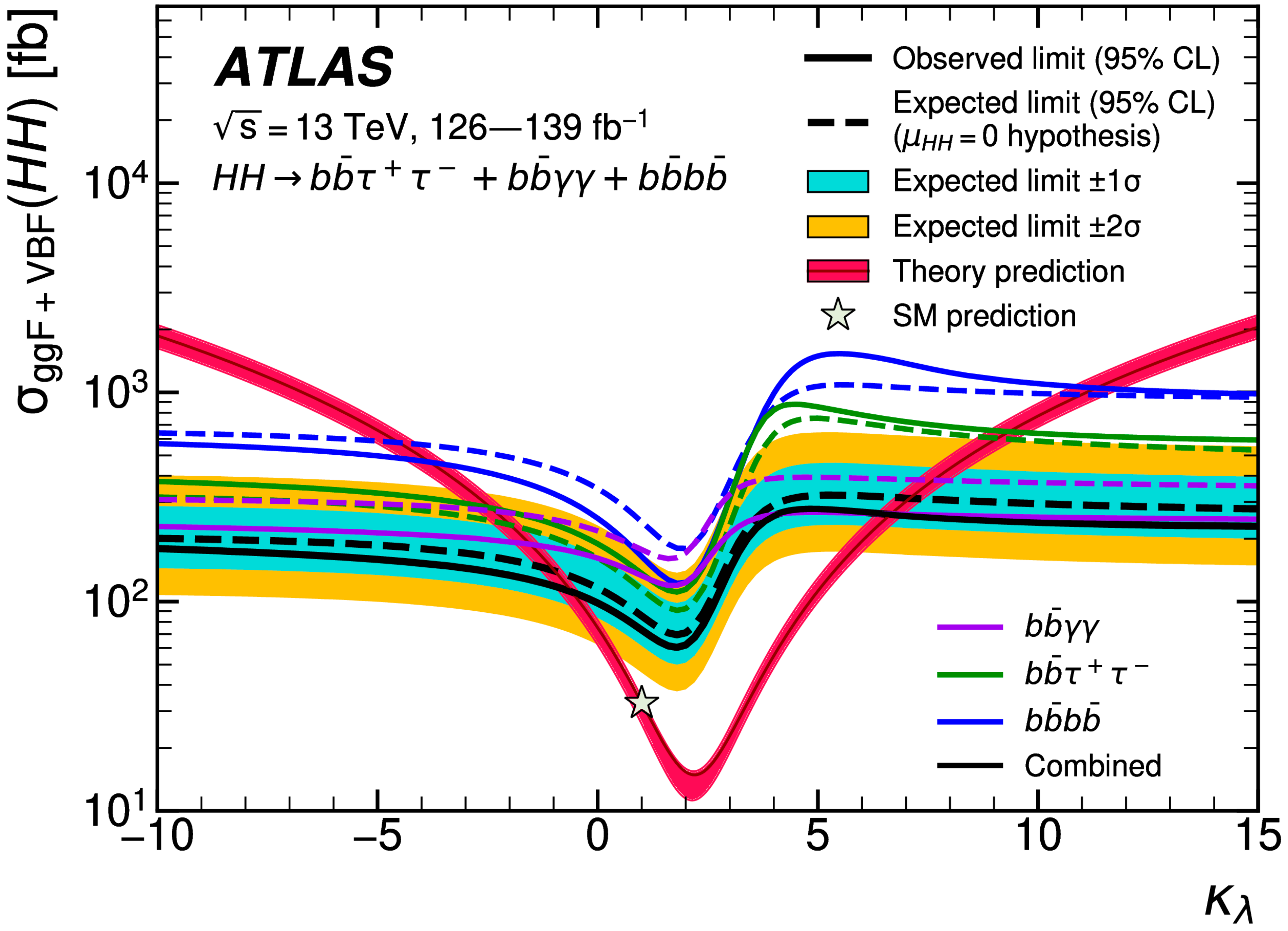
Multi-Higgs Production & Electroweak Phase Transition

Osama Karkout

Working with
Jorinde van de Vis, Marieke Postma, Andreas Papaefstathiou, Gilberto Tetlalmatzi, Tristan du Pree

Project: ATLAS Higgs results → matter-antimatter asymmetry



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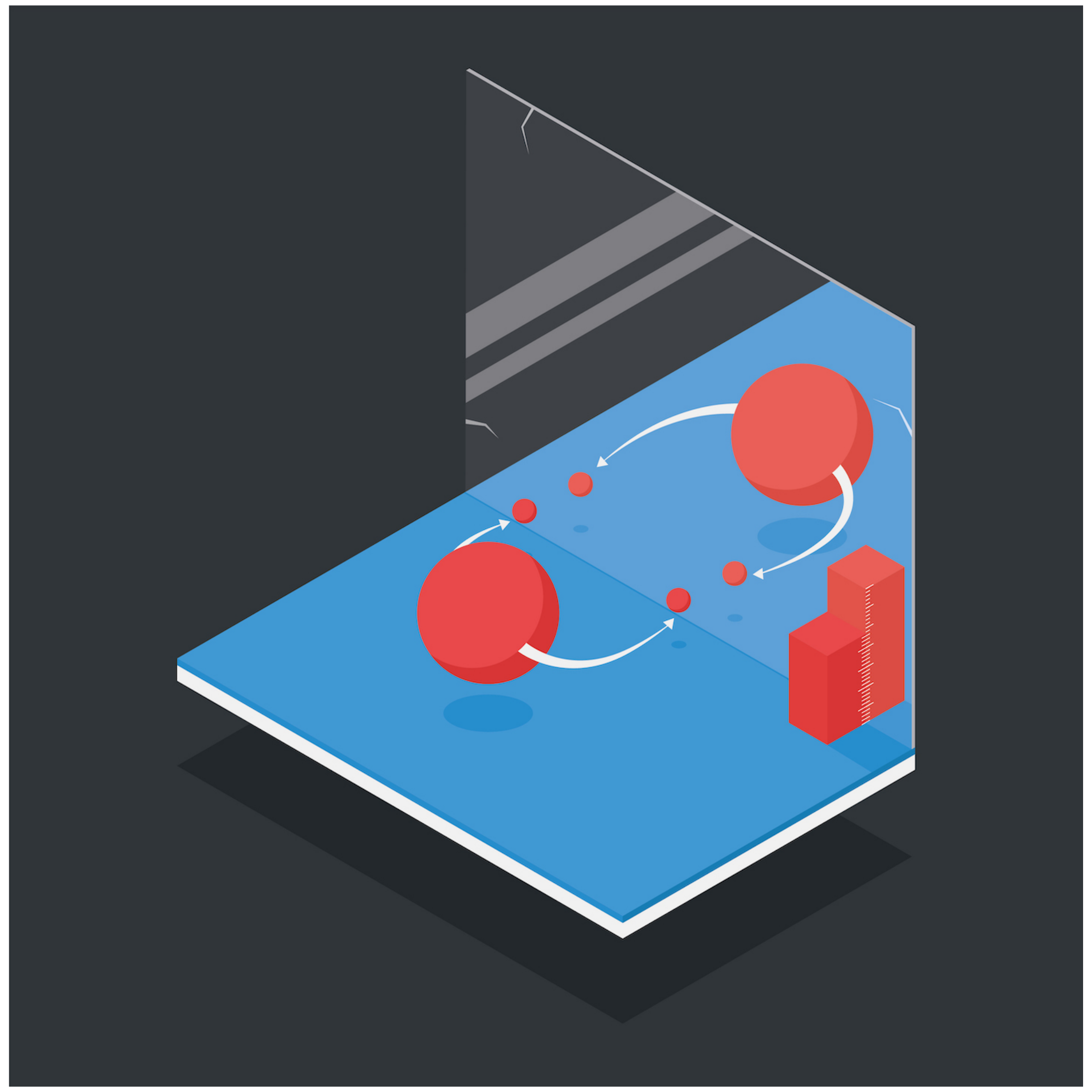


matter-antimatter asymmetry

Cosmic rays: $\bar{p}/p = 10^{-4}$

= no ambient antiprotons ($\bar{p}$)

BIG DEAL!

Lorentz invariance

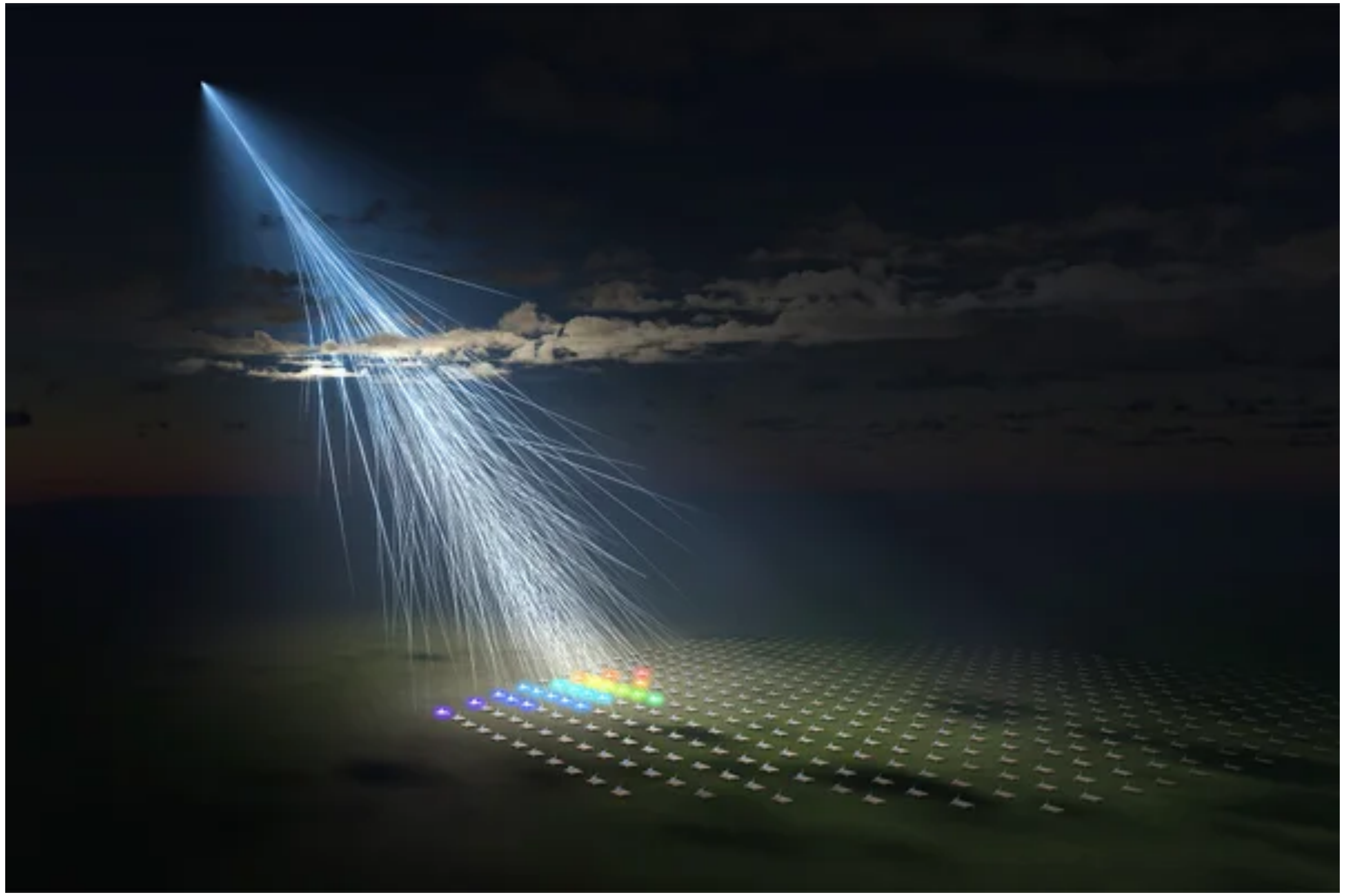
+

Hermitian Hamiltonian (physical observables are real)

=

matter-antimatter symmetry (CPT) is conserved!

True in SM and any BSM!!!!



Baryogenesis (matter-antimatter asymmetry)

Problem: we exist :(

(CPT) is conserved => need for **dynamical** mechanism to generate matter-antimatter asymmetry.

Sakharov conditions:

- Baryon number violation
- Loss of thermal equilibrium
- Break C and CP symmetries

In SM: all related to the Higgs field

https://arxiv.org/pdf/hep-ph/0609145.pdf
https://arxiv.org/pdf/2301.05197.pdf
http://www.laine.itp.unibe.ch/cosmology/lec09.pdf

BARYOGENESIS

James M. Cline

Baryogenesis (matter-antimatter asymmetry)

Sakharov is mostly known for his political activism for individual freedom, human rights, civil liberties

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BARYOGENESIS

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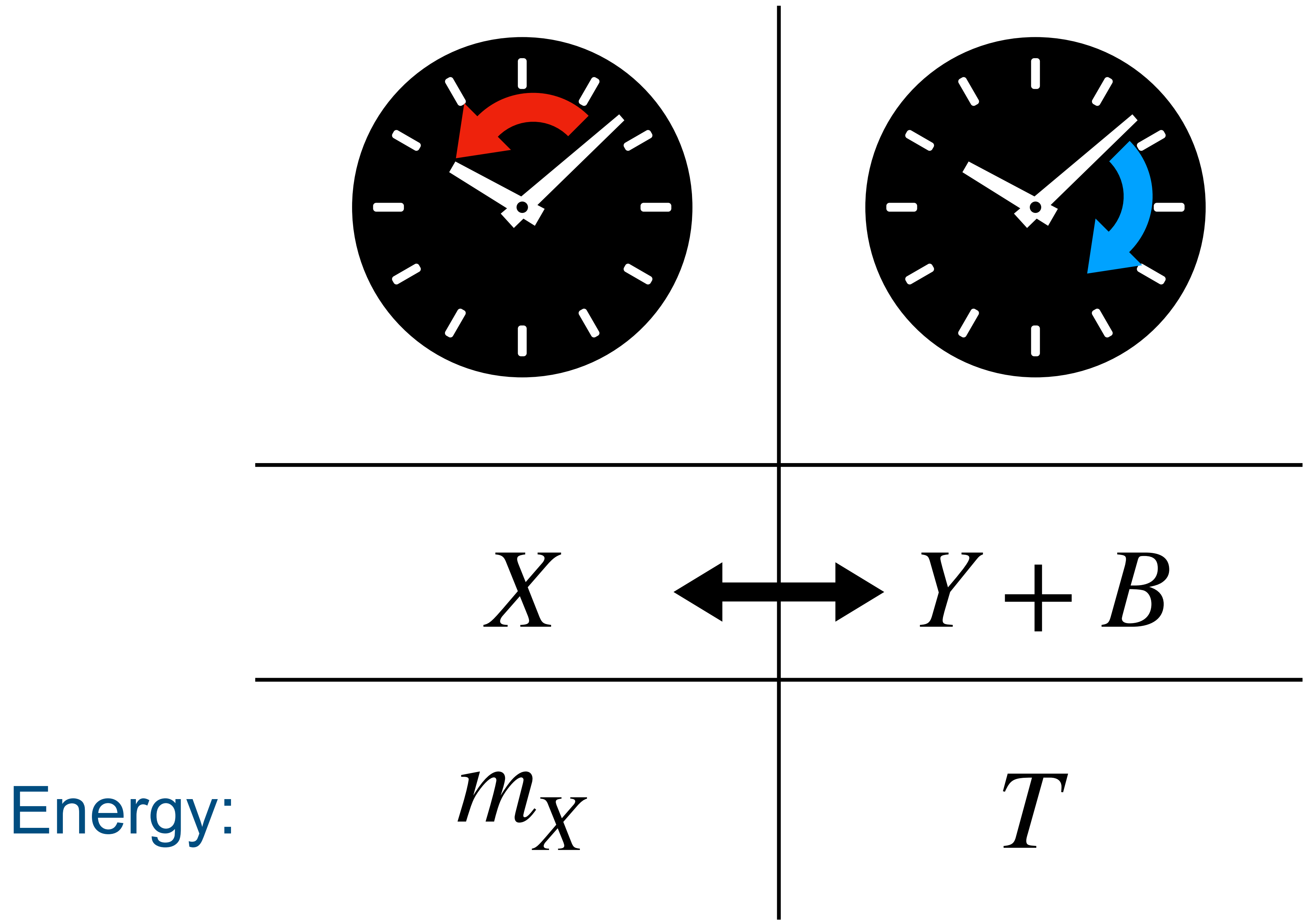
Electroweak Baryogenesis

Out of thermal equilibrium

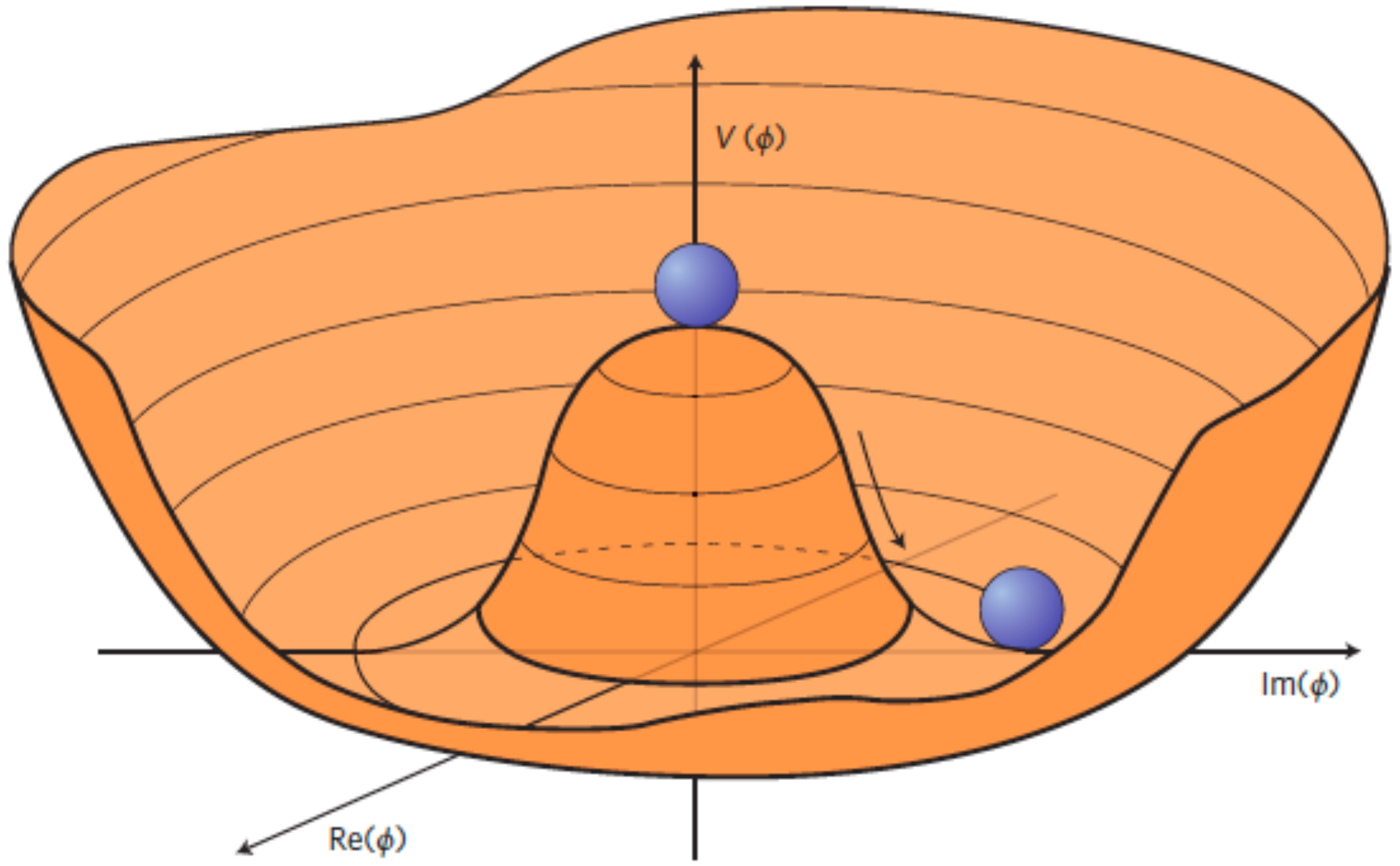
In thermal equilibrium:
any process that generates some extra B
$X \rightarrow Y + B$
comes with the inverse process at the same rate
$Y + B \rightarrow X$

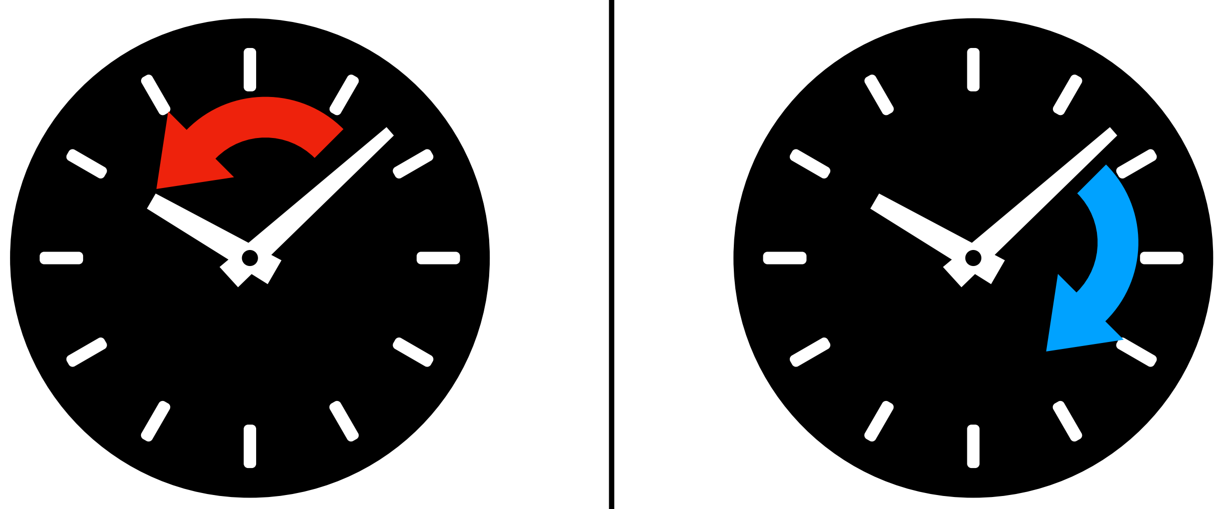
Out of thermal equilibrium if for example $T < m_X$
$Y + B \rightarrow X$ surpassed by $e^{-m_X/T}$



Electroweak Baryogenesis

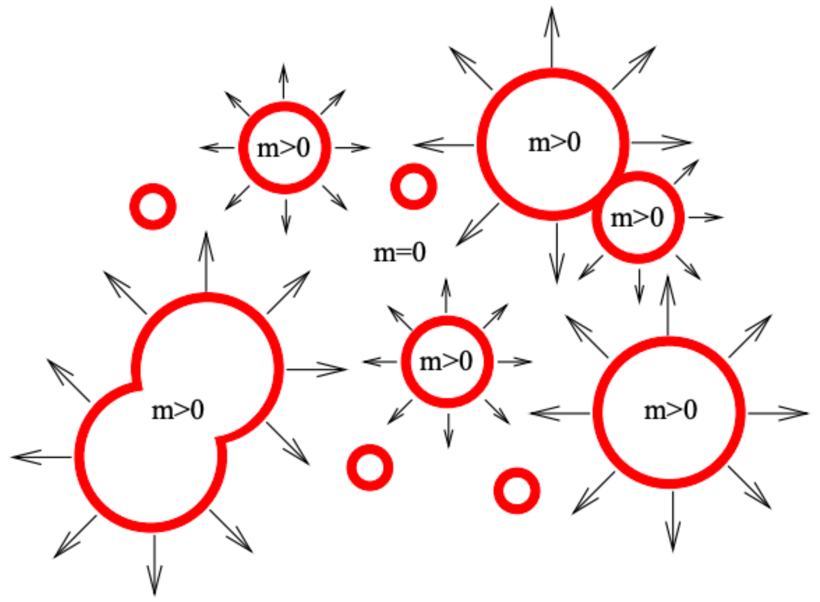
Out of thermal equilibrium



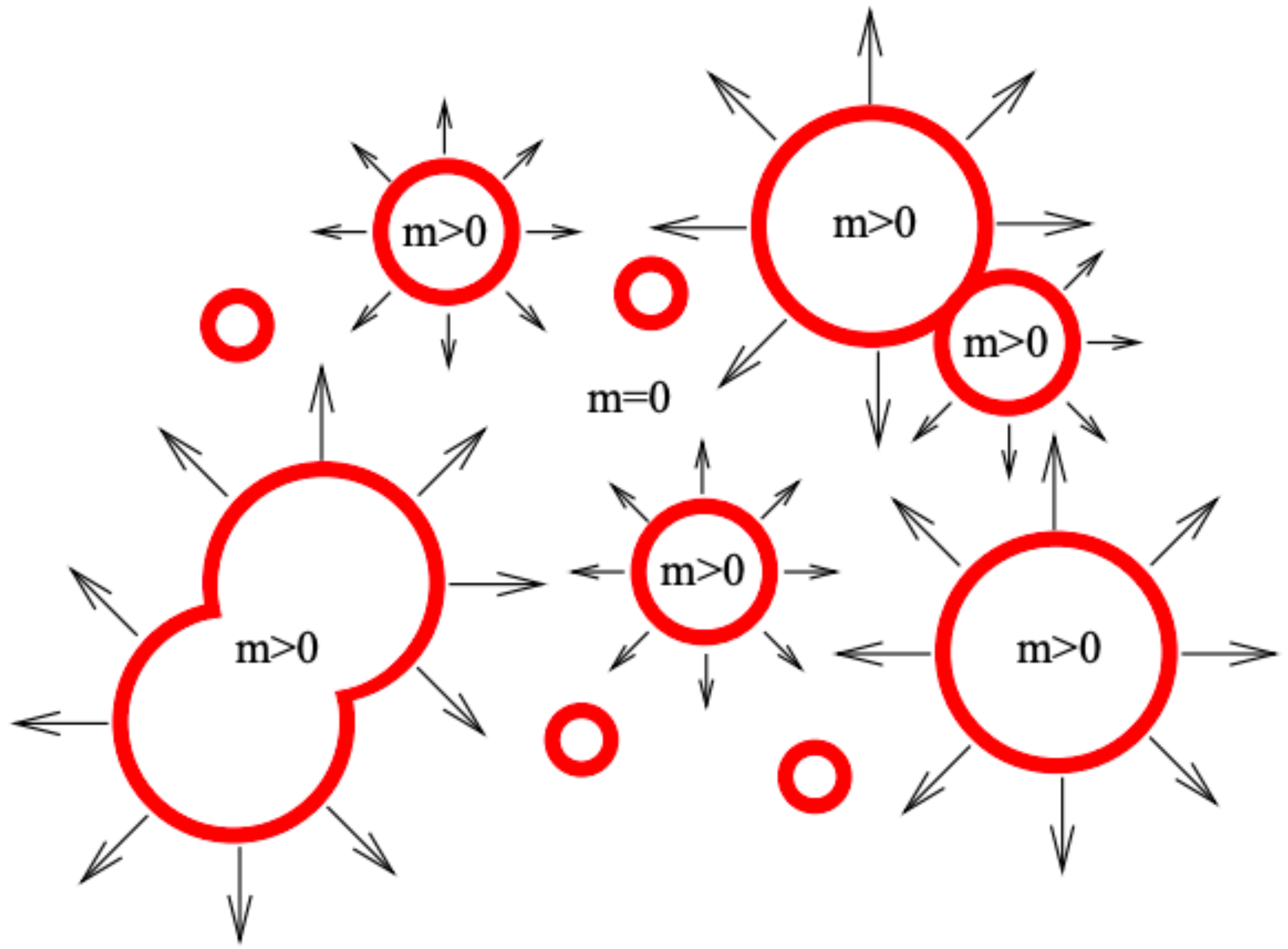


	X	$\longleftrightarrow$	$Y + B$
Energy:	m_X		T

Electroweak symmetry breaking (EWSB) is a phase transition!
It can cause loss of thermal equilibrium if it is a First Order Phase Transition (FOPT)



Electroweak Baryogenesis



First Order Phase Transition (FOPT)

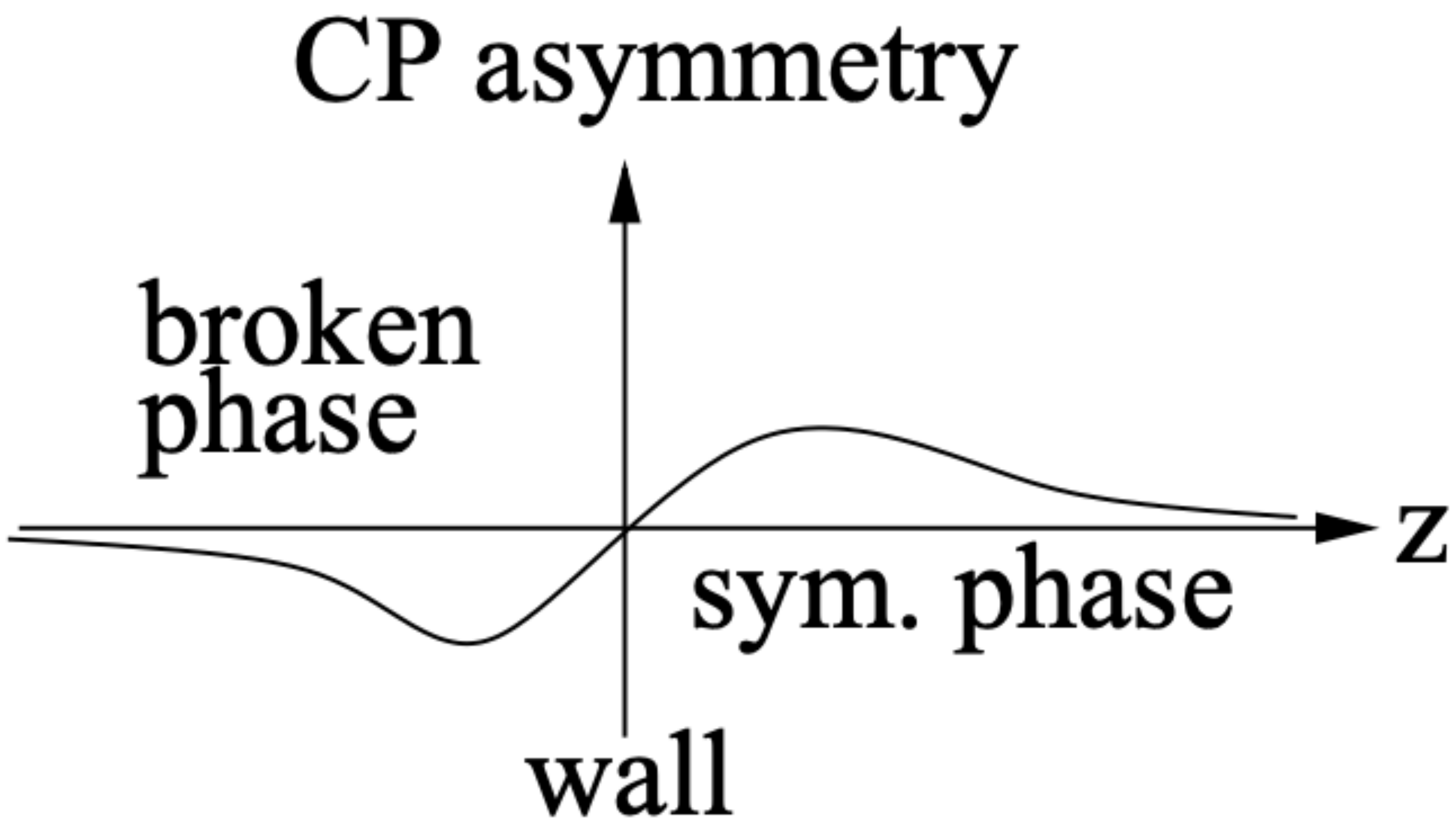
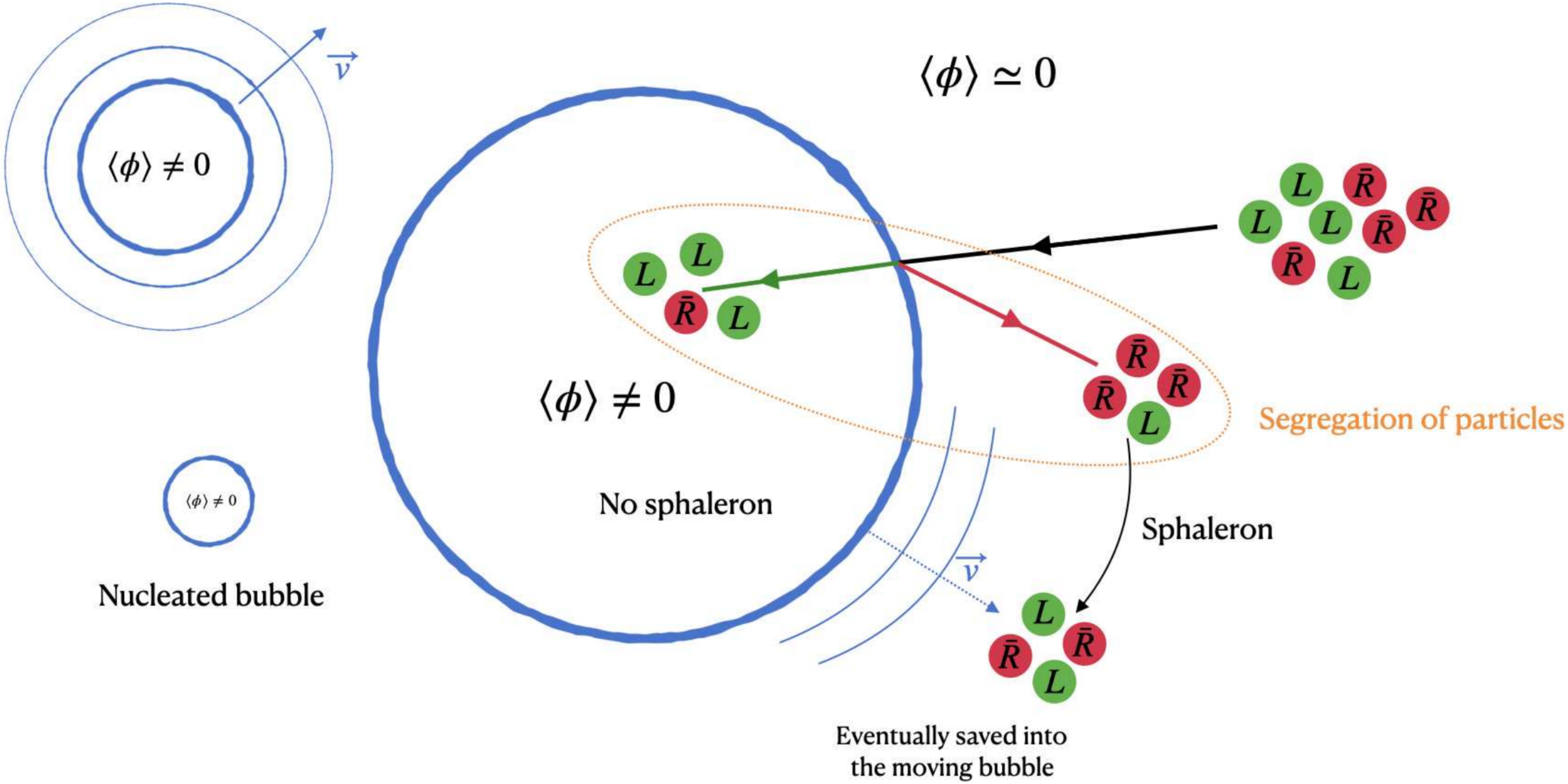


Fig. 13. The CP asymmetry which develops near the bubble wall.

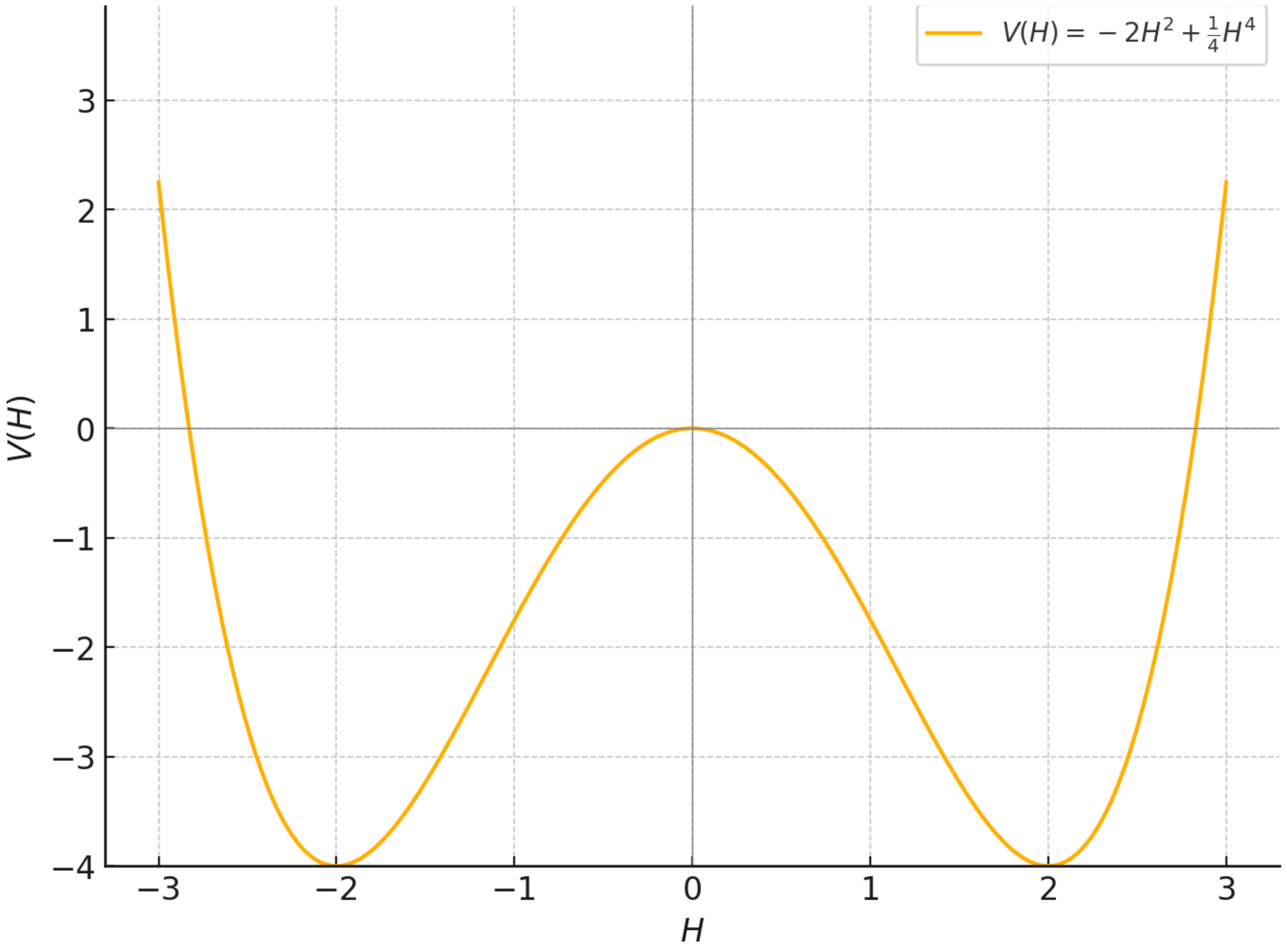
Electroweak Baryogenesis



Electroweak Phase Transition: Thermal QFT

Before symmetry breaking, Higgs potential is:

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4$$



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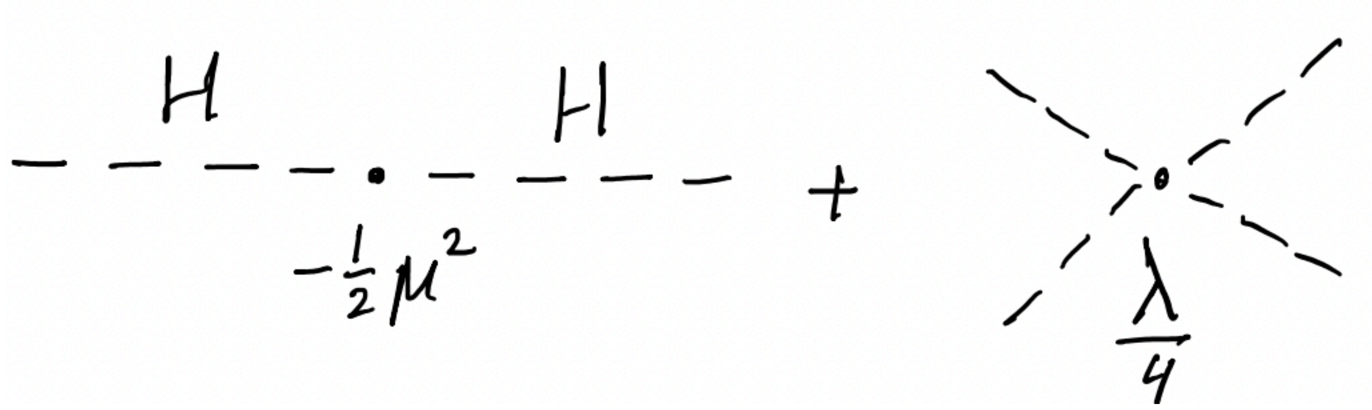
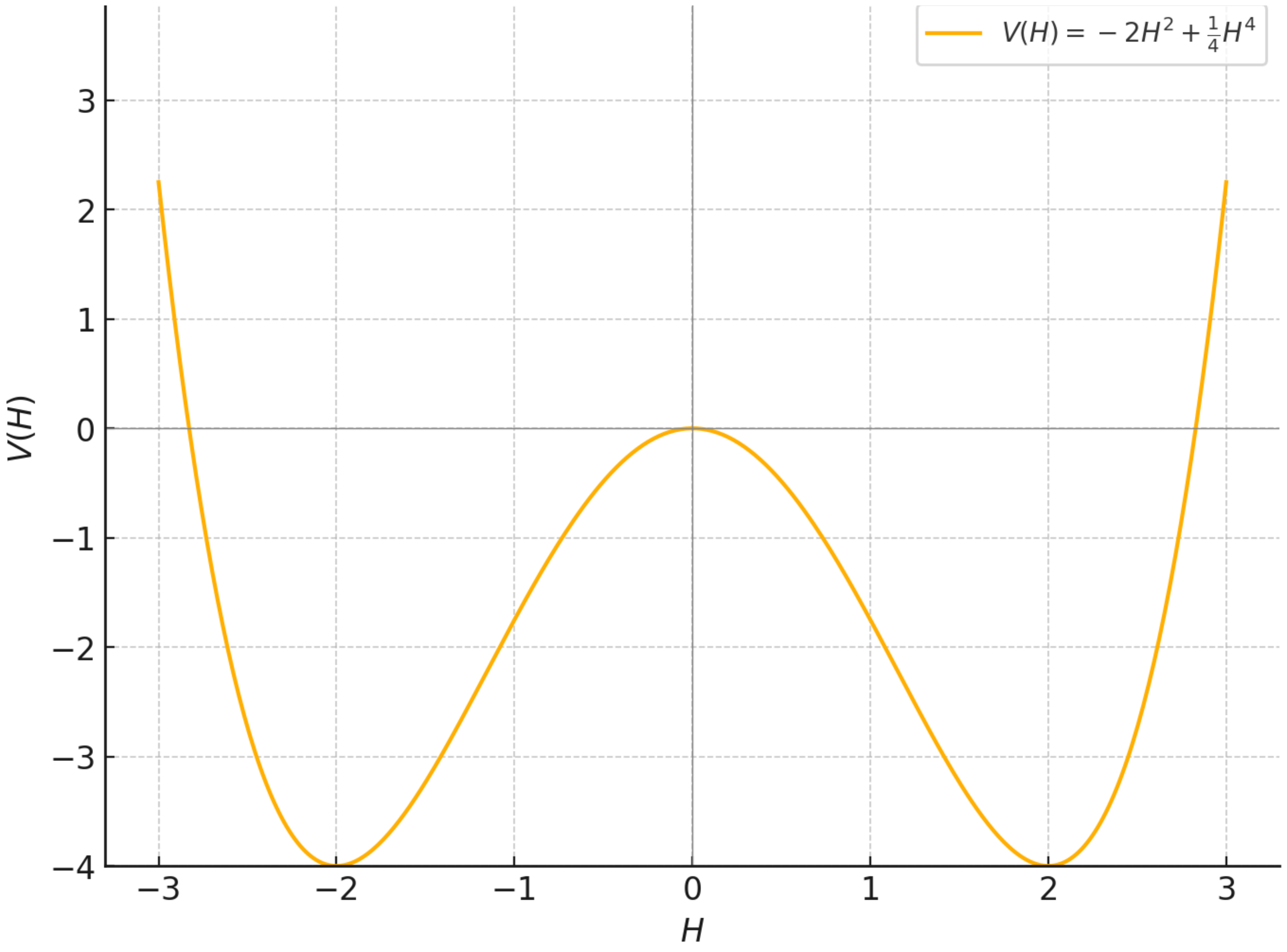
$V(H) =$ 



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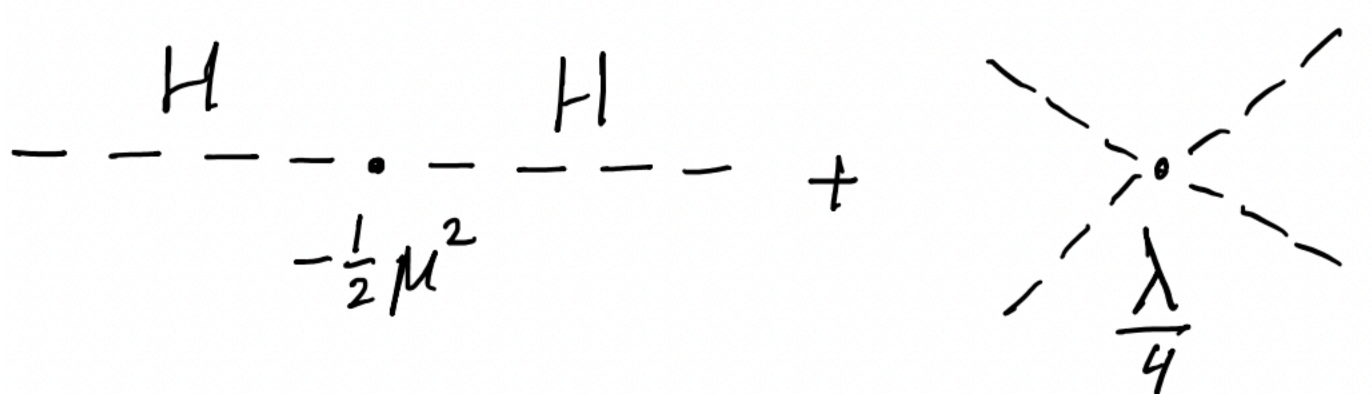
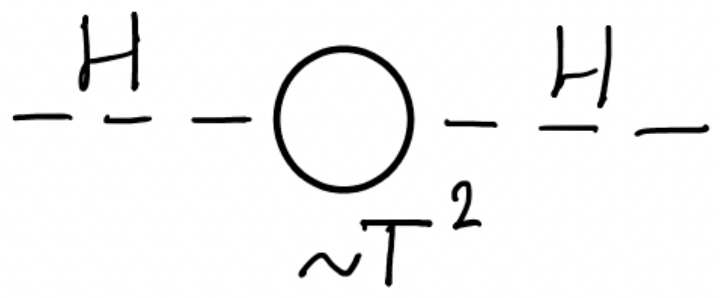
$V(H) =$ 

Higgs field is coupled to a thermal bath of fields.
at LO, this looks like:



$$V_{eff}(H,T) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + \frac{\alpha}{2}T^2 H^2$$

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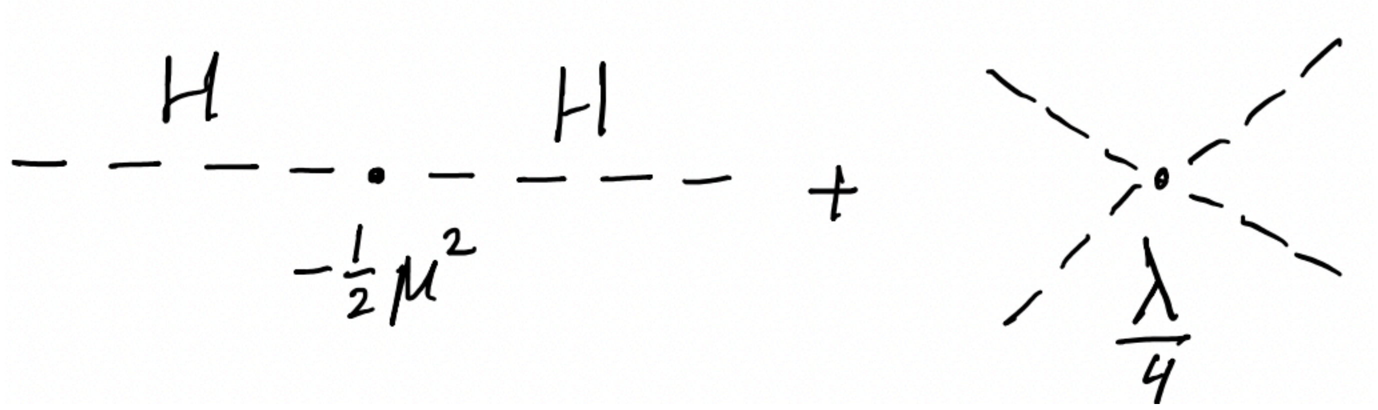

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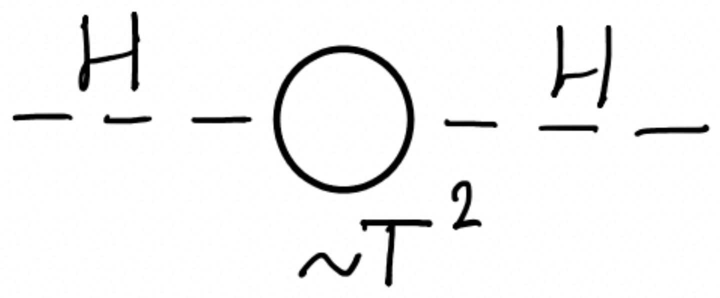

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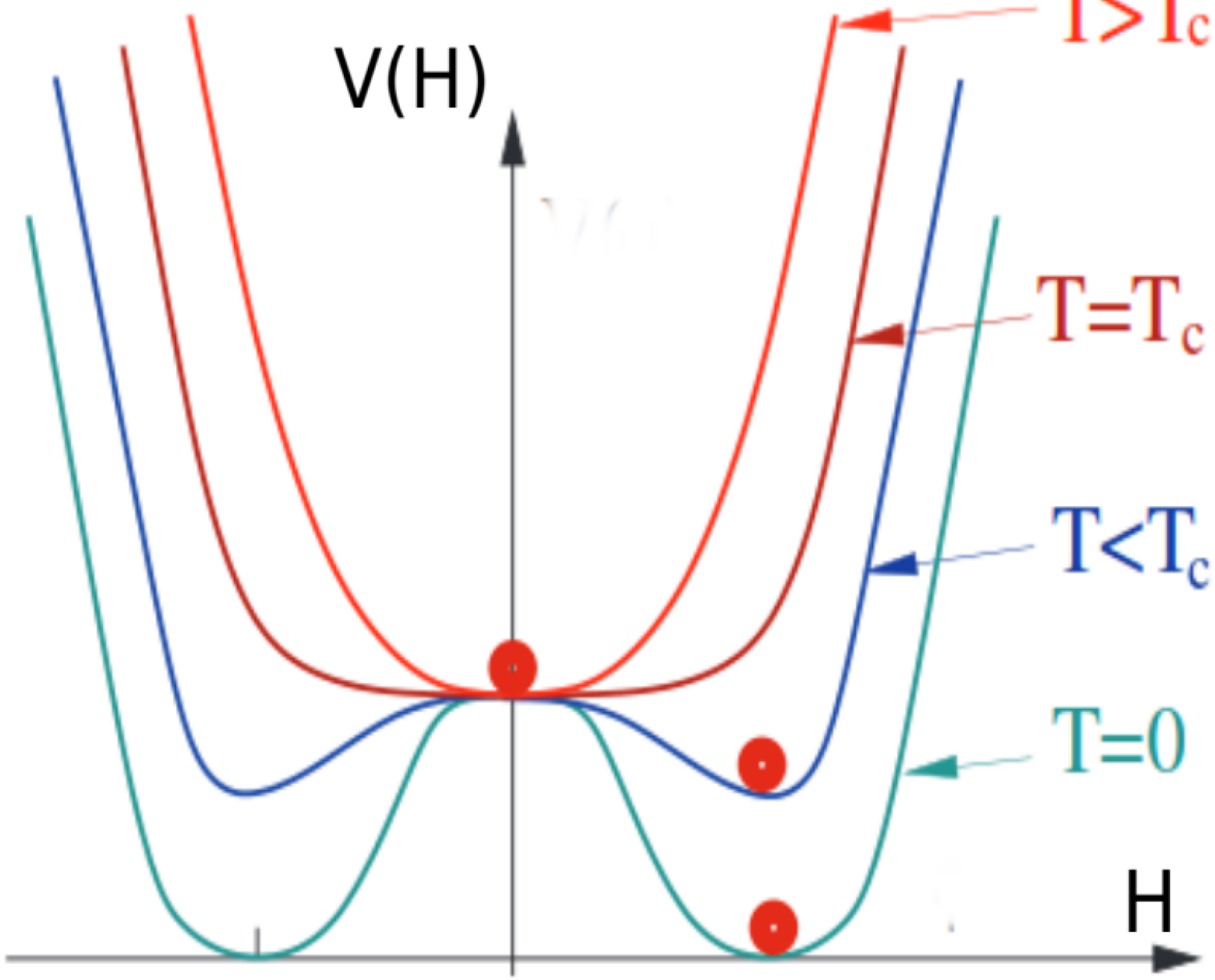


Electroweak Phase Transition: Thermal QFT

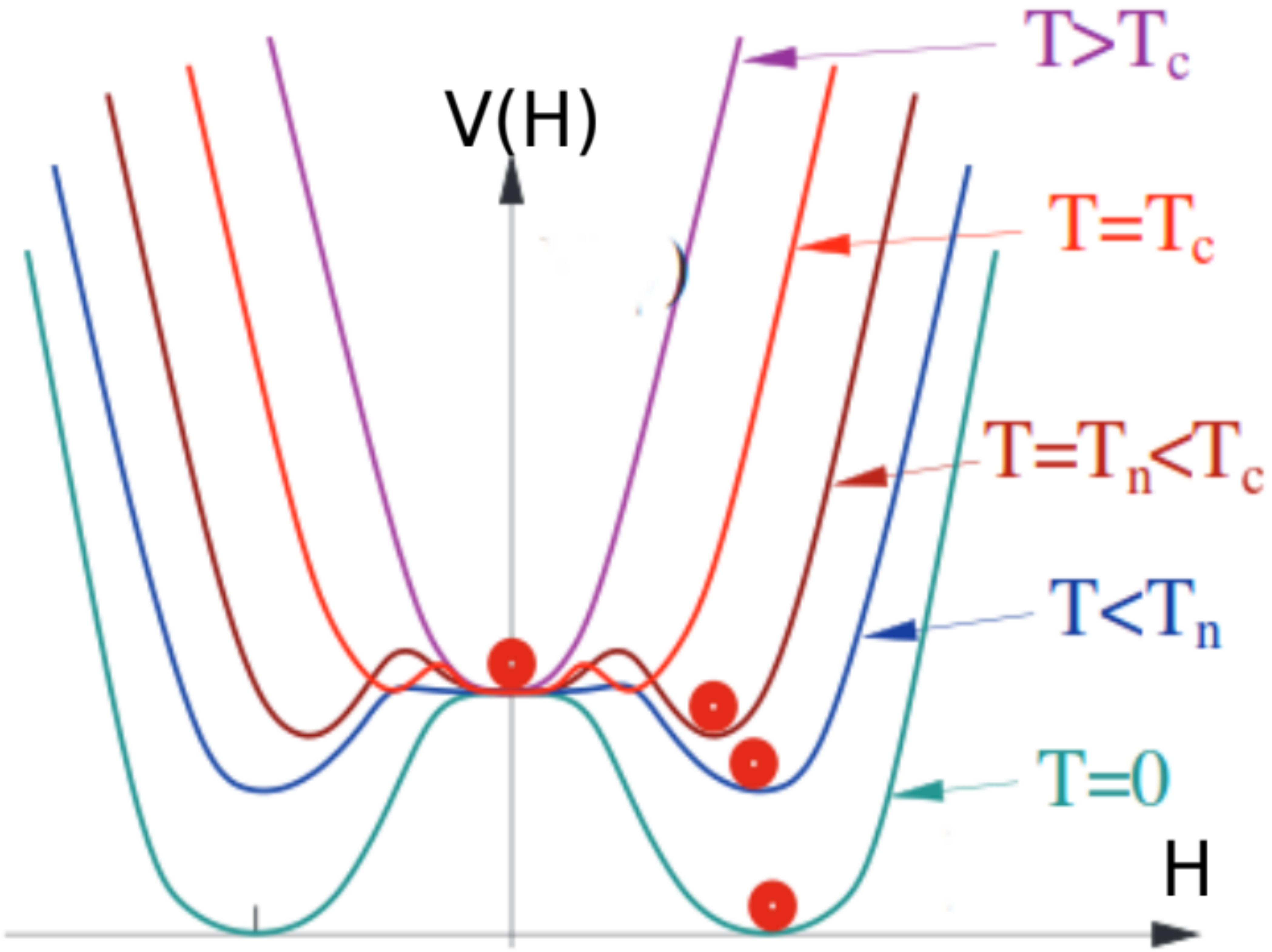
at NLO, the effective potential gets a cubic term

$$V_{eff}(H,T) = \frac{1}{2}(-\mu^2 + \alpha T^2)H^2 - \beta T(-\mu^2 + \gamma H^2)^{3/2} + \frac{1}{4}\lambda H^4$$

The values of (α, β, γ) depend on your theory

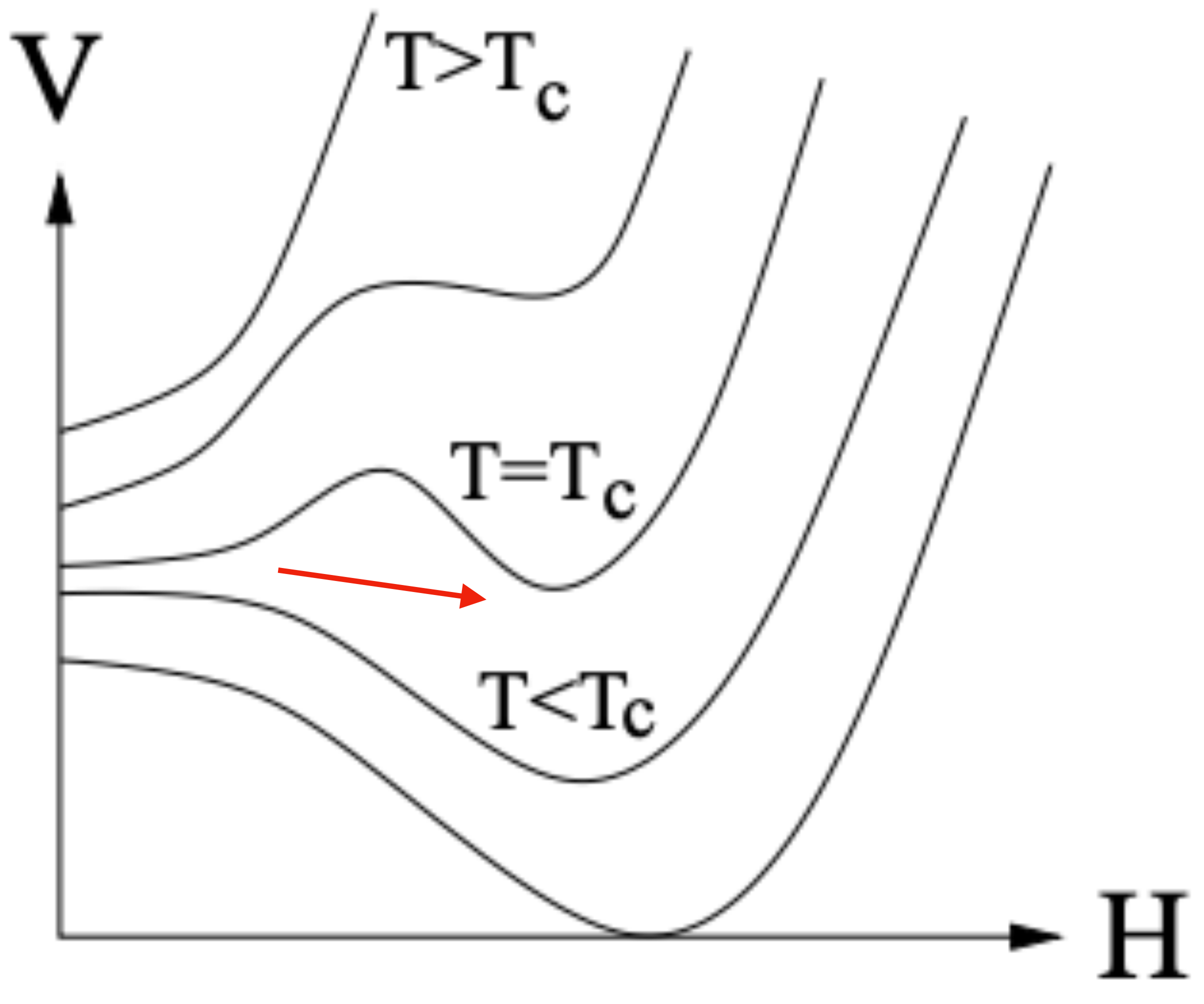
If you're lucky, you can get a barrier between two minima at some critical temperature T_c

Then at some random point in space, the VEV tunnels
a bubble forms around it and expands!

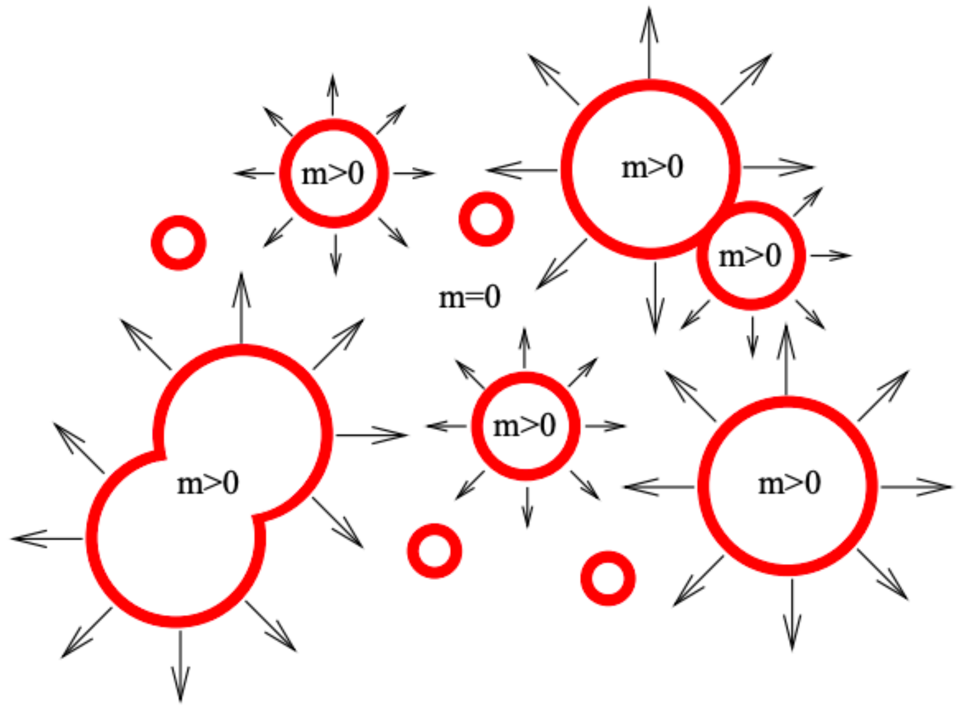


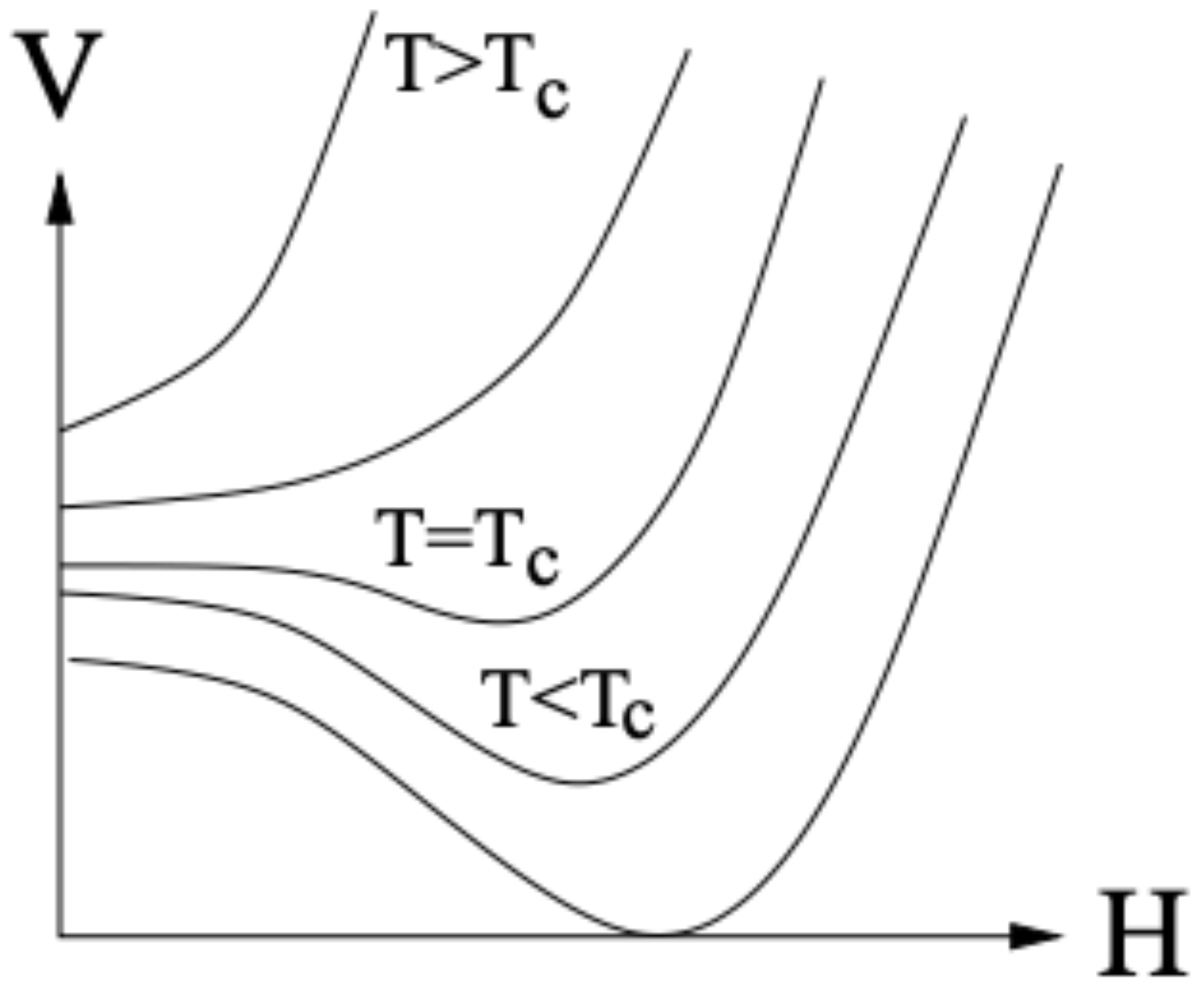
T_n = temperature of bubble nucleation

Electroweak Phase Transition: Thermal QFT



first order phase transition
FOPT





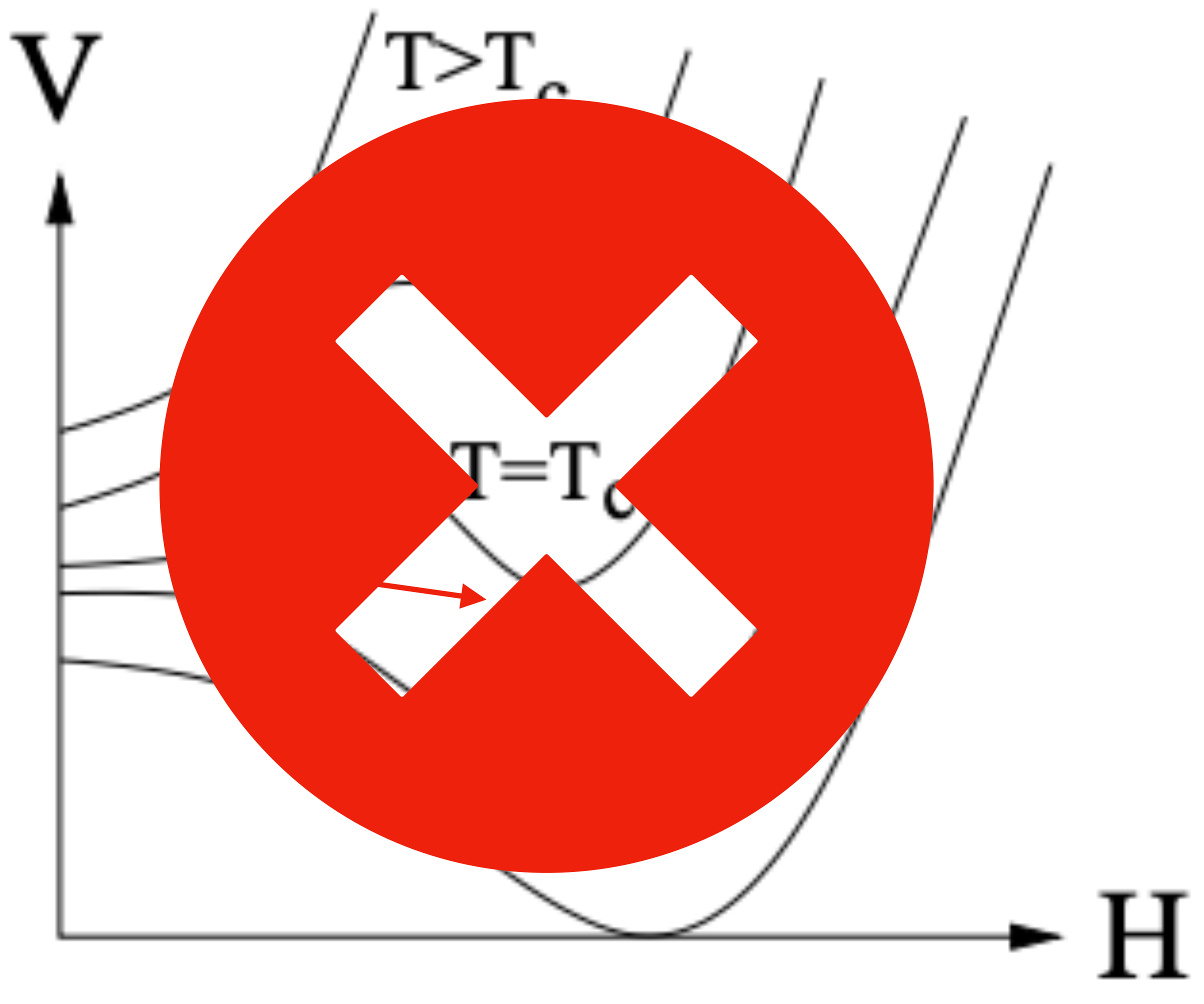
second order phase transition
SOPT
(or crossover)

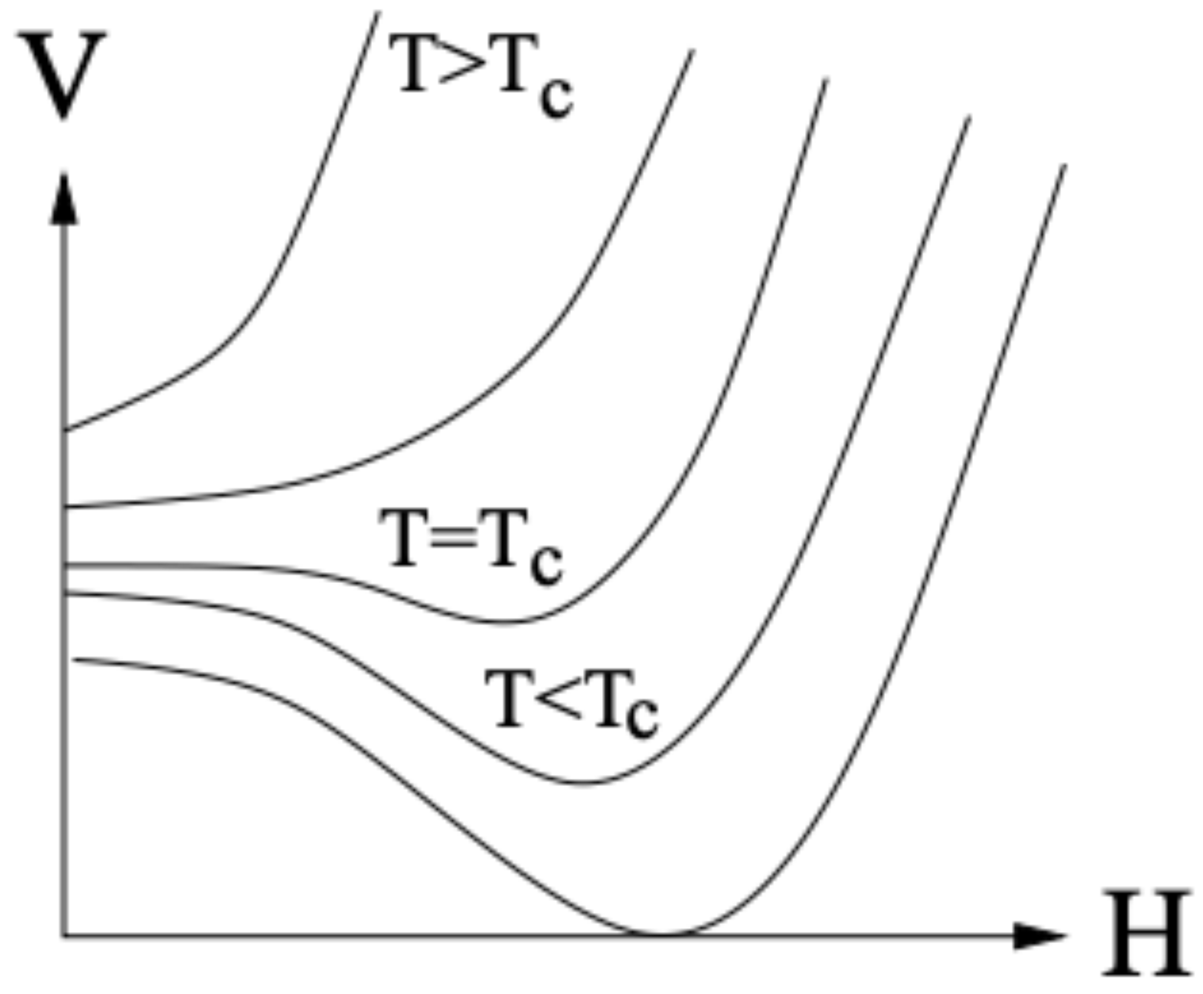
$V_{eff}(H,T) =$  $+ \ldots$

$$\sim \frac{1}{2}(-m^2 + \alpha T^2)H^2 - \beta T H^3 + \frac{1}{4}\lambda H^4$$

WE WANT TO BE FIRST!!!

Electroweak Phase Transition: SM



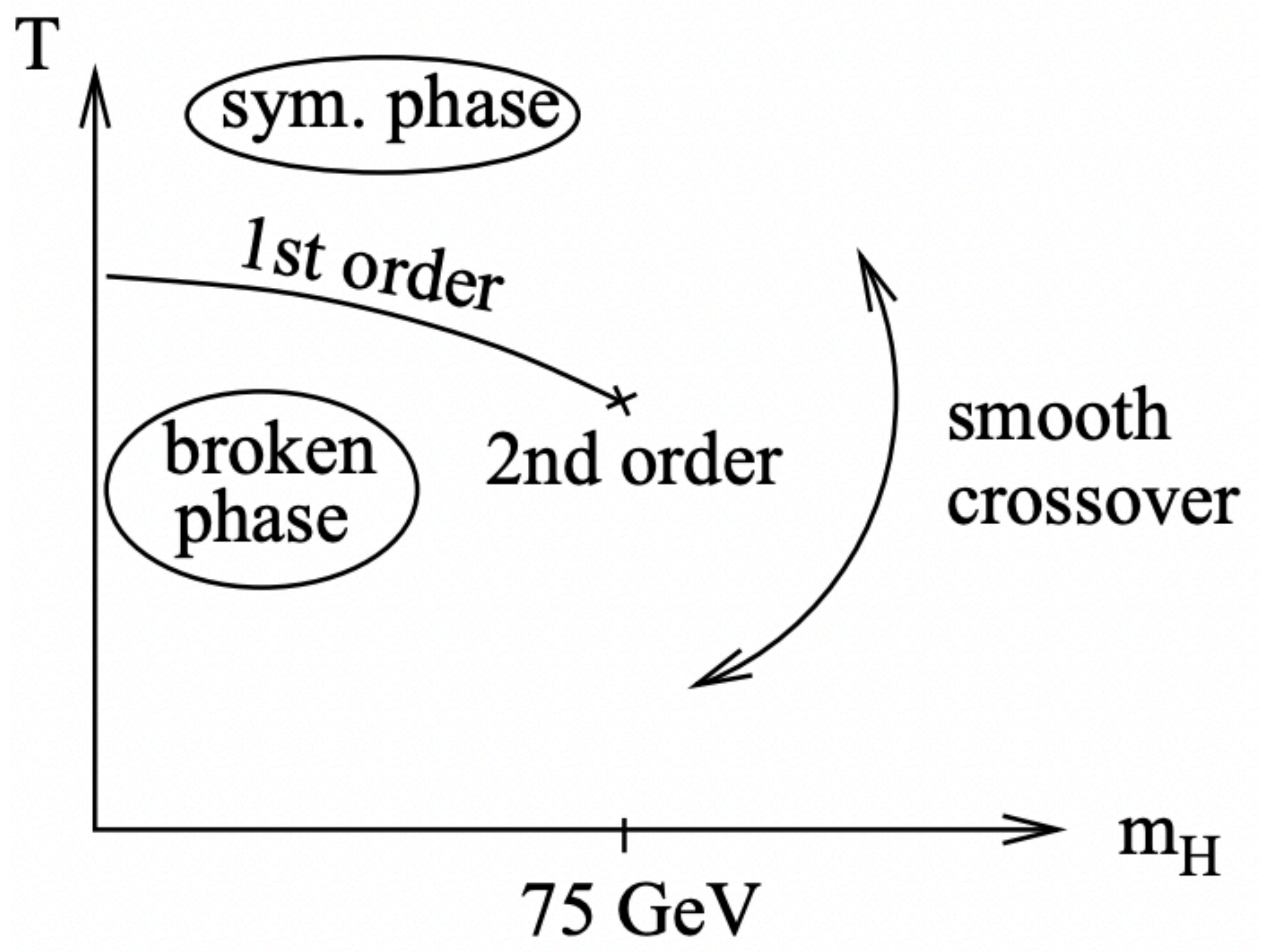


second order phase transition
SOPT
(or crossover)

$$V_{eff}(H,T) = \bigcirc + \{\bigcirc\!\bigcirc + \ominus\} + \ldots$$
$$\sim \frac{1}{2}(-m^2 + \alpha T^2)H^2 - \beta T H^3 + \frac{1}{4}\lambda H^4$$

WE WANT TO BE FIRST!!!

But we're not…



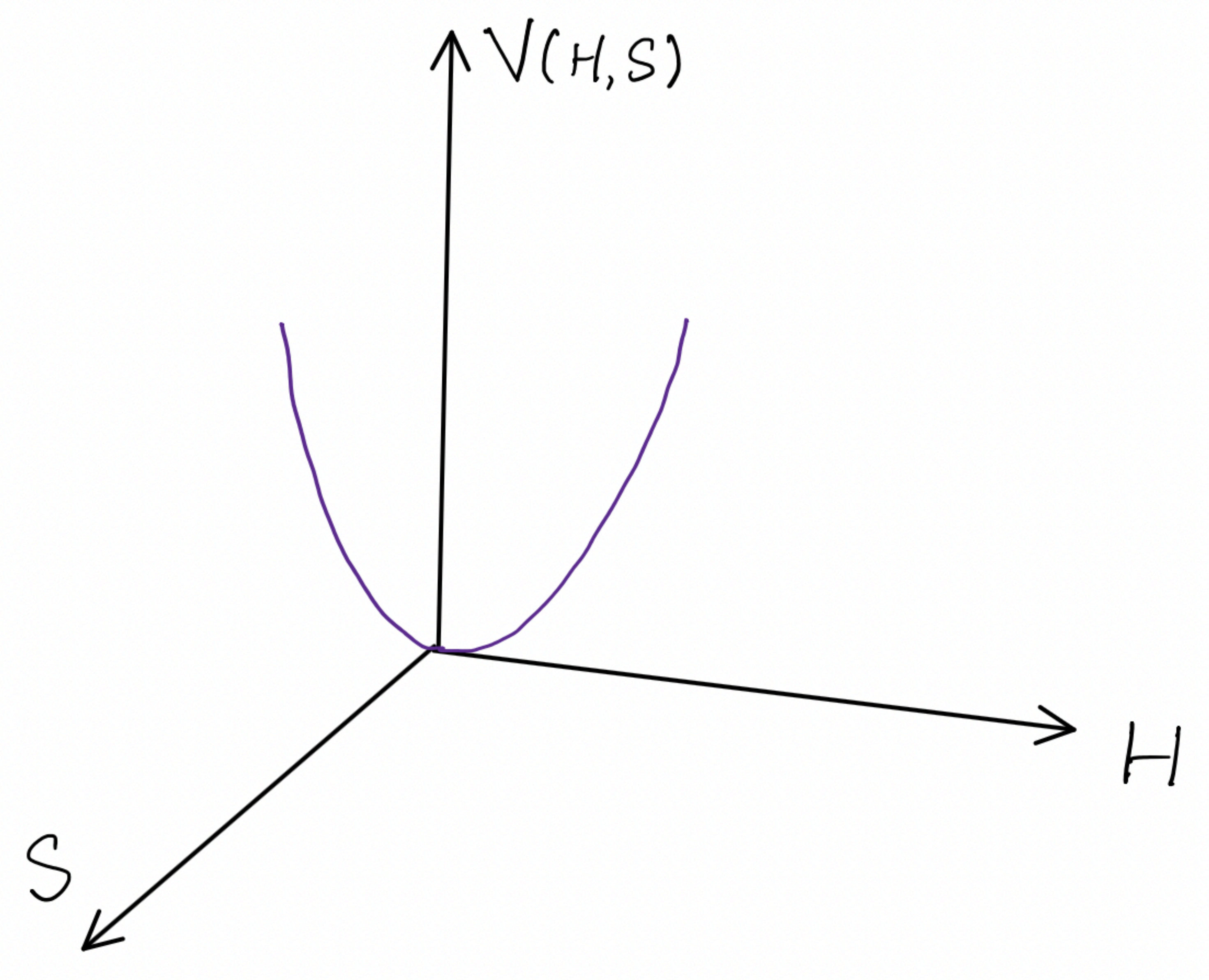
No first order phase transition in SM

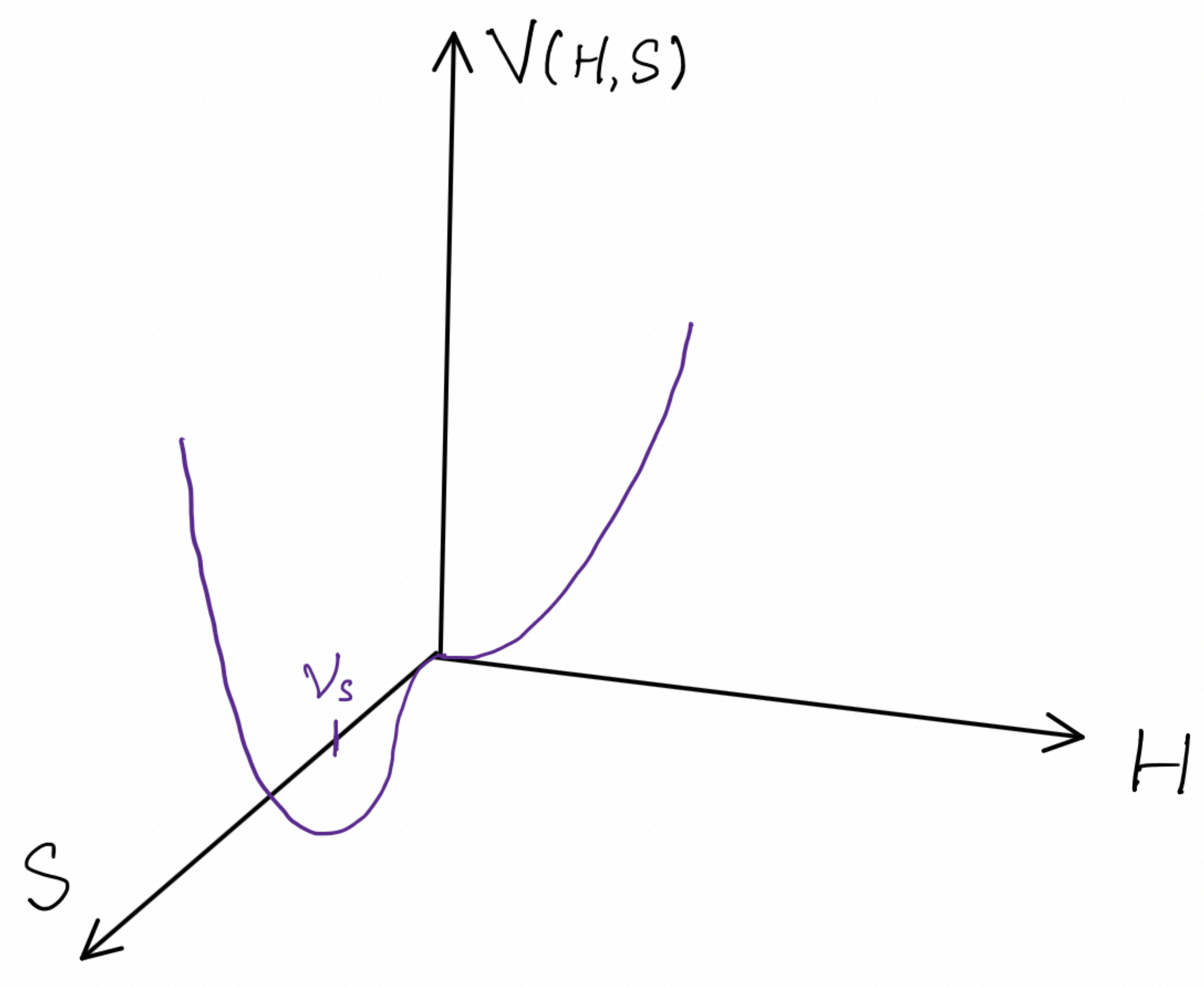
Electroweak Phase Transition: BSM

Idea:

1. add a scalar field S which couples to the Higgs.
2. This scalar field also has a phase transition! Going to a VEV for S

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + V(H,S)$$



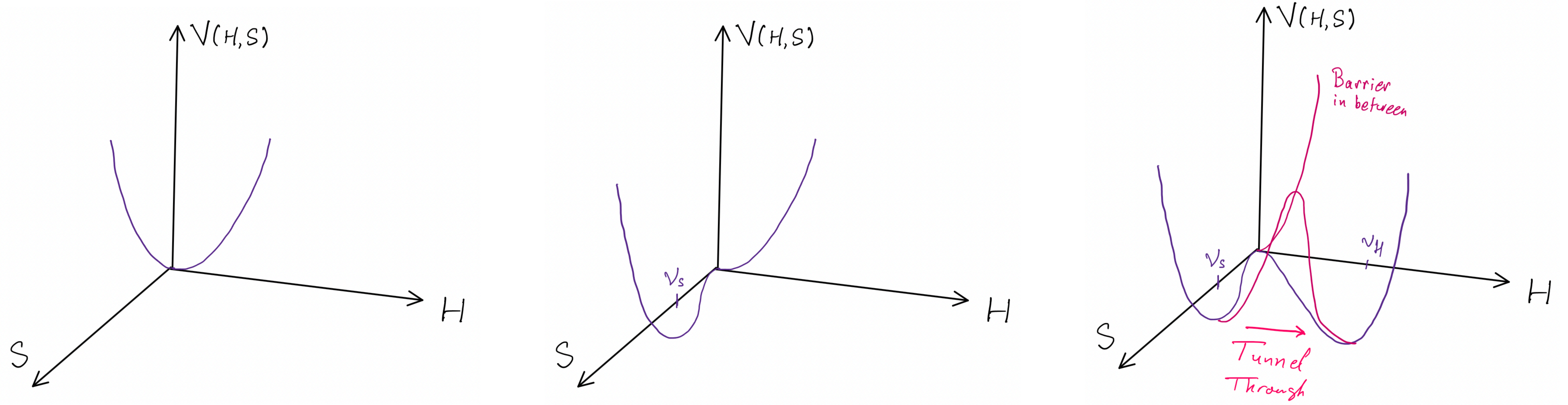


Electroweak Phase Transition: BSM

Idea:

1. add a scalar field S which couples to the Higgs.
2. This scalar field also has a phase transition! Going to a VEV for S
3. Form a potential barrier between the VEV of S and the VEV of H
4. Tunnel to the VEV of H: This is FOPT

$$V(H) = -\frac{1}{2}\mu^2 H^2 + \frac{1}{4}\lambda H^4 + V(H,S)$$

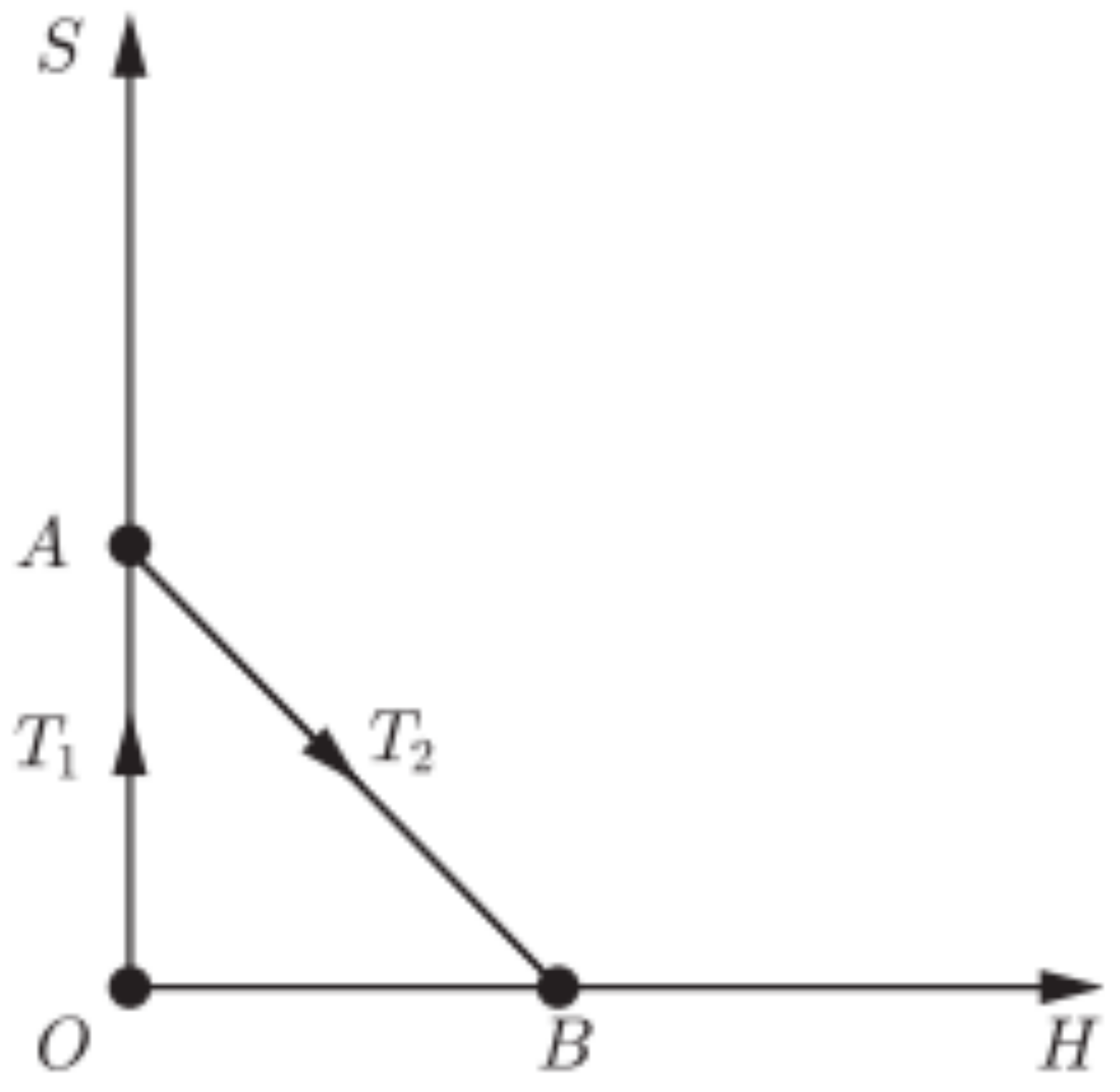


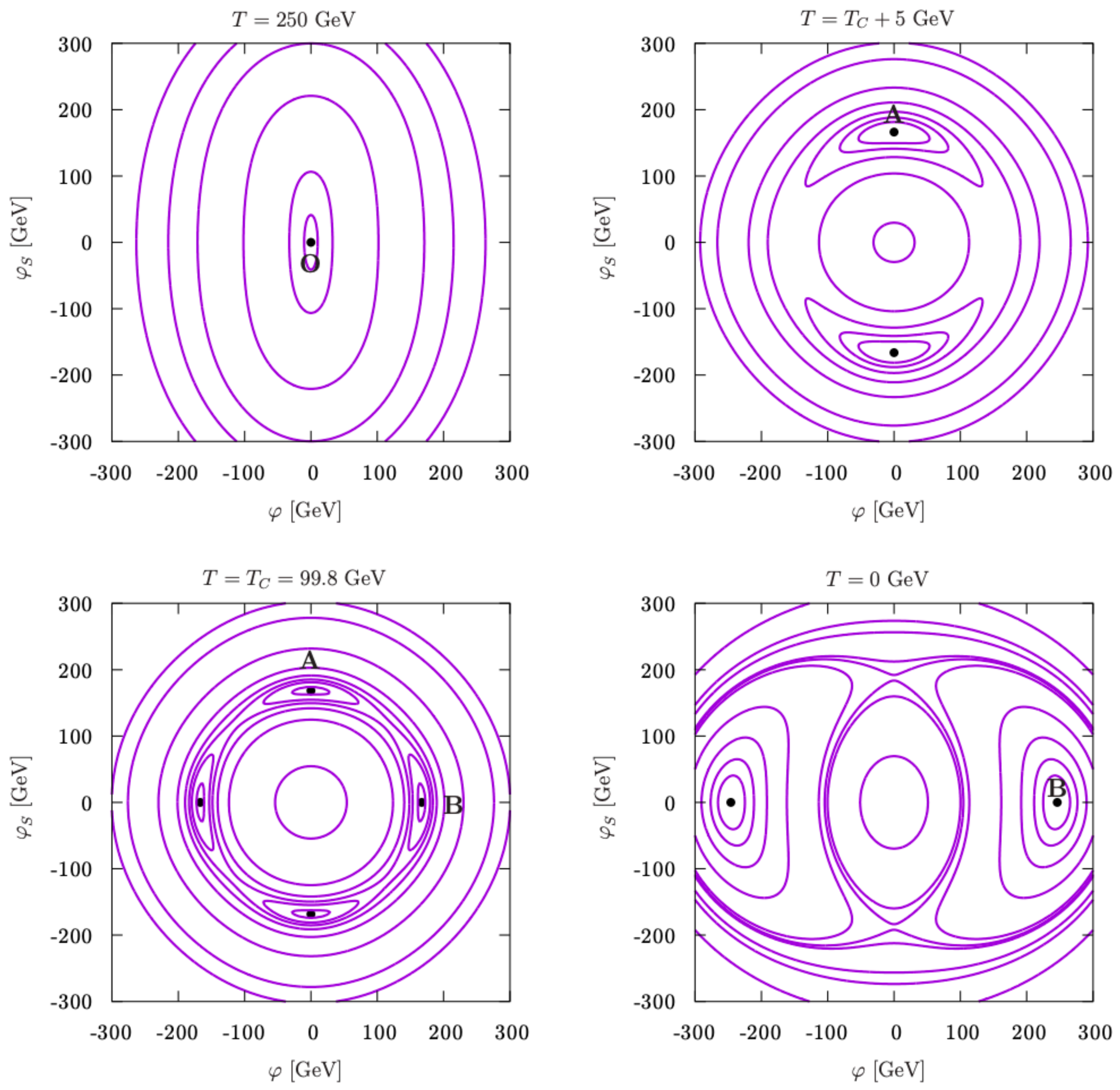
Electroweak Phase Transition: BSM

Adding a scalar field can make a two-step FOPT!

$$V = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\mu_m s h^2 + \frac{1}{4}\lambda_m s^2 h^2 + \mu_1^3 s + \frac{1}{3}\mu_3 s^3$$

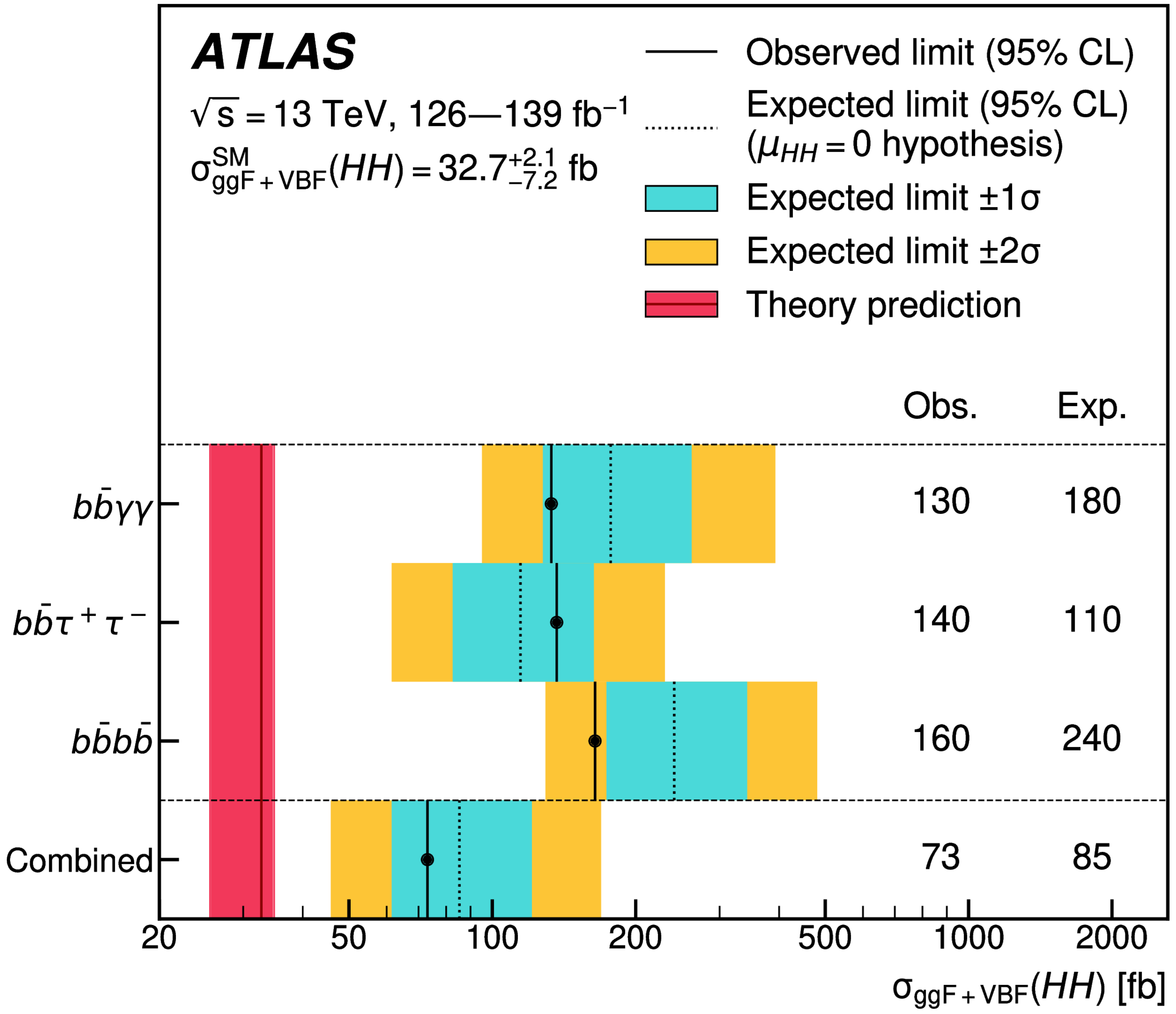
$$V^{\text{high-}T}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S) + \frac{1}{2}(\Sigma_H \varphi^2 + \frac{1}{2}\Sigma_S \varphi_S^2)T^2$$

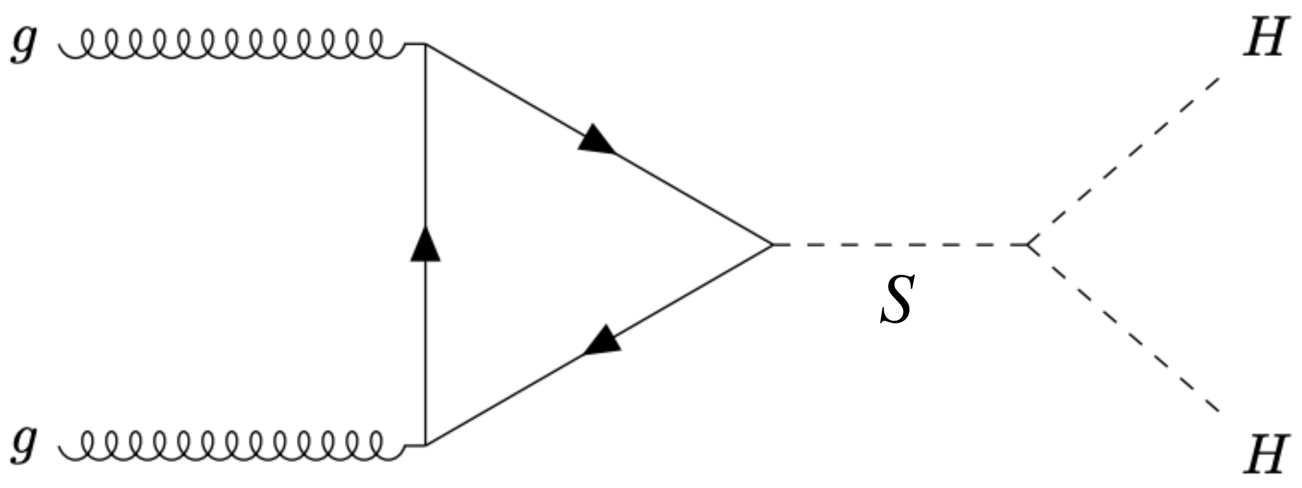




Cheng-Wei Chiang,[1,2,3,4,*] Michael J. Ramsey-Musolf,[5,6,†] and Eibun Senaha[1,7,‡]

ATLAS Higgs results → Higgs phase transition





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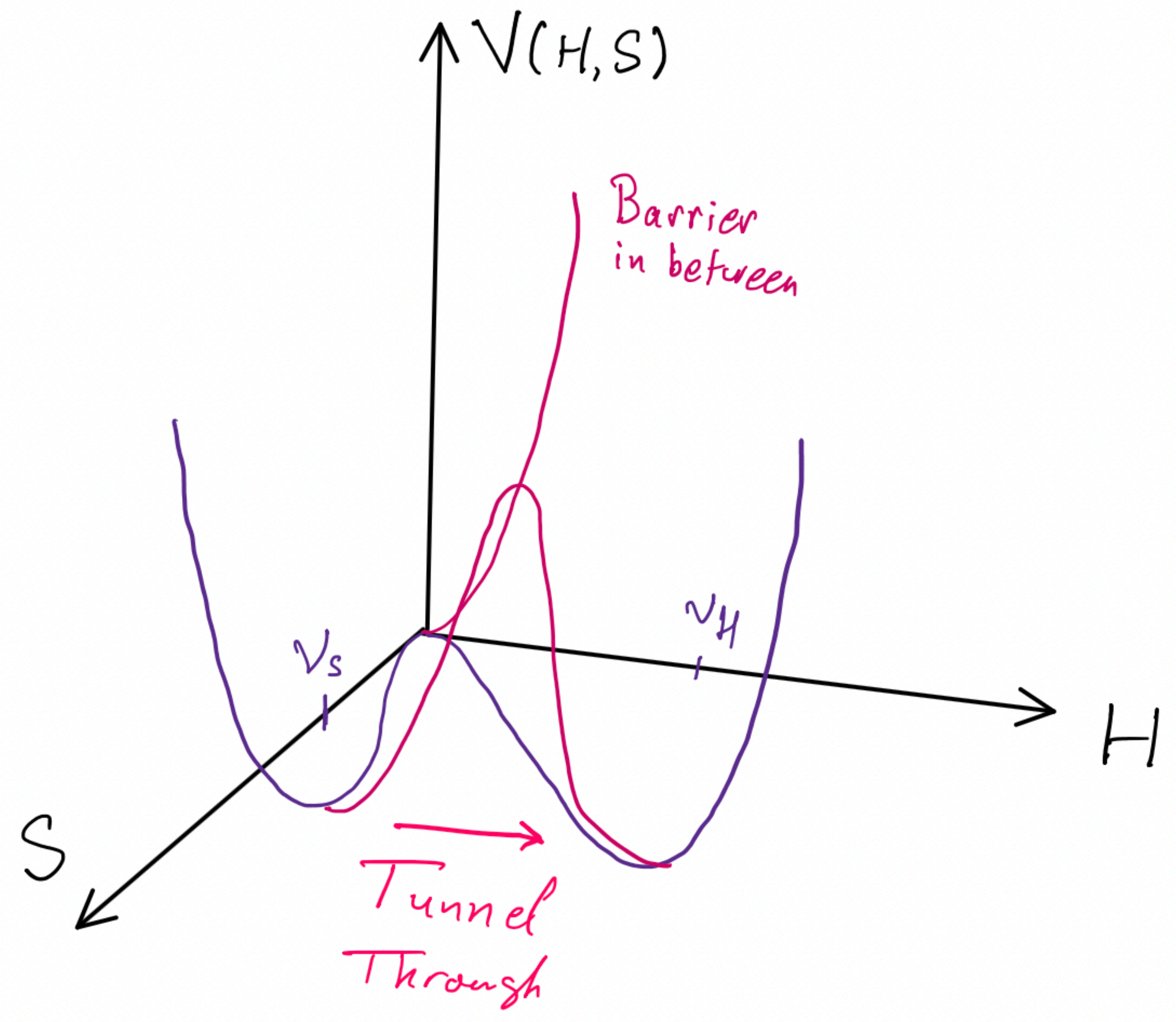
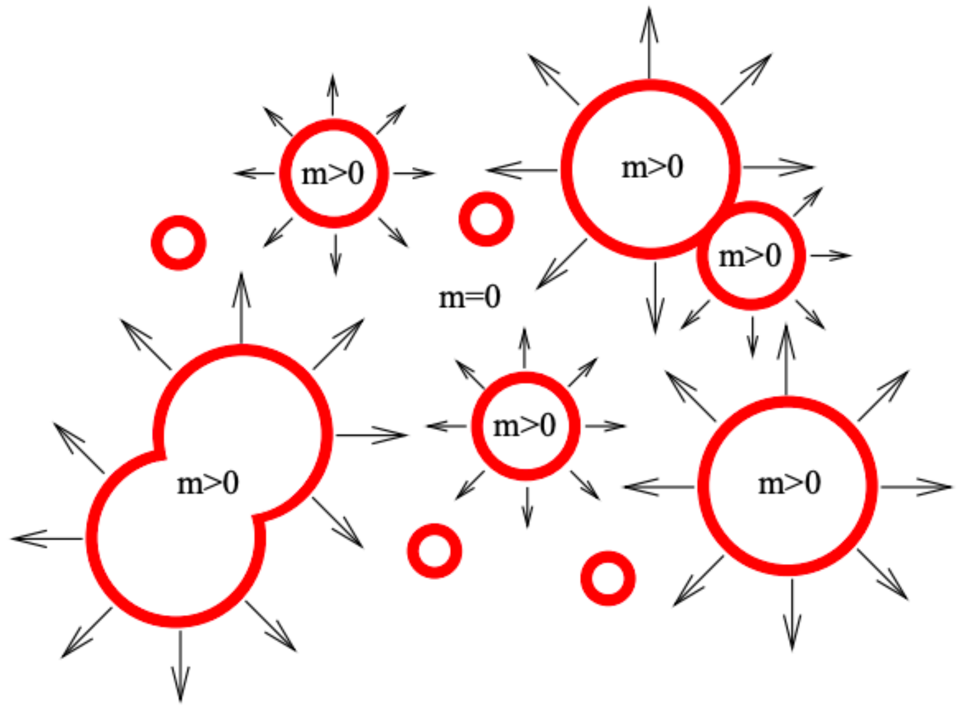
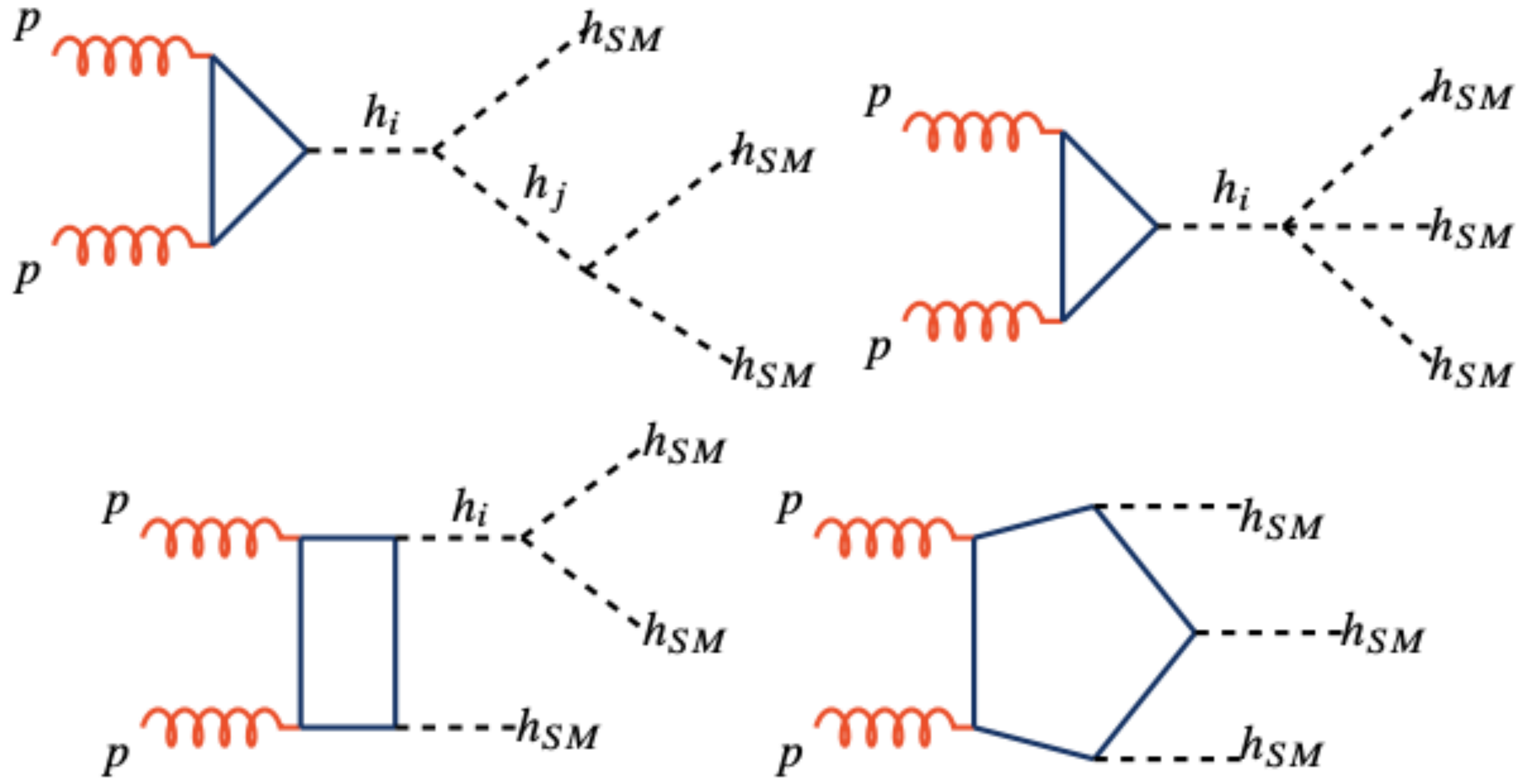




explored for one added scalar, answer will come

ATLAS Higgs results → Higgs phase transition

Maybe we will see enhancement of HHH production!



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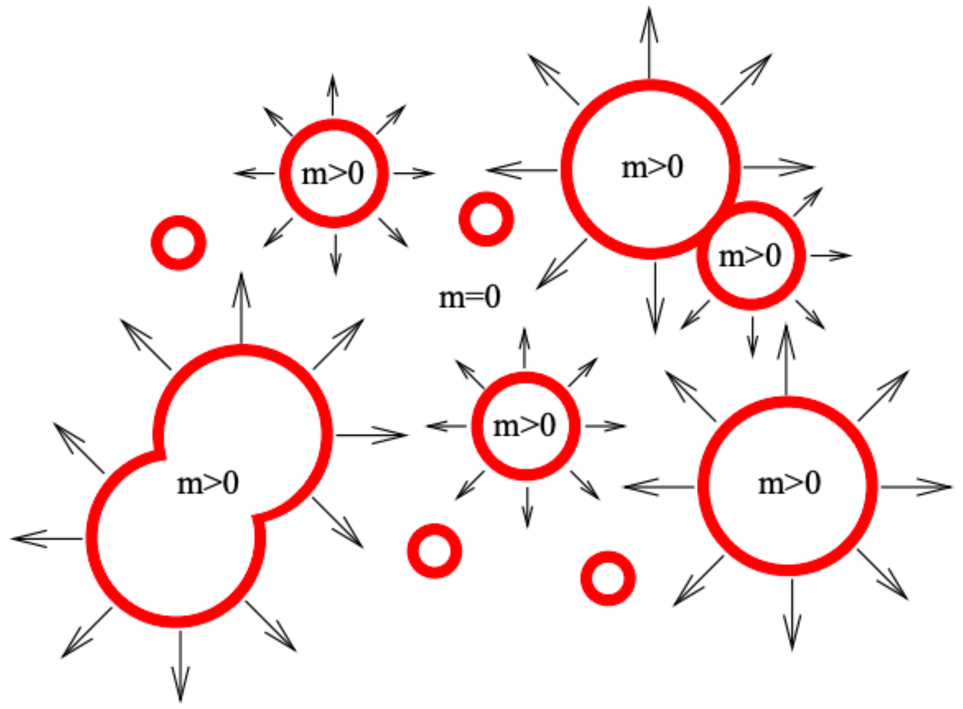

How to enhance HHH

Simplified BSM model predicting large HHH: **TRSM**.

SM + two singlets coupling to the Higgs.

$$V = \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \mu_S^2 S^2 + \lambda_S S^4 + \mu_X^2 X^2 + \lambda_X X^4 \\ + \lambda_{\Phi S} \Phi^\dagger \Phi S^2 + \lambda_{\Phi X} \Phi^\dagger \Phi X^2 + \lambda_{SX} S^2 X^2 \,.$$

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Scalars get VEVs! → Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

h1 can be our scalar particle of 125 GeV

Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

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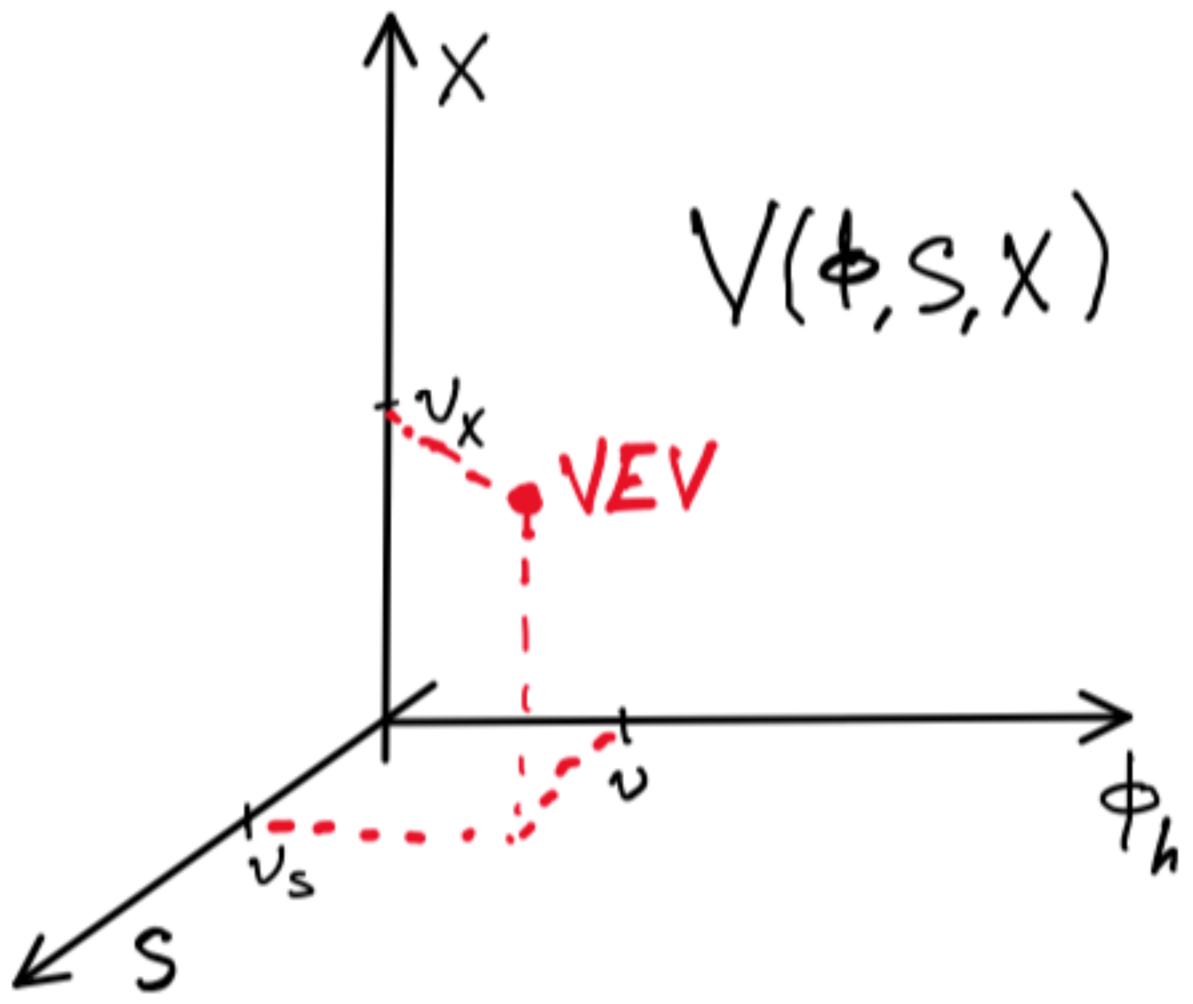

Remember:
Mixing requires nonzero VEV
For added scalars

h1 can be our scalar particle of 125 GeV

Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

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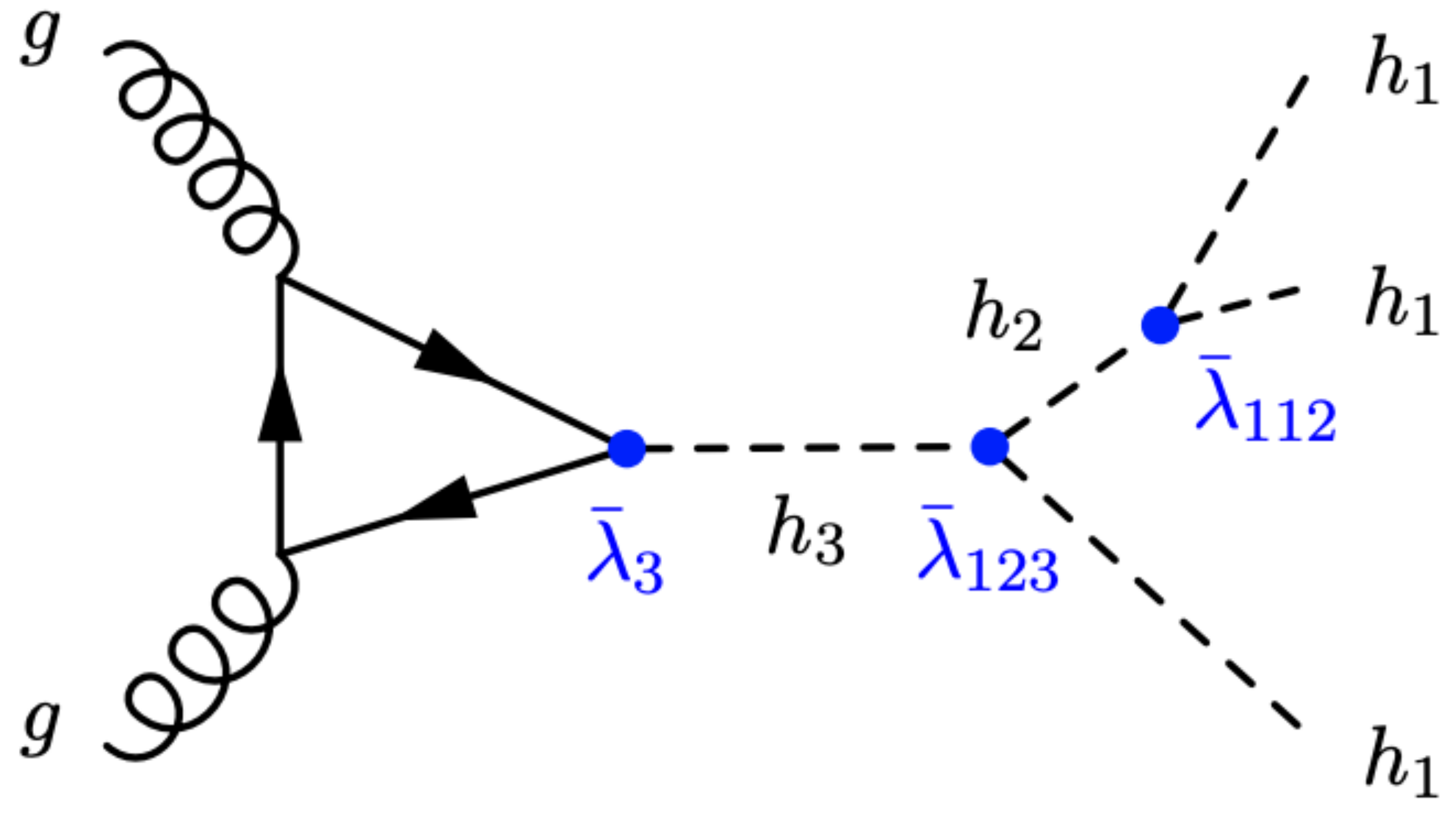
Tania Robens,[1,*] Tim Stefaniak,[2,†] and Jonas Wittbrodt[2,‡]

HHH production is enhanced through **resonance**
xsec ~ 30 fb (~ HH production in SM)



We updated this conclusion using better theoretical bounds (perturbativity) and newest experimental bounds!

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425

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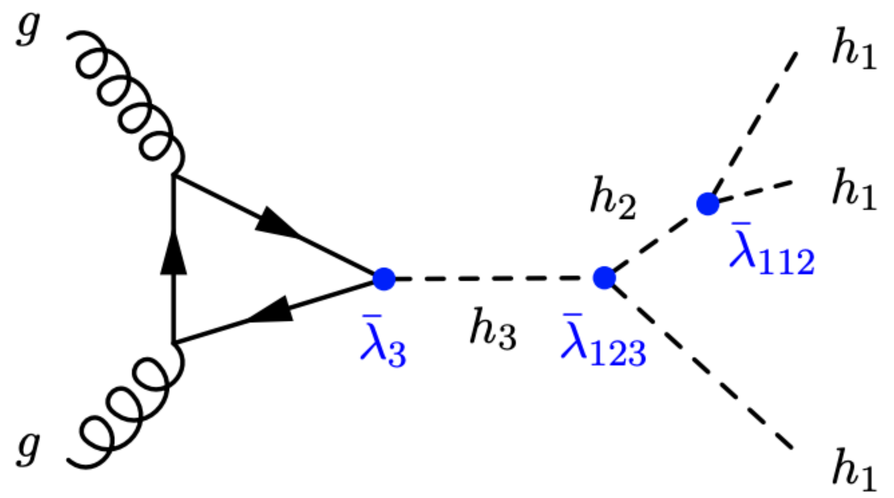

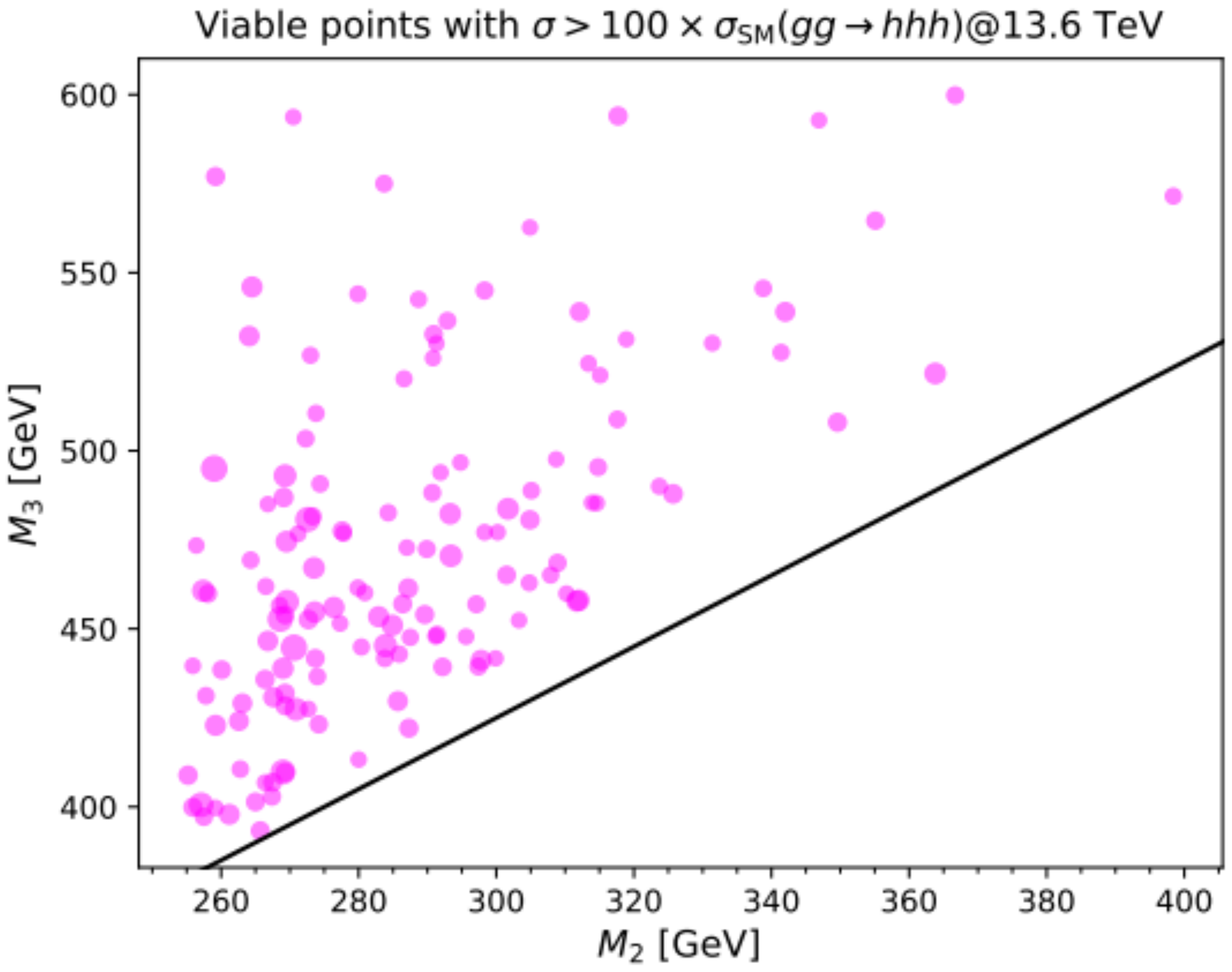

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Electroweak Phase Transition: TRSM

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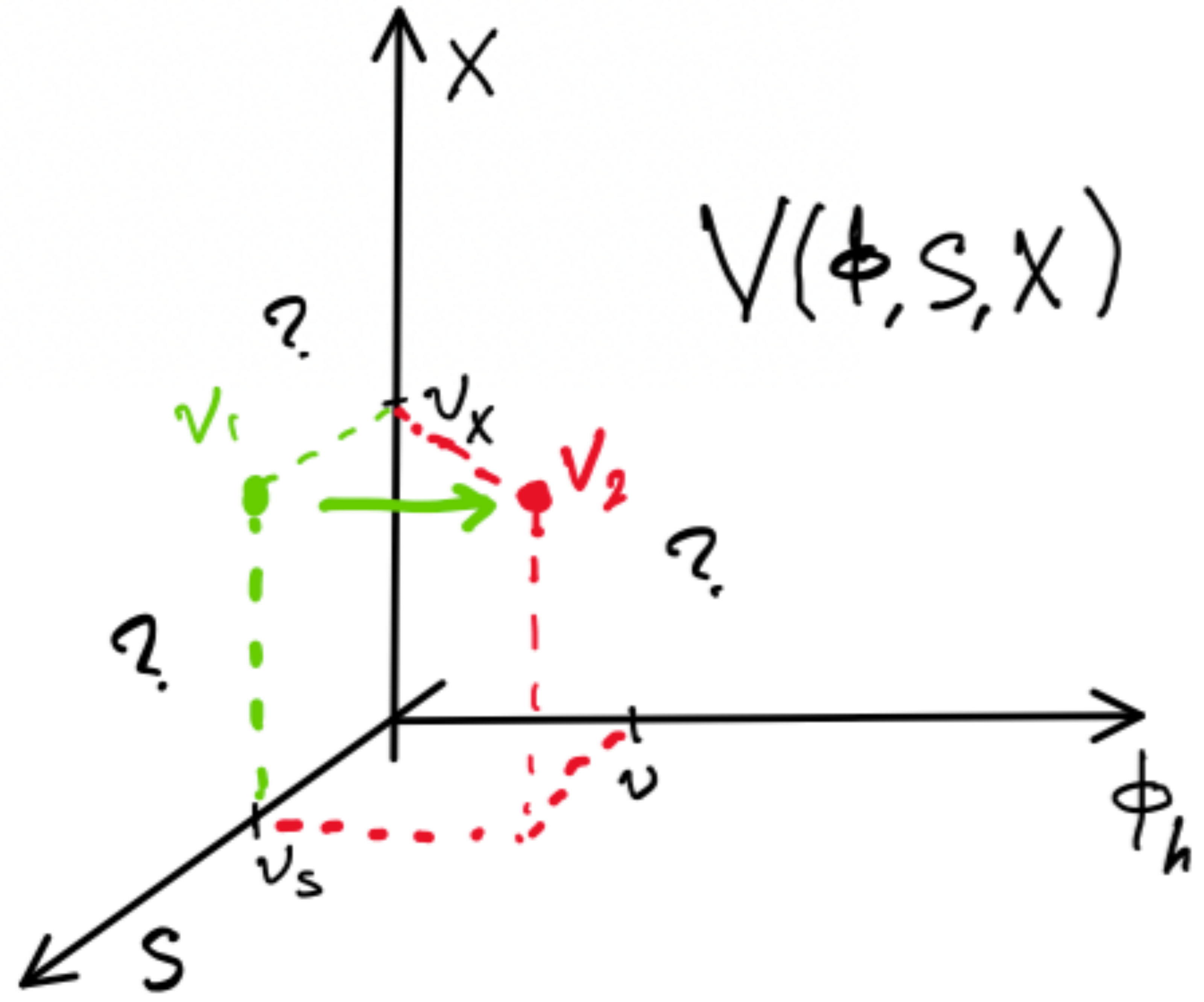


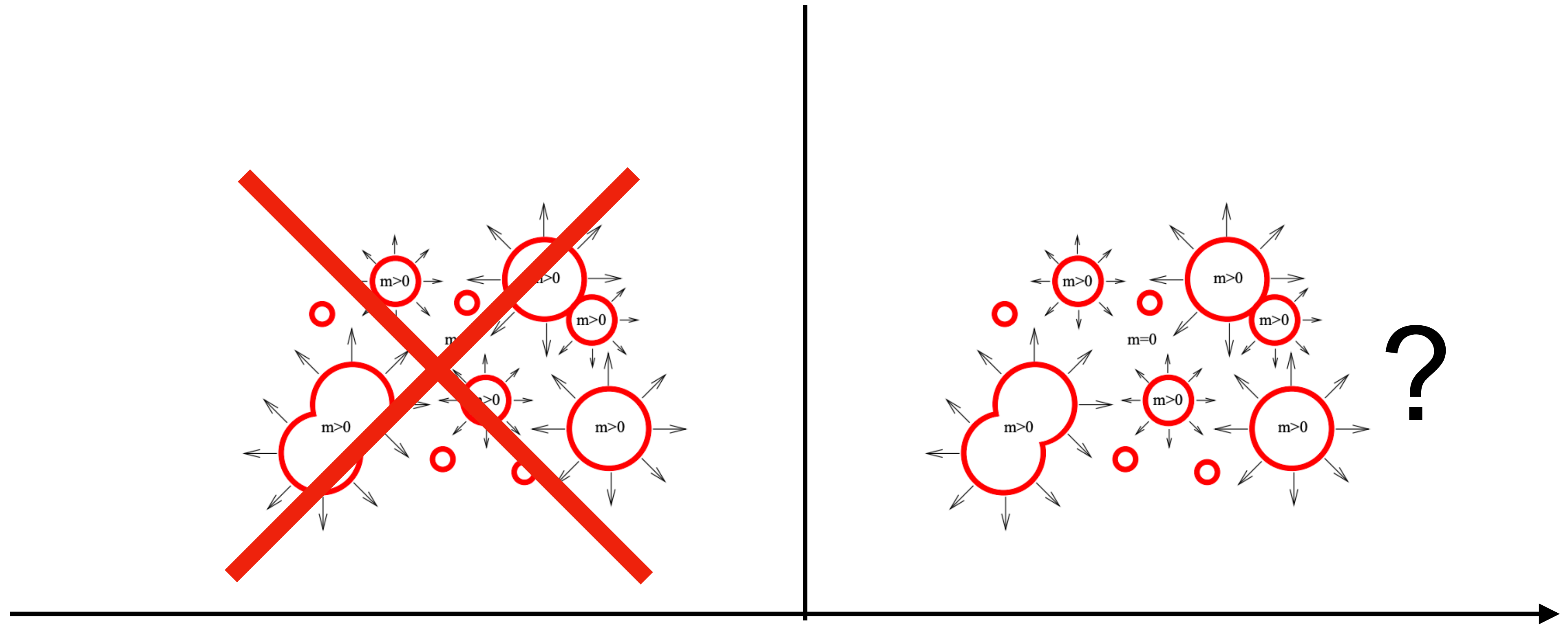
Mixing:

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi_h + v}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{\phi_S + v_S}{\sqrt{2}}, \quad X = \frac{\phi_X + v_X}{\sqrt{2}} \qquad \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \phi_h \\ \phi_S \\ \phi_X \end{pmatrix}$$

Physical parameter space:

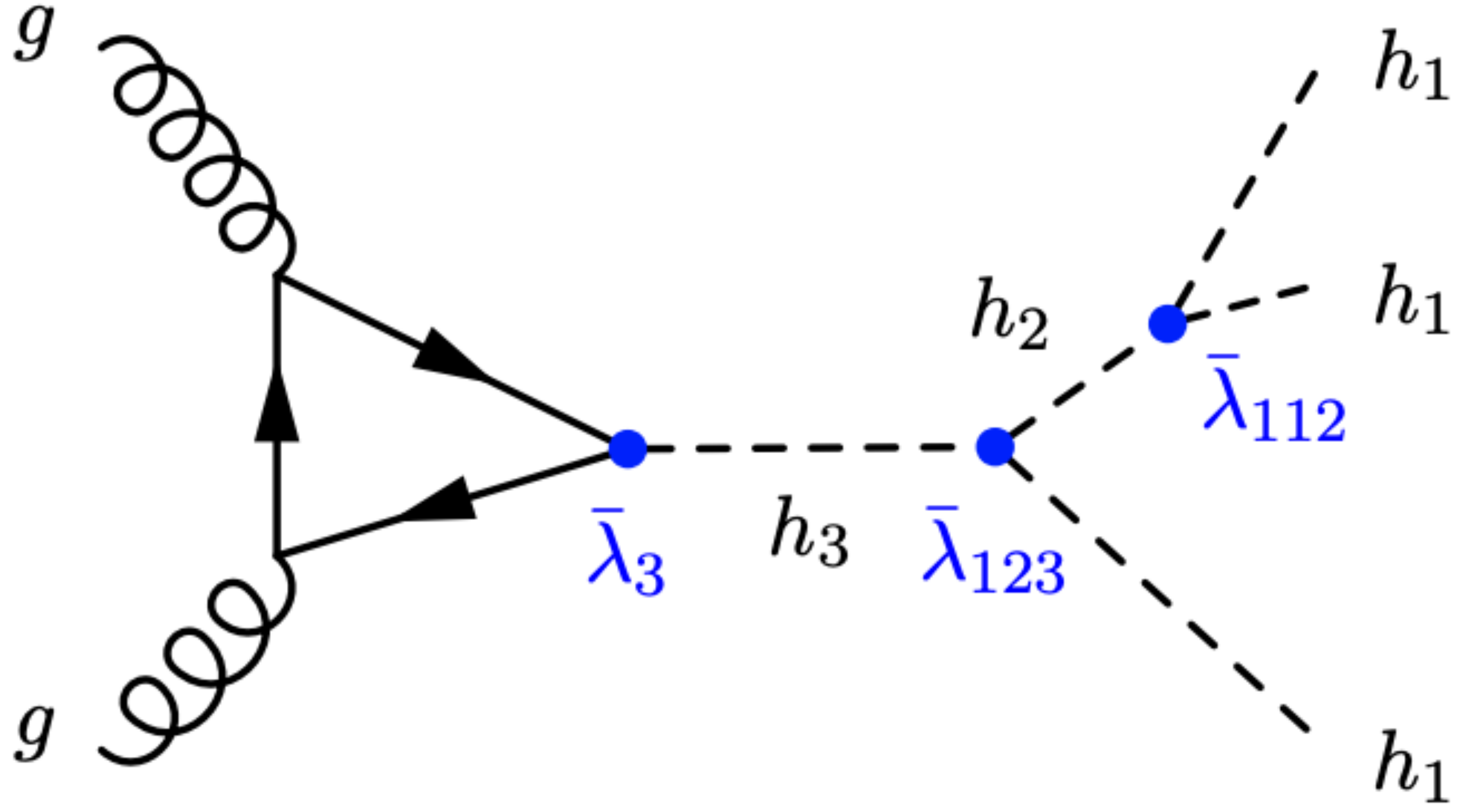
$$\{M_2, M_3, v_s, v_x, \theta_1, \theta_2, \theta_3\}$$

$M_1 = 125\ GeV,\ v = 246\ GeV$

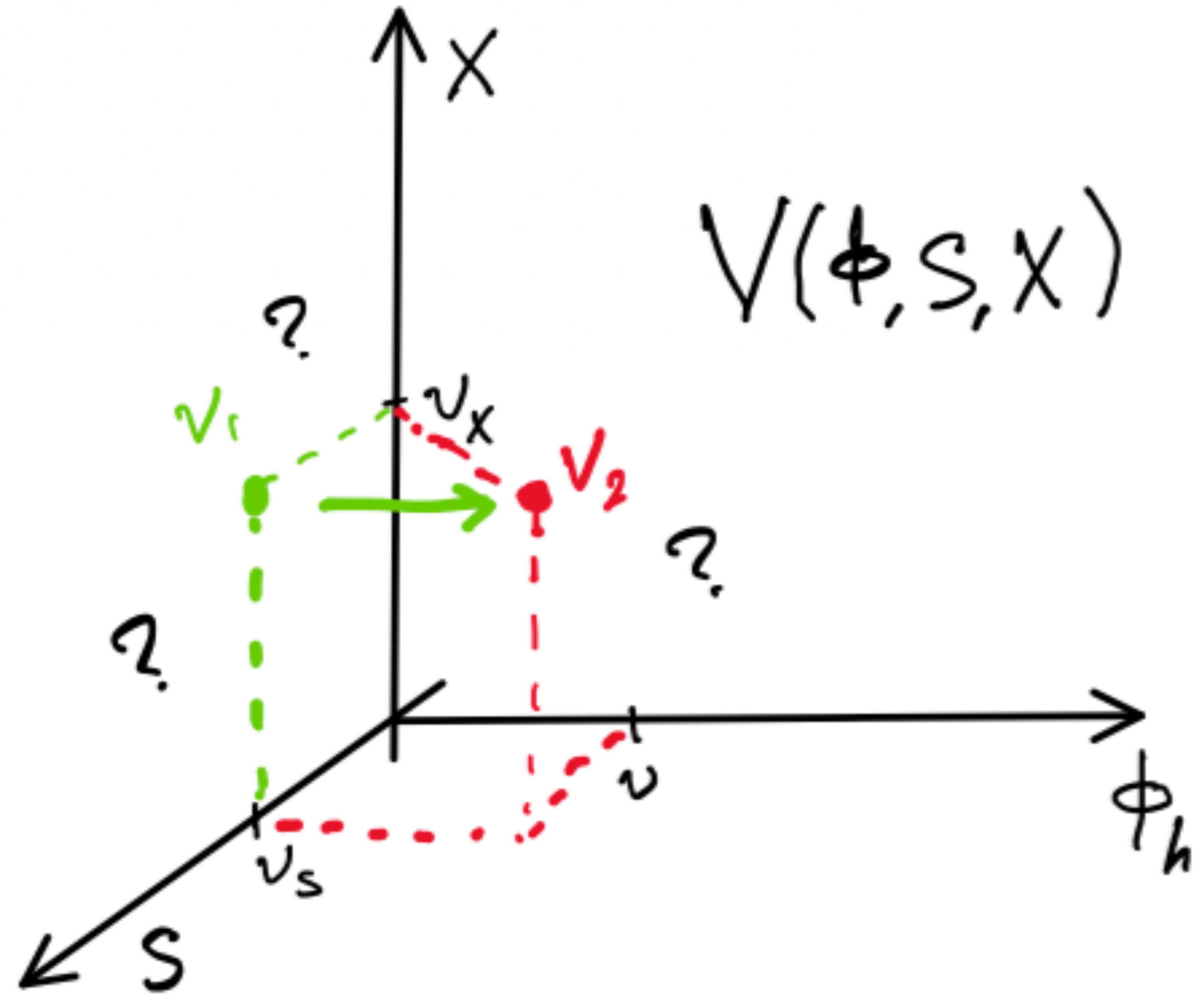


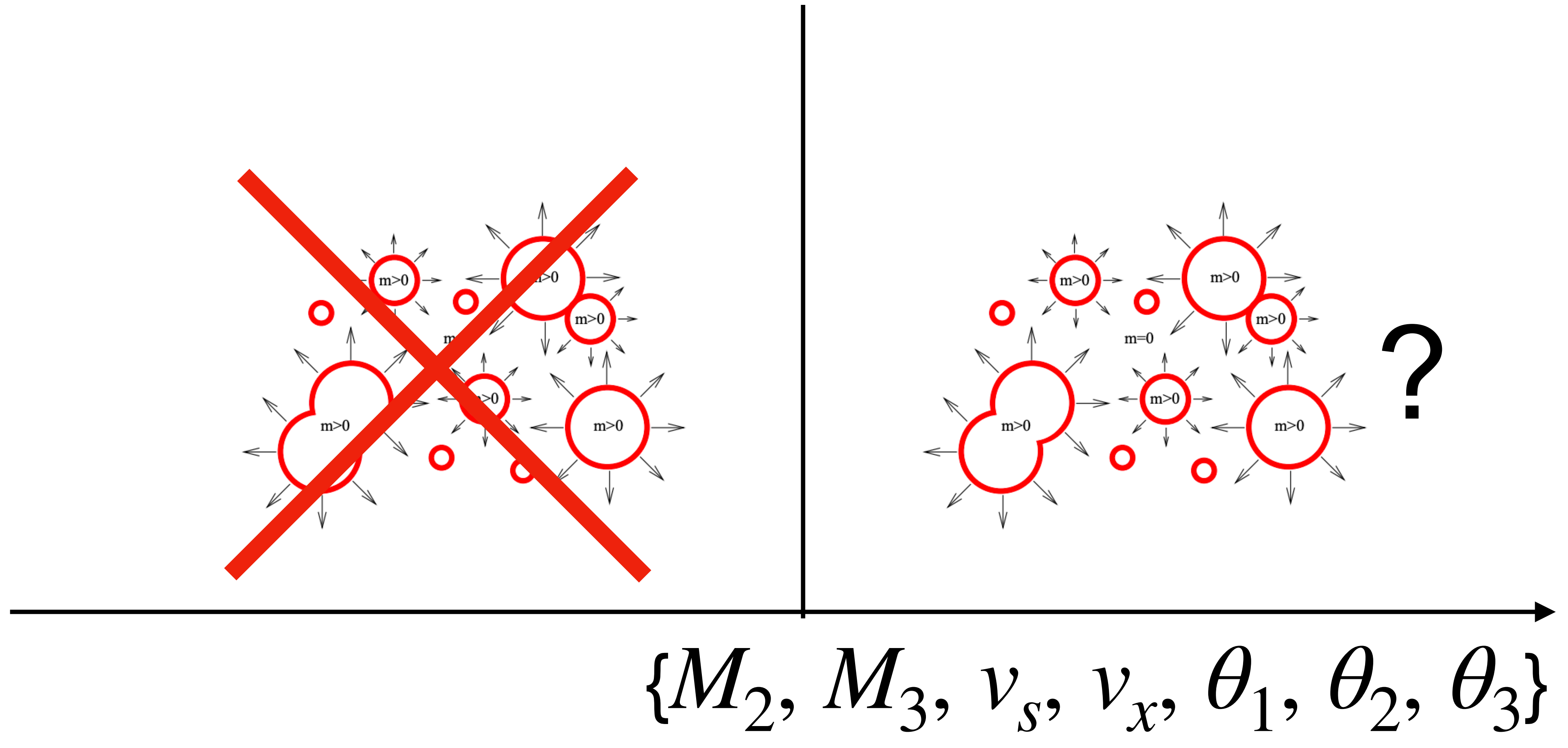
Can we have First-Order Phase Transition (FOPT)?
For which parameters?
Does it come with HHH enhancement?

TRSM: HHH production and Higgs FOPT



Need nonzero VEV
for two added scalars
for double resonance

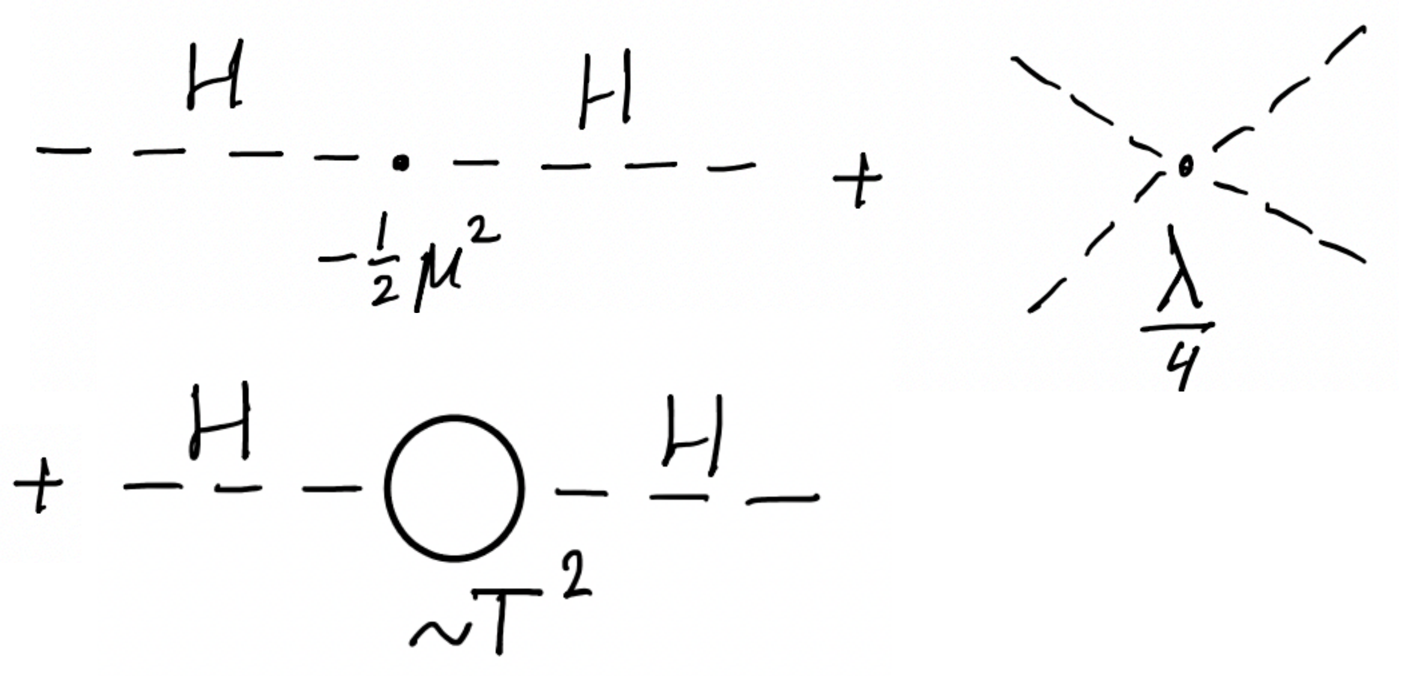




PT in TRSM: start with thermal QFT

At LO: only masses get T contribution

$V(H) =$



$$m_1^2(T) = -\mu_1^2 + \frac{T^2}{48}\left(3g_1^2 + 9g_2^2 + 2(6y_t^2 + 12\lambda_1 + \lambda_{12} + \lambda_{13})\right),$$

$$m_2^2(T) = -\mu_2^2 + \frac{T^2}{24}\left(4\lambda_{12} + \lambda_{23} + 6\lambda_2\right),$$

$$m_3^2(T) = -\mu_3^2 + \frac{T^2}{24}\left(4\lambda_{13} + \lambda_{23} + 6\lambda_3\right),$$

resulting in an *effective* finite-temperature potential:

$$V_{\text{eff,LO}}(\phi_i, T) = \frac{1}{2}\sum_i m_i^2(T)\phi_i^2 + \frac{1}{4}\sum_{i\leqslant j}\lambda_{ij}\phi_i^2\phi_j^2.$$

PT in TRSM: start with thermal QFT

At LO: only masses get T contribution

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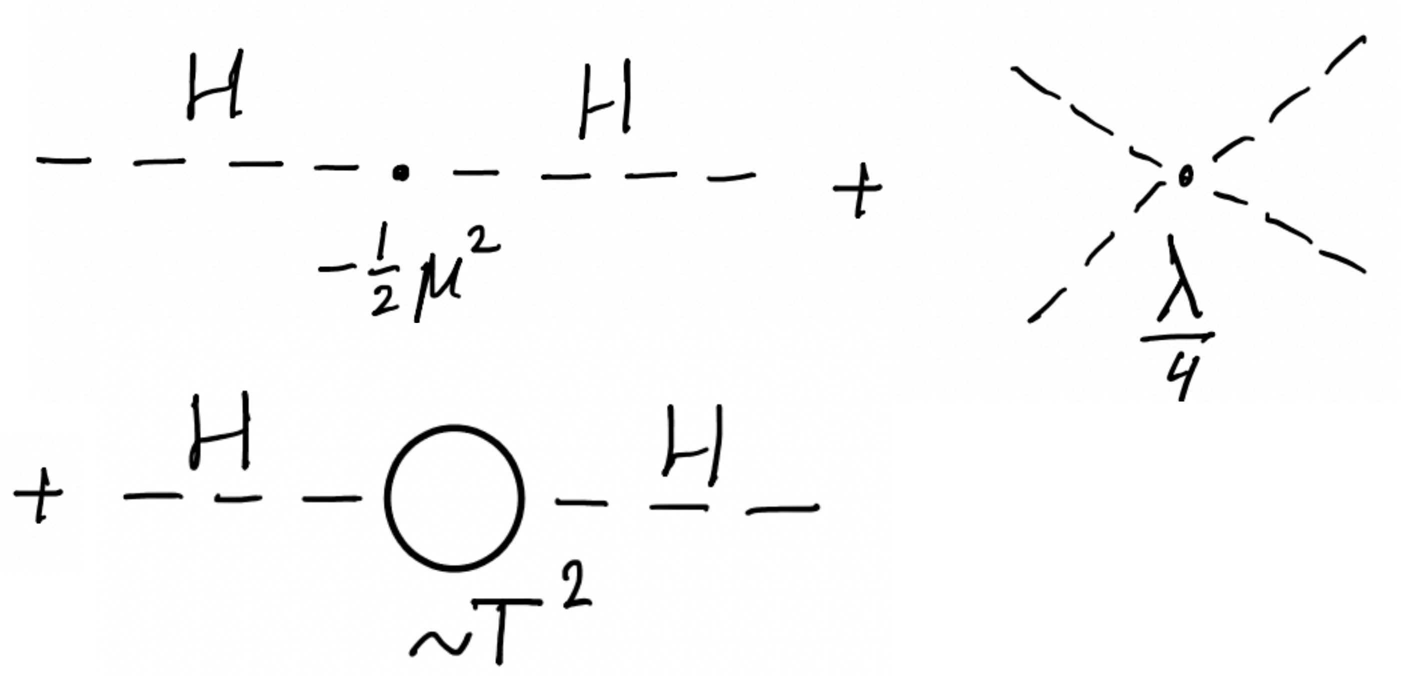

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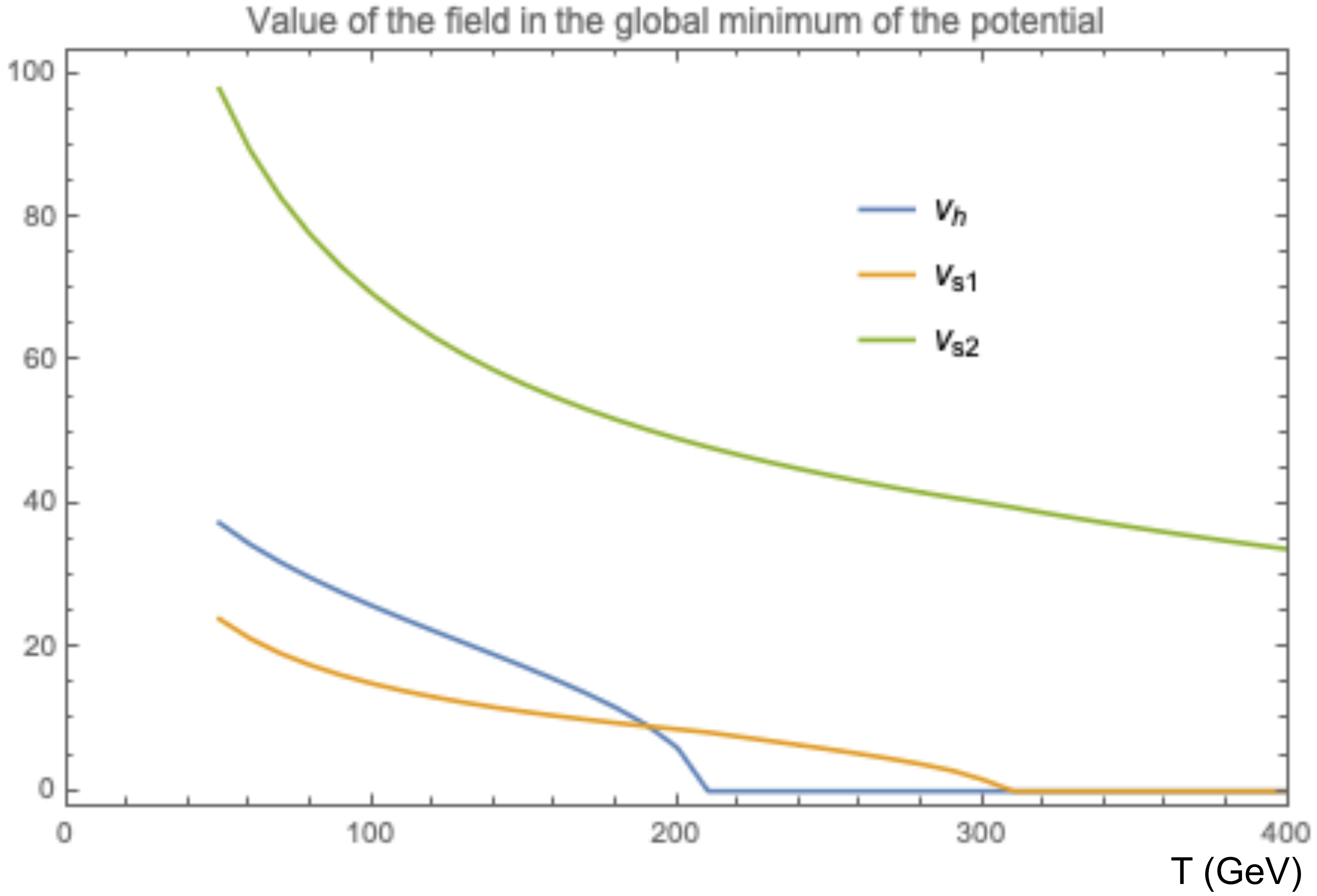
Started using Mathematica to numerically solve RGEs (differential equations as a function of T)

We tried points with large HHH xsec: No FOPT!

Intuition: I don't think there will be FOPT…
Can we prove it?



PT in TRSM: start with only one scalar

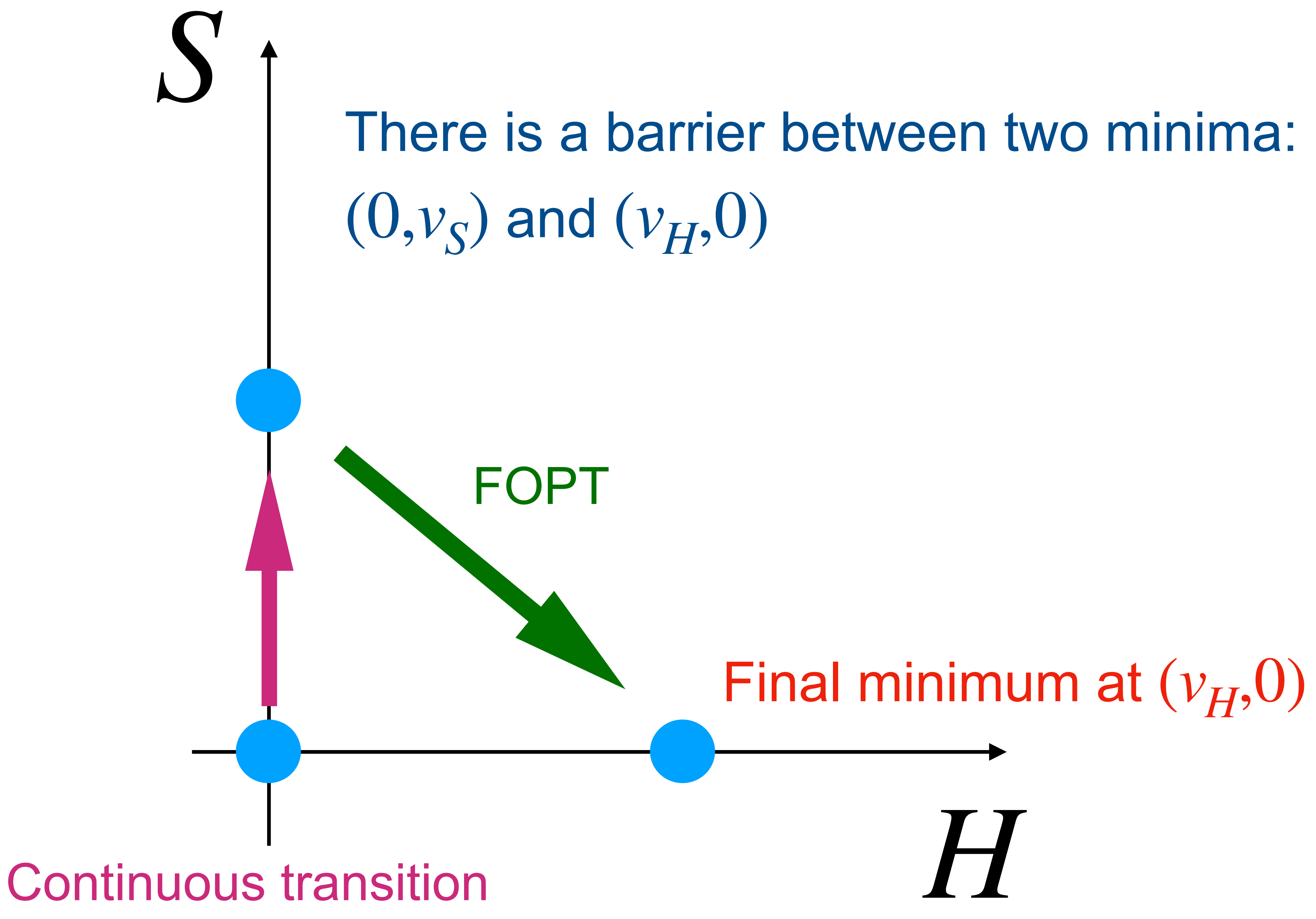
Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425

$$V \in \frac{1}{2}m^2(T)H^2 + \frac{1}{4}\lambda H^4 + \frac{1}{2}m_S^2(T)S^2 + \frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{HS}H_S^2 S^2$$

Extrema at: $\partial_H V = 0, \quad \partial_S V = 0$

Case 1: $\lambda\lambda_s - \lambda_{HS}^2 < 0$



Takeaway: Since there is a barrier between the two axes (fields)
You cannot put a minimum there!
So the field S must end up with a zero VEV
Therefor: No Mixing! No resonant HHH production!

PT in TRSM: start with only one scalar

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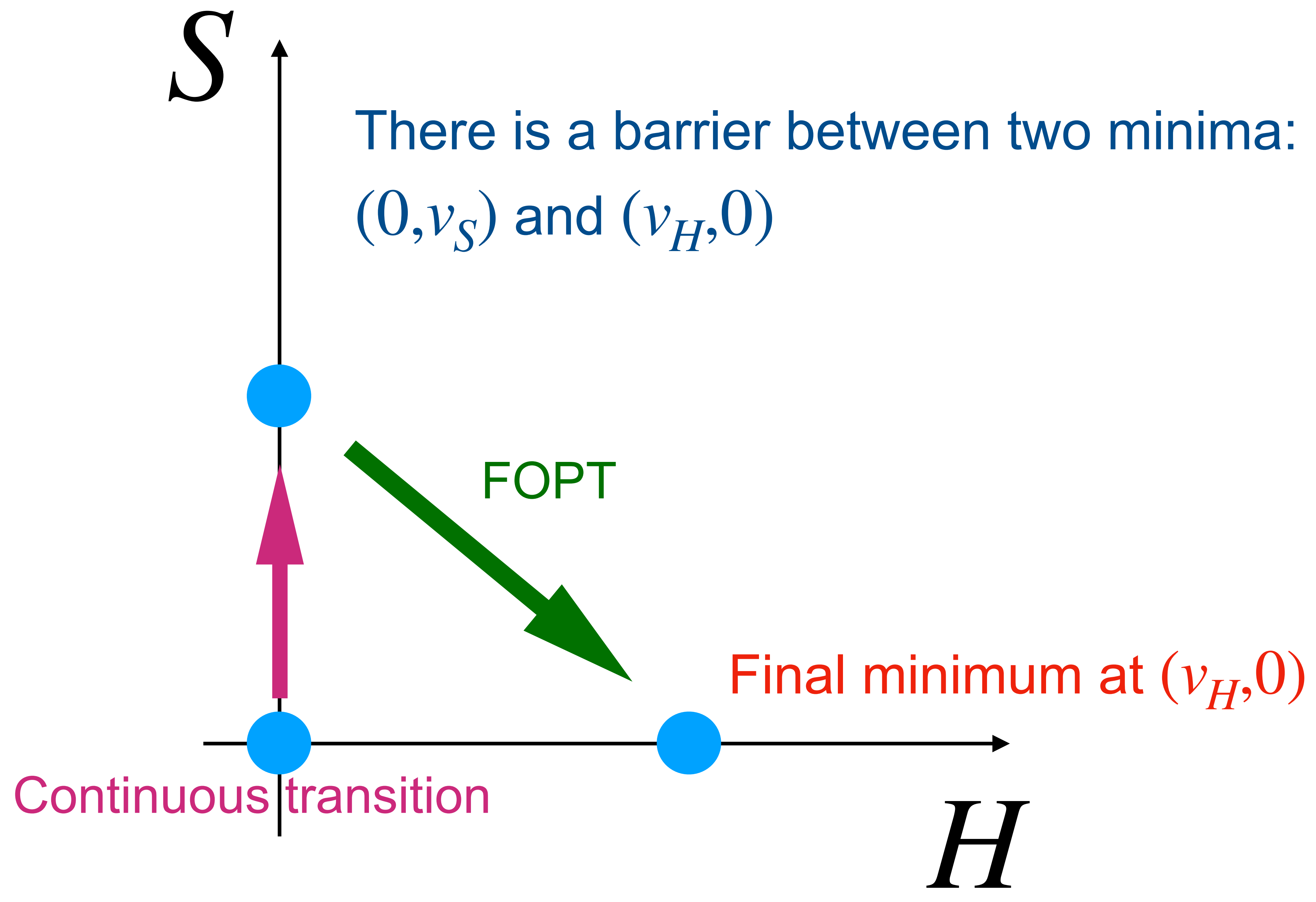
$$V \in \frac{1}{2}m^2(T)H^2 + \frac{1}{4}\lambda H^4 + \frac{1}{2}m_S^2(T)S^2 + \frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{HS}H_S^2S^2$$

Extrema at: $\partial_H V = 0, \quad \partial_S V = 0$

Case 1: $\lambda\lambda_s - \lambda_{HS}^2 < 0$

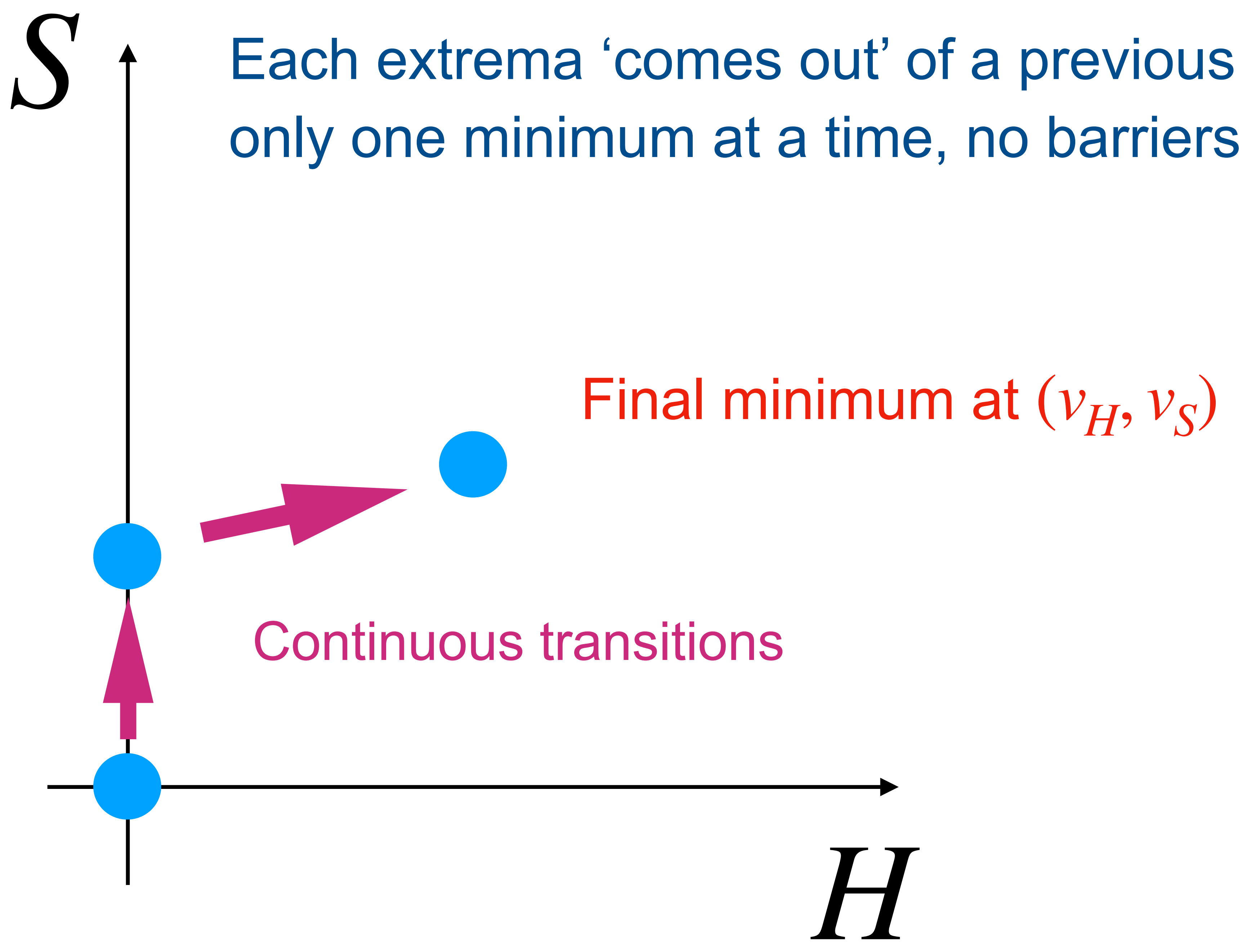
There is a barrier between two minima: $(0, v_S)$ and $(v_H, 0)$



Case 1: Yes FOPT! No resonant HHH production!

Case 2: $\lambda\lambda_s - \lambda_{HS}^2 > 0$

Each extrema 'comes out' of a previous one: only one minimum at a time, no barriers



Case 2: NO FOPT! Yes resonant HHH production!

PT in TRSM: with both scalars

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Call the fields x_i

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

Find all extrema by taking $\partial_i V = 0$

- Origin: $\mathbf{x}_0 \equiv (0,0,0)$.
- Axial extremum $\mathbf{x}_1 \equiv (x_1, 0, 0)$ with

$$x_1 = \sqrt{-m_1^2/c_{11}}.$$

- Planar extremum $\mathbf{x}_{12} \equiv (x_1, x_2, 0)$ with

$$x_1 = \sqrt{\frac{c_{12}m_2^2 - c_{22}m_1^2}{c_{11}c_{22} - c_{12}^2}}, \quad x_2 = \sqrt{\frac{c_{12}m_1^2 - c_{11}m_2^2}{c_{11}c_{22} - c_{12}^2}}.$$

- Bulk extremum $\mathbf{x}_{123} \equiv (x_1, x_2, x_3)$ with

$$x_1 = \frac{\sqrt{(c_{23}^2 - c_{22}c_{33})m_1^2 + (c_{12}c_{33} - c_{13}c_{23})m_2^2 + (c_{13}c_{22} - c_{12}c_{23})m_3^2}}{\sqrt{D}},$$

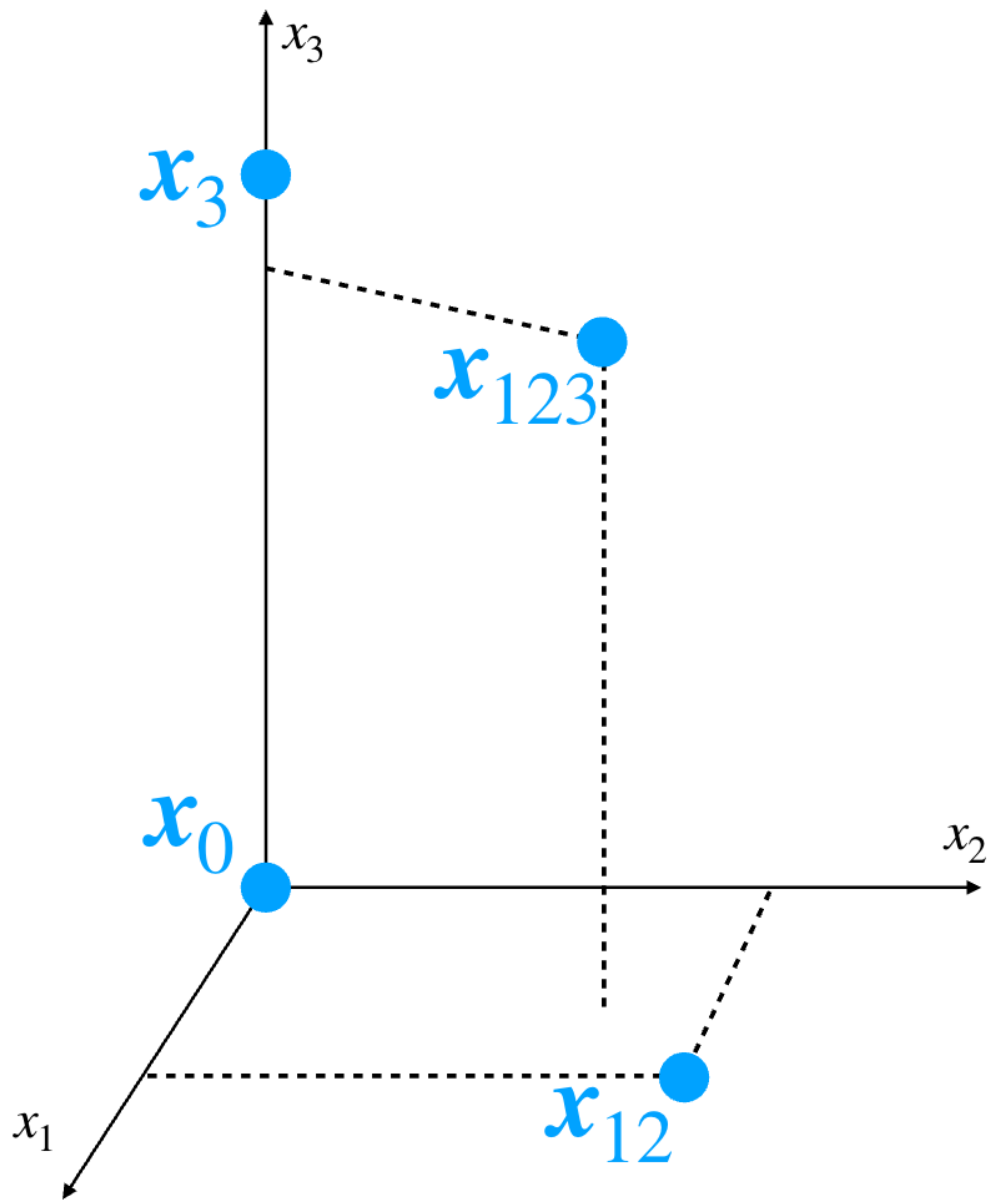
$$x_2 = \frac{\sqrt{(c_{12}c_{33} - c_{13}c_{23})m_1^2 + (c_{13}^2 - c_{11}c_{33})m_2^2 + (c_{11}c_{23} - c_{12}c_{13})m_3^2}}{\sqrt{D}},$$

$$x_3 = \frac{\sqrt{(c_{13}c_{22} - c_{12}c_{23})m_1^2 + (c_{11}c_{23} - c_{12}c_{13})m_2^2 + (c_{12}^2 - c_{11}c_{22})m_3^2}}{\sqrt{D}},$$

where

$$D = c_{11}c_{22}c_{33} + 2c_{12}c_{13}c_{23} - c_{13}^2 c_{22} - c_{11}c_{23}^2 - c_{12}^2 c_{33},$$

is the determinant of c_{ij}.



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https://arxiv.org/pdf/2404.12425

The extremum is a minimum if the eigenvalues of the Hessian of the potential h_{kl}, i.e. the mass matrix, evaluated at the extremum are all positive, with

$$h_{kl}(x_1,x_2,x_3) \equiv \partial_{x_k}\partial_{x_l}V(x_1,x_2,x_3) = (m_k^2 + \sum_i c_{ik}x_i^2)\delta_{kl} + 2c_{kl}x_kx_l. \qquad (4.16)$$

```
In[]:= curve = Simplify[Eigenvalues[Simplify[hessian /. Solutions[[16]]]]]

Out[]= {(4 (-2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh + c22 (ch1^2 - c11 chh)) Root[c22^4 ch1^8 ch2^3 Sqrt[-(c22 ch1^2 - 2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh - c11 c22 chh)^2 m1^6 -
    7 c12 c22^3 ch1^7 ch2^4 Sqrt[-(c22 ch1^2 - 2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh - c11 c22 chh)^2 m1^6 + ≪1527≫ + #1^3 &, 1]) / (-(-2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh + c22 (ch1^2 - c11 chh))^2)^(3/2),
 4 (-2 c12 ch1 ch2 + ≪2≫ + c22 (≪1≫)) Root[c22^4 ch1^8 ch2^3 Sqrt[-(c22 ch1^2 - 2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh - c11 c22 chh)^2 m1^6 - 7 c12 c22^3 ch1^7 ch2^4 Sqrt[-(≪1≫)^2 m1^6 + ≪1526≫ + (≪26≫ + ≪1≫) #1^2 + #1^3 &, 2]
 / (-(≪1≫)^2)^(3/2),
 (4 (-2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh + c22 (ch1^2 - c11 chh)) Root[c22^4 ch1^8 ch2^3 Sqrt[-(c22 ch1^2 - 2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh - c11 c22 chh)^2 m1^6 -
    7 c12 c22^3 ch1^7 ch2^4 Sqrt[-(c22 ch1^2 - 2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh - c11 c22 chh)^2 m1^6 + ≪1527≫ + #1^3 &, 3]) / (-(-2 c12 ch1 ch2 + c11 ch2^2 + c12^2 chh + c22 (ch1^2 - c11 chh))^2)^(3/2)}

Full expression not available (original memory size: 5.8 MB)
```

Not even mathematica could help… insight needed.

PT in TRSM: with both scalars

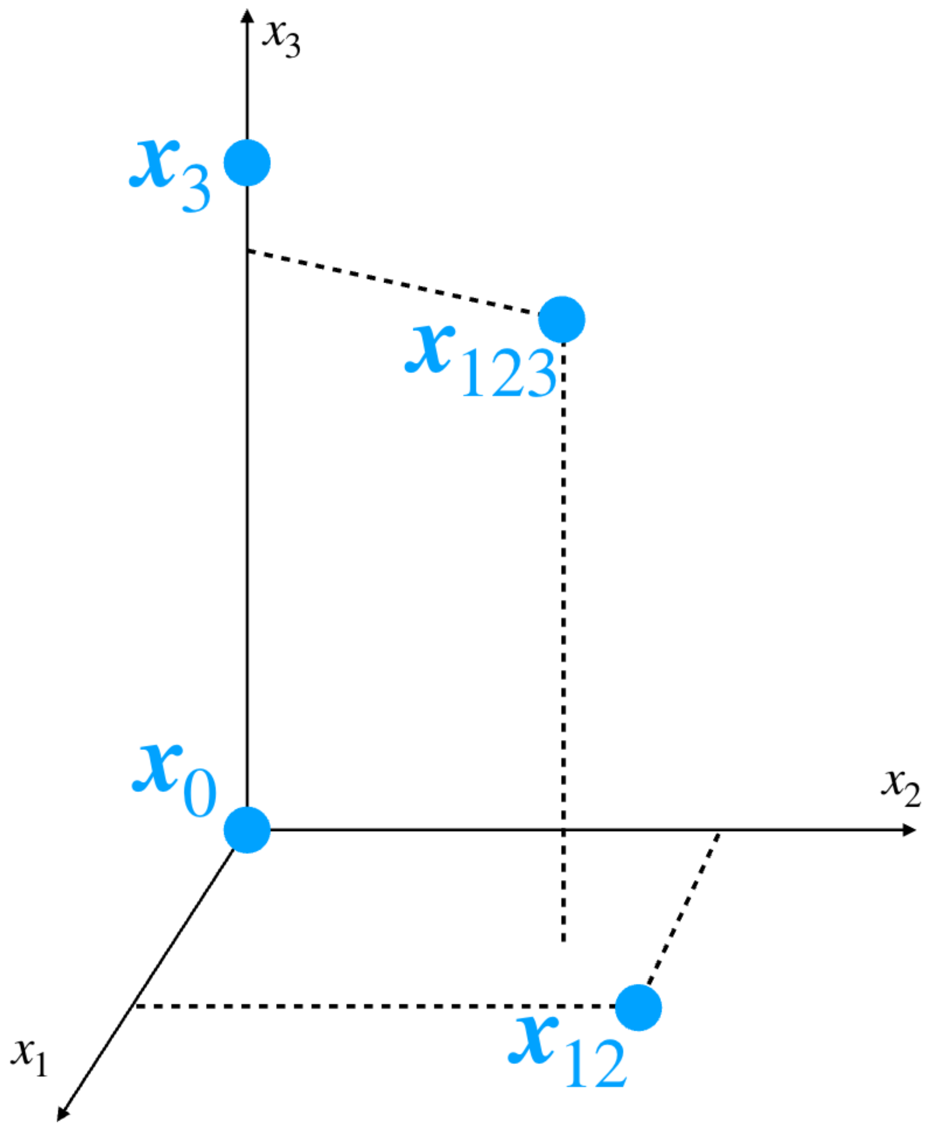
Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]
https://arxiv.org/pdf/2404.12425

$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

Insights:

- Z_2 symmetry: $(x \rightarrow -x)$ does not change the potential! I can focus on the positive x_i and generalise.
- The shape of the potential (whether an extremum is minimum) does not change if I scale the axes: $x^2 \rightarrow x$

Now the Hessian is simple: $$h_{kl}(x_1, x_2, x_3) \equiv \partial_{x_k}\partial_{x_l} V(x_1, x_2, x_3) = \frac{1}{2} c_{kl}.$$



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For resonant HHH:

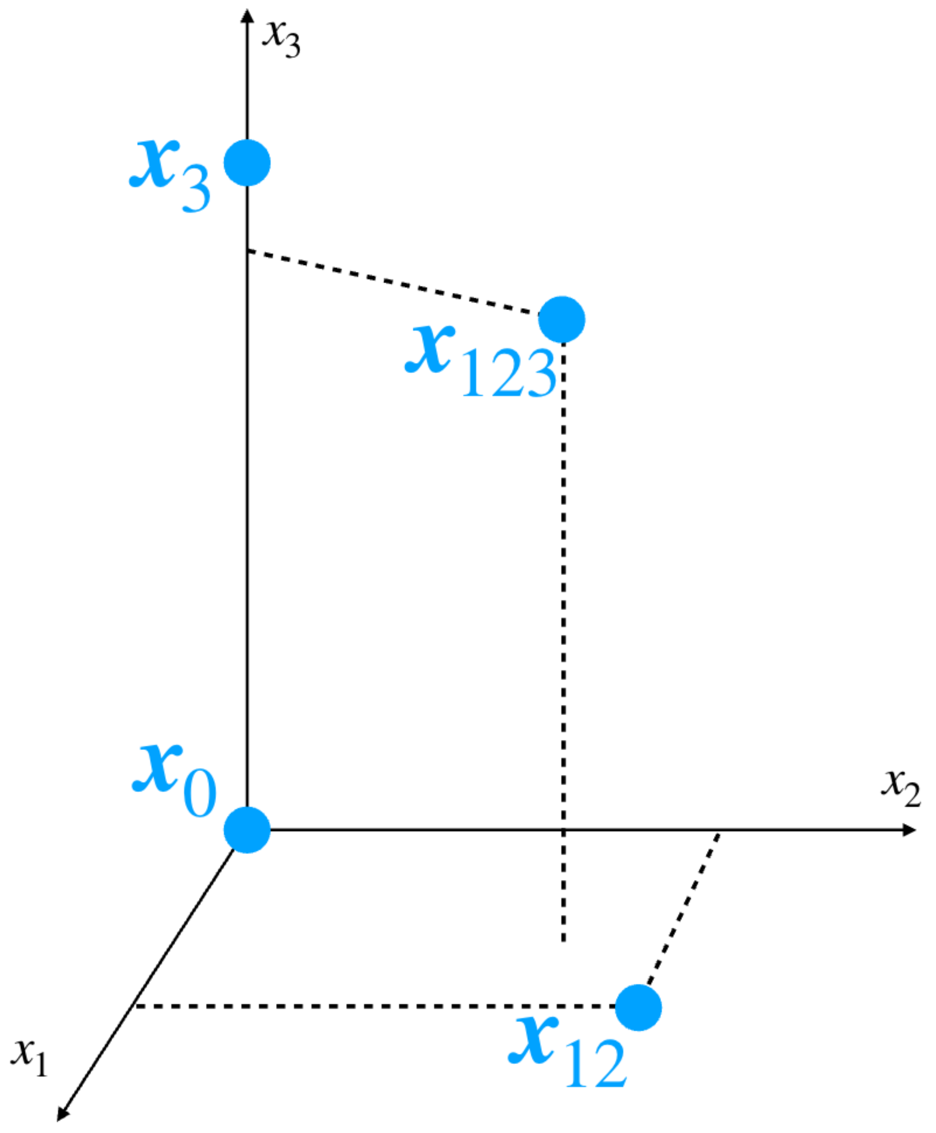
We demand that $\mathbf{x}_{123}$ is today's vacuum. The eigenvalues of the rescaled Hessian should then be positive. Sylvester's criterion, stating that a square Hermitian matrix is positive definite if *and only if* all the leading principal minors are positive, then gives

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0, \qquad (4.18)$$

where

$$D = c_{11}c_{22}c_{33} + 2c_{12}c_{13}c_{23} - c_{13}^2 c_{22} - c_{11}c_{23}^2 - c_{12}^2 c_{33},$$

is the determinant of c_{ij}.



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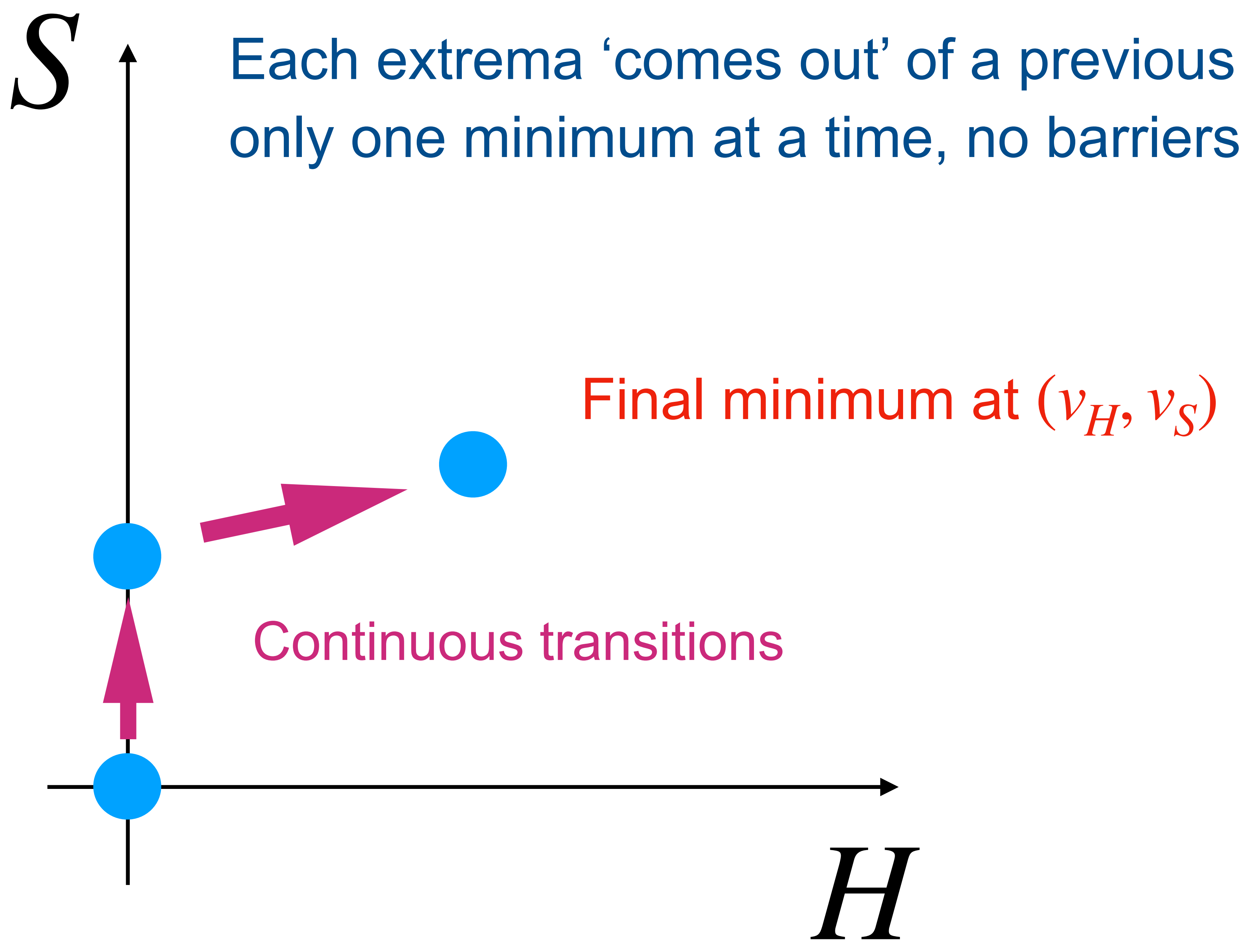
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad \boxed{C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0,} \quad \& \quad D > 0,$$

Case 2: $\lambda\lambda_s - \lambda_{HS}^2 > 0$

Each extrema 'comes out' of a previous one: only one minimum at a time, no barriers



Case 2: NO FOPT! Yes resonant HHH production!

PT in TRSM: with both scalars

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$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

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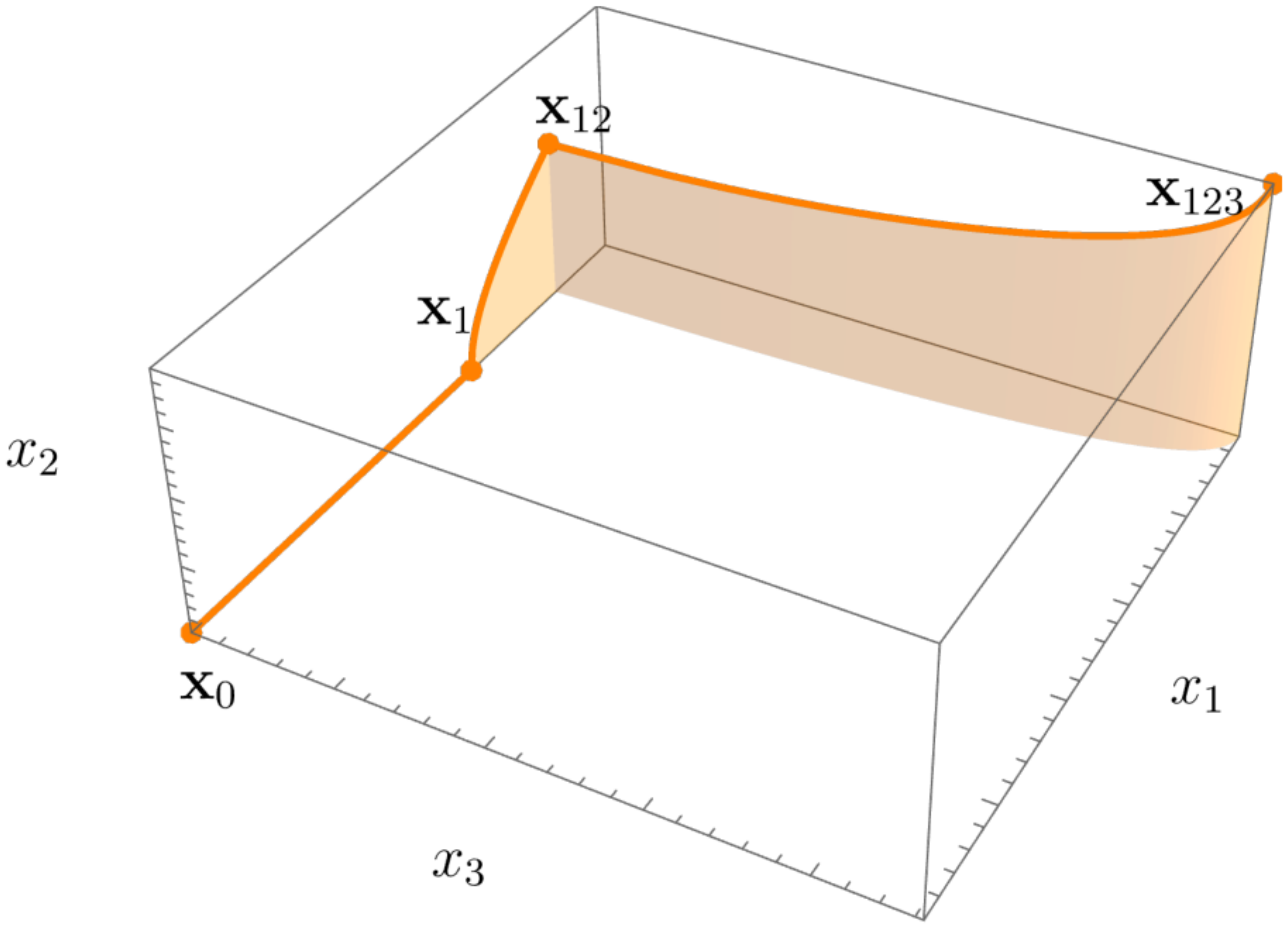

Each extrema ‘comes out’ of a previous one: only one minimum at a time, no barriers

Final minimum at (v_H, v_S, v_x)

Continuous transitions

Case 2: NO FOPT! Yes resonant HHH production!

PT in TRSM: with both scalars

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

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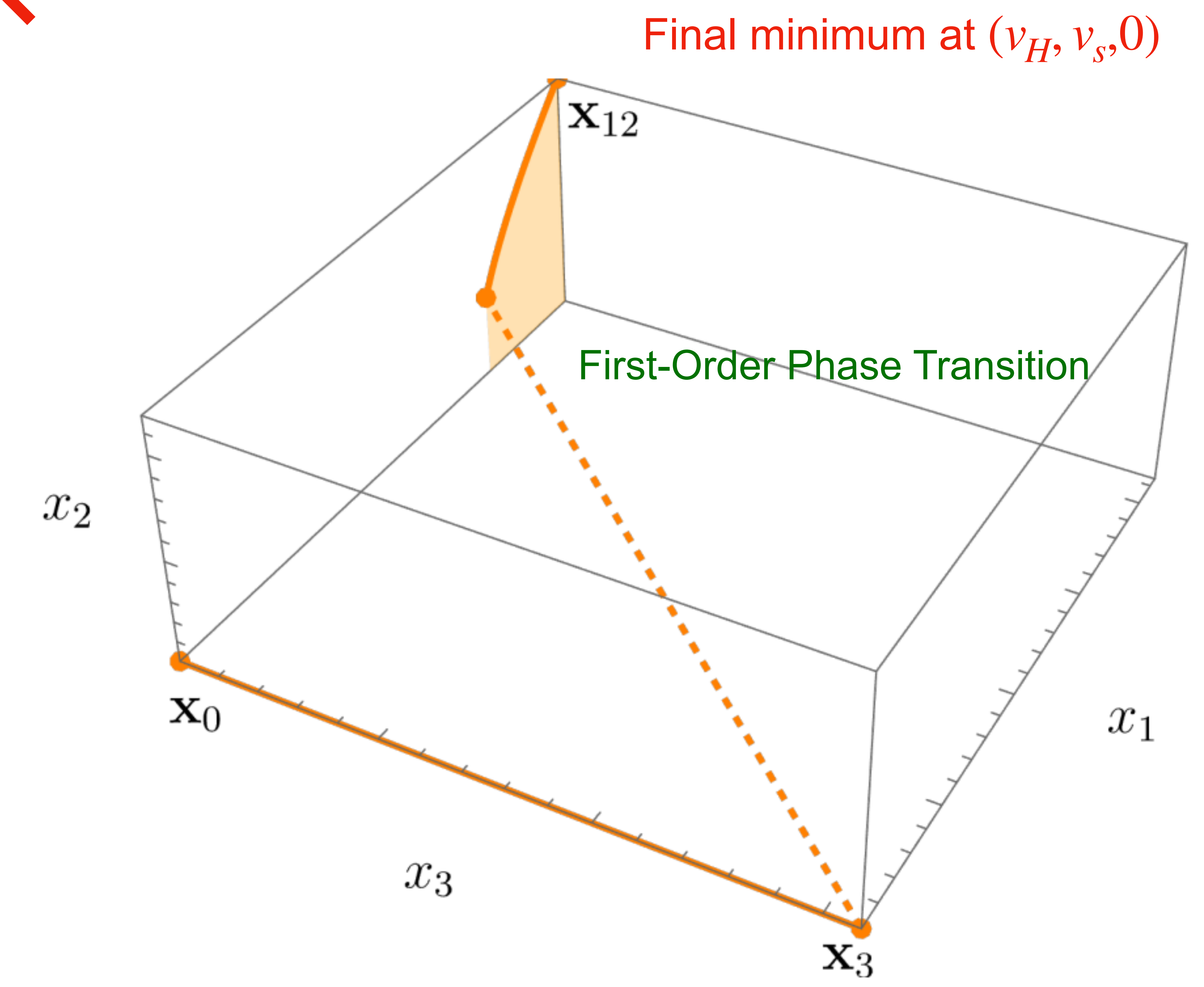
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,$$

$D < 0$



Case 1: Yes FOPT! No resonant HHH production!

PT in TRSM: with both scalars

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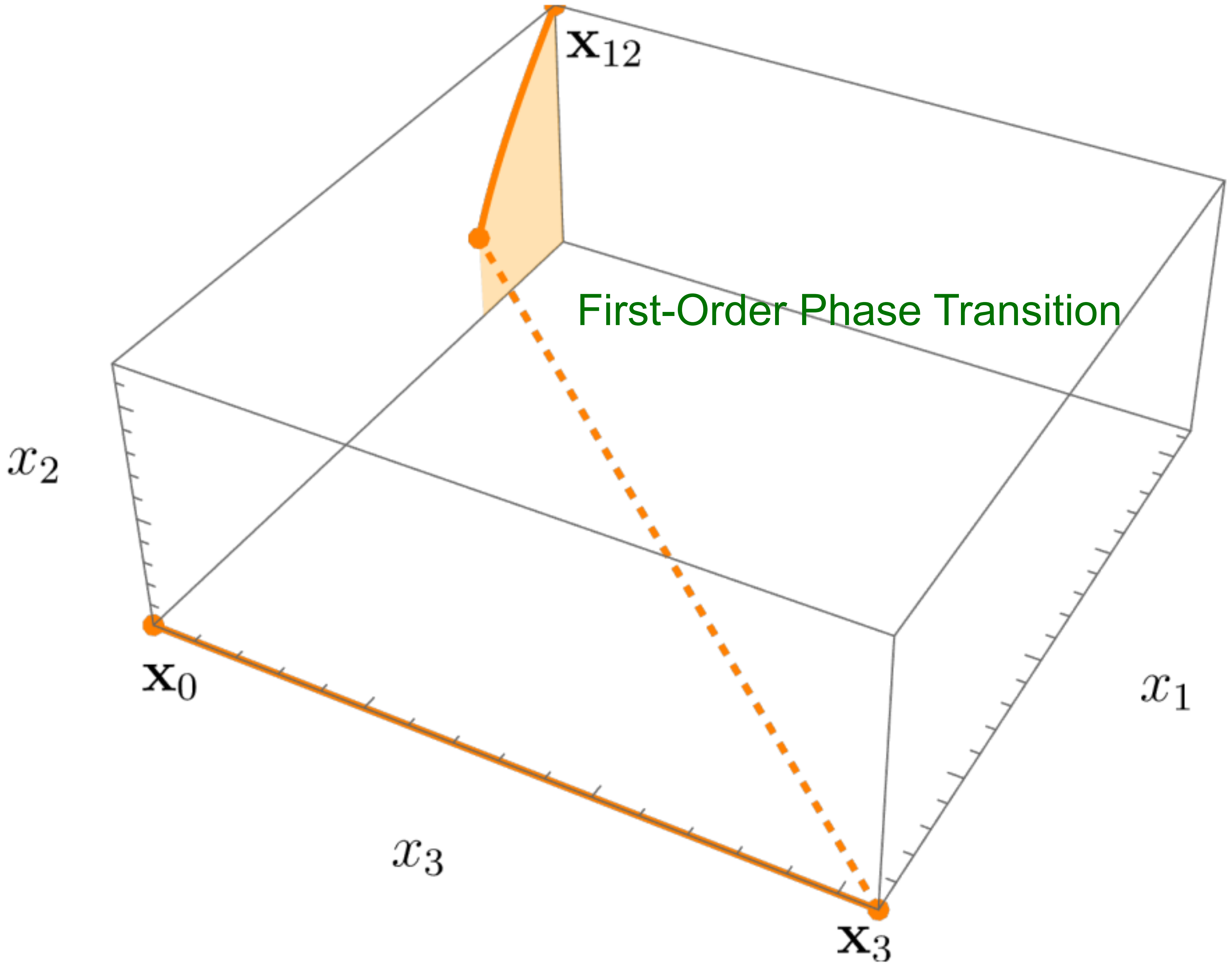
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,$$

$D < 0$

Nightmare!
If we want FOPT,
we cannot detect it with HHH



Case 1: Yes FOPT! No resonant HHH production!

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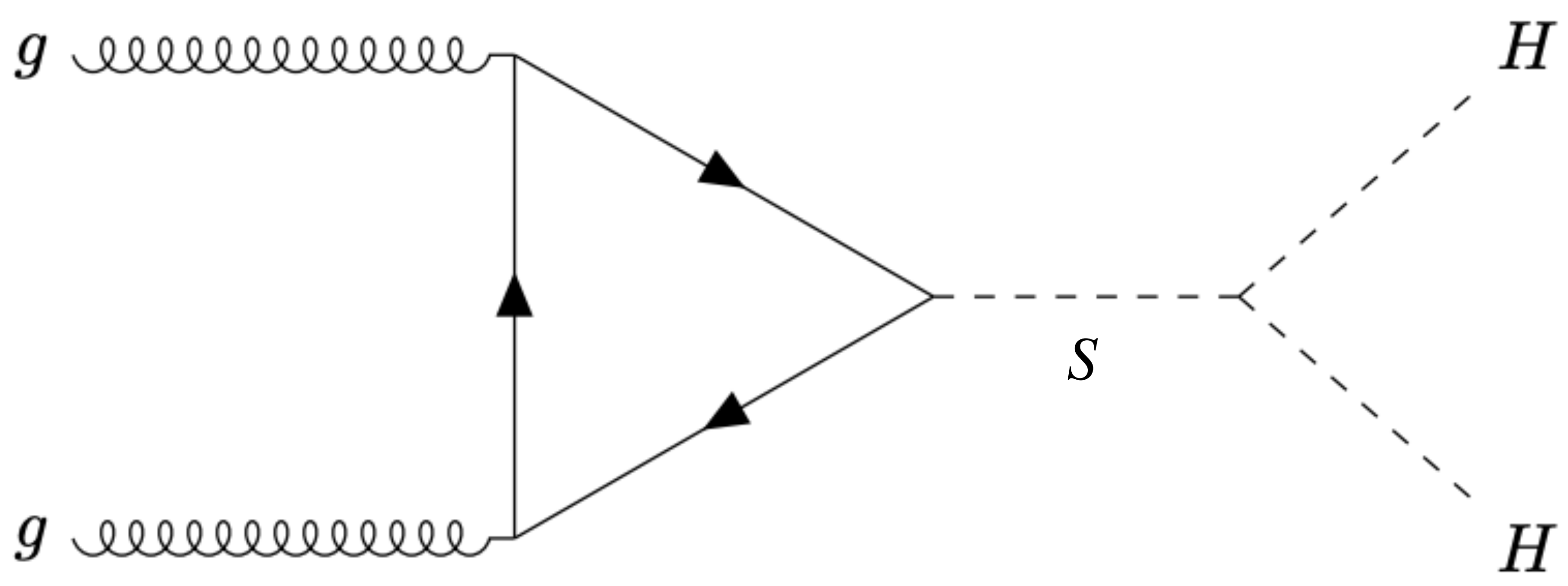
$$V(x_1, x_2, x_3) = \frac{1}{2}\sum_i m_i^2 x_i^2 + \frac{1}{4}\sum_{i,j} c_{ij} x_i^2 x_j^2,$$

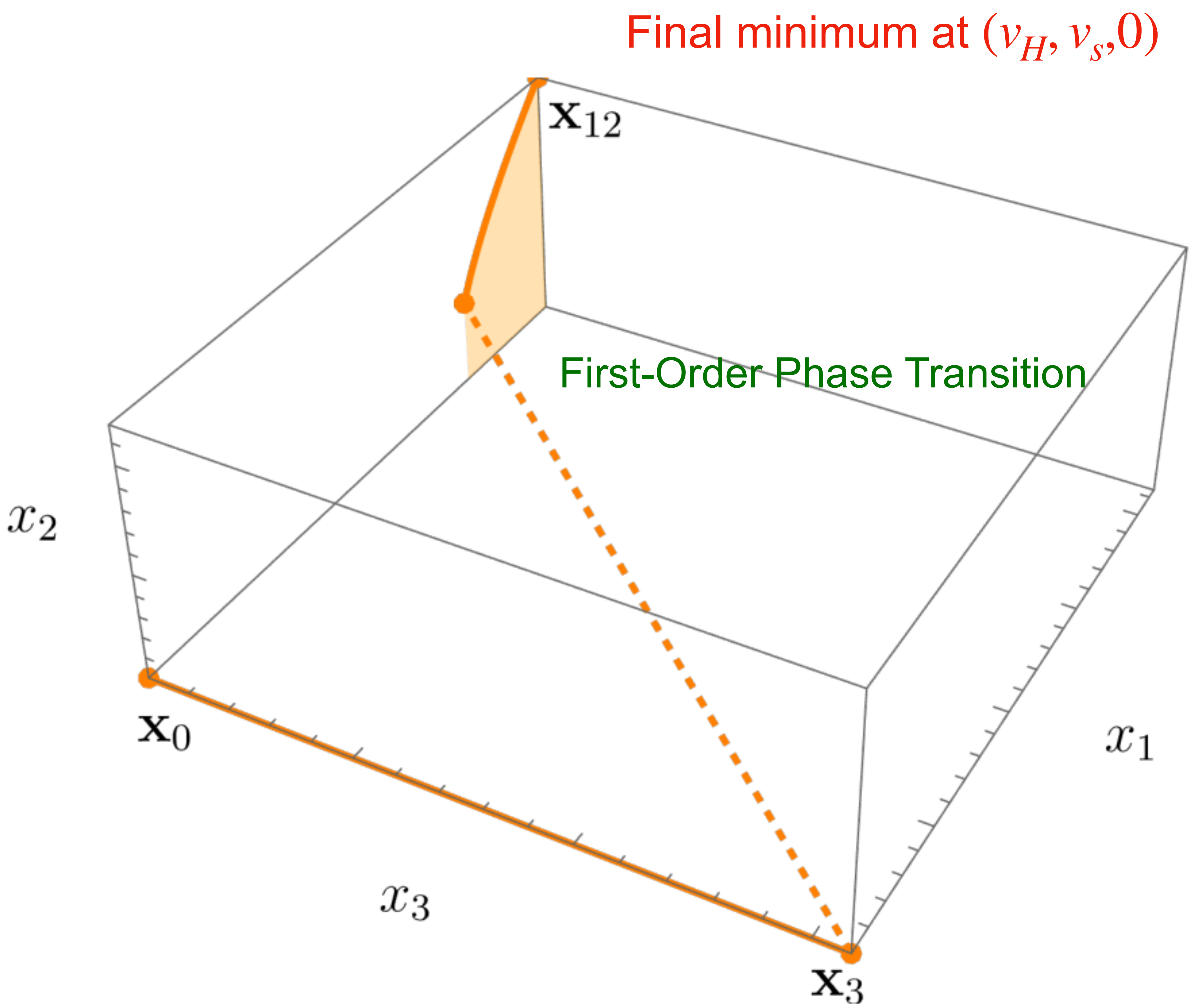
$$c_{ii} > 0, \quad \& \quad C_{ij} \equiv c_{ii}c_{jj} - c_{ij}^2 > 0, \quad \& \quad D > 0,$$

$D < 0$

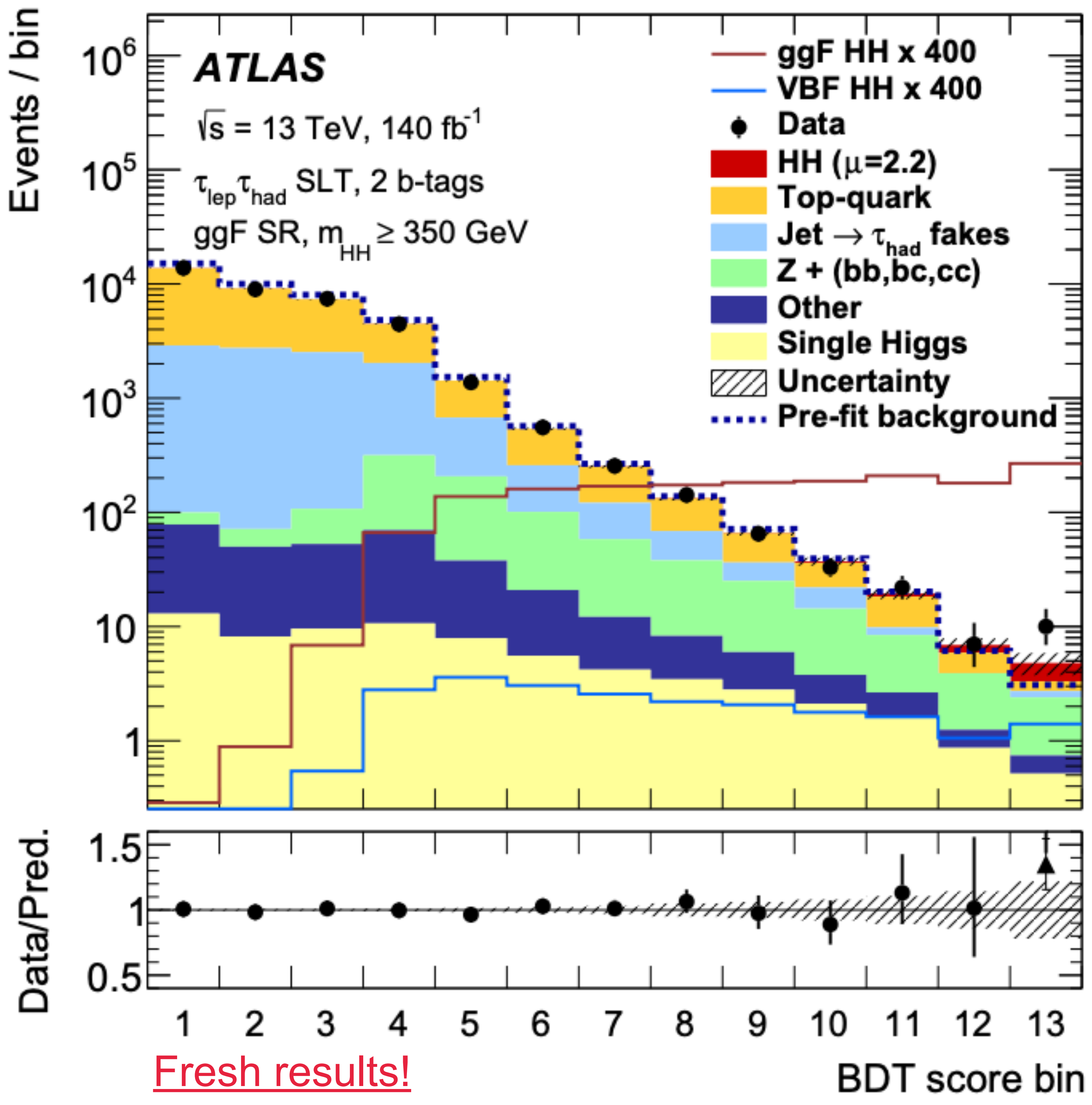
Silver lining:

Final minimum at $(v_H, v_s, 0)$ → resonant HH production!!



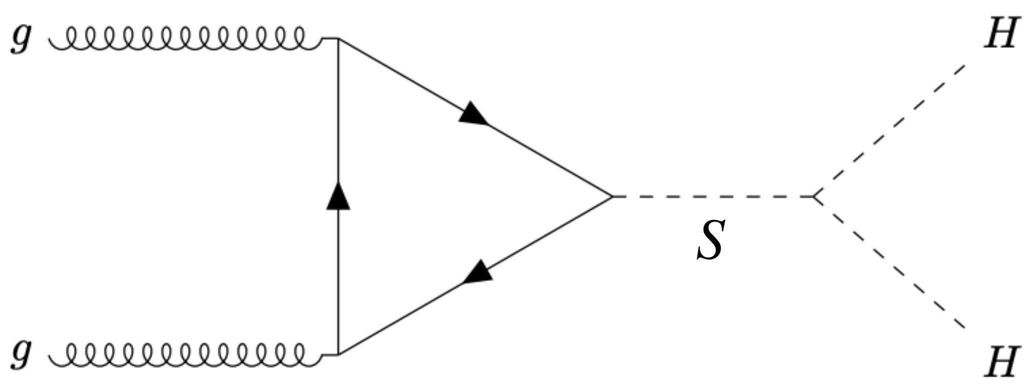


TRSM can accommodate both HH enhancement and FOPT!



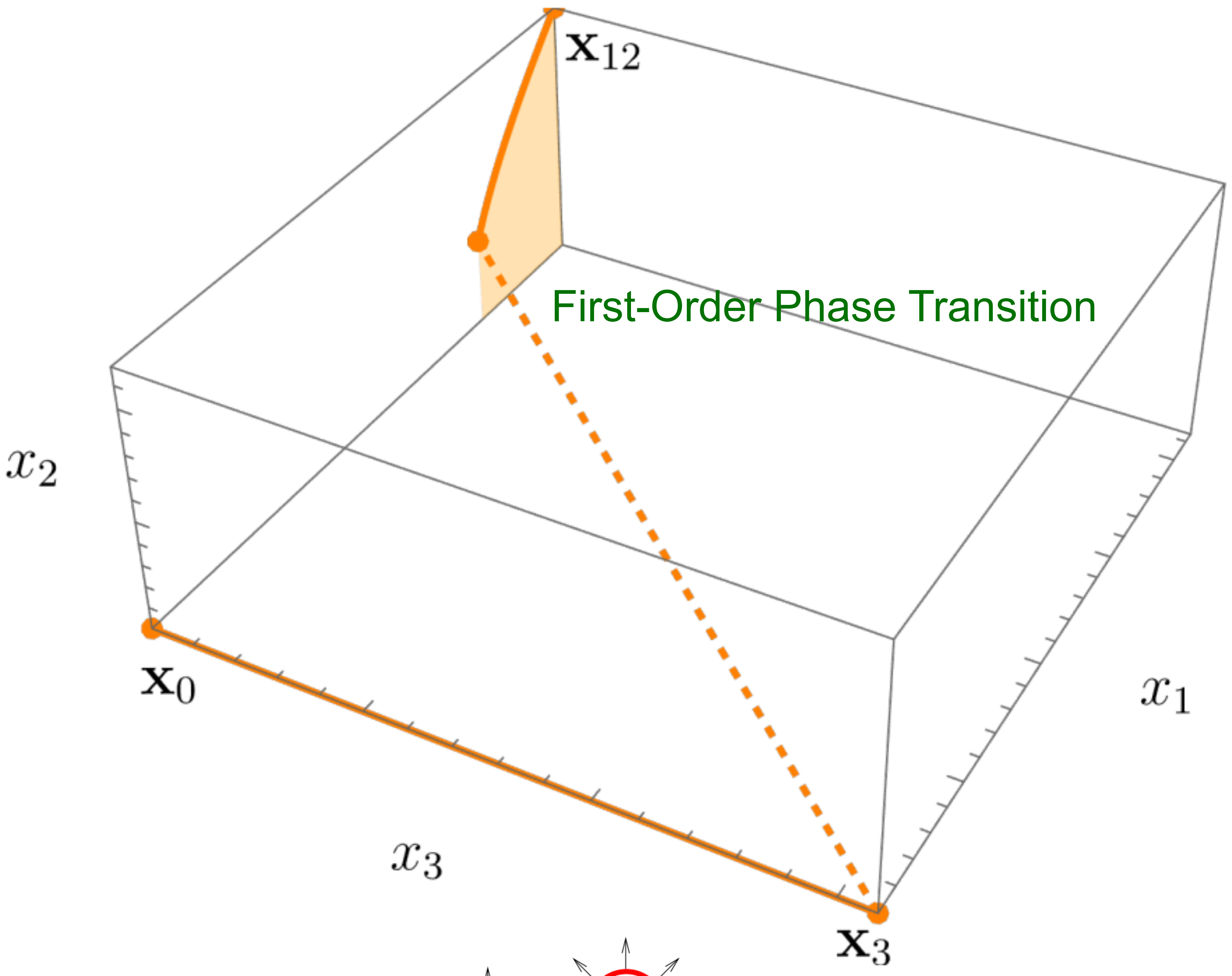
;)

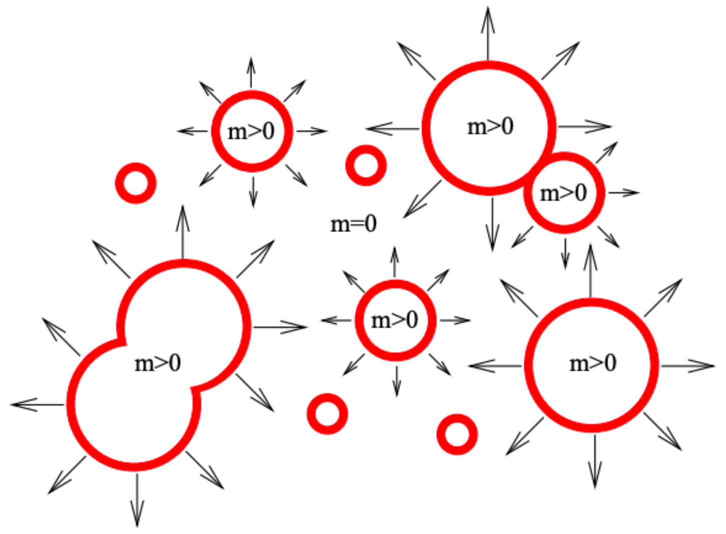
Fresh results!



?

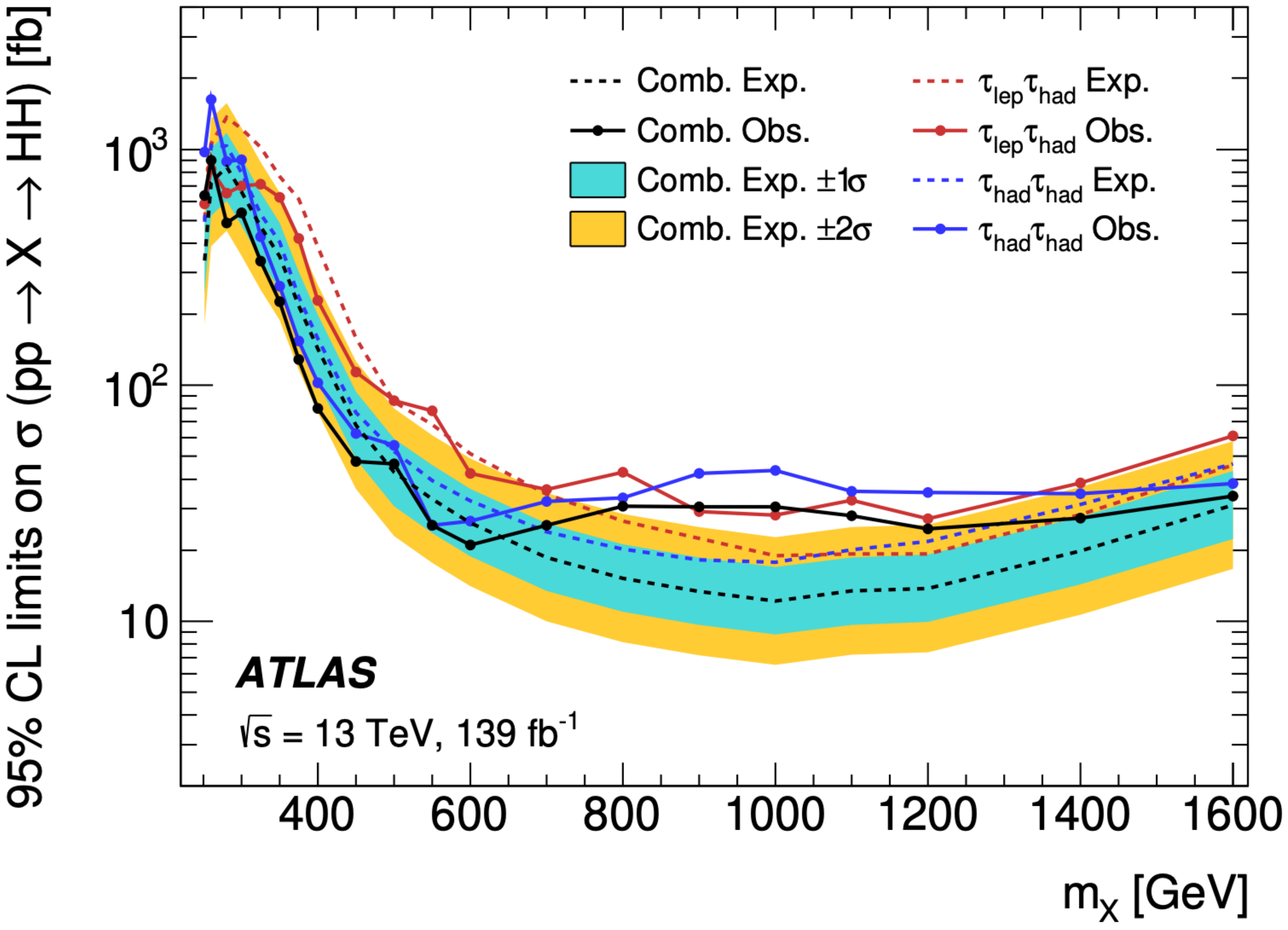
Final minimum at $(v_H, v_s, 0)$

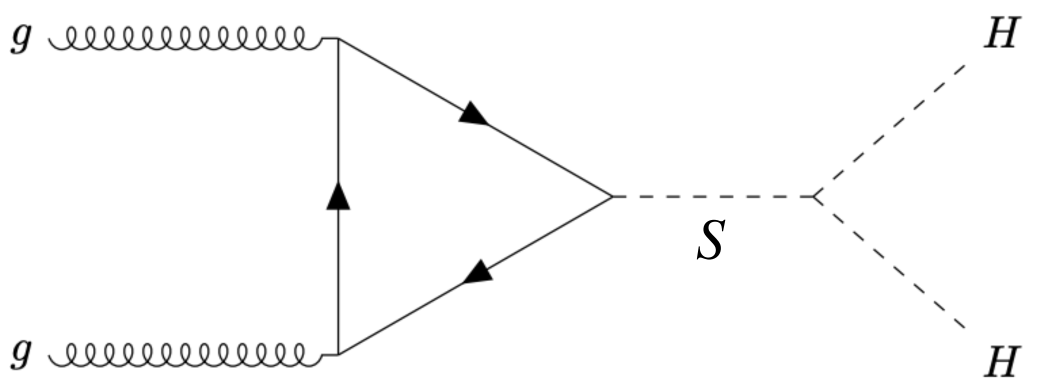




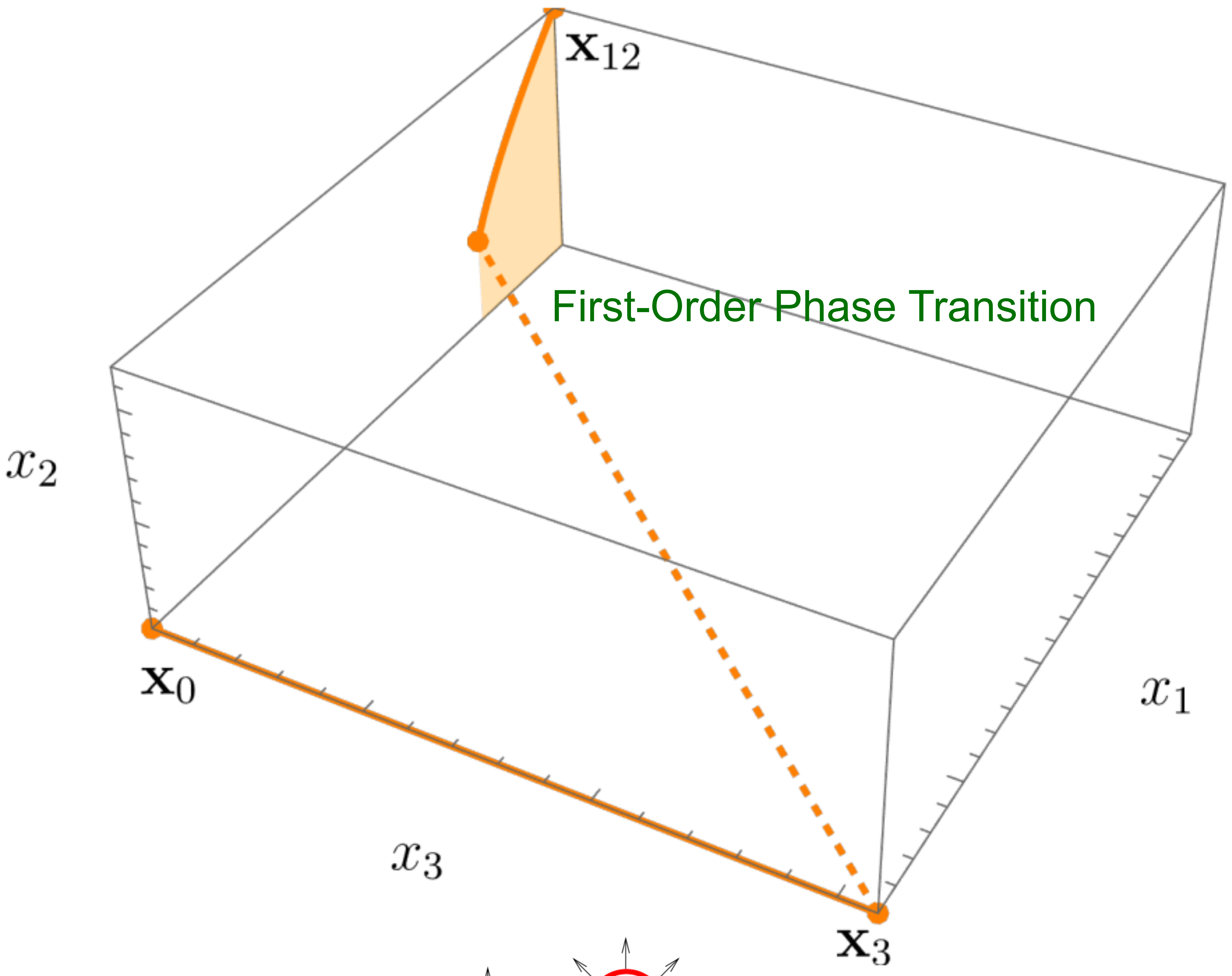
Impossible for only one added scalar

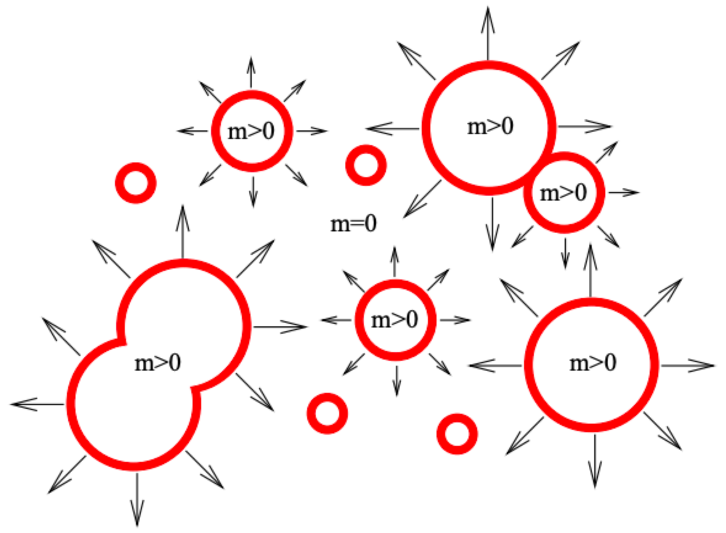
TRSM can accommodate both HH enhancement and FOPT!





Final minimum at $(v_H, v_s, 0)$





Impossible for only one added scalar

Final notes

Osama Karkout,[1] Andreas Papaefstathiou,[2] Marieke Postma,[1,3] Gilberto Tetlalmatzi-Xolocotzi,[4,5] Jorinde van de Vis,[6] Tristan du Pree[1]

https://arxiv.org/pdf/2404.12425

- Z_2 symmetric TRSM can **enhance HHH** if both scalars have nonzero VEVs at zero temperature (today)
- Z_2 symmetric TRSM can accommodate **First Order Phase Transitions** (desired for matter-antimatter asymmetry)
- Z_2 symmetric TRSM **cannot accommodate both** at the same time! Zero scalar VEV required for FOPT

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- Z_2 symmetric TRSM **cannot accommodate both** at the same time! Zero scalar VEV required for FOPT

Ideas to achieve both FOPT and HHH:

- Add terms that break Z_2 symmetry
- Add yet another scalar ;)

I presented analytic analysis for LO effective thermal potential.
Going to NLO numerically showed us the same conclusion

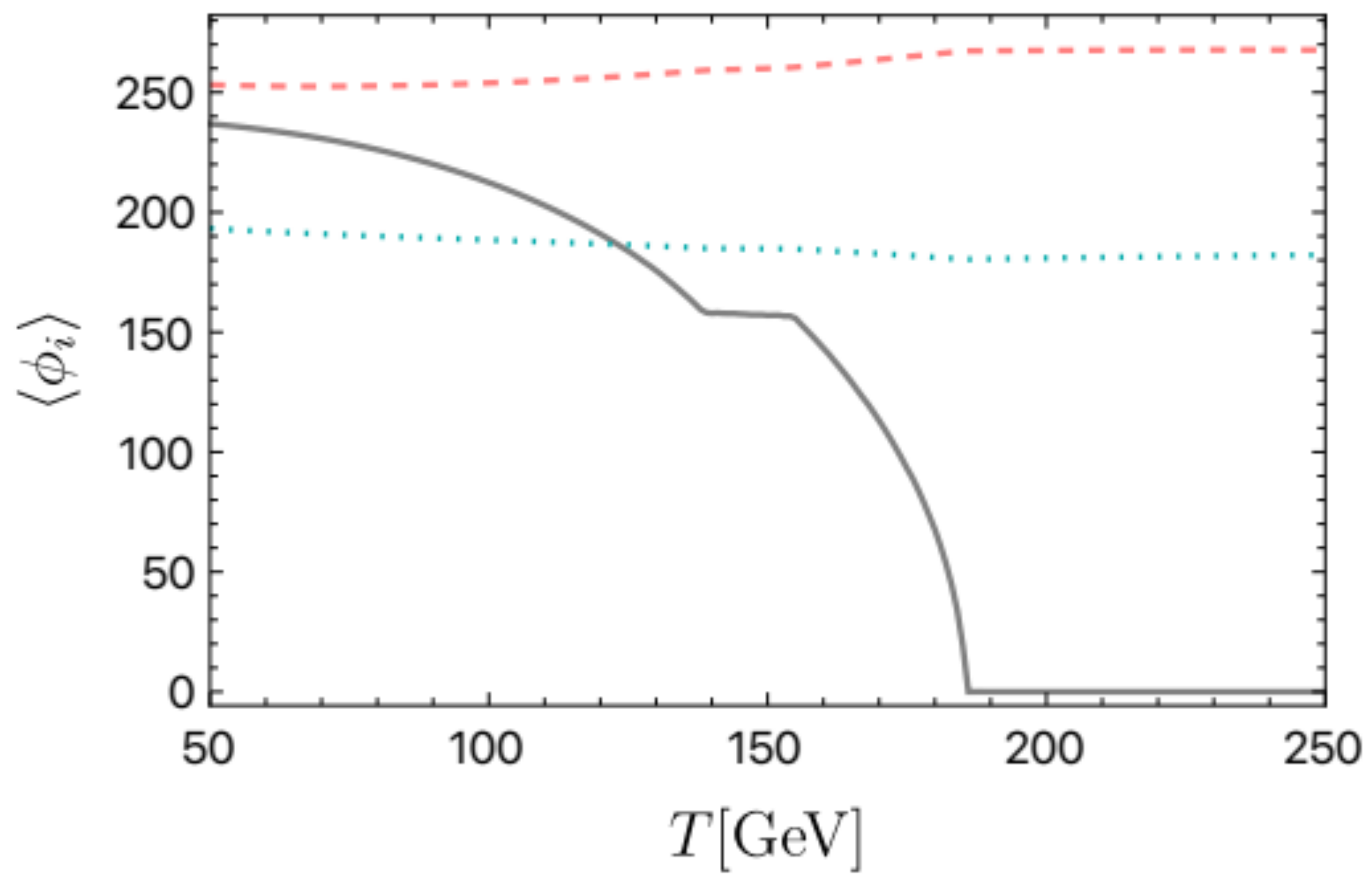

Figure 5: Evolution of the field expectation values in the minimum of the potential for the third BM point in Table 2. The Higgs field is represented by gray solid, ϕ_2 by dashed pink, and ϕ_3 by dotted cyan.

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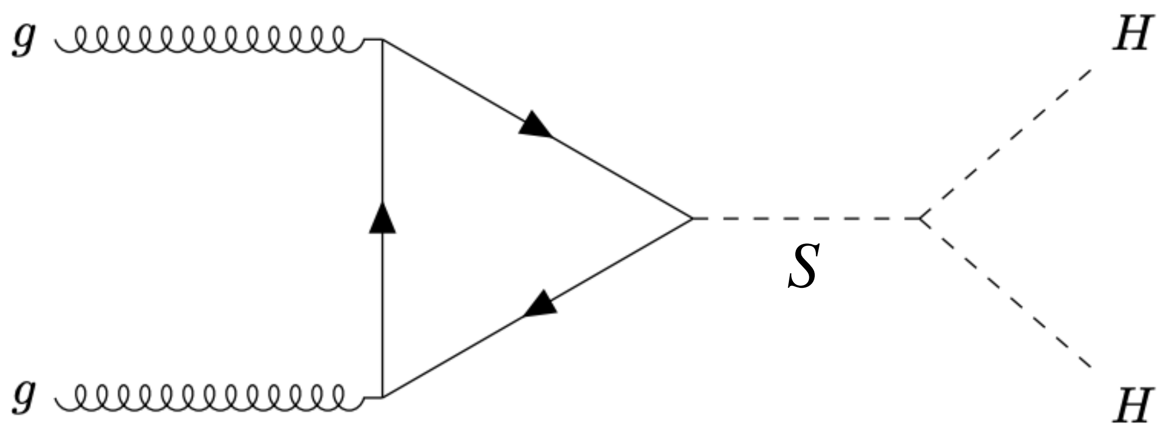
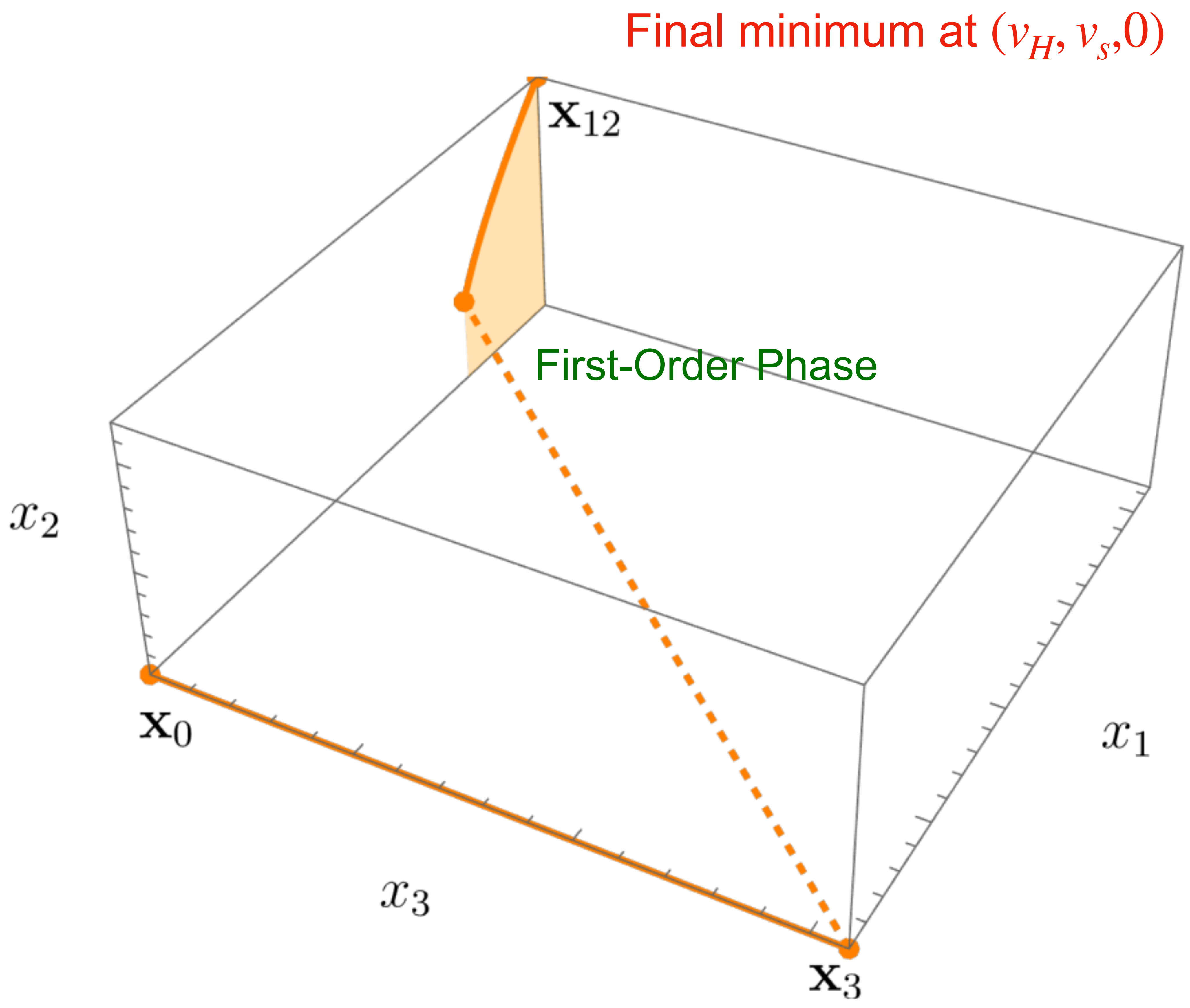
- Z_2 symmetric TRSM can **enhance HH** and accommodate a **First Order Phase Transition (FOPT):**
  one added scalar gets a vev, mixes with Higgs, and enhances HH production, while the other provides a barrier for FOPT
- Further studies can include gravitational waves and dark matter constraints for TRSM benchmark points for HH enhancement



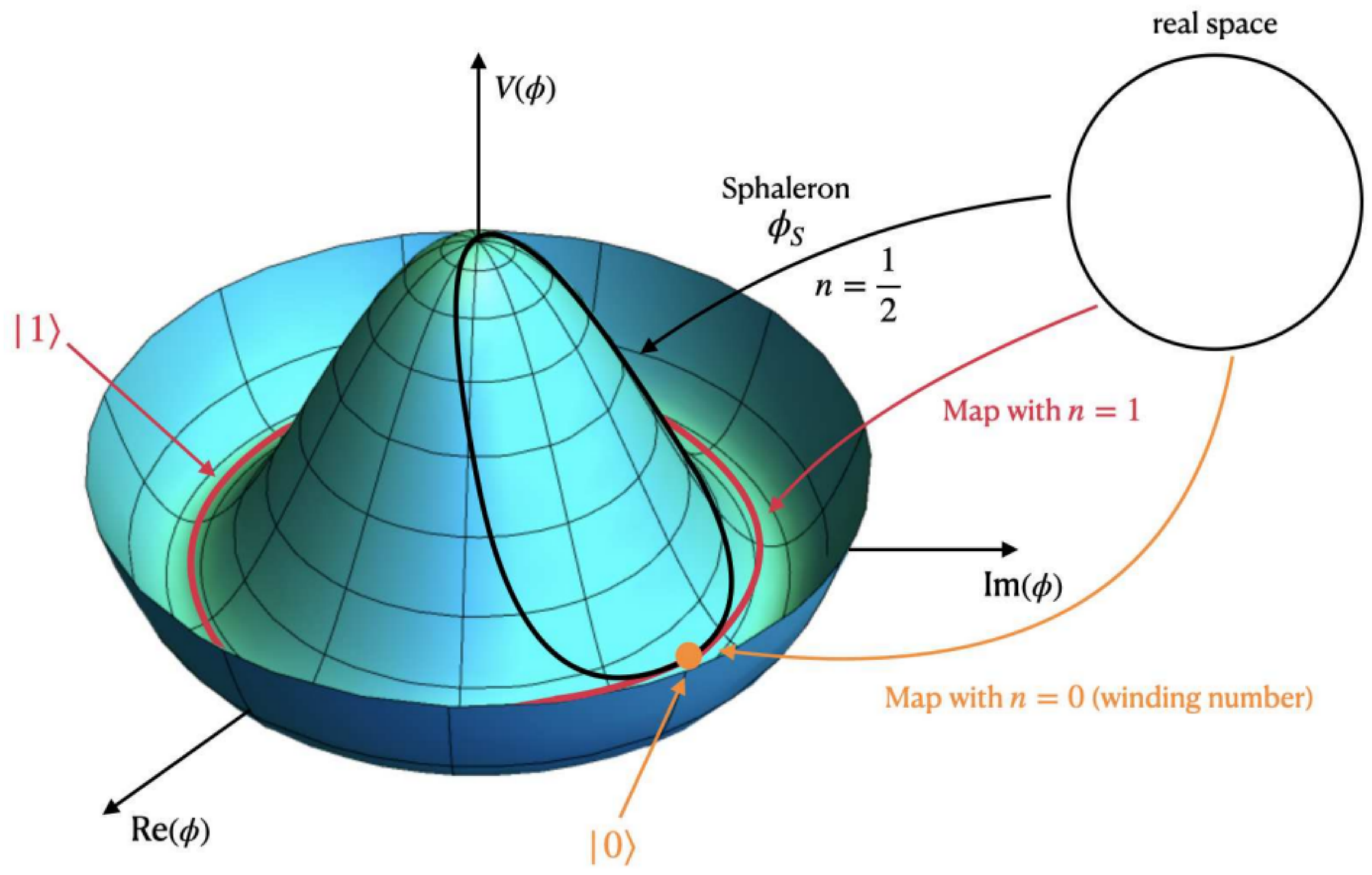


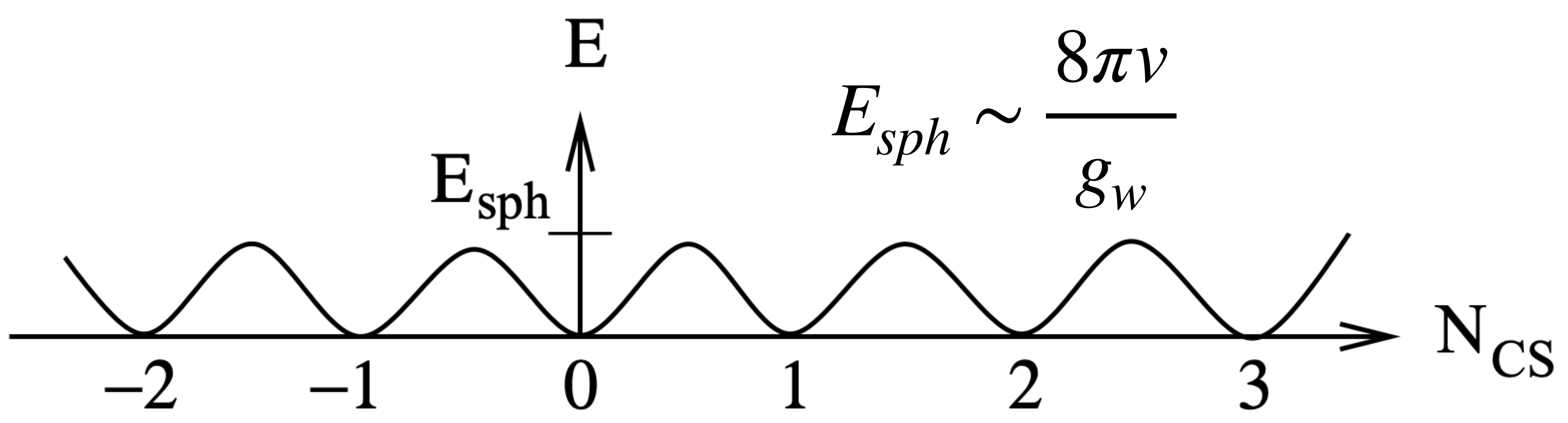

YOU CALLED FOR BACKUP?

Electroweak Baryogenesis

Baryon number violation

In SM: left handed B+L violated!





$$E_{sph} \sim \frac{8\pi v}{g_w}$$

Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

v is the Higgs VEV

Electroweak Baryogenesis

Baryon number violation

In SM: left handed B+L violated!

$$\partial_\mu J^\mu_{B_L+L_L} = \frac{3g^2}{32\pi^2}\epsilon_{\alpha\beta\gamma\delta}W_a^{\alpha\beta}W_a^{\gamma\delta}$$

where $W_a^{\alpha\beta}$ is the SU(2) field strength.

$$\Delta B = \Delta L = \pm 3 \qquad (2.2)$$

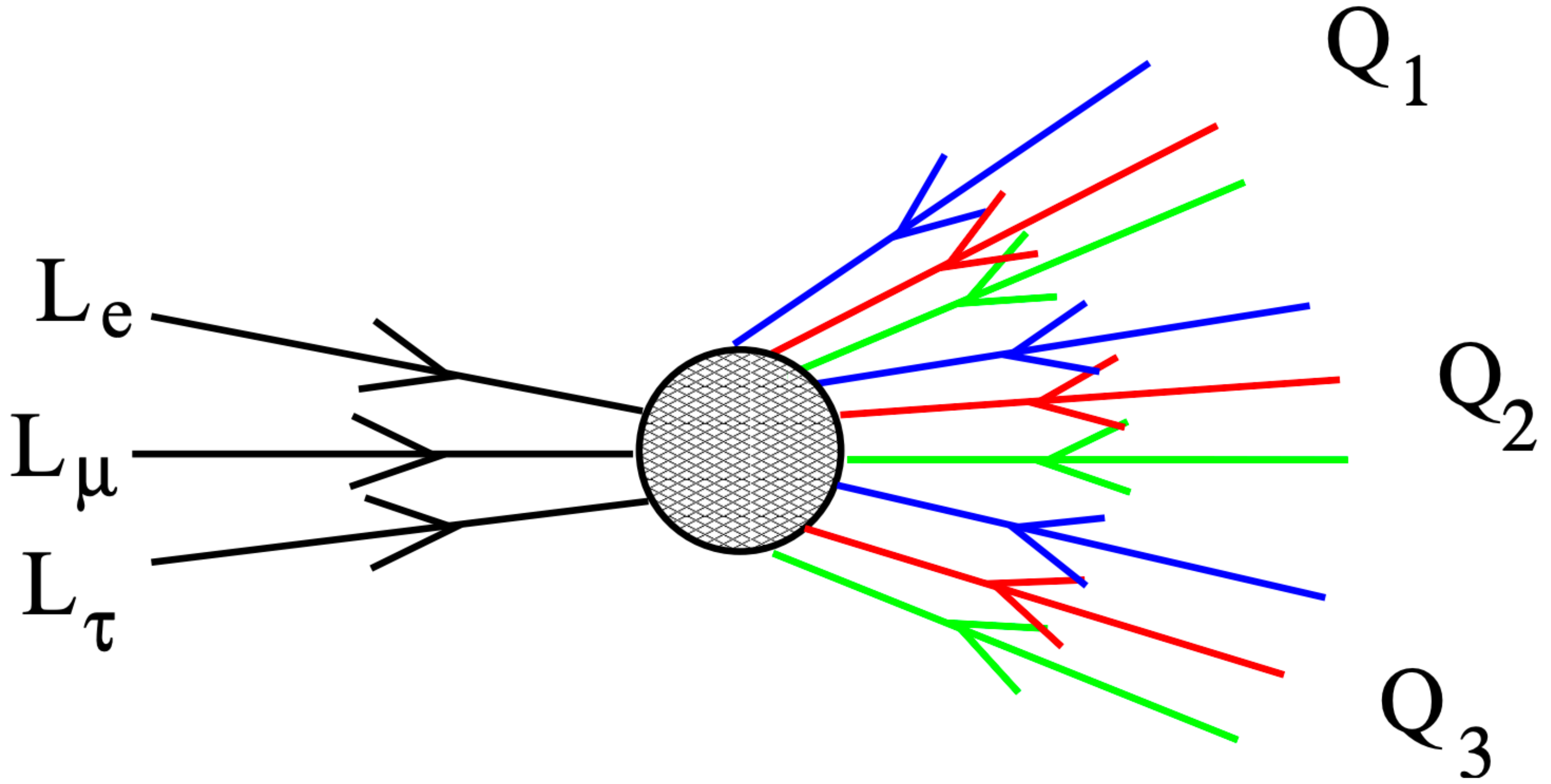
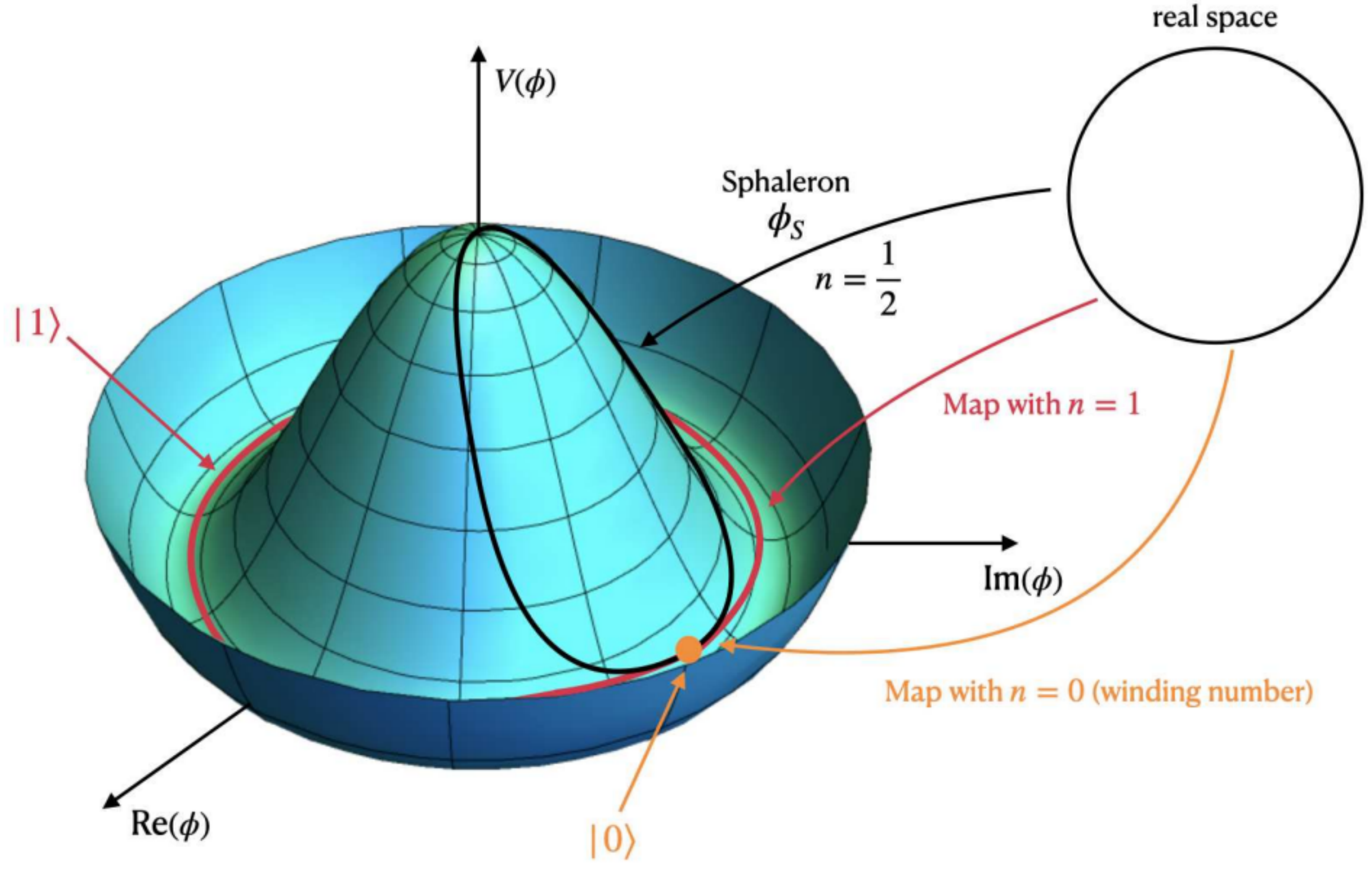


Fig. 4. The sphaleron.



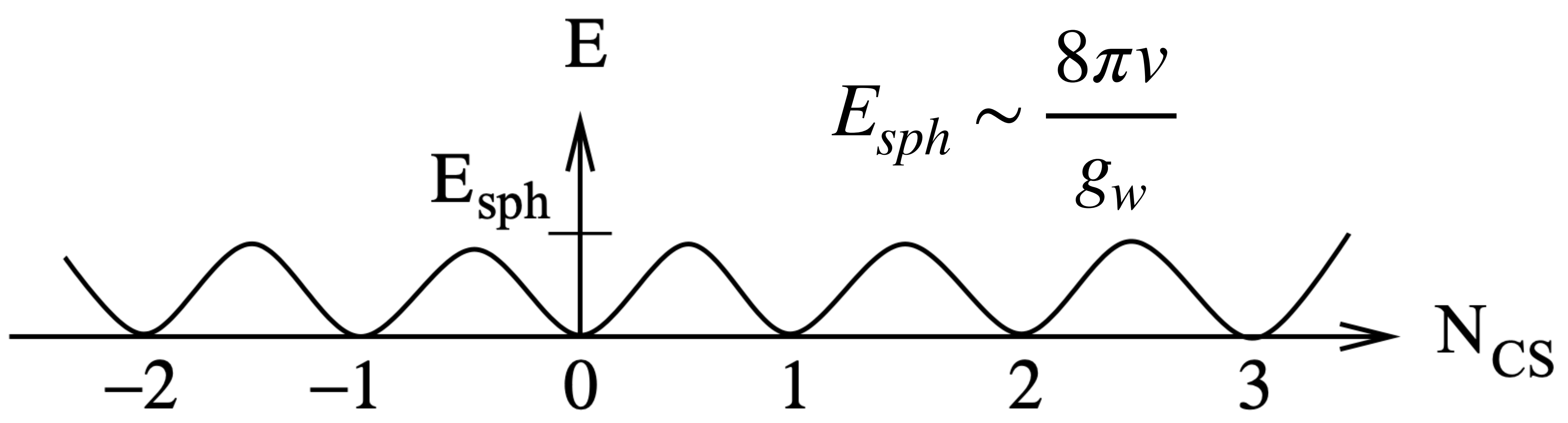


Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

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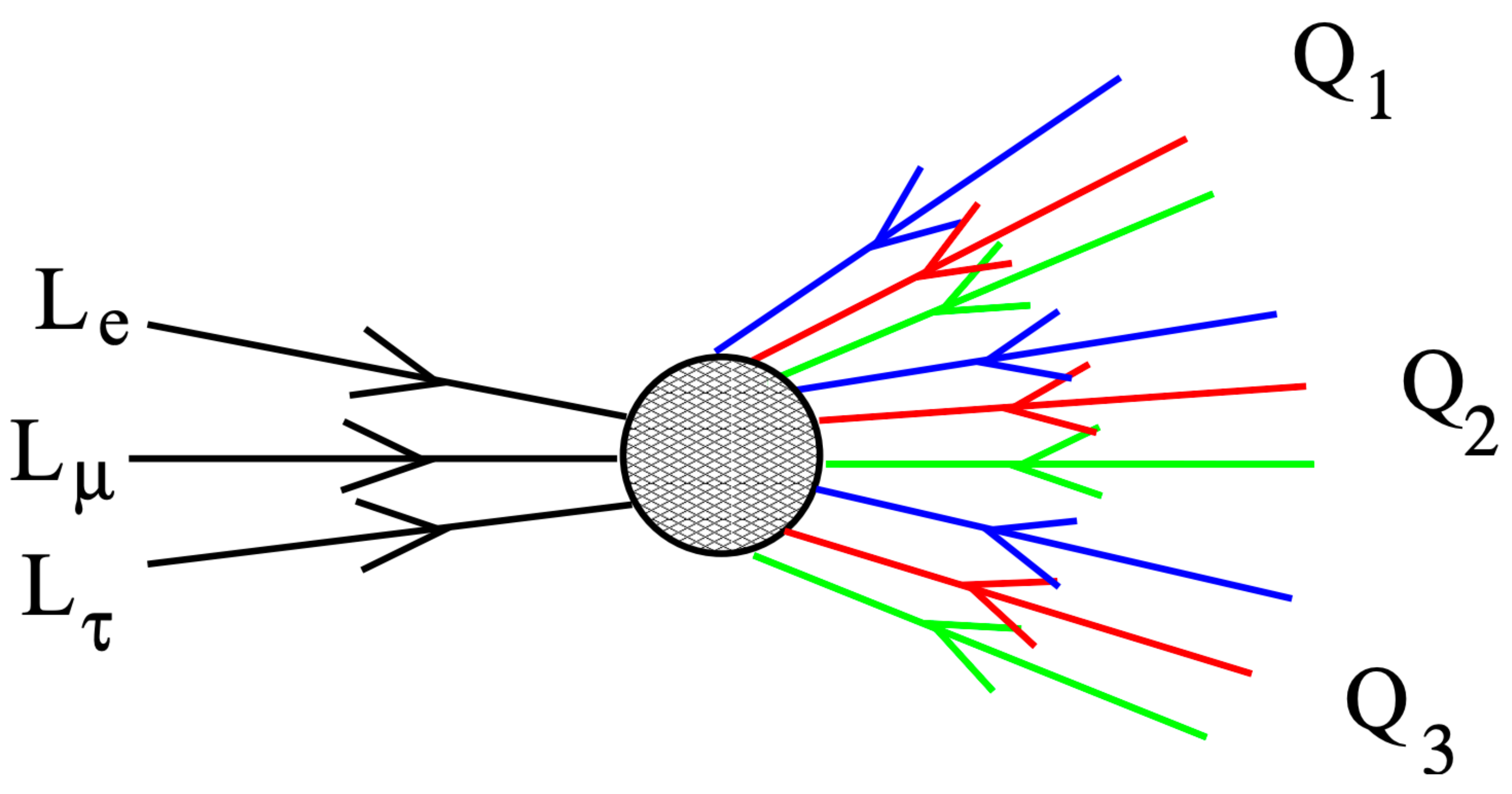
Electroweak Baryogenesis

Baryon number violation

Rate of tunnelling to another vacuum:

$$\Gamma_{sph}(T) \sim e^{-E_{sph}/T} \sim e^{-v/T}$$

$\Delta B = \Delta L = \pm 3$ (2.2)



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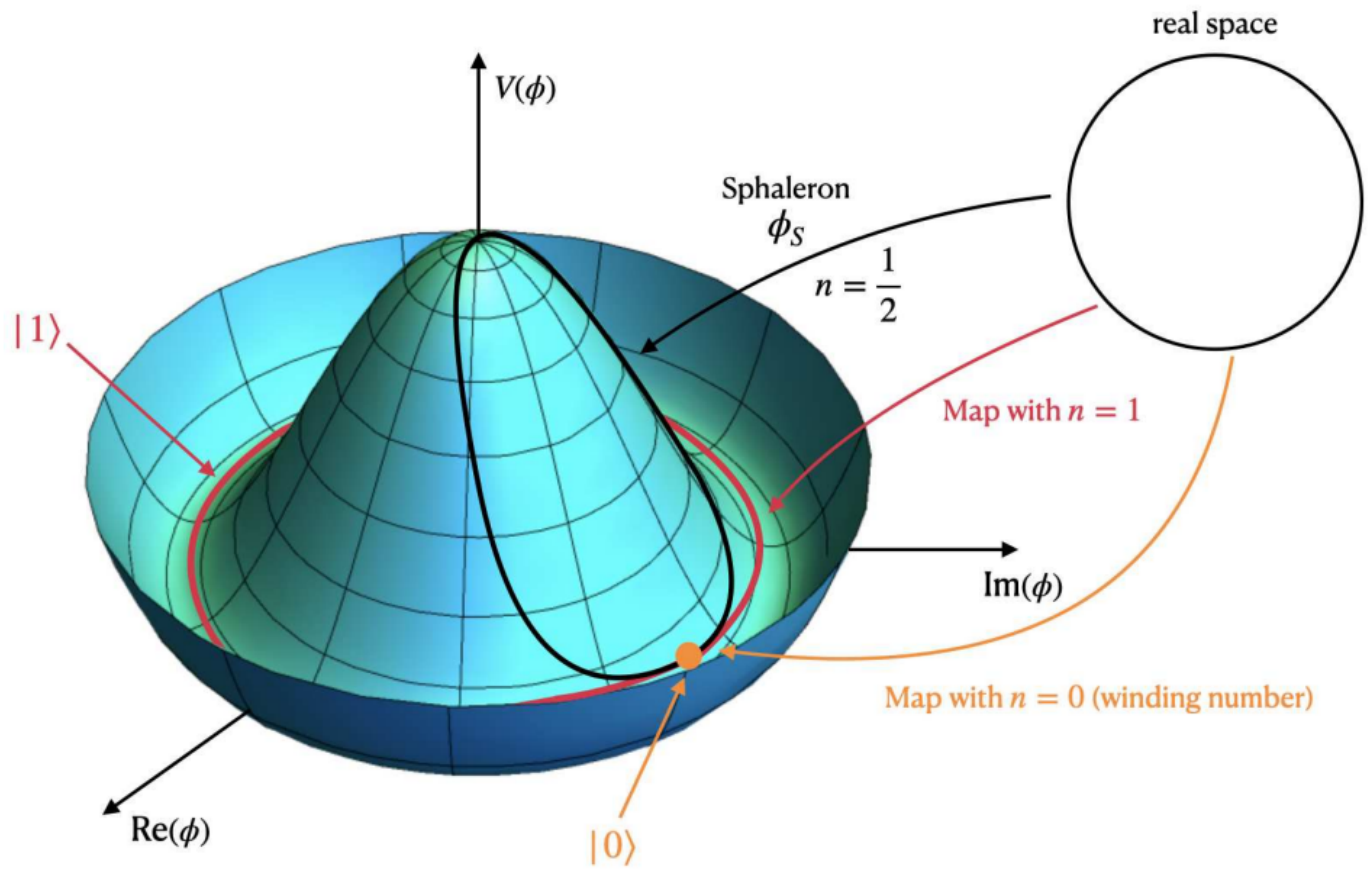

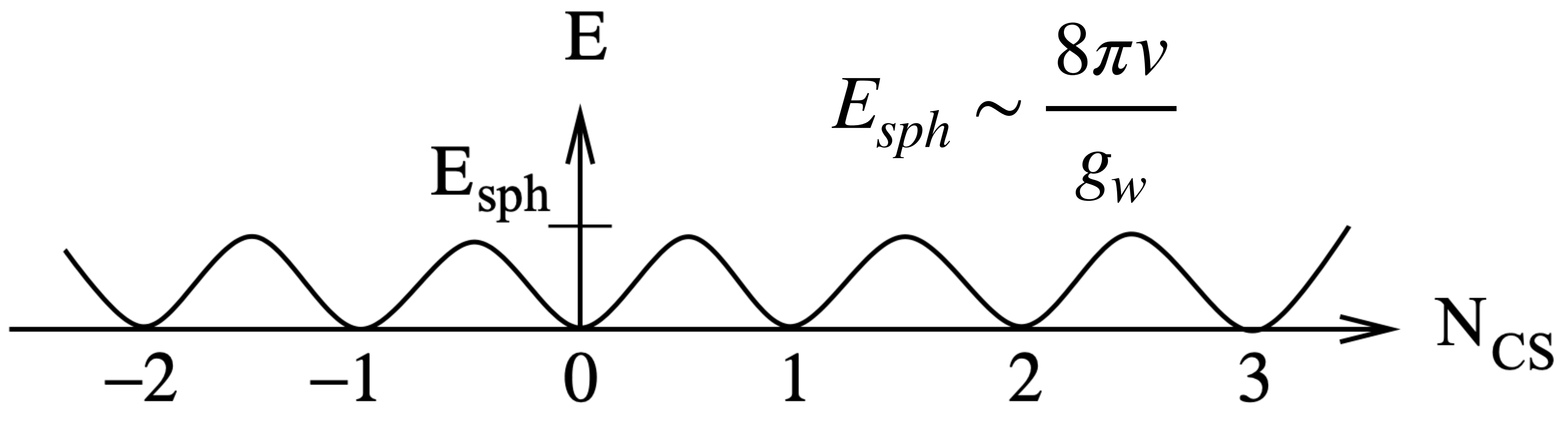

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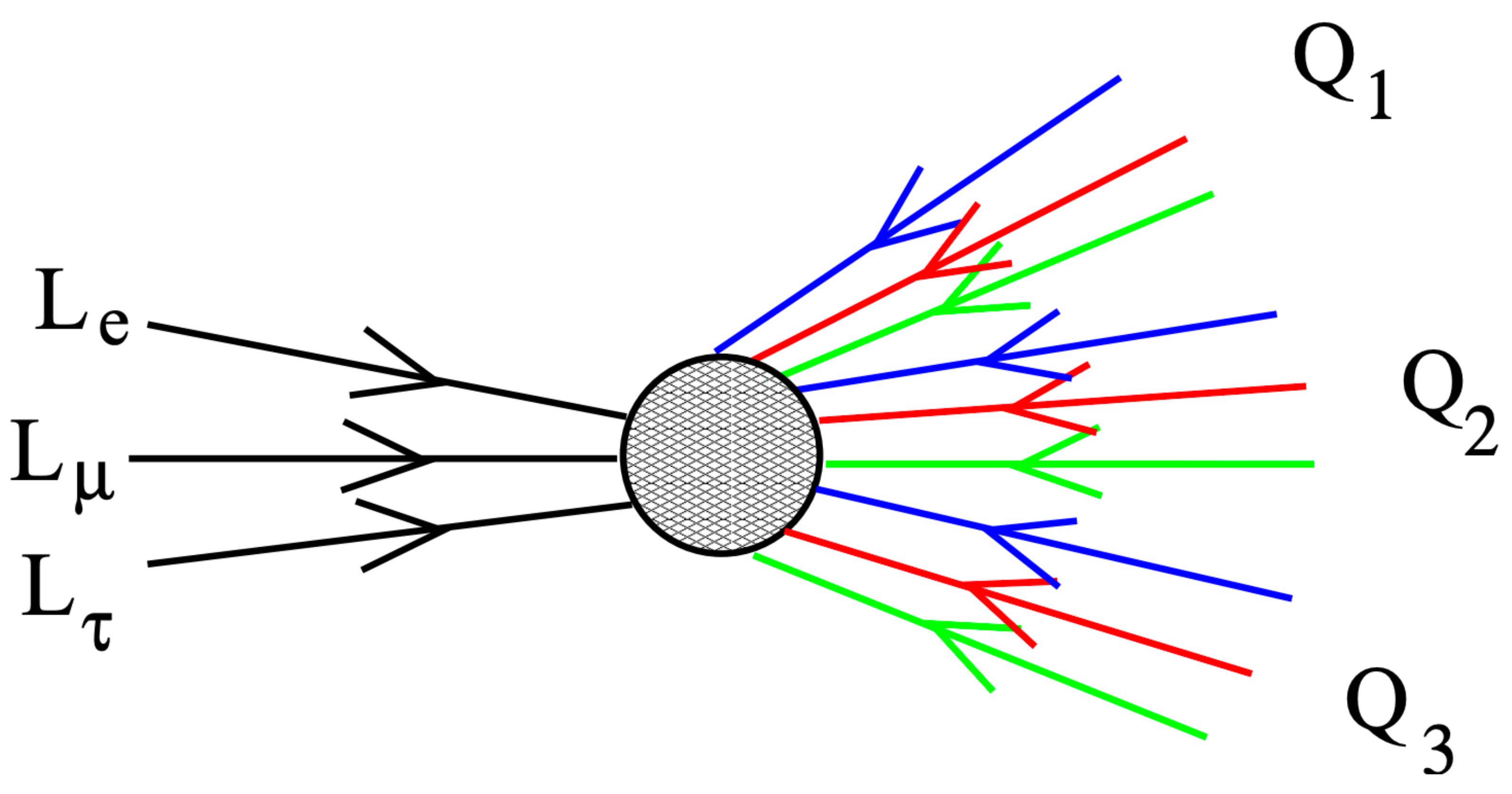
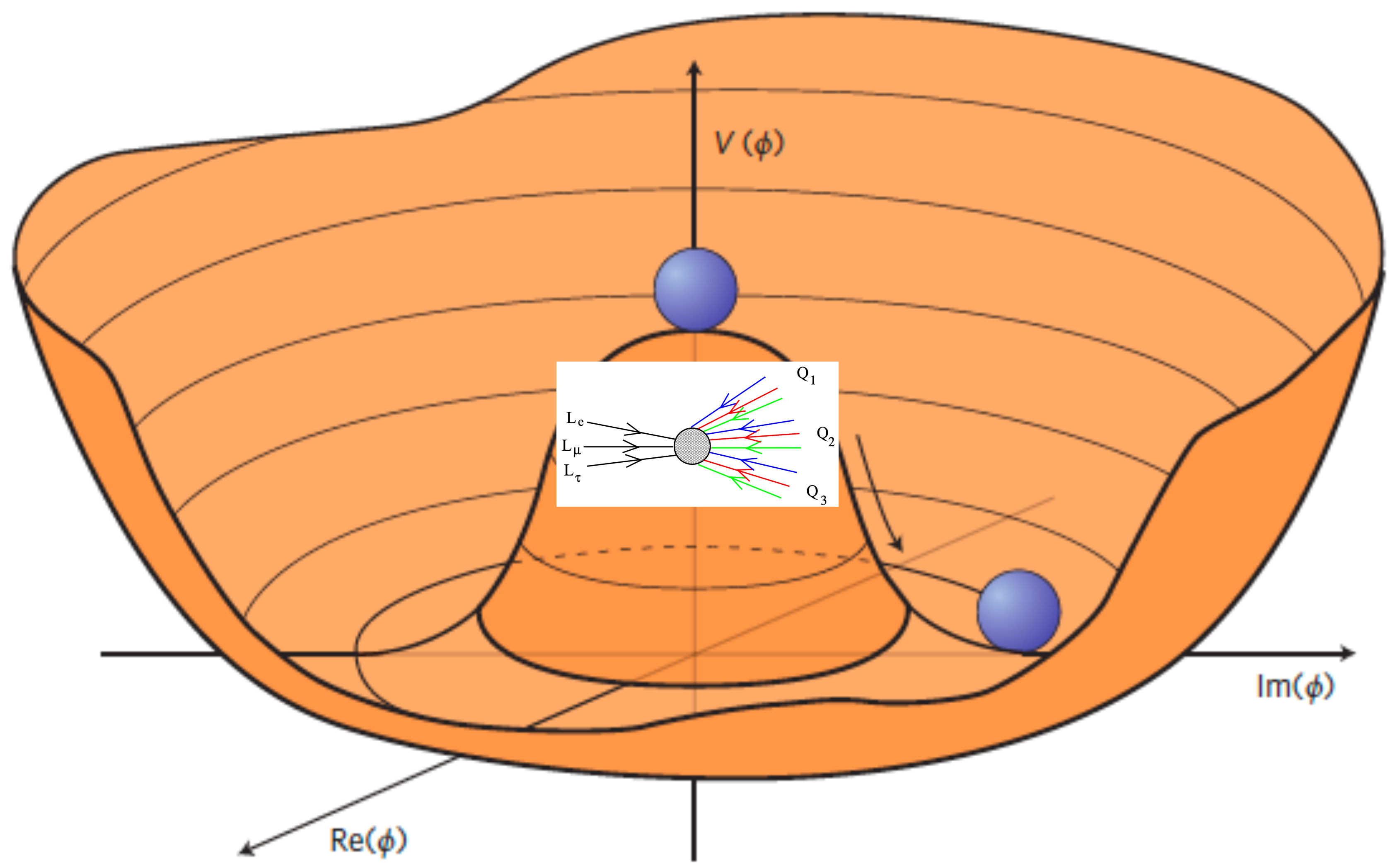


Fig. 4. The sphaleron.



If EW symmetry is restored (VEV=0)
Sphalerons everywhere!

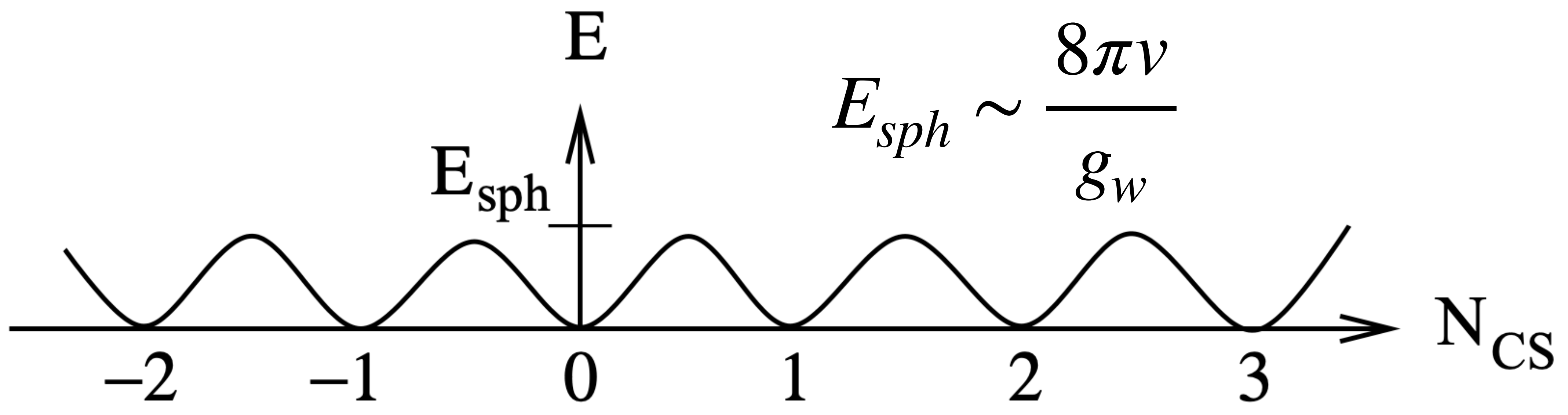


Fig. 8. Energy of gauge field configurations as a function of Chern-Simons number.

v is the Higgs VEV

Electroweak Baryogenesis

Charge and Charge+Parity symmetries (C and CP violation)

$$\begin{aligned} C: &\quad q_L \longrightarrow \bar{q}_L \\ CP: &\quad q_L \longrightarrow \bar{q}_R \end{aligned}$$

Under C conservation:

$X \to Y + B$ comes with $\bar{X} \to \bar{Y} + \bar{B}$

$$\Gamma(\bar{X} \to \bar{Y} + \bar{B}) = \Gamma(X \to Y + B)$$

The net rate of baryon production goes like the difference of these rates,

$$\frac{dB}{dt} \propto \Gamma(\bar{X} \to \bar{Y} + \bar{B}) - \Gamma(X \to Y + B)$$

CP violation is a longer story but also needed

Electroweak Baryogenesis

Charge and Charge+Parity symmetries (C and CP violation)

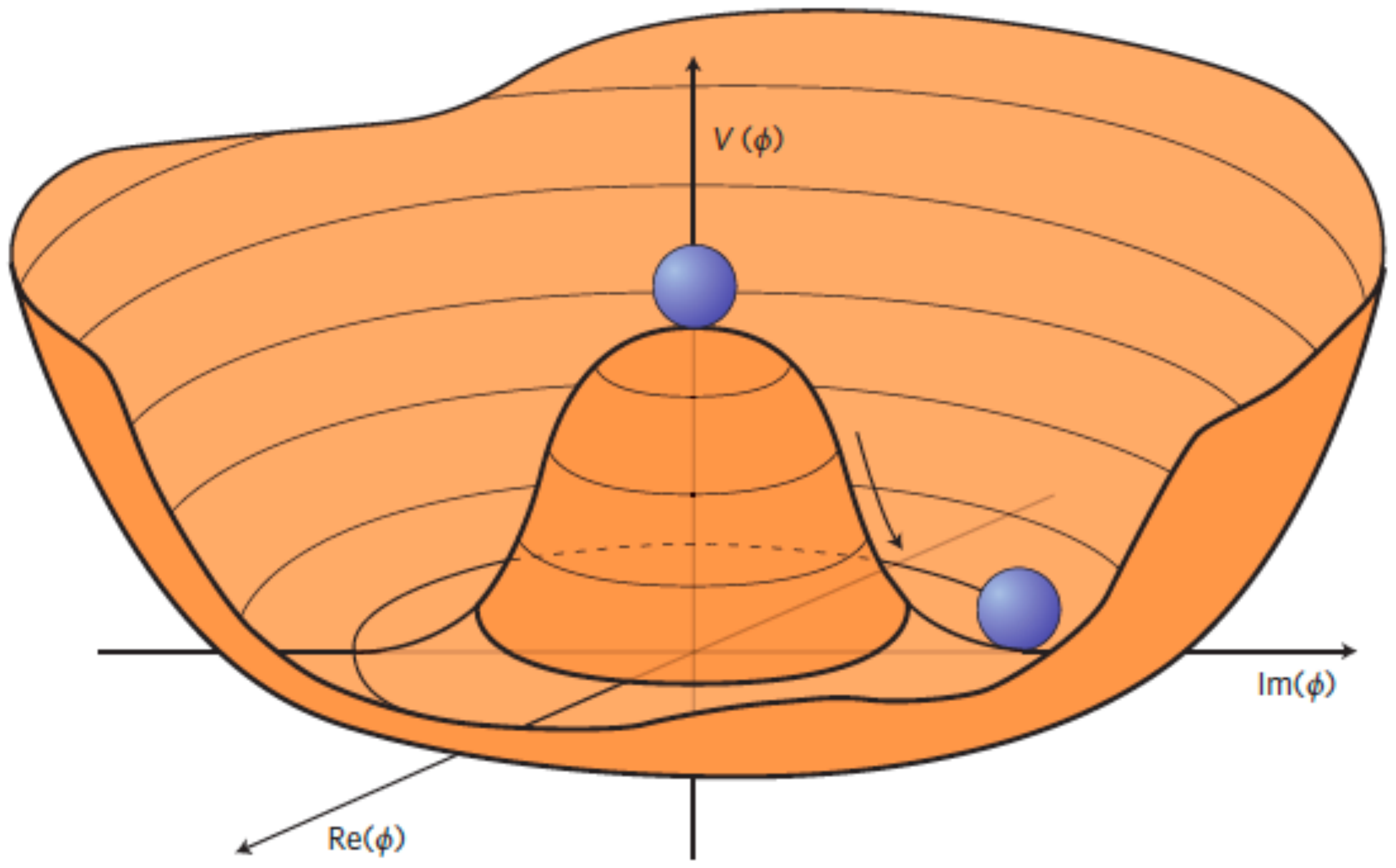
$$
\begin{aligned}
C: &\quad q_L \to \bar{q}_L \\
CP: &\quad q_L \to \bar{q}_R
\end{aligned}
$$

In SM: CP violation in CKM matrix. Not enough though! BSM CP violation is more than welcomed.

$$
V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_2 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}
$$

Electroweak Baryogenesis

Charge and Charge+Parity symmetries (C and CP violation)



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Backup

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https://arxiv.org/pdf/2404.12425

For the actual scan we have generated 530,000 random points over the phase space defined by $M_2, M_3, v_2, v_3, \theta_{12}, \theta_{13}, \theta_{23}$. The ranges considered are as follows:

$$M_2 \in [255, 700]\ \text{GeV}, \qquad M_3 \in [350, 900]\ \text{GeV},$$
$$v_2 \in [0, 1000]\ \text{GeV}, \qquad v_3 \in [50, 1000]\ \text{GeV}. \tag{3.1}$$

For the mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ we impose the following limits on the scaling factors [38, 68] of eq. (2.4):

$$0.95 \leqslant \kappa_1 \leqslant 1.00, \quad 0.0 \leqslant \kappa_2 \leqslant 0.25, \quad 0.0 \leqslant \kappa_3 \leqslant 0.25. \tag{3.2}$$

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stan du Pree[1]

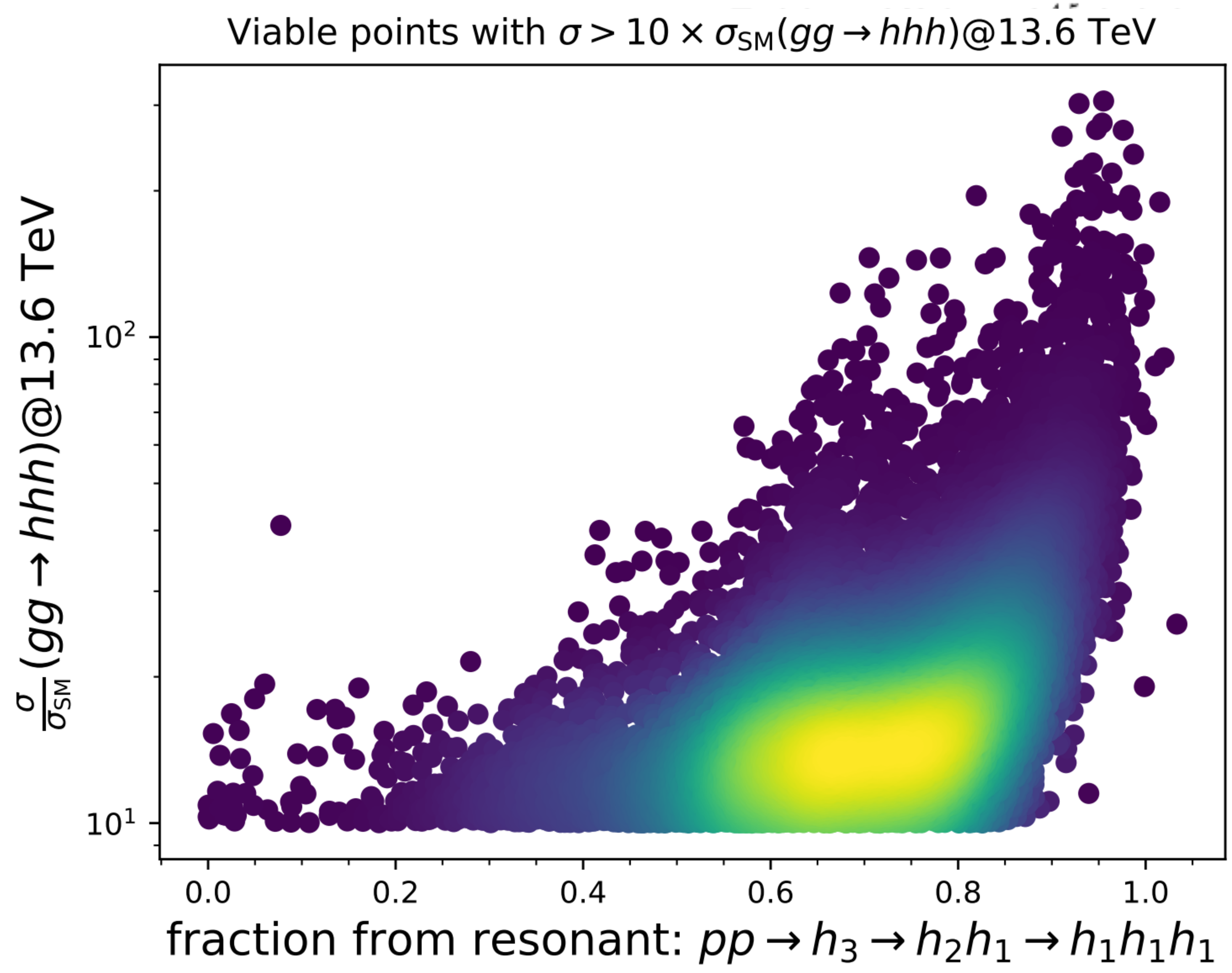

Figure 2: Enhancement of the triple Higgs boson production cross section $\sigma(pp \to h_1 h_1 h_1)$ at 13.6 TeV, given in terms of multiples of the SM value, and the resonant fraction contribution from $pp \to h_3 \to h_2 h_1 \to h_1 h_1 h_1$. Only points with a factor 10 enhancement or greater are shown. The density of points increases from the dark blue to yellow shade.

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Benchmark points for enhanced triple Higgs production

M_2	M_3	v_2	v_3	θ_{12}	θ_{13}	θ_{23}	$\frac{\sigma}{\sigma_{SM}}$	Res. Frac.	μ_{pert}	$\frac{\mu_{\text{pert}}}{\mu_{\text{pole}}}$
259.0	495.0	215.8	180.8	6.191	0.163	5.691	306.025	0.955	2.7×10^2	7.3
270.6	444.7	122.4	847.2	0.268	0.030	0.522	302.361	0.929	1.8×10^2	7.3
268.6	452.7	137.8	784.8	0.263	0.023	0.645	275.616	0.954	2.4×10^2	7.3
272.6	480.7	928.3	143.7	3.098	2.9	2.375	267.245	0.948	1.4×10^2	7.2
269.0	409.8	138.0	599.4	0.244	0.004	0.773	266.439	0.976	2.4×10^2	7.2
269.1	486.9	227.5	307.9	0.074	6.149	2.631	157.583	0.956	4.3×10^2	8.0
259.2	577.0	289.0	275.6	0.137	6.148	2.324	145.470	0.781	1.2×10^4	7.2
283.7	575.0	259.4	330.4	0.137	6.152	2.299	122.546	0.779	3.0×10^3	7.2
264.3	469.3	207.3	359.5	0.285	6.277	0.692	119.121	0.999	5.4×10^3	7.3
266.5	461.9	653.1	229.0	2.889	3.046	1.015	112.794	0.863	5.3×10^4	8.0
259.2	399.7	444.5	217.0	2.917	3.046	1.047	103.717	0.973	1.2×10^5	8.0

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The one-loop TRSM effective potential at finite temperature is:

$$V_T(\phi_i, T) = V(\phi_i) + V_{\mathrm{CW}}(\phi_i) + V_{\mathrm{c.t.}}(\phi_i) + V_{T,\,1-\mathrm{loop}}(\phi_i, T), \tag{4.2}$$

with ϕ_i the field values defined in eq. (2.2) (with $\phi_i = v_i$ in the vacuum today). $V(\phi_i)$ is the tree-level potential of eq. (2.1), V_{CW} the standard zero-temperature one-loop 'Coleman-Weinberg' potential and $V_{\mathrm{c.t.}}$ the corresponding counterterms. The temperature-corrections are captured by $V_{T,\,1-\mathrm{loop}}$, which is given by

$$V_{T,1-\mathrm{loop}}(\phi, T) = \frac{T^4}{2\pi^2}\left[\sum_{\alpha=\Phi_i,W,Z} n_\alpha J_B[m_\alpha^2(\phi_i)/T^2] + n_t J_F[m_t^2(\phi_i)/T^2]\right]. \tag{4.3}$$

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At temperatures large compared to the mass, the functions $J_{B,F}$ can be expanded in $m_\alpha^2(\phi_i)/T^2$ as

$$J_B(m_\alpha^2/T^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{24}\frac{m_\alpha^2}{T^2} - \frac{\pi}{6}\frac{m_\alpha^3}{T^3} - \frac{1}{32}\frac{m_\alpha^4}{T^4}\left(\log\frac{m_\alpha^2}{16\pi^2 T^2} - \frac{3}{2} + 2\gamma_E\right)\cdots ,$$

$$J_F(m_\alpha^2/T^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}\frac{m_\alpha^2}{T^2} - \frac{1}{32}\frac{m_\alpha^4}{T^4}\left(\log\frac{m_\alpha^2}{\pi^2 T^2} - \frac{3}{2} + 2\gamma_E\right)\cdots , \qquad (4.5)$$

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A.3 RGEs

The one-loop RGEs for the quartic couplings are

$$
\begin{aligned}
(4\pi)^2\beta_{\lambda_{11}} &= 24\lambda_{11}^2 + \frac{\lambda_{22}^2}{2} + \frac{\lambda_{33}^2}{2} + \frac{3}{8}g_1^4 + \frac{9}{8}g_2^4 + \frac{3}{4}g_1^2g_2^2 - 6y_t^4 - 4\lambda_{11}\gamma_{\Phi_1}, \\
(4\pi)^2\beta_{\lambda_{22}} &= 18\lambda_{22}^2 + 2\lambda_{12}^2 + \frac{\lambda_{23}^2}{2}, \\
(4\pi)^2\beta_{\lambda_{33}} &= 18\lambda_{33}^2 + 2\lambda_{13}^2 + \frac{\lambda_{23}^2}{2}, \\
(4\pi)^2\beta_{\lambda_{12}} &= 4\lambda_{12}^2 + 12\lambda_{12}\lambda_{11} + 6\lambda_{12}\lambda_{22} + \lambda_{13}\lambda_{23} - 2\lambda_{12}\gamma_{\Phi_1}, \\
(4\pi)^2\beta_{\lambda_{13}} &= 4\lambda_{13}^2 + 12\lambda_{13}\lambda_{11} + 6\lambda_{13}\lambda_{33} + \lambda_{12}\lambda_{23} - 2\lambda_{13}\gamma_{\Phi_1}, \\
(4\pi)^2\beta_{\lambda_{23}} &= 4\lambda_{23}^2 + 6\lambda_{23}\lambda_{22} + 6\lambda_{23}\lambda_{33} + 4\lambda_{12}\lambda_{13},
\end{aligned}
\qquad (A.5)
$$

with $\beta_\lambda = \mu\partial\lambda/\partial\mu$ and $\gamma_{\Phi_1} = \left(\frac{3g_1^2}{4} + \frac{9g_2^2}{4} - 3y_t^2\right)$. The running of the gauge couplings and the top quark is as in the SM

$$
\begin{aligned}
(4\pi)^2\beta_{g_i} &= b_i g_i^3, \\
(4\pi)^2\beta_{y_t} &= \frac{9}{2}y_t^3 - y_t(\frac{2}{3}g_1^2 + 9g_3^2) - y_t\gamma_{\Phi_1},
\end{aligned}
\qquad (A.6)
$$

with $b_i = (41/6, -19/6, -7)$ for $i = 1, 2, 3$.

How to enhance HHH

$$\mathcal{L} = -\bar{\lambda}_{abc} h_a h_b h_c - \frac{1}{2}\bar{\lambda}_{aab} h_a^2 h_b - \frac{1}{3!}\bar{\lambda}_{aaab} h_a^3 h_b + \dots \,, \tag{2.5}$$

with

$$\begin{aligned}\bar{\lambda}_{abc} &= (M_a^2 + M_b^2 + M_c^2) \sum_j \frac{R_{aj} R_{bj} R_{cj}}{v_j}, \\ \bar{\lambda}_{aaab} &= (3!) \sum_{ijk} \frac{M_k^2}{v_i v_j} R_{ki} R_{kj} (R_{ai}^2 R_{aj} R_{bj} + R_{ai} R_{bi} R_{aj}^2),\end{aligned} \tag{2.6}$$

and R the mixing matrix of eq. (A.3). The tree-level amplitudes can then be written as (up to symmetry factors)

$$\mathcal{A}_1 \sim (\mathcal{A}^{\rm SM}_{pp\to h_1} \kappa_3) \times \frac{\bar{\lambda}_{321} \bar{\lambda}_{211}}{D_3(p) D_2(p')}, \qquad \mathcal{A}_2^{(a)} \sim (\mathcal{A}^{\rm SM}_{pp\to h_1} \kappa_a) \times \frac{\bar{\lambda}_{a111}}{D_a(p)}. \tag{2.7}$$

The inverse propagators are $D_a(p) = p^2 - M_a^2 + iM_a\Gamma_a$, with p the momentum flowing through the propagator, and Γ_a the decay width of h_a. On resonance, we have $|p^2 - M_a^2| \ll |M_a\Gamma_a|$.