Antenna subtraction for the production of massive final state fermions at hadron colliders

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Motivations

Why are we interested in top quarks?

- Very large cross section at the LHC:
  $\sigma_{t\bar{t}}(14\mathrm{TeV}, \mathrm{p_T^{top}} > 700\mathrm{GeV}) \approx 700\mathrm{fb}$
- Large Yukawa coupling. Sensitivity to the mechanism of electroweak symmetry breaking
- Background to various New Physics searches
- Preferred channel for the decay of potential new heavy resonances

Motivations

A few facts about $t\bar{t}$ production

- At the LHC an experimental error of $\sim$ 5% is expected for $\sigma_{t\bar{t}}$
- Theoretically, NLO[1]+NLL[2] calculations for the LHC give an uncertainty of $\sim$ 10%

- [1] Nason, Dawson, Ellis '88-'90; Beenakker, Kuijf, van Neerven, Smith '89-'91
- [2] Kidonakis, Sterman '97; Bonciani et al. '98, Cacciari et al. '08; Moch, Uwer '08; Kidonakis '08
- Most recently completed NNLL resummation: Ahrens et al. '11

To match the theoretical and experimental accuracies, a full NNLO calculation is needed

- 2-Loop corrections: Czakon '08; Bonciani et al. '08-'10
- 1$\times$1-loop corrections: Korner et al. '05, Anastasiou, Aybat '08; Kniehl et al. '08
- Real-Virtual corrections: Bierenbaum, Czakon, Mitov, '11
- Real corrections:
  - Subtraction methods at NNLO (massless): Daleo et al. '09; Boughezal, Gehrmann-De Ridder, Ritzmann '10; Glover, Pires '10
  - New NNLO methods (massive): Czakon '11; Anastasiou, Herzog, Lazopoulos '10

Motivations

Full NNLO calculation is not completed yet

$\Rightarrow$ Our aim: compute real radiation corrections

Towards this goal we

- Fully extended the NLO antenna subtraction method for hadronic collisions to incorporate massive particles in the final state [1]
- Computed NLO subtraction terms for $\sigma_{t\bar{t}}$ and $\sigma_{t\bar{t}+jet}$ [1]
    - $\sigma^{NLO}_{t\bar{t}+jet}$ needed for $t\bar{t}$ at NNLO (same matrix elements, same single unresolved limits)
- Computed NNLO subtraction terms for the following processes contributing to the double real emission corrections to $t\bar{t}$ production at the LHC:

$$q\bar{q} \to t\bar{t}q'\bar{q}' \quad q\bar{q} \to t\bar{t}q\bar{q} \quad qq' \to t\bar{t}qq' \quad qq \to t\bar{t}qq$$

[1] G.A and A. Gehrmann-De Ridder, JHEP **1104**, 063 (2011)

Subtraction at NLO for hadronic processes

Symbolically, we can write

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{NLO} &= \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{R}_{NLO} - \mathrm{d}\hat{\sigma}^{S}_{NLO} \right) J_m^{(m+1)} \\ &+ \int_{\mathrm{d}\Phi_m} \left(\mathrm{d}\hat{\sigma}^{V}_{NLO} + \mathrm{d}\hat{\sigma}^{MF}_{NLO} + \int_1 \mathrm{d}\hat{\sigma}^{S}_{NLO} \right) J_m^{(m)}, \end{aligned}$$

Subtraction term:

- Approximation to the $(m+1)$-particle matrix element in its single unresolved limits
    - Soft limits: $E_g \to 0$
    - Collinear limits: for example $q||g \Rightarrow s_{qg} \to 0$
- Can be integrated over a factorized form of the $(m+1)$-particle phase space and added to the 1-loop m-particle contribution

$\Rightarrow$ $\mathrm{d}\hat{\sigma}^{R}_{NLO} - \mathrm{d}\hat{\sigma}^{S}_{NLO}$ is numerically finite
$\mathrm{d}\hat{\sigma}^{V}_{NLO} + \mathrm{d}\hat{\sigma}^{MF}_{NLO} + \int_1 \mathrm{d}\hat{\sigma}^{S}_{NLO}$ is free of divergencies

Real radiation for $t\bar{t} + jets$ production at NLO

For $p_1 + p_2 \to k_Q + k_{\bar{Q}} + (m-2) jets$

$$\begin{aligned} \mathrm{d}\hat{\sigma}^R_{NLO}(p_1, p_2) &= \mathcal{N} \sum \mathrm{d}\Phi_{m+1}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_{m-1}; p_1, p_2) \\ &\quad \times \frac{1}{S_{m+1}} |\mathcal{M}_{m+1}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_{m-1}; p_1, p_2)|^2 \\ &\quad \times J_m^{(m+1)}(k_Q, k_{\bar{Q}}, k_1, \ldots, k_{m-1}). \end{aligned}$$

Knowing the factorization properties of $\mathcal{M}$ in its infrared limits, we can construct $\mathrm{d}\hat{\sigma}^S_{NLO}$ that reproduces all the configurations in which a parton j becomes unresolved between the hard radiators i and k.

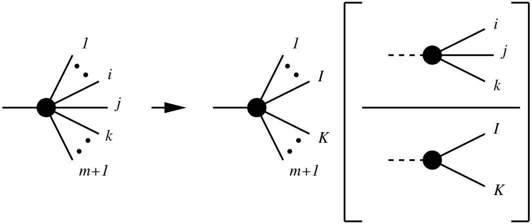
Unresolved limits of $\mathcal{M}$:

- Soft gluon limits $\to$ ϵ poles in $\mathrm{d}\hat{\sigma}$
- Collinear limits (massless partons) $\to$ ϵ poles in $\mathrm{d}\hat{\sigma}$
- Quasi-collinear limits (with at least one massive final state fermion) $\to$ $\ln(Q^2/M_Q^2)$ in $\mathrm{d}\hat{\sigma}$

NLO Antenna subtraction

Subtraction terms for m-jet production (massless case):

- Product of reduced matrix elements with m particles and antenna functions
- Antenna functions $X_3^0(i,j,k)$: normalized three-particle matrix element
  - Two hard particles (hard radiators)
  - One particle soft, or collinear to either of the radiators



$$|\mathcal{M}_{m+1}(\ldots,p_i,p_j,p_k,\ldots)|^2 \longrightarrow X_3^0(i,j,k)\cdot|\mathcal{M}_m(\ldots,p_I,p_K,\ldots)|^2$$

For $t\bar{t}$ and $t\bar{t} + jet$ at NLO...

We need

- Three types of subtraction terms
  - Final-final: both hard radiators in the final state
  - Initial-final: one hard radiator in the initial state and one in the final state
  - Initial-initial: both hard radiators in the initial state
- Massive final-final and initial-final antennae as well as massless initial-final and initial-initial three parton antennae

Example: To account for the unresolved limits of a gluon between a massless $q\bar{q}$ pair we need

$$A_3^0(q, g, \bar{q}), \text{ which is generated from } (\gamma^* \to qg\bar{q})/(\gamma^* \to q\bar{q})$$

If we have a massive $Q\bar{Q}$ pair instead, we need

$$A_3^0(Q, g, \bar{Q}), \text{ which is generated from } (\gamma^* \to Qg\bar{Q})/(\gamma^* \to Q\bar{Q})$$

Initial-final and initial- initial antennae (massless or massive) are obtained by appropriate crossings of their final-final counterparts.

$t\bar{t}$ production at NLO

For $t\bar{t}$ production at NLO

$$\mathrm{d}\sigma^R = \int \frac{\mathrm{d}\xi_1}{\xi_1}\frac{\mathrm{d}\xi_2}{\xi_2}\Big\{ \sum_q \Big[f_q(\xi_1) f_{\bar{q}}(\xi_2) \mathrm{d}\hat{\sigma}_{q\bar{q}\to Q\bar{Q}g} + f_q(\xi_1) f_g(\xi_2) \mathrm{d}\hat{\sigma}_{qg\to Q\bar{Q}q} + f_{\bar{q}}(\xi_1) f_g(\xi_2) \mathrm{d}\hat{\sigma}_{\bar{q}g\to Q\bar{Q}\bar{q}} \Big] + f_g(\xi_1) f_g(\xi_2) \mathrm{d}\hat{\sigma}_{g\bar{g}\to Q\bar{Q}g} \Big\}$$

For the color decomposition of the amplitudes needed for the partonic cross-sections we consider the fictitious processes

- $0 \to Q\bar{Q}q\bar{q}g$
- $0 \to Q\bar{Q}ggg$

Antenna subtraction for $t\bar{t}$ production at NLO

- Colour decomposed amplitude squared:

$$
\begin{aligned}
|M_5^0(3_{\bar{q}}4_q \to 1_Q, 2_{\bar{Q}}, 5_g)|^2 = {} & \frac{g^6(N_c^2-1)}{8} \\
& \times \Big[N_c \left(|\mathcal{M}_5^0(1_Q, 5_g, \hat{4}_q;;\hat{3}_{\bar{q}}, 2_{\bar{Q}})|^2 + |\mathcal{M}_5^0(1_Q, \hat{4}_q;;\hat{3}_{\bar{q}}, 5_g, 2_{\bar{Q}})|^2 \right) \\
& + \frac{1}{N_c} \Big(|\mathcal{M}_5^0(1_Q, 5_g, 2_{\bar{Q}};;\hat{3}_{\bar{q}}, \hat{4}_q)|^2 + |\mathcal{M}_5^0(1_Q, 2_{\bar{Q}};;\hat{3}_{\bar{q}}, 5_g, \hat{4}_q)|^2 \\
& -2|\mathcal{M}_5^0(1_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q, 5_\gamma)|^2 \Big) \Big].
\end{aligned}
$$

- Subtraction term:

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^S_{q\bar{q}\to Q\bar{Q}g} = {} & \frac{g^6(N_c^2-1)}{8} \mathrm{d}\phi_3(k_{1Q}, k_{2\bar{Q}}, k_{5g}; p_{4q}, p_{3\bar{q}}) \\
& \times \Big\{ N_c \Big[A_3^0(4_q; 1_Q, 5_g)|\mathcal{M}_4^0((\widetilde{15})_Q, 2_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{\hat{4}}_{\bar{q}})|^2 J_2^{(2)}(K_{\widetilde{15}}, k_2) \\
& + A_3^0(3_{\bar{q}}; 2_{\bar{Q}}, 5_g)|\mathcal{M}_4^0(1_Q, (\widetilde{25})_{\bar{Q}}, \hat{\hat{3}}_{\bar{q}}, \hat{4}_q)|^2 J_2^{(2)}(k_1, K_{\widetilde{25}}) \Big] \\
& - \frac{1}{N_c} \Big[A_3^0(1_Q, 5_g, 2_{\bar{Q}})|\mathcal{M}_4^0((\widetilde{15})_Q, (\widetilde{25})_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q)|^2 J_2^{(2)}(k_{\widetilde{15}}, k_{\widetilde{25}}) \\
& + A_3^0(4_q, 3_{\bar{q}}; 5_g)|\mathcal{M}_4^0(\tilde{1}_Q, \tilde{2}_{\bar{Q}}, \hat{\hat{3}}_{\bar{q}}, \hat{\hat{4}}_q)|^2 J_2^{(2)}(\tilde{k}_1, \tilde{k}_2) \Big] \Big\}.
\end{aligned}
$$

$t\bar{t} + jet$ production at NLO

For $t\bar{t} + jet$ production the unphysical processes are:

- $0 \rightarrow Q\bar{Q}q\bar{q}q'\bar{q}'$
- $0 \rightarrow Q\bar{Q}q\bar{q}gg$
- $0 \rightarrow Q\bar{Q}gggg$

Calculations are more involved because

- More partial amplitudes
- More unresolved limits to subtract
- Identical flavour contributions
- Colour interferences

This calculation is our first step towards $t\bar{t}$ at NNLO [1]

- Same $|M|^2$ and same colour decomposition
- Same single unresolved limits as the real corrections to $t\bar{t}$ at NNLO

[1]G.A and A. Gehrmann-De Ridder, JHEP **1104**, 063 (2011)

$t\bar{t}$ production at NNLO

At NNLO we can symbolically write

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} \quad = \quad & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\hat{\sigma}^{R}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO} \right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\hat{\sigma}^{V,1}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,1}_{NNLO} - \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO} \right) \\
& + \int_{\mathrm{d}\Phi_{m}} \left(\mathrm{d}\hat{\sigma}^{V,2}_{NNLO} + \mathrm{d}\hat{\sigma}^{MF,2}_{NNLO} \right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{VS,1}_{NNLO}
\end{aligned}
$$

- With the subtraction term each line is free of singularities and can be integrated numerically
- We focus on double the real radiation piece: $\mathrm{d}\hat{\sigma}^{R}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}$

Double real radiation for $t\bar{t}$ production at NNLO

For $p_1 + p_2 \to k_Q + k_{\bar{Q}}$

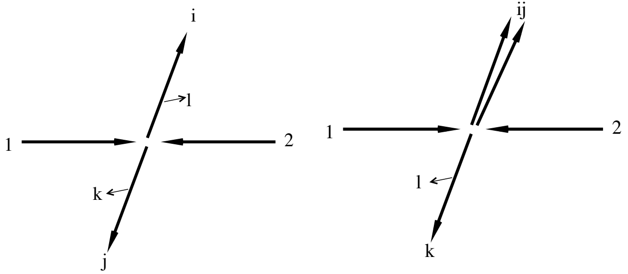
$$\begin{aligned} \mathrm{d}\hat{\sigma}^{R}_{NNLO}(p_1, p_2) &= \mathcal{N} \sum \mathrm{d}\Phi_4(k_Q, k_{\bar{Q}}, k_1, k_2; p_1, p_2) \\ &\quad \times \frac{1}{S_4} \left|\mathcal{M}_6(k_Q, k_{\bar{Q}}, k_1, k_2; p_1, p_2)\right|^2 \\ &\quad \times J_2^{(4)}(k_Q, k_{\bar{Q}}, k_1, k_2). \end{aligned}$$

Construct $\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ that reproduces the behaviour of $|\mathcal{M}_6|^2$ in all

- Single unresolved limits
- Double unresolved limits

Double unresolved limits in $t\bar{t}$ production at NNLO

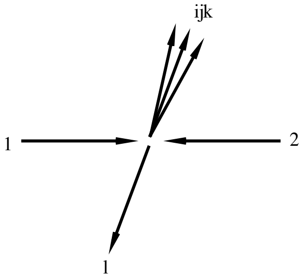
- Limits that involve the top quark mass (explicit m_t dependance in soft factors):



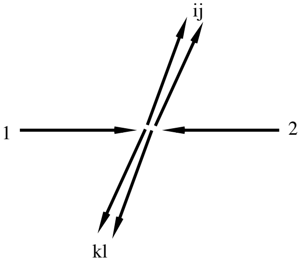
(a) Double soft limits (b) Soft-collinear limits

Double unresolved limits in $t\bar{t}$ production at NNLO

- Limits that do not involve the top quark mass (massless limits):



(c) Triple collinear limits



(d) Double collinear limits

NOTE: Collinearities involving a massive (anti) quark are regulated by m_t, i.e we do not subtract quasi-collinear limits at NNLO

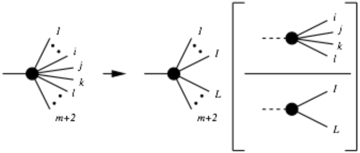
Antenna subtraction for double real radiation in $t\bar{t}$ production at NNLO

Four parton antenna functions $X_4^0(i,j,k,l)$

- Two hard particles (massless or massive)
- Two unresolved particles (massless)

Subtraction terms for m-jet production

- Product of reduced matrix elements with m particles and antenna functions:
  - 1 particle unresolved $\Rightarrow$ 3 parton antennae (as NLO)
  - 2 particles unresolved $\Rightarrow$ 4 parton antennae (genuine NNLO) (colour connected)



Antenna subtraction for double real radiation in $t\bar{t}$ production at NNLO

For example, the partial amplitude squared

$$|\mathcal{M}_6^0(1_Q, 6_{\bar{q}'};; 5_{q'} 2_{\bar{Q}};; \hat{3}_{\bar{q}}, \hat{4}_q)|^2$$

(contributing to $q\bar{q} \to t\bar{t}q'\bar{q}'$) contains the $5_{q'}, 6_{\bar{q}'}$ double soft limit.

To subtract this limit we use the following combination of antenna functions

$$(B_4^0(1_Q, 5_{q'}, 6_{\bar{q}'}, 2_{\bar{Q}}) - E_3^0(1_Q, 5_{q'}, 6_{\bar{q}'}) A_3^0((\widetilde{15})_Q, (\widetilde{56})_g, 2_{\bar{Q}})) |\mathcal{M}_4^0((\widetilde{156})_Q, (\widetilde{256})_{\bar{Q}}, \hat{3}_{\bar{q}}, \hat{4}_q)|^2$$

- $B_4^0(1_Q, 5_{q'}, 6_{\bar{q}'}, 2_{\bar{Q}})$
  - Subtracts the $5_{q'}, 6_{\bar{q}'}$ double soft limit
  - Introduces a spurious collinear singularity $5_{q'} || 6_{\bar{q}'}$
- $E_3^0(1_Q, 5_{q'}, 6_{\bar{q}'}) A_3^0(\widetilde{15}_Q, \widetilde{56}_g, 2_{\bar{Q}})$
  - Subtracts the spurious $5_{q'} || 6_{\bar{q}'}$ limit introduced by $B_4^0(1_Q, 5_{q'}, 6_{\bar{q}'}, 2_{\bar{Q}})$

Antenna subtraction for double real radiation in $t\bar{t}$ production at NNLO

So far we have

- Computed massive four-parton final-final and inital-final antenna functions required for partonic processes involving quarks
- Used
  - Massless four-parton antennae (final-final, initial-final, initial-initial)
  - Massive and massless three parton antenna functions
- Constructed subtraction terms for all partonic processes involving quarks :

$$q\bar{q} \to t\bar{t}q'\bar{q}' \quad q\bar{q} \to t\bar{t}q\bar{q} \quad qq' \to t\bar{t}qq' \quad qq \to t\bar{t}qq$$

- Performed non trivial numerical tests on these subtraction terms in all single and double unresolved limits (more on this in the next transparencies)

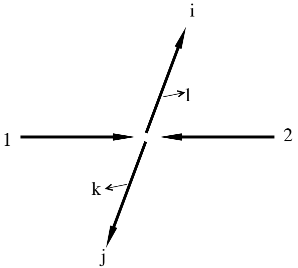
Check of the subtraction terms for $t\bar{t}$ at NNLO for partonic processes involving quarks

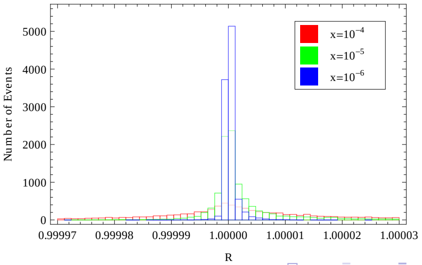
To check our subtraction terms

- Choose scaling parameter x for each limit
- Generate phase space trajectories approaching each limit
- Require the $t\bar{t}$ pair to be in separate hard jets
- Compute the ratio $R =$(amplitude squared)/(subtraction term)

Example: $q\bar{q} \to t\bar{t}q'\bar{q}'$

- Double soft limit: $x = (s - s_{t\bar{t}})/s$

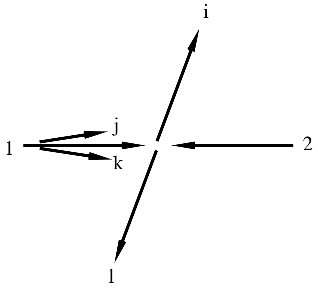


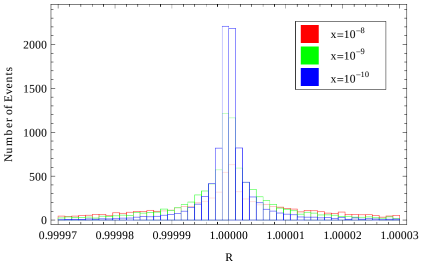


Check of the subtraction terms for $t\bar{t}$ at NNLO for partonic processes involving quarks

Example: $q\bar{q} \to t\bar{t}q'\bar{q}'$

- Triple collinear limit: $x = -s_{qq'\bar{q}'}/s$





Summary and conclusions

- We fully extended the antenna subtraction method at NLO for hadronic processes with massive final state fermions:
    - Computed and integrated massive initial-final antenna functions relevant for $t\bar{t}$ and $t\bar{t} + jet$
    - Generalized phase space mapping and factorization formulae for the massive case
- We constructed NLO subtraction terms for all partonic processes involved in $t\bar{t}$ and $t\bar{t} + jet$ production

These results are our first step towards an NNLO calculation for $t\bar{t}$ production at the LHC and have been published in G.A and A. Gehrmann-De Ridder, JHEP **1104**, 063 (2011)

Summary and conclusions

Towards $t\bar{t}$ at NNLO (double real radiation)

- We constructed NNLO subtraction terms for all partonic processes involving quarks : $q\bar{q} \to t\bar{t}q'\bar{q}'$ $q\bar{q} \to t\bar{t}q\bar{q}$ $qq' \to t\bar{t}qq'$ $qq \to t\bar{t}qq$
- Performed non trivial numerical tests on these subtraction terms in all single and double unresolved limits
- NEXT: We shall compute subtraction terms for the remaining processes involving gluons in initial and final state

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THANK YOU!