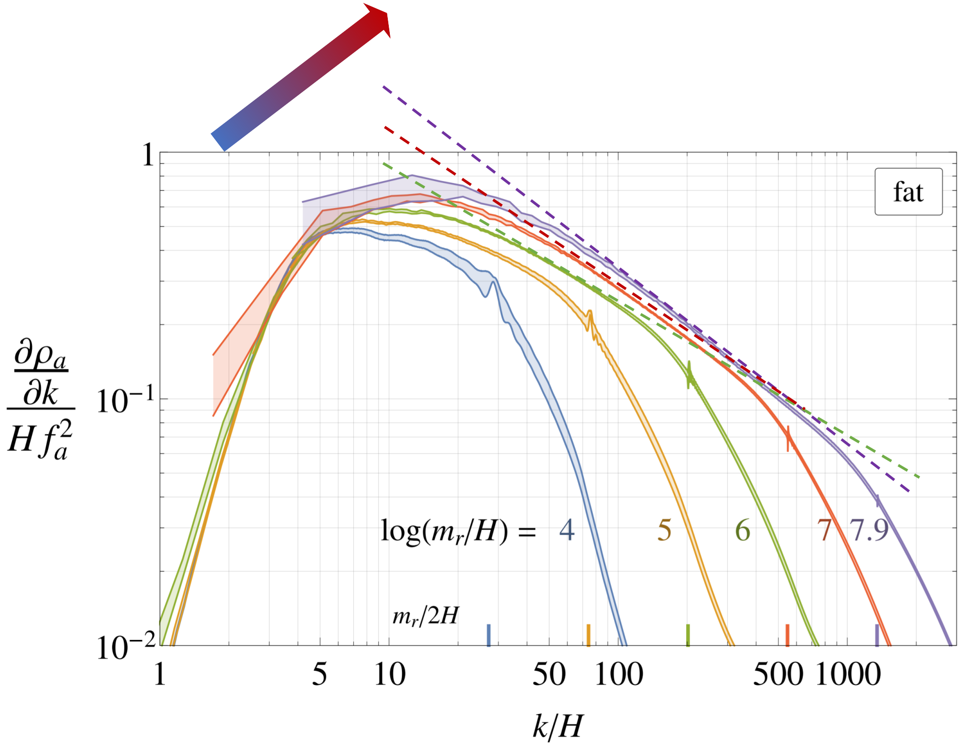


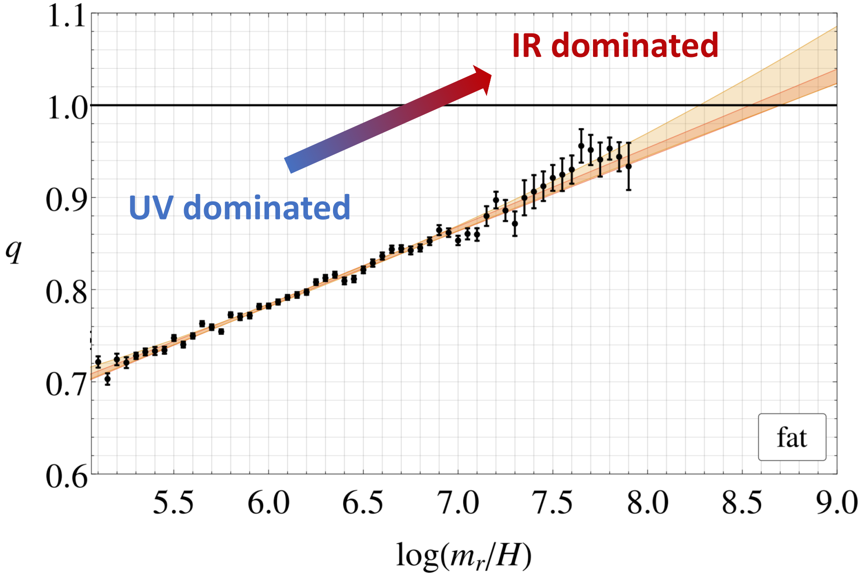
The Spectral Index





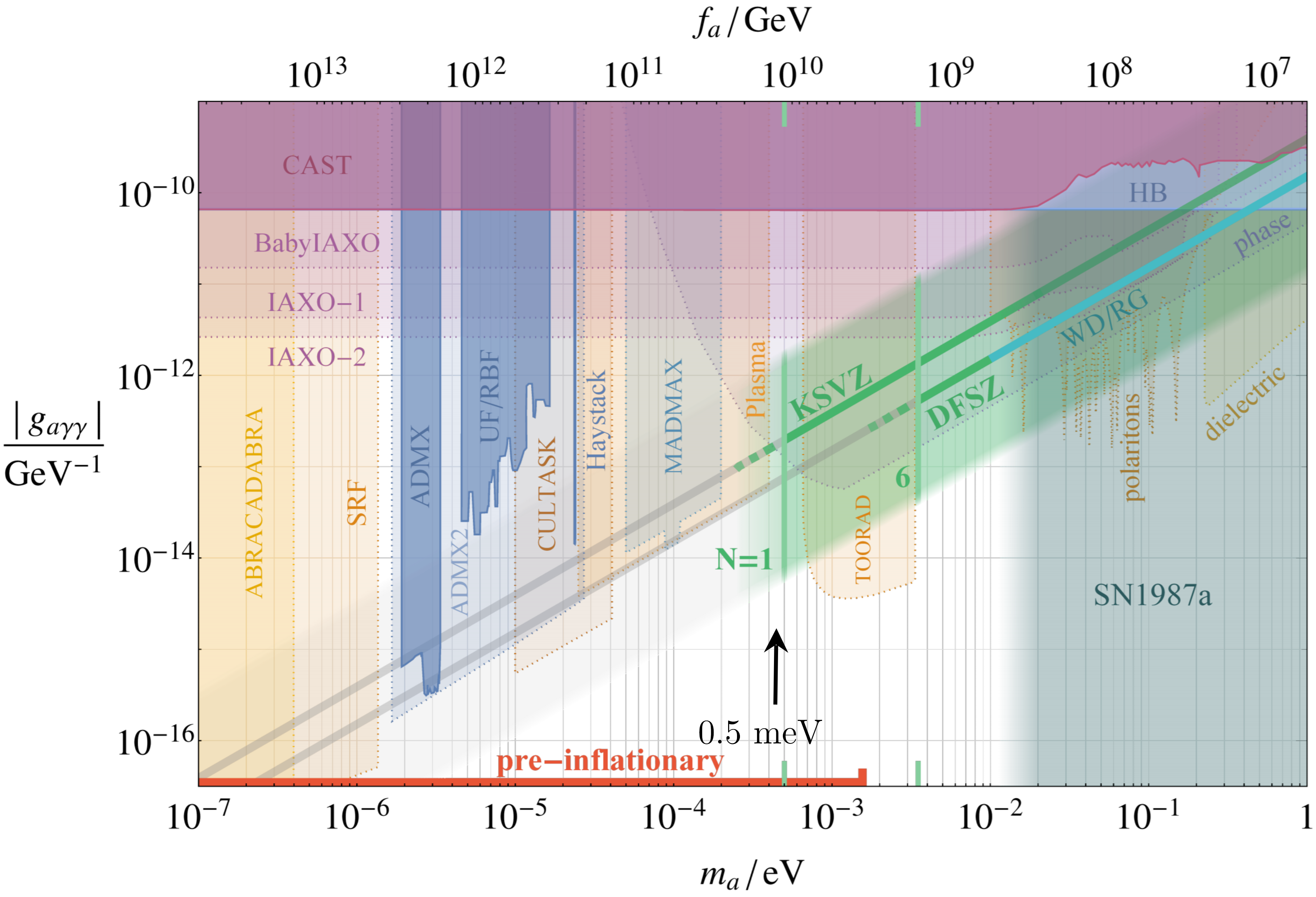
The Spectral Index





Running of *q* → $\boxed{\overset{\log \to 70}{q\ >\ 1}}$

$$f_a \simeq (\overset{q>1}{1} \div \overset{q=1}{6}) \cdot 10^{10}\ \text{GeV} \quad + \text{DW?}$$



$\underbrace{\qquad}_{q \simeq 1}$ $\underbrace{\qquad}_{q > 1}$

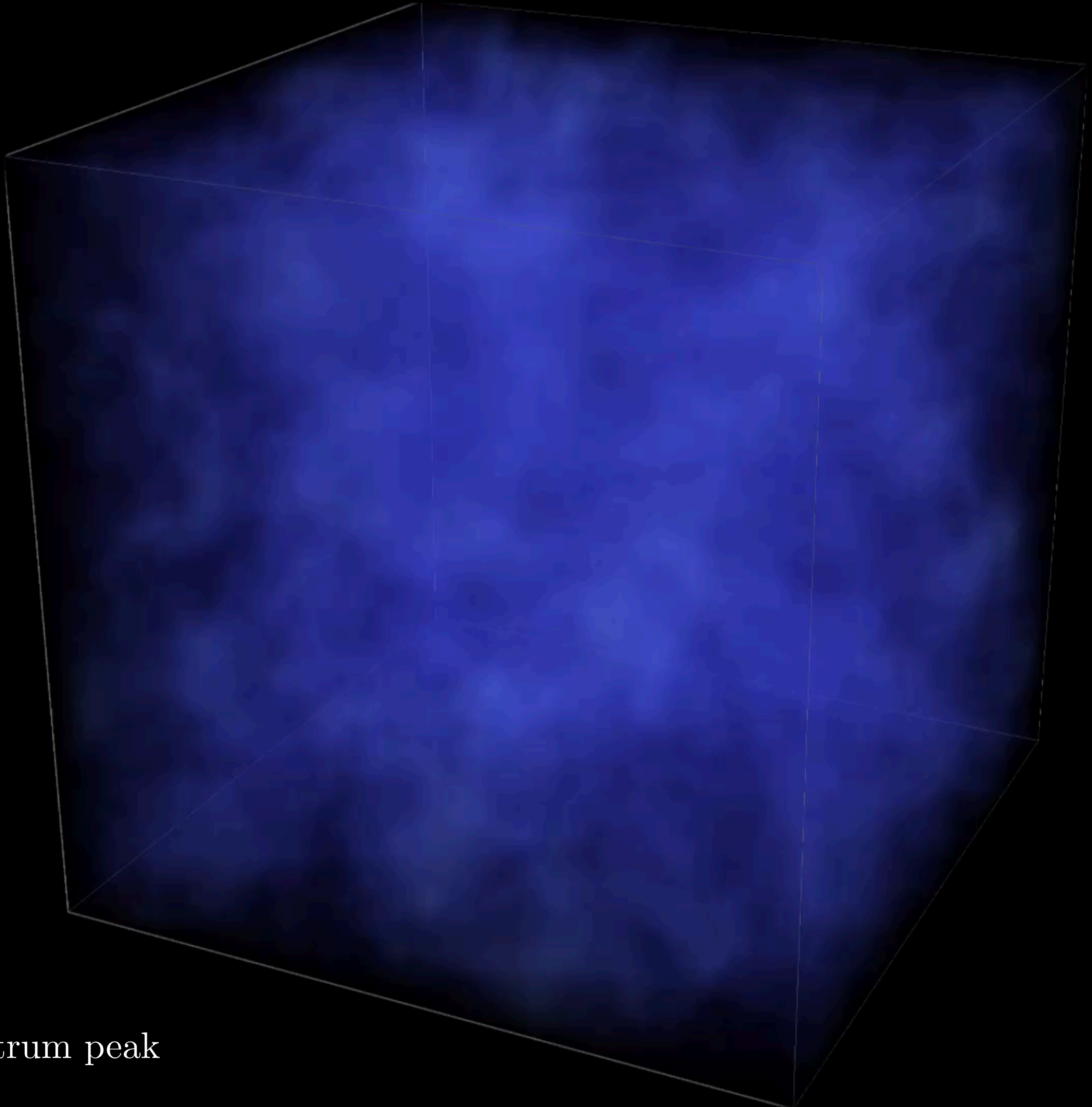
$f_a \lesssim 10^{10}$ GeV from DM overproduction

$\simeq$ if domain walls negligible

Formation of structures

after wall decay, $T \ll \Lambda_{\mathrm{QCD}}$

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

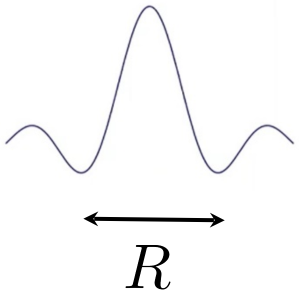


$H_\star^{-1}$

$\sim k_p^{-1}$ spectrum peak

Gravitational collapse *vs* quantum Jeans scale

overdensity ρ



spatial size of the overdensity

$$R$$

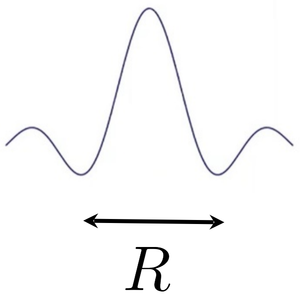
$\longleftrightarrow$

de Broglie wavelength of the particles
in the resulting clump

$$\frac{1}{mv}$$

Gravitational collapse *vs* quantum Jeans scale

overdensity ρ



spatial size of the overdensity

$$R$$

$\longleftrightarrow$

de Broglie wavelength of the particles in the resulting clump

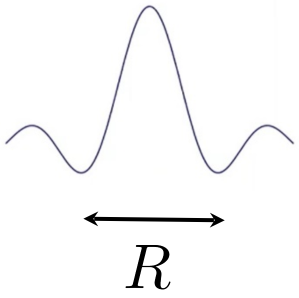
$$\frac{1}{mv}$$

$$\simeq \frac{1}{m(\frac{GM}{R})^{1/2}} \simeq \frac{1}{R(4\pi G\rho m^2)^{1/2}}$$

$$M \to 4\pi\rho R^3/3$$

Gravitational collapse *vs* quantum Jeans scale

overdensity ρ



spatial size of the overdensity

$$R$$

$$\longleftrightarrow$$

de Broglie wavelength of the particles in the resulting clump

$$\frac{1}{mv}$$

$$\simeq \frac{1}{m(\frac{GM}{R})^{1/2}} \simeq \frac{1}{R(4\pi G\rho m^2)^{1/2}}$$

$$4\pi\rho R^3/3$$

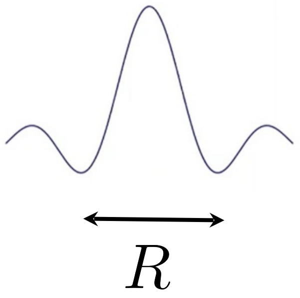
$$\downarrow$$

$$R_{\text{crit}} \simeq \lambda_J \simeq (16\pi G\rho m^2)^{-1/4}$$

quantum Jeans length $\lambda_J = 2\pi/k_J$ $\equiv$ smallest scale an overdensity can have before wave effects (quantum pressure) have to be considered

Gravitational collapse *vs* quantum Jeans scale

overdensity ρ



spatial size of the overdensity

$$R$$

⟷

de Broglie wavelength of the particles
in the resulting clump

$$\frac{1}{mv}$$

$$\simeq \frac{1}{m(\frac{GM}{R})^{1/2}} \simeq \frac{1}{R(4\pi G\rho m^2)^{1/2}}$$

$4\pi\rho R^3/3$

↓

$$R_{\rm crit} \simeq \lambda_J \simeq (16\pi G\rho m^2)^{-1/4}$$

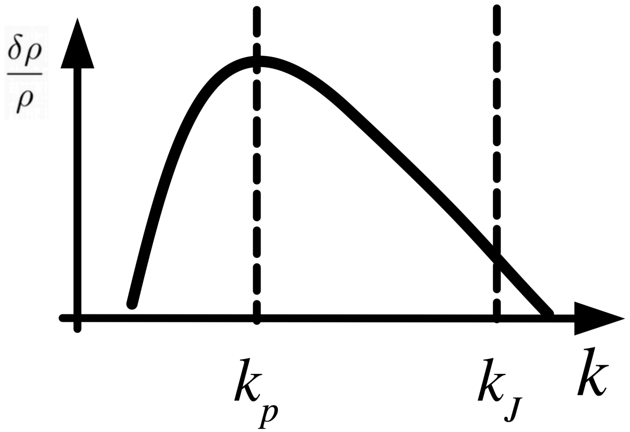
quantum Jeans length $\lambda_J = 2\pi/k_J$ $\equiv$ smallest scale an overdensity can have before wave effects (quantum pressure) have to be considered

The standard lore after DW decay

quantum Jeans scale

$$k_J \equiv (16\pi G \rho m^2)^{\frac{1}{4}}$$

@MRE



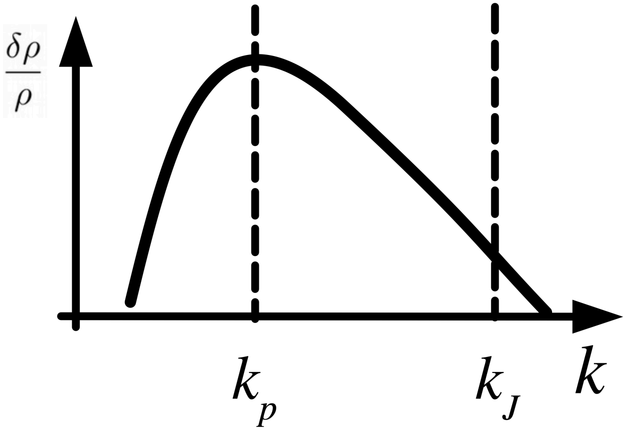
$$\left.\frac{k_p}{k_J}\right|_{\mathrm{MRE}} \simeq \left(\frac{f_a}{M_p}\right)^{1/3} \frac{k_{p\star}}{H_\star} \sim 10^{-3}\,\frac{k_{p\star}}{H_\star}$$

The standard lore after DW decay

quantum Jeans scale

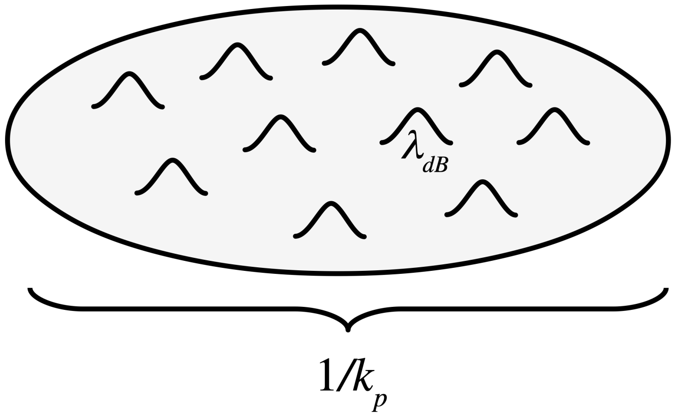
$$k_J \equiv (16\pi G\rho m^2)^{\frac{1}{4}}$$

@MRE



axion minicluster

$\lambda_{\rm dB} \ll 1/k_p$



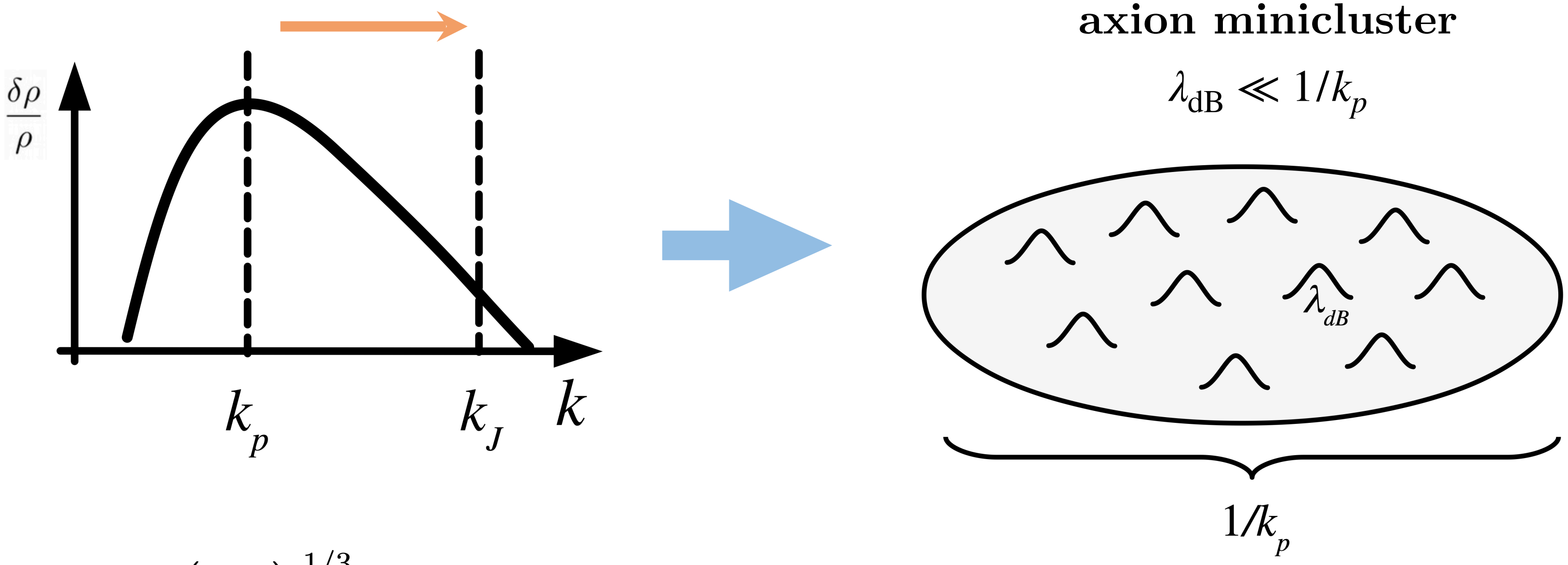
$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} \simeq \left(\frac{f_a}{M_p}\right)^{1/3} \frac{k_{p\star}}{H_\star} \sim 10^{-3}\,\frac{k_{p\star}}{H_\star}$$

The standard lore after DW decay

quantum Jeans scale

$$k_J \equiv (16\pi G\rho m^2)^{\frac{1}{4}}$$

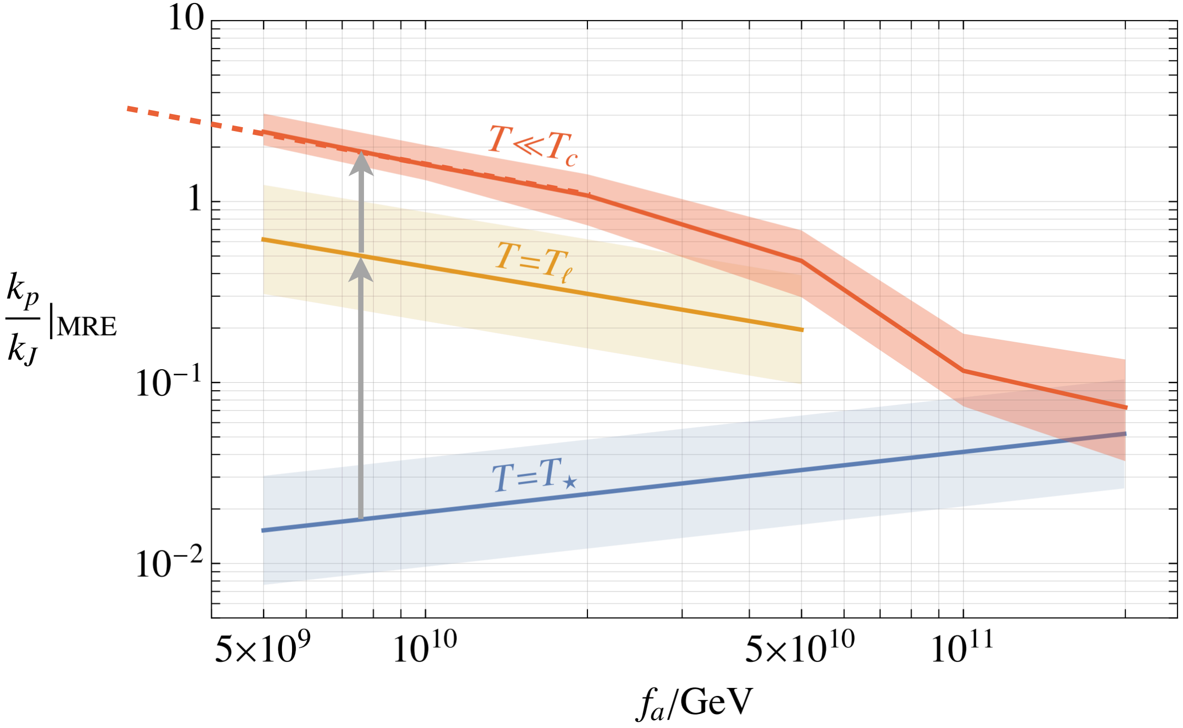
@MRE

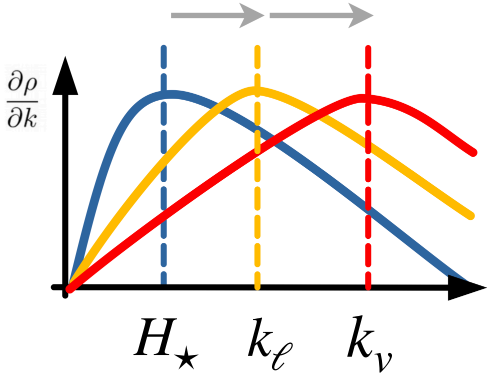


$$\left.\frac{k_p}{k_J}\right|_{\rm MRE} \simeq \left(\frac{f_a}{M_p}\right)^{1/3} \frac{k_{p\star}}{H_\star} \sim 10^{-3}\,\frac{k_{p\star}}{H_\star}$$

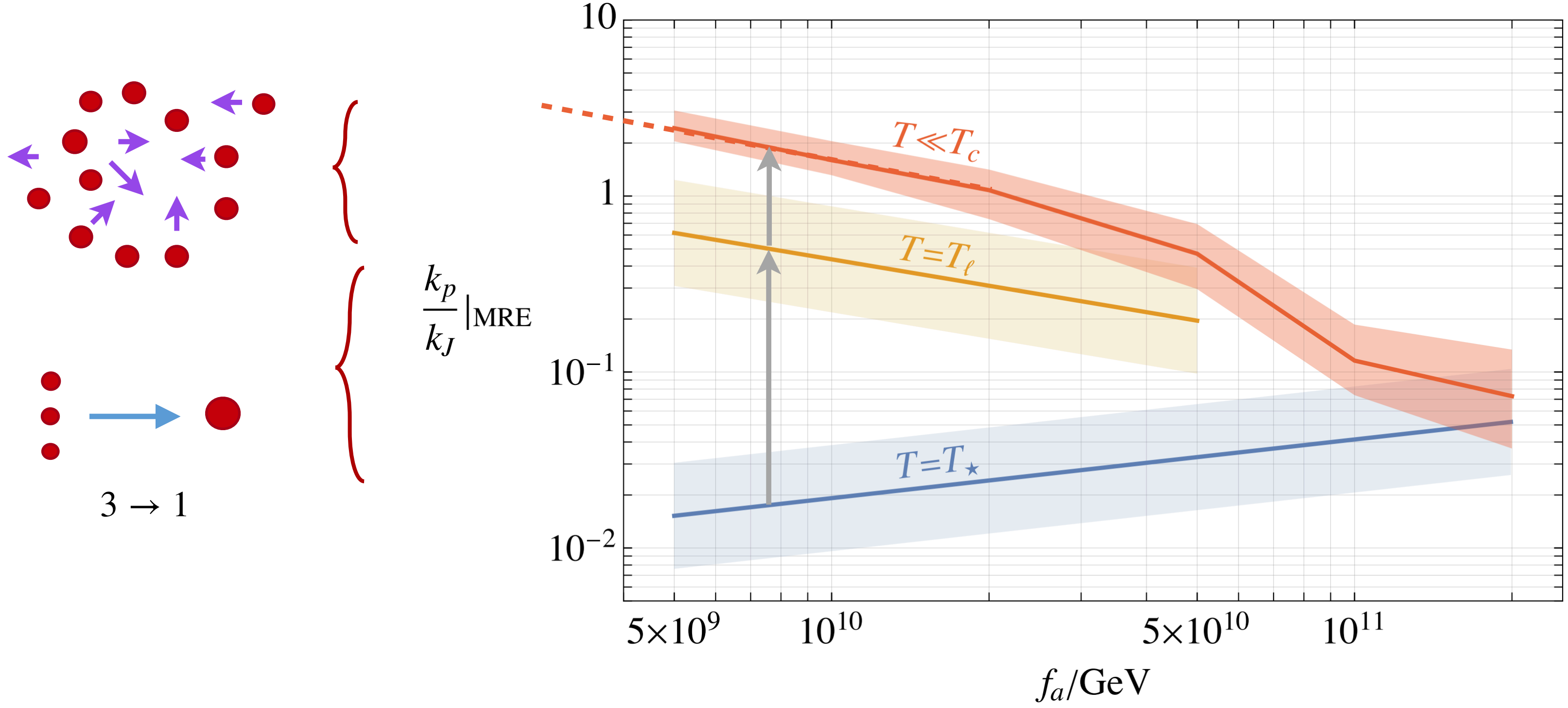
Naive because k_p increases due to the self-interactions and becomes of order k_J

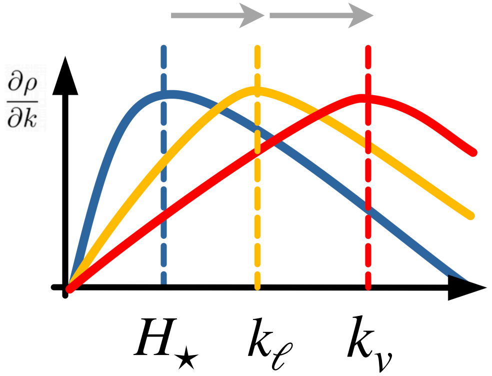
The remarkable coincidence



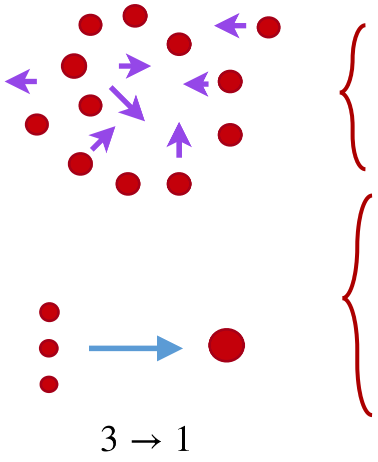


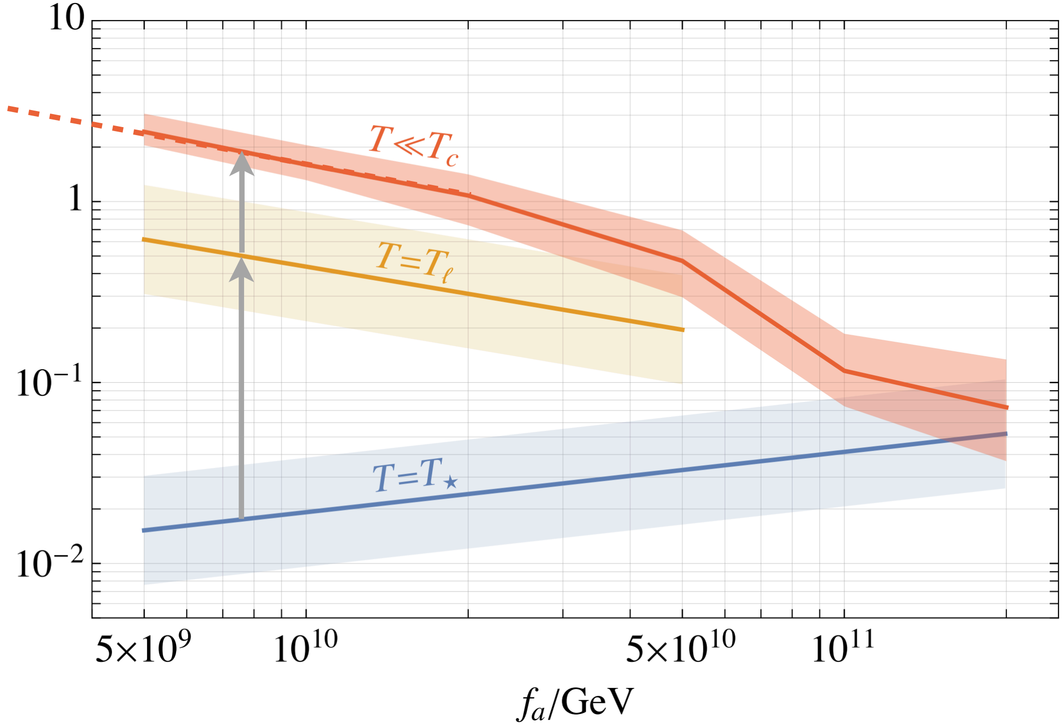
The remarkable coincidence

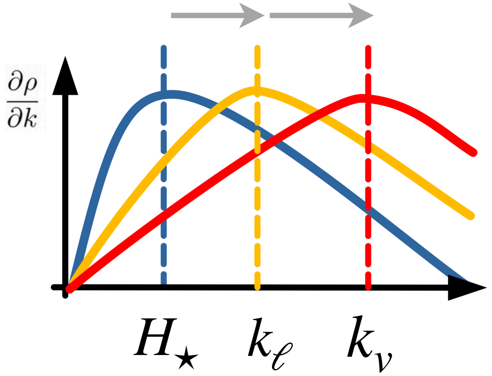




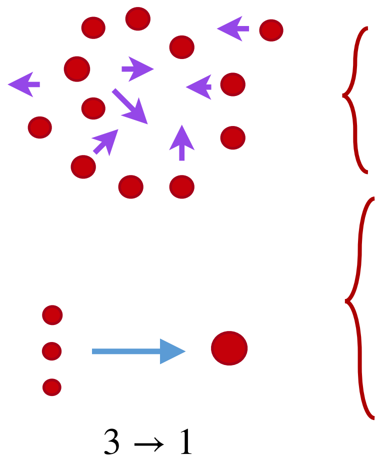
The remarkable coincidence

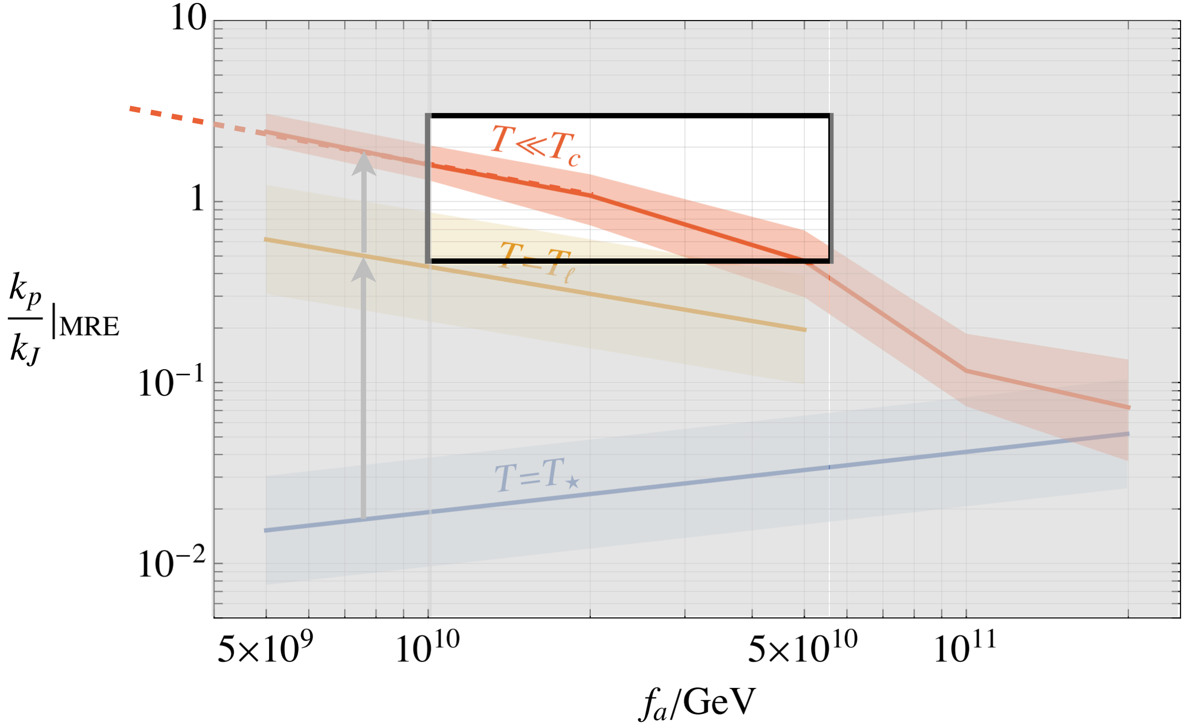


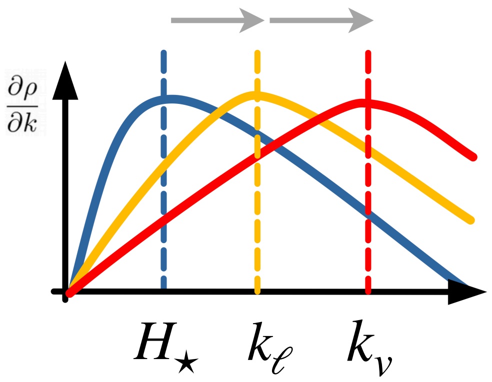




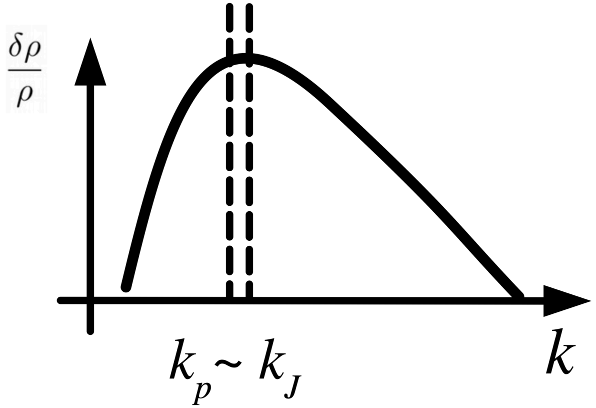
The remarkable coincidence

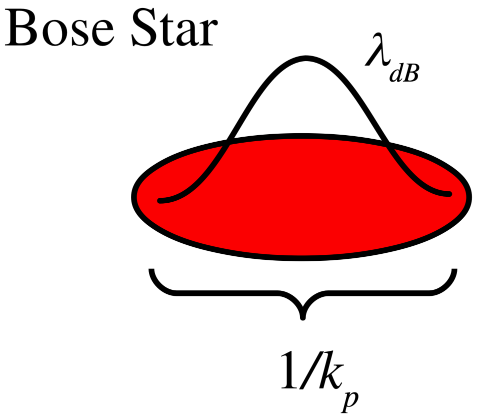




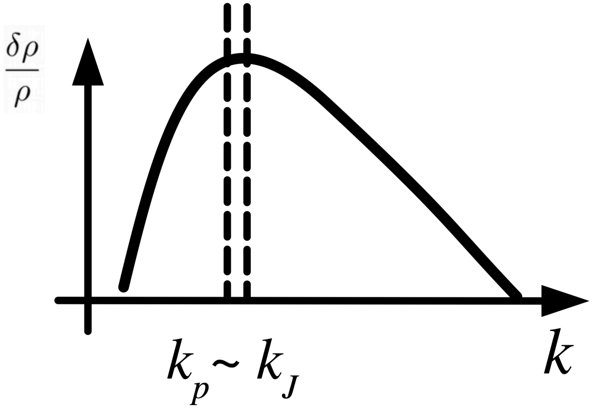


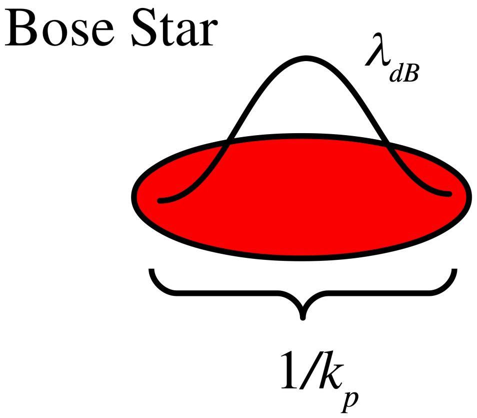
Axion stars:



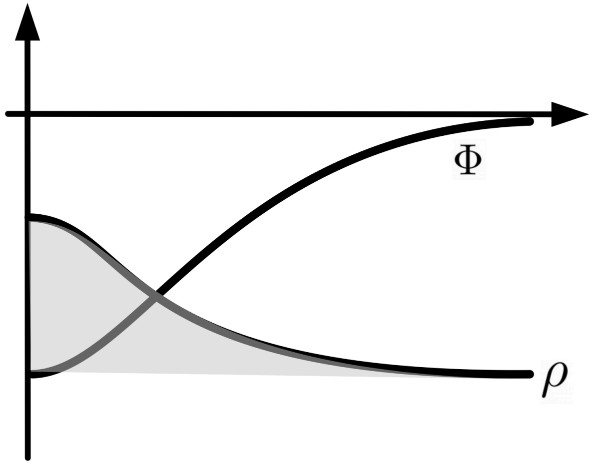


Axion stars:

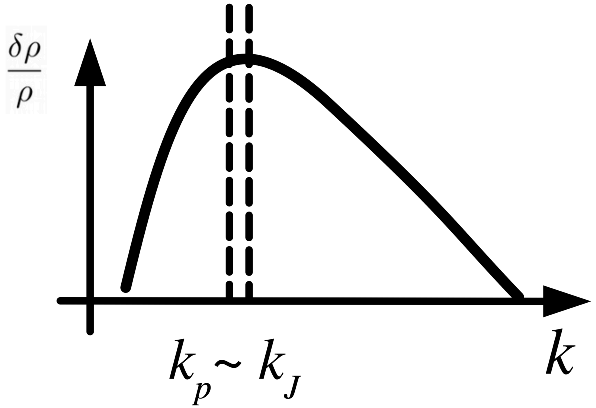


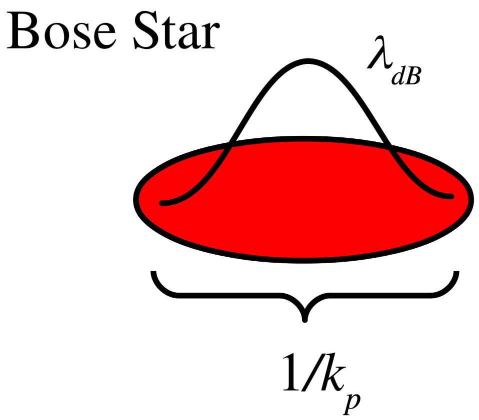


$$\begin{cases}\dot{\psi} + \frac{\nabla^2}{2m}\psi + m\Phi\psi = 0\\ \nabla^2\Phi = 4\pi G|\psi|^2\end{cases} \rightarrow \begin{cases}\nabla^2\sqrt{\rho} = 2m^2\Phi\sqrt{\rho}\\ \nabla^2\Phi = 4\pi G\rho\end{cases} \qquad \rho = |\psi|^2$$



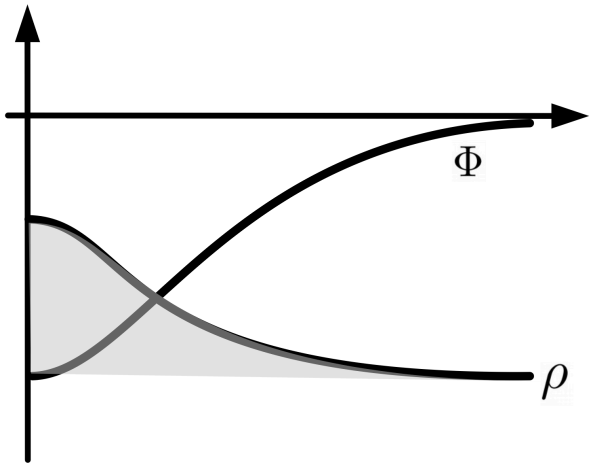
Axion stars:





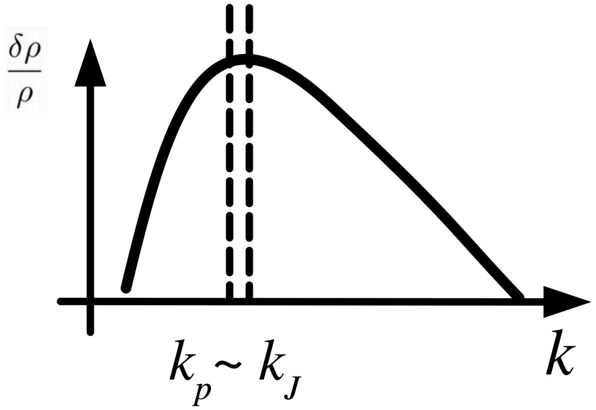
$$\left\{\begin{array}{l}\dot{\psi} + \frac{\nabla^2}{2m}\psi + m\Phi\psi = 0 \\ \nabla^2\Phi = 4\pi G|\psi|^2\end{array}\right. \rightarrow \left\{\begin{array}{l}\nabla^2\sqrt{\rho} = 2m^2\Phi\sqrt{\rho} \\ \nabla^2\Phi = 4\pi G\rho\end{array}\right.$$

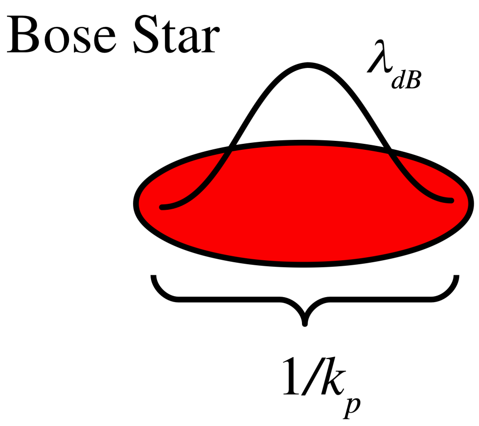
$$\rho = |\psi|^2$$



$$M_s R_s \sim \frac{1}{Gm^2}$$

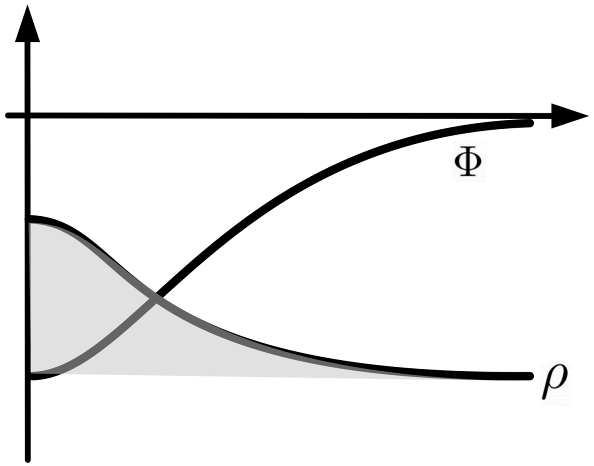
Axion stars:



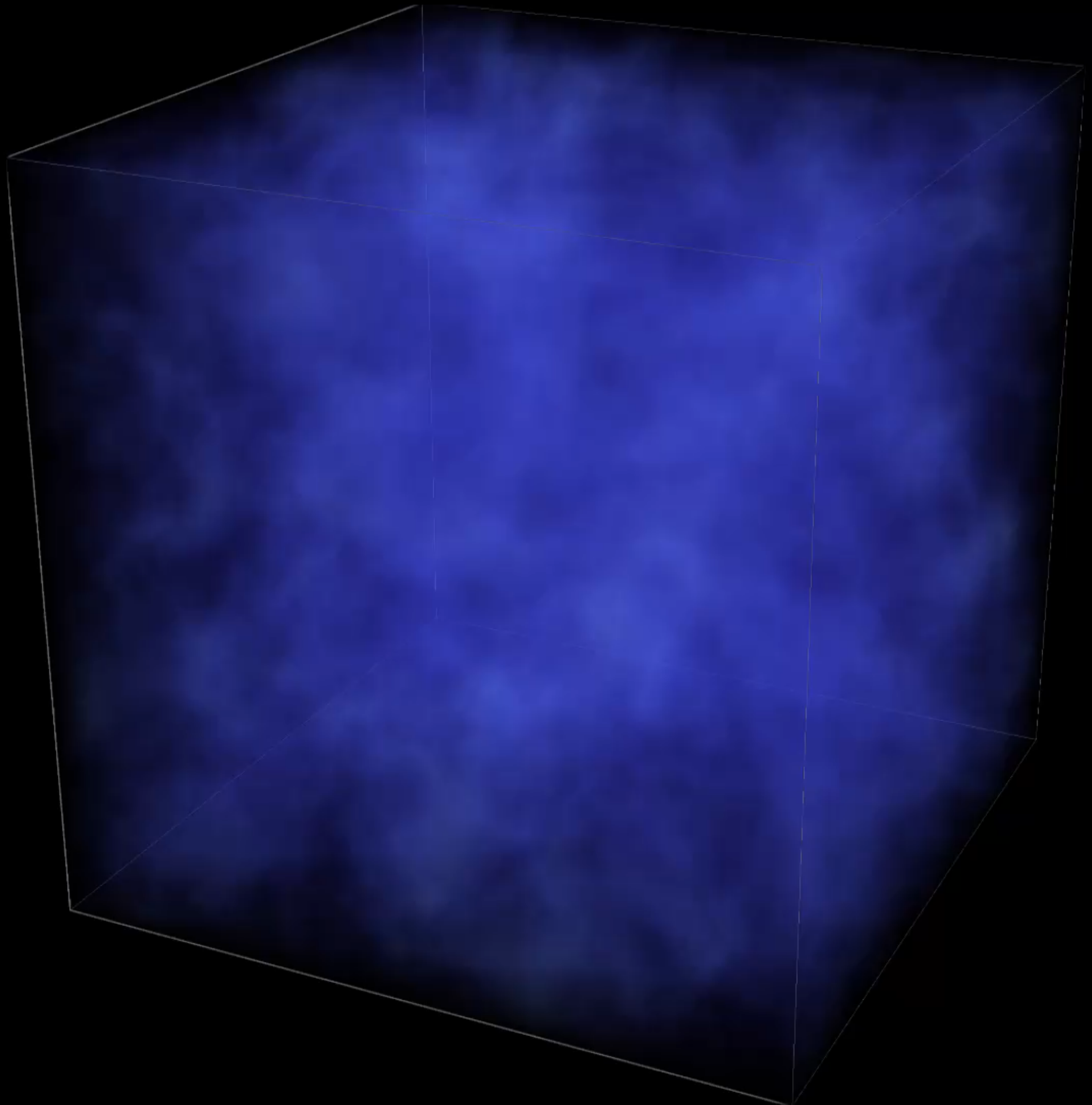


$$\begin{cases} \dot{\psi} + \frac{\nabla^2}{2m}\psi + m\Phi\psi = 0 \\ \nabla^2\Phi = 4\pi G|\psi|^2 \end{cases} \rightarrow \begin{cases} \nabla^2\sqrt{\rho} = 2m^2\Phi\sqrt{\rho} \\ \nabla^2\Phi = 4\pi G\rho \end{cases}$$

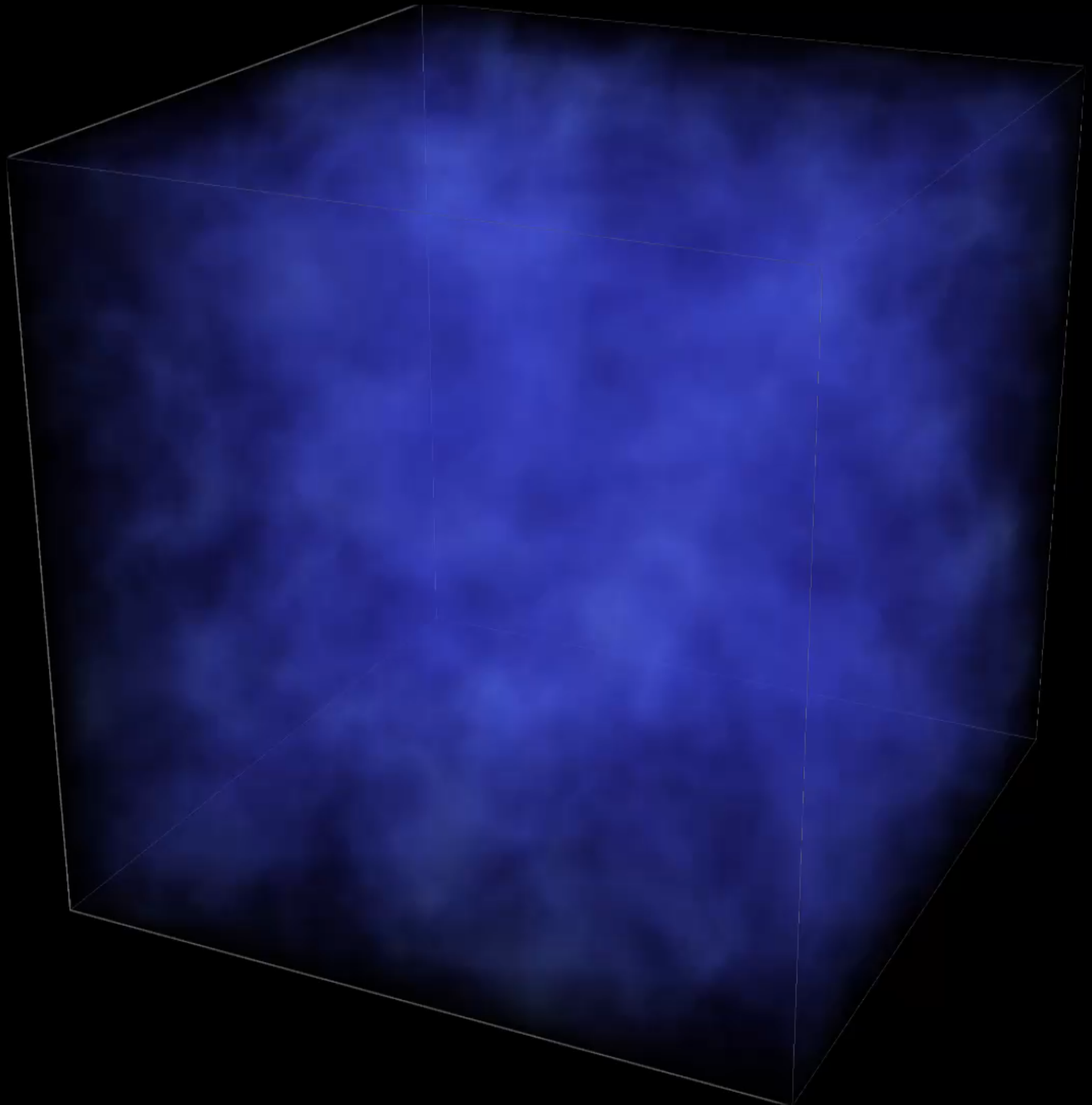
$$\rho = |\psi|^2$$



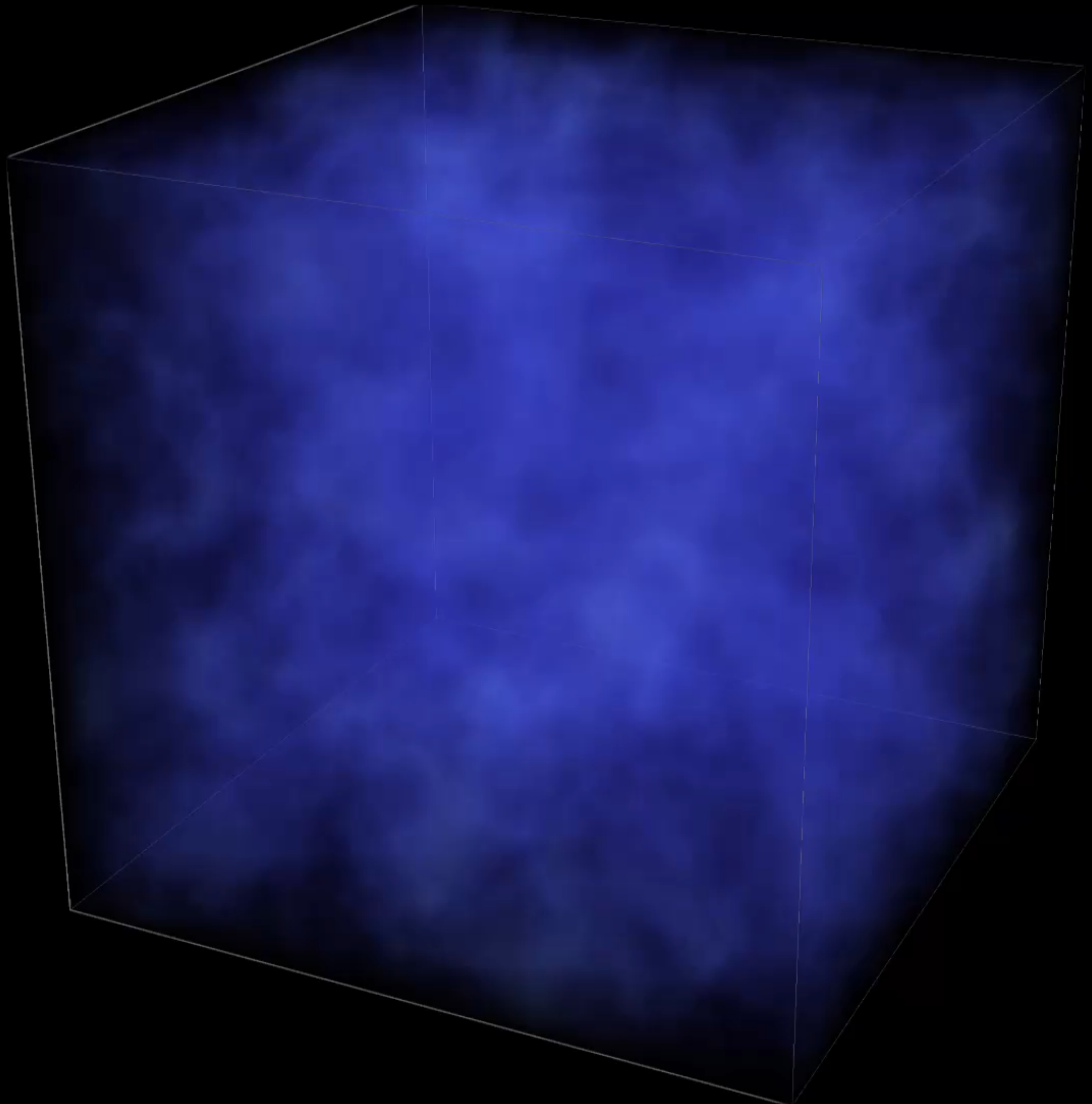
$$M_s R_s \sim \frac{1}{Gm^2}$$



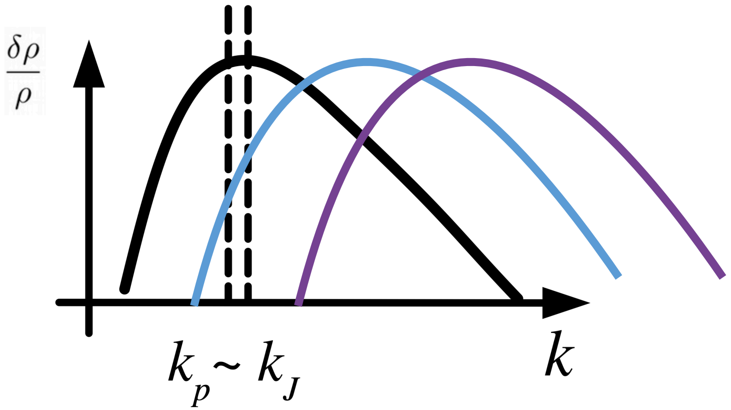
$0.5 < \frac{a}{a_{\rm eq}} < 7$

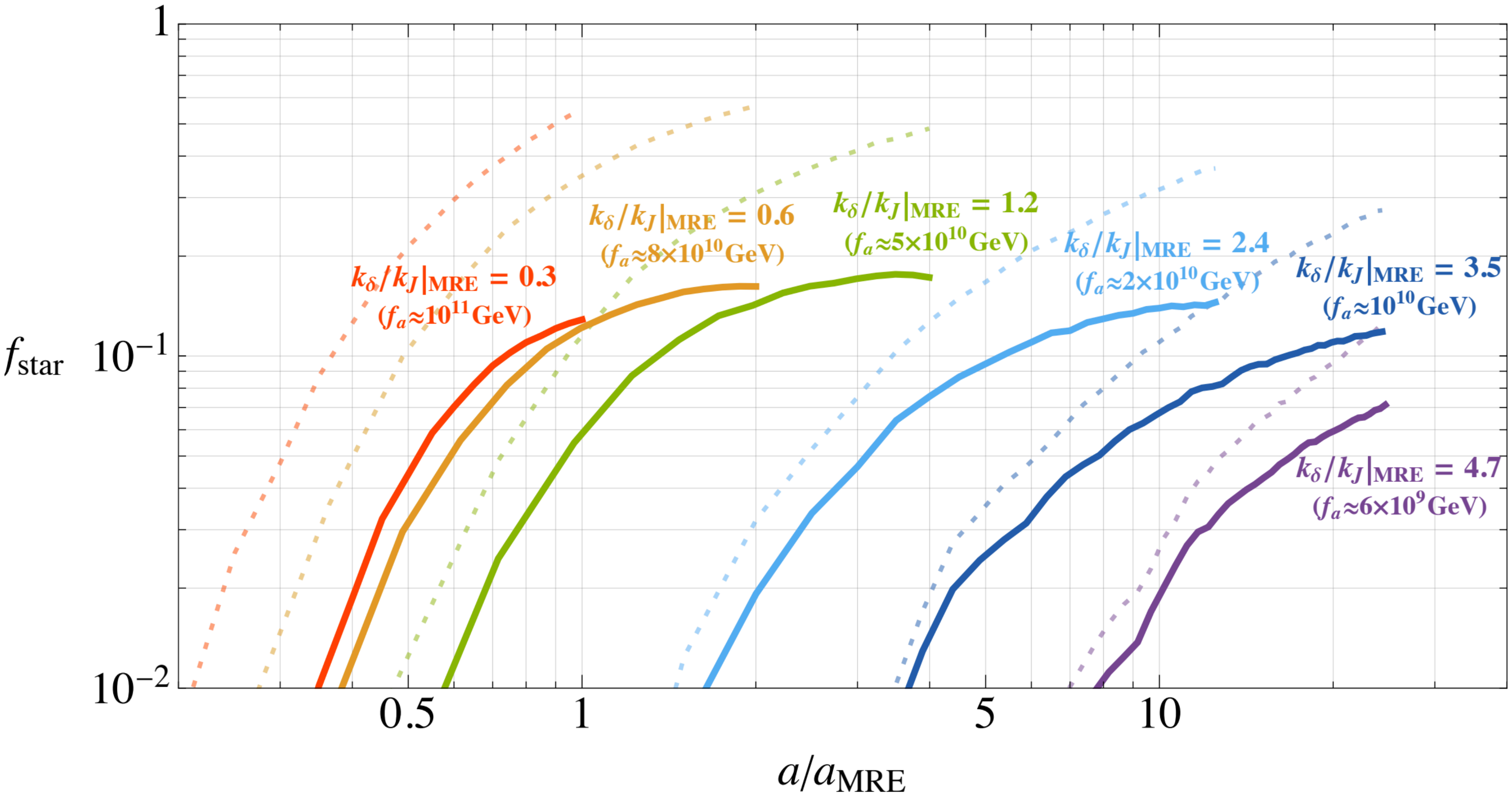


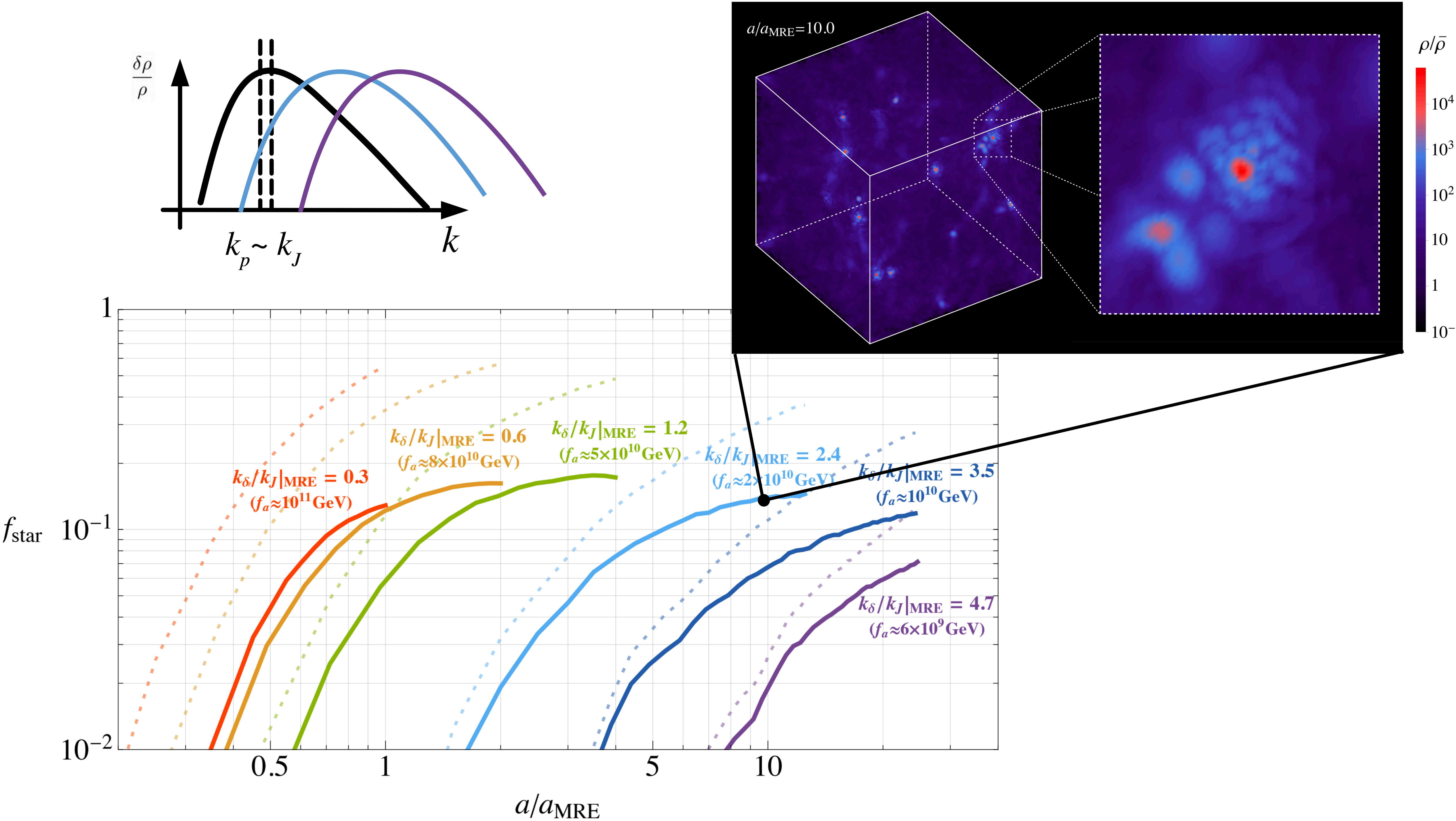
$$0.5 < \frac{a}{a_{\rm eq}} < 7$$

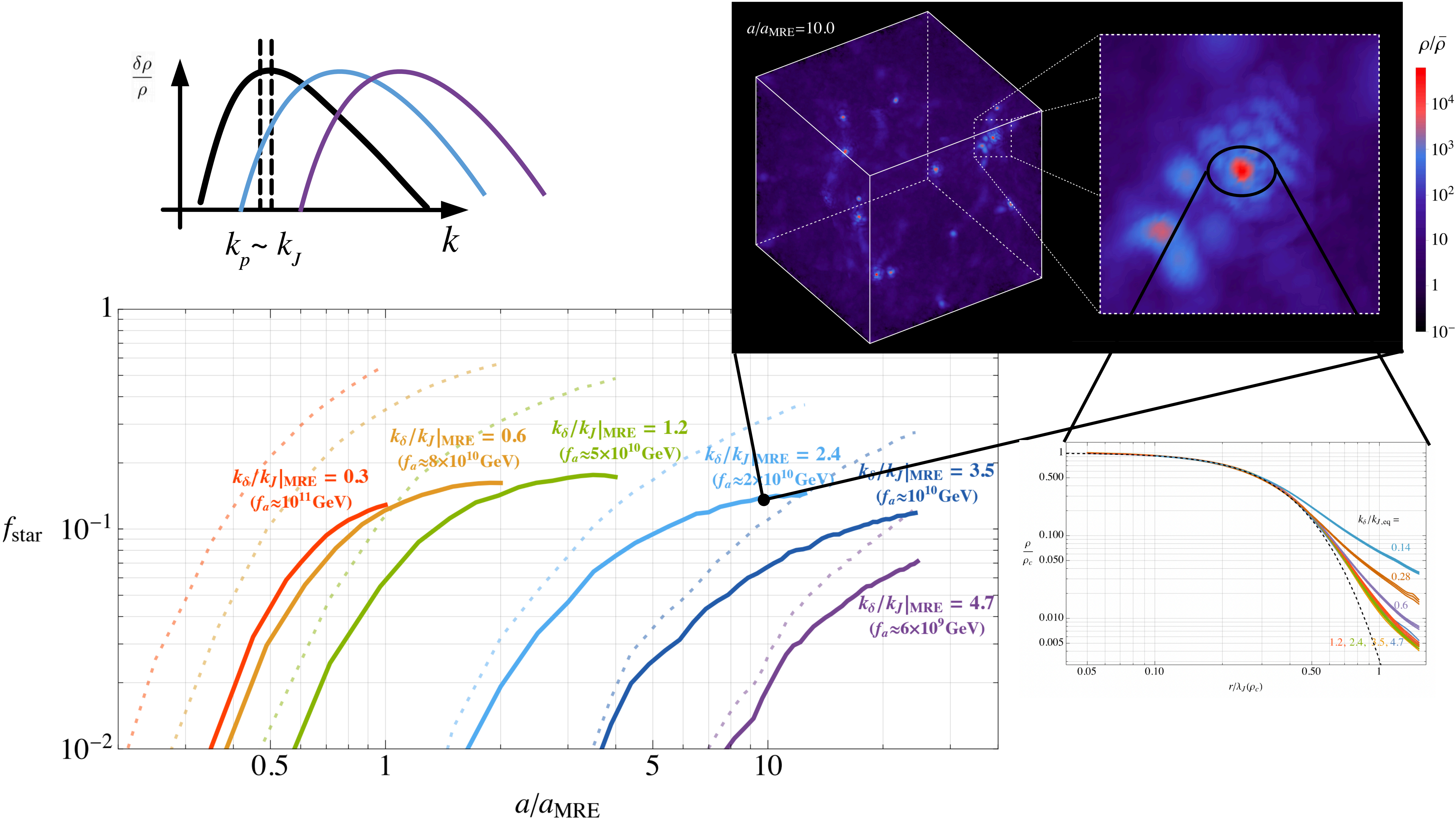


$$0.5 < \frac{a}{a_{\rm eq}} < 7$$

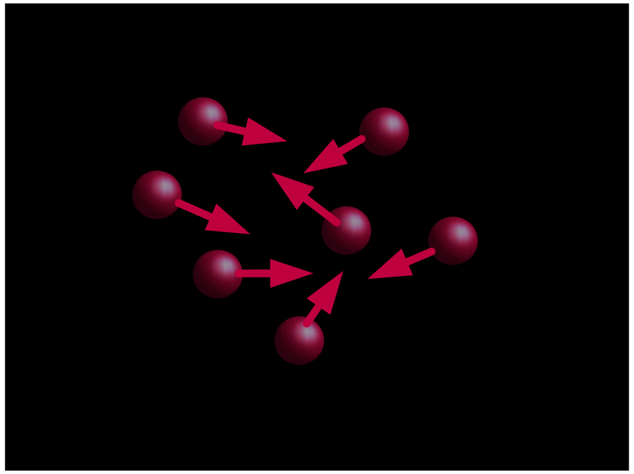

$\frac{\delta\rho}{\rho}$
$k_p \sim k_J$
k


$f_{\rm star}$
1
10^{-1}
10^{-2}
0.5
1
5
10
$a/a_{\rm MRE}$
$k_\delta/k_J|_{\rm MRE}$ = 0.3
($f_a \approx 10^{11}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 0.6
($f_a \approx 8\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 1.2
($f_a \approx 5\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 2.4
($f_a \approx 2\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 3.5
($f_a \approx 10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 4.7
($f_a \approx 6\times10^{9}$GeV)

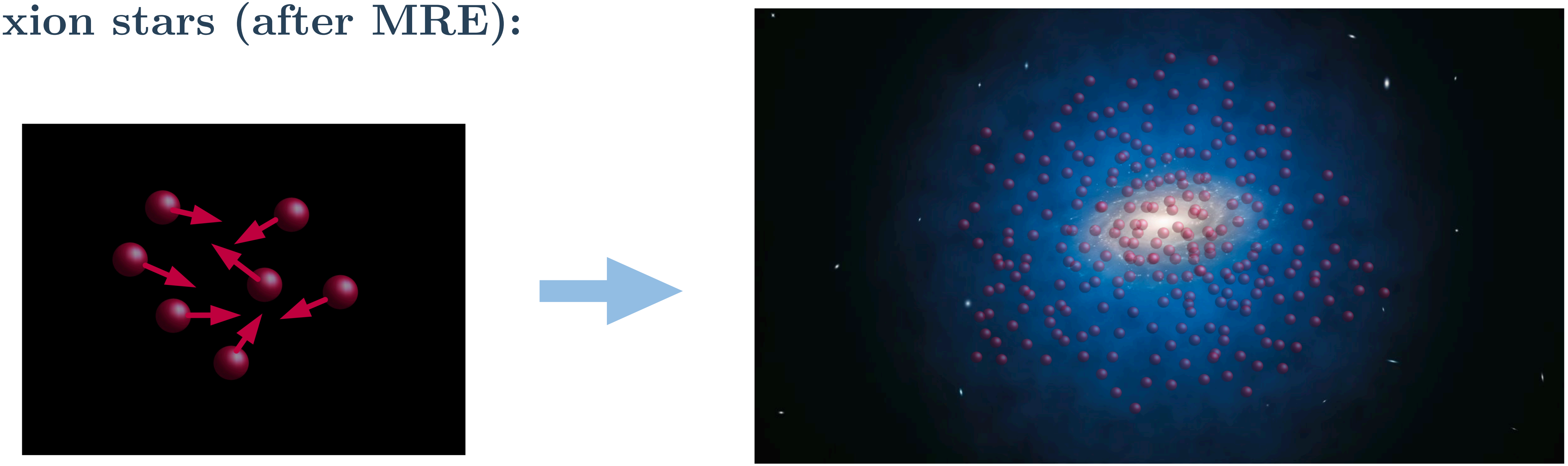

$\frac{\delta\rho}{\rho}$
$k_p \sim k_J$
k
$a/a_{\rm MRE}$=10.0
$\rho/\bar{\rho}$
10^4
10^3
10^2
10
1
10^-
$f_{\rm star}$
1
10^{-1}
10^{-2}
0.5
1
5
10
$a/a_{\rm MRE}$
$k_\delta/k_J|_{\rm MRE}$ = 0.3
(f_a≈10^{11}GeV)
$k_\delta/k_J|_{\rm MRE}$ = 0.6
(f_a≈8×10^{10}GeV)
$k_\delta/k_J|_{\rm MRE}$ = 1.2
(f_a≈5×10^{10}GeV)
$k_\delta/k_J|_{\rm MRE}$ = 2.4
(f_a≈2×10^{10}GeV)
$k_\delta/k_J|_{\rm MRE}$ = 3.5
(f_a≈10^{10}GeV)
$k_\delta/k_J|_{\rm MRE}$ = 4.7
(f_a≈6×10^9GeV)

$\frac{\delta\rho}{\rho}$
$k_p \sim k_J$
k
$a/a_{\rm MRE}$=10.0
$\rho/\bar{\rho}$
10^4
10^3
10^2
10
1
10^{-}
$k_\delta/k_J|_{\rm MRE}$ = 0.3
($f_a \approx 10^{11}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 0.6
($f_a \approx 8\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 1.2
($f_a \approx 5\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 2.4
($f_a \approx 2\times10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 3.5
($f_a \approx 10^{10}$GeV)
$k_\delta/k_J|_{\rm MRE}$ = 4.7
($f_a \approx 6\times10^{9}$GeV)
$f_{\rm star}$
1
10^{-1}
10^{-2}
0.5
1
5
10
$a/a_{\rm MRE}$
1
0.500
0.100
0.050
0.010
0.005
$\frac{\rho}{\rho_c}$
$k_\delta/k_{J,\rm eq}$ =
0.14
0.28
0.6
1.2, 2.4, 3.5, 4.7
0.05
0.10
0.50
1
$r/\lambda_J(\rho_c)$

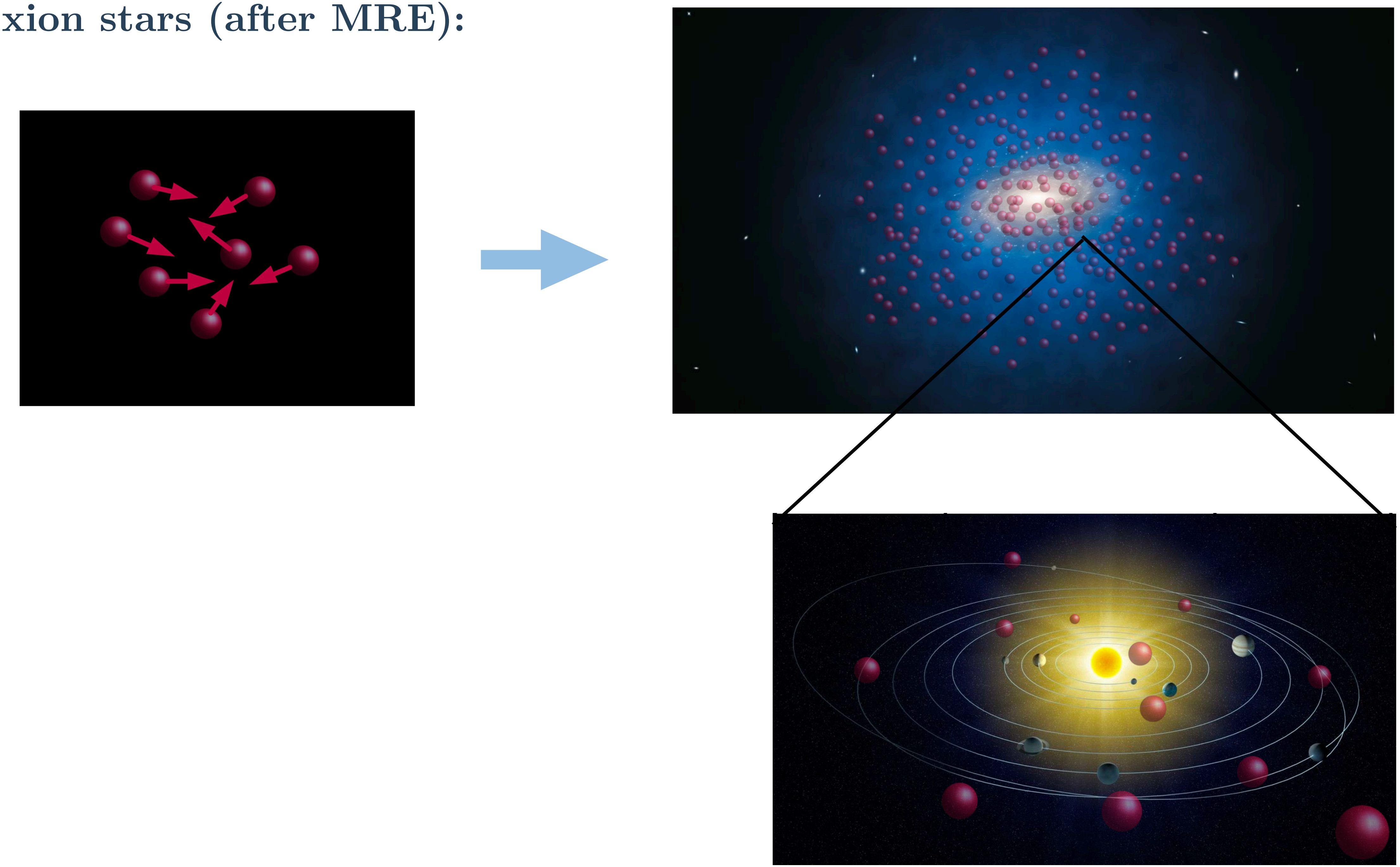
Axion stars (after MRE):



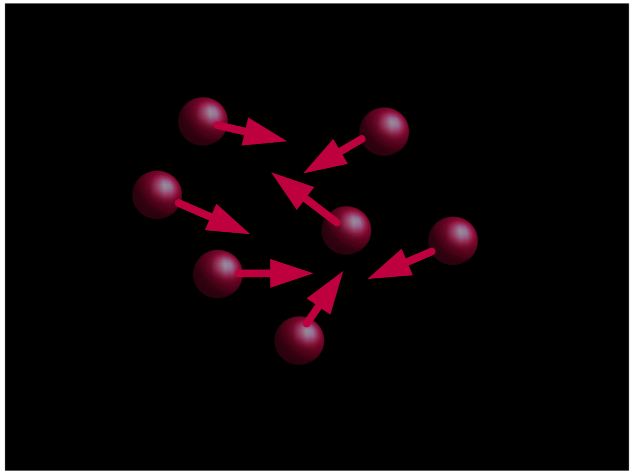
Axion stars (after MRE):

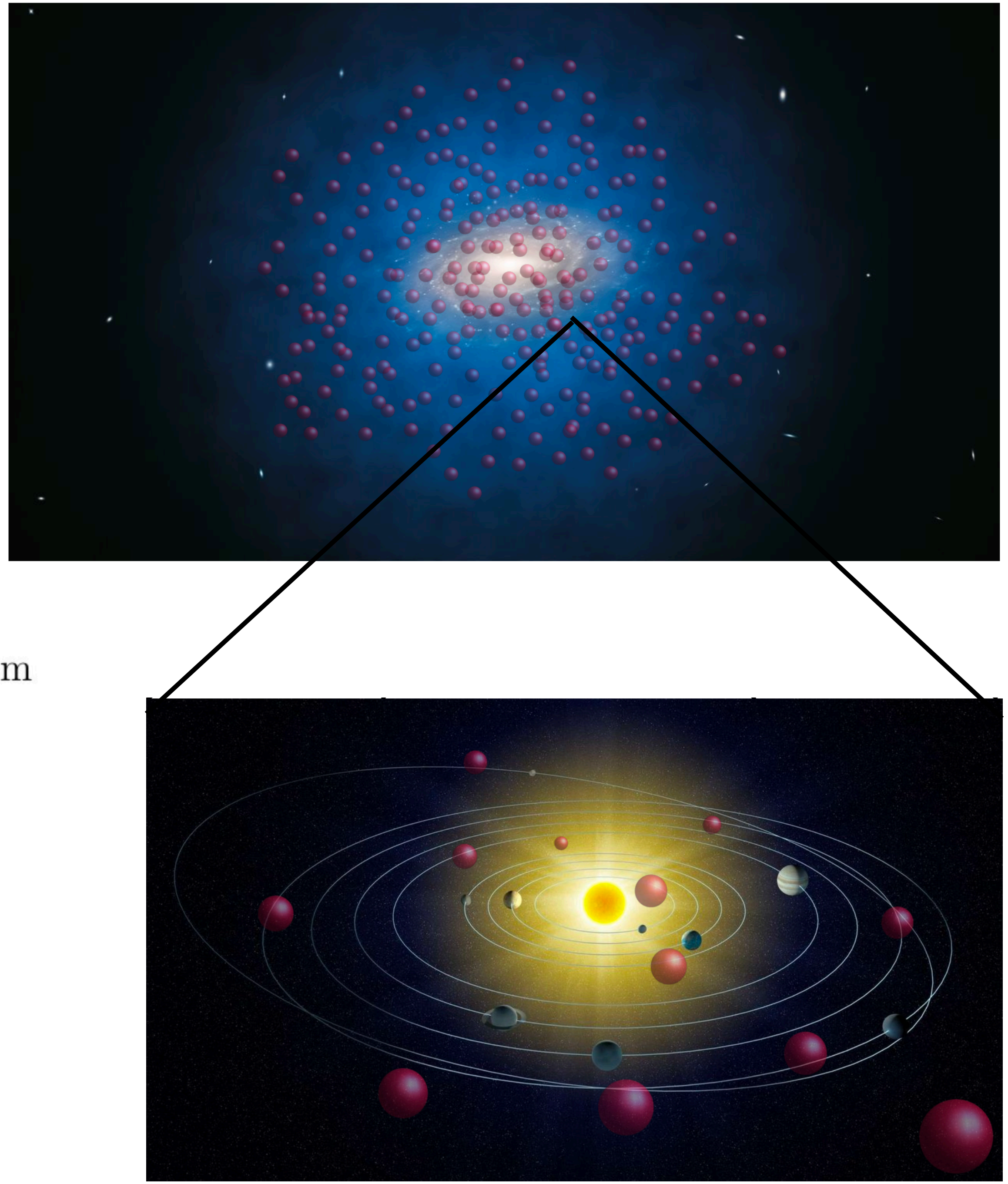


Axion stars (after MRE):



Axion stars (after MRE):





e.g. for $\begin{cases} M_s = 10^{-19} M_\odot \\ f_a = 10^{10}\ \text{GeV} \\ f_s = 0.1 \end{cases} \rightarrow \begin{cases} n_s^{-1/3} = 1.4 \cdot 10^8\,\text{km} \\ \tau_\oplus = 5\ \text{yrs} \\ \Delta t \simeq 8\ \text{hrs} \end{cases}$

Conclusions

- Post-inflationary abundance still **uncertain**, despite progress

$$f_a \lesssim 10^{10}\,\mathrm{GeV} \quad \text{or} \quad m_a \gtrsim 0.5\,\mathrm{meV} \quad \text{from dark matter over-production}$$

- **Axion star** formation enhanced at MRE

- Potential new observational opportunities

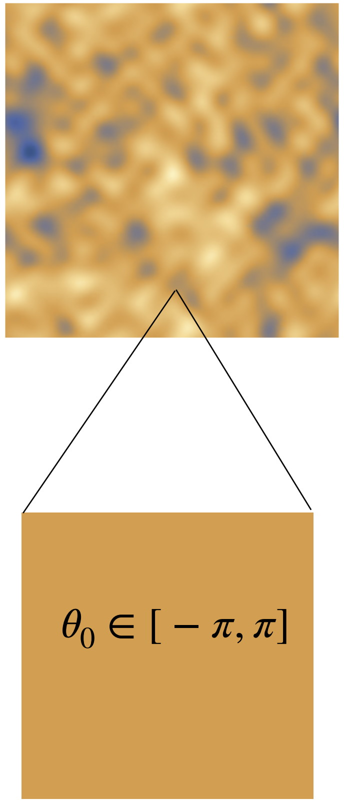
Thanks!

Backup

Axion dark matter

Pre-inflationary scenario

$$\ddot{a} + 3H(T)\dot{a} + m_a^2(T)\, a = 0$$

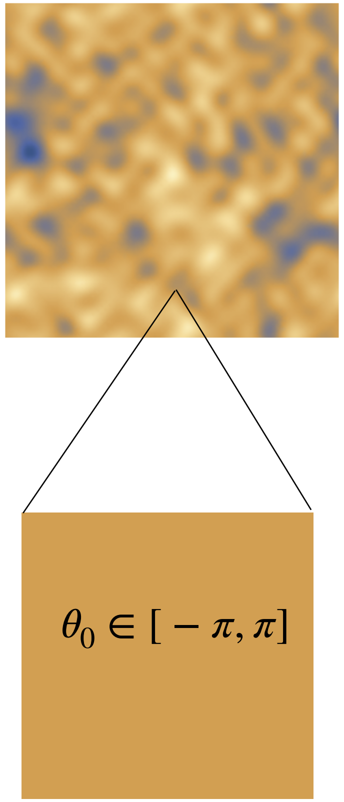


$\theta_0 \in [-\pi, \pi]$

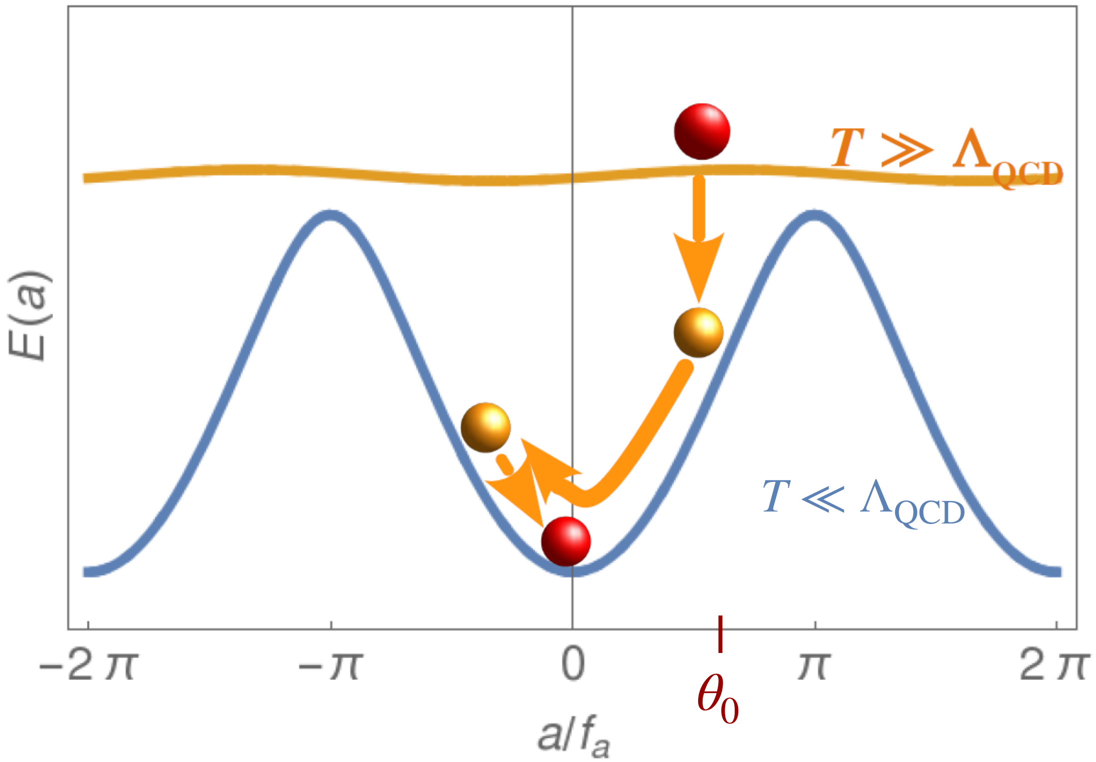
$a(t) =$ const in space

Pre-inflationary scenario

$$\ddot{a} + 3H(T)\dot{a} + m_a^2\,(T)\,a = 0$$



$a(t) = \text{const in space}$



$$a(t) \simeq \frac{1}{R(t)^{\frac{3}{2}}} \cos m_a t$$

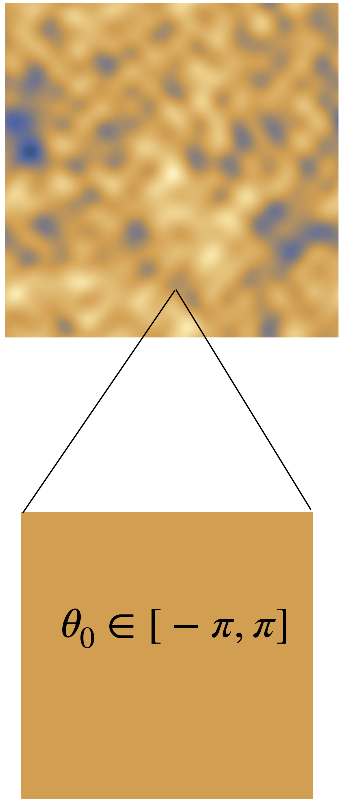
scale factor

Energy density:

$$\rho_a(t) \propto R(t)^{-3}$$

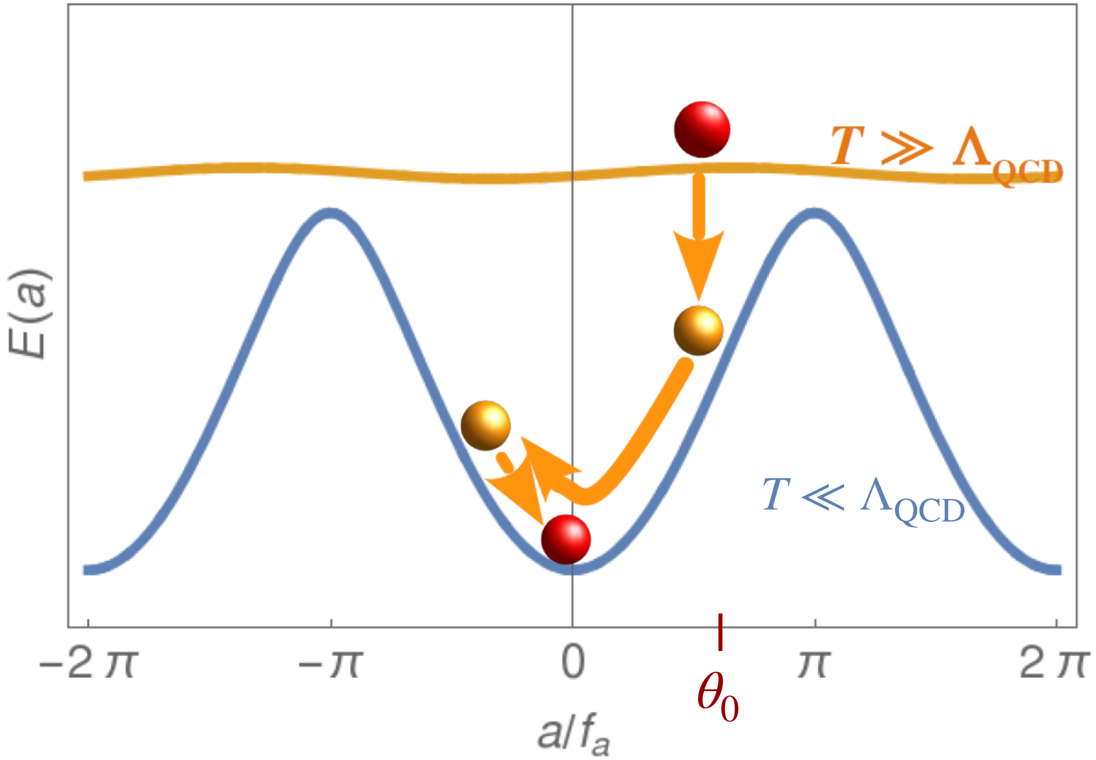
Pre-inflationary scenario

$$\ddot{a} + 3H(T)\dot{a} + m_a^2(T)\,a = 0$$



$\theta_0 \in [-\pi, \pi]$

$a(t) = \text{const in space}$



$$a(t) \simeq \frac{1}{R(t)^{\frac{3}{2}}} \cos m_a t$$

scale factor

Energy density:

$$\rho_a(t) \propto R(t)^{-3}$$

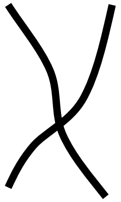
$$\boxed{\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\text{GeV}}\right]^{1+\epsilon}}$$

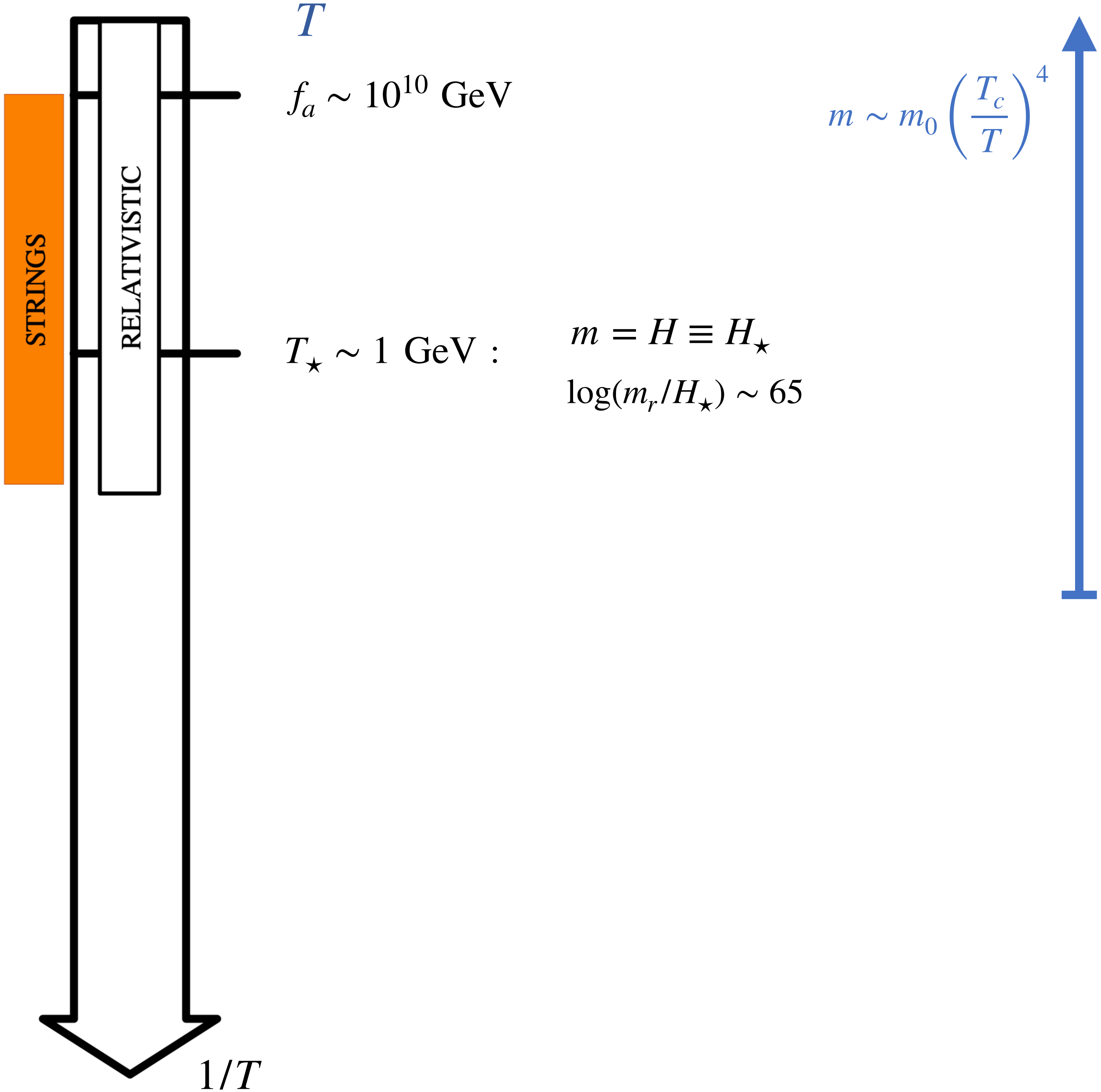
→

$$0 \leq |\theta_0| < \pi$$

$$10^{18} \gtrsim f_a/\text{GeV} \gtrsim 4 \cdot 10^9$$

$$10^{-11} \lesssim m_a/\text{eV} \lesssim 1.5 \cdot 10^{-3}$$





T

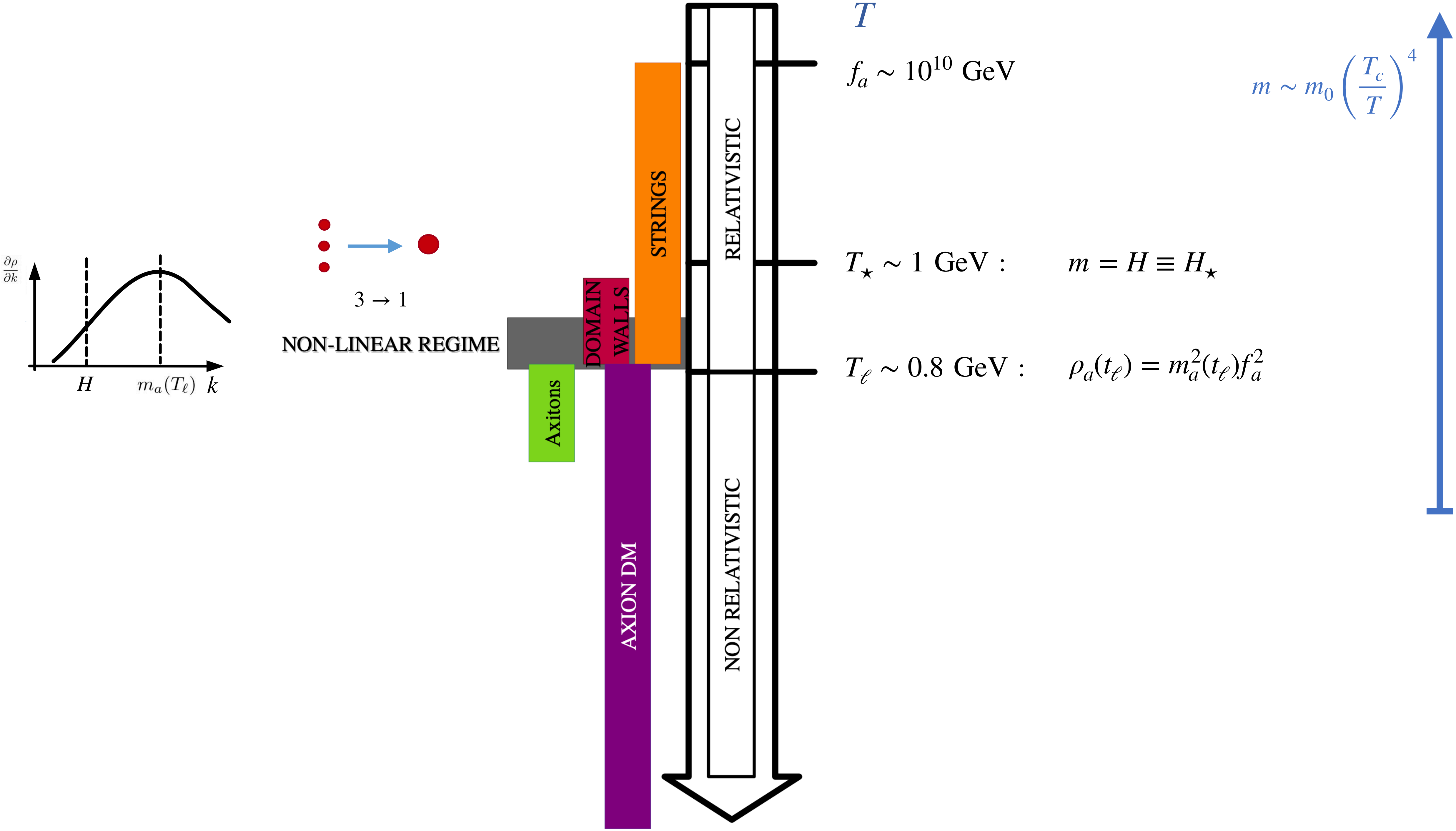
$f_a \sim 10^{10}$ GeV

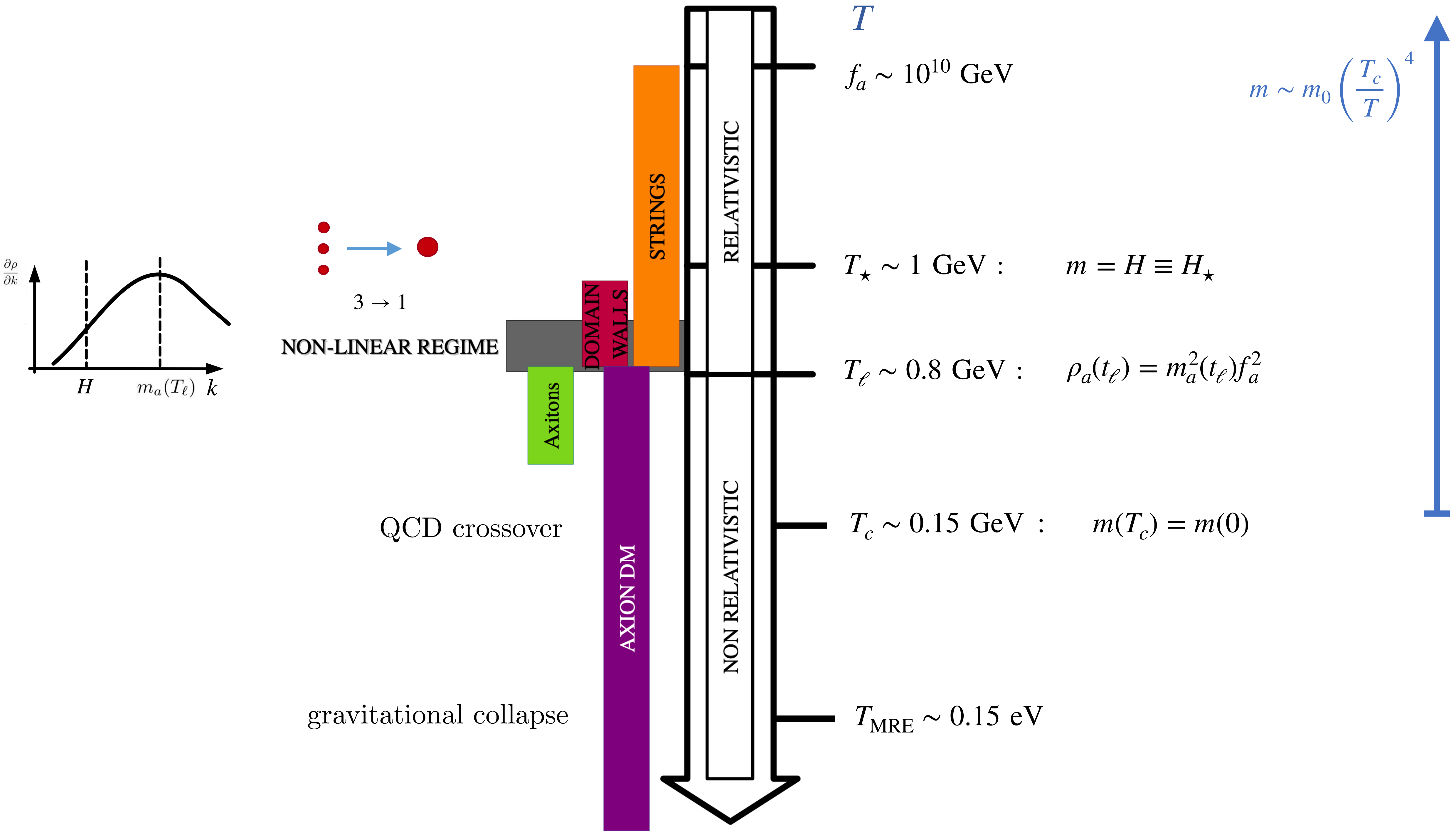
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$

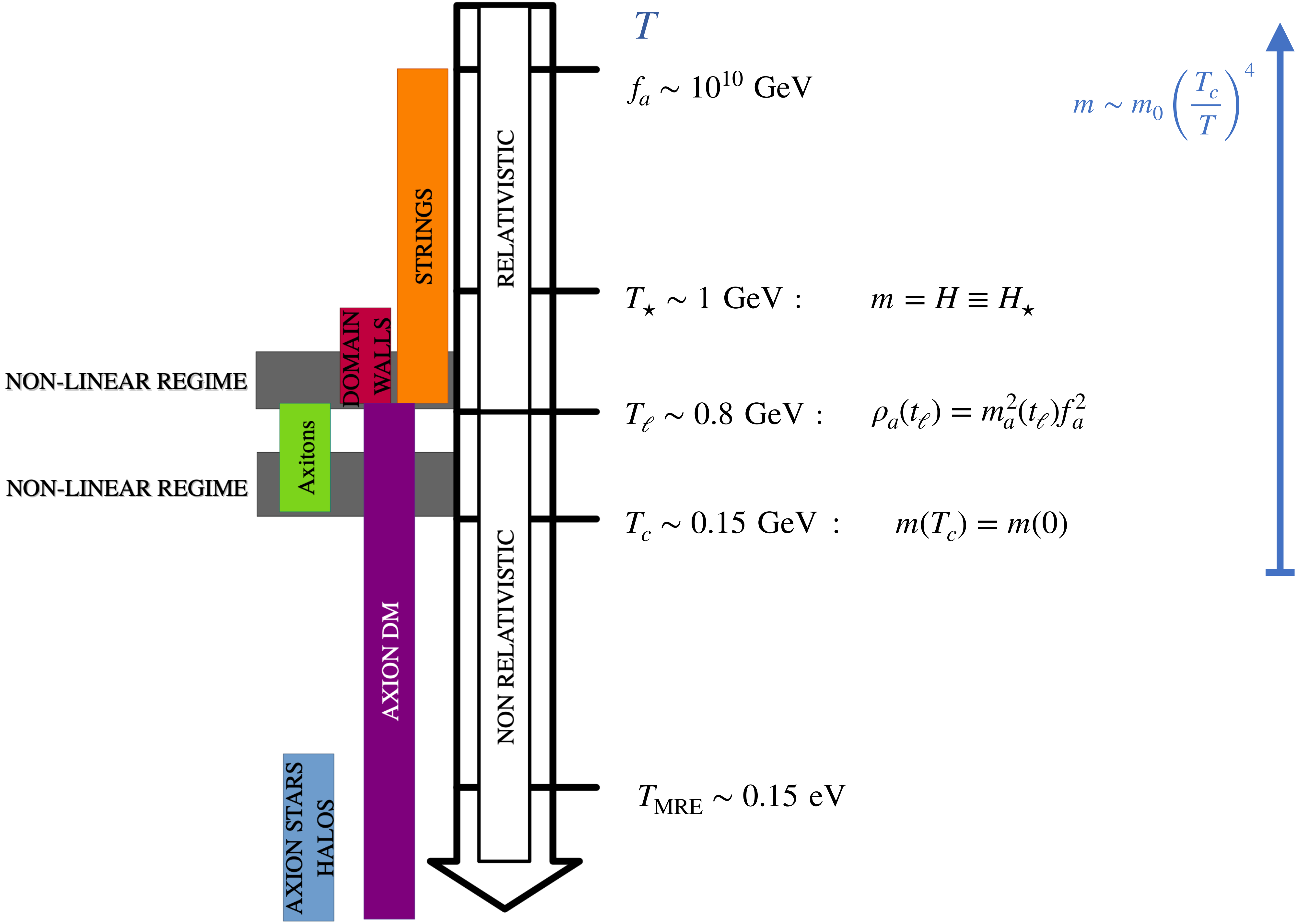
$T_\star \sim 1$ GeV : $m = H \equiv H_\star$

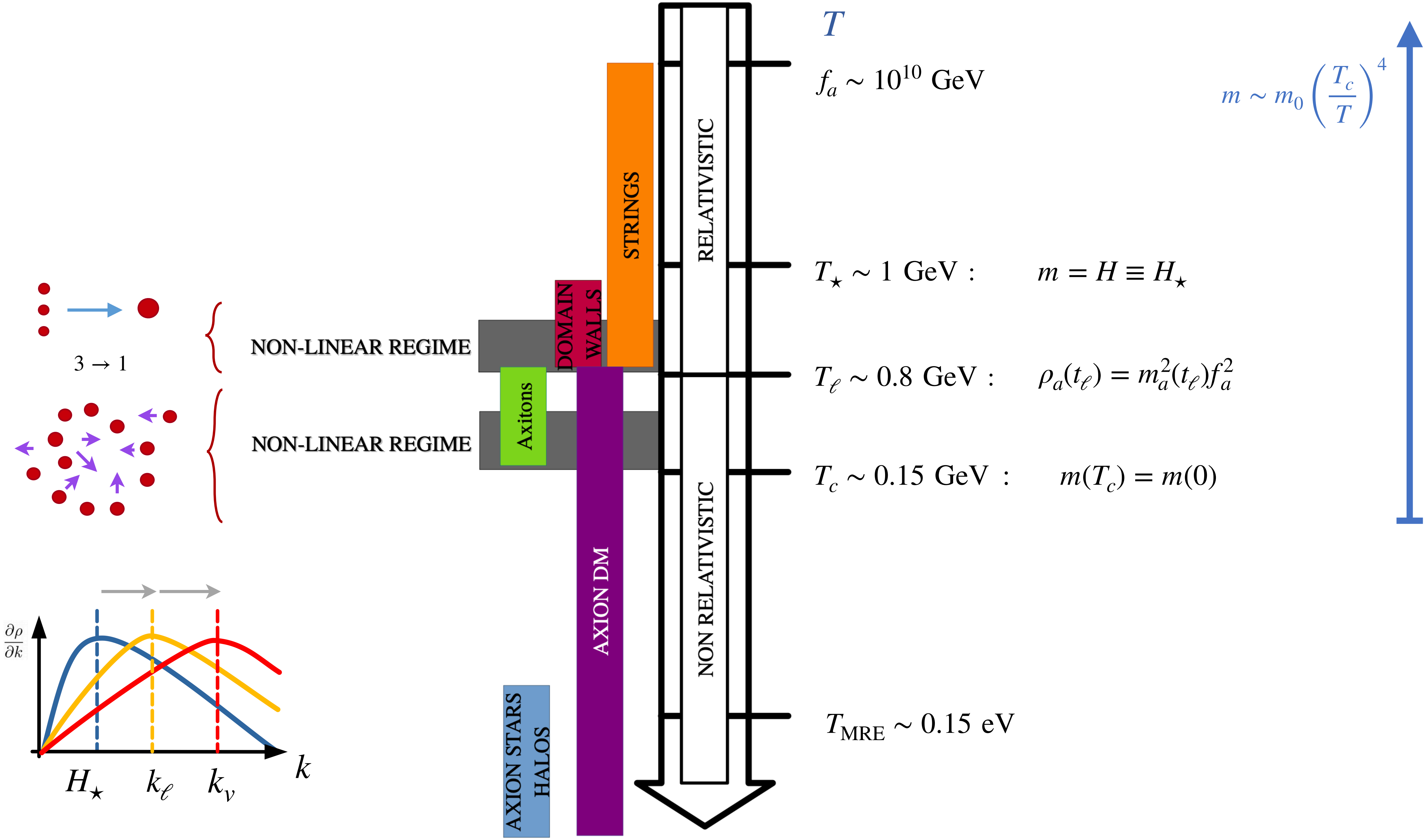
$\log(m_r/H_\star) \sim 65$

$1/T$


$\frac{\partial\rho}{\partial k}$
H
$m_a(T_\ell)$
k
$3 \rightarrow 1$
NON-LINEAR REGIME
Axitons
DOMAIN WALLS
AXION DM
STRINGS
RELATIVISTIC
NON RELATIVISTIC
T
$f_a \sim 10^{10}$ GeV
$T_\star \sim 1$ GeV : $m = H \equiv H_\star$
$T_\ell \sim 0.8$ GeV : $\rho_a(t_\ell) = m_a^2(t_\ell) f_a^2$
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$

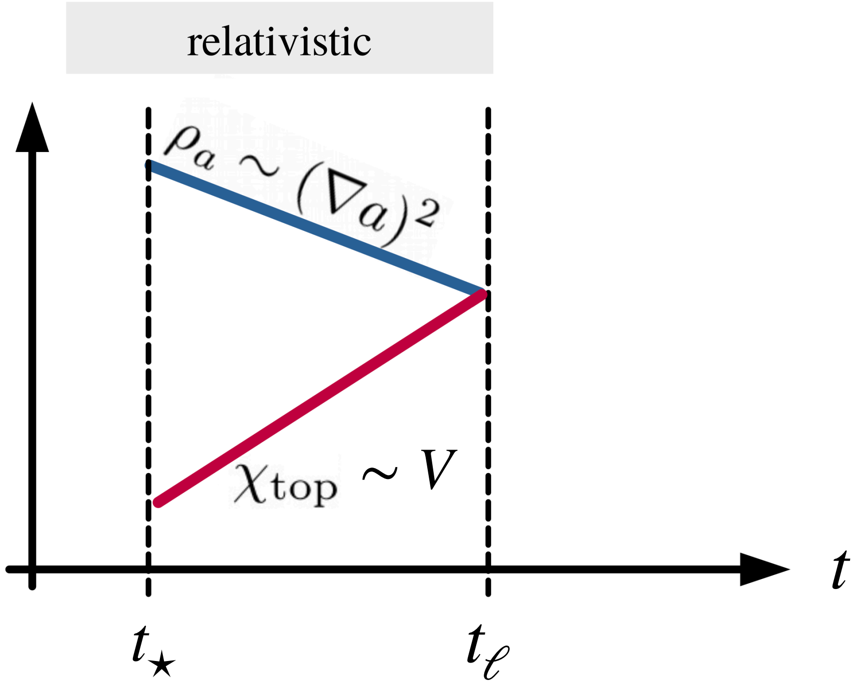

T
$f_a \sim 10^{10}$ GeV
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$
$T_\star \sim 1$ GeV : $m = H \equiv H_\star$
$T_\ell \sim 0.8$ GeV : $\rho_a(t_\ell) = m_a^2(t_\ell) f_a^2$
$T_c \sim 0.15$ GeV : $m(T_c) = m(0)$
$T_{\text{MRE}} \sim 0.15$ eV
RELATIVISTIC
NON RELATIVISTIC
STRINGS
DOMAIN WALLS
NON-LINEAR REGIME
Axitons
AXION DM
QCD crossover
gravitational collapse
$3 \to 1$
$\frac{\partial \rho}{\partial k}$
H
$m_a(T_\ell)$
k




T
$f_a \sim 10^{10}$ GeV
$m \sim m_0 \left(\frac{T_c}{T}\right)^4$
STRINGS
RELATIVISTIC
$T_\star \sim 1$ GeV : $m = H \equiv H_\star$
DOMAIN WALLS
NON-LINEAR REGIME
$3 \rightarrow 1$
$T_\ell \sim 0.8$ GeV : $\rho_a(t_\ell) = m_a^2(t_\ell) f_a^2$
Axitons
NON-LINEAR REGIME
$T_c \sim 0.15$ GeV : $m(T_c) = m(0)$
AXION DM
NON RELATIVISTIC
$\frac{\partial \rho}{\partial k}$
$H_\star$
k_ℓ
k_v
k
AXION STARS HALOS
$T_{\rm MRE} \sim 0.15$ eV

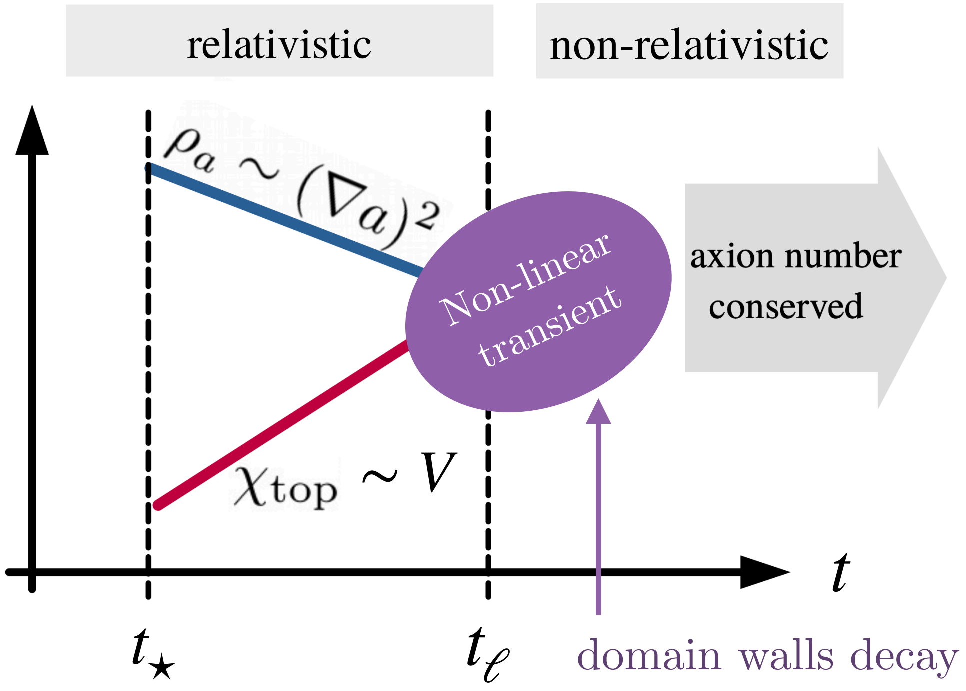
Effect of non-linearities (I)

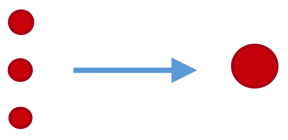
If $q \geq 1$: $\quad \rho_a(t_\star) \gg \rho^{\rm mis} \sim m_\star^2 f_a^2 = \chi_{\rm top}(T_\star)$



Effect of non-linearities (I)

If $q \geq 1$: $\rho_a(t_\star) \gg \rho^{\rm mis} \sim m_\star^2 f_a^2 = \chi_{\rm top}(T_\star)$





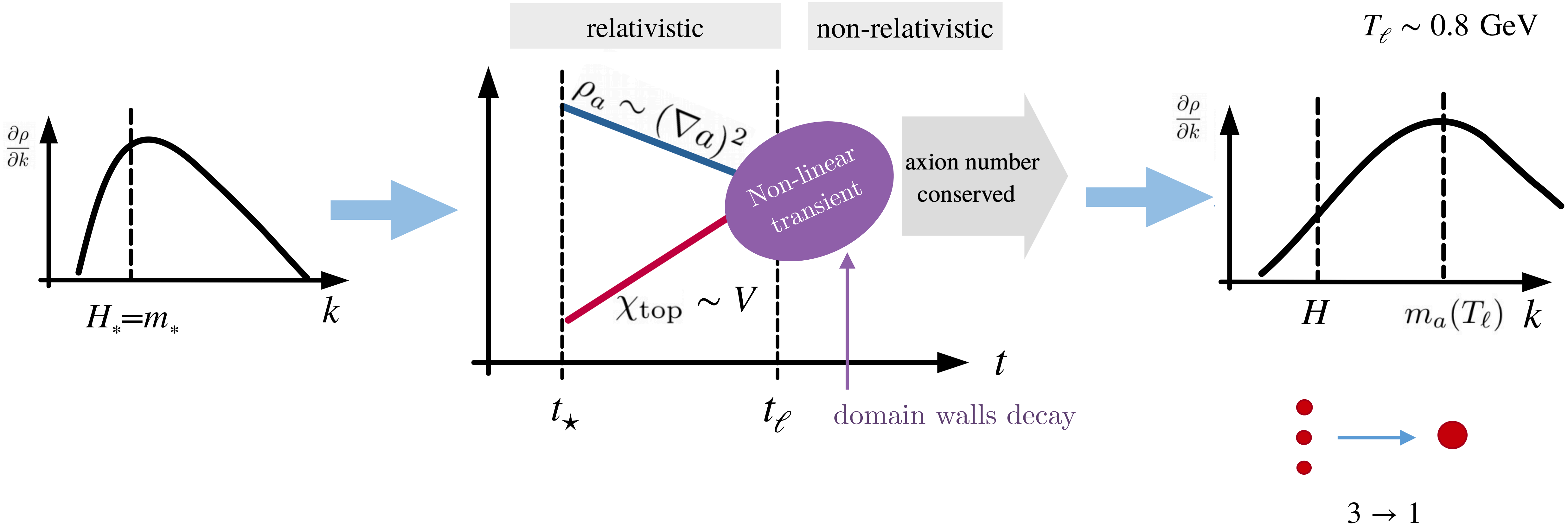
$3 \to 1$

$$f_a \simeq (\overset{q>1}{1} \div \overset{q=1}{6}) \cdot 10^{10}\ \text{GeV} \quad + \text{DW?}$$

Effect of non-linearities (I)

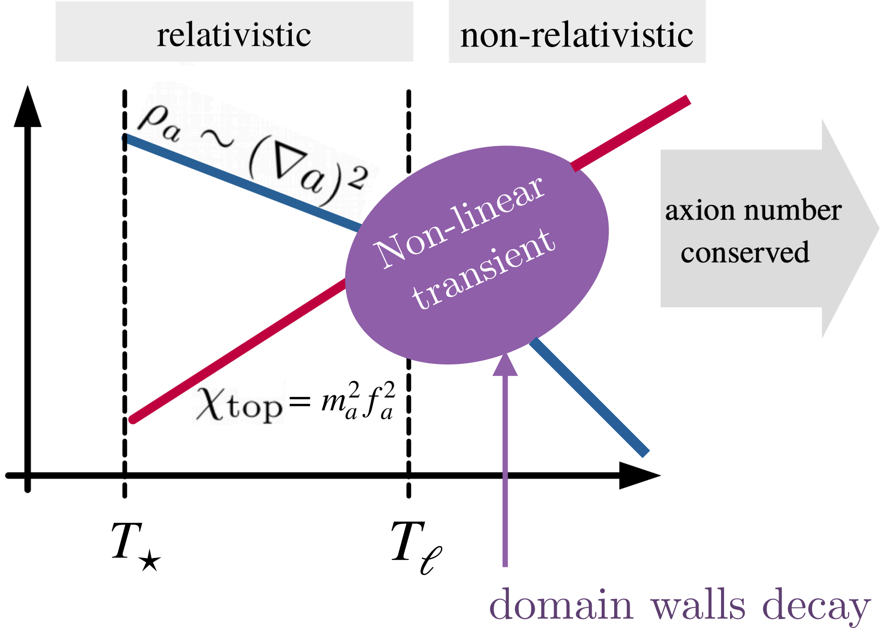
If $q \geq 1$: $\rho_a(t_\star) \gg \rho^{\text{mis}} \sim m_\star^2 f_a^2 = \chi_{\text{top}}(T_\star)$

$T_\ell \sim 0.8$ GeV

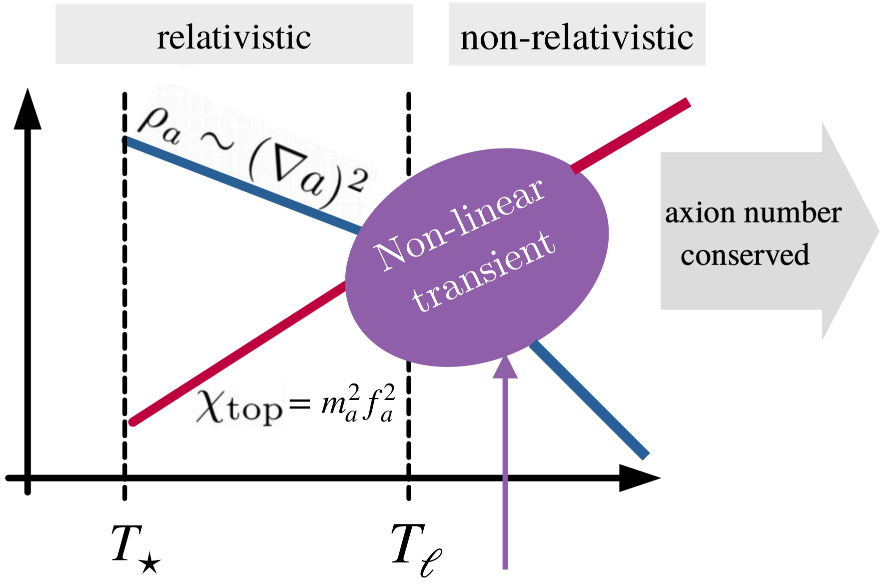


$$f_a \simeq (\overset{q>1}{1} \div \overset{q=1}{6}) \cdot 10^{10}\ \text{GeV} \quad + \text{DW?}$$

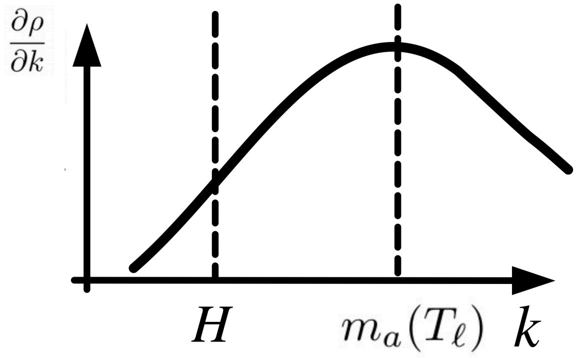
After DW decay: the standard lore



After DW decay: the standard lore



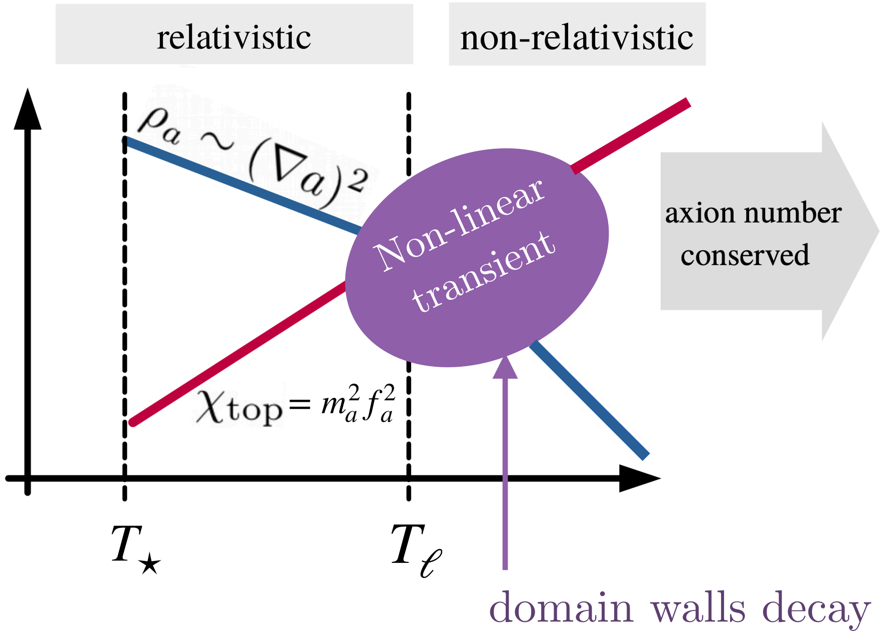
domain walls decay



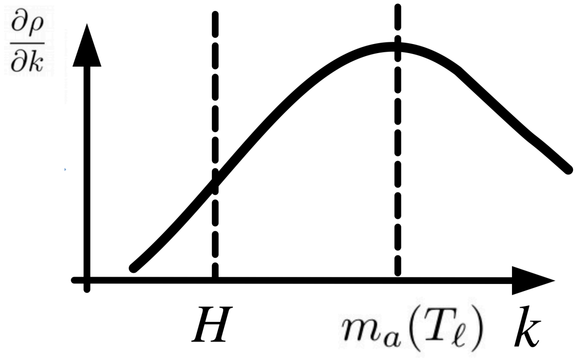
$m_a(T) \gg k_p$ → axions become non-relativistic

$$m \sim m_0 \left(\frac{T_c}{T}\right)^4$$

After DW decay: the standard lore



$m_a(T) \gg k_p$ → axions become non-relativistic



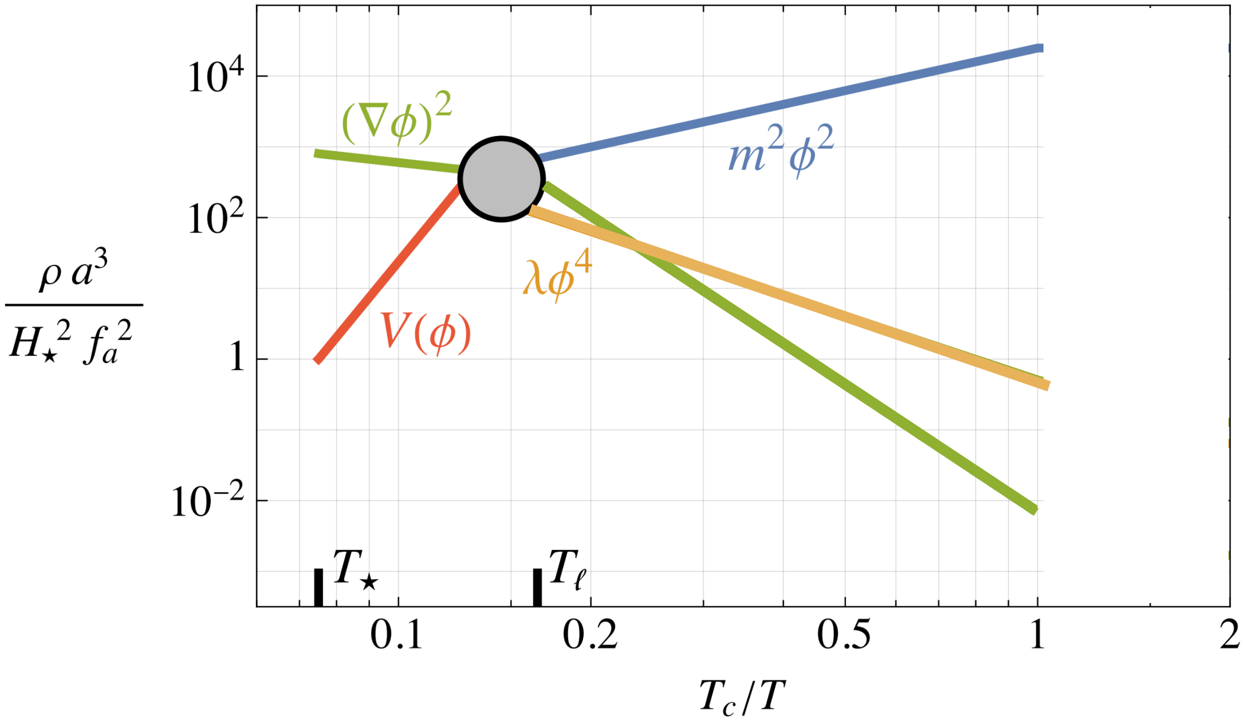
$$m \sim m_0 \left(\frac{T_c}{T}\right)^4$$

$V(a) \simeq \frac{1}{2} m^2 a^2$ axions become free

$\Longrightarrow$ the field redshifts like CDM until MRE

@ MRE, fluctuations $\delta\rho/\rho \sim 1$ gravitationally collapse in objects of size $\sim 1/k_p$

However: effect of non-linearities (II)



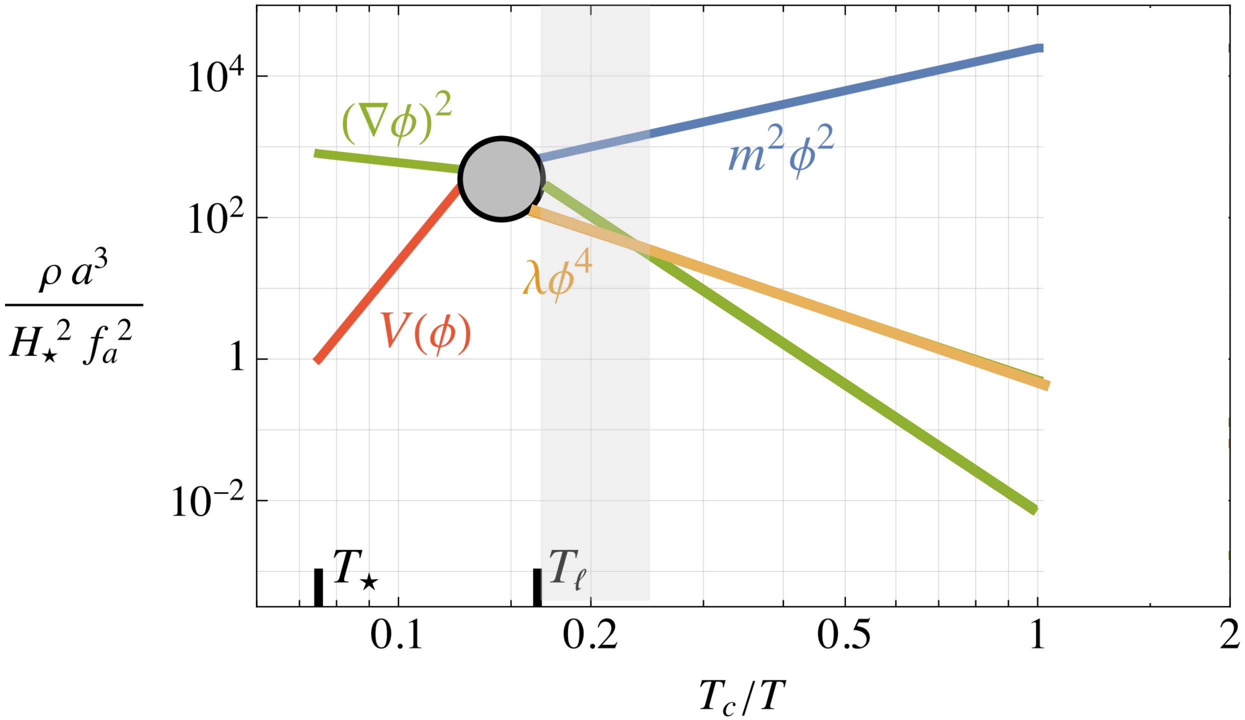
$$\rho \sim \dot{\phi}^2 + m^2\phi^2 + (\nabla\phi)^2 + \lambda\phi^4$$

$$\phi \sim \psi e^{-imt}$$

$$T_c \sim 0.15\,\mathrm{GeV}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \xcancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

However: effect of non-linearities (II)



$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

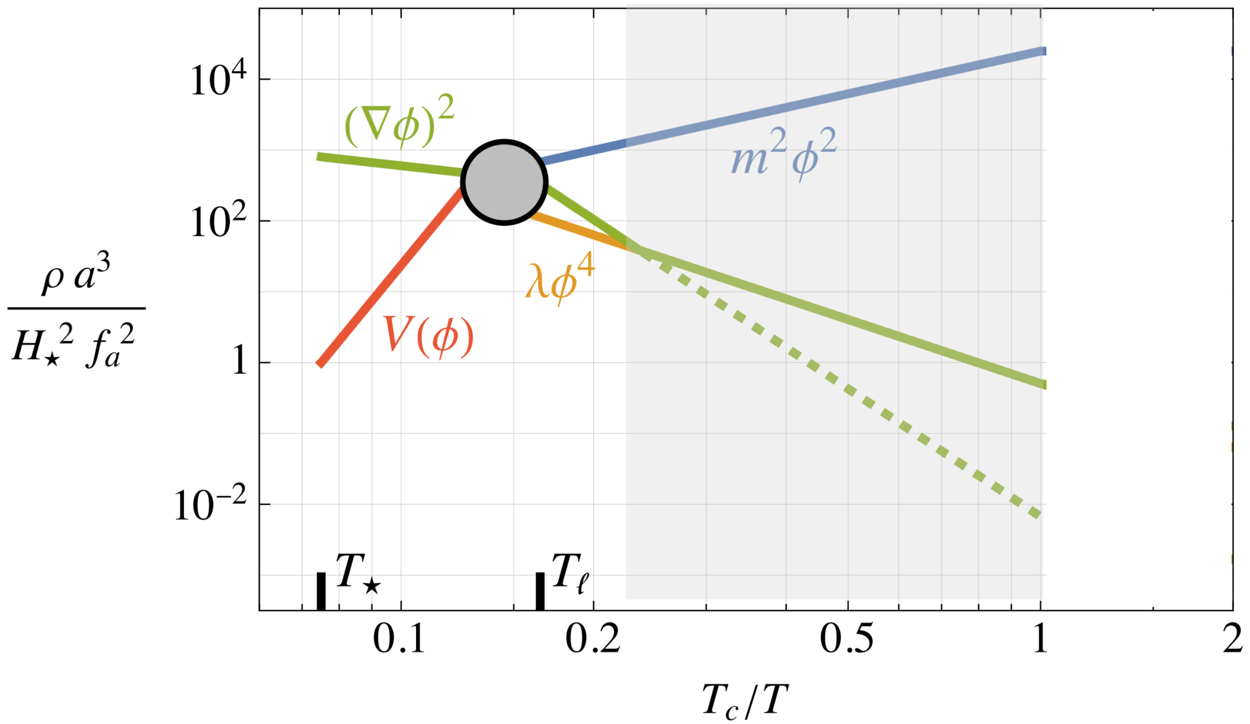
$$(\nabla\phi)^2 \;+\; \boxed{\lambda}\phi^4$$

grows

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \xcancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2} \right)\psi = 0$$

perturbation

However: effect of non-linearities (II)



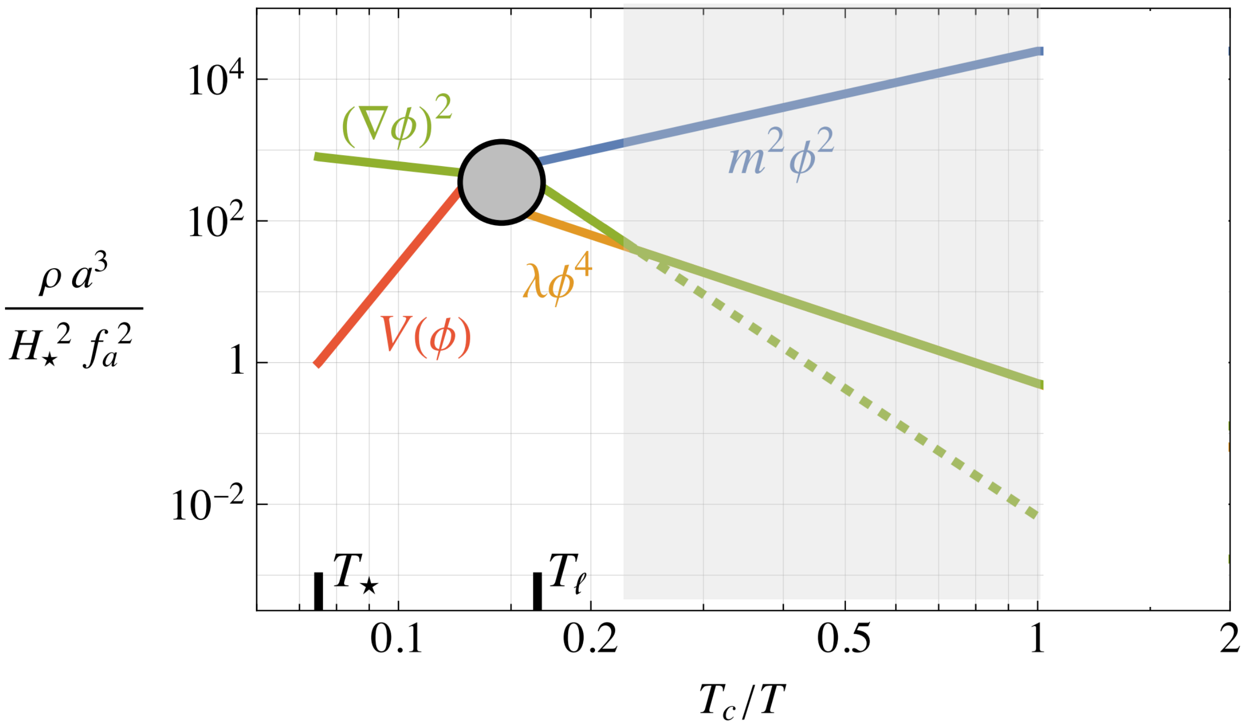
$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

$$(\nabla\phi)^2 + \underset{\downarrow \atop \text{grows}}{\lambda\phi^4}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \xcancel{m\Phi} + \underset{\downarrow}{\frac{\lambda|\psi|^2}{8a^3 m_0 m^2}}\right)\psi = 0$$

same order as the others

However: effect of non-linearities (II)



$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

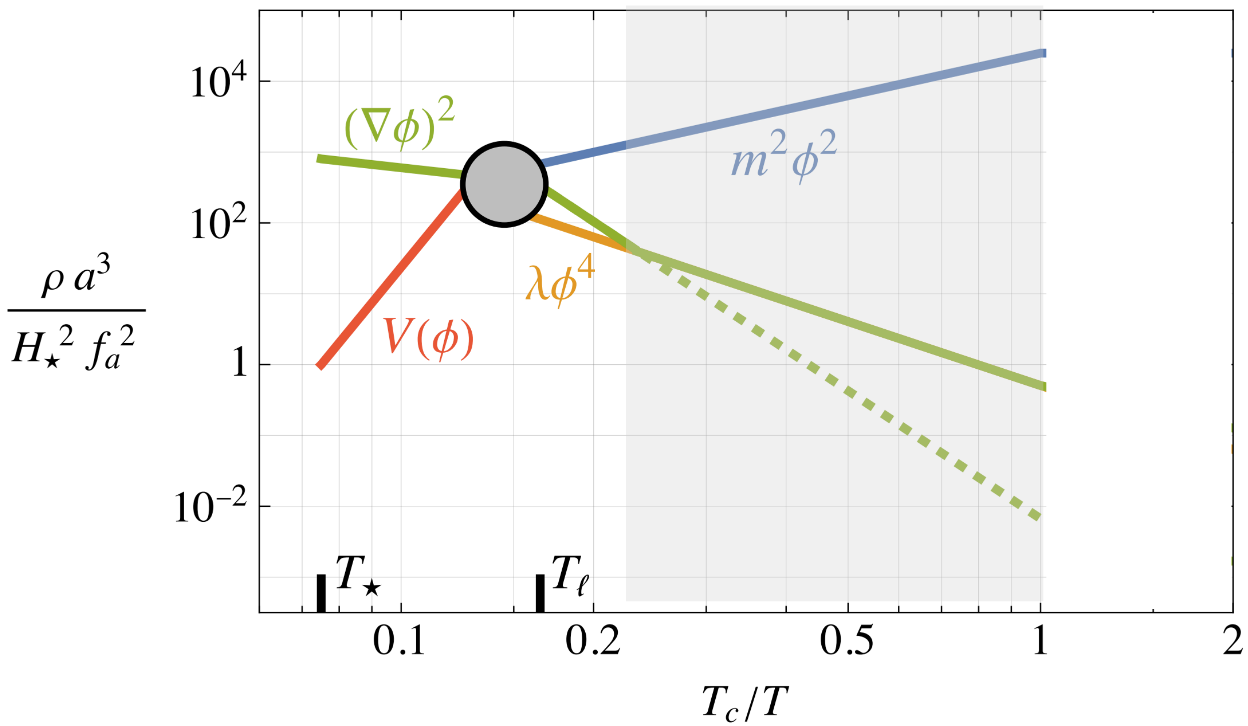
$$(\nabla\phi)^2 + \boxed{\lambda}\phi^4$$

grows

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \cancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

same order as the others

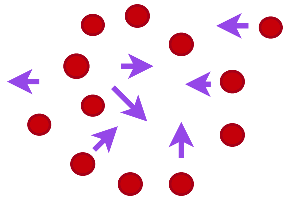
However: effect of non-linearities (II)



$$\rho \sim \dot\phi^2 + m^2\phi^2 + (\nabla\phi)^2 + \boxed{\lambda}\phi^4$$

$\downarrow$

grows



$$\left(i\partial_t + \frac{\nabla^2}{2m} - \cancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

$\downarrow$

same order as the others

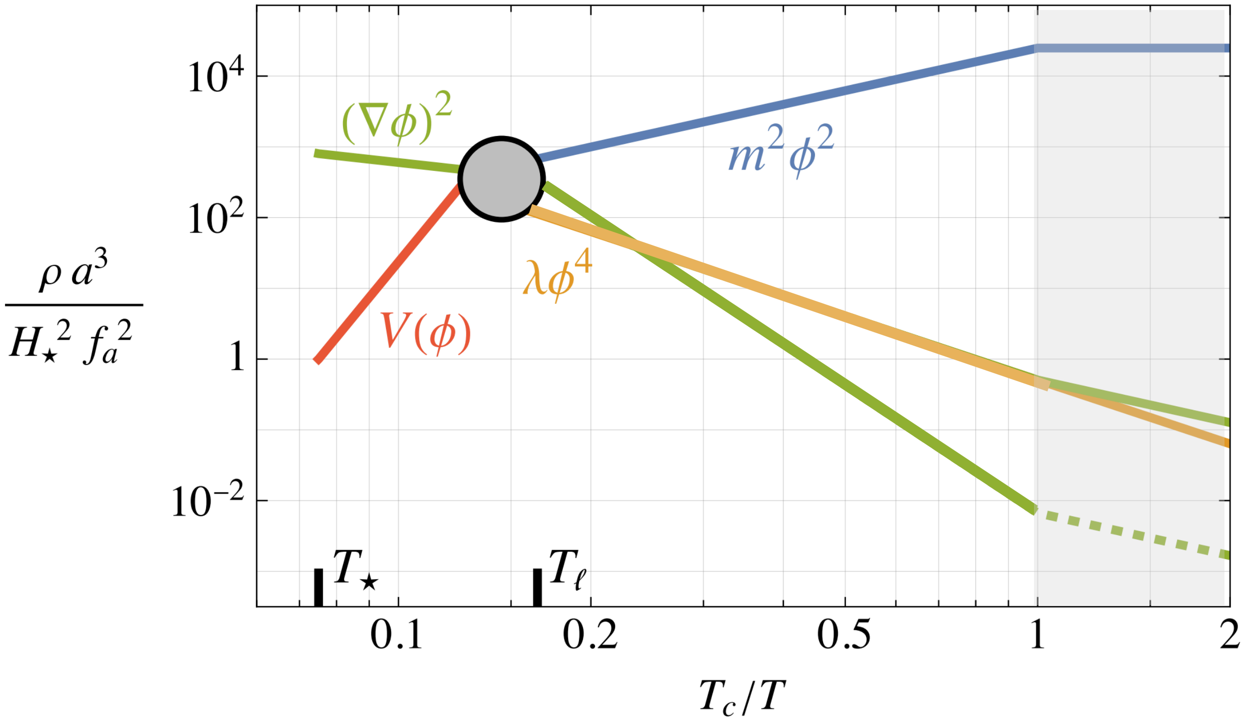
$$k_p \to k_v = \sqrt{\lambda\langle\phi^2\rangle} \simeq \sqrt{\rho}/f_a$$

$\updownarrow$

$$(\nabla\phi)^2 \sim \lambda\phi^4$$

$$\tau_v = 8m/(\lambda\phi^2)$$

However: effect of non-linearities (II)



$$\rho \sim \dot{\phi}^2 + m^2\phi^2 +$$

$$(\nabla\phi)^2 \; + \; \lambda\phi^4$$

$$\downarrow$$

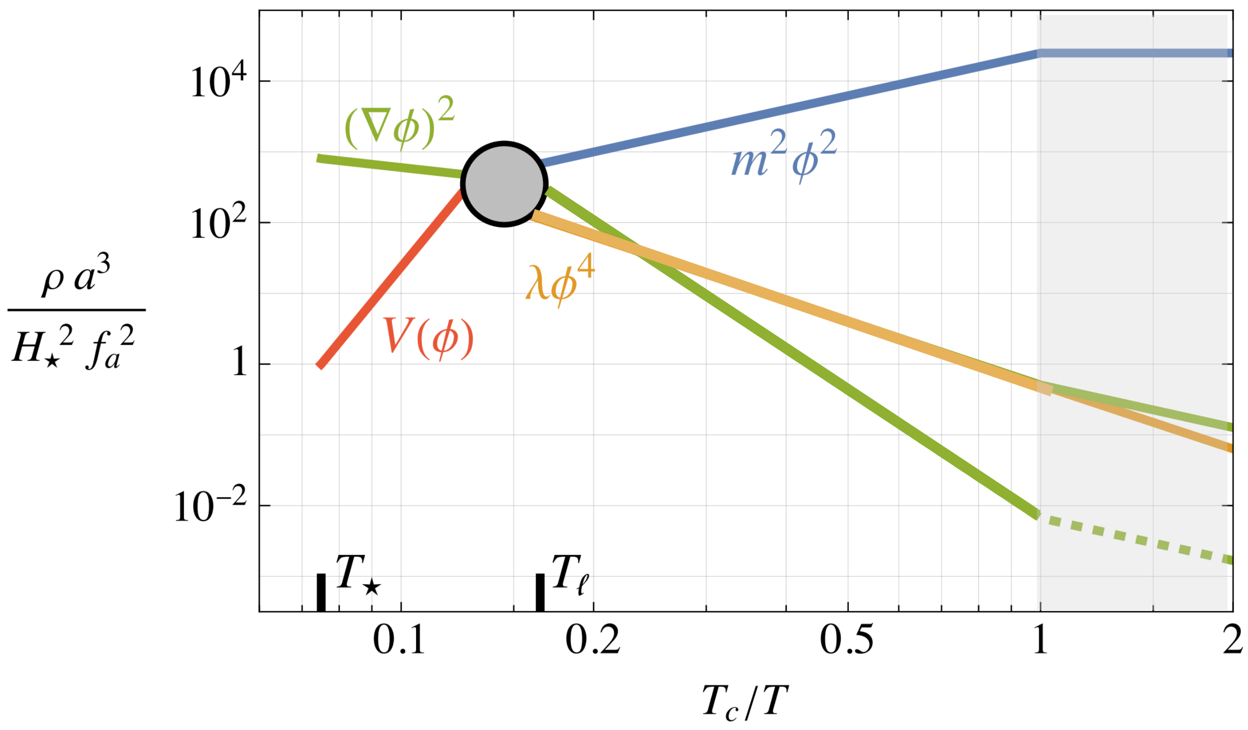
constant

$$\left(i\partial_t + \frac{\nabla^2}{2m} - \xcancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

$$\downarrow$$

perturbation

However: effect of non-linearities (II)



$$\rho \sim \dot\phi^2 + m^2\phi^2 +$$

$$(\nabla\phi)^2 + \boxed{\lambda}\phi^4$$

$$\downarrow$$

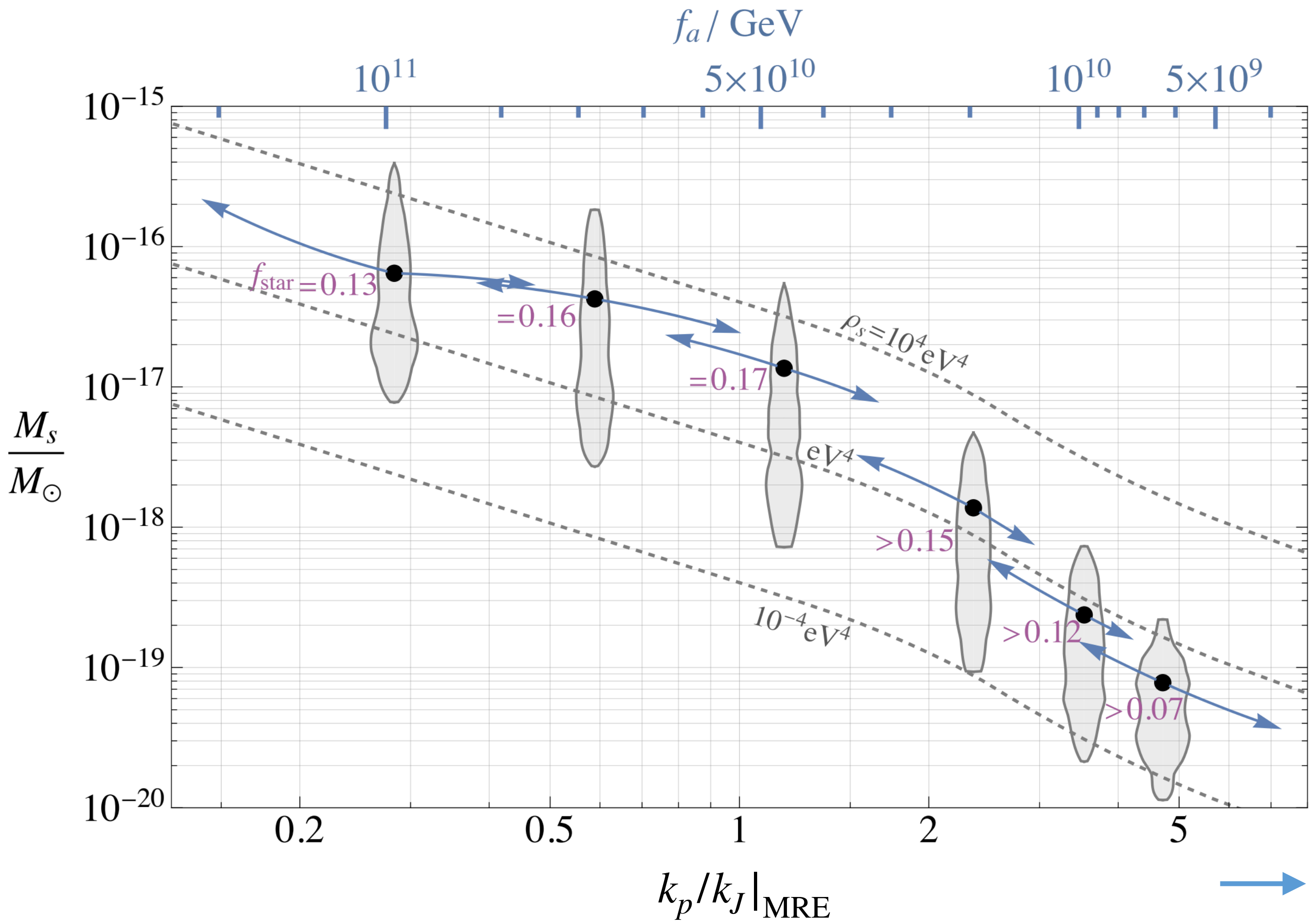
constant

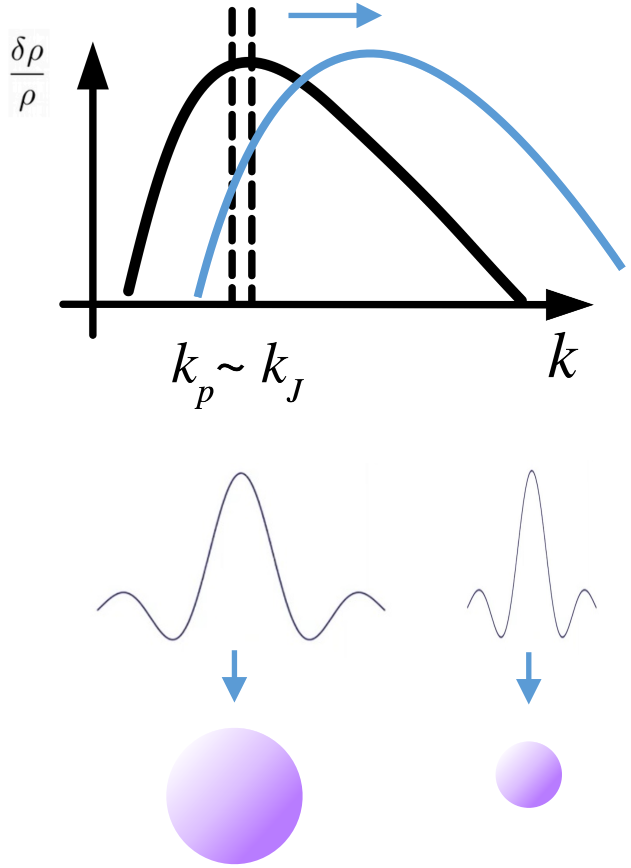
$$\left(i\partial_t + \frac{\nabla^2}{2m} - \cancel{m\Phi} + \frac{\lambda|\psi|^2}{8a^3 m_0 m^2}\right)\psi = 0$$

$$\downarrow$$

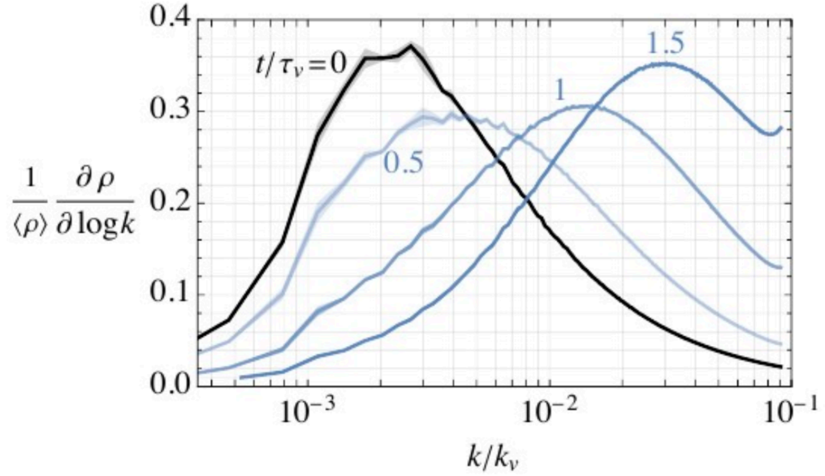
perturbation

Axion stars properties:

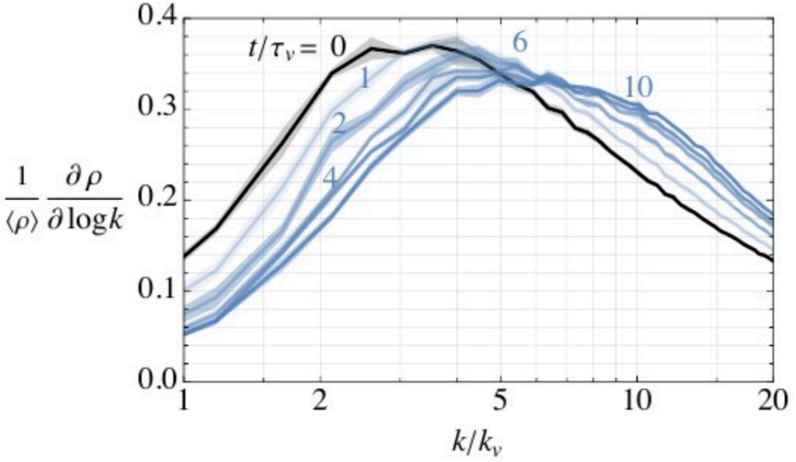




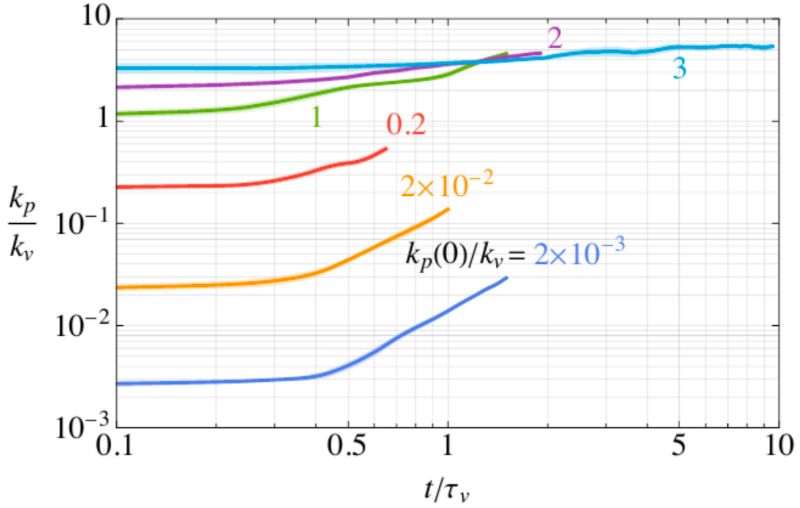
$$\bar{R}_{0.1} \approx 2.1 \cdot 10^6\ \text{km} \left(\frac{10^{10}\ \text{GeV}}{f_a}\right)^{\frac{1}{2}} \qquad v_a \approx \text{mm/s}$$



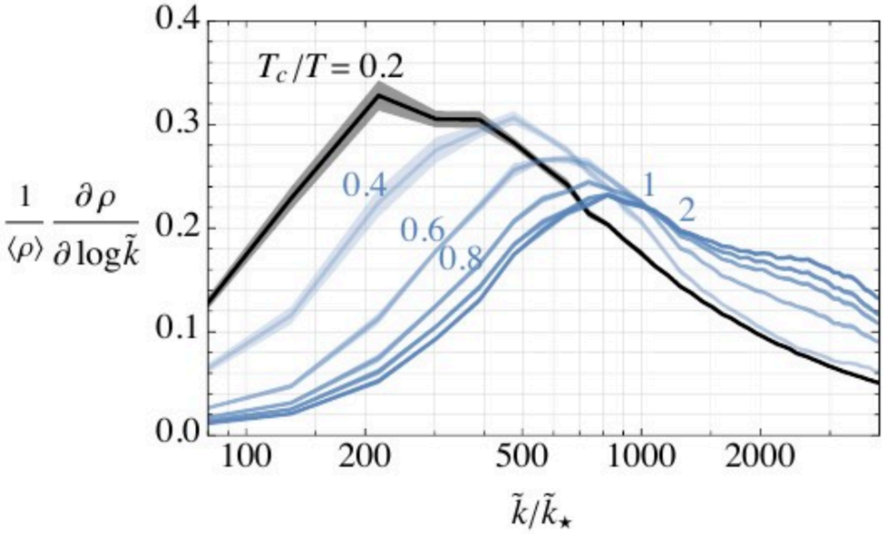
0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log k}$
$t/\tau_v = 0$
0.5
1
1.5
10^{-3}
10^{-2}
10^{-1}
k/k_v

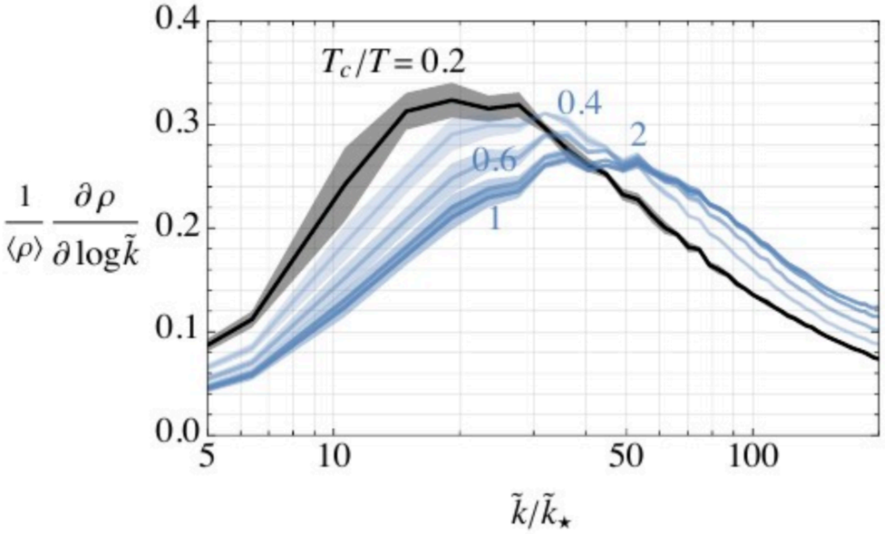


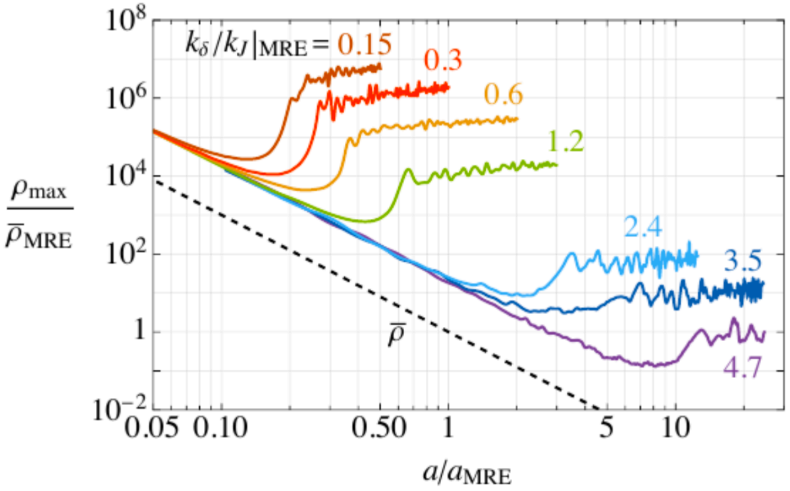
0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log k}$
$t/\tau_v = 0$
1
2
4
6
10
1
2
5
10
20
k/k_v

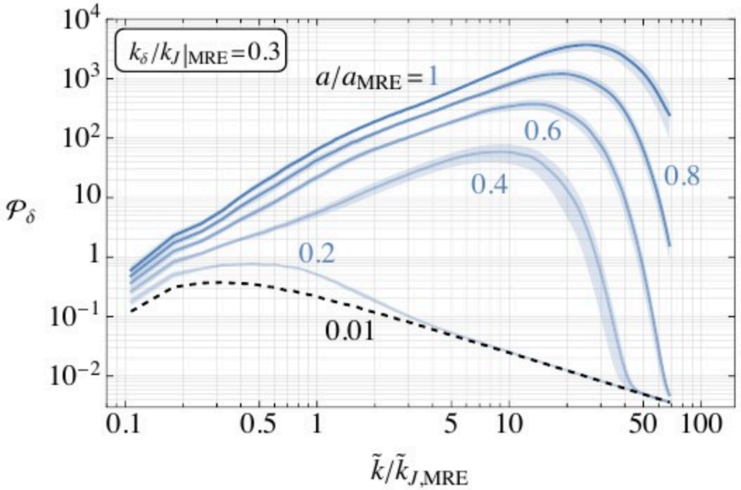


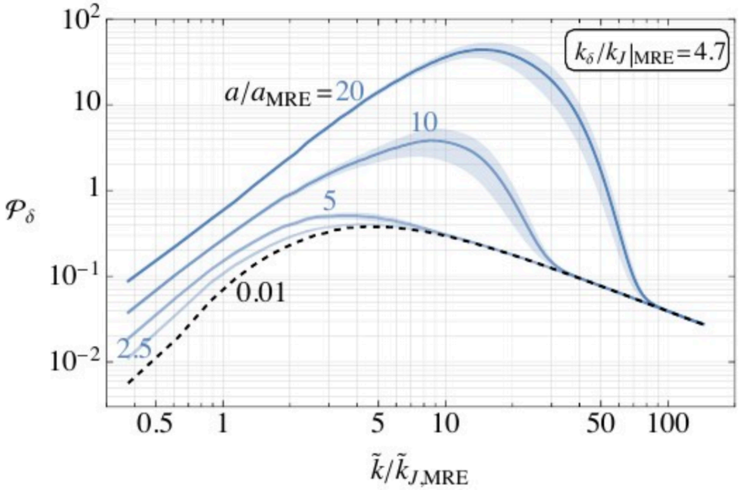
10
1
10^{-1}
10^{-2}
10^{-3}
$\frac{k_p}{k_v}$
2
3
1
0.2
2×10^{-2}
$k_p(0)/k_v = 2\times10^{-3}$
0.1
0.5
1
5
10
t/τ_v

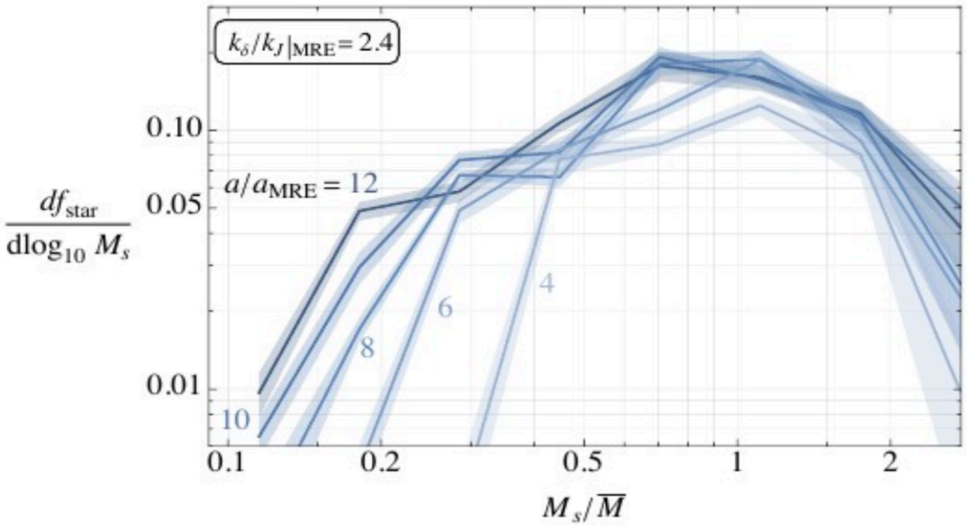

0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log\tilde{k}}$
$T_c/T = 0.2$
0.4
0.6
0.8
1
2
100
200
500
1000
2000
$\tilde{k}/\tilde{k}_\star$


0.4
0.3
0.2
0.1
0.0
$\frac{1}{\langle\rho\rangle}\frac{\partial\rho}{\partial\log\tilde{k}}$
$T_c/T = 0.2$
0.4
2
0.6
1
5
10
50
100
$\tilde{k}/\tilde{k}_\star$

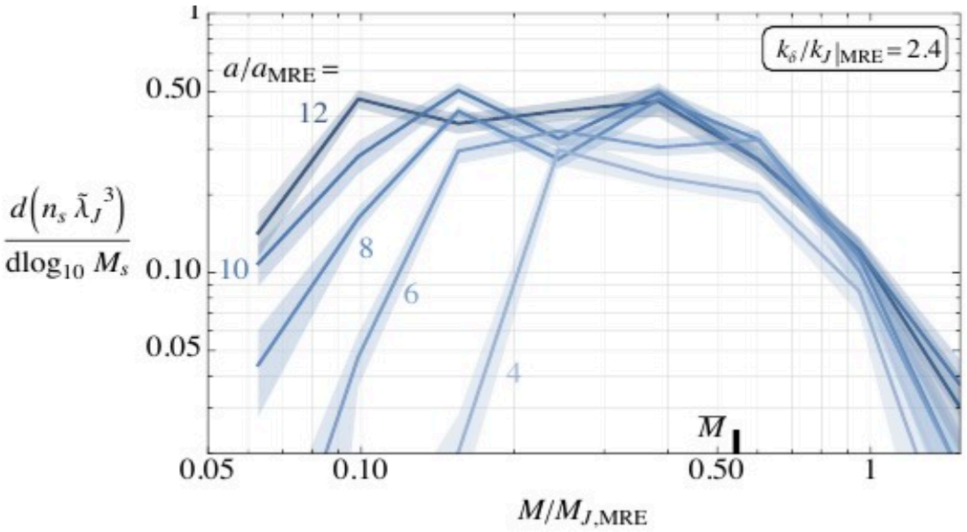

$k_\delta/k_J|_{\rm MRE}=$ 0.15
0.3
0.6
1.2
2.4
3.5
4.7
$\frac{\rho_{\rm max}}{\bar\rho_{\rm MRE}}$
$\bar\rho$
10^8
10^6
10^4
10^2
1
10^{-2}
0.05
0.10
0.50
1
5
10
$a/a_{\rm MRE}$


$k_\delta/k_J|_{\rm MRE}=0.3$
$a/a_{\rm MRE}=1$
0.8
0.6
0.4
0.2
0.01
$\mathcal{P}_\delta$
10^4
10^3
10^2
10
1
10^{-1}
10^{-2}
0.1
0.5
1
5
10
50
100
$\tilde k/\tilde k_{J,\rm MRE}$

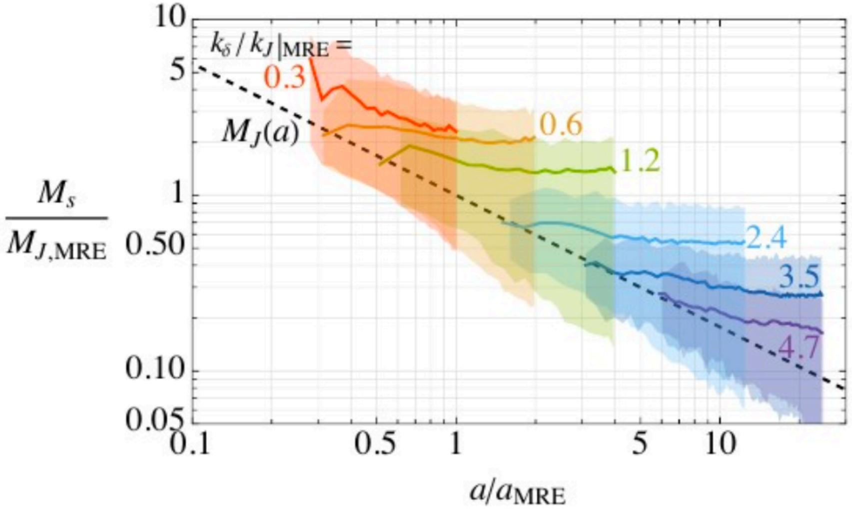

$k_\delta/k_J|_{\rm MRE}=4.7$
$a/a_{\rm MRE}=20$
10
5
2.5
0.01
$\mathcal{P}_\delta$
10^2
10
1
10^{-1}
10^{-2}
0.5
1
5
10
50
100
$\tilde k/\tilde k_{J,\rm MRE}$



$k_\delta/k_J|_{\rm MRE} = 2.4$
$a/a_{\rm MRE} = 12$
10
8
6
4
$\frac{df_{\rm star}}{d\log_{10} M_s}$
0.10
0.05
0.01
0.1
0.2
0.5
1
2
$M_s/\overline{M}$



1
$k_\delta/k_J|_{\rm MRE} = 2.4$
$a/a_{\rm MRE} =$
12
10
8
6
4
$\overline{M}$
$\frac{d\left(n_s\,\tilde{\lambda}_J{}^3\right)}{d\log_{10} M_s}$
0.50
0.10
0.05
0.05
0.10
0.50
1
$M/M_{J,\rm MRE}$



$k_\delta/k_J|_{\rm MRE} =$
0.3
0.6
1.2
2.4
3.5
4.7
$M_J(a)$
$\frac{M_s}{M_{J,\rm MRE}}$
10
5
1
0.50
0.10
0.05
0.1
0.5
1
5
10
$a/a_{\rm MRE}$

,

The collapsed objects should be captured by a stationary solution of the Schroedinger–Poisson eq. (for $a = 1$)

$$\psi_i = \sqrt{\rho_i} e^{i\theta_i}$$
$$\vec{v}_i = \tfrac{1}{m}\nabla\theta_i$$

$$\longrightarrow$$

$$\begin{aligned}\partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) &= 0\\ \partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i &= -\,a^{-1}(\nabla\Phi + \nabla\Phi_{Qi})\\ \nabla^2\Phi &= 4\pi G a^2(\rho - \overline{\rho})\ ,\end{aligned}$$

The collapsed objects should be captured by a stationary solution of the Schroedinger–Poisson eq. (for $a = 1$)

$$\begin{aligned} \psi_i &= \sqrt{\rho_i} e^{i\theta_i} \\ \vec{v}_i &= \tfrac{1}{m} \nabla \theta_i \end{aligned} \longrightarrow \begin{aligned} \partial_t \rho_i + 3H\rho_i + a^{-1}\nabla \cdot (\rho_i \vec{v}_i) &= 0 \\ \partial_t \vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i \cdot \nabla)\vec{v}_i &= -\, a^{-1}(\nabla\Phi + \nabla\Phi_{Qi}) \\ \nabla^2\Phi &= 4\pi G a^2 (\rho - \overline{\rho}) \ , \end{aligned}$$

Halos

Solitons

The collapsed objects should be captured by a stationary solution of the Schroedinger–Poisson eq. (for $a = 1$)

$$\psi_i = \sqrt{\rho_i}e^{i\theta_i}$$

$$\vec{v}_i = \frac{1}{m}\nabla\theta_i$$

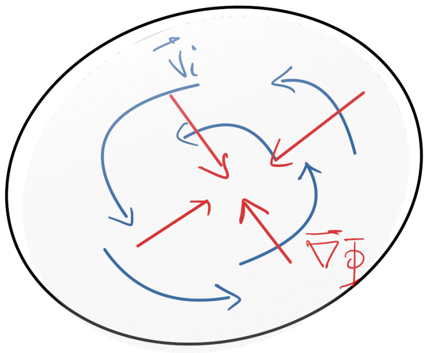
$$\longrightarrow$$

$$\begin{aligned} \partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) &= 0 \quad \textbf{halos}\\ \boxed{\partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i} &= -\,a^{-1}(\boxed{\nabla\Phi} + \nabla\Phi_{Qi})\\ \nabla^2\Phi &= 4\pi G a^2(\rho - \bar{\rho})\ , \end{aligned}$$

Halos

$$\Phi_Q = 0$$

→ gravitational potential balanced by the velocity terms



angular momentum ‘supports’ the gravitational potential

Solitons

The collapsed objects should be captured by a stationary solution of the Schroedinger–Poisson eq. (for $a = 1$)

$$\psi_i = \sqrt{\rho_i}e^{i\theta_i}$$
$$\vec{v}_i = \frac{1}{m}\nabla\theta_i$$

$$\longrightarrow$$

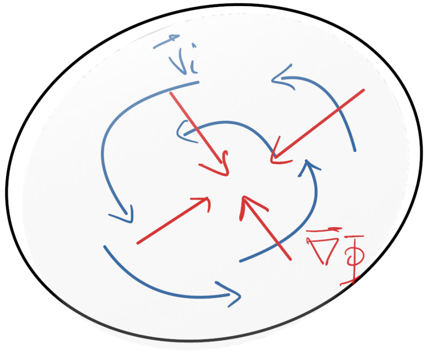
halos solitons

$$\partial_t\rho_i + 3H\rho_i + a^{-1}\nabla\cdot(\rho_i\vec{v}_i) = 0$$
$$\boxed{\partial_t\vec{v}_i + H\vec{v}_i + a^{-1}(\vec{v}_i\cdot\nabla)\vec{v}_i} = -a^{-1}(\boxed{\nabla\Phi} + \boxed{\nabla\Phi_{Qi}})$$
$$\nabla^2\Phi = 4\pi G a^2(\rho - \bar{\rho}) \; ,$$

Halos

$$\Phi_Q = 0$$

→ gravitational potential balanced by the velocity terms



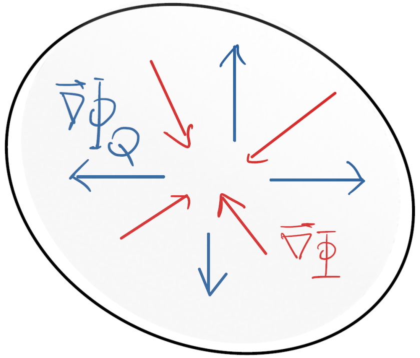
halo

angular momentum ‘supports’ the gravitational potential

Solitons

$$\Phi_Q = -\Phi \quad \longleftrightarrow \quad \vec{v}_i = 0$$

→ gravitational potential balanced by the quantum pressure

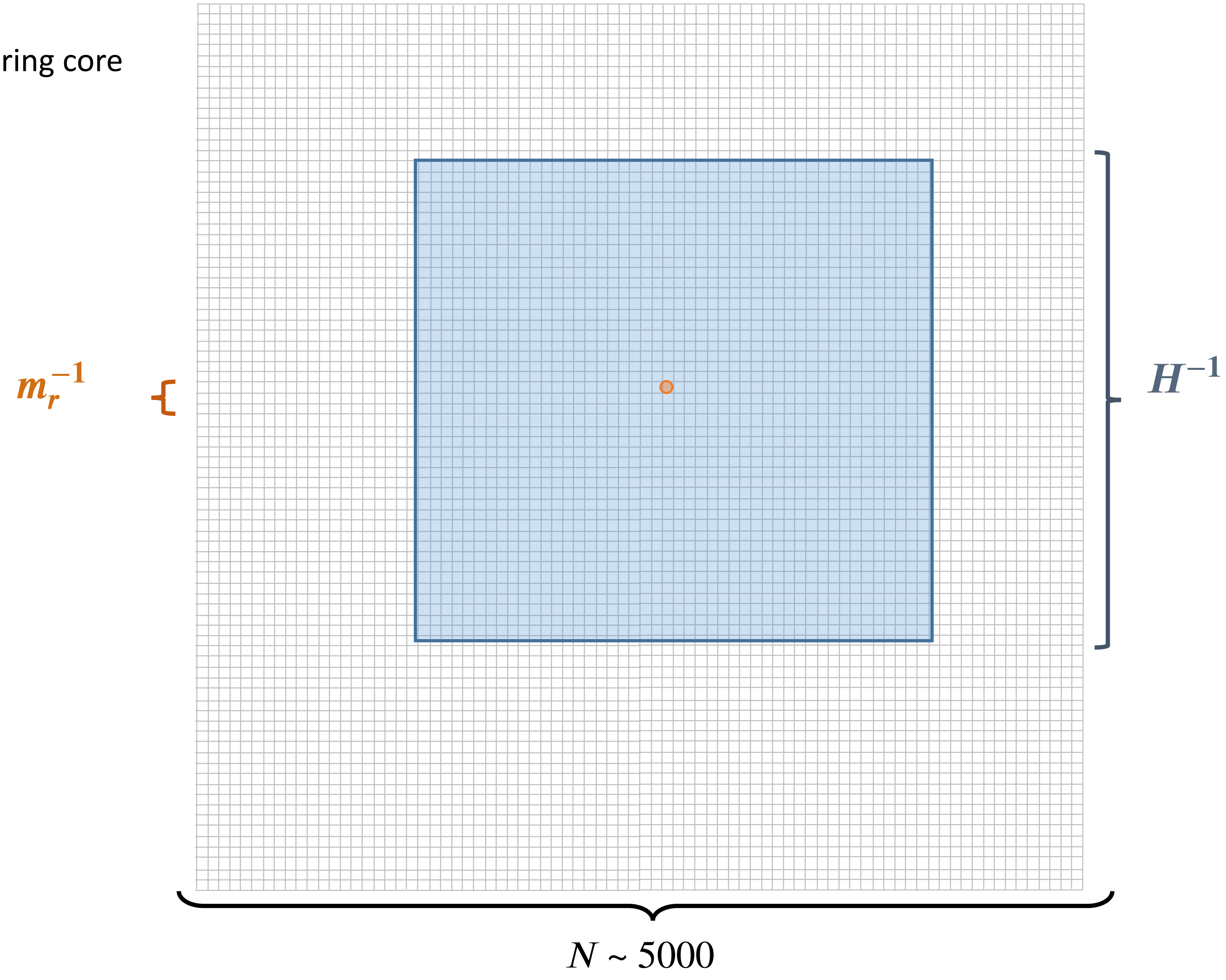


soliton

quantum pressure ‘supports’ the gravitational potential

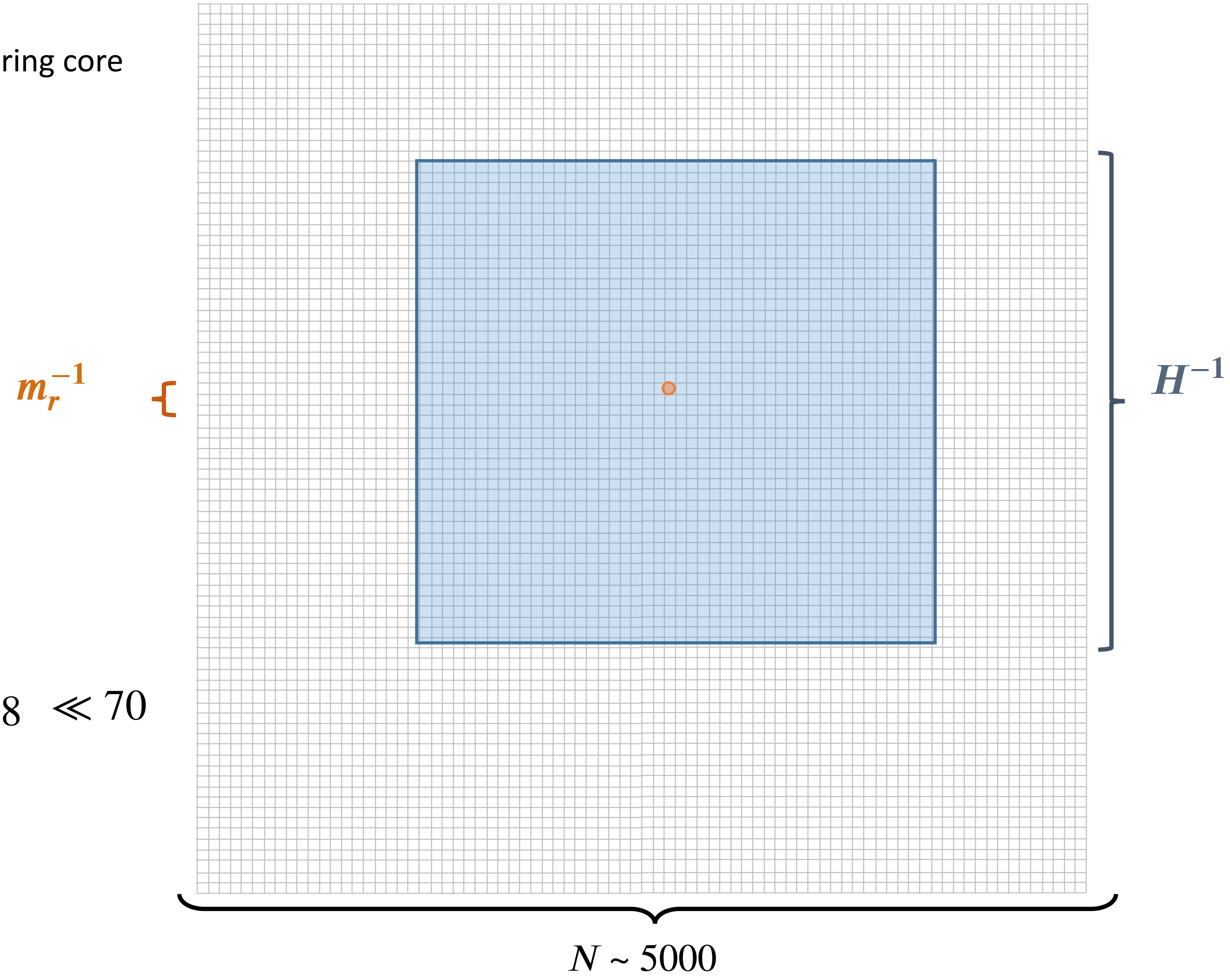
The Bottle Neck

- a few lattice points per string core
- a few Hubble patches



The Bottle Neck

- a few lattice points per string core
- a few Hubble patches



$$\log\frac{m_r}{H} \leq \log\left(\frac{\square}{\bigcirc}\right) \sim 8 \ll 70$$