Neutrino Texture Definitions and Phenomenology

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Abstract

We draw attention to the role the unphysical phases play in the definition of a neutrino texture, and apply this to a texture defined by one-equality and one-antiequality.
(Reference: Chamoun & Lashin , e-Print: 2308.10985 [hep-ph], submitted to JHEP)

Motivation

- Many studies related to neutrino textures involve scanning over the free parameters of the neutrino mass matrix
- since the unphysical phases have no physical meaning in Standard Model supplemented by neutrino masses, one may expect that putting them equal to zero has no effect on the phenomenology
- This is the case for zero textures, but not in general true. WHY ?

- Majorana neutrinos can, within seesaw scenarios, interpret the smallness of neutrino masses.
- Majorana mass term $({\nu_L}^T M_\nu \mathcal{C} \nu_L)$ (with $\mathcal{C}$ the charge conjugation matrix), with the 12-parameters complex symmetric matrix M_ν analysed into a 3-mass matrix $M_\nu^{\text{diag.}}$ and a unitary 9-parameters matrix V
- $$M_\nu = V_\nu^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V_\nu^\dagger$$
- $$V = P_\phi \; U_\delta \; P^{\text{Maj.}} \quad :$$
$$P_\phi = \text{diag}\left(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\right) \quad , \quad P^{\text{Maj.}} = \text{diag}\left(e^{i\rho}, e^{i\sigma}, 1\right),$$

Measurable Mixing

- In the flavor basis V is the measurable V_{PMNS}
- $$U_{\text{PMNS}} \;=\; U_\delta \; P^{\text{Maj.}}$$

$$U_\delta =$$

$$\begin{pmatrix} c_{12}\,c_{13}e^{i\rho} & s_{12}\,c_{13}e^{i\sigma} & s_{13} \\ (-c_{12}\,s_{23}\,s_{13} - s_{12}\,c_{23}\,e^{-i\delta})e^{i\rho} & (-s_{12}\,s_{23}\,s_{13} + c_{12}\,c_{23}\,e^{-i\delta})e^{i\sigma} & s_{23}\,c_{13} \\ (-c_{12}\,c_{23}\,s_{13} + s_{12}\,s_{23}\,e^{-i\delta})e^{i\rho} & (-s_{12}\,c_{23}\,s_{13} - c_{12}\,s_{23}\,e^{-i\delta})e^{i\sigma} & c_{23}\,c_{13} \end{pmatrix}$$

UnPhysical Phases & Texture Definition

- The flavor basis of the charged leptons is defined up to a 3-dim phase matrix $F = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$, where (ϕ_1, ϕ_2, ϕ_3) are the unphysical charged lepton phases.
- One can absorb F by redefining equally the neutrino fields, upon which M_ν is "phased" as

$$(\nu \to F\nu) \quad \Rightarrow \quad \left(M_\nu \to M'_\nu = F^* M_\nu F^*\right)$$

- For a fixed parametrization of the $V^{phys.}_{\text{PMNS}}$, any M_ν can be decomposed uniquely as :

$$M_\nu \quad = \quad (FV^{phys.}_{\text{PMNS}})M^d(FV^{phys.}_{\text{PMNS}})^T,$$

where M^d is diagonal with positive masses,

- $$F^* M_\nu F^* = M_\nu^{\text{phys}} \quad = \quad V_{\text{PMNS}}^{phys.} M^d (V_{\text{PMNS}}^{phys.})^T$$
- The two matrices M_ν and M_ν^{phys} differ only in the nonphysical phases but have the same physics and no way to distinguish one from the other by physical SM measurements.
- Any texture definition should be a characteristic of M_ν^{phys}, such that two matrices differing only in the unphysical phases should together, either both satisfy the texture definition or neither does.

- we define an equivalence relation on the 12-dim space $\mathcal{M}$ of complex symmetric 3×3 matrices A by:

$$A \sim A' \quad \Leftrightarrow \exists \text{phase matrix } F : \quad A' = F.A.F$$

- the texture is defined actually on the set of equivalence classes $\mathcal{M}/\sim \equiv \{[M]\}$.
-

$$A'_{ij} \quad = \quad e^{i(\phi_i+\phi_j)} A_{ij}$$

Three common ways to define a texture

- $$\text{'Mathematical' def. :} \quad M_\nu \in \text{texture} \Leftrightarrow g(M_\nu) = 0$$
  However, it may be sensitive to unphysical phases and may not be rephasing-invariant. ($M_{\nu 11} = 0$) is insensitive to unphysical phases ($M'_{\nu 11} = 0$), the zero-texture is indeed rephasing-invariant.
  However, a texture definition given by ($M_{\nu 11} = M_{\nu 22}$), then ($M'_{\nu 11} = e^{2i\phi_1} M_{\nu 11}, M'_{\nu 22} = e^{2i\phi_2} M_{\nu 22}$), and so we get ($M'_{\nu 11} = e^{2i(\phi_1 - \phi_2)} M'_{\nu 22} \neq M'_{\nu 22}$). Thus, the texture definition is met for M_ν whereas it is not met for M'_ν.
- The correct way to define a texture is to define it on the equivalence classes $\mathcal{M}/\sim$, in that two equivalent matrices either both satisfy the texture definition or both fail it, such that the definition would be invariant under "rephasing".

Re-Phasing-Invariance

Here, two common ways to meet this:

$$\text{`Generalized' def.}: \quad M_\nu \in \text{texture} \Leftrightarrow \quad \exists M'_\nu \sim M_\nu : g(M'_\nu) = 0$$
$$\text{`Specific' def.}: \quad M_\nu \in \text{texture} \Leftrightarrow \quad g(M_\nu^{\text{phys.}}) = 0$$

Past studies:

- We stress also that all past studies, which restricted the analysis to the vanishing unphysical phases slice, should be looked at as being carried out within ('Specific' def.), otherwise their analysis would have been susceptible to weaknesses having picked up a subset of the admissible parameter space.

Parametrization and unphysical phases

- $$U_\delta^{\text{PDG}} = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}e^{i\delta} & s_{23}\,c_{13} \\ s_{12}s_{23} - c_{12}\,c_{23}\,s_{13}e^{i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}e^{i\delta} & c_{23}\,c_{13} \end{pmatrix},$$
  $$P_{\text{PDG}}^{\text{Maj.}} = \text{diag}\left(e^{i\rho}, e^{i\sigma}, 1\right), P_\phi^{\text{PDG}} = \text{diag}\left(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\right)$$
- $$\text{Adopted parametrization: } (\phi_1, \sigma, \rho)$$
  $$\text{PDG parametrization: } (\phi'_1 = \phi_1 + \delta, \sigma' = \sigma - \delta, \rho' = \rho - \delta)$$
  A vanishing unphysical phases slice in one parametrization does NOT correspond to a constant, nor -a fortiori- a vanishing, unphysical phases slice in another parametrization.
- A second requirement for a consistent texture definition is to be paramterization-independent.

Comparison

$M_\nu \in$ texture $\Leftrightarrow$	$\phi^{\text{unphys.}}$-invariance	Parametrization independence	Physicality	ϕ^{unphys} correlations	realizability
$g(M_\nu) = 0$ ('Mathematical' def.)	$\times$	$\surd$	$\times$	not trivial	$\surd$
$g(M_\nu^{\text{phys}}) = 0$ ('Specific' def.)	$\surd$	$\times$	$\times$	trivial	$\times$
$\exists M'_\nu \sim M_\nu : g(M'_\nu) = 0$ ('Generalized' def.)	$\surd$	$\surd$	$\surd$	trivial	$\surd$

Table: Properties of the three different texture definitions. ϕ^{unphys}-invariance means that the definition is defined for the equivalence class of matrices, where $M'_\nu \sim M_\nu$ means that both matrices have the same 9 physical observables and where M_ν^{phys} is equivalent to M_ν but with vanishing ϕ^{unphys}. Because ϕ^{unphys}'s are sensitive to the PMNS paramterization, then 'Physicality' requires both ϕ^{unphys}-invariance and PMNS parametrization-independence. By realizability we mean whether the model leading to a texture of the specified form can embody or not the definition.

S_4-motivated Texture:

$$
\begin{aligned}
I: &\quad M_{\nu 22} = -M_{\nu 33} \quad \& \quad M_{\nu 11} = +M_{\nu 23}, \\
II: &\quad M_{\nu 11} = -M_{\nu 33} \quad \& \quad M_{\nu 22} = +M_{\nu 13}, \\
III: &\quad M_{\nu 11} = -M_{\nu 22} \quad \& \quad M_{\nu 33} = +M_{\nu 12}
\end{aligned}
$$

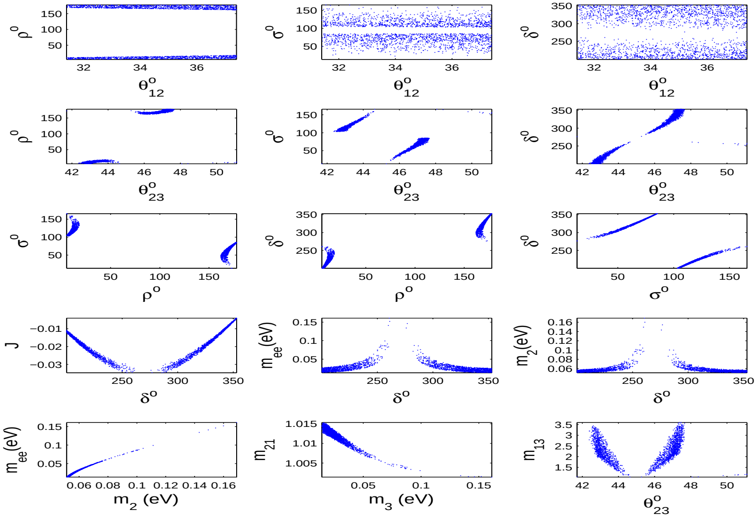

Figure: Texture I ($M_{22} + M_{33} = 0$ & $M_{11} - M_{23} = 0$) at vanishing unphysical phases slice, **IH**.

Pattern I: at non-vanishing unphysical phases

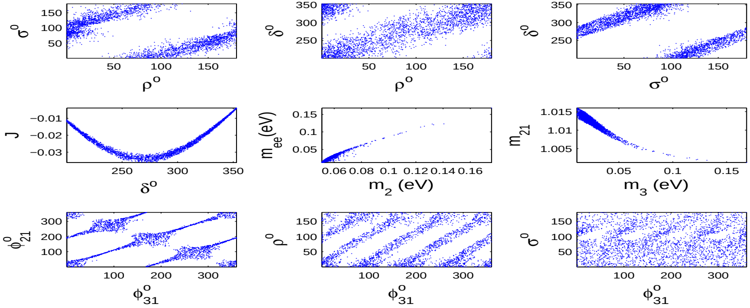

Figure: Texture I ($M_{22} + M_{33} = 0$ & $M_{11} - M_{23} = 0$) at nonvanishing unphysical phases slice, **IH**.

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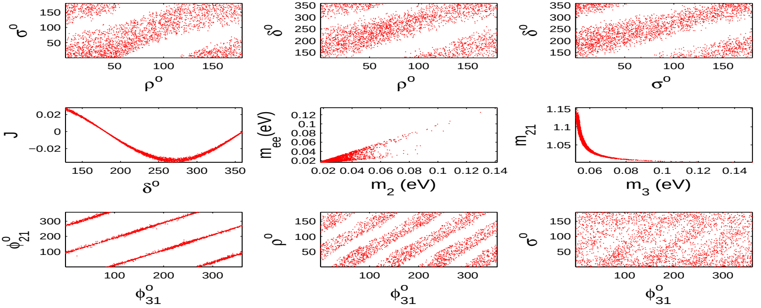

Figure: Texture I ($M_{22} + M_{33} = 0$ & $M_{11} - M_{23} = 0$) at nonvanishing unphysical phases slice, **NH**.

Summary

- Single out the role of the unphysical phases in the texture definition. All past studies restricted to vanishing unphysical phases case should be looked upon as textures defined not merely by a mathematical constraint, but rather via a constraint defined on the vanishing unphysical phases slice depending in turn on the taken parametrization.
- Three different definitions of a given texture: "Mathematical", "Specific" and "Generalized". Only the third definition is insensitive to the unphysical phases and is independent of the PMNS parametrization.
- Different Phenomenologies when switching on/off the unphysical phases are actually corresponding to different textures.