Bayesian analys

Neutron skin 00000 A nuclear bowling pin 000000000

The intersection of heavy ions and nuclear structure from the neutron skin of ²⁰⁸Pb to the unexpected uses of a nuclear bowling pin

Govert Nijs

October 1, 2024

Based on:

- GN, van der Schee, 2312.04623
- Giacalone, GN, van der Schee, 2305.00015
- Giacalone, Bally, GN, Shen, Duguet, Ebran, Elhatisari, Frosini, Lähde, Lee, Lu, Ma, Meißner, Noronha-Hostler, Plumberg, Rodríguez, Roth, van der Schee, Somà, 2402.05995



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The status of the field

- The general picture of the stages of a heavy ion collision is known.
- Theoretical modelling follows these stages:
 - T_RENTo or IP-Glasma for the initial state.
 - Free streaming for the pre-hydrodynamic stage.
 - Viscous hydrodynamics with temperature dependent shear and bulk viscosity.
 - SMASH or UrQMD as a hadronic afterburner.
- Bayesian analysis gives a data-driven approach to understand each stage in more detail.



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Uses of Bayesian analysis: viscosities

- We know the QGP phase is described by viscous hydrodynamics.
 - We know exactly what the free parameters are, i.e. η/s, ζ/s, ...
- We can use Bayesian analysis to find data-preferred values for these parameters.
- The values of the parameters provide an interface with microscopic theories of the QGP.



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Uses of Bayesian analysis: parameterized phenomenology



- For the initial state, there is no single widely accepted model.
- With a phenomenological model such as T_RENTo, aspects of microscopic models can be tested, such as the scaling shown here, parameterized by p.
 - IP-Glasma and EKRT are ruled in.
 - KLN and wounded nucleon are ruled out.



[Bernhard, 1804.06469]

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Uses of Bayesian analysis: deciding between models

- One can take this idea a step further, and actually compare different models.
- Here shown are different particlization schemes.
- By taking into account how well each model fits, one can even take a weighted average over models, known as Bayesian model averaging.



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Model used: Trajectum

- New heavy ion code developed in Utrecht/MIT/CERN.
 - Trajectum is the old Roman name for Utrecht.
- Contains initial stage, hydrodynamics and freeze-out, as well as an analysis suite.
- Easy to use, example parameter files distributed alongside the source code.
- Fast, fully parallelized.
 - Figure (20k oversampled PbPb events at 2.76 TeV) computes on a laptop in 21h.
 - Bayesian analysis requires O(1000) similar calculations to this one.
- Publicly available at sites.google.com/ view/govertnijs/trajectum/.





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Some simple intuition



- Model details are not necessary to understand the contents of this talk.
 - Some details are available in the backup.
- Hydrodynamics can be intuitively understood:
 - Pressure gradients drive expansion.
 - Hotter systems expand faster, resulting in more transverse momentum.
 - Spatially anisotropic systems expand preferentially along the short axis, resulting in momentum anisotropy in the final state.

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Bayesian analysis workflow

- In principle, Bayesian analysis is simply a fit to data.
- In practice the process is more complicated:
 - Generate a large number of randomly chosen parameter sets called *design points*.
 - Run the model for each one to obtain the prior.
 - Train the emulator.
 - Run the MCMC to obtain the posterior.
- The posterior then is a list of likely parameter sets.



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Image: A matrix and a matrix



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Data used: 670 individual data points

✓: data used				.				.	/
😂: data available 🛛	Pł	oPb 2.	76 Te\	/	Pt	oPb 5.	02 Te\	/	<i>p</i> Pb 5.02 TeV
🗡: data unavailable	incl.	π^{\pm}	$ K^{\pm} $	p	incl.	π^{\pm}	$ K^{\pm} $	p	incl.
σ	×	X	×	X	 Image: A start of the start of	X	X	X	 ✓
dN/dy	1	1	1	1	1	1	1	1	()
$\langle p_T \rangle$	X	1	1	1	1	1	1	1	9
$dE_T/d\eta$	 ✓ 	X	×	X	X	X	×	X	×
$\delta p_T / \langle p_T \rangle$	1	×	×	X	×	×	×	X	×
$v_{2,3,4}{2}$	1	B	B	9	1	B	(1)	9	(5
v ₂ {4}	1	X	X	X	1			9	8
$d^2N/dp_T dy$	×	1	1	1	×	1	1	1	×
$v_2{2}(p_T)$	X	1	1	1	X	1	1	1	9
$v_3{2}(p_T)$	×	1	B	9	X	1	B	9	Ð
NSC(2,3)	B	×	×	X	1	×	×	×	0
NSC(2,4)	B	×	×	×	1	×	×	×	9
$\rho(v_2\{2\}^2, \langle p_T \rangle)$	×	X	X	X	 Image: A set of the set of the	X	X	X	×



The intersection of heavy ions and nuclear structure

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Using the posterior parameter values to make predictions

- The posterior parameter values can be used to make predictions for new observables.
 - When using multiple samples from the posterior, this includes systematic uncertainty from the parameter estimation.
- Here shown is the prediction for ultracentral (p_T).





[GN, van der Schee, 2312.04623; CMS, 2401.06896]

Using the posterior parameter values to make predictions

- The posterior parameter values can be used to make predictions for new observables.
 - When using multiple samples from the posterior, this includes systematic uncertainty from the parameter estimation.
- Here shown is the prediction for ultracentral (p_T).
- Precise agreement between theory and experiment.



Neutron skin

- In a ²⁰⁸Pb nucleus, neutrons sit further from the center than protons.
 - This is quantified by the *neutron skin*:

$$\Delta r_{np} = \langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2},$$

i.e. the *difference* in RMS radii of the neutron and proton distributions.

- Heavy nuclei and neutron stars are sensitive to the same nuclear interactions.
 - A constraint on Δr_{np} translates directly into a constraint on the radius of a $1.4M_{\odot}$ neutron star.
 - We can learn something about the low *T*, high μ_B region even at LHC energies!



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How to measure neutron skin?



- To measure the neutron skin, we need the distributions of protons and neutrons inside the nucleus.
 - The proton distribution distribution is well-known from electron scattering.
- Several different methods are in use for the neutron distribution:
 - Polarized electron scattering off ²⁰⁸Pb (PREX).
 - Photon tomography of ¹⁹⁷Au (STAR).
- Heavy ion collisions provide a completely orthogonal method.
 - Sensitive to the total matter distribution inside the nucleus.

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Purely gluonic measurement.



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The Woods-Saxon distribution

 Nucleon positions are drawn from a Woods-Saxon distribution:

$$ho_{\mathrm{WS}}(r) \propto rac{1}{1+\exp\left(rac{r-R}{a}
ight)}.$$

- We fix R for both protons and neutrons.
- We fix *a* for protons, while varying *a_n* as a parameter.
- Neutron skin $\Delta r_{np} = \langle r^2 \rangle_n^{1/2} \langle r^2 \rangle_p^{1/2}$ strongly depends on a_n :

$$\langle r^2 \rangle_{\rm WS} = \frac{12a^2\operatorname{Li}_5\left(-e^{R/a}\right)}{\operatorname{Li}_3\left(-e^{R/a}\right)}.$$

[Giacalone, GN, van der Schee, 2305.00015]

The intersection of heavy ions and nuclear structure



	proton	neutron
<i>R</i> [fm]	6.68	6.69
<i>a</i> [fm]	0.447	an

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[Giacalone, GN, van der Schee, 2305.00015]

The intersection of heavy ions and nuclear structure



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Do we have observables sensitive to a_n ?

- Initial geometry is sensitive to a_n.
 Larger nuclei lead to:
 - Larger hadronic PbPb cross-section,
 - Larger initial QGP size,
 - Smaller initial QGP eccentricity.
- Final state observables are in turn sensitive to initial geometry. Larger Δr_{np} leads to:
 - Larger hadronic PbPb cross-section,
 - Smaller charged particle yield,
 - Smaller mean transverse momentum,
 - Smaller elliptic flow.



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Do we have observables sensitive to a_n ?

- Initial geometry is sensitive to a_n . Larger nuclei lead to:
 - Larger hadronic PbPb cross-section.
 - Larger initial QGP size,
 - Smaller initial QGP eccentricity.
- Final state observables are in turn sensitive to initial geometry. Larger Δr_{nn} leads to:
 - Larger hadronic PbPb cross-section.
 - Smaller charged particle yield,
 - Smaller mean transverse momentum.
 - Smaller elliptic flow.



PbPb. $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$

[Giacalone, GN, van der Schee, 2305.00015] The intersection of heavy ions and nuclear structure

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Bayesian analysis result using LHC data

- Resulting posterior for Δr_{np} is compatible with PREX II and ab initio nuclear theory.
- Slightly stronger constraint than PREX II ($\Delta r_{np} = 0.283 \pm 0.071$).
- Result is in principle improvable with better Bayesian analyses.
 - May be hard to do in practice.
 - The current analysis already took 2M CPUh.





[Giacalone, GN, van der Schee, 2305.00015; PREX, 2102.10767; Hu et al., Nat. Phys. 18, 1196-1200 (2022)]

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One fluid to rule them all?



- Anisotropic flow is present in a great range of system sizes:
 - PbPb,
 - High multiplicity pPb,
 - High multiplicity pp,
 - . . .
- Is this a sign of hydrodynamics?
 - Hydrodynamical simulations seem to work reasonably well.
 - But can a system that small really behave hydrodynamically?
 - Initial state geometry is poorly understood.

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 We need a precision test of hydrodynamics in small systems.



[ALICE, 1903.01790]

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Recap: why do we believe PbPb is hydrodynamic?



- Not just the presence of $v_n\{k\}$.
- We understand where the $v_n\{k\}$ come from!
 - Hydrodynamics converts initial state anisotropic geometry into final state momentum anisotropy.
 - We understand very well what the initial geometry looks like!
- For *p*Pb this is not the case.
 - There is $v_n\{k\}$ measured.
 - But we do not understand the initial geometry.

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 No clear interpretation of experimental results.



[ALICE, 1602.01119]

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Posing a precise question



- Can we describe PbPb and a small system in a hydrodynamical model with the same settings?
 - Hydro model used should describe a wide range of PbPb observables.
- Can we find a quantity to predict which does not suffer from huge theoretical uncertainties? Wishlist:
 - Initial geometry under control.
 - Small sensitivity to proton substructure.
 - No longitudinal structure issues.
 - Quantifiable and small theory uncertainty.



[ALICE, 1903.01790]

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Can ¹⁶O¹⁶O collisions help?

- ¹⁶O¹⁶O collisions are planned at the LHC for 2025.
- Shape of the proton and longitudinal structure are not an issue, but...



Image: A matrix

[Giacalone, Bally, GN, Shen et al., 2402.05995]

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Image: A matrix and a matrix

A nuclear bowling pin

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- ¹⁶O¹⁶O collisions are planned at the LHC for 2025.
- Shape of the proton and longitudinal structure are not an issue, but...
- Magnitude of fluctuations in the initial state is poorly constrained.
- Different nuclear structure calculations give different answers!
- We have a handle on systematics, but errors are substantial.



Image: A matrix and a matrix



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The nuclear bowling pin: ²⁰Ne

- We use both the PGCM and NLEFT frameworks for our nuclear structure input.
 - PGCM computes the average deformed densities.
 - NLEFT simulates an effective theory on a lattice.
- ¹⁶O is shaped like an irregular tetrahedron.
- ²⁰Ne is close in size, but has the most extreme shape in the Segrè chart.
- Can we take a ratio between systems to cancel the uncertainties?



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A careful look at uncertainties

- *Trajectum* systematic uncertainty contains contributions from:
 - Uncertainties in parameters.
 - Extrapolation to zero grid spacing.
- PGCM systematic uncertainty contains contributions from:
 - Sampling method: how to convert a density into a configuration.
 - Constraint application: order of operations in the PGCM computation.
- NLEFT systematic uncertainty contains contributions from:
 - Resolution of ambiguities from periodicity of the lattice.



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NLEFT

PGCM

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<u>Comparing ²⁰Ne to ¹⁶O significantly reduces errors!</u>

- NLEFT and PGCM are consistent within uncertainties.
- Ratio of v_2 {2} reaches percent level precision from 5% to 20% centrality!
- Difference of $\rho(v_2\{2\}^2, \langle p_T \rangle)$ has uncertainty reduced by up to a factor 6
- Larger PGCM uncertainty is mostly due to ambiguity in how to generate configurations from densities.

 $v_2\{2\}_{NeNe}/v_2\{2\}_{OO}$



[Giacalone, Bally, GN, Shen et al., 2402.05995] The intersection of heavy ions and nuclear structure

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$$\begin{array}{|c|c|c|c|c|c|c|} \hline 0-1\% & v_2\{2\}_{NeNe}/v_2\{2\}_{OO} & \rho_{2,NeNe}-\rho_{2,OO} \\ \hline \text{NLEFT} & 1.170(8)_{\text{stat.}}(30)_{\text{syst.}}^{\text{Traj.}}(0)_{\text{syst.}}^{\text{str.}} & -0.121(14)_{\text{stat.}}(10)_{\text{syst.}}^{\text{Traj.}}(0)_{\text{syst.}}^{\text{str.}} \\ \hline \text{PGCM} & 1.139(6)_{\text{stat.}}(27)_{\text{syst.}}^{\text{Traj.}}(28)_{\text{syst.}}^{\text{str.}} & -0.124(10)_{\text{stat.}}(10)_{\text{syst.}}^{\text{Traj.}}(29)_{\text{syst.}}^{\text{str.}} \\ \hline 0.124(10)_{\text{stat.}}(10)_{\text{syst.}}^{\text{syst.}}(29)_{\text{syst.}}^{\text{syst.}} & \hline 0.124(10)_{\text{str.}}(29)_{\text{syst.}}^{\text{syst.}} \\ \hline 0.124(10)_{\text{str.}}(29)_{\text{syst.}}^{\text{syst.}} & \hline 0.124(10)_{\text{str.}}(20)_{\text{str.}}^{\text{syst.}} & \hline 0.124(10)_{\text{str.}}(20)_{\text{str.}}^{\text{syst.}} & \hline 0.124(10)_{\text{str.}}(20)_{\text{str.}}^{\text{syst.}} & \hline 0.124(10)_{\text{str.}}^{\text{str.}} & \hline 0.124(10)_{\text{str.}^{\text{str.}}(20)_{\text{str.}^{\text{str.}}} & \hline 0.124(10)_{\text{str.}}^{\text{str.}}$$

0 - 1%

NLEFT

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Conclusions

• Let us take another look at our wishlist:

$v_n\{k\}$ in	<i>p</i> Pb	00	NeNe/OO
Initial geometry under control	×	1	 Image: A set of the set of the
Small sensitivity to proton substructure	×	1	✓
No longitudinal decorrelation issues	×	1	1
Quantifiable theory uncertainty	×	1	1
Small theory uncertainty	×	\geq 4%	$\geq 1\%$

- Theory has a much better handle on ${}^{16}O{}^{16}O$ compared to pPb.
- Theory uncertainties can be substantially reduced by supplementing ¹⁶O¹⁶O collisions with ²⁰Ne²⁰Ne collisions.
 - v₂{2} ratio can be predicted to 1% precision between 5% and 20% centrality.
 - Different nuclear structure calculations give consistent results.



Image: A matrix and a matrix

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TH Institute: Light lons at the LHC





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Bayesian analysis details

- 3000 design points.
- 18k events per design point.
- Every 15th design point has $10 \times$ more statistics, enabling to emulate 'hard' observables such as SC(n, m) and $\rho(v_2\{2\}^2, \langle p_T \rangle)$.



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Bayesian analysis details

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Error budget



[GN, van der Schee, 2110.13153]

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Nucleon width and $ho(v_n \{2\}^2, \langle p_T \rangle$

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Posterior observables (1/3)





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Posterior observables (2/3)





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Nucleon width and $\rho(v_n \{2\}^2, \langle p_T \}$

Weights 0000

Posterior observables (3/3)





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T_RENTo initial conditions

Nucleons A and B become wounded with probability

$$P_{ ext{wounded}} = 1 - \exp\left(-\sigma_{gg}\int d extbf{x}\,
ho_A(extbf{x})
ho_B(extbf{x})
ight), \quad
ho_A \propto \exp\left(rac{-| extbf{x} - extbf{x}_A|^2}{2w^2}
ight).$$

Each wounded nucleon desposits energy into its nucleus's *thickness function* $T_{A/B}$:

$$\mathcal{T}_{A/B} = \sum_{i \in ext{wounded } A/B} \gamma \exp(-|\mathbf{x}-\mathbf{x}_i|^2/2w^2),$$

with γ drawn from a gamma distribution with mean 1 and standard deviation $\sigma_{\rm fluct}.$

• Actual formulas slightly modified because each nucleon has n_c constituents.

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Nucleon width and $\rho(v_n \{2\}^2, \langle p_T \rangle 000$

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The TRENTo phenomenological ansatz

• The standard T_RENTo formula combines thickness functions of the two nuclei T_A and T_B into a *reduced thickness* T, interpreted as an energy density:



[Moreland, Bernhard, Bass, 1412.4708]

Trajectum details

Free streaming pre-hydrodynamic stage

• T_RENTo creates matter at proper time $\tau = 0^+$.

Propagate the matter using free streaming:

$$T^{\mu
u}(x, y, au_{\text{hyd}}) = rac{1}{ au_{ ext{hyd}}} \int d\phi \, \hat{
ho}^{\mu} \hat{
ho}^{
u} \mathcal{T}(x - au_{ ext{hyd}} \cos \phi, y - au_{ ext{hyd}} \sin \phi),$$

with

$$\hat{p}^{\mu} = \left(\begin{array}{cc} 1 & \cos \phi & \sin \phi \end{array}
ight),$$

giving us the stress tensor $T^{\mu\nu}$ at proper time $\tau = \tau_{hvd}$.

- Here τ_{hvd} is the time at which hydrodynamics is started.
- The factor $1/\tau_{hvd}$ is due to longitudinal expansion.

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Basics of hydrodynamics

- Hydrodynamics is the ultimate effective theory. Knowledge of the underlying microscopic theory is completely summarized in transport coefficients.
- Only conservation laws survive: equation of motion is simply

$$\partial_{\mu} T^{\mu\nu} = 0.$$

- Not enough equations to close the system. Need additional assumption of *local thermal equilibrium*.
- We write $T^{\mu\nu}$ in terms of building blocks T, u^{μ} , $g^{\mu\nu}$ and ∂_{μ} .



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Hydrodynamics in the 14-moment approximation

Define $(g^{\mu\nu} = \text{diag}(1, -1, -1, -1))$:

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}, \quad \nabla^{\mu} = \Delta^{\mu\nu}\partial_{\nu}, \quad D = u^{\mu}\nabla_{\mu}, \quad \sigma^{\mu\nu} = \nabla^{\langle\mu}u^{\nu\rangle},$$

with $\langle \rangle$ symmetrizing and removing the trace.

• We solve viscous hydrodynamics without currents, i.e.

$$\partial_{\mu}T^{\mu\nu} = 0, \quad T^{\mu\nu} = eu^{\mu}u^{\nu} - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu},$$

• $\pi^{\mu\nu}$ and Π follow the 14-moment approximation:

$$-\tau_{\pi}\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}D\pi^{\alpha\beta} = \pi^{\mu\nu} - 2\eta\sigma^{\mu\nu} + \delta_{\pi\pi}\pi^{\mu\nu}\nabla \cdot u - \phi_{7}\pi^{\langle\mu}_{\alpha}\pi^{\nu\rangle\alpha} + \tau_{\pi\pi}\pi^{\langle\mu}_{\alpha}\sigma^{\nu\rangle\alpha} - \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} -\tau_{\Pi}D\Pi = \Pi + \zeta\nabla \cdot u + \delta_{\Pi\Pi}\nabla \cdot u\Pi - \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}.$$



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[Denicol, Jeon, Gale, 1403.0962]

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Particlization

- At the freeze-out temperature T_{sw} , we turn the fluid back into particles.
- Particles are sampled thermally, and boosted with the fluid velocity u^{μ} .
- We use the PTB prescription to match $\pi^{\mu\nu}$ and Π across the transition, so that $T^{\mu\nu}$ is smooth.
- After particlization, we use SMASH as a hadronic afterburner.

Fitting to the *p*Pb and PbPb cross sections

- In the T_RENTo model, the nucleon size is described by the Gaussian radius w.
- Previous analyses favored $w \approx 1 \, \text{fm}$.
 - This leads to a 3σ discrepancy in $\sigma_{\text{PbPb.}}$
- Fitting to the *p*Pb and PbPb cross sections lowers *w* to 0.6 fm.
 - σ_{PbPb} discrepancy is reduced to 1σ .
 - Many other observables fit slightly worse.
- Smaller width is now compatible with our knowledge of the gluonic structure of the proton at low x.



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Implication for $\rho(v_n\{2\}^2, \langle p_T \rangle)$

- Pearson correlation coefficient $\rho(v_n\{2\}^2, \langle p_T \rangle)$ between $v_n\{2\}^2$ and $\langle p_T \rangle$ is sensitive to the nucleon size.
- Postdiction without fitting to σ_{PbPb} and σ_{pPb} is qualitatively wrong:
 - ρ(v₂{2}², ⟨p_T⟩) goes negative already at 30% centrality.
 - $\rho(v_3\{2\}^2, \langle p_T \rangle)$ has the wrong sign.
- Fitting to σ_{PbPb} and σ_{pPb} results in a much improved agreement.



Nucleon width and $ho(v_n \{2\}^2, \langle p_T \rangle$ 00● Extra NeNe/OO 000 Weights 0000

Nucleon width and $\rho(v_n\{2\}^2, \langle p_T \rangle)$





Image: A matrix and a matrix

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[Giacalone, Schenke, Shen, 2111.02908]

lucleon width and $\rho(v_n \{2\}^2, \langle p_T \}$

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- We show the NLEFT densities for ¹⁶O and ²⁰Ne.
- Densities are computed from configurations, requiring translation and rotation.
- This introduces biases, so we also show spherical configurations rotated in the same way to illustrate the size of this effect.



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Bayesian analysis details 000		Extra NeNe/OO O●O	Weight: 0000

Other observables

- We show the NeNe/OO ratios for $\langle p_T \rangle$, $\delta p_T / \langle p_T \rangle$ and $v_3 \{2\}$.
- Discrepancy in (p_T) between PGCM and NLEFT is due to the different nuclear charge radius.
- δp_T/(p_T) has interesting non-monotonic behavior for central collisions.





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Bayesian analysis details 000		Extra NeNe/OO 00●	Weights 0000

PGCM error ratios



[Giacalone, Bally, GN, Shen et al., 2402.05995]

The intersection of heavy ions and nuclear structure

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Nucleon width and $ho(v_n \{2\}^2, \langle p_T \rangle$

Why weights?

- Higher p_T, higher centralities are harder to model theoretically.
- Experimental correlation matrix is not available.
 - Figure shows 1σ and 2σ regions for $\rho \in \{0, 0.9, -0.9, 0.99\}$, with standard deviations the same.
 - Same difference between theory and experiment can be within 1σ or outside of 2σ depending on ρ.
 - Correlated observable classes can be over/underimportant for the Bayesian analysis.





Definition of weights

In the bayesian analysis, the probability of the data given the parameter point x is given by:

$$P(D|x) = \frac{1}{\sqrt{(2\pi)^m \det \Sigma}} \exp\left(-\frac{1}{2}(y - y_{\exp})^T \Sigma^{-1}(y - y_{\exp})\right),$$

- with y the vector of observables computed from x, y_{exp} the vector of the corresponding experimental data, and Σ the combined theory/experiment covariance matrix.
- We define weights by replacing

$$P(D|x) = \frac{1}{\sqrt{(2\pi)^m \det \Sigma}} \exp\left(-\frac{1}{2}(y - y_{\exp})^T \omega \Sigma^{-1} \omega (y - y_{\exp})\right),$$

where ω is the diagonal matrix containing the weight for each observable.

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Choice of weights

• We choose for weights ω :

- 1/2 for every particle identified observable.
- 1/2 for *p*_T-differential observables, and an additional
 - $(2.5 p_T[GeV])/1.5$ if $p_T > 1 \text{ GeV}.$
- (100 c[%])/50 if the centrality class *c* is beyond 50%.
- Weighting only worsens the average discrepancy slightly.
- Distribution of discrepancies makes more sense.

	$\langle (y_{theo}) \rangle$	ry — Yex	periment	$)/\sigma\rangle$	$\bar{\omega}$
	$\sigma_{\rm AA}$ & ω	ω	σ_{AA}	neither	
$dN_{\rm ch}/d\eta$	0.55	0.60	1.23	1.22	1.00
$dN_{\pi^{\pm},k^{\pm},p^{\pm}}/dy$	0.76	0.70	0.60	0.57	0.48
$dE_T/d\eta$	1.59	1.51	0.82	0.77	0.48
$\langle p_T \rangle_{ch,\pi^{\pm},K^{\pm},p^{\pm}}$	0.66	0.60	0.88	0.72	0.46
$\delta p_T / \langle p_T \rangle$	0.56	0.62	0.51	0.58	0.49
$v_n\{k\}$	0.58	0.51	0.54	0.49	1.00
$d^2 N_{\pi^{\pm}}/dy dp_T$	1.19	1.07	0.86	0.92	0.20
$d^2 N_{K^{\pm}}/dy dp_T$	1.41	1.27	0.79	0.73	0.20
$d^2 N_{p^{\pm}}/dy dp_T$	1.35	1.21	0.73	0.67	0.25
$v_2^{\pi^{\pm}}(p_T)$	0.81	0.74	0.46	0.44	0.19
$v_{2}^{K^{\pm}}(p_{T})$	0.92	0.89	0.55	0.55	0.19
$v_2^{p^{\pm}}(p_T)$	0.49	0.47	0.34	0.35	0.25
$v_{3}^{\pi^{\pm}}(p_{T})$	0.65	0.57	0.69	0.57	0.24
average	0.89	0.83	0.69	0.66	
σΔΔ	1.13	3.80	1.53	3.40	1.00

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Trajectum details

Nucleon width and $ho(v_n \{2\}^2, \langle p_T \}$

Extra NeNe/OO 000 Weights 0000

How much do weights change the posteriors?



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