PARTICLES TRAJECTORIES AROUND MAGNETIZED BLACK HOLES EMBEDDED IN QUINTESSENTIAL MATTER

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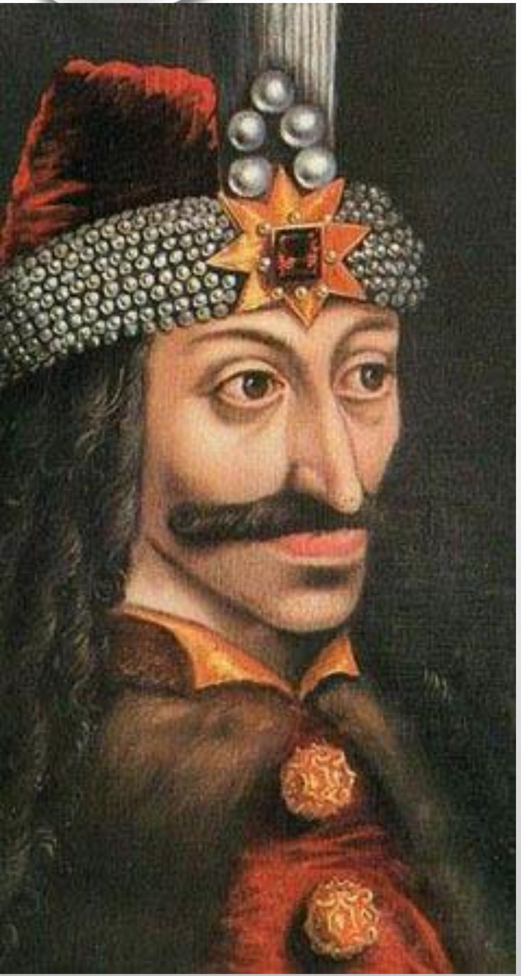


HARTA ROMÂNIEI
LEGENDĂ
DRUM EUROPEAN
DRUM NAȚIONAL
LIMITĂ DE JUDEȚ
ARAD REȘEDINȚĂ DE JUDEȚ
ORAȘ
ALTE LOCALITĂȚI
PUNCT VAMAL
REZERVAȚII, MONUMENTE ALE NATURII
PRACTICAREA SPORTURILOR DE IARNĂ
MĂNĂSTIRI
UCRAINA
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MAREA NEAGRĂ
Spre Budapesta, Viena
E 60
Spre Beograd, Zagreb, Milano
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Spre Chișinău, Moscova
Spre Sofia, Istanbul, Atena
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Spre Varna, Istanbul
E 87
SATU MARE
BAIA MARE
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ORADEA
CLUJ NAPOCA
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RÂMNICU VÂLCEA
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DROBETA TURNU SEVERIN
CRAIOVA
SLATINA
BUCUREȘTI
SLOBOZIA
CĂLĂRAȘI
CONSTANȚA
ALEXANDRIA
GIURGIU
BEOGRAD
ARHI DESIGN
ARVES

Accelerating expansion of our universe

Astrophysical observations from supernovae (Type Ia), cosmic microwave background radiation (CMBR), Baryon acoustic oscillations (BAO) and the Hubble measurements are suggesting an accelerating expansion of our universe, which may be explained by the presence of dark energy.

DARK ENERGY

One of the candidates for dark energy is ***the quintessence***.

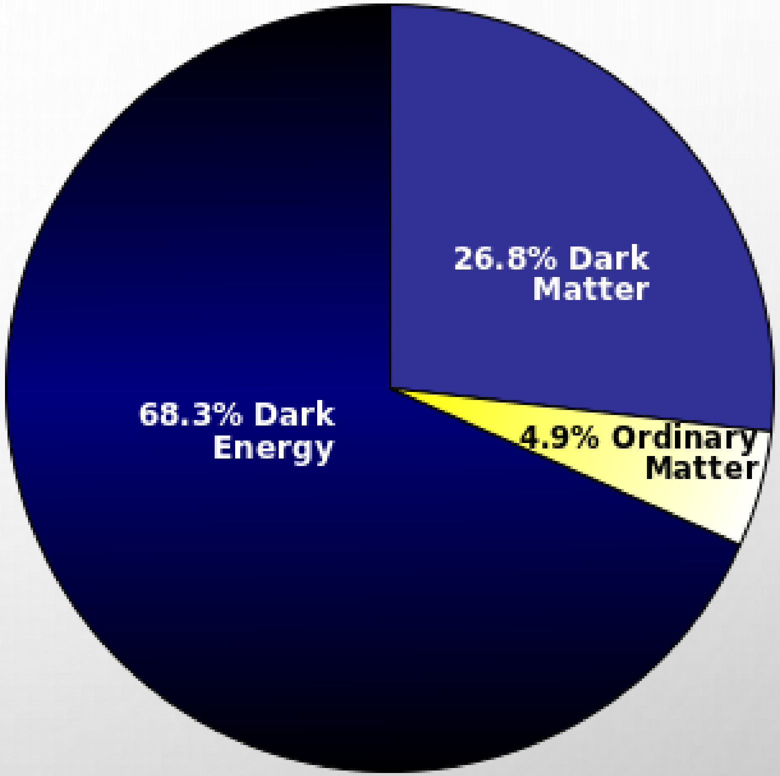
AN ANISOTROPIC FLUID

EQUATION OF STATE $$p = w\rho$$

IN ORDER TO CAUSE THE ACCELERATED EXPANSION OF THE UNIVERSE:

$$-1 < w < -1/3$$

COMOLOGICAL CONSTANT $$w = -1$$



Kiselev SOLUTION

The Kiselev geometry is sourced by an anisotropic fluid.

The static four-dimensional line-element

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

The metric function, k is a positive quintessence parameter

$$g(r) = 1 - \frac{2M}{r} - \frac{k}{r^{3w+1}}$$

$$w \in [-1, -1/3]$$

$$\rho = -\frac{3kw}{r^{3(w+1)}} = -p_r$$

$$p_\theta = p_\varphi = -\frac{3(3w+1)kw}{2r^{3(w+1)}}$$

Kiselev V. V. (2003). Quintessence and black holes. Classical and Quantum Gravity, 20(6), 1187–1197.

$$g(r) = 1 - \frac{2M}{r} - \frac{k}{r^{3w+1}}$$

$$\boxed{w = -\frac{1}{3}} \Rightarrow g(r) = 1 - k - \frac{2M}{r} \Rightarrow r_b = \frac{2M}{1-k}$$

$$\boxed{w = -1} \Rightarrow g(r) = 1 - \frac{2M}{r} - kr^2$$

$$-kr^3 + r - 2M = 0$$

$$\boxed{w = -\frac{2}{3}} \Rightarrow g(r) = 1 - \frac{2M}{r} - kr$$

$$r_\pm = \frac{1 \pm \sqrt{1 - 8kM}}{2k}, \quad r \in [r_-, r_+]$$

$$r_- \approx 2M(1 + 2kM), \quad r_+ \approx \frac{1}{k} - 2M$$

KISELEV SOLUTION IN POWER-MAXWELL ELECTRODYNAMICS

$$L = -F^2 \rightarrow L = -\alpha (F_{ij}F^{ij})^q$$

$$G_{ij} = T_{ij}$$

$$\partial_i (\sqrt{-g} F^{ij} F^{q-1}) = 0$$

$$T_{ij} = 2\alpha \left[qF_{ik}F_j^{\,k}F^{q-1} - \frac{1}{4} g_{ij} F^q \right], \quad \rho = \frac{\alpha F^q (1-2q)}{2} > 0$$

$$\boxed{g = 1 - \frac{2M}{r} - kr^p}$$

$$p = -(3w+1)$$

$$p \in \; 0,2$$

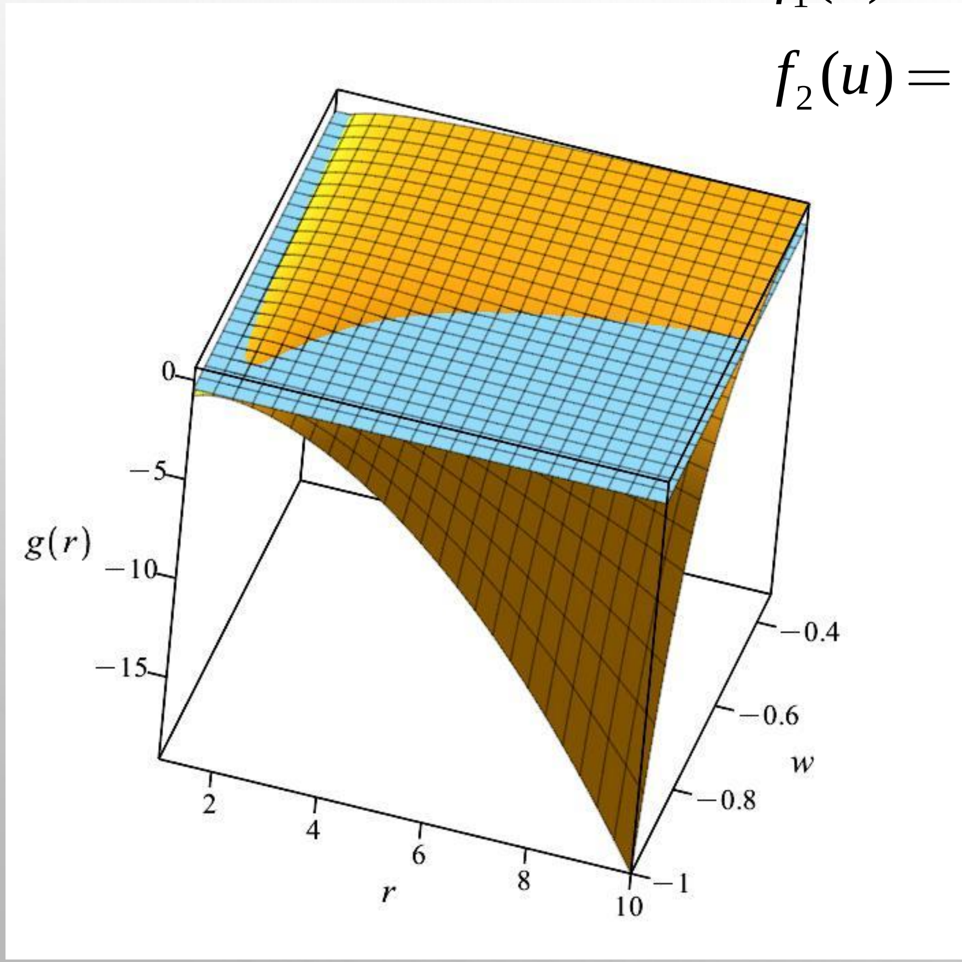
Dariescu M.-A., Dariescu C., Lungu V. & Stelea C. (2022).
Kiselev solution in power-Maxwell Electrodynamics. Physical Review D, *106*(6), 064017.

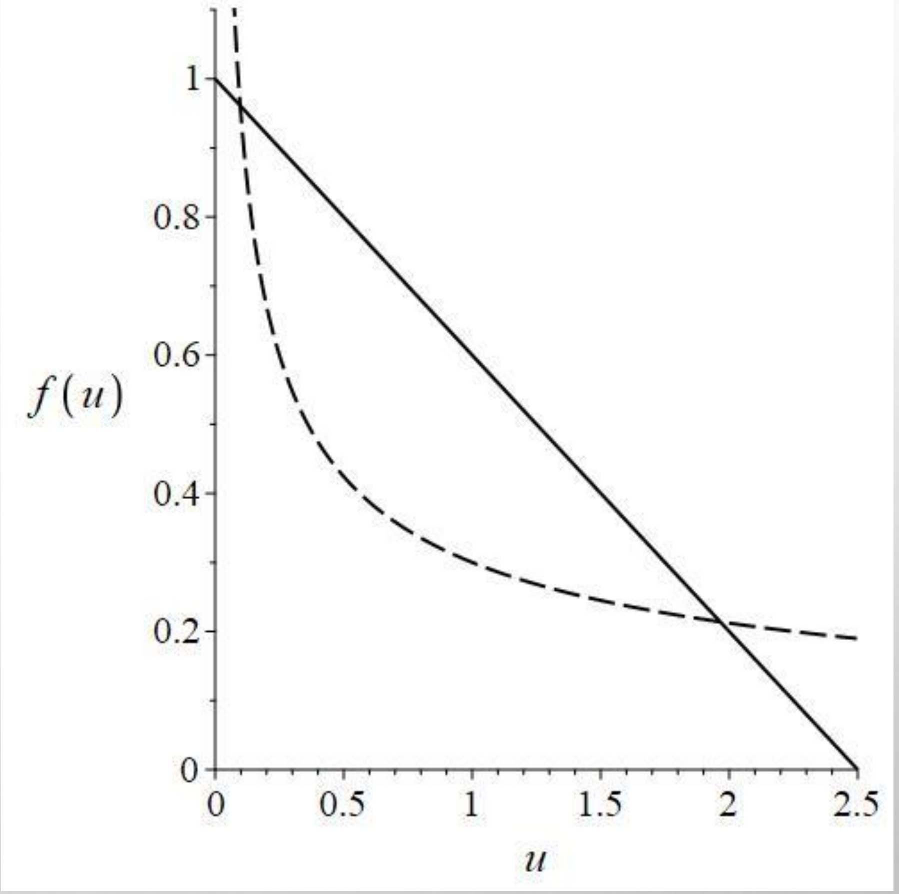
THE LOCATION OF THE HORIZONS

$$g = 1 - \frac{2M}{r} - kr^{p}$$

$$f_1(u) = 1 - 2Mu$$

$$f_2(u) = ku^{-p}$$





Intersection of $f_1(u)$ and $f_2(u)$ with *k=0.3* and *p=0.5*.

ELECTRIC ANSATZ

Nonlinear electric fields with the potential

$$g(r) = 1 - \frac{2M}{r} - kr^p$$

$$A_i = (\chi(r), 0, 0, 0)$$

$$\chi(r) = C_1 + C_2 r^{p+1}$$

$$C_2 = \frac{Q_e^{-p/2}}{2^{(p+2)/4}(p+1)}$$

$$k = \frac{2^{(2-p)/4} Q_e^{(2-p)/2}}{p(p+1)}$$

$$q = \frac{p-2}{2p} < 0$$

MAGNETIC ANSATZ

a magnetic monopole ansatz for the electromagnetic potential

$$A_i = (0,0,0,Q_m(1-\cos\theta))$$

$$Q_m = \frac{1}{4\pi}\int_{S^2} F$$

$$\rho = \frac{\alpha F^q}{2}$$

$$k = \frac{(2Q_m^2)^{(2-p)/4}}{2(p+1)}$$

$$q = \frac{2-p}{4}$$

THE MAGNETIC ANSATZ

$$\mathrm{L} = \frac{1}{2} g_{ij}\dot{x}^j \dot{x}^k + \varepsilon Q_m (1 - \cos\theta)\dot{\varphi}$$

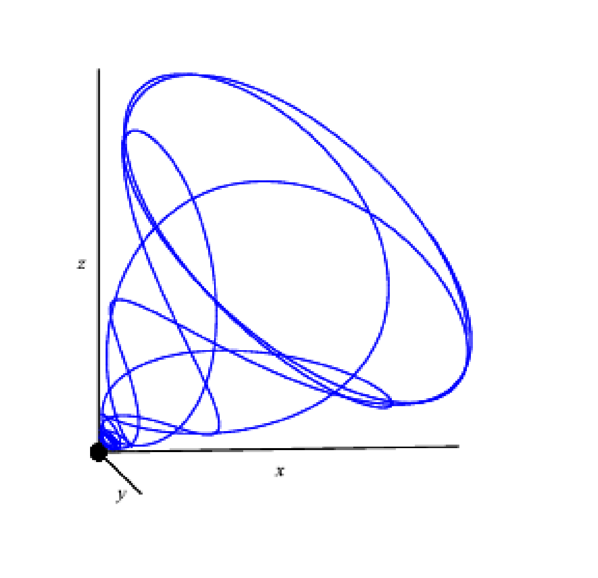
$$\frac{dt}{d\tau} = \frac{E}{g(r)}$$

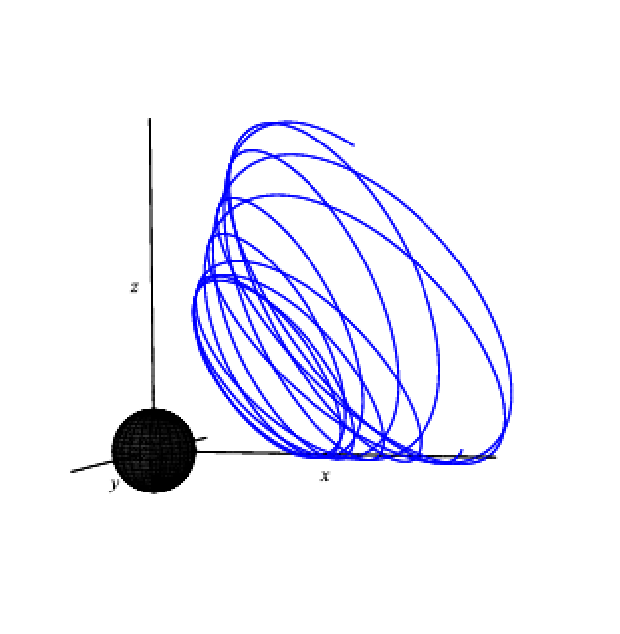
$$\frac{d\varphi}{d\tau} = \frac{1}{r^2 \sin^2\theta}[L - \varepsilon Q_m (1 - \cos\theta)]$$

$$K = r^4 \left(\frac{d\theta}{d\tau}\right)^2 + r^4 \sin^2\theta \left(\frac{d\varphi}{d\tau}\right)^2 \Rightarrow \left(\frac{d\theta}{d\tau}\right)^2 = \frac{1}{r^4}\left[K - \frac{[L - \varepsilon Q_m (1 - \cos\theta)]^2}{\sin^2\theta}\right]$$

$$\ddot{r} = \frac{\dot{r}^2 f'}{2f} - \frac{f' E^2}{2f} + r f \dot{\theta}^2 + \frac{f\left[L - \varepsilon Q_m (1-\cos\theta)\right]^2}{r^3 \sin^2\theta},$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} + \frac{\cos\theta\left[L - \varepsilon Q_m (1-\cos\theta)\right]^2}{r^4 \sin^3\theta} + \frac{\varepsilon Q_m\left[L - \varepsilon Q_m (1-\cos\theta)\right]}{r^4 \sin\theta},$$





M. A. Dariescu, C. Dariescu, V. Lungu and C. Stelea, Phys. Rev. D 106 (6) (2022) 064017

CHARGED PARTICLE DYNAMICS AROUND BLACK HOLE WITH QUINTESSENCE SURROUNDED BY A MAGNETIC FIELD

- THERE ARE EVIDENCE OF THE EXISTENCE OF LARGE SCALE MAGNETIC FIELDS.
- THE LARGE SCALE MAGNETIC FIELDS INFLUENCES THE FORMATION OF GALAXIES AND LARGE SCALE STRUCTURES.
- ELECTRICALLY/MAGNETICALLY CHARGED BLACK HOLES PLAY A SIGNIFICANT ROLE IN THE PARTICLE DYNAMICS.
- THE STRONG ELECTROMAGNETIC FIELDS MAY AFFECT THE SPACETIME GEOMETRY AROUND BLACK HOLES.
- IN THE CASE OF WEAK FIELD REGIME, ONE EXPECTS THAT THE CHARGED PARTICLES DYNAMICS WOULD BE AFFECTED BY THE ELECTROMAGNETIC FORCES.

KISELEV SOLUTION

THE KISELEV GEOMETRY IS CHARACTERIZED BY THE LINE ELEMENT

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$f(r) = 1 - \frac{2M}{r} - \frac{k}{r^{3w+1}}$$

$$f(r) = 1 - \frac{2M}{r} - kr$$

$$r_{\pm} = \frac{1 \pm \sqrt{1-8kM}}{2k}$$

MAGNETIZED BLACK HOLE SOLUTIONS
(AXIAL SYMMETRIC SOLUTION)

$$ds^2 = \Lambda_m^2\left[-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2\right] + \frac{r^2\sin^2\theta}{\Lambda_m^2}d\varphi^2$$

$$\Lambda_m = 1 + B_0^2 r^2 \sin^2\theta, \quad f(r) = 1 - \frac{2M}{r} - kr$$

THE SOURCE IS AN ANISOTROPIC (CHARGED) FLUID WITH THE CURRENT

$$J^i = \frac{2\rho B_0 r^2 \sin^2\theta}{\Lambda_m^2}\delta_\varphi^i$$

THE EINSTEIN-MAXWELL EQUATIONS

$$G_{ij} = 8\pi T_{ij}^{em} + 8\pi T_{ij}^{fluid}, \ \nabla_j F^{ij} = 4\pi J^i, \ \nabla_j(\star F)^{ij} = 0$$

$$T_{ij}^{em} = \frac{1}{4\pi}\left(F_{ik}F_j^l - \frac{1}{4}F_{kl}F^{kl}g_{ij}\right)$$

$$F_{ij} = \nabla_i A_j - \nabla_j A_i$$

$$\star F^{ij} = \frac{1}{2}\epsilon_{ijkl}F^{kl}$$

$$T_{ij}^{fluid} = \rho u_i u_j + p_r\chi_i\chi_j + p_\theta\xi_i\xi_j + p_\varphi\zeta_i\zeta_j + 2p_{r\theta}\chi_i\xi_j$$

C. Stelea, M. A. Dariescu and C. Dariescu, Phys. Rev. D 98(12) (2018) 124022

C. Stelea, M. A. Dariescu and C. Dariescu Phys. Lett. B, 847 (2023) 138275

- THE ELECTRIC POTENTIAL VANISHES AND REMAINS ONLY THE MAGNETIC ONE
- THE COMPONENTS OF THE POLOIDAL MAGNETIC FIELD

$$B_r = -\frac{2B_0 r \sin\theta}{\Lambda_m}, \quad B_\varphi = \frac{2B_0 r \sin^2\theta}{\Lambda_m}$$

- THE MAGNETIC POTENTIAL

$$A_i = \left(0, 0, 0, \frac{B_0 r^2 \sin^2\theta}{\Lambda_m} \right)$$

EQUATIONS OF MOTION AND ORBITS

$$\mathcal{L}=\frac{1}{2}\left[-f\Lambda_m^2\dot{t}^2+\frac{\Lambda_m^2}{f}\dot{r}^2+\Lambda_m^2r^2\dot{\theta}^2+\frac{r^2\sin^2\theta}{\Lambda_m^2}\dot{\varphi}^2\right]+\frac{\varepsilon B_0r^2\sin^2\theta}{\Lambda_m}\dot{\varphi}$$

$$\dot{t}=\frac{E}{f\Lambda_m^2},\quad \dot{\varphi}=\frac{\Lambda_m^2}{r^2\sin^2\theta}\left(L-\frac{\varepsilon B_0r^2\sin^2\theta}{\Lambda_m}\right)$$

- THE RADIAL EQUATION:

$$\ddot{r}=\left(\frac{f'}{2f}-\frac{\partial_r\Lambda_m}{\Lambda_m}\right)\dot{r}^2-\frac{E^2}{\Lambda_m^4}\left(\frac{f'}{2f}+\frac{\partial_r\Lambda_m}{\Lambda_m}\right)-\frac{2\partial_\theta\Lambda_m}{\Lambda_m}\dot{\theta}\dot{r}+fr\left(1+\frac{r\partial_r\Lambda_m}{\Lambda_m}\right)\dot{\theta}^2+\frac{f}{r^3\sin^2\theta}\left(1-\frac{r\partial_r\Lambda_m}{\Lambda_m}\right)\left(L-\frac{\varepsilon B_0r^2\sin^2\theta}{\Lambda_m}\right)^2+\frac{f\varepsilon B_0}{\Lambda_mr}\left(2-\frac{r\partial_r\Lambda_m}{\Lambda_m}\right)\left(L-\frac{\varepsilon B_0r^2\sin^2\theta}{\Lambda_m}\right)$$

- THE ANGULAR EQUATION

$$\ddot{\theta}=\frac{\partial_\theta\Lambda_m}{\Lambda_m}\left(\frac{\dot{r}^2}{fr^2}-\dot{\theta}^2\right)-\frac{2}{r}\left(1+\frac{r\partial_r\Lambda_m}{\Lambda_m}\right)\dot{\theta}\dot{r}-\frac{E^2\partial_\theta\Lambda_m}{f\Lambda_m^5r^2}+\frac{1}{r^4\sin^2\theta}\left(\cot\theta-\frac{\partial_\theta\Lambda_m}{\Lambda_m}\right)\left(L-\frac{\varepsilon B_0r^2\sin^2\theta}{\Lambda_m}\right)^2+\frac{\varepsilon B_0}{r^2\Lambda_m}\left(2\cot\theta-\frac{\partial_\theta\Lambda_m}{\Lambda_m}\right)\left(L-\frac{\varepsilon B_0r^2\sin^2\theta}{\Lambda_m}\right)$$

TIMELIKE TRAJECTORIES

$$ds^2 = -\frac{f(r)}{\Lambda_m^2}dt^2 + \Lambda_m^2\left[\frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] = -c^2 d\tau^2$$

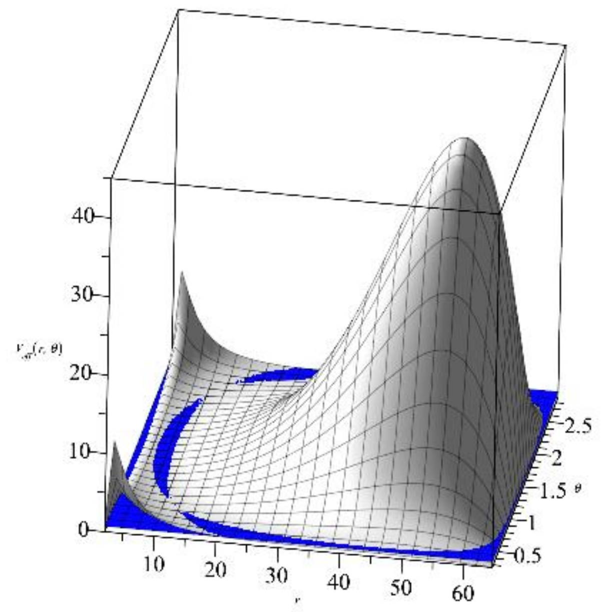
$$\frac{\Lambda_m^2}{f}\dot{r}^2 + \Lambda_m^2 r^2\dot{\theta}^2 + \frac{\Lambda_m^2}{r^2\sin^2\theta}\left(L - \frac{\varepsilon B_0 r^2\sin^2\theta}{\Lambda_m}\right)^2 - \frac{E^2}{f\Lambda_m^2} = -1$$

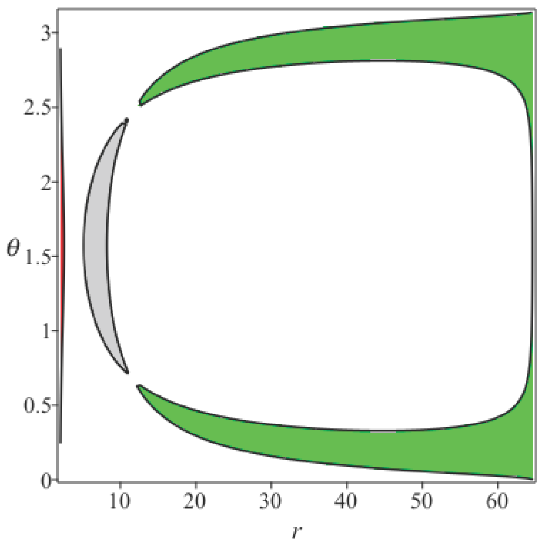
$$\Lambda_m^4\left(\dot{r}^2 + fr^2\dot{\theta}^2\right) + V_{eff}(r) = E^2$$

$$\Downarrow$$

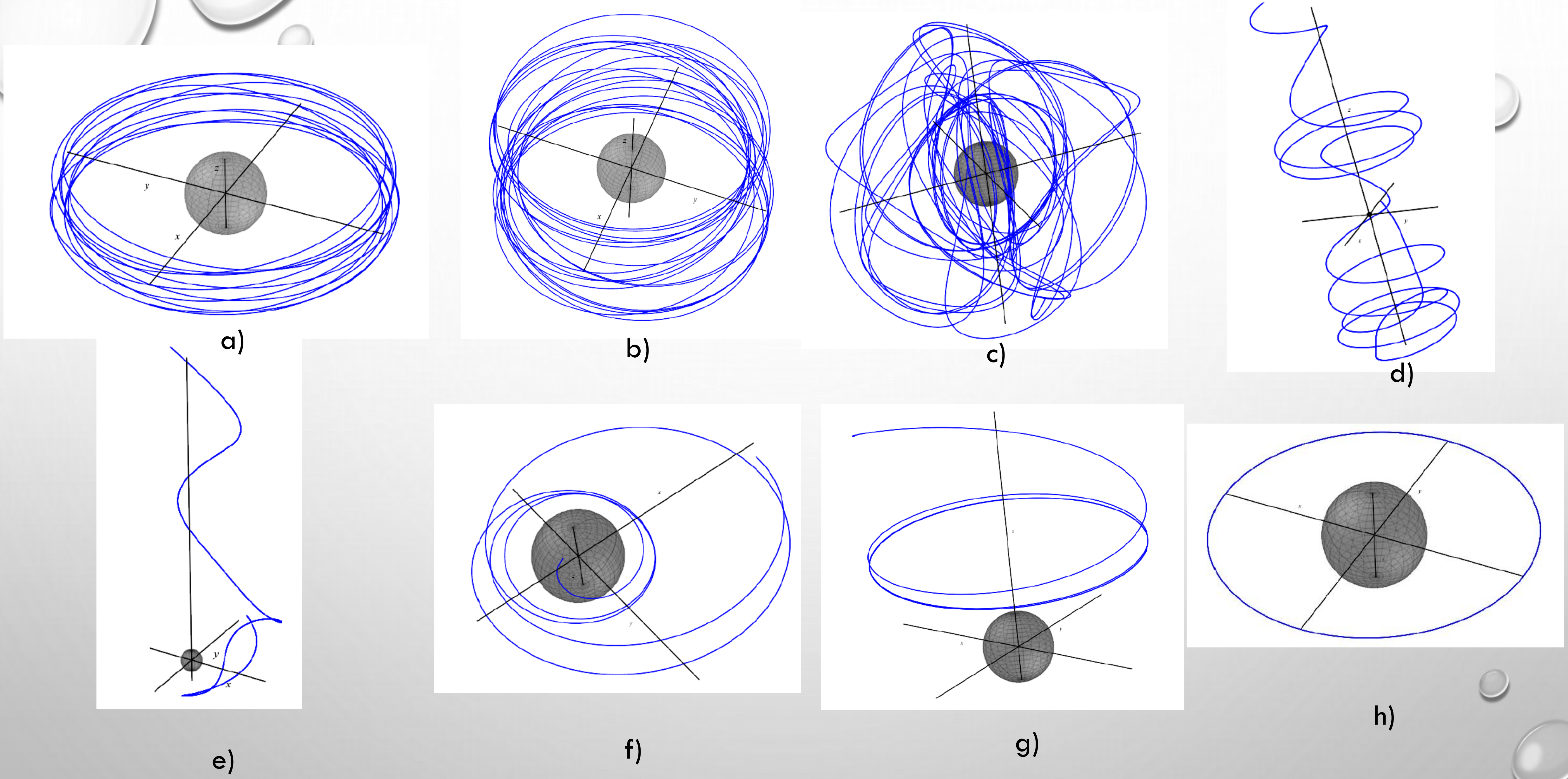
$$\boxed{V_{eff}(r,\theta) = f\Lambda_m^2\left[1 + \frac{\Lambda_m^2}{r^2\sin^2\theta}\left(L - \frac{\varepsilon B_0 r^2\sin^2\theta}{\Lambda_m}\right)^2\right]}$$

$$\boxed{E^2 \geq V_{eff}(r,\theta)}$$

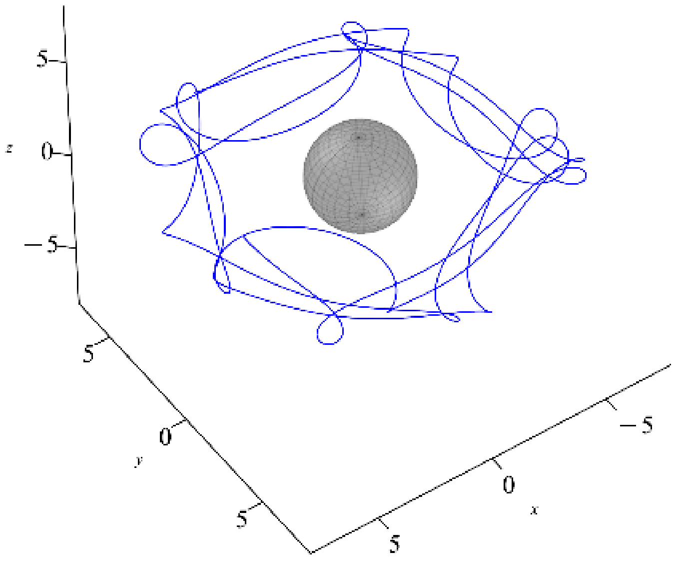




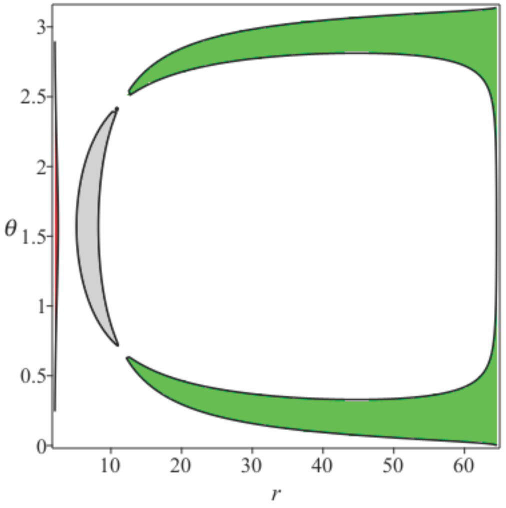
$$V_{eff}(r,\theta) = f\,\Lambda_m^2\left[1+\frac{\Lambda_m^2}{r^2\sin^2\theta}\left(L-\frac{\varepsilon B_0 r^2\sin^2\theta}{\Lambda_m}\right)^2\right]$$



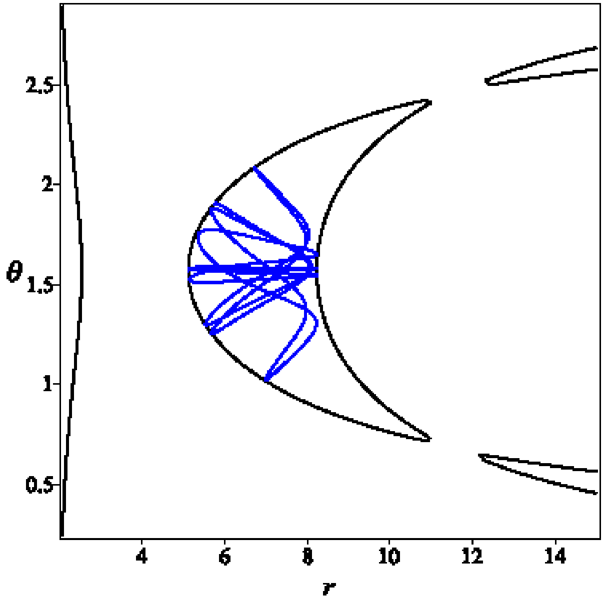
Different type of trajectories of charged particles. a)-c) Bound orbits, d)-g) Escape orbits, h) Stable circular orbit.



5
0
−5
z
5
0
y
5
0
−5
x
5



3
2.5
2
θ 1.5
1
0.5
0
10
20
30
40
50
60
r



2.5
2
θ 1.5
1
0.5
4
6
8
10
12
14
r

THE MOTION IN THE EQUATORIAL PLANE
ERNST, MELVIN, MAGNETIZED KISELEV SOLUTIONS

$$\theta = \pi / 2$$

$$\mathcal{L}_0 = \frac{1}{2}\left[-f\Lambda_0^2 \dot{t}^2 + \frac{\Lambda_0^2}{f}\dot{r}^2 + \frac{r^2}{\Lambda_0^2}\dot{\varphi}^2 \right] + \frac{\varepsilon B_0 r^2}{\Lambda_0}\dot{\varphi}$$

$$\Lambda_0 = 1 + B_0^2 r^2$$

- THE RADIAL EQUATION

$$\dot{t} = \frac{E}{f\Lambda_0^2}, \quad \dot{\varphi} = \frac{\Lambda_0^2}{r^2}\left(L - \frac{\varepsilon B_0 r^2}{\Lambda_0} \right)$$

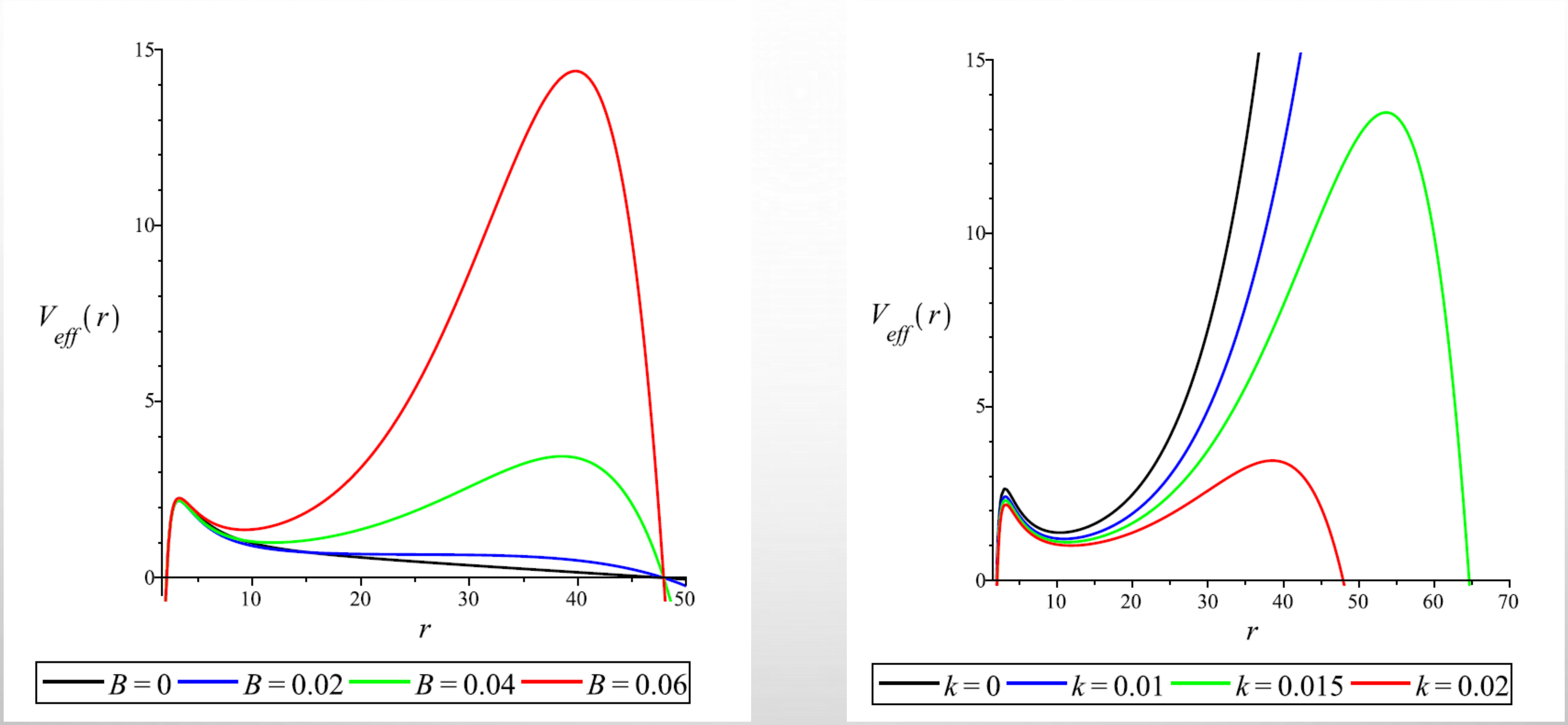
$$\ddot{r} = \left(\frac{f'}{2f} - \frac{\Lambda'_{m0}}{\Lambda_{m0}} \right)\dot{r}^2 - \frac{E^2}{\Lambda_{m0}^4}\left(\frac{f'}{2f} + \frac{\Lambda'_{m0}}{\Lambda_{m0}} \right) + \frac{f}{r^3}\left(1 - \frac{r\Lambda_{m0}}{\Lambda_{m0}} \right)\left(L - \frac{\varepsilon B_0 r^2}{\Lambda_{m0}} \right)^2 + \frac{f\varepsilon B_0}{\Lambda_{m0} r}\left(2 - \frac{r\Lambda'_{m0}}{\Lambda_{m0}} \right)\left(L - \frac{\varepsilon B_0 r^2}{\Lambda_{m0}} \right)$$

- THE EFFECTIVE POTENTIAL

$$V_{eff\,0}(r) = f\Lambda_{m0}^2 \left[1 + \frac{\Lambda_{m0}^2}{r^2}\left(L - \frac{\varepsilon B_0 r^2}{\Lambda_{m0}} \right)^2 \right]$$

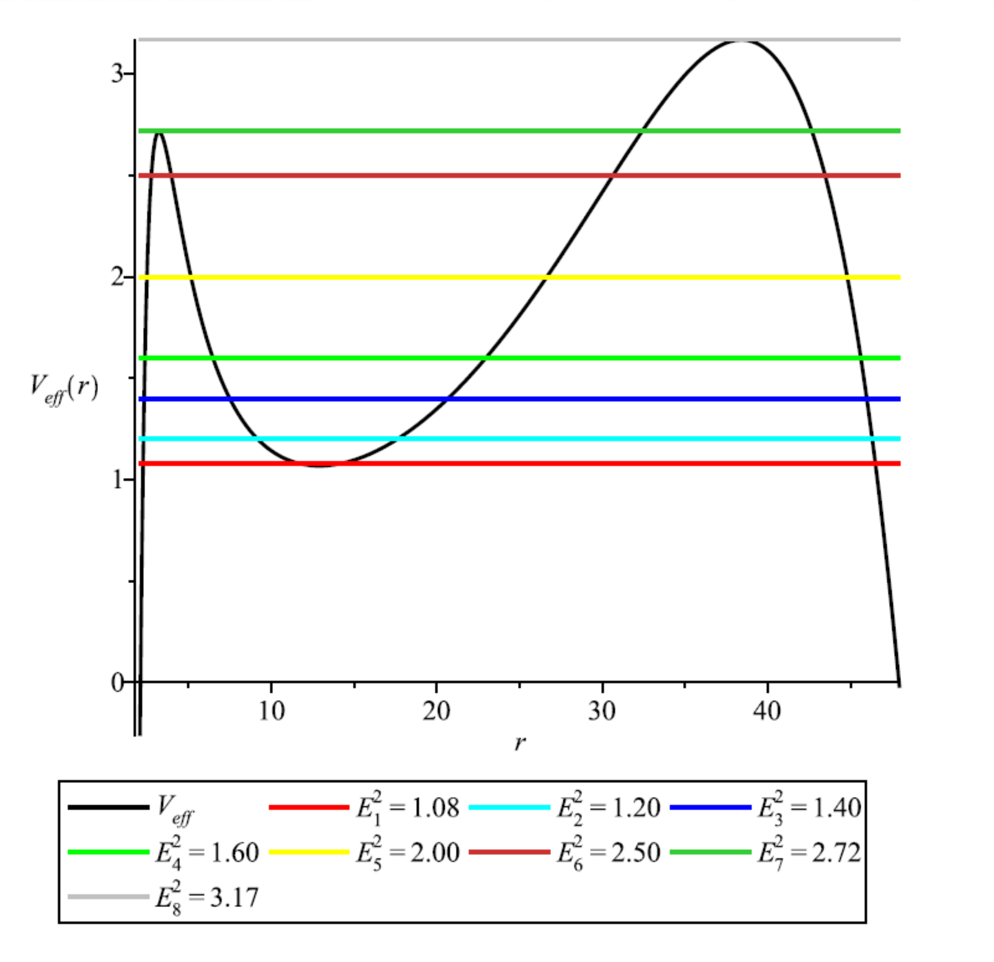
- TROCHOID-LIKE TRAJECTORIES AS $\dot{\varphi}$ CHANGES SIGN IN

$$\dot{\varphi} = \frac{\Lambda_0^2}{r^2}\left(L - \frac{\varepsilon B_0 r^2}{\Lambda_0} \right) = 0 \Rightarrow r_* = \sqrt{\frac{L}{B_0(\varepsilon - LB_0)}}$$

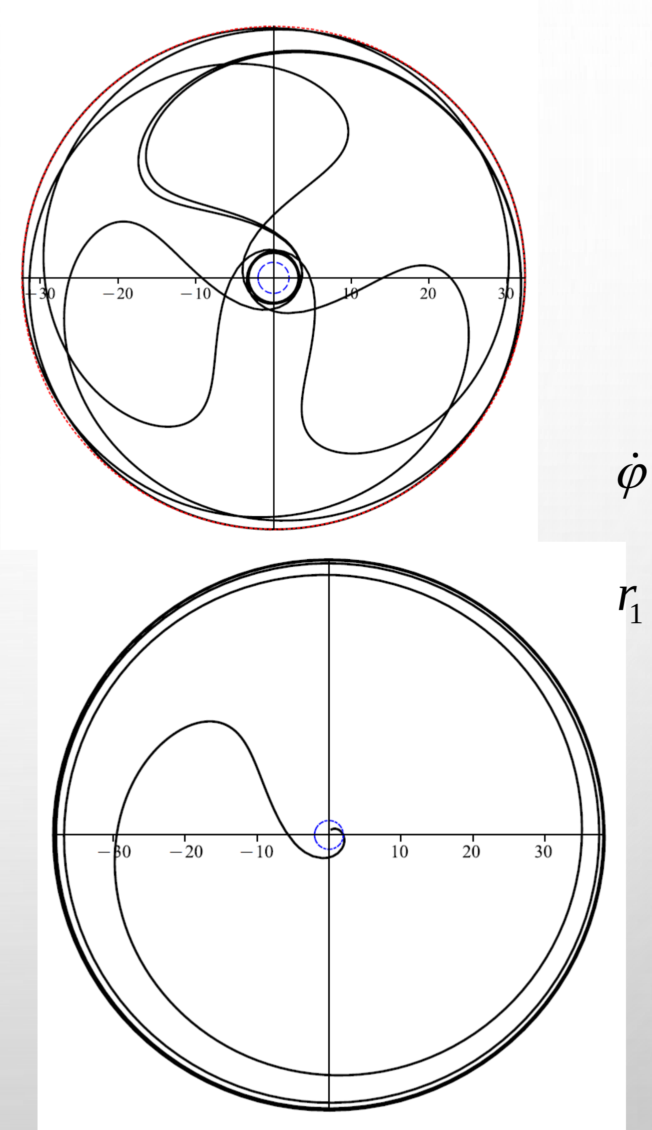


***Left panel**. Effective potential for different values of B and k = 0.02. **Right panel**. Effective potential for different values of k and B=0.05.*

TYPE OF ORBITS: CAPTURE, BOUNDED OR ESCAPE



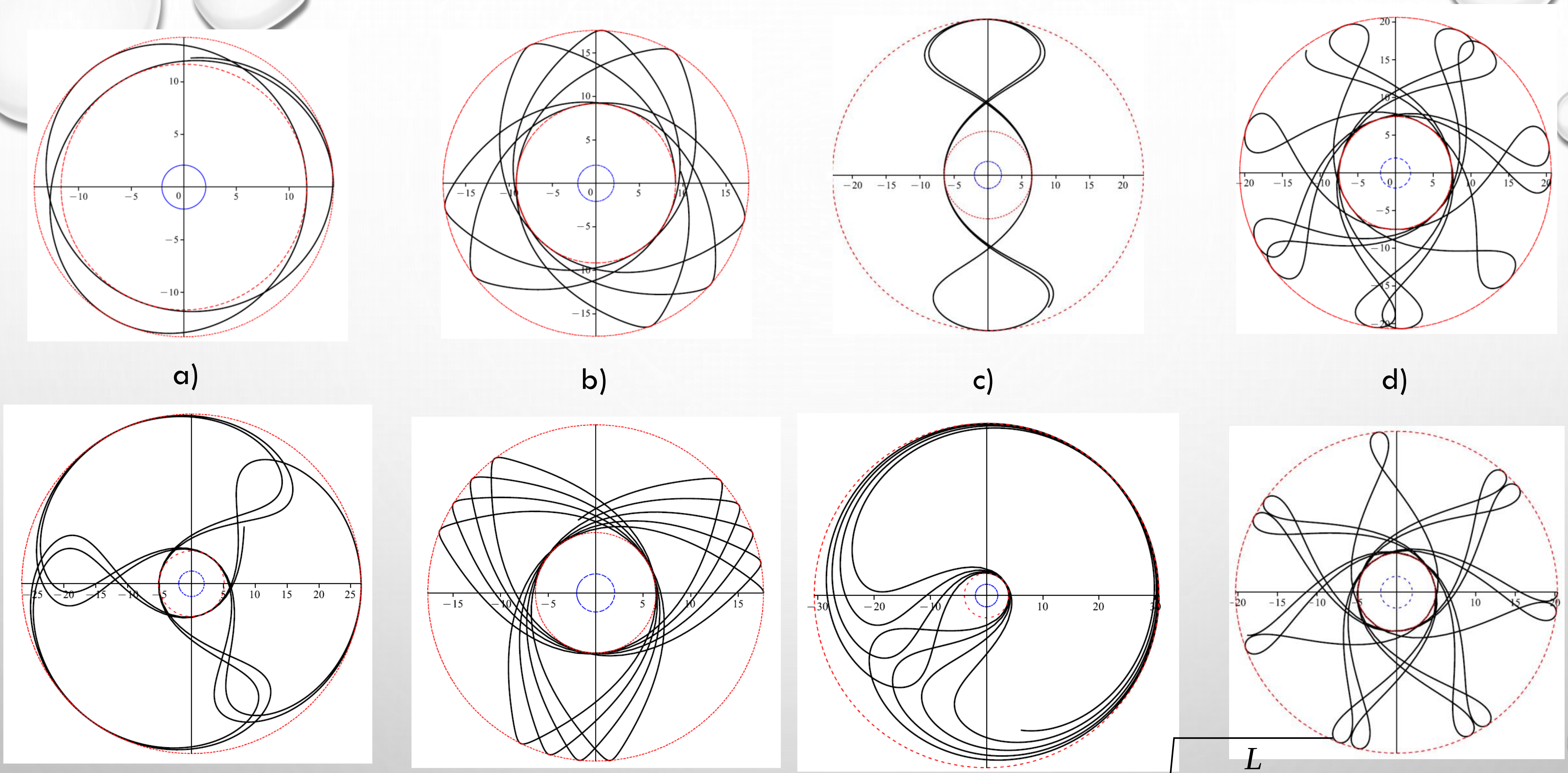
Effective potential for M = 1,k = 0.02, ε = 1, B = 0.04 and L = 9.



$$\dot{\varphi} = 0 \Rightarrow r_* = \sqrt{\frac{L}{B_0(\varepsilon - LB_0)}}$$

$$r_1 < r_* < r_+$$

Top: first unstable circular orbit (E^2=2.72).
Bottom: Second unstable circular orbit(E^2=3.17).



e) f) g) h)

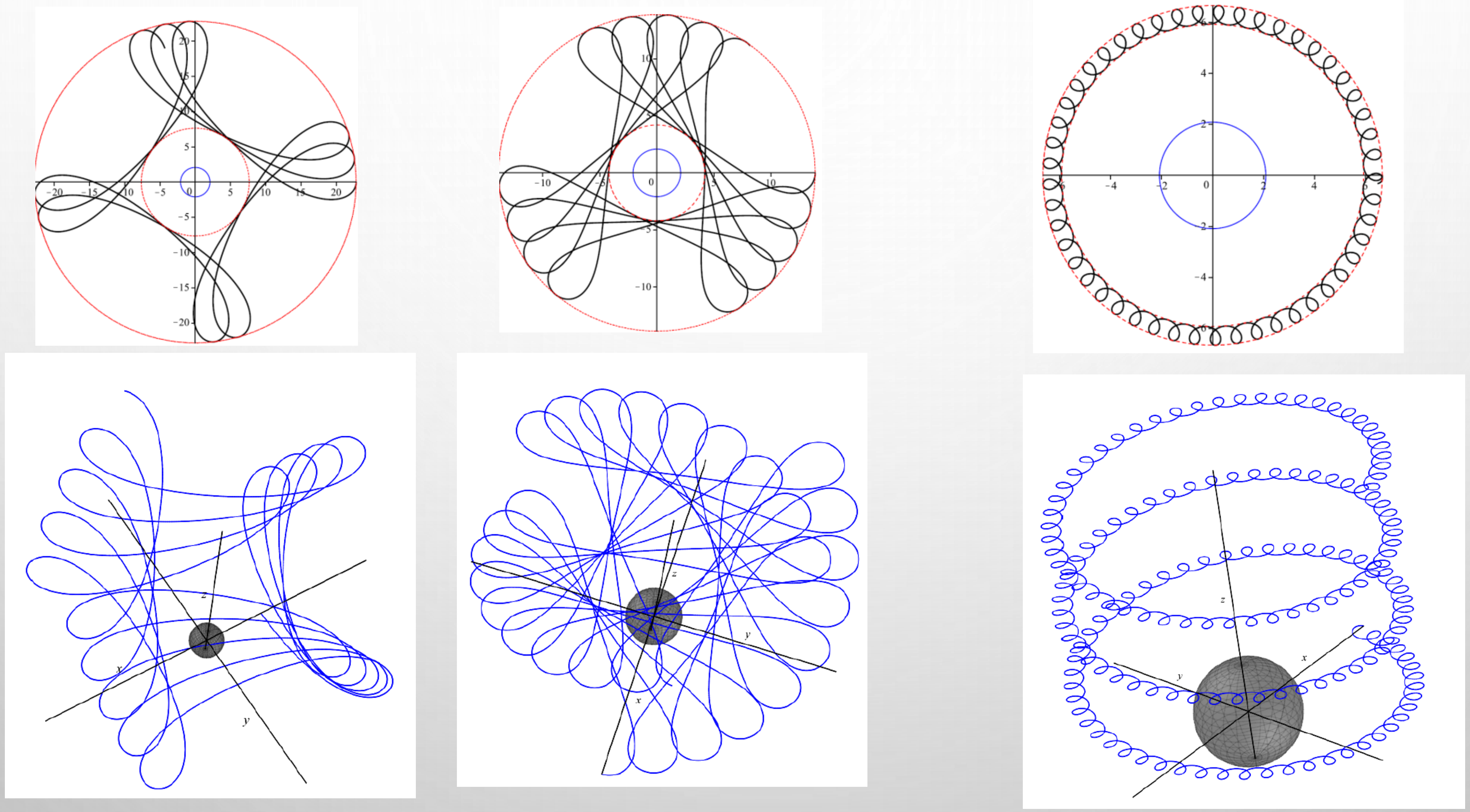
$$r_* = \sqrt{\frac{L}{B_0(\varepsilon - LB_0)}}$$

Bound orbits. a) E^2=1.08, b) E^2=1.20, c) E^2 = 1.40, d) E^2 = 1.60, e) E^2 = 2.00, f) E^2 = 2.00 and k=0, g) E^2 = 2.50, h) E^2 = 2.50 and k=0.

MOTION IN THE MELVIN MAGNETIC UNIVERSE

THERE IS NOT A BLACK HOLE NOR THE QUINTESSENTIAL FLUID.

$$\Lambda_m = 1 + B_0^2 r^2 \sin^2\theta, \quad f(r) = 1$$



Bound orbits of charged particles in Melvin magnetic universe.

Y. K. Lim, Physical Review D, 91(2) (2015) 91.024048

THE WEAK FIELD REGIME. ANALYTIC ANALYSIS

$$B_G \sim \frac{c^4}{G^{3/2}M_S}\cdot\frac{M_S}{M} \sim 10^{19} M_S/M \sim 10^{10}\ (G)$$

$$f(r) = 1 - \frac{2M}{r} - kr, \qquad \Lambda_m \simeq 1$$

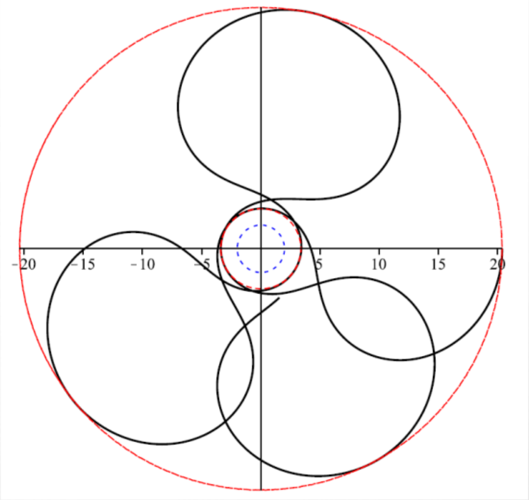
$$\mathcal{L} = \frac{1}{2}\left[-f\dot t^2 + \frac{1}{f}\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\dot\varphi^2\right] + \varepsilon B_0 r^2 \sin^2\theta\dot\varphi$$

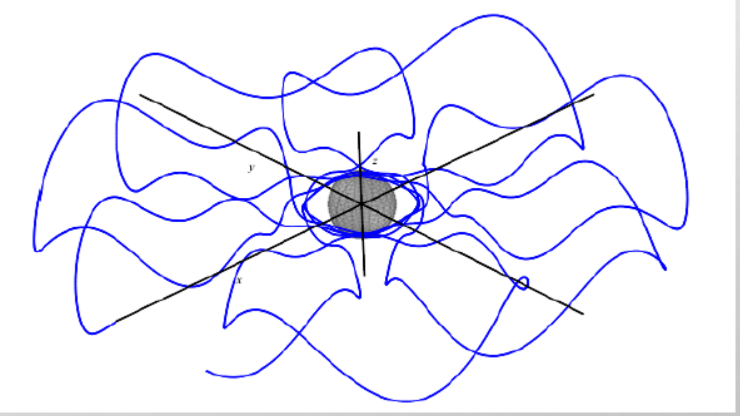
$$\Downarrow$$

$$\ddot r = r\left(\frac{L}{r^2\sin^2\theta} - \varepsilon B_0\right)\left[\sin^2\theta\left(\frac{L}{r^2\sin^2\theta} - \varepsilon B_0\right)\left(f - \frac{f'r}{2}\right) + 2f\varepsilon B_0\right] - \frac{f'}{2}$$

$$\ddot\theta = \frac{1}{2}\sin(2\theta)\left[\frac{L^2}{r^4\sin^4\theta} - \varepsilon^2 B_0^2\right]$$

$$\dot\varphi = \frac{L}{r^2\sin^2\theta} - \varepsilon B_0$$





THE EFFECTIVE POTENTIAL:

$$V_{eff}(r) = f\left[1 + r^2\sin^2\theta\left(\frac{L}{r^2\sin^2\theta} - \varepsilon B_0\right)^2\right]$$

Bound orbits in the weak field regime. The numerical values: ε = 1, B = 0.06, L = 6, M = 1, k = 0.01, E^2 = 1.30.

For k=0, see V. P. Frolov and A. A. Shoom, Physical Review D, 82(8) (2010) 084034

A SMALL PERTURBATION ON THE CIRCULAR ORBIT:

$$r \approx r_0 + r_1, \quad \varphi \approx \varphi_0 + \varphi_1, \quad \theta \approx \theta_0 + \theta_1$$

THE ZEROTH ORDER

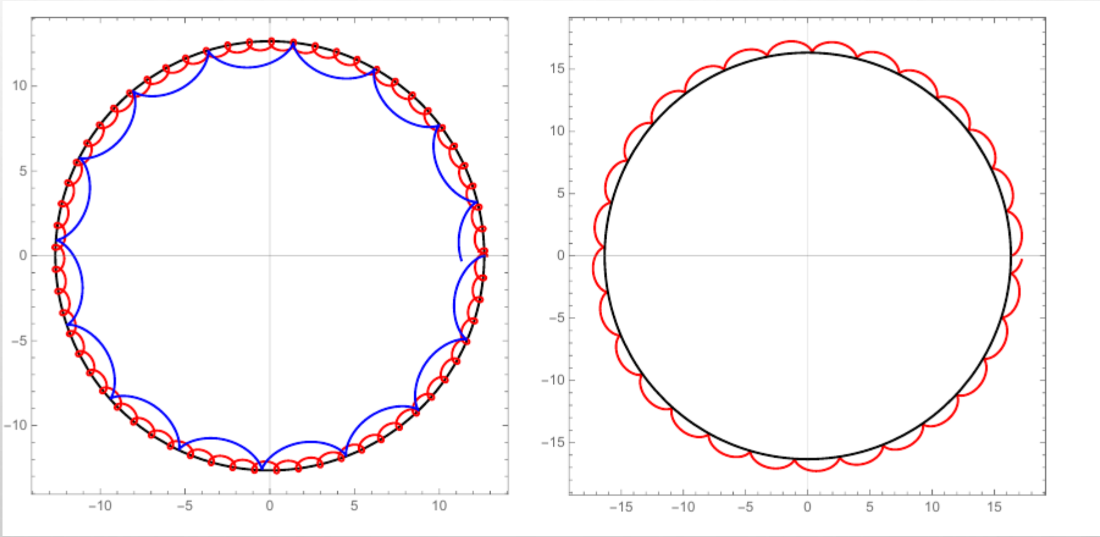
$$L = \varepsilon B_0 r_0^2 \Rightarrow r_0 = \sqrt{\frac{L}{\varepsilon B_0}}, \quad \theta_0 = \frac{\pi}{2}$$

$$\omega s = 2\tau$$

$$\ddot{r}_1 + \omega^2 r_1 = -\frac{M}{r_0^2} + \frac{k}{2}, \quad \omega = 2\varepsilon B_0 \sqrt{1 - \frac{2M}{r_0} - k r_0} \Rightarrow r_1 = \alpha \cos(\omega s) - \frac{2M - k r_0^2}{2\omega^2 r_0^2}$$

THE FIRST ORDER

$$\dot{\varphi}_1 = -\frac{2\varepsilon B_0 r_1}{r_0} \Rightarrow \varphi_1 = -\frac{2\varepsilon B_0 \alpha}{\omega r_0} \sin(\omega s) + \frac{\varepsilon B_0 (2M - k r_0^2)}{\omega^2 r_0^3} s$$



DEFINE THE PARAMETER WHICH DICTATES THE SHAPE OF THE ORBITS:

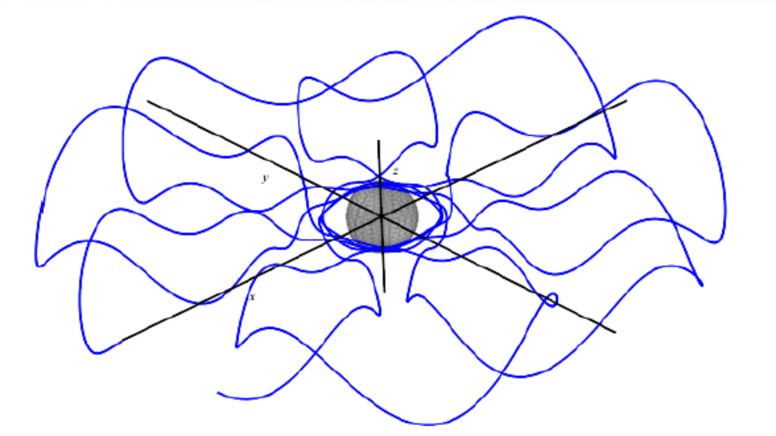
$$\eta = \frac{2M - k r_0^2}{2\omega^2 r_0^2 \alpha}$$

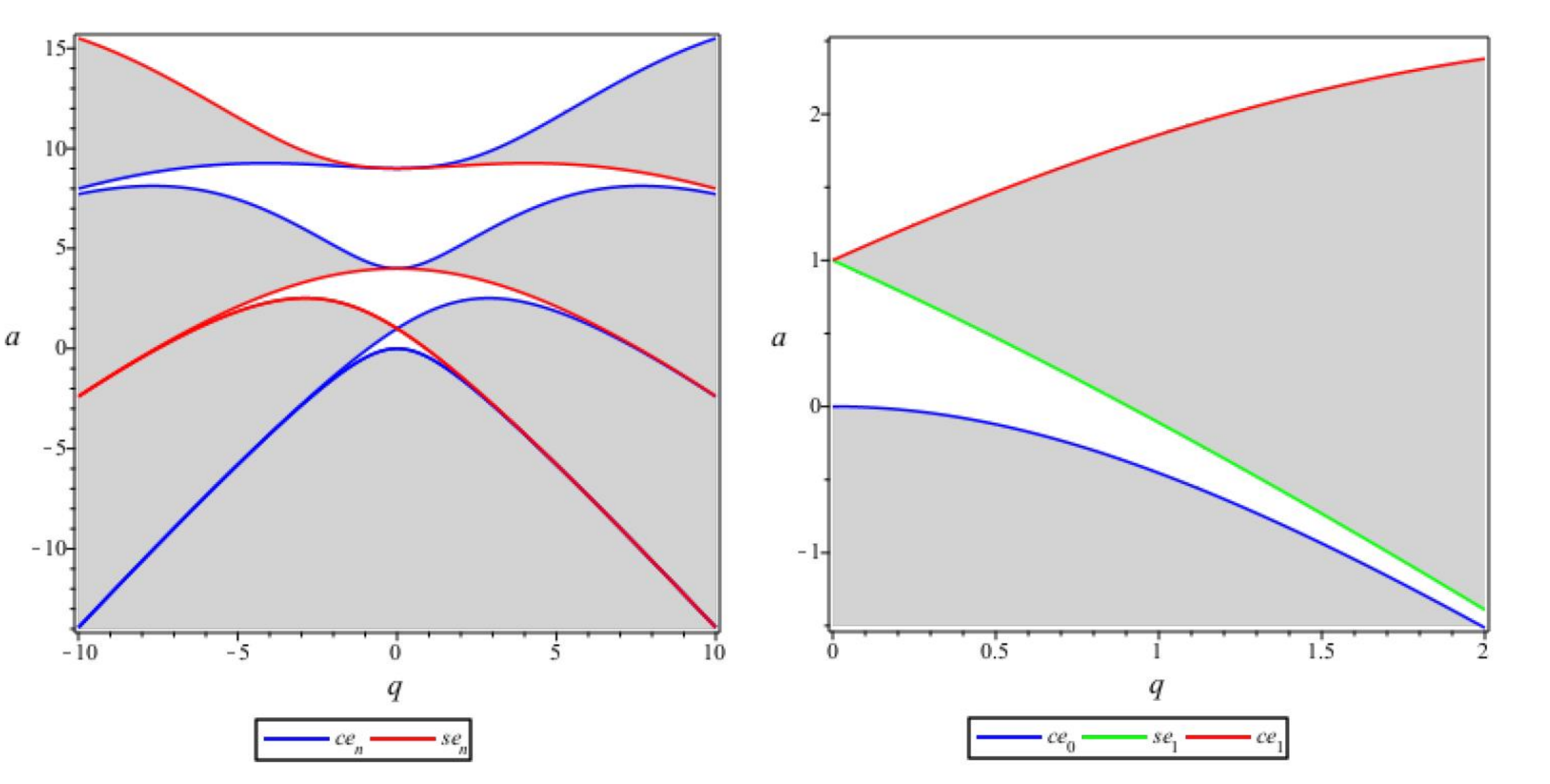
THE ORBIT WOULD CURL TOWARD THE BLACK HOLE, AS THE ATTRACTIVE TERM DOMINATES OVER THE REPULSIVE TERM.

$$\ddot{\theta}_1 + \left[a - 2q\cos(2\tau)\right]\theta_1 = 0, \quad \omega s = 2\tau$$

$$\theta_1 = C_1 MathieuC(a, q, \tau) + C_2 MathieuS(a, q, \tau)$$

$$a = \frac{8\varepsilon^2 B_0^2 (2M - kr_0^2)}{\omega^4 r_0^3}, \quad q = \frac{8\varepsilon^2 B_0^2 (2M - kr_0^2)\alpha}{\omega^4 r_0^3}$$

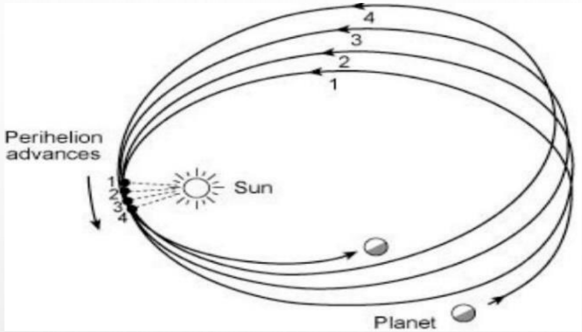




Stability chart. Stability regions are white, instability regions are gray

$$a = \frac{8\varepsilon^2 B_0^2 (2M - kr_0^2)}{\omega^4 r_0^3}, \quad q = \frac{8\varepsilon^2 B_0^2 (2M - kr_0^2)\alpha}{\omega^4 r_0^3}$$

CIRCULAR ORBITS AND PERIAPSIS SHIFT



$$r = r_c + r_1$$

$$\ddot{r}_1 + \omega_r^2 r_1 = 0$$

$$\omega_\varphi = \dot{\varphi} = \frac{\Lambda_0^2}{r^2}\left(L - \frac{\varepsilon B_0 r^2}{\Lambda_0}\right)$$

$$A = \left(\frac{\omega_r}{\omega_\varphi}\right)^2 \Rightarrow \boxed{\Delta\phi = 2\pi\left[\frac{\omega_\varphi - \omega_r}{\omega_r}\right] = 2\pi\left[\frac{1}{\sqrt{A}} - 1\right]},$$

$$\begin{cases} \Delta\phi_S \approx \dfrac{6\pi M}{r} > 0 \\ \Delta\phi_{RN} \approx \dfrac{6\pi M}{r} - \dfrac{\pi Q^2}{rM} > 0 \end{cases}$$

S2 is a star in a cluster close to the supermassive black hole in the galactic center.
Orbiting it with a period of aprox 16 years
Its changing apparent position has been monitored since 1995,
2018 pericentre passage
In 2020, the GRAVITY collaboration released an analysis showing full agreement with GR (the Schwarzschild precession).

CONCLUSIONS

- PARTICLES TRAJECTORIES ARE DETERMINED BY THE MAGNETIC FIELD.
- THE MAGNETIC FIELD MAY DISTURB THE SPACETIME AROUND THE BH.
- IN THE CASE OF THE WEAK MAGNETIC FIELD, THE TEST PARTICLE MOTION IS AFFECTED BY THE LORENTZ FORCE AND THE TRAJECTORY MAY BECOME UNSTABLE FOR INCREASING QUINTESSENCE PARAMETER
- THE PERIAPSIS SHIFT (VALUE AND SIGN) DEPENDS ON THE MAGNETIC INDUCTION AND ON THE QUINTESSENCE PARAMETER

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