

**PSI** Center for Neutron and  
Muon Sciences



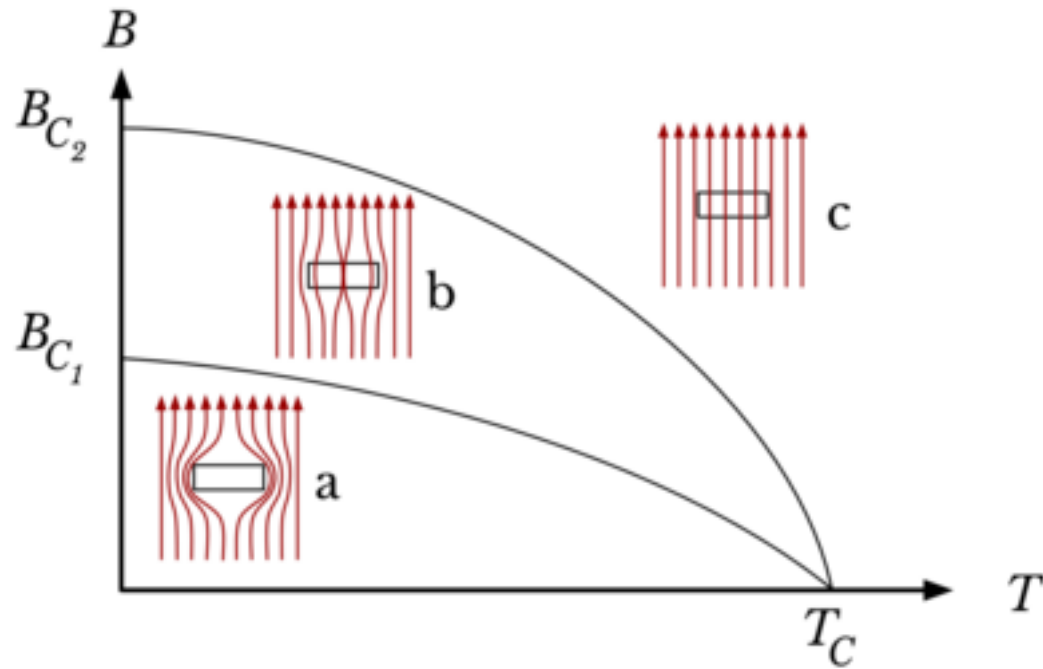
**University of  
Zurich** <sup>UZH</sup>

# Resonant Ultrasound Spectroscopy (RUS) A Study of the Vortex Lattice Phase Diagram of Niobium

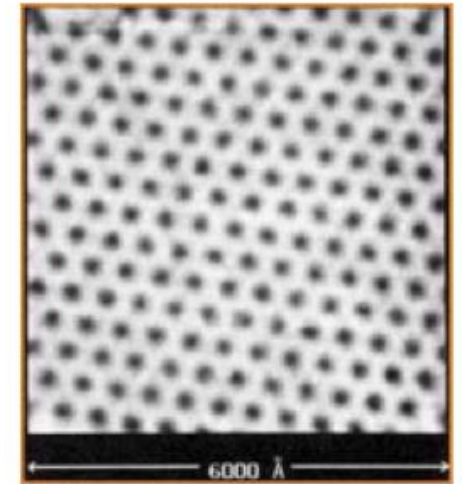
**Swiss Physical Society meeting 2024**

Dang Xuan Dang :: Ph.D. student  
ETH Zurich, 10 September 2024

# Vortex Matter in Superconductors



Repulsive interaction between vortices.



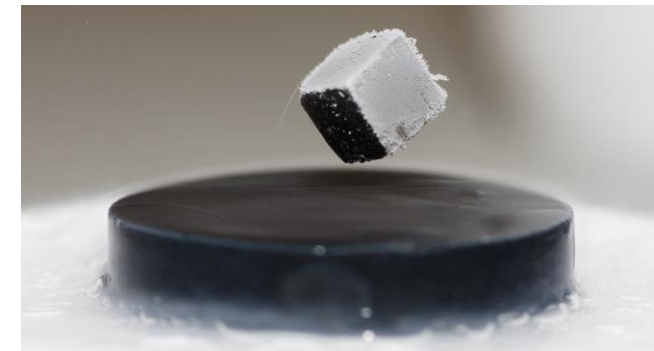
Scanning Tunneling Microscopy  
NbSe<sub>2</sub>, 1T, 1.8K

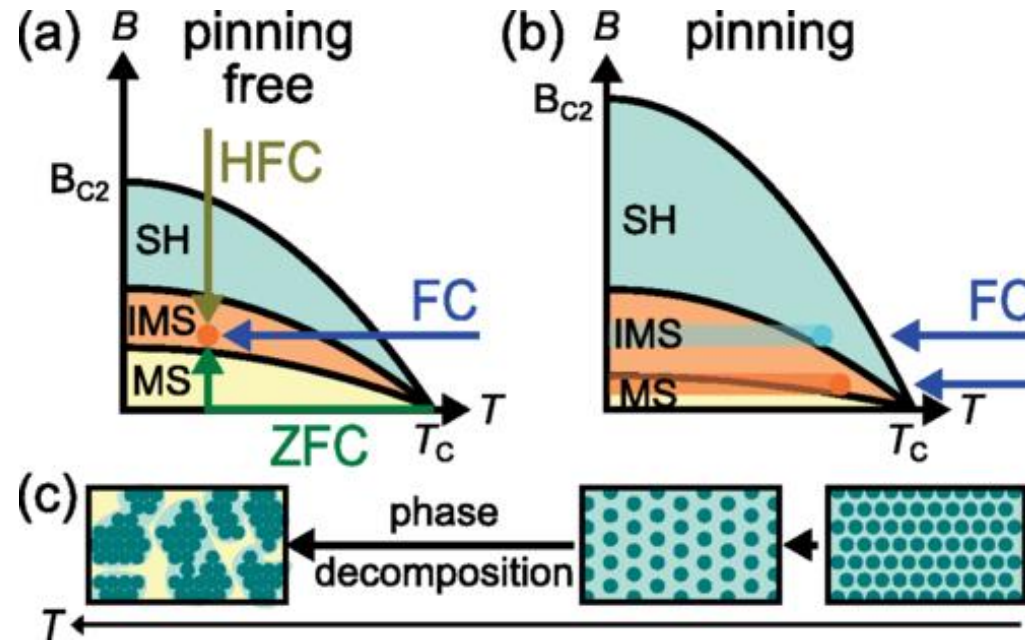
H. F. Hess et al., Bell Labs

## Type II superconductor

- $\kappa > 1/\sqrt{2}$
- External magnetic field allows passing via quantised vortex lines

Pinning of vortex lines.





T. Reimann et al Physical Review B 96, 144506 (2017)

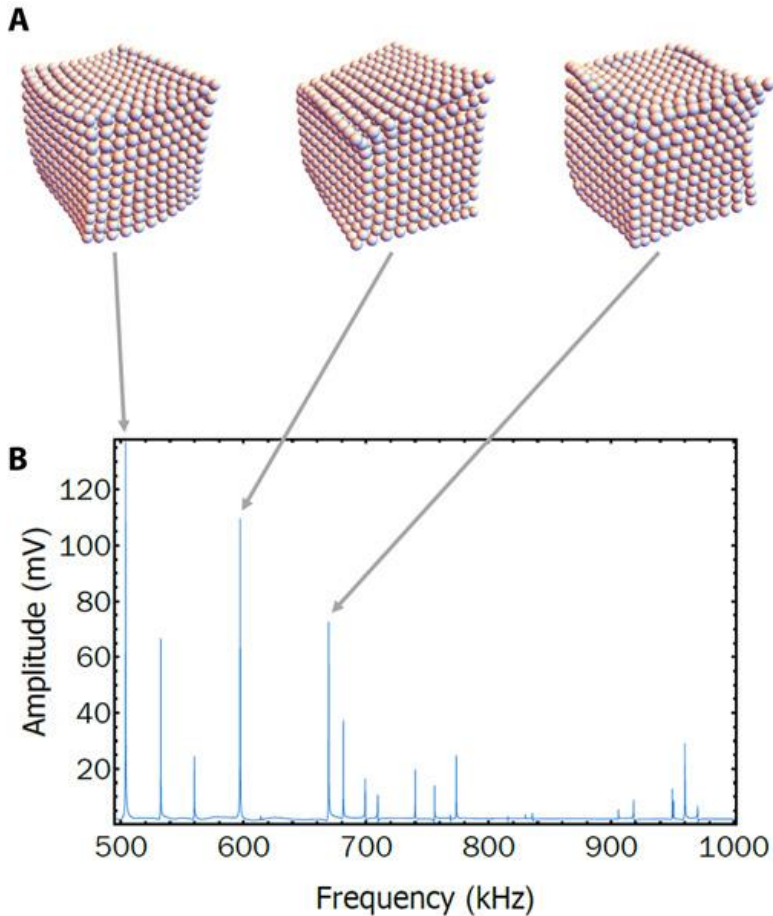
## Type II Superconductor

- Superconducting Transition Temperature 9.2K
- $B_{c2} \sim 0.5T$

## Vortex matter in Niobium

- Immediate Mixed phase  $0.1T < B < 0.2T$
- Shubnikov phase

# Resonant Ultrasound Spectroscopy (RUS) and Elasticity of Crystals



S. Ghosh et al, Science advance. 6, 10 (2020)

- RUS measures Eigenmodes of the sample
- Depend on sample dimensions, density and elastic modulus
- RUS measures all elastic moduli  $c_{ij}$  simultaneously.
- RUS is highly sensitive to changes in lattice strength.

**For example:**

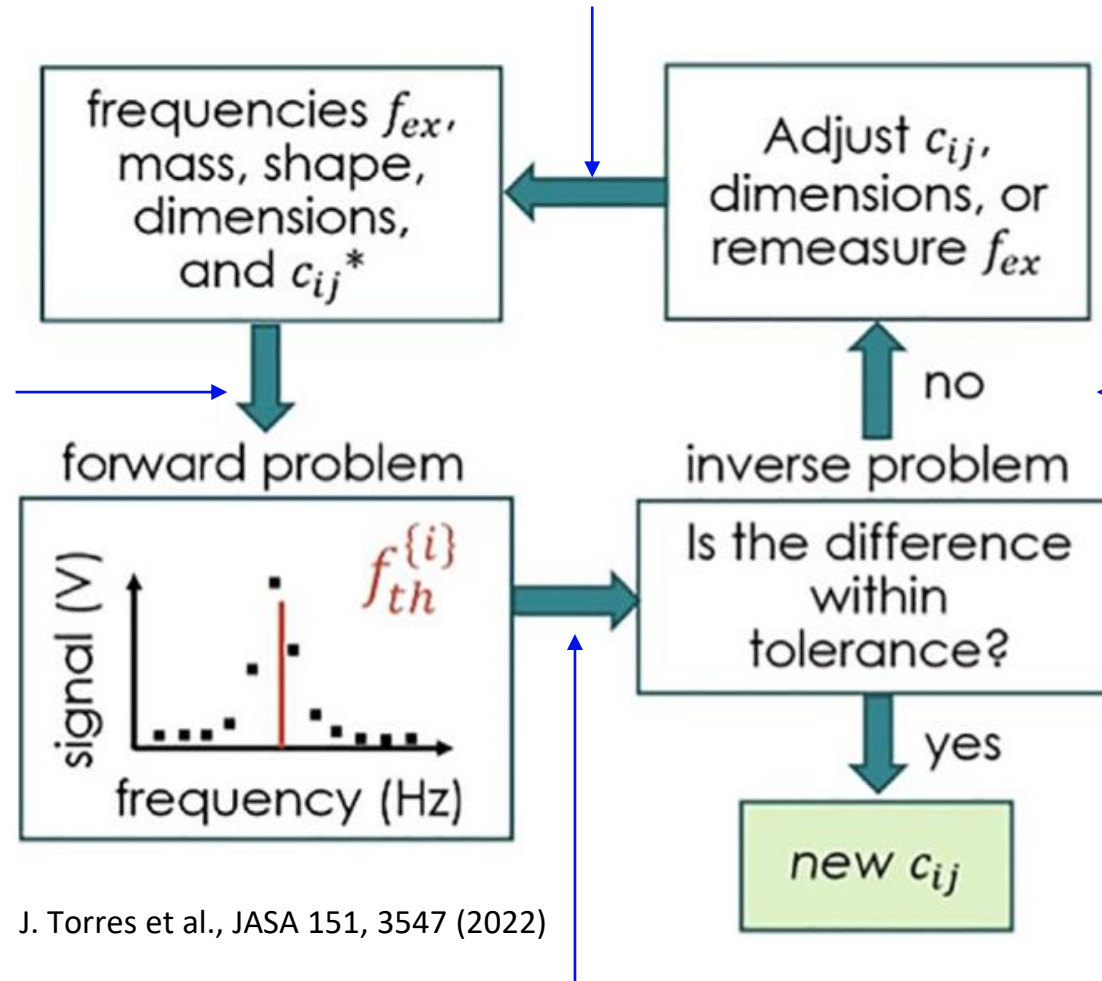
BCC crystal has 3 components to the elastic tensor

$$\begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{pmatrix}$$

$c_{11}$  .. compression  
 $c_{12}$  .. torsion  
 $c_{44}$  .. shear

# Spectrum computation

Adapt input parameters



J. Torres et al., JASA 151, 3547 (2022)

Compare calculation to experiment

Calculate Eigen frequencies by Rayleigh-Ritz method.

Get the new initial  $c_{ij}^*$

# Spectrum computation

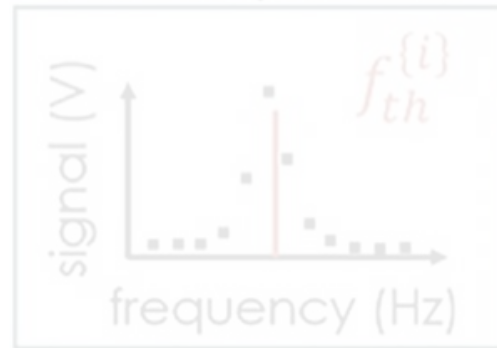
Adapt input parameters

frequencies  $f_{ex}$ ,  
mass, shape,  
dimensions,  
and  $c_{ij}^*$

Adjust  $c_{ij}$ ,  
dimensions, or  
remeasure  $f_{ex}$

**We use the package *RUScal***

<https://github.com/RUS-ORNL/RUScal>



J. Torres et al., JASA 151, 3547 (2022)

Is the difference  
within  
tolerance?

yes

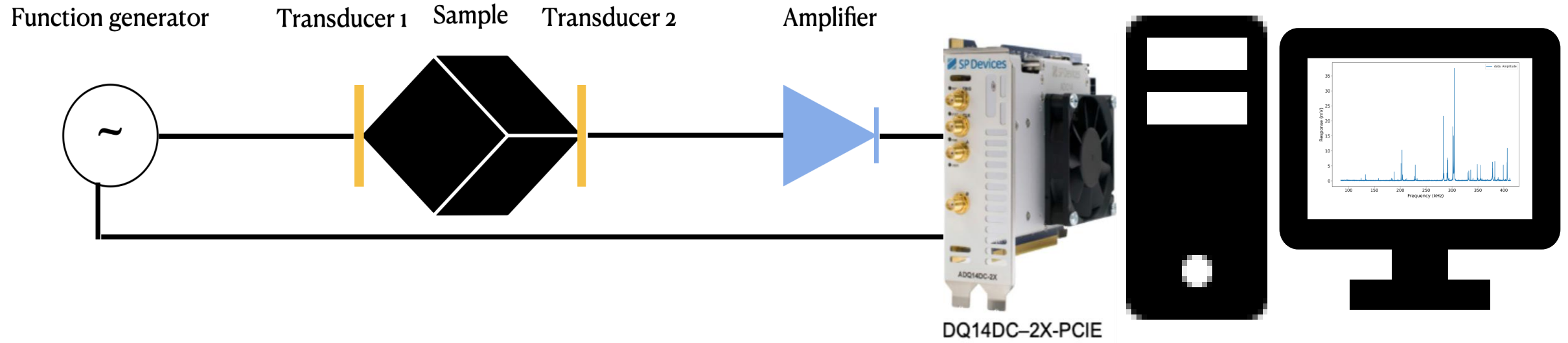
new  $c_{ij}$

Calculate Eigen frequencies  
by Rayleigh-Ritz method.

Get the new initial  $c_{ij}^*$

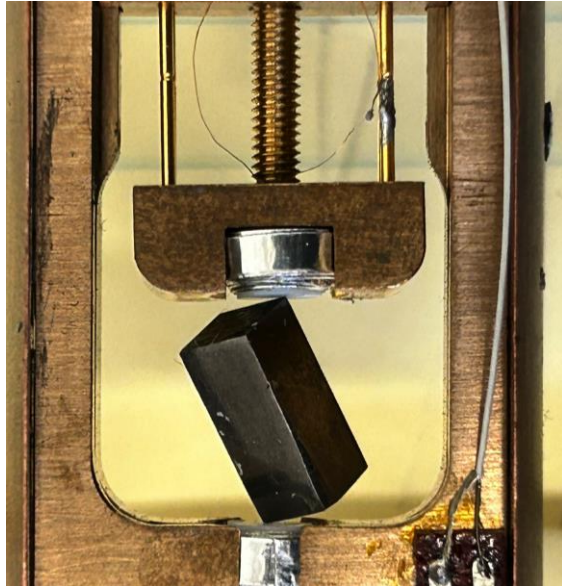
Compare calculation to  
experiment

# RUS experimental setup



- Crystal is clamped by transmitter and receiver transducers.
- F.G controls the frequency of excitation.
- Amplifier gain the response signal.
- DAQ records data and plotting in screen of PC.

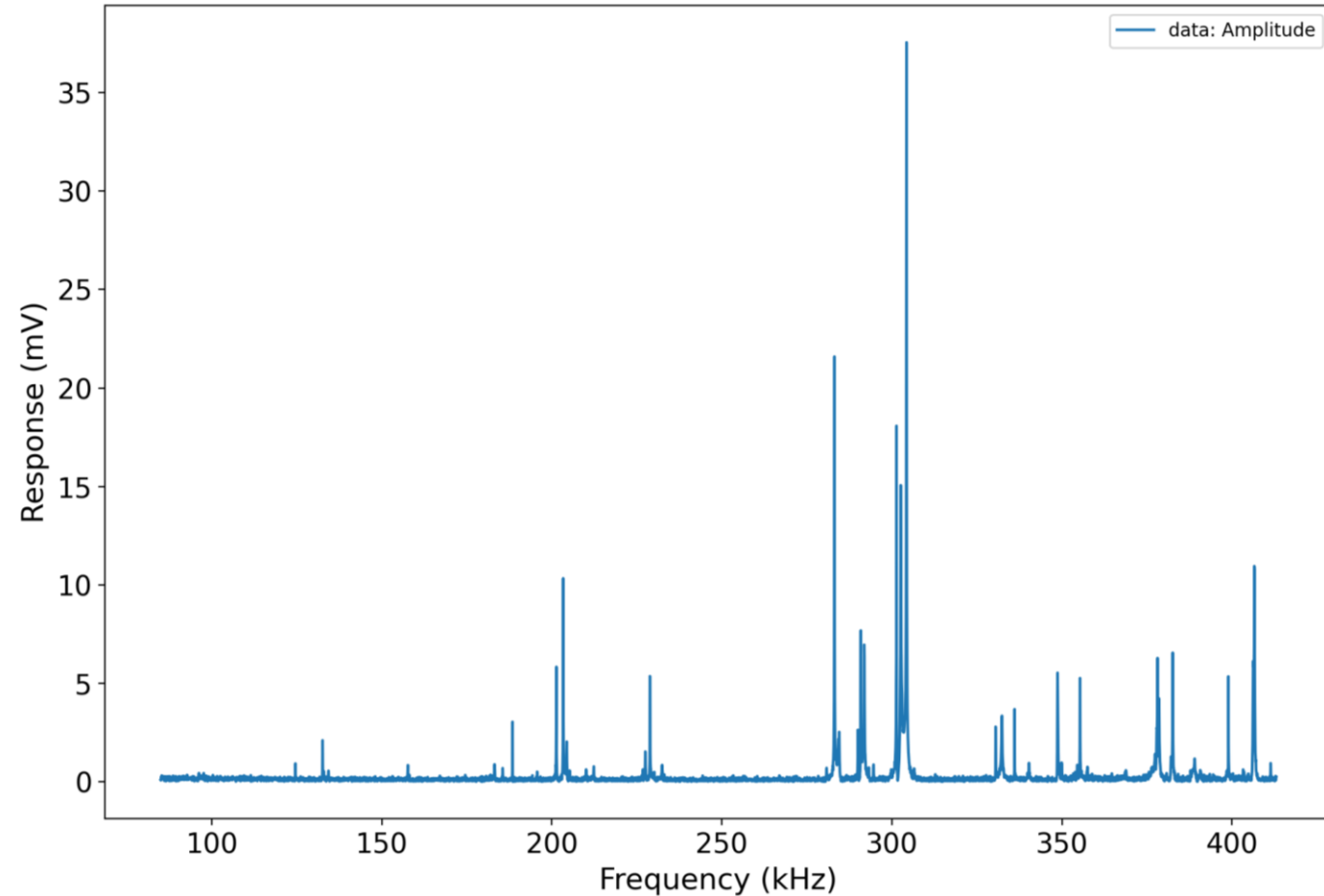
# RUS Spectrum of Niobium (Nb)



Nb crystal in stick

- Rectangular shape.
- Dimensional: 3.84 x 4.10 x 9.40 mm<sup>3</sup>
- Mass: 1.2528 g
- $c_{11} = 245.6$  GPa,  $c_{12} = 138.7$  GPa,  $c_{44} = 29.3$  GPa (\*)

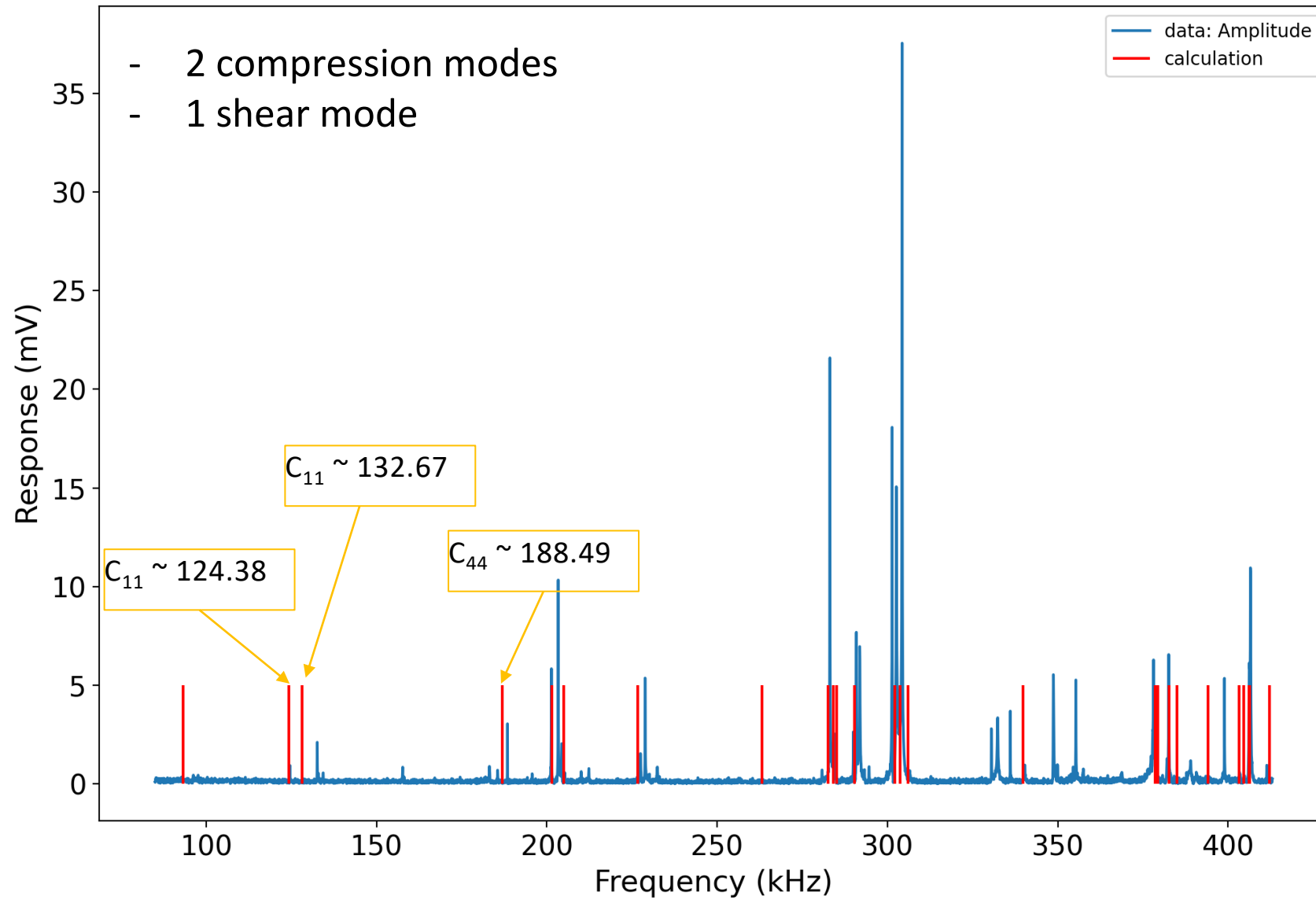
(\*): By K. J. Carroll thesis, 1962



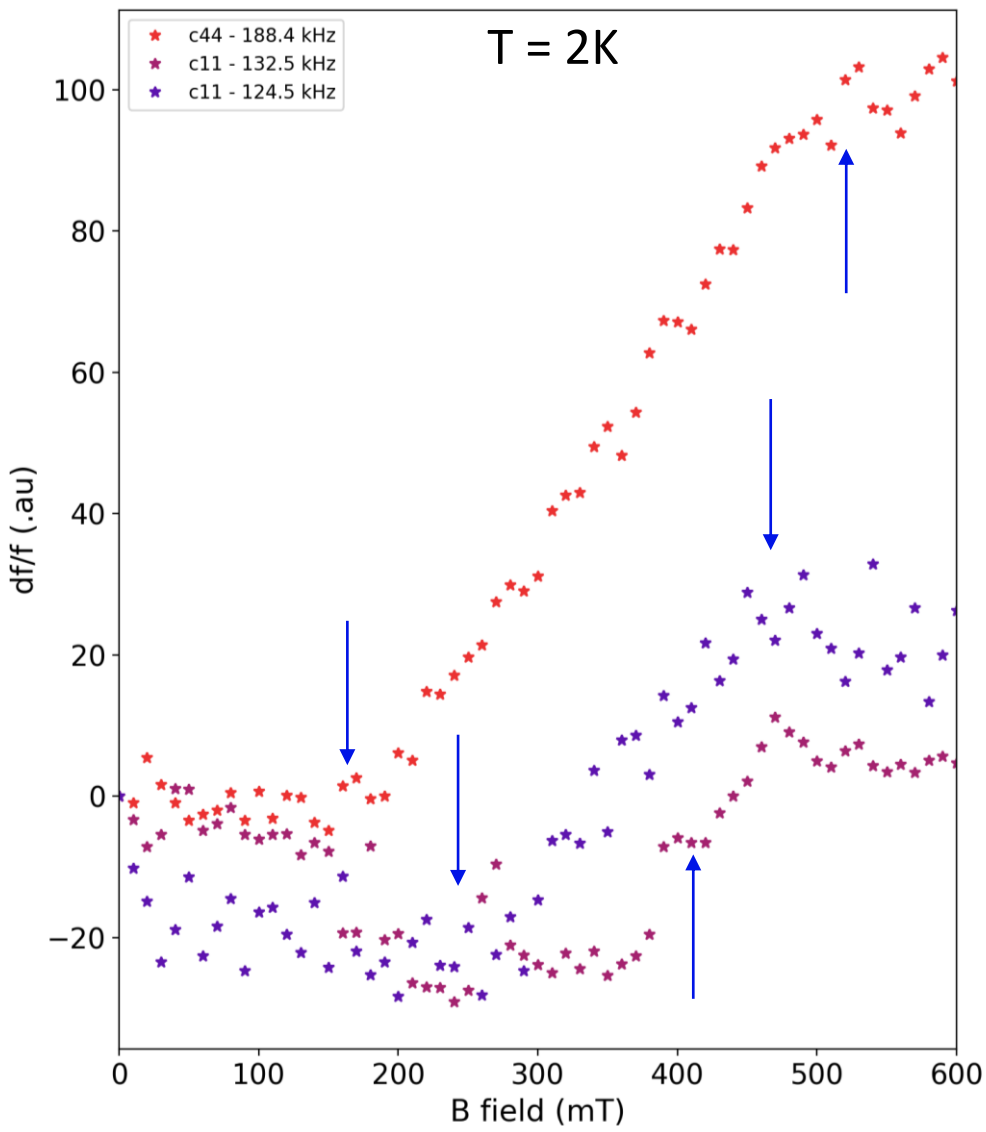
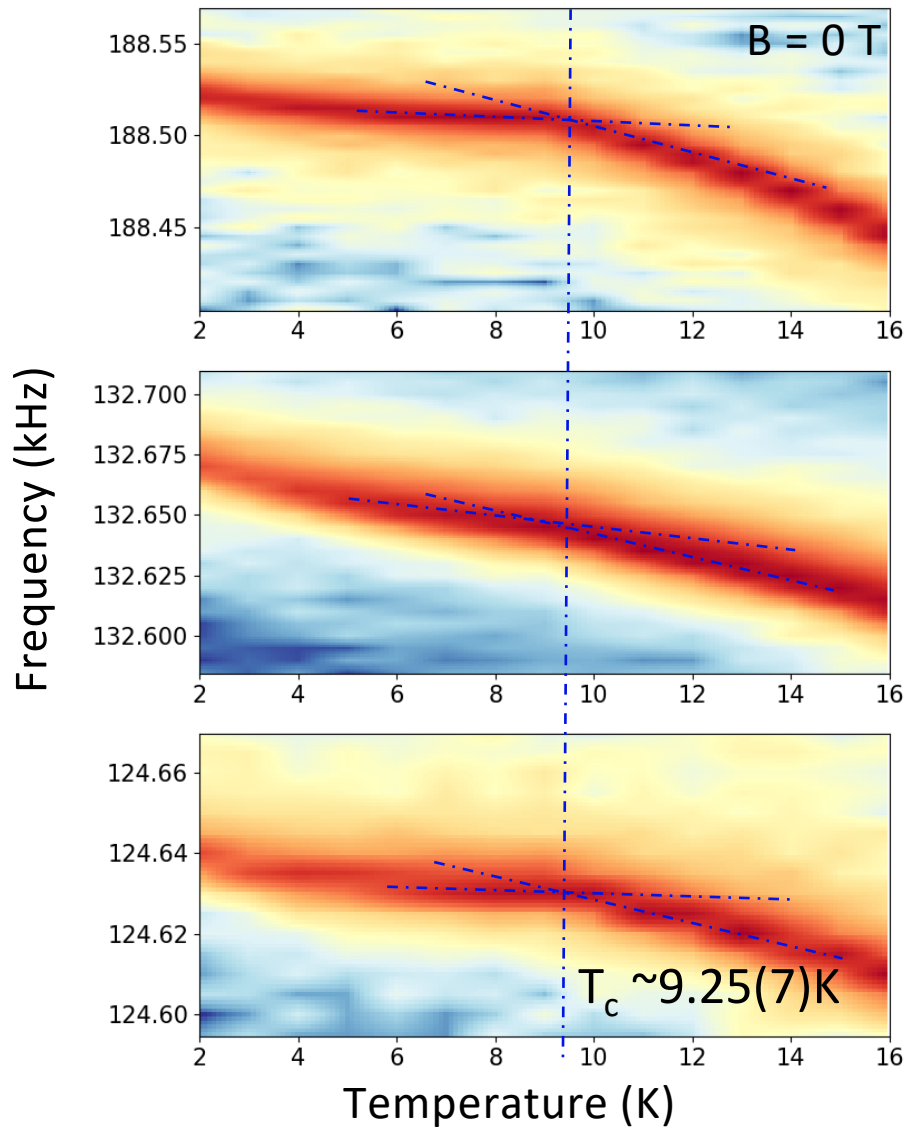
RUS spectrum of Nb at T = 16K



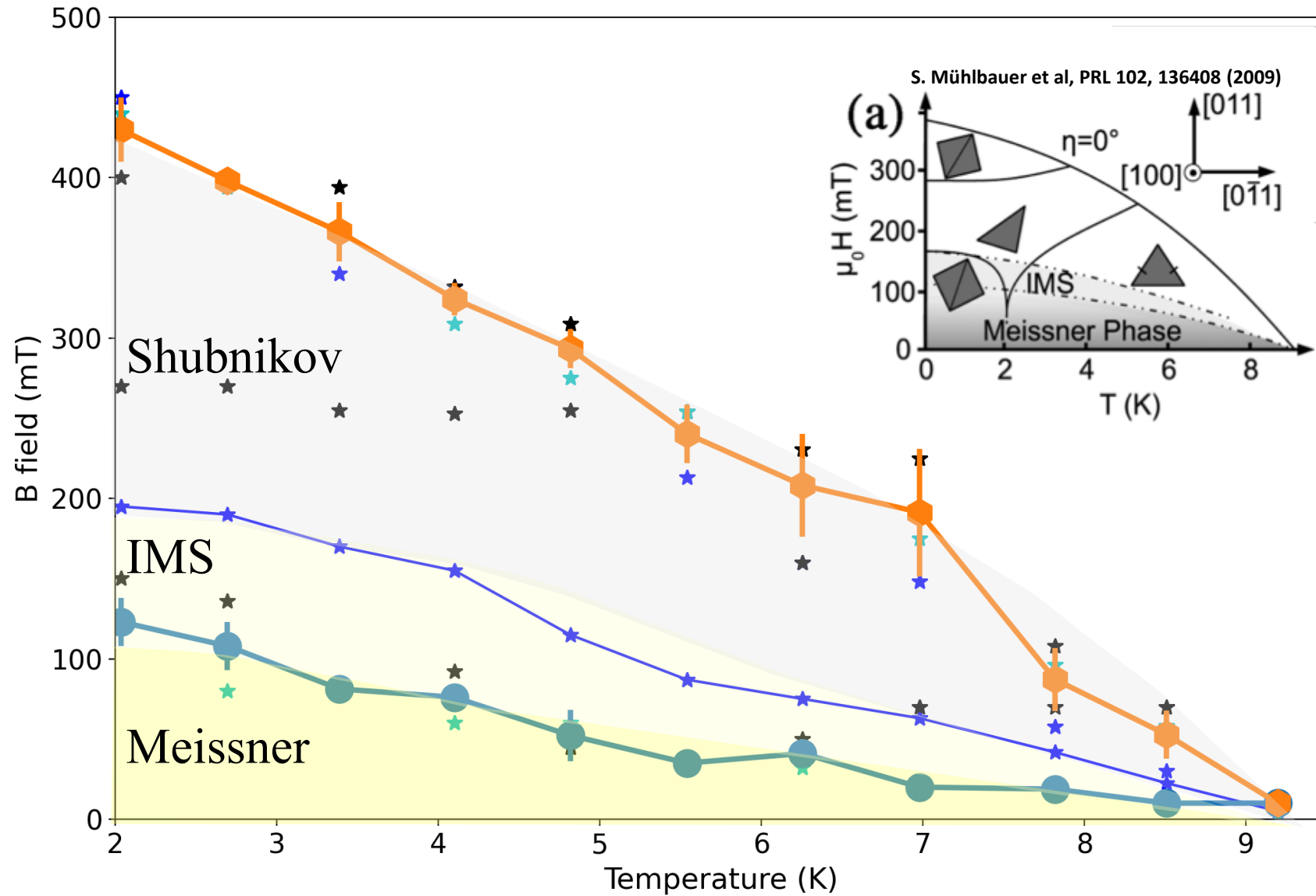
# Comparison measured and calculated Spectrum



# Eigenmodes as function of tuning parameters



# Phase transition of Niobium by RUS measurement.





Developed RUS experiment successful investigate the phase transition of Niobium cause the change of elasticity.



RUS useful tool for the investigation vortex phases and phase transitions

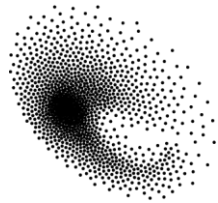
# Acknowledgments



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**PSI** Center for Neutron and Muon Sciences



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C. Klauser



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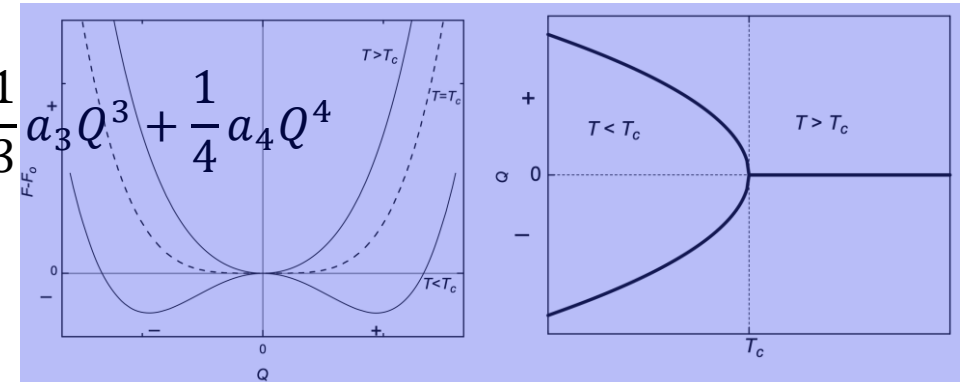


F. Fautz

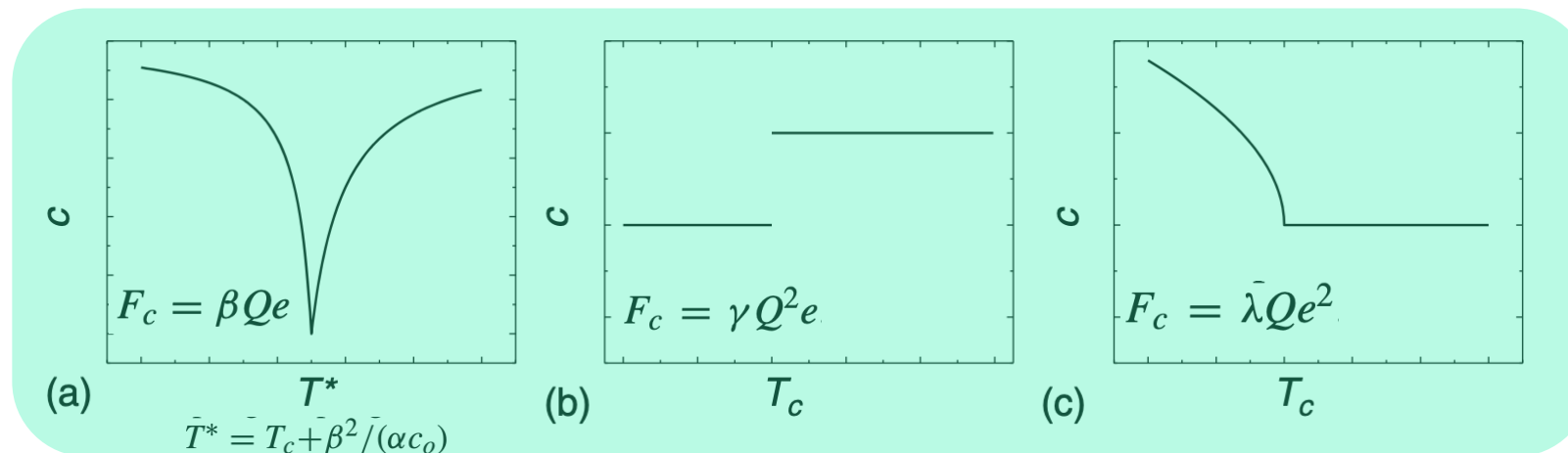
# Supplementary Information

In the second-order phase transition, Landau theory:

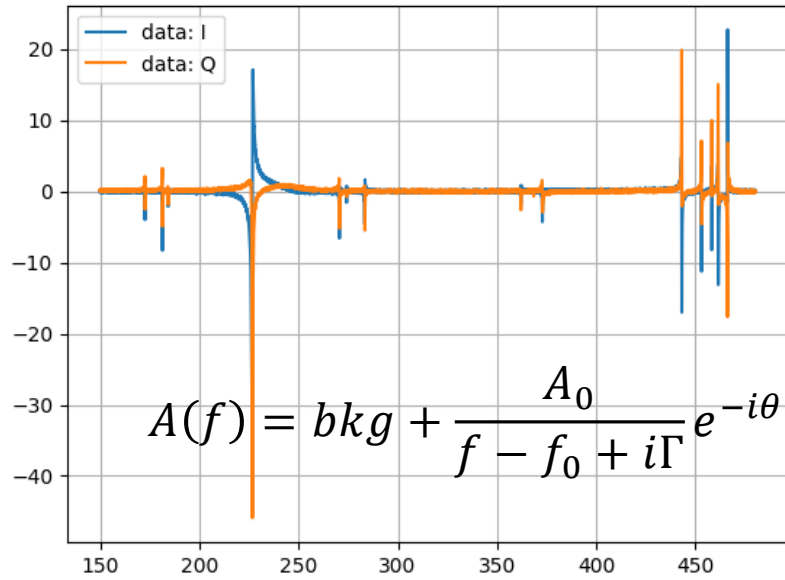
$$F(Q, T) = F_0(T) + a_1 Q + \frac{1}{2} a_2 Q^2 + \frac{1}{3} a_3 Q^3 + \frac{1}{4} a_4 Q^4 + F_c + \frac{1}{2} c_0 e^2$$



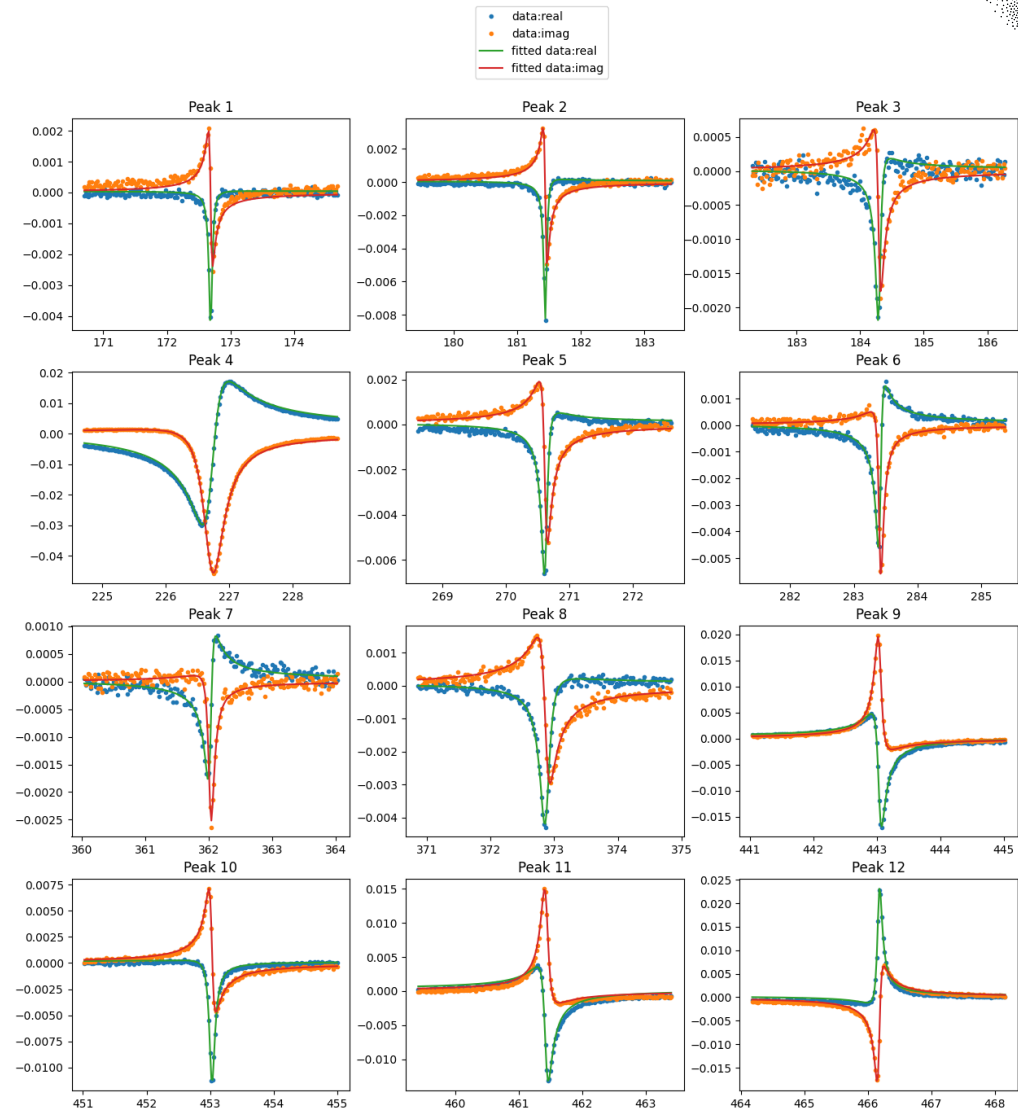
Adding the term for order parameter (Q) and elastic strain (e) coupling ( $F_c$ ) and elastic energy



# Supplementary Information



	a	f0	gamma	psi	bkg	ampl
0	0.138932	172688.774584	31.568018	-1.486012	0.000039	0.006952
1	0.243688	181442.693651	29.099660	-1.359570	0.000063	0.013217
2	0.114655	184296.355237	48.180994	-1.056724	0.000028	0.003766
3	9.189233	226718.308593	195.685216	-0.336902	0.001378	0.075141
4	0.407319	270620.070202	56.616396	-1.081214	0.000086	0.011387
5	0.270674	283413.107904	43.745395	-0.573973	0.000063	0.009782
6	0.144141	362026.436337	54.879735	-0.406547	0.000034	0.004160
7	0.408352	372867.927439	92.077797	-1.224203	0.000076	0.007042
8	1.397320	443042.657529	64.441224	-2.511157	0.000286	0.034347
9	0.626876	453024.626812	53.159098	-1.776801	0.000133	0.018657
10	1.163777	461432.159021	69.262600	-2.487403	0.000238	0.026631
11	1.127121	466174.870531	45.675569	1.085989	0.000263	0.039025



n	fexp(MHz)	fcalc(MHz)	%err	wt	dlnf/dlnc11	dlnf/dlnc12	dlnf/dlnc44	sum
1	0.0966910	0.0936072	-3.19	0.00	0.01058	-0.00545	0.99460	0.99974
2	0.1243830	0.1238165	-0.46	1.00	1.29546	-0.64098	0.34314	0.99762
3	0.1326780	0.1276039	-3.82	1.00	1.25959	-0.62301	0.36101	0.99759
4	0.1884900	0.1877040	-0.42	1.00	0.04728	-0.02415	0.97655	0.99968
5	0.2013840	0.2016217	0.12	1.00	0.38046	-0.18661	0.80493	0.99858
6	0.2045440	0.2049890	0.22	1.00	0.47008	-0.23143	0.75972	0.99837
7	0.2268960	0.2247837	-0.93	1.00	2.02363	-1.02535	0.00003	0.99832
8	0.2808700	0.2642724	-5.91	1.00	0.11519	-0.06003	0.94423	0.99939
9	0.2831800	0.2849427	0.62	1.00	0.25013	-0.12331	0.87235	0.99917
10	0.2845050	0.2852637	0.27	1.00	0.58337	-0.29020	0.70588	0.99905
11	0.2846990	0.2860489	0.47	1.00	0.09084	-0.04699	0.95574	0.99958
12	0.2909300	0.2912385	0.11	1.00	0.20617	-0.10177	0.89490	0.99930
13	0.3028560	0.3025830	-0.09	1.00	0.80270	-0.40820	0.60459	0.99910
14	0.3044030	0.3041980	-0.07	1.00	0.79020	-0.40455	0.61326	0.99892
15	0.3068540	0.3069968	0.05	1.00	0.04265	-0.01419	0.97126	0.99972
16	0.3406100	0.3410209	0.12	1.00	0.20467	-0.08898	0.88340	0.99909
17	0.3808760	0.3811810	0.08	1.00	0.58716	-0.26817	0.67908	0.99807
18	0.3823940	0.3821377	-0.07	1.00	0.86308	-0.43581	0.57010	0.99738
19	0.3827420	0.3841636	0.37	1.00	0.14384	-0.07397	0.92952	0.99939
20	0.3893330	0.3893532	0.01	1.00	0.42145	-0.21570	0.79335	0.99909
21	0.3908650	0.3965475	1.45	1.00	0.55082	-0.27360	0.72143	0.99864
22	0.3989490	0.4019516	0.75	1.00	1.18078	-0.62251	0.43689	0.99516
23	0.4036940	0.4021275	-0.39	1.00	1.60025	-0.84621	0.24342	0.99747
24	0.4064160	0.4078617	0.36	1.00	0.36414	-0.17471	0.81002	0.99945
25	0.4067130	0.4084954	0.44	1.00	0.28952	-0.14000	0.84954	0.99906

c11	c22	c33	c23	c13	c12	c44	c55	c66
250.28	250.28	250.28	133.68	133.68	133.68	31.172	31.172	31.172

dx(cm) dy(cm) dz(cm)      Shape set to rectangular parallelepiped.  
 0.3840 0.4100 0.9400