

# Grand Unification at LHC?

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# Why Grand Unification

Grand unification is one of the most appealing candidates for physics beyond Standard Model



Charge quantization  
Existence of magnetic monopoles

Proton decay

Unification of gauge forces

Typical scales at around  $10^{14} - 10^{16}$  GeV. Is there any hope to probe it?

# Magnetic Monopoles

Existence of magnetic monopoles implies quantization of charge

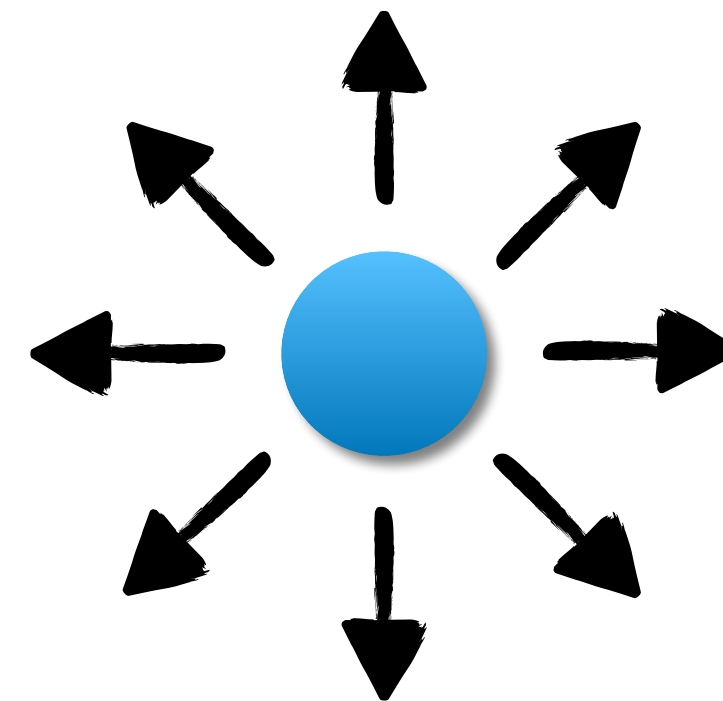
*Dirac '31*

$$g_m e_q = 2\pi n$$

*Wu, Yang '68*

Grand Unified theories charge is quantized and leads to the existence of magnetic monopoles

First explicit solution in  $SU(2)$ :  
*'t Hooft '74, Polyakov '74* monopoles



No monopole has been observed so far. However, one would expect them to be produced in the early Universe.

*Kibble '76*



Monopole problem

*Zel'dovich, Khlopov '78*  
*Preskill '79*

# Minimal $SU(5)$ model: an instructive predictive failure

*Georgi Glashow model '74 is the minimal theory*

Gauge Sector

$$\begin{aligned} 24_{\text{gauge}} &= 8_{\mathbf{C}} + 3_{\mathbf{L}} + 1 + (3_{\mathbf{C}}, 2_{\mathbf{L}}) + (\bar{3}_{\mathbf{C}}, 2_{\mathbf{L}}) \\ &= \text{gluons} + W's + B + (X, Y) + (\bar{X}, \bar{Y}) \end{aligned}$$

Fermion Sector

$$\begin{aligned} \bar{5}_{\mathbf{F}} &= (d^{r,g,b}, \nu, e)_{\mathbf{L}}, \\ 10_{\mathbf{F}} &= (u^c, u, d, e^c)_{\mathbf{L}} \end{aligned}$$

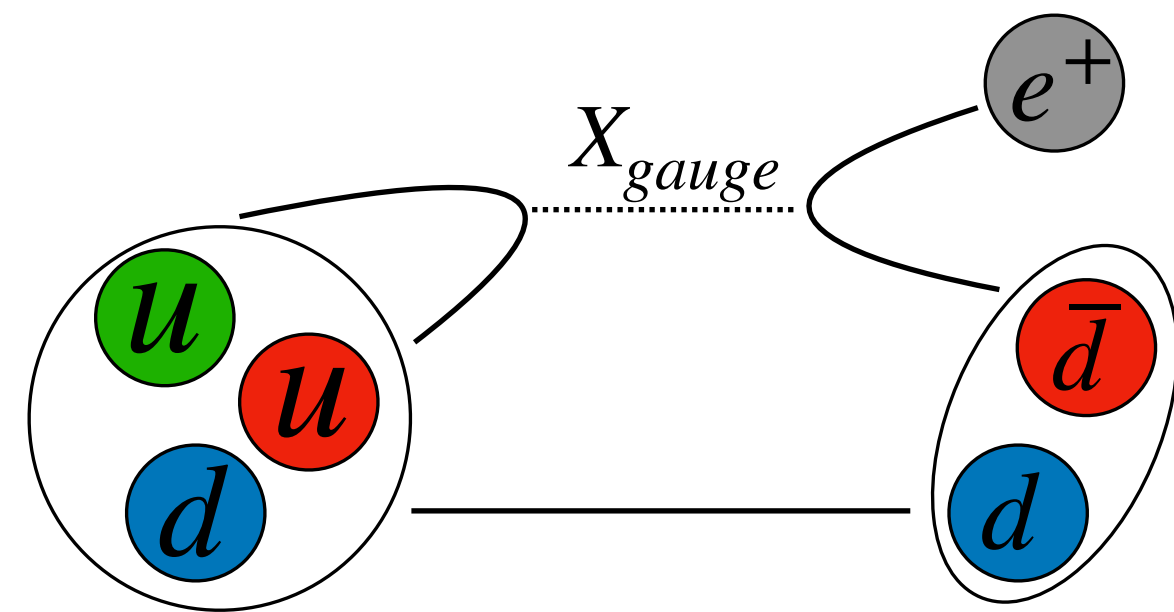
*Baryon number violating interaction*

$$\mathcal{L}_{\mathbf{B}} = g_{\text{GUT}} \left[ X (\bar{u}^c u) (\bar{e}^c d + \bar{d}^c e) + Y (\bar{u}^c d) (\bar{e}^c u + \bar{d}^c \nu) \right]$$



*Leads to  $p$ -decay*

# Minimal $SU(5)$ model: Proton decay



| Channel                   | Lifetime ( $10^{30}$ yrs)  |
|---------------------------|----------------------------|
| $N \rightarrow e^+ \pi$   | 5300 (n), <u>16000 (p)</u> |
| $N \rightarrow \mu^+ \pi$ | 3500 (n), 7700 (p)         |
| $N \rightarrow \nu \pi$   | 1100 (n), 390 (p)          |
| $N \rightarrow e^+ K$     | 17 (n), 1000 (p)           |
| $N \rightarrow \mu^+ K$   | 26 (n), 1600 (p)           |
| $N \rightarrow \nu K$     | 86 (n), 5900 (p)           |

Proton lifetime from Super-Kamiokande requires

$$\tau_p = \simeq C \frac{M_{\text{GUT}}^4}{\alpha_{\text{GUT}}^2} (m_{\text{proton}})^{-5} \gtrsim 10^{34} \text{ yrs} \quad \longrightarrow \quad M_{\text{GUT}} \gtrsim 4 \cdot 10^{15} \text{ GeV} \left( \frac{\alpha_{\text{GUT}}}{40^{-1}} \right)^{1/2}$$

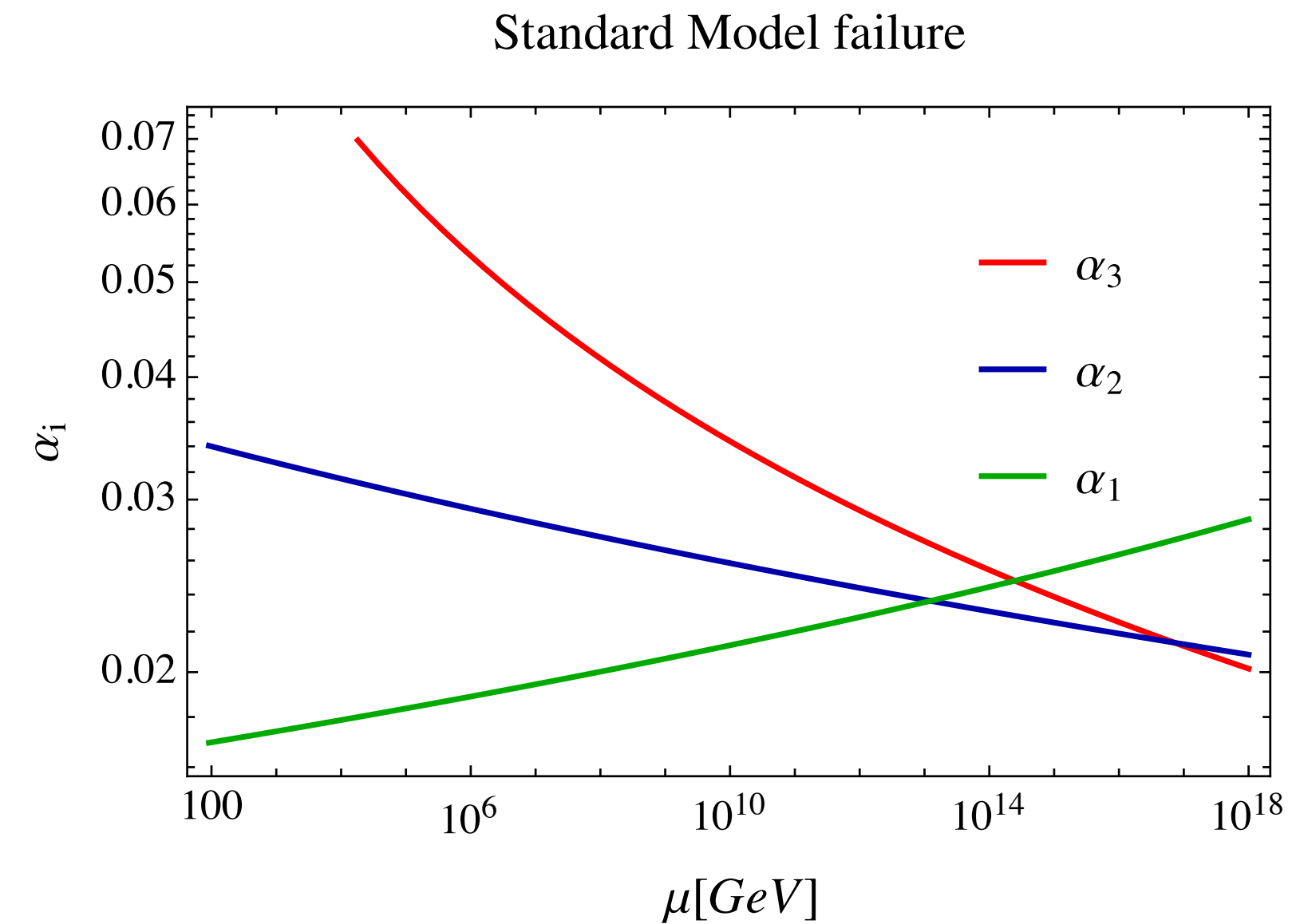
$C$  is of order one and branching ratio are predicted *Mohapatra '79*. The same is not true in extensions.  
*Crucial*: it can spoil relation between  $\tau_p$  and  $M_{\text{GUT}}$

# Scales in minimal $SU(5)$

*Georgi, Quinn, Weinberg '74*: Unification of gauge couplings in SM

Values found by GQW:

| $g_{30}^2/4\pi$ | $M$<br>(GeV)       | $\sin^2\theta$ |
|-----------------|--------------------|----------------|
| 0.5             | $2 \times 10^{17}$ | 0.175          |
| 0.2             | $2 \times 10^{16}$ | 0.187          |
| → 0.1           | $5 \times 10^{14}$ | 0.207          |
| 0.05            | $2 \times 10^{11}$ | 0.248          |



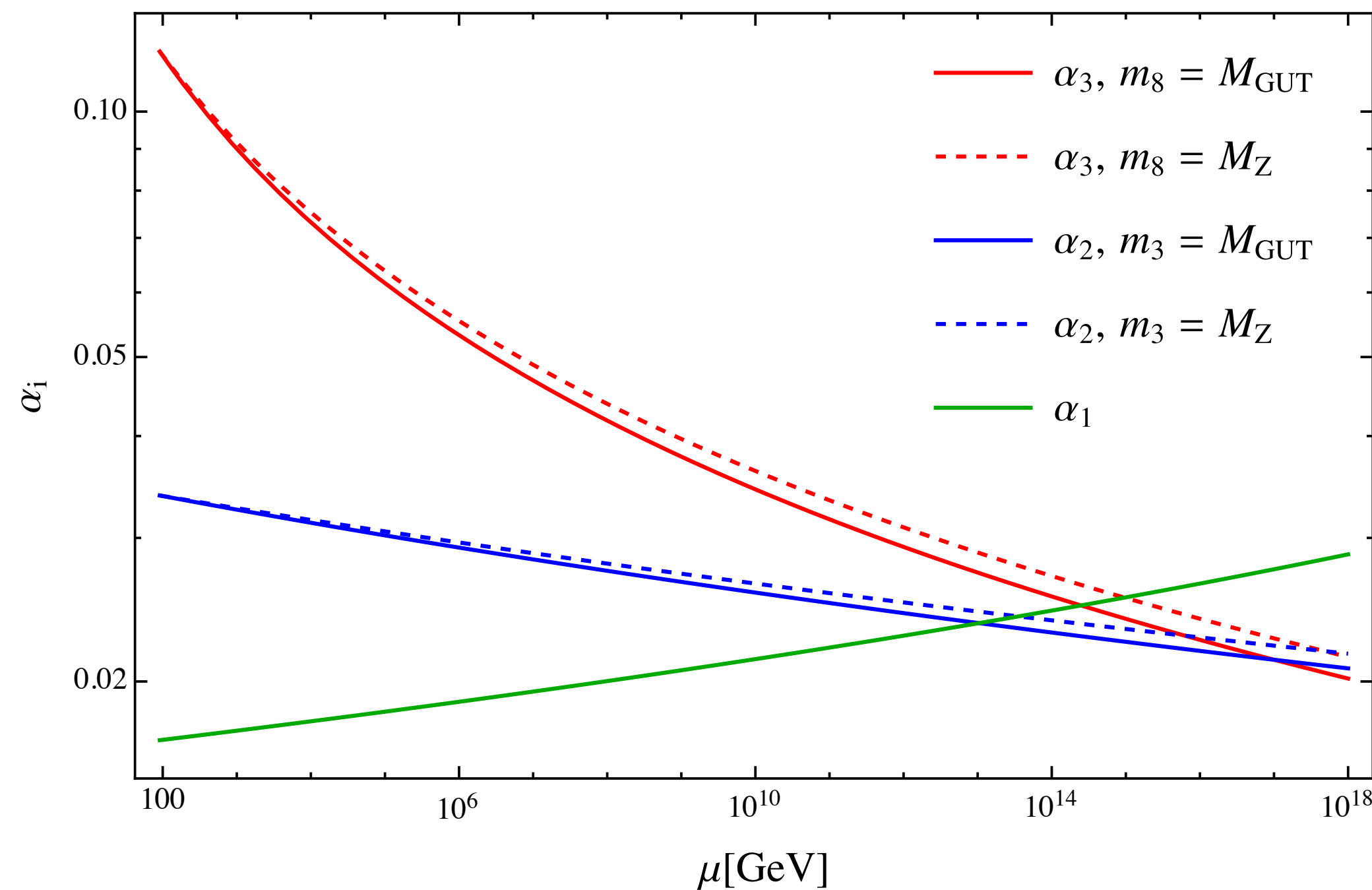
However, new particle states can aid unification:

GUT symmetry breaking realized via  $24_H = 8_C + 3_L + 1 + \cancel{(3_C, 2_L)} + \cancel{(\bar{3}_C, 2_L)}$   
 Eaten by  $X, Y$

# Scales in minimal $SU(5)$

GUT symmetry breaking realized via  $24_H = 8_C + 3_L + 1 + \cancel{(3_C, 2_L)} + \cancel{(\bar{3}_C, 2_L)}$   
 Eaten by  $X, Y$

EW symmetry breaking realized via  $5_H = 3_C + 2_L = T_C + H_{SM}$   
 $m_T = M_{GUT}$



- Color octet  $8_C$  of mass  $m_8$
- Weak triplet  $3_L$  of mass  $m_3$
- Color triplet  $T_C$  of mass  $m_T$
- Singlet  $1$  irrelevant

More is needed

$T_C$  normally required heavy due to proton decay. See *Dvali '92* for the light case scenario.

# Failures of minimal $SU(5)$

No gauge coupling unification

Neutrino massless (Right-handed neutrino missing)

Just as in the Standard Model, one could add a sterile neutrino. Admittedly, not the best solution.

Predicts wrong Yukawa relations

$$m_d = m_e$$

This is due to an accidental  $SU(4)$  symmetry

What to do?



# What to do?

## Three possible roads to follow

1) Give up on grand unification.

Aka give up on: p-decay, existence of monopoles, charge quantization, unification of forces

2) Add new representations.

For example, addition of  $45_H$  can address Yukawa and unification.

Yet, this implies lots of new degrees of freedom → simplicity and predictivity lost?

Change symmetry group? ( $SO(10)$ ? More later.)

3) Go effective.

Treat Georgi-Glashow model as an effective theory and see if higher-dimensional operators can salvage it. In this approach, minimal model believed to be ruled out. (See *Doršner, Fileviez Perez '05, Bajc, Senjanović '07* adding new representations)

They do.

*Senjanović, MZ, '24*

*Babu, Ma, '84  
See also Murayama,  
Yanagida '92,  
Doršner, Fileviez  
Perez '05,  
Haba et al '24 ...*

# Minimal $SU(5)$ on the edge: *the importance of being effective*

**Idea:** Keep the particle content of minimal  $SU(5)$  model, but allow for  $d > 4$  operators.

## **Perturbativity requirement:**

All higher-dimensional operators suppressed by high scale  $\Lambda$  with

$$\Lambda \gtrsim 10 M_{\text{GUT}}$$

## **Contra:**

$SU(5)$  features are altered by unknown physics (gravity?). Not ideal.

## **Pro:**

Small number of degrees of freedom  $\rightarrow$  simplified - and possible - analysis of scales

**Simplicity  $\rightarrow$  correlations between  $\tau_p$  and the scale of the new scalar particle (light)**

# Minimal $SU(5)$ on the edge: *the importance of being effective*

Predicts wrong Yukawa relations

$$m_d = m_e$$

$$\mathcal{L}_y^{d=4} = \bar{5}_F Y_d 5_H^* 10_F + 10_F Y_u 5_H 10_F$$

This can be easily corrected by the addition of  $d = 5$  Yukawa couplings

$$\mathcal{L}_y^{d=5} = \frac{1}{\Lambda} \bar{5}_F 24_H 5_H^* 10_F + \dots$$

*Ellis, Gaillard, '79*

$\langle 24_H \rangle \propto \text{diag} (2, 2, 2, -3, -3)$  breaks the accidental symmetry  
 $SU(3)_C$   $SU(2)_L$

This makes the p-decay branching ratio unpredictable, as opposed to the minimal case.  
(More on this later.)

# Minimal $SU(5)$ on the edge: *the importance of being effective*

Particle thresholds insufficient



include  $d = 5$  gauge boson kinetic energy

$$\Delta \mathcal{L}_{\text{kin}} = \frac{1}{\Lambda} \text{Tr} F_{\mu\nu} \langle 24_{\text{H}} \rangle F^{\mu\nu}$$

*Shafi, Wetterich '84*

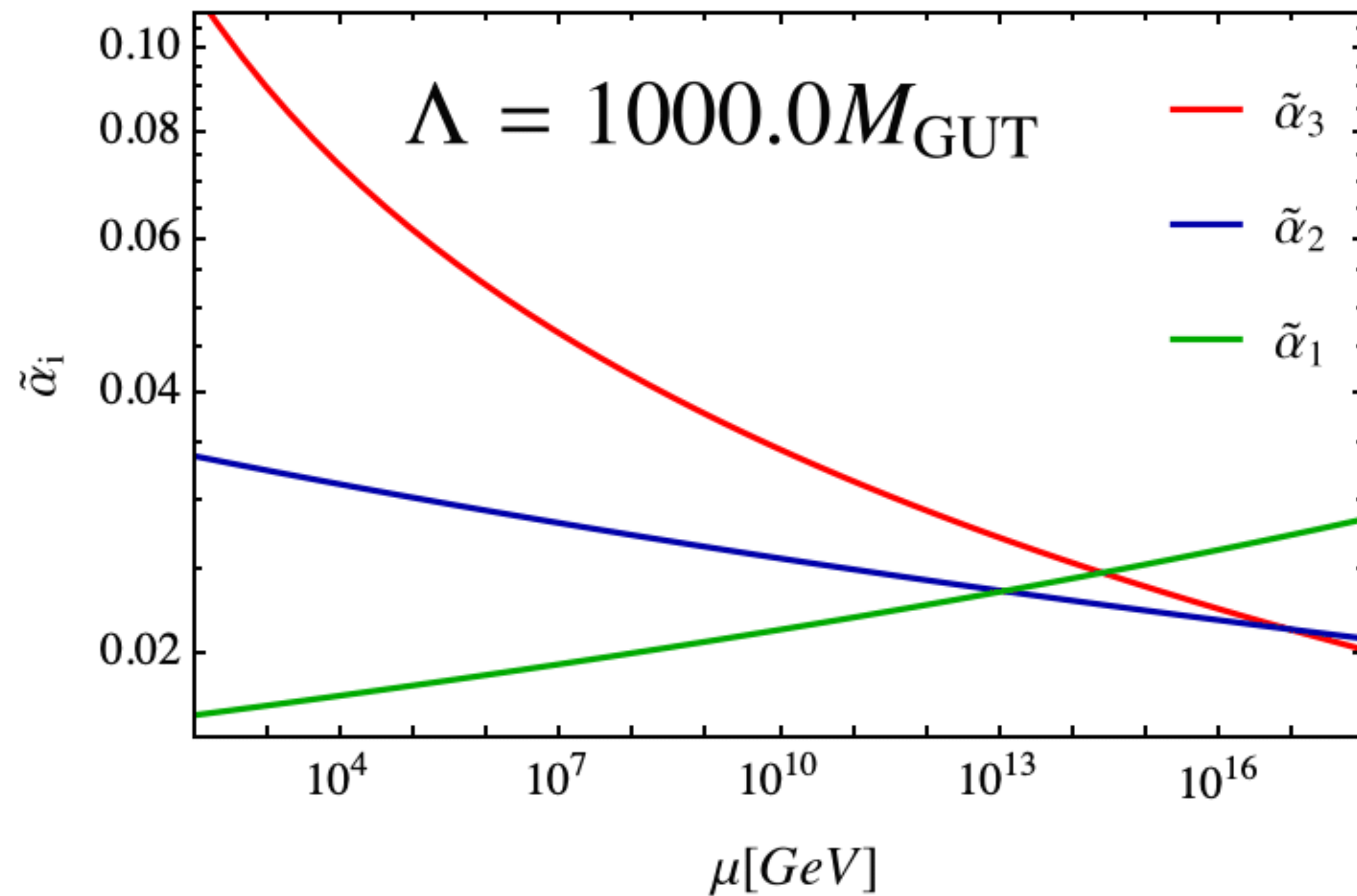
$$\langle 24_{\text{H}} \rangle = M_{\text{GUT}} \text{diag} (2, 2, 2, -3, -3)$$

$$\alpha_3 \rightarrow \tilde{\alpha}_3 = \left( 1 - \frac{M_{\text{GUT}}}{\Lambda} \right) \alpha_3,$$

$$\alpha_2 \rightarrow \tilde{\alpha}_2 = \left( 1 + \frac{3 M_{\text{GUT}}}{2 \Lambda} \right) \alpha_2,$$

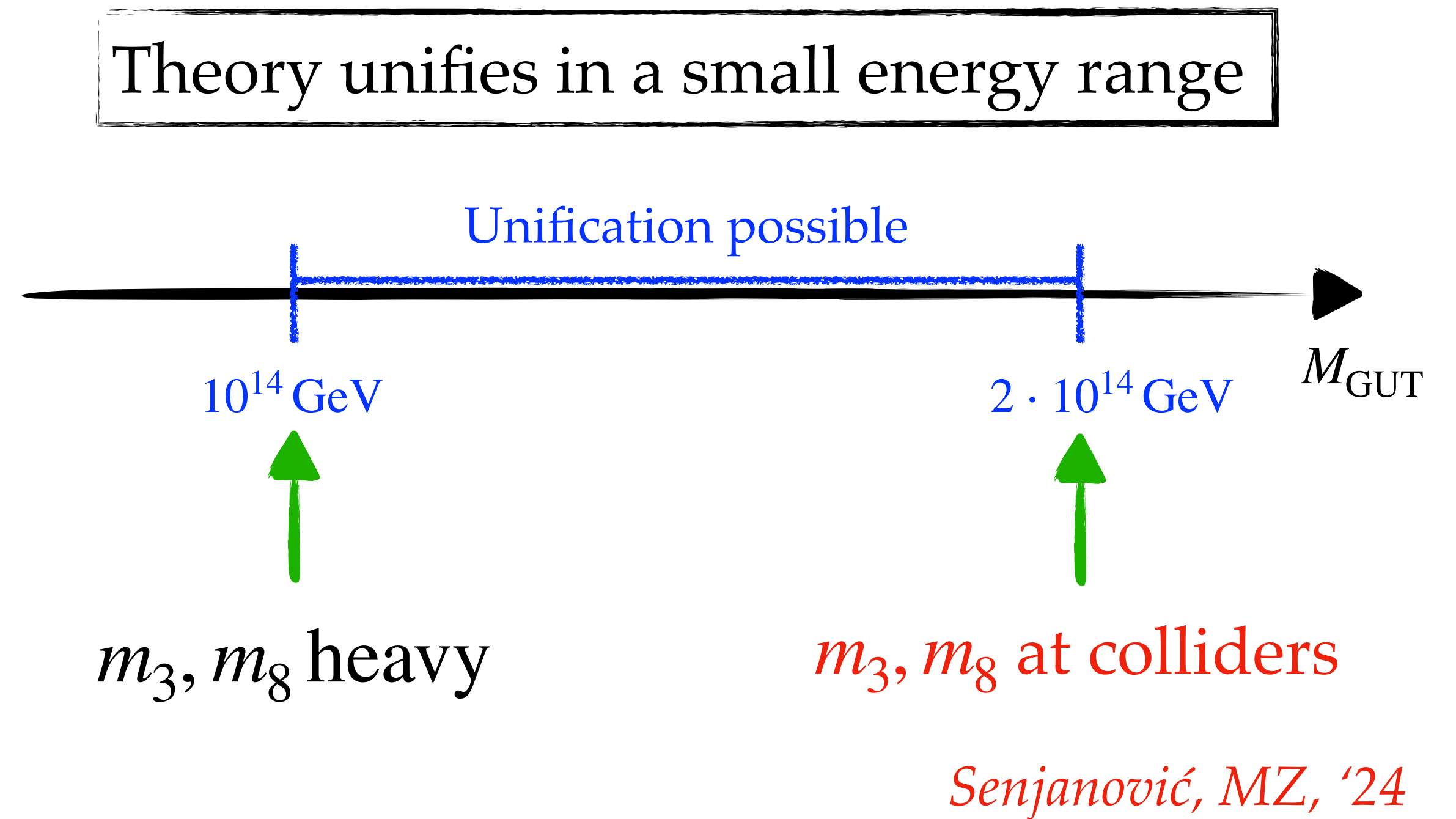
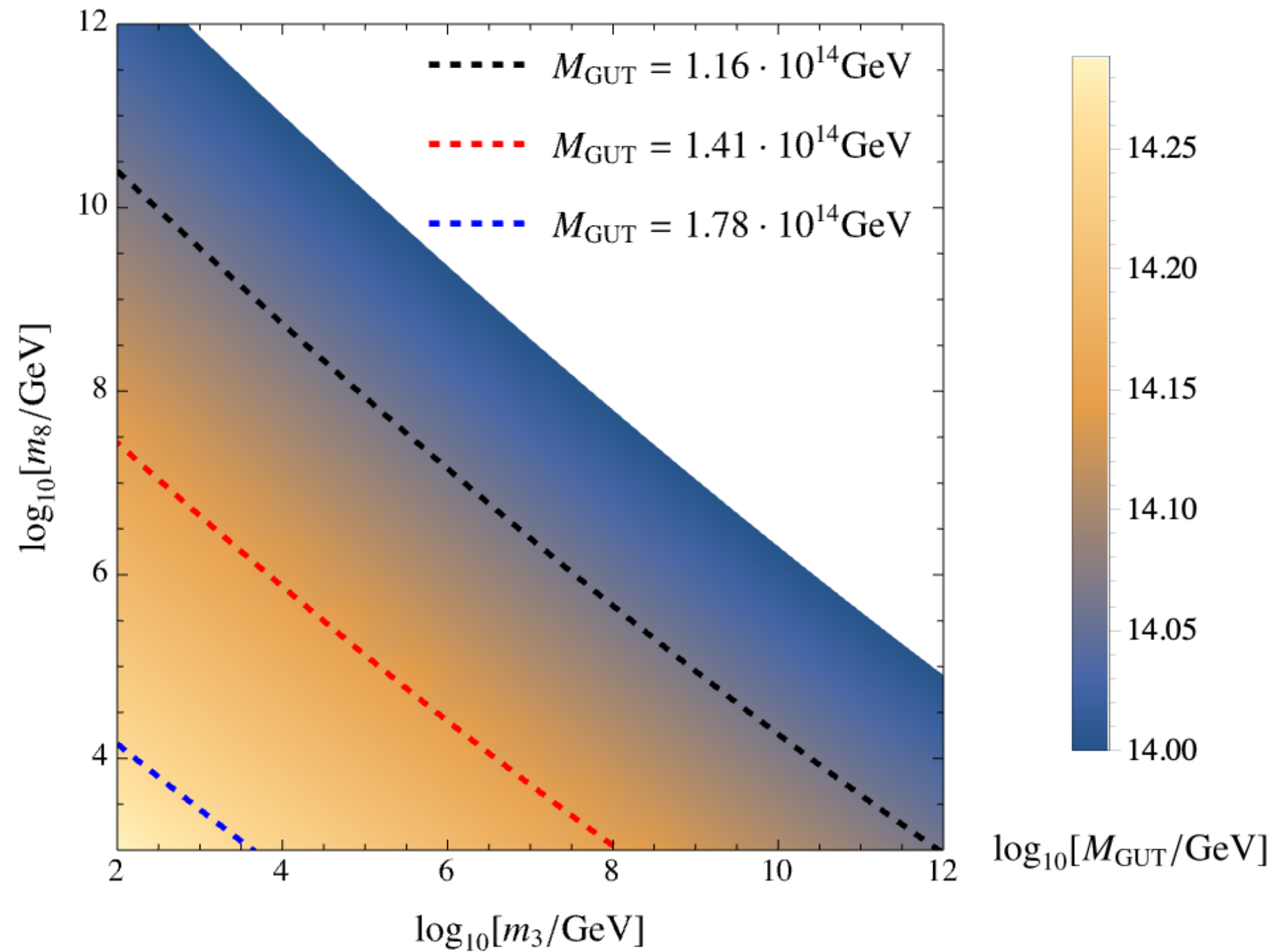
$$\alpha_1 \rightarrow \tilde{\alpha}_1 = \left( 1 + \frac{M_{\text{GUT}}}{2 \Lambda} \right) \alpha_1,$$

$$\text{Unification condition: } \tilde{\alpha}_3 (M_{\text{GUT}}) = \tilde{\alpha}_2 (M_{\text{GUT}}) = \tilde{\alpha}_1 (M_{\text{GUT}})$$



$\Lambda \gtrsim 20$  compatible with perturbativity. But  $M_{\text{GUT}} \simeq 10^{14} \text{ GeV}$  leads to too short  $\tau_p$ ?

# Minimal $SU(5)$ on the edge: *the importance of being effective*



$$\tau_p = \simeq C \frac{M_{\text{GUT}}^4}{\alpha_{\text{GUT}}^2} \left(m_{\text{proton}}\right)^{-5} \gtrsim 10^{34} \text{ yrs} \quad \longleftrightarrow \quad M_{\text{GUT}} \gtrsim 4 \cdot 10^{15} \text{ GeV} \left(\frac{\alpha_{\text{GUT}}}{40^{-1}}\right)^{1/2} \gg 10^{14} \text{ GeV}$$

$C$  is not fixed, and this spoils the relation between  $M_{\text{GUT}}$  and  $\tau_p$

*Nandi, Stern, Sudarschan '82*

# Minimal $SU(5)$ on the edge: *the importance of being effective*

$$\mathcal{L}_{\mathbf{B}} = (\bar{u}^c u) (\bar{e}^c d + \bar{d}^c e) + (\bar{u}^c d) (\bar{e}^c u + \bar{d}^c \nu)$$



Fermion rotation to mass basis

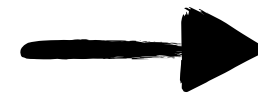
$$f \rightarrow Ff; \quad f^c \rightarrow F_c f^c; \quad FF^\dagger = F_c F_c^\dagger = 1$$

$$\mathcal{L}_{\mathbf{B}} = (\bar{u}^c U_c^\dagger U u) (\bar{e}^c E_c^\dagger D d + \bar{d}^c D_c^\dagger E e) + (\bar{u}^c U_c^\dagger D d) (\bar{e}^c E_c^\dagger U u + \bar{d}^c D_c^\dagger N \nu)$$

The unitary matrices determine  $C$ . Notice,  $V_{\text{CKM}} = U^\dagger D$

$$\tau_p \simeq C \frac{M_{\text{GUT}}^4}{\alpha_{\text{GUT}}^2} \left( m_{\text{proton}} \right)^{-5}$$

$$C = \infty \implies \theta_{13} = 0$$



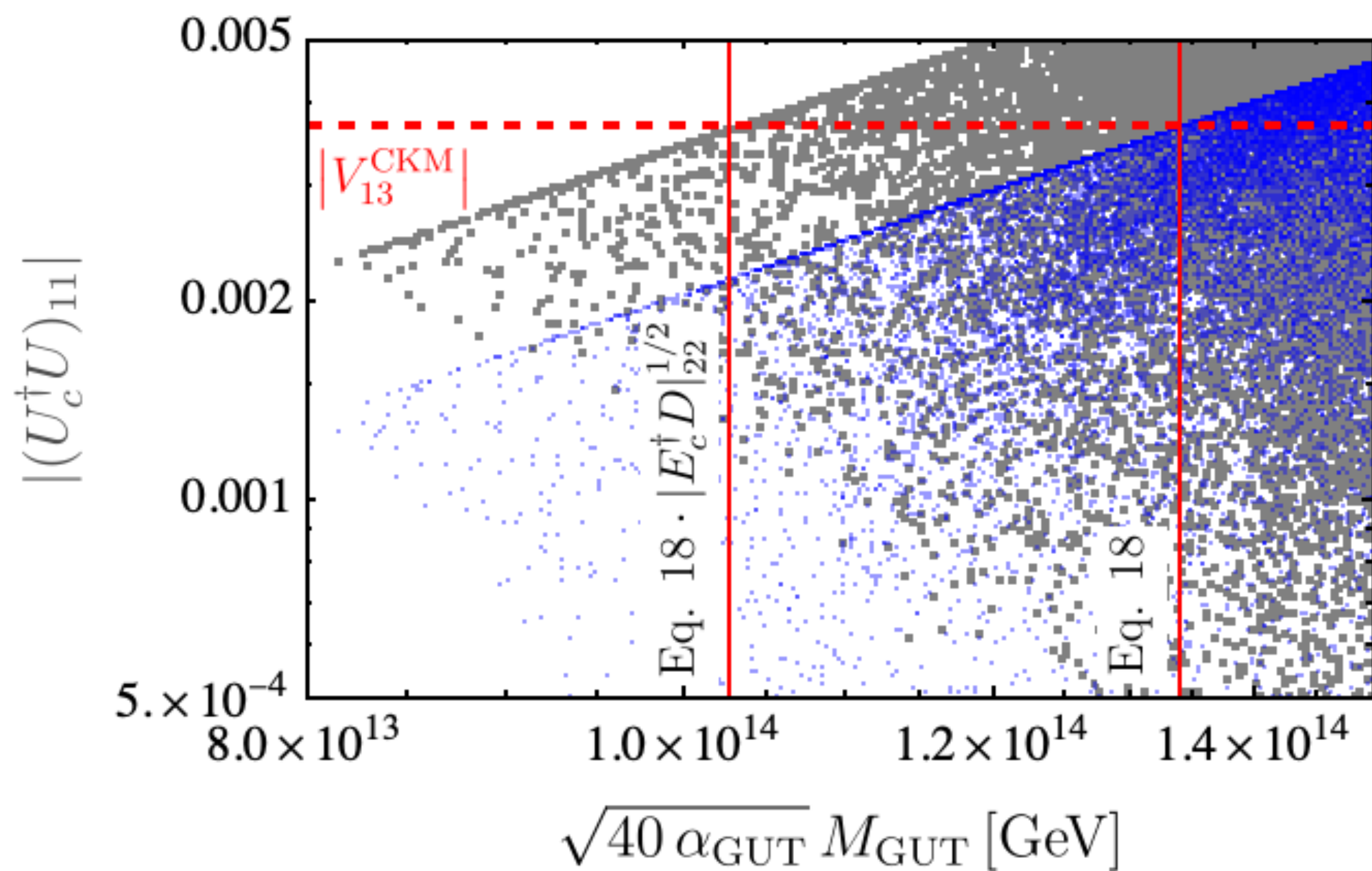
No p-decay!

$$\tau_p = \infty$$

*Nandi, Stern, Sudarschan '82*

# Minimal $SU(5)$ on the edge: *the importance of being effective*

$$\tau_p \simeq C \frac{M_{\text{GUT}}^4}{\alpha_{\text{GUT}}^2} \left(m_{\text{proton}}\right)^{-5} \simeq (\theta_{13})^{-2} \frac{M_{\text{GUT}}^4}{\alpha_{\text{GUT}}^2} \left(m_{\text{proton}}\right)^{-5} \rightarrow M_{\text{GUT}} \text{ can be way smaller}$$



$|(U_c^\dagger D)_{11}|$

| Channel                   | Lifetime ( $10^{30}$ yrs) |
|---------------------------|---------------------------|
| $N \rightarrow e^+ \pi$   | 5300 (n), 16000 (p)       |
| $N \rightarrow \mu^+ \pi$ | 3500 (n), 7700 (p)        |
| $N \rightarrow \nu \pi$   | 1100 (n), 390 (p)         |
| $N \rightarrow e^+ K$     | 17 (n), 1000 (p)          |
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Monte Carlo analysis to find the lowest possible leads to

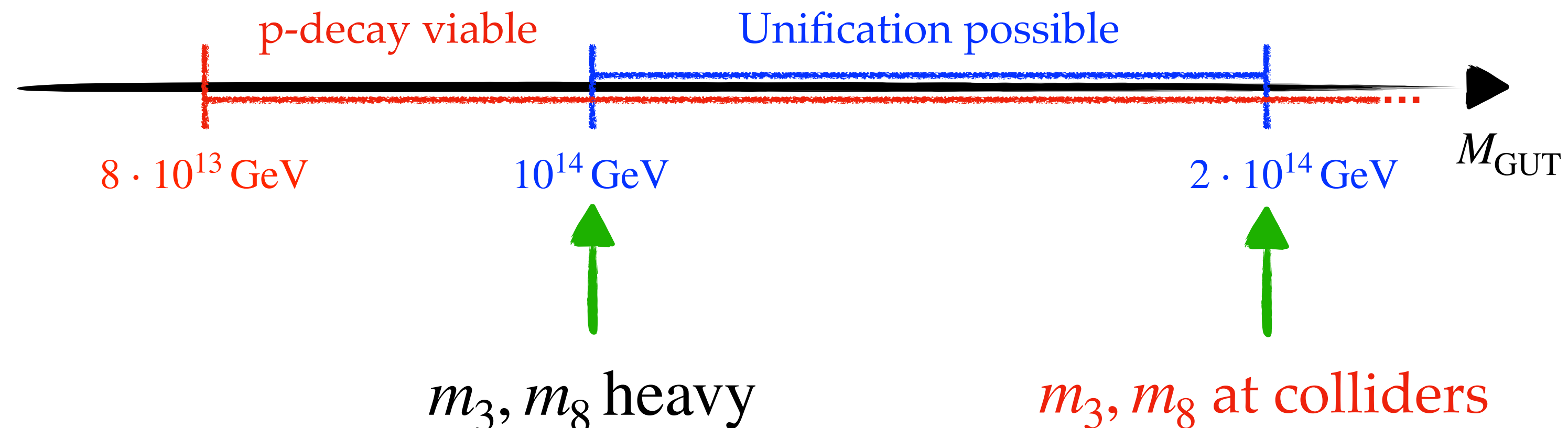
$$M_{\text{GUT}} \gtrsim 8 \cdot 10^{13} \text{ GeV}$$

*Senjanović, MZ, '24*

*Similar bound in Doršner, Fileviez Perez '05*

# Minimal $SU(5)$ on the edge: *the importance of being effective*

Theory unifies in a small energy range, near the lower bound from p-decay



Improvement of p-lifetime by a factor **10** (**20**)  $\implies m_3$  or  $m_8 \lesssim$  **100** (**10**) TeV

Improvement by factor  $\sim 50$  would rule out the model.

*Senjanović, MZ, '24*

\* improvement by factor 10 will be achieved in the near future.



# Minimal $SU(5)$ on the edge: *the importance of being effective*

Gauge gauge coupling unification compatible with p-decay

Neutrino massless (Right-handed neutrino missing)

Just as in the Standard Model, one could add a sterile neutrino. Admittedly, not the best solution.

Yukawa relations can be corrected by higher-dimensional operators

$$m_d \neq m_e$$

# Minimal $SU(5)$ on the edge: *the importance of being effective*

Neutrino massless (Right-handed neutrino missing)

The leading contribution comes from higher-dimensional operators, via *Weinberg operator '79*

$$\mathcal{L}_{\mathbb{L}} = \frac{c_{\nu}}{\Lambda} \bar{5}_{\text{F}} 5_{\text{H}} 5_{\text{H}} \bar{5}_{\text{F}} \longrightarrow m_{\nu} \simeq \frac{c_{\nu}}{2} \frac{(v_{\text{SM}})^2}{\Lambda}$$

$$m_{\nu} \gtrsim 0.1 \text{ eV} \implies \Lambda \lesssim c_{\nu} 3 \cdot 10^{14} \text{ GeV}$$



Borderline with perturbativity requirement:  $c_{\nu} \sim 3$  (but compatible with unitarity)

Just as in the Standard Model, one could add a sterile neutrino, and realize the seesaw mechanism.

*Minkowski '77*

*Mohapatra, Senjanović '79*

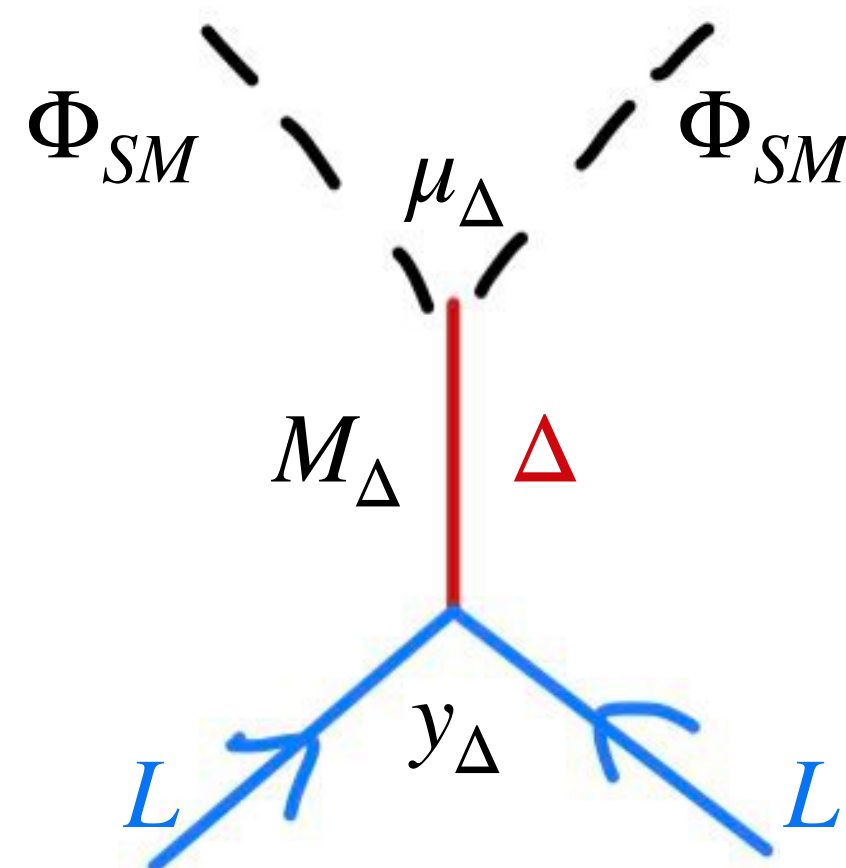
*Yanagida '79*

# Implementation of neutrino mass

Another possibility is adding new representations

Minimal SU(5) +  $15_H$  *Doršner, Fileviez Perez '05*

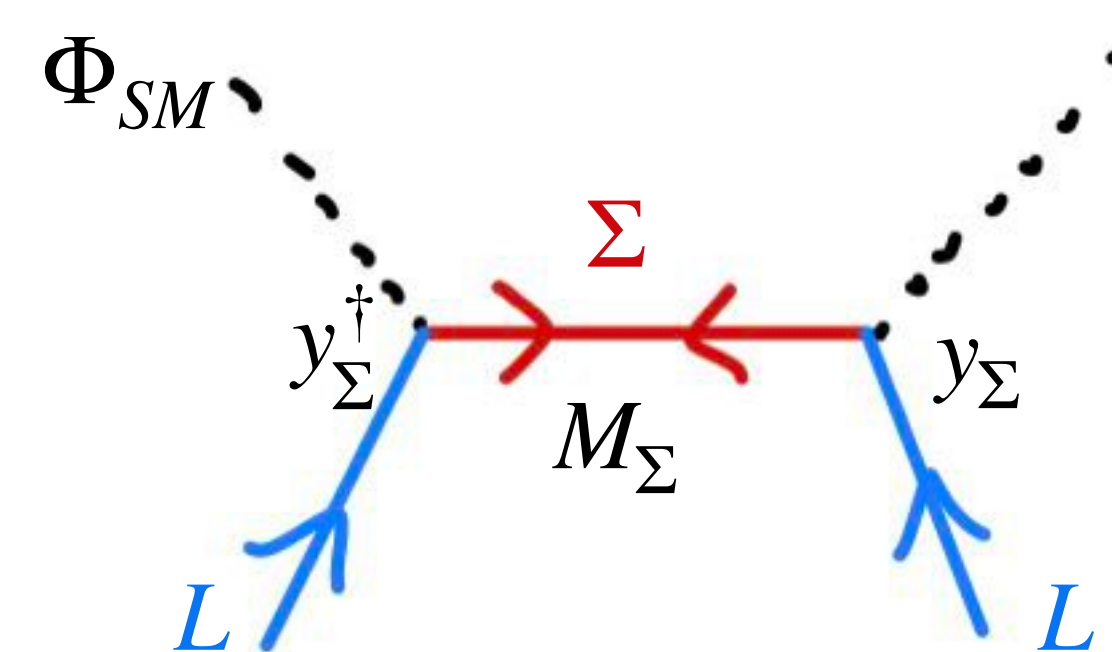
$15_H$  contains a scalar weak triplet  $\Delta$  with  $Y=2$



$$m_\nu \simeq y_\Delta \mu_\Delta \frac{v^2}{M_\Delta^2}$$

Minimal SU(5) +  $24_F$  *Bajc, Senjanović '07*

$24_F$  contains a fermionic weak triplet  $\Sigma$  with  $Y=0$



$$m_\nu \simeq v^2 \left( y_\Sigma^T \frac{1}{M_\Sigma} y_\Sigma \right)$$

These theories still require non-renormalizable operators. However, the tight correlations on the scales are lost due to the higher number of d.o.fs

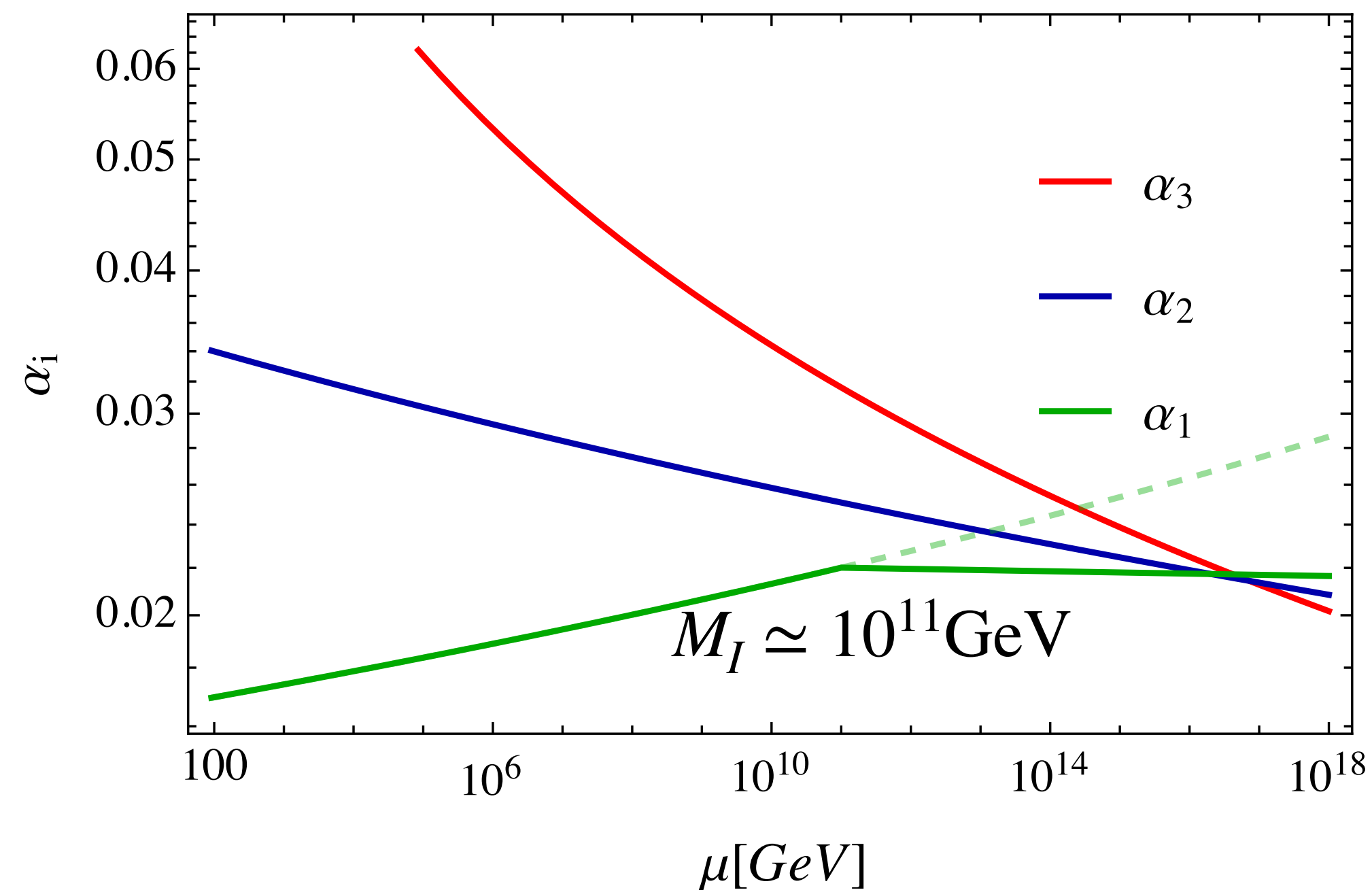
# $SO(10)$ : a true GUT for neutrino mass

*Georgi '74*

*Fritzsch, Minkowski, '74*

*Del Aguila, Ibanez '80*

Adding  $W_R$  *Rizzo, Senjanović '80*



$$16_F = \underline{10 + 5 + 1} =$$

Just as in  $SU(5)$

$$\begin{pmatrix} u_\alpha \\ \nu \\ d_\alpha \\ e \\ e^c \\ d_\alpha^c \\ \nu^c \\ u_\alpha^c \end{pmatrix}$$

Unified generation in same multiplet



Right handed neutrino already there  
Charge conjugation is gauge symmetry

In generic  $SO(10)$  model, intermediate scale can always be chosen to ensure unification.  
No need for new light particle states due to gauge coupling unification?

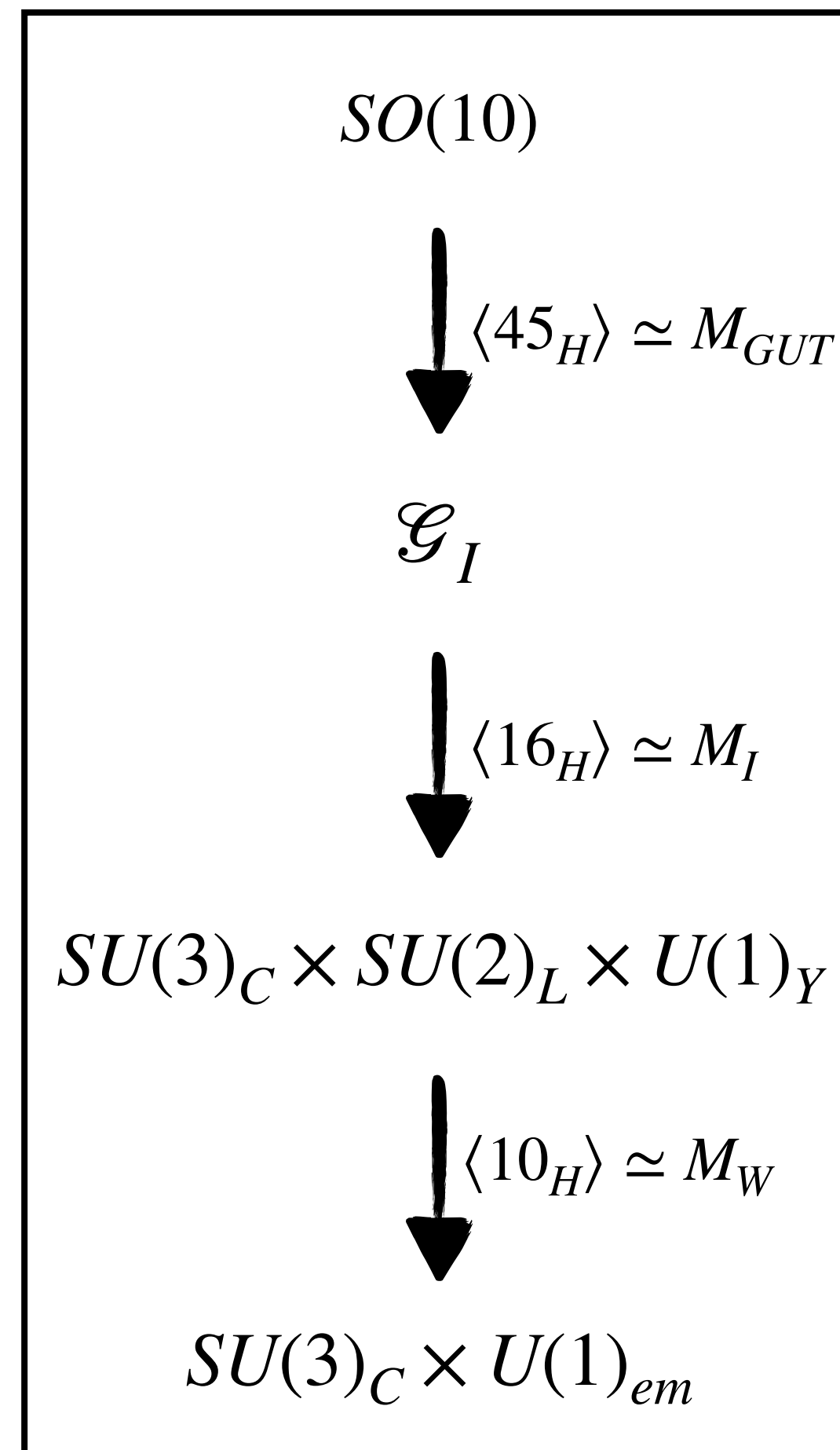
# $SO(10)$ : model building approach

At the renormalizable level, the theory requires large representations such as  $120_H$ ,  $126_H$  which introduce lots of new particle thresholds. Hard to make predictions. Moreover, even the minimal model seems to be ruled out *See Susić, Jarkovska, Malinsky '24*

What happens if, just as in the  $SU(5)$ , we go effective and choose the smallest possible representations allowing for a realistic model?

*A. Preda, G. Senjanović, MZ, '22*

# $SO(10)$ : still important to be effective?



Scalar sector with small representations:

Adjoint  $45_H$ , Spinor  $16_H$ , Fundamental  $10_H$

At renormalizable level, only  $10_H$  couples to fermions

$$16_F 16_F \langle 10_H \rangle \implies m_d = m_e, \quad m_u = m_D \quad \text{Neutrino Dirac mass}$$



$$m_{3D} = m_t$$

**This is crux of it all!**

# $SO(10)$ : still important to be effective?

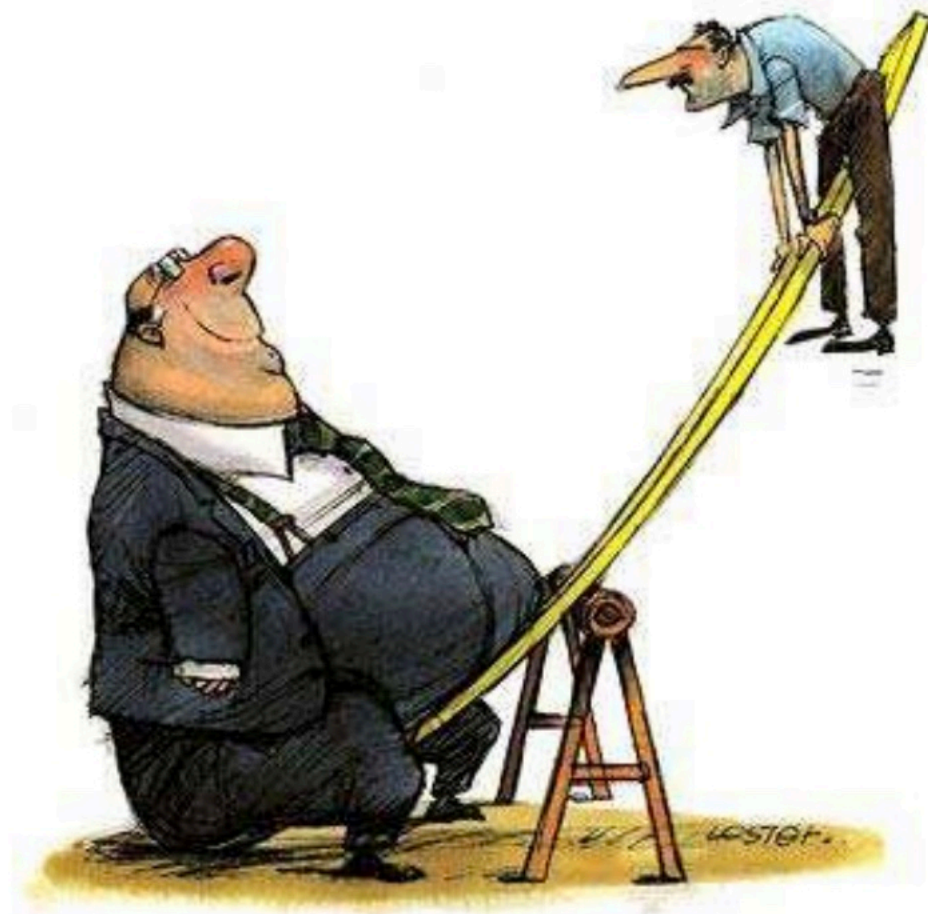
The right handed neutrino  $N$  obtains mass from  $d = 5$  operator

$$16_F 16_F 16_H^* 16_H^* / \Lambda$$

$$m_N \simeq \frac{M_I^2}{\Lambda}$$



$$\begin{pmatrix} 0 & m_D^T \\ m_D & m_N \end{pmatrix} \begin{matrix} \nu \\ N \end{matrix}$$



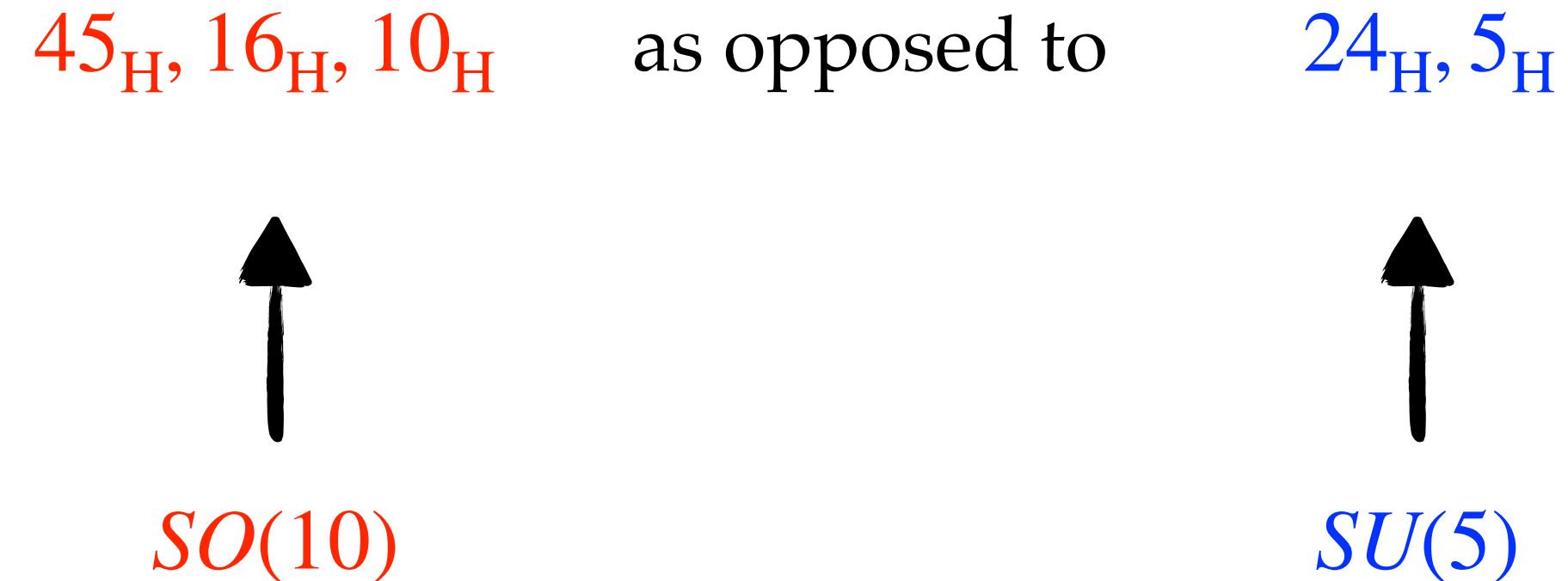
$$m_\nu \simeq \frac{(m_{3D})^2}{m_N} \simeq \frac{m_t^2 \Lambda}{M_I^2} \simeq \text{eV} \left( \frac{m_t}{100 \text{GeV}} \frac{4 \cdot 10^{14} \text{Gev}}{M_I} \right)^2 \left( \frac{\Lambda}{4 \cdot 10^{16} \text{GeV}} \right) \lesssim \text{eV}$$

$$\Rightarrow M_I \sim M_{GUT}$$

# $SO(10)$ : still important to be effective?

Due to the choice of scalar sector, we are pushed into single step breaking, just as in  $SU(5)$ .

However, more thresholds states i.e., more freedom in unification?



Is the  $SO(10)$  model still capable of informing us about scales?



# $SO(10)$ : still important to be effective?

Key-point: Different gauge kinetic operators are at play

$$\Delta \mathcal{L}_{\text{kin}} : \frac{1}{\Lambda^2} \text{Tr} F_{\mu\nu} \langle 45_{\text{H}}^2 \rangle F^{\mu\nu} \propto \frac{M_{\text{GUT}}^2}{\Lambda^2} \lesssim 10^{-2}$$



$SO(10)$

Here the first operator is  $d = 6$ . Negligible!  
**Unification can fail!**

$$\frac{1}{\Lambda} \text{Tr} F_{\mu\nu} \langle 24_{\text{H}} \rangle F^{\mu\nu} \propto \frac{M_{\text{GUT}}}{\Lambda} \lesssim 10^{-1}$$



$SU(5)$

This operator was **always ensuring unification**,  
regardless of particle scales

$SO(10)$  has more particle states. However, they need to need to ensure unification as opposed to  $SU(5)$ . Therefore, more is demanded from them.

# $SO(10)$ : still important to be effective

Bottom line: Surprisingly, something can still be said about scales.

Under the assumption of no rotation in p-decay ( $C = 1$ ):

Scalar weak triplet  $3_L$ , a squark  $(2_L, 3_C)$  and an octet scalar gluon  $8_C$ , need to lie below 10 TeV. Moreover, p-decay has to be found before 2030.

*A. Preda, G. Senjanović, MZ, '22*

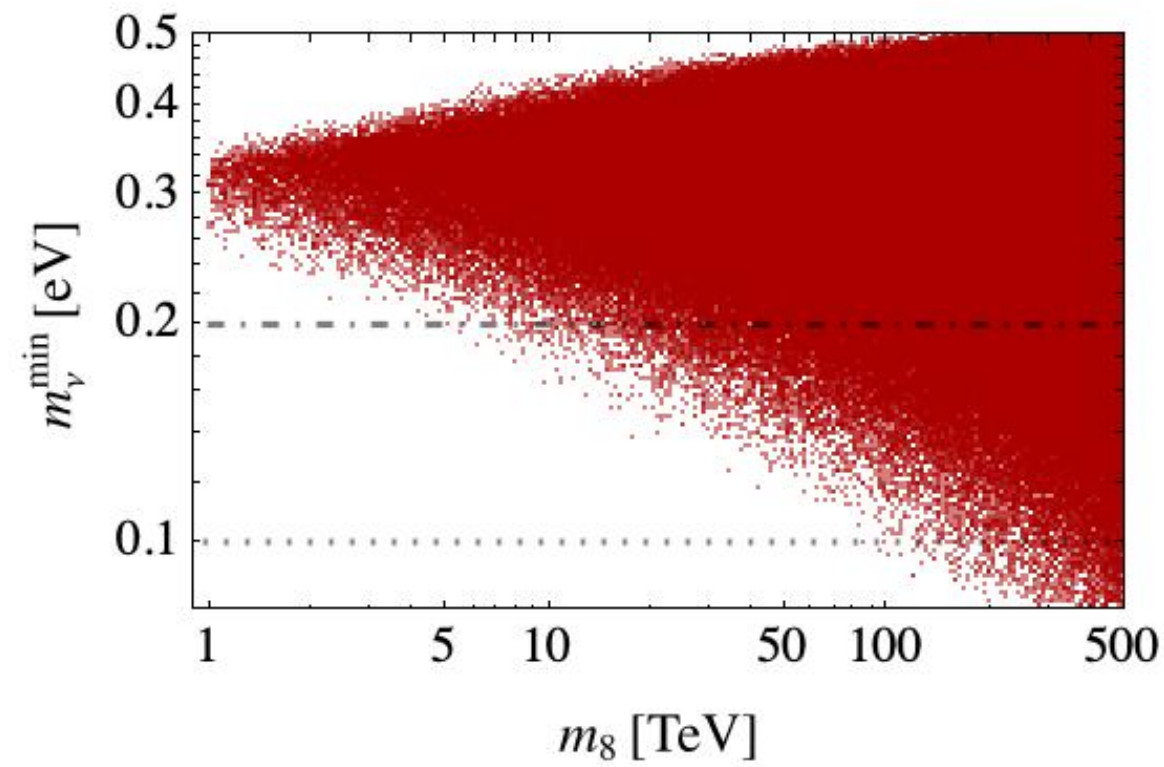
The assumption of no p-decay cancellation is commonly accepted by the community.

However, this would not exhaust all parameter space of the theory. Theories live on a point. Nevertheless, result is still surprising.

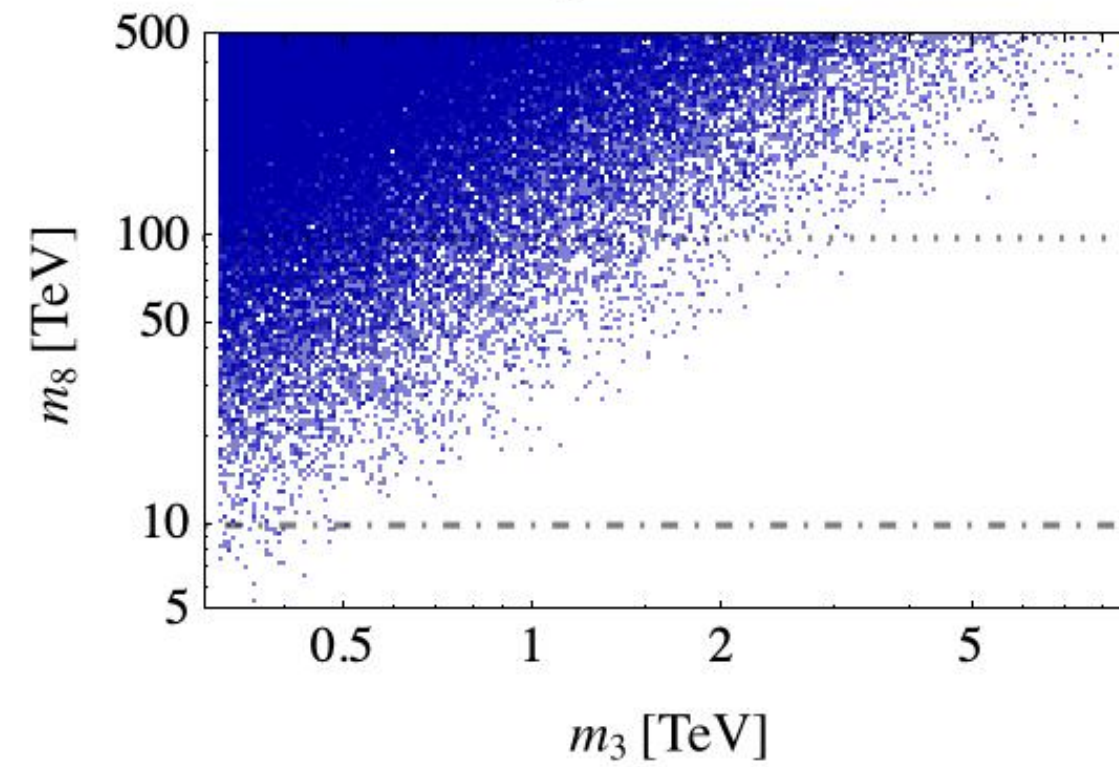
# $SO(10)$ : still important to be effective

The full parameter space allows still for a relaxed statement. If we allow for cancellations in p-decay amplitudes:

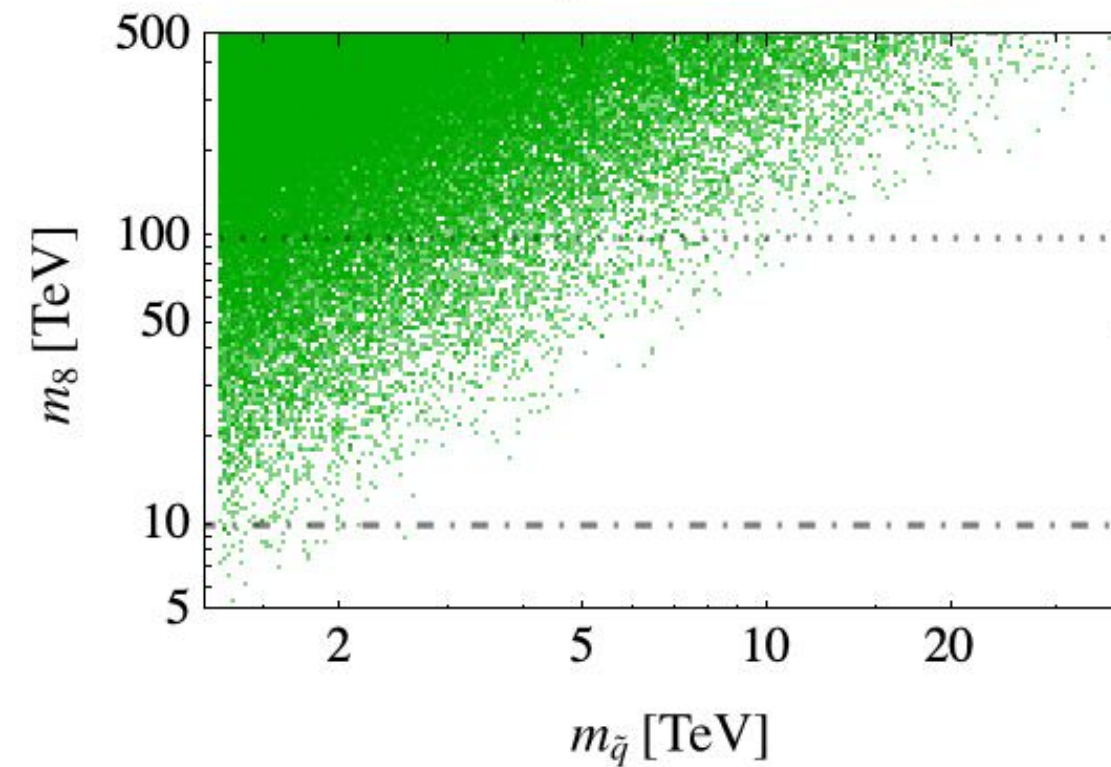
$$\langle 16_H \rangle \gtrsim v_{LR}/3$$



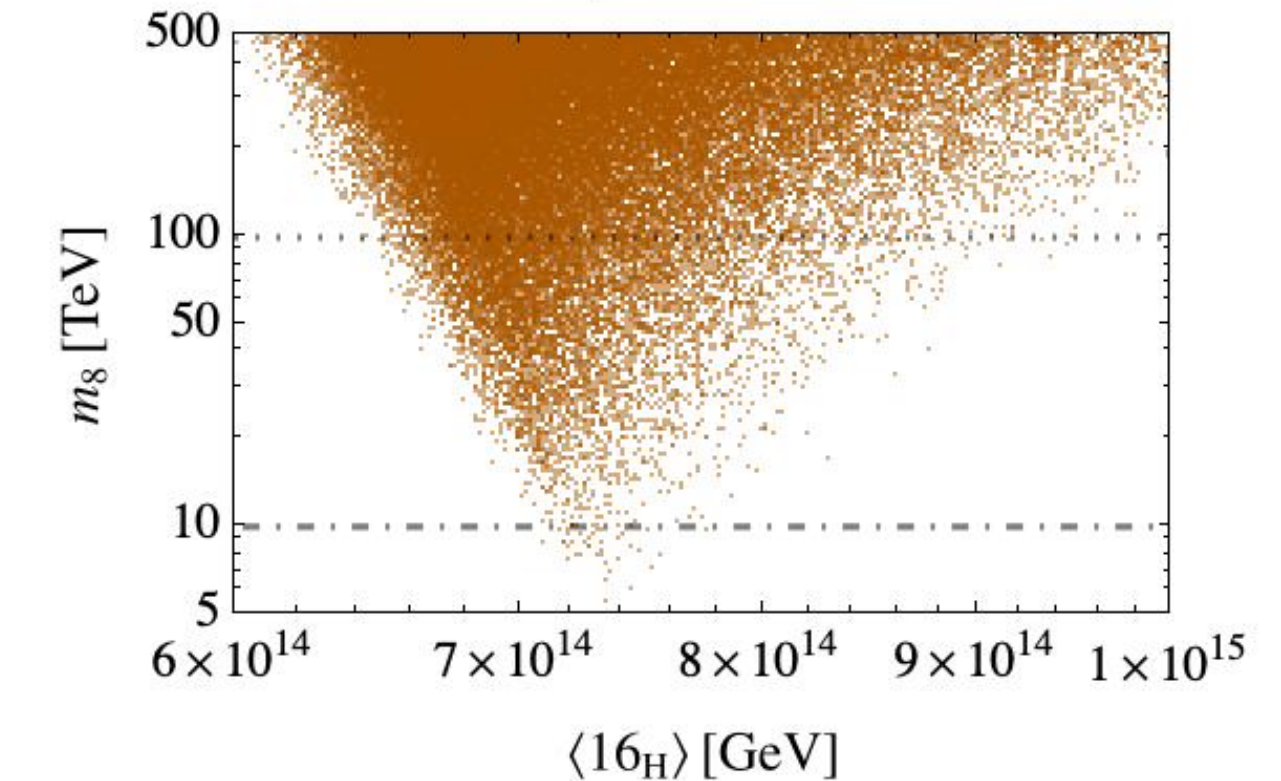
$$m_v^{\min} \gtrsim 0.2 \text{ eV}$$



$$m_v^{\min} \gtrsim 0.2 \text{ eV}$$



$$m_v^{\min} \gtrsim 0.2 \text{ eV}$$



If the colour octet  $8_C$  lies below 10 TeV  $\implies$

- $m_\nu \gtrsim 2 \cdot 10^{-1} \text{ eV}$
- Weak triplet below 500 GeV
- Squark below 2 TeV

*Potentially within  
experimental reach*

*At collider energies*

*A. Preda, G. Senjanović, MZ, to appear*

Example of what happens when we analyze a GUT theory in its full parameter space.

# Conclusions

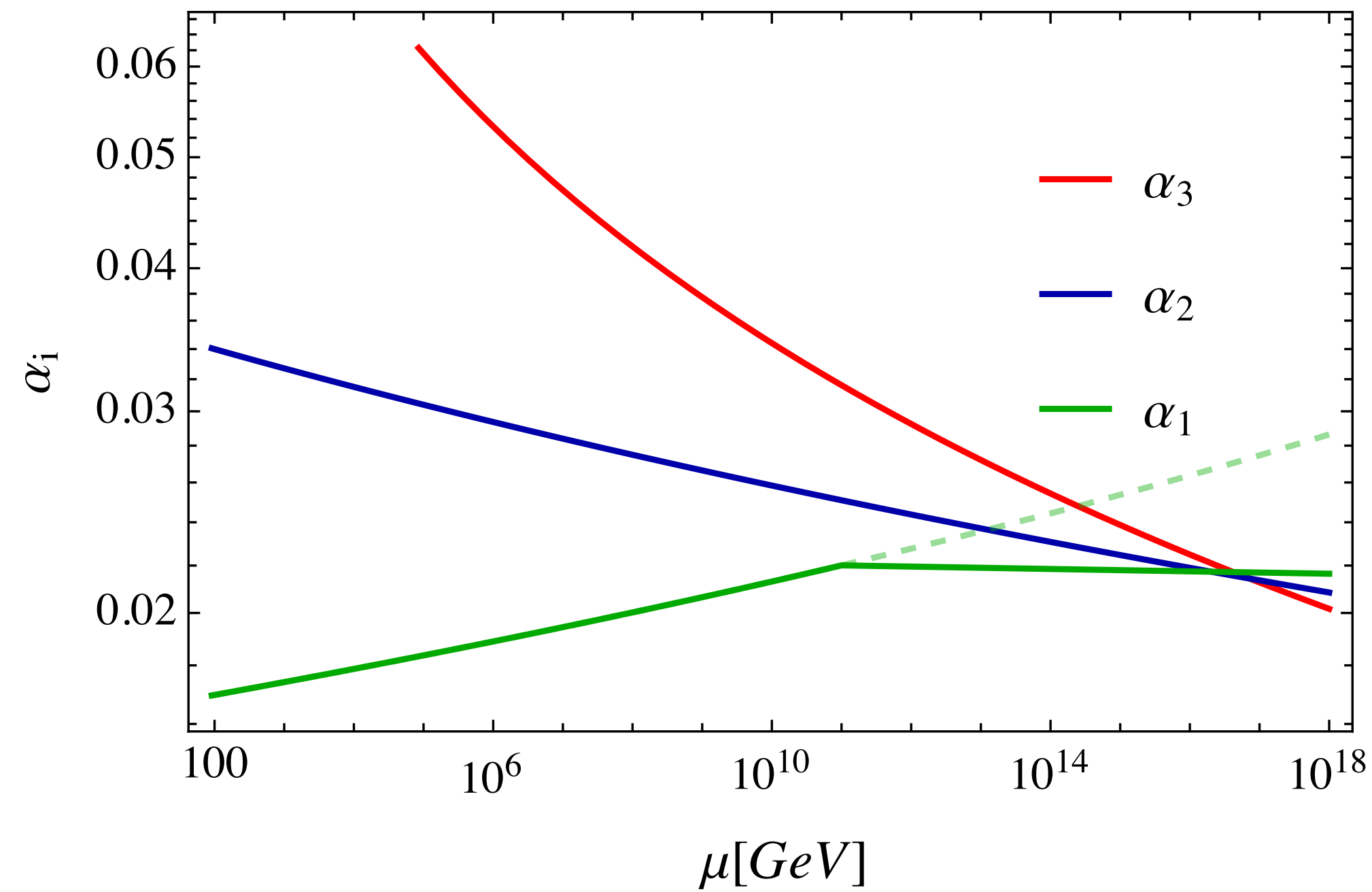
- We showed that the original  $SU(5)$  model by Georgi-Glashow, known to be ruled out, is viable as an effective theory
- Its advantage lies in the **small number of new degrees of freedom** introduced, which leads to constraint in the particle spectrum
- New particle states, a **weak triplet** and a **color octet need to be light** for the theory to be viable
- **Their lightness is correlated with bounds on proton lifetime.** An improvement in current bounds by a factor of order 10 would imply these particle are at collider energies
- **A similar situation** emerges in minimal  $SO(10)$  based on small representation if treated as an effective theory.
- Because  $SO(10)$  naturally predicts neutrino mass, the **potentially light particle states imply a lower bound on neutrino mass**, within the reach of current experiments.
- **Both theories, however, cannot be fully ruled out within the near future. What to do?**

*Thank you!*

*Backup*

## More on $SO(10)$ ?

Adding  $W_R$



- Unification ensured by intermediate scale  $M_I$
- Right-handed neutrino gets a mass — fermions unified in  $16_F$
- Can realize seesaw mechanism

Higgs sector:

$45_H; 16_H; 10_H$

SM undergoes usual breaking with  $10_H$

- $10_H$  **real** does not work since  $m_u = m_d$  hence the need for complex  $10_H$
- At **renormalizable** level still  $m_d = m_e$ , so higher dimensional operators are needed for Yukawa

In what follows, for validity of perturbativity, we will require cutoff

$$\Lambda \gtrsim 10 M_{GUT}$$

**Crux:**  $m_{3D} = m_t$

$16_H$ 

| $4_C 2_L 2_R$     | $4_C 2_L 1_R$                | $3_c 2_L 2_R 1_X$               | $3_c 2_L 1_R 1_X$                          | $3_c 2_L 1_Y$                | 5         | $5' 1_{Z'}$     | $1_{Y'}$       |
|-------------------|------------------------------|---------------------------------|--|------------------------------|-----------|-----------------|----------------|
| (4, 2, 1)         | (4, 2, 0)                    | $(3, 2, 1, +\frac{1}{6})$       | $(3, 2, 0, +\frac{1}{6})$                  | $(3, 2, +\frac{1}{6})$       | 10        | (10, +1)        | $+\frac{1}{6}$ |
|                   |                              | $(1, 2, 1, -\frac{1}{2})$       | $(1, 2, 0, -\frac{1}{2})$                  | $(1, 2, -\frac{1}{2})$       | $\bar{5}$ | $(\bar{5}, -3)$ | $-\frac{1}{2}$ |
| $(\bar{4}, 1, 2)$ | $(\bar{4}, 1, +\frac{1}{2})$ | $(\bar{3}, 1, 2, -\frac{1}{6})$ | $(\bar{3}, 1, +\frac{1}{2}, -\frac{1}{6})$ | $(\bar{3}, 1, +\frac{1}{3})$ | $\bar{5}$ | (10, +1)        | $-\frac{2}{3}$ |
|                   | $(\bar{4}, 1, -\frac{1}{2})$ |                                 | $(\bar{3}, 1, -\frac{1}{2}, -\frac{1}{6})$ | $(\bar{3}, 1, -\frac{2}{3})$ | 10        | $(\bar{5}, -3)$ | $+\frac{1}{3}$ |
|                   |                              | $(1, 1, 2, +\frac{1}{2})$       | $(1, 1, +\frac{1}{2}, +\frac{1}{2})$       | (1, 1, +1)                   | 10        | (1, +5)         | 0              |
|                   |                              |                                 | $(1, 1, -\frac{1}{2}, +\frac{1}{2})$       | (1, 1, 0)                    | 1         | (10, +1)        | +1             |

 $45_H$ 

| $4_C 2_L 2_R$ | $4_C 2_L 1_R$          | $3_c 2_L 2_R 1_X$               | $3_c 2_L 1_R 1_X$                          | $3_c 2_L 1_Y$                | 5          | $5' 1_{Z'}$      | $1_{Y'}$       |
|---------------|------------------------|---------------------------------|--|------------------------------|------------|------------------|----------------|
| (1, 1, 3)     | (1, 1, +1)             | (1, 1, 3, 0)                    | (1, 1, +1, 0)                              | (1, 1, +1)                   | 10         | (10, -4)         | +1             |
|               | (1, 1, 0)              |                                 | (1, 1, 0, 0)                               | (1, 1, 0)                    | 1          | (1, 0)           | 0              |
|               | (1, 1, -1)             |                                 | (1, 1, -1, 0)                              | (1, 1, -1)                   | $\bar{10}$ | $(\bar{10}, +4)$ | -1             |
| (1, 3, 1)     | (1, 3, 0)              | (1, 3, 1, 0)                    | (1, 3, 0, 0)                               | (1, 3, 0)                    | 24         | (24, 0)          | 0              |
| (6, 2, 2)     | $(6, 2, +\frac{1}{2})$ | $(3, 2, 2, -\frac{1}{3})$       | $(3, 2, +\frac{1}{2}, -\frac{1}{3})$       | $(3, 2, \frac{1}{6})$        | 10         | (24, 0)          | $-\frac{5}{6}$ |
|               | $(6, 2, -\frac{1}{2})$ |                                 | $(3, 2, -\frac{1}{2}, -\frac{1}{3})$       | $(3, 2, -\frac{5}{6})$       | 24         | (10, -4)         | $+\frac{1}{6}$ |
|               |                        | $(\bar{3}, 2, 2, +\frac{1}{3})$ | $(\bar{3}, 2, +\frac{1}{2}, +\frac{1}{3})$ | $(\bar{3}, 2, +\frac{5}{6})$ | 24         | $(\bar{10}, +4)$ | $-\frac{1}{6}$ |
|               |                        |                                 | $(\bar{3}, 2, -\frac{1}{2}, +\frac{1}{3})$ | $(\bar{3}, 2, -\frac{1}{6})$ | $\bar{10}$ | (24, 0)          | $+\frac{5}{6}$ |
| (15, 1, 1)    | (15, 1, 0)             | (1, 1, 1, 0)                    | (1, 1, 0, 0)                               | (1, 1, 0)                    | 24         | (24, 0)          | 0              |
|               |                        | $(3, 1, 1, +\frac{2}{3})$       | $(3, 1, 0, +\frac{2}{3})$                  | $(3, 1, +\frac{2}{3})$       | $\bar{10}$ | $(\bar{10}, +4)$ | $+\frac{2}{3}$ |
|               |                        | $(\bar{3}, 1, 1, -\frac{2}{3})$ | $(\bar{3}, 1, 0, -\frac{2}{3})$            | $(\bar{3}, 1, -\frac{2}{3})$ | 10         | (10, -4)         | $-\frac{2}{3}$ |
|               |                        | (8, 1, 1, 0)                    | (8, 1, 0, 0)                               | (8, 1, 0)                    | 24         | (24, 0)          | 0              |



## More on the breaking Pattern

### Content

At tree level only one VEV allowed for

$$\langle 45_H \rangle^{SU(5)} = v_{GUT} \sigma_2 \otimes \text{diag}(1, 1, 1, \pm 1, \pm 1),$$

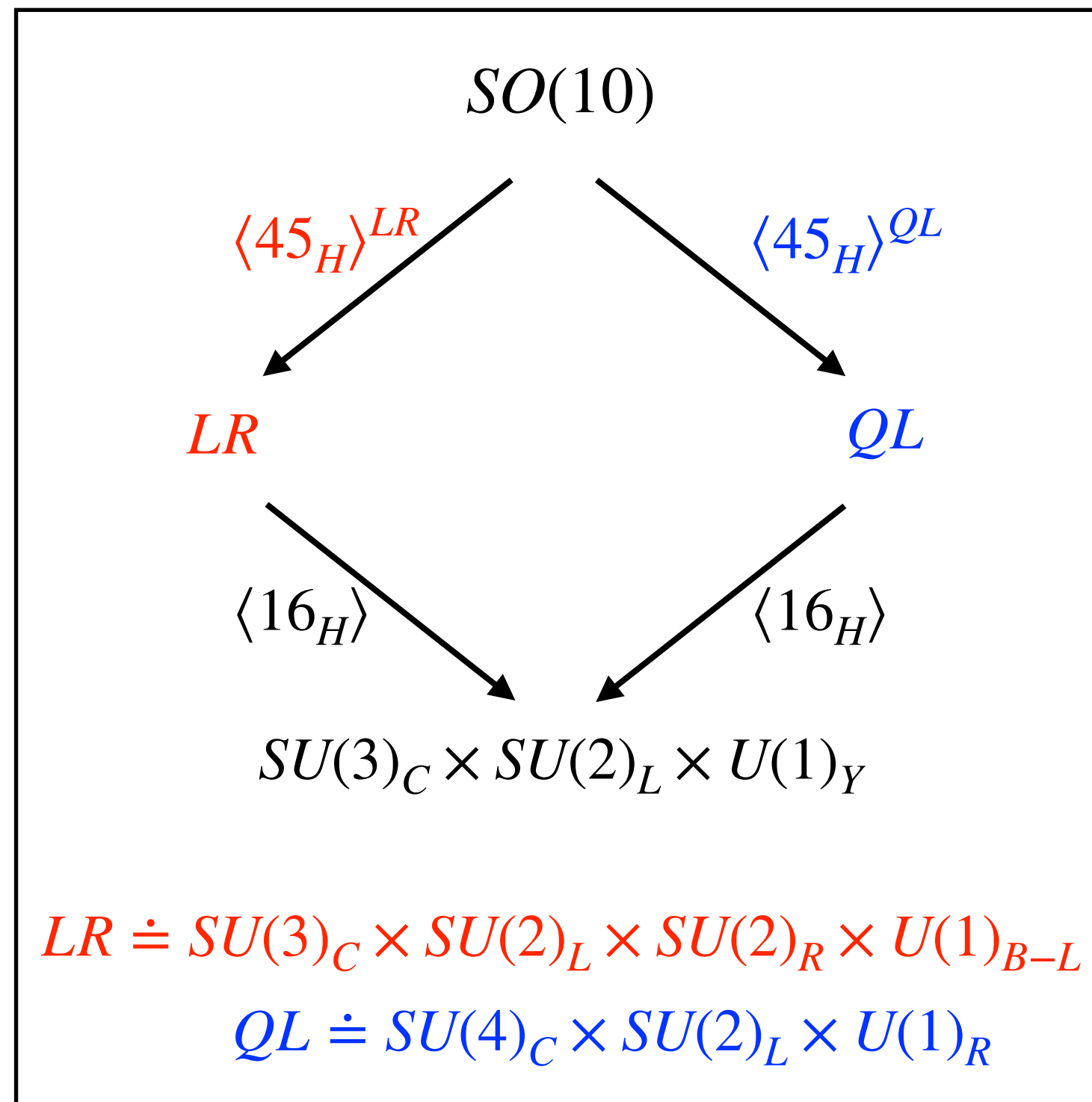
Non viable for unification

Including higher dimensional operators, new directions are possible

$$\begin{aligned} \langle 45_H \rangle^{LR} &= v_{GUT} \sigma_2 \otimes \text{diag}(1, 1, 1, 0, 0), \\ \langle 45_H \rangle^{QL} &= v_{GUT} \sigma_2 \otimes \text{diag}(0, 0, 0, 1, 1) \end{aligned}$$

→ Breaking pattern possible also at tree-level via inclusion of radiative corrections (see Bertolini, Di Luzio, Malinsky 0912.1796)

→ Higher-dimensional operators give more freedom in the spectrum



$\langle 16_H \rangle \simeq M_I$  realizes tadpole for vanishing component when acquiring VEV via  $16_H 45_H 16_H^*$

# Seesaw

The right handed neutrino  $N$  obtains mass from  $d = 5$  operator

$$16_F 16_F 16_H^* 16_H^* / \Lambda$$

$$m_N \simeq \frac{M_I^2}{\Lambda}$$

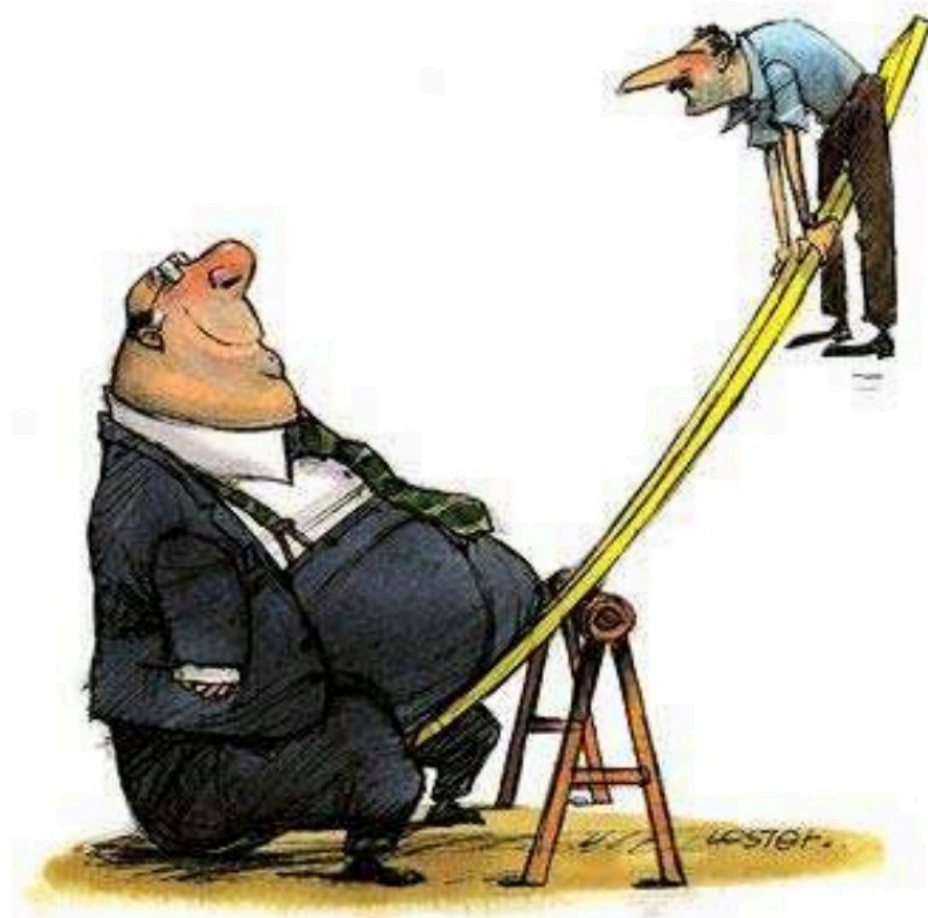
$$\downarrow \begin{pmatrix} 0 & m_D^T \\ m_D & m_N \end{pmatrix} \begin{matrix} \nu \\ N \end{matrix}$$

$$m_\nu \simeq \frac{(m_{3D})^2}{m_N} \simeq \frac{m_t^2 \Lambda}{M_I^2} \simeq \text{eV} \left( \frac{m_t}{100 \text{ GeV}} \frac{4 \cdot 10^{14} \text{ GeV}}{M_I} \right)^2 \left( \frac{\Lambda}{4 \cdot 10^{16} \text{ GeV}} \right) \simeq \text{eV}$$

$$\Rightarrow M_I \gtrsim 0.1 M_{GUT}$$

But in the absence of an intermediate scale, we learnt from  $SU(5)$  that **new light states are necessary**

Scalar weak triplet  $3_W$ , a squark  $(2_W, 3_C)$  and an octet scalar gluon  $8_C$



|        | $4_C 2_L 2_R$                | $3_c 2_L 1_Y$                | $m_i$     |
|--------|------------------------------|------------------------------|-----------|
| $16_H$ | $(4, 2, 1)$                  | $(3, 2, +\frac{1}{6})$       | $m_{sq}$  |
|        |                              | $(1, 2, -\frac{1}{2})$       |           |
|        | $(\bar{4}, 1, 2)$            | $(\bar{3}, 1, +\frac{1}{3})$ | $m_{sd}$  |
|        |                              | $(\bar{3}, 1, -\frac{2}{3})$ | $m_{sup}$ |
|        |                              | $(1, 1, +1)$                 | $m_{sel}$ |
|        |                              | $(1, 1, 0)$                  |           |
|        | $4_C 2_L 2_R$                | $3_C 2_L 1_Y$                | $m_i$     |
| $45_H$ | $(1, 1, 3)$                  | $(1, 1, +1)$                 | $m_{sel}$ |
|        |                              | $(1, 1, 0)$                  |           |
|        |                              | $(1, 1, -1)$                 |           |
|        | $(1, 3, 1)$                  | $(1, 3, 0)$                  | $m_3$     |
|        | $(6, 2, 2)$                  | $(3, 2, \frac{1}{6})$        | $m_{sq}$  |
|        |                              | $(3, 2, -\frac{5}{6})$       |           |
|        |                              | $(\bar{3}, 2, +\frac{5}{6})$ |           |
|        |                              | $(\bar{3}, 2, -\frac{1}{6})$ |           |
|        | $(15, 1, 1)$                 | $(1, 1, 0)$                  |           |
|        |                              | $(3, 1, +\frac{2}{3})$       |           |
|        | $(\bar{3}, 1, -\frac{2}{3})$ | $m_{sup}$                    |           |
|        | $(8, 1, 0)$                  | $m_8$                        |           |

$10_H$  contains 2 doublets (one of which is SM one). Also, it contains 2 coloured triplets mediating p-decay  $\rightarrow$  heavy or decoupled *Dvali '92*

One-loop RG leads to

$$\frac{M_{GUT}}{M_Z} \simeq \exp \left\{ \frac{\pi}{10} (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})_{M_Z} \right\} \left[ \left( \frac{M_Z}{M_I} \right)^{22} \left( \frac{M_Z^2 m_{sel} m_{sup}}{m_3 m_8 m_{sq}^2} \right) \right]^{\frac{1}{20}}$$

- p-decay  $\rightarrow M_{GUT} \gtrsim 4 \cdot 10^{15} \text{GeV}$

-  $\nu$  mass  $\rightarrow M_I \gtrsim 10^{14} \text{GeV}$  and  $\Lambda \sim 10 M_{GUT}$

Typically  $M_I \sim 10^{12} \text{GeV}$   $\rightarrow$  scalars must be light

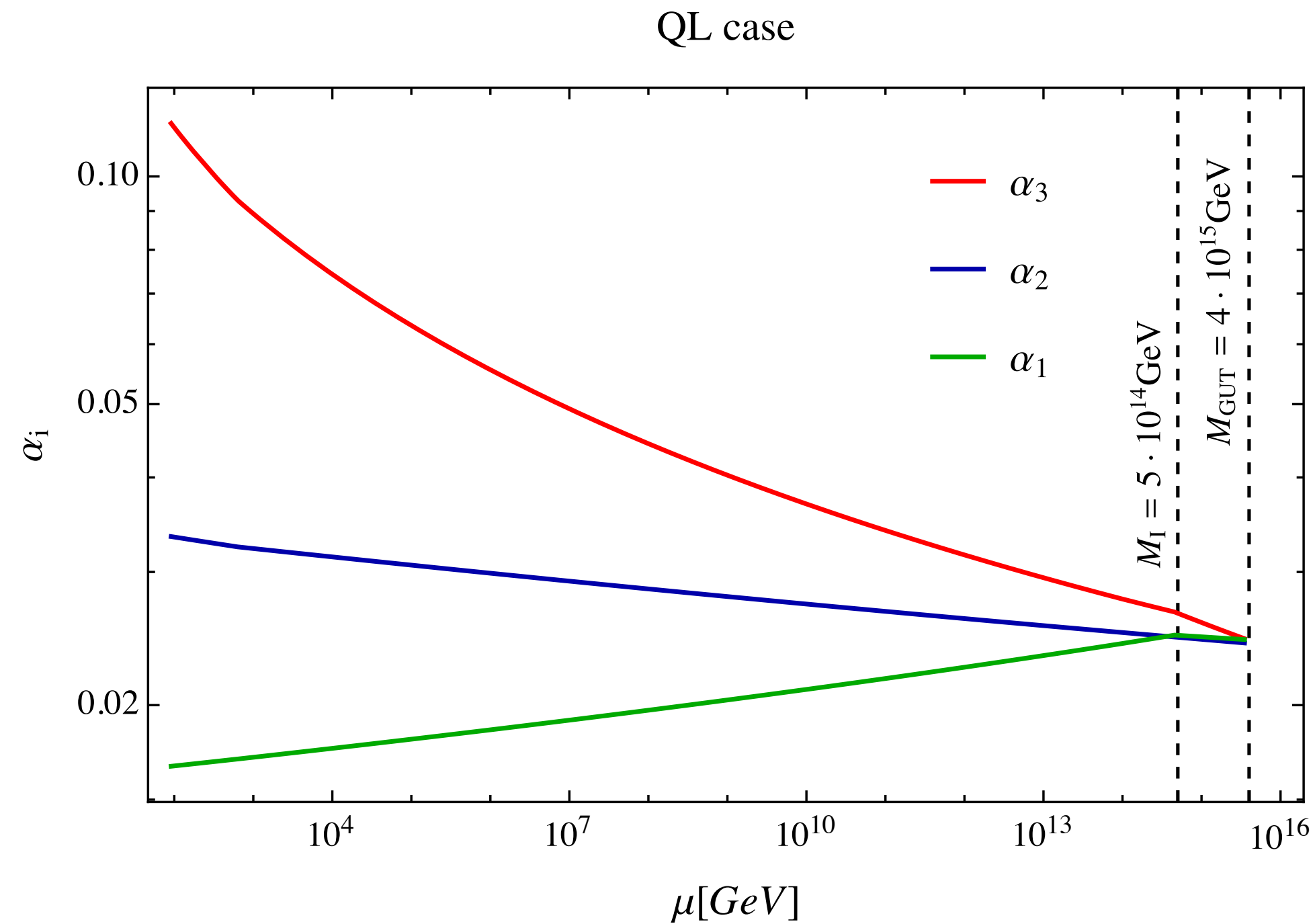
Cutoff not so far from  $M_{GUT}$

$$\frac{1}{\Lambda^2} FF 45_H^2$$

$$\delta\alpha_1^{LR} = \delta\alpha_1^{QL} = \left( \frac{M_{GUT}}{\Lambda} \right)^2 ; \quad \delta\alpha_3^{LR} = \frac{5}{2} \left( \frac{M_{GUT}}{\Lambda} \right)^2 ; \quad \delta\alpha_2^{QL} = \frac{5}{3} \left( \frac{M_{GUT}}{\Lambda} \right)^2$$

## 2-loops RG analysis

We varied all particle masses and took into account the effects of higher dimensional operators



Example of realization:

$$m_3 = m_8 = m_{sq} = \text{TeV}$$

### Spectrum

| Particle                     | Mass range                             |
|------------------------------|--|
| scalar quark doublet         | $m_{sq} \lesssim 10\text{TeV}$         |
| weak triplet                 | $m_3 \lesssim 10\text{TeV}$            |
| color octet                  | $m_8 \lesssim 10\text{TeV}$            |
| scalar lepton doublet        | $10^3 \text{ GeV} - M_I$               |
| second Higgs doublet         | $10^3 \text{ GeV} - M_{\text{GUT}}$    |
| scalar down quark            | $10^{12} \text{ GeV} - M_{\text{GUT}}$ |
| color triplet Higgs partners | $10^{12} \text{ GeV} - M_{\text{GUT}}$ |
| scalar up quark              | $10^{14} \text{ GeV} - M_{\text{GUT}}$ |
| scalar electron              | $10^{14} \text{ GeV} - M_{\text{GUT}}$ |

- $m_3, m_{sq}, m_8$  always lie below 10TeV to ensure unification, p-lifetime and neutrino mass
- $M_{\text{GUT}} < 10^{16} \text{ GeV}$  always implying  $\tau_p < 10^{35} \text{ yrs}$

# The problem of scale in supersymmetry

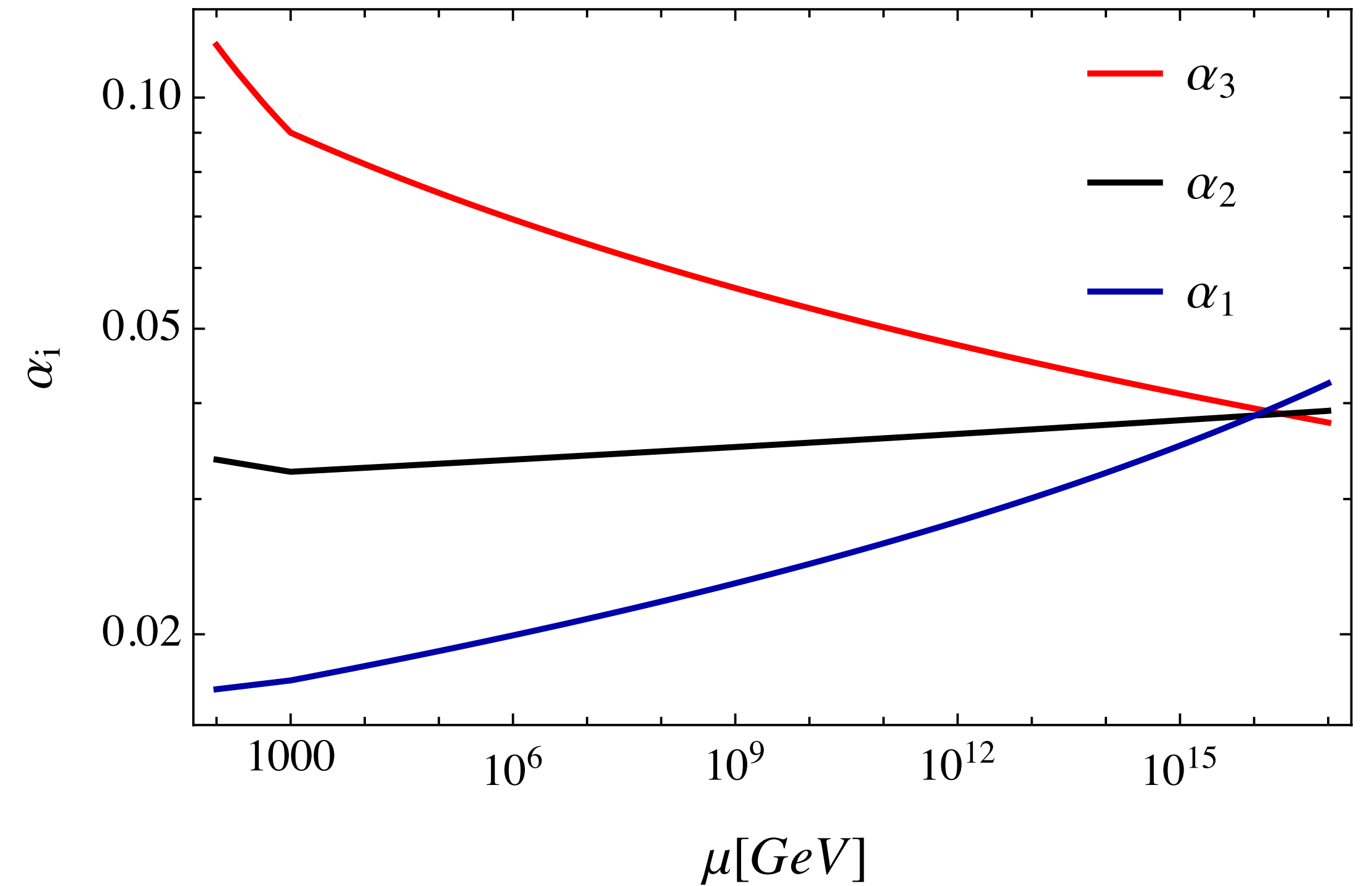
Supersymmetry: particle  $p \longrightarrow$  sparticle  $\tilde{p}$

$$m_h^2 = m_0^2 + \frac{y_t}{16\pi^2} \Lambda^2 + m_t^2 + \dots \quad \text{SM Higgs correction}$$

$$-\frac{y_t}{16\pi^2} \Lambda^2 - m_{\tilde{t}}^2 + \dots \quad \text{MSSM addition}$$

$$m_{\tilde{p}} \simeq TeV$$

$$\Lambda^{\text{MSSM}} \sim TeV, \quad M_{\text{GUT}} \simeq 10^{16} GeV$$



# The problem of scale in supersymmetry

*Senjanović, MZ '23*

$$\Lambda_s \sim m_8 \sim 10^{11} \text{ GeV}, \quad m_3 \sim 10^9 \text{ GeV}, \quad M_G \simeq 10^{16} \text{ GeV}$$

$$\Lambda = \Lambda^{\text{MSSM}} \left( \frac{M_{\text{GUT}}^2}{m_3 m_8} \right)^{3/4}.$$

For more, tune in to G. Senjanović  
closing talk on Friday

