Supersymmetric solitons, ground states and higher dimensional interpretation

Antonio Gallerati

Politecnico di Torino - Dipartimento di Scienza Applicata e Tecnologia - Torino, Italy

Outline

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1. Introduction

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2. **The model**
   - Explicit solutions
   - Thermodynamics
   - BPS solutions

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3. **Conclusions**

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Solitons: special role in classical physics as well as in quantum and string theory, determining a richer structure of the full non-perturbative regime:

- originally used as “bounce solutions” to discuss the possible instability of the pure Kaluza-Klein vacuum ground state;
- generalizations of these soliton solutions have been also considered in the analysis of the semiclassical stability of non-susy AdS gravity;
- soliton configurations can turn out to be the lowest energy solution with chosen boundary conditions, leading to a new kind of positive energy conjecture;

- it is possible to consider configuration featuring Wilson loops:
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- BPS configurations preserving some of the supercharges can be obtained analysing the explicit form of the Killing spinors equations.

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4. We consider an explicit solutions in the T^3 model, the latter resulting in a single dilaton truncations of the maximal $SO(8)$ gauged supergravity in $D = 4$.

We consider electrically charged solutions in a purely magnetic gauging. The action has the explicit form:

$$\mathscr{S} = \frac{1}{8\pi G}\int d^4x\,\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}(\partial\phi)^2 + \frac{3}{L^2}\cosh\left(\sqrt{\frac{2}{3}}\phi\right) - \frac{1}{4}e^{3\sqrt{\frac{2}{3}}\phi}\left(F^1\right)^2 - \frac{1}{4}e^{-\sqrt{\frac{2}{3}}\phi}\left(F^2\right)^2\right).$$

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- in the model we consider there are two Wilson lines,

$$\Phi^1_M = \int F^1, \qquad \Phi^2_M = \int F^2,$$

  and there is a one-parameter family of values of the Wilson lines which give supersymmetric solitons;

- the explicit solution has the schematic form

$$\phi = \pm\ell^{-1}\ln(x)\,, \qquad F^{\Lambda}_{\mu\nu}(x,\Gamma^{\Lambda})\,,$$

$$ds^2 = \Upsilon(x)\left(L^2\,dt^2 - \frac{\eta^2}{f(x)}\,dx^2 - f(x)\,d\psi^2 - L^2\,dz^2\right)\,;$$

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$\Longrightarrow$ surprisingly, there is a family of non-susy solutions of lower energy and free energy than the susy ones.

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- Regularity of the metric at $x = x_0$ requires $\varphi \in [0,\Delta]$ where

$$\Delta^{-1} = \left|\frac{1}{4\pi\eta}\frac{df}{dx}\right|_{x=x_0} \;;$$

- Solutions with non-zero charges have net magnetic fluxes at infinity

$$\Phi_M^1 = \int F^1 = \oint A^1 = Q_1\,\Delta\left(1 - x_0^{-2}\right) \equiv 2\pi L\,\psi_1,$$
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- The scalar field induces a vev of an operator in the dual theory,

$$\langle \mathcal{O} \rangle = \phi_0 = \pm\frac{\sqrt{6}}{2}\,\frac{\pi\,x_0\left|\psi_1^2\,(1+2\,x_0^2) - \psi_2^2\right|}{\Delta}\,.$$

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- for fixed charge boundary conditions there are 2 distinct susy soliton configurations (degeneracy of supersymmetric solutions);
- for the same fixed charge boundary conditions, surprisingly a family of non-susy solutions of lower energy and free energy than the supersymmetric ones can be found.

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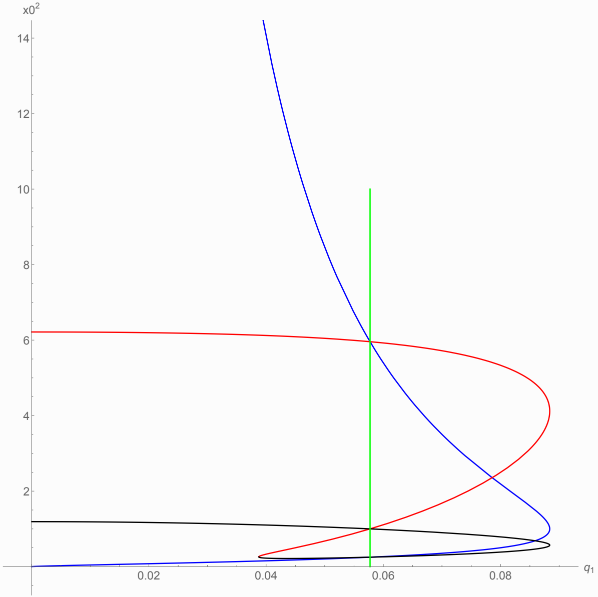

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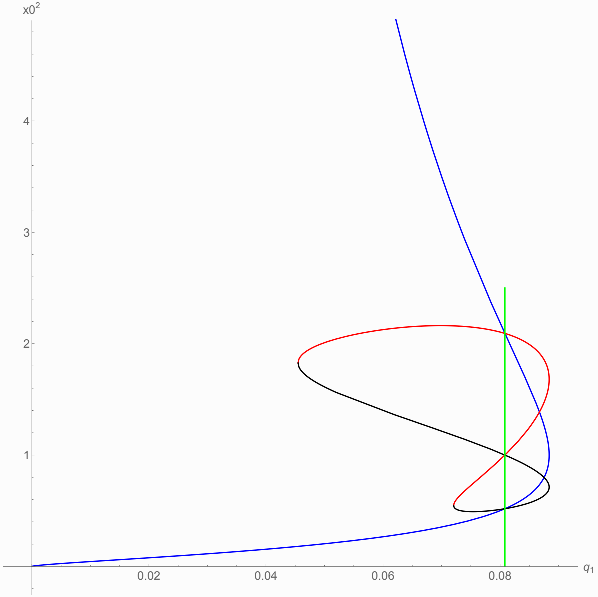

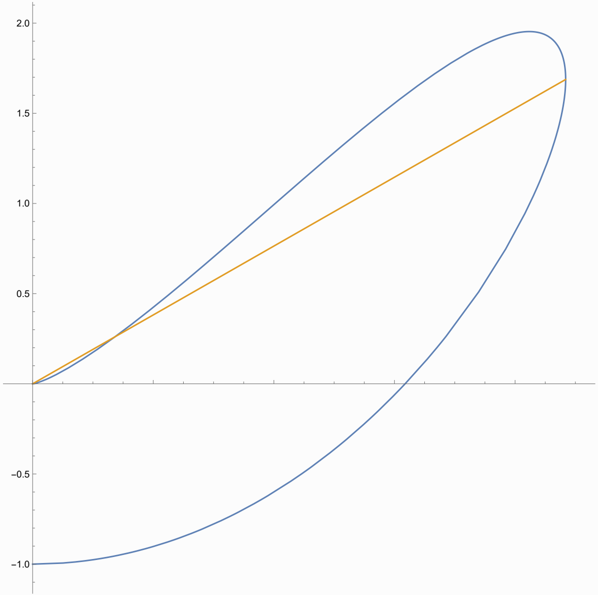
2.0
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Conclusions Distinctive features

1. We studied some soliton configuration in a gauged supergravity framework, constructing solutions of its T^3 model truncation.
2. In the model under consideration there are two Wilson lines, with a one-parameter family of the latter which give supersymmetric solitons.
3. For supersymmetry-preserving fixed charge boundary conditions there are two distinct soliton solutions.

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4. The new solutions require a more in-depth study of the degeneracy of the susy configurations in the presence of generic boundary conditions.
5. One branch of susy solutions has higher energy than a non-susy one with the same boundary conditions.

Thank you for listening!