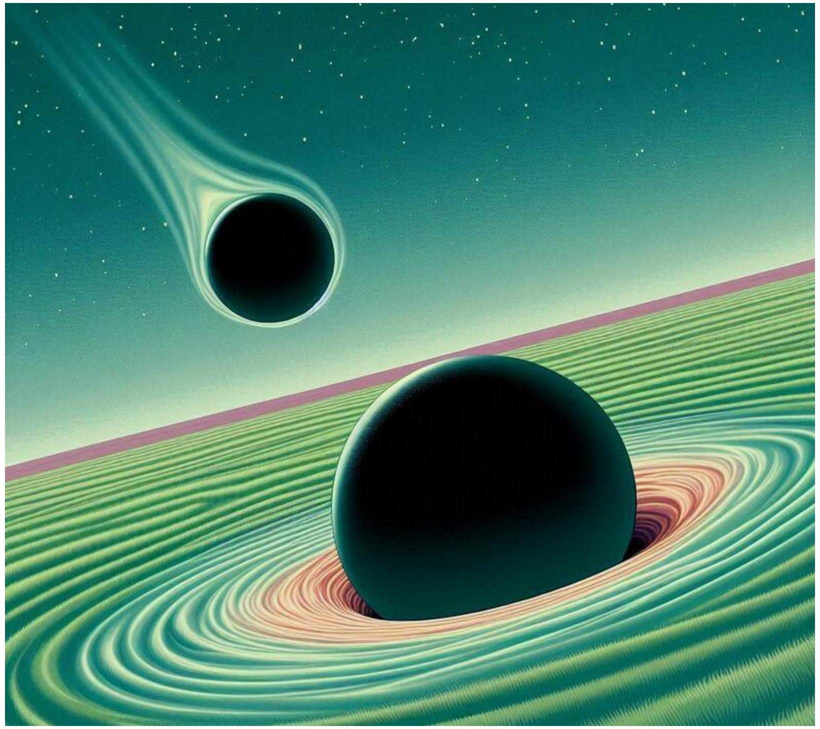
Waveforms in the Post-Minkowskian Expansion

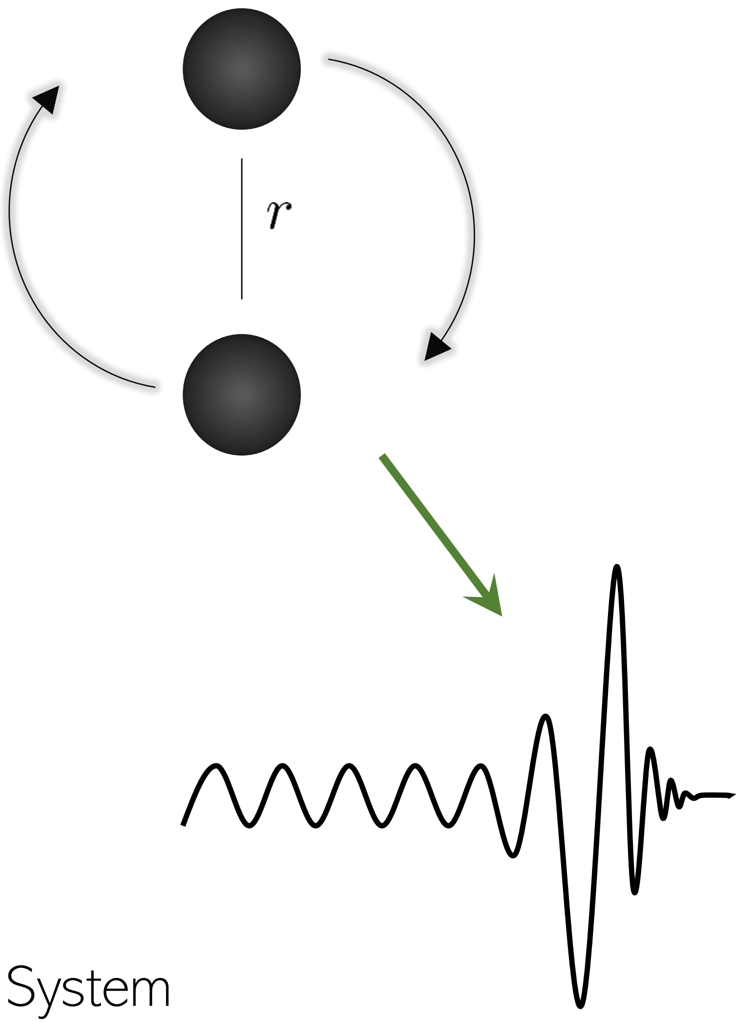


Based on 2312.14859 with Harald Ita, Manfred Kraus and Johannes Schlenk

SPS Annual Meeting 2024
Lara Bohnenblust
11. September 2024

University of Zurich[UZH]

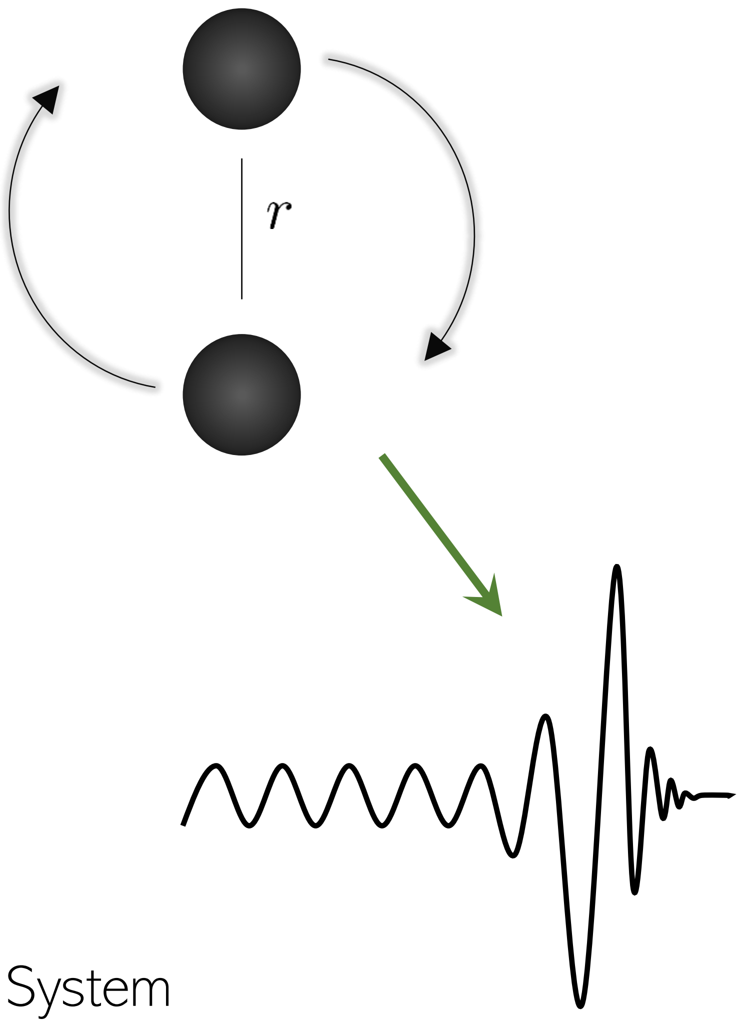
Gravitational Scattering



Bound System

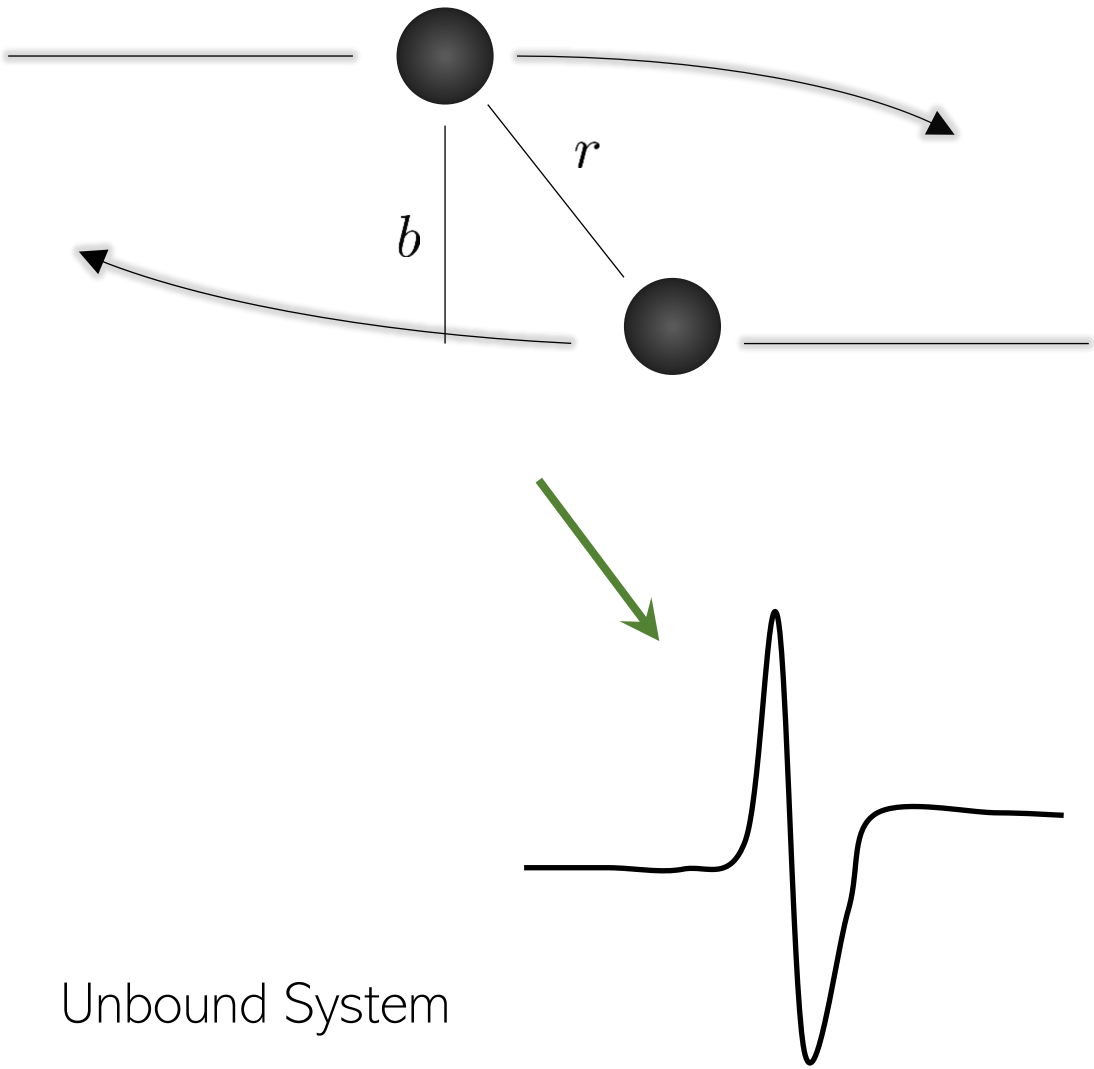
$$v^2 \sim \frac{Gm}{r} \ll 1$$

Gravitational Scattering



Bound System

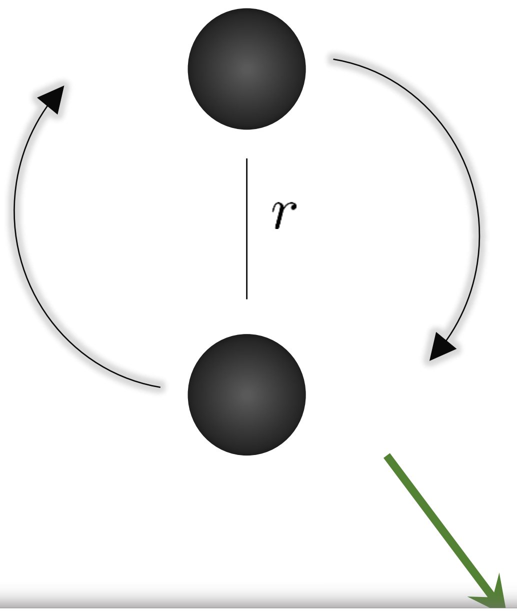
$v^2 \sim \frac{Gm}{r} \ll 1$



Unbound System

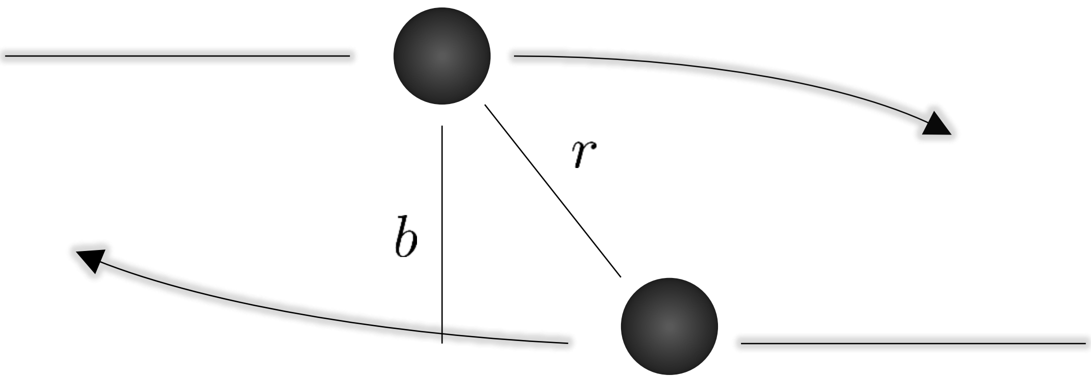
$\frac{Gm}{r} \ll 1$

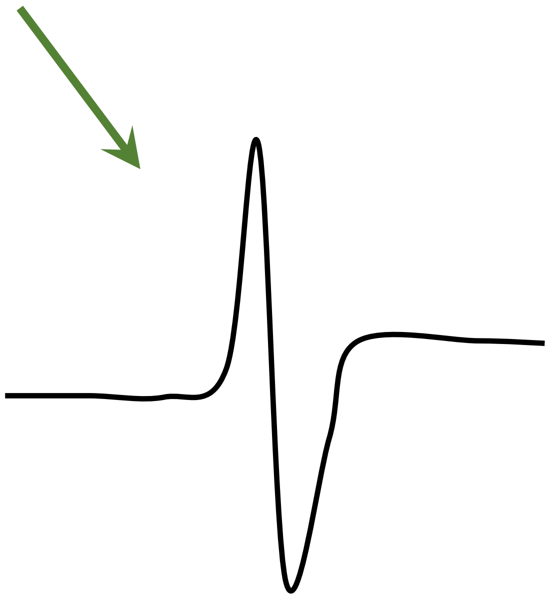
Gravitational Scattering



LVK Collaboration [1706.01812]

$$v^2 \sim \frac{Gm}{r} \ll 1$$

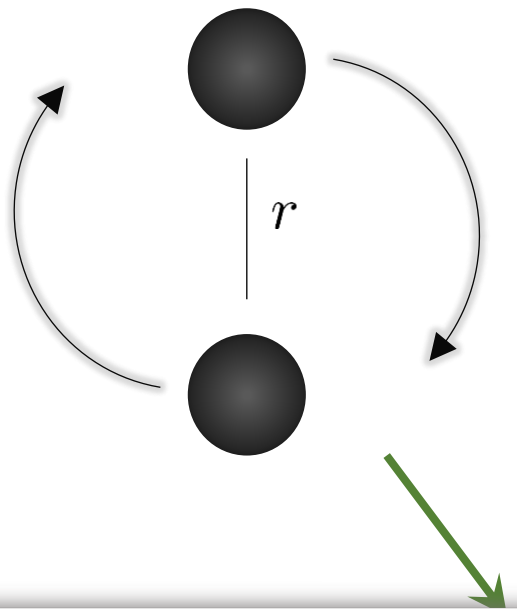


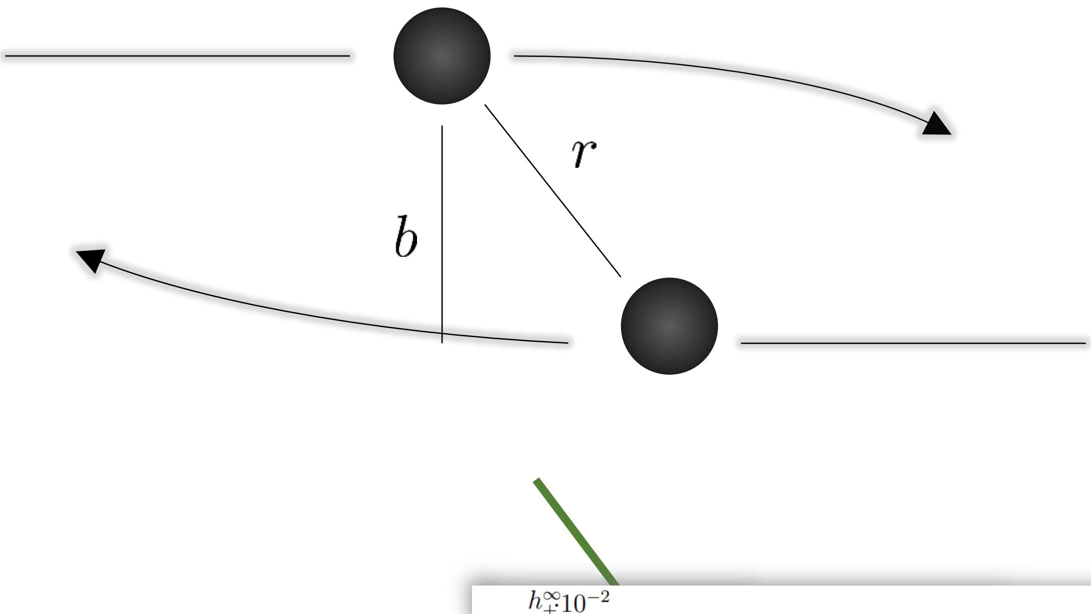


Unbound System

$$\frac{Gm}{r} \ll 1$$

Gravitational Scattering

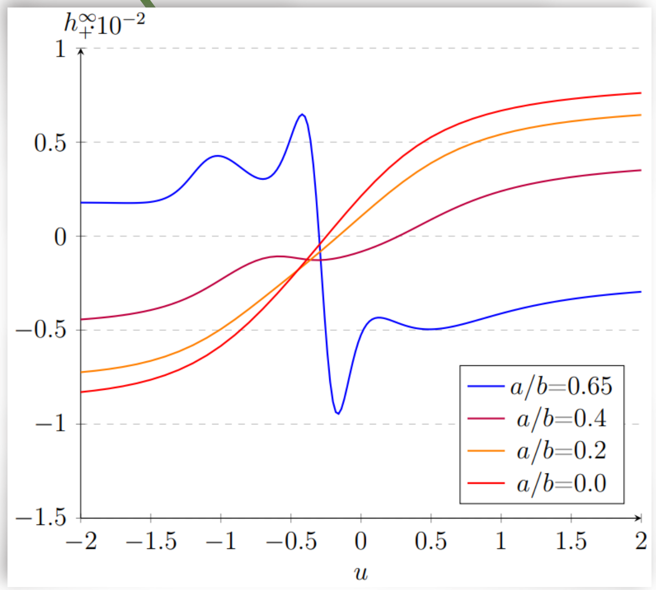




LVK Collaboration [1706.01812]

$$v^2 \sim \frac{Gm}{r} \ll 1$$

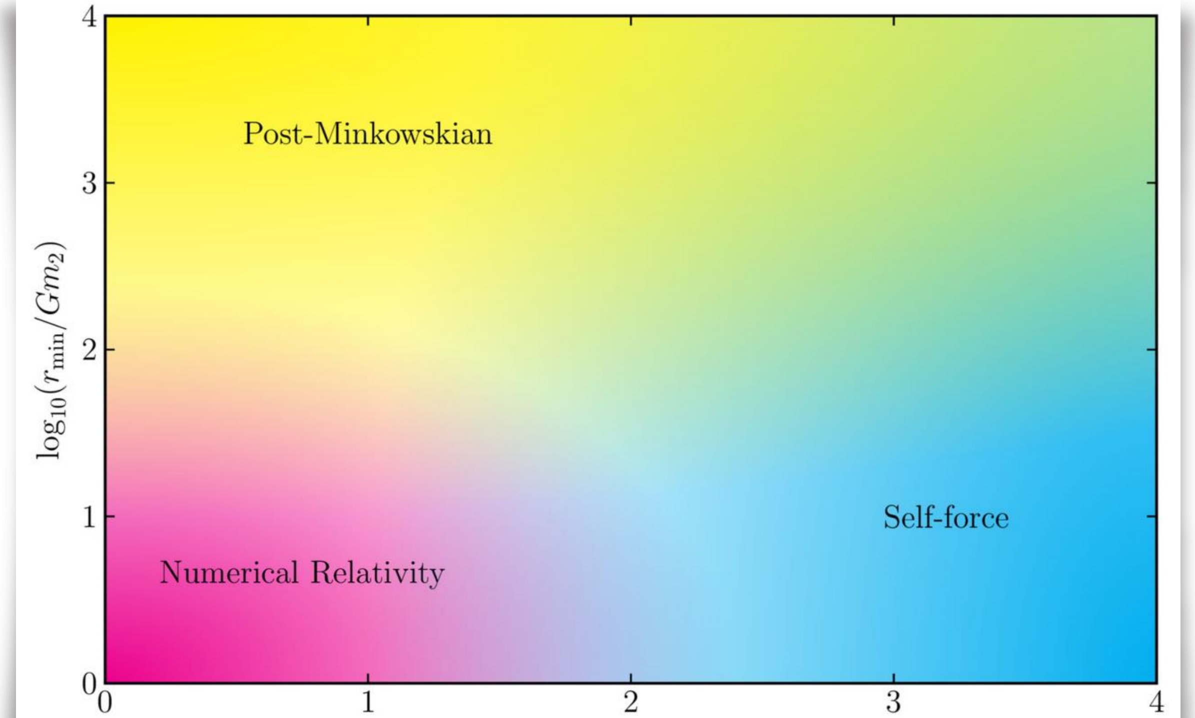
Unbound System



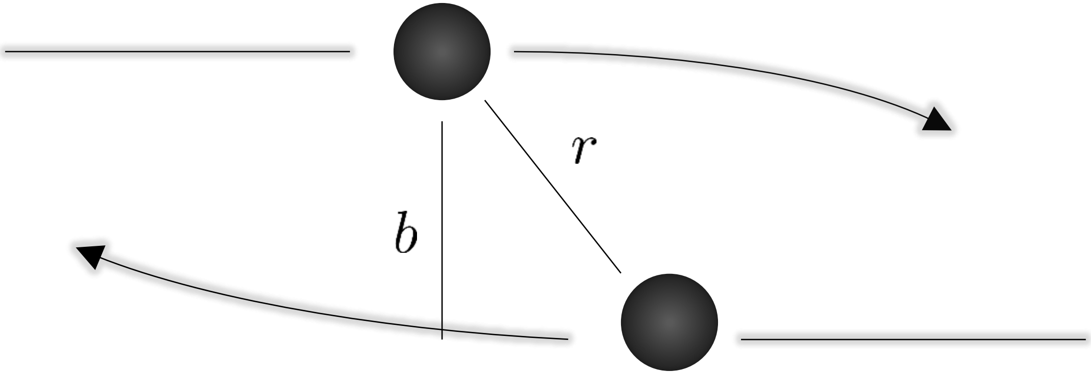
$$\frac{Gm}{r} \ll 1$$

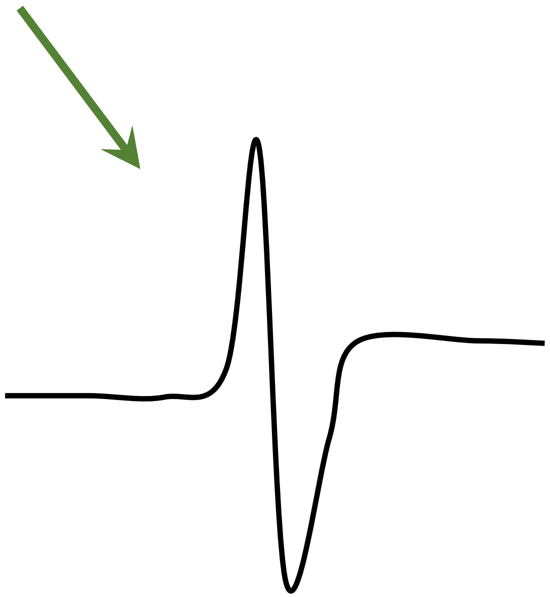
Brandhuber, Brown, Chen, Gowdy, Travaglini [2310.04405]

Gravitational Scattering



Barack et.al [2304.09200]





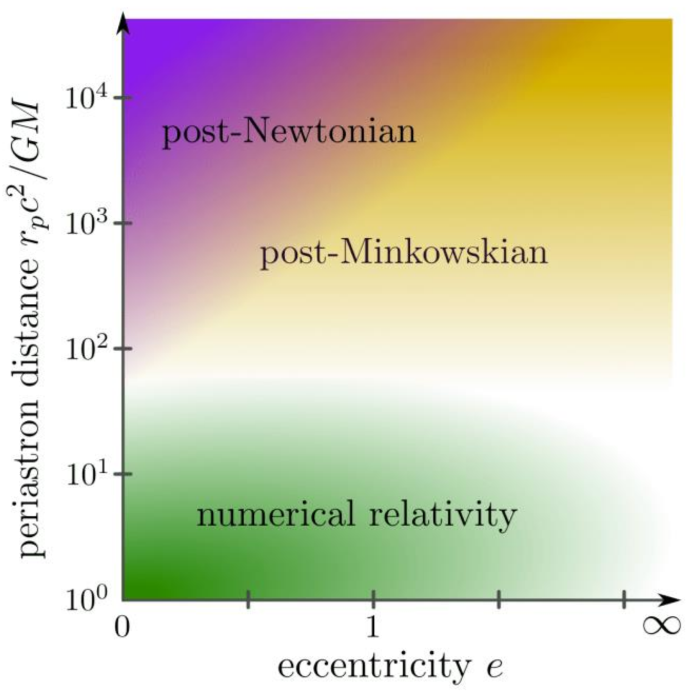
Unbound System

$$\frac{Gm}{r} \ll 1$$

PN vs PM

		0PN	1PN	2PN	3PN	4PN	5PN	6PN	
0PM		$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$v^{12}+$	$v^{14}+\cdots)$
1PM	G		$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$v^{12}+\cdots)$
2PM	$+G^2$			$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+\cdots)$
3PM	$+G^3$				$(1+$	v^2+	v^4+	v^6+	$v^8+\cdots)$
4PM	$+G^4$					$(1+$	v^2+	v^4+	$v^6+\cdots)$
5PM	$+G^5$						$(1+$	v^2+	$v^4+\cdots)$
6PM	$+G^6$							$(1+$	$v^2+\cdots)$
7PM	$+G^7$								$(1+\cdots)$
	$+\cdots$								

Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng [2204.05194]

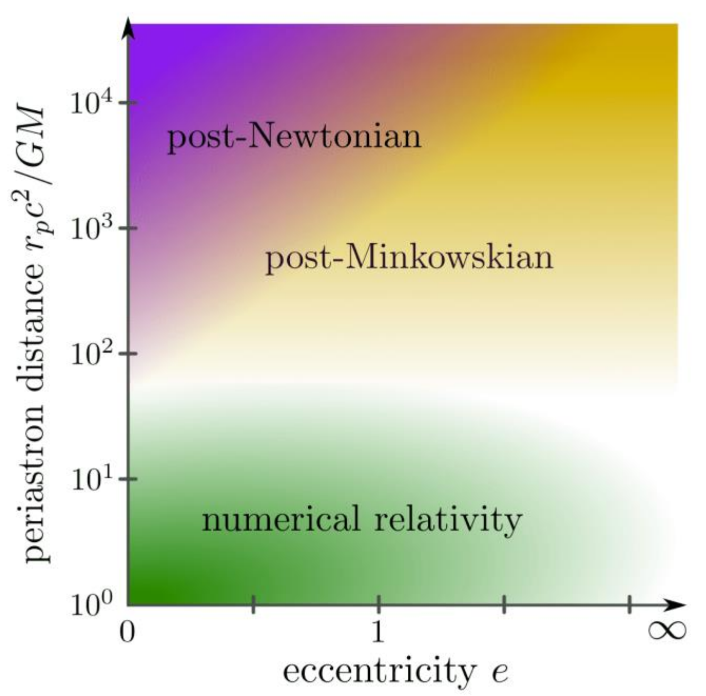


Khalil, Buonanno, Steinhoff, Vines [2204.05047]

PN vs PM

		0PN	1PN	2PN	3PN	4PN	5PN	6PN	
0PM	$(1 +$	$v^2 +$	$v^4 +$	$v^6 +$	$v^8 +$	$v^{10} +$	$v^{12} +$	$v^{14} +$	$\cdots)$
1PM	G	$(1 +$	$v^2 +$	$v^4 +$	$v^6 +$	$v^8 +$	$v^{10} +$	$v^{12} +$	$\cdots)$
2PM	$+G^2$		$(1 +$	$v^2 +$	$v^4 +$	$v^6 +$	$v^8 +$	$v^{10} +$	$\cdots)$
3PM	$+G^3$			$(1 +$	$v^2 +$	$v^4 +$	$v^6 +$	$v^8 +$	$\cdots)$
4PM	$+G^4$				$(1 +$	$v^2 +$	$v^4 +$	$v^6 +$	$\cdots)$
5PM	$+G^5$					$(1 +$	$v^2 +$	$v^4 +$	$\cdots)$
6PM	$+G^6$						$(1 +$	$v^2 +$	$\cdots)$
7PM	$+G^7$							$(1 +$	$\cdots)$
	$+\cdots$								

Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng [2204.05194]

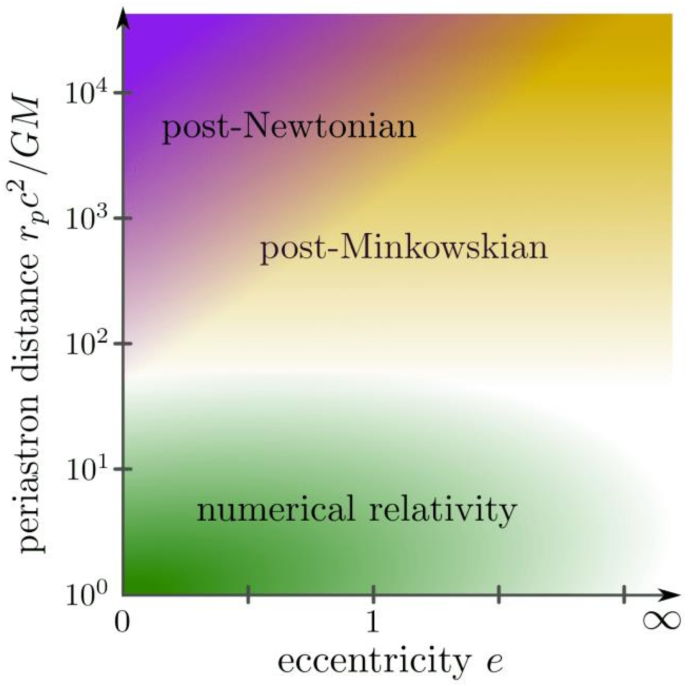


Khalil, Buonanno, Steinhoff, Vines [2204.05047]

PN vs PM

			0PN	1PN	2PN	3PN	4PN	5PN	6PN	
0PM		$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$v^{12}+$	$v^{14}+$	$\cdots)$
1PM	G		$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$v^{12}+$	$\cdots)$
2PM	$+G^2$			$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$\cdots)$
3PM	$+G^3$				$(1+$	v^2+	v^4+	v^6+	v^8+	$\cdots)$
4PM	$+G^4$					$(1+$	v^2+	v^4+	v^6+	$\cdots)$
5PM	$+G^5$						$(1+$	v^2+	v^4+	$\cdots)$
6PM	$+G^6$							$(1+$	v^2+	$\cdots)$
7PM	$+G^7$								$(1+$	$\cdots)$
	$+\cdots$									

Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng [2204.05194]

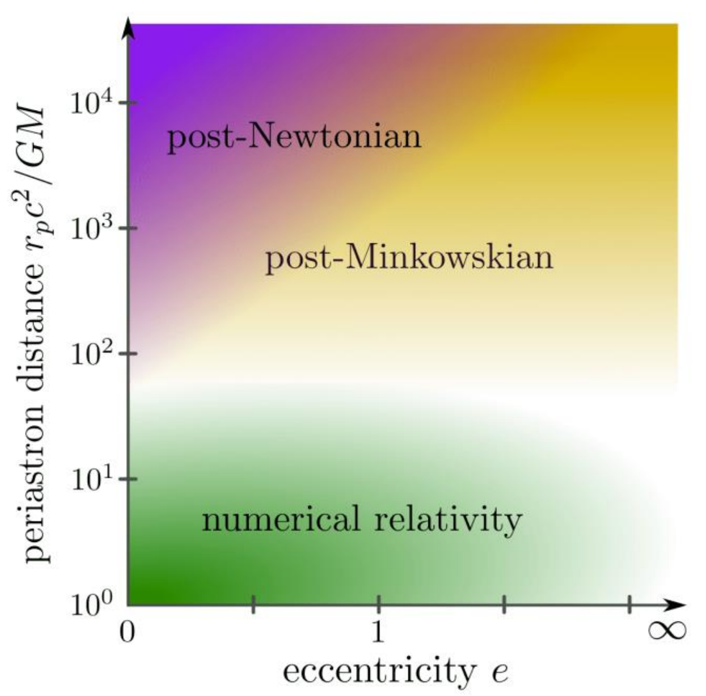


Khalil, Buonanno, Steinhoff, Vines [2204.05047]

PN vs PM

			0PN	1PN	2PN	3PN	4PN	5PN	6PN	
0PM		$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$v^{12}+$	$v^{14}+$	$\cdots)$
1PM	G		$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$v^{12}+$	$\cdots)$
2PM	$+G^2$			$(1+$	v^2+	v^4+	v^6+	v^8+	$v^{10}+$	$\cdots)$
3PM	$+G^3$				$(1+$	v^2+	v^4+	v^6+	v^8+	$\cdots)$
4PM	$+G^4$					$(1+$	v^2+	v^4+	v^6+	$\cdots)$
5PM	$+G^5$						$(1+$	v^2+	v^4+	$\cdots)$
6PM	$+G^6$							$(1+$	v^2+	$\cdots)$
7PM	$+G^7$								$(1+$	$\cdots)$
	$+\cdots$									

Buonanno, Khalil, O’Connell, Roiban, Solon, Zeng [2204.05194]



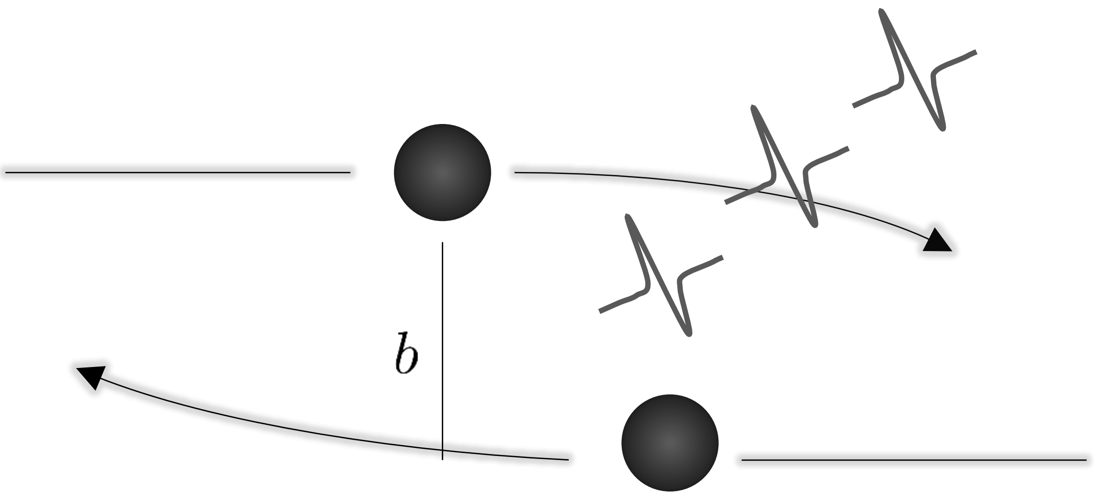
Khalil, Buonanno, Steinhoff, Vines [2204.05047]

The Post-Minkowskian (PM) Expansion

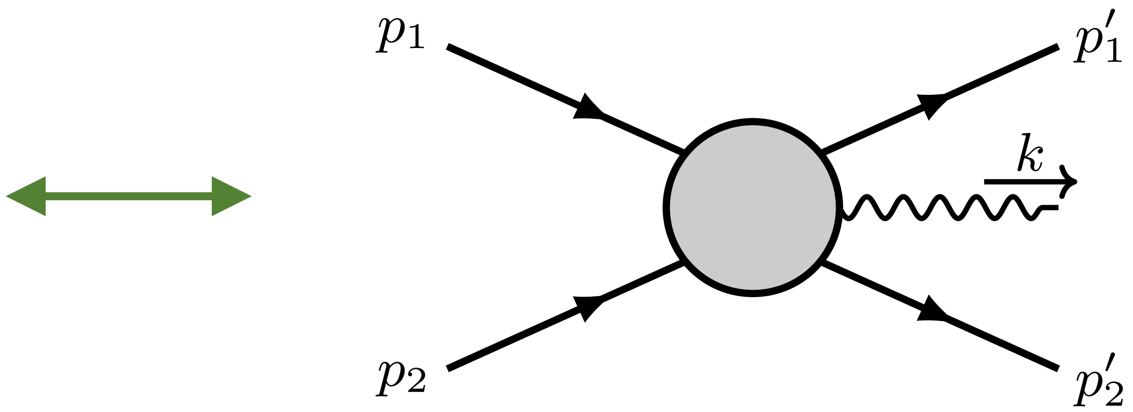
$$\mathcal{O} \sim \sum_i c_i(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i$$

The Post-Minkowskian (PM) Expansion

$$\mathcal{O} \sim \sum_i c_i(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i$$



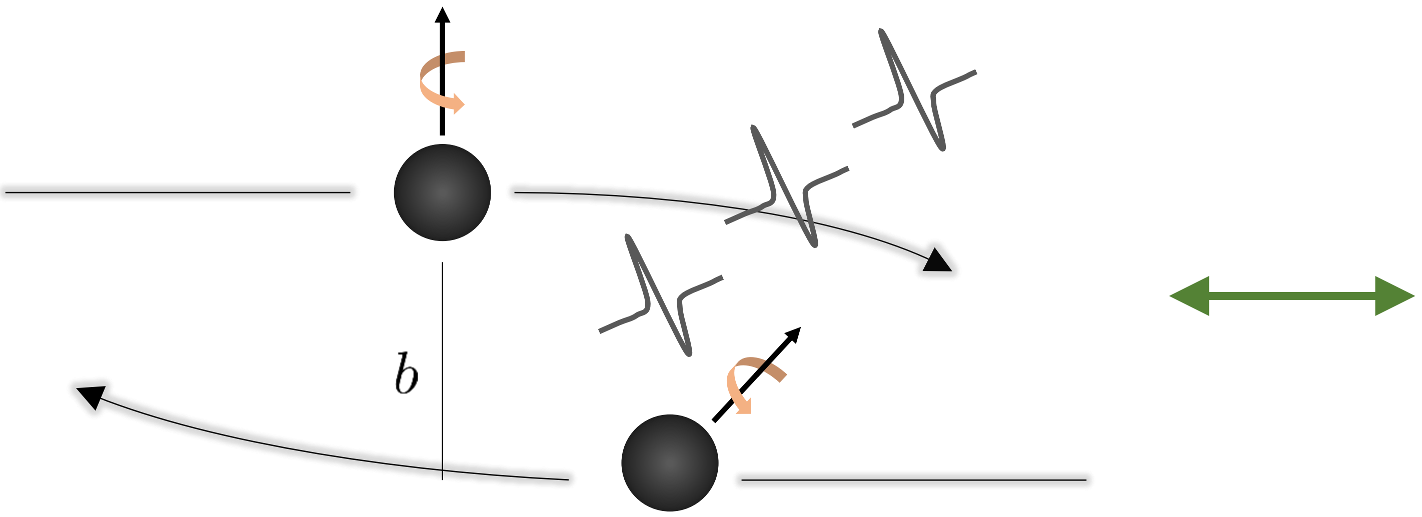
Scattering encounter



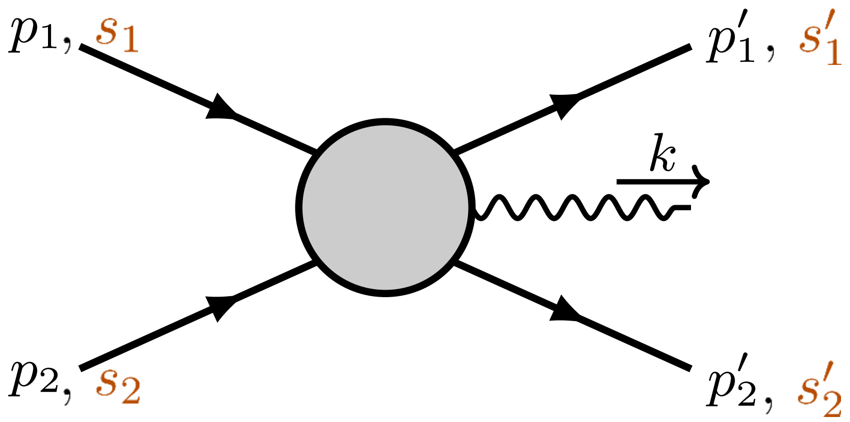
Scattering amplitude

The Post-Minkowskian (PM) Expansion

$$\mathcal{O} \sim \sum_{ijk} c_{ij}(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i \left(\frac{S_1}{m_1 b}\right)^j \left(\frac{S_2}{m_2 b}\right)^k$$



Scattering encounter



Scattering amplitude

Theories

Vaidya [1410.5348]

Minimally-coupled **scalar field**

$$\mathcal{L}_{(\phi,m)} = \frac{1}{2}\sqrt{|g|}\Big[g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - m^2\phi^2\Big]$$

Minimally-coupled **Proca field**

$$\mathcal{L}_{(V,m)} = -\frac{1}{4}\sqrt{|g|}\Big[g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - 2m^2 g^{\mu\nu}V_\mu V_\nu\Big]\,, \qquad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

Black-hole scattering

$$\mathcal{L}_{\text{scalar}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{(\phi_1,m_1)} + \mathcal{L}_{(\phi_2,m_2)}$$

$$\mathcal{L}_{s-1} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{(V_1,m_1)} + \mathcal{L}_{(\phi_2,m_2)}$$

PM waveforms

$$\mathcal{O} \sim \sum_{ijk} c_{ij}(p_1, p_2, k, b) \cdot \left(\frac{Gm}{b}\right)^i \left(\frac{S_1}{m_1 b}\right)^j \left(\frac{S_2}{m_2 b}\right)^k$$

	S^0	S^1	S^2	S^3	S^4	S^∞
Tree (G^2)	Kovacs & Thorne '78 Jakobsen, Mogull, Plefka, Steinhoff '21		Jakobsen, Mogull, Plefka, Steinhoff '21		De Angelis, Gonzo, Novichkov '23	Brandhuber, Brown, Chen, Gowdy, Travaglini '23 Aoude, Haddad, Heissenberg, Helset '23
1-loop (G^3)	Brandhuber, Brown, Chen, De Angelis, Gowdy '23 Herderschee, Roiban, Teng '23 Georgoudis, Heissenberg, Vazquez-Holm '23	LB, Ita, Kraus, Schlenk '23				

Comparison to the well-established MPM waveforms:

Bini, Damour, De Angelis, Geralico, Herderschee, Roiban, Teng [2402.06604]

Georgoudis, Heissenberg, Russo [2402.06361]

Waveform from Amplitudes

Kosower, Maybee, O'Connell [1811.10950]
Cristofoli, Gonzo, Kosower, O'Connell [2107.10193]

$$h^{(\eta)} = \lim_{r\to\infty} r\,\mathrm{FT}\Big[i\langle p_1', p_2', k|T|p_1, p_2\rangle + \langle p_1', p_2'|T^\dagger|r_1 r_2\rangle\langle r_1, r_2, k|T|p_1, p_2\rangle + \text{c.c.}\Big]$$

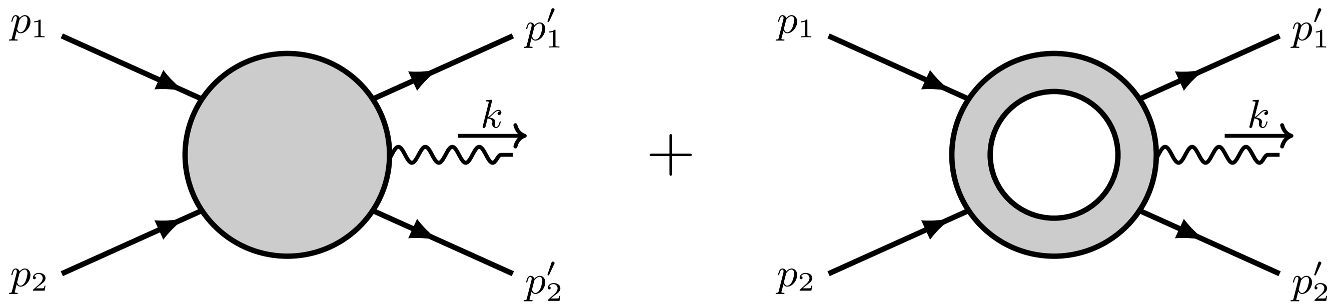
Waveform from Amplitudes

Kosower, Maybee, O'Connell [1811.10950]
Cristofoli, Gonzo, Kosower, O'Connell [2107.10193]

$$h^{(\eta)} = \lim_{r\to\infty} r\,\mathrm{FT}\Big[i\langle p_1', p_2', k|T|p_1, p_2\rangle + \langle p_1', p_2'|T^\dagger|r_1 r_2\rangle\langle r_1, r_2, k|T|p_1, p_2\rangle + \text{c.c.}\Big]$$

Up to order $G^{5/2}$:

$$i\langle p_1' p_2', k|T|p_1 p_2\rangle =$$



$$= \mathcal{M}^{(\eta)}_{\text{amp}}$$

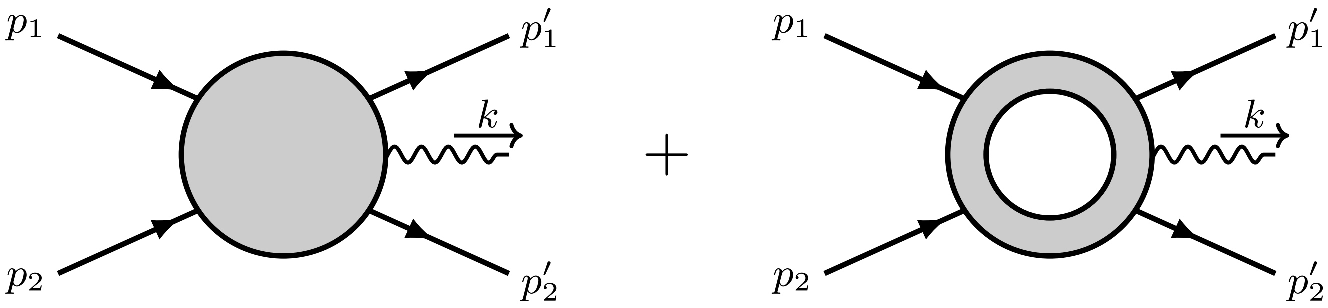
Waveform from Amplitudes

Kosower, Maybee, O'Connell [1811.10950]
Cristofoli, Gonzo, Kosower, O'Connell [2107.10193]

$$h^{(\eta)} = \lim_{r\to\infty} r\,\mathrm{FT}\left[i\langle p_1', p_2', k|T|p_1, p_2\rangle + \langle p_1', p_2'|T^\dagger|r_1 r_2\rangle\langle r_1, r_2, k|T|p_1, p_2\rangle + \text{c.c.}\right]$$

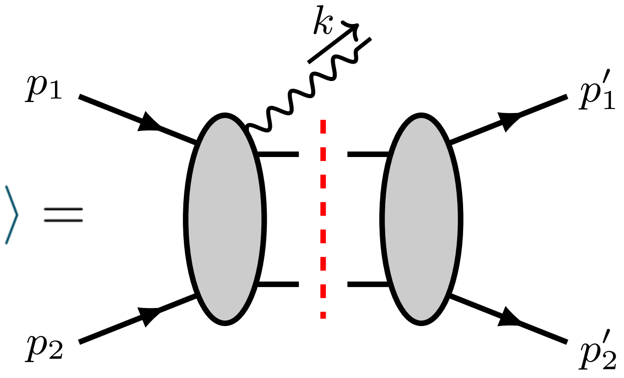
Up to order $G^{5/2}$:

$$i\langle p_1' p_2', k|T|p_1 p_2\rangle =$$



$$= \mathcal{M}^{(\eta)}_{\text{amp}}$$

$$\langle p_1', p_2'|T^\dagger|r_1 r_2\rangle\langle r_1, r_2, k|T|p_1, p_2\rangle =$$



$$= \mathcal{M}^{(\eta)}_{\text{cut}}$$

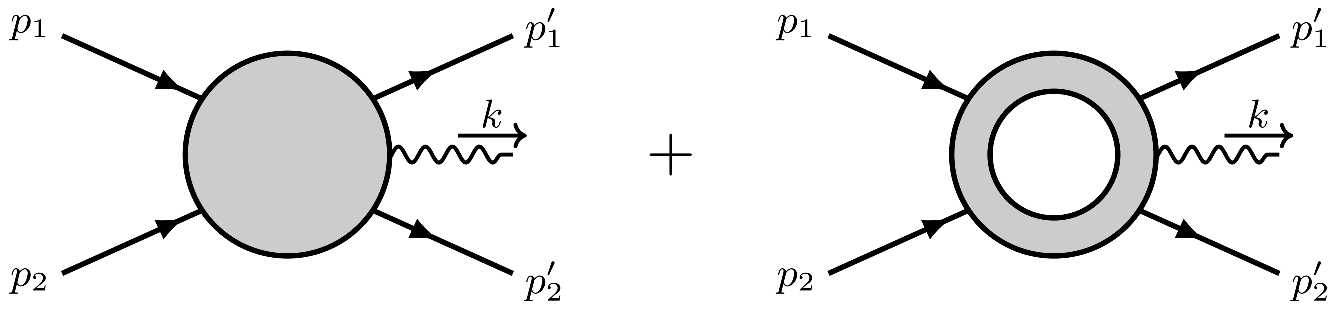
Waveform from Amplitudes

Kosower, Maybee, O'Connell [1811.10950]
Cristofoli, Gonzo, Kosower, O'Connell [2107.10193]

$$h^{(\eta)} = \lim_{r\to\infty} r\,\mathrm{FT}\Big[i\langle p_1', p_2', k|T|p_1, p_2\rangle + \langle p_1', p_2'|T^\dagger|r_1 r_2\rangle\langle r_1, r_2, k|T|p_1, p_2\rangle + \text{c.c.}\Big]$$

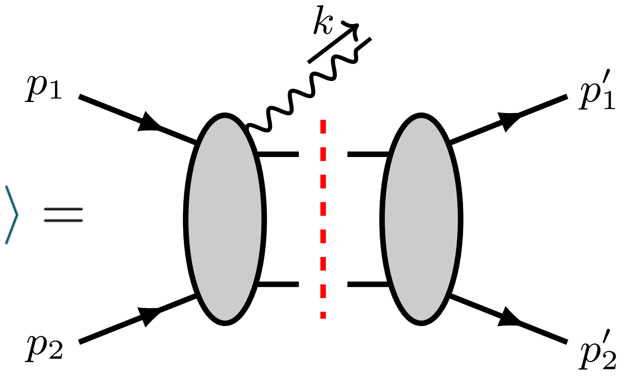
Up to order $G^{5/2}$:

$$i\langle p_1' p_2', k|T|p_1 p_2\rangle =$$



$$= \mathcal{M}^{(\eta)}_{\text{amp}}$$

$$\langle p_1', p_2'|T^\dagger|r_1 r_2\rangle\langle r_1, r_2, k|T|p_1, p_2\rangle =$$



$$= \mathcal{M}^{(\eta)}_{\text{cut}}$$

$$h^{(\eta)}(\tau) \sim \int_0^\infty d\omega e^{-i\omega\tau}\mathrm{FT}\Big[\Big(\mathcal{M}^{(\eta)}_{\text{amp}} + \mathcal{M}^{(\eta)}_{\text{cut}} + \text{c.c.}\Big)\ \delta^{(D)}(q_1 + q_2 + k)\Big]$$

Waveform

LB, Ita, Kraus, Schlenk [2312.14859]

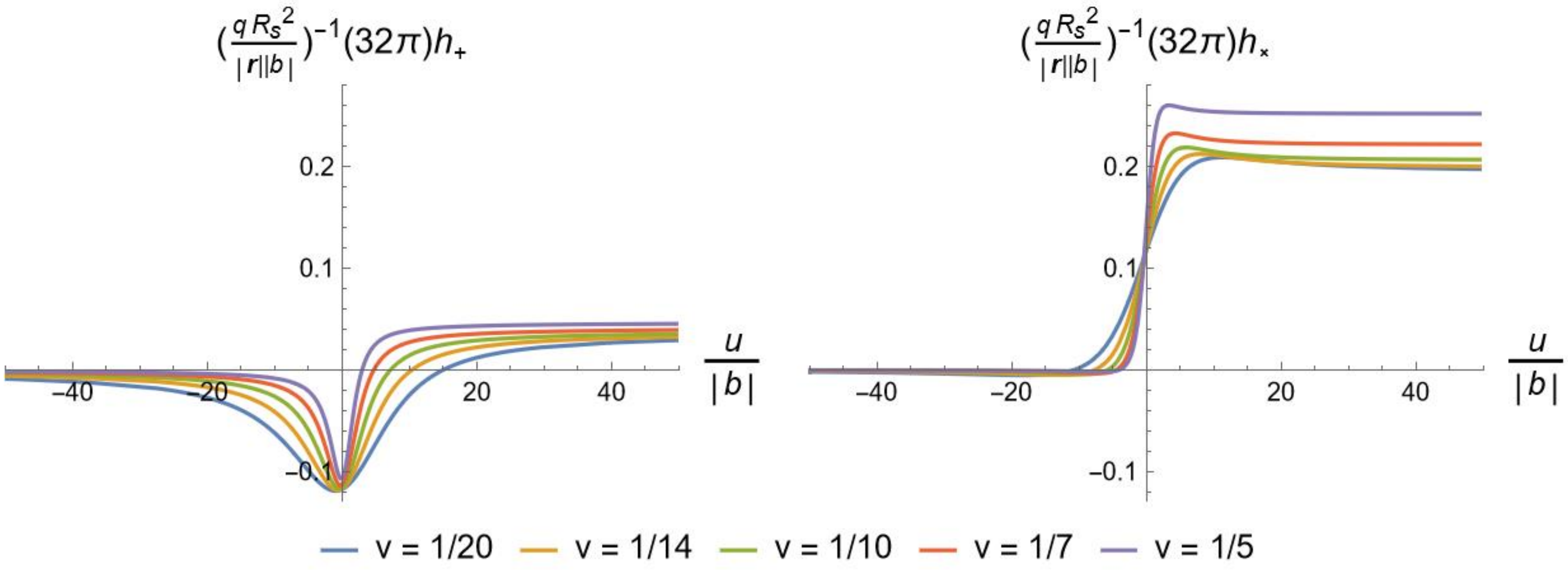
$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

Scalar case checked against

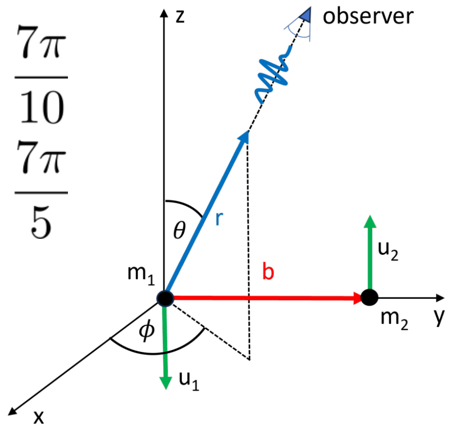
Brandhuber, Brown, Chen, De Angelis, Gowdy [2303.06111]
Herderschee, Roiban, Teng [2303.06112]
Georgoudis, Heissenberg, Vazquez-Holm [2312.14710]

Waveform: Time-domain

$\mathcal{M}_{\rm tree}$ (LO) contribution:

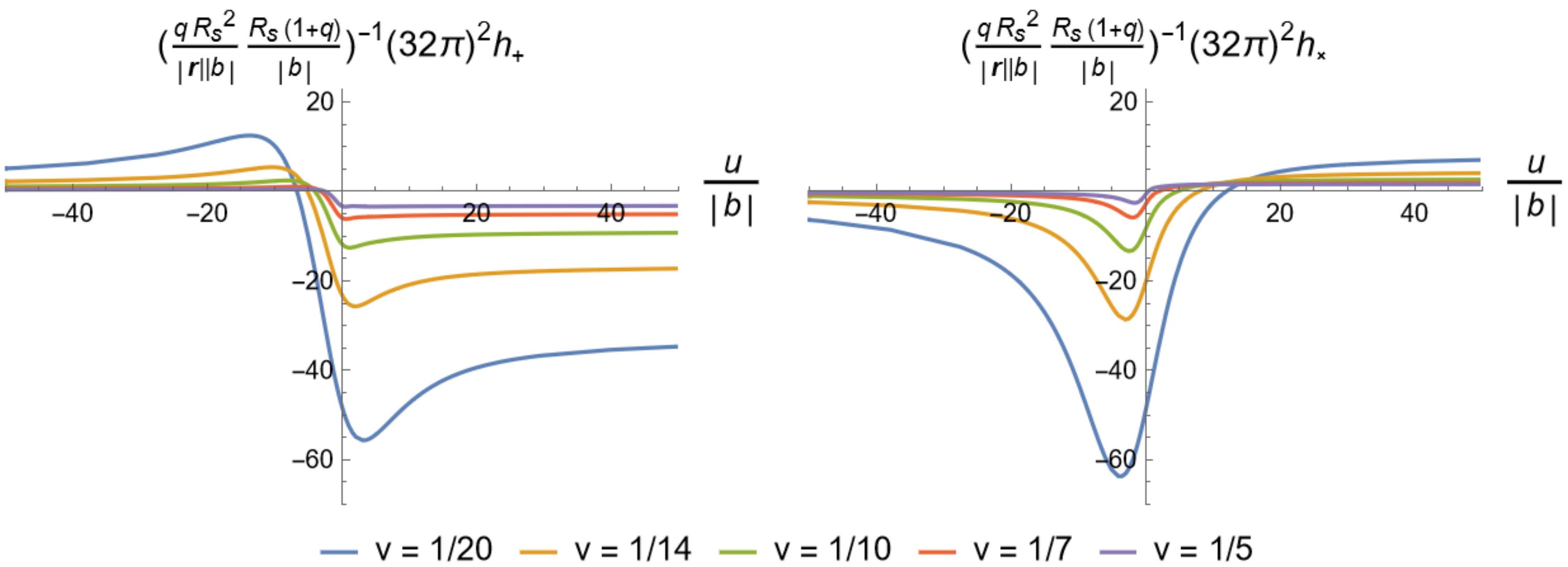


$$\phi = \frac{7\pi}{10}$$
$$\theta = \frac{7\pi}{5}$$



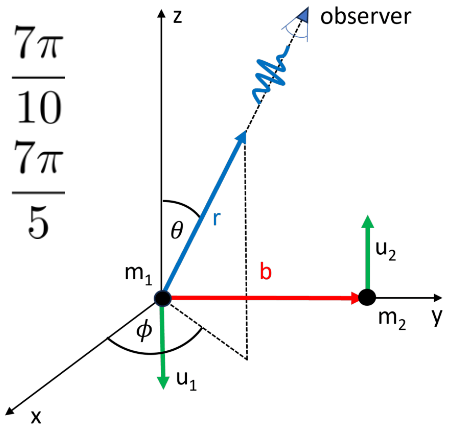
Waveform: Time-domain

$\mathcal{M}_{\text{finite}}$ (NLO) contribution:



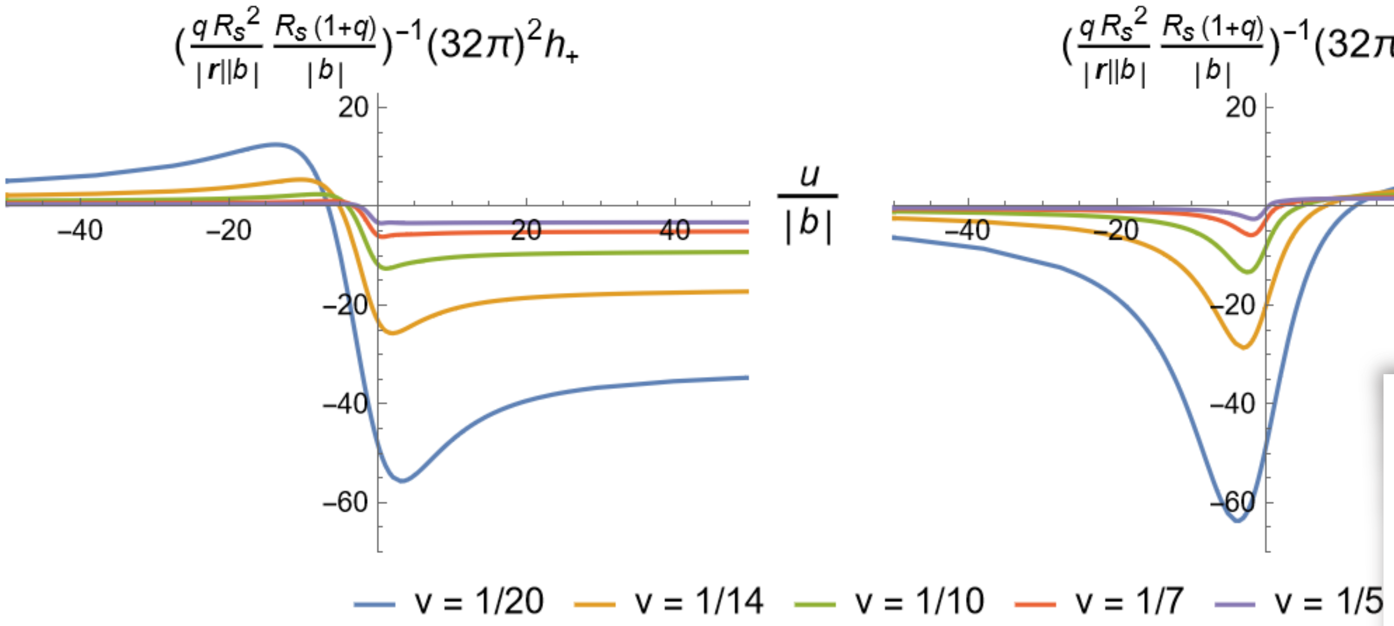
$$\phi = \frac{7\pi}{10}$$

$$\theta = \frac{7\pi}{5}$$

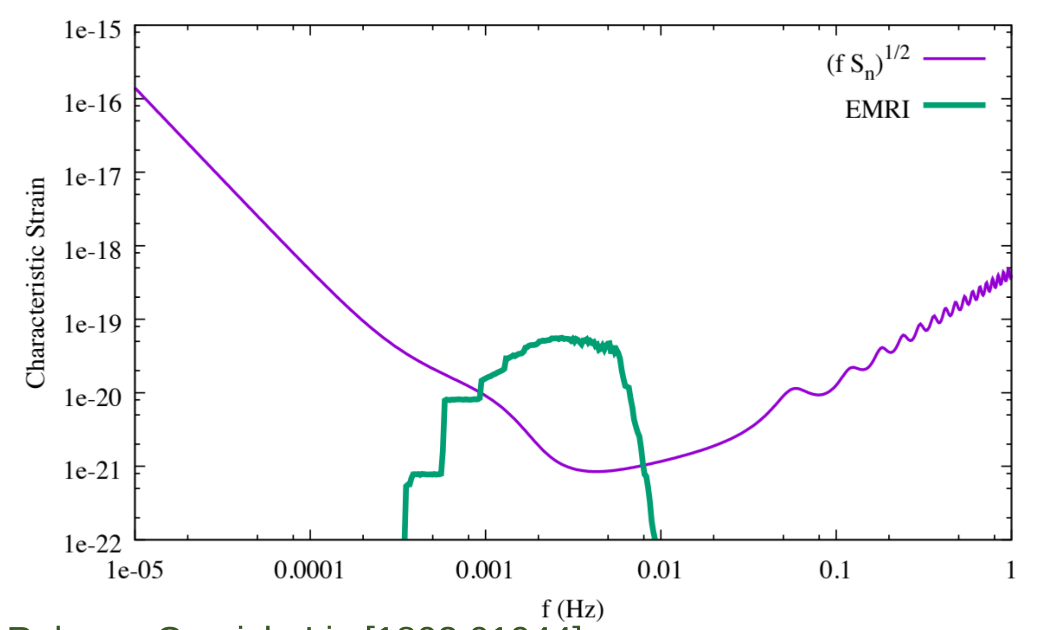


Waveform: Time-domain

$\mathcal{M}_{\rm finite}$ (NLO) contribution:



LISA sensitivity curve



Robson, Cornish, Liu [1803.01944]

Detectability example

$$|b| = 1000\, r_S^1\,,$$
$$m_1 = 5\cdot 10^4\, M_\odot, m_2 = 50\, M_\odot\,,$$
$|\boldsymbol{r}| \approx 10^3\,\mathrm{kpc}$, distance to Andromeda

$$T \sim 10^4\,\mathrm{s}\,,$$
$$f \sim 10^{-4}\,\mathrm{Hz}\,,$$
$$h_{\rm LO} \sim 10^{-13},$$
$$h_{\rm NLO} \sim 10^{-16}$$

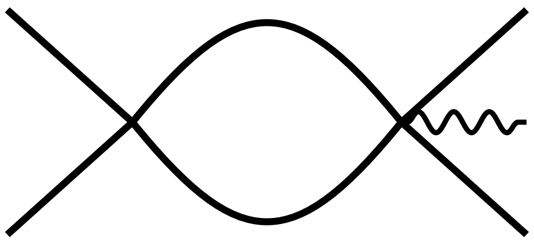
Outlook

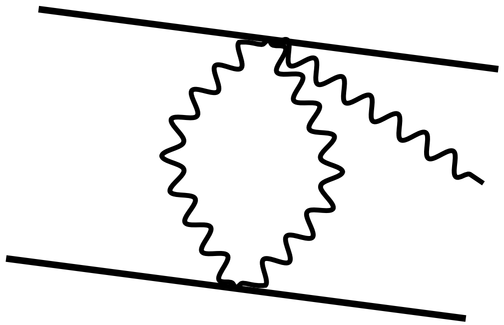
Outlook

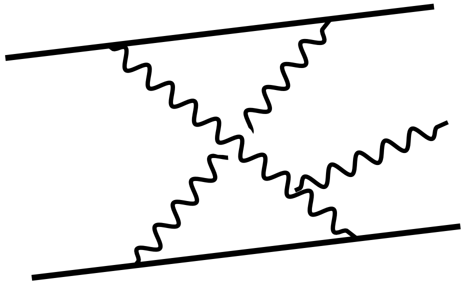
- G^4 and S^2 spin corrections
- Rate and parameter space of scattering?
- Bound system Adamo, Gonzo, Ilderton [2402.00124]
- Fast and stable Fourier transformation required Brunello, De Angelis [2403.08009]

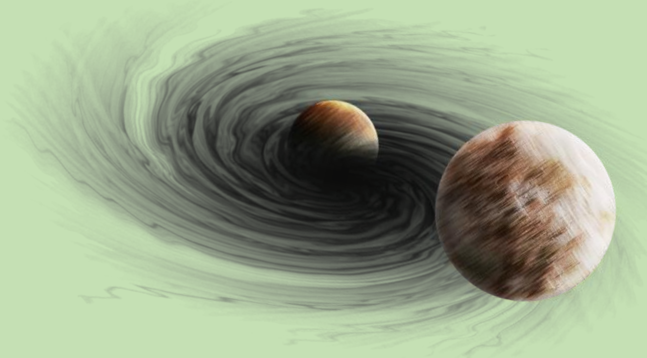
University of Zurich[UZH]

Thanks!









Backup Slides

PM waveforms

$$
\begin{aligned}
& G^2 \left(1 + S + S^2 + S^3 + S^4 + S^5 + S^6 + S^7 + \cdots\right) \\
+ & G^3 \left(1 + S + S^2 + S^3 + S^4 + S^5 + S^6 + \cdots\right) \\
+ & G^4 \left(1 + S + S^2 + S^3 + S^4 + S^5 + \cdots\right) \\
+ & G^5 \left(1 + S + S^2 + S^3 + S^4 + \cdots\right) \\
+ & G^6 \left(1 + S + S^2 + S^3 + \cdots\right) \\
+ & G^7 \left(1 + S + S^2 + \cdots\right) \\
+ & G^8 \left(1 + S + \cdots\right) \\
+ & \cdots
\end{aligned}
$$

Waveform: Tree

LB, Ita, Kraus, Schlenk [2312.14859]

$$\mathcal{M} = \mathcal{M}^{\rm tree} + \mathcal{M}^{\rm IR} + \mathcal{M}^{\rm UV} + \mathcal{M}^{\rm tail} + \mathcal{M}^{\rm finite}$$

Tree (including ϵ - corrections)

$$\mathcal{M}_D^{\rm tree} = -\frac{\kappa^3 \bar{m}_1^2 \bar{m}_2^2}{4}\, \varepsilon^{(\eta)}_{\mu\nu} \left[\frac{4P^\mu P^\nu}{q_1^2 q_2^2} + \frac{2y}{q_1^2 q_2^2}(Q^\mu P^\nu + P^\mu Q^\nu) + \left(y^2 - \frac{1}{D_s - 2} \right) \left(\frac{Q^\mu Q^\nu}{q_1^2 q_2^2} - \frac{P^\mu P^\nu}{\omega_1^2 \omega_2^2} \right) \right]$$
$$= \mathcal{M}^{\rm tree}_{D=4} + \epsilon \mathcal{M}^{\rm tree}_{\epsilon} + \mathcal{O}(\epsilon)\,,$$

$$P^\mu = -\omega_1 u_2^\mu + \omega_2 u_1^\mu\,, \qquad Q^\mu = (q_1 - q_2)^\mu + \frac{q_1^2}{\omega_1} u_1^\mu - \frac{q_2^2}{\omega_2} u_2^\mu\,,$$

$$D_s = 4 - 2\epsilon$$

Waveform: Finite Contribution

LB, Ita, Kraus, Schlenk [2312.14859]

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

Finite amplitude contribution

$$\begin{aligned}
\mathcal{M}^{\text{amp}}_{\text{finite}} =& i\Bigg[r_1 + \frac{r_2}{\sqrt{y^2-1}} + \frac{r_3}{\sqrt{\omega_2^2 - q_1^2}} + \frac{r_4}{\sqrt{\omega_1^2 - q_2^2}} + \frac{r_5}{\sqrt{-q_1^2}} + \frac{r_6}{\sqrt{-q_2}}\Bigg] \\
&+ r_7 + \frac{r_8}{\sqrt{\omega_1^2 - q_2^2}} \log\left(\frac{\sqrt{\omega_1^2 - q_2^2} - \omega_1}{\sqrt{-q_2^2}}\right) + \frac{r_9}{\sqrt{\omega_2^2 - q_1^2}} \log\left(\frac{\sqrt{\omega_2^2 - q_1^2} - \omega_2}{\sqrt{-q_1^2}}\right) \\
&+ \frac{r_{10}}{\sqrt{y^2-1}} \log\left(y + \sqrt{y^2-1}\right) + r_{11} \log\left(\frac{\omega_2^2}{\omega_1^2}\right) + r_{12} \log\left(\frac{q_1^2}{q_2^2}\right)
\end{aligned}$$

Waveform: Finite Contribution

LB, Ita, Kraus, Schlenk [2312.14859]

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

Finite cut contribution

$$\mathcal{M}^{\text{cut}}_{\text{finite}} = \frac{1}{\sqrt{y^2-1}}\Big[r_{13} + r_{14}\log\left(y^2-1\right) + r_{15}\log\left(\frac{\omega_1}{\omega_2}\right) + r_{16}\log\left(\frac{-q_1^2}{\omega_1\omega_2}\right) + r_{17}\log\left(\frac{-q_2^2}{\omega_1\omega_2}\right)\Big]$$
$$+ r_{18}\log\left(y + \sqrt{y^2-1}\right)$$

Waveform: Tail Contribution

LB, Ita, Kraus, Schlenk [2312.14859]

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

Tail contribution (scale-dependent)

$$\mathcal{M}^{\text{tail}} = -\log\left(\frac{\omega_1\omega_2}{\mu_{\text{IR}}^2}\right)\mathcal{W}_S\,\mathcal{M}_{D=4}^{\text{tree}} - \log\left(\frac{\omega_1\omega_2}{\mu_{\text{UV}}^2}\right)\overline{\mathcal{M}}^{\text{UV}}$$

Waveform: IR Divergence

LB, Ita, Kraus, Schlenk [2312.14859]

$$\mathcal{M} = \mathcal{M}^{\text{tree}} + \mathcal{M}^{\text{IR}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}$$

IR divergence

$$\mathcal{M}^{\text{IR}} = \left[\frac{1}{\epsilon} - \log\left(\frac{\mu_{\text{IR}}^2}{\mu^2}\right)\right] \mathcal{W}_S \, \mathcal{M}^{\text{tree}}_{D=4}$$

Weinberg soft factor

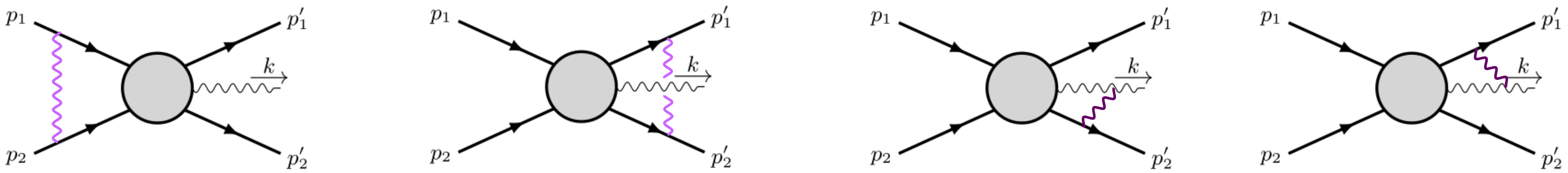
$$\mathcal{W}_S = iG(\bar{m}_1\omega_1 + \bar{m}_2\omega_2)\left(1 + \frac{y(2y^2-3)}{2(y^2-1)^{3/2}}\right)$$

Cut contribution

Waveform: IR Divergence

Weinberg soft theorem

Weinberg, 1965



$$\mathrm{Re}\Big[W_S\Big] = 0$$

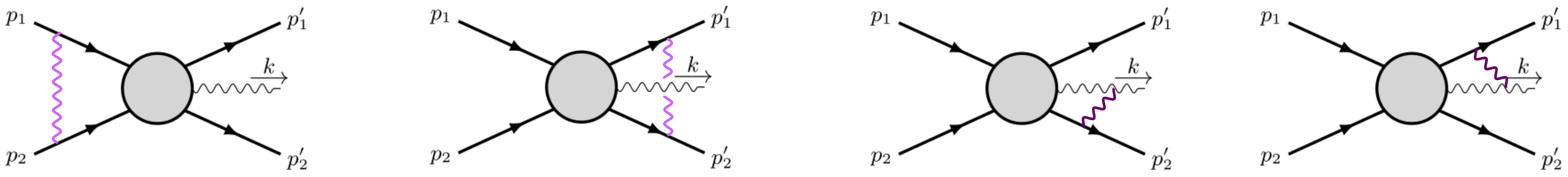
$$\mathrm{Im}\Big[W_S\Big] = \frac{1}{4}\sum_{\substack{i,j=1\\ i\neq j}}^{n} c_{ij}\,\mathrm{Im}[f_{ij}] = -\frac{\pi}{2}\left(2c_{12} + 2c_{1'2'} + c_{1'k} + c_{2'k}\right),$$

$$\mathrm{Im}[f_{ij}] = \begin{cases} -\pi\Theta[(p_i\cdot p_j)]\,, & m_{i/j} = 0\,, \\ -2\pi\Theta[(p_i\cdot p_j)], & \text{else}\,. \end{cases}$$

Waveform: IR Divergence

Weinberg soft theorem

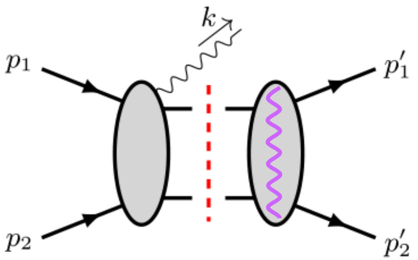
Weinberg, 1965



$$\mathrm{Re}\Big[W_S\Big] = 0$$

$$\mathrm{Im}\Big[W_S\Big] = \frac{1}{4}\sum_{\substack{i,j=1\\ i\neq j}}^{n} c_{ij}\,\mathrm{Im}[f_{ij}] = -\frac{\pi}{2}\left(2c_{12} + 2c_{1'2'} + c_{1'k} + c_{2'k}\right), \qquad \mathrm{Im}[f_{ij}] = \begin{cases} -\pi\Theta[(p_i\cdot p_j)]\,, & m_{i/j} = 0\,,\\ -2\pi\Theta[(p_i\cdot p_j)], & \text{else}\,.\end{cases}$$

Subtract cut: $-i$



$\Longrightarrow$ $\mathrm{Cut}[f_{1'2'}] = 2\,\mathrm{Im}[f_{1'2'}]$

$$\mathrm{Im}\Big[\mathcal{W}_S\Big] = \mathrm{Im}\Big[W_S\Big] - \mathrm{Cut}_{1'2'}(W_S) = -\frac{\pi}{2}\left(2c_{12} - 2c_{1'2'} + c_{1'k} + c_{2'k}\right).$$

Waveform: IR Divergence

Exponentiation

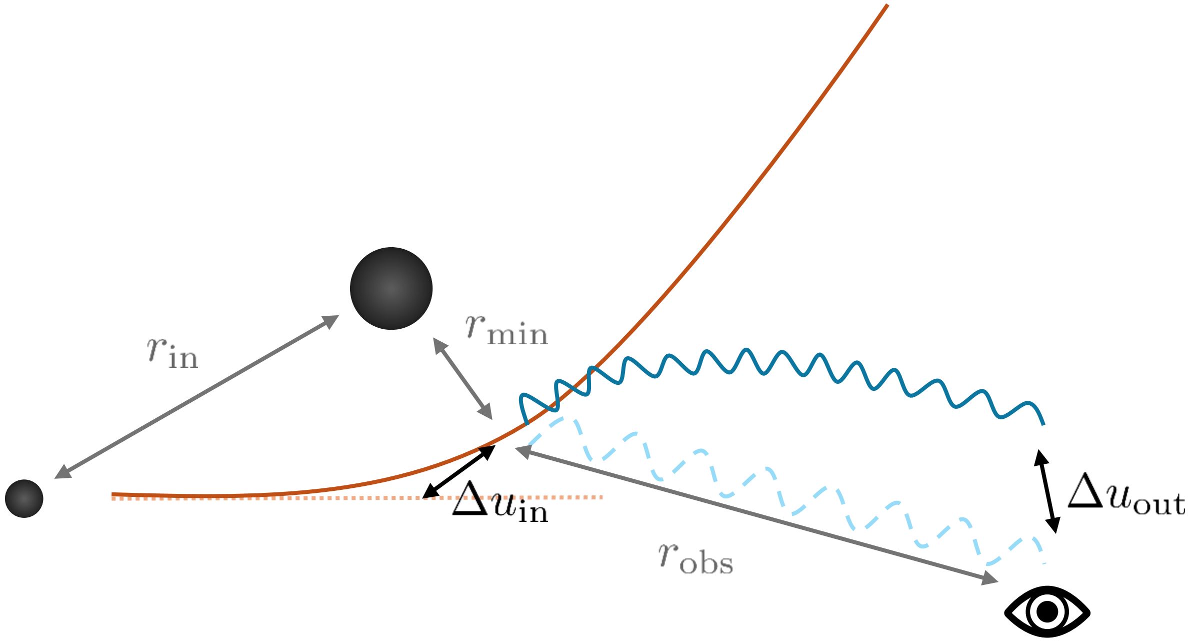
$$e^{-i\omega u}\mathcal{M} = e^{-i\omega\left[u-\left(\frac{1}{\epsilon}-\log\frac{\mu_{\mathrm{IR}}^2}{\mu^2}\right)\frac{\mathcal{W}_S}{i\omega}\right]}\left(\mathcal{M}^{\mathrm{tree}} - \mathcal{W}_S\mathcal{M}_{\epsilon}^{\mathrm{tree}} + \mathcal{M}^{\mathrm{UV}} + \mathcal{M}^{\mathrm{tail}} + \mathcal{M}^{\mathrm{finite}}\right)+\mathcal{O}(G^3)$$

Waveform: IR Divergence

Exponentiation

$$e^{-i\omega u}\mathcal{M} = e^{-i\omega\left[u-\left(\frac{1}{\epsilon}-\log\frac{\mu_{\text{IR}}^2}{\mu^2}\right)\frac{\mathcal{W}_S}{i\omega}\right]}\left(\mathcal{M}^{\text{tree}} - \mathcal{W}_S\mathcal{M}_\epsilon^{\text{tree}} + \mathcal{M}^{\text{UV}} + \mathcal{M}^{\text{tail}} + \mathcal{M}^{\text{finite}}\right) + \mathcal{O}(G^3)$$

Classical interpretation



Shapiro delay

$$\Delta u_{\text{out}} = \frac{2G}{\omega}(\bar{m}_1\omega_1 + \bar{m}_2\omega_2)\log\left(\frac{r_{\text{obs}}}{r_{\text{min}}}\right)$$

Deflection of incoming trajectory

$$\Delta u_{\text{in}} = \frac{2G}{\omega}(\bar{m}_1\omega_1 + \bar{m}_2\omega_2)\frac{y(2y^2-3)}{2(y^2-1)^{3/2}}\log\left(\frac{r_{\text{in}}}{r_{\text{min}}}\right)$$

Cut contribution

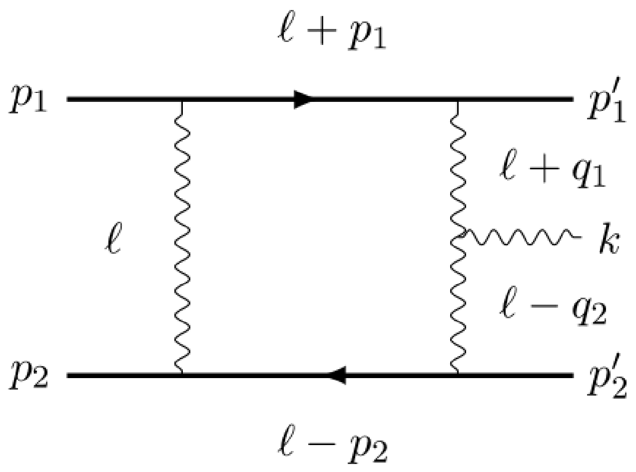
Caron-Huot, Giroux, Hannesdottir, Mizera [2308.02125]

Waveform: UV Divergence

$$\mathcal{M}^{\rm UV} = -\left(\frac{\kappa}{2}\right)^5 \frac{i}{8\pi\epsilon}(\bar{m}_1\omega_1 + \bar{m}_2\omega_2)\frac{\bar{m}_1^2\bar{m}_2^2(\omega_1^2+\omega_2^2+y\omega_1\omega_2)(1-2y^2)^2}{\omega_1\omega_2^3(y^2-1)^{3/2}}\ F_1^{(2h)}$$

Contact term that integrates to $\delta(|b|)$ in the FT and does not contribute to the far-field waveform!

Where does the UV pole come from?



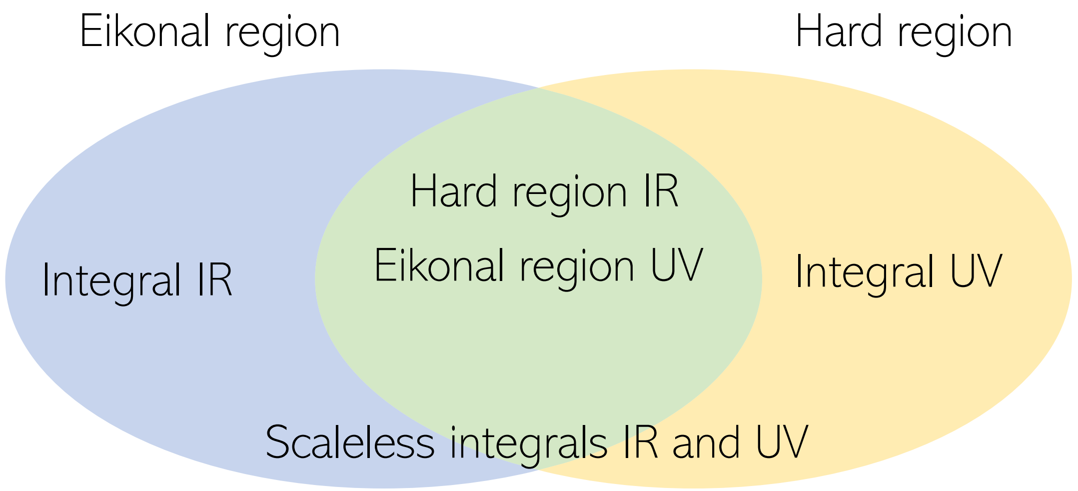
$\Longrightarrow$

$$G^{\rm eik} \sim e^{\epsilon\gamma_E}\int \frac{d^D\ell}{i\pi^{D/2}} \frac{1}{\ell^2[2\ell\cdot p_1](\ell+q_1)^2(\ell-q_2)^2[-2\ell\cdot p_2]}\,, \qquad \ell^\mu \ll \bar{m}_i$$

$$G^{\rm hard} \sim e^{\epsilon\gamma_E}\int \frac{d^D\ell}{i\pi^{D/2}} \frac{1}{[\ell^2]^3[(\ell+p_1)^2-m_1^2][(\ell-p_2)^2+m_2^2]}\,, \qquad \ell^\mu \sim \bar{m}_i$$

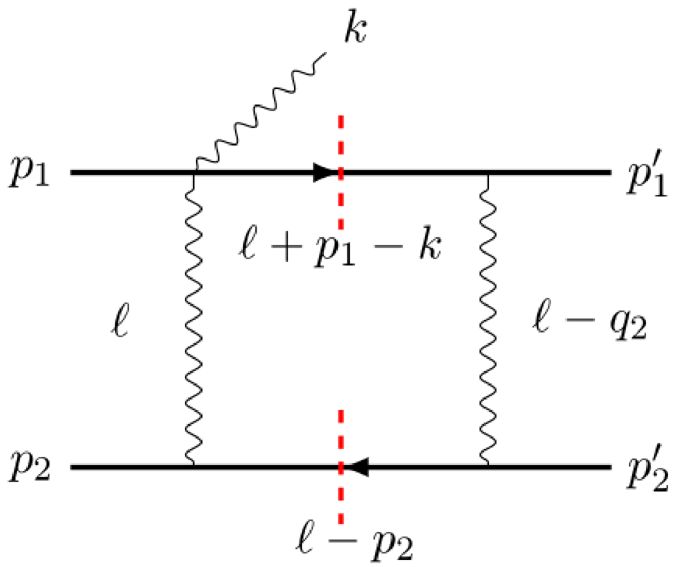
Contributions polynomial in q_i

Waveform: UV Divergence



Full quantum amplitude

$$\mathcal{I}_{\text{box}} = \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{\ell^2\,[(\ell+p_1-k)^2-m_1^2](\ell-q_2)^2[(\ell-p_2)^2-m_2^2]}$$



Eikonal expansion of second massive propagator

$$\frac{1}{(\ell-p_2)^2-m_2^2+i\epsilon} = \underbrace{\frac{1}{-2\bar{m}_2 u_2\cdot\ell+i\epsilon}}_{\sim\,\ell^{-1}} - \underbrace{\frac{\ell\cdot(\ell-q_2)}{[-2\bar{m}_2 u_2\cdot\ell+i\epsilon]^2}}_{\sim\,\ell^{0}} + \dots$$

$$\ell \to \infty$$