GUP Correction for the Hawking Temperature of a (2+1) Dimensional Black Hole

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Outline

Introduction

Black holes are among the most intriguing objects in theoretical physics, especially due to their unique properties in both

- general relativity
- quantum mechanics

Figure: Black Hole

Introduction

Key Questions:

- How does the GUP modify the Hawking temperature of Kerr-like black holes?
- Can the formation of remnants due to GUP corrections help preserve quantum information?

Introduction: Kerr-like Black Holes

Kerr-like rotating black holes are often studied in $3D$ gravity within the framework of Topologically Massive Gravity (TMG).

Metric:

$$ds^2 = -\left(\frac{\rho^2 - \rho_0^2}{r^2}\right) dt^2 + \frac{d\rho^2}{\rho^2 - \rho_0^2} + r^2 \left(d\phi - \frac{2\rho + 3\omega_{BH}}{r^2} dt\right)^2$$

$$r^2 = \rho^2 + 4\omega_{BH}\rho + 3\omega_{BH}^2 + \frac{\rho_0^2}{3}$$

ρ_0 and ω_{BH} are constants related to Kerr-like black hole properties.

Black Hole Parameters

Key Parameters of Kerr-like Black Holes:

- **Mass** (M): Total mass of the black hole.
- **Event Horizons** :
  - (ρ_0) : outer event horizons.
  - ($-\rho_0$): inner event horizons.
- **Ergosphere**: Region extends to infinity for $\omega_{BH} > 0$.

Spacetime Structure of Kerr-like Black Holes

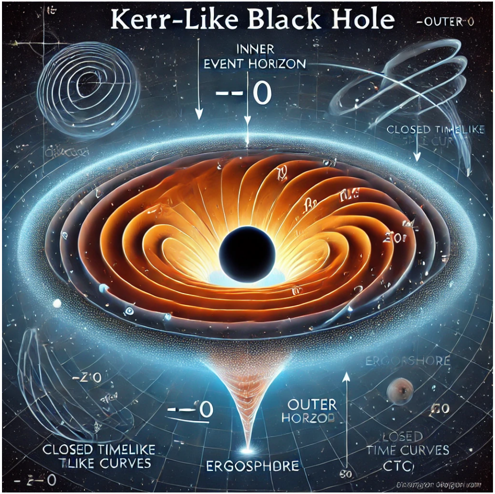

Figure: Spacetime structure and ergosphere

Superradiance Condition

Superradiance is a process in which particles with angular momentum interact with a rotating black hole, extracting energy from it. This occurs when:

$$\omega < m\Omega_H$$

where ω is the energy of the particle and $\Omega_H = \frac{J}{2M\rho_0}$ is the angular velocity at the outer horizon.

Superradiance Phenomenon

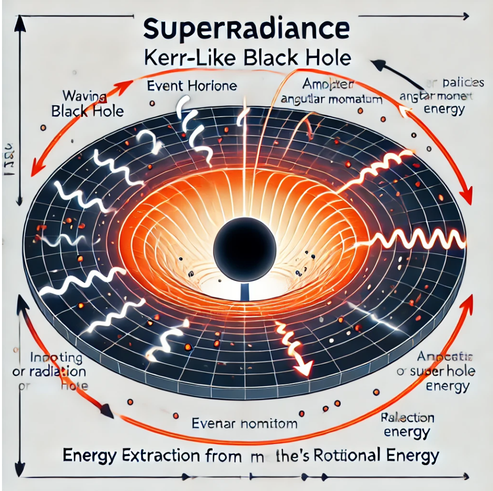

Figure: Superradiance process

Superradiance Phenomenon

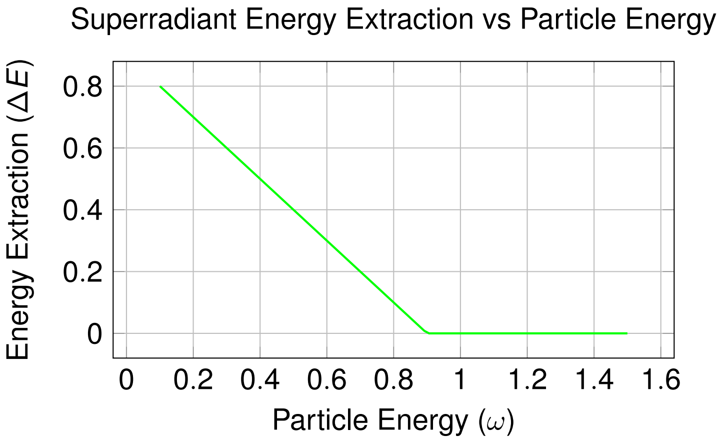

Figure: Superradiant Energy Extraction

Superradiance Phenomenon

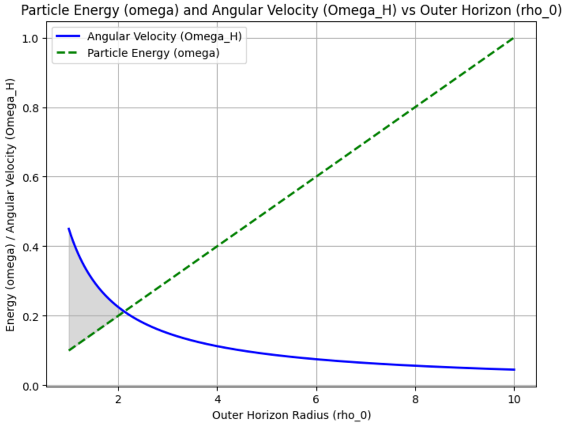

Figure: Particle's energy and angular velocity vs ρ_0

Superradiance Phenomenon

- The number of emitted particles is described by:

$$\langle N \rangle \propto \frac{1}{e^{(\omega - m\Omega_H)/T_H} - 1}$$

- As particle's energy increases, number of emitted particles decreases exponentially

Superradiance Phenomenon

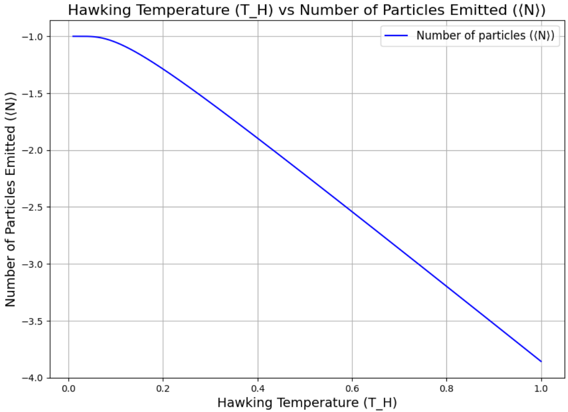

Figure: Number of emitted particles and Hawking radiation.

Hawking Radiation process

- Black holes emit radiation due to quantum effects near the event horizon.
- Particle-antiparticle pairs are created, with one particle escaping as Hawking radiation.

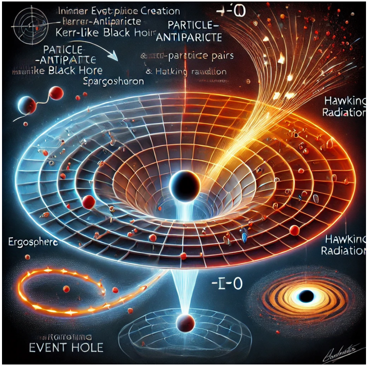

Figure: Hawking Radiation process

Hawking Radiation and Black Hole Evaporation

- This causes the black hole to lose mass over time, leading to evaporation.
- For large black holes, this process is slow, but as the black hole becomes smaller, Hawking radiation becomes more intense, potentially leading to complete evaporation.

Fermion Tunneling Process

Fermion tunneling describes how particles with spin (fermions) escape the black hole via quantum tunneling.

- Fermions tunnel through the event horizon by solving the Dirac equation in curved spacetime.
- The tunneling probability is proportional to the imaginary part of the action I:

$$P \propto \exp(-2\text{Im}(I))$$

Dirac Equation and Tunneling

The Dirac equation in the Kerr-like black hole metric is:

$$i\gamma^a e_a^\mu \nabla_\mu \psi - m\psi = 0$$

The spinor field ψ is expressed as an ansatz:

$$\psi = \begin{pmatrix} A(t,\rho,\phi) \\ B(t,\rho,\phi) \end{pmatrix} e^{-\frac{i}{\hbar} I(t,\rho,\phi)}$$

$A(t,\rho,\phi)$ and $B(t,\rho,\phi)$ are the spinor components that depend on spacetime coordinates.

Surface gravity and Hawking Temperature

Solving this equation using the WKB approximation leads to the calculation of Hawking temperature from tunneling.

$$T_H = \frac{\kappa}{2\pi}$$

For Kerr-like black holes, the surface gravity is given by:

$$\kappa = \frac{\rho_0}{\rho_0^2 + 4\omega_{BH}\rho_0 + 3\omega_{BH}^2 + \rho_0^2}$$

This shows that the temperature depends on both mass and angular momentum of the black hole.

Hawking temperature via Fermion Tunneling

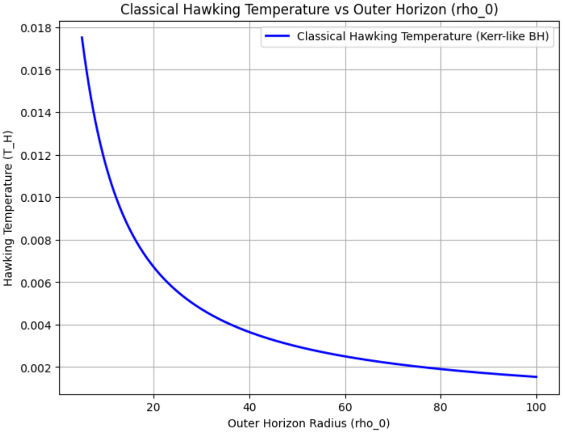

Figure: Hawking temperature Vs event horizon Kerr-like Black Hole

Generalized Uncertainty Principle (GUP)

Generalized Uncertainty Principle (GUP):

- GUP modifies the Heisenberg Uncertainty Principle by introducing a minimal length scale Δx_{min}.
- This is particularly relevant for small black holes where quantum gravity effects are significant.

Concept of the Generalized Uncertainty Principle (GUP)

- "GUP" is a modification of the Heisenberg Uncertainty Principle due to quantum gravity effects.
- GUP introduces a minimal length scale Δx_{min}, which affects the behavior of small black holes and high-energy particles :

$$\Delta x \Delta p \geq \hbar \left(1 + \frac{\alpha \hbar}{(\Delta p)^2} \right)$$

  - α is a constant related to quantum gravity.
- GUP implies that there is a minimal measurable length, preventing arbitrarily small distances.

Role of GUP in Black Hole Remnants

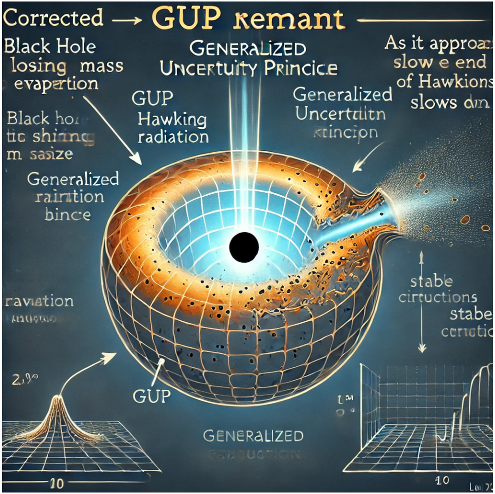

Figure: Effect of GUP on Black Hole Evaporation and Remnant Formation

GUP and Hawking Radiation

The GUP-modified Hawking temperature is given by:

$$T_H^{\mathrm{GUP}} = T_H \left(1 - \frac{\alpha \hbar}{M^2}\right)$$

As the black hole's mass decreases, evaporation slows down, potentially leading to the formation of a stable remnant.

GUP and Hawking Radiation

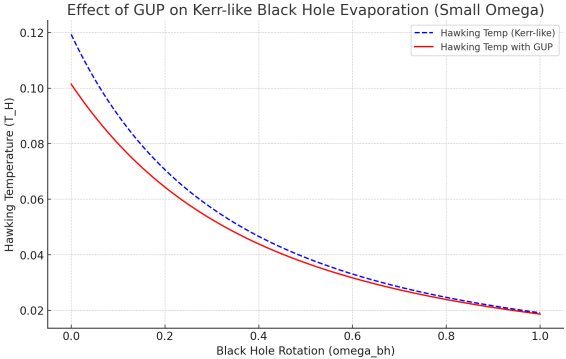

Figure: Hawking temperature vs Kerr-like black hole's rotation parameter (classical and GUP cases)

GUP and Hawking Radiation

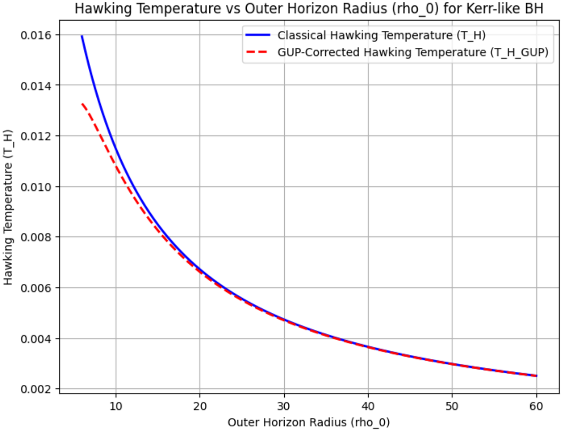

Figure: Hawking temperature vs Kerr-like black hole's event horizon (classical and GUP cases)

GUP and Hawking radiation

GUP correction becomes stronger as α increases, and the Hawking temperature decreases more slowly for small ρ_0

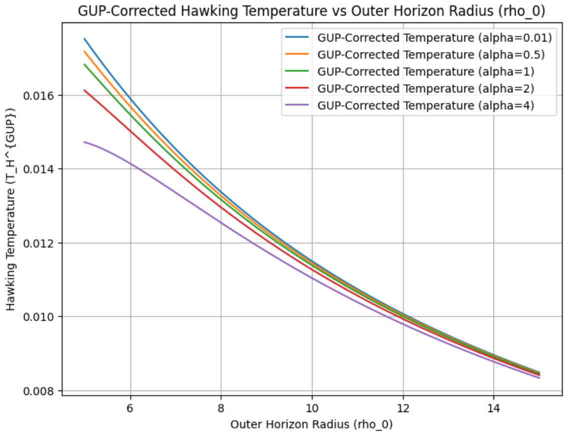

Figure: Effect of GUP on Black Hole radiation for several values of α

Number of Emitted Particles: GUP case

The number of emitted particles for a Kerr-like black hole depends on the temperature at the outer horizon (ρ_0) and the influence of the black hole's angular momentum.

$$\langle N_{\text{GUP}} \rangle \propto \frac{1}{e^{\frac{(\omega - m\Omega_H)}{T_H\left(1 - \frac{\alpha\hbar}{M^2}\right)}} - 1}$$

The GUP correction introduces a term that reduces the number of emitted particles as the black hole approaches the remnant phase.

Number of Emitted Particles: Classical vs GUP

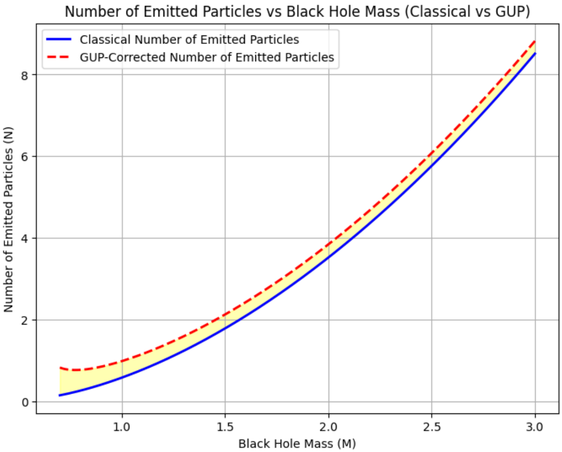

Figure: Number of Emitted Particles vs Black Hole Mass (Classical vs GUP)

Black Hole Evaporation and Remnants

- Without GUP, combined with superradiance, Kerr-like black hole evaporates completely via Hawking radiation.
- However, GUP introduces a minimum size remnant, preventing complete evaporation:

$$M_{\text{remnant}} \sim \sqrt{\alpha\hbar}$$

- This remnant mass represents the smallest possible stable black hole, preserving quantum information and preventing the total loss of entropy.

Resolving the Information Paradox

GUP in Kerr-like black holes addresses the information paradox:

- Without GUP, Hawking radiation and superradiance cause total evaporation and information loss.
- With GUP, black holes stabilize as remnants, conserving quantum information and entropy.
- This aligns black hole thermodynamics with quantum mechanics.

Conclusion

- Kerr-like black holes exhibit key quantum effects such as **superradiance** and **fermion tunneling**, which are crucial in their energy dissipation and evaporation process.
- The tunneling mechanism provides insights into the relationship between the event horizon's **surface gravity** and the black hole's **Hawking temperature.**
- Through tunneling, particles extract energy from the black hole, accelerating its evaporation.

Conclusion

- GUP modifies this process, potentially halting full evaporation and leaving a stable **remnant.**
- GUP-induced remnants preserve both **quantum information** and angular momentum, addressing the long-standing black hole **information paradox.**
- The remnants act as quantum information reservoirs, offering a resolution to the paradox and bridging the gap between **quantum mechanics** and **general relativity** in black hole physics.

Perspectives

- Advanced numerical simulations of black hole remnants formed through GUP, providing a clearer picture of their stability and potential observational signatures.
- Applications to other rotating systems and extending the study to other rotating compact objects or exotic stars, to examine whether GUP could influence their thermodynamic properties or stability.

Thank you

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