CP sources in electroweak baryogenesis

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The aim:

Clarify the **importance of modified dispersion relations** for consistent derivations of **CP-violating sources** in **electroweak baryogenesis**

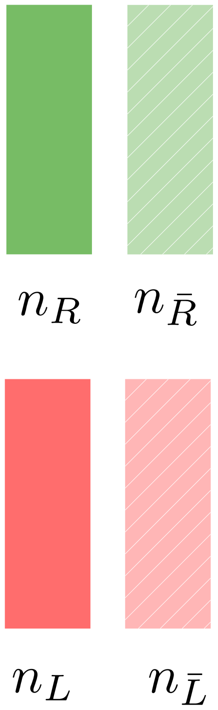
Check if **sources** induced by **mixing fermions** can be computed with **different methods** (**spinor decomposition** vs **vev-insertion approximation (VIA)**)

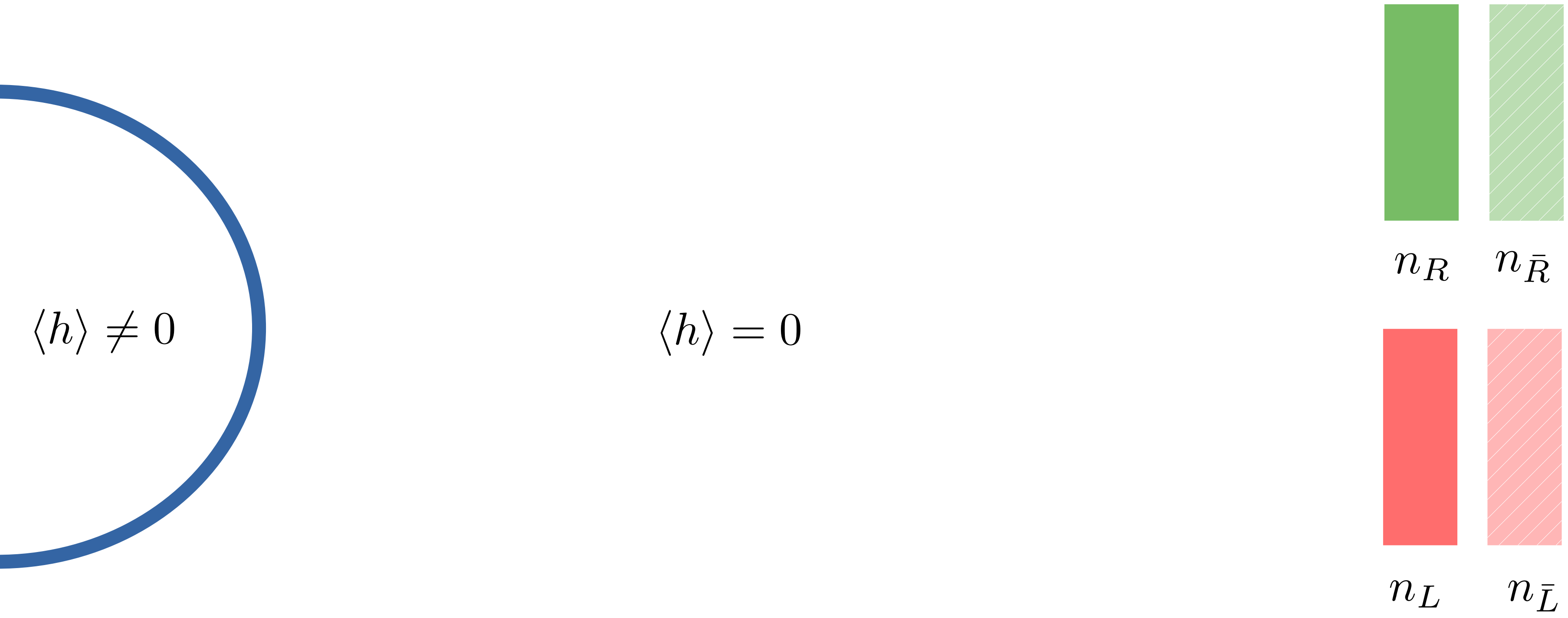
The plan:

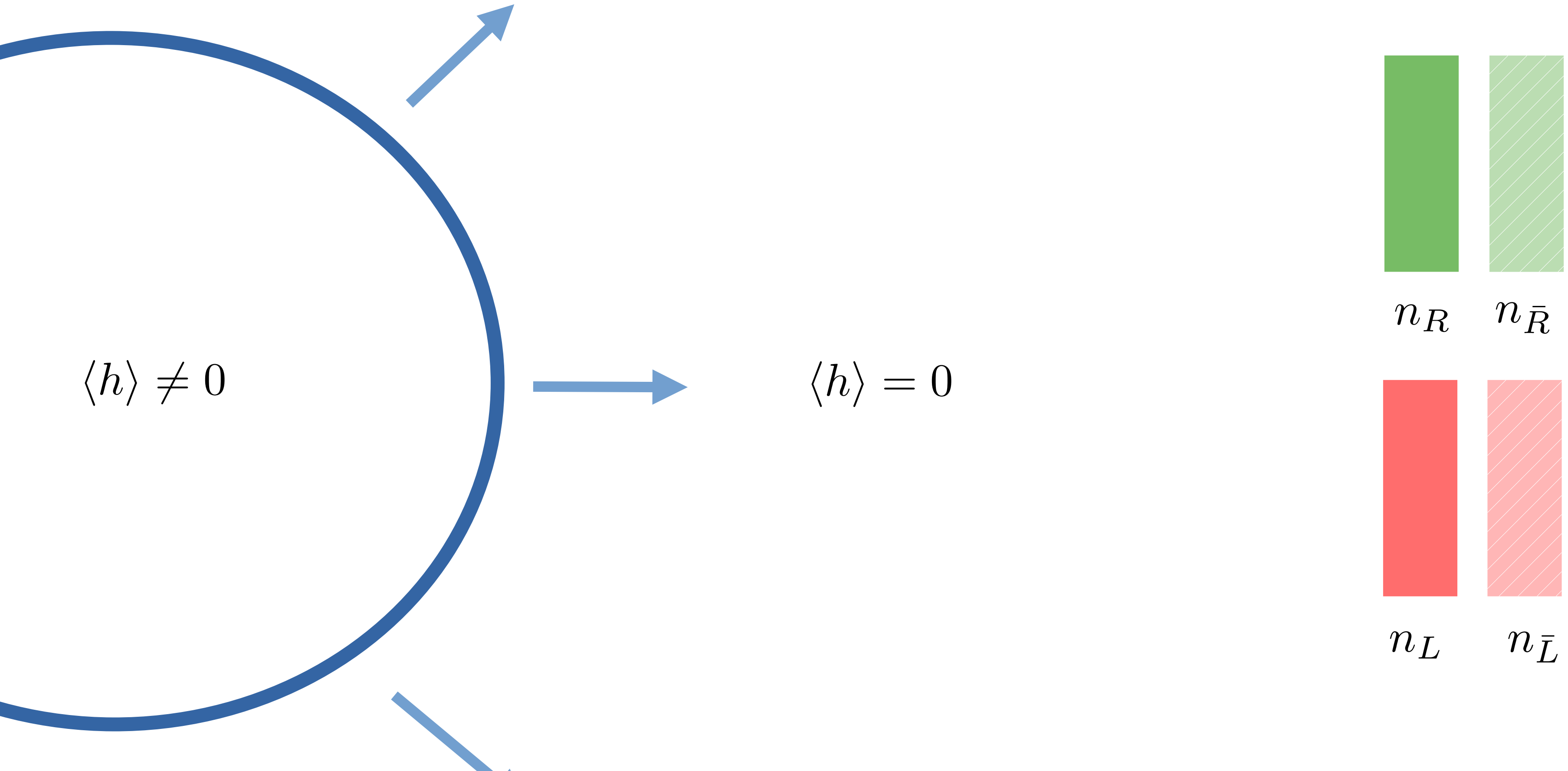
1. Basics of electroweak baryogenesis
2. Methods to compute the CP-violating source
3. Conditions for consistent fluid equations
4. Case study: 2-fermion system

1. Basics of electroweak baryogenesis

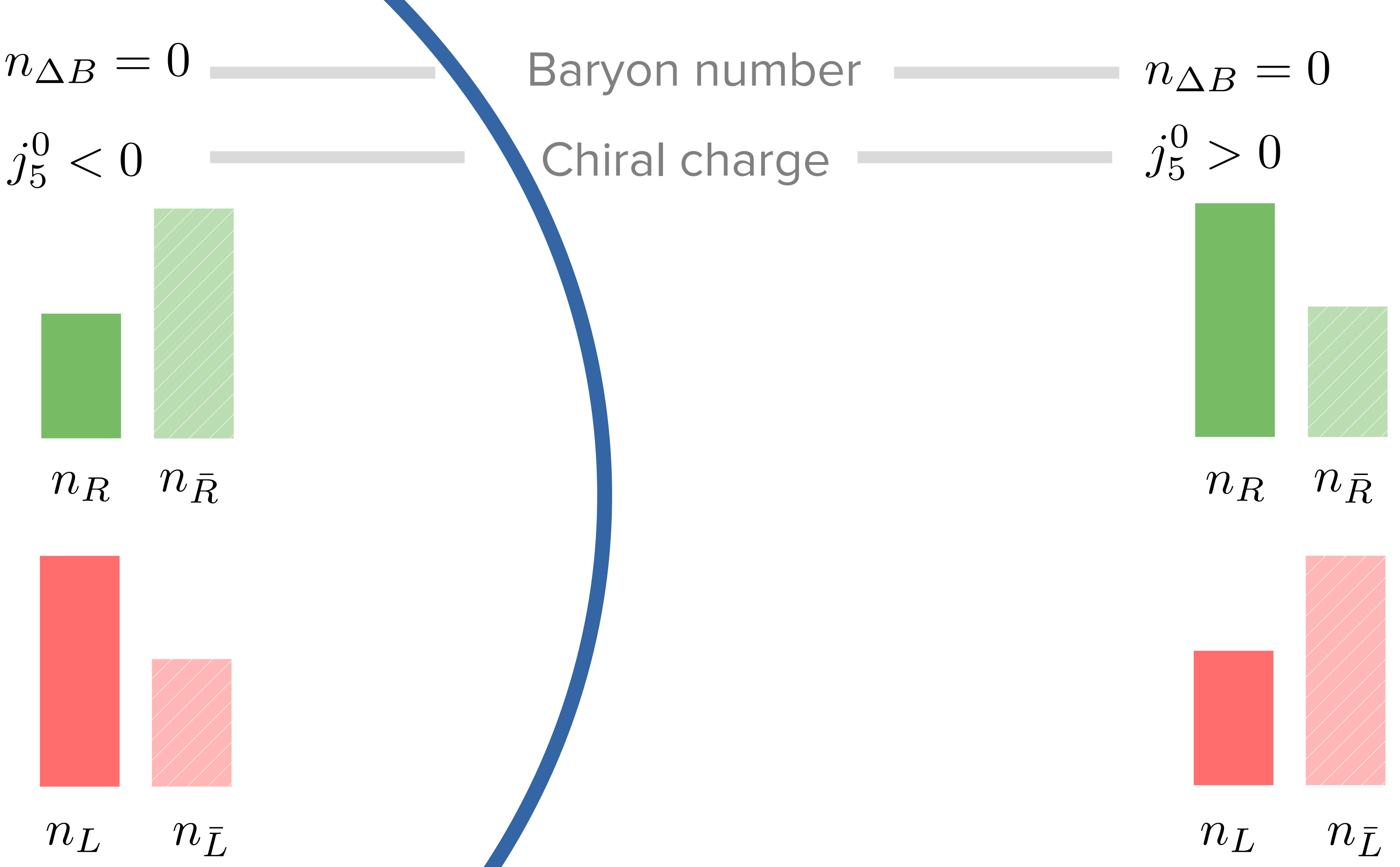
Thermal plasma, symmetric phase $\langle h \rangle = 0$



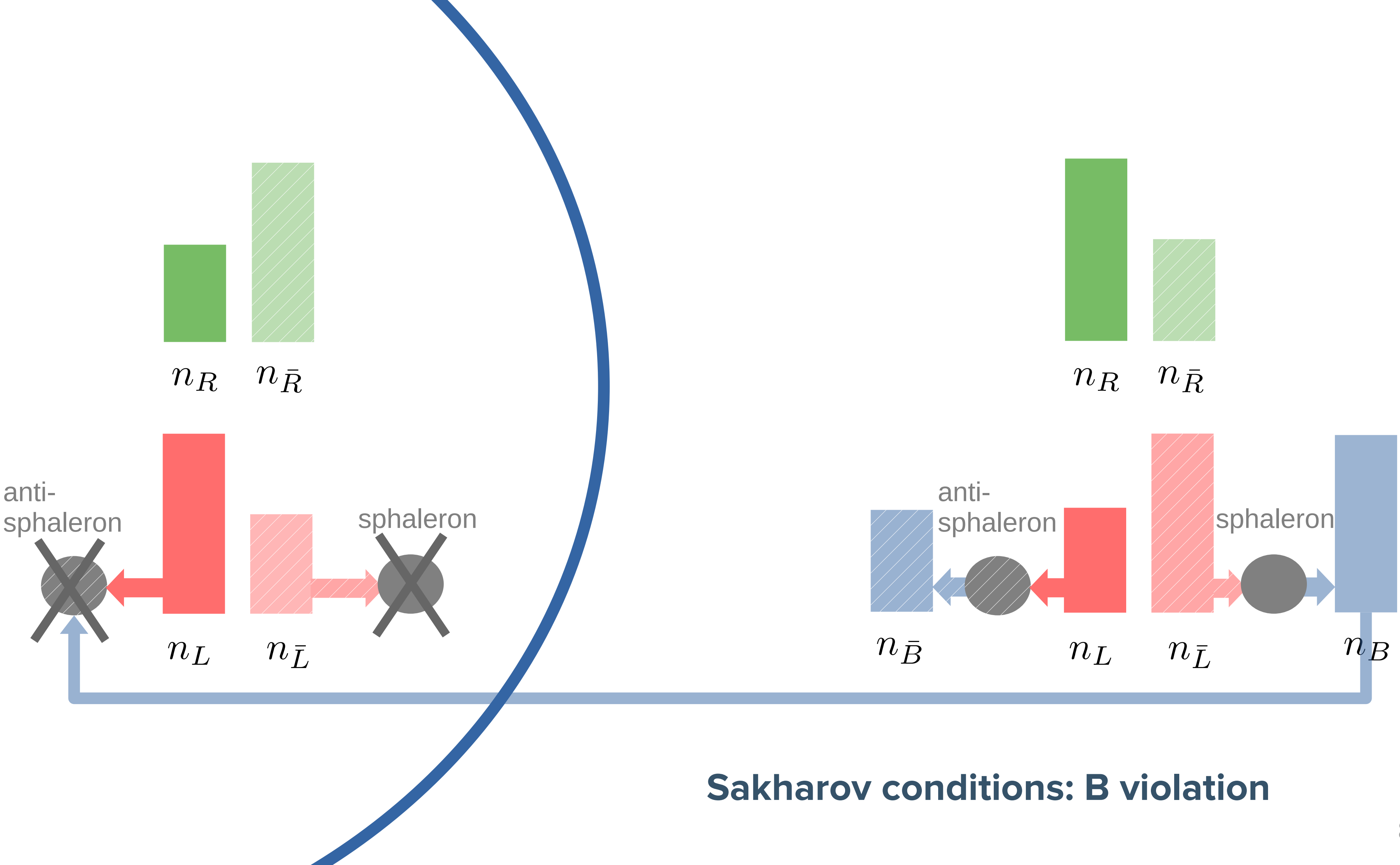

$\langle h \rangle \neq 0$
$\langle h \rangle = 0$
n_R
$n_{\bar{R}}$
n_L
$n_{\bar{L}}$



Sakharov conditions: Departure from equilibrium



Sakharov conditions: C and CP violation



Sakharov conditions: B violation

2. Methods to compute CP-violating sources

WKB method

[Joyce, Prokopec, Turok] [Cline, Joyce, Kainulainen, Prokopec]

- Based on **solving Dirac equation** in the **WKB approximation**

$$\Psi_s = e^{-\mathrm{i}\omega t} \begin{bmatrix} \Psi_{L,s} \\ \Psi_{R,s} \end{bmatrix} \otimes \xi_s, \quad \sigma_3 \xi_s = s\chi_s, \quad s = \pm 1$$

$$\Psi_{L/R,s} = \omega e^{\mathrm{i}\int_0^z \hat{p}_{L/R,s}(z')dz'}$$

- Dirac equation fixes $\hat{p}_{L/R.s}$ in terms of ω → **dispersion relation**

- Particle **momentum** assumed to be related to **group velocity** of spinor:

$$\mathbf{p} = \omega \mathbf{v}_g = \omega \nabla_{\hat{\mathbf{p}}} \omega$$

- Particle **forces** computed by considering constant ω and

$$\mathbf{F}_{\mathbf{L/R,s}} = \frac{d\mathbf{p}_{L/R,s}}{dt}$$

Fluid equations with the WKB method

- **Boltzmann equations** assumed to be of the form

$$(\partial_t + \mathbf{v}_g \cdot \nabla_{\mathbf{x}} + \mathbf{F} \cdot \nabla_{\mathbf{p}})\mathbf{f}(\mathbf{x}, \mathbf{p}) = \mathbf{C}[\mathbf{f}]$$

- **Particles** and **antiparticles** treated separately, resulting in **different forces** that **depend on spin: CP violation**

- Taking moments of Boltzmann equations one derives fluid / diffusion equations for the particle asymmetries of different species

WKB method: key aspects

- **Modified dispersion relations essential** to compute the forces / **CP violating sources**
- **Advantage**. Relative simplicity
- **Disadvantage**. Boltzmann equations not derived from 1^{st} principles

Close timepath (CTP) formalism

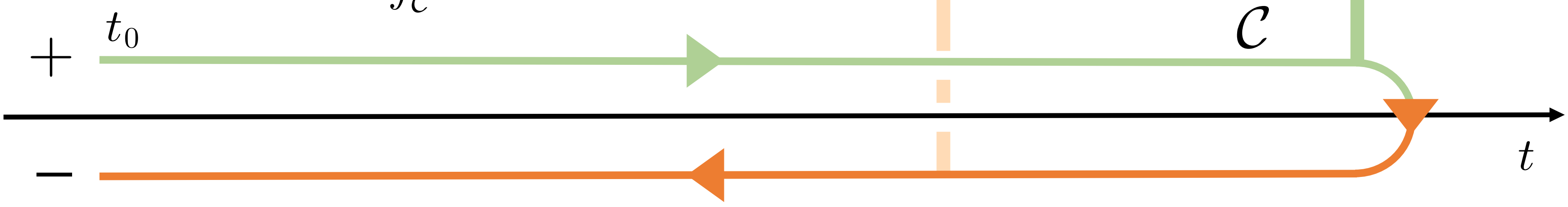
- **Time-dependent observables** in QFT can be related to a **path integration** over a **closed time path**

$$\langle S(t)|\hat{O}(t)|S(t)\rangle = \int \mathcal{D}\phi(t) \langle S(t_0)|e^{-i\hat{H}(t-t_0)}\hat{O}(t)|\phi\rangle_t \, {}_t\langle\phi|e^{i\hat{H}(t-t_0)}|S(t_0)\rangle.$$

Path integral from t to t_0

Path integral from t_0 to t

$$= \int_{\mathcal{C}} \mathcal{D}\phi e^{iS[\phi]}$$



CTP propagators

- **Propagators** carry indices $a,b = \pm$ from the time branches of the field insertions

$$iS_{ab}(x,y) = \langle T_{\mathcal{C}}\psi_a(x)\bar{\psi}_b(y)\rangle \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\, iS_{ab}\left(k, \frac{x+y}{2}\right)$$

Wigner transf.

- Contain **info** about the **shell** and **number densities** of propagating d.o.f.s

$$iS^{+-}_{\rm free}(k) \equiv iS^{<}_{\rm free}(k) = -2\pi\delta(k^2-m^2)(\not{k}+m)\left[\theta(k^0)f(\mathbf{k}) - \theta(-k^0)(1-\bar{f}(-\mathbf{k})\right]$$

- They satisfy quantum equations of motion: **Schwinger-Dyson** eqs. in contour C

$$(i\not{\partial}-m)\, iS^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y) - i\sum_{c=\pm} c\int^4 z\, i\Sigma^{ac}(x,z)\, iS^{cb}(z,y)$$

Self energy (1PI)

This leads to **Boltzmann / fluid equations from first principles!**

CTP: spin decomposition

[Kainulainen, Prokopec, Schmidt, Weinstock]

- For a **planar wall** in x,y directions, S_z **is conserved**
- Can **expand** propagators in **structures that commute with S_z**

$$iS_s^< = -\frac{1}{2}(\mathbf{1} + sS^z)\left[s\gamma^3\gamma^5 g_0^{s<} - s\gamma^3 g_3^{s<} + \mathbf{1} g_1^{s<} - i\gamma^5 g_2^{s,<}\right]$$

- **Schwinger-Dyson equations** solved in terms of functions g_i
- Combining eqs. → **algebraic constraints** that determine **modified mass shells**

$$(k^2 - m_{\text{eff}}^2(x))g_i^<(x,k) = 0 \Rightarrow g_i^<(x,k) \propto \delta(k^2 - m_{\text{eff}}^2(x))$$

CTP: VEV insertion approximation

- Consider **Schwinger-Dyson equation without a 1PI self-energy** and $m = m_0 + \delta m(x)$

$$(i\partial\!\!\!/ - m_0 - \delta m(x))\, iS^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y)$$

- Start with **solutions** iS_0^{ab} to the **homogeneous** case $m = m_0$

$$(i\partial\!\!\!/ - m_0)\, iS_0^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y)$$

- The first equation is solved then by a **geometric series** [Postma]

$$S^{ab}(x,y) = S_0^{ab}(x,y) + \int d^4z\, c\, S_0^{ac}(x,z)\, \delta M(z)\, S_0^{cb}(z,y)$$
$$+ \int d^4z\, d^4w\, cd\; S_0^{ac}(x,z)\, \delta M(z)\, S_0^{cd}(z,w)\, \delta M(w)\, S_0^{db}(w,y) + \cdots$$

CTP: VEV insertion approximation (VIA)

- $\delta m(x)$ **effects** can be **absorbed** into a **"self-energy-like"** contribution in eq. for S^{ab}

$$(i\partial\!\!\!/ - m_0 + \delta m(x))\, iS^{ab}(x,y) = a\delta_{ab} i\delta^{(4)}(x-y)$$

$$\Updownarrow$$

$$(i\partial\!\!\!/ - m)S^{ab}(x,y) = a\delta^{ab}\delta^4(x-y) + \int d^4z\, c\, \delta\Sigma_0^{ac}(x,z) S^{cb}(z,y),$$

$$\delta\Sigma_0^{ab}(x,y) = a\delta^{ab}\,\delta M\,\delta(x-y) + \delta M(x) S_0^{ab}(x,y)\,\delta M(y) + \int d^4z\, d\, \delta M(x) S_0^{ad}(x,z)\,\delta M(z)\, S_0^{db}(z,y)\,\delta M(y) + \ldots$$

- Schwinger Dyson equations can be solved in terms of **powers of $S_0, \delta M$**
- $S_0 \propto \delta(k^2 - m_0^2)$ so the **full mass shell cannot be directly recovered**

CTP: key aspects

Spin decomposition

Separation into spin functions

Can compute full mass shell

VEV-insertions

Expansion in terms of $S_0^{ab}, \delta m$

Does not give full mass shell

In both cases one can compute CPV observables like $\partial_\mu j_5{}^\mu$ without knowing the full mass shell. This seems to be at odds with WKB method

CPV sources in the literature

Source type	Methods	Resonant enhancement?	Gradient order	
Single flavour	WKB	No	2	[Cline, Joyce, Kainulainen, Prokopec]
	spin dec.	No	2	[Kainulainen, Prokopec, Schmidt, Weinstock]
			AGREE!	
Multi-flavour	WKB	No	2	[Cline, Joyce, Kainulainen]
	spin dec.	Diag. sources are resonant but effect compensated by flavour oscillations	1+2	[Konstandin, Prokopec, Schmidt, Seco]
	VIA	Yes	1	[Riotto][Carena, Moreno, Quiros, Seco, Wagner] [Lee,Cirigliano, Ramsey-Musolf]

Open questions

- Do the **different multiflavour sources** correspond to **independent** physical **effects**, or different **approximations to the same**?

  Is the **resonantly enhanced VIA source** correct?
  [Kainulainen][Postma, van de Vis, White…]
  [see Postma's talk in this workshop]

  Can one use **different methods** to get **equivalent answers** for the same multiflavour source?

- In the **CTP** methods, **does one need to know the modified shell** in order to extract CP-violating effects, as is the case in the WKB approximation?

3. Conditions for consistent Boltzmann equations

Kinetic description

- Assume a **classical/quantum kinetic description** of system with particles of mass $\bar{m}(x)$
- The **phase-space/spectral density** will be of the form

$$g(x,k) \sim 2\pi\, \delta(k^2 - \bar{m}^2(x))\, f(\mathbf{k}, x).$$

- A **dynamical/kinetic equation** in the static wall frame will read

$$\partial_z g(x,k) = c(x,k)\, \partial_{k_z} g(x,k) + \ldots\ldots$$

- To get a **Boltzmann eqs.**, must take moments and integrate over k^0 to eliminate δ

Boltzmann equations

$$\int \frac{dk^0}{(2\pi)} \sqrt{\mathbf{k}^2 + \bar{m}^2} \left(\frac{k^0}{\sqrt{\mathbf{k}^2 + \bar{m}^2}} \right)^{2l} \times \left(\partial_z g(x,k) = c(x,k)\, \partial_{k_z} g(x,k) + \dots \right)$$

contains terms with $\delta(k^2 - \bar{m}^2(x)), \delta'(k^2 - \bar{m}^2(x))$

$$\int \frac{dk^0}{(2\pi)} \sqrt{\mathbf{k}^2 + \bar{m}^2} \left(\frac{k^0}{\sqrt{\mathbf{k}^2 + \bar{m}^2}} \right)^{2l} \delta(k^2 - \bar{m}^2) = \frac{1}{2\pi} \text{ is } l\text{-independent,}$$

$$\int \frac{dk^0}{(2\pi)} \sqrt{\mathbf{k}^2 + \bar{m}^2} \left(\frac{k^0}{\sqrt{\mathbf{k}^2 + \bar{m}^2}} \right)^{2l} \delta'(k^2 - \bar{m}^2) = -\frac{2l-1}{4\pi(\mathbf{k}^2 + \bar{m}^2)} \text{ is } l\text{-dependent.}$$

Unambiguous Boltzmann equations require **cancellation of δ' terms**

This might only be possible if $\bar{m}(x)$ is correlated with $c(x,k)$

A new consistency check

- In the quantum case, the **mass** of **on-shell excitations** can be **computed** with **constrain eqs.**

- **Cancellation of** δ' in the **kinetic equations** is then a **nontrivial consistency check**

▶ **Unambiguous Boltzmann equations** including CPV sources **in the CTP formalism** **require** to compute the **modified dispersion relation**, similar to the **WKB** case

Trivial shell: classical particle

- Consider a **classical point particle** with **space-time dependent mass**. E.o.m.s are

$$k^\mu = m(x)\frac{dx^\nu}{d\tau} \Rightarrow \frac{dk^\nu}{d\tau} = \frac{\partial m(x)}{\partial x_\nu}$$

- With $g(x,k) = \delta(k^2 - m^2(x)) f(x, \mathbf{k})$ the classical **phase-space density**, **Liouville's theorem** gives

$$\frac{d}{d\tau} g(x,k) = \delta(k^2 - m^2(x)) \left(\frac{k^\mu}{m} \frac{\partial f(\mathbf{k},x)}{\partial x^\mu} + \frac{dk^\mu}{d\tau} \frac{\partial f(\mathbf{k},x)}{\partial k^\mu} \right)$$

$$+ 2k_\mu f(\mathbf{k},x)\delta'(k^2 - m^2(x)) \left(\frac{dk^\mu}{d\tau} - \frac{\partial m}{\partial x_\mu} \right)$$

Cancels! Consistent Boltzmann eq.

Nontrivial shell: quantum fermion

- Consider a **quantum fermion** with

$$\mathcal{L} \supset -\bar{\psi}(M^{\mathrm{H}}(x) + \mathrm{i}\gamma^5 M^{\mathrm{A}}(x))\psi$$

- The **spin decomposition method** gives a **CPV source** in static bubble frame:

$$\partial_z j_5^z(k,z) = \left(M^H M^{H\prime} + M^A M^{A\prime}\right)\frac{1}{k^z}\partial_{k^z} j_5^z(k,z) + \left(M^H(\partial_z^2 M^A) - M^A(\partial_z^2 M_H)\right)\frac{1}{2k^z}\partial_{k^z}\left(\frac{j_N^3(k,z)}{k^z}\right)$$

as well as a **modified shell**

$$m_s^2(z) = m^2(z) - \frac{s}{\tilde{k}^0}\left(M^H(z)\partial_z M^A(z) - M^A(z)\partial_z M^H(z)\right), \quad \tilde{k}^0 \equiv \mathrm{sign}(k^0)\sqrt{(k^0)^2 - (k^1)^2 - (k^2)^2}$$

[Kainulainen, Prokopec, Schmidt, Weinstock]

- **Our input**: **Modified shell** precisely leads to **cancellation of δ' terms** !

4. Case study: 2-fermion mixing

The model

- We consider a **2 fermion system** with **CP-odd phases** present in **mixing terms**

$$M = \begin{bmatrix} m_1 & e^{i\varphi} v_b(z) \\ v_a(z) e^{i\gamma} & m_2 \end{bmatrix}$$

$$\mathcal{L} \supset -\bar{\psi}(M^{\mathrm{H}}(x) + \mathrm{i}\gamma^5 M^{\mathrm{A}}(x))\psi \qquad \begin{cases} M^H = \frac{1}{2}(M + M^\dagger) \\ M^A = \frac{1}{2}(M - M^\dagger) \end{cases}$$

- Have computed **CPV source** $\partial_\mu j_5^\mu$ with **two different methods**:
  - **spin decomposition**
  - **VIA expansion**

Agreement up to $O(v^3\ ,\ \partial_z^3)$

2 fermion mixing: CPV source

Mixing effect. Add up to zero

Involve CP odd phases. Contribute to total source.
Resonance compensated in sum by
$j^z_{1,1} - j^z_{2,2} = \mathcal{O}(m_1 - m_2)$

$$(\partial_z j^z_5)_{1,1} = -\frac{(v_a v'_a - v_b v'_b)}{m_1^2 - m_2^2}\left(j^z_{1,1} - j^z_{2,2}\right) + \frac{\sin(\varphi+\gamma)\, m_1 m_2}{k_z(m_1^2 - m_2^2)}(2v'_a v'_b + v_b v''_a + v_a v''_b)\left[\frac{1}{2k_z^2}\left(j^z_{1,1} - k_z \partial_{k_z} j^z_{1,1}\right) - \frac{1}{m_1^2 - m_2^2}\left(j^z_{2,2} - j^z_{1,1}\right)\right] + \mathcal{O}(v^3, vv''', v'v''),$$

$$(\partial_z j^z_5)_{2,2} = \frac{(v_a v'_a - v_b v'_b)}{m_1^2 - m_2^2}\left(j^z_{1,1} - j^z_{2,2}\right) - \frac{\sin(\varphi+\gamma)\, m_1 m_2}{k_z(m_1^2 - m_2^2)}(2v'_a v'_b + v_b v''_a + v_a v''_b)\left[\frac{1}{2k_z^2}\left(j^z_{2,2} - k_z \partial_{k_z} j^z_{2,2}\right) + \frac{1}{m_1^2 - m_2^2}\left(j^z_{1,1} - j^z_{2,2}\right)\right] + \mathcal{O}(v^3, vv''', v'v'')$$

Mixing effect. Add up to zero

Include δ' contributions

2 fermion mixing: Consistency check

The **consistency check** requires to compute the **modified dispersion relation**

$$k^2 = (m_i + \delta m_i)^2$$

$$\delta m_1^s = -\frac{s m_2 \sin(\gamma+\phi)\,(v_b v_a' + v_a v_b')}{2\,(m_1^2 - m_2^2)\sqrt{{k_z}^2 + m_1^2}} + (s\text{-independent}) + \mathcal{O}(v^3, vv'', v'v'),$$

$$\delta m_2^s = -\frac{s m_1 \sin(\gamma+\phi)\,(v_b v_a' + v_a v_b')}{2\,(m_2^2 - m_1^2)\sqrt{{k_z}^2 + m_2^2}} + (s\text{-independent}) + \mathcal{O}(v^3, vv'', v'v'),$$

This results lead to the cancellation of δ' terms!! ▶ Consistent Boltzmann eq.

▶ Terms that give nonzero total source lead to force terms in Boltzmann eqs. $\propto m_i \partial_z(\delta m_i)$ exactly as in the WKB formalism

Conclusions

- We have obtained a **new consistency condition** for the derivation of **Boltzmann equations**: all **terms proportional to δ' must cancel**
  - For quantum fermions, this requires **modified dispersion relations**
  - **Mass shell** can be inferred from **constrain equations** → **nontrivial check**
  - Computing the modified **dispersion relation essential** in CTP as in WKB, and **cannot be done with the VIA approach**
- We have derived **CPV sources** in a system of 2 **mixing fermions** in the CTP formalism
  - **VIA** and **spin decomposition** calculations agree, consistency condition satisfied
  - **Resonance not present** when **summing** over **flavours**
  - Sources contributing to **flavour sum** have the structure of **WKB semiclassical force**

Thank you!