Theoretical constraints on Wilson coefficients following from the three top quark production process under partial unitarity requirements

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SMEFT

- Currently, no experimental evidence of physics beyond the Standard Model has been observed.
- Given that BSM physics is on the scale beyond our reach at the moment, it can still be accessed on lower scales by analysing modified SM interactions.
- Convenient framework for such kind of analyzes is Standard Model of Effective Field Theory (SMEFT).

As such, SMEFT provides a model-independent way to discover, constrain and parametrise potential deviations from the Standard Model.

SMEFT

If only operators of dimension six are preserved, then the SMEFT Lagrangian reads as follows:

$$L = L_{SM} + \sum \frac{c_i}{\Lambda^2} O_i^{d=6}, \quad (1)$$

where L_{SM} is the SM Lagrangian, Λ - hypothetical scale of the BSM physics, $O_i^{d=6}$ - local composite SMEFT operators of dimension six, c_i - dimensionless Wilson coefficients. We consider here only terms of dimension six since in the processes under study they are leading in the expansion on inverse scale Λ^{-1}.

Impact on observables

In the SMEFT framework any observable, in particular the cross-section, can be parametrized in the following form:

$$\sigma = \sigma_{SM} + \sum_{k} \frac{c_i}{\Lambda^2} \sigma_k^{(1)} + \sum_{j<=k} \frac{c_j c_k}{\Lambda^4} \sigma_{k,j}^{(2)}, \qquad (2)$$

where σ_{SM} is the SM value, $\sigma^{(1)}$ and $\sigma^{(2)}$ - coefficients, representing linear and quadratic (in terms of EFT coupling) contributions of the SMEFT operators.

Given the measurement of σ and having calculated values of σ_{SM}, $\sigma^{(1)}$ and $\sigma^{(2)}$, one can estimate constrains on Wilson coefficients c_i. These constraints can be used to calculate limits on physical parameters in various SM extensions.

Four top quark production

There are two recent analyses with an observation of the four top production:

$$\sigma = 22.5^{+6.6}_{-5.5} \text{ fb (arXiv:2303.15061, ATLAS)}$$

$$\sigma = 17.7^{+4.4}_{-4.0} \text{ fb (arXiv:2305.13439, CMS)}$$

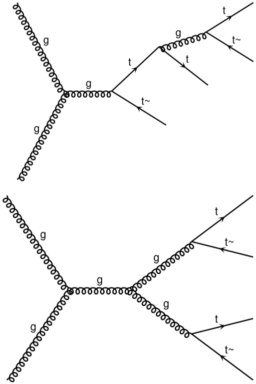

Figure 1: Representative diagrams of the four top quark hadroproduction process

Four top quark production

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Current experimental constraints on four-fermion SMEFT operators (arXiv:2303.15061 [hep-ex]):

Operator	Observed $C_k/\Lambda^2(TeV^{-2})$
Q^1_{tt}	[-3.5, 4.1]
Q^1_{QQ}	[-3.5, 3.0]
Q^1_{Qt}	[-1.7, 1.9]
Q^8_{Qt}	[-6.2, 6.9]

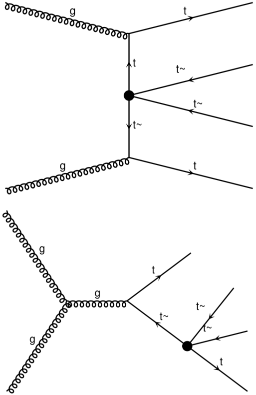

Figure 2: Examples of modified vertices in four top quark hadroproduction

Triple top quark production

Three top quark production has LO SM cross-section of 1.9 fb (arXiv:2107.07629 [hep-ph]) and has not been experimentally observed yet

- Like the four top production, triple top production is also potentially sensitive to a potential contribution of BSM physics

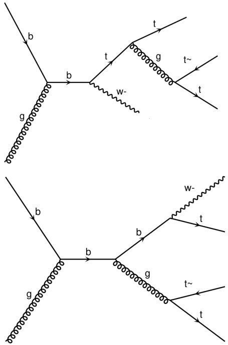

Figure 3: Representative diagrams of three top quark hadroproduction

Triple top quark production

- One can estimate possible constraints on relevant Wilson coefficients by comparing SM and SMEFT theoretical cross sections.
- We use the SM cross section as an effective "measurement" and compare it with the SMEFT cross-section as given in Eq. 2.

To compare expected sensitivities obtained from the three and four top quark production processes, the same analysis procedure is applied to both processes to determine the limits.

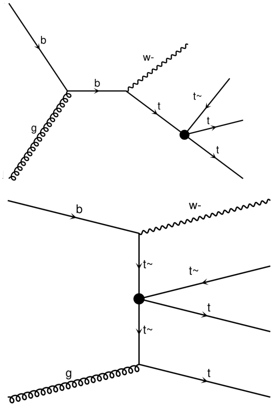

Figure 4: Examples of modified vertices in three top quark hadroproduction

Possible unitarity violation

- `Contributions of EFT operators grow too fast with energy`
- `Extra care should be taken and one should check to not consider kinematic regions where unitarity is violated for calculations to be self-consistent (arXiv:2004.14498 [hep-ph])`

In the current work, we study the effect of the partial unitarity requirement on the limits of the extracted Wilson coefficients. As a result, the corresponding kinematic cutoffs are obtained, which are used in simulations to ensure partial unitarity.

Goals of this study

- Obtain theoretical (or phenomenological) constraints on Wilson coefficients of relevant dimension-6 EFT operators from process of three top quark production.
- Conduct similar procedure for the four top production and compare processes in terms of sensitivity to the possible contribution of the New Physics.
- Estimate the requirements for the partial unitarity non-violation and assess the effect of these restrictions on the accuracy of obtained limits on anomalous parameters.

Simulation setup

- MC events generator: *Madgraph5* (results cross checked with *CompHEP* package)
- PDF set: *NNPDF31_nlo_as_0118_luxqed*
- LO PDF set: *NNPDF31_lo_as_0118*
- Factorization/renormalization scale: *172.5 GeV*
- FeynRules EFT model: *SMEFTatNLO*
- Statistical analysis tool: *Chi-squared*, *EFTfitter*, *SMEFiT*

Four top SM production, 13 TeV

Order	LO		
Scale, $\mu_{F/R}$	cross-section, σ pb	$\delta_{\rm scale}$	δ_{PDF}
m_{top}	9.03	+73% -39%	±7.3%
$2m_{top}$	5.47	+65% -37%	± 6.8%
$H_t/2$	3.93	+60% -35%	± 6.4%
Order	**QCD NLO**		
Scale, $\mu_{F/R}$	cross-section, σ pb	δ_{scale}	δ_{PDF}
m_{top}	13.2	+9.5% -20.1%	± 2.7%
$2m_{top}$	11.2	+26.7% -25.0%	± 2.6%
$H_t/2$	8.4	+29.6% -25.1%	± 2.5%

Table 1: Cross-sections of the four top quarks hadroproduction with corresponding scale/PDF uncertainties.

C.o.m. energy, TeV	SM cros.-sect. σ_{SM}, fb	scale uncert., %	PDF uncert., %
4 top production, LO			
13	9.02	73.4	7.32
14	12.0	72.6	7.28
4 top production, NLO			
13	13.2	20.1	2.5
14	17.8	20.3	2.5

C.o.m. energy, TeV	SM cros.-sect. σ_{SM}, fb	scale uncert., %	PDF uncert., %
3 top production, LO			
13	1.16	30.1	7.1
14	1.5	29.2	6.64
3 top production, NLO			
13	1.78	20.0	3.3
14	2.3	19.4	3.1

Table 2: SM cross-sections for processes $pp \to t\bar{t}t\bar{t}$ and $pp \to t\bar{t}t(\bar{t}\bar{t}t)$

SMEFT operators

There are a total of 11 four-quark EFT operators of dimension 6. Among them, only the following five operators will contribute to both processes of four and three top quark production (arXiv:2208.04962 [hep-ph]):

$$
\begin{aligned}
O^1_{tt} &= (\bar{t}_R \gamma^\mu t_R)(\bar{t}_R \gamma_\mu t_R), \\
O^1_{QQ} &= (\bar{Q}_L \gamma^\mu Q_L)(\bar{Q}_L \gamma_\mu Q_L), \\
O^1_{Qt} &= (\bar{Q}_L \gamma^\mu Q_L)(\bar{t}_R \gamma_\mu t_R), \\
O^8_{Qt} &= (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{t}_R \gamma_\mu T^A t_R), \\
O^8_{QQ} &= (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{Q}_L \gamma_\mu T^A Q_L)
\end{aligned}
$$

where Q_L is the left-handed third generation quark doublet, t_R - the right-handed top quark singlet, T^A - $SU(3) generators$

Since much of this analysis is based on comparisons between the four and the three top quark production processes, we will consider only this set of five operators in what follows.

Pipeline for extraction of Wilson coefficients

To obtain limits on Wilson coefficients we first need to extract values for parameterization coefficients $\sigma^{(1)}$ and $\sigma^{(2)}$. The whole procedure of obtaining restrictions on Wilson coefficients can be described as follows:

1. Calculate the cross-sections with opposite values of anomalous coupling c_i (i.e. 1 and -1)
2. Solve a linear equation set obtained from 2 for $\sigma^{(1)}$ and $\sigma^{(2)}$
3. Having the values of coefficients $\sigma^{(1)}$ and $\sigma^{(2)}$ and their uncertainties, one can use a suitable statistical model to obtain limits on c_i/Λ^2

As a statistical model, we tested several choices including a self-made model based on the chi-squared distribution (in the following noted as χ^2), EFTfitter package (arXiv:1605.05585 [hep-ex]), and SMEFiT package (arXiv:2302.06660 [hep-ph]).

Sources of uncertainty

In the following, we consider three types of uncertainties:

- Theory uncertainty, which follows from simulation, the numbers are provided in Tables 1, 2
- Experimental systematic uncertainty
- Statistical uncertainty

The last two uncertainties are extrapolated from the recent measurements of four top-quark production in CMS (arXiv:2305.13439 [hep-ex])
The integrated luminosity (L) for the calculations has been taken as 138 fb^{-1} for 13 TeV (available experimental results), 3 ab^{-1} for 14 TeV (HL-LHC), 15 ab^{-1} for 27 TeV (HE-LHC) and 25 ab^{-1} for 100 TeV (FCC).

		$pp \to 4top$		$pp \to 3top + X$	
Energy	int. Luminosity	δsys, %	$\delta stat$, %	δsys, %	$\delta stat$, %
13 TeV	138 fb^{-1}	13	24	13	76
14 TeV	3 ab^{-1}	13	5	13	14
27 TeV	15 ab^{-1}	13	0.6	13	2
100 TeV	25 ab^{-1}	13	0.1	13	0.3

Table 3: Estimated experimental systematic and statistical uncertainties for processes of three and four top quark production at energies of 13, 14, 27, and 100 TeV

Cross checks of the methodology

Stat. model	C_{tt}^1	C_{QQ}^1	C_{Qt}^1	C_{Qt}^8	C_{QQ}^8
χ^2,1D,4t	[-1.3,1.2]	[-2.5,2.3]	[-2.2,2.2]	[-6.3,5.2]	[-5.6,5.5]
EFTfitter,1D,4t	[-1.1,1.1]	[-2.2,2.1]	[-2.0,2.0]	[-5.7,4.6]	[-5.1,4.9]
SMEFiT,1D,4t	[-1.1,1.1]	[-2.2,2.1]	[-2.0,2.0]	[-5.7,4.6]	[-5.0,4.8]
χ^2,1D,3t	[-4.2,4.1]	[-2.9,3.2]	[-2.9,3.0]	[-6.0,6.3]	[-5.8,6.8]
EFTfitter,1D,3t	[-3.8,3.7]	[-2.5,2.9]	[-2.6,2.7]	[-5.4,5.6]	[-5.2,6.1]
SMEFiT,1D,3t	[-3.7,3.7]	[-2.5,2.9]	[-2.6,2.7]	[-5.3,5.6]	[-5.1,6.1]
χ^2,1D,3+4t	[-1.3,1.2]	[-2.2,2.2]	[-2.1,2.1]	[-5.2,4.7]	[-4.8,5.0]
EFTfitter,1D,3+4t	[-1.1,1.1]	[-2.0,2.0]	[-1.8,1.9]	[-4.7,4.2]	[-4.3,4.5]
SMEFiT,1D,3+4t	[-1.1,1.0]	[-2.0,2.0]	[-1.8,1.8]	[-4.7,4.2]	[-4.2,4.5]
EFTfitter,5D,4t	[-0.94,0.92]	[-2.3,2.2]	[-1.7,1.6]	[-4.9,3.7]	[-5.2,5.1]
SMEFiT,5D,4t	[-0.95,0.90]	[-1.8,1.7]	[-1.6,1.6]	[-4.8,3.6]	[-4.2,4.0]
EFTfitter,5D,3t	[-3.3,3.7]	[-2.1,2.4]	[-2.2,2.5]	[-4.6,5.4]	[-4.3,5.5]
SMEFiT,5D,3t	[-3.1,3.0]	[-2.0,2.4]	[-2.1,2.2]	[-4.3,4.6]	[-4.2,5.1]
EFTfitter,5D,3+4t	[-0.98,0.92]	[-1.8,1.8]	[-1.5,1.6]	[-4.0,3.4]	[-3.9,4.0]
SMEFiT,5D,3+4t	[-0.95,0.90]	[-1.6,1.6]	[-1.5,1.5]	[-4.0,3.3]	[-3.5,3.7]

Table 4: Comparison of the expected limits on $C_k/\Lambda^2[\mathrm{TeV}^{-2}]$ for different statistical models.

Table 4 demonstrates good agreement between SMEFiT, EFTfitter, and χ^2 results. The theoretical constraints seem to coincide pretty well with experimental limits obtained from four top-quark production in CMS (arXiv:1906.02805 [hep-ex]) and ATLAS (arXiv:2303.15061 [hep-ex]) experiments.

Electroweak and QCD contributions in triple top-quark production

The importance of electroweak contribution (EW) to the triple top quark production processes has been shown in arXiv:2107.07629 [hep-ph]. If one takes into account only QCD diagrams the total cross section will be almost correct due to the cancellation of negative interference terms and EW contribution, but the kinematic properties can be significantly different.

model	C^1_{tt}	C^1_{QQ}	C^1_{Qt}	C^8_{Qt}	C^8_{QQ}
3t,QCD,1D	[-3.6,3.7]	[-2.6,2.5]	[-2.5,2.5]	[-6.3,4.8]	[-6.5,5.3]
3t,QCD+EW,1D	[-3.4,3.4]	[-2.3,2.7]	[-2.4,2.5]	[-5.0,5.2]	[-4.8,5.6]
3t,QCD,5D	[-3.0,3.0]	[-2.2,2.1]	[-2.0,2.1]	[-5.3,3.9]	[-5.5,4.2]
3t,QCD+EW,5D	[-2.8,2.8]	[-1.9,2.2]	[-1.9,2.1]	[-4.1,4.3]	[-3.8,4.7]

Table 5: Comparison of the expected limits on $C_k/\Lambda^2[\mathrm{TeV}^{-2}]$. Two simulation models are considered, the only QCD diagrams and QCD plus EW diagrams and their interference.

Direct comparison of the expected limits in Table 5 demonstrate notable improvement in the sensitivity for the calculations with the correct simulation of all QCD, EW and interference contributions.

Optical theorem and perturbative unitarity

To estimate the admissible range of parameters, we apply the optical theorem, which follows from the unitarity of the S matrix. The optical theorem states that the imaginary part of the forward scattering amplitude is proportional to the total cross section of the process:

$$\sigma = \frac{1}{s}\mathrm{Im}\left(A(\theta = 0)\right) = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2, \tag{3}$$

where a_l – is the amplitude of the partial wave. Therefore, $\mathrm{Im}a_l = |a_l|^2$ and

$$|\mathrm{Re}(\mathrm{a_1})|^2 + \left[\mathrm{Im}(\mathrm{a_l}) - \frac{1}{2}\right]^2 = \frac{1}{4}. \tag{4}$$

$$|\mathrm{Re}(\mathrm{a_0})| < \frac{1}{2}. \tag{5}$$

Optical theorem and perturbative unitarity

$$a_0 = \frac{1}{16\pi\lambda}\left|\int_{t_-}^{t_+} dt \cdot A\right| \quad (6)$$

where λ is the kinematic function of the triangle and A is the amplitude of the process.

Amplitudes are calculated by representing a four-particle vertex with three-particle vertices and an auxiliary massive vector field. Using the Weyl representation for spinors and the method of helicity amplitudes, for each case of an anomalous operator, the amplitudes $2 \to 2$ of $tt \to tt$ and $t\bar{t} \to t\bar{t}$ processes were calculated.

The resulting helicity amplitudes were integrated over the two-particle phase volume to calculate the corresponding partial amplitudes.

Unitary limitations for O^1_{tt}, O^1_{QQ}, Q^1_{Qt} and Q^8_{Qt} operators

Based on the calculated partial amplitudes for the operators under study, we plotted graphs of the boundary of partial unitarity, where the invariant mass of a pair of top quarks is plotted along the x-axis, and the current upper experimental limits for the Wilson coefficients of anomalous operators are plotted along the y-axis:

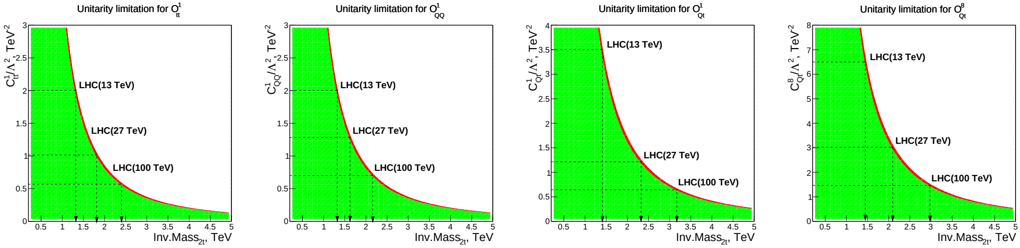

Figure 5: Partial unitarity limit $a_0 = \frac{1}{2}$(red line) for various anomalous operators. Leftmost: O^1_{tt}, middle left: O^1_{QQ}, middle right: O^1_{Qt}, rightmost: O^8_{Qt}.

In Fig. 5, the red color marks the values at which the partial amplitude $a_0 = \frac{1}{2}$. The green color marks the zone of parameters allowed from the point of view of unitarity. The dashed horizontal line indicates the experimental limits on the corresponding Wilson coefficients for different LHC operating modes. Dashed vertical lines with an arrow indicate the limits of the invariant mass of two t-quarks for corresponding LHC modes.

Impact of restrictions following from partial unitarity requirement

model	C^1_{tt}	C^1_{QQ}	C^1_{Qt}	C^8_{Qt}	C^8_{QQ}
4t,nocut,1D	[-1.1,1.1]	[-2.2,2.1]	[-2.0,2.0]	[-5.7,4.6]	[-5.0,4.8]
4t,cut,1D	[-1.2,1.2]	[-2.4,2.3]	[-2.2,2.2]	[-6.8,5.0]	[-6.0,5.7]
3t,nocut,1D	[-3.7,3.7]	[-2.5,2.9]	[-2.6,2.7]	[-5.3,5.6]	[-5.1,6.1]
3t,cut,1D	[-4.3,4.2]	[-2.9,3.2]	[-3.1,3.2]	[-6.9,7.3]	[-6.4,7.7]
3+4t,nocut,1D	[-1.1,1.0]	[-2.0,2.0]	[-1.8,1.8]	[-4.7,4.2]	[-4.2,4.5]
3+4t,cut,1D	[-1.2,1.2]	[-2.2,2.2]	[-2.1,2.1]	[-5.8,4.8]	[-5.2,5.4]
4t,nocut,5D	[-0.95,0.90]	[-1.8,1.7]	[-1.6,1.6]	[-4.8,3.6]	[-4.2,4.0]
4t,cut,5D	[-1.0,1.0]	[-2.0,1.9]	[-1.8,1.9]	[-5.7,4.1]	[-4.6,4.4]
3t,nocut,5D	[-3.1,3.0]	[-2.0,2.4]	[-2.1,2.2]	[-4.3,4.6]	[-4.2,5.1]
3t,cut,5D	[-3.5,3.4]	[-2.3,2.7]	[-2.5,2.7]	[-5.6,6.1]	[-5.1,6.5]
3+4t,nocut,5D	[-0.95,0.90]	[-1.6,1.6]	[-1.5,1.5]	[-4.0,3.3]	[-3.5,3.7]
3+4t,cut,5D	[-1.0,1.0]	[-1.8,1.8]	[-1.7,1.7]	[-4.8,3.8]	[-4.1,4.3]

Table 6: Comparison of the expected limits on $C_k/\Lambda^2[\mathrm{TeV}^{-2}]$. The limits are shown for the case of unitarity bound cuts applied at the simulation level (cut) and without such cuts (nocut)

Expected limits in Table 6 achieved with additional unitarity bound cuts worse than without such cuts, but not significantly. Since the unitarity is necessary requirement, all simulations in the following obey the calculated unitarity bound cuts.

Summary table

Energy, model	C_{tt}^{1}	C_{QQ}^{1}	C_{Qt}^{1}	C_{Qt}^{8}	C_{QQ}^{8}
13 TeV, 4t	[-1.2, 1.2]	[-2.4, 2.3]	[-2.2, 2.2]	[-6.8, 5.0]	[-6.0, 5.7]
13 TeV, 3t	[-4.3, 4.2]	[-2.9, 3.2]	[-3.1, 3.2]	[-6.9, 7.3]	[-6.4, 7.7]
13 TeV, 3+4t	[-1.2, 1.2]	[-2.2, 2.2]	[-2.1, 2.1]	[-5.8, 4.8]	[-5.2, 5.4]
14 TeV, 4t	[-1.1, 1.0]	[-2.1, 2.0]	[-1.9, 1.9]	[-5.8, 4.2]	[-5.2, 4.9]
14 TeV, 3t	[-2.5, 2.5]	[-1.6, 2.0]	[-1.8, 1.9]	[-3.9, 4.4]	[-3.7, 5.1]
14 TeV, 3+4t	[-1.1, 1.0]	[-1.5, 1.7]	[-1.5, 1.6]	[-3.8, 3.6]	[-3.5, 4.3]
27 TeV, 4t	[-0.90, 0.83]	[-1.7, 1.6]	[-1.6, 1.6]	[-4.9, 3.6]	[-4.4, 4.2]
27 TeV, 3t	[-2.0, 2.0]	[-1.3, 1.5]	[-1.4, 1.6]	[-3.3, 3.9]	[-2.7, 4.1]
27 TeV, 3+4t	[-0.88, 0.83]	[-1.2, 1.3]	[-1.3, 1.3]	[-3.2, 3.2]	[-2.6, 3.5]
100 TeV, 4t	[-0.68, 0.66]	[-1.3, 1.3]	[-1.2, 1.2]	[-3.8, 3.0]	[-3.7, 3.6]
100 TeV, 3t	[-1.3, 1.4]	[-0.89, 1.0]	[-1.0, 1.1]	[-2.1, 2.6]	[-1.8, 2.7]
100 TeV, 3+4t	[-0.67, 0.64]	[-0.85, 0.94]	[-0.93, 0.94]	[-2.1, 2.3]	[-1.8, 2.5]

Table 7: Comparison of the expected limits on $C_k/\Lambda^2[\mathrm{TeV}^{-2}]$ estimated for $pp \to 4top$ (4t) and $pp \to 3top + X$ (3t) cross sections. The limits are shown with applied unitarity bound requirements. The results have been achieved in a one-dimensional statistical model with a variation of each coefficient separately.

Summary

- Numerical simulations of processes of three and four top quark production in Standard Model and dimension six SMEFT were carried out. Simulation results were used to derive theoretical constraints on Wilson coefficients $C_k/\Lambda^2(TeV^{-2})$ of respective SMEFT operators
- Both processes were compared in terms of their sensitivity to a potential contribution of the New Physics.
- The analysis of the impact of cuts, following from the partial unitarity requirement, on the accuracy of extracted Wilson coefficients was conducted

Overall, the production of three top quarks seems to be quite an interesting target for BSM studies. Despite the lower cross-section (as compared to the more widely discussed four top quark production), it can potentially provide some interesting opportunities for constraining the New Physics.

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