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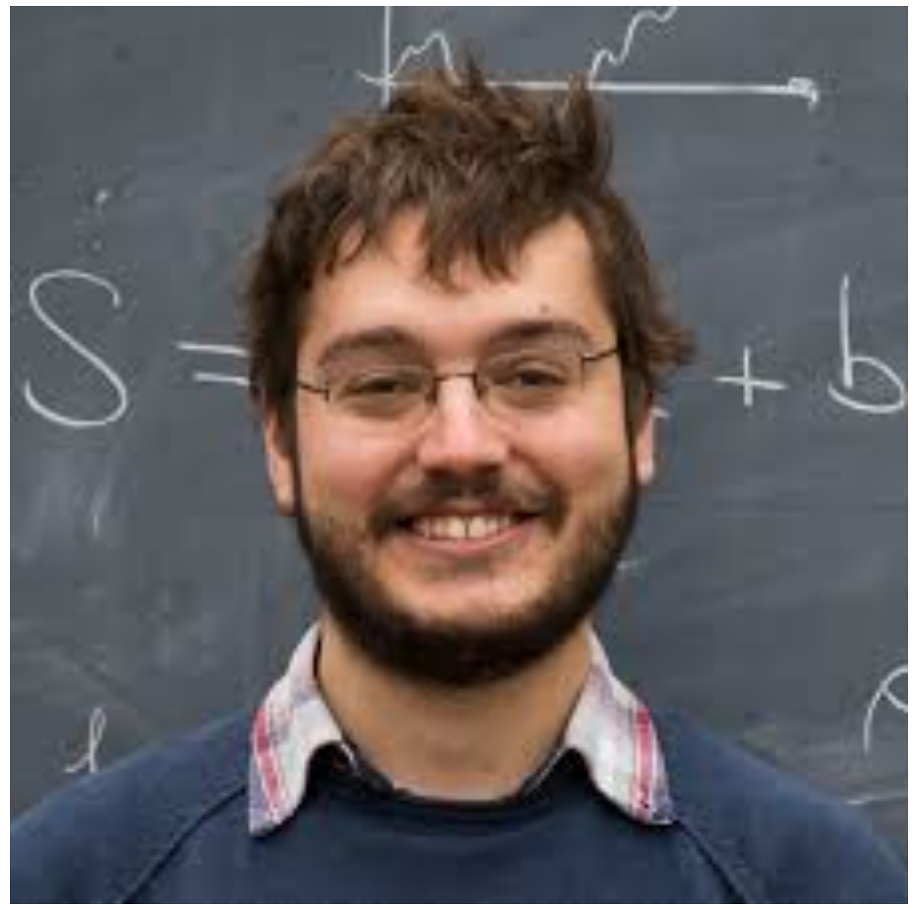
An entanglement asymmetry study of black hole radiation

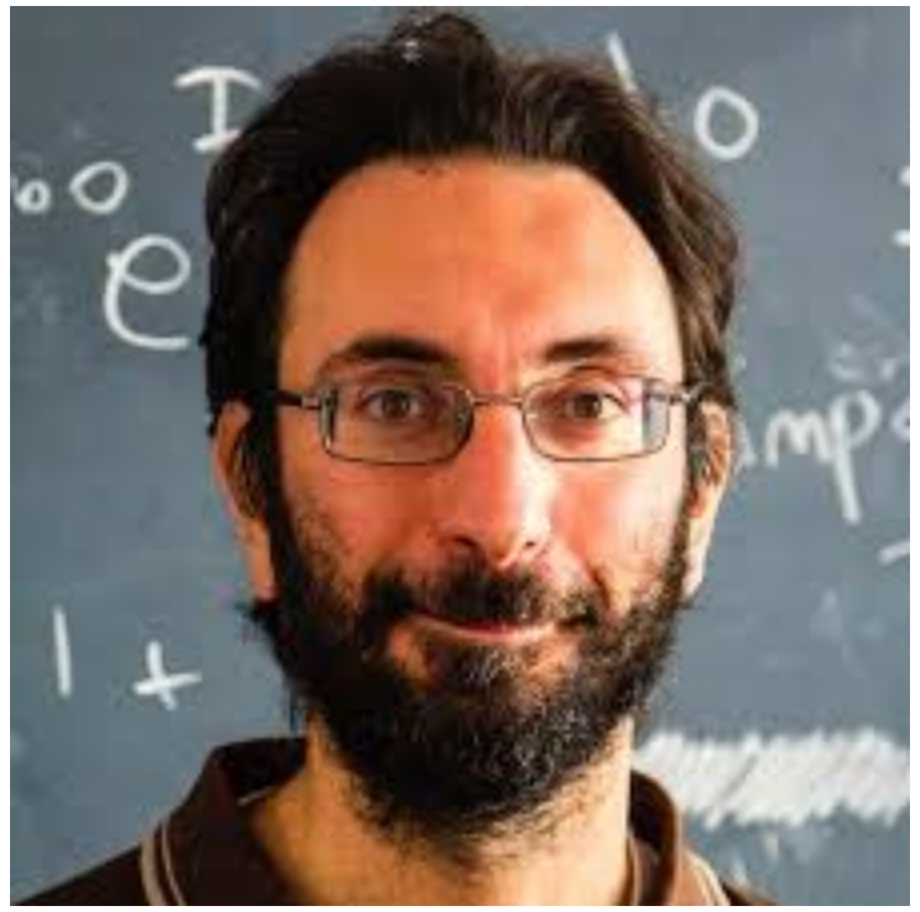
Sara Murciano

CERN String Theory Seminar, 21 May 2024



Filiberto Ares

Lorenzo Piroli



Pasquale Calabrese

arXiv:2311.12683

Entanglement: from an obstruction to a resource

'30s

spooky action
at distance

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‘30s

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Bell inequalities

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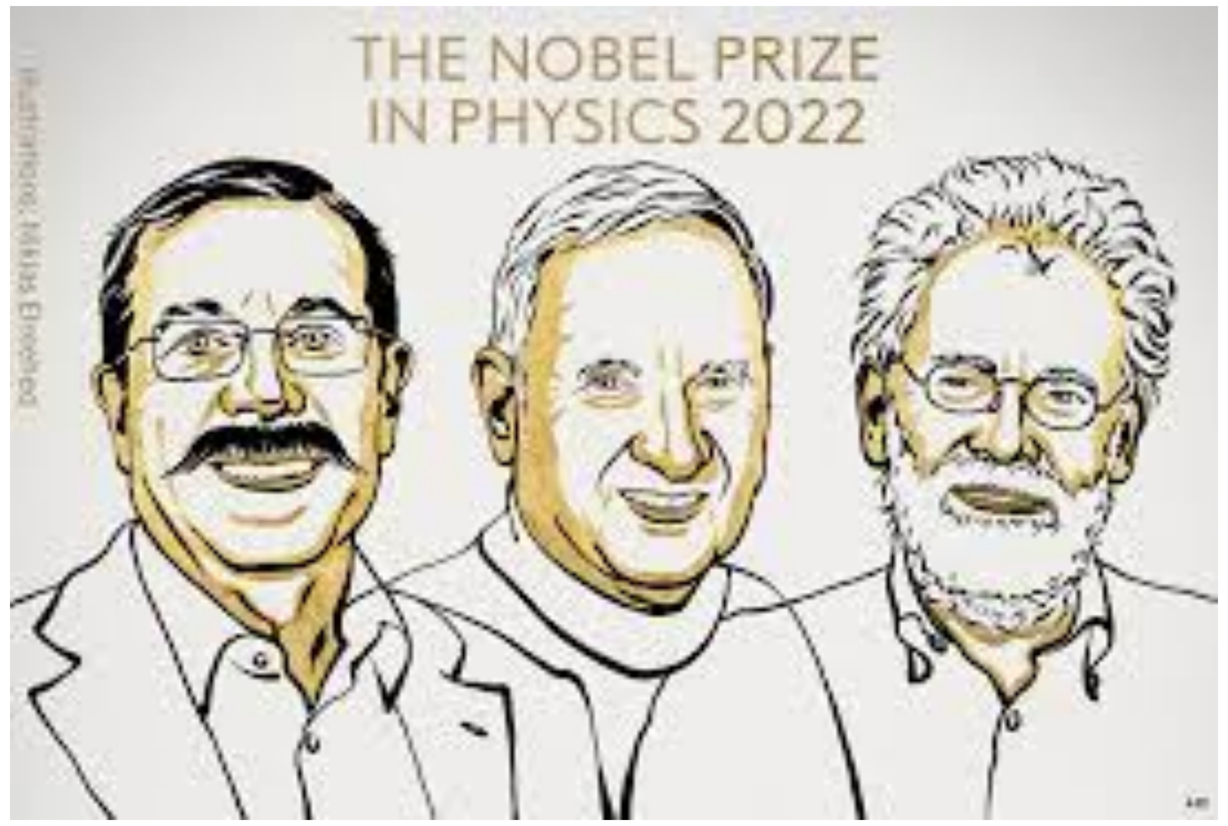

Entanglement: from an obstruction to a resource

‘30s ‘60s today

Black hole information paradox

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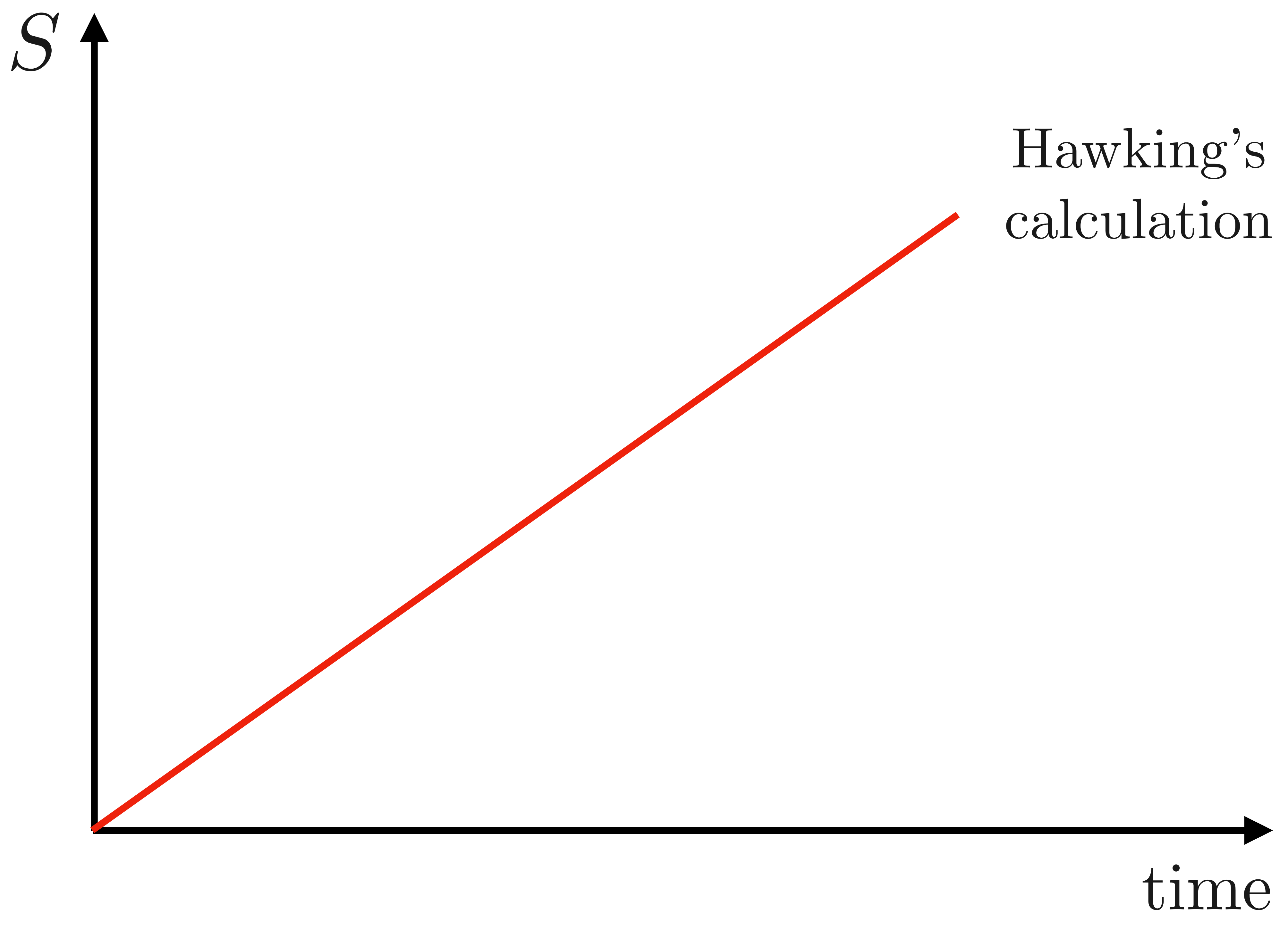
Black hole information paradox



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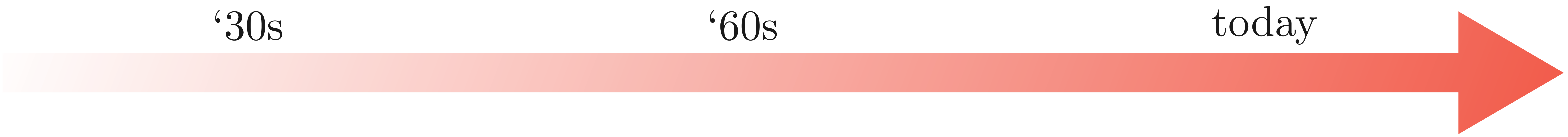

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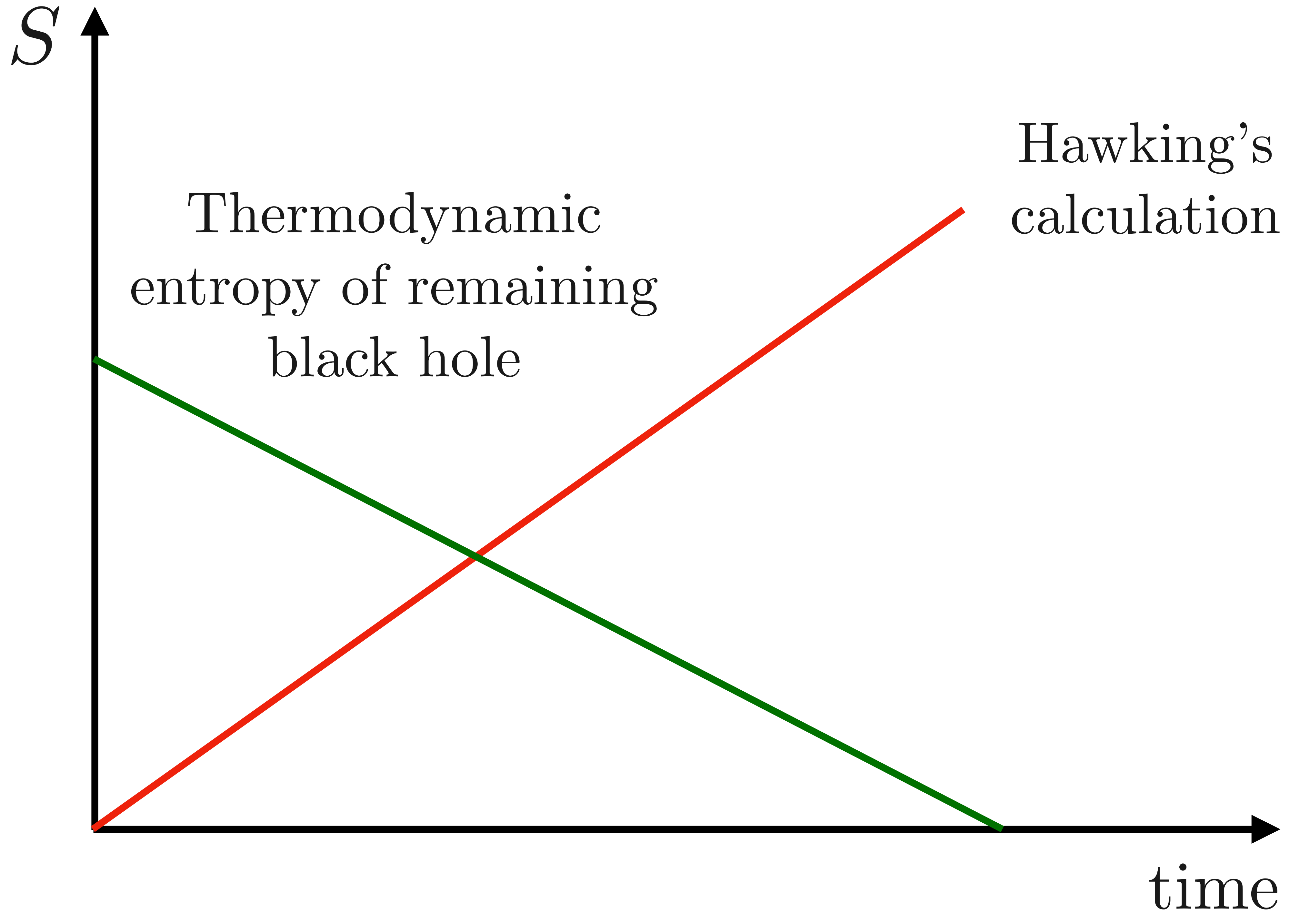

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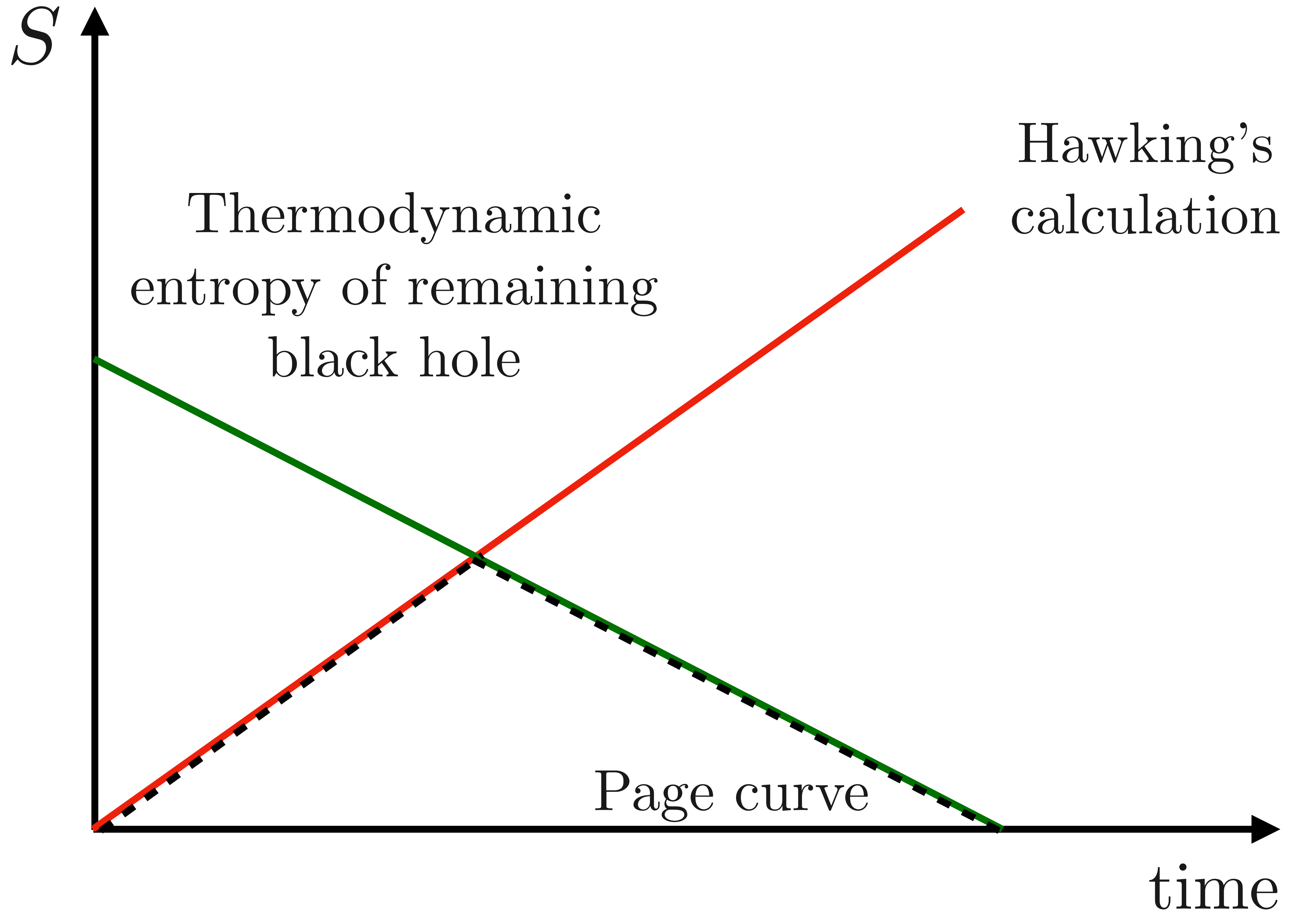

The black hole emits radiation containing entangled particles,
inducing an entanglement reduction

D. N. Page, Phys. Rev. Lett. 71, 3743 (1993)

What are the implications of unitarity to symmetries in the evaporating black holes?

No global symmetries in evaporating black holes.

C. Misner, J. Wheeler, Ann. Phys. 2, 525 (1957).
T. Banks, L. Dixon, Nucl. Phys. B 307, 93 (1988).
R. Kallos, L. Susskind, Phys. Rev. D 52, 912 (1995).
T. Banks, N. Seiberg, Phys. Rev. D 83, 084019 (2011).

Long history

D. Harlow and H. Ooguri, Phys. Rev. Lett. 122, 191601 (2019). **AdS-CFT**

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Goal of this talk: How a broken global $U(1)$ symmetry evolves during the black hole evaporation, modelled by random pure states.

Outline:

- Entanglement entropy and the computation by Page
- How to quantify the symmetry breaking in a subsystem: technical details and physical interpretation
- Conclusions & outlook

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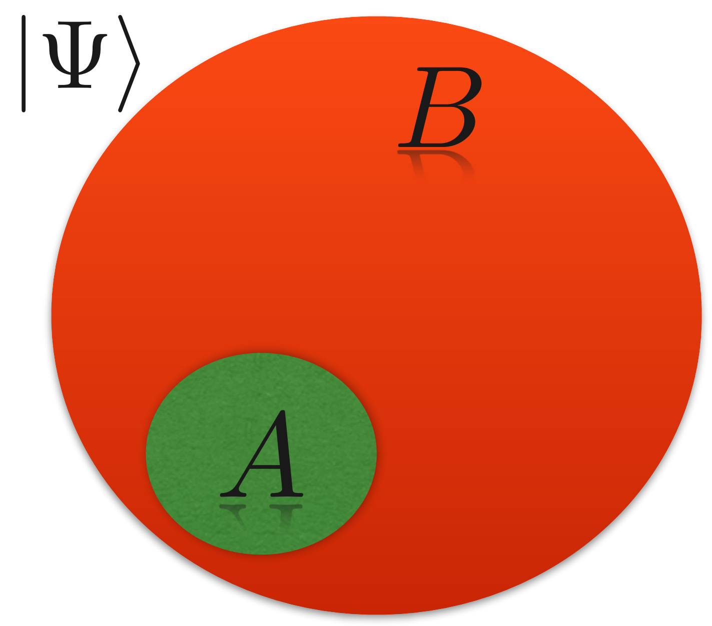
How to quantify entanglement

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B, \qquad \rho = |\Psi\rangle\langle\Psi| \qquad \rho_A = \mathrm{Tr}_B \rho$$



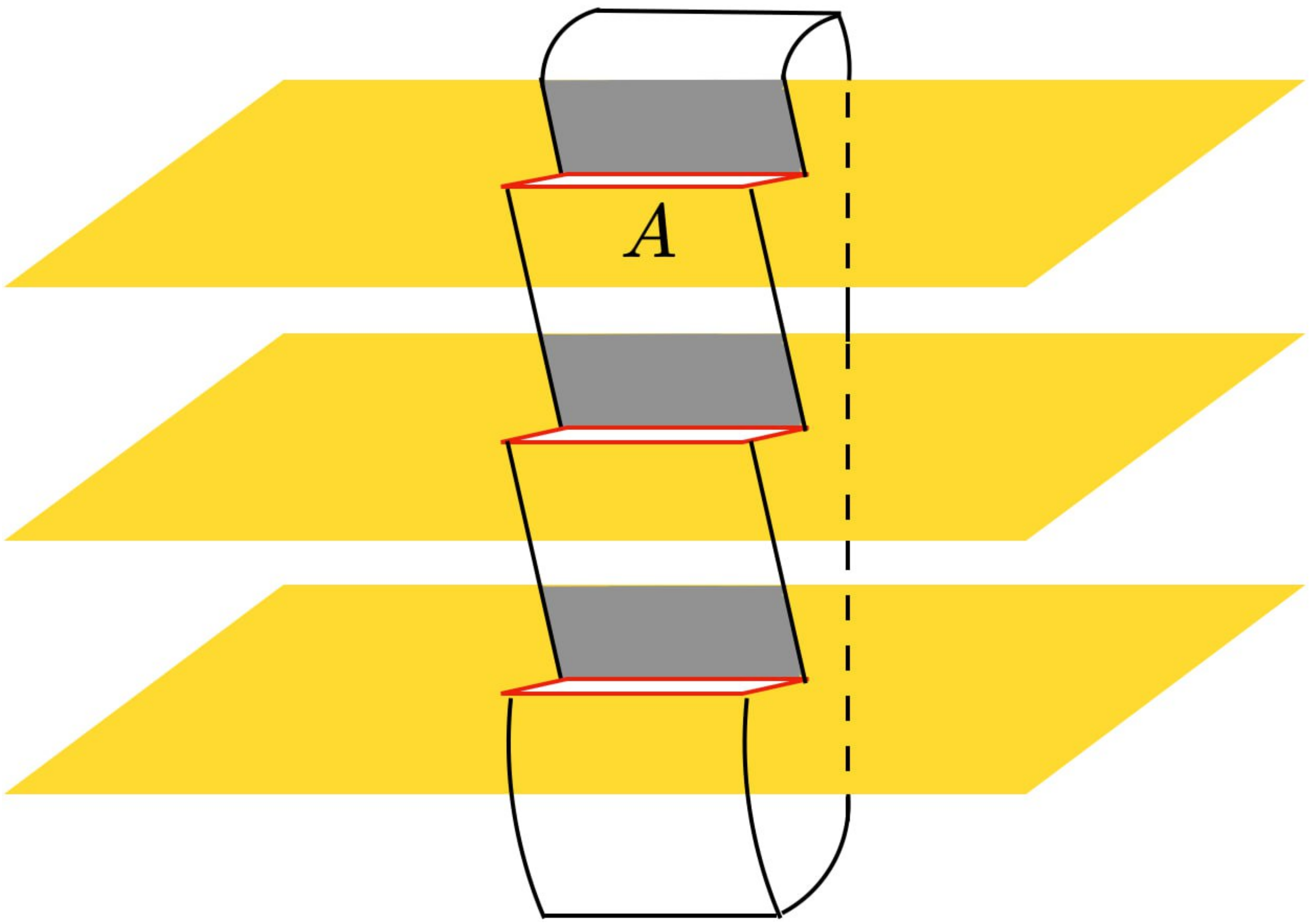
A measure of the entanglement between A and B is the von Neumann entropy

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$$

A replica approach

Rényi entropies: $$S_A^{(n)} = \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n$$

Analytic continuation in n and limit $n \to 1$ gives S_A



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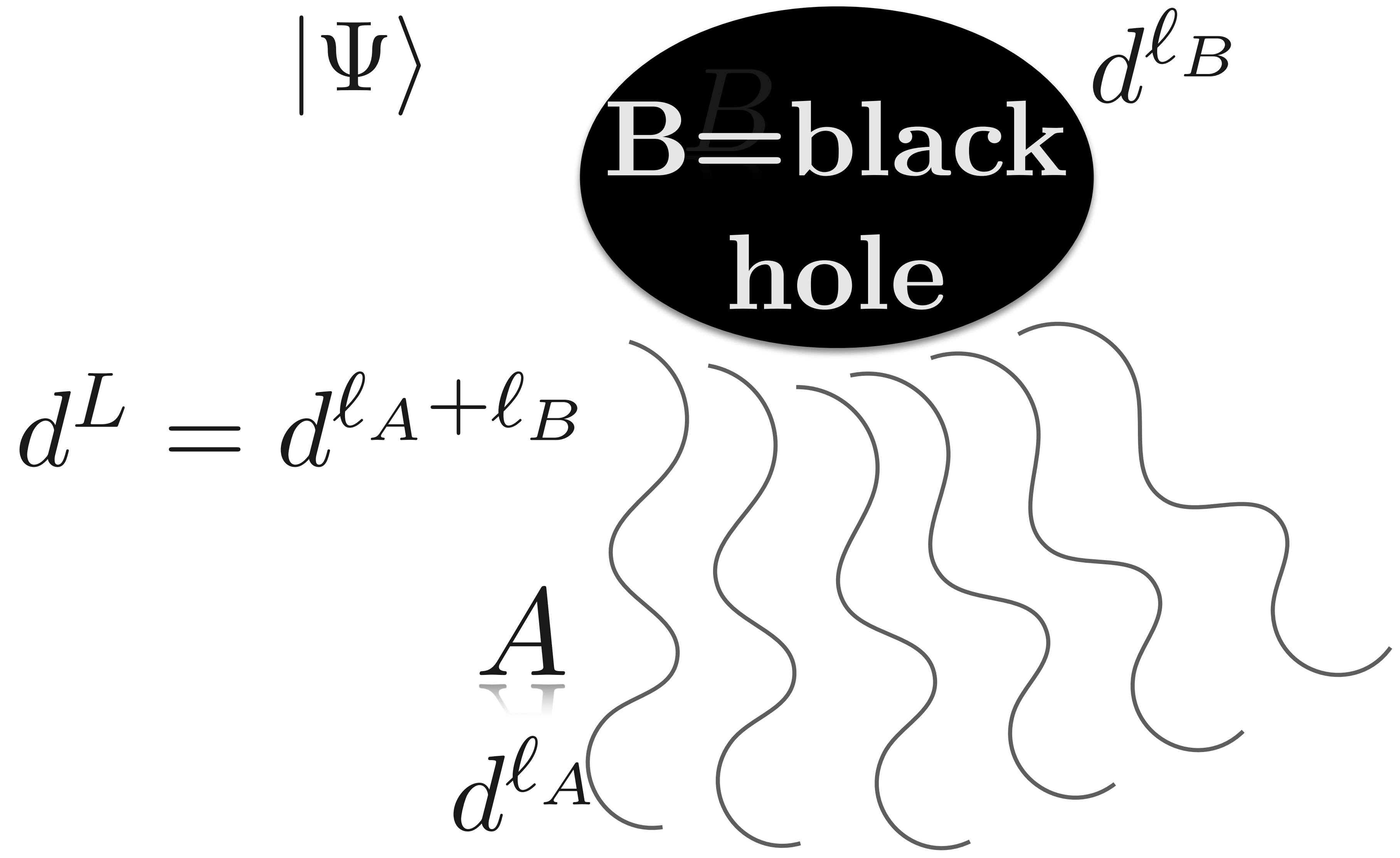
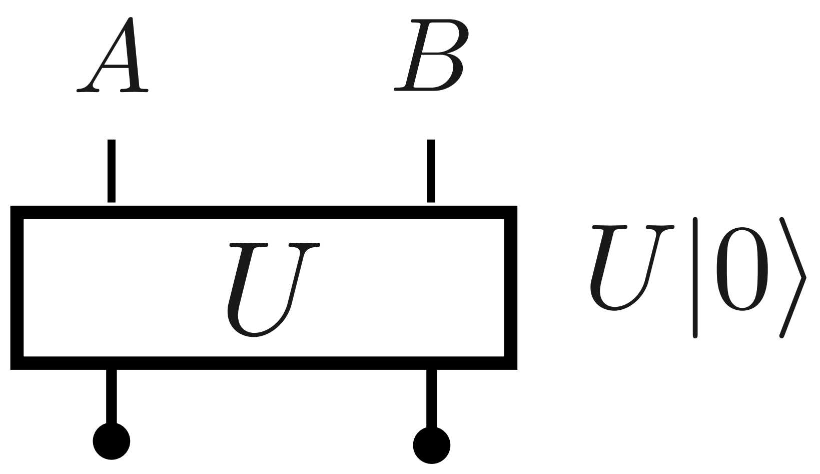


Fraction of radiated volume plays the role of time

Warm-up: The Page curve

$\{U\,|0\rangle\}$: ensemble of Haar random states



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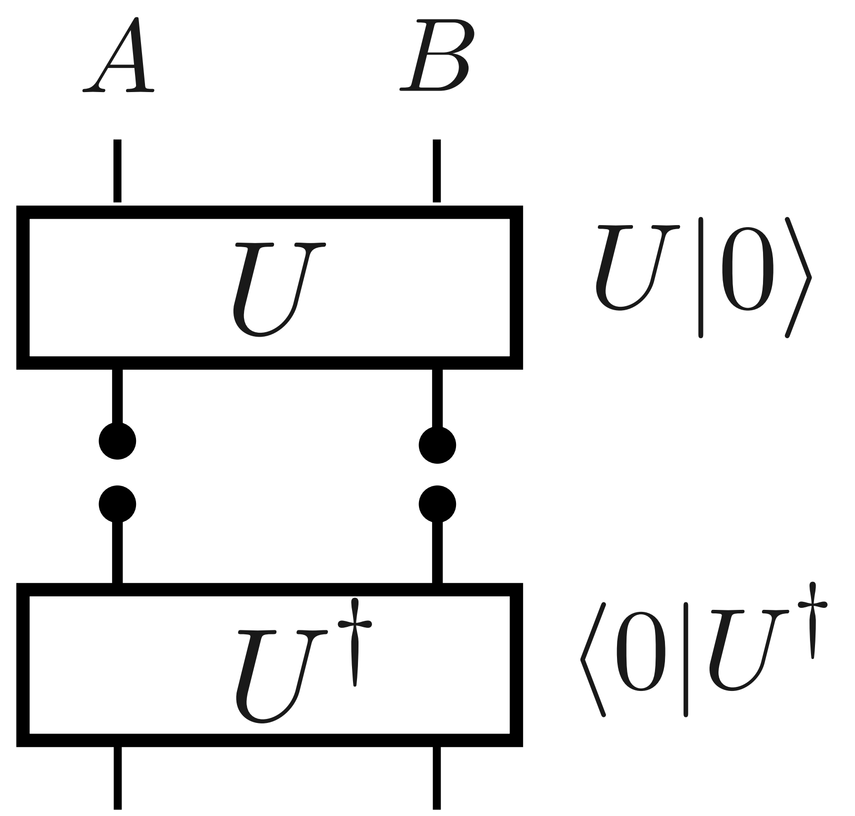
$U|0\rangle\langle 0|U^\dagger$: total density matrix



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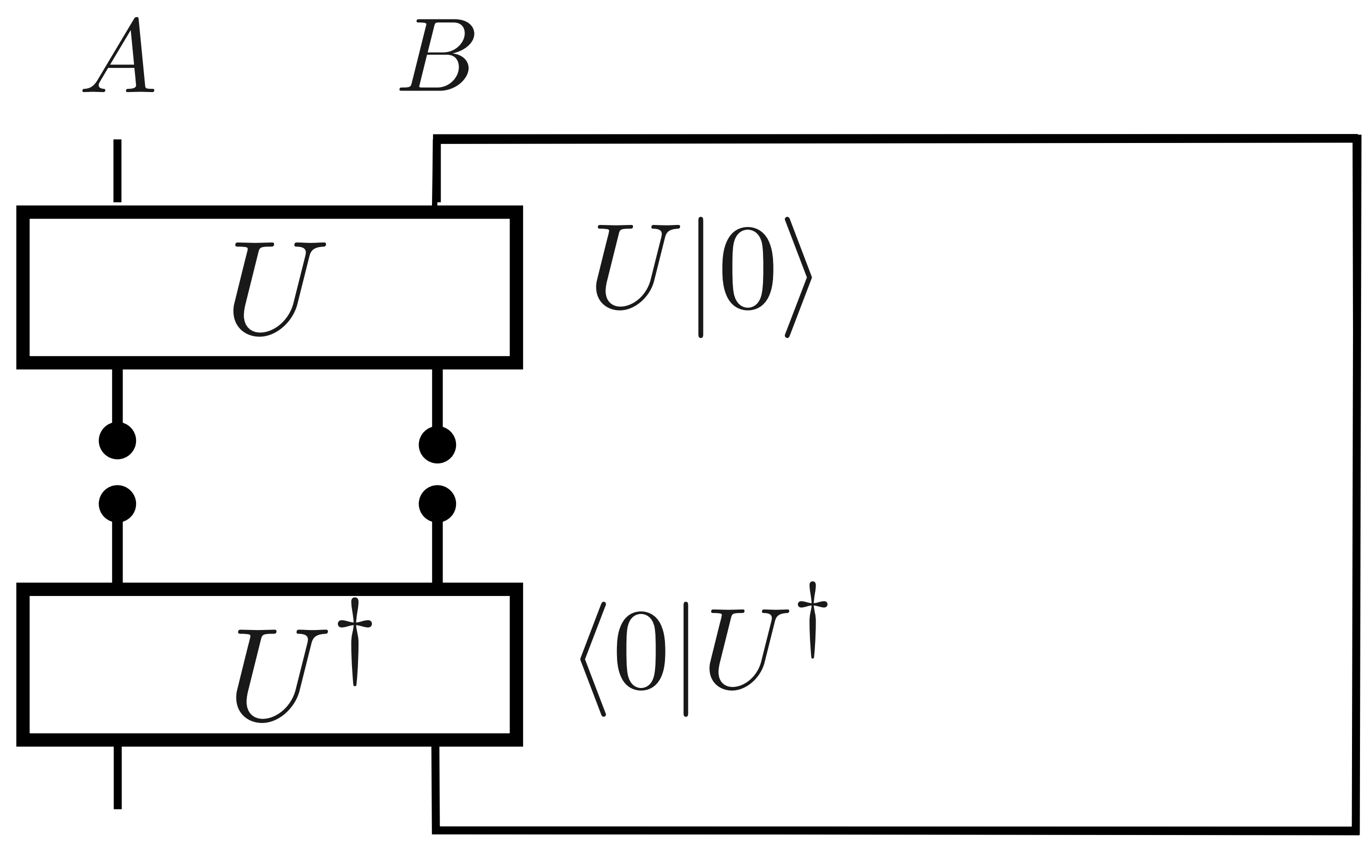
$\mathrm{Tr}_B(U|0\rangle\langle 0|U^\dagger)$



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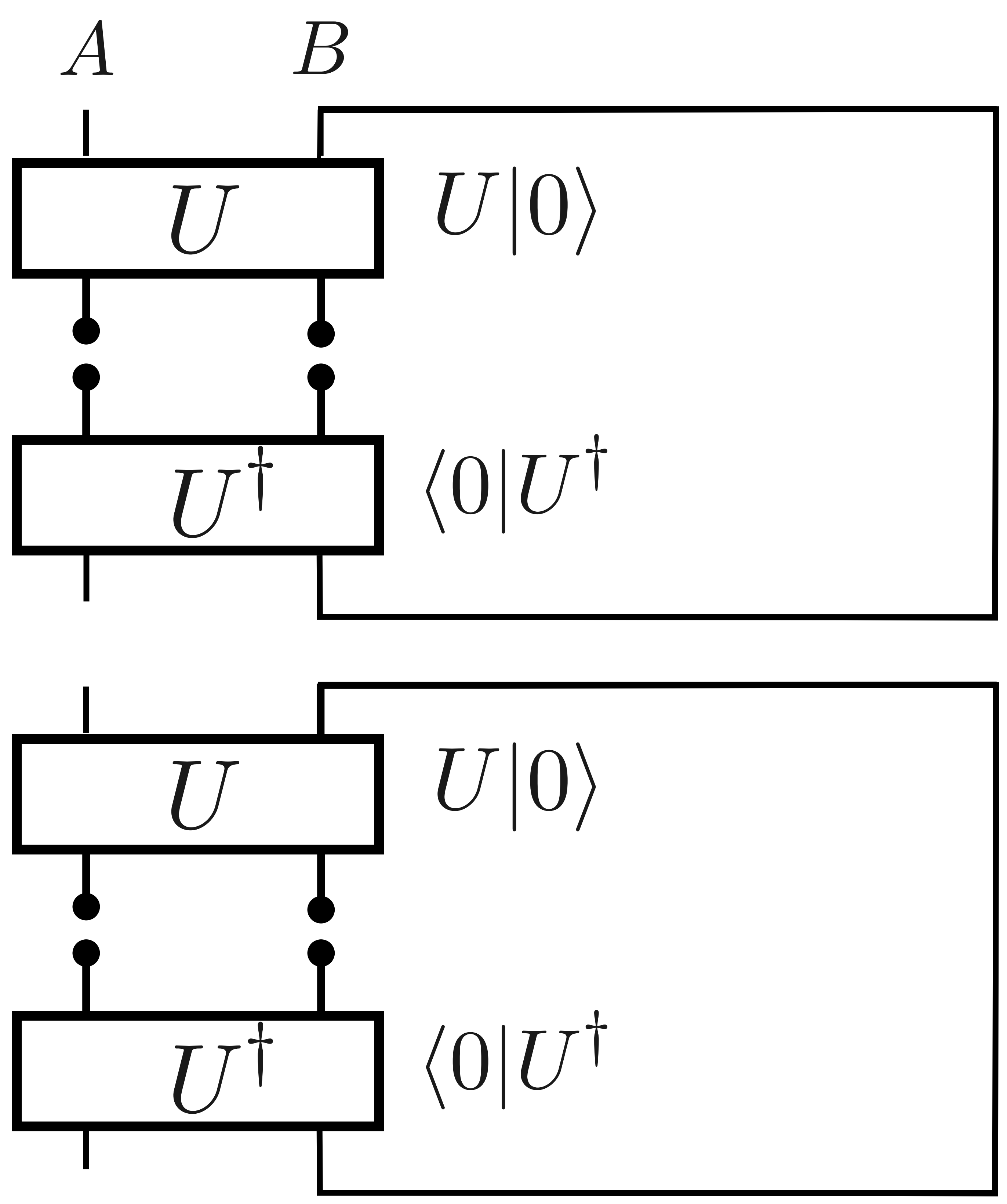
$[\mathrm{Tr}_B(U|0\rangle\langle 0|U^\dagger)]^2$



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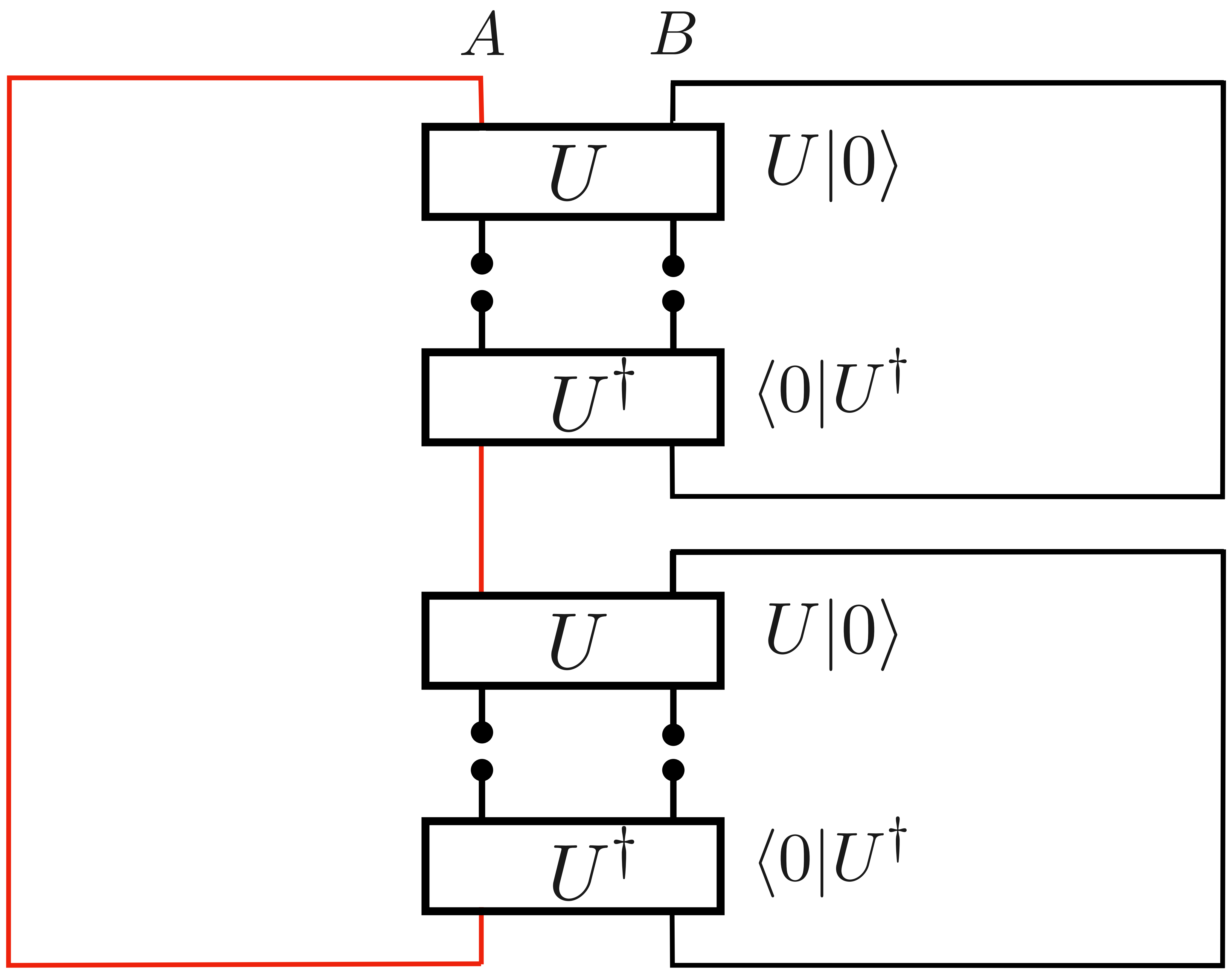

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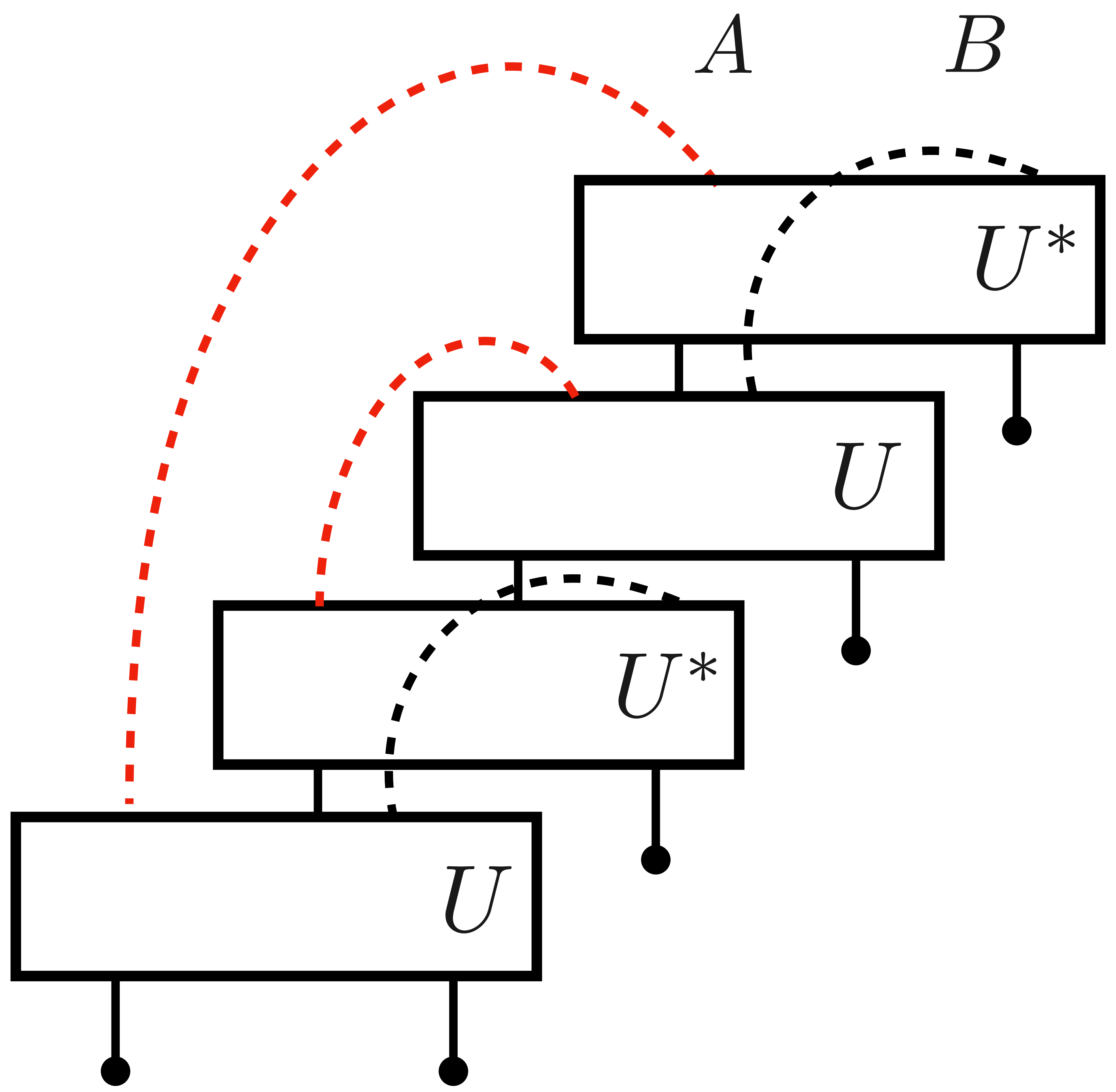

Choi-Jamiolkowski mapping:
$\rho_A \otimes \rho_A \in \mathcal{H}_A \otimes \mathcal{H}_{\mathcal{A}} \to |\rho_A \otimes \rho_A\rangle \in \mathcal{H}_A^{\otimes 4}$

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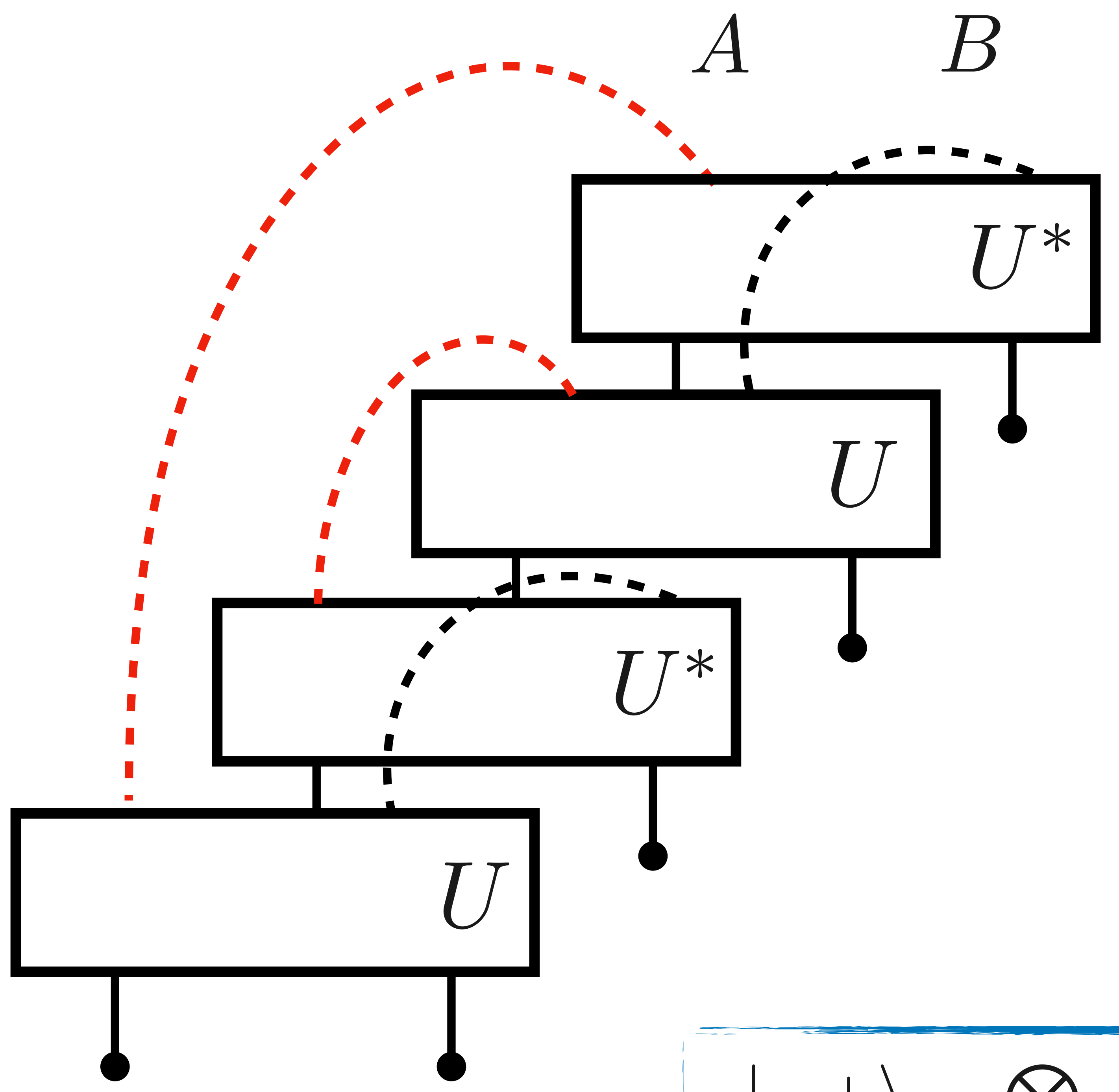

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$$|-+\rangle = \bigotimes_{k\in A} |-\rangle_k \bigotimes_{k\in B} |+\rangle_k$$

$$|+\rangle_k = \sum_{a_1,a_2=0}^{d-1} (|a_1\rangle_k \otimes |a_1\rangle_k)(|a_2\rangle_k \otimes |a_2\rangle_k)$$

$$|-\rangle_k = \sum_{a_1,a_2=0}^{d-1} (|a_1\rangle_k \otimes |a_2\rangle_k)(|a_2\rangle_k \otimes |a_1\rangle_k)$$

Goal: $\mathbb{E}[\mathrm{Tr}\rho_A^2] = \langle -+| \, \mathbb{E}[U^{\otimes 2} \otimes (U^*)^{\otimes 2}] \, |0\rangle^{\otimes 4}$

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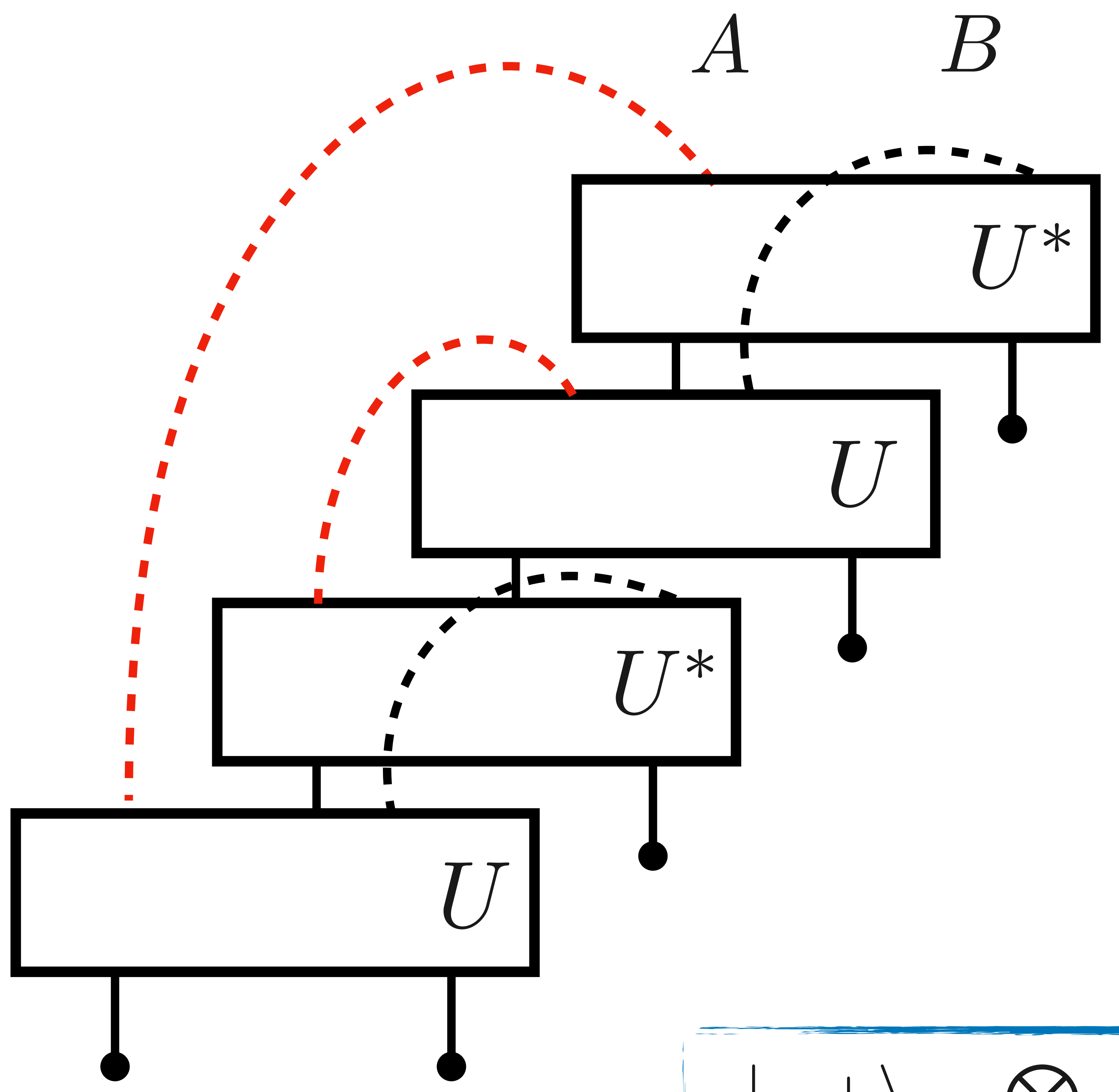

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$$\mathbb{E}[U^{\otimes 2} \otimes (U^*)^{\otimes 2}] = \frac{1}{d^{2L}-1}\left[|{+}{+}\rangle\langle{+}{+}| + |{-}{-}\rangle\langle{-}{-}| - \frac{1}{d^L}\left(|{+}{+}\rangle\langle{-}{-}| + |{-}{-}\rangle\langle{+}{+}|\right)\right]$$

Weingarten formula

D. Weingarten, J. Math. Phys. 19, 999 (1978).

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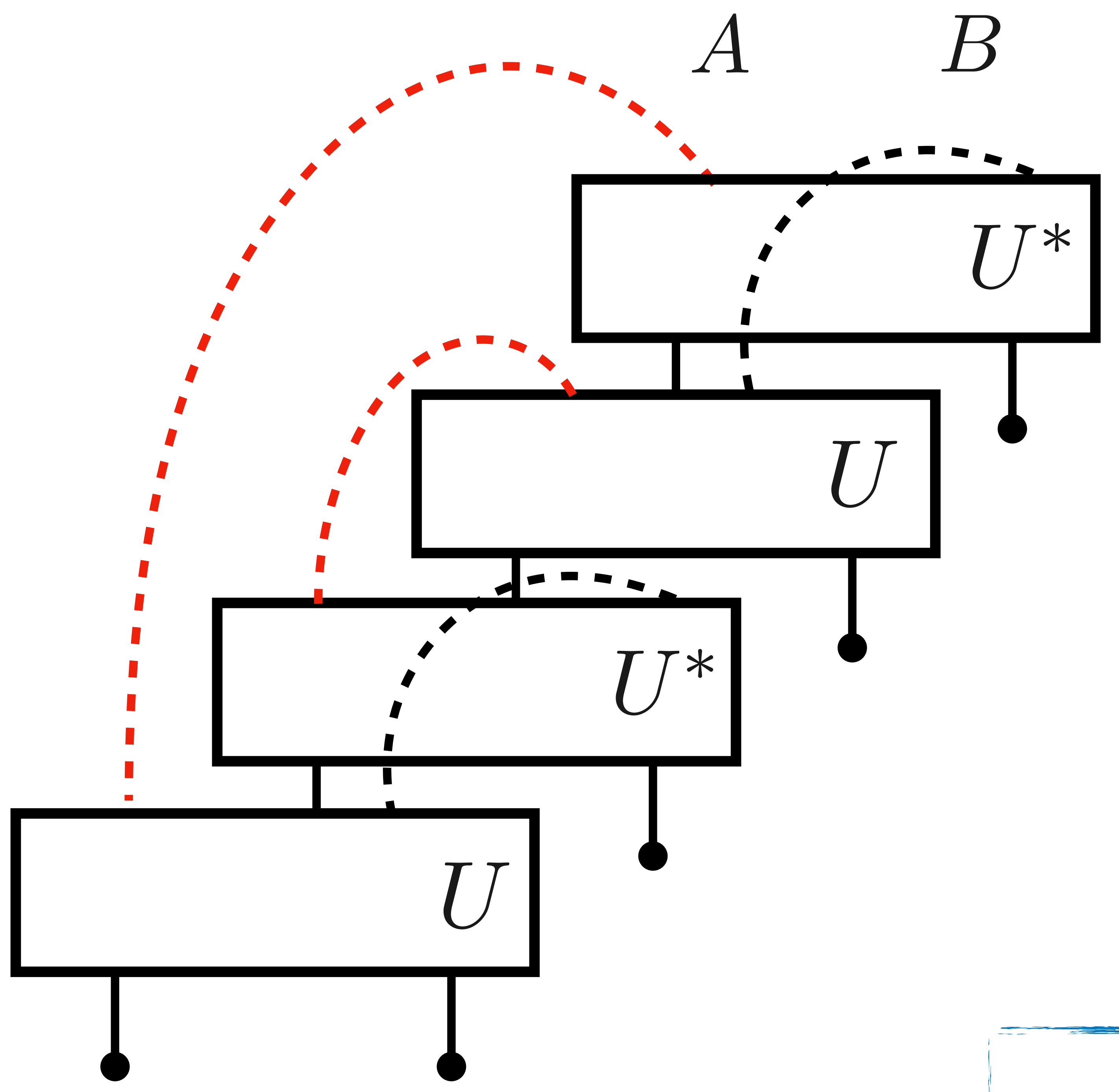

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$${}_j\langle \pm|\pm\rangle_k = d^2\delta_{jk}$$

$${}_j\langle \mp|\pm\rangle_k = d\delta_{jk}$$

$${}_j\langle \pm|0\rangle_k = \delta_{jk}$$

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$$\mathbb{E}[U^{\otimes 2}\otimes (U^*)^{\otimes 2}] = \frac{1}{d^{2L}-1}\left[|{++}\rangle\langle{++}| + |{--}\rangle\langle{--}| - \frac{1}{d^L}\left(|{++}\rangle\langle{--}| + |{--}\rangle\langle{++}|\right)\right]$$

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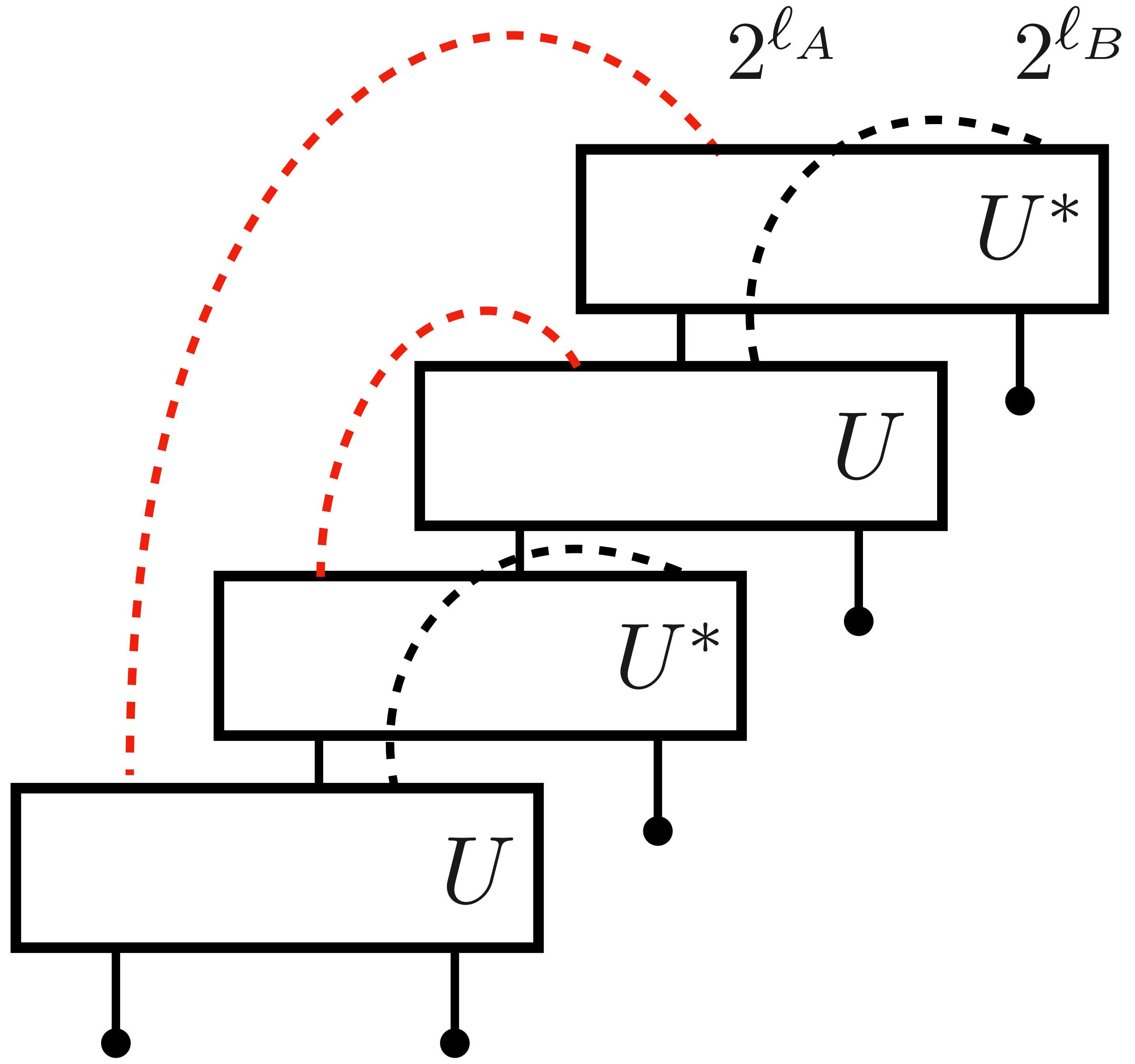

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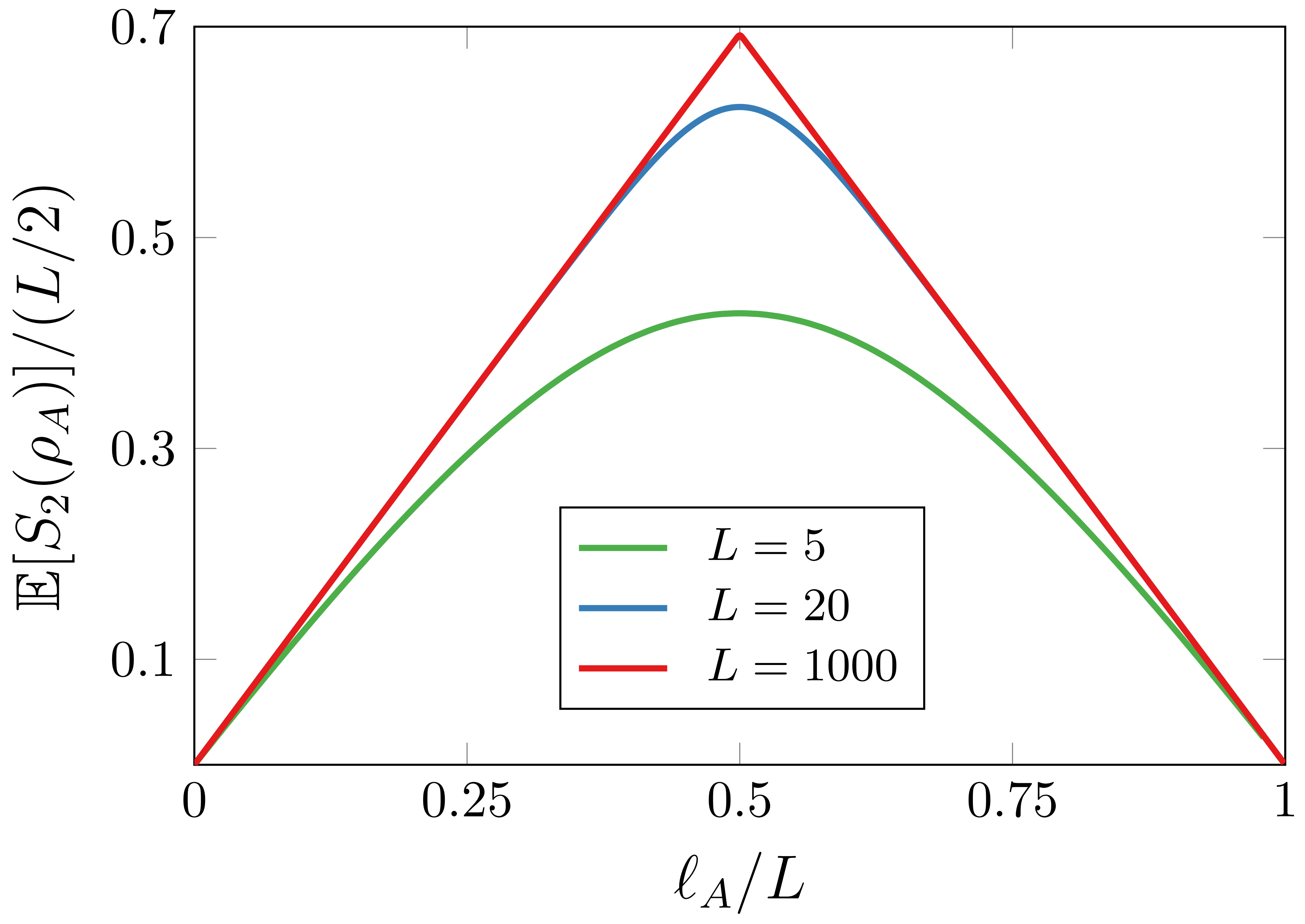
$d = 2$

$$\mathbb{E}[\mathrm{Tr}\rho_A^2] = \frac{2^{\ell_A} + 2^{\ell_B}}{2^{\ell_A+\ell_B} + 1}$$

Page curve n=2

Assumption: $\mathbb{E}[\log \mathrm{Tr}(\rho_A^2)] \simeq \log \mathbb{E}[\mathrm{Tr}(\rho_A^2)]$

$$\mathbb{E}[\Delta S_A^{(2)}] = -\mathbb{E}[\log \mathrm{Tr}\rho_A^2] \simeq -\log \frac{2^{\ell_B} + 2^{\ell_A}}{2^{\ell_A+\ell_B} + 1}$$



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- How to quantify the symmetry breaking in a subsystem: technical details and physical interpretation
- Conclusions & outlook

What happens in a symmetric state

Q: charge operator generating a $U(1)$ symmetry

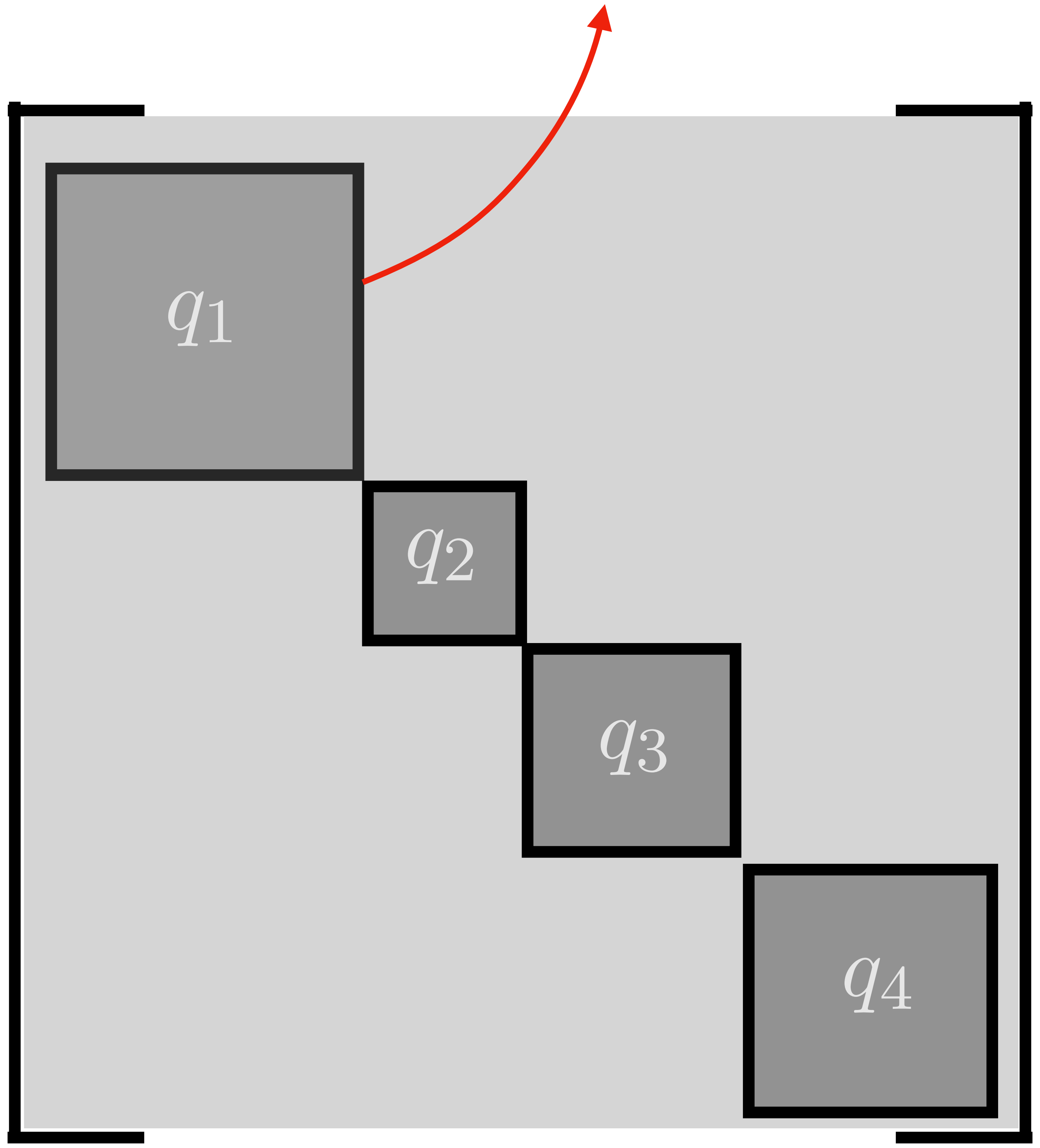
The charge is local:

$$Q = Q_A + Q_B$$

$$[\rho, Q] = 0 \xrightarrow{\mathrm{Tr}_B} [\rho_A, Q_A] = 0$$

Symmetry-resolved Renyi entropies

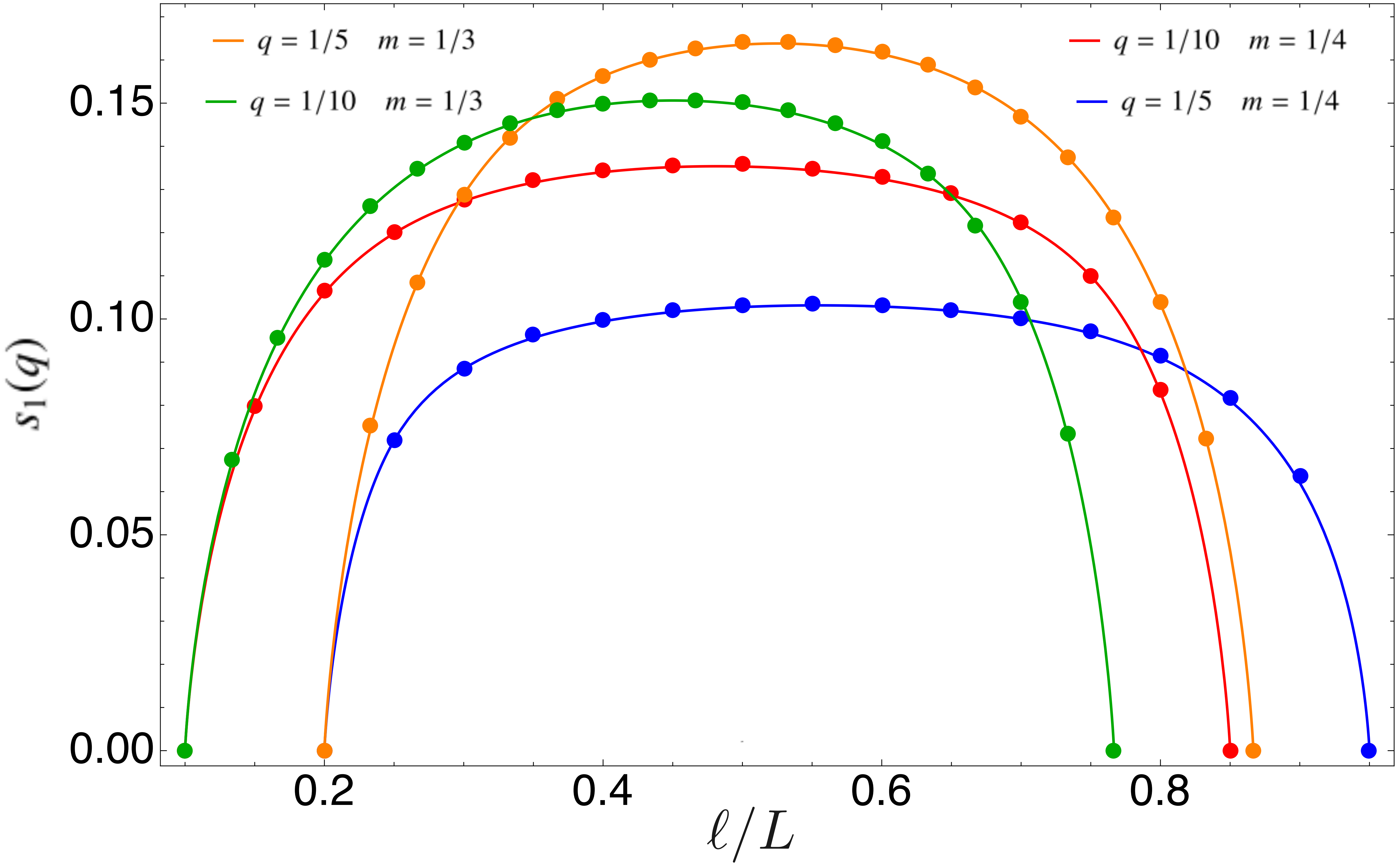
$$S_A^{(n)}(q) = \frac{1}{1-n} \log \mathrm{Tr}(\rho_A^n(q))$$

$\rho_A =$ 

M. Goldstein and E. Sela, Phys. Rev. Lett. 120, 200602 (2018)

What happens in a symmetric state

Symmetry-resolved Page curve in
Gaussian Haar random ensemble with $U(1)$ symmetry



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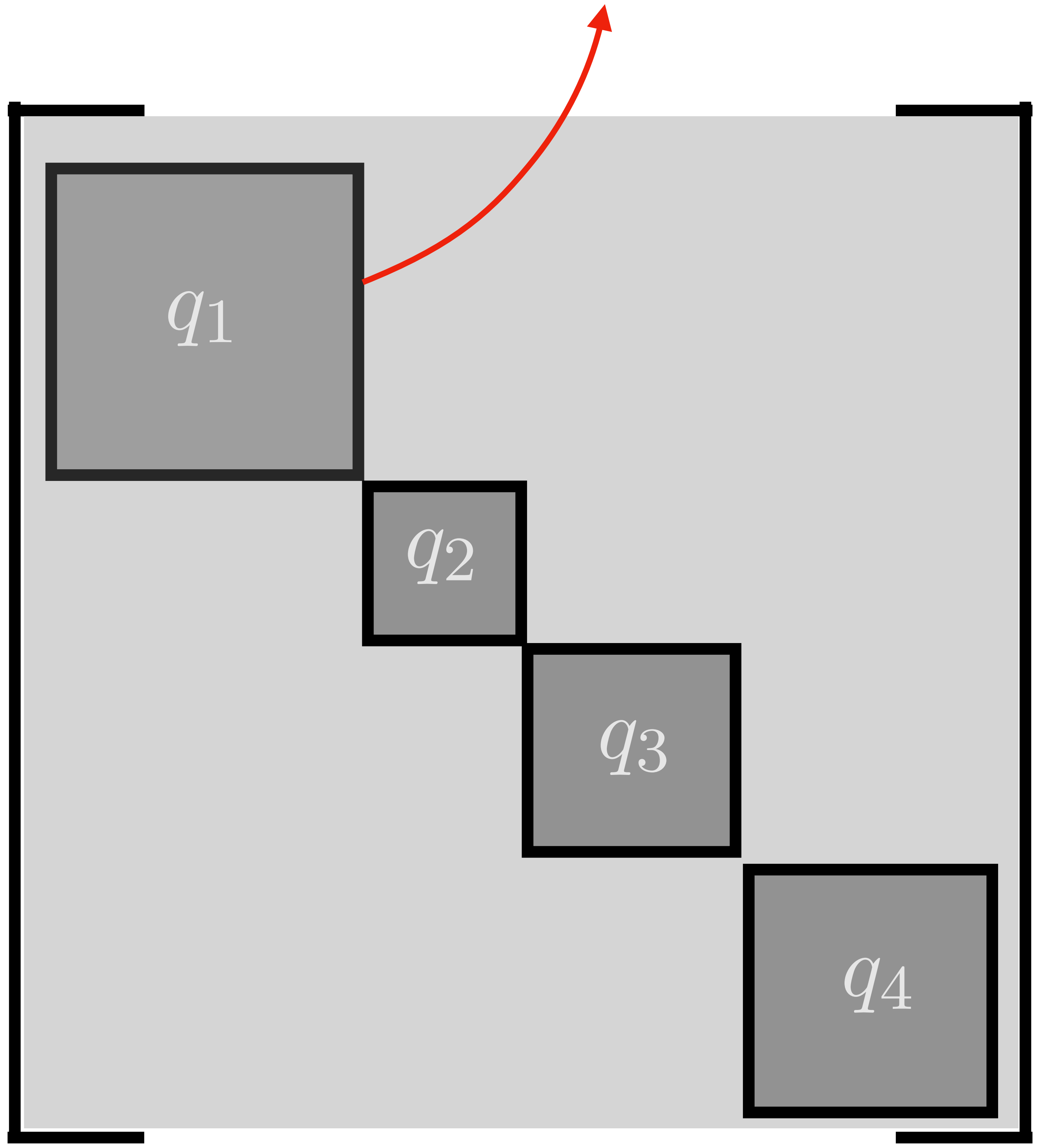
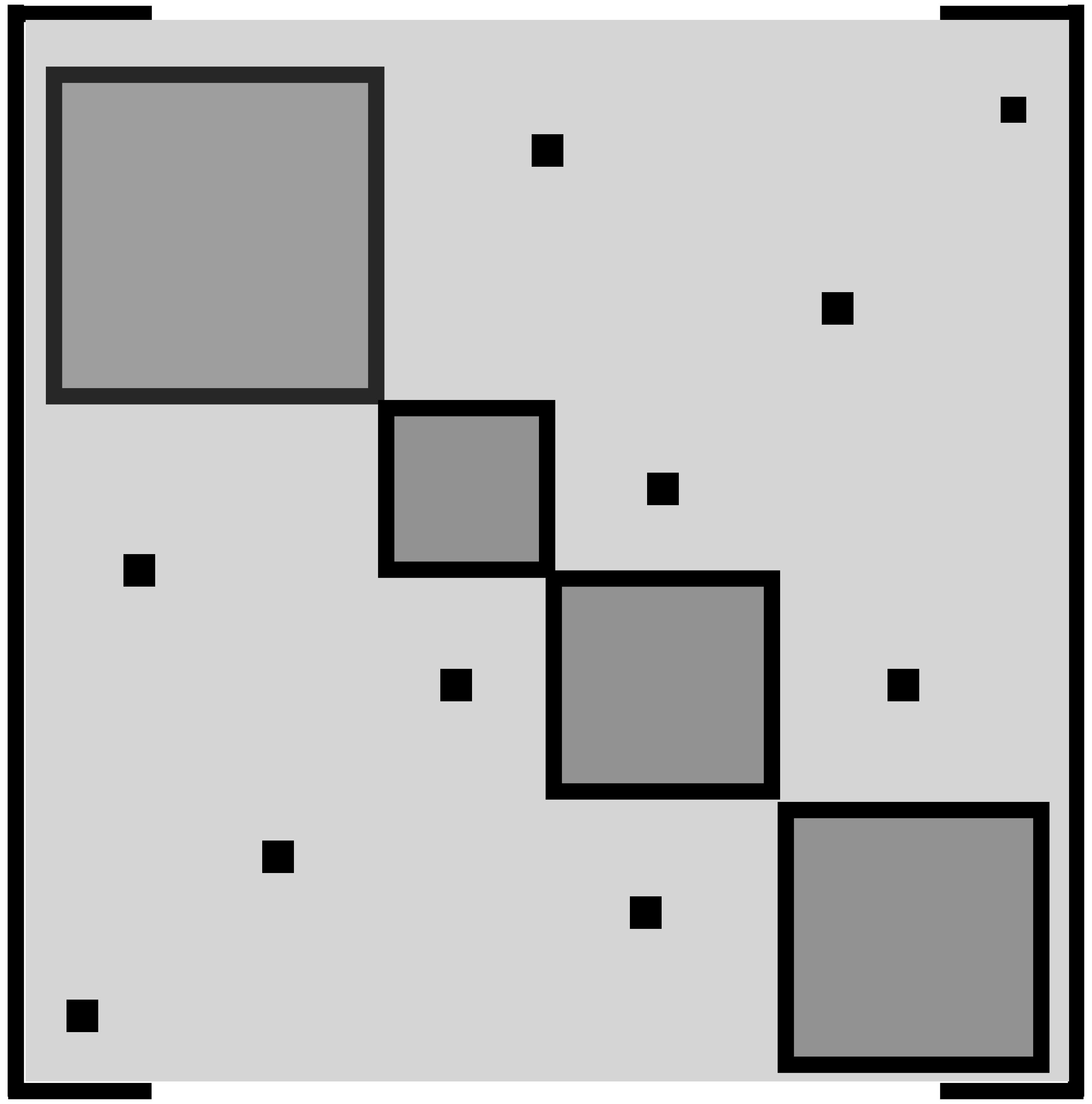


SM, P. Calabrese, L. Piroli, Phys. Rev. D 106, 046015 (2022).

Entanglement asymmetry as a probe of symmetry breaking

$$[\rho_A, Q_A] \neq 0$$

$$\rho_A =$$

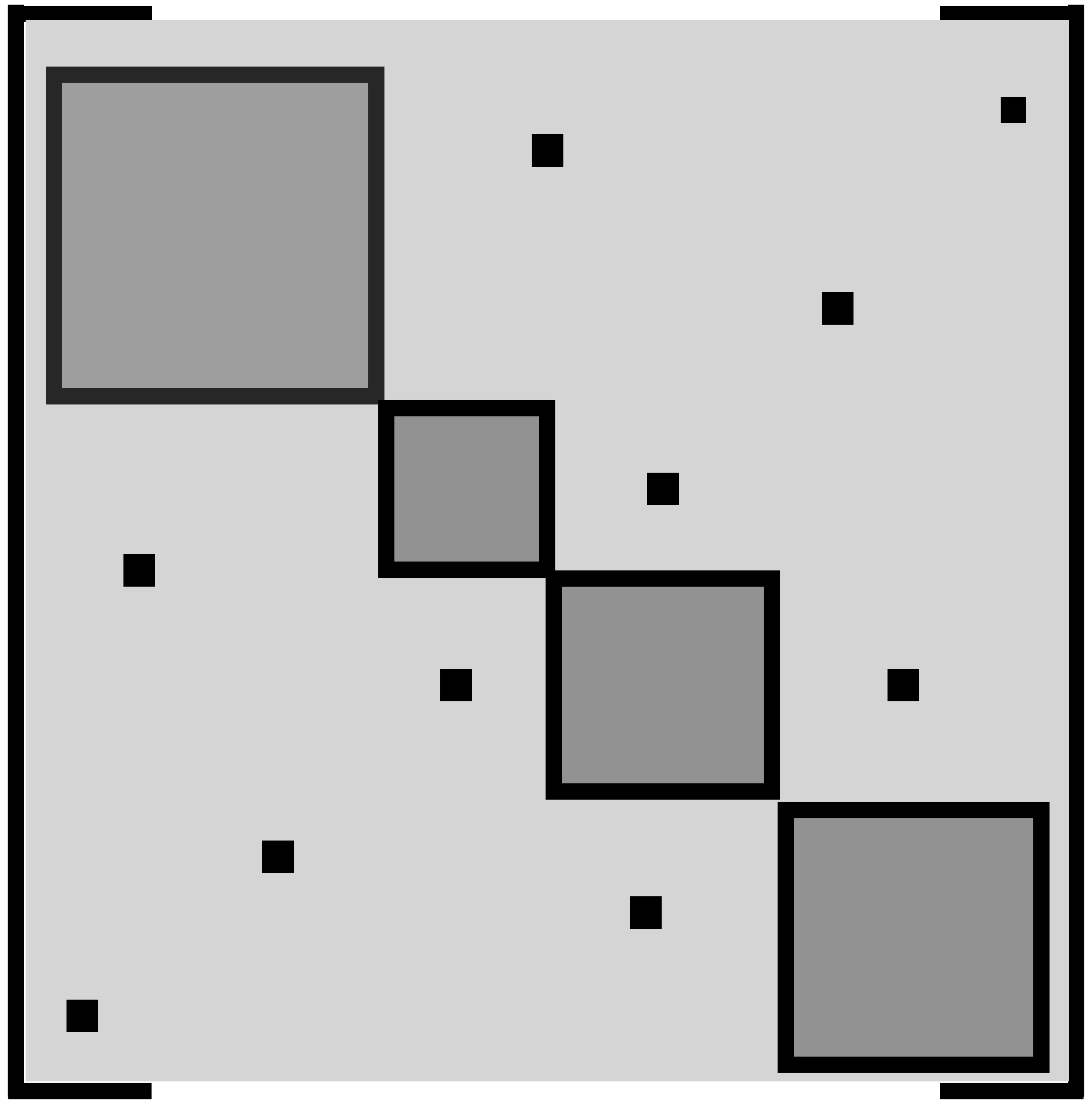


Entanglement asymmetry as a probe of symmetry breaking

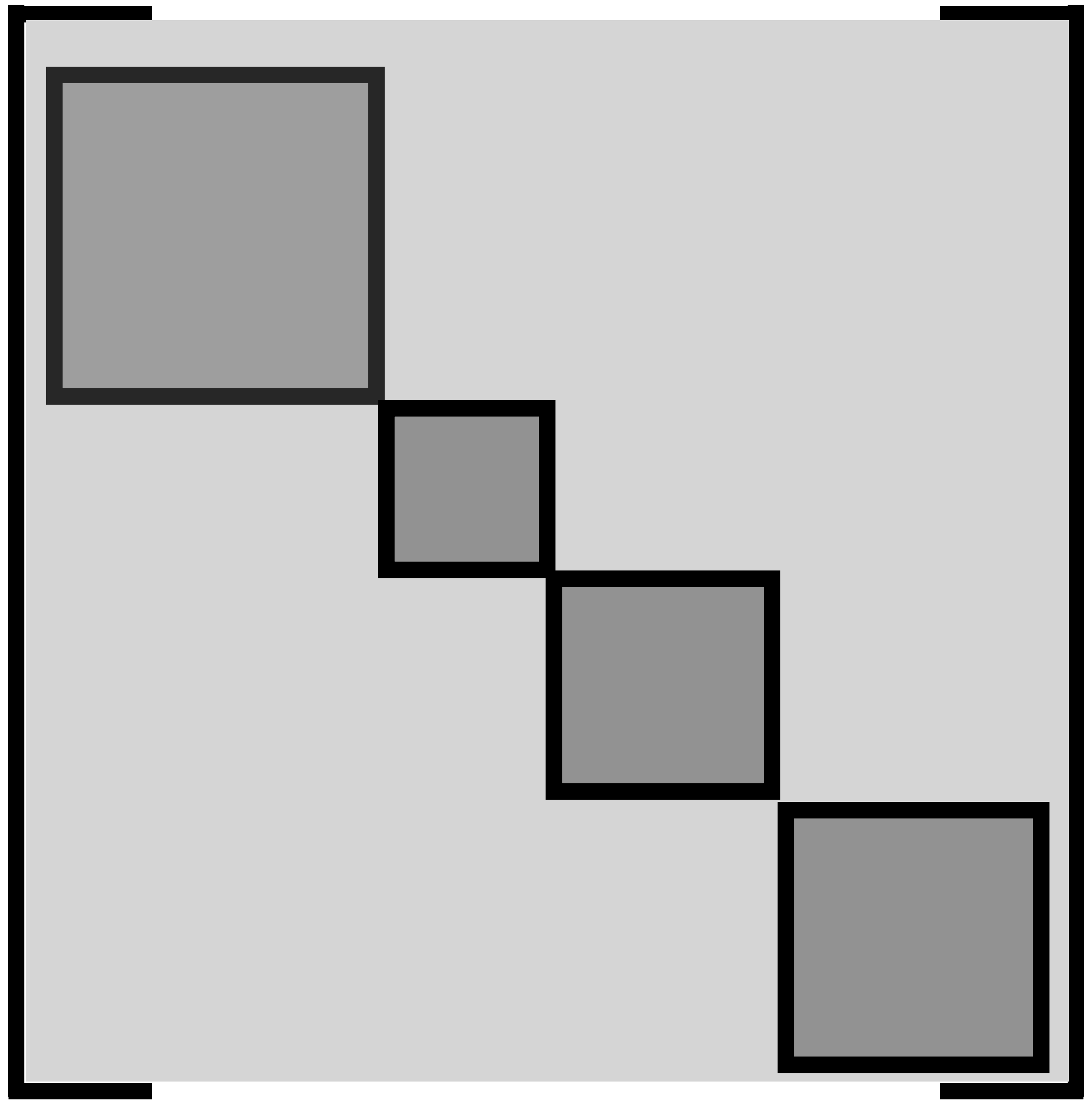
$[\rho_A, Q_A] \neq 0$

$\rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q$

$\rho_A =$



$\rho_{A,Q} =$



$$\Delta S_A = S(\rho_{A,Q}) - S(\rho_A)$$

$\Delta S_A \geq 0$

$\Delta S_A = 0 \Leftrightarrow [\rho_A, Q_A] = 0$

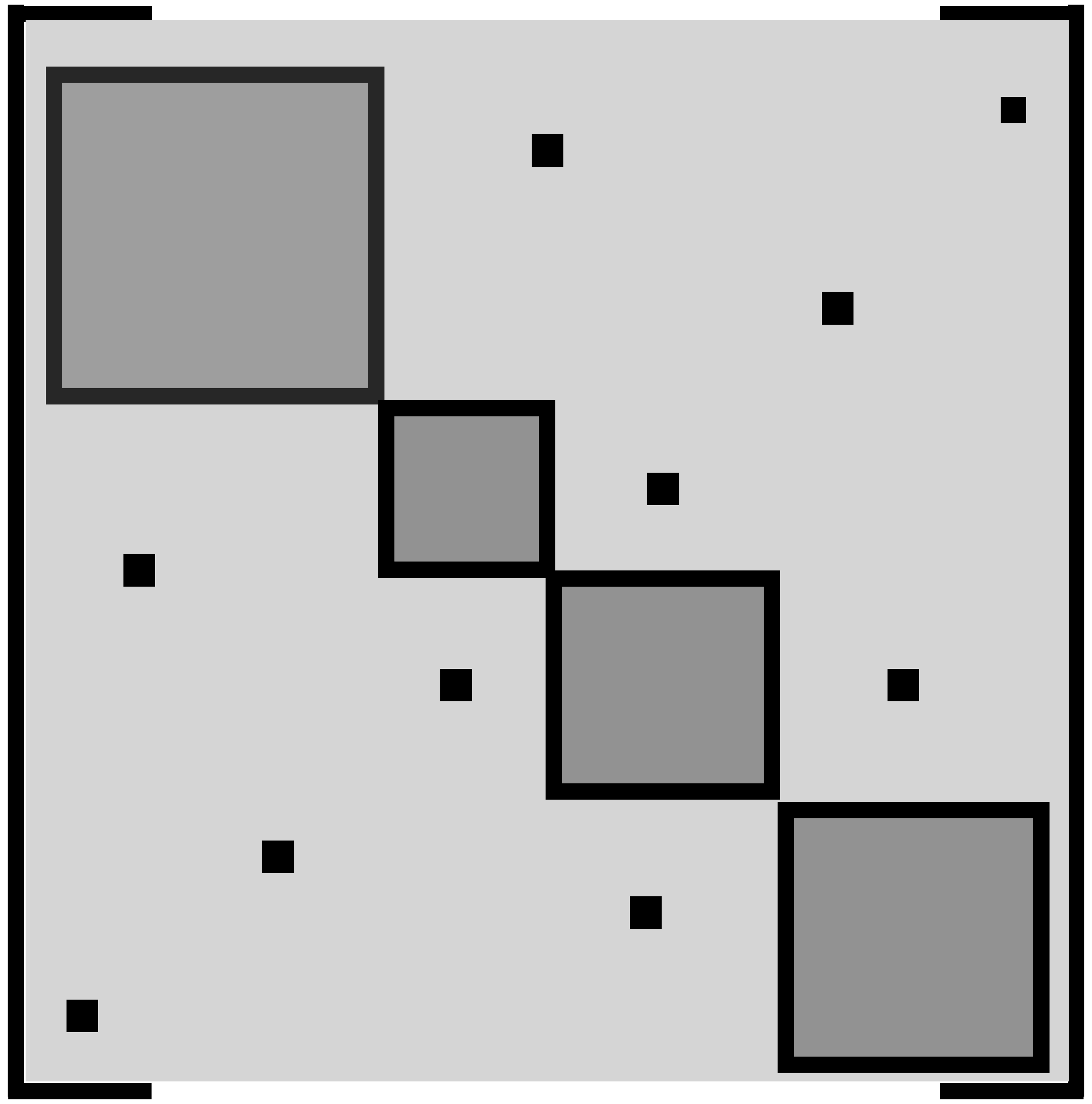
F. Ares, S. M., P. Calabrese, Nat. Comm. 14, 2036 (2023).

Entanglement asymmetry as a probe of symmetry breaking: a replica trick

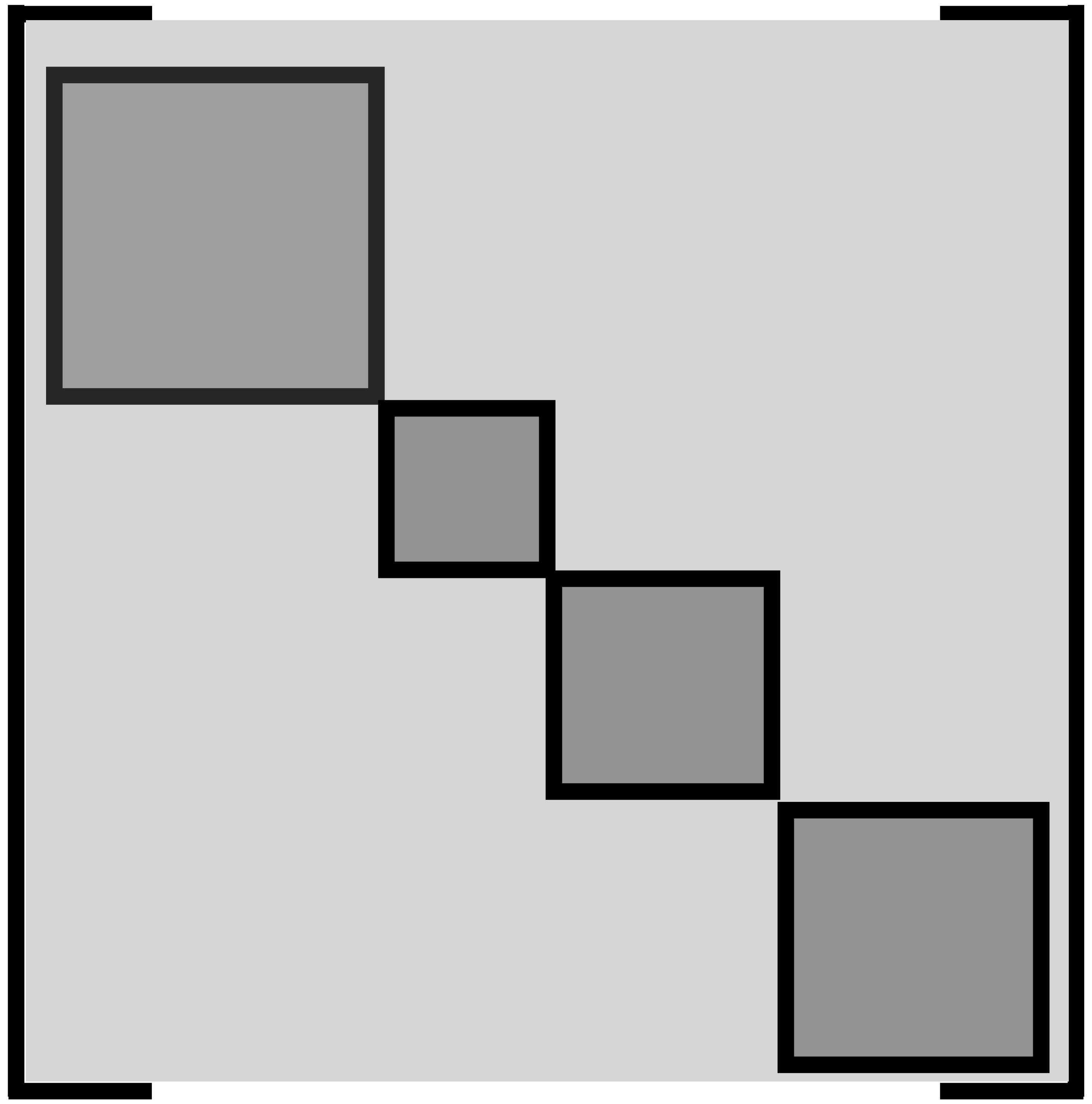
$[\rho_A, Q_A] \neq 0$

$\rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q$

$\rho_A =$



$\rho_{A,Q} =$



Experimentally accessible

$$\Delta S_A^{(n)} = S^{(n)}(\rho_{A,Q}) - S^{(n)}(\rho_A)$$

$$\Delta S_A^{(n)} \geq 0$$

$$\Delta S_A^{(n)} = 0 \Leftrightarrow [\rho_A, Q_A] = 0$$

F. Ares, S. M., P. Calabrese, Nat. Comm. 14, 2036 (2023).

L. Joshi, J. Franke, A. Rath, F. Ares, SM *et al.*, arXiv.2401.04270 (2024).

Computation of the asymmetry

$$\Delta S_A^{(n)} = \frac{1}{1-n}\left[\log \mathrm{Tr}(\rho_{A,Q}^n) - \log \mathrm{Tr}(\rho_A^n)\right] \qquad \rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q$$

Using the Fourier transform of the projector Π_q for $q \in \mathbb{Z}$: $\rho_{A,Q} = \int \frac{d\alpha}{2\pi} e^{-i\alpha Q_A} \rho_A e^{i\alpha Q_A}$

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$$\mathrm{Tr}(\rho_{A,Q}^n) = \int_{-\pi}^{\pi} \frac{\mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_n}{(2\pi)^n} Z_n(\boldsymbol{\alpha})$$

where $Z_n(\boldsymbol{\alpha})$ are the *charged moments*

$$Z_n(\boldsymbol{\alpha}) = \mathrm{Tr}\left[\rho_A e^{i(\alpha_1-\alpha_2)Q_A} \rho_A e^{i(\alpha_2-\alpha_3)Q_A} \ldots \rho_A e^{i(\alpha_n-\alpha_1)Q_A}\right]$$

$$\alpha_{j,j+1} = \alpha_j - \alpha_{j+1}$$

$$[\rho_A, Q_A] = 0 \Rightarrow Z_n(\boldsymbol{\alpha}) = Z_n(\mathbf{0}) \Rightarrow \mathrm{Tr}(\rho_{A,Q}^n) = \mathrm{Tr}(\rho_A^n), \Delta S_A^{(n)} = 0$$

Charged moments at n=2

$\{U|0\rangle\}$: ensemble of Haar random states

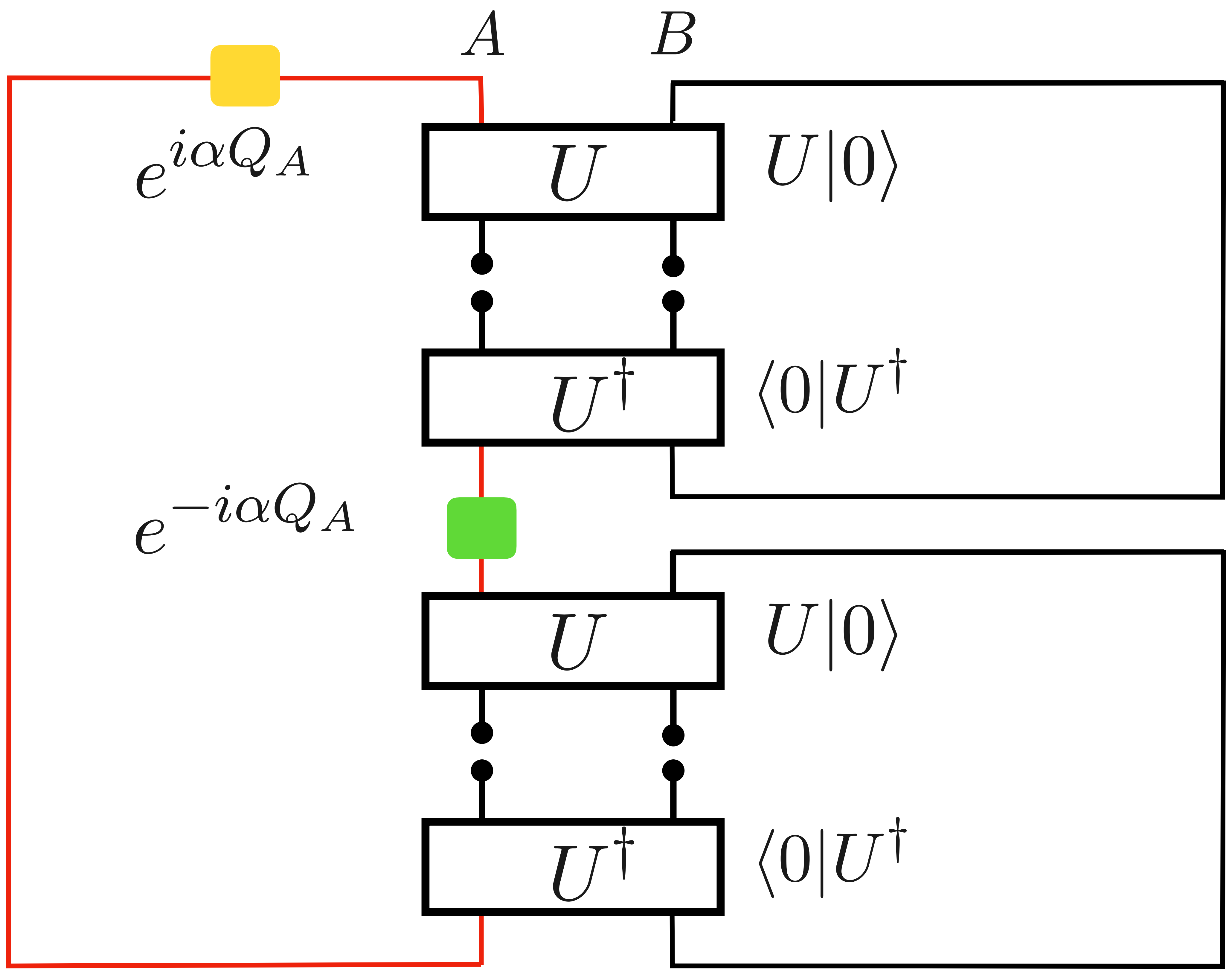
$\alpha_{12} = -\alpha_{21} = \alpha$

$U|0\rangle\langle 0|U^\dagger$: total density matrix

$\mathrm{Tr}_B(U|0\rangle\langle 0|U^\dagger)$

$\rho_A e^{i\alpha Q_A}\rho_A e^{-i\alpha Q_A}$

$\mathrm{Tr}_A[\rho_A e^{i\alpha Q_A}\rho_A e^{-i\alpha Q_A}]$



charge operator:
$Q = \sum_{k=1}^{L} |1\rangle_k \langle 1|_k$

Charged moments at n=2

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$U\left|0\right\rangle\left\langle 0\right|U^\dagger$: total density matrix

$\mathrm{Tr}_B(U\left|0\right\rangle\left\langle 0\right|U^\dagger)$

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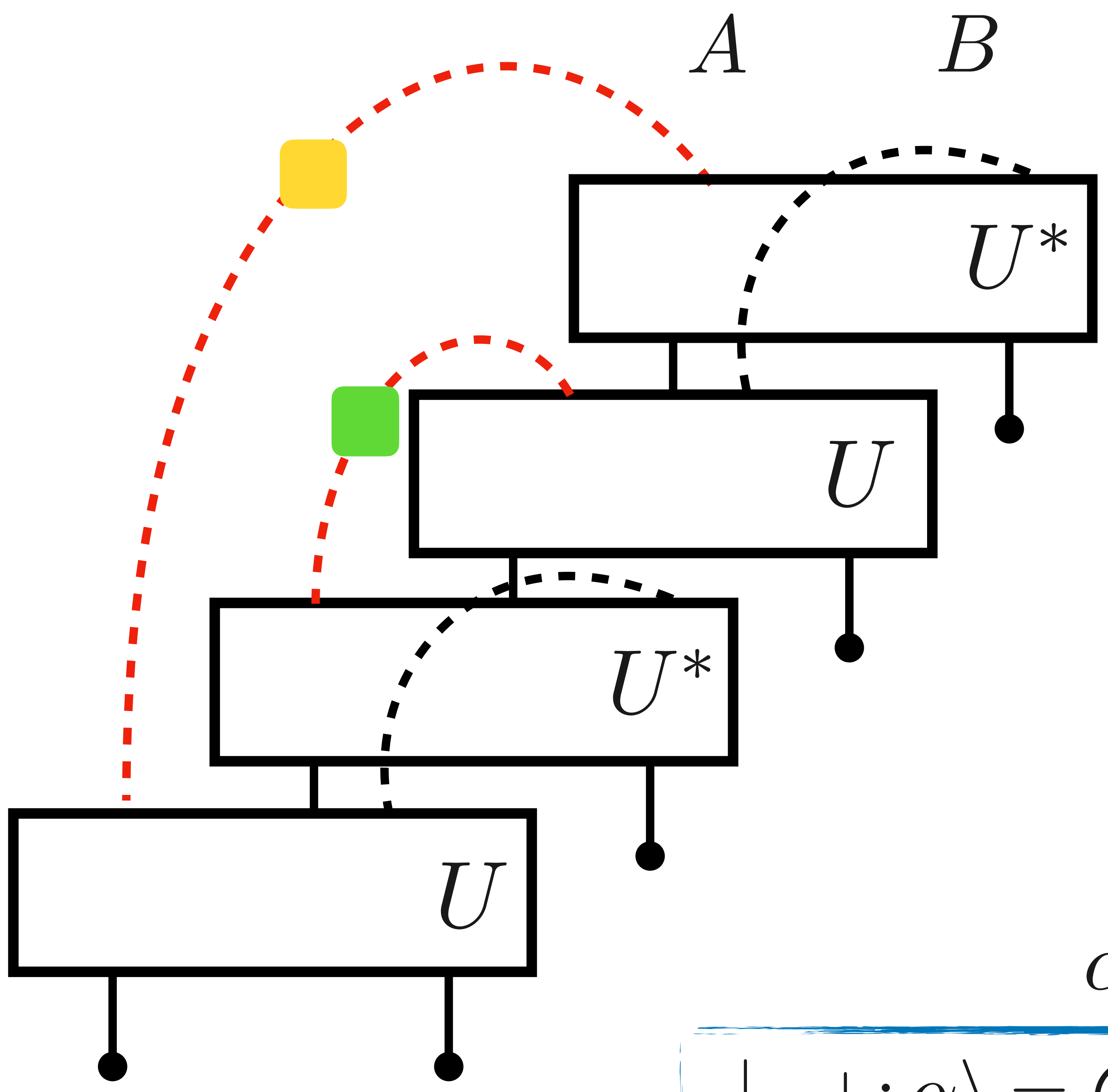


$\alpha_{12}=-\alpha_{21}=\alpha$

$$\left|-+;\alpha\right\rangle=\bigotimes_{k\in A}\left|-;\alpha\right\rangle_k\bigotimes_{k\in B}\left|+\right\rangle_k$$

$$\left|+\right\rangle_k=\sum_{a_1,a_2=0}^{1}(\left|a_1\right\rangle_k\otimes\left|a_1\right\rangle_k)(\left|a_2\right\rangle_k\otimes\left|a_2\right\rangle_k)$$

$$\left|-;\alpha\right\rangle_k=\sum_{a_1,a_2=0}^{1}(e^{i\alpha_{12}a_1}\left|a_1\right\rangle_k\otimes e^{i\alpha_{21}a_2}\left|a_2\right\rangle_k)(e^{i\alpha_{21}a_2}\left|a_2\right\rangle_k\otimes e^{i\alpha_{12}a_1}\left|a_1\right\rangle_k)$$

Goal: $\mathbb{E}[Z_2(\alpha)]=\left\langle -+;\alpha\right|\mathbb{E}[U^{\otimes 2}\otimes(U^*)^{\otimes 2}]\left|0\right\rangle^{\otimes 4}$

Charged moments at n=2

$\{U|0\rangle\}$: ensemble of Haar random states

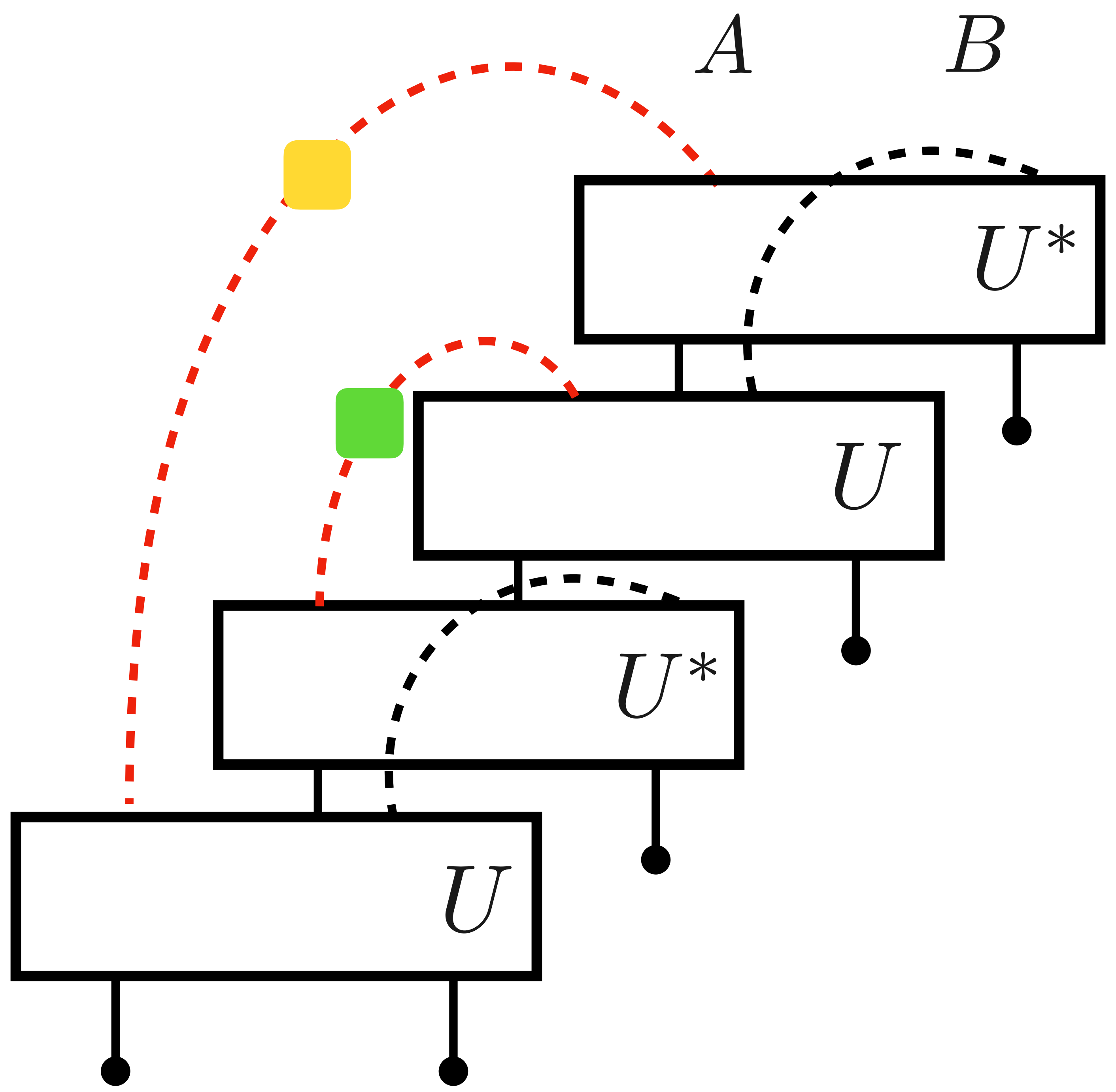
$U|0\rangle\langle 0|U^\dagger$: total density matrix

$\mathrm{Tr}_B(U|0\rangle\langle 0|U^\dagger)$

$\rho_A e^{i\alpha Q_A}\rho_A e^{-i\alpha Q_A}$

$\mathrm{Tr}_A[\rho_A e^{i\alpha Q_A}\rho_A e^{-i\alpha Q_A}]$

Choi-Jamiolkowski mapping:
$\rho_A\otimes\rho_A \in \mathcal{H}_A\otimes\mathcal{H}_A \to |\rho_A\otimes\rho_A\rangle \in \mathcal{H}_A^{\otimes 4}$



$$\mathbb{E}[Z_2(\alpha)] = \frac{2^{\ell_B} + 2^{\ell_A}\cos(\alpha)^{2\ell_A}}{2^{\ell_A+\ell_B}+1}$$

Entanglement asymmetry at n=2

Assumption: $\mathbb{E}[\log \mathrm{Tr}(\rho_{A,Q}^n)] \simeq \log \mathbb{E}[\mathrm{Tr}(\rho_{A,Q}^n)]$

$$\mathbb{E}[\Delta S_A^{(2)}] = \mathbb{E}[\log \mathrm{Tr}\rho_A^2] - \mathbb{E}[\log \mathrm{Tr}\rho_{A,Q}^2]$$

$$\downarrow$$

$$\log \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} \mathbb{E}[Z_2(\alpha)]$$

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$$\mathbb{E}[\Delta S_A^{(2)}] = \mathbb{E}[\log \mathrm{Tr}\rho_A^2] - \mathbb{E}[\log \mathrm{Tr}\rho_{A,Q}^2] \simeq -\log\left[\frac{1}{2^{2\ell_A - L} + 1}\left(1 + 2^{-L}\frac{(2\ell_A)!}{(\ell_A!)^2}\right)\right]$$

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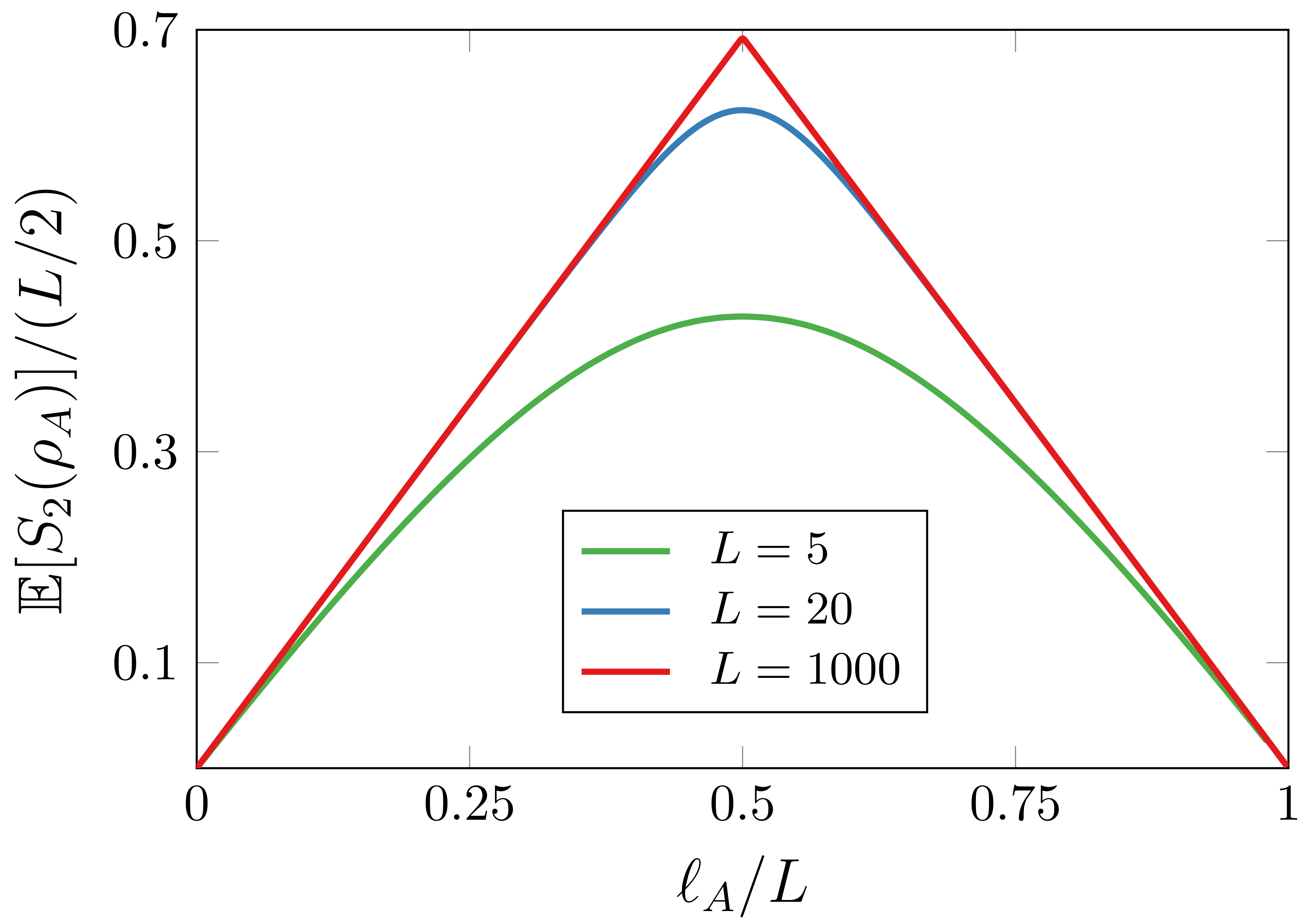

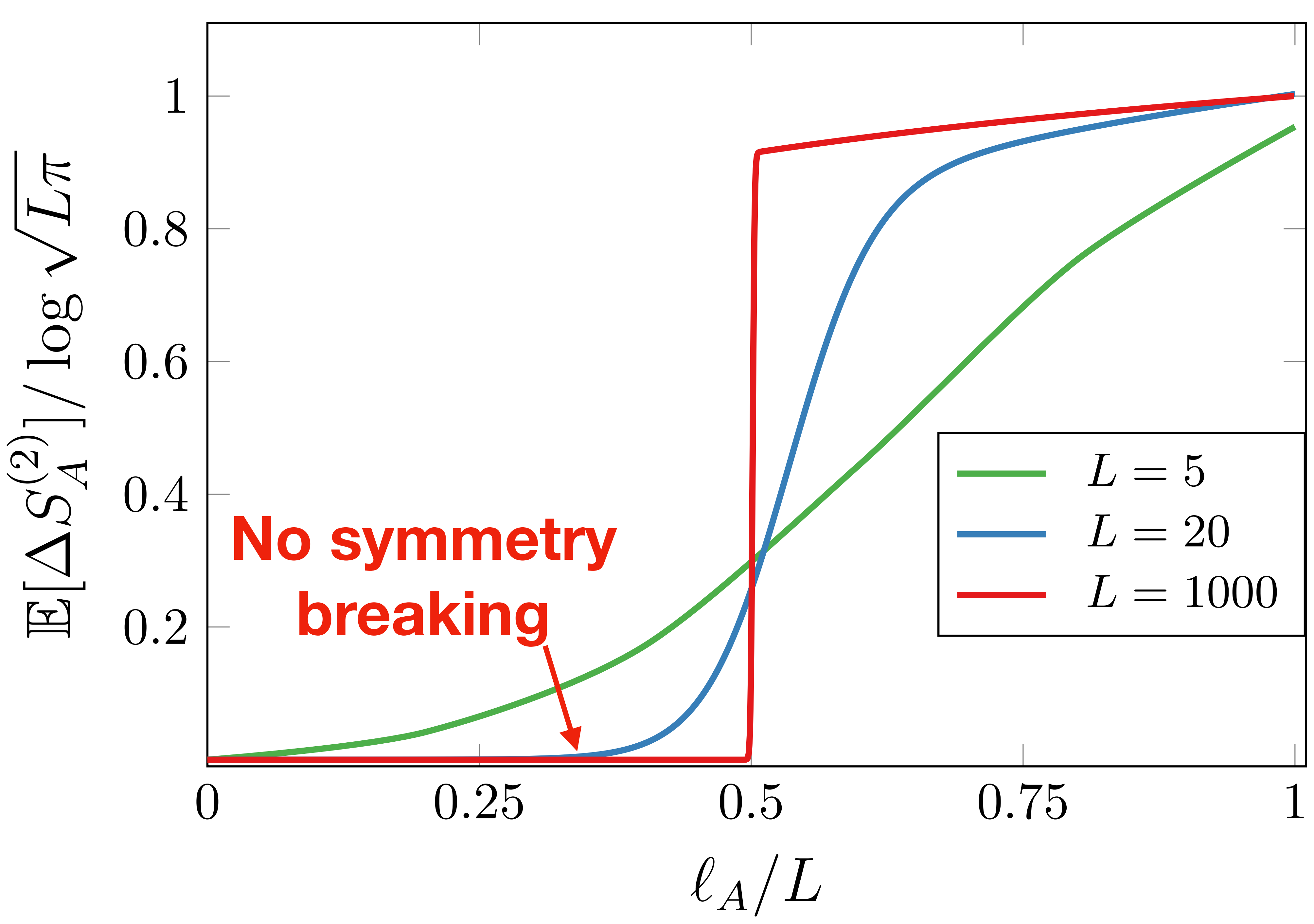

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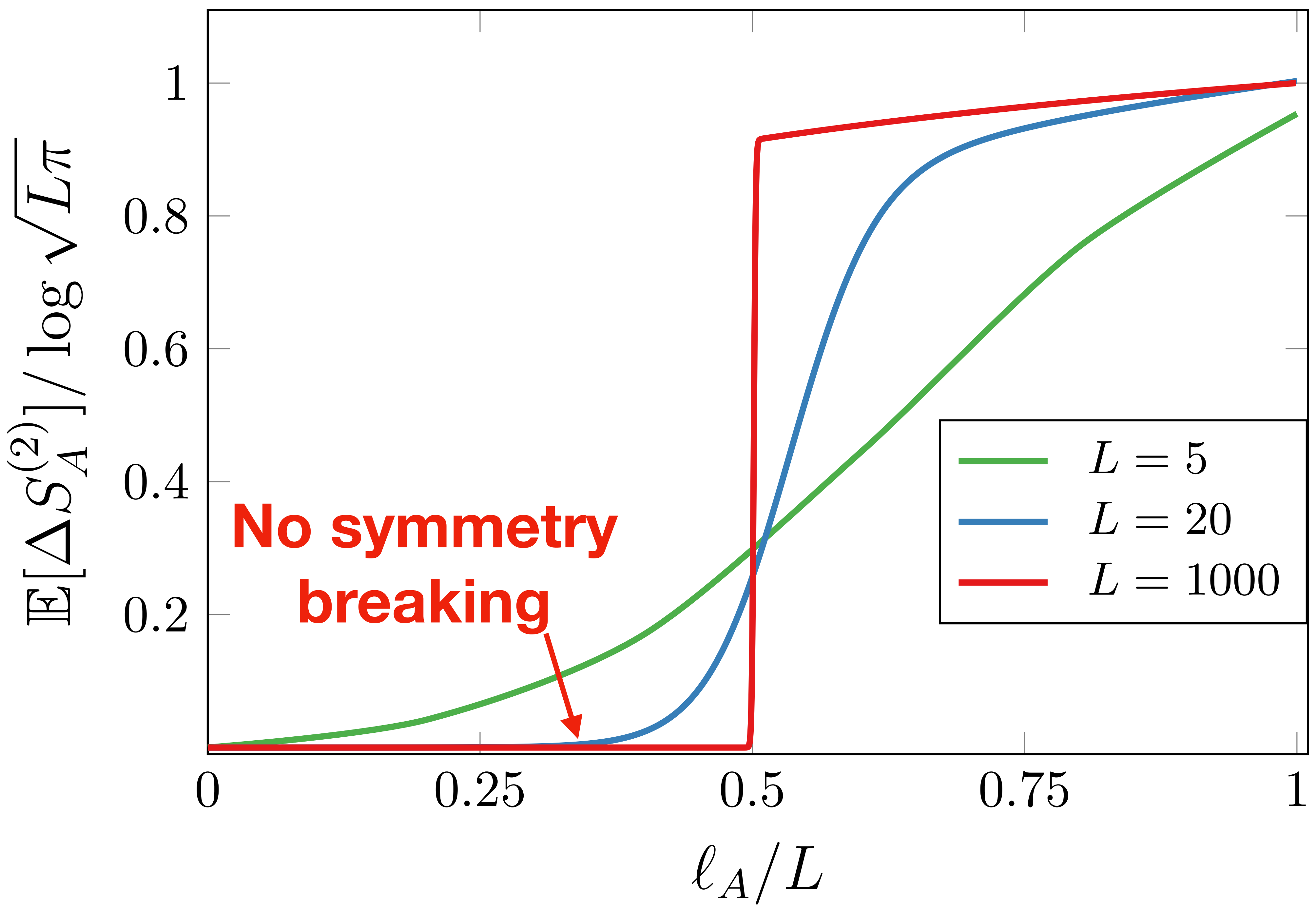
Any signature of the symmetry breaking in radiation cannot be detected before the Page time



Charged moments at generic n

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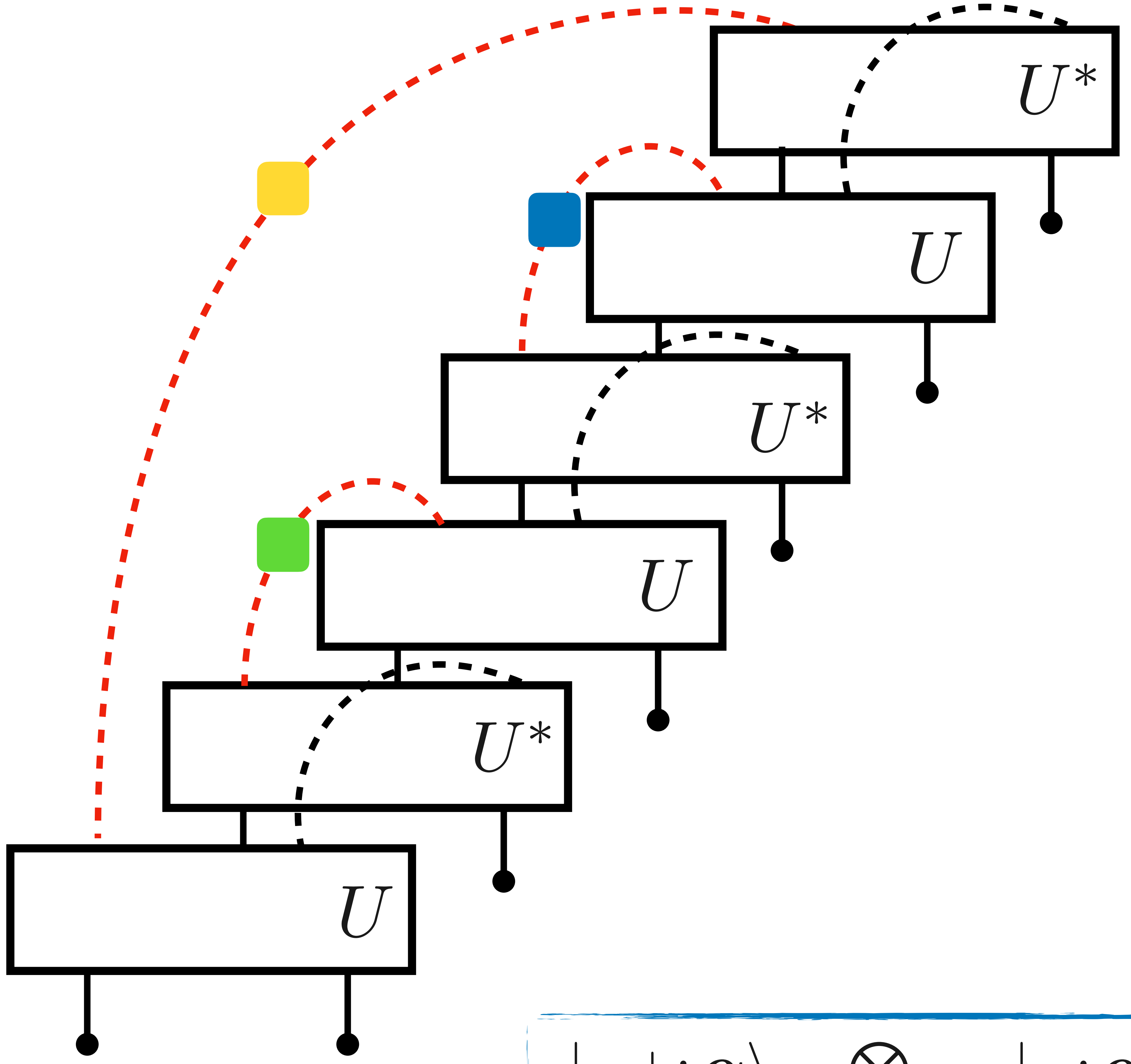
$U|0\rangle\langle 0|U^\dagger$: total density matrix

$\mathrm{Tr}_B(U|0\rangle\langle 0|U^\dagger)$

$\rho_A e^{i\alpha_{12}Q_A}\rho_A e^{i\alpha_{23}Q_A}\rho_A e^{i\alpha_{31}Q_A}$

$Z_n(\boldsymbol{\alpha}) = \mathrm{Tr}(\prod_{j=1}^n \rho_A e^{i\alpha_{jj+1}Q_A})$



Weingarten formula

$$\mathbb{E}[U^{\otimes n}\otimes (U^*)^{\otimes n}] = \sum_{\sigma_1,\sigma_2\in\mathcal{S}_n} \mathrm{Wg}(\sigma_1\sigma_2^{-1})\,|\sigma_1\rangle\langle\sigma_2|$$

symmetric group

$$|\sigma\rangle = \otimes_{k=1}^L |\sigma\rangle_k$$

$$|\sigma\rangle_k = \sum_{\{a_j=0\}}^1 \bigotimes_{j=1}^n (|a_j\rangle_k \otimes |a_{\sigma(j)}\rangle_k)$$

$$|-+;\boldsymbol{\alpha}\rangle = \bigotimes_{k\in A}|-;\boldsymbol{\alpha}\rangle_k \bigotimes_{k\in B}|+\rangle_k$$

$$|+\rangle_k = \sum_{\{a_j=0\}}^1 \bigotimes_{j=1}^n (|a_j\rangle_k \otimes |a_j\rangle_k)$$

$$|-;\boldsymbol{\alpha}\rangle_k = \sum_{\{a=0\}}^1 \bigotimes_{j=1}^n (e^{i\alpha_{j,j+1}a_1}|a_j\rangle_k \otimes e^{i\alpha_{j+1,j+2}a_{j+1}}|a_{j+1}\rangle_k)$$

D. Weingarten, J. Math. Phys. 19, 999 (1978).

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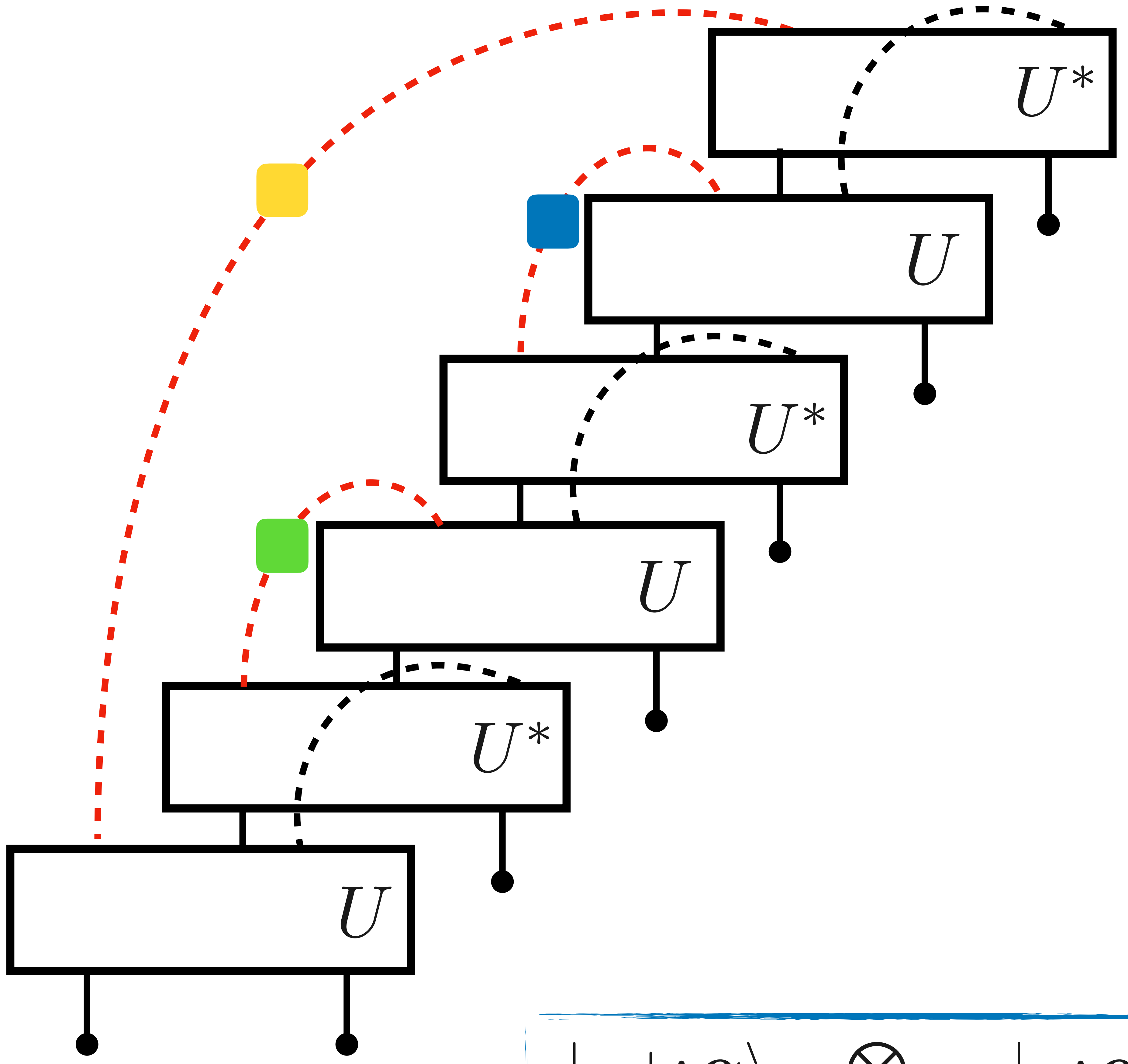

Weingarten formula (large system size L)

$$\mathbb{E}[U^{\otimes n}\otimes(U^*)^{\otimes n}] = \frac{1}{2^{nL}}\sum_{\sigma\in\mathcal{S}_n}|\sigma\rangle\langle\sigma|$$

$$|-+;\boldsymbol{\alpha}\rangle = \bigotimes_{k\in A}|-;\boldsymbol{\alpha}\rangle_k \bigotimes_{k\in B}|+\rangle_k$$

$$|+\rangle_k = \sum_{\{a_j=0\}}^{1}\bigotimes_{j=1}^{n}(|a_j\rangle_k\otimes|a_j\rangle_k)$$

$$|-;\boldsymbol{\alpha}\rangle_k = \sum_{\{a=0\}}^{1}\bigotimes_{j=1}^{n}(e^{i\alpha_{j,j+1}a_1}|a_j\rangle_k \otimes e^{i\alpha_{j+1,j+2}a_{j+1}}|a_{j+1}\rangle_k)$$

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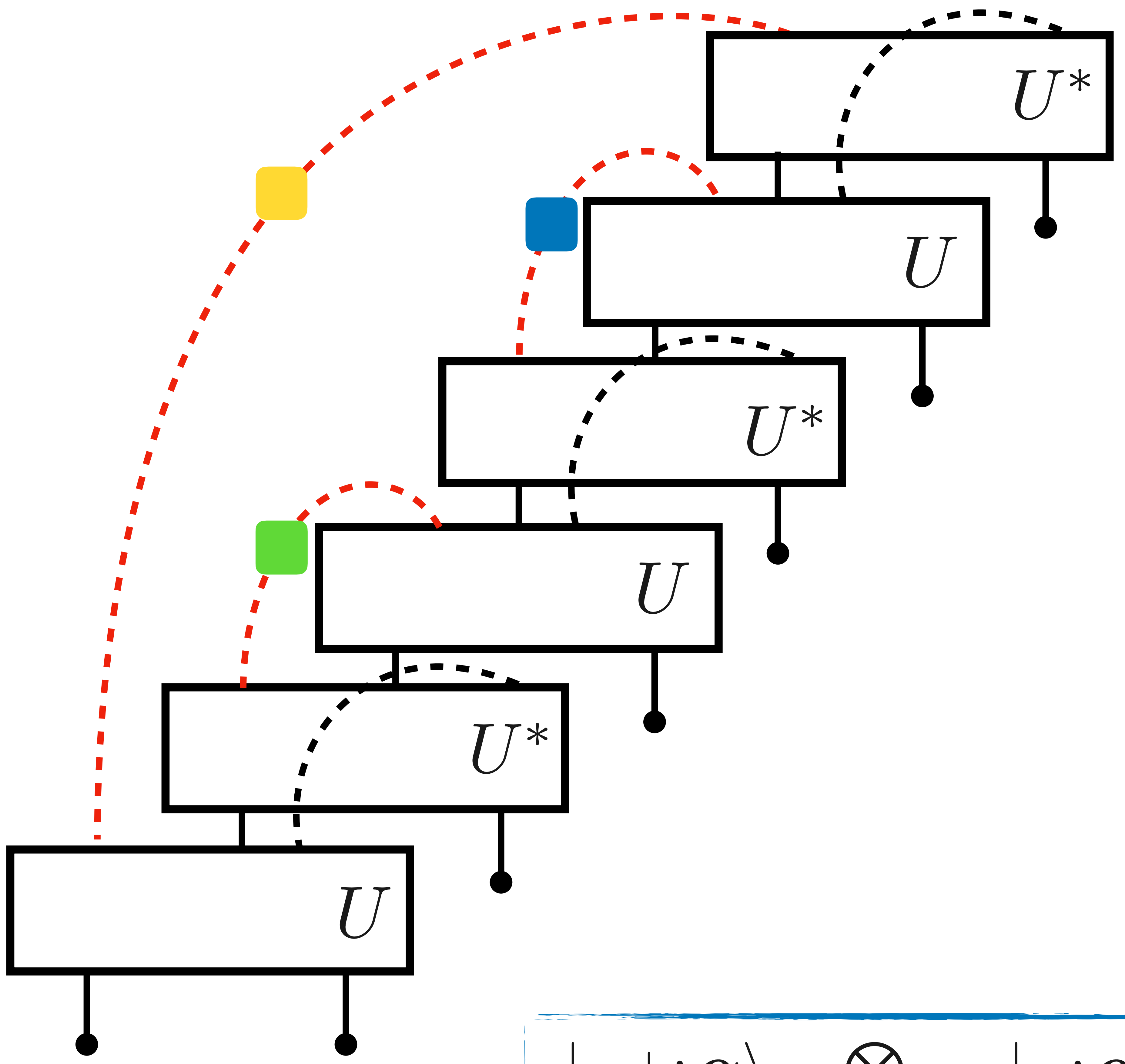

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$$\mathbb{E}[Z_n(\boldsymbol{\alpha})] = \frac{1}{2^{nL}}\sum_{\sigma\in\mathcal{S}_n}\left(\sum_{\{a_j=0\}}^1\prod_{j=1}^n\delta_{a_j,a_{\sigma(j)}}\right)^{\ell_B}\times \left(\sum_{\{a_j=0\}}^1\prod_{j=1}^n e^{i2\alpha_{jj+1}a_j}\delta_{a_{j+1},a_{\sigma(j)}}\right)^{\ell_A}$$

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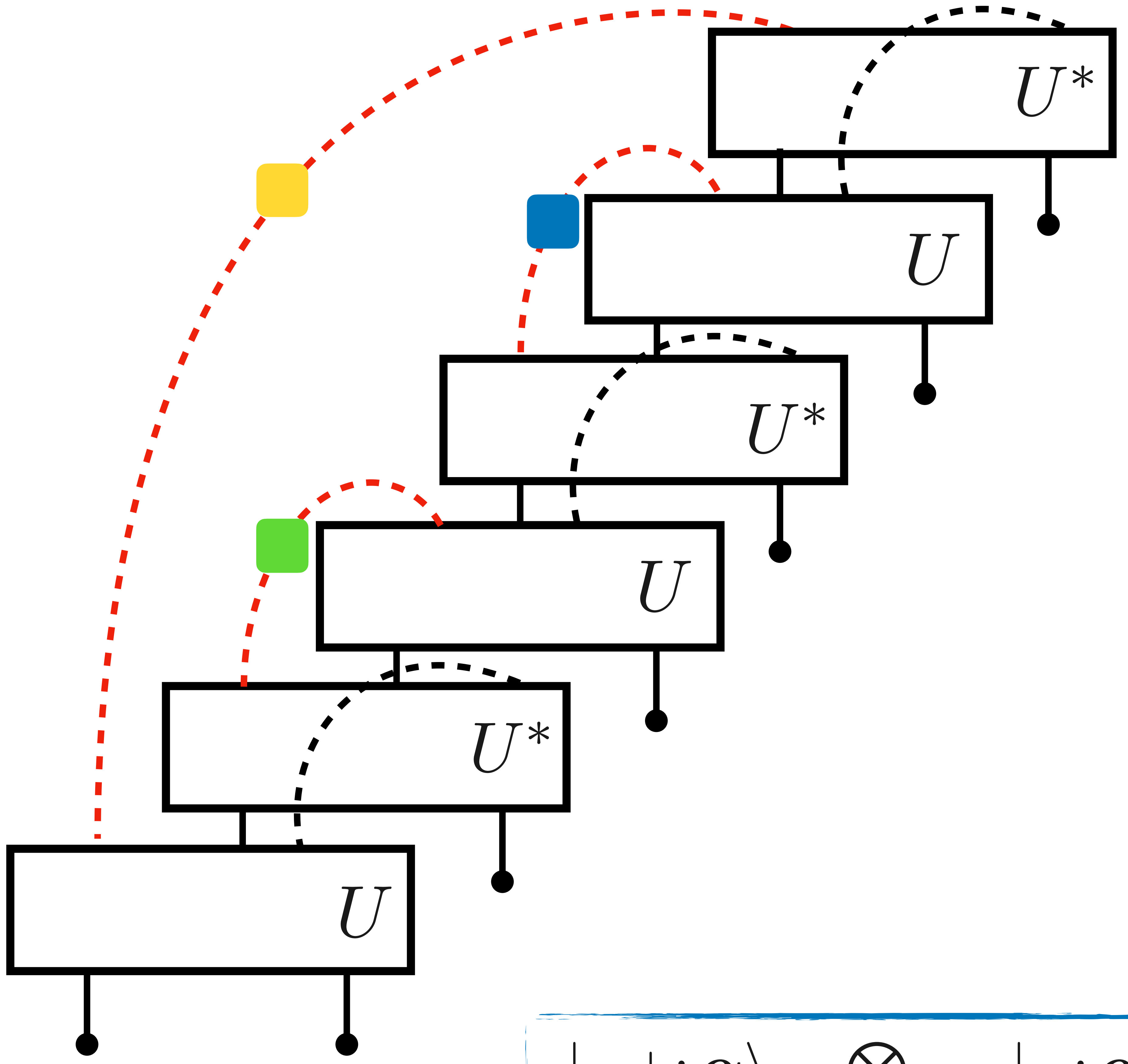

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$$\mathbb{E}[Z_n(\boldsymbol{\alpha})] \sim 2^{(1-n)\ell_A} + 2^{(1-n)(L-\ell_A)}\prod_{j=1}^n \cos(\alpha_{jj+1})^{\ell_A}$$

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Assumption: $\mathbb{E}[\log \mathrm{Tr}(\rho_{A,Q}^n)] \simeq \log \mathbb{E}[\mathrm{Tr}(\rho_{A,Q}^n)]$

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$$\log \int_{-\pi}^{\pi} \frac{\mathrm{d}\alpha_1 \ldots \mathrm{d}\alpha_n}{(2\pi)^n} \mathbb{E}[Z_n(\boldsymbol{\alpha})]$$

$$\mathbb{E}[Z_n(\boldsymbol{\alpha})] = \tfrac{1}{2^{nL}} \textstyle\sum_{\sigma\in\mathcal{S}_n} \left(\sum_{\{a_j=0\}}^{1} \prod_{j=1}^{n} \delta_{a_j, a_{\sigma(j)}}\right)^{\ell_B} \times \left(\sum_{\{a_j=0\}}^{1} \prod_{j=1}^{n} e^{i2\alpha_{jj+1}a_j} \delta_{a_{j+1}, a_{\sigma(j)}}\right)^{\ell_A}$$

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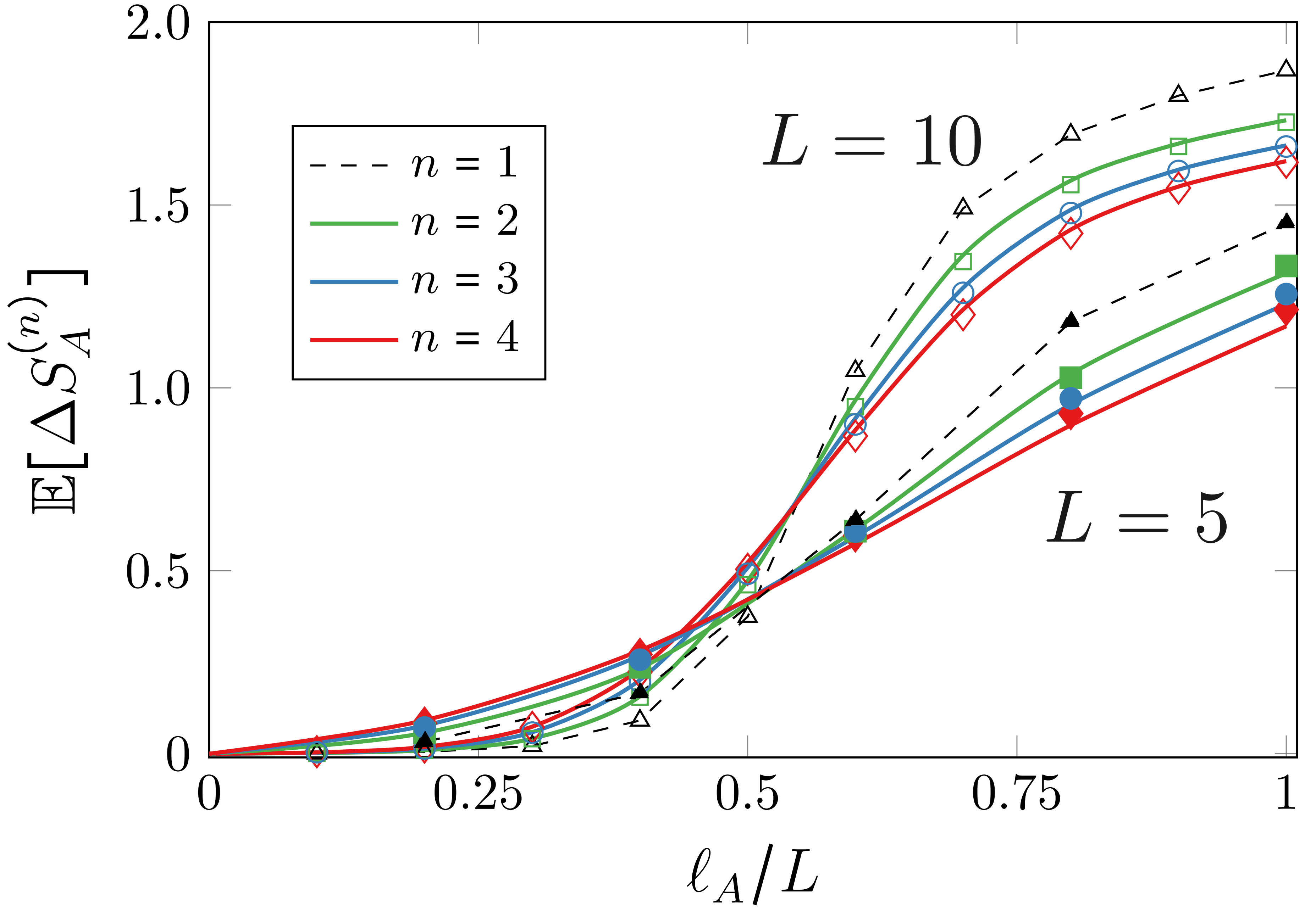

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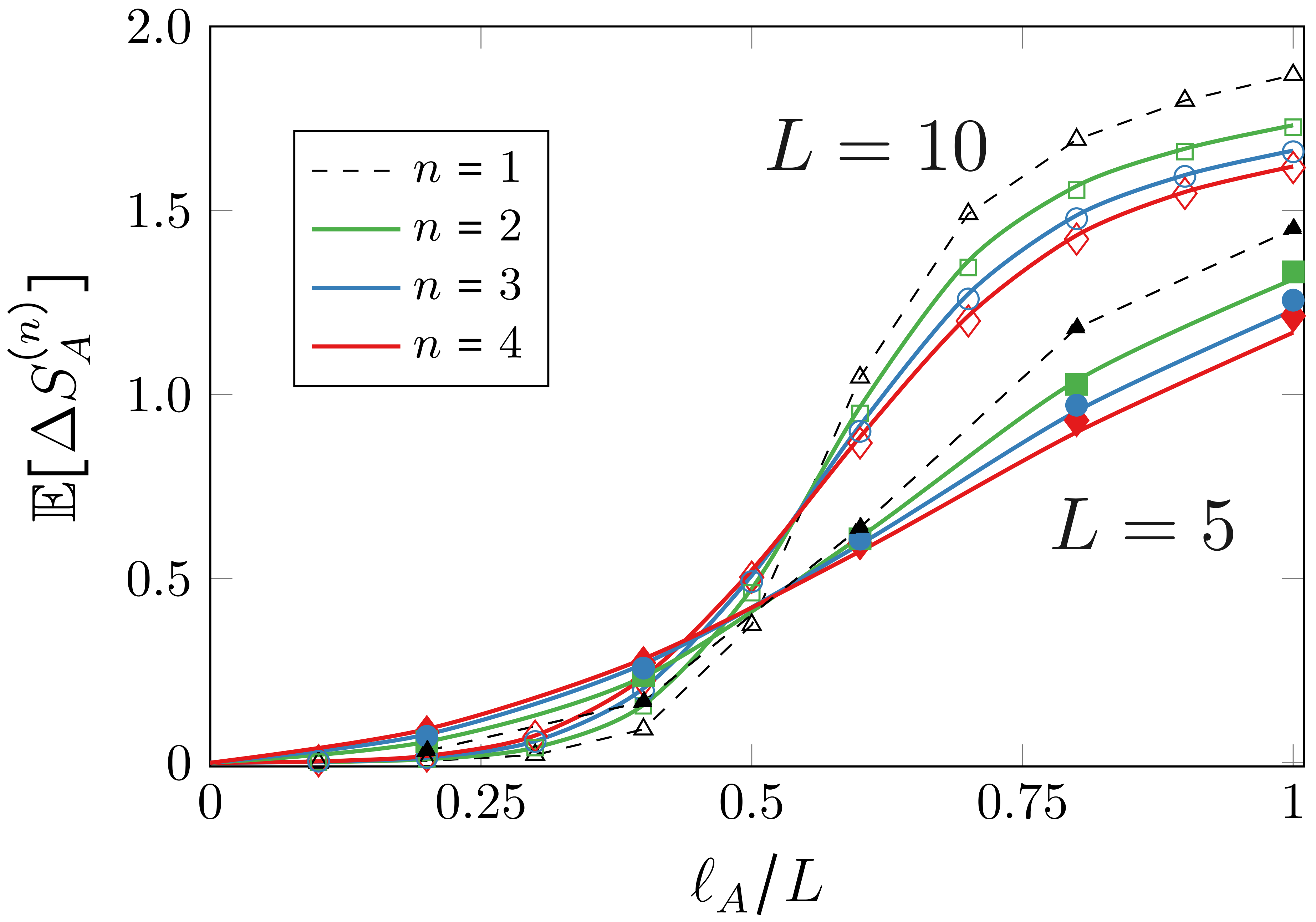
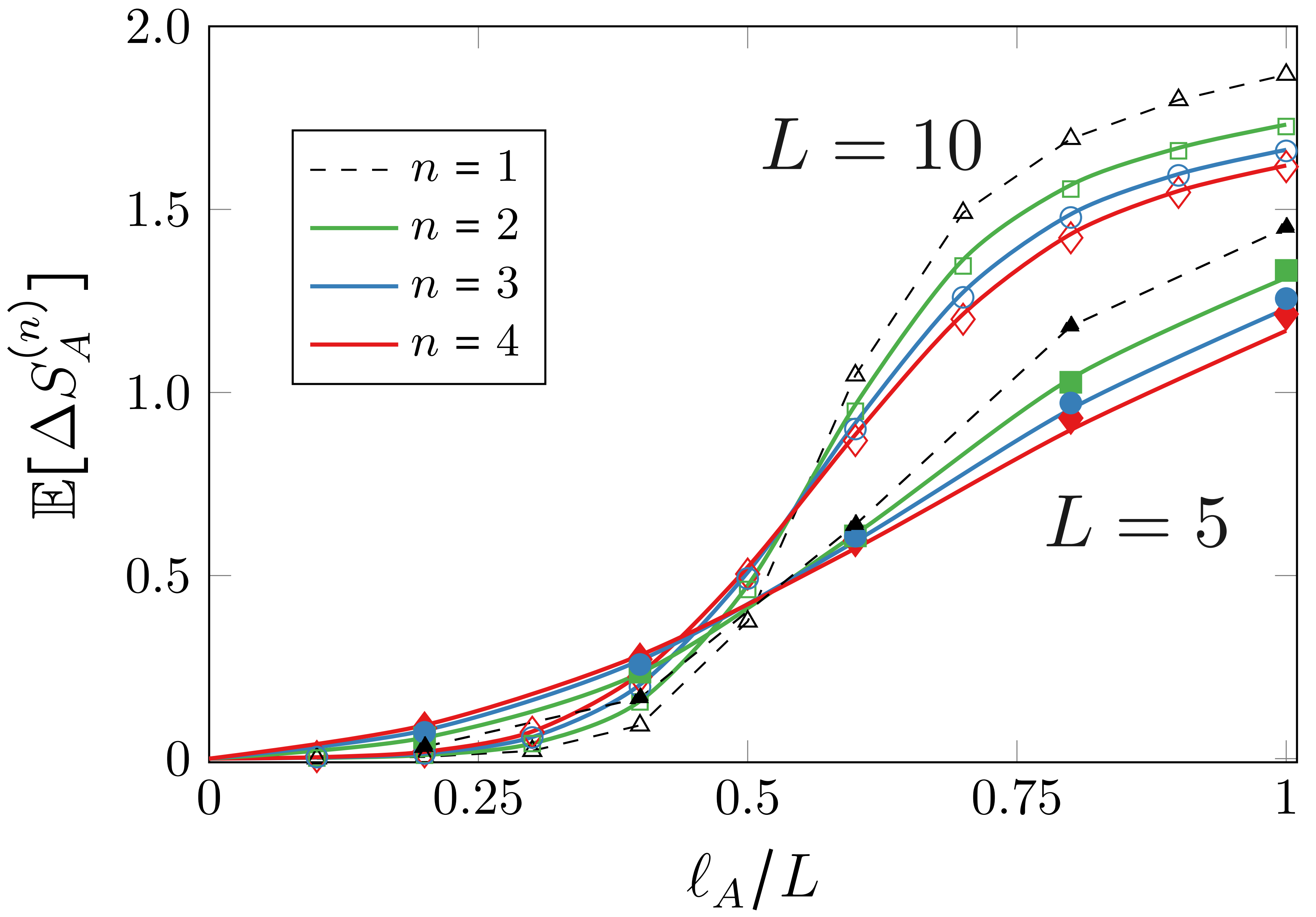


Analytic continuation

Assumption: $\mathbb{E}[\log \mathrm{Tr}(\rho_{A,Q}^n)] \simeq \log \mathbb{E}[\mathrm{Tr}(\rho_{A,Q}^n)]$

$$\mathbb{E}[\Delta S_A] \sim \begin{cases} 0, & \ell_A < L/2, \\ 1/2 \log(\ell_A \pi/2), & \ell_A > L/2. \end{cases}$$



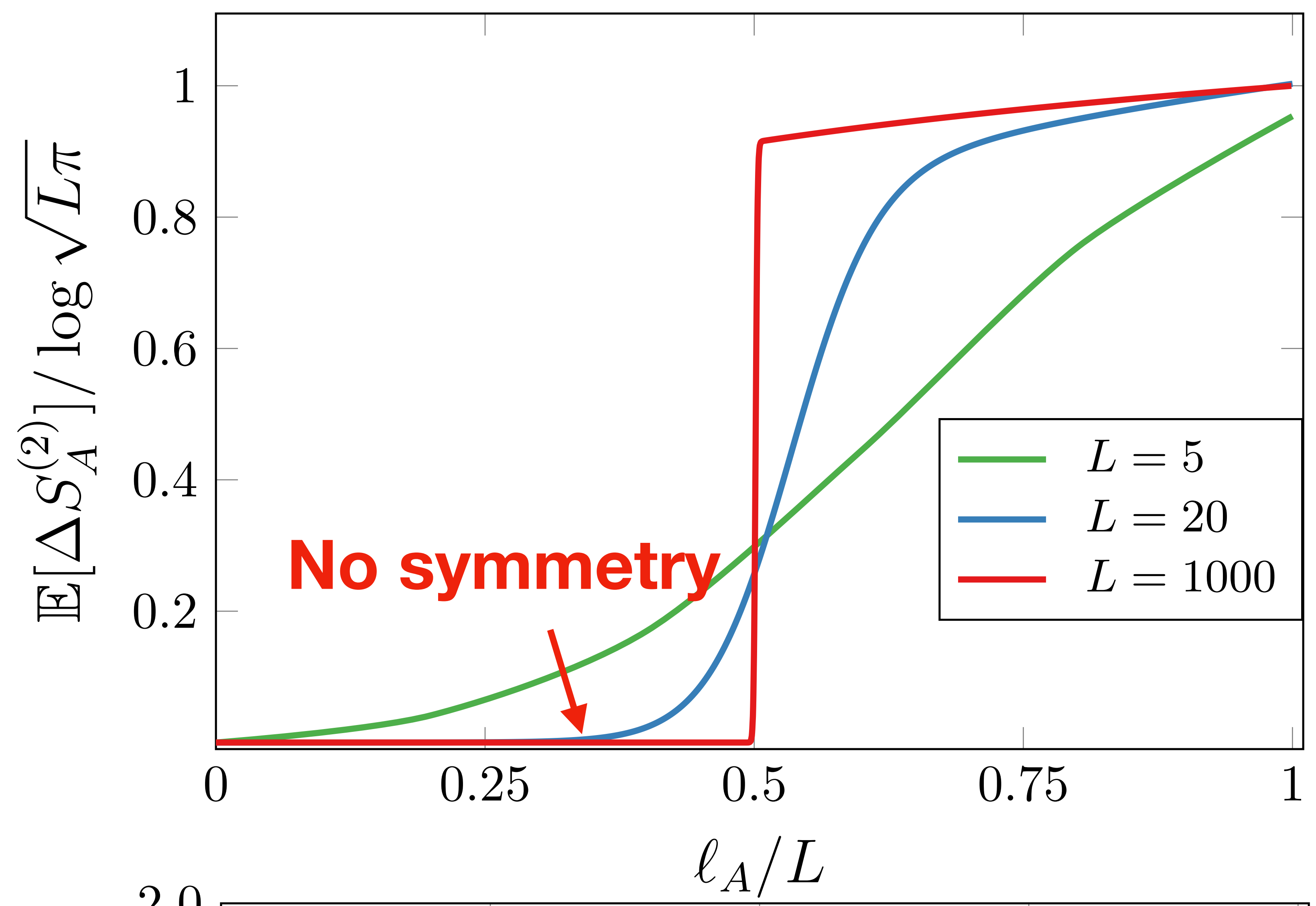
Physical interpretation

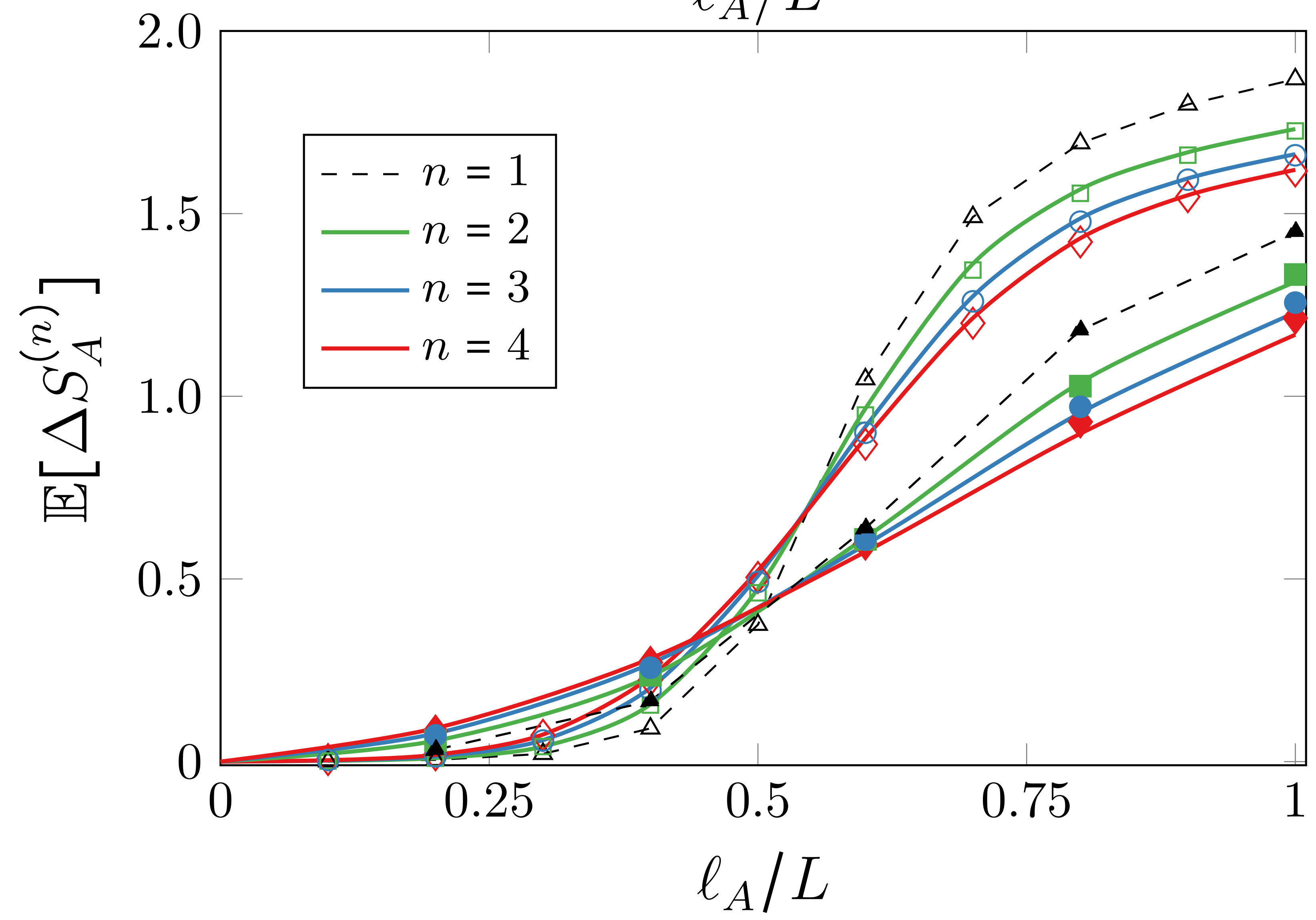
$$\mathbb{E}[\Delta S_A] \sim \begin{cases} 0, & \ell_A < L/2, \\ 1/2\log(\ell_A \pi/2), & \ell_A > L/2. \end{cases}$$

- Decoupling inequality:

$$\mathbb{E}\left[\left|\left|\rho_A - \frac{\mathbf{1}}{2^{\ell_A}}\right|\right|_1\right]^2 \leq 2^{2(\ell_A - L/2)}$$

ρ_A is exponentially close to the (normalized) identity for $\ell_A < L/2$





P. Hayden and J. Preskill, JHEP 09 (2007) 120.

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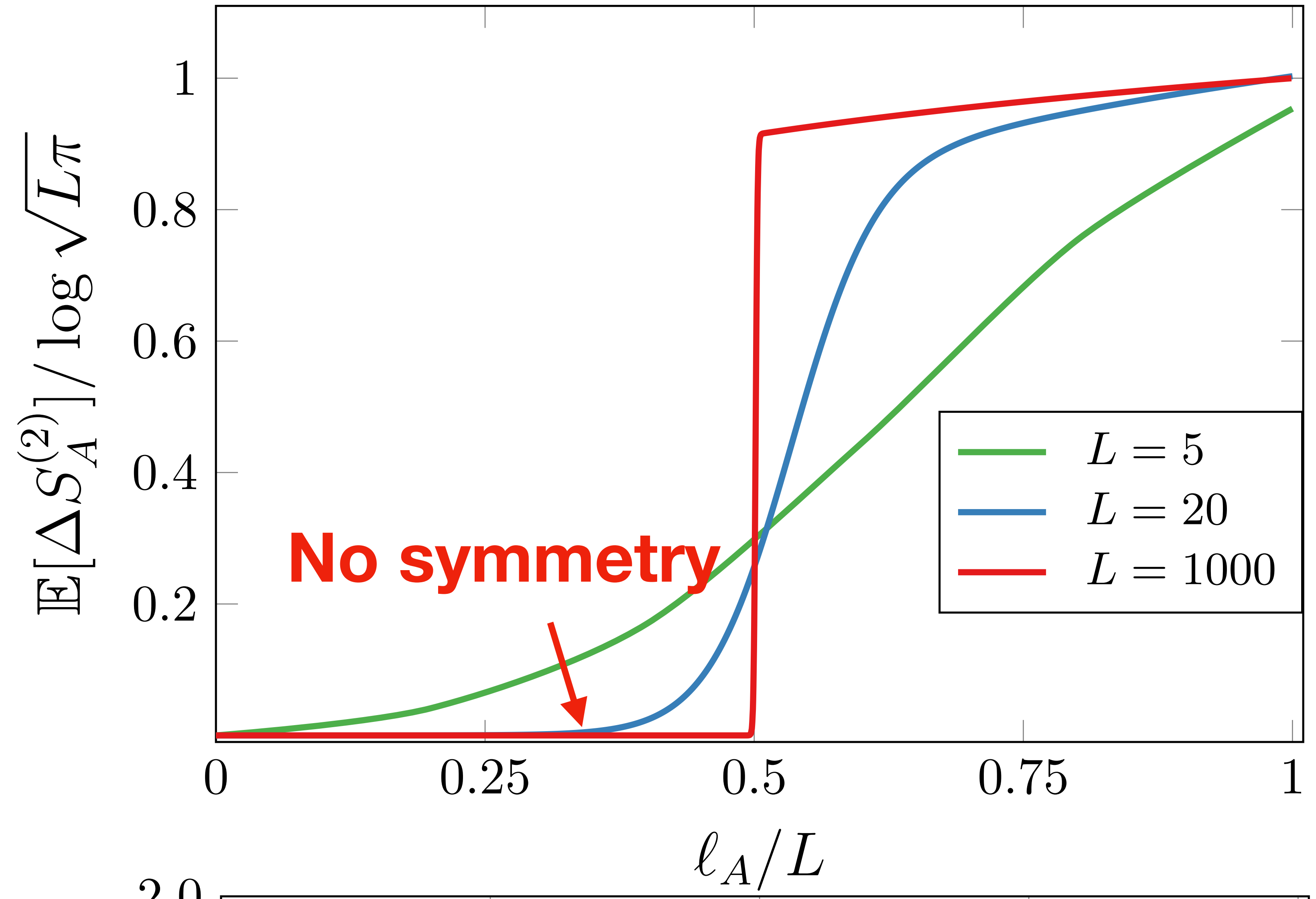

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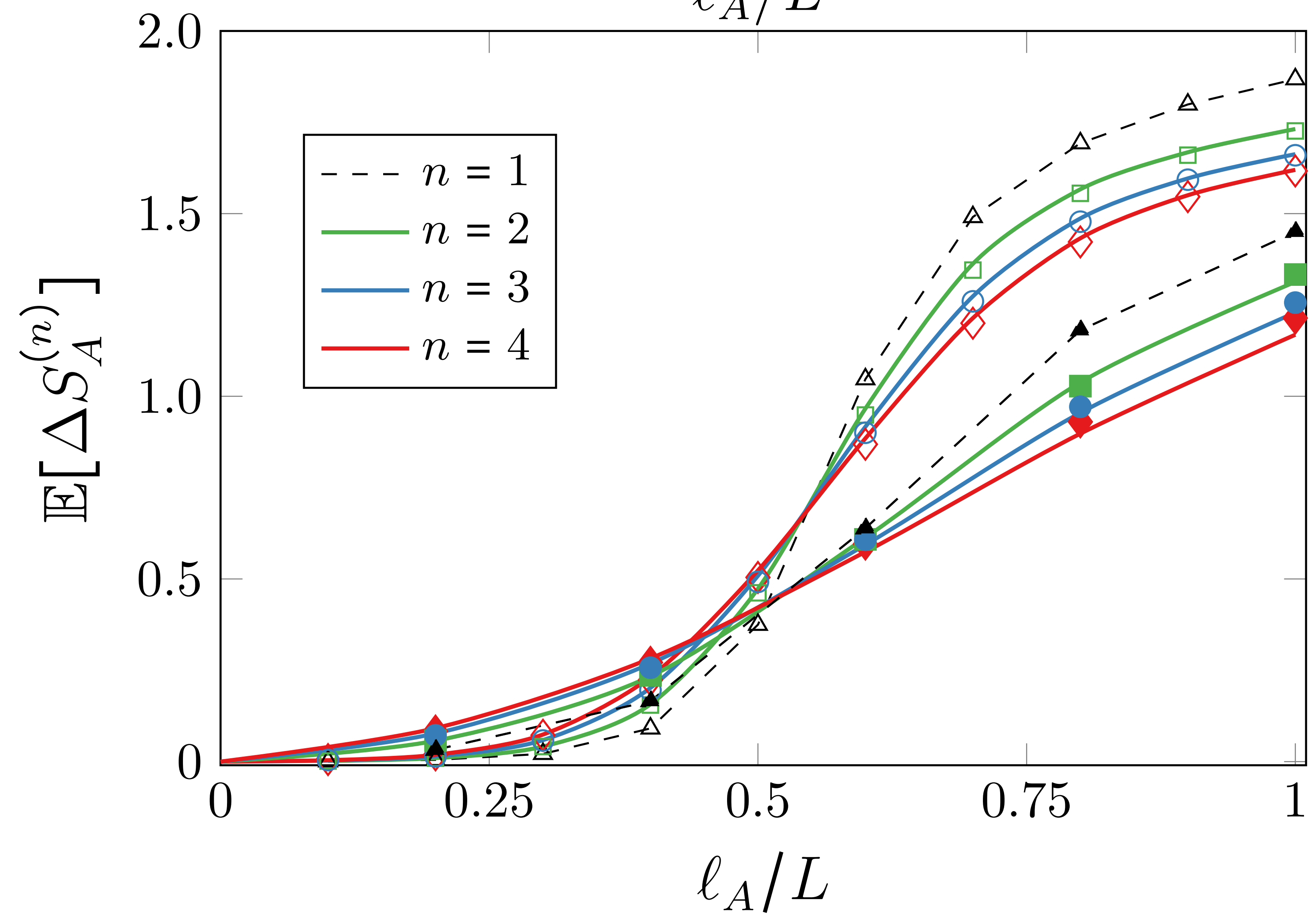


- Recovers semi-classical gravity computation using “island” formulas

Y. Chen, H. W. Lin, JHEP 03 (2021) 040.

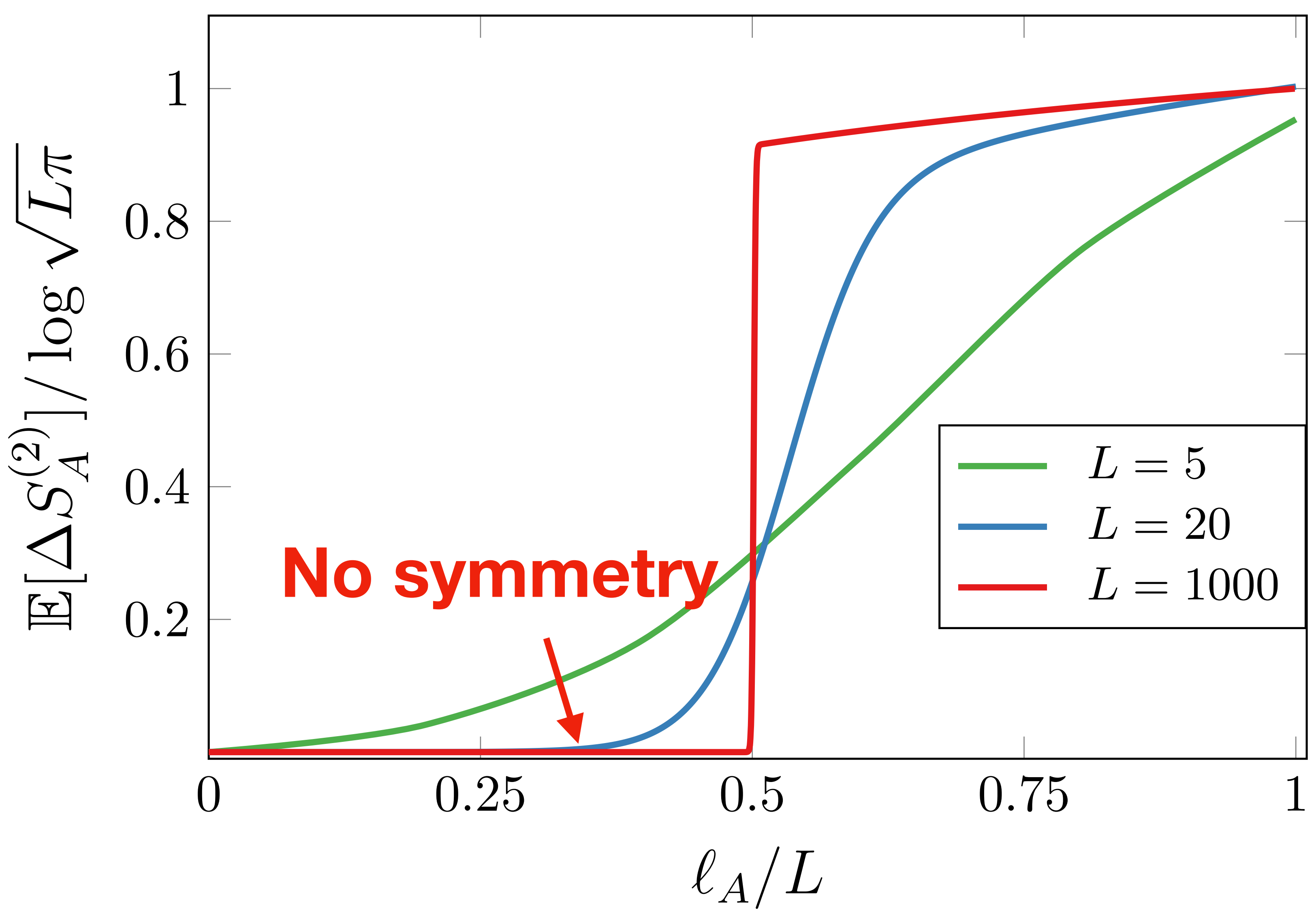
Outline:

- Entanglement entropy and the computation by Page
- How to quantify the symmetry breaking in a subsystem: technical details and physical interpretation
- Conclusions & outlook

Outlook & Future directions

- For $\ell_A < L/2$, the asymmetry vanishes, i.e. the $U(1)$ symmetry is typically restored in A.

- At the Page time, $\ell_A = L/2$, the asymmetry shows a sharp jump and then it increases logarithmically for $\ell_A > L/2$.



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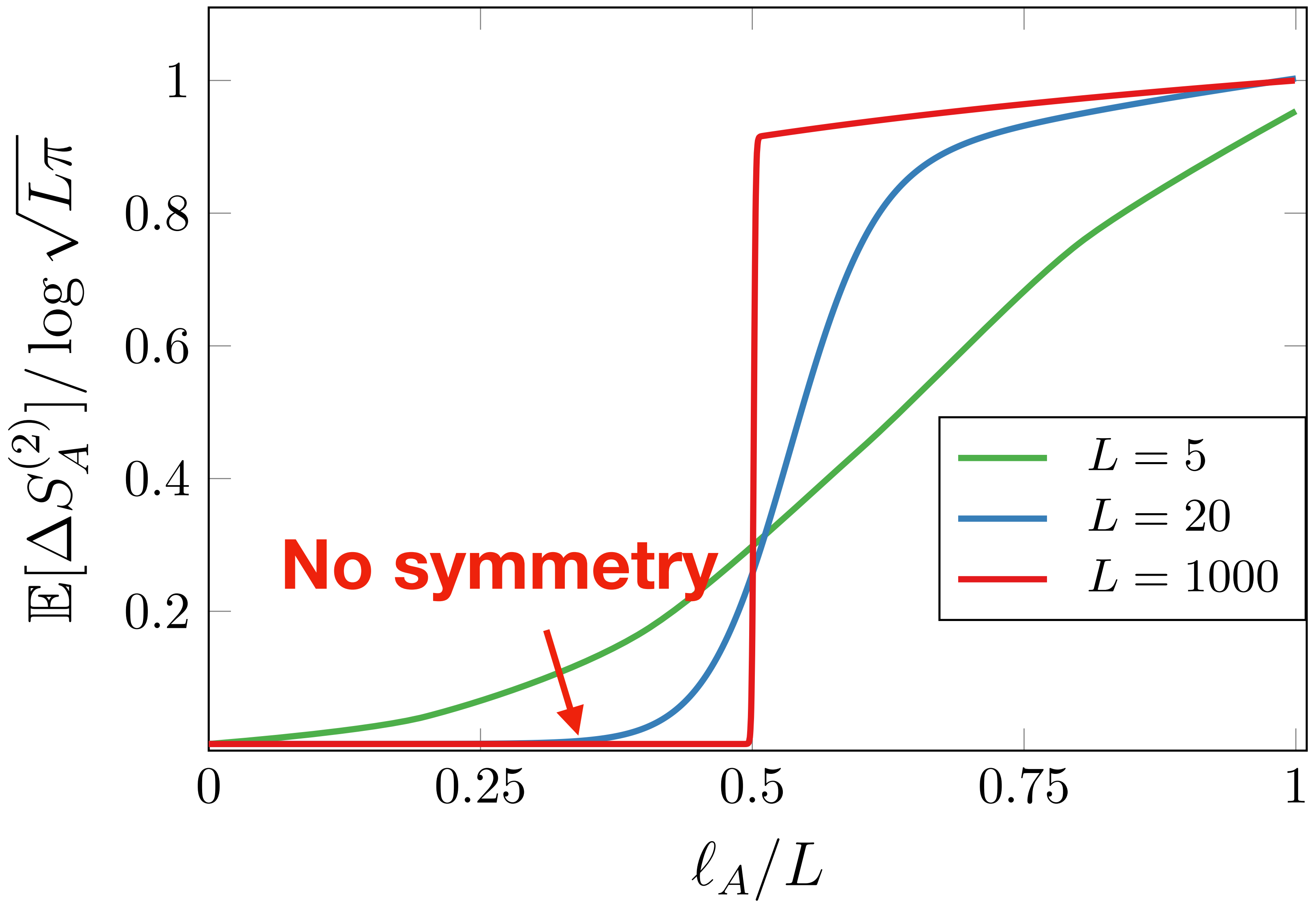
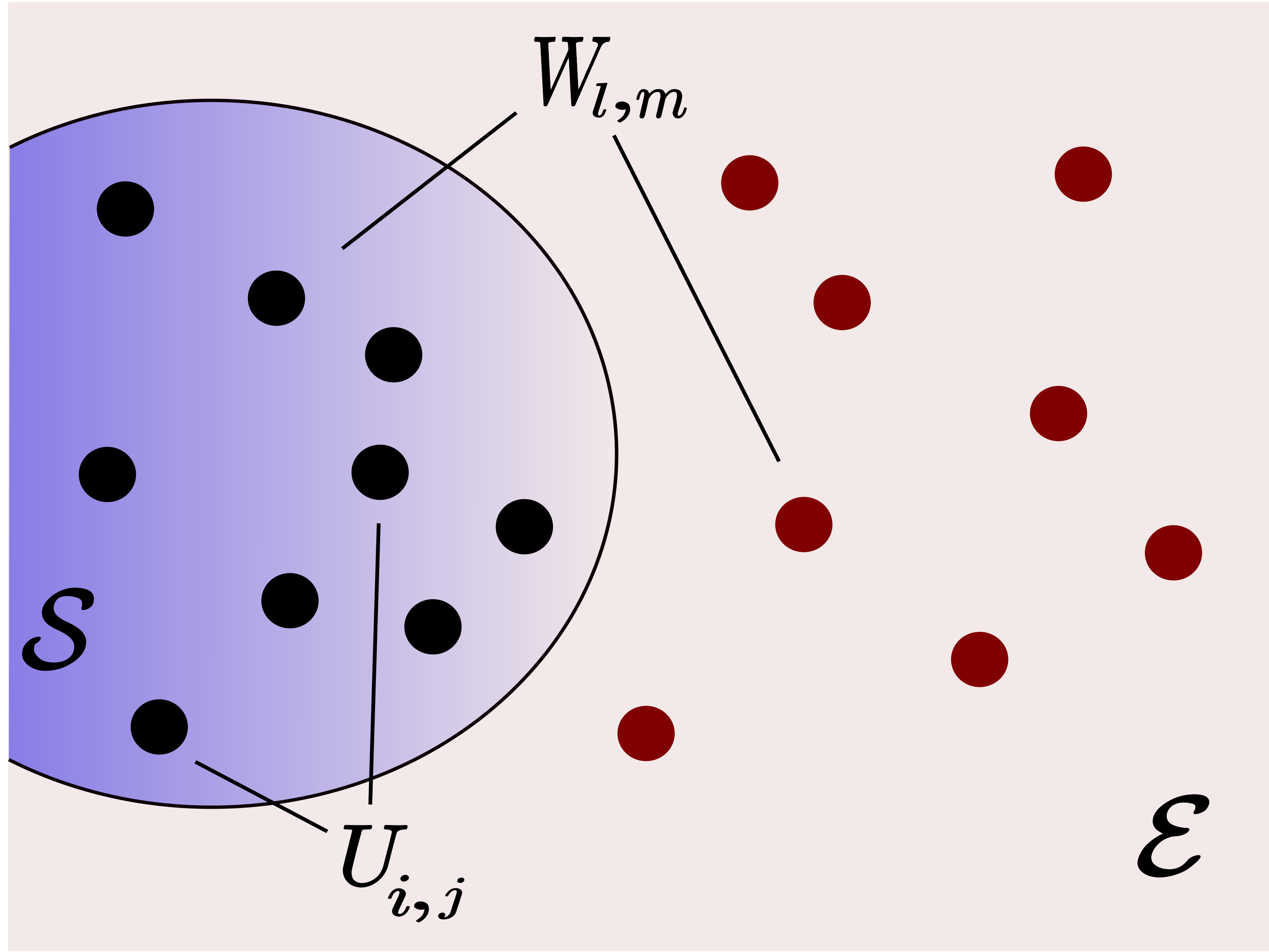


- Is the symmetry maximally broken for $\ell > L/2$?
- Evolution of entanglement asymmetry in maximally scrambling dynamics.
- Dynamical settings for black hole evaporation.



L. Piroli, C. Sünderhauf, X.-L. Qi, JHEP (2020) 2020 63.

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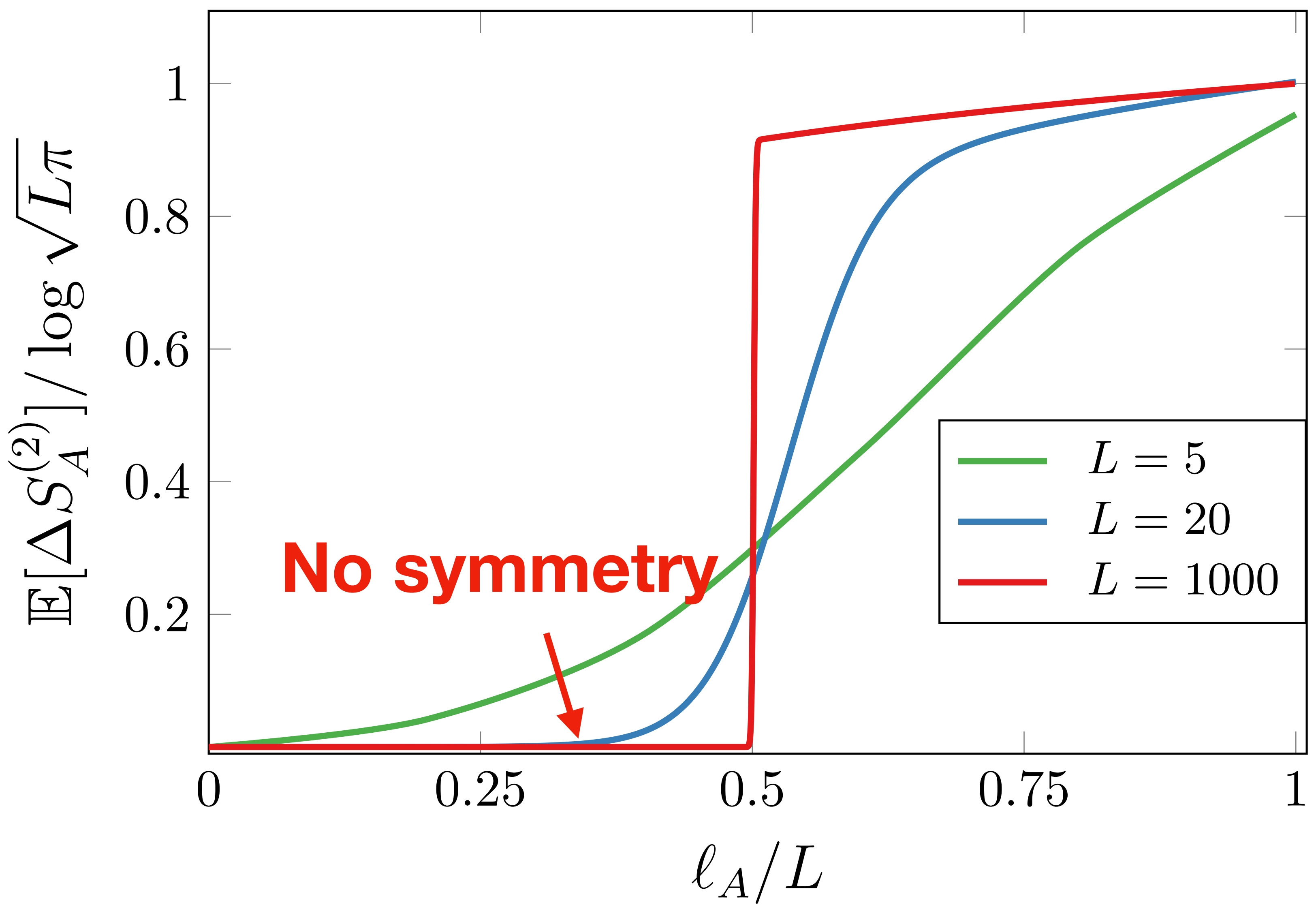

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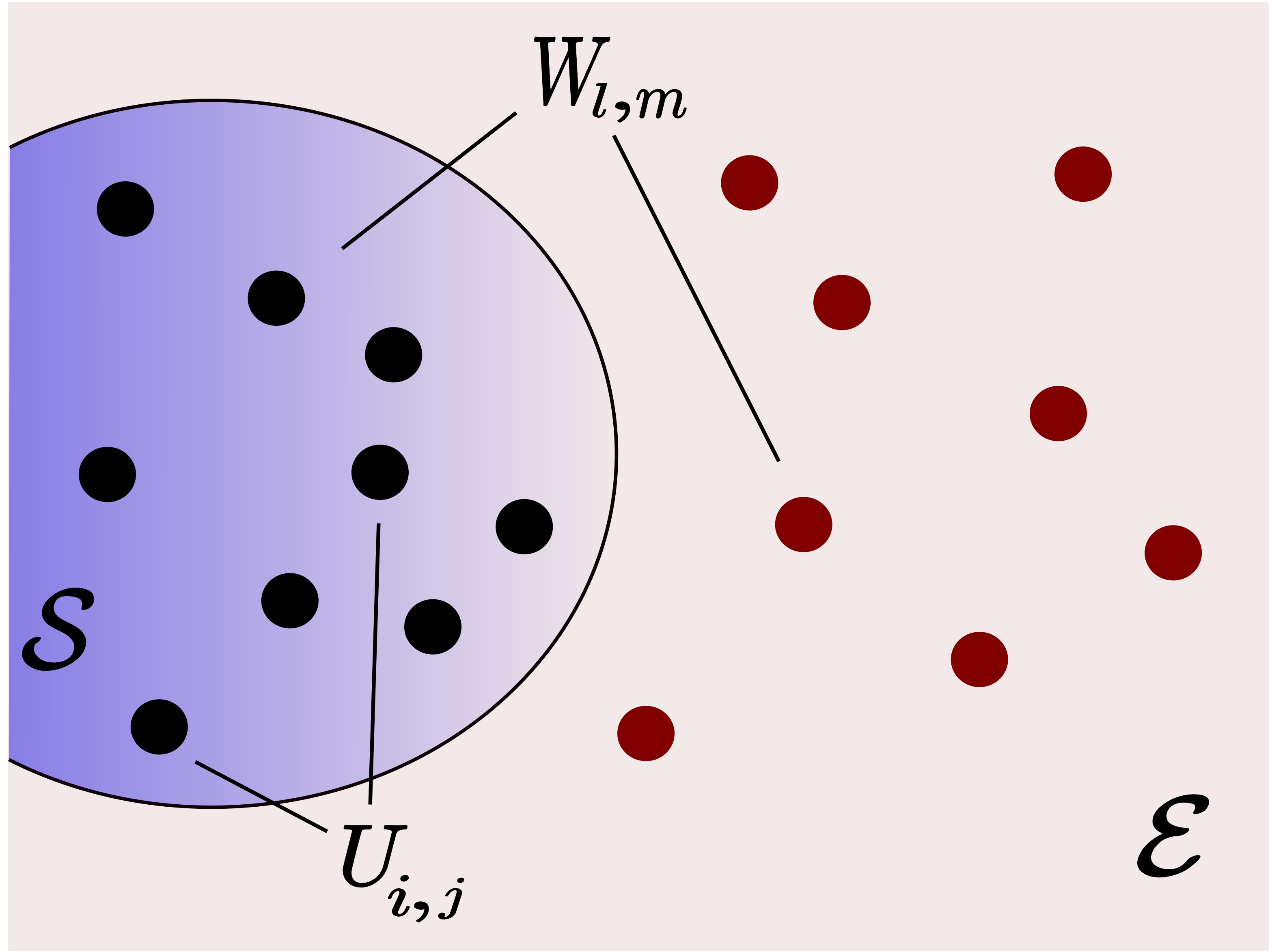

L. Piroli, C. Sünderhauf, X.-L. Qi, JHEP (2020) 2020 63.

Thanks!