



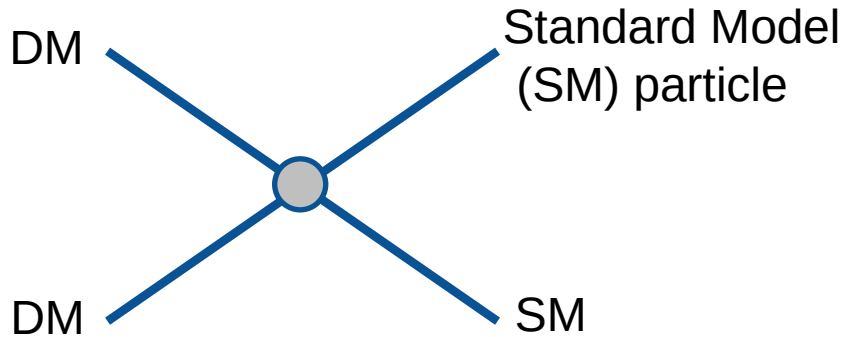
Precise Estimation of Charged Higgsino/Wino Decay Rates

Satoshi Shirai (Kavli IPMU)

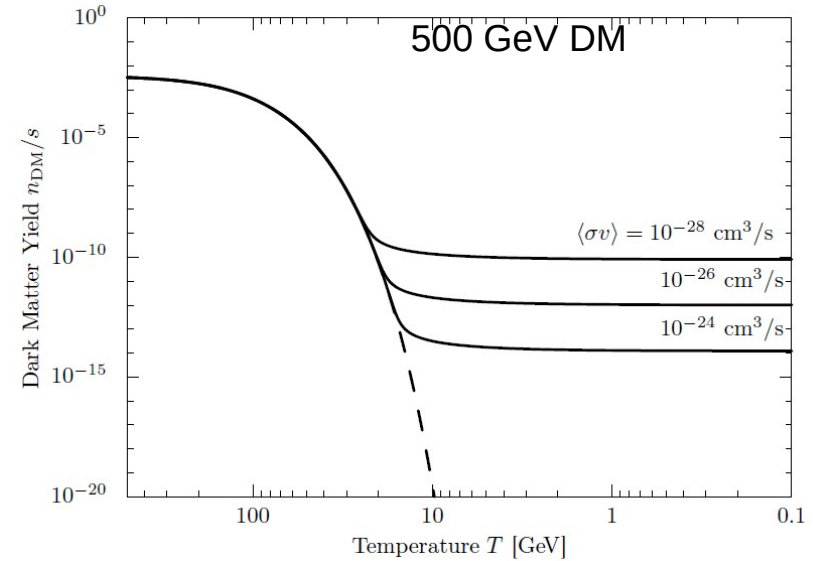
Based on [arXiv:2210.16035](#) and [2312.08087](#) with
Masahiro Ibe, Masataka Mishima, and Yuhei Nakayama

WIMP Dark Matter

Weakly Interacting Massive Particle



DM abundance



Time

Minimal **WIMP** Model

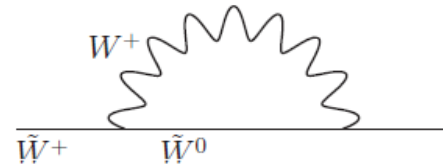
Gauge Portal dark matter

- DM charged weak interaction.
- Annihilation via electroweak interaction.
- **Wino** or **Higgsino** in SUSY model with R-parity.
- 5plet fermion.

Example: Wino

- Majorana fermion \widetilde{W}
- Hypercharge $Y=0$
- $SU(2)_L$ triplet $\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{W}^0 \\ \widetilde{W}^- \end{pmatrix}$

Wino Spectrum



\tilde{W}^\pm _____

\tilde{W}^0 _____

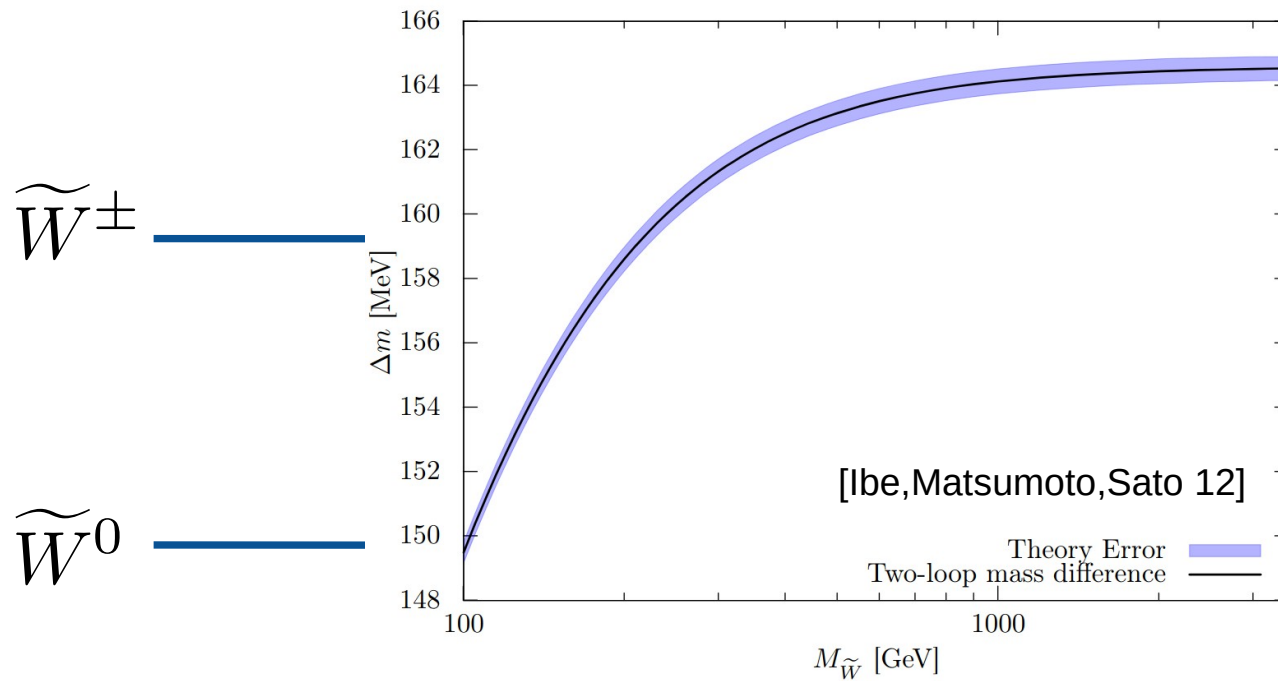
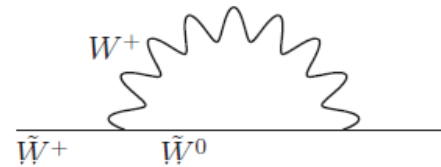


Radiative correction

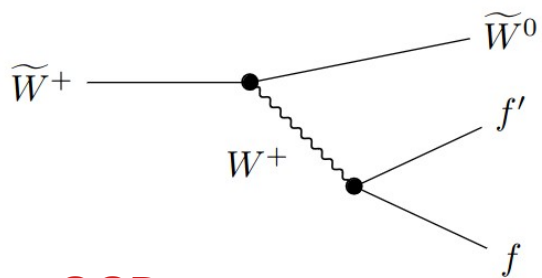
$$\Delta m \simeq 160 \text{ MeV}$$

$$\mathcal{O}\left(\frac{\alpha}{4\pi} m_Z\right)$$

Wino Spectrum



Charged Wino Decay

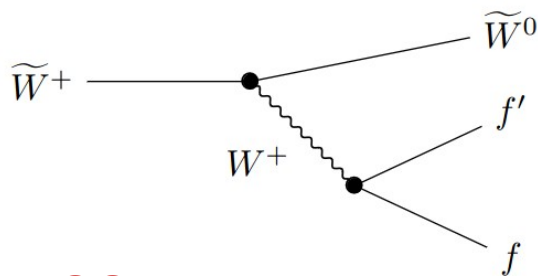


Without QCD

$$\Gamma = \frac{2(G_F)^2 \Delta m^5}{15\pi^3}$$

$$\longrightarrow c\tau \simeq 1 \text{ m} \left(\frac{\Delta m}{160 \text{ MeV}} \right)^{-5}$$

Charged Wino Decay

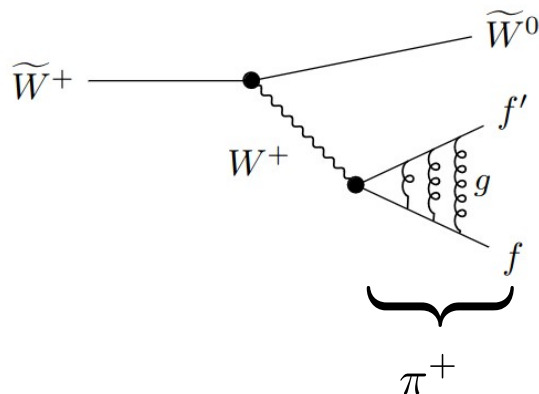


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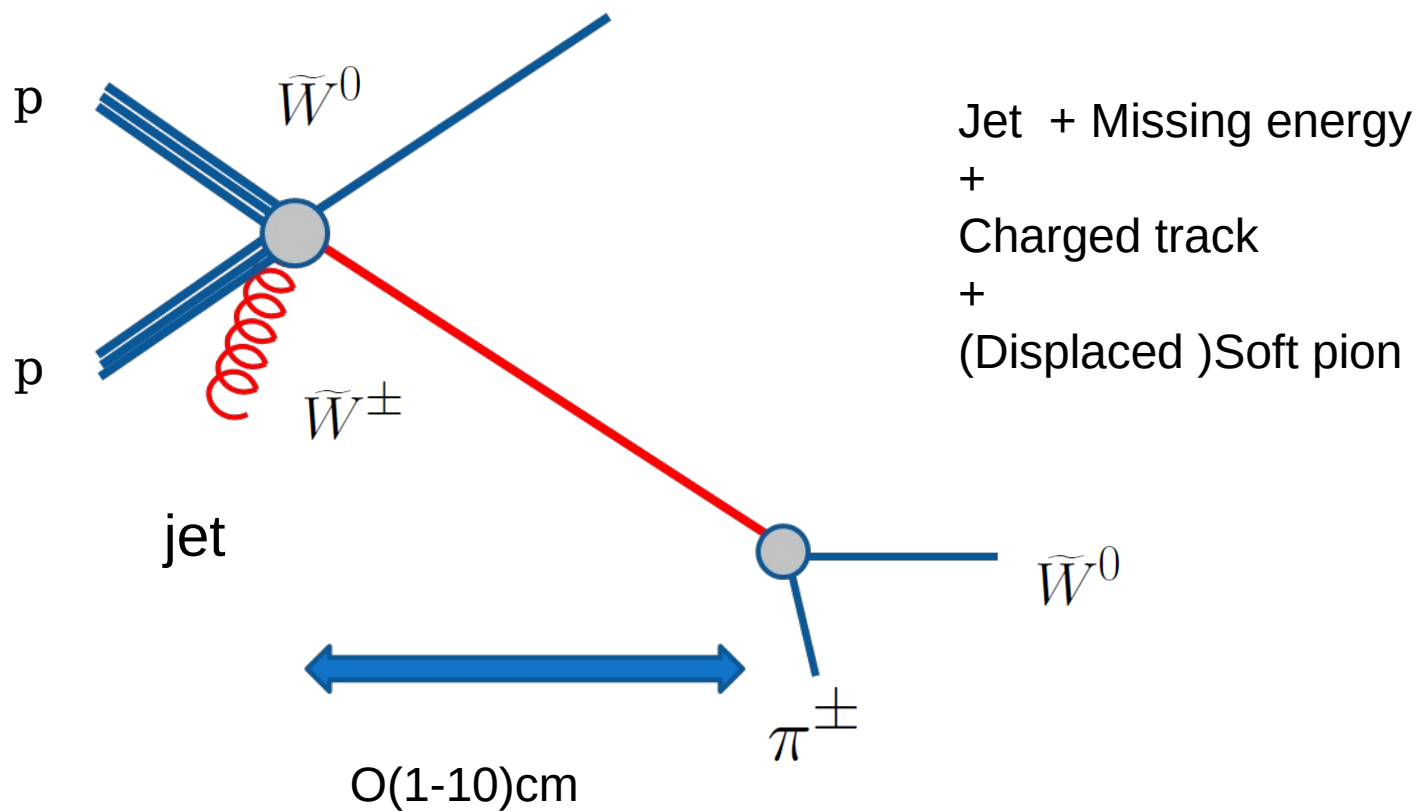
With QCD



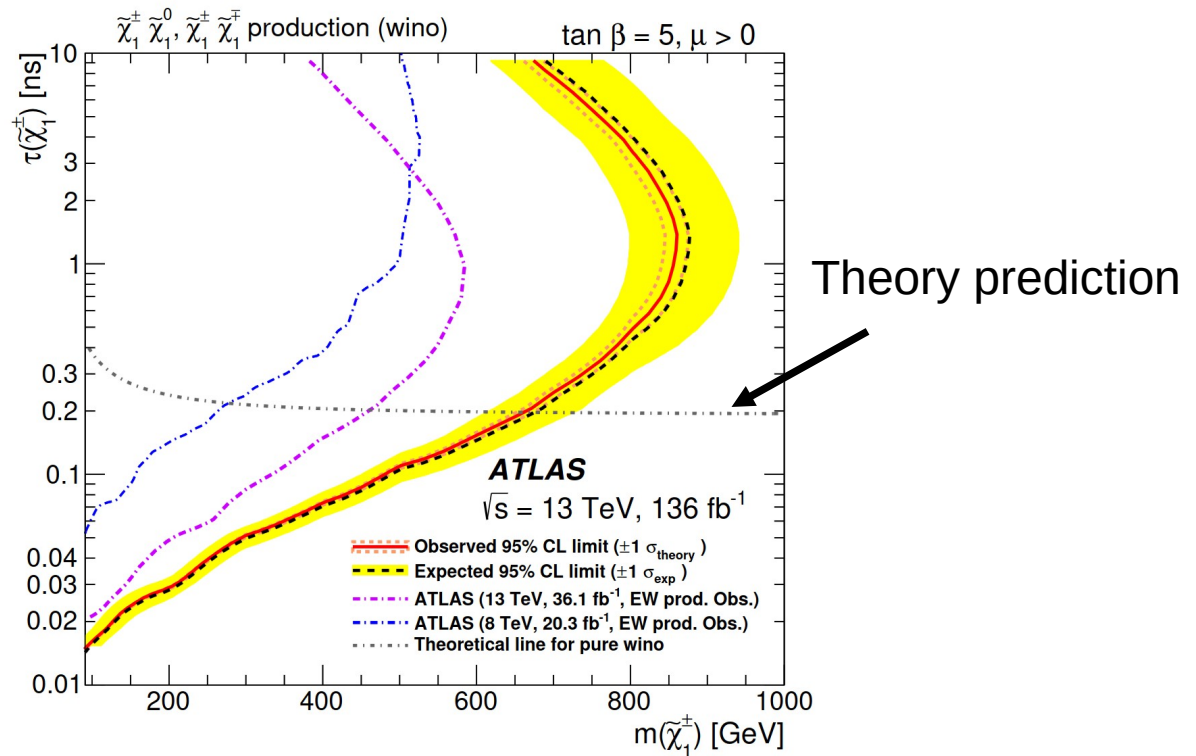
$$\Gamma = \frac{4}{\pi} F_\pi^2 (G_F)^2 \Delta m^3 \left(1 - \frac{m_\pi^2}{\Delta m^2} \right)^{1/2}$$

$$\longrightarrow c\tau \simeq 5 \text{ cm} \left(\frac{\Delta m}{160 \text{ MeV}} \right)^{-3}$$

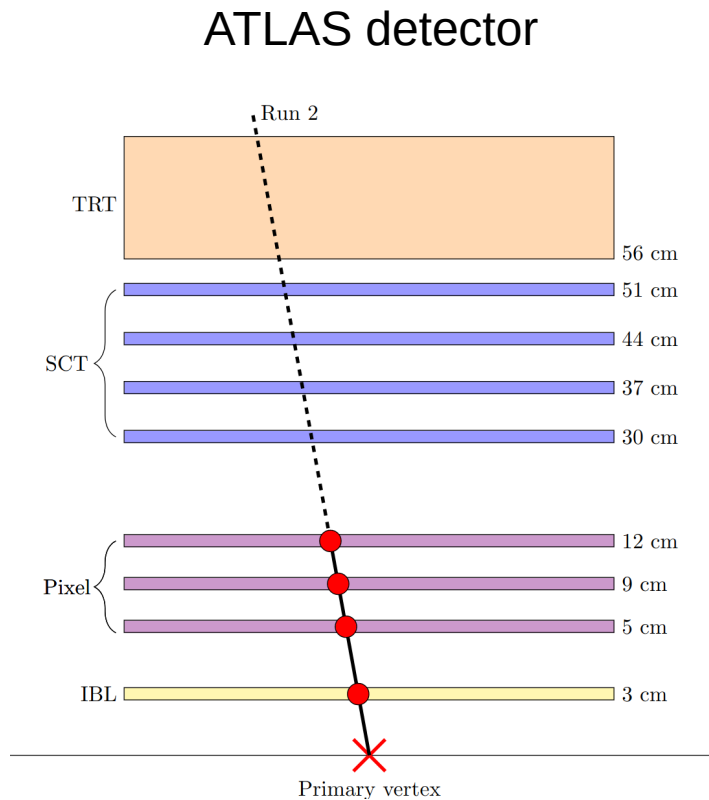
Direct LHC Signals



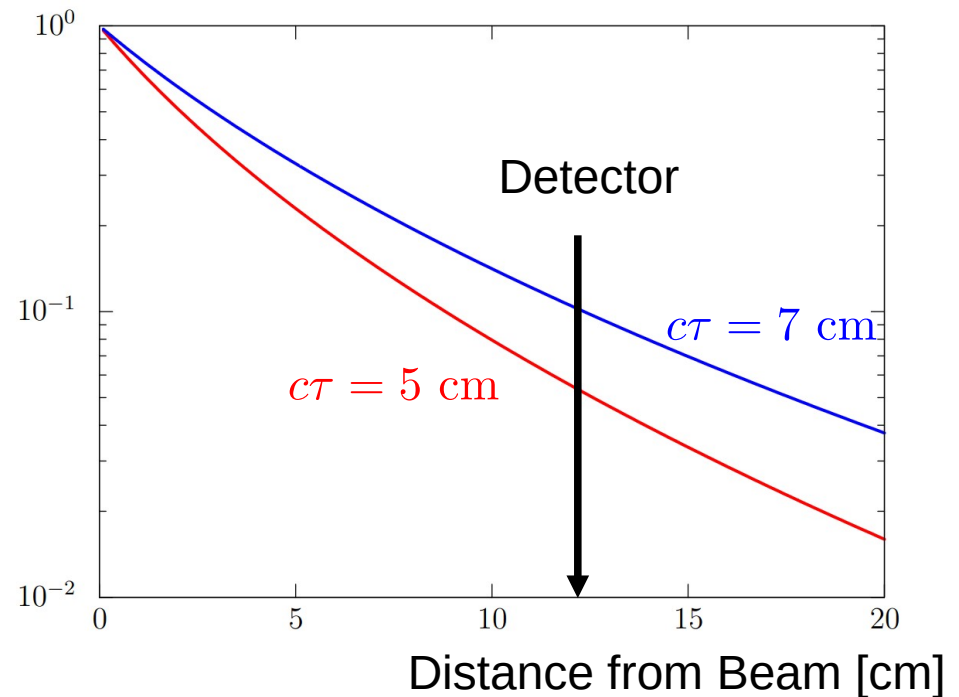
LHC Search



Decay Rate and Signals

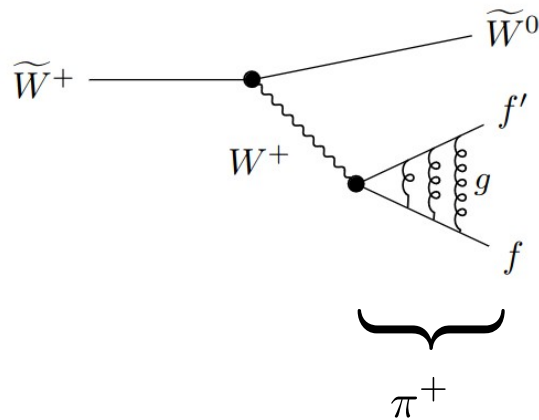


Survival Probability



10% error of lifetime \rightarrow 50% error of signal.

Tree-level Wino Decay

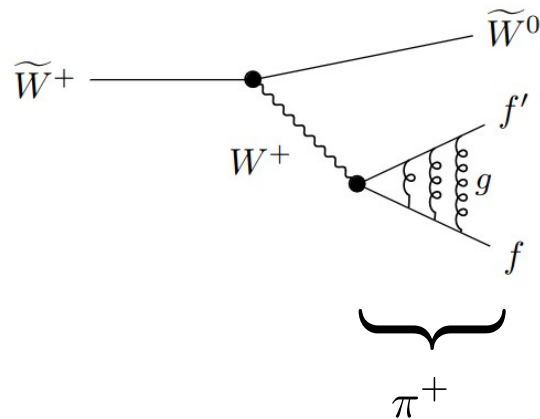


$$\Gamma = \frac{4}{\pi} F_{\pi}^2 (G_F)^2 \Delta m^3 \left(1 - \frac{m_{\pi}^2}{\Delta m^2} \right)^{1/2}$$

$$G_F = \frac{\sqrt{2}}{8} \frac{g^2}{m_W^2}$$

Multi scale problem: $m_{\widetilde{W}} \gg m_W \gg \Lambda_{\text{QCD}} \sim \Delta m$

Tree-level Wino Decay



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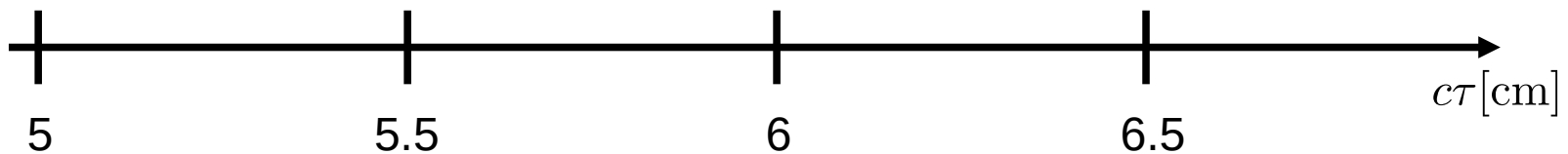
Which gauge coupling should be used?

$$g(\mu = m_{\widetilde{W}}), \quad g(\mu = m_W), \quad g(\mu = \Delta m) \quad ?$$

Tree-level Wino Decay

1 TeV Wino, $\Delta m = 164.11$ MeV

$$G_F \simeq \frac{\sqrt{2}}{8} \frac{g^2}{m_W^2} \simeq \frac{e^2}{4\sqrt{2}s_W^2 m_W^2} \simeq \frac{e^2}{4\sqrt{2}m_W^2} \left(1 - \frac{m_W^2}{m_Z^2}\right)^{-1}$$



Tree-level Wino Decay

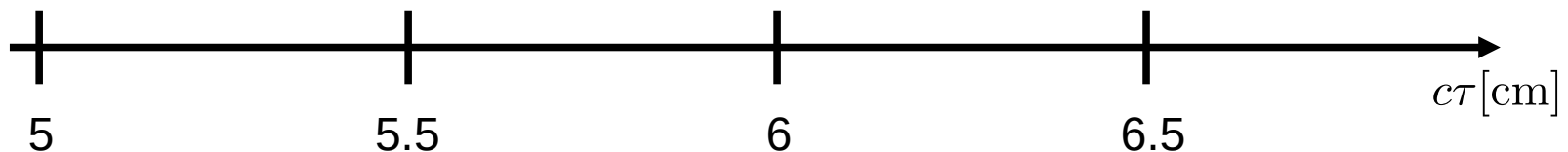
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$1.166 \times 10^{-5} \text{ GeV}^{-2}$ (PDG)

$1.211 \times 10^{-5} \text{ GeV}^{-2}$ $\alpha(m_Z)=1/128$

$1.126 \times 10^{-5} \text{ GeV}^{-2}$ $\alpha=1/137$



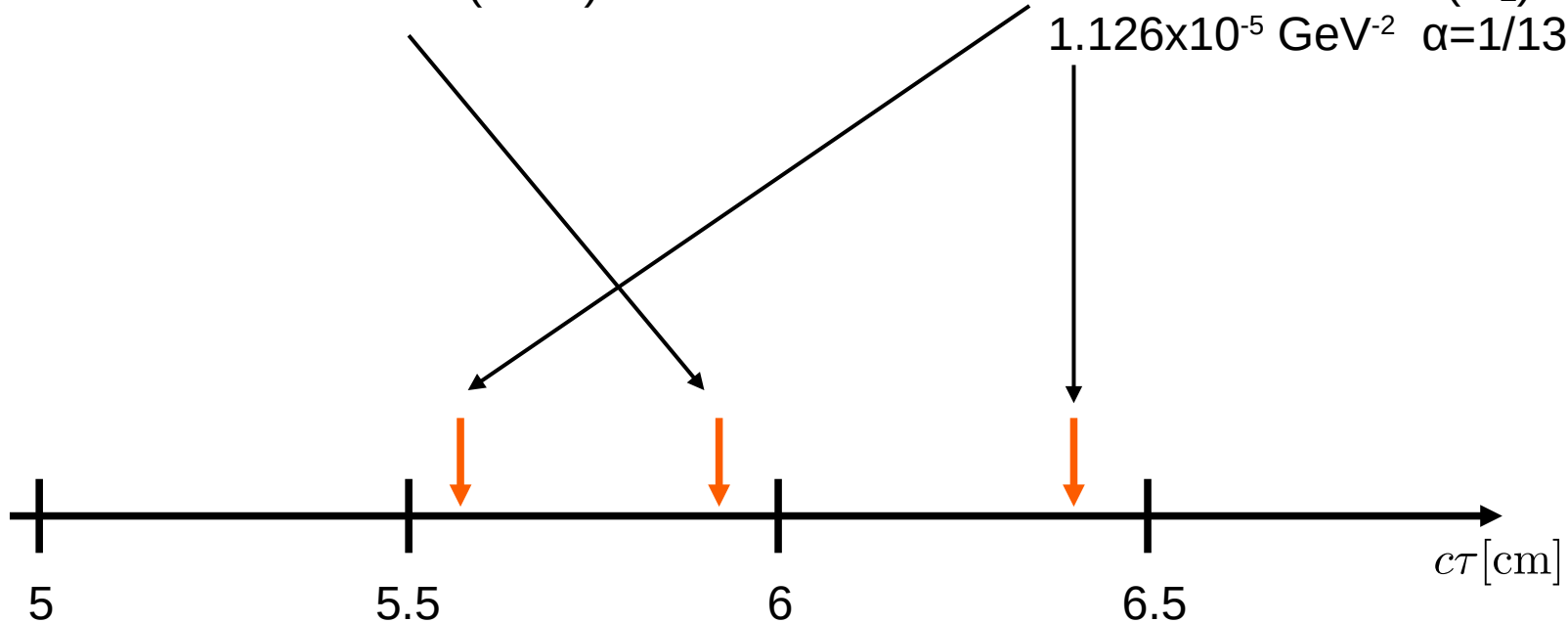
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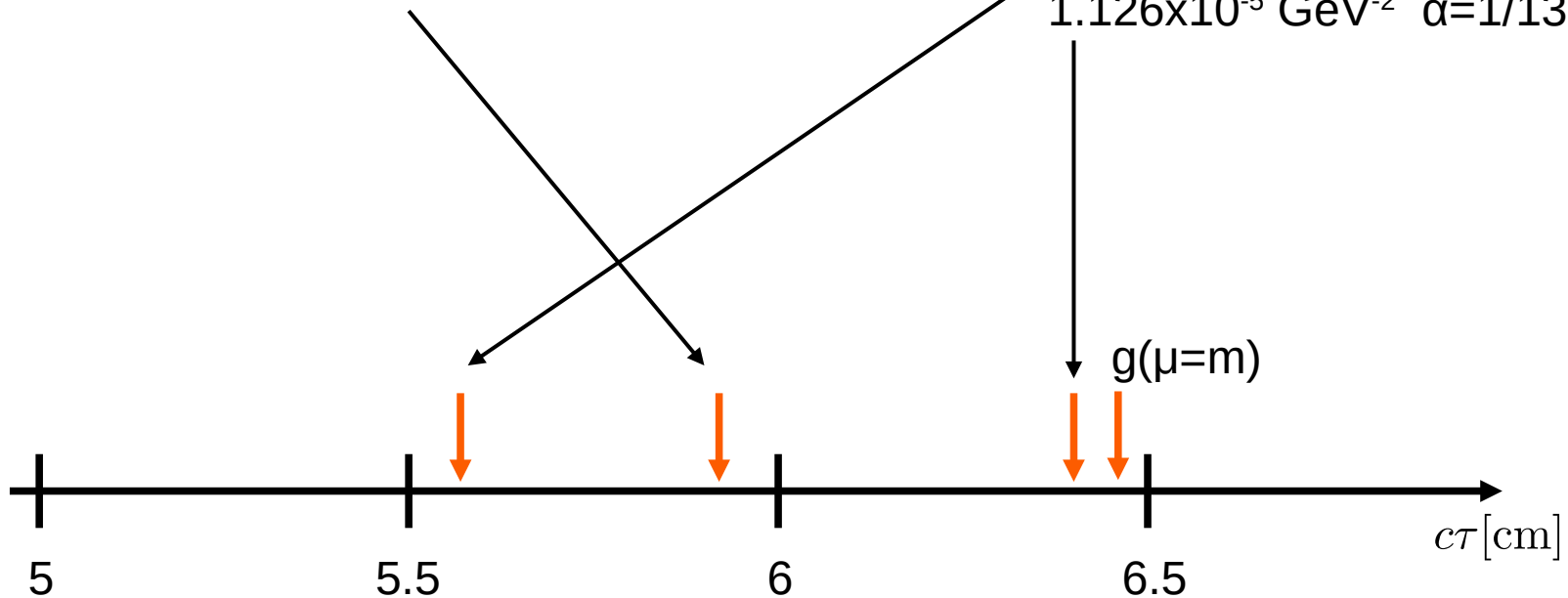
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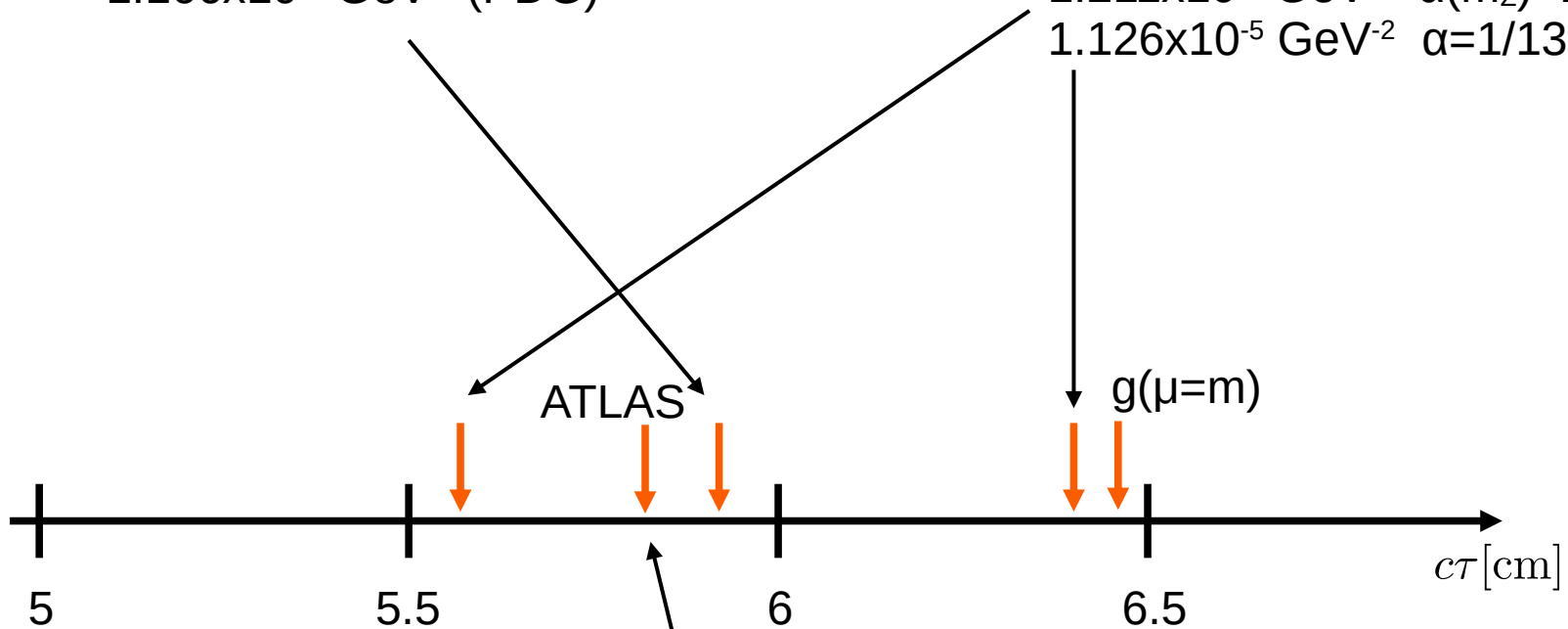
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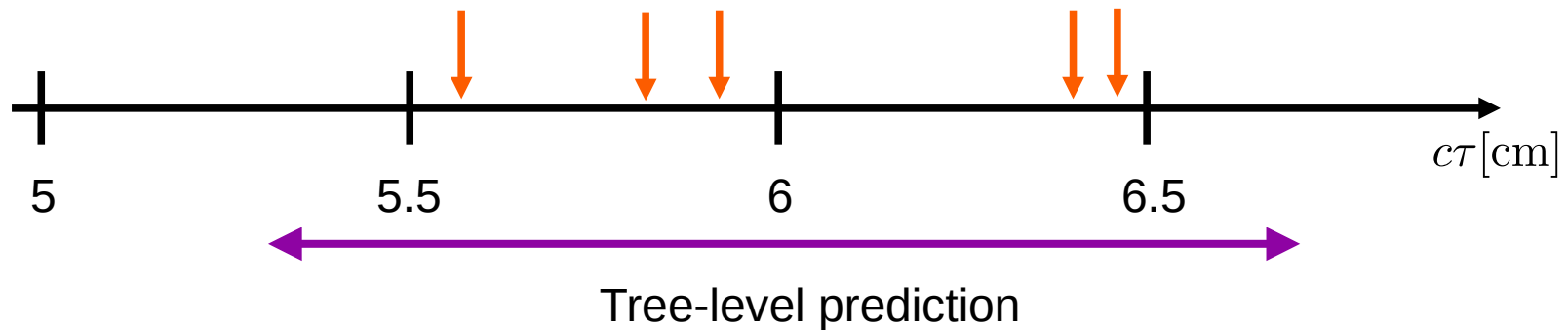
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 $1.126 \times 10^{-5} \text{ GeV}^{-2}$ $\alpha = 1/137$



$$\Gamma \simeq \Gamma(\pi^- \rightarrow \mu^- + \bar{\nu}_\mu) \times \frac{4\Delta m^3}{m_\pi m_\mu^2} \left(1 - \frac{m_\pi^2}{\Delta m^2}\right)^{1/2} \left(1 - \frac{m_\mu^2}{m_{\pi^\pm}^2}\right)^{-2}$$

Tree-level Wino Decay



All the values are correct as far as we consider tree level.

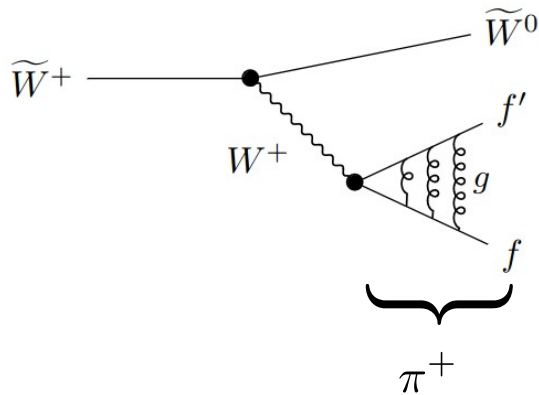


Need of next-to-leading order calculation!



Charged Wino Decay at NLO

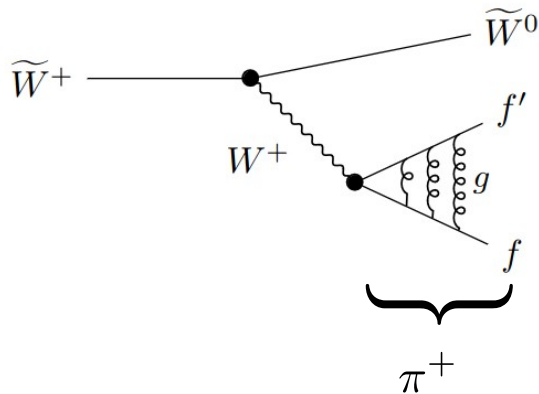
Charged Wino Decay



$$m_{\tilde{W}} \gg m_W \gg \Lambda_{\text{QCD}} \sim \Delta m$$

EW correction includes multi-scale physics.

Charged Wino Decay

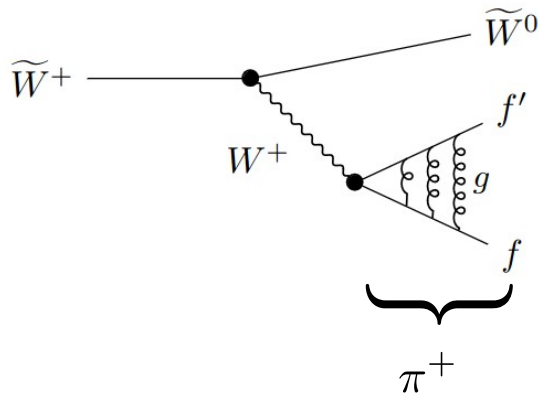


$$m_{\tilde{W}} \gg m_W \gg \Lambda_{\text{QCD}} \sim \Delta m$$

EW correction includes multi-scale physics.

Large logarithm? e.g., $\frac{\alpha}{4\pi} \log \left(\frac{m_{\tilde{W}}}{\Lambda_{\text{QCD}}} \right)$?

Charged Wino Decay



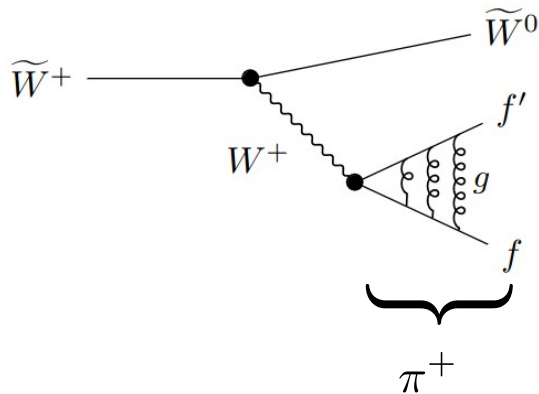
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EW correction includes multi-scale physics.

Large logarithm? e.g., $\frac{\alpha}{4\pi} \log \left(\frac{m_{\tilde{W}}}{\Lambda_{\text{QCD}}} \right)$?

Which energy scale parameter? $\alpha(m_{\tilde{W}})$? $\alpha(m_W)$? $\alpha(\Lambda_{\text{QCD}})$?

Charged Wino Decay



$$m_{\widetilde{W}} \gg m_W \gg \Lambda_{\text{QCD}} \sim \Delta m$$

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EW next-to-leading order calculation.

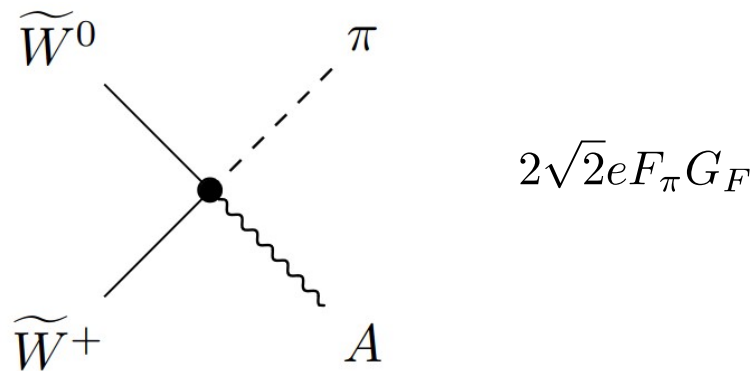
Inclusion of QED in Pion

Coupling Wino and pion

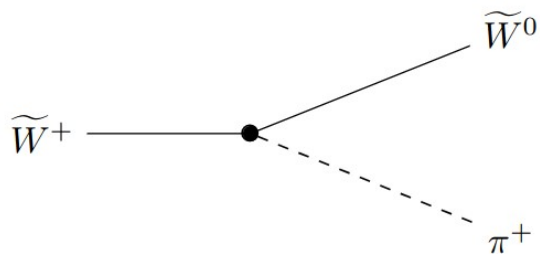
covariant derivative

$$2\sqrt{2}F_\pi G_F(\partial_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0) \quad \longrightarrow \quad 2\sqrt{2}F_\pi G_F(D_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0)$$

$$D_\mu = \partial_\mu + ieA_\mu$$



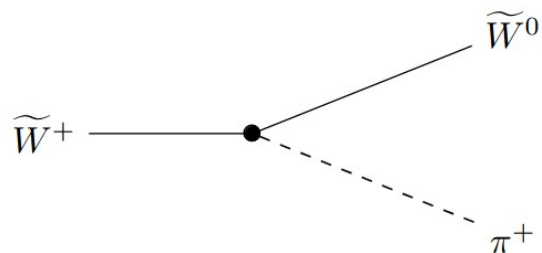
Photon Loop Example



$$\mathcal{M}_{\text{tree}} \simeq F_\pi G_F m_{\widetilde{W}} \Delta m$$

$$\text{c.f., } \Delta m \sim \frac{\alpha}{4\pi} m_Z$$

Photon Loop Example



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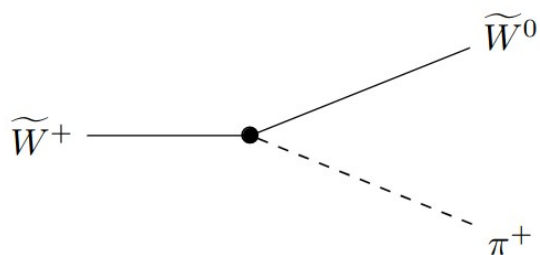
c.f., $\Delta m \sim \frac{\alpha}{4\pi} m_Z$

$$2\sqrt{2}F_\pi G_F (\partial_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0)$$



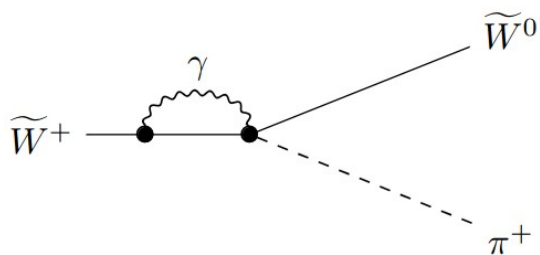
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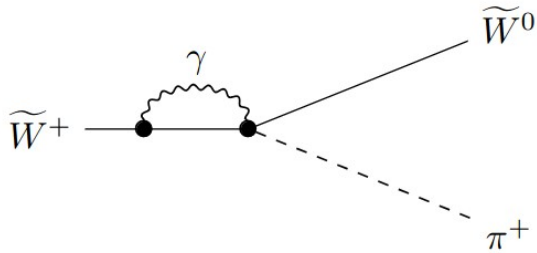
c.f., $\Delta m \sim \frac{\alpha}{4\pi} m_Z$



$$\mathcal{M}_{\text{loop}} \simeq F_\pi G_F m_{\widetilde{W}}^2 \times \frac{\alpha}{4\pi} \left(\frac{1}{\bar{\epsilon}} - 2 \log \frac{m_{\widetilde{W}}}{\mu} + \frac{4}{3} \right)$$

Loop is much larger than tree?

To get observable effect



$$\mathcal{M}_{\text{loop}} \simeq F_\pi G_F m_{\widetilde{W}}^2 \times \frac{\alpha}{4\pi} \left(\frac{1}{\epsilon} - 2 \log \frac{m_{\widetilde{W}}}{\mu} + \frac{4}{3} \right)$$

UV divergent

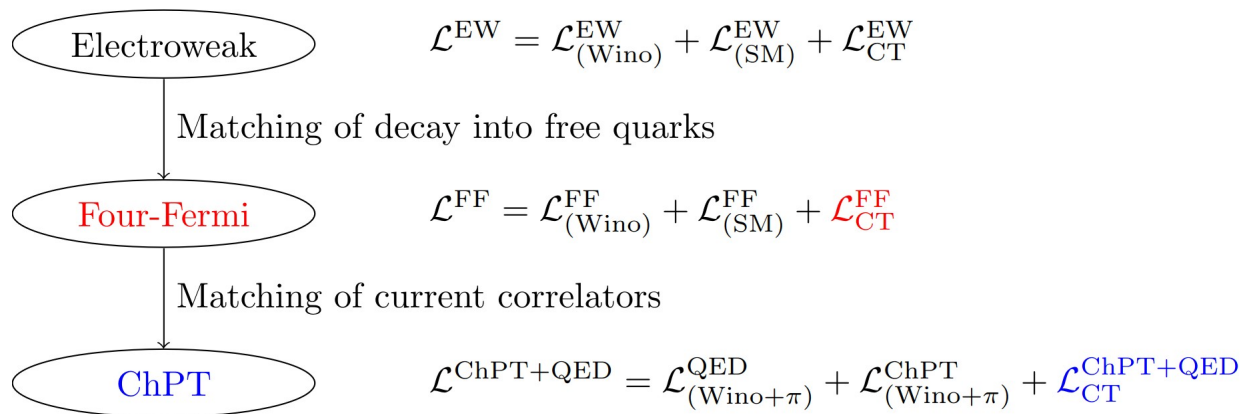
We need specify counter-terms relevant for Wino decay.

Matching with electroweak theory and chiral perturbation (ChPT)

Matching procedure

Strategy is similar to precision of pion decay calculation in SM.

[Descotes-Genon & Moussallam 2005]



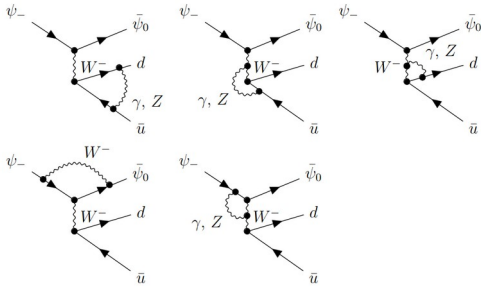
Compute both Wino decay and pion decay with EW/QCD corrections.

Relating

$$\Gamma_{\text{loop}}(\pi^+ \rightarrow \mu^+ \nu(\gamma)) \quad \longleftrightarrow \quad \Gamma_{\text{loop}}(\widetilde{W}^+ \rightarrow \widetilde{W}^0 \pi^+(\gamma))$$

Computing...

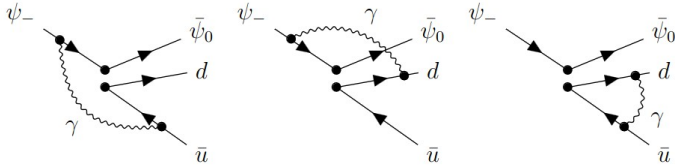
$$\frac{\mathcal{M}_{\text{tree}}^{\text{WF,Box,Vertex(EW)}}}{\mathcal{M}_{\text{tree}}^{\text{quark}}} = \frac{\alpha}{4\pi} \left[\frac{3}{2} \log \frac{M_Z^2}{m_\gamma^2} - \left(\frac{1}{s_W^2} - \frac{4}{s_W^4} \right) \log c_W + \frac{3}{s_W^2} \right] - \frac{\alpha(4 + 6c_W + c_W^2) M_W}{4(1 + c_W)s_W^2 m_\chi} + \mathcal{O}(M_W^2/m_\chi^2).$$



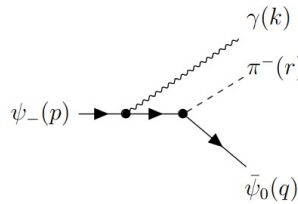
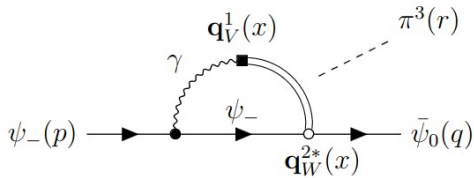
$$\hat{\mathcal{M}}_{\text{tree}}^{\text{quark}} \times \frac{\alpha}{8\pi} \left[-Q_\chi^2 \left(\frac{1}{\epsilon_{\text{FF}}} + \log \frac{\mu_{\text{FF}}^2}{m_\chi^2} + 2 \log \frac{m_\gamma^2}{m_\chi^2} + 4 \right) - (Q_d^2 + Q_u^2) \left(\frac{1}{\epsilon_{\text{FF}}} + \log \frac{\mu_{\text{FF}}^2}{m_\gamma^2} - \frac{1}{2} \right) \right]$$

$$F_{VW}^{\text{(loop,FF)}} = \frac{\alpha}{16\pi\sqrt{2}} F_0 G_F \left[\frac{5}{\epsilon_{\text{ChPT}}} + 5 \log \frac{\mu_{\text{FF}}^2}{m_\chi^2} + \log \frac{m_\chi^2}{m_\gamma^2} + \log \frac{m_\chi^2}{M_V^2} - \frac{5\pi^2}{3} - 4 \log \frac{m_\gamma^2}{M_V^2} - \log^2 \frac{m_\gamma^2}{4\Delta m^2} + \frac{6M_A^2 - 9M_V^2}{M_A^2 - M_V^2} + 3 \frac{M_A^4}{(M_A^2 - M_V^2)^2} \log \frac{M_A^2}{M_V^2} - \frac{4}{\Delta m} \frac{\pi M_A M_V}{M_A + M_V} \right] \times \bar{u}_0(q) \not{f} u_-(p).$$

$$\begin{aligned} \mathcal{L}_K = e^2 F_0^2 \left\{ \frac{1}{2} K_1 \langle (\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2 \rangle \langle u_\mu u^\mu \rangle + K_2 \langle \mathcal{Q}_L \mathcal{Q}_R \rangle \langle u_\mu u^\mu \rangle \right. \\ - K_3 [\langle \mathcal{Q}_L u_\mu \rangle \langle \mathcal{Q}_L u^\mu \rangle + \langle \mathcal{Q}_R u_\mu \rangle \langle \mathcal{Q}_R u^\mu \rangle] + K_4 \langle \mathcal{Q}_L u_\mu \rangle \langle \mathcal{Q}_R u^\mu \rangle \\ + K_5 \langle [(\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2] u_\mu u^\mu \rangle + K_6 \langle (\mathcal{Q}_L \mathcal{Q}_R + \mathcal{Q}_R \mathcal{Q}_L) u_\mu u^\mu \rangle \\ + \frac{1}{2} K_7 \langle (\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2 \rangle \langle \chi_+^{(\text{sp})} \rangle + K_8 \langle \mathcal{Q}_L \mathcal{Q}_R \rangle \langle \chi_+^{(\text{sp})} \rangle \\ + K_9 \langle [(\mathcal{Q}_L)^2 + (\mathcal{Q}_R)^2] \chi_+^{(\text{sp})} \rangle + K_{10} \langle (\mathcal{Q}_L \mathcal{Q}_R + \mathcal{Q}_R \mathcal{Q}_L) \chi_+^{(\text{sp})} \rangle \\ - K_{11} \langle (\mathcal{Q}_L \mathcal{Q}_R - \mathcal{Q}_R \mathcal{Q}_L) \chi_+^{(\text{sp})} \rangle \\ - i K_{12} \langle [(\mathcal{Q}_{L\mu} \mathcal{Q}_L - \mathcal{Q}_L \mathcal{Q}_{L\mu}) - (\mathcal{Q}_{R\mu} \mathcal{Q}_R - \mathcal{Q}_R \mathcal{Q}_{R\mu})] u^\mu \rangle \\ \left. + K_{13} \langle \mathcal{Q}_{L\mu} \mathcal{Q}_R^\mu \rangle + K_{14} \langle (\mathcal{Q}_{L\mu} \mathcal{Q}_L^\mu) + (\mathcal{Q}_{R\mu} \mathcal{Q}_R^\mu) \rangle \right\}. \end{aligned}$$



$$\begin{aligned} \mathcal{L}_Y = e^2 \left\{ \sqrt{2} F_0^2 G_F \left[Y_1 \bar{\psi}_- \gamma_\mu \psi_0 \langle u^\mu \{ \mathcal{Q}_R, \mathcal{Q}_W \} \rangle + Y_1 \psi_- \gamma_\mu \psi_0 \langle u^\mu \{ \mathcal{Q}_L, \mathcal{Q}_W \} \rangle \right. \right. \\ + Y_2 \bar{\psi}_- \gamma_\mu \psi_0 \langle u^\mu [\mathcal{Q}_R, \mathcal{Q}_W] \rangle + Y_2 \psi_- \gamma_\mu \psi_0 \langle u^\mu [\mathcal{Q}_L, \mathcal{Q}_W] \rangle \\ + Y_3 m_\chi \bar{\psi}_- \psi_0 \langle \mathcal{Q}_R \mathcal{Q}_W \rangle \\ + i Y_4 \bar{\psi}_- \gamma_\mu \psi_0 \langle \mathcal{Q}_L^\mu \mathcal{Q}_W \rangle + i Y_5 \psi_- \gamma_\mu \psi_0 \langle \mathcal{Q}_R^\mu \mathcal{Q}_W \rangle + h.c. \left. \right] \\ + \hat{Y}_6 \bar{\psi}_- (i \not{\partial} - e \not{A}) \psi_- + \hat{Y}_7 m_\chi \bar{\psi}_- \psi_- \left. \right\}. \end{aligned}$$



$$\begin{aligned} \frac{\delta \Gamma_\chi}{\Gamma_\chi} = - \frac{\alpha M_A M_V}{\Delta m (M_A + M_V)} + \frac{\alpha}{16\pi} g_\chi \left(\frac{M_V}{M_A}, \frac{\Delta m}{M_A} \right) + \frac{\alpha}{\pi} f_\chi \left(\frac{m_\pi}{\Delta m} \right) \\ + e^2 (2f_{\chi\chi}^r(\mu_{\text{FF}}) + f_{\chi d}^r(\mu_{\text{FF}}) - f_{\chi\bar{u}}^r(\mu_{\text{FF}}) + 2f_{d\bar{u}}^r(\mu_{\text{FF}})) \\ + \frac{8}{3} e^2 (K_1^r(\mu_{\text{ChPT}}) + K_2) + \frac{20}{9} e^2 (K_5^r(\mu_{\text{ChPT}}) + K_6) \\ + \frac{3\alpha}{8\pi} \log \frac{\mu_{\text{ChPT}}^2}{M_V^2} + \frac{3\alpha}{4\pi} \log \frac{\mu_{\text{FF}}^2}{\mu_{\text{ChPT}}^2} + \frac{\alpha}{4\pi} \log \left(\frac{\Delta m^2 M_V^4}{m_\pi^6} \right) + \frac{2\alpha}{\pi} \log 2 \end{aligned}$$

Final Result

$$\Gamma_{\widetilde{W}^\pm}^{\text{loop}} = \Gamma_{\widetilde{W}^\pm}^{\text{tree}} \left\{ 1 + \frac{\alpha}{4\pi} \left[c_{-2} \left(\frac{m_{\widetilde{W}}}{\Delta m} \right)^2 + c_{-1} \left(\frac{m_{\widetilde{W}}}{\Delta m} \right) + c_{\log} \log \left(\frac{m_{\widetilde{W}}}{\Delta m} \right) + c_0 + \dots \right] \right\}$$

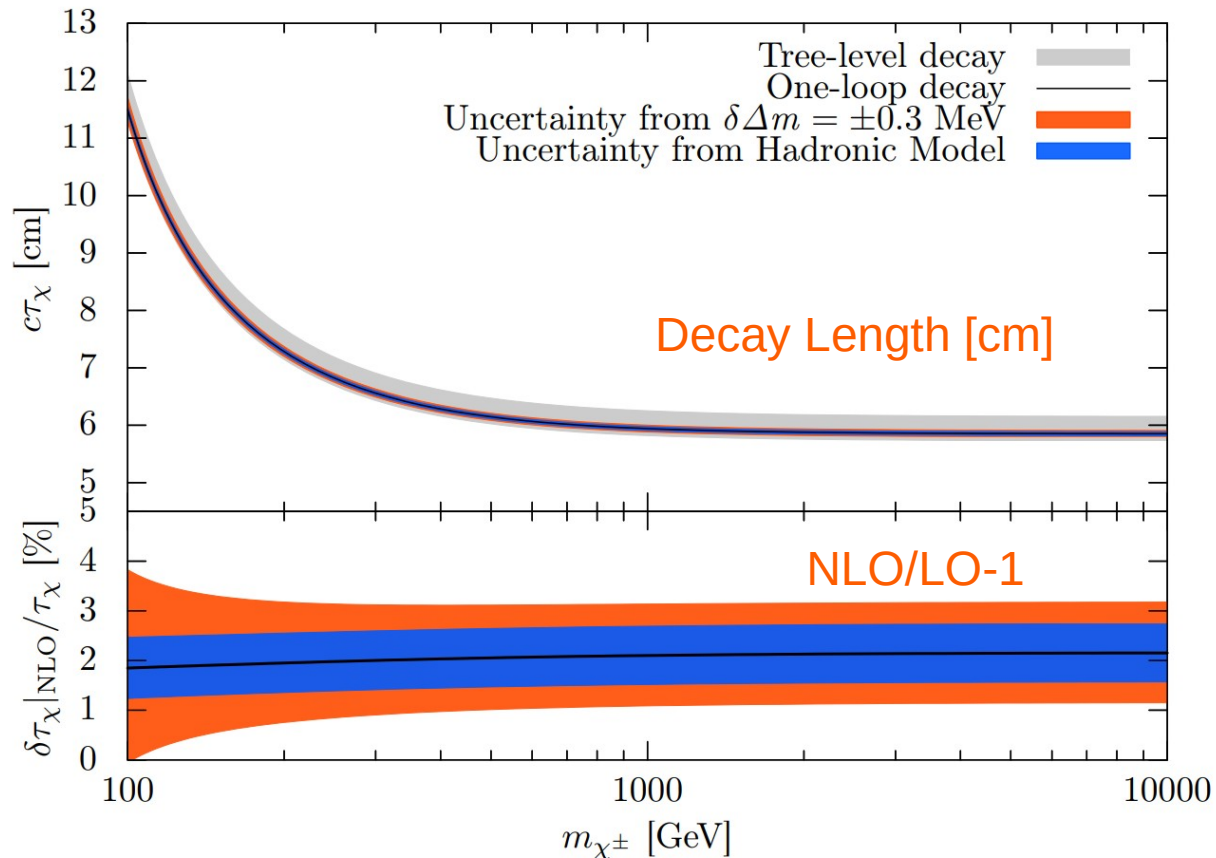
Final Result

$$\Gamma_{\widetilde{W}^\pm}^{\text{loop}} = \Gamma_{\widetilde{W}^\pm}^{\text{tree}} \left\{ 1 + \frac{\alpha}{4\pi} \left[\underbrace{c_{-2} \left(\frac{m_{\widetilde{W}}}{\Delta m} \right)^2}_{=0} + \underbrace{c_{-1} \left(\frac{m_{\widetilde{W}}}{\Delta m} \right)}_{=0} + \underbrace{c_{\log} \log \left(\frac{m_{\widetilde{W}}}{\Delta m} \right)}_{=0} + c_0 + \dots \right] \right\}$$

~ -0.02

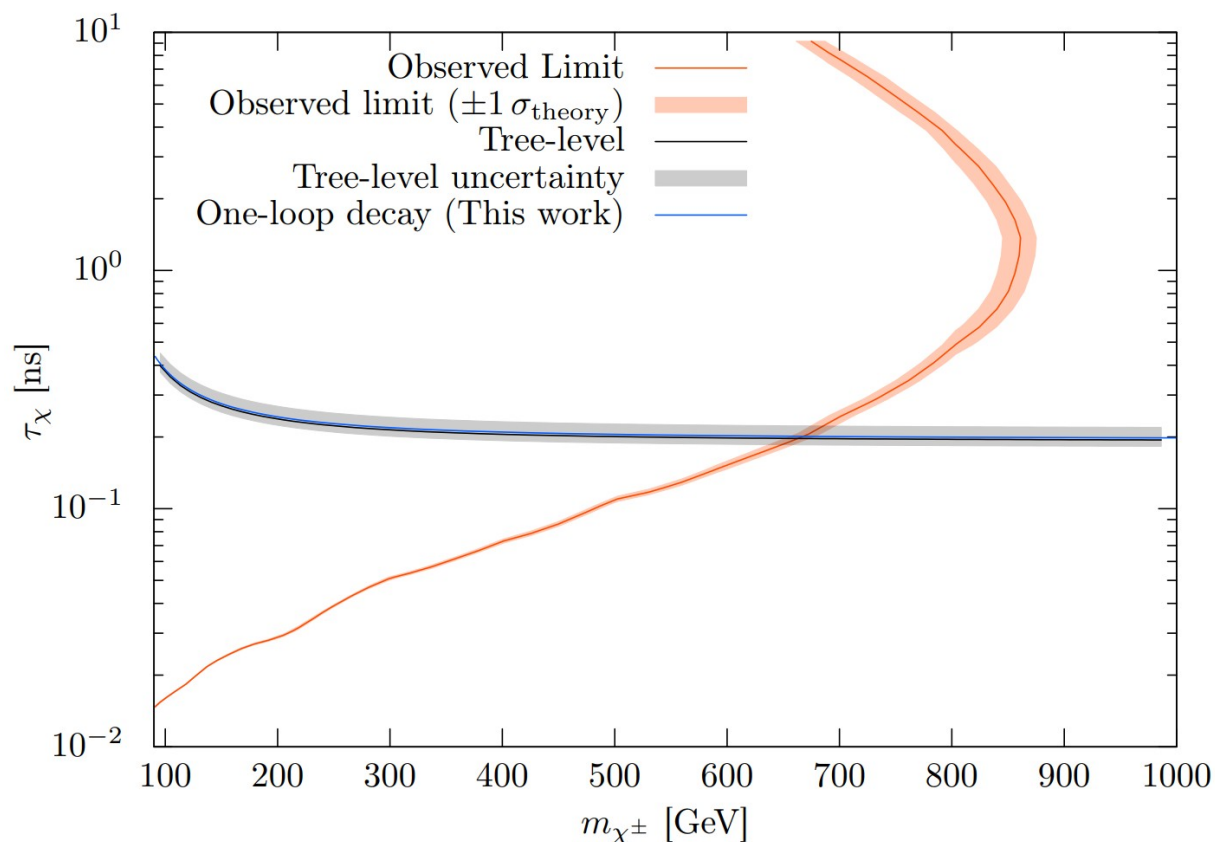
No Wino mass enhancement effect is found!

Final Result

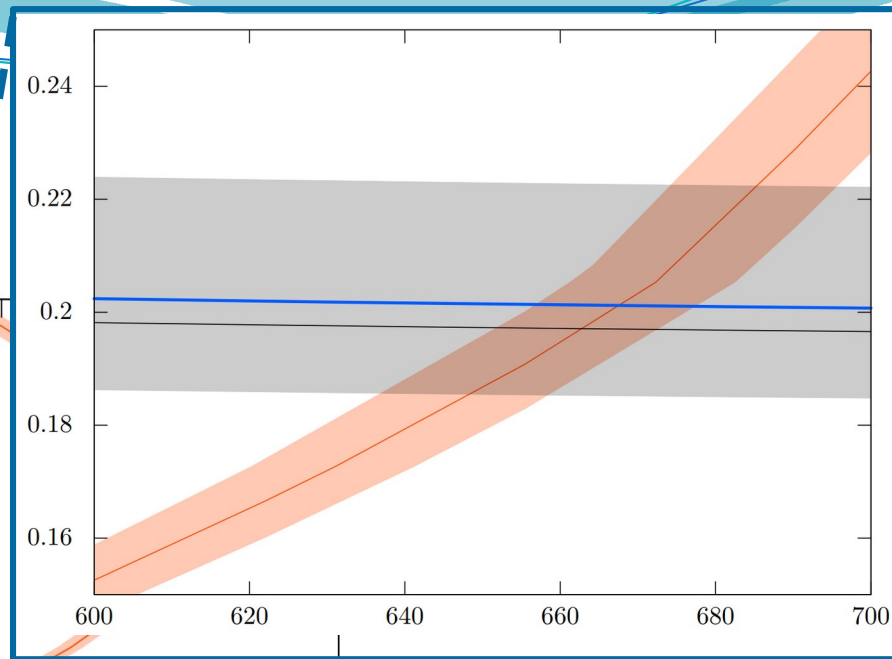
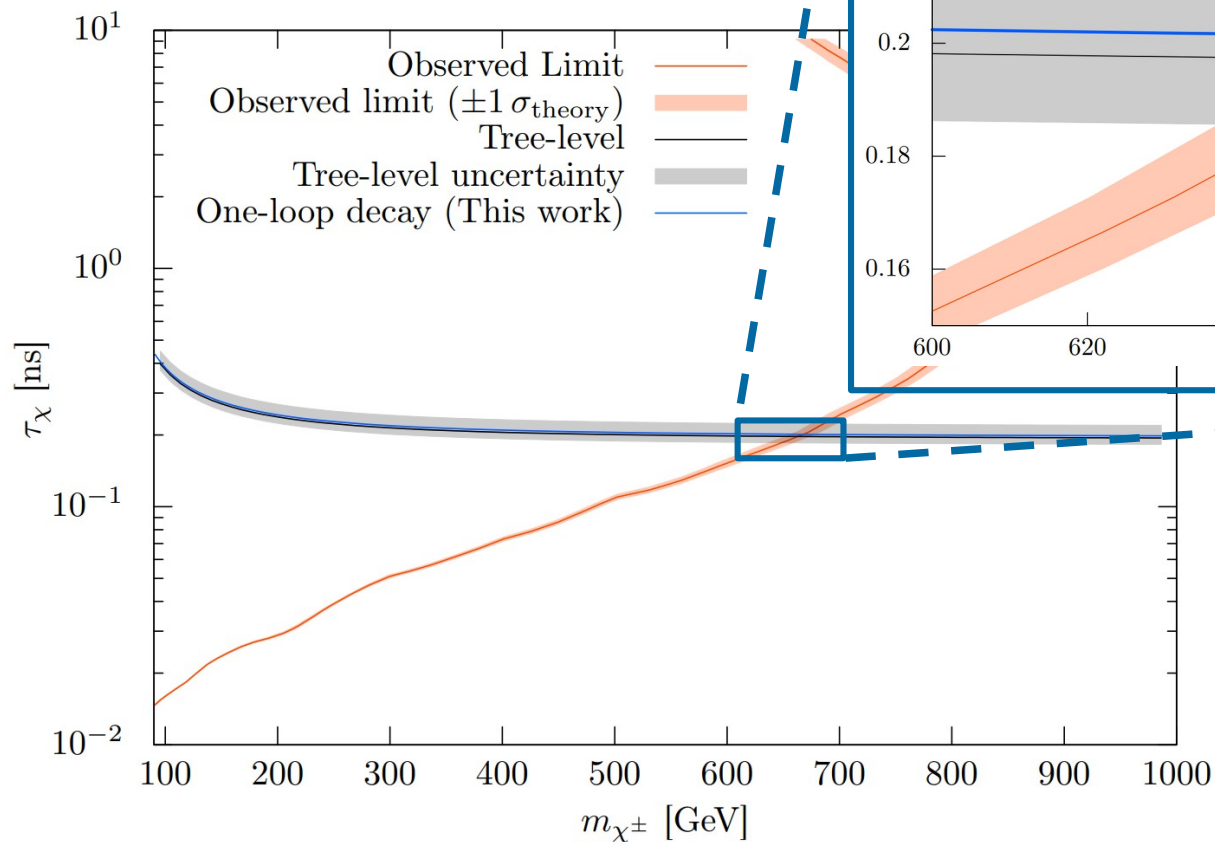


Main theory errors from unknown piece of three-loop mass difference $\Delta_{3\text{-loop}}m$

Impact on LHC Search



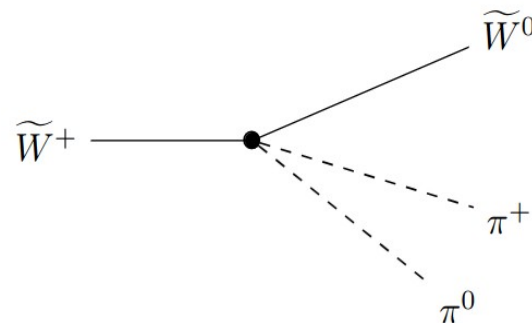
Impact on LHC



General Mass Difference

	Δm
Pure Wino DM	~ 160 MeV
Higgsino-like DM	~ 300 MeV – 2 GeV from gaugino mixing
5-plet DM	~ 160 MeV and ~ 500 MeV

For larger mass difference, lepton and multi-meson decay modes are dominant.

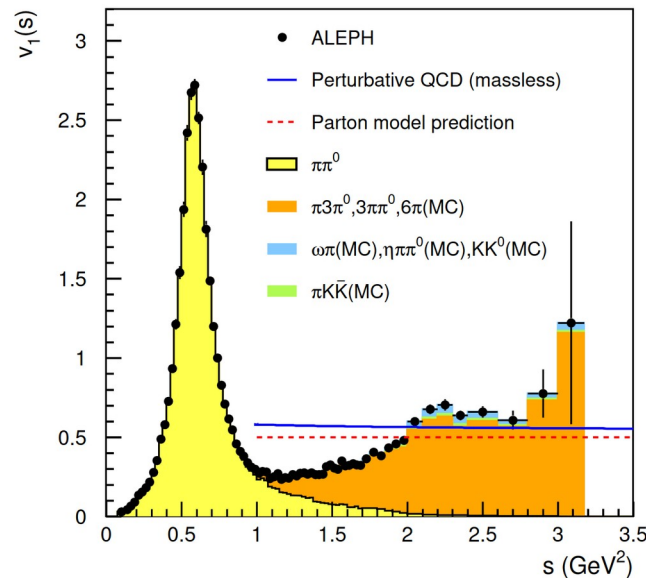


Tau Decay



Tau decay has similar structure.

Hadron data for tau decay is available for BSM particle decay.



[Chen, Drees & Gunion, 96]

Higgsino Decay

Single meson:

EW corrections.

Lepton:

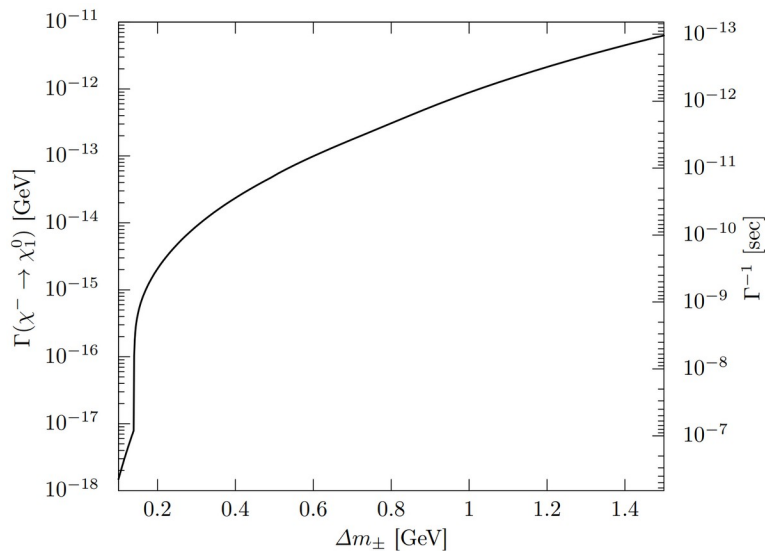
EW corrections.

Multi-meson:

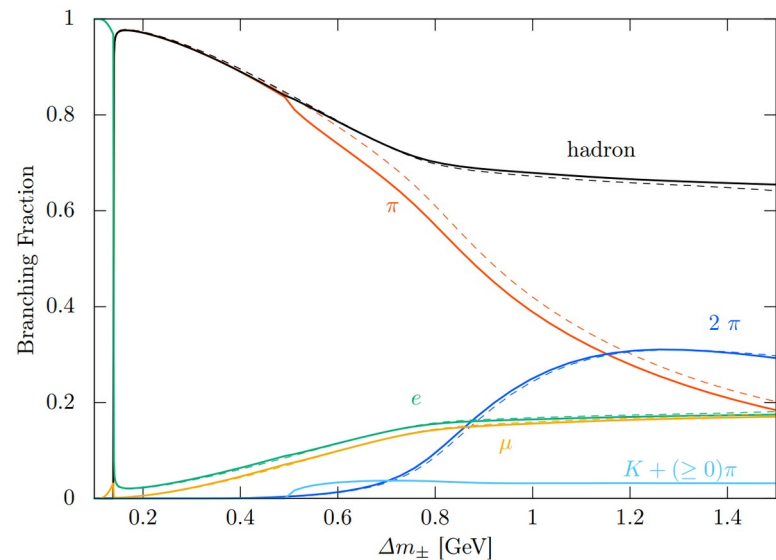
EW + leading QED+ Latest hadron data
(Belle and Aleph)

Charged Higgsino Decay

Decay Rate



Branching Fraction



Dashed line: previous work

5-10% change from work by Chen, Drees & Gunion.

Summary

- Precise estimation of EW charged fermion is crucial for LHC search.
- All the large enhancements from heavy DM are completely canceled.
 - Non-relativistic version of Appelquist-Carazzone's decoupling theorem.
- EW correction to lepton and pion mode, achieving $O(0.1)\%$ precision.
- Leading EW correction + latest hadron rate for multi-meson mode.
- Data is available: https://member.ipmu.jp/satoshi.shirai/Chargino_Decay/