Higgs Self-Couplings in the MSSM

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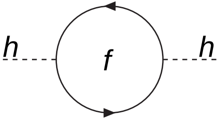
Outline

Hierarchy Problem in the Standard Model

Embedding the SM in a grand unified theory (GUT)
$\rightarrow$ introduce UV cutoff Λ

- Loop corrections to the Higgs mass generate quadratic divergence:



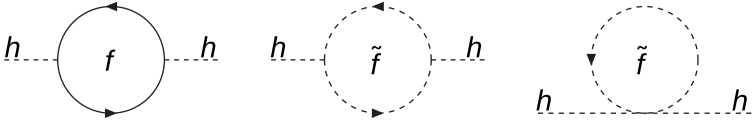
$$\Delta m_H^2 \sim \Lambda^2 \sim \mathcal{O}(M_{GUT}) \gg m_H^2$$

absorb in counterterm: $m_H^2 \rightarrow m_H^2 + \Delta m_H^2 - \delta m_H^2$

$\Rightarrow$ unnatural fine tuning $\sim$ 28 digits

Motivations for the MSSM

- Supersymmetry: fermions $\leftrightarrow$ bosons
- Quadratic divergencies are cancelled:



$$\Delta m_H^2 \sim (\tilde{m}^2 - m^2) \log \frac{\Lambda^2}{m^2} \Rightarrow \text{no fine-tuning for } \tilde{m} \lesssim \mathcal{O}(1 TeV)$$

$\Rightarrow$ Hierarchy problem solved

- SUSY-$SU(5)$-GUT predicts: $\sin^2 \Theta_W = 0.2334 \pm 0.0026$
  LEP measures: $\sin^2 \Theta_W = 0.2317 \pm 0.0002$

Electro Weak Symmetry Breaking in the MSSM

Two complex Higgs doublets: H_1, H_2

$\rightarrow$ 8 degrees of freedom (DoF)

Longitudinal polarizations of the $W^{\pm}$ and Z bosons eat 3 DoF:

$\Rightarrow$ 5 physical Higgs bosons

- 2 scalar Higgs bosons h, H
- 1 pseudoscalar Higgs boson A
- 2 charged Higgs bosons H^{+}, H^{-}

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Higgs Boson Masses in the MSSM at Tree Level

The MSSM Higgs potential is

$$V_0 = (m_1^2 + \mu^2)|H_1|^2 + (m_2^2 + \mu^2)|H_2|^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + h.c.) + \\ + \frac{g^2 + g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2.$$

diagonalize mass matrix $\rightarrow$ physical Higgs boson masses

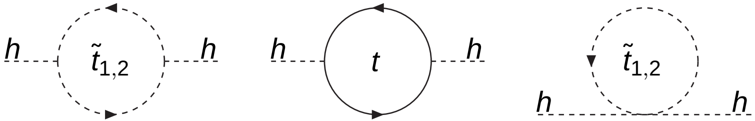
$$m_{H^\pm}^2 = m_A^2 + m_W^2 \qquad m_A^2 = \frac{2B\mu}{\sin(2\beta)}$$

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2(2\beta)}\right]$$

$$\Rightarrow m_h < m_Z$$

One-Loop Corrections to Higgs Masses

Top-Yukawa-coupling h_t is large
$\rightarrow$ get large corrections from top/stop loops $\mathcal{O}(\alpha_t)$, $\alpha_t = \frac{h_t^2}{4\pi}$:



$$m_h^2 \leq m_Z^2 + \frac{3G_F}{\sqrt{2}\pi^2}\frac{m_t^4}{sin^2(\beta)}\left[\log\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \frac{X_t^2}{M_{SUSY}^2}\left(1 - \frac{X_t^2}{12 M_{SUSY}^2}\right)\right]$$

$X_t = A_t - \mu \cot(\beta)$

Ellis,$\cdots$ (1991)
Haber,$\cdots$ (1990)
etc.

$\Rightarrow$ m_h is easily pushed beyond m_Z bound

Further Corrections to Higgs Masses

- QCD corrections are dominant at two-loop: $\mathcal{O}(\alpha_t \alpha_s)$

Hempfling,Hoang (1994)
Heinemeyer,$\cdots$ (1998)
Zhang (1999)
Slavich,$\cdots$ (2001)
etc.

- $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_b \alpha_s)$ have been calculated.

Espinosa, Zhang (2000)
Slavich,$\cdots$ (2001)
Heynemeyer,$\cdots$ (2005)
etc.

- First 3-loop results: $\mathcal{O}(\alpha_t \alpha_s^2)$

Martin (2007)
Harlander,$\cdots$ (2010)

$$\Rightarrow m_h \lesssim 135 \text{ GeV}$$

Higgs couplings to SM particles

Modified couplings (tree level):

ϕ	g_u^ϕ	g_d^ϕ	g_V^ϕ
H_{SM}	1	1	1
h	$\cos(\alpha)/\sin(\beta)$	$-\sin(\alpha)/\cos(\beta)$	$\sin(\beta-\alpha)$
H	$\sin(\alpha)/\sin(\beta)$	$\cos(\alpha)/\cos(\beta)$	$\cos(\beta-\alpha)$
A	$\tan^{-1}(\beta)$	$\tan(\beta)$	0

mixing angle α: $\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}$

$$t_\beta \uparrow \quad \Rightarrow \quad g_u^\phi \downarrow \quad g_d^\phi \uparrow$$

Higgs Boson Decay

- low tan(β)

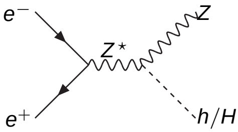
mainly: $h \to \tau^+\tau^- / b\bar{b}$
large m_h: $h \to gg/\gamma\gamma/WW/ZZ$
low m_A: $H, A \to \tau^+\tau^- / b\bar{b}$
large m_A: $H, A \to t\bar{t}$ $H \to WW/ZZ/hh/\tilde{f}\tilde{f}$ $A \to Zh$

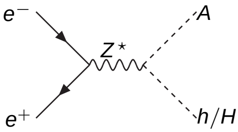
- large tan(β)

$$h, H, A \to \tau^+\tau^- / [\mu^+\mu^-] \qquad h, H, A \to b\bar{b}$$

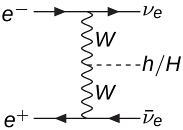
Higgs Production at LEP

- Higgs-strahlung and pair production





- Fusion process



Higgs Production at LEP

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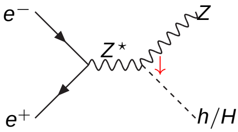

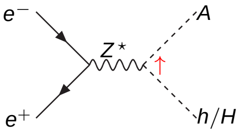

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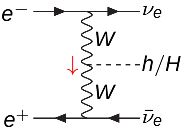
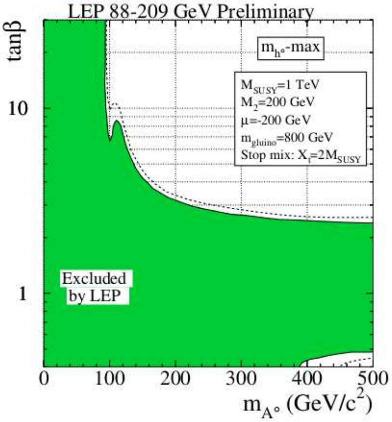


$\tan(\beta) \uparrow$

Exclusion Limits from LEP



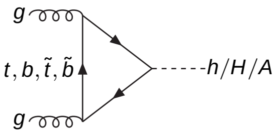
$E_{CMS} = 209$ GeV

- $m_A < 93.4$ GeV and $m_{h,H} < 92.8$ GeV excluded
- $0.7 < \tan(\beta) < 2.4$ excluded

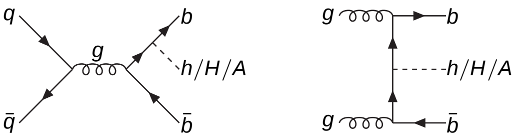
A linear collider with $E_{CMS} \gtrsim 250$ GeV could access the whole parameter space.

Higgs Production at the LHC

- Gluon fusion



- Higgs bremsstrahlung off bottom quarks



Higgs Production at the LHC

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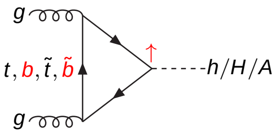

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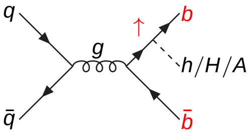
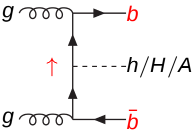
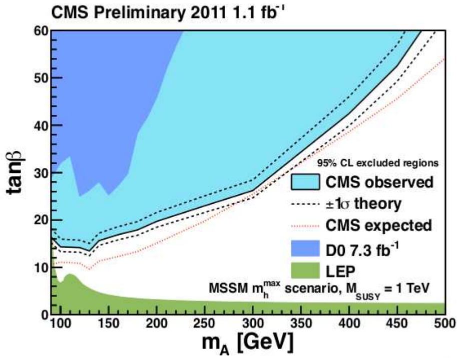




$\tan(\beta) \uparrow$

Exclusion Limits from the LHC



Cross section is enhanced for

- large $\tan(\beta)$
- low m_A

High m_A and moderate $\tan(\beta) \to h = H_{SM}$
$\Rightarrow$ hard to detect because of huge $b\bar{b}$ background

Motivation for Calculating Two-Loop Corrections to the Couplings

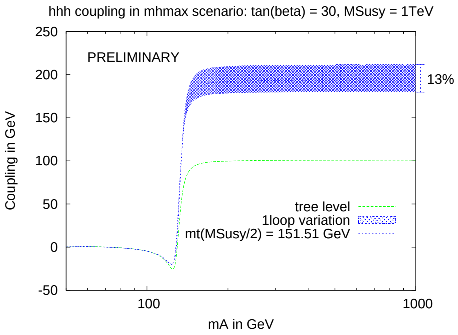
- Higgs self-couplings determine Higgs potential
- Higgs potential is responsible for Electro Weak Symmetry Breaking (EWSB)

$\Rightarrow$ need to measure Higgs self-interactions to understand EWSB

(very difficult at LHC, linear collider needed)

$\Rightarrow$ need high-precision predictions for trilinear couplings

Existing One-Loop Calculation



Barger,··· (1992)

$$\lambda_{hhh}^{tree} = \frac{3m_Z^2}{v} c_{2\alpha} s_{\alpha+\beta}$$

- large corrections
- sizable uncertainties

$\Rightarrow$ two-loop calculation needed.

Effective Potential Method

Effective Potential V^{eff}:

- Non-derivative part of the effective action
  $\rightarrow$ correct in the limit of vanishing external momenta
- Generating functional of 1PI Greens functions with no external legs (vacuum diagrams)
- n-th derivative of V^{eff}:
  sum of all 1PI diagrams with n external legs

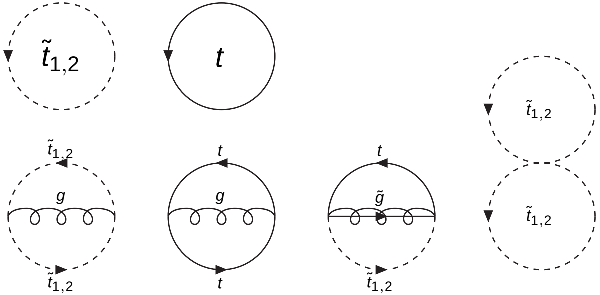
$$\Rightarrow \lambda(h_1, h_2, h_3) = \left.\frac{\partial^3 V^{eff}}{\partial h_1 \partial h_2 \partial h_3}\right|_{min}$$

Computing the Effective Potential

First step: calculate ΔV^{α_t} and $\Delta V^{\alpha_t \alpha_s}$

<u>1loop:</u>
$\mathcal{O}(\alpha_t)$

<u>2loop:</u>
$\mathcal{O}(\alpha_t \alpha_s)$



Zhang,· · · (1999)
Slavich,· · · (2001)

Renormalization

The fully renormalized coupling can be calculated by

$$\lambda_{\alpha_t \alpha_s}(h_1, h_2, h_3) = \left. \frac{\partial^3 (V_0 + \Delta V^{\alpha_t} + \Delta V^{\alpha_t \alpha_s})}{\partial h_1 \partial h_2 \partial h_3} \right|_{min} + \delta\lambda_{CT}.$$

The counterterm is obtained from derivatives

$$\delta\lambda_{CT}^{(2)} = \sum_i \frac{\partial \Delta\lambda_1}{\partial x_i} \delta x_i ,$$

where $x_i = \left\{ m_t^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, A_t \right\}$ are all parameters of the one-loop couplings that are renormalized at $\mathcal{O}(\alpha_s)$.

Cancellation of Divergences

For simplicity, start with $\overline{DR}$-scheme:

- $\overline{DR}$-counterterms $\delta^{\overline{DR}}$ are $\frac{1}{\epsilon}$-divergences
- $\mathcal{O}(\epsilon)$-terms in $\Delta V_1^{\alpha_t}$ give finite contributions
- $\mathcal{O}(\epsilon^0)$-terms in $\Delta V_1^{\alpha_t}$ give $\frac{1}{\epsilon}$ poles

Non-trivial consistency check: All $\frac{1}{\epsilon^2}$ and $\frac{1}{\epsilon}$ poles cancel.

$\Rightarrow \lambda_{\alpha_t \alpha_s}$ is finite

Renormalization Scheme

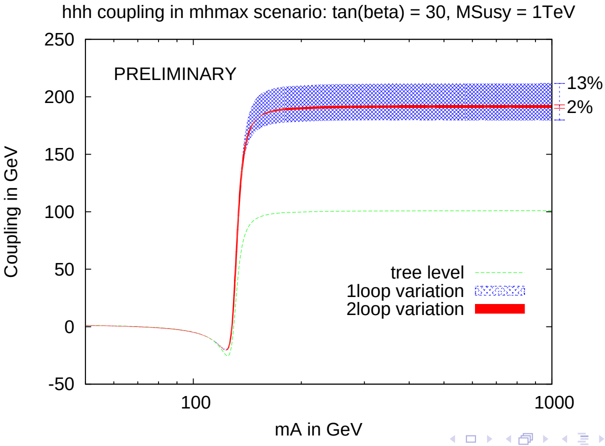
Can shift to any other scheme by adding finite counterterm.
e.g. on-shell-scheme:

$$\lambda_{\alpha_t\alpha_s}^{OS} = \lambda_{\alpha_t\alpha_s}^{\overline{DR}} + \Delta\lambda_{CT}^{OS}$$

- $\Delta\lambda_{CT}^{OS} = \sum_i \frac{\partial \Delta\lambda_1}{\partial x_i} \Delta^{OS} x_i$
- $\Delta^{OS} x_i$: finite part of on-shell counterterm

$\Rightarrow \lambda_{\alpha_t\alpha_s}^{OS}$ is independent of the 't Hooft scale Q.

Results: hhh


hhh coupling in mhmax scenario: tan(beta) = 30, MSusy = 1TeV
PRELIMINARY
13%
2%
Coupling in GeV
mA in GeV
250
200
150
100
50
0
-50
100
1000
tree level
1loop variation
2loop variation

Summary

- The effective potential method provides an efficient way to calculate two-loop corrections to Higgs self-interactions.
- The $\mathcal{O}(\alpha_t \alpha_s)$ corrections to the hhh-coupling are small at the central scale $M_{SUSY}/2$ and the theoretical uncertainty is reduced from $\sim 15\%$ to $\sim 2\%$.

$\Rightarrow$ stabilization

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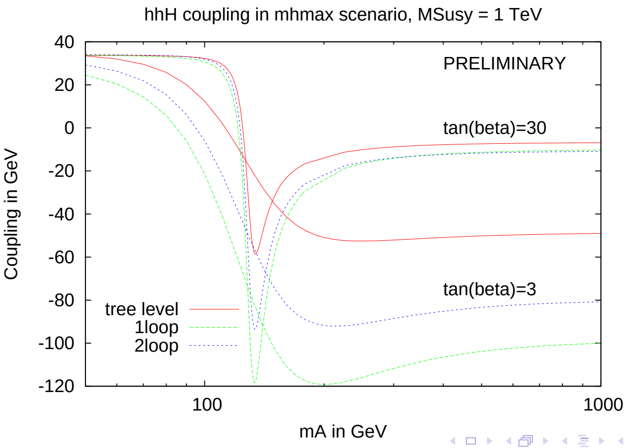
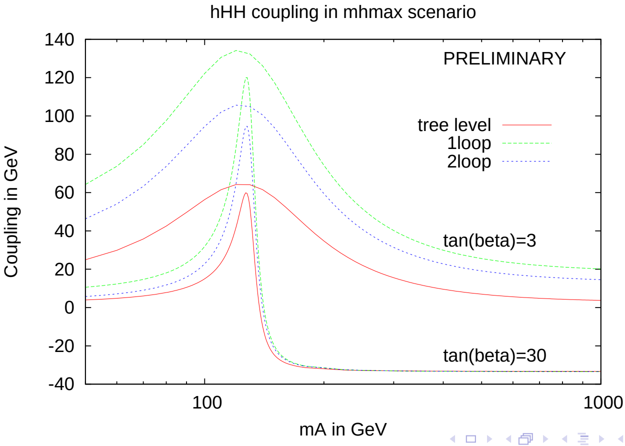
- Outlook:
  - quartic couplings $\rightarrow$ done ✓
  - $\mathcal{O}(\alpha_t^2)$ corrections $\rightarrow$ in progress
  - analytic formulae, public code

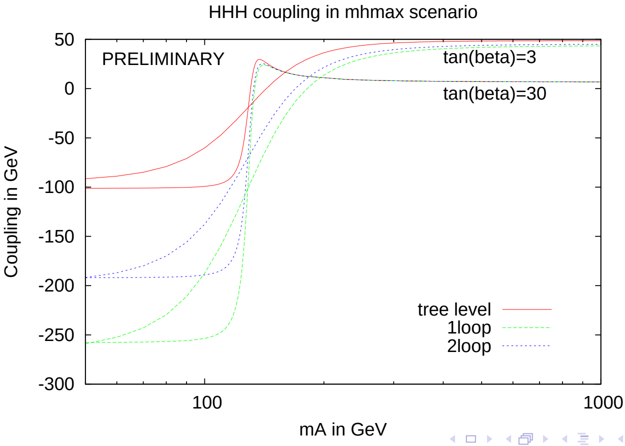
Results: hhH


hhH coupling in mhmax scenario, MSusy = 1 TeV
PRELIMINARY
tan(beta)=30
tan(beta)=3
tree level
1loop
2loop
Coupling in GeV
mA in GeV
40
20
0
-20
-40
-60
-80
-100
-120
100
1000

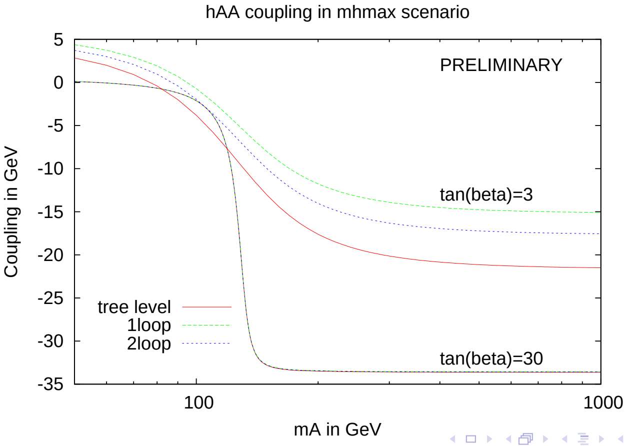
Results: hHH


hHH coupling in mhmax scenario
PRELIMINARY
tree level
1loop
2loop
tan(beta)=3
tan(beta)=30
Coupling in GeV
mA in GeV
140
120
100
80
60
40
20
0
-20
-40
100
1000

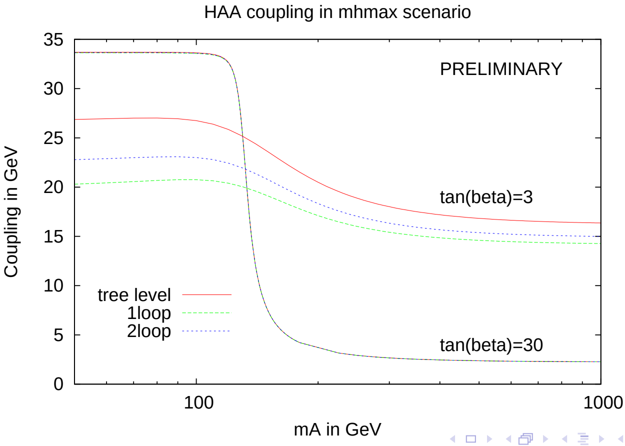
Results: HHH


HHH coupling in mhmax scenario
PRELIMINARY
tan(beta)=3
tan(beta)=30
Coupling in GeV
50
0
-50
-100
-150
-200
-250
-300
100
1000
mA in GeV
tree level
1loop
2loop

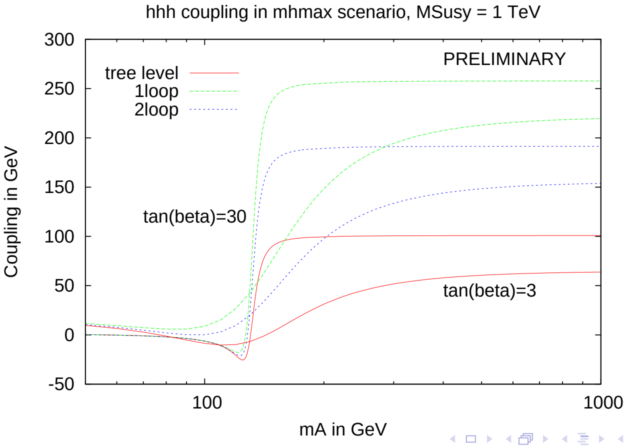
Results: hAA


hAA coupling in mhmax scenario
PRELIMINARY
tan(beta)=3
tan(beta)=30
tree level
1loop
2loop
Coupling in GeV
mA in GeV
5
0
-5
-10
-15
-20
-25
-30
-35
100
1000

Results: HAA


HAA coupling in mhmax scenario
PRELIMINARY
Coupling in GeV
35
30
25
20
15
10
5
0
100
1000
mA in GeV
tan(beta)=3
tan(beta)=30
tree level
1loop
2loop

Results: hhh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1loop
2loop
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tan(beta)=3
Coupling in GeV
mA in GeV
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200
150
100
50
0
-50
100
1000