Skyrmions in Composite Higgs Models

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in collaboration with
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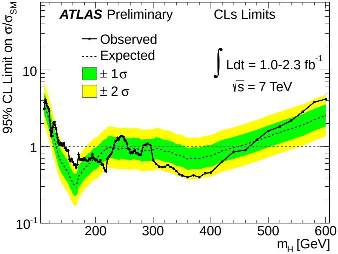
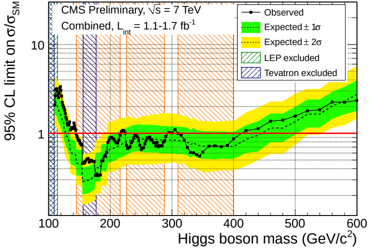
Introduction



Atlas collaboration (Lepton-Photon 2011)



CMS collaboration (Lepton-Photon 2011)

LHC data (and indirect evidence from previous experiments) points towards a light Higgs in the mass range 100–150 GeV.

Introduction

Any new physics above this scale contributes to the Higgs mass through radiative corrections.

⇒ need for a symmetry to protect it:

1. SUSY: requires the introduction of a superpartner for each SM field.
2. Composite Higgs: the Higgs is a bound state of fermions from some strongly interacting sector, and it is light since it arises as a pseudo-Goldstone boson
   → technicolor, little Higgs, holographic models, . . .

Introduction

The Higgs sector of composite models is described at energies below the cutoff $\Lambda \sim 4\pi f$ by a σ-model,

$$\mathscr{L} = \frac{f^2}{4} \operatorname{Tr}\left(D_\mu \Phi \, D^\mu \Phi^\dagger \right),$$

with two consequences:

1. The fields described by $\Phi(x)$ are much lighter than the symmetry breaking scale f (they are the "pions" of the theory, in analogy with QCD)
2. The model may also contain topological solitons called skyrmions (the "baryons" of the theory)

Outline

1. The skyrmion in QCD
2. Topology of composite Higgs models
3. An example: the littlest Higgs model
4. Relic density of skyrmions

The chiral effective theory

In the limit of massless quarks, QCD has a global $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry, broken down to $SU(N_f)_V$ by a non-zero quark condensate $\langle \overline{q} q \rangle \neq 0$.

The low-energy degrees of freedom of QCD are the pions, described by a σ-model

$$\mathscr{L}_{\chi PT} = \frac{f_\pi^2}{4} \operatorname{Tr} \left(\partial_\mu \Phi \, \partial^\mu \Phi^\dagger \right), \qquad \Phi(x) \in SU(N_f).$$

In the two-flavour case $N_f = 2$, $\Phi = \exp\left(i \, \boldsymbol{\pi} \cdot \boldsymbol{\sigma} / f_\pi \right)$ and

$$\mathscr{L}_{\chi PT} = \frac{1}{2} \left| \partial_\mu \boldsymbol{\pi} \right|^2 + \frac{1}{6 f_\pi^2} \left[\left| \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} \right|^2 - \boldsymbol{\pi}^2 \left| \partial_\mu \boldsymbol{\pi} \right|^2 \right] + \ldots$$

Topological solitons

The σ-model also contains topological solitons, the skyrmions:

- classical static solutions of the field equations, with finite size and finite energy,
- topologically stable, since they cannot be deformed into the true vacuum by infinitesimal transformation.

The presence of skyrmions in a theory depends on the topology of the vacuum manifold, described by the third homotopy group π_3 of the target space. For QCD,

$$\pi_3(SU(N_f)) = \mathbb{Z}\,.$$

Winding number integral:

$$B(\Phi) = \frac{1}{24\pi^2}\epsilon_{ijk}\int d^3x\,\mathrm{Tr}\left(\Phi^\dagger\partial_i\Phi\right)\left(\Phi^\dagger\partial_j\Phi\right)\left(\Phi^\dagger\partial_k\Phi\right) \in \mathbb{Z}\,.$$

The Skyrme model of baryons

B is actually the baryon number of the theory, and the skyrmions represent baryon states. Witten (1983) Adkins, Nappi, Witten (1983)

Higher-derivative terms are needed to stabilize the skyrmion.

G.H. Derrick (1964)

→ consider the Skyrme Lagrangian:

$$\mathscr{L} = \frac{f_\pi^2}{4}\,\mathrm{Tr}\left(\partial_\mu\Phi\,\partial^\mu\Phi^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left|\left[\Phi^\dagger\partial_\mu\Phi, \Phi^\dagger\partial_\nu\Phi\right]\right|^2$$

T.H.R. Skyrme (1961)

With $e \sim 5$, the baryons properties are reproduced with about 20% precision: mass, radius, magnetic moment, . . .

Topology of composite Higgs models

The skyrmions in composite Higgs models can be fundamentally different from the QCD ones, because of the different topology of the vacuum manifold.

Symmetry	Models	π_3
$\frac{SU(N)\times SU(N)}{SU(N)}$	"Minimal moose" models Arkani-Hamed *et al.* (2002) – with exact DM parity Freitas, Schwaller, Wyler (2009)	$\mathbb{Z}$
$\frac{SO(N)\times SO(N)}{SO(N)}$	– with custodial symmetry Chang, Wacker (2004) "Bestest LH" Schmaltz, Stolarski, Thaler (2010)	$\mathbb{Z}$
$SU(2N)/Sp(2N)$	"Antisymmetric cond." Low, Skiba, Tucker-Smith (2002)	0
$SU(N)/SU(N-1)$	LH from a simple group Kaplan, Schmaltz (2003)	0
$SO(N)/SO(N-1)$	Min. Composite Higgs Agashe, Contino, Pomarol (2004)	$0^{(a)}$
$SU(N)/SO(N)$	Littlest Higgs Arkani-Hamed *et al.* (2002) – with T-parity Cheng, Low (2003)	$\mathbb{Z}_2{}^{(b)}$

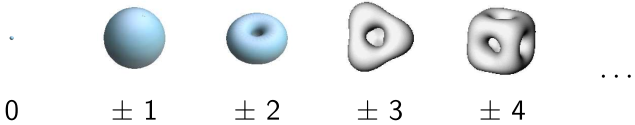
Bryan, Carroll, Pyne (1993)

$^{(a)}$ $\mathbb{Z}$ for $N \leq 4$, $^{(b)}$ $\mathbb{Z}_4$ for $N = 3$.

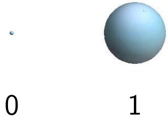
Topology of composite Higgs models

The skyrmion symmetry depends on its winding number:

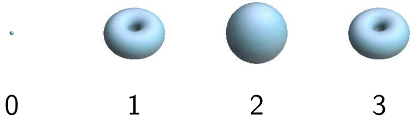
$\mathbf{SU(N) \times SU(N)/SU(N)}$ N. Manton (2011)



$\mathbf{SU(N)/SO(N)}, \quad \mathbf{N \geq 4}$ MG, von Manteuffel, Schwaller, Wyler (2010)



$\mathbf{SU(3)/SO(3)}$ MG (2011)



An example: the littlest Higgs model

The littlest Higgs is described by a field $\Sigma(x)$ in the two-index symmetric representation of $SU(5)$, with a vev $\langle\Sigma\rangle = \mathbb{1}$ breaking the symmetry down to $SO(5)$:

$$\mathscr{L}_{LH} = \frac{f^2}{4}\,\mathrm{Tr}\left(\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger\right) + \frac{1}{32e^2}\,\mathrm{Tr}\left|\left[\Sigma^\dagger\partial_\mu\Sigma, \Sigma^\dagger\partial_\nu\Sigma\right]\right|^2$$

→ we add the Skyrme term as in QCD.

The field configuration of minimal energy with $B = 1$ is obtained using the Cartan embedding $\Sigma(x) = \Phi(x)\Phi(x)^T$ and a “hedgehog ansatz”

$$\Phi(x) = \exp\left[i\,F(r)\,\hat{x}_i\,T_i\right], \qquad T_i = \frac{1}{4}\begin{pmatrix} \sigma_i & i\,\sigma_i & 0 \\ -i\sigma_i & \sigma_i & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

An example: the littlest Higgs model

The energy density is a functional of F:

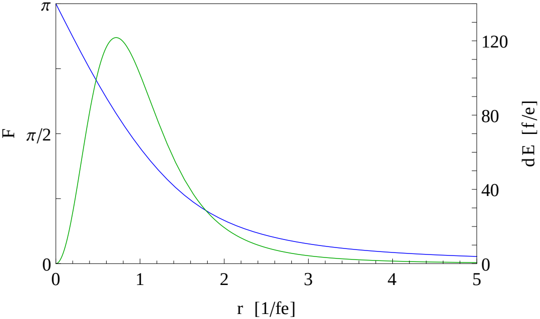
$$E[F] = 4\pi \frac{f}{e} \int_0^\infty d\tilde{r} \left[\left(\tilde{r}^2 + 2\sin^2 F\right) F'^2 + \left(2\tilde{r}^2 + \sin^2 F\right) \frac{\sin^2 F}{\tilde{r}^2} \right]$$

minimized
by solving the
Euler-Lagrange
equation for F

$$\Downarrow$$

$$M = 145.8\ \frac{f}{e}$$

$$\langle r^2 \rangle = \left(\frac{1.058}{f\, e} \right)^2$$



The effects of gauge fields

In the littlest Higgs model, the global $SU(5)$ symmetry is explicitly broken by gauging a $[SU(2) \times U(1)]^2$ subgroup.

$\Rightarrow$ the skyrmion can decay with the help of an instanton.

D'Hoker, Farhi (1984)

Still, the extremely small tunneling probability makes the skyrmion quasi-stable, with lifetime

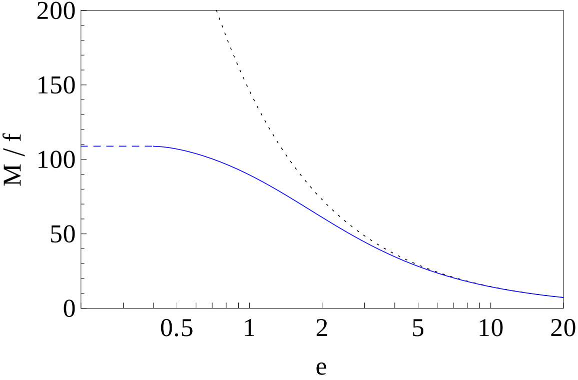
$$\tau \sim \frac{e^{16\pi^2/g^2}}{M} \gg \tau_{\text{universe}}$$

Also, the gauge fields lower the mass of the skyrmion, without spoiling its spherical symmetry.

Classical mass of the skyrmion

- Tends to $M_\infty = 145.8\, f/e$ at large e
- Upper bound

$$M_{e\to 0} = 16\sqrt{2}\,\pi\, f/g \cong 108.9\, f$$



Relic density of skyrmions

The peculiar $\mathbb{Z}_2$ topology of the littlest Higgs implies that:

- skyrmions are produced and annihilated in pairs of identical particles,
- no need to generate an asymmetry to obtain a skyrmion relic density in the universe.

Skyrmion pair-production and annihilation cannot be computed perturbatively. A naive estimate for the cross-section:

$$\sigma \sim \pi \langle r^2 \rangle \cong \frac{\pi}{(f\, e)^2}$$

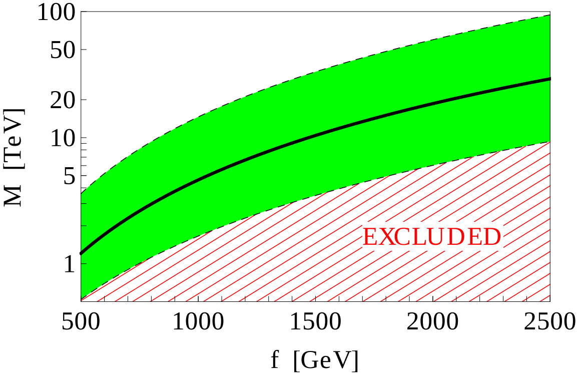
The WIMP relic density $\Omega h^2 \cong \frac{3 \cdot 10^{-27} \mathrm{cm}^3/\mathrm{s}}{\langle \sigma v \rangle} \cong 0.1$ is satisfied provided $f\, e \sim 35\ \mathrm{TeV}$

Griest, Kamionkowski (1990)

→ a very natural choice of parameters!

Relic density of skyrmions

Implementing an effective skyrmion-to-matter coupling in microMEGAs: Bélanger *et al.* (2011) Belyaev *et al.* (2006)



Coupling the skyrmion both to the scalar sector and to the fermions yield comparable bounds.

Quantum properties of skyrmions

The physical skyrmion states are obtained by canonical quantisation of small oscillations around the classical solution.

The fermionic/bosonic nature of the skyrmion and its charge under the electroweak gauge groups depend on the UV completion of the model, and in particular on the number of colors of the underlying strongly coupled theory (if any).

The lightest skyrmion state is stable, so it should better be electrically neutral!

$\Rightarrow$ constraints for model building

(work in progress)

Conclusions and outlook

Skyrmions are present in many realisations of the composite Higgs idea, even when the global symmetry is explicitly broken by gauge interactions.

Skyrmions are expected to be heavy, above the symmetry breaking scale f, but are still surprisingly good dark matter candidates due to their naturally small cross-section.

The quantum properties of skyrmions are quite dependent on the UV completion of the model:

- exact mass including quantum loop corrections,
- spin statistics and charge under the EW gauge group,
- possible collider signature at the LHC?