



# Diboson Polarization Measurements Latest updates from ATLAS

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# What is diboson polarization?

#### **Polarization:**

Polarization describes the alignment of a particle's spin with its momentum. Quantified using the helicity:

$$h = \vec{S} \cdot \frac{\vec{p}}{|p|}$$

**Transversal polarization (T):** the spin and momenta are (anti-)aligned (h = 1, -1)

**Longitudinal polarization (L or 0):** spin orthogonal to the momenta (h = 0)

# Transversal Right-handed: P SLongitudinal P

#### **Diboson polarization:**

Refers to the study of the polarization states of pairs of bosons (e.g., *WW*, *WZ*, *ZZ*) produced in high-energy particle collisions, namely 00, 0T, T0, TT

## What is diboson polarization?

Joint Polarization:

- Joint polarization involves the simultaneous consideration of the polarization states of two bosons produced in the same event.
- In diboson events, the polarizations of both bosons can be correlated  $\rightarrow$  providing information about the vector boson couplings.

uncorrelated:  $P(h_{V1}h_{V2}) = P(h_{V1}) \cdot P(h_{V2})$ correlated:  $P(h_{V1}h_{V2}) \neq P(h_{V1}) \cdot P(h_{V2})$ 

Joint polarization is described by a combined spin density matrix:

$$\begin{split} \rho_{\lambda_{W}\lambda'_{W}\lambda_{Z}\lambda'_{Z}} &\equiv \frac{1}{C} \times \sum_{\mu_{q}\mu_{\bar{q}}} F_{\lambda_{W}\lambda_{Z}}^{(\mu_{q}\mu_{\bar{q}})} F_{\lambda'_{W}\lambda'_{Z}}^{(\mu_{q}\mu_{\bar{q}})*} & f_{00} &= \rho_{0000} \,, \\ f_{\mathrm{TT}} &= \rho_{++--} + \rho_{--++} + \rho_{---+} + \rho_{++++} \,, \\ \text{where} \quad C &= \sum_{\mu_{q}\mu_{\bar{q}}\lambda_{W}\lambda_{Z}} \left| F_{\lambda_{W}\lambda_{Z}}^{(\mu_{q}\mu_{\bar{q}})} \right|^{2} & f_{0T} &= \rho_{00--} + \rho_{00++} \,, \\ f_{\mathrm{TO}} &= \rho_{--00} + \rho_{++00} \,. \end{split}$$

### Why study diboson polarization?

In standard model, the Higgs mechanism provides mass to the *W* and *Z* bosons through spontaneous symmetry breaking of the electroweak symmetry:

- Spontaneous breaking of a continuous symmetry gives rise to a massless Goldstone boson (ξ)
- □ By choosing a proper gauge, the Goldstone bosons is absorbed into the gauge fields → forming new massive gauge bosons
- None-zero mass indicates gauge fields with three polarization states (two transverse and one longitudinal)

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = e^{i\vec{\tau} \cdot \vec{\xi}/2v} \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix}$$

$$\begin{aligned} U(\xi) &= e^{-i\vec{\tau}\cdot\vec{\xi}/2v} \\ \phi' &= U(\xi)\phi = \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}(v+H)\chi \\ \vec{A}'_{\mu} &= U(\xi)\vec{A}_{\mu}U(\xi)^{-1} - \frac{i}{g}(\partial_{\mu}U(\xi))U^{\dagger}(\xi) \end{aligned}$$

#### Thus,

#### gauge boson polarization is strongly related to the electroweak theory in the standard model!

## Why study diboson polarization?

#### What we can learn from diboson polarization study?

Validation of Theoretical Predictions:

By studying diboson polarization, we can validate the Standard Model electroweak thoeries

#### **Probing New physics:**

- Discrepancies between observed polarization patterns and SM predictions could indicate beyond standard model (BSM) physics.
- New physics might behave differently with different polarization, which might have been overlooked by those unpolarized study

	SM	BSM
$q_{L,R}\bar{q}_{L,R} \to V_L V_L(h)$	$\sim 1$	$\sim E^2/M^2$
$q_{L,R}\bar{q}_{L,R} \to V_{\pm}V_L(h)$	$\sim m_W/E$	$\sim m_W E/M^2$
$q_{L,R}\bar{q}_{L,R} \to V_{\pm}V_{\pm}$	$\sim m_W^2/E^2$	$\sim E^2/M^2$
$q_{L,R}\bar{q}_{L,R} \to V_{\pm}V_{\mp}$	$\sim 1$	$\sim 1$
JHEP	<u>02 (2018) 11</u>	<u>1</u>
high-energy behavior of am helicity configurations, in th	plitudes with die SM and in ge	ifferent diboson eneric BSM (meaning

helicity configurations, in the SM and in generic BSM (meaning the maximal effect that can be achieved with an insertion of any d = 6 operator) with cutoff scale M

In addition to the single boson polairzation study, diboson joint polarization study provides an chance to inspect correlation between the helicity of interacting gauge bosons

# How to study diboson polarization?

Angular variables between the bosons and the decay products are typically used to measure the weak bosons polarizations

- Choosing proper variables can effectively distinguish different polarizations
- Templates of different polarization process can be made utilizing the distribution of those variables

The fraction of each polarization state can be extracted by performing fits to data (with background subtracted) distributions using those polarized templates



Polarization measurements are frame dependent  $\rightarrow$  For all measurements you need to define a frame

#### Variables sensitive to diboson polarization

The vector boson scattering angle:  $\theta_v$ 



#### Variables sensitive to diboson polarization

Child lepton decay angle of vector bosons:  $\theta_{\rho}^{*}$ :

$$\frac{1}{\sigma_{W^{\pm}Z}} \frac{\mathrm{d}\sigma_{W^{\pm}Z}}{\mathrm{d}\cos\theta^{*}_{\ell^{Z}}} = \frac{3}{8} f_{\mathrm{L}} (1 + 2\alpha\cos\theta^{*}_{\ell^{Z}} + \cos^{2}\theta^{*}_{\ell^{Z}}) + \frac{3}{8} f_{\mathrm{R}} (1 - 2\alpha\cos\theta^{*}_{\ell^{Z}} + \cos^{2}\theta^{*}_{\ell^{Z}}) + \frac{3}{4} f_{0}\sin^{2}\theta^{*}_{\ell^{Z}}$$

$$\frac{1}{\sigma_{W^{\pm}Z}} \frac{\mathrm{d}\sigma_{W^{\pm}Z}}{\mathrm{d}\cos\theta_{\ell W}^{*}} = \frac{3}{8} f_{\mathrm{L}} [(1 - q_{W} \cdot \cos\theta_{\ell W}^{*})^{2}] + \frac{3}{8} f_{\mathrm{R}} [(1 + q_{W} \cdot \cos\theta_{\ell W}^{*})^{2}] + \frac{3}{4} f_{0} \sin^{2}\theta_{\ell W}^{*}$$

Phys. Rev. D 99, 055001 (2019)

\*Defined in the modified helicity frame as the angle between "the negatively (positively for  $W^+$ ) charged lepton as seen in the W(Z) rest frame" and "the direction of the W(Z) which is given in the WZ centre-of-mass frame".

![](_page_7_Figure_6.jpeg)

#### Variables sensitive to diboson polarization

#### MVA variables:

- BDT, DNN...
- □ In general provides better separating power than using single variable

![](_page_8_Figure_4.jpeg)

#### **Polarized diboson samples**

Recent developments make it possible to separate diboson production into different polarisations  $\rightarrow$  Input templates for fitting!

Some available polarized samples:

- □ LO polarised  $qq \rightarrow ZZ$  samples: MadGraph5\_aMC@NLO 2.7.3, interfaced to Pythia 8.240 for parton shower, hadronisation, and underlying event
- □ LO polarised  $qq \rightarrow WZ$  sample: MadGraph5\_aMC@NLO 2.7.3, interfaced to Pythia 8.244 for parton shower, hadronisation and the underlying event

Currently polarized samples are only available at Leading Order (LO)

Official high-order corrections (k-factors) not yet available from ATLAS Metadata Interface (AMI) for polarized samples  $\rightarrow$  High-order corrections are made by each analysis team

## **High order correction**

High-order corrections include contributions from both real emissions (additional particles) and virtual particles (loops):

- □ To account for the real part of NLO QCD corrections, events were simulated with either 0,1 jets in the matrix element at LO, and merged with Pythia parton shower using the CKKW-L scheme
- Electroweak and virtual part corrections still missing.

![](_page_10_Figure_4.jpeg)

### **High order correction**

Several ways to achieve full (real+virtual, QCD+EWK+Other) NLO corrections:

- Reweight NLO inclusive diboson sample to NLO polarized diboson samples using MVA method
  - Reweighting factors obtained from LO polarized sample to LO inclusive sample
  - □ Four sets (00, 0T, T0, TT) of DNN-based event-by-event reweighting factors as functions of variables sensitive to polarization
  - Adopted by <u>Phys. Lett. B 843 (2023) 137895</u>
- Reweight LO polarized sample (0+1 jet) to NLO polarized sample according to theoretical calculation (QCD+EWK+Other)
  - $\square \quad \text{Theorists provide NLO distribution in fiducial regions} \rightarrow \text{compared with LO 0+1 distribution} \rightarrow \text{k-factor!}$

![](_page_11_Figure_8.jpeg)

- Additional corrections (e.g, interference effect, higher order effects...) are also considered
- Adopted by JHEP 12 (2023) 107 and Phys. Rev. Lett. 133 (2024) 101802 (as systematic uncertainty)

## Analyses of diboson polarization

Measurements at LEP:

 $e^{-}e^{+} \rightarrow WW$ :

OPAL @189 GeV 183 pb<sup>-1</sup> [Eur. Phys. J. C 19, 229-240 (2001)]

DELPHI @189-209 GeV 520 pb<sup>-1</sup> [Eur. Phys. J. C 63, 611 (2009)]

Measurements at the LHC:

 $pp 
ightarrow W^{\pm}W^{\pm}jj$ 

CMS @13TeV 137 fb<sup>-1</sup> (VBS phase space) <u>Phys. Lett. B 812 (2020) 136018</u>  $\rightarrow$  The electroweak  $W^{\pm}W^{\pm}$  production with at least one longitudinal W bosons measured with observed (expected) significance of 2.3 (3.1) standard deviations.

 $pp \rightarrow ZZ$ 

ATLAS @13TeV 140 fb<sup>-1</sup> (inclusive phase space) <u>JHEP 12 (2023) 107</u>  $\rightarrow$  Simultaneously longitudinal ZZ production measured with observed (expected) significances of 4.3 (3.8) standard deviations

 $pp \rightarrow WZ$ 

CMS @13TeV 137 fb<sup>-1</sup> (inclusive phase space) <u>JHEP 07 (2022) 032</u>  $\rightarrow$  Simultaneously longitudinal *WZ* production observed at the level of five standard deviations for the first time

ATLAS @13TeV 139 fb<sup>-1</sup> (inclusive phase space) <u>Phys. Lett. B 843 (2023) 137895</u>  $\rightarrow$  Simultaneously longitudinal *WZ* production measured with observed (expected) significances of 7.1 (6.2) standard deviations

ATLAS @13TeV 140 fb<sup>-1</sup> (high pT(Z) phase space) Phys. Rev. Lett. 133 (2024) 101802 Just published!

Studies of the energy dependence of diboson polarization fractions and the Radiation Amplitude Zero (RAZ) effect in WZ production with the ATLAS detector (Phys. Rev. Lett. 133 (2024) 101802)

**RAZ effect:** At leading-order, the dominant helicity amplitude with two transversely-polarized bosons is exactly zero when the scattering angle of the W boson in the WZ rest frame with respect to the incoming antiquark direction approaches 90°

$$|\mathcal{M}_{WZ}^{TT}|^2 = \frac{g^4}{32} \left(\frac{s}{t} - \frac{s}{u}\right)^2 \sin^2 \theta_V (1 + \cos^2 \theta_V), \qquad |\mathcal{M}_{WZ}^{LL}|^2 = \frac{g^4}{16} \sin^2 \theta_V.$$

RAZ effect behaves differently between different polarization states  $\rightarrow$  inspires two topics of this analysis:

- □ *WZ* joint polarization fraction at 00 enhanced region
- Direct quantification of RAZ effect

Joint polarization fraction measurement:

**Event selection starts from inclusive WZ region, and:** 

#### pT(Z) > 100 (200) GeV to select *s*-channel events

- At leading-order, WZ production is dominated by the TT polarization due to the presence of t- and u-channel
- t-channel significantly enhances the forward-scattering amplitude
- □ High pT(Z) cut tamed *t*-channel contribution and hence enhance 00 fraction! → A better chance to observe 00 polarization states and measure the low 00 fraction!
- □ s-channel  $\rightarrow$  probing "joint" polarization fraction where both bosons come from the same vertex

#### pT(WZ) < 70 GeV to suppress jet multiplicity

- RAZ effect significant only at LO
- Reducing jet activity enhances RAZ effect, hence enhances separation power between polarization
- $\Box$  Achieved by requiring low pT(*WZ*)

![](_page_14_Figure_12.jpeg)

Use BDT variable to perform the fit and extract the joint polarization fraction

RAZ effect leads to a dip around 0 in the  $\Delta Y$  (*WZ*) and  $\Delta Y$  ( $\ell_W Z$ ) distributions  $\rightarrow$  contributing separation power to BDT

Variable $\Delta Y(\ell_W Z)$  $p_T(WZ)$  $p_T^{\ell}(W)$ Subleading  $p_T^{\ell}(Z)$  $E_T^{miss}$  $\cos \theta^{\ell}(Z)$  $\cos \theta^{\ell}(W)$ 

![](_page_15_Figure_4.jpeg)

Process	$100 < p_T^Z \le 200 \text{ GeV}$	$p_T^Z > 200 \text{ GeV}$
$W_0Z_0$	$222 \pm 5$	$47.6 \pm 1.5$
$W_0 Z_T + W_T Z_0$	$323 \pm 12$	$23.7 \pm 0.8$
$W_T Z_T$	$856 \pm 31$	$124 \pm 4$
Prompt background	$169 \pm 18$	$24.1 \pm 2.7$
Non-prompt background	$68 \pm 29$	$2.8 \pm 1.1$
Total Expected	$1640 \pm 60$	222 ± 8
Data	1740	236

Phys. Rev. Lett. 133 (2024) 101802

The measured fractions are found to be consistent with the SM predictions.

A non-zero fraction of events where both bosons are longitudinally polarized is measured with an observed significance of  $5.2\sigma$  (1.6 $\sigma$ ) in the phase space with 100 < pT(Z) ≤ 200 GeV (pT(Z) > 200 GeV)

	Measurement			Prediction		
	$100 < p_T^Z \le 200 \text{ GeV}$	$p_T^Z > 200 \text{ GeV}$	100 < p	$P_T^Z \le 200 \text{ GeV}$	$p_T^Z > 200 \text{ GeV}$	
$f_{00}$	$0.19 \pm _{0.03}^{0.03} (\text{stat}) \pm _{0.02}^{0.02} (\text{syst})$	$0.13 \pm _{0.08}^{0.09} (\text{stat}) \pm _{0.02}^{0.02} (\text{syst}) \parallel f_0$	0.15	$52 \pm 0.006$	$0.234 \pm 0.007$	
$f_{0T+T0}$	$0.18 \pm_{0.08}^{0.07} (\text{stat}) \pm_{0.06}^{0.05} (\text{syst})$	$0.23 \pm _{0.18}^{0.17} (\text{stat}) \pm _{0.10}^{0.06} (\text{syst})  f_0$	0.12	$20 \pm 0.002$	$0.062 \pm 0.002$	
ftt	$0.63 \pm_{0.05}^{0.05} (\text{stat}) \pm_{0.04}^{0.04} (\text{syst})$	$0.64 \pm _{0.12}^{0.12} (\text{stat}) \pm _{0.06}^{0.06} (\text{syst}) \int f_{12}^{0.06} f_{12}^{0.06} (\text{syst}) \int f_{12}^{0.06} (\text{syst})$	ro 0.10	$9 \pm 0.001$	$0.058\pm0.001$	
$f_{00}$ obs (exp) sig.	5.2 (4.3) $\sigma$	$1.6 (2.5) \sigma$    $f_1$	<i>TT</i> 0.61	$9 \pm 0.007$	$0.646 \pm 0.008$	

**Direct RAZ measurement:** 

RAZ effect leads to a dip around 0 in the  $\triangle Y$  (*WZ*) and  $\triangle Y$  ( $\ell_W Z$ ) distributions  $\rightarrow$  Makes it possible to quantify RAZ effect by measuring the depth of the dip in  $\triangle Y$  (*WZ*) and  $\triangle Y$  ( $\ell_W Z$ ) distributions

- **The RAZ effect only happens for**  $W_{\gamma}$  and  $W_{Z}$  production
- Observed in  $W_{\gamma}$  events at both  $p\overline{p}$  and pp colliders.
- □ Never measured in WZ before  $\rightarrow$  This analysis is the first study of RAZ effect in WZ channel!

#### Difficulty to observe RAZ effect and the treatment in this analysis:

□ 00, 0T and T0 events make the dip less significant

 $\rightarrow$ Solution: None-TT contributions are subtracted from data to check TT dip only. WZ 00, 0T, and T0 contributions are normalized to the SM predicted cross sections

 $\mathcal{D} = 1 - 2 \times N_{\text{central}}^{\text{unf}} / N_{\text{sides}}^{\text{unf}}$ 

RAZ effect significant only at LO, higher order effects dilute the RAZ effect

→**Solution:** Requiring low pT(WZ) to suppress jet multiplicity and hence high-order effect → pT(WZ) < 20, 40, or 70 GeV is applied to define three regions to measure the depth

#### Unfolded $|\Delta Y(\boldsymbol{\ell}_{w} Z)|$ and $|\Delta Y(WZ)|$ distributions are measured and compared to theoretical predictions

The depth of the RAZ dip, represented by the variable  $\mathcal{D} = 1 - 2 \times N_{\text{central}}^{\text{unf}} / N_{\text{sides}}^{\text{unf}}$ 

where  $N_{\text{central}}^{\text{unf}}(N_{\text{sides}}^{\text{unf}})$  indicates the number of events with  $|\Delta Y(WZ)| < 0.5$  (0.5 <  $|\Delta Y(\ell_WZ)| < 1.5$ ) after the unfolding. A positive value of  $\mathcal{D}$  indicates the existence of a dip.

![](_page_18_Figure_3.jpeg)

Significant dips are observed in the  $\Delta Y(\ell_w Z)$  and  $\Delta Y(WZ)$  distributions!

### **Prospects for diboson polarization**

Possible to probe not only the breaking of but also the restoration of electroweak symmetry through diboson polarization study

n<sup>V</sup> [GeV]

- Goldstone boson equivalence theorem (GBET): At high energy, the longitudinal degree of freedom of a massive vector boson is equivalent to the Goldstone boson (*G*)
- At high energy scales, the effects of symmetry breaking become less dominant → Restoration of electroweak symmetry!
  - In this limit the EW gauge bosons become massless. That is, only the transverse polarizations persist and the longitudinal polarizations are replaced by their associated Goldstone bosons
  - **G** Symmetry of the terms involving  $G^{\pm}$ ,  $G^{0}$ , and *h*

#### What can be observed at high energy:

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix}$$

 $\begin{array}{c} \widehat{W}_{0} = \widehat{U}_{0,3,5}^{2} \\ \widehat{W}_{0,3,5} = \\ 2.5 \\ 2.5 \\ 2.5 \\ 2.5 \\ 2.5 \\ 2.5 \\ 0.5 \\$ 

PTLOCI	[0, 200]	[200, 100]	[100, 000]	[000,000]	[000, 1000]	[1000, 1000]
$\sigma(W_0Z_0)$ (fb)	784	58.5	4.84	0.799	0.188	0.0749
$\sigma(W_0h)$ (fb)	387	46.5	4.30	0.726	0.169	0.0671
$\sigma(Z_0h)$ (fb)	198	24.5	2.24	0.367	0.0835	0.0327
$p_{\pi}^{V}$ [GeV]	[0, 20	001 [200, 400	01 [400, 600	1 [600, 800]	[800, 1000]	[1000, 1500]
$\frac{\sigma(W_0Z_0)}{\sigma(W_0)}$	h) 2.02	3 1.26	1.13	1.10	1.11	1.11
$\sigma(W_0Z_0)/\sigma(Z_0)$	h) 3.90	6 2.39	2.16	2.18	2.25	2.29

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In the Higgs mechanism, a massive W boson acquired its longitudinal component by absorbing a Goldstone boson from the Higgs sector. When the W is at rest, it is not so clear which polarization state comes from the original vector boson and which comes from the Higgs boson. However, for a highly boosted W, there is a clear distinction between the transverse and longitudinal polarization states. Then,

$$\mathcal{M}(X \to Y + W_0^+(p)) = \mathcal{M}(X \to Y + \pi^+(p)) \ (1 + \mathcal{O}(m_W/E_W))$$

Helicity cross sections (in fb)

[0 200] [200 400] [400 600] [600 800] [800 1000] [1000 1500]

#### **Prospects for diboson polarization**

Prospects of VBS diboson polarization measurement in HL-LHC: Reviews in Physics 8 (2022) 100071

#### CMS:

- $\square$   $W^{\pm}W^{\pm}jj$ : The expected significance for an integrated luminosity of 3000 fb<sup>-1</sup> is expected to reach 2.7 standard deviations, and exceed  $3\sigma$  when combining CMS and ATLAS results.
- $\Box$  *ZZjj*: The expected significance for selecting the VBS  $Z_L Z_L$  event fraction is 1.4 $\sigma$  with 3000 fb<sup>-1</sup> at the CMS experiment. Comparable results are anticipated for the ATLAS experiment.

#### ATLAS:

- □  $W^{\pm}W^{\pm}jj$ : The significance is expected to be 1.8 $\sigma$ . The expected precision of the measurement of the LL  $W^{\pm}W^{\pm}$  scattering cross section is 47%.
- $W^{\pm}Zjj$ : Different selections are tested, namely the nominal selection requiring  $m_{jj} > 500, 600, 1100$  GeV, as well as two different BDT cuts. The luminosity is doubled to emulate the combination of the ATLAS and CMS data samples. The expected significance for the polarization signal is  $0.5-3.5\sigma$  depending on the selection

![](_page_20_Figure_8.jpeg)

## Summary

- □ The study of diboson polarization is interesting for both validating standard model electroweak theory and probing BSM new physics
- □ New techniques makes it possible to better study diboson polarization
  - MC generation could provide polarized samples in LO, with NLO corrections available with helps from the theorists
  - Utilization of MVA methods provides better separation between different polarization states
- □ Multiple measurements of diboson polarization has been done with LEP and LHC. The latest ATLAS diboson polarization analysis was introduced today → First study of the energy-dependence of diboson polarization fractions in  $WZ \rightarrow \ell v \ell \ell$  and the first study of the Radiation Amplitude Zero effect in WZ process
- Still rich physics potential to be discovered with diboson polarization
  - Prospects of VBS diboson polarization measurement in HL-LHC: <u>Reviews in Physics 8 (2022)</u> <u>100071</u>

#### back up

## **High order correction**

Formulation of reweighting LO polarized sample (0+1 jet) to NLO polarized sample according to theoretical calculation (QCD+EWK+Other)

$$k_{factor} = \frac{MoCaNLO_{Polarized}^{Parton}}{MadGraph01LO_{Polarized}^{Particle}} \times \frac{Sherpa\_NLO_{Inclusive}^{Particle}}{MoCaNLO_{Inclusive}^{Parton}} \xrightarrow{\text{Theoretical polarized NLO calculation}}_{\rightarrow reweight it to get particle level (after PS) values}$$

If the theory calculation for combined QCD+EWK+Other is not available, one can get QCD correction and EWK correction individually, then combine them in either additive way or multiplicative way:

$$d\sigma_{\rm NLO_{+}} = d\sigma_{\rm LO} \left(1 + \delta_{\rm QCD} + \delta_{\rm EW}\right) + \text{other corrections}$$
$$d\sigma_{\rm NLO_{\times}} = d\sigma_{\rm LO} \left(1 + \delta_{\rm QCD}\right) \left(1 + \delta_{\rm EW}\right) + \text{other corrections}$$

The difference can be taken as uncertainty

#### **High order correction**

![](_page_24_Figure_1.jpeg)

![](_page_24_Figure_2.jpeg)