New Physics effects in semileptonic $\bar{B}_s \to K^{*+}(\to K\pi)l^-\bar{\nu}_l$ decays

SHABANA KHAN

UNIVERSITY OF RAJASTHAN, JAIPUR, RAJASTHAN

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In Collaboration with **Dr. DINESH KUMAR**

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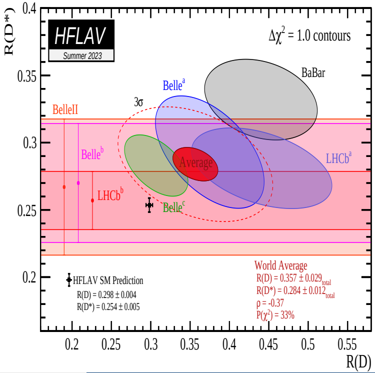
Introduction

The semileptonic decays are interesting avenue to look for the New Physics beyond the Standard Model.

- Several analysis with New Physics have performed which can explain the observed discrepancy. (Very recent arXiv 2405.06062)
- We analyzed the allowed New Physics constrained by the available $b \to ul\nu$ data.
- We aim to provide a comprehensive analysis of the $\bar{B}_s \to K^{*+}(\to K\pi)l^- \bar{\nu}_l$ decays process, focusing particularly on its sensitivity to NP effects.



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Effective Field Theory

The effective Hamiltonian for the transition governed by $b \to u l \nu$ is given by:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ub} \left[(1 + C_{V_L}) O_{V_L} + C_{V_R} O_{V_R} + C_{S_L} O_{S_L} + C_{S_R} O_{S_R} + C_T O_T \right],$$

where the operators are:

$$\begin{aligned} O_{V_L} &= (\bar{u}\gamma_\mu P_L b)(\bar{l}\gamma^\mu P_L \nu) \\ O_{V_R} &= (\bar{u}\gamma_\mu P_R b)(\bar{l}\gamma^\mu P_L \nu), \\ O_{S_R} &= (\bar{u} P_R b)(\bar{l} P_L \nu), \\ O_{S_L} &= (\bar{u} P_L b)(\bar{l} P_L \nu), \\ O_T &= (\bar{u}\sigma^{\mu\nu} P_L b)(\bar{l}\sigma_{\mu\nu} P_L \nu). \end{aligned}$$

We assume the lepton flavour universal NP couplings for light leptons (l = μ or e) :

$$C_i^l = \frac{(C_i^e + C_i^\mu)}{2}.$$

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New Physics Constraints

We constraint the New Physics by the available $b \to ul\nu$ data :

- For the decay mode $\bar{B}^0 \to \pi^+ l^- \bar{\nu}$, we have utilized the globally averaged q^2- binned branching ratio spectrum published by the HFLAV collaboration.arXiv:2206.07501
- We use the world average of the differential branching fractions in different q^2 bins for the decay $B \to \rho\, l\nu$ published by the HFLAV collaboration.arXiv:2206.07501
- We use the world average of the differential branching fractions in different q^2 bins for the decay $B \to \omega\, l\nu$ published by the HFLAV collaboration.arXiv:2206.07501
- The measurement of leptonic decay $B \to \mu\nu$ from Belle is also used to constraint the NP parameters.arXiv:1911.03186

New Physics contribution in $B \to P l \nu$

The differential decay rate of semileptonic decay of B → P can be written in term of NP WCs as:

$$\frac{d\Gamma(B \to P l \nu)/dq^2}{d\Gamma(B \to P l \nu)^{SM}/dq^2} = \left|1 + C^l_{V_L} + C^l_{V_R}\right|^2 \left[\left(1 + \frac{m_l^2}{2q^2}\right) H^{s^2}_{V,0} + \frac{3}{2}\frac{m_l^2}{q^2} H^{s^2}_{V,t}\right]$$
$$+ \frac{3}{2}|C^l_{S_L} + C^l_{S_R}|^2 H^{s^2}_S + 8|C^l_T|(1 + \frac{2m_l^2}{q^2}) H^{s^2}_T$$
$$+3\,Re[(1 + C^l_{V_L} + C^l_{V_R})(C^{l*}_{S_L} + C^{l*}_{S_R})] \frac{m_l}{\sqrt{q^2}} H^s_S H^s_{V,t}$$
$$-12\,Re[(1 + C^l_{V_L} + C^l_{V_R}) C^{l*}_T] \frac{m_l}{\sqrt{q^2}} H^s_T H^s_{V,0}$$

Hadronic matrix elements can be written in terms of Form Factors which have been determined by using combined LCSR + Lattice fit. [arXiv:1205.6245, 1911.03186]

New Physics contribution in $B \to V l\nu$

Similarly for B → V can be written in terms of NP WCs as :

$$\frac{d\Gamma(B\to V l\nu)/dq^2}{d\Gamma(B\to V l\nu)^{SM}/dq^2} = \Big(|1+C^l_{V_L}|^2+|C^l_{V_R}|^2\Big)\Big[\Big(1+\frac{m_l^2}{2q^2}\Big)(H^2_{V,+}+H^2_{V,-}+H^2_{V,0})+\frac{3}{2}\frac{m_l^2}{q^2}H^2_{V,t}\Big]$$
$$-2\,Re\Big[(1+C^l_{V_L})C^{l*}_{V_R}\Big]\Big[\Big(1+\frac{m_l^2}{2q^2}\Big)(H^2_{V,0}+2H_{V,+}H_{V,-})+\frac{3}{2}\frac{m_l^2}{q^2}H^2_{V,t}\Big]$$
$$+\frac{3}{2}|C^l_{S_R}-C^l_{S_L}|^2H^2_S+8|C^l_T|(1+\frac{2m_l^2}{q^2})(H^2_{T,+}+H^2_{T,-}+H^2_{T,0})$$
$$+3\,Re[(1-C^l_{V_R}+C^l_{V_L})(C^{l*}_{S_R}-C^{l*}_{S_L})]\frac{m_l}{\sqrt{q^2}}H_S H_{V,t}$$
$$-12\,Re[(1+C^l_{V_L})C^{l*}_T]\frac{m_l}{\sqrt{q^2}}(H_{T,0}H_{V,0}+H_{T,+}H_{V,+}-H_{T,-}H_{V,-})$$
$$-12\,Re[C^l_{V_R}C^{l*}_T]\frac{m_l}{\sqrt{q^2}}(H_{T,0}H_{V,0}+H_{T,+}H_{V,+}-H_{T,-}H_{V,-})$$

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Methodology

- We perform the χ^2 analysis to constraints the NP parameter space and use MINUIT for the χ^2 analysis.

The χ^2 in our analysis is defined as :

$$\chi^2(C_i) = \sum_{m,n} \left(O^{th}(C_i) - O^{exp}\right)_m C^{-1}_{mn} \left(O^{th}(C_i) - O^{exp}\right)_n$$

where C^{-1}_{mn} is the covariance matrix which includes both experimental and theoretical uncertainties. O^{exp} and O^{th} are the experimental measurement and theoretical predictions, respectively.

- We consider the NP in 1D and 2D scenarios.The best fit values for the NP parameters are obtained by minimizing the χ^2.
- We also get the allowed parameter space of new physics Wilson coefficients for 2-D scenarios based on $\Delta\chi^2$ values. ($\Delta\chi^2 = \chi^2$-χ^2_{min})

Best fit of 1D New Physics Scenario

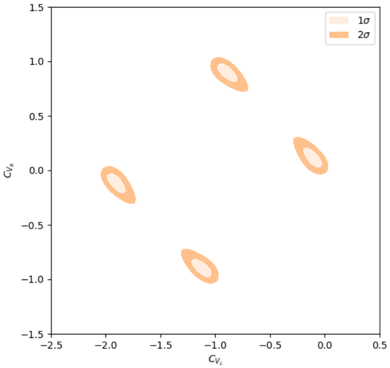
Scenarios	best fit point	χ^2_{min}
SM	-	24.34
$S1 : C_{V_L}$	-0.032(47)	23.87
$S2 : C_{V_R}$	0.069(47)	22.31
$S3 : C_{S_L}$	-0.003(4)	23.85
$S4 : C_{S_R}$	0.003(4)	23.85
$S5 : C_T$	0.005(49)	24.33
$\mathrm{S6} : C_{V_L} = -C_{V_R}$	-0.093(54)	20.61

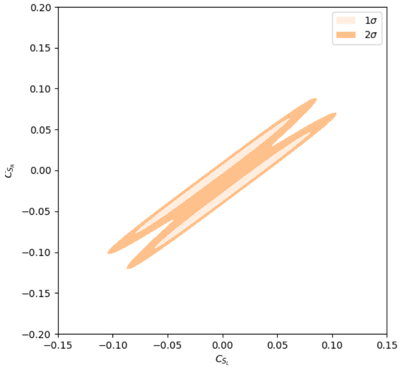
Best fit of 2D New Physics Scenario

Scenarios	best fit point	χ^2_{min}
$S7:(C_{V_L},C_{V_R})$	S7a : $[-0.079(56), 0.115(62)]$	20.21
	S7b: $[-0.892(60), 0.928(56)]$	20.21
	S7c: $[-1.122(63), -0.928(57)]$	20.21
	S7d: $[-1.934(58), -0.115(62)]$	20.21
$S8:(C_{V_L},C_{S_L})$	$[-0.038(48), -0.003(4)]$	23.22
$S9:(C_{V_L},C_{S_R})$	$[-0.038(48), 0.004(4)]$	23,21
$S10:(C_{V_L},C_T)$	$[-0.032(47), 0.006(57)]$	23.85
$S11:(C_{V_R},C_{S_L})$	$[0.075(48), -0.004(4)]$	21.44
$S12:(C_{V_R},C_{S_R})$	$[0.075(48), 0.004(4)]$	21.46
$S13:(C_{V_R},C_T)$	$[0.068(48), 0.0007(50)]$	22.31
$S14:(C_{S_L},C_{S_R})$	$[0.008(121), 0.011(120)]$	23.85
$S15:(C_{S_L},C_T)$	$[-0.003(4), 0.005(49)]$	23.85
$S16:(C_{S_R},C_T)$	$[0.003(4), 0.005(49)]$	23.85
$S17:(C_{V_L}=-C_{V_R},C_{S_L}=-C_{S_R})$	$[-0.116(59), 0.015(2)]$	18.84

ALLOWED NEW PHYSICS PARAMETER SPACE

We plot the 1 σ and 2 σ contours in the 2-D WC's plane.





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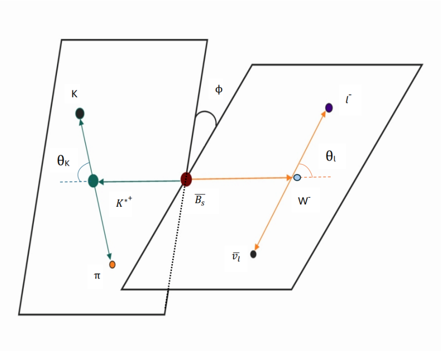
Kinematics of $\bar{B}_s \to K^{*+}(\to K\pi) l^- \bar{\nu}_l$ decay

- In our work we provide comprehensive analysis of the $\bar{B}_s \to K^{*+}(\to K\pi) l^- \bar{\nu}_l$ decay.



The four body decays distribution for $\bar{B}_s \to K^{*+}(\to K\pi) l^- \bar{\nu}_l$ decay can be characterized by four kinematic variables : q^2 , θ_l, θ_{K^*} and ϕ.

Angular Distribution for $\bar{B}_s \to K^{*+}(\to K\pi) l^- \bar{\nu}_l$ decay

The four fold differential distribution for this decay is given by arXiv : 1212.2231 :

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*}\, d\phi} = \frac{8\pi}{3}\Big[(J_{1s} + J_{2s} + J_3 \cos 2\phi + J_{6s} \cos\theta_l + J_9 \sin 2\phi) + (J_{1c} + J_{2c}) + (J_4 \cos\phi + J_5 \sin_{\theta_l} \cos\phi + J_7 \sin\theta_l \sin\phi + J_8 \sin\phi) J_{6c} \cos\theta_l\Big]$$

Here $J_i(q^2)$ are the angular coefficient . These coefficients contains the form factors and are sensitive to different new physics.

- The angular coefficients can be written as:

$$J_{1s} = \frac{3}{16}\left[3|\mathscr{A}_\perp^L|^2 + 3|\mathscr{A}_\parallel^L|^2 + 16|\mathscr{A}_{0\parallel}|^2 + 16|\mathscr{A}_{t\perp}|^2\right]$$

$$J_{1c} = \frac{3}{4}\left[|\mathscr{A}_0^L|^2 + 2|\mathscr{A}_t^L|^2 + 8|\mathscr{A}_{\parallel\perp}|^2\right]$$

$$J_{2s} = \frac{3}{16}\left[|\mathscr{A}_\perp^L|^2 + |\mathscr{A}_\parallel^L|^2 - 16|\mathscr{A}_{0\parallel}|^2 - 16|\mathscr{A}_{t\perp}|^2\right]$$

$$J_{2c} = -\frac{3}{4}\left[|\mathscr{A}_0^L|^2 - 8|\mathscr{A}_{\parallel\perp}|^2\right]$$

$$J_3 = \frac{3}{8}\left[|\mathscr{A}_\perp^L|^2 - |\mathscr{A}_\parallel^L|^2 + 16|\mathscr{A}_{0\parallel}|^2 - 16|\mathscr{A}_{t\perp}|^2\right]$$

$$J_4 = \frac{3}{4\sqrt{2}}\left[|\mathscr{A}_0^L||\mathscr{A}_\parallel^L|^* - 8\sqrt{2}|\mathscr{A}_{\parallel\perp}||\mathscr{A}_{0\parallel}|^*\right]$$

$$J_5 = \frac{3}{2\sqrt{2}}Re\left[|\mathscr{A}_0^L||\mathscr{A}_\perp^L| + 2\sqrt{2}|\mathscr{A}_{0\parallel}||\mathscr{A}_t^L|^*\right]$$

$$J_{6s} = \frac{3}{2}Re\left[|\mathscr{A}_\parallel^L||\mathscr{A}_\perp^L|^*\right]$$

$$J_{6c} = -6Re\left[|\mathscr{A}_{\parallel\perp}||\mathscr{A}_t^L|^*\right]$$

$$J_7 = \frac{3}{2\sqrt{2}}Im\left[|\mathscr{A}_0^L||\mathscr{A}_\parallel^L|^* - 2\sqrt{2}|\mathscr{A}_{t\perp}||\mathscr{A}_t^L|^*\right]$$

$$J_8 = \frac{3}{4\sqrt{2}}Im\left[|\mathscr{A}_0^L||\mathscr{A}_\perp^L|^*\right]$$

$$J_9 = \frac{3}{4}Im\left[|\mathscr{A}_\perp^L||\mathscr{A}_\parallel^L|^*\right]$$

Observables in $\bar{B}_s \to K^{*+}(\to K\pi) l^- \bar{\nu}_l$ decay

The differential decay rate :

$$\frac{d\Gamma}{dq^2} = \left[2J_{1s} + J_{1c} - \frac{1}{3}\left(2J_{2s} + J_{2c}\right)\right]$$

The forward-backward asymmetry for lepton can be written in terms of the angular coefficients as :

$$A_{FB} = \frac{J_{6s} + \frac{1}{2}J_{6c}}{\left[2J_{1s} + J_{1c} - \frac{1}{3}\left(2J_{2s} + J_{2c}\right)\right]}$$

The Longitudenal Polarization of K^* meson can be written as :

$$F_L = \frac{J_{1c} - \frac{1}{3}J_{2c}}{J_{tot}} \quad , J_{tot} = \frac{(2J_{1s} + J_{1c}) - (2J_{2s} + J_{2c})}{3}$$

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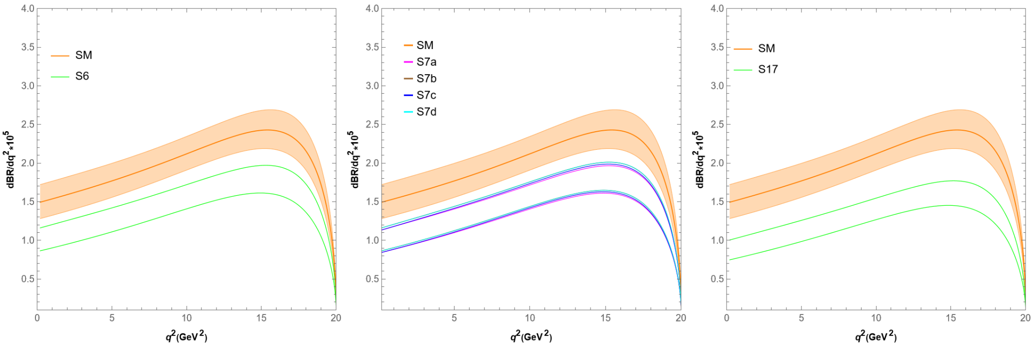
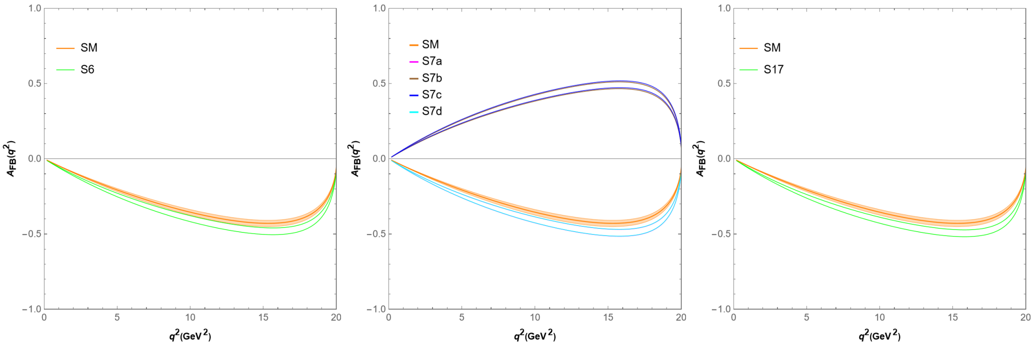
Predictions for the Differential Branching Fraction



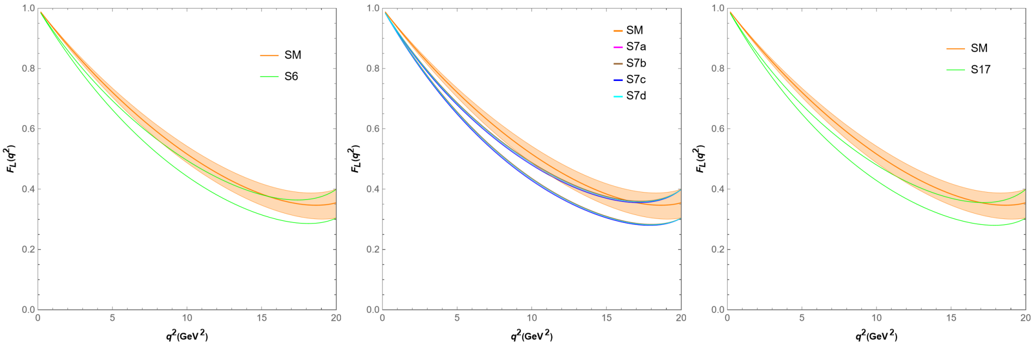
- Scenarios S6,S7 and S17 show the deviation from SM in the Branching fraction.
- The four different cases in S7 scenario can not be distinguished based on the Branching fraction.

Predictions for the Forward-Backward Asymmetry



- In AFB S6,S7 and S17 show the deviation from SM.
- S7b and S7c Scenarios can be distinguish from S7a and S7d Scenarios.

Predictions for the Logitudnal Polarization of K^* meson



- Longitudinal polarization of K^* meson shows the similar kind of deviation as in Branching Fraction.

Prediction for the INTIGRATED ANGULAR OBSERVABLES

Normalized angular observables defined as :

$$\tilde{J}_i = \frac{\int_{q^2_{min}}^{q^2_{max}} J_i(q^2)\, dq^2}{\int_{q^2_{min}}^{q^2_{max}} \frac{d\Gamma}{dq^2}\, dq^2}$$

Scenario	$\tilde{J}_{1s}$	$\tilde{J}_{1c}$	$\tilde{J}_{2s}$	$\tilde{J}_{2c}$	$\tilde{J}_3$	$\tilde{J}_4$	$\tilde{J}_5$	$\tilde{J}_{6s}$
SM	0.255(35)	0.409(47)	0.085(12)	-0.409(47)	-0.059(23)	0.194(7)	-0.283(23)	-0.311(40)
$S1$	0.247(36)	0.420(48)	0.082(12)	-0.420(48)	-0.071(23)	0.199(7)	-0.266(22)	-0.286(38)
$S2$	0.258(37)	0.405(49)	0.086(12)	-0.405(49)	-0.055(26)	0.192(10)	-0.292(30)	-0.314(45)
$S3$	0.247(36)	0.420(48)	0.082(12)	-0.420(48)	-0.071(23)	0.199(7)	-0.266(22)	-0.286(38)
$S4$	0.247(36)	0.420(48)	0.082(12)	-0.420(48)	-0.071(23)	0.199(7)	-0.266(22)	-0.286(38)
$S5$	0.247(36)	0.420(48)	0.082(13)	-0.420(49)	-0.070(23)	0.199(11)	-0.266(23)	-0.286(38)
$S6$	0.267(38)	0.395(50)	0.089(13)	-0.395(50)	-0.043(30)	0.187(12)	-0.308(34)	-0.331(49)

Scenario	$\tilde{J}_{1s}$	$\tilde{J}_{1c}$	$\tilde{J}_{2s}$	$\tilde{J}_{2c}$	$\tilde{J}_3$	$\tilde{J}_4$	$\tilde{J}_5$	$\tilde{J}_{6s}$
$S7a$	0.270(38)	0.390(51)	0.090(13)	-0.390(51)	-0.039(31)	0.185(12)	-0.314(35)	-0.338(50)
$S7b$	0.270(39)	0.390(51)	0.090(13)	-0.390(52)	-0.039(33)	0.185(13)	0.313(38)	-0.337(52)
$S7c$	0.272(39)	0.387(52)	0.091(13)	-0.387(52)	-0.035(34)	0.184(14)	0.318(38)	0.342(52)
$S7d$	0.270(38)	0.390(52)	0.090(13)	-0.390(52)	-0.039(33)	0.185(14)	-0.313(38)	-0.337(52)
$S8$	0.247(36)	0.420(48)	0.082(12)	-0.420(48)	-0.071(23)	0.199(07)	-0.266(22)	-0.286(38)
$S9$	0.247(36)	0.420(48)	0.082(12)	-0.420(48)	-0.071(23)	0.199(07)	-0.266(22)	-0.286(38)
$S10$	0.248(36)	0.419(48)	0.081(15)	-0.418(51)	-0.070(23)	0.198(16)	-0.265(23)	-0.285(38)
$S11$	0.260(37)	0.404(49)	0.087(12)	-0.404(47)	-0.053(27)	0.191(10)	-0.294(30)	-0.317(45)
$S12$	0.260(37)	0.404(49)	0.087(12)	-0.404(47)	-0.053(27)	0.191(10)	-0.294(30)	-0.317(45)
$S13$	0.258(36)	0.405(49)	0.086(12)	-0.405(49)	-0.055(26)	0.192(10)	-0.292(30)	-0.314(45)
$S14$	0.247(36)	0.420(48)	0.082(12)	-0.420(48)	-0.071(23)	0.199(07)	-0.266(22)	-0.286(38)
$S15$	0.247(36)	0.420(48)	0.082(13)	-0.419(49)	-0.070(23)	0.199(11)	-0.266(23)	-0.286(38)
$S16$	0.247(36)	0.420(48)	0.081(13)	-0.419(49)	-0.070(23)	0.199(11)	-0.267(23)	-0.286(38)
$S17$	0.273(39)	0.386(52)	0.091(13)	-0.385(52)	-0.033(34)	0.183(14)	-0.329(37)	-0.344(52)

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Conclusion

- We investigated the New Physics in the semileptonic decay $\bar{B}_s \to K^{*+}(\to K\pi)l^-\bar{\nu}_l$ induced by the quark level transition $b \to ul\nu$.
- We considered the most general effective Hamiltonian with the different possible Lorentz structures.
- The different NP wilson coefficients are constrainted by the available measurements of branching ratios of $\bar{B}^0 \to \pi^+ l^- \bar{\nu}$, $B \to \rho\, l\nu$, $B \to \omega\, l\nu$ and $B \to \mu\nu$ decays.
- We investigated the NP effects in $\bar{B}_s \to K^{*+}(\to K\pi)l^-\bar{\nu}_l$ by predicting the q^2 spectrum of Branching Ratio, Forward-Backward asymmetry and polarization fraction of K^* meson F_L. And also provide predictions for the Integrated Angular Observables.

Thank you for listening !

Appendix

The hadronic matrix elements for $B_s \to K^*$ can be written in terms of seven form factors namely $V, A_0, A_1, A_{12}, T_1, T_2$ and T_{23}. The form factors are defined by simplified series expansion in z given by Bharucha-Straub-Zwicky as

$$f_i(q^2) = \frac{1}{(1-q^2/m^2_{R,i})} \sum_k \alpha^i_k [z(q^2) - z(0)]^k, \qquad \text{Where } z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

with $t_\pm = (m_{B_s} \pm m_{K^*})$ and $t_0 = (m_{B_s} + m_{K^*})(\sqrt{m_{B_s}} - \sqrt{m_{K^*}})^2$.

f_i	J^P	$m_{R,i}/GeV$
A_0	0^-	5.279
V, T_1	1^-	5.325
A_1, T_2, A_{12}, T_{23}	1^+	5.724

Table 1: Masses of resonances required for form factor parameterizations

f_i	α^i_0	α^i_1	α^i_2
V	0.28 ± 0.02	-0.82 ± 0.19	5.08 ± 1.42
A_0	0.36 ± 0.02	-0.36 ± 0.20	8.03 ± 2.07
A_1	0.22 ± 0.01	0.24 ± 0.16	1.77 ± 0.85
A_{12}	0.27 ± 0.02	1.12 ± 0.11	3.43 ± 0.78
T_1	0.24 ± 0.01	-0.75 ± 0.15	2.49 ± 1.37
T_2	0.24 ± 0.01	0.31 ± 0.15	1.58 ± 0.93
T_{23}	0.60 ± 0.04	2.40 ± 0.27	9.64 ± 2.03

Table 2: Simplified series expansion coefficients α^i_k for parameterising the $B_s \to K^*$ form factors using the combined LCSR + Lattice fit

Appendix

The form factors for vector currents, axial vector currents and tensor currents in the helicity basis can be written as :

- Vector current

$$\mathcal{F}_\perp(q^2) = \frac{\sqrt{2\lambda}}{M_{B_s}(M_{B_s} + M_{K^*})} V(q^2)$$

- Axial vector current

$$\mathcal{F}_t(q^2) = \frac{\sqrt{\lambda}}{M_{B_s}{}^2} A_0(q^2)$$

$$\mathcal{F}_\parallel(q^2) = \sqrt{2}\,\frac{M_{B_s} + M_{K^*}}{M_{B_s}} A_1(q^2)$$

$$\mathcal{F}_0(q^2) = \frac{8 M_{K^*} A_{12}(q^2)}{M_{B_s}}$$

- Tensor current

$$\mathcal{F}_\perp^T(q^2) = \frac{\sqrt{2\lambda}}{M_{B_s}{}^2} T_1(q^2)$$

$$\mathcal{F}_\parallel^T(q^2) = \frac{\sqrt{2}(M_{B_s}{}^2 - M_{K^*}{}^2)}{M_{B_s}{}^2} T_2(q^2)$$

$$\mathcal{F}_0^T(q^2) = \frac{4 M_{K^*} T_{23}(q^2)}{M_{B_s} + M_{K^*}}$$

Appendix

The contribution from helicity amplitudes can be given as

$$\mathcal{A}_0^L = -4 \frac{M_{B_s}^2 (1 + C_{V_L} - C_{V_R}) \mathcal{F}_0(q^2)}{\sqrt{q^2}}$$

$$\mathcal{A}_\perp^L = 4 M_{B_s} (1 + C_{V_L} + C_{V_R}) \mathcal{F}_\perp(q^2)$$

$$\mathcal{A}_\parallel^L = -4 M_{B_s} (1 + C_{V_L} - C_{V_R}) \mathcal{F}_\parallel(q^2)$$

$$\mathcal{A}_t^L = -4 \Big[\frac{m_l M_{B_s}^2}{\sqrt{q^2}} (1 + C_{V_L} - C_{V_R}) + \frac{M_{B_s}^2}{m_b} (C_{S_L} - C_{S_R}) \Big] \mathcal{F}_t(q^2)$$

$$\mathcal{A}_{\parallel\perp} = +8 M_{B_s} C_T \mathcal{F}_0^T(q^2)$$

$$\mathcal{A}_{t\perp} = 4\sqrt{2} \frac{M_{B_s}^2}{\sqrt{q^2}} C_T \mathcal{F}_\perp^T(q^2)$$

$$\mathcal{A}_{0\parallel} = 4\sqrt{2} \frac{M_{B_s}^2}{\sqrt{q^2}} C_T \mathcal{F}_\parallel^T(q^2)$$