Annual Meeting of the Swiss Physical Society 2024, ETH Zürich

Radiative corrections and Monte Carlo tools for low-energy e^+e^- experiments

McMule's contributions to the Muon $g-2$ puzzle

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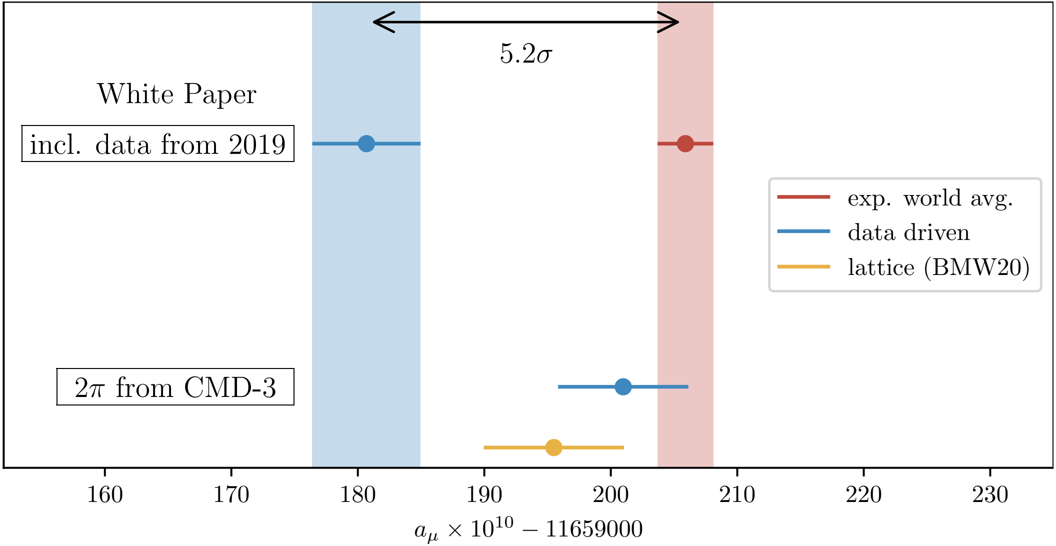
11 September 2024

the present situation in Muon $g-2$



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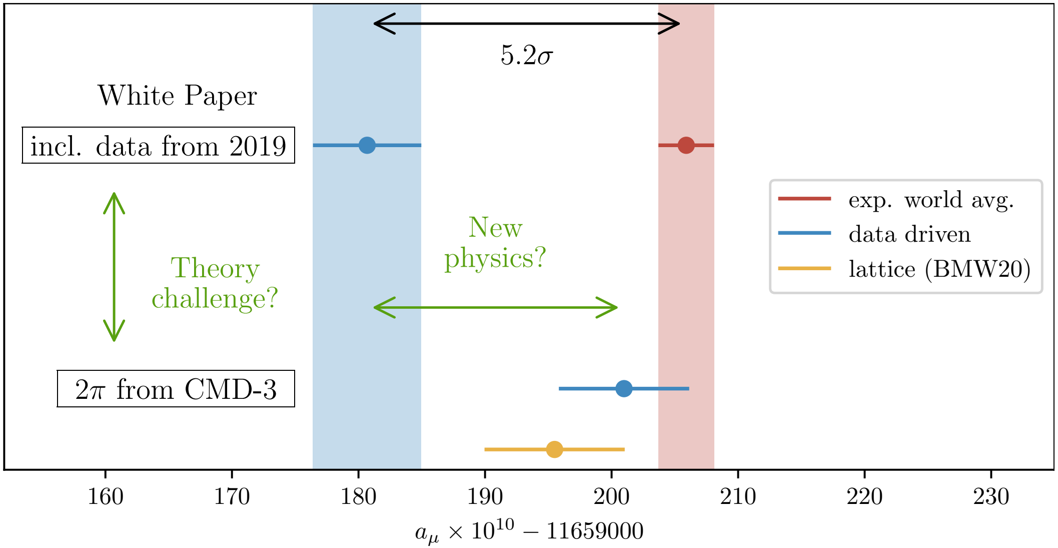
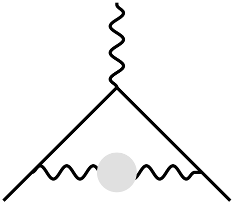


[white paper 20] [BMW 20] [CMD-3 23]

what is the problem with the theory prediction?



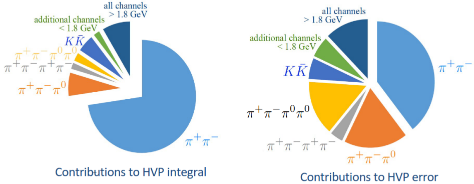


figure from [A. Denig]

a) data driven (dispersion relation) $\rightarrow$ does rely on other experiments

$$a_\mu^{\text{HVP}} \sim \int \mathrm{d}s\, K(s)\, \sigma_{e^+e^-\rightarrow \text{hadrons}}(s)$$

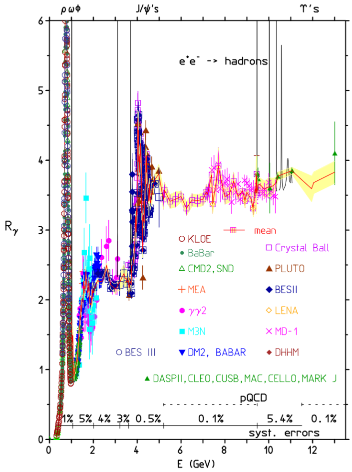
$K(s)$ known kernel function

b) lattice QCD

$\Longrightarrow$ disagreement between the two methods! the problem is bigger than $g-2$!

$e^+e^- \to$ hadrons experiments at a few GeV

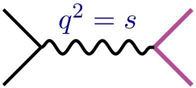
figure from [1511.04473]



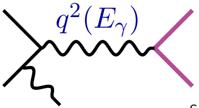
$$R(s) = \frac{\sigma_{e^+e^-\to \text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

experiments need to cover huge range of s

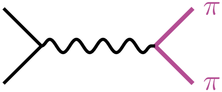
a) scan experiments: vary s manually (eg. CMD)



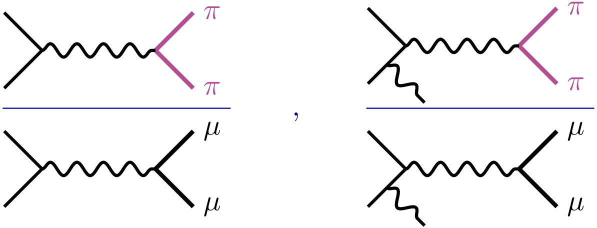
b) radiative return experiments: vary s via photon emission (eg. KLOE)



what we want

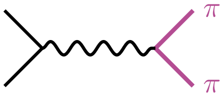


what we actually want



measuring hadrons: how theorists can help

what we want



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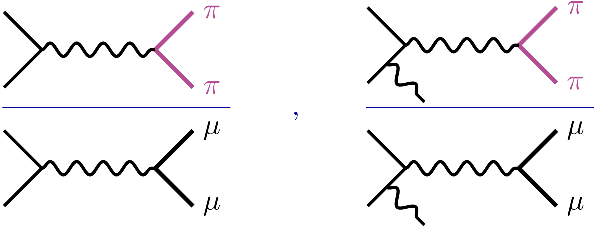

what we need **Monte Carlo codes** !

important processes : $e^+e^- \to e^+e^-(\gamma)$ $e^+e^- \to \mu^+\mu^-(\gamma)$ $e^+e^- \to \pi^+\pi^-(\gamma)$

facing theory challenge: RadioMonteCarLow 2

ongoing community effort for better Monte Carlo codes at $s \sim (\text{few GeV})^2$

- inspired by similar effort in 2009 [0912.0749]
- improve SM precision at low energy & provide input for $g-2$ puzzle

  $g-2$ disclaimer: see [official talk at 7th g-2 Plenary Workshop]
- study the importance of different contributions in realistic-ish environments
- start of comparisons of codes in Phase I
- today McMule's contributions & lessons learned

radiomontecarlow2.gitlab.io

Members during Phase I:
Riccardo Aliberti, Paolo Beltrame, Ettore Budassi, Carlo M. Carloni Calame, Gilberto Colangelo, Lorenzo Cotrozzi, Achim Denig, Anna Driutti, Tim Engel, Lois Flower, Andrea Gurgone, Martin Hoferichter, Fedor Ignatov, Sophie Kollatzsch, Bastian Kubis, Andrzej Kupsc, Fabian Lange, Alberto Lusiani, Guido Montagna, Stefan E. Müller, Oreste Nicrosini, Jérémy Paltrinieri, Pau Petit Rosàs, Fulvio Piccinini, Alan Price, Lorenzo Punzi, Marco Rocco, Kay Schönwald, Olga Shekhovtsova, Andrzej Siódmok, Adrian Signer, Giovanni Stagnitto, Peter Stoffer, Thomas Teubner, William J. Torres Bobadilla, Francesco P. Ucci, Yannick Ulrich, Graziano Venanzoni

our code: McMule

Monte **C**arlo for **Mu**ons and other **le**ptons

- integrator for fixed-order QED up to NNLO for $ee \to XX$ and others such as $\ell p \to \ell p$
- electroweak effects at NLO for selected processes
- builds upon methods developed for higher-order QCD calculations $\to$ adapted to QED with massive fermions

McMule

highlight for $e^+e^- \to$ hadrons

- $e^+e^- \to \ell^+\ell^-$ with $\ell \in \{e, \mu\}$ at NNLO
- $e^+e^- \to \ell^+\ell^-\gamma$ with $\ell \in \{e, \mu\}$ at NLO

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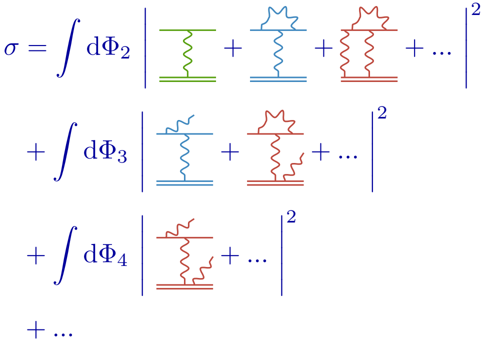


challenges to overcome

- divergent phase space integration

⇒ FKS$^\ell$ subtraction

- numerical instabilities in phase space integration

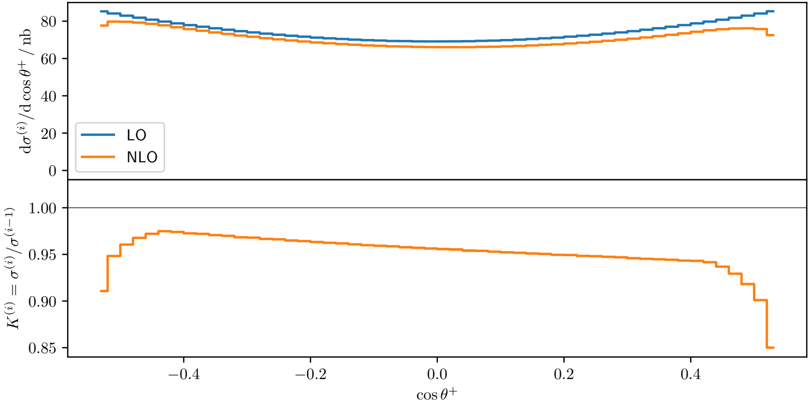
⇒ next-to-soft stabilisation

- virtual amplitudes with $m \neq 0$

⇒ OpenLoops (one-loop) massification (two-loop)

fixed order vs. approx. higher orders I

$e^+e^- \to \mu^+\mu^-$ @ CMD-like $(\sqrt{s} = 700\text{MeV})$ (more details `radiomontecarlow2.gitlab.io/plots/CMD/mm`)



up to 10 % NLO effects due to large logarithms!

$\rightarrow$ resummation via parton shower (PS), or NNLO?

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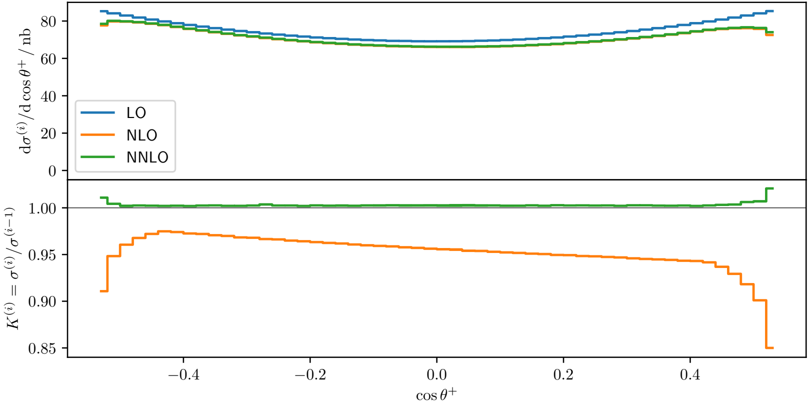

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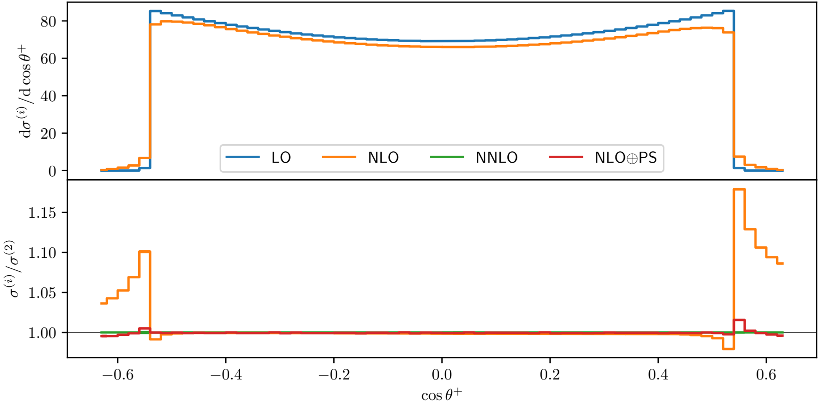

$\Longrightarrow$ multiphoton effects ([NLO ⊕ PS from BabaYaga@NLO]) needed but seems to be dominated by one extra photon

fixed order vs. approx. higher orders I

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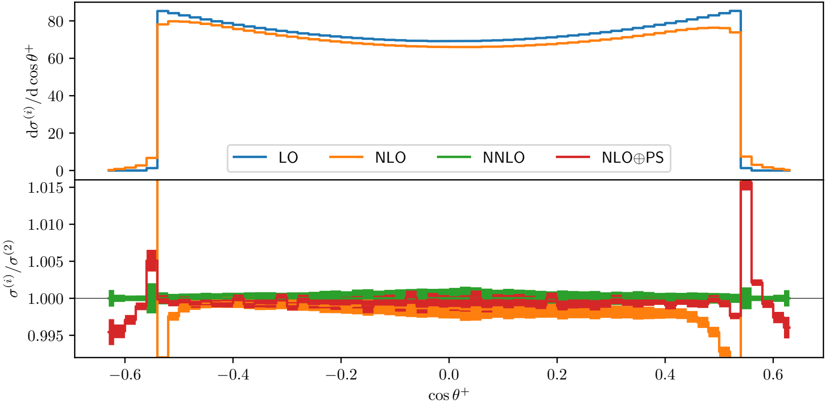
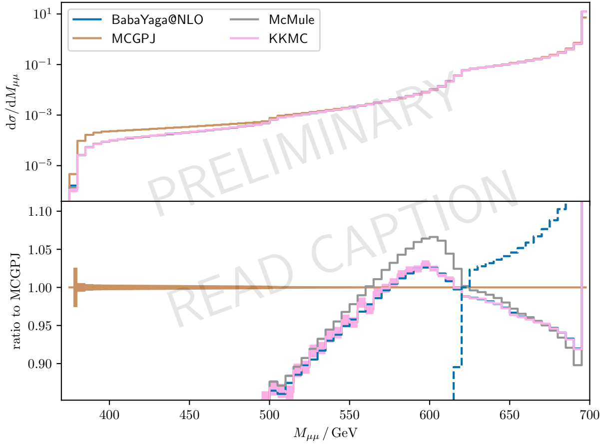


$\Longrightarrow$ multiphoton effects ([NLO $\oplus$ PS from BabaYaga@NLO]) needed but seems to be dominated by one extra photon

fixed order vs. approx. higher orders II

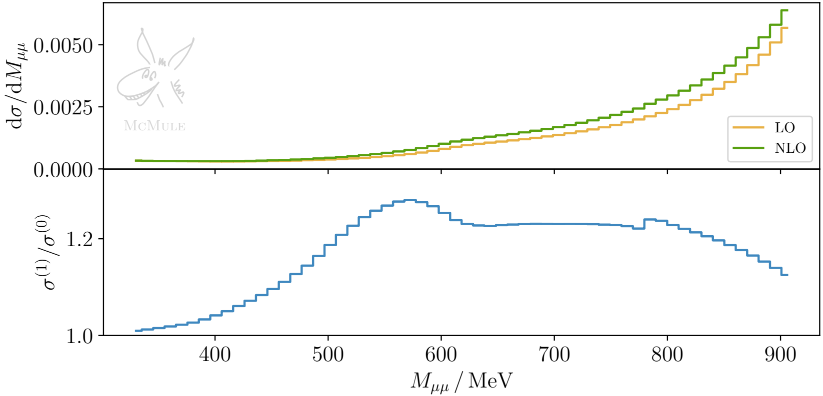
$e^+e^- \to \mu^+\mu^-$ @ CMD-like $(\sqrt{s} = 700\text{MeV})$ (more details `radiomontecarlow2.gitlab.io/plots/CMD/mm`)



- fixed order NLO (blue dashed line): BabaYaga@NLO = McMule
- McMule's NNLO ≈ BabaYaga@NLO's and KKMC's higher order effects

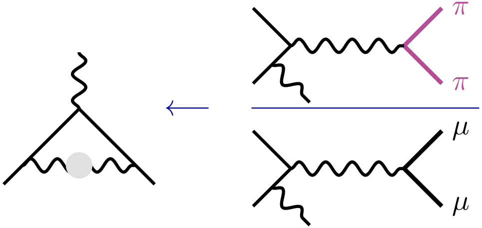
⟹ higher order effects dominated by one extra emission

$e^+e^- \to \mu^+\mu^-\gamma$ @ KLOE-like $(\sqrt{s} = 1\text{GeV})$ (more details `radiomontecarlow2.gitlab.io/plots/KLOE-LA/mm/`)



NLO corrections large $\Longrightarrow$ we need NNLO for $2 \to 3$!

take-home messages



$\longleftarrow$ Monte Carlo codes

$g-2$ issues are involved

- exciting $e^+e^- \to$ hadrons developments ahead!
- comparison of Monte Carlo codes & effects included ongoing

McMule goals for 2025+

- NNLO for $ee \to \mu\mu\gamma$
- approximate higher orders
- NNLO initial state radiation for $ee \to \pi\pi\gamma$

McMule team

f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)
not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (Zurich & PSI), S.Gündogdu (Zurich & PSI)

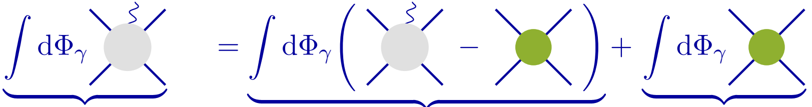
McMule

mule-tools.gitlab.io

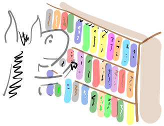
McMule in 3min

Monte **C**arlo for **MU**ons and other **LE**ptons (`mule-tools.gitlab.io`)

- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS$^\ell$ subtraction with massive fermions



- challenge virtual amplitudes with $m \neq 0 \implies$ massification
- challenge numerical instabilities $\implies$ next-to-soft stabilisation



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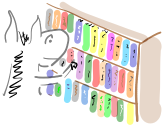
$$\mathcal{A}(m) = \Big(\prod_j \sqrt{Z(m)}\Big) \times S \times \mathcal{A}(m=0) + \mathcal{O}(m)$$

- challenge numerical instabilities $\implies$ next-to-soft stabilisation

McMule in 3min



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