

# **Transverse Beam Shaping with RF Cavities**

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#### Uniformization of the transverse beam profile by means of nonlinear focusing method

Yosuke Yuri, Nobumasa Miyawaki, Tomihiro Kamiya, Watalu Yokota, and Kazuo Arakawa Takasaki Advanced Radiation Research Institute, Japan Atomic Energy Agency, 1233 Watanuki-machi, Takasaki, Gunma 370-1292, Japan

Mitsuhiro Fukuda

Research Center for Nuclear Physics, Osaka University, 10-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan (Received 16 April 2007; published 29 October 2007)

### Initially?

Beam with transverse Gaussian distribution goes
 though multipole magnet

### Then?

• Accelerator optics, as described by **M** 

### Goal?

• Beam with uniform distribution at target





### Thin lens magnet Start with a Gaussian distribution:

Large  $\beta$ -function:

All particles conserved:

Decoupled transverse motion:

**Brief Theory** 

want uniform:

**Derive**:

**Assuming:** 

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Using an even-multipole magnet of:

 $p_0 = -\alpha_0 / \beta_0 x_0$ 

|y/x| << 1

 $dN = \rho_0 dx_0 = \rho_t dx_t$ 





$$\rho_{0} = \frac{N}{2\pi\sigma_{0}} \exp\left(-\frac{x_{0}^{2}}{2\sigma_{0}^{2}}\right)$$

$$\rho_{t} = \frac{N}{2r_{t}}$$

$$r_{t} = \sqrt{2\pi}\sqrt{\epsilon_{0}\beta_{0}}|\cos(\phi)|$$

$$K_{2(n-1)} = 0;$$

$$(n = 3,5,7,...)$$

$$K_{2n} = \frac{(n-2)!}{\left(\frac{n}{2}-1\right)!} \frac{(-1)^{\frac{n}{2}}}{(2\epsilon_{0}\beta_{0})^{\frac{n}{2}-1}} \frac{1}{\beta_{0}\tan(\phi)};$$

$$(n = 4,6,8,...)$$

$$K_{2(n+2)} = -\frac{n-1}{\epsilon_{0}\beta_{0}}K_{2n};$$

$$(n = 4,6,8,...)$$



# **Investigate this with RF-Track**

- Create a test beamline of *L* = 1.7 m
  - Initial conditions  $p_0 = 20 m_e \sim 10 \text{ MeV} \text{ electrons}$  $\beta_0 = 15 \text{ m}, \alpha_0 = 15, \epsilon_0 = 21 \text{ mm} \text{ mrad}$
  - Initially a 1D beam (only direction in x)!
  - Add in a quadrupole to amplify  $\beta$  and set  $\phi \sim 0$  $s_q = 0.2 \text{ m}, L_q = 0.3 \text{ m}, K_q = -4.25 \text{ m}^{-1}$
  - So, obviously, Gaussian distribution goes to Gaussian...





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# **Investigate this with RF-Track**

### $r_t = \sqrt{2\pi} \sqrt{\epsilon_0 \beta_0} |\cos(\phi)|$

• Create a test beamline of *L* = 1.7 m

•

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- Initial conditions  $p_0 = 20 m_e \sim 10 \text{ MeV}$  electrons  $\beta_0 = 15 \text{ m}, \alpha_0 = 15, \epsilon_0 = 21 \text{ mm}$  mrad
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- So, obviously, Gaussian distribution goes to Gaussian...





## **Ideal Multipole Magnet**

- Create a test beamline of *L* = 1.7 m
  - Add in multipole magnet  $s_M = 0.0 \text{ m}, L_M = 4 \text{ cm}, K_{2n} = \text{formula}$
- Ideal field ( $\sigma_0 = 17.75$  mm):









## Just an octupole

$$K_8 = \frac{(4-2)!}{\left(\frac{4}{2}-1\right)!} \frac{(-1)^{\frac{2}{2}}}{(2\epsilon_0\beta_0)^{\frac{2}{2}-1}} \frac{1}{\beta_0 \tan(\phi)} = 3115 \text{ m}^2$$

- Create a test beamline of *L* = 1.7 m
  - Add in multipole magnet  $s_M = 0.0 \text{ m}, L_M = 4 \text{ cm}, K_8 = \text{formula}$
- First test, just put in an octupole term
  - $K_8 = 3115 \text{ m}^{-2}$

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## Just an octupole

- Create a test beamline of *L* = 1.7 m
  - Add in multipole magnet  $s_M = 0.0 \text{ m}, L_M = 4 \text{ cm}, K_8 = \text{formula}$
- First test, just put in an octupole term
  - $K_8 = 3115 \text{ m}^{-2}$



$$K_8 = \frac{(4-2)!}{\left(\frac{4}{2}-1\right)!} \frac{(-1)^{\frac{2}{2}}}{(2\epsilon_0\beta_0)^{\frac{2}{2}-1}} \frac{1}{\beta_0 \tan(\phi)} = 3115 \text{ m}^2$$





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### But we can get flatter with just octupole

- Analytical formula used to calculate  $K_8$  is dependent on an infinite number of  $K_{2n}$
- In the case of a finite number, the optimum K<sub>8</sub> differs and can be solved for by RF-Track optimization
- Find:  $K_8 = 2200 \text{ m}^{-2}$  (compared to  $K_8 = 3115 \text{ m}^{-2}$ )







# Let's add more multipoles!

- Optimum of octupole and dodecupole:
  - $K_8 = 3200 \text{ m}^{-2}, K_{12} = -2.1\text{e7} \text{ m}^{-4}$

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## Let's add more + more multipoles!

- Optimum of octupole and dodecupole and hexadecupole:
  - $K_8 = 3.3 \text{e3} \text{ m}^{-2}$ ,  $K_{12} = -2.1 \text{e7} \text{ m}^{-4}$ ,  $K_{16} = -2.1 \text{e7} \text{ m}^{-4}$



# **Conclusion from Magnet Study**

- **Analytical prediction + RF-Track** • optimization works well
- More multipoles = flatter beam •
  - BUT octupole + dodecupole + higher order • magnetic fields are hard to incorporate in practice
- For 2D flat beam, need two minima to • place two multipole magnets
  - This has been shown to work experimentally with • octupoles - see right
  - Horns can be collimated but lose charge •
- How about using RF cavities instead...? •





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25

30

Path length s [m]

35

40

40

20

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# **RF Cavity Instead**

RF cavity use not just limited to accelerating beams in the longitudinal direction

Also use to transversely deflect/kick/rotate/focus a particle beam

So let's explore using it instead of the multipolar magnet





# How? Look at pillbox cavity

- RF cavity pillbox supports TM<sub>mnp</sub> modes (and also TE<sub>mnp</sub> discussed later)
  - m = azimuthal order
  - n = radial order
  - p = longitudinal order



# **Tests with RF cavity fields**

### • Same test beamline as before but..

• Add in multipolar RF cavity field  $s_M = 0.0 \text{ m}, L_{RF} = 4 \text{ cm}, \tilde{g}_M = \text{formula} \mid \text{optimized}$ 

Fields as per below. Assume a static field with  $k = \frac{\omega}{c} = \frac{2\pi f}{c}$ , = 62.9 m<sup>-1</sup> (f = 3 GHz)

$$E_r = E_\theta = B_z = 0,$$

$$E_{z}(r,\theta) = \sum_{\{M\}} \tilde{g}_{M} J_{M}(kr) \cos\left(M\theta - \phi_{M}\right),$$
$$B_{r}(r,\theta) = \frac{i}{c} \sum_{\{M\}} \frac{M}{kr} \tilde{g}_{M} J_{M}(kr) \sin\left(M\theta - \phi_{M}\right),$$
$$B_{\theta}(r,\theta) = \frac{i}{c} \sum_{\{M\}} \tilde{g}_{M} J'_{M}(kr) \cos\left(M\theta - \phi_{M}\right),$$



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## First Test - TM<sub>410</sub> mode

$$B_r(r,\theta) = \frac{i}{c}\tilde{g}_4 \frac{4}{kr}J_4(kr)\sin(4\theta)$$
$$B_\theta(r,\theta) = \frac{i}{c}\tilde{g}_4 J'_4(kr)\cos(4\theta)$$





## Just use an RF octupole - TM<sub>410</sub> mode



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## Why so much better than magnetic octupole?

• Ideal magnetic field looks as follows



$$K_{2n} = \frac{(n-2)!}{\left(\frac{n}{2}-1\right)!} \frac{(-1)^{\frac{n}{2}}}{(2\epsilon\beta_0)^{\frac{n}{2}-1}} \frac{1}{\beta_0 \tan(\phi)};$$

$$B_{y}^{\text{ideal}}(x,0) = \sum_{n=4}^{\infty} \frac{K_{4n-8}}{Lc} \frac{P_{0}}{q} \frac{x^{2n-5}}{(2n-5)!}$$
$$= \frac{P_{0}}{Lcq} \frac{1}{\beta_{0} \tan \phi} \sum_{n=4}^{\infty} \frac{(-1)^{n}}{(n-3)!(2\epsilon\beta_{0})^{n-3}} \frac{x^{2n-5}}{(2n-5)!}$$



## Why so much better than magnetic octupole?

• Comparing the RF octupolar field to the magnetic octupole field



$$B_{y}^{\text{ideal}}(x,0) = \frac{P_{0}}{Lcq} \frac{1}{\beta_{0} \tan \phi} \sum_{n=4}^{\infty} \frac{(-1)^{n}}{(n-3)!(2\epsilon\beta_{0})^{n-3}} \frac{x^{2n-5}}{(2n-5)}$$

$$B_{y}^{\text{magnet octupole}}(x,0) = \frac{P_{0}}{Lcq} K_{8}^{*} \frac{x^{3}}{3!}$$

$$J'_{m}(kx) = \frac{1}{2} \sum_{q=0}^{\infty} \frac{(-1)^{q} (m+2q)}{q! (m+q)!} \left(\frac{kx}{2}\right)^{m+2q-1}$$

$$B_{y}^{\text{rf octupole}}(x,0) = \frac{\tilde{g}_{4}}{c} J'_{4}(kx)$$

$$= \frac{\tilde{g}_{4}}{c} \sum_{n=4}^{\infty} \frac{(-1)^{n} (n-2)}{(n-4)!n!} \left(\frac{k}{2}\right)^{2n-5} x^{2n-5}$$



## Why so much better than magnetic octupole?



- *k* scales the radial variation in Bessel function,  $\epsilon \& \beta_0$  scale the beam size
- In the perfect (obviously impossible) case, the ratio being 1 for all *n* would ensure  $B_{y}^{\text{rfoctupole}} = B_{y}^{\text{ideal}}$
- In any case, there is a clear dependence of the flatness of the beam that can be achieved with just an rf octupole, depending on the interplay of  $\epsilon$ ,  $\beta_0$ , k





- f = 300 MHz
- $(\tilde{g}_4 = 33.9 \text{ GV/m})$

4.0000e+00	2.4468e-03	1.0000e+00
5.0000e+00	7.6175e-06	3.1132e-03
6.0000e+00	1.1067e-08	4.5230e-06
7.0000e+00	1.0547e-11	4.3106e-09
8.0000e+00	7.5249e-15	3.0754e-12

x^3 ratio is too strong relative to others



- f = 3 GHz
- $(\tilde{g}_4 = 49.9 \text{ MV/m})$

4.0000 2	.4468 1	.0000
5.0000 0	.7618 0	.3113
6.000000	0.110670	0.045230
7.0000e+00	1.0547	e-02 4.3106e-03
8.0000e+00	7.5249	e-04 3.0754e-04

• x^3 ratio is just right relative to others



- f = 6 GHz
- $(\tilde{g}_4 = 7.0 \text{ MV/m})$

4.0000	19.5746	1.0000
5.0000	24.3760	1.2453
6.0000	14.1658	0.7237
7.0000	5.4002 0	9.2759
8.00000	1.541100	0.078730

 x^3 ratio is too weak relative to others



# Method 1: Optimising $\epsilon$ , $\beta_0$ , k for maximal flatness in the TM<sub>410</sub> mode

- RF-Track to use k as an optimizing variable, as well as  $\tilde{g}_4$  (Practically, would fix k and vary  $\beta_0$  but simpler analysis here doing opposite)
- For the test beam line, find the optimum to be:



- $E_z^{peak} = 18.3 \text{ MV/m}$
- $\tilde{g}_4 = 4.60$ e7 V/m
- *f* = 3.12 GHz

• Already very close!

# Method 2: Adding additional multipoles to refine the flatness in a $TM_{4,6,8,...}$ mode

- To create uniformizing field, we need magnetic field like:
- So we need an RF cavity that supports a  $TM_{\{4,6,8,10,12,\ldots\}n0}$  mode. How?
- Method exists for doing so...! Change the cavity cross-section Fully described in PhD thesis, here examples:



Novel Hybrid Multipolar RF Cavities for Transverse Beam Manipulations





# Method 2: Adding additional multipoles to refine the flatness in a $TM_{4,6,8,...}$ mode

• To illustrate, use f = 300 MHz case where RF octupole is not so well-matched





# Method 3 (Briefly): Use a TE mode

- Can also use a TE mode to provide the EM field that uniformises the beam
- Here we note that TE modes must have a longitudinal component

$$k_{1} = \frac{2\pi}{L},$$

$$\kappa_{1} = \sqrt{k^{2} - k_{1}^{2}} = \sqrt{k^{2} - \left(\frac{2\pi}{L}\right)^{2}}$$

$$E_{r}(r,\theta,z) = -\sin\left(k_{1}z\right)\frac{k_{1}}{\kappa_{1}^{2}r}\sum_{\{M\}}M\tilde{g}_{M}J_{M}(\kappa_{1}r)\cos\left(M\theta - \psi_{M}\right),$$

$$E_{\theta}(r,\theta,z) = \sin\left(k_{1}z\right)\frac{k_{1}}{\kappa_{1}}\sum_{\{M\}}\tilde{g}_{M}J_{M}'(\kappa_{1}r)\sin\left(M\theta - \psi_{M}\right),$$

$$E_{z}(r,\theta,z) = 0,$$

$$B_{r}(r,\theta,z) = -\frac{i}{\omega_{l}}\cos\left(k_{1}z\right)\frac{k_{1}^{2}}{\kappa_{1}}\sum_{\{M\}}M\tilde{g}_{M}J_{M}'(\kappa_{1}r)\sin\left(M\theta - \psi_{M}\right),$$

$$B_{\theta}(r,\theta,z) = \frac{i}{\omega_{l}}\cos\left(k_{1}z\right)\frac{k_{1}^{2}}{\kappa_{1}^{2}r}\sum_{\{M\}}M\tilde{g}_{M}J_{M}(\kappa_{1}r)\cos\left(M\theta - \psi_{M}\right),$$

$$B_{z}(r,\theta,z) = \frac{i}{\omega_{l}}\sin\left(k_{1}z\right)k_{1}\sum_{\{M\}}\tilde{g}_{M}J_{M}(\kappa_{1}r)\sin\left(M\theta - \psi_{M}\right).$$

$$E_x(x,0,z) = -\sin(k_1 z) rac{k_1}{\kappa_1^2 r} \sum_{\{M\}} M \tilde{g}_M J_M(\kappa_1 r),$$
  
 $E_x(x,0) = -rac{k_1}{\kappa_1^2 r} \sum_{\{M\}} M \tilde{g}_M J_M(\kappa_1 r).$ 

### Caveats:

- Ignore longitudinal term which acts as a transit time factor
- TE mode must be used at nonrelativistic energy (else magnetic force cancels the electric force)



# Method 3 (Briefly): Use a TE mode

• Longer RF cavity (15 cm) so TE<sub>411</sub> mode existsat 3 GHz



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# **Uniform Beam with RF Cavity Stocktake**

### • Three methods:

- 1. Tailor  $\epsilon$ ,  $\beta_0$ , k in a TM<sub>410</sub> mode to get as flat as beam as possible
- 2. If  $\epsilon$ ,  $\beta_0$ , k are fixed and beam is not sufficiently flat, add more multipoles to make a TM<sub>{4,6,8,...}10</sub> that can be supported by an azimuthally modulated cavity
- 3. Either of the above but with a TE mode

### • Method 1 works nicely noting:

- $P \sim 10$  MeV/c electron beam requires cavity w/  $E_z^{\text{peak}} = 20$  MV/m in a L = 4 cm structure with no transit time factor
  - Expect transit time factor increase  $E_z^{\text{peak}}$  by a third
  - Longer structure reduce  $E_z^{\text{peak}}$  by ratio of new length to 4 cm
- Space charge turned off, on-crest through a static field so E field is ignored, 1D uniform beam, no realistic cavity design with couplers etc

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# Implementing a field map

- No beam pipe, just a 3 GHz TM<sub>410</sub> mode
- Test was to check implementation of field map in RF-Track – coarse mesh and export but result is flat beam







# Adding a beam pipe

- Now add a  $r_{\text{pipe}} = 48 \text{ mm} (\sigma_0 = 17.75 \text{ mm})$  beam pipe
- Field leaks into pipes but still get flat beam
- Interesting that radius is smaller





## **Next Steps - Create a 2D uniform beam**

- No reason it shouldn't work, but I need to develop the code to optimise for 2D flatness
  - Adapt Andrea's DEFT code
- Plan for this proof of concept:
  - Build a FODO testline
  - Insert a multipole cavity in each of the x and y beta function minima
  - Optimise for flatness
  - Analyse the effect of finite y-dimension on the flatness of the beam that can be achieved





### **RF-Track Improvements**

- Andrea is implementing the ability to model TM<sub>{M}np</sub> modes, including with a beam pipe
- Ping has method for interfacing RF-Track with CST so can directly optimize RF-Track beam dynamics simulations with CST cavity designing



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# **Uniform beam applications**

### **Uniform irridation**

- Radiotherapy
- Sterilisation
- Targets (spallation, isotope production, particle production)

### Halo reduction of nanobeam collision

• ^Use octupoles to fold tails of beam in CLIC final focus system



## **Other beam shaping**

- The basis of the EM modes in RF cavities is well-described by Bessel functions
- In theory, one can build any polynomial map out of Bessel functions as:

$$x^{n} = \begin{cases} J_{0}(x) + 2\sum_{k=0}^{\infty} J_{2k}(x); & n = 0, \\\\ 2^{n}n! \sum_{k=0}^{\infty} \binom{n+k}{n} \frac{n+2k}{n+k} J_{n+2k}(x); & n \ge 1. \end{cases}$$

• One can then determine the azimuthally modulated cavity that supports it



# Other transverse beam shaping with RF cavity ideas



- e.g. generate more muons by generating pions close to a target surface
- Formation of 'linear' beam
  - e.g. provide a linear dose for testing detector?
- Formation of uniform momentum beam

- Tailoring of flat accelerating, dipole, quadrupole fields
  - e.g. optimal flat accelerating field for cyclotron, tailored fields for FFAs/non-linear accelerators



Prog. Theor. Exp. Phys. 2019, 053G01 (11 pages) DOI: 10.1093/ptep/ptz024

### Formation of hollow ion beams of various shapes using multipole magnets

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<sup>1</sup>Takasaki Advanced Radiation Research Institute, National Institutes for Quantum and Radiological Science and Technology, 1233 Watanuki-machi, Takasaki 370-1292, Japan <sup>2</sup>Research Center for Nuclear Physics, Osaka University, 10-1 Mihogaoka, Ibaraki 567-0047, Japan \*E-mail: yuri.yosuke@gast.go.jp

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This study describes a novel and simple beam-focusing method for the formation of a chargedparticle beam with a hollow transverse intensity distribution using the nonlinear focusing force of multipole magnets is na beam transport line. The dynamic behavior of the beam focused by multipole magnets is theoretically investigated to predict the phase-space shape and spatial profile of the hollow beam. It is shown numerically and experimentally that the hollow beam has a steep peak at the peripheral edge and high contrast between the edge peak intensity and the intensity near the beam center. Depending on the order and strength of the applied nonlinear multipole field, the cross-sectional shape of the hollow beam can be diversely transformed, e.g., into an ellipse, a rounded rectangle, a rhombus for octupole focusing, or a triangle for sextupole focusing. The present beam-formation method, applicable to various charged-particle beams of different parameters such as the particle species, kinetic energy, and time structure, enables the shaping of the transverse profile that can never be realized through conventional linear beam optics.







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## **Next steps and discussion**

Uniform beam:

- Finish 2D proof of concept
- Make a more realistic cavity?
- Detail an application of the idea in specific concept/accelerator
  - What? Where?
- Write into conference & journal paper

Any questions etc

