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EW Sudakov Logarithms in EW Vector-Boson Production

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Outline

- With increasing experimental precision EW corrections become more and more important
- automated tools for NLO EW corrections exist (GoSam, MadGraph5_aMC@NLO, NLOX, OpenLoops, Recola)
- full EW NLO corrections for multiboson processes demanding
- EW NLO corrections not included in standard tool chains of LHC experiments
- dominant EW corrections at high energies result from enhanced logarithms $\Rightarrow$ Sudakov logarithms

Idea: use logarithmic EW corrections

- as approximation to full NLO corrections
- to resum leading corrections beyond NLO

Origin of leading EW corrections

Leading EW corrections for energies $Q \lesssim 300\,\mathrm{GeV}$:

- corrections originating from soft photons or collinear massless fermion–antifermion or (anti)fermion–photon pairs $\propto \alpha \ln(m_f/Q)$
  QED corrections: typical size $\mathcal{O}(10\%)$
  - YFS resummation (Yennie–Frautschi–Suura)
  - electromagnetic parton showers
- corrections related to the running of the electromagnetic coupling $\alpha(Q)$ $\propto \alpha \ln(m_f/Q)$, typical size $\sim 6\%$
  $\Rightarrow$ incorporated by suitable choice of renormalisation of α
  - $\alpha(0)$ for external isolated photons
  - $\alpha(M_\mathrm{Z})$ or α_{G_μ} otherwise

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)$$

- top-mass corrections $\propto \alpha m_\mathrm{t}^2/(M_\mathrm{W}^2 s_\mathrm{w}^2)$, typical size $\sim 3\%$
  $\Rightarrow$ (partially) incorporated by using α_{G_μ}

Origin of leading EW corrections

For energies $Q \gtrsim 300\,\mathrm{GeV}$ in addition:

- logarithmic EW corrections involving $\alpha\ln(Q/M_\mathrm{W})$ and $\alpha\ln^2(Q/M_\mathrm{W})$
  typical size of double logarithms (always negative)

$$\frac{\alpha}{4\pi s_\mathrm{w}^2}\ln^2\frac{s}{M_\mathrm{W}^2} = 6.6\%\ (18\%)\ @\,1\,\mathrm{TeV}\ (5\,\mathrm{TeV})$$

  typical size of single logarithms (in general positive, larger coefficients)

$$\frac{\alpha}{4\pi s_\mathrm{w}^2}\ln\frac{s}{M_\mathrm{W}^2} = 1.3\%\ (2.1\%)\ @\,1\,\mathrm{TeV}\ (5\,\mathrm{TeV})$$

  double and single EW logarithms dominant in TeV range

  beware:
  - typically cancellations between leading and subleading logarithms
  - additional sources of large logarithms for ratios of invariants $\alpha\ln(s/t)$ and $\alpha\ln^2(s/t)$ in specific regions of phase space

Outline

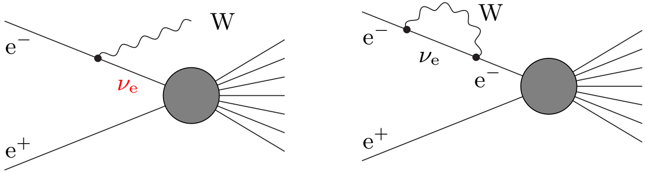
History of EW Sudakov logarithms

- Large EW corrections in tails of distributions realised since the 1990ies
  Kuroda, Moultaka, Schildknecht 1991; Beenakker et al. 1993; Denner et al. hep-ph/9503442, hep-ph/9612390; Beccaria et al. hep-ph/9805250
- resummation of EW double logarithms Fadin et al. hep-ph/9910338
- resummation of subleading logarithms using infrared evolution equations
  Kühn et al. hep-ph9912503, hep-ph/0106298; ... Melles hep-ph/0104232, ...
- Bloch–Nordsieck violation of EW corrections
  M. Ciafaloni, P. Ciafaloni, Comelli, hep-ph/0001142
- general framework for one-loop virtual EW logarithms
  Denner, Pozzorini, hep-ph/0010201, hep-ph/00104127
- results EW two-loop Sudakov logarithms Hori et al. hep-ph/0007329; Beenakker, Werthenbach hep-ph/0120030; Denner et al. hep-ph/0301241; Jantzen et al. hep-ph/0504111, ...
- EW Sudakov corrections using Soft Collinear Effective Theory (SCET)
  Chiu, Golf, Kelley, Manohar 0709.2377, 0806.1240, ...

Bloch–Nordsieck violation of EW logarithms

Initial states carry non-abelian charges (weak isospin)
⇒ EW Sudakov double logarithms do not cancel in inclusive observables!
M. Ciafaloni, P. Ciafaloni, Comelli, hep-ph/0001142

example: W emission from incoming electron



⇒ different hard matrix elements in real and virtual corrections

$$\Delta\sigma_{\mathrm{e^+e^-}} \propto \left(|\mathcal{M}_0(\mathrm{e}^+\nu_\mathrm{e})|^2 - |\mathcal{M}_0(\mathrm{e}^+\mathrm{e}^-)|^2\right)$$

result

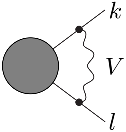
$$\Delta\sigma^{\mathrm{LL}}_{\mathrm{e^+e^-}} = \left(\sigma^{\mathrm{LL}}_{\mathrm{e}^+\nu_\mathrm{e}} - \sigma^{\mathrm{LL}}_{\mathrm{e^+e^-}}\right)\frac{\alpha}{4\pi s_\mathrm{w}^2}\ln^2\frac{s}{M_\mathrm{W}^2} = -\Delta\sigma^{\mathrm{LL}}_{\mathrm{e}^+\nu_\mathrm{e}}$$

cancellation recovered by summing over complete multiplets
(colour average in QCD!)

General framework for all mass-singular EW logarithms
in the Sudakov limit: $r_{kl} = (p_k + p_l)^2 \gg M_{\mathrm{W}}^2$ for all k, l
for arbitrary non-mass-suppressed virtual matrix elements
Denner, Pozzorini hep-ph/0010201

Soft-collinear logarithms

$$\sum_{V=\gamma,\mathrm{Z},\mathrm{W}} \sum_{k} \sum_{l\neq k}$$

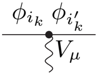


$k, l = 1, \ldots n$ external legs

eikonal approximation, high-energy limit $\Rightarrow$

$$\delta\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\frac{1}{2}\sum_{k=1}^{n}\sum_{l\neq k}\sum_{V=\gamma,Z,W^\pm} I^V_{i'_k i_k}(k) I^V_{i'_l i_l}(l) \mathcal{M}_0^{i_1\ldots i'_k \ldots i'_l \ldots i_n} \ln^2\frac{-r_{kl}-\mathrm{i}\epsilon}{M_V^2}$$

with gauge coupling matrices $I^V_{i'_k i_k}$



General formalism for one-loop EW logarithms (cont.)

split soft-collinear logarithms in universal and angular-dependent part

$$\ln^2 \frac{-r_{kl} - \mathrm{i}\epsilon}{M_V^2} = \underbrace{\ln^2 \frac{s}{M_V^2}}_{\text{leading}} + \underbrace{2\ln\frac{s}{M_V^2}\ln\frac{|r_{kl}|}{s} - 2\mathrm{i}\pi\theta(r_{kl})\ln\frac{|r_{kl}|}{s}}_{\text{subleading}} + \underbrace{\ln^2\frac{|r_{kl}|}{s}}_{\text{regular}}$$

leading soft-collinear logarithms

- gauge invariance ⇒ single sum over external legs

subleading soft-collinear logarithms

- given by sum over pairs of external legs
- angular-dependent $\left(\ln\frac{|r_{kl}|}{s} = \ln\frac{1+\cos\theta_{kl}}{2}\right)$
- depend on $\mathrm{SU(2)}\times\mathrm{U(1)}$ rotated Born matrix elements since $I^{W^\pm}_{i'_k i_k}$ non-diagonal (e.g. $\mathrm{e}\to\nu_\mathrm{e}$)

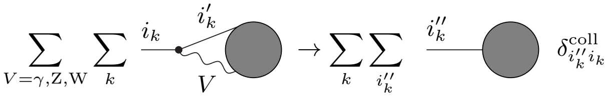
regular logarithms

- not part of a consistent expansion, but potentially numerically relevant

General formalism for one-loop EW logarithms (cont.)

Collinear logarithms

collinear-singular diagrams factorize into collinear factor times Born matrix element owing to gauge invariance/Ward identities hep-ph/0014127



combine with external-line wave-function renormalisation

$$\Rightarrow \qquad \delta^{\mathrm{C}}\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\mathcal{M}_0^{i_1..i'_k..i_n}\,\delta^{\mathrm{C}}_{i'_k i_k}$$

universal factors for external particles:

- associated with single external lines (factorisation)
- δ^{C} in general non-diagonal owing to mixing
- longitudinal gauge bosons behave as Goldstone bosons (Ward identity)

logarithms from parameter renormalisation

- contribute as counterterms (in high-energy approximation)

Establishment of SCET$_{\mathrm{EW}}$ Chiu, Golf, Kelley, Manohar 0709.2377, 0806.1240, ...

treatment of virtual EW corrections in SCET$_{\mathrm{EW}}$

- provides resummation of EW logarithms
- includes finite terms in high-energy limit via matching
- neglects power-suppressed terms of order M_{W}^2/r_{kl}

implementation in Monte Carlo code Denner, Rode 2402.10503
master formula (high scale $\mu_{\mathrm{h}} \sim \sqrt{s}$, low scale $\mu_{\mathrm{h}})) = M_{\mathrm{W}}$

$$\mathcal{M}_{\mathrm{SCET}} = \sum_{l,j} D_{1l}(\mu_{\mathrm{l}}) \left[\hat{\mathrm{P}} \exp\left(-\int_{\mu_{\mathrm{l}}}^{\mu_{\mathrm{h}}} \gamma(\mu)\,\mathrm{d}\ln\mu\right)\right]_{lj} \mathcal{M}_j(\mu_{\mathrm{h}})$$

with (l, j run over all processes related by SU(2) transformations)

- $\mathcal{M}_j(\mu_{\mathrm{h}})$: matrix elements in high-energy limit process dependent
  contain no mass logarithms but logarithms $\ln^2(r_{kl}/s)$, $\ln(r_{kl}/s)$
- $\gamma(\mu)$: anomalous dimension matrix, universal
  provides EW Sudakov logarithms and their resummation
- $D_{1l}(\mu_{\mathrm{l}})$: low-scale SCET$_{\mathrm{EW}}$ corrections, universal
  involve a single logarithm and finite mass-dependent terms

Logarithms/singularities in real corrections
arise from integration over phase space $\Rightarrow$ implicit logarithms

real photon radiation

- full inclusion   Denner, Rode 2402.10503
  - complexity of LO calculation
  - cancels IR singularities
  - integrated subtraction terms must be expanded in high-energy limit
- omission of logarithms related to photon   Pagani, Zaro 2110.03714
  - in sufficiently inclusive quantities real and virtual QED logarithms cancel
    $\Rightarrow$ discard photon contribution from virtual Sudakov logarithms
  - consistent at NLO but inconsistent for resummation

real emission of EW vector bosons

- separate IR-finite contribution, experimentally identifiable
- can be included as extra LO process if needed
- at LHC typically only a small fraction remains unresolved and compensates part of virtual corrections   Baur hep-ph/0611241.

Logarithmic approximation for real corrections

At very high energies (in particular for lepton colliders),
real EW corrections could/should be treated in analogy to QCD and QED
(with additional ingredients/difficulties, e.g. from longitudinal polarisation)
⇒ EW splitting functions, EW jets, EW parton showers

Some existing work

- EW splitting functions
  Chen, Han, Tweedie 1611.00788, Nardi, Ricci, Wulzer 2405.08220
- EW Parton Distribution and fragmentation functions
  Bauer, Ferland, Webber 1703.08562, Bauer, Provasoli, Webber 1806.10157,
  Fornal, Manohar, Waalewijn 1803.06347, Bauer, Webber 1806.10157
- EW showers
  Christiansen, Sjöstrand 1401.5238, Kleiss, Verheyen 2002.09248,
  Brooks, Skands, Verheyen 2108.10786, Masouminia, Richardson 2108.10817
- EW parton distributions for lepton colliders
  Han, Ma, Xie 2007.14300, 2103.09844, Ruiz, Costantini, Maltoni, Mattelaer,
  2111.02442, Garosi, Marzocca, Trifinopoulos 2303.16964

multiple emission of heavy vector bosons not relevant for LHC!

Outline

General results for virtual EW logarithmic corrections to arbitrary non-mass suppressed processes in Sudakov limit from Denner, Pozzorini hep-ph/0010201 implemented in

- ALPGEN (specific processes) Chiesa et al. 1305.6837
- MCFM (specific processes) Campbell et al. 1608.03356
- SHERPA (general processes) Bothmann, Napoletano 2006.14635
- MADGRAPH5_AMC@NLO (general processes) Pagani, Zaro 2110.03714
  Pagani, Vitos, Zaro 2309.00452; Ma, Pagani, Zaro 2409.09129
- OPENLOOPS (general processes) Lindert, Mai 2312.07927

optionally including some universal subsubleading non-mass-singular terms

Non-logarithmic terms can be consistently included via SCET_{EW}.
Chiu, Manohar et al. 1409.1918 and refs. therein.
Non-logarithmic terms are process dependent!
Recent implementation of SCET approach for di-boson processes in Monte Carlo integrator based on RECOLA2 Denner, Rode 2402.10503

Features of logarithmic approximation (LA)

- Algorithm/automation of leading EW logarithms only applies to virtual corrections
- simple formulas, complexity of tree-level calculation
- not directly applicable to processes with resonances $r_{kl} \sim M_\mathrm{W}^2$
  Sudakov limit requires $r_{kl} \gg M_\mathrm{W}^2$ for all k, l
- non-logarithmic terms neglected $\Rightarrow$ typical accuracy of few percent
- non-logarithmic terms may reach up to 10%
  (e.g. for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}_\mathrm{L}^+\mathrm{W}_\mathrm{L}^-$ for $\sqrt{s} = 3\,\mathrm{TeV}$)
- logarithmic approximation (LA) often not useful for inclusive quantities
  dominated by small scales, small EW corrections of $\mathcal{O}(\alpha/(s_\mathrm{w}^2\pi)) \sim 1\%$
- quality of LA needs to be checked case by case
  depends on process, distribution, and phase-space region

Modification/extension of logarithmic approximation

- Denner, Pozzorini hep-ph/0010201: imaginary part neglected in

$$\ln^2 \frac{-r_{kl} - \mathrm{i}\epsilon}{M^2} = \ln^2 \frac{|r_{kl}|}{M^2} - 2\mathrm{i}\pi\theta(r_{kl}) \ln \frac{|r_{kl}|}{M^2}$$

  since irrelevant for $2 \to 2$ processes (real amplitudes!)
  needed for more complicated processes

- implementations in MadGraph5_aMC@NLO and OpenLoops contain in addition to logarithms of hep-ph/0010201
  $\ln^2 \frac{r_{kl}}{s}$ terms resulting from

$$\ln^2 \frac{|r_{kl}|}{M^2} = \ln^2 \frac{s}{M^2} + 2 \ln \frac{s}{M^2} \ln \frac{|r_{kl}|}{s} + \ln^2 \frac{|r_{kl}|}{s}$$

  improve approximation in many cases,
  but not a result of a consistent expansion

- implementation in MadGraph5_aMC@NLO contains logarithmic corrections from QCD corrections (translated from QED results) relevant for processes with contributions of different orders in α_s at LO

Large NLO EW corrections to VBS processes

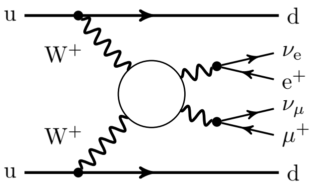
process	$\sigma_{\mathrm{LO}}^{\mathcal{O}(\alpha^6)}$ [fb]	$\Delta\sigma_{\mathrm{NLO,EW}}^{\mathcal{O}(\alpha^7)}$ [fb]	δ_{EW} [%]
Biedermann et al. 1708.00268	(Dittmaier et al. 2308.16716)		
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ ($\mathrm{W}^+\mathrm{W}^+$)	$1.4178(2)$	$-0.2169(3)$	-15.3
Denner et al. 1904.0088			
$\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$ (ZW^+)	$0.25511(1)$	$-0.04091(2)$	-16.0
Denner et al. 2009.00411			
$\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\mathrm{e}^-\mathrm{jj}$ (ZZ)	$0.097681(2)$	$-0.015573(5)$	-15.9
Denner et al. 2202.10844			
$\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\mathrm{e}^-\mathrm{jj}$ ($\mathrm{W}^+\mathrm{W}^-$)	$2.6988(3)$	$-0.307(1)$	-11.4

- EW corrections similar for all processes and rather independent of cuts
  $\Rightarrow$ intrinsic feature of VBS process
- smaller corrections to $\mathrm{W}^+\mathrm{W}^-$ due to Higgs resonance in fiducial phase space
  (Higgs contribution about 25%, corresponding EW corrections -6.5%)
- σ^{LO} receives sizeable contributions involving large invariants $r_{kl} \gg M_\mathrm{W}^2$

Source of large EW corrections for VBS

Double-pole approximation (DPA) for outgoing W bosons
effective vector-boson approximation (EVBA) for incoming W bosons

- DPA and EVBA reduce discussion to $V_1 V_2 \to V_3 V_4$
- DPA accurate for cross section within 1%
- EVBA crude approximation ($\sim 50\%$)
  Kuss, Spiesberger '96, Dittmaier et al. '23
  sufficient to understand dominant effects



high-energy, logarithm. approximation (LA) for $V_1 V_2 \to V_3 V_4$ Denner, Pozzorini '00

$$\mathrm{d}\sigma_{\mathrm{LL}} = \mathrm{d}\sigma_{\mathrm{LO}} \left[1 - \frac{\alpha}{4\pi} 4 C_{\mathrm{W}}^{\mathrm{EW}} \ln^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{\alpha}{4\pi} 2 b_{\mathrm{W}}^{\mathrm{EW}} \ln\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) \right]$$

$$C_{\mathrm{W}}^{\mathrm{EW}} = \frac{2}{s_{\mathrm{w}}^2}, \quad b_{\mathrm{W}}^{\mathrm{EW}} = \frac{19}{6 s_{\mathrm{w}}^2} \qquad \text{for transverse W bosons}, \qquad Q \to M_{4\ell}$$

(double EW logs, collinear single EW logs, and single logs from parameter renormalisation included) (angular-dependent logarithms omitted, $\ln \frac{t}{u} \ln \frac{Q}{M_{\mathrm{W}}}$)

large NLO EW corrections intrinsic feature of VBS

Simple formula for total cross section

$$\mathrm{d}\sigma_{\mathrm{LL}} = \mathrm{d}\sigma_{\mathrm{LO}}\left[1 - \frac{\alpha}{4\pi} 4C_{\mathrm{W}}^{\mathrm{EW}} \ln^2\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) + \frac{\alpha}{4\pi} 2b_{\mathrm{W}}^{\mathrm{EW}} \ln\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right)\right]$$

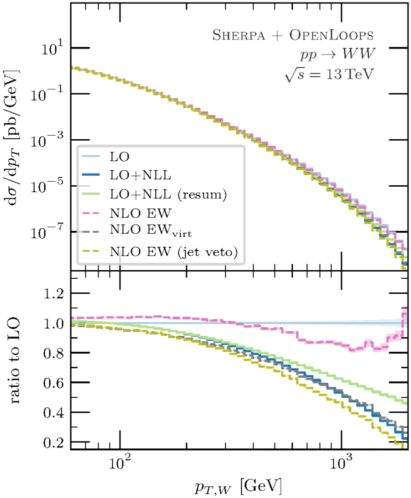
process	δ_{EW} [%]	$\delta_{\mathrm{EW}}^{\mathrm{log,diff}}$ [%]	$\delta_{\mathrm{EW}}^{\mathrm{log,int}}$ [%]	$\langle M_{4\ell}\rangle$ [GeV]
$\mathrm{pp} \to \mu^+\nu_\mu \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$	-16.0	-15.0	-16.1	390
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\nu_\mathrm{e}\mathrm{jj}$	-16.0	-16.4	-17.5	413
$\mathrm{pp} \to \mu^+\mu^- \mathrm{e}^+\mathrm{e}^-\mathrm{jj}$	-15.9	-14.8	-15.8	385

- surprisingly good agreement with complete calculation
- large EW corrections are due to large gauge couplings of vector bosons (C^{EW}) and large scale $Q \sim \langle M_{4\ell}\rangle \sim 400\,\mathrm{GeV}$
- angular-dependent logarithms depend on process ~ 1–2% owing to cancellations

large NLO EW corrections intrinsic feature of VBS

NLO EW corrections to distribution in transverse momentum of W boson

Bothmann, Napoletano 2006.14635 (Sherpa)



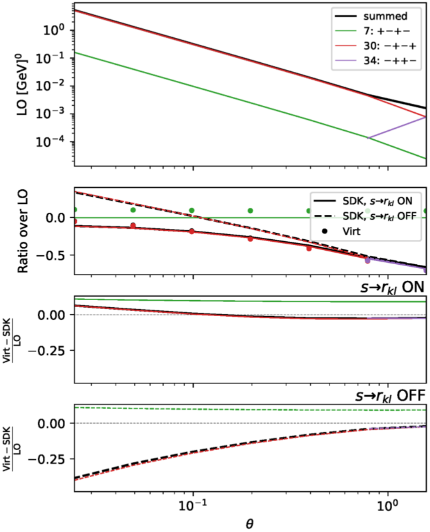
- NLO $\mathrm{EW_{virt}}$ contains only virtual EW corrections with IR subtraction using Catani–Seymour dipoles
- LO+NLL: EW logarithms in $\mathcal{O}(\alpha)$ via factor $(1 + K_{\mathrm{NLL}})$
- LO+NLL (resum): EW logarithms naively resummed $\exp(1 + K_{\mathrm{NLL}})$
- NLO EW contains photon-induced contributions and real corrections
- NLO EW (jet veto) contains veto on real radiation

- few percent difference between LO+NLL and $\mathrm{EW_{virt}}$ approximation up to $1\,\mathrm{TeV}$
- real contributions enhance corrections
- can be eliminated by jet veto

Logarithmic approximation for $u\bar{u} \to ZZ$ (10 TeV)

Virtual EW corrections to polarised squared matrix elements with IR-scale set to $\sqrt{s}$ as function of the scattering angle

Pagani, Zaro 2110.03714 (Madgraph)



Results for different polarisations of quarks and (on-shell) Z bosons (different colours)

Virt (dots)
full virtual corrections

$s \to r_{kl}$ ON (solid):
LA with $\ln^2(t/s)$

$s \to r_{kl}$ OFF (dashed):
LA without $\ln^2(t/s)$ terms

observations:

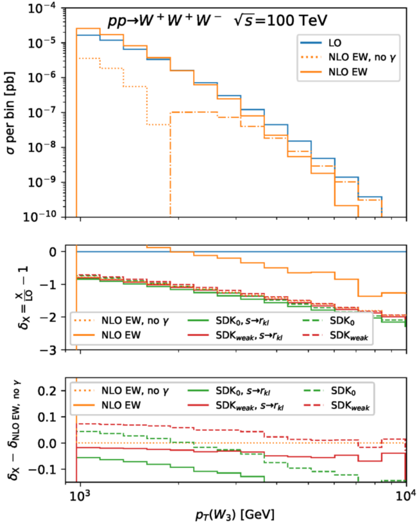
- for small angles $\alpha \ln^2(t/s)$ terms contribute 35%
- finite terms amount to 10%

$\Rightarrow$ inclusion of $\ln^2(t/s)$ terms improves approximation

Logarithmic approximation for $\mathrm{pp} \to \mathrm{W}^+\mathrm{W}^+\mathrm{W}^-$ (100 TeV)

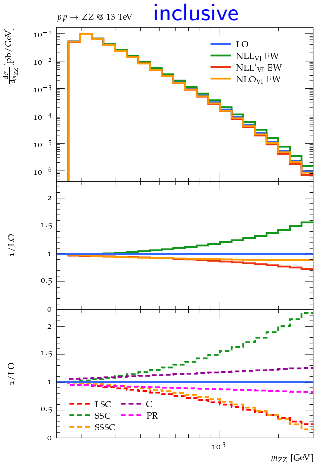
NLO EW corrections for distribution in the transverse momentum of the third leading W boson

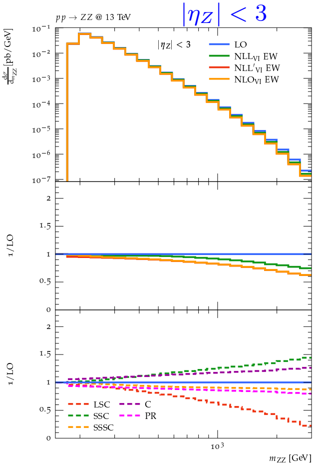
Pagani, Zaro 2110.03714 (Madgraph)



- photon-induced processes not included in LA as in NLO EW, no γ (dashed)
- $s \to r_{kl}$ (solid) contains squared angular logarithms $\ln^2(t/s)$, (dashed) does not
- SDK_0: photon treated as massive with mass M_W
  $\mathrm{SDK}_\mathrm{weak}$: all electromagnetic logarithms removed (apart from parameter renormalisation)
- EW corrections reach -200%
- LA $\mathrm{SDK}_\mathrm{weak}$ with $s \to r_{kl}$ reproduces relative EW corrections (excluding photon-induced) within 10%

Distribution in invariant mass of Z-boson pair   Lindert, Mai 2312.07927 (OpenLoops)
virtual EW corr. with IR poles subtracted via Catani–Seymour I operator $\mathrm{NLO_{VI}}$ EW
$\mathrm{NLL'_{VI}}$ EW contains $\ln^2(t/s)$ terms, $\mathrm{NLL_{VI}}$ EW does not





large effect of $\ln^2(t/s)$ terms (SSSC) reduced to 10% in restricted $|\eta_Z|$ range

LA requires $r_{kl} \gg M_{\mathrm{W}}^2$ for all invariants
$\Rightarrow$ not directly applicable to processes with resonances

solution: narrow-width approximation or pole approximation

$\mathrm{pp} \to \mathrm{e}^+\nu_\mathrm{e}\mu^-\nu_\mu \quad \Rightarrow \quad \mathrm{pp} \to \mathrm{W}^+\mathrm{W}^-,\ \mathrm{W}^+ \to \mathrm{e}^+\nu_\mathrm{e},\ \mathrm{W}^- \to \mu^-\nu_\mu$
(and resonant W-boson propagators)

- LA applicable to $\mathrm{pp} \to \mathrm{W}^+\mathrm{W}^-$
- no large logarithms in $\mathrm{W}^- \to \mu^-\nu_\mu$ and $\mathrm{W}^+ \to \mathrm{e}^+\nu_\mathrm{e}$

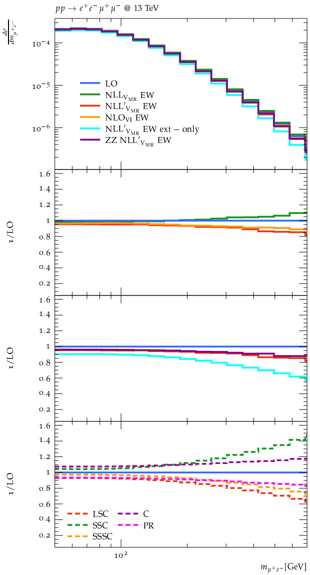
problem: restricted accuracy of narrow-width/pole approximation

proposal by Lindert, Mai 2312.07927
combine off-shell and on-shell processes via probabilities based on kinematic projectors $P(k)$ to include logarithms for both simultaneously
probability for on-shell boson:

$$P(k) = \left| \frac{\mu^2 - w^2 M^2 \Gamma^2}{(k^2 - M^2 + \mathrm{i} w M \Gamma)^2 + \mu^2} \right| = \begin{cases} 1 & \text{if } k^2 \to M^2 \\ 0 & \text{if } k^2 \to \infty \end{cases}$$

($w = 10$ scaling factor, $\mu^2 = M^2 - \mathrm{i} M \Gamma$)

Logarithmic approximation for $\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^-$



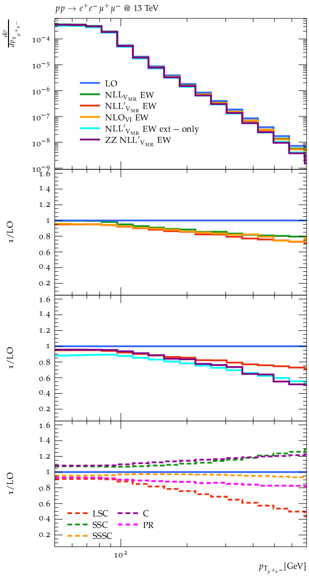
Distribution in invariant mass $m_{\mu^+\mathrm{e}^-}$

LA for processes with resonances using kinematic projectors   Lindert, Mai 2312.07927 (OpenLoops)

- $\mathrm{NLO_{VI}EW}$: IR-subtracted virtual NLO EW corrections
- $\mathrm{NLL'_{V_{MR}}EW}$: LA for combined full and on-shell process
  $\Rightarrow$ approximates within few %
- $\mathrm{NLL'_{V_{MR}}EW}$ ext-only : LA for full process
  $\Rightarrow$ deviates up to 20%
- ZZ $\mathrm{NLL'_{V_{MR}}}$: LA for on-shell process
  $\Rightarrow$ approximates within few %

Quality of on-shell approximation depends on distribution!

Logarithmic approximation for $\mathrm{pp} \to \mathrm{e}^+\mathrm{e}^-\mu^+\mu^-$



Distribution in transverse momentum $p_{\mathrm{T},\mu^+\mathrm{e}^-}$

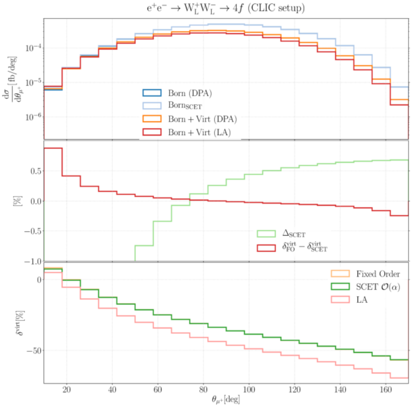
LA for processes with resonances using kinematic projectors   Lindert, Mai 2312.07927 (OpenLoops)

- $\mathrm{NLO_{VI}EW}$: IR-subtracted virtual NLO EW corrections
- $\mathrm{NLL'_{V_{MR}}EW}$: LA for combined full and on-shell process
  $\Rightarrow$ approximates within few %
- $\mathrm{NLL'_{V_{MR}}EW}$ ext-only : LA for full process
  $\Rightarrow$ deviates up to 20%
- ZZ $\mathrm{NLL'_{V_{MR}}}$: LA for on-shell process
  $\Rightarrow$ deviates up to 20%

Quality of on-shell approximation depends on distribution!

$e^+e^- \to W_L^+W_L^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^-$ for $\sqrt{s} = 3\,\mathrm{TeV}$ Denner, Rode 2402.10503

virtual EW corrections to distribution in μ production angle for longitudinal W bosons

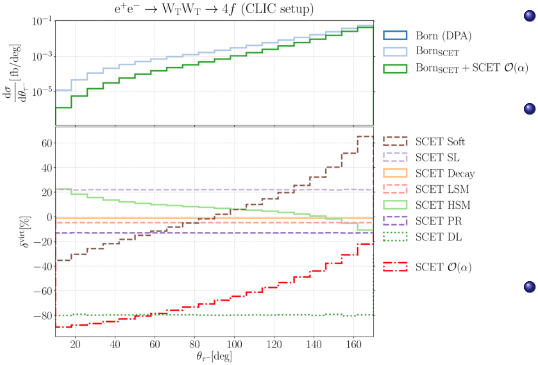


- SCET neglects all power-suppressed corrections $\propto M_W^2/s$
- SCET deviates by up to 5% at LO (Δ_{SCET})
- SCET $\mathcal{O}(\alpha)$ reproduces full $\mathcal{O}(\alpha)$ to better than 1% ($\delta^{\mathrm{virt}}_{\mathrm{FO}} - \delta^{\mathrm{virt}}_{\mathrm{SCET}}$) (relative corrections in SCET)
- LA deviates by up to 13% from full NLO result

$e^+e^- \to W_T^+ W_T^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^-$ for $\sqrt{s} = 3\,\text{TeV}$

Denner, Rode 2402.10503

individual SCET$_{\text{EW}}$ virtual corrections to distribution in τ production angle for transverse W bosons



- SCET neglects all power-suppressed corrections
- SCET $\mathcal{O}(\alpha)$ reproduces full $\mathcal{O}(\alpha)$ to better than 0.5%
- $\mathcal{O}(\alpha)$ corrections dominated by double logarithms (DL) and angular-dep. logarithms (Soft)
- non-logarithmic corrections in high-scale matching (HSM), in low-scale matching (LSM), and in corrections to boson decay (Decay)

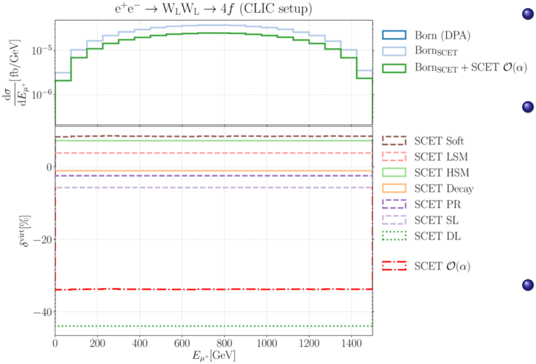
- 20% corrections in HSM [contains all(!) $\ln^2(s/t)$ and $\ln(s/t)$ terms]
- -4% corrections in LSM

W-pair production at CLIC using $\mathrm{SCET_{EW}}$

$\mathrm{e^+e^-} \to \mathrm{W_L^+W_L^-} \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^-$ for $\sqrt{s} = 3\,\mathrm{TeV}$

Denner, Rode 2402.10503

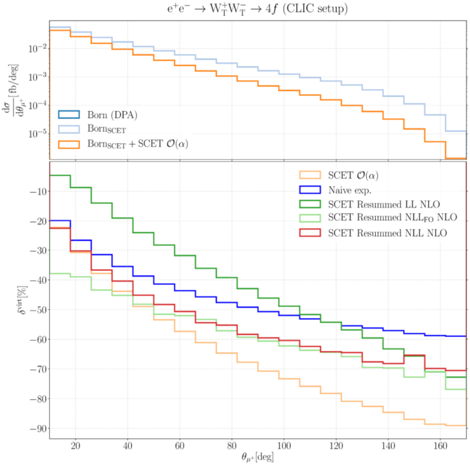
individual $\mathrm{SCET_{EW}}$ virtual corrections to distribution in μ energy for longitudinal W bosons



- SCET neglects all power-suppressed corrections
- SCET $\mathcal{O}(\alpha)$ reproduces full $\mathcal{O}(\alpha)$ to better than 1%
- $\mathcal{O}(\alpha)$ corrections dominated by double logarithms (DL) and angular-dep. logarithms (Soft)
- non-logarithmic corrections in high-scale matching (HSM), in low-scale matching (LSM), and in corrections to boson decay (Decay)
- 7% constant corrections in HSM [contains all(!) $\ln^2(s/t)$ and $\ln(s/t)$ terms]
- 4% constant corrections in LSM

$\mathrm{e^+e^-} \to \mathrm{W_L^+W_L^-} \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^-$ for $\sqrt{s} = 3\,\mathrm{TeV}$ Denner, Rode 2402.10503

virtual EW corrections to distribution in μ production angle for transverse W bosons



resummation of EW logarithms

- resummed LL NLO: $\exp(\alpha L^2) + \alpha L + \alpha$
- resummed $\mathrm{NLL_{FO}}$ NLO: $\exp(\alpha L^2)(1 + \alpha L) + \alpha$
- resummed NLL NLO: $\exp(\alpha L^2 + \alpha L) + \alpha$
- naive exp.: $\exp(\delta^{\mathrm{virt}}_{\mathrm{FO}})$ (as in Sherpa)

- complete NLL exponentiation important, effects of 20%
- naive exponentiation deviates by 5–10%

Outline

Status of EW logarithmic Sudakov corrections

- EW corrections at high energies dominated by large logarithms
  $\ln^{(2)}(E/M_\mathrm{W}) \Rightarrow \mathcal{O}(20\text{–}50\%)$ at LHC
- simple generic results exist for virtual EW logarithmic corrections
  to non-mass suppressed matrix elements
  with complexity of tree-level calculation
- large cancellations between leading and subleading logarithms
- EW corrections in logarithmic approximation (LA) (plus improvements)
  implemented in automated tools
  Sherpa, MadGraph5_aMC@NLO, OpenLoops:
  - LA can describe virtual EW corrections within 10%.
  - Photon-induced channels (opening at NLO) have to be treated separately.
  - Results from LA should be checked against full calculation if available.
- SCET$_\mathrm{EW}$ provides a consistent framework to include finite terms as well.
  (in the high-energy limit, involving process-dependent contributions!)
- LA useful to resum corrections beyond NLO.

Thank You!