Eigenstates and decay of a positronium in a strong magnetic field

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Introduction

The problem is important in the astrophysics when the electromagnetic signals, whose energy is about $0.511 MeV$, coming from the astrophysical objects:

i) pulsars
ii) neutron stars.

Low energy positronium states in a magnetic field

We direct the magnetic field $\boldsymbol{B}$ along the OZ axis.

$$a \ll a_B$$

$$\left(\triangle + \frac{2(4\pi\alpha)E_w}{r} + \frac{(4\pi\alpha)^2}{r^2} + i\sqrt{\pi\alpha}\nabla(\boldsymbol{B}\times\boldsymbol{\rho}) - 4\pi\alpha B^2\rho^2 - m_w^2 + E_w^2\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\Psi(E_w,\boldsymbol{r})$$
$$= -\sqrt{\pi\alpha}\begin{pmatrix}\sigma_{z,1}+\sigma_{z,2} & 0\\ 0 & \sigma_{z,1}+\sigma_{z,2}\end{pmatrix}\Psi(E_w,\boldsymbol{r}), \qquad (1)$$

Low energy positronium states in a magnetic field

$$\Psi(E_w, \boldsymbol{r}) = \frac{\psi_{m=0,n=0}(\boldsymbol{\rho})}{\sqrt{2}} \sum_{S=0}^{1} \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 + (-1)^{S+1} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right) \psi_S(z), \tag{2}$$

$$\psi_{m=0,n=0}(\boldsymbol{\rho}) = \frac{1}{a\sqrt{2\pi}} e^{-\frac{\rho^2}{4a^2}}, \tag{3}$$

Low energy positronium states in a magnetic field

$$\Big(\frac{d^2}{d\xi^2} + 2^{1/2}\sqrt{\pi}(4\pi\alpha)m_w\varepsilon_w a e^{\xi^2/2} erfc(|\xi|/\sqrt{2}) + \frac{(4\pi\alpha)^2 e^{\xi^2/2}}{2}E_1(\xi^2/2) - \varepsilon_S^2\Big)\psi_S(\xi) = -2\sqrt{\pi\alpha}\psi_{1-S}(\xi), \quad (4)$$

$$z/a \equiv \xi, \quad (m_w^2 - E_w^2)a^2 + 1 + (-1)^{S+1} \equiv \varepsilon_S^2, \quad \varepsilon_w = \frac{E_w}{m_w}. \quad (5)$$

Low energy positronium states in a magnetic field

$$\psi_S(\xi) = \sqrt{k_S} \exp(-k_S|\xi|), \tag{6}$$

$$k_S = (4\pi\alpha)^2 \int_0^\infty d\xi \exp\left(\xi^2/2 - 2k_S\xi\right) E_1(\xi^2/2) +$$
$$2^{3/2}\sqrt{\pi}(4\pi\alpha) m_w \varepsilon_w a \int_0^\infty d\xi \exp\left(\xi^2/2 - 2k_S\xi\right) erfc(\xi/\sqrt{2}) \tag{7}$$

Low energy positronium states in a magnetic field

i) Non relativistic case.

$$m_e a \gg \alpha$$

$$E_w = \frac{m_e}{2}\left(1 + \frac{2(1+(-1)^{S+1})}{(m_e a)^2} - 2(4\pi\alpha/3)^2 \ln^2(a_B/a)\right) (8)$$

Low energy positronium states in a magnetic field

ii) Relativistic case.

$$m_e a \ll \alpha$$

$$E_w \simeq \frac{1}{a^2}((1+(-1)^{S+1}) - (2\pi)^7\alpha^4/162). \qquad (9)$$

Positronium decay in a magnetic field

$$\Gamma \simeq \frac{|\psi(0)|^2}{M^2}, \tag{10}$$

$$|\psi_{B=0}(0)|^2 \sim a_B^{-3} \tag{11}$$

$$|\psi_{B\neq 0}(0)|^2 \sim \frac{1}{a_B a^2}. \tag{12}$$

A strong magnetic field leads to sufficient increasing, in $(a_B/a)^2 \gg 1$ times, the decay width as compared with the case $B = 0$.

Conclusion

1. The eigenvalue and eigenstate of a positronium in a strong magnetic field are studied.
2. A strong magnetic field decreases the positronium lifetime.

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