Search for the $B^+ \to \tau^+ \nu$ decay channel at the LHCb experiment at CERN

SPS Annual Meeting 2024

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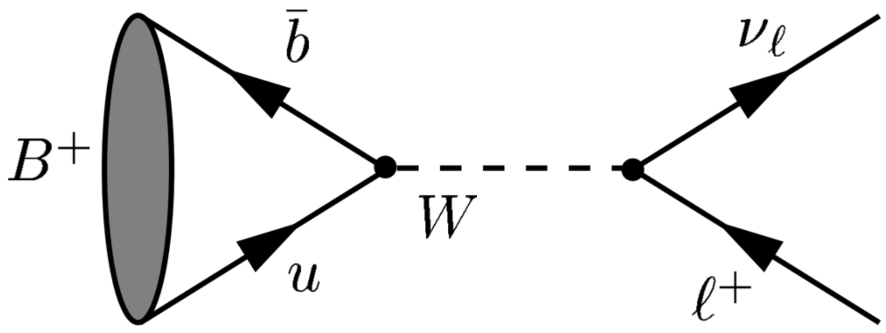
Motivation to study $B_{(c)}^{+} \to \tau^{+}\nu$

- This pure leptonic B decay allows for precise tests of the Standard Model predictions
  - Enables a clean direct experimental determination of V_{ub} (V_{cb})



$$\mathcal{B}(B^+ \to \tau^+\nu) = \frac{G_F^2\, m_B}{8\pi} m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_b^2\, |V_{ub}|^2 \tau_B$$

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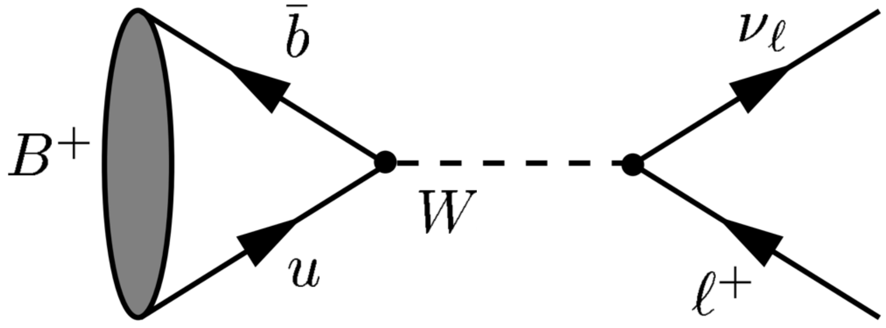

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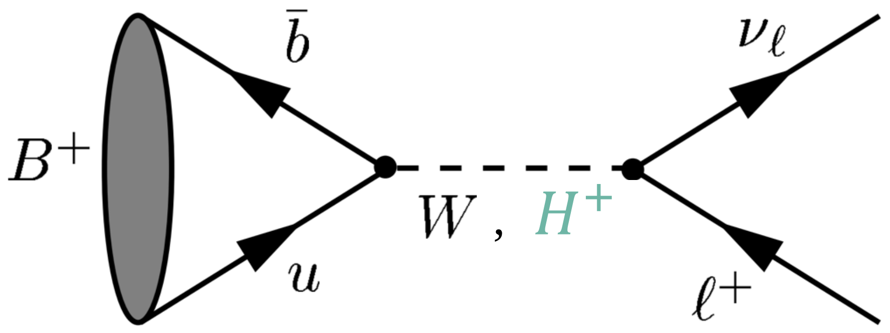
  - Supersymmetry, **two-Higgs-doublet models**, etc.



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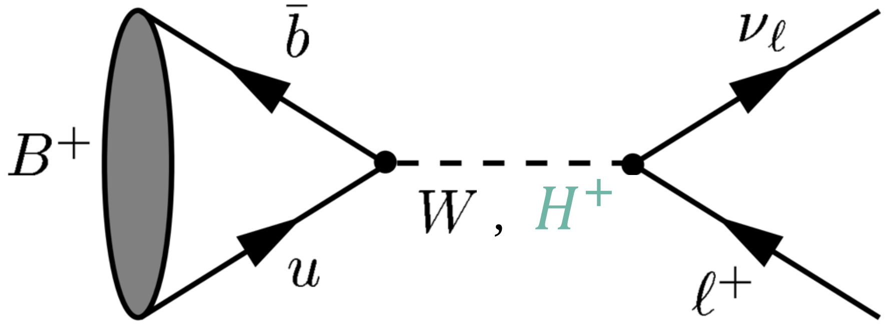
- Serves as a valuable probe for searching for new physics:
  - Supersymmetry, **two-Higgs-doublet models**, etc.
- $B_c^+ \to \tau^+ \nu$ has never been observed.



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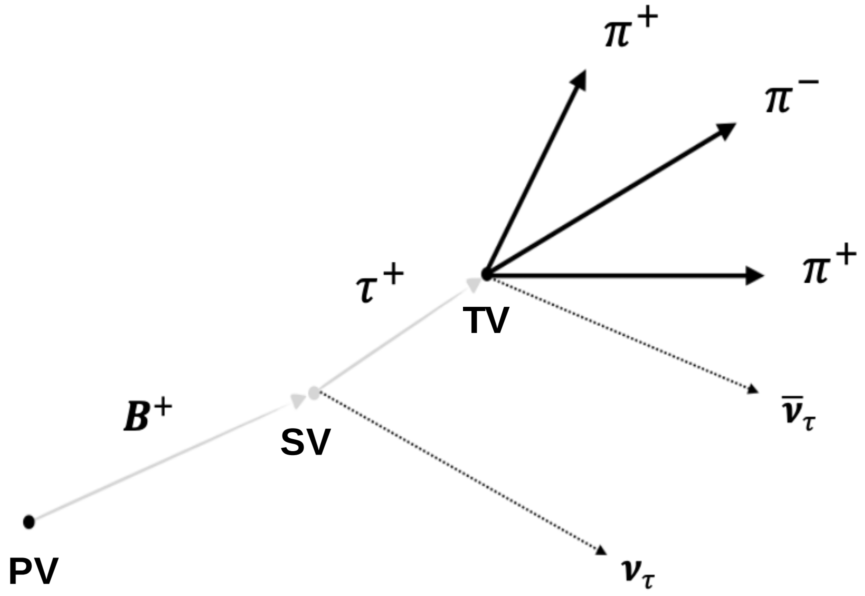
Reconstruction strategy for $B^+ \to \tau^+ \nu$

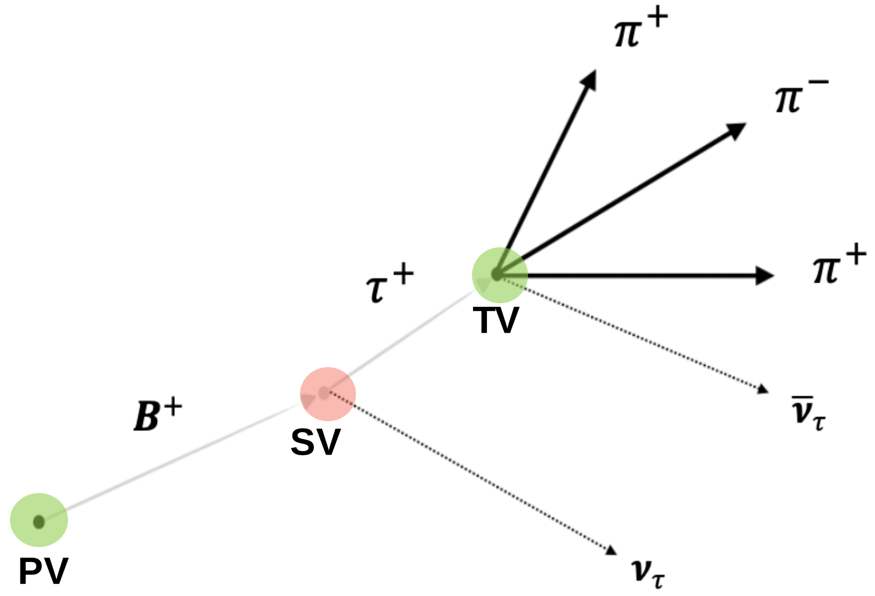
- We aim to reconstruct the decay mode $\tau^+ \to \pi^+ \pi^- \pi^+ \overline{\nu_\tau}$

$$\mathcal{B}(\tau^+ \to \pi^+ \pi^- \pi^+ \overline{\nu_\tau}) = (9.31 \pm 0.05)\ \%$$



Main Challenges

- We cannot directly reconstruct the secondary vertex (SV):
  - Only three pion tracks (TV) and the primary vertex (PV) are available.
  - Two neutrinos in the final state.

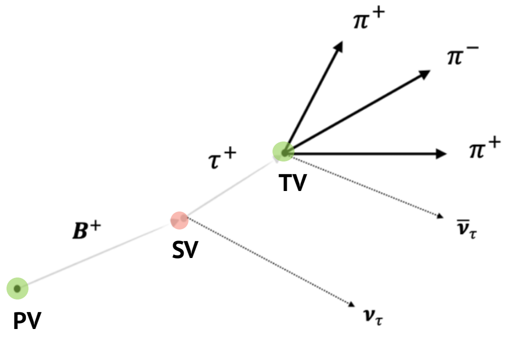


- Numerous physics processes can contribute to the background.
  - Challenging hadronic environment at LHCb.

Simulation

- RapidSim:
  - It is a fast Monte Carlo generator for simulation of heavy-quark hadron decays.
  - Does not contain the simulation of the detectors and their response.
  - Generated the decay $B^+ \to (\tau^+ \to \pi^+\pi^-\pi^+ \bar{\nu_\tau})\, \nu$.
  - Simulated 100k events.

Different strategies to calculate SV



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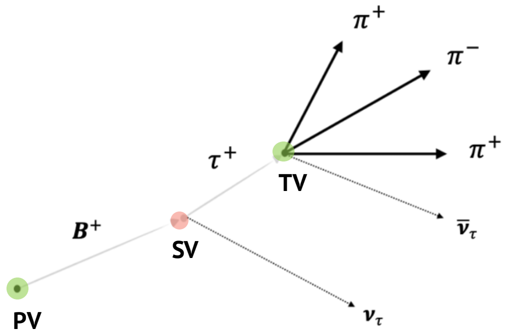

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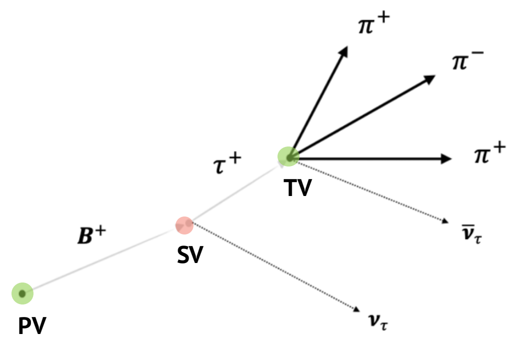

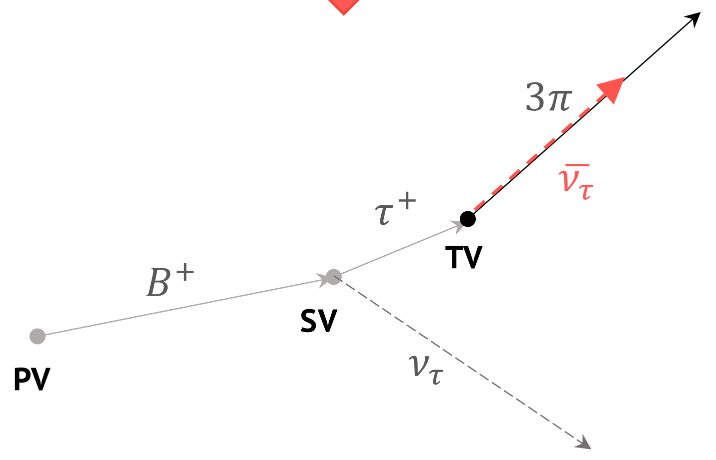

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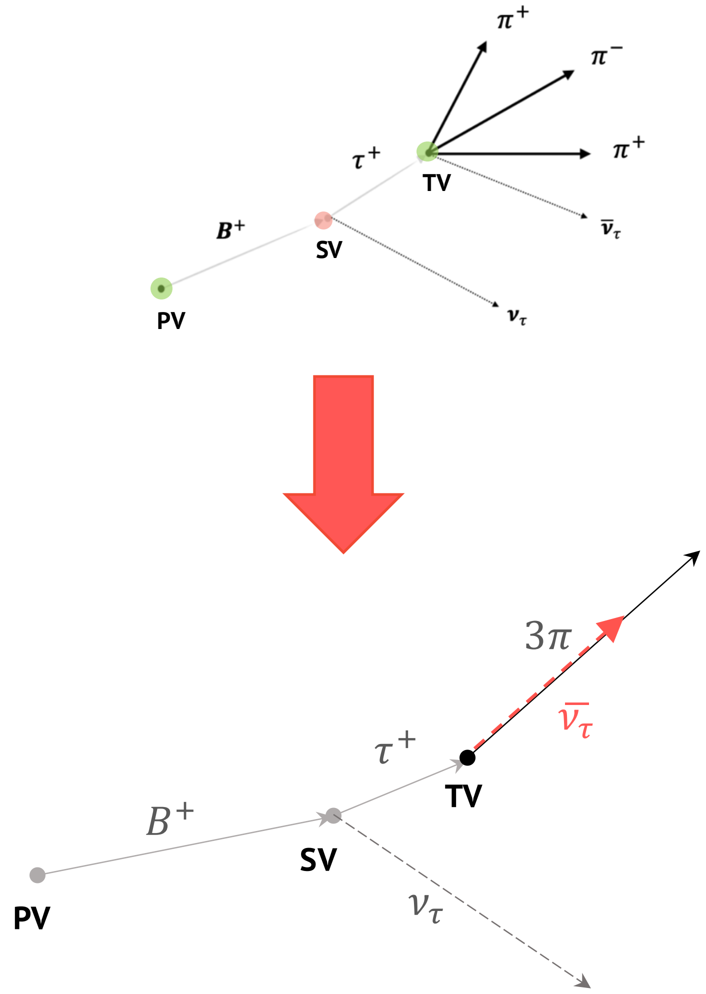

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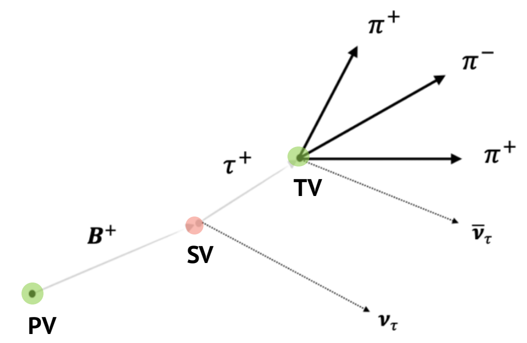

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    - Assume $\vec{p}_\nu$ is in the same direction as $\vec{p}_{3\pi}$
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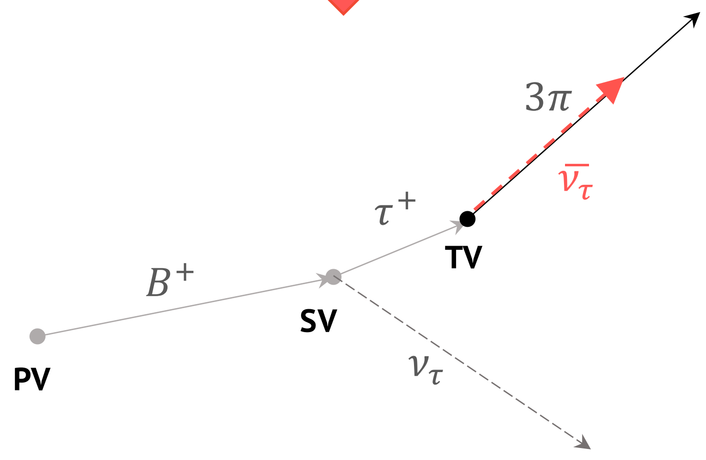

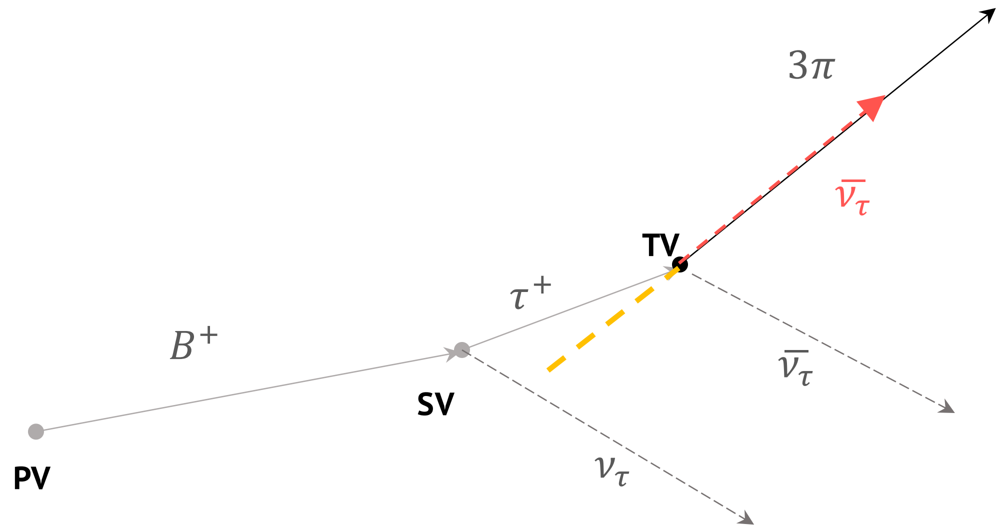

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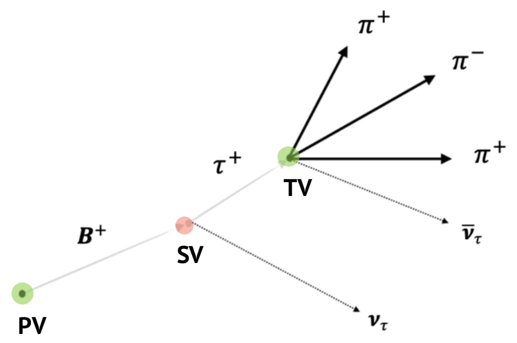

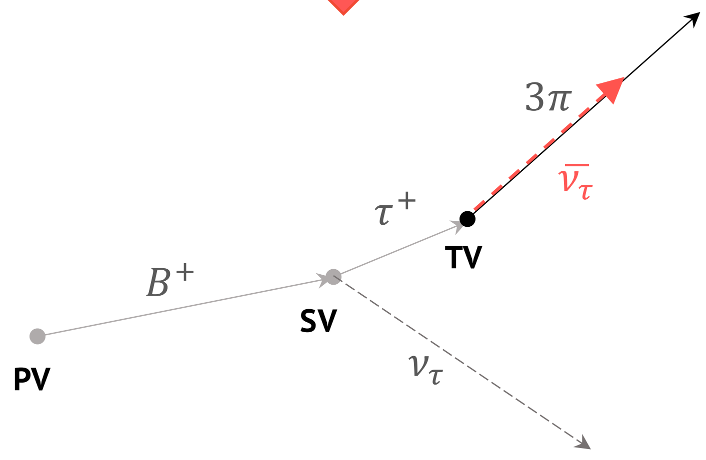

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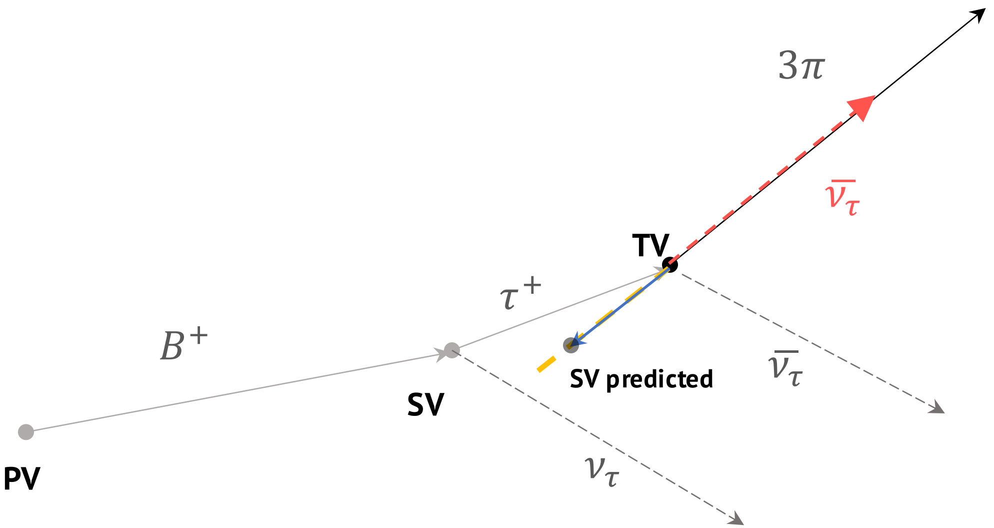

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   - Assume $\vec{p}_\nu$ is in the same direction as $\vec{p}_{3\pi}$
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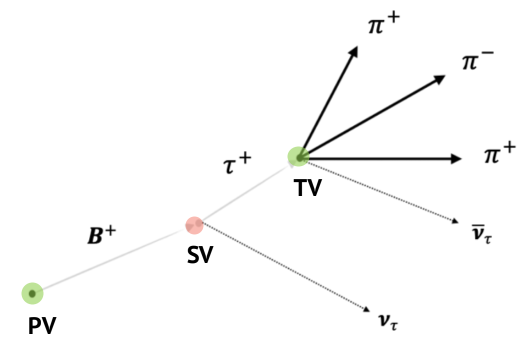
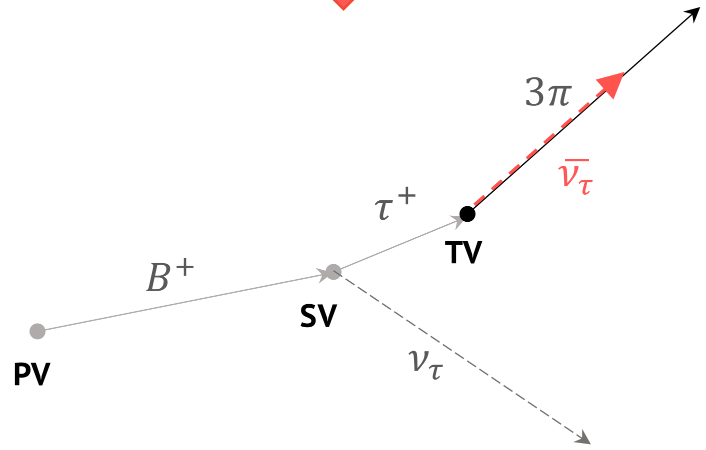
   - Estimate B^+ corrected mass





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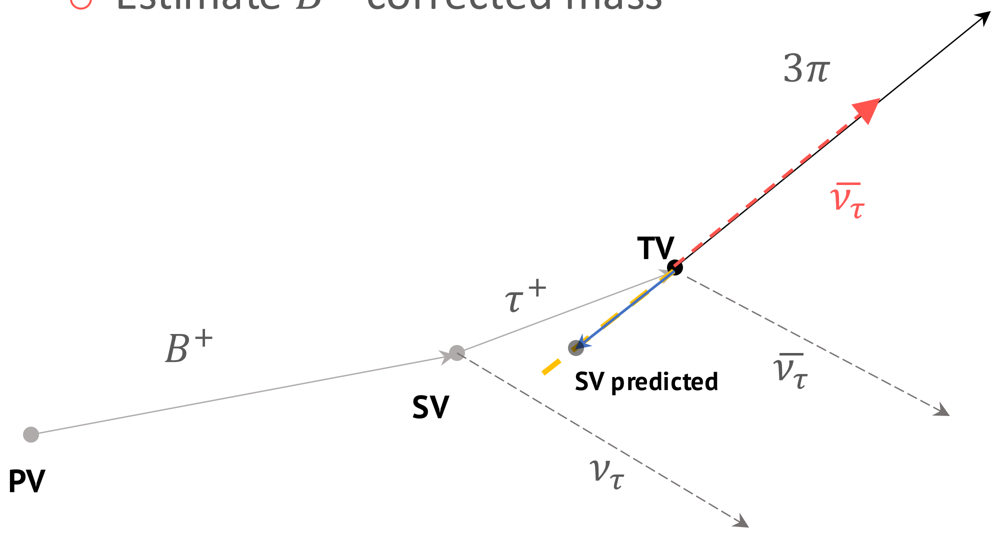

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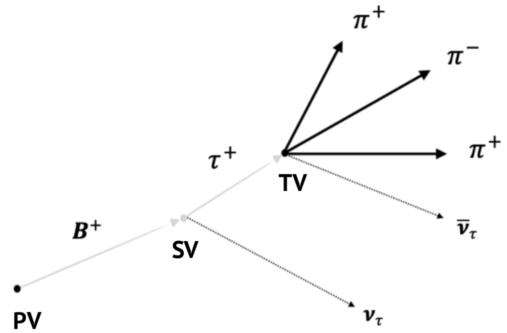

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2. Second Strategy:
   - Use Machine Learning (NN) to predict SV, using RapidSim dataset
     - Input variables: $\vec{p}_{\pi^+}$, $\vec{p}_{\pi^-}$, $\vec{p}_{\pi^+}$, PV, TV, $m_{3\pi}$
   - Estimate B^+ corrected mass, with $\vec{p}_\nu \parallel \vec{p}_{3\pi}$ to reconstruct $\vec{p}_\tau$

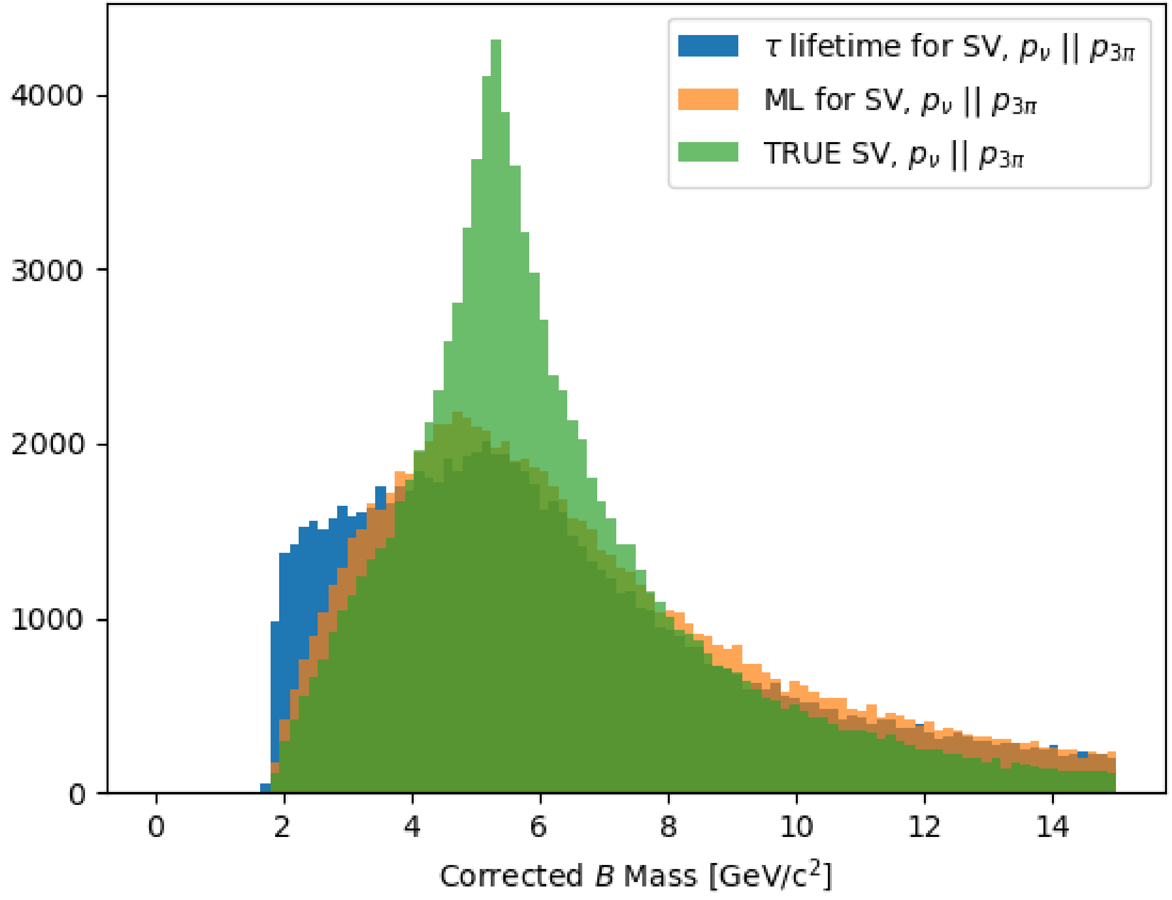
B corrected mass distribution

$$M_{corr}^{B} = \sqrt{m_\tau^2 + p_\perp^2} + p_\perp$$

Green: assuming $\vec{p}_\nu \parallel \vec{p}_{3\pi}$ and using true SV

Blue: assuming $\vec{p}_\nu \parallel \vec{p}_{3\pi}$ and using tau lifetime to calculate SV

Orange: assuming $\vec{p}_\nu \parallel \vec{p}_{3\pi}$ and using ML technique to obtain SV



Kinematic strategy to obtain SV

$\hat{f}$: tau flight direction

$$\hat{f} = \frac{\overrightarrow{TV} - \overrightarrow{SV}}{\left|\overrightarrow{TV} - \overrightarrow{SV}\right|}$$

Orthogonal basis

$$\hat{x} = \left(-\hat{f}_y, \hat{f}_x, 0\right)$$

$$\hat{y} = \hat{f} \times \hat{x}$$

$$\hat{z} = \hat{f}$$

In this orthogonal basis:

$$\vec{p}'_{3\pi}$$

$$\vec{p}'_{\nu} = \left(-p'_{3\pi\, x}, -p'_{3\pi\, y}, p'_L\right)$$

$$p'_T = \sqrt{p'^{\,2}_{3\pi\, x} + p'^{\,2}_{3\pi\, y}}$$

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We constrain the τ mass (M_τ) and use energy conservation:

$$M_\tau^2 = (E_{3\pi} + E_\nu)^2 - (\vec{p}'_{3\pi} + \vec{p}'_\nu)^2$$

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$$A = \frac{M_\tau^2 - M_{3\pi}^2}{2}$$

$$p'_L = \frac{p'_{3\pi\,z}(A - p'^2_T) \pm E_{3\pi}\sqrt{A^2 - p'^2_T M_\tau^2}}{E_{3\pi}^2 - p'^2_{3\pi\,z}}$$

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$$A^2 - p\!\!/'^{\,2}_T M_\tau^2 > 0$$

$$p\!\!/'_T < \frac{M_\tau^2 - M_{3\pi}^2}{2M_\tau}$$

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same condition for the first neutrino

$$p\!\!/'_T < \frac{M_B^2 - M_\tau^2}{2M_B}$$

Steps to obtain SV

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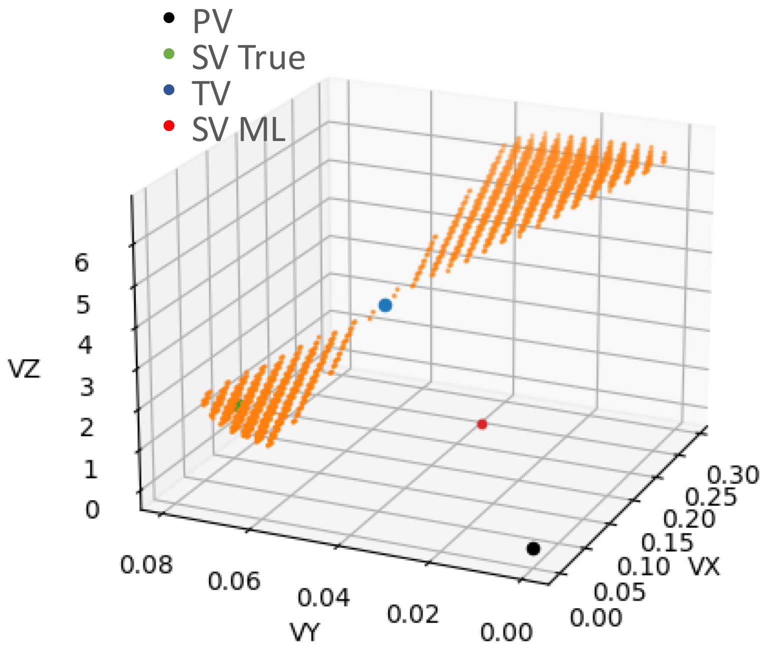
1. Create a grid around TV, select the points that pass the conditions



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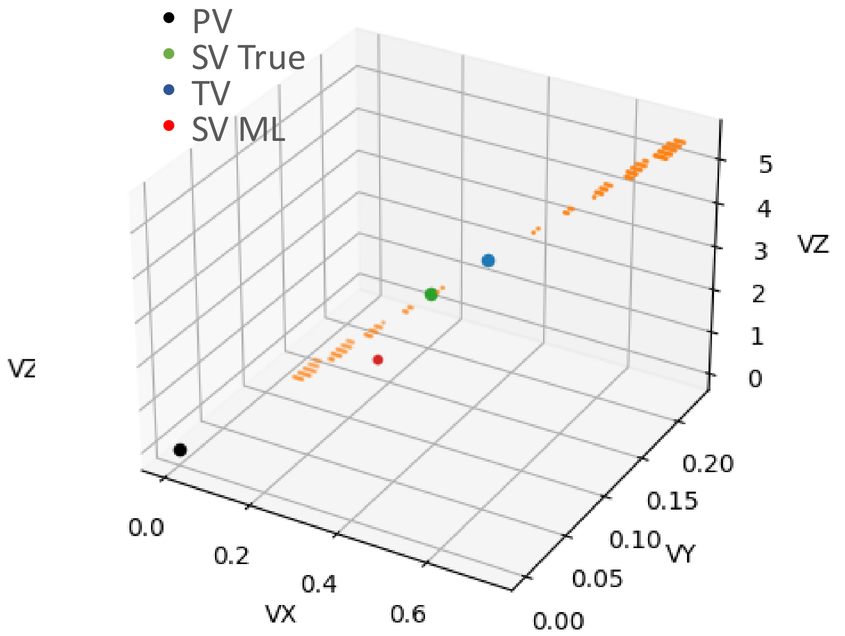

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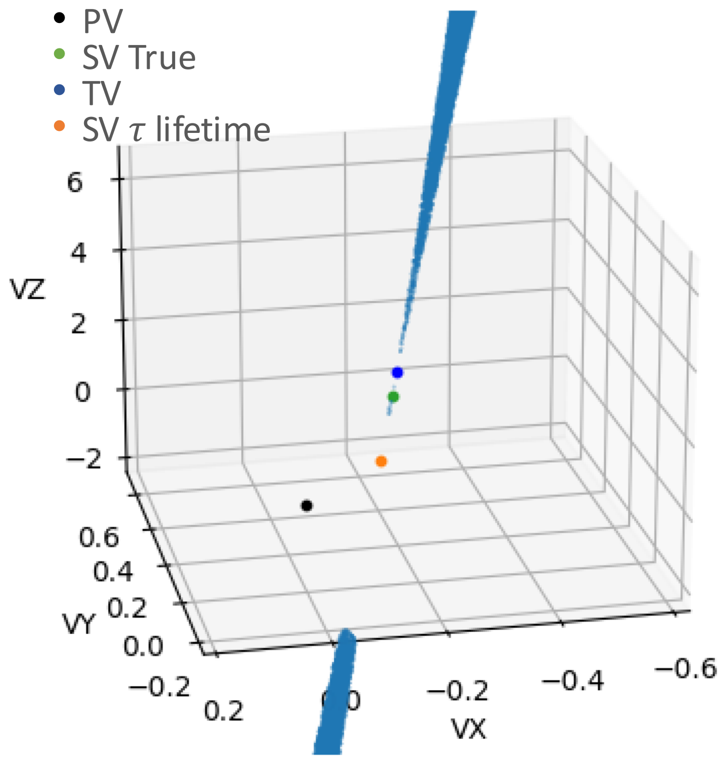
   - 50% of the events showed a well separated cluster between TV and PV.



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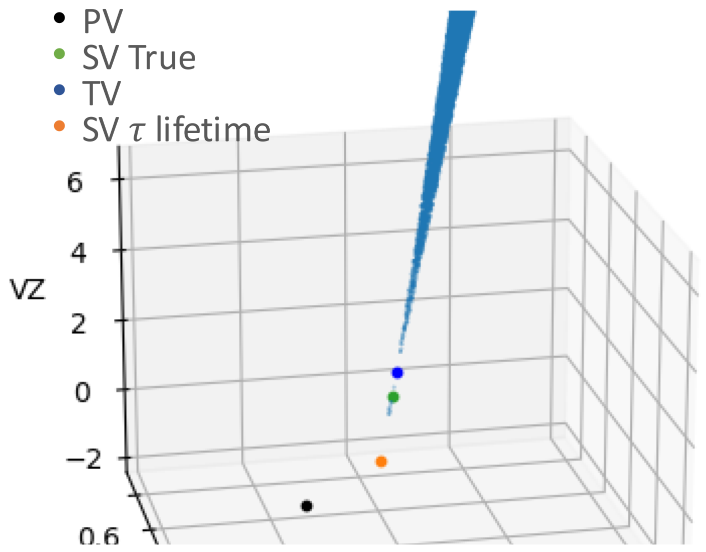

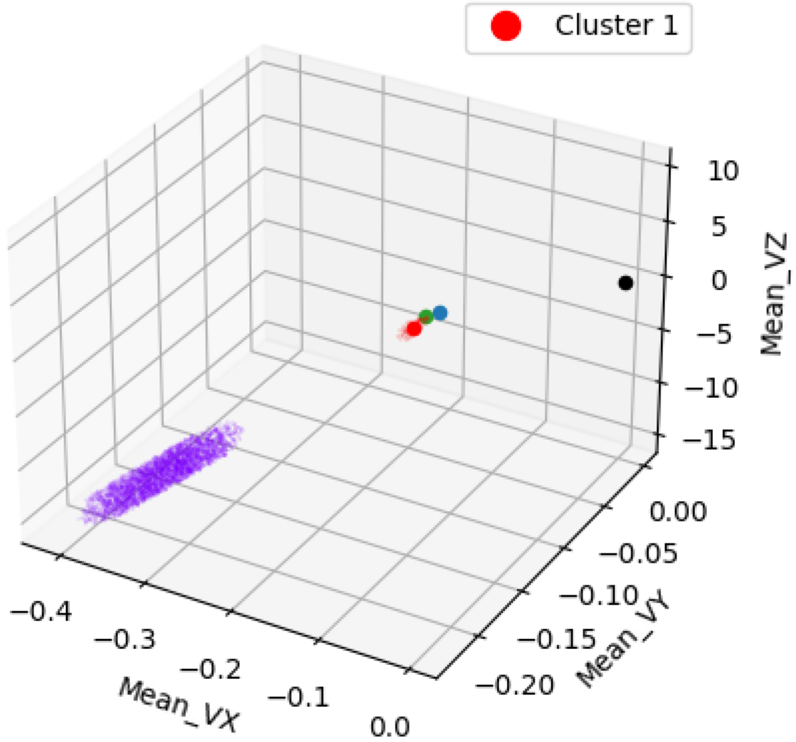

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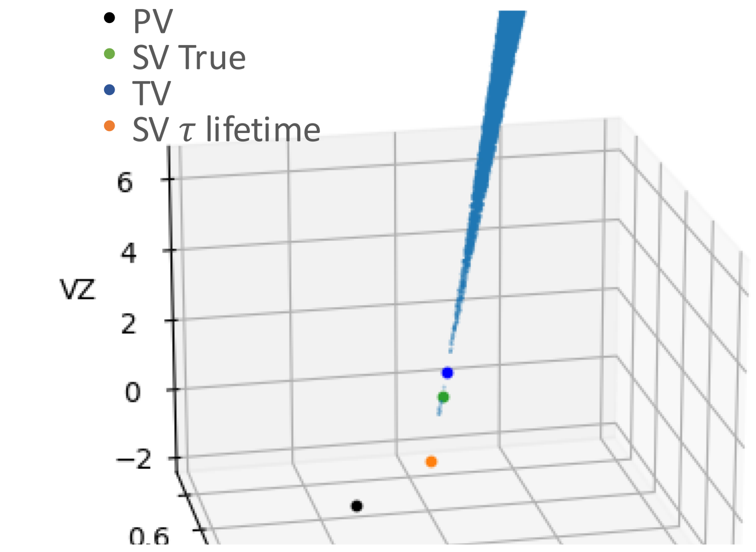
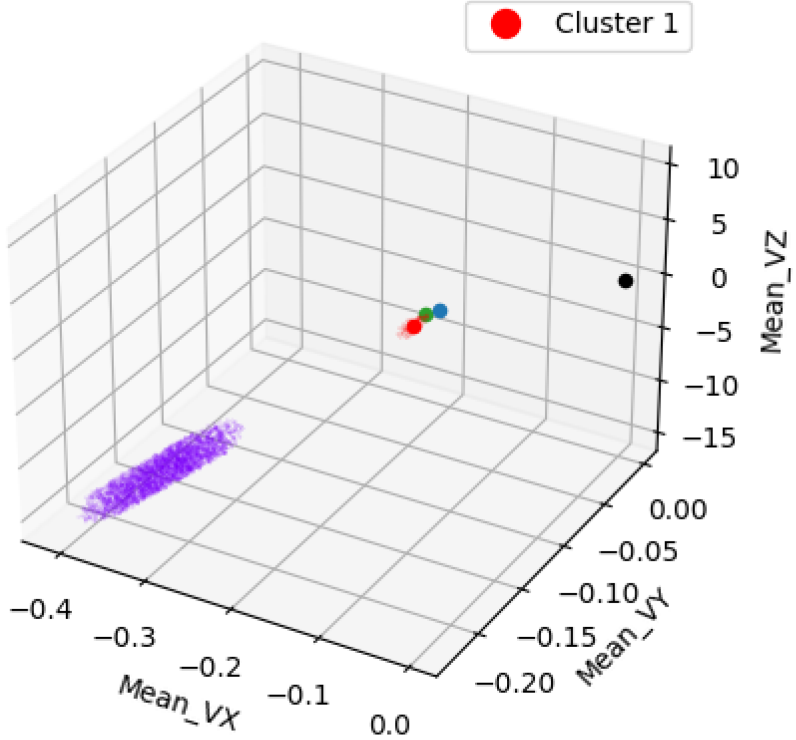
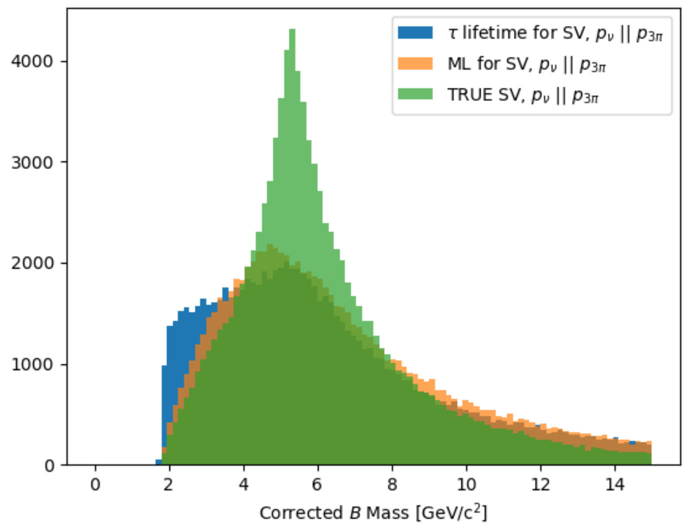
```
########################
Ev. 59
########################
Centroid cluster 0: [-2.909 -0.965 -8.558] mm
Centroid cluster 1: [1.662 -1.124 3.516] mm
SV True: [1.655 -1.146 3.517] mm
########################
```





B corrected mass distribution with predicted SV



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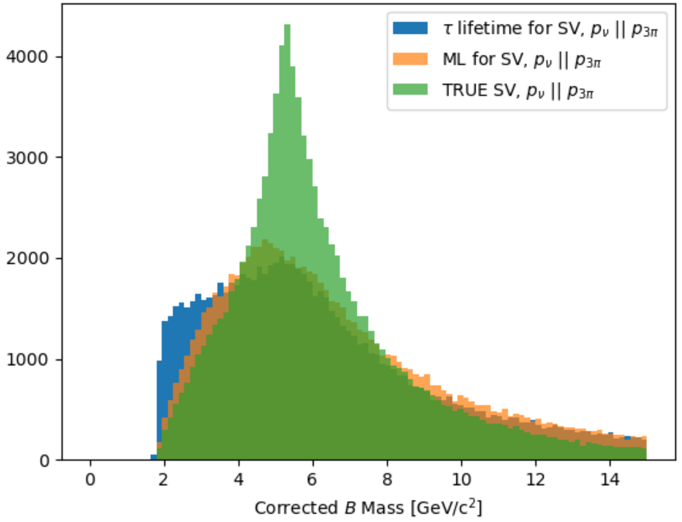

By using only
3π tracks, TV and PV

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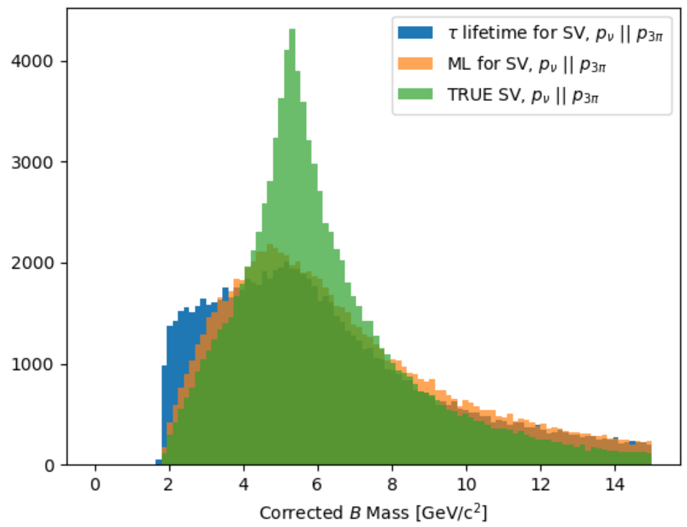

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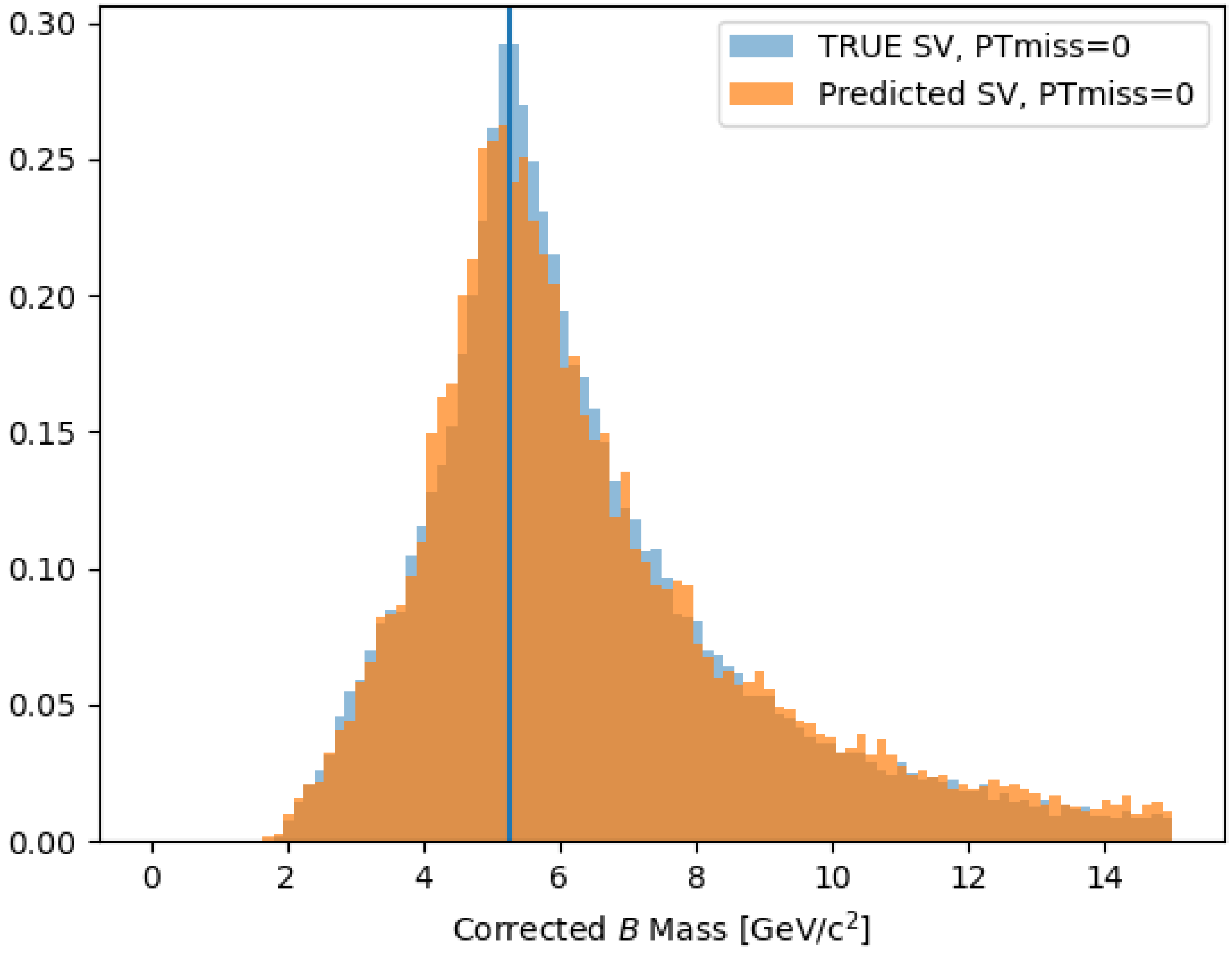

Background Study

- Minimum Bias MC 2018 (MU/MD) Sample
  - Simulation that is similar to real data
- 1.5k events passing offline selection
  - In 31% of events, the three pions decayed from the same particle

Process	% observed	
Combinatorial Background	30.5%	
$Mother \rightarrow (X \rightarrow \pi^+\pi^-)\,\pi$	38.2%	
$B^+ \rightarrow (X \rightarrow \pi^+\pi^-)\,\pi^+$	7.9%	
$D^+ \rightarrow (X \rightarrow \pi^+\pi^-)\,\pi^+$	6.4%	
$X \rightarrow \pi^+\,\pi^-\,\pi^+$	31.3%	**BR**
$D^+ \rightarrow \pi^+\pi^-\pi^+$	6.1%	3.27×10^{-3}
$B^+ \rightarrow \pi^+\pi^-\pi^+$	1.4%	1.52×10^{-5}
$D_s^+ \rightarrow \tau^+\nu$	0.5% → 8 ev	5.32 %
$\boldsymbol{B^+ \rightarrow \tau^+\nu}$	0.3% → 4 ev	1.09×10^{-4}
$B^0 \rightarrow \tau^+\,\nu\,X$	0.1% → 2 ev	1.05 %

Background Study

- Minimum Bias MC 2018 (MU/MD) Sample
  - Simulation that is similar to real data
- 1.5k events passing offline selection
  - In 31% of events, the three pions decayed from the same particle
  - 14 events involving a τ lepton

Process	% observed	BR
Combinatorial Background	30.5%	
$Mother \rightarrow (X \rightarrow \pi^+\pi^-)\,\pi$	38.2%	
$B^+ \rightarrow (X \rightarrow \pi^+\pi^-)\,\pi^+$	7.9%	
$D^+ \rightarrow (X \rightarrow \pi^+\pi^-)\,\pi^+$	6.4%	
$X \rightarrow \pi^+\,\pi^-\,\pi^+$	31.3%	BR
$D^+ \rightarrow \pi^+\pi^-\pi^+$	6.1%	3.27×10^{-3}
$B^+ \rightarrow \pi^+\pi^-\pi^+$	1.4%	1.52×10^{-5}
$D_s^+ \rightarrow \tau^+\nu$	0.5% → 8 ev	5.32 %
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Background Study

- Minimum Bias MC 2018 (MU/MD) Sample
  - Simulation that is similar to real data
- 1.5k events passing offline selection
  - In 31% of events, the three pions decayed from the same particle
  - 14 events involving a τ lepton
  - Only 4 events of signal

Process	% observed	BR
Combinatorial Background	30.5%	
$Mother \to (X \to \pi^+\pi^-)\,\pi$	38.2%	
$B^+ \to (X \to \pi^+\pi^-)\,\pi^+$	7.9%	
$D^+ \to (X \to \pi^+\pi^-)\,\pi^+$	6.4%	
$X \to \pi^+\,\pi^-\,\pi^+$	31.3%	
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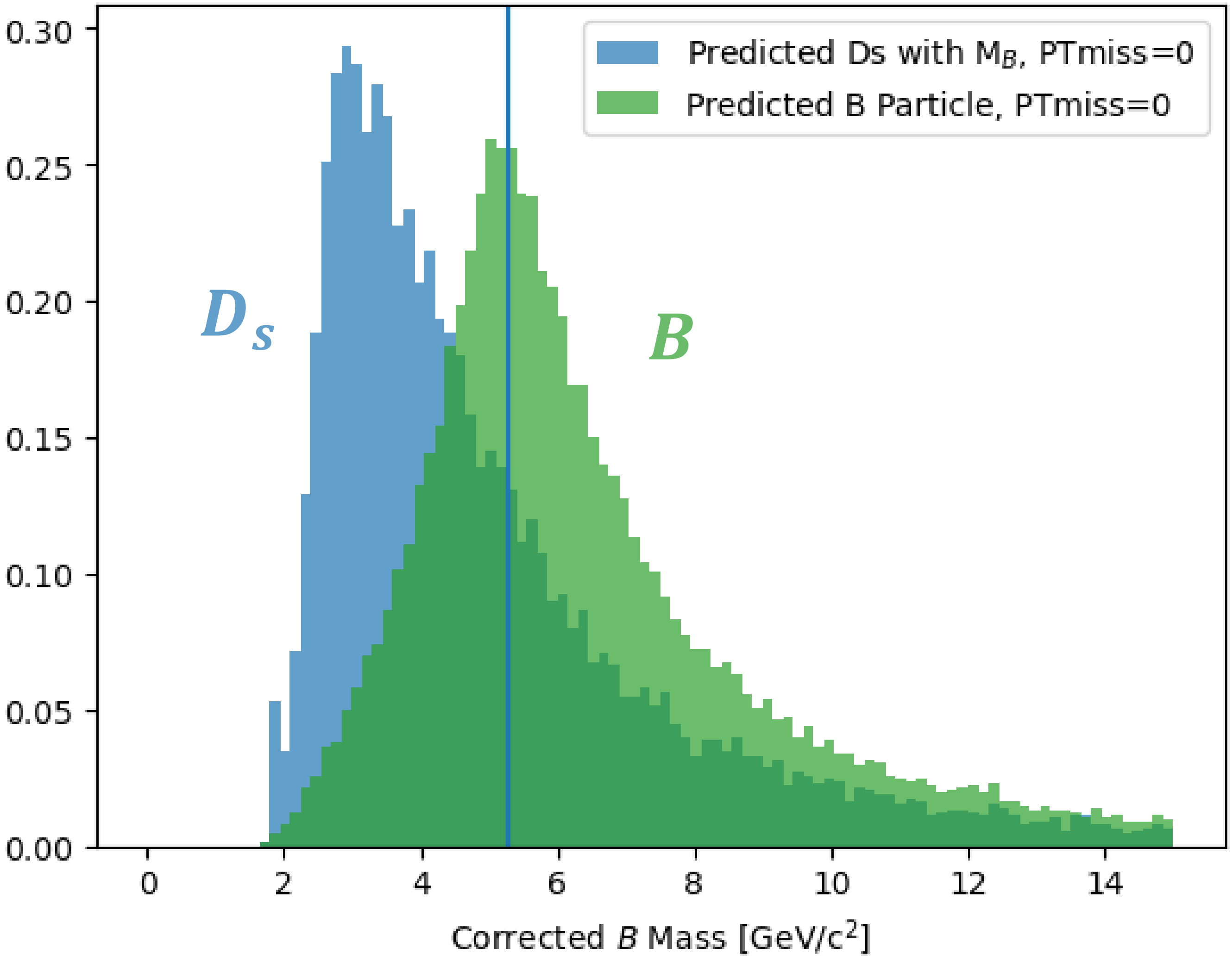
Signal →

Background Study: $D_s^+ \to \tau^+\nu$

- We generated 100k events of $D_s^+ \to \tau^+\nu$ using RapidSim.
- Applied the same strategy: only 20k events passed the selection.



Conclusions

- We propose to measure the branching fraction of the $B^{\pm} \rightarrow \tau^{+}\nu$ process.
- We found a strategy that solves the kinematics in our signal.
- There are a lot of processes that will contribute as background
  - Already showed that the strategy allows to separate $D_s^+ \; -> \tau^+ \, \nu$ and the signal.

Next Steps:

- Test the strategy on Full LHCb MC simulation.
- Design a strategy to separate the signal from the background.

Conclusions

- We propose to measure the branching fraction of the $B^{\pm} \rightarrow \tau^{+}\nu$ process.
- We found a strategy that solves the kinematics in our signal.
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  - Already showed that the strategy allows to separate $D_s^+ \; -> \tau^+ \, \nu$ and the signal.

Next Steps:

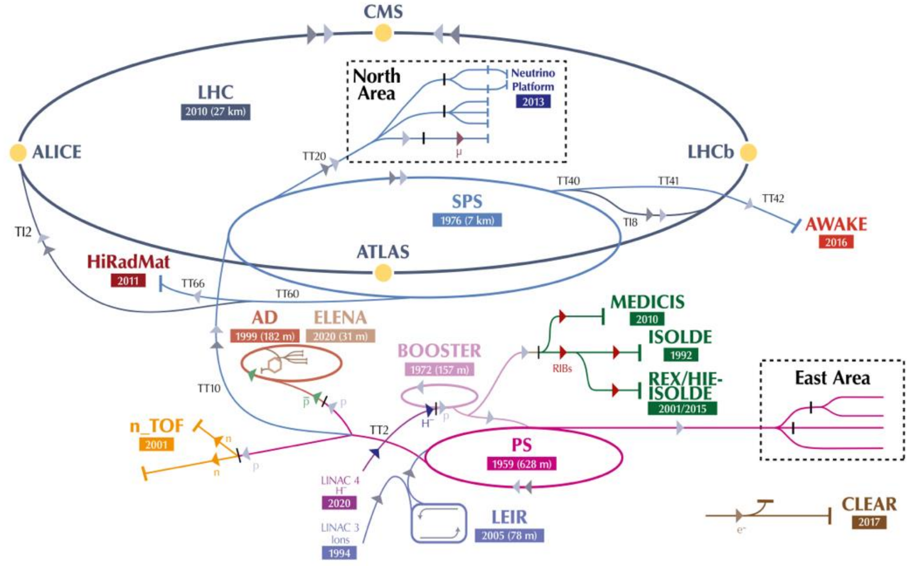
- Test the strategy on Full LHCb MC simulation.
- Design a strategy to separate the signal from the background.

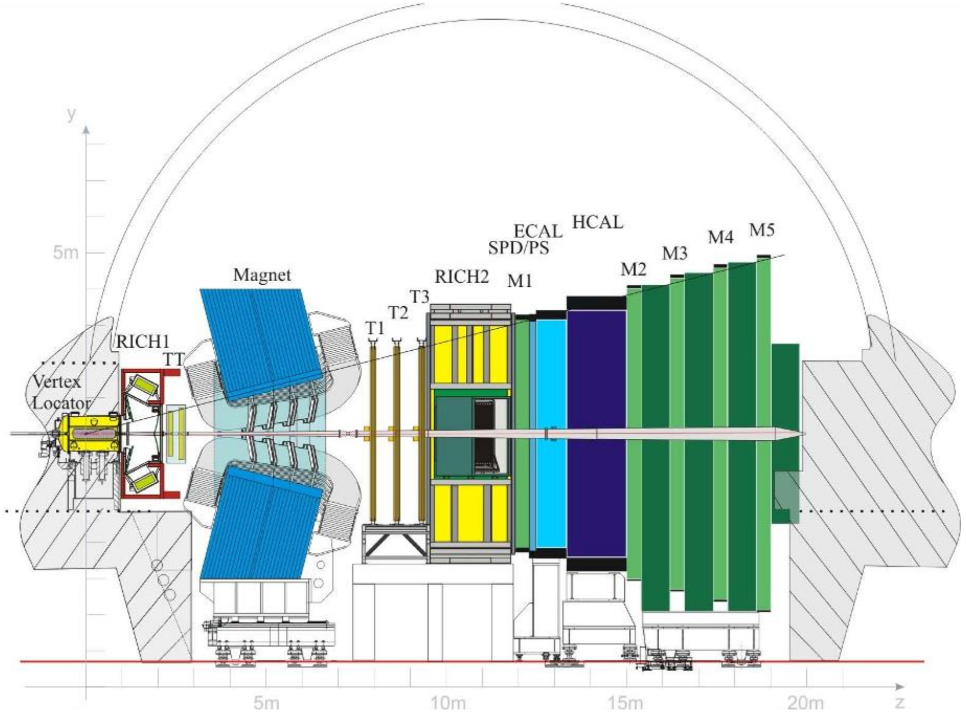
Thank you for your attention!

Backup slides

LHCb Experiment

- Located at the Large Hadron Collider (LHC) at CERN.
- Proton-proton collisions at 13 TeV.
- Single-arm forward spectrometer covering $2 < \eta < 5$.
- Tracking: VELO, a high-precision tracking system (TT, T1, T2, T3);
- Particle Identification: RICH; ECAL, HCAL, Muon stations.





Efficiency of the Stripping line

- The stripping line has an efficiency of $(0.84 \pm 0.42)\%$
- Taking into account the LHCb B^+ production cross section and the decay branching fraction:

Total amount of B^+ in Run 2: $(1.17 \pm 0.09) \times 10^{12}$

Expected amount of signal in Run2: $(1.27 \pm 0.30) \times 10^{8}$

Expected amount of signal after stripping: $(1.1 \pm 0.6) \times 10^{6}$

Proof: First strategy – neutrino aligned with 3pi

1.2 Considering the case of $\not{p}_T = 0$

$$M_\tau^2 = (E_{\pi\pi\pi} + E_{\bar{\nu}})^2 - |\vec{p}_{\pi\pi\pi} + \vec{p}_{\bar{\nu}}|^2 = \\ E_{\pi\pi\pi}^2 + E_{\bar{\nu}}^2 + 2E_{\pi\pi\pi}.E_{\bar{\nu}} - p_{\pi\pi\pi}^2 - p_{\bar{\nu}}^2 - 2\vec{p}_{\pi\pi\pi}.\vec{p}_{\bar{\nu}} = \\ M_{\pi\pi\pi}^2 + \cancel{p_{\pi\pi\pi}^2} + \cancel{p_{\bar{\nu}}^2} + 2E_{\pi\pi\pi}.E_{\bar{\nu}} - \cancel{p_{\pi\pi\pi}^2} - \cancel{p_{\bar{\nu}}^2} - 2\vec{p}_{\pi\pi\pi}.\vec{p}_{\bar{\nu}} \tag{11}$$

$$M_\tau^2 = M_{\pi\pi\pi}^2 + 2.(E_{\pi\pi\pi}.E_\nu - \vec{p}_{\pi\pi\pi}.\vec{p}_{\bar{\nu}}) \tag{12}$$

where

$$E_{\pi\pi\pi} = \sqrt{M_{\pi\pi\pi}^2 + |\vec{p}\,'_{\pi\pi\pi}|^2} \tag{13}$$

and

$$E_{\bar{\nu}} = p_{\bar{\nu}} \tag{14}$$

Taking into account the 3-pion momenta and the neutrino momentum are parallel

$$\vec{p}_{\pi\pi\pi}.\vec{p}_{\bar{\nu}} = p_{\pi\pi\pi}.p_{\bar{\nu}} \tag{15}$$

We obtain

$$M_\tau^2 = M_{\pi\pi\pi}^2 + 2.(E_{\pi\pi\pi}.p_{\bar{\nu}} - p_{\pi\pi\pi}.p_{\bar{\nu}}) \tag{16}$$

As a result

$$p_{\bar{\nu}} = \frac{M_\tau^2 - M_{\pi\pi\pi}^2}{2.(E_{\pi\pi\pi} - p_{\pi\pi\pi})} \tag{17}$$

Machine Learning Model for SV Reconstruction

- **Algorithm and Model Architecture**:
  - Neural network for regression
  - Architecture:
    - Input layer: Xtrain_scaled*X*train_scaled features
    - Hidden layers: 2 layers with 64 neurons each, ReLU activation
    - Output layer: 3 neurons, linear activation (for 3 coordinates)
- **Data Processing**:
  - Input features scaled using StandardScaler
- **Training Details**:
  - Optimizer: Adam
  - Loss Function: Mean Squared Error (MSE)
  - Validation Split: 20%
  - Epochs: 100
  - Batch Size: 32
- **Performance Evaluation**:
  - Model evaluated on test data
- **Predictions**:
  - Model predicts three coordinates for the secondary vertex

Proof: conditions from strategy

$\hat{f}$: flight direction

$$\hat{f} = \frac{\vec{TV} - \vec{SV}}{|\vec{TV} - \vec{SV}|}$$

Orthogonal basis

$$\hat{x} = (-\hat{f}_y, \hat{f}_x, 0)$$
$$\hat{y} = \hat{f} \times \hat{x}$$
$$\hat{z} = \hat{f}$$

In this orthogonal basis:

$$\vec{p}'_{3\pi}$$
$$\vec{p}'_\nu = (-p'_{3\pi\,x}, -p'_{3\pi\,y}, p'_L)$$
$$p'_T = \sqrt{p'^2_{3\pi\,x} + p'^2_{3\pi\,y}}$$

We impose the τ mass:

$$\begin{aligned} M_\tau^2 &= (E_{3\pi} + E_\nu)^2 - (\vec{p_{3\pi}} + \vec{p_\nu})^2 \\ M_\tau^2 &= E_{3\pi}^2 + E_\nu^2 + 2E_{3\pi}E_\nu - p'^2_{3\pi} - p_\nu^2 - 2\vec{p}'_{3\pi} \cdot \vec{p}_\nu \\ M_\tau^2 &= M_{3\pi}^2 + p'^2_{3\pi} + M_\nu^2 + p_\nu^2 + 2E_{3\pi}E_\nu - p'^2_{3\pi} - p_\nu^2 - 2\vec{p}'_{3\pi} \cdot \vec{p}_\nu \\ M_\tau^2 &= M_{3\pi}^2 + 2E_{3\pi}E_\nu - 2\vec{p}'_{3\pi} \cdot \vec{p}_\nu \\ M_\tau^2 &= M_{3\pi}^2 + 2(E_{3\pi}E_\nu - \vec{p_{3\pi}} \cdot \vec{p_\nu}) \end{aligned} \tag{1}$$

using:

$$\begin{aligned} \vec{p_{3\pi}}' &= \left(p_{3\pi x}\hat{x}, p_{3\pi y}\hat{y}, p_{3\pi z}\hat{f}\right) \\ \vec{p_\nu} &= (-p_{3\pi x}\hat{x}, -p_{3\pi y}\hat{y}, p_L) \end{aligned} \tag{2}$$

we can rewrite this as:

$$\begin{aligned} \vec{p}'_{3i} \cdot p'_y &= -p_{3+2}^2 - p_{3\pi}^{2\,2} + p'_{3\pi 2}p_2 = - p_t^2 + p'_{3in} = p_2 \\ \frac{M_\tau^2 - M_{3\pi}^2}{2} &= E_{3\pi}E_\nu + P_T^2 - P'_{3\pi z}P_L \end{aligned} \tag{3}$$

Assuming $M_\nu = 0$ $(E_\nu = p_\nu)$, we define A as:

$$A = \frac{M_\tau^2 - M_{3\pi}^2}{2} \tag{4}$$

with this we have:

$$A = E_{3\pi}\sqrt{p_T^2 + p_L^2} + p_T^2 + p_{3\pi z}p_L \tag{5}$$

Definying now B, C, D and x as:

$$\begin{aligned} B &= p_T \\ C &= p_{3\pi z} \\ D &= E_{3\pi} \\ x &= p_L \end{aligned} \tag{6}$$

Proof: conditions from strategy

We impose the τ mass:

$$
\begin{aligned}
M_\tau^2 &= (E_{3\pi} + E_\nu)^2 - (\vec{p_{3\pi}} + \vec{p_\nu})^2 \\
M_\tau^2 &= E_{3\pi}^2 + E_\nu^2 + 2E_{3\pi}E_\nu - p_{3\pi}^{'2} - p_\nu^2 - 2\vec{p}_{3\pi}^{\,'} \cdot \vec{p}_\nu \\
M_\tau^2 &= M_{3\pi}^2 + p_{3\pi}^{'2} + M_\nu^2 + p_\nu^2 + 2E_{3\pi}E_\nu - p_{3\pi}^{'2} - p_\nu^2 - 2\vec{p}_{3\pi}^{\,'} \cdot \vec{p}_\nu \\
M_\tau^2 &= M_{3\pi}^2 + 2E_{3\pi}E_\nu - 2\vec{p}_{3\pi}^{\,'} \cdot \vec{p}_\nu \\
M_\tau^2 &= M_{3\pi}^2 + 2\left(E_{3\pi}E_v - \vec{p_{3\pi}} \cdot \vec{p_\nu}\right)
\end{aligned} \tag{1}
$$

using:

$$
\begin{aligned}
\vec{p_{3\pi}}^{\,'} &= \left(p_{3\vec{\pi} x}\hat{x}, p_{3\vec{\pi} y}\hat{y}, p_{3\vec{\pi} z}\hat{f}\right) \\
\vec{p_\nu} &= (-p_{3\vec{\pi} x}\hat{x}, -p_{3\vec{\pi} y}\hat{y}, p_L)
\end{aligned} \tag{2}
$$

we can rewrite this as:

$$
\begin{aligned}
\vec{p}_{3i}^{\,'} \cdot p_y' &= -p_{3+2}^2 - {p_{3\pi}^2}^2 + p_{3\pi 2}'p_2 = - \not{p}_i^2 + p_{3in}' = p_2 \\
\frac{M_\tau^2 - M_{3\pi}^2}{2} &= E_{3\pi}E_\nu + P_T^2 - P_{3\pi z}^{'}P_L
\end{aligned} \tag{3}
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Assuming $M_\nu = 0$ $(E_\nu = p_\nu)$, we define A as:

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A = \frac{M_\tau^2 - M_{3\pi}^2}{2} \tag{4}
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with this we have:

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A = E_{3\pi}\sqrt{p_T^2 + p_L^2} + p_T^2 + p_{3\pi z}p_L \tag{5}
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Definying now B, C, D and x as:

$$
\begin{aligned}
B &= p_T \\
C &= p_{3\pi z} \\
D &= E_{3\pi} \\
x &= p_L
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
A &= D\sqrt{B^2 + x^2} + B^2 - Cx \\
\left(A - B^2 + Cx\right)^2 &= \left(D\sqrt{B^2 + x^2}\right)^2 \\
A^2 + B^4 + C^4x^2 - 2AB^2 + 2ACx - 2B^2Cx &= D^2\left(B^2 + x^2\right)
\end{aligned} \tag{7}
$$

After some manipulation we arrive at an expression for x, i.e. p_L:

$$
p_L = \frac{p_{3\pi z}\left(A - p_T^2\right) \pm E_{3\pi}\sqrt{A^2 - p_T^2 M_\tau^2}}{E_{3\pi}^2 - p_{3\pi z}^2} \tag{8}
$$

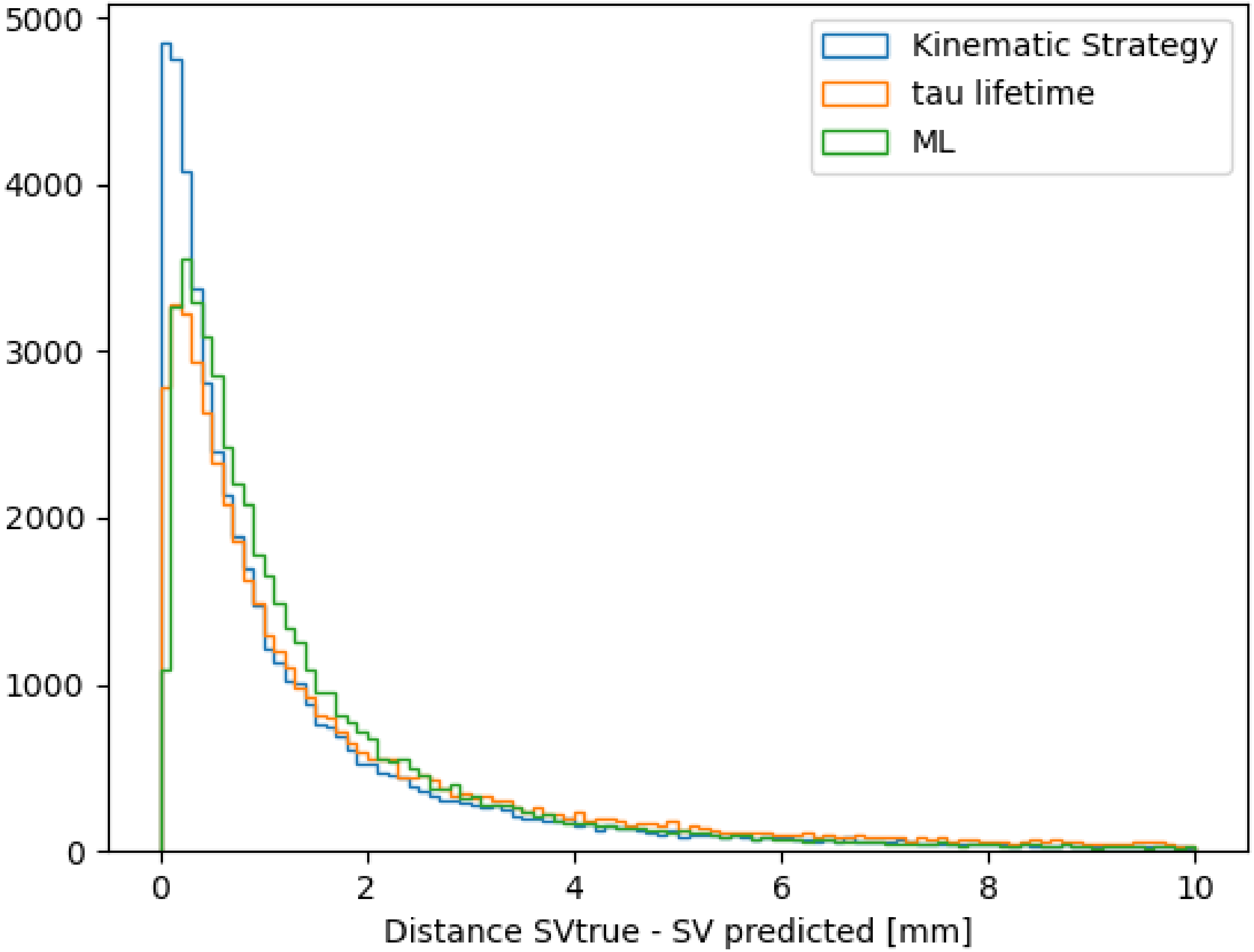
Allowing us to arrive at the conditions:

$$
p_T^2 < \frac{M_\tau^2 - M_{3\pi}^2}{2M_\tau} \tag{9}
$$

and for the other vertex:

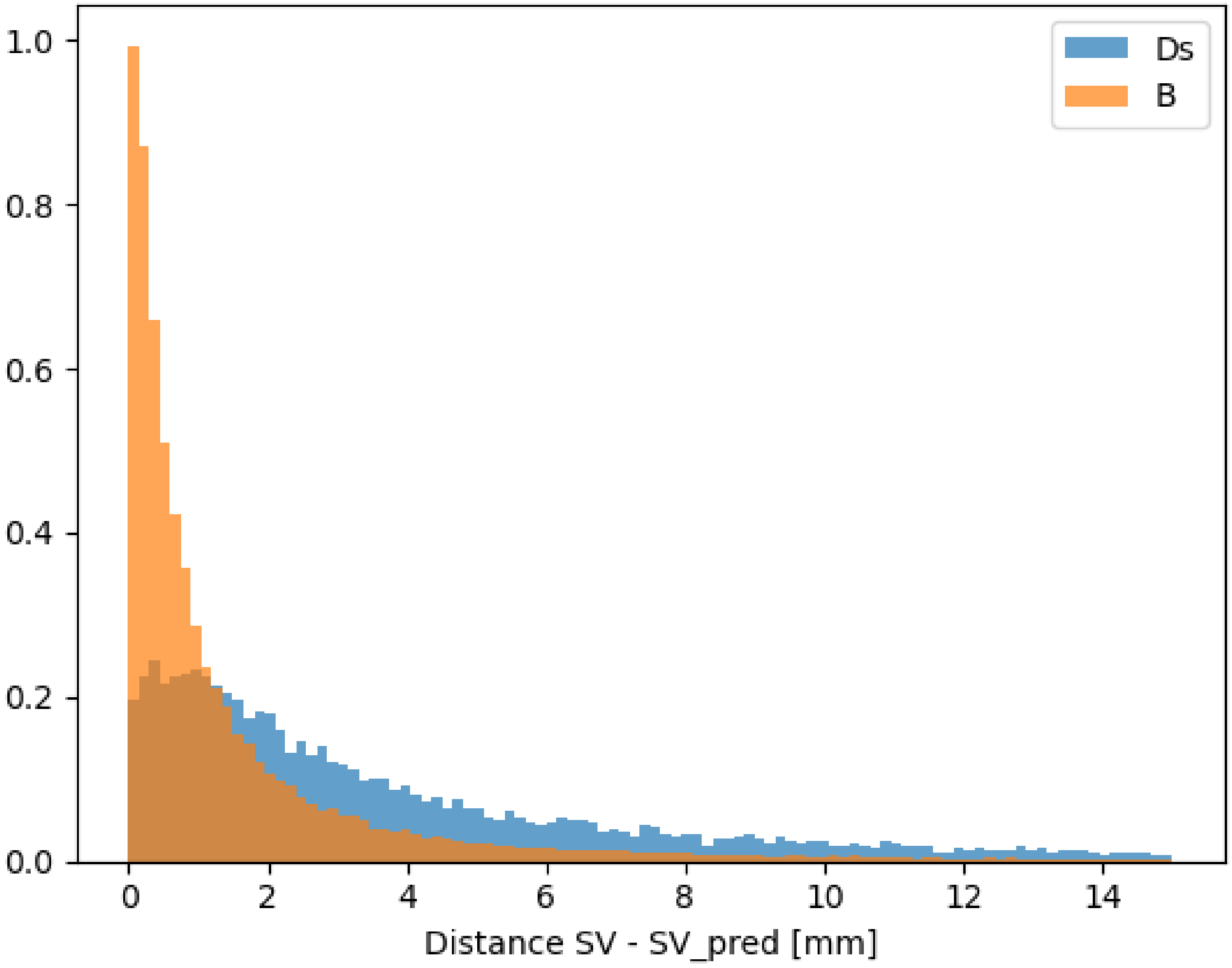
$$
p_T^2 < \frac{M_B^2 - M_\tau^2}{2M_B} \tag{10}
$$

Distance SV true – SV predicted

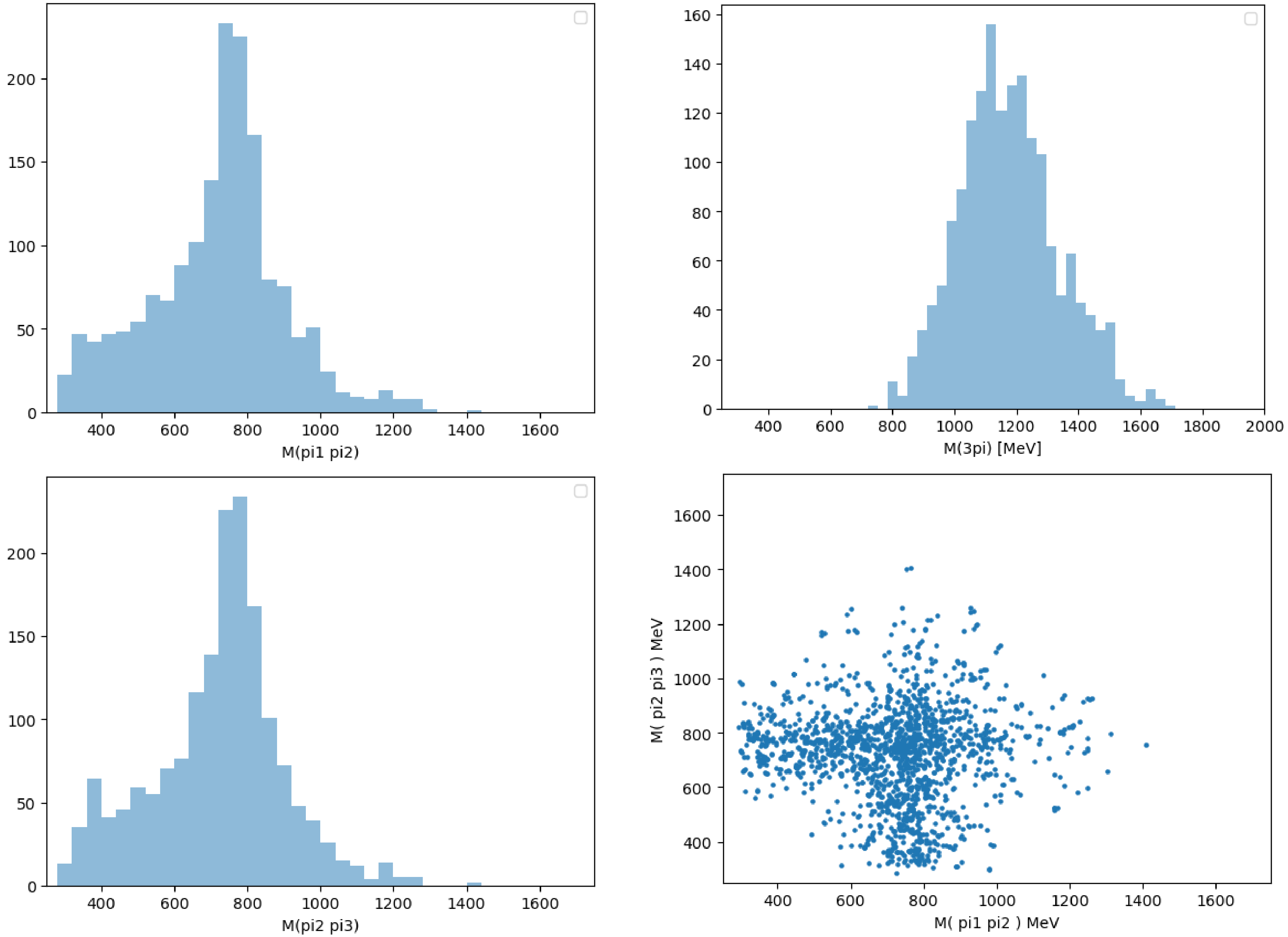


Background Study: $D_s^+ \to \tau^+ \nu$

Difference between SV TRUE and SV predicted by the strategy



Full Simulation



Preliminary Full Simulation