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Neutrino Oscillations in a Quantum Walk Framework

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HIGH ENERGY PHYSICS MODELING & SIMULATION

Objectives

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Neutrino oscillation

2

Quantum Simulation: A Powerful Tool

3

Project

Neutrino Oscillation

Unveiling the Flavor Flippers: The Secret Lives of Neutrinos

- Ever wondered what zips through you undetected? Neutrinos do, and they're tricksters!
- Tiny, ghostly particles that barely interact with matter
- Electron, Muon, Tau
- Neutrinos can change flavors as they travel!

The Neutrino Flavor Trio:

- Neutrinos come in three flavors: electron (ν_e), muon (ν_μ), and tau (ν_τ).
- These flavors are linked to the charged leptons they interact with (electron, muon, tau).
- Each flavor interacts differently with matter.
- Neutrino oscillation = mass.

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$$

What Makes Them Flip?

- Distance Traveled: More travel time means more chances to switch flavors!
- Neutrino Mass: Different weights (masses) lead to different flip frequencies.
- Neutrino Energy: High-energy neutrinos are less likely to change flavors.

It's like a pachinko machine!!

$$P(\nu_\alpha \to \nu_\beta; t) = \left| \sum_j U^*_{\alpha j} U_{\beta j} e^{-iE_j t} \right|$$

Quantum Simulation: A Powerful Tool

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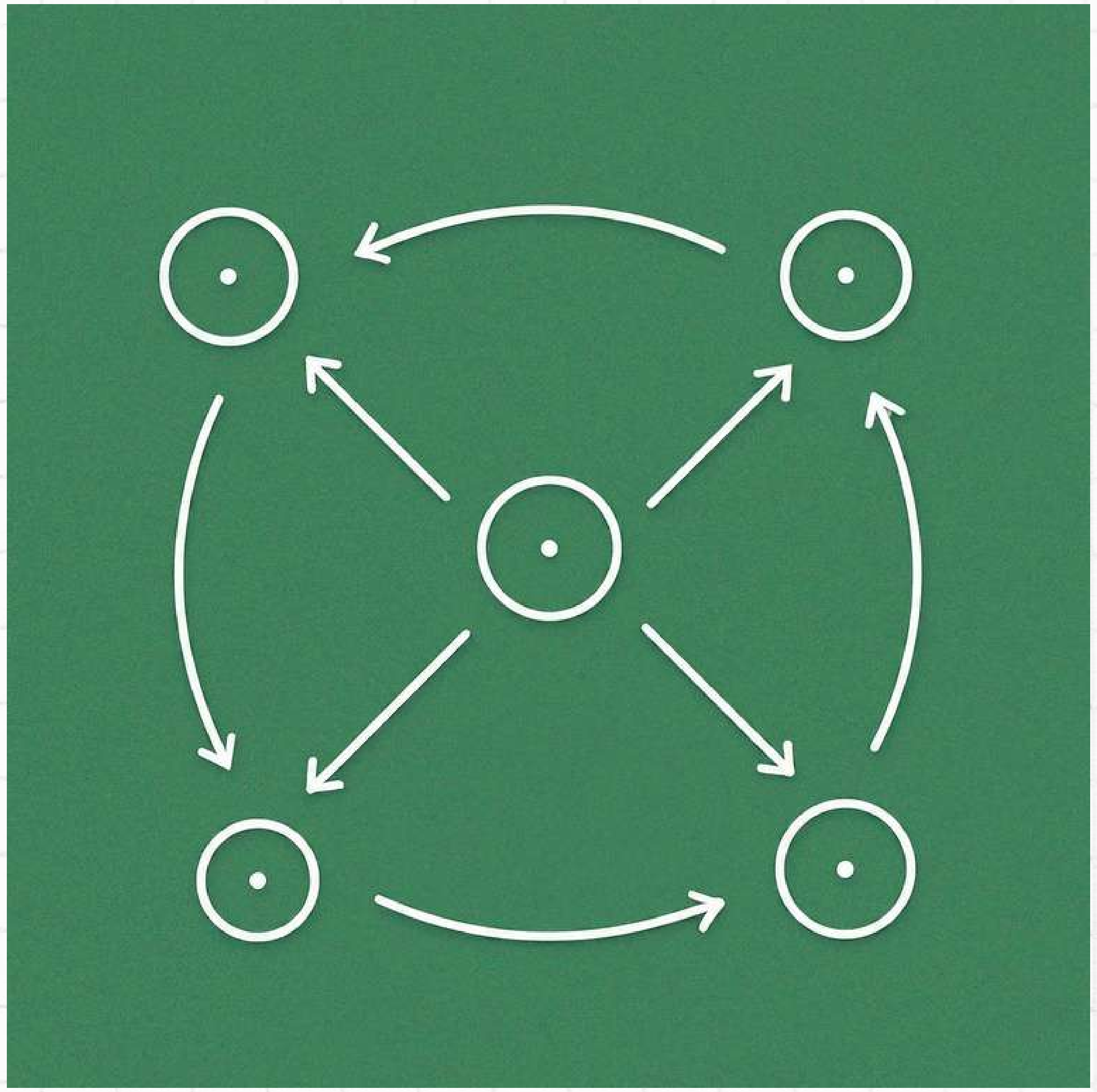
Discrete-Time Quantum Walks (DTQW)

$$|\Psi(t)\rangle = |\uparrow\rangle \otimes |\Psi^{\uparrow}(t)\rangle + |\downarrow\rangle \otimes |\Psi^{\downarrow}(t)\rangle = \sum_x \begin{bmatrix} \psi^{\uparrow}_{x,t} \\ \psi^{\downarrow}_{x,t} \end{bmatrix}. \quad (1)$$



Hilbert space H = Hc ⊗ Hp;
Hc is coin Hilbert space,
Hp is the position Hilbert space.

Hc is spanned by the basis set | ↑ 〉 & | ↓ 〉
representing the internal degree of the walker,
Hp is spanned by the basis state of the position |x〉.

The state at time (t + τ) where τ is the time required to implement one step of the walk

$$|\Psi(t+\tau)\rangle = S(C \otimes I)|\Psi(t)\rangle = W|\Psi(t)\rangle \qquad (2)$$

The coin operator

$$\begin{aligned} C = C(\xi,\theta,\varphi,\delta) &= e^{i\xi}e^{-i\theta\sigma_x}e^{-i\varphi\sigma_y}e^{-i\delta\sigma_z} \\ &= e^{i\xi}\begin{pmatrix} e^{-i\delta}(c_\theta c_\varphi - i s_\theta s_\varphi) & -e^{i\delta}(c_\theta s_\varphi + i s_\theta c_\varphi) \\ e^{-i\delta}(c_\theta s_\varphi - i s_\theta c_\varphi) & e^{i\delta}(c_\theta c_\varphi + i s_\theta s_\varphi) \end{pmatrix} \\ &= e^{i\xi}\begin{pmatrix} F_{\theta,\varphi,\delta} & G_{\theta,\varphi,\delta} \\ -G^*_{\theta,\varphi,\delta} & F^*_{\theta,\varphi,\delta} \end{pmatrix} \end{aligned} \qquad (3)$$

where ξ is global phase angle, 2θ, 2φ, 2δ are the angles of rotations along x, y and z axes respectively and σi is the ith component of the Pauli spin matrices

The shift operator

$$S = |\downarrow\rangle\langle\downarrow| \otimes T_+ + |\uparrow\rangle\langle\uparrow| \otimes T_- = \begin{bmatrix} T_+ & 0 \\ 0 & T_- \end{bmatrix} \quad (4)$$

where

$$T_\pm = \sum_{x \in \mathbb{Z}} |x \pm a\rangle\langle x| \quad (5)$$

The initial state of the walker

$$|\Psi(0)\rangle = |\chi\rangle \otimes |\psi_x\rangle. \quad (6)$$

The internal spin state

$$|\chi\rangle \in \mathcal{H}_c = \{a_\uparrow|\uparrow\rangle + a_\downarrow|\downarrow\rangle : a_{\uparrow/\downarrow} \in \mathbb{C}\} \quad (7)$$

The position state

$$|\psi_x\rangle \in \mathcal{H}_p = \left\{\sum_{x\in\mathbb{Z}} c_x|x\rangle : \sum_{x\in\mathbb{Z}} |c_x|^2 < \infty\right\}. \quad (8)$$

After t steps

$$|\Psi(t)\rangle = W^t|\Psi(0)\rangle = W^t\left(|\chi\rangle \otimes |\psi_x\rangle\right) \quad (9)$$

The density matrix representation of the state

$$\begin{aligned}\rho(t) &= |\Psi(t)\rangle\langle\Psi(t)| \\ &= W^t|\Psi(0)\rangle\langle\Psi(0)|(W^\dagger)^t.\end{aligned} \quad (10)$$

we can trace over the position space to get

$$\begin{aligned}\rho_c(t) &= \sum_{x=-t}^{t} \langle x|W^t|\Psi(0)\rangle\langle\Psi(0)|(W^\dagger)^t|x\rangle \\ &= \sum_{x=-t}^{t} \langle x|W^t|\psi_x\rangle|\chi\rangle\langle\chi|\langle\psi_x|(W^\dagger)^t|x\rangle \\ &= \sum_{x=-t}^{t} \tilde{K}_x(t)\rho_c(0)\tilde{K}_x^\dagger(t)\end{aligned} \quad (11)$$

where

$$\tilde{K}_x(t) \equiv \langle x|W^t|\psi_x\rangle \quad (12)$$

we break down the evolution operator W in shift and coin operator

$$\begin{aligned} W &= S(C \otimes I) \\ &= [|\uparrow\rangle\langle\uparrow| \otimes T_- + |\downarrow\rangle\langle\downarrow| \otimes T_+][C \otimes 1] \\ &= |\uparrow\rangle\langle\uparrow|C \otimes T_- + |\downarrow\rangle\langle\downarrow|C \otimes T_+ \\ &= C_\uparrow \otimes T_- + C_\downarrow \otimes T_+. \end{aligned} \quad (13)$$

Simulating the Neutrino Dance

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Two-flavor neutrino oscillations

In this case, we need to mimic the dynamics of two Dirac particles and therefore the coin hilbert space is four dimensional space spanned by :

$$\bigoplus_{f=1,2}\{|f,\uparrow\rangle,|f,\downarrow\rangle\}.$$

The evolution operator W has a block diagonal form

$$W=\bigoplus_{f=1,2} W_f = S(B_2\otimes I)=\bigoplus_{f=1,2} S_f(B_f\otimes I) \quad (14)$$

$$B_f=\cos\theta_f|f,\uparrow\rangle\langle f,\uparrow|+\sin\theta_f\,(|f,\uparrow\rangle\langle f,\downarrow|-|f,\downarrow\rangle\langle f,\uparrow|)+\cos\theta_f|f,\downarrow\rangle\langle f,\downarrow|. \quad (15)$$

$B \equiv C(0, \theta, 0, 3\pi/2)$

$$S_f=T_+\otimes|f,\uparrow\rangle\langle f,\uparrow|+T_-\otimes|f,\downarrow\rangle\langle f,\downarrow|. \quad (16)$$

The mass eigenstates

$$|\nu_1\rangle = \begin{bmatrix} f(\theta_1,k) & g(\theta_1,k) & 0 & 0 \end{bmatrix}^T \otimes |k\rangle \equiv |\nu_1\rangle_c \otimes |k\rangle$$
$$|\nu_2\rangle = \begin{bmatrix} 0 & 0 & f(\theta_2,k) & g(\theta_2,k) \end{bmatrix}^T \otimes |k\rangle \equiv |\nu_2\rangle_c \otimes |k\rangle \quad (17)$$

The initial state $|\Psi(0)\rangle$ of the neutrino corresponding to a flavor using the mixing matrix acting on each sector

$$|\Psi(0)\rangle = |\nu_\alpha\rangle = \sum_{i=1,2} U_{\alpha i}|\nu_i\rangle. \quad (18)$$

The associated reduced coin density matrix

$$\rho_c(0) = \sum_{i,j} U_{\alpha i} U^*_{\alpha j} |\nu_i\rangle_c \langle \nu_j|_c. \quad (19)$$

The Kraus operator for two particle

$$\tilde{\mathcal{K}}_x(t) = \bigoplus_{f=1,2} \langle x|W_f|\psi_x\rangle = \bigoplus_{f=1,2} \tilde{K}_x(\theta_f, t) \quad (20)$$

where state $|\psi x\rangle$ is momentum eigenstate k in position space representation.

The reduced density matrix

$$\rho_c(t) = \sum_x \tilde{\mathcal{K}}_x^{(2)}(t) \rho_c(0) (\tilde{\mathcal{K}}_x^{(2)})^\dagger(t) \quad (21)$$

The probability of the $\nu\alpha \rightarrow \nu\beta$ transition after a time t

$$P(\nu_\alpha \rightarrow \nu_\beta; t) = \mathrm{Tr}\left[|\nu_\beta\rangle_c \langle \nu_\beta|_c \rho_c(t)\right] \quad (22)$$

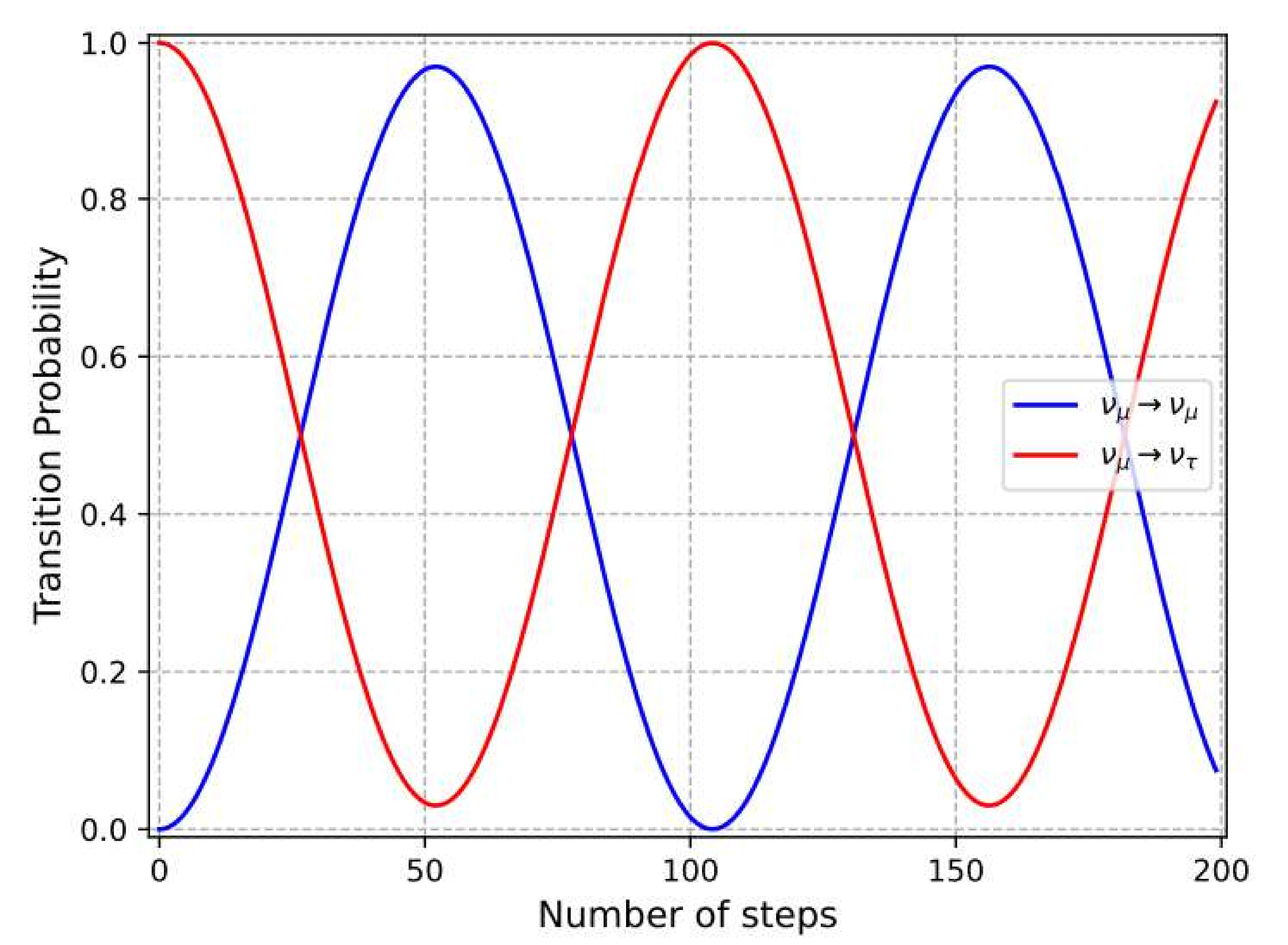
where

$$|\nu_\beta\rangle_c = \sum_{i=1,2} U_{\beta i} |\nu_i\rangle_c \quad (23)$$

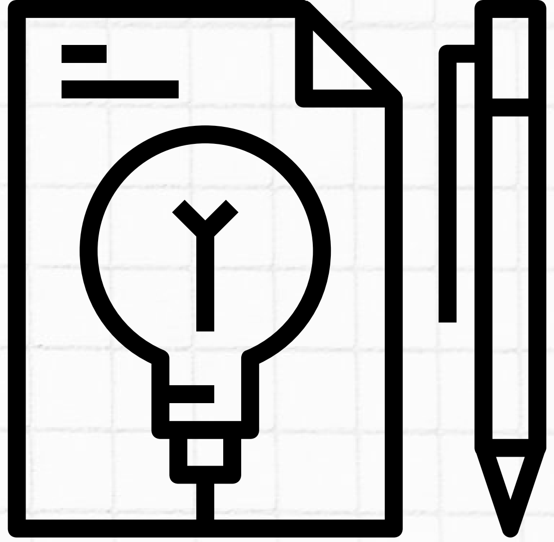
so that

$$|\nu_\beta\rangle_c \langle \nu_\beta|_c = \sum_{i,j=1,2} U_{\beta i} U^*_{\beta j} |\nu_i\rangle_c \langle \nu_j|_c . \quad (24)$$

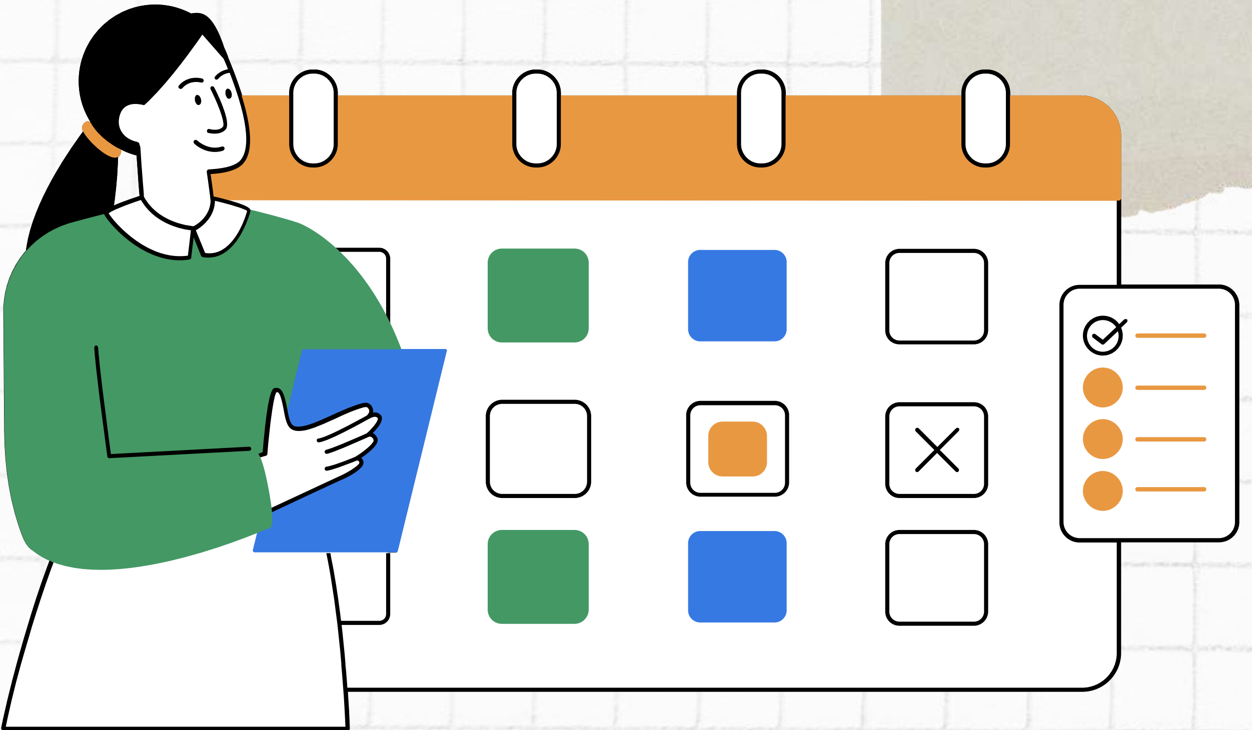
Results



Transition probabilities of two flavor neutrino oscillation obtained from numerical simulation using the Kraus operator associated with the DTQW with initial state $|\nu\mu\rangle$. The coin angles are $\theta 1 = 0.001$ rad., $\theta 2 = 0.0986$ rad., and the mixing angle $\phi = 0.698$ rad. with $\tilde{k} = 0.05$.



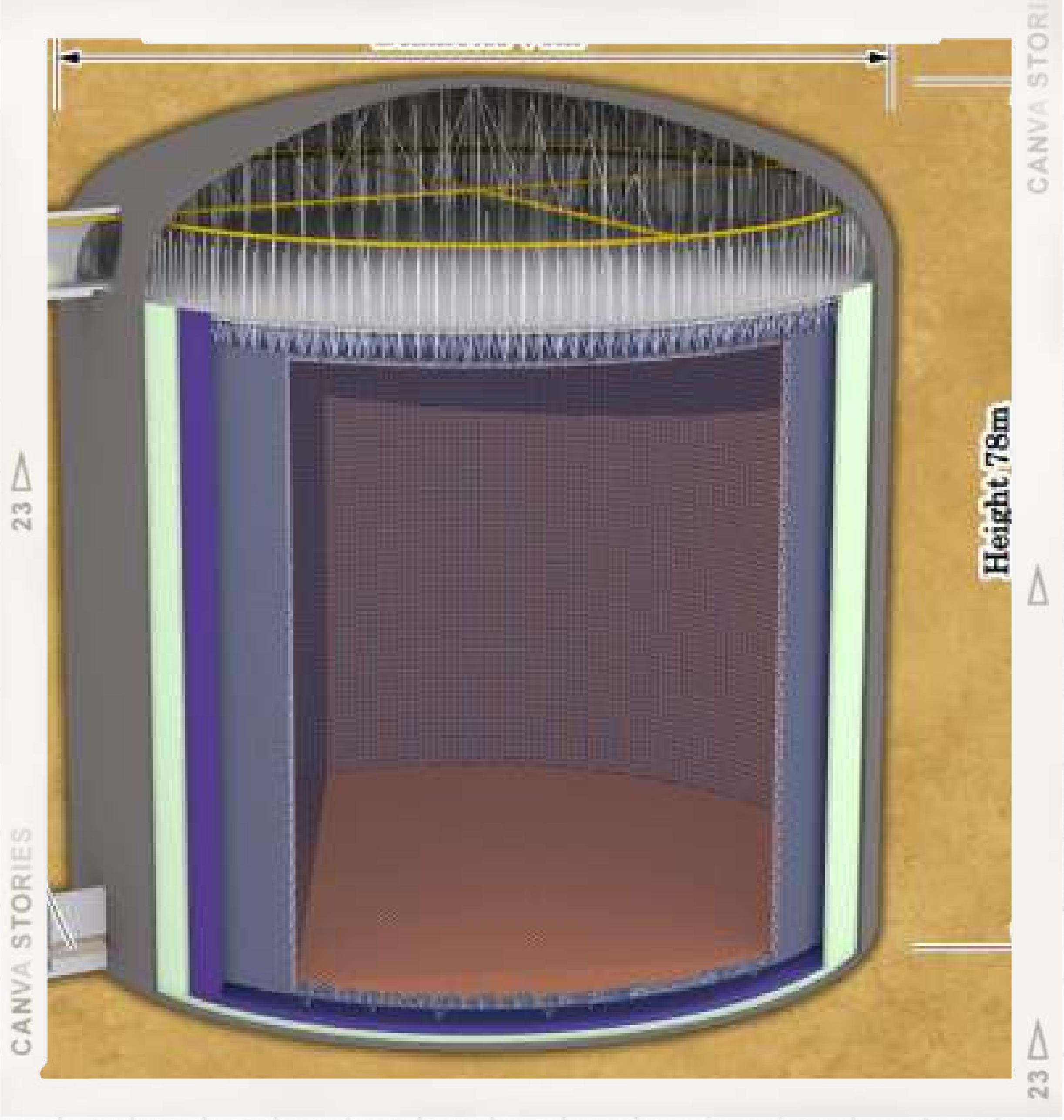
The Project



Hyper-kamiokade

Hyper-Kamiokande

- Building on the success of Super-K
- Larger and more precise
- Study ν oscillations
- Determine the exact masses of ν



ν oscillations by Super-K

Prize Nobel !!!

1987	2015	2027	2030
		Expected	Expected

Kamiokande detects ν from Supernova 1987A

Hyper-K operations

Conclusion

Unveiling the Future - Exploring the Potential

- Novel approach to simulating neutrino oscillations by leveraging the power of DTQW.
- Open quantum walk framework unlocks efficient calculations of neutrino oscillations through reduced coin space dynamics.
- Emphasize the potential of this approach for studying neutrino oscillations.

Future Directions

- Extend the model to describe the dynamics of multiple interacting neutrinos.
- Investigating the application to study more complex neutrino interactions.
- Exploring the potential for designing new neutrino oscillation experiments.

Thank you