DUPDFs

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- DUPDFs (Double unintegrated PDFs)
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Introduction

THE Standard MODEL
Quarks
Bosons
Leptons
γ Photon
g Gluon
u Up quark
c Charm quark
t Top quark
d Down quark
s Strange quark
b Bottom quark
H Higgs boson
Z Z boson
W W boson
e Electron
μ Muon
τ Tau
Electron neutrino
Muon neutrino
Tau neutrino

Factorization Theorem

Factorization Theorem: Factorization means the ability to separate a hadronic cross section into the convolution of parton distribution functions (PDFs) and partonic cross section.

1. Collinear factorization: This framework assumes the elementary constituents of the hadrons, partons (quarks and gluons), move collinear to the hadron, i.e. : In this framework for hadronic cross section one needs what we call collinear PDFs.

2. -factorization: This framework assumes the elementary constituents of the hadrons also have transverse momentum in addition to the momentum along the hadron, i.e.: . In this framework for hadronic cross section needs Transverse Momentum Dependent PDFs (TMD-PDFs) or Unintegrated Parton Distribution Functions (UPDFs).

Collinear factorization framework

Collinear factorization : in the collinear factorization, it is assumed that the parton moves collinear to the proton and enters into the hard collision. Therefore in the collinear factorization theorem, it is supposed that the constituent partons can only have a fraction of the proton momentum:

$$\boldsymbol{\sigma} = \sum_{i,j\epsilon q,g} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\hat{\sigma}_{ij}$$

To describe the distribution of these partons inside the proton, one should use the PDFs.

k_t-factorization framework

k_t-factorization: One can write the p-p collision cross section in this framework as follows:

$$\sigma = \sum_{a,b=q,g} \int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} \frac{\mathrm{d}k_{1t}^2}{k_{1t}^2} \frac{\mathrm{d}k_{2t}^2}{k_{2t}^2} f_a(x_1, k_{1t}^2, \mu^2)\, f_b(x_2, k_{2t}^2, \mu^2)\, \hat{\sigma}_{ab}^*$$

First step is obtaining suitable UPDFs that is a hard task. Fortunately some approaches based on the DGLAP evolution equation exist which allow to obtain UPDFs simply with powerful predicting power, for example the **KMR**, **MRW** and **PB** (Parton Branching) approaches.

(z, k_t) -factorization framework

In new formalism the UPDFs become *z* dependent, (DUPDFs), and hence one needs to modify hadronic factorization formula for calculating the cross section, compared to the k_t-factorization approach. Therefore, by generalizing the k_t-factorization framework, one can write the general p–p cross section formula in the *(z, k_t)*-factorization as:

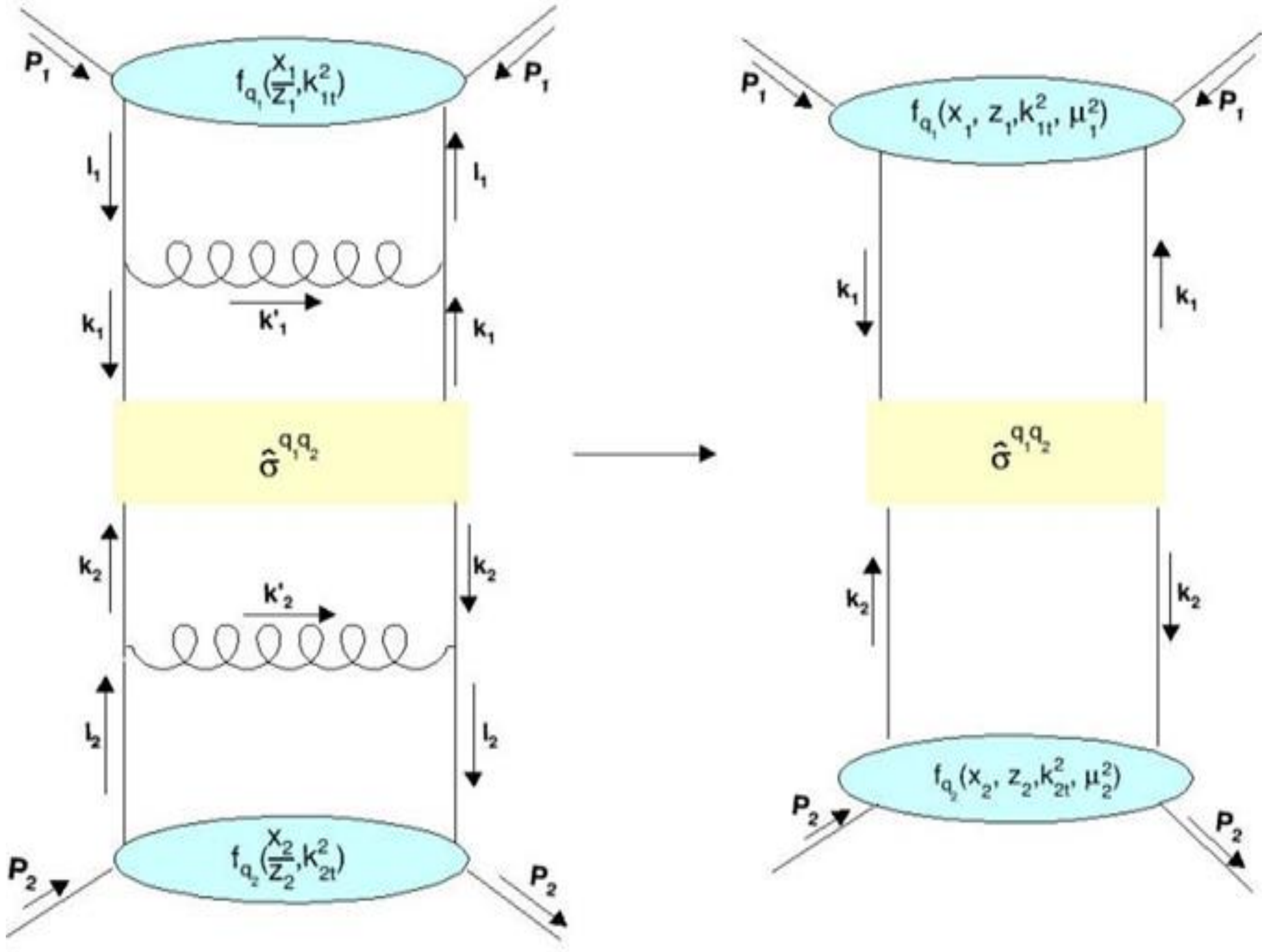
$$\boldsymbol{\sigma = \sum_{a,b=q,g} \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2 \int_0^1 \frac{dx_1}{x_1} \int_0^1 \frac{dx_2}{x_2} \int_0^\infty \frac{dk_{1t}^2}{k_{1t}^2} \int_0^\infty \frac{dk_{2t}^2}{k_{2t}^2} f_a(x_1, z_1, k_{1t}^2, \mu^2) f_b(x_2, z_2, k_{2t}^2, \mu^2) \hat{\sigma}_{ab}^*}$$

In this framework due to considering the z, fractional momenta of parent parton in the last step, one can have the full kinematics of the last step. Therefore the last step emitted parton comes directly into the calculation.

In the small x limit, the transverse momentum of parton ($\boldsymbol{k_t}$), becomes comparable against the collinear component, i.e., x P (P is the proton momentum). Therefore, the evolution equation of collinear parton, should be generalized by the fact that, parton can also have transverse momentum.

In contrast to the $\boldsymbol{k_t}$-factorization, where the full kinematics of hard parton is not considered, in the (z, $\boldsymbol{k_t}$)-factorization the full kinematics is taken into account. In this approach, the last step emitted parton plays an important role in the DCS calculation.

Illustration of the (z, k_t)- factorization at hadron-hadron collision. In the left panel of this figure, the transverse momentum of each incoming parton into the sub-process is generated by a single parton emission in the last evolution step. While, in the right panel of this figure, the last evolution step is factorized into the DUPDFs, i.e., $f_{q_i}(x_i, z_i, k_{it}^2,\ \mu^2)$, where i= 1, 2.

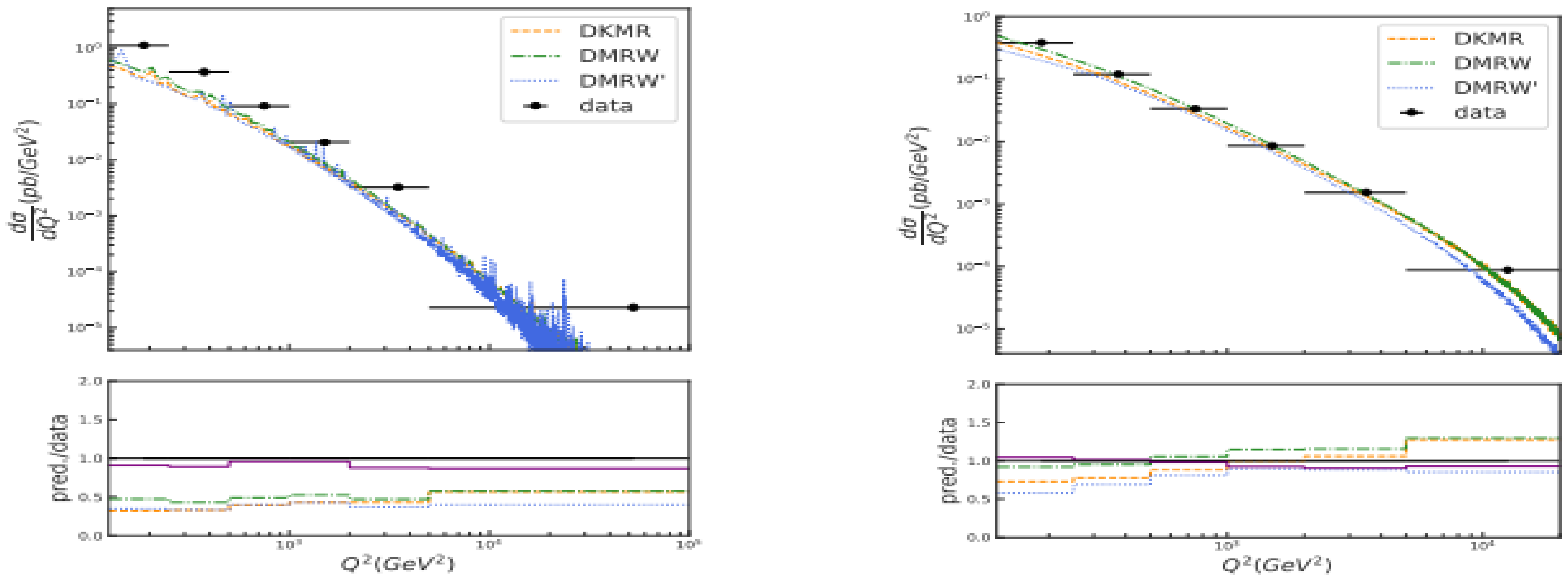


DUPDFs Methods

- **DKMR based on KMR**
- **DMRW based on LO-MRW**
- **DMRW′ based on NLO- MRW**

- We investigate electron-proton inclusive jet and dijet productions at 300 GeV and 319 GeV in the ZEUS experiment within the (z, k_t)-factorization.
- We use the MRW DUPDFs at LO and NLO levels (DMRW and DMRW'), in addition to the KMR DUPDF (DKMR).
- The cross section calculation is performed directly.
- The DUPDFs are unintegrated over both $k_t$and z.

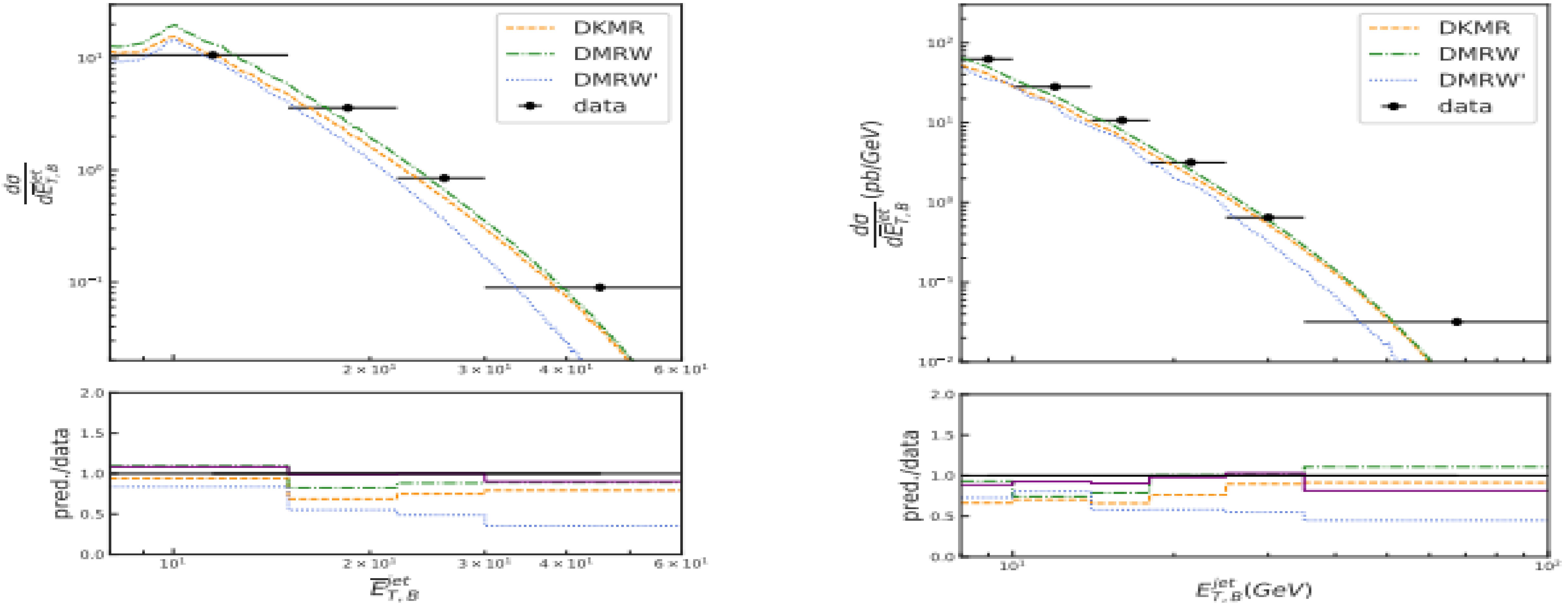
R. Kord Valeshabadi, M. Modarres, S .Rezaie, R. Aminzadeh Nik, Inclusive jet and dijet productions using k_t and (z, k_t)-factorizations versus ZEUS collaboration data, J.Phys.G 48 (2021) 8, 085009.



- All the DUPDFs predictions undershoot the data of $d\sigma/dQ^2$. We also see the result of the DMRW' is smaller with respect to the predictions of DKMR and DMRW due to the cutoff and virtuality.

- In contrast to the corresponding data of inclusive jet production, it can be observed that the results of the dijet subprocesses, Using DUPDFs have excellent agreement with the data.

J. Phys. G 48, 085009 (2021)



- The predictions of the DKMR and DMRW are in relatively good agreement with the data of $d\sigma/dE_{T,B}^{jet}$ channels of the inclusive jet and dijet prediction.
- We also see the result Of the DMRW' is smaller with respect to the predictions of DKMR and DMRW due to the cutoff and virtuality. Because we are working in the small center of mass energy, k^2 becomes large. As a result of this, DMRW' predictions become smaller than other DUPDFs .

J. Phys. G 48, 085009 (2021)

Numerical Methods

- We study the Z boson production via the proton-proton (p-p) collisions within the k_t and (z, k_t)-factorization frameworks, using the Martin-Ryskin-Watt (MRW) unintegrated parton distribution functions (UPDFs) and double unintegrated parton distribution functions (DUPDFs), respectively.

- For calculation of the differential cross section (DCS) within the k_t-factorization (k_t is the partonic transverse momentum), the KATIE parton level event generator is used, while for the (z, k_t)-factorization, the DCS is directly computed.

- We perform these calculations for the first three quark flavors, i.e., up, down, strange, and their anti-quarks.

- $\mu_f = \mu_r = \sqrt{{p_t^{ll}}^2 + {m^{ll}}^2}$ as the factorization and renormalization scales is chosen, in which p_t^{ll} and m^{ll} are the transverse momentum and the invariant mass of the output dilepton, respectively.

- We compare our results with the 13 TeV data of the ATLAS, LHCb, CMS collaborations and the corresponding collinear factorization prediction.

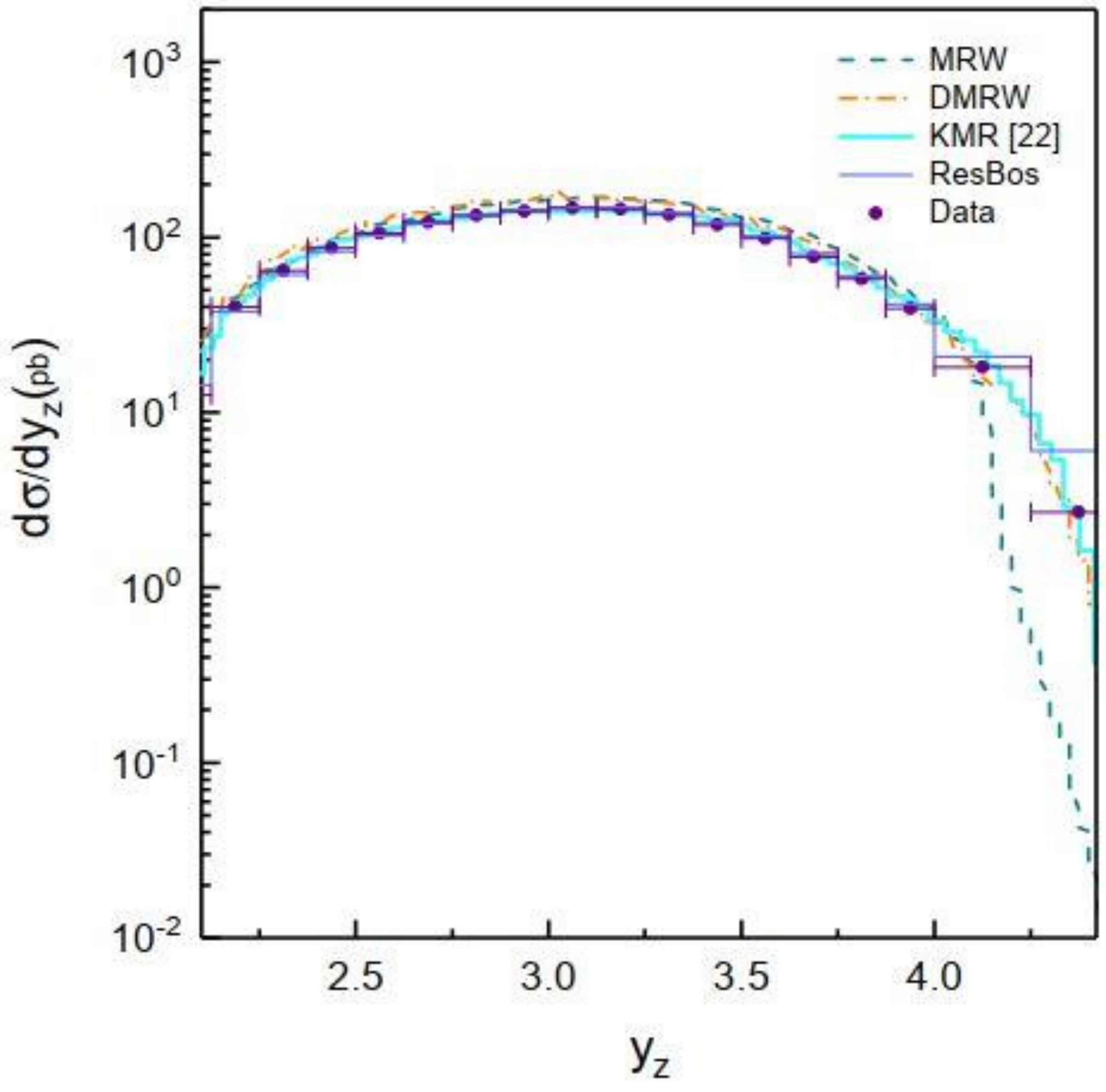
S. Rezaie, M. Modarres, Investigation of the Z boson production via hadron–hadron collisions in the k_t and (z, k_t)-factorization frameworks., Eur.Phys.J.C (2023) 83, 678.

KaTie parton level event generator

- KaTie is the first parton level event generator which calculates cross section for different parton level processes in collinear and k_t-factorization frameworks.
- This library is mostly written in Fortran.
- One can use TMDLib libraries for input UPDFs. However, when UPDFs are not available in TMDLIB, one should generate grid files for those UPDFs.
- KaTie can also generate LHEF event files, which late can feed into CASCADE to generate hadronic events.

The LHCb experiment

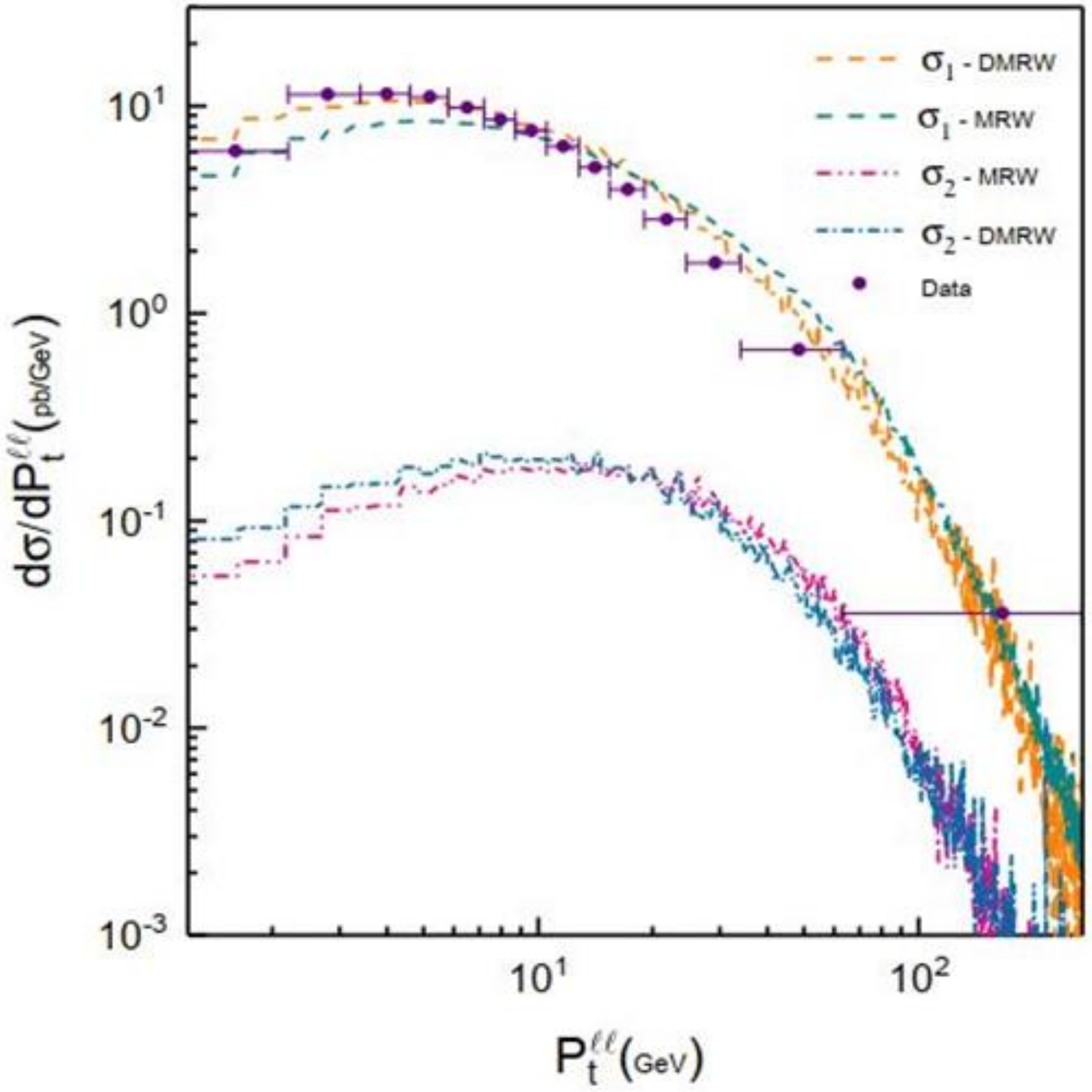
In this figure, one observes the variation of KMR, MRW and DMRW approaches with respect to the y_z. It is evident from this figure that, their results are close to each other in the Z boson rapidity region of $y_z < 4$, while they become separate from each other in the $y_z > 4$, wherein the MRW fails to describe the data well in that region. Additionally, despite the fact that the ResBos result, can cover the data well within all rapidity regions, but it tends to overestimate the data at large y_z limit.



Eur. Phys. J. C 83, 678 (2023)

The LHCb experiment

In this figure, a comparison between the contribution of the higher order sub-processes, denoted by σ_2, and lower order sub-processes, denoted by σ_1, for the MRW and the DMRW are compared. As it is obvious from this figure, the role of higher order sub-processes, is negligibly relative to the lower order sub-processes. Therefore one can safely ignore their contributions into our calculation.



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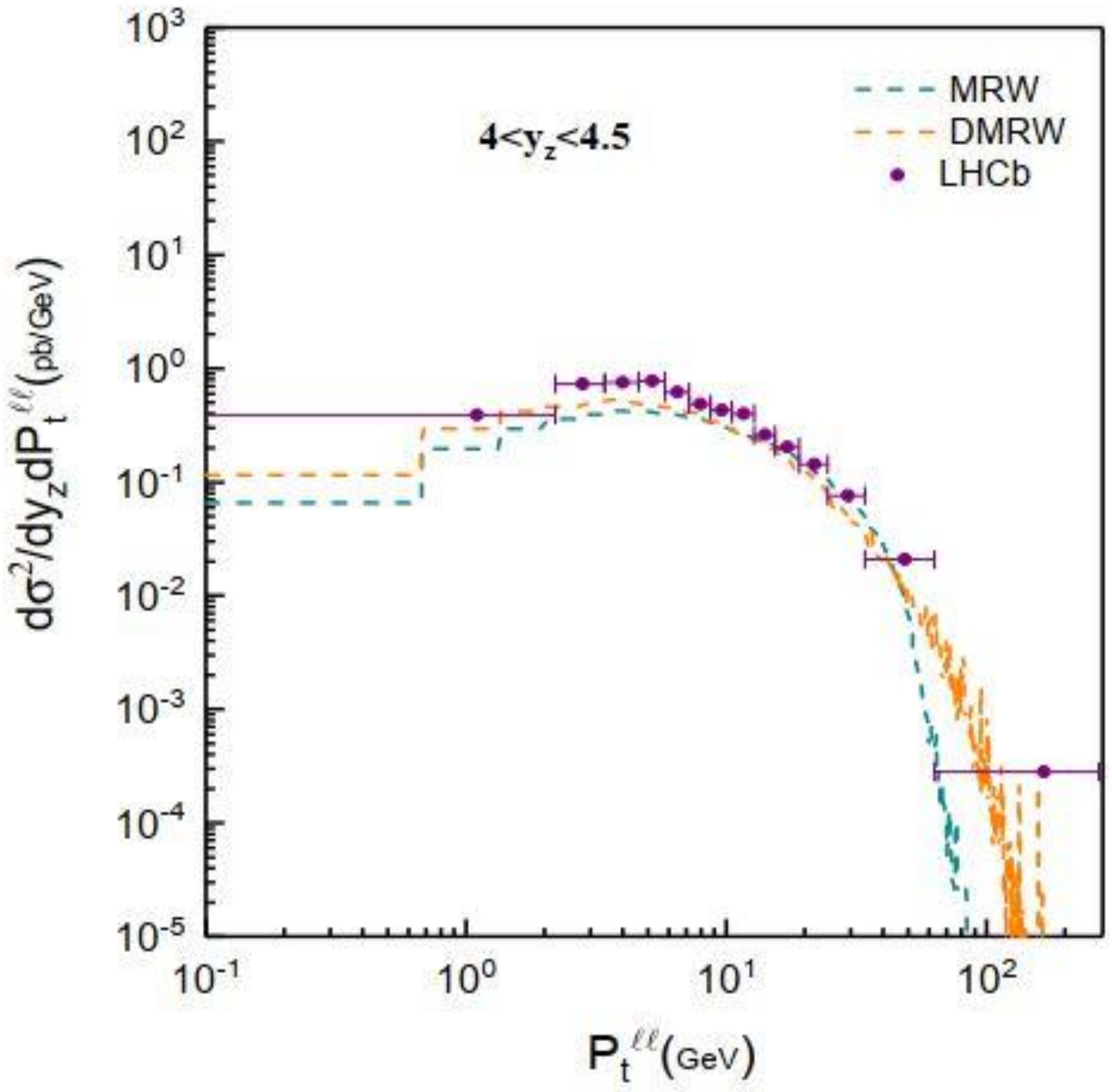
The LHCb experiment

In this figure, it can be observed the double DCS with respect to p_t^{ll} in various rapidity regions of the produced Z boson. Similar to our previous results for the cross section with respect to p_t^{ll}, it can be obtained relatively the same behavior in all of the regions except where $4 < y_z < 4.5$. In fact as we move toward large rapidity regions, the DMRW becomes much better relative to the MRW, especially in small and large dilepton transverse momentum regions.



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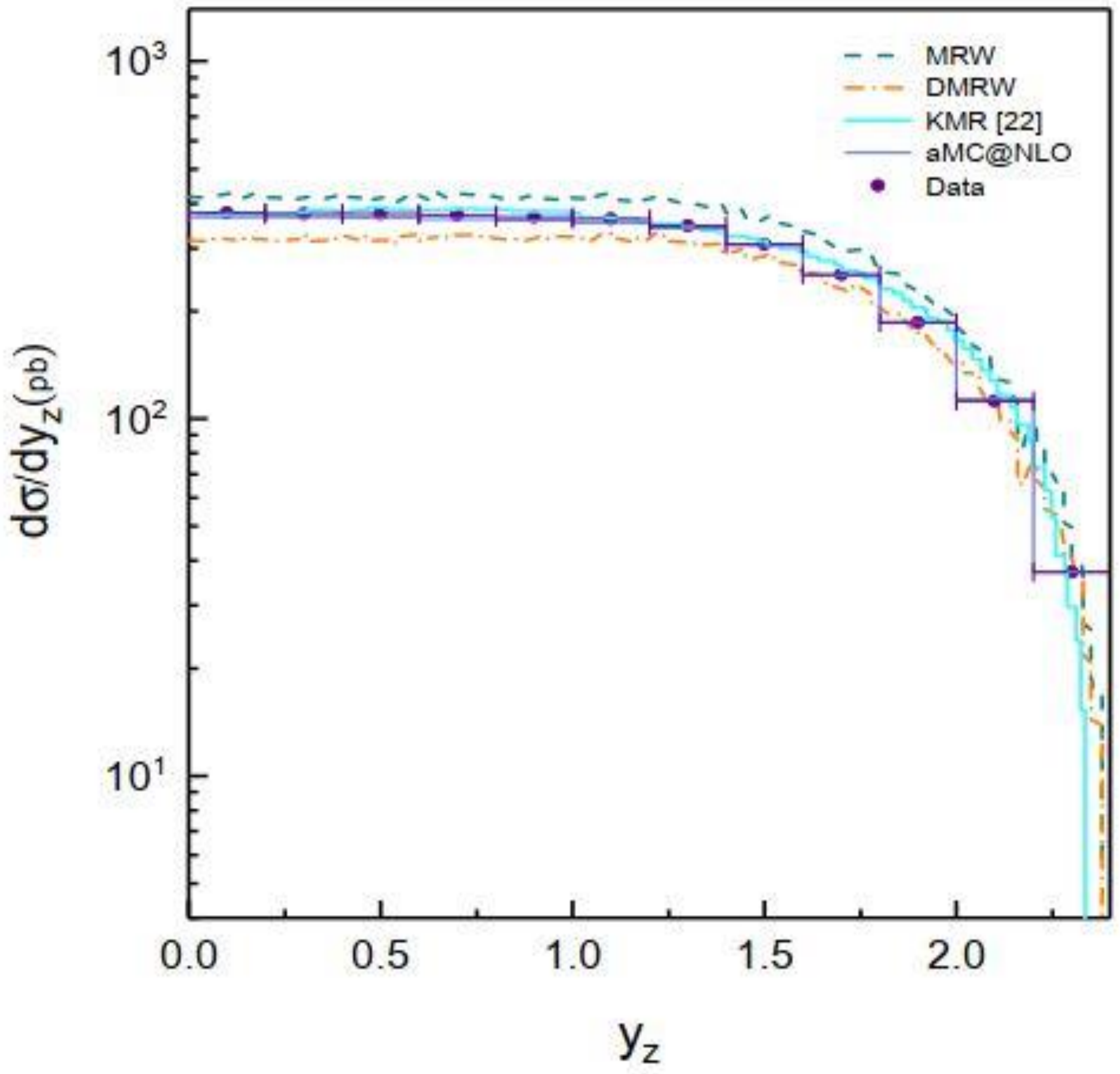
The CMS experiment

Similar to our results of the MRW and the DMRW for the LHCb experiment, it can also be observed relatively the same behavior for this data. It should be mentioned that because in this experiment the forward regions, i.e., $2.5 \leq y_z \leq 4.5$, do not play any role in calculation, so similar behavior can be seen in both frameworks.



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Conclusion

- Finally, it should be clarified that although the collinear framework can describe experimental data well with respect to the $k_t$and $(z,\, k_t)$- factorizations, it should be noted that these two frameworks can also exhibit remarkable results of exclusive processes with suitable UPDFs and DUPDFs.
- For instance, our recent works on three-photon productions and Drell-Yan processes have demonstrated that NLO-MRW and PB UPDFs can yield remarkable results. However, in this work, we aimed to provide a one-to-one comparison between the k_t and $(z,\, k_t)$-factorizations to better understand the difference between these two frameworks.
- As we have demonstrated in this work, the k_t and $(z,\, k_t)$ frameworks exhibit relatively similar behavior in all regions except for large rapidity limits. Nevertheless, it is an important goal to generate these DUPDFs and better describe the results.

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THANK YOU!

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Backup

TMDs

KMR

KMR is based on DGLAP evolution equation. The main assumption of the DGLAP evolution (Strong Ordering) has been altered on the “last step emission”, i.e.

$$\mathbf{f_q^{KMR}}\left(\mathbf{x}, \mathbf{k_t^2}, \boldsymbol{\mu}^2\right) = T_q^{KMR}(k_t^2, \mu^2)$$

$$\frac{\alpha_s\left(k_t^2\right)}{2\pi}\int_x^{1-\Delta}\left[P_{qq}^{LO}(z)\frac{x}{z}q^{LO}\left(\frac{x}{z}, k_t^2\right) + P_{qg}^{LO}(z)\frac{x}{z}g^{LO}\left(\frac{x}{z}, k_t^2\right)\right] dz$$

$$f_g^{KMR}\left(x, k_t^2, \mu^2\right) = T_g^{KMR}(k_t^2, \mu^2)$$

$$\frac{\alpha_s\left(k_t^2\right)}{2\pi}\int_x^{1-\Delta}\left[P_{gg}^{LO}(z)\frac{x}{z}g^{LO}\left(\frac{x}{z}, k_t^2\right) + \sum_{\mathrm{q}} P_{gq}^{LO}(z)\frac{x}{z}q^{LO}\left(\frac{x}{z}, k_t^2\right)\right] dz$$

$$\mathrm{T_q^{KMR}}\left(\mathrm{k_t^2}, \mu^2\right) = \exp\left(-\int_{\mathrm{k_t^2}}^{\mu^2}\frac{d\kappa_t^2}{\kappa_t^2}\frac{\alpha_s\left(\mathrm{k_t^2}\right)}{2\pi}\int_0^{1-\Delta}\mathrm{P_{qq}^{LO}}(\xi)d\xi\right)$$

$$\mathrm{T_g^{KMR}}\left(\mathrm{k_t^2}, \mu^2\right) = \exp\left(-\int_{\mathrm{k_t^2}}^{\mu^2}\frac{d\kappa_t^2}{\kappa_t^2}\frac{\alpha_s\left(\mathrm{k_t^2}\right)}{2\pi}\int_0^{1-\Delta}\left(\left[\xi\mathrm{P_{gg}^{LO}}(\xi) + n_f\mathrm{P_{qg}^{LO}}(\xi)\right]\right)d\xi\right)$$

LOMRW

$$f_q(x,,k_t^2,\mu^2) = T_q(k_t^2,\mu^2)\frac{\alpha_s(k_t^2)}{2\pi}\int_x^1 \left[P_{qq}(z)f_q\left(\frac{x}{z},k_t^2\right)\Theta(\mathbf{z}-\mathbf{1}+\boldsymbol{\Delta}) + P_{qg}(z)f_g\left(\frac{x}{z},k_t^2\right)\right] dz$$

$$f_g(x,k_t^2,\mu^2) = T_g(k_t^2,\mu^2)\frac{\alpha_s(k_t^2)}{2\pi}\int_x^1 \left[P_{gg}(z)f_g\left(\frac{x}{z},k_t^2\right)\Theta(\mathbf{z}-\mathbf{1}+\boldsymbol{\Delta}) + \sum_q P_{gq}(z)f_q\left(\frac{x}{z},k_t^2\right)\right] dz$$

$$T_q(k_t^2,\mu^2) = exp\left(-\int_{k_t^2}^{\mu^2}\frac{d\kappa_t^2}{\kappa_t^2}\frac{\alpha_s(\kappa_t^2)}{2\pi}\int_0^1 \mathbf{P}_{\mathbf{qq}}^{\mathbf{LO}}(\xi)\Theta(\xi-\mathbf{1}+\boldsymbol{\Delta})\mathbf{d}\xi\right)$$

$$T_g(k_t^2,\mu^2) = \mathbf{exp}\left(-\int_{\mathrm{k}_\mathrm{t}^2}^{\mu^2}\frac{\mathrm{d}\kappa_t^2}{\kappa_t^2}\frac{\alpha_s(\kappa_t^2)}{2\pi}\int_0^1 [\xi\mathbf{P}_{\mathbf{gg}}^{\mathbf{LO}}(\xi)\Theta(\xi-\mathbf{1}+\boldsymbol{\Delta})\Theta(\xi-\boldsymbol{\Delta}) + n_f\mathbf{P}_{qg}^{\mathbf{LO}}(\xi)]\mathbf{d}\xi\right)$$

In the above equations, the cutoff $\Theta(\mathbf{z}-\mathbf{1}+\boldsymbol{\Delta})$ is imposed in order to avoid soft gluon emissions. This cutoff can be determined according to the angular ordering in the last evolution step, i.e.:

$$\boldsymbol{\Delta} = \frac{k_t}{\mu + k_t}$$

NLOMRW

- In the NLO-MRW scale is set: $k^2 = -\frac{k_t^2}{1-z}$.
- Can be approximated with the LO splitting functions.

$$f_q^{NLO-MRW}(x, k_t^2, \mu^2) =$$

$$\int_x^1 \frac{\alpha_s^{NLO}(k^2)}{2\pi} T_q^{NLO-MRW}(k^2, \mu^2) \left[P_{qq}^{LO}(z) \frac{x}{z} q^{NLO}\left(\frac{x}{z}, k^2\right) \Theta(\mathrm{z} - 1 + \Delta) + P_{qg}^{LO}(z) \frac{x}{z} g^{NLO}\left(\frac{x}{z}, k^2\right)\right] \Theta(\mu^2 - k^2)\, dz$$

$$f_g^{NLO-MRW}(x, k_t^2, \mu^2) =$$

$$\int_x^1 \frac{\alpha_s^{NLO}(k^2)}{2\pi} T_g^{NLO-MRW}(k^2, \mu^2) \left[P_{gg}^{LO}(z) \frac{x}{z} g^{NLO}\left(\frac{x}{z}, k^2\right) \Theta(\mathrm{z} - 1 + \Delta) + \sum_q P_{gq}^{LO}(z) \frac{x}{z} q^{NLO}\left(\frac{x}{z}, k^2\right)\right] \Theta(\mu^2 - k^2)\, dz$$

$$T_q^{NLO-MRW}(k^2, \mu^2) = exp\left(-\int_{k^2}^{\mu^2} \frac{d\kappa^2}{\kappa^2} \frac{\alpha_s^{NLO}(\kappa^2)}{2\pi} \int_0^1 \xi[\mathrm{P}_{\mathrm{qq}}^{\mathrm{LO}}(\xi)\Theta(\xi - 1 + \Delta) + \mathrm{P}_{gq}^{\mathrm{LO}}(\xi)]\mathrm{d}\xi\right)$$

$$T_g^{NLO-MRW}(k^2, \mu^2) = exp\left(-\int_{k^2}^{\mu^2} \frac{d\kappa^2}{\kappa^2} \frac{\alpha_s^{NLO}(\kappa^2)}{2\pi} \int_0^1 \xi[\mathrm{P}_{\mathrm{gg}}^{\mathrm{LO}}(\xi)\Theta(\xi - 1 + \Delta) + 2n_f \mathrm{P}_{qg}^{\mathrm{LO}}(\xi)]\mathrm{d}\xi\right)$$

DUPDFs

(z, k_t)-factorization framework

$$f_q^{DMRW}(x, \mathrm{z}, k_t^2, \mu^2) = T_q^{DMRW}(k_t^2, \mu^2)\frac{\alpha_s(k_t^2)}{2\pi}\left[P_{qq}(z)f_q\left(\frac{x}{z}, k_t^2\right)\Theta(\mathrm{z} - 1 + \Delta) + P_{qg}(z)f_g\left(\frac{x}{z}, k_t^2\right)\right]$$

$$f_g^{DMRW}(x, z, k_t^2, \mu^2) = T_g^{DMRW}(k_t^2, \mu^2)\frac{\alpha_s(k_t^2)}{2\pi}\left[P_{gg}(z)f_g\left(\frac{x}{z}, k_t^2\right)\Theta(\mathrm{z} - 1 + \Delta) + \sum_q P_{gq}(z)f_q\left(\frac{x}{z}, k_t^2\right)\right]$$

$$T_q^{DMRW}(k_t^2, \mu^2) = exp\left(-\int_{k_t^2}^{\mu^2}\frac{\mathrm{d}\kappa_t^2}{\kappa_t^2}\frac{\alpha_s(\kappa_t^2)}{2\pi}\int_0^1 d\xi P_{qq}^{LO}(\xi)\Theta(1 - \xi - \Delta)\right)$$

$$T_g^{DMRW}(k_t^2, \mu^2) = exp(-\int_{k_t^2}^{\mu^2}\frac{\mathrm{d}\kappa_t^2}{\kappa_t^2}\frac{\alpha_s(\kappa_t^2)}{2\pi}\int_0^1 d\xi[\xi P_{gg}^{LO}(\xi)\Theta(1 - \xi - \Delta)$$
$$\Theta(\xi - \Delta) + n_f P_{qg}^{LO}(\xi)])$$

End