Multi-parton interactions at hadron colliders

Luca Rottoli

University of Zurich^UZH

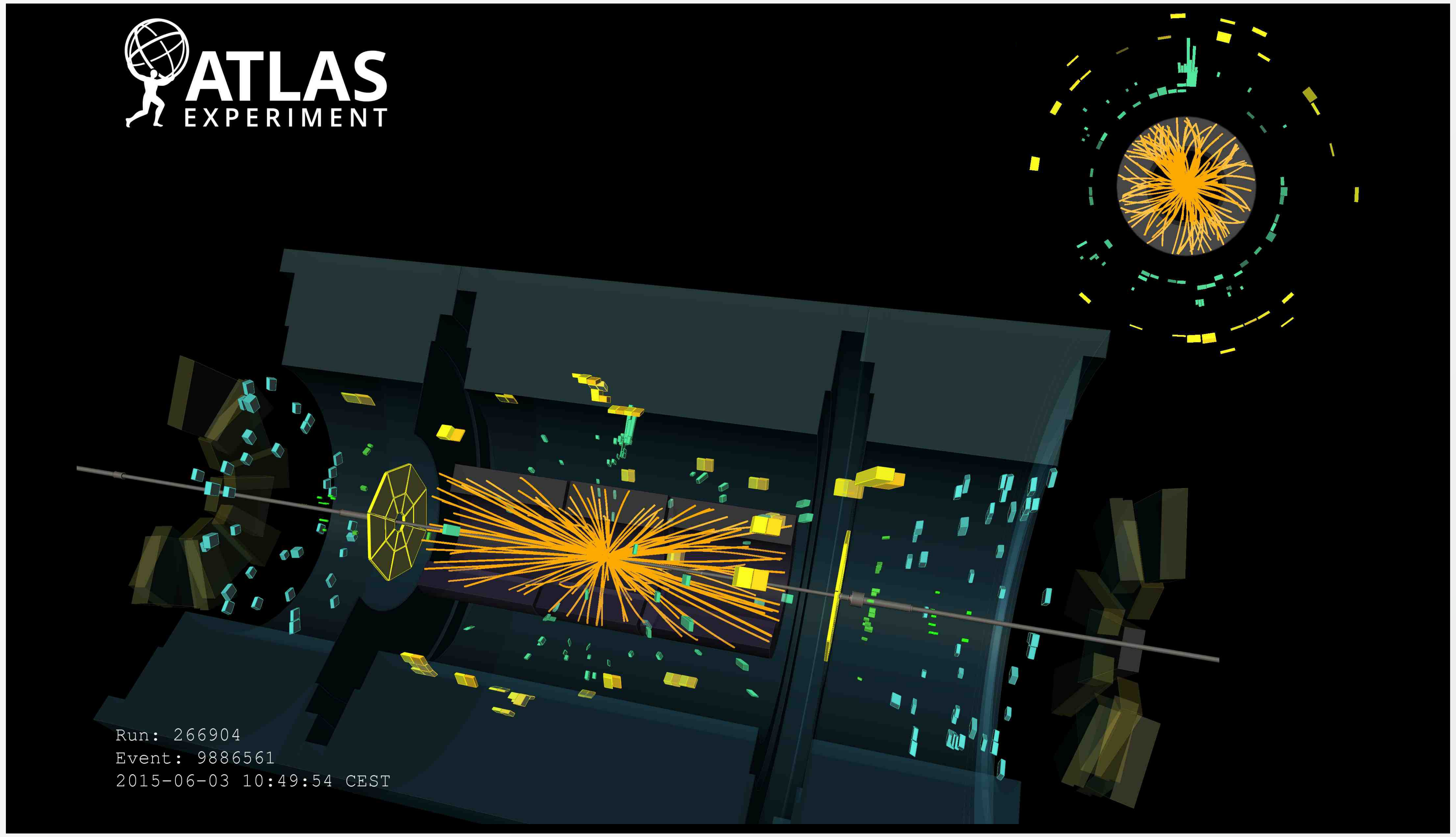
FNSNF

Swiss National Science Foundation

Multi-parton interactions

Inelastic hadron-hadron collisions are dominated by **soft** (low-p_T) interactions, also known as **minimum bias (MB)**

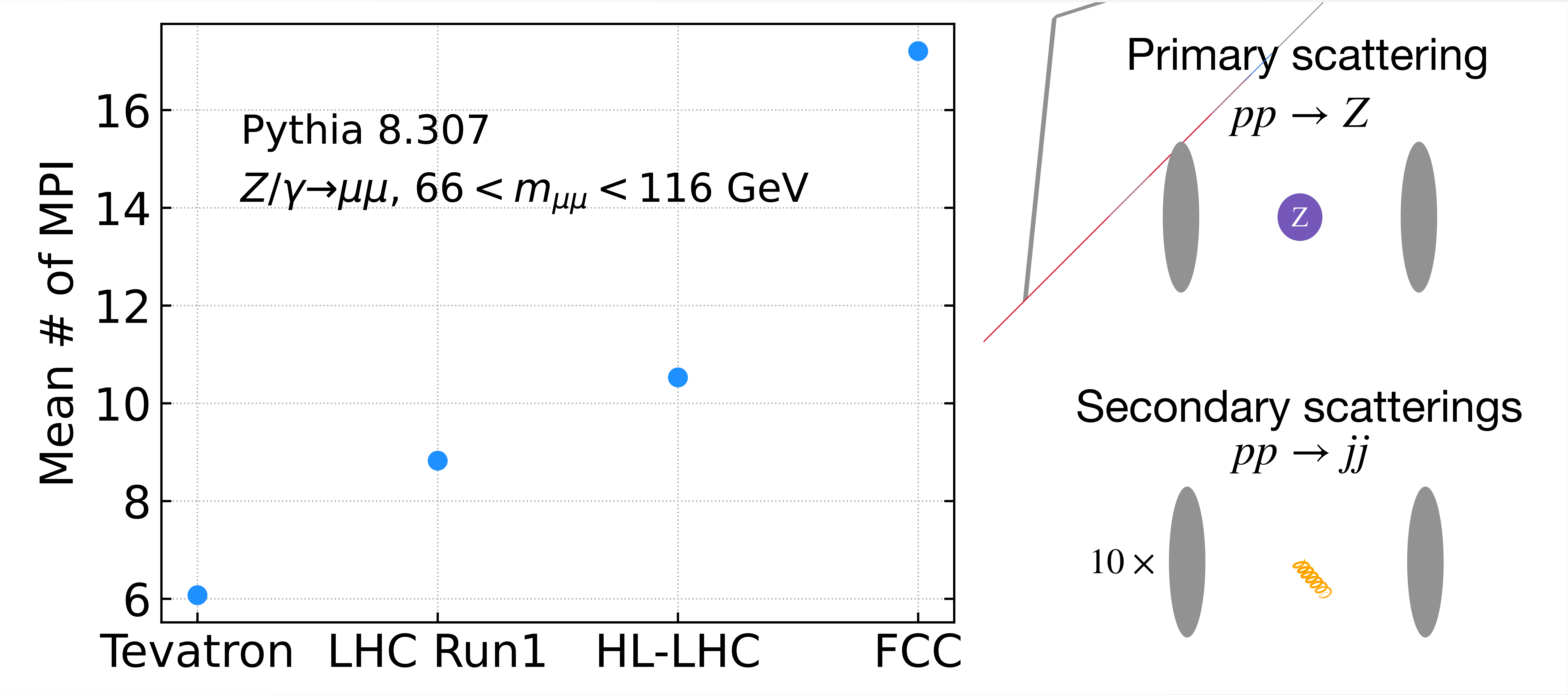
Typically associated with the **inelastic non-diffractive component** σ_{nd} of the total cross section σ_{tot}



Composite nature of hadrons implies that in such generic collisions **multi-parton interactions (MPI) are inevitable**

Underlying event at hadron colliders

This **remains true also when selecting events where a hard collisions takes place**: these collisions at the LHC can be interpreted as a hard scattering between partons accompanied by an “**underlying event” (UE)** of additional soft interactions



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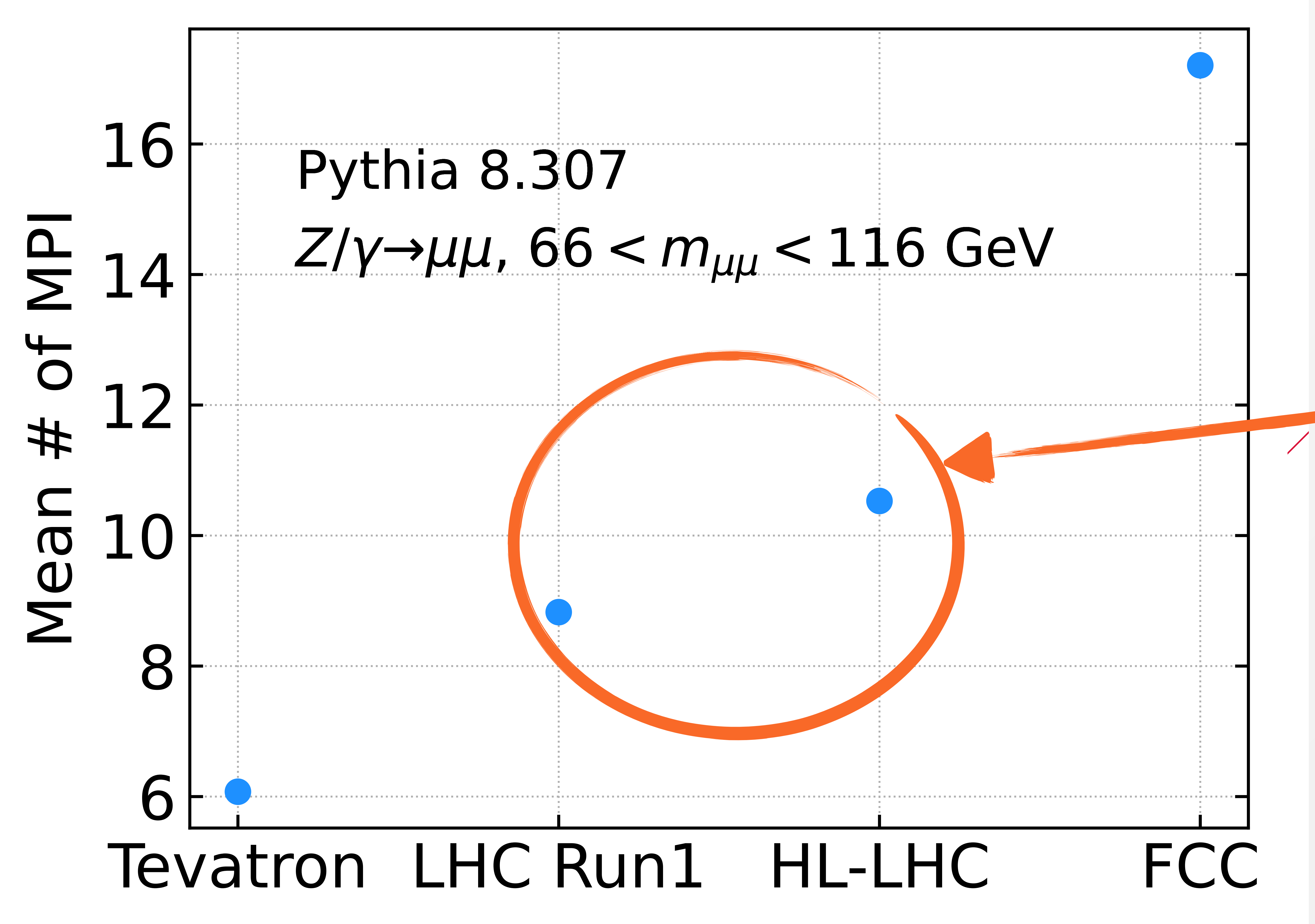

$\mathcal{O}(10)$ **additional parton-parton collisions** per Drell-Yan event at the LHC

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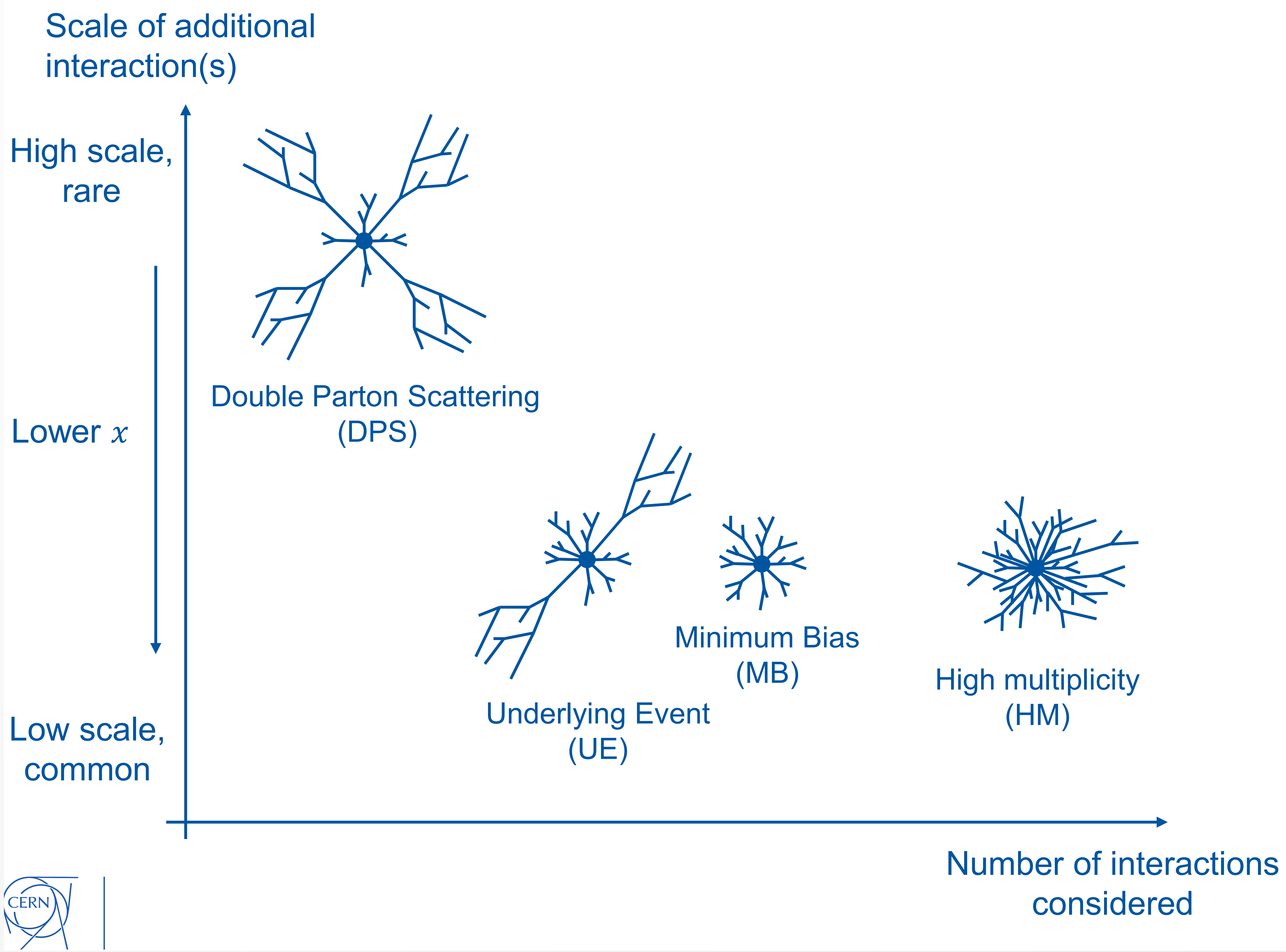
MPI ‘landscape’



Scale of additional interaction(s)

High scale, rare

Lower x

Low scale, common

Double Parton Scattering (DPS)

Underlying Event (UE)

Minimum Bias (MB)

High multiplicity (HM)

Number of interactions considered

Gaunt, Bartalini MPI@LHC 2018

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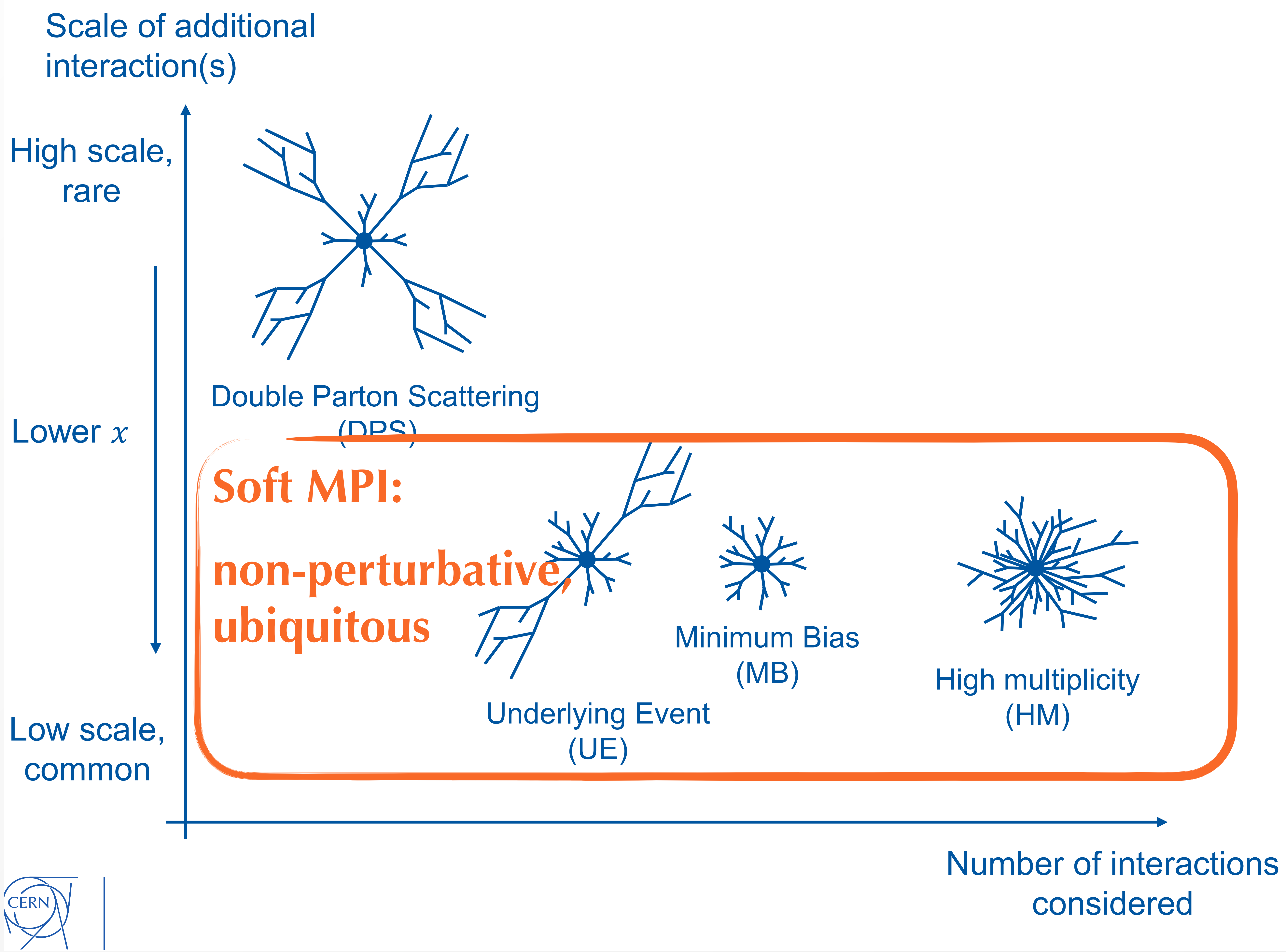

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Double Parton Scattering (DPS)

Soft MPI:

non-perturbative, ubiquitous

Underlying Event (UE)

Minimum Bias (MB)

High multiplicity (HM)

Number of interactions considered

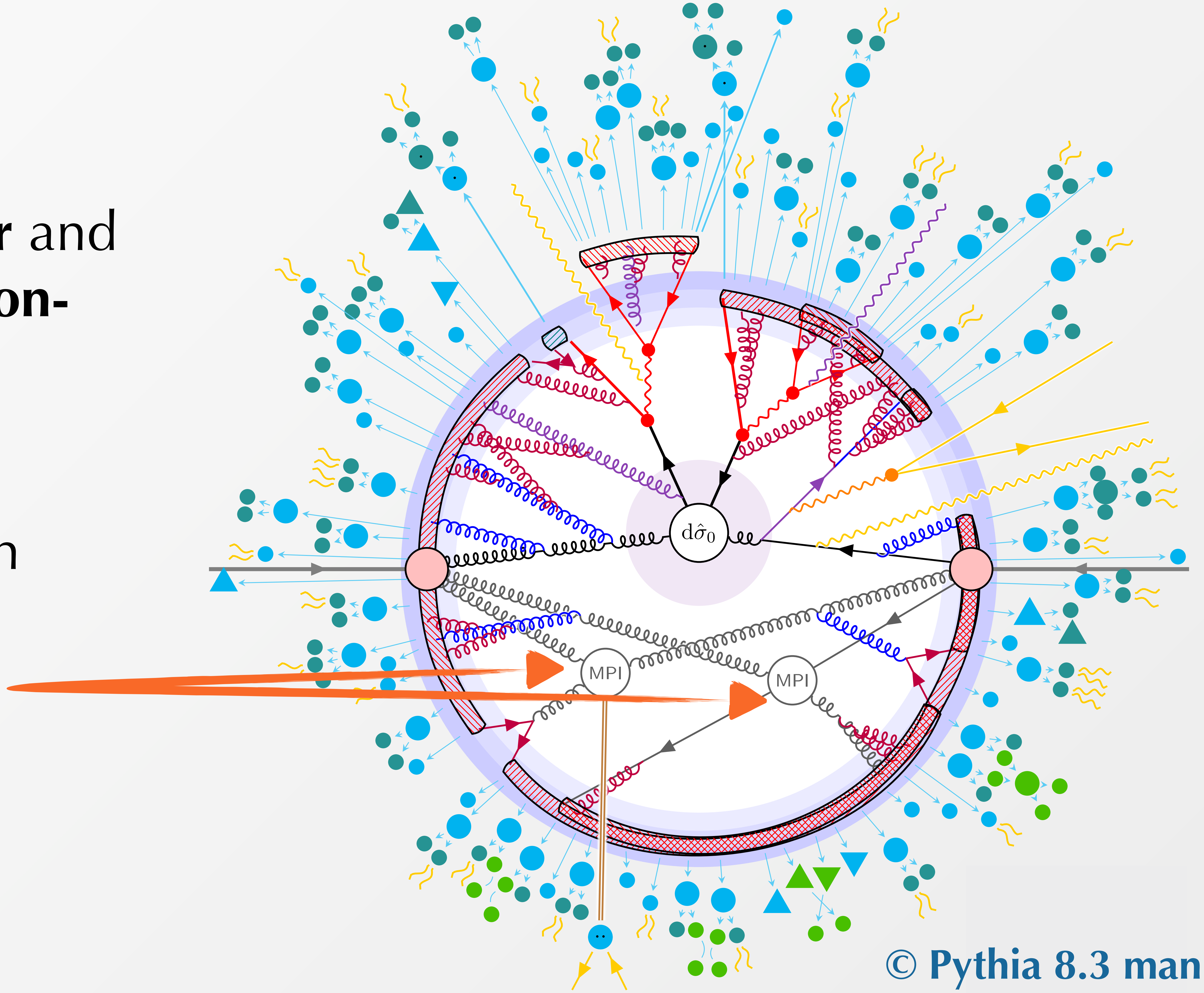
Gaunt, Bartalini MPI@LHC 2018

MPI modelling in MC event generators

Accurate modeling of additional (**soft**) scatters is **crucial to ensure proper interpretation of hard events at hadronic colliders**

A complete description of the MPI is challenging due to their **large number** and complex interplay and their **chiefly non-perturbative nature**

At present soft MPI are modelled in an approximate way by **general-purpose Monte Carlo (MC) event generators**



© Pythia 8.3 manual

Basics of MPI modelling

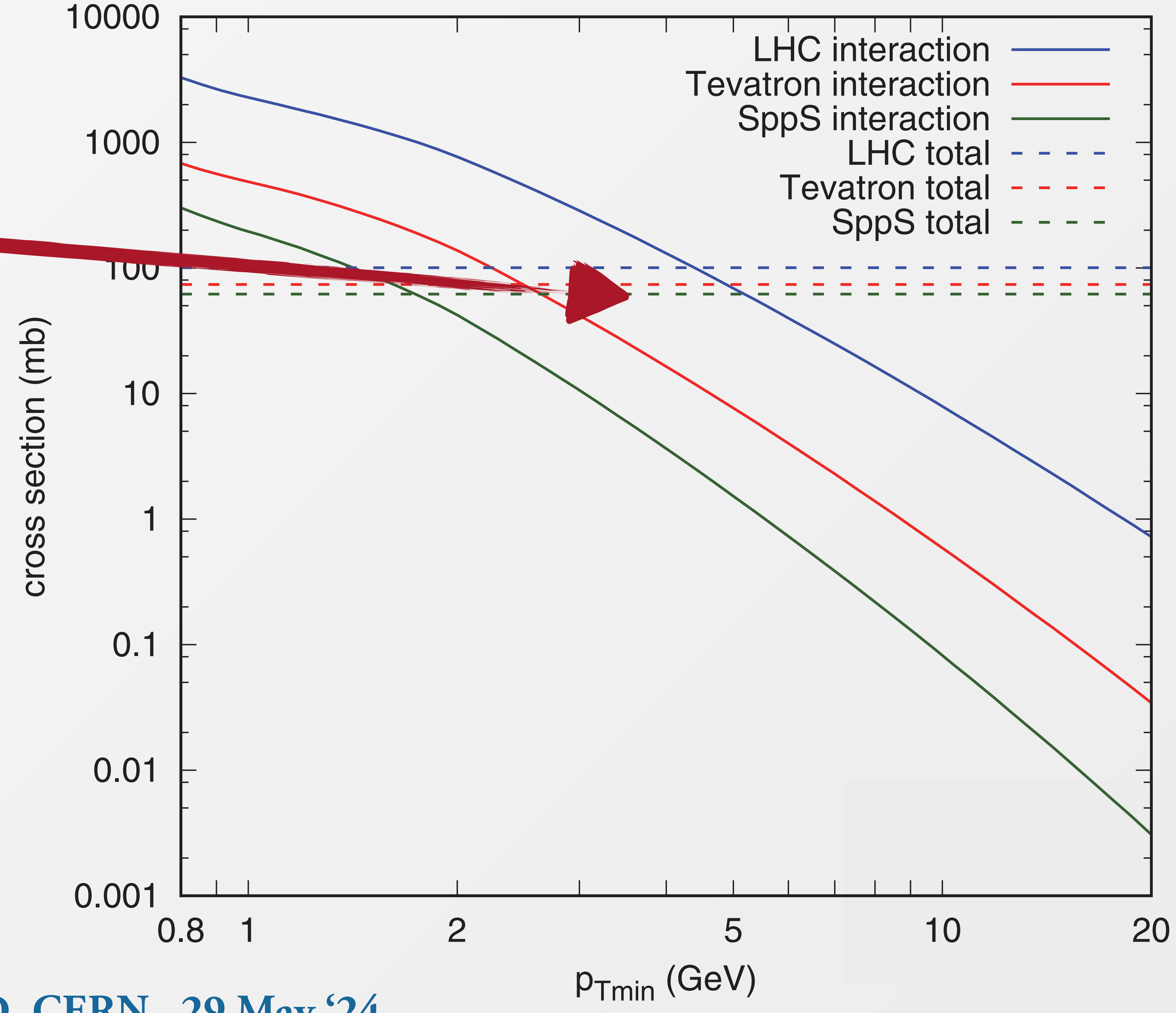
MC event generators such as Pythia, Herwig and Sherpa have refined and extended the modelling of MPI during the years, but all share the **same fundamental model** developed by T. Sjostrand and M. van Zijl in 1987 in the effort of improving the description of S$\bar{p}p$S data

Starting point of the model is the differential perturbative QCD 2 → 2 dijet cross section

$$\frac{d\sigma}{dp_T^2} = \sum_{i,j} \int f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}} \delta(p_T^2 - \hat{t}\hat{u}/\hat{s}) dx_1 dx_2 d\hat{t}$$

The integrated cross section depends on the chosen $p_{T,\text{min}}$ scale and ultimately **exceeds the total cross section** even when $p_{T,\text{min}} \gg \Lambda_{\text{QCD}}$

$$\sigma_{\text{int}}(p_{T,\text{min}}) = \int_{p_{T,\text{min}}}^{\sqrt{s}/2} \frac{d\sigma}{dp_T} dp_T \sim \frac{1}{p_{T,\text{min}}^2}$$



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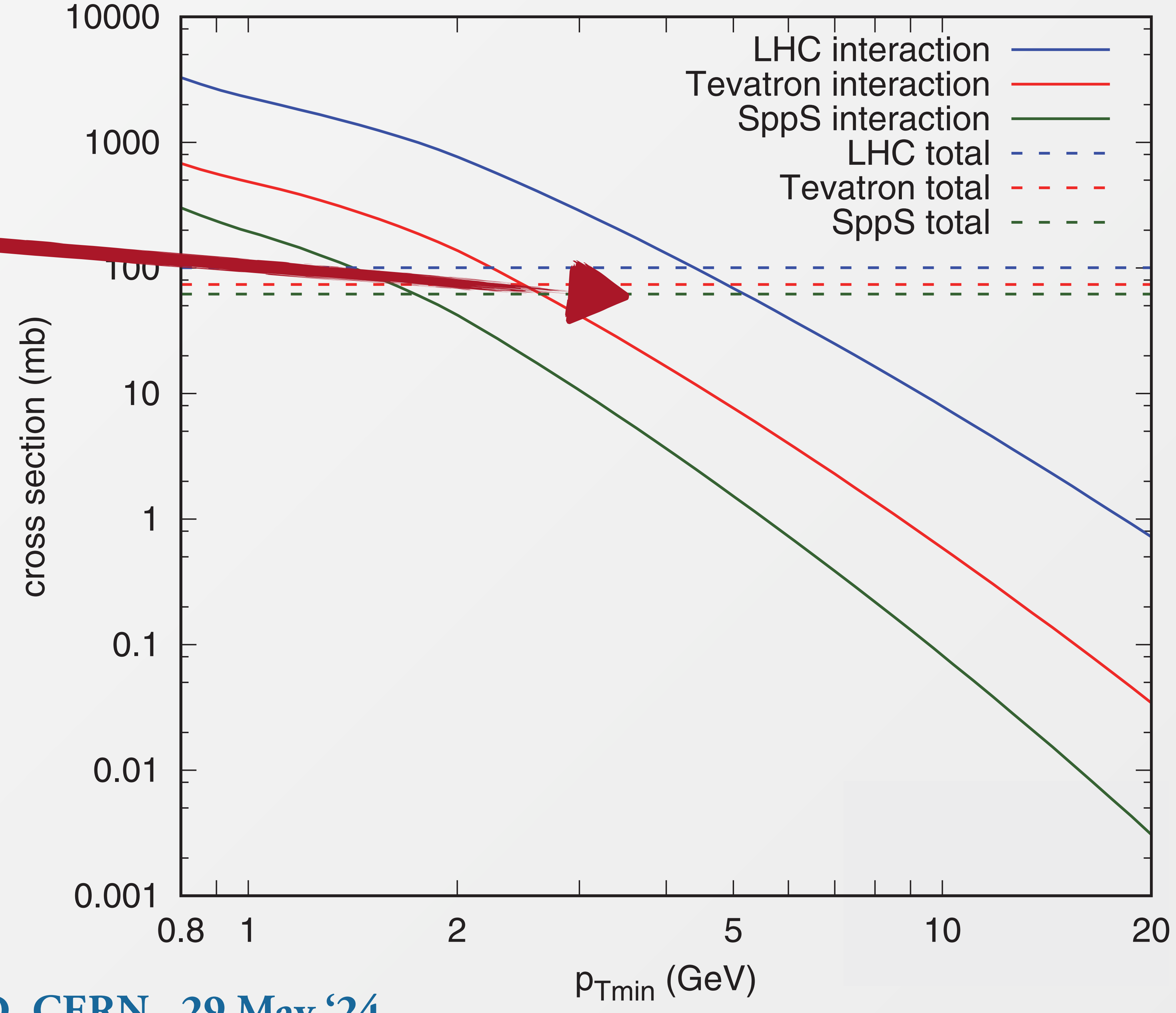


Interpretation: several parton-parton interactions per hadron-hadron interactions

$$\langle n_{\mathrm{MPI}}(p_{T,\mathrm{min}}) \rangle \simeq \frac{\sigma_{\mathrm{int}}(p_{T\mathrm{min}})}{\sigma_{\mathrm{tot}}}$$

Example: MPI modelling in Pythia

Assuming that each interaction is independent: $\langle n_{\rm MPI}(p_{T,\rm min})\rangle$ follows a Poissonian distribution

A naive approach does not take into account even basic correlations (e.g. energy–momentum conservation): the sum of interaction energies may exceed the total CM energy

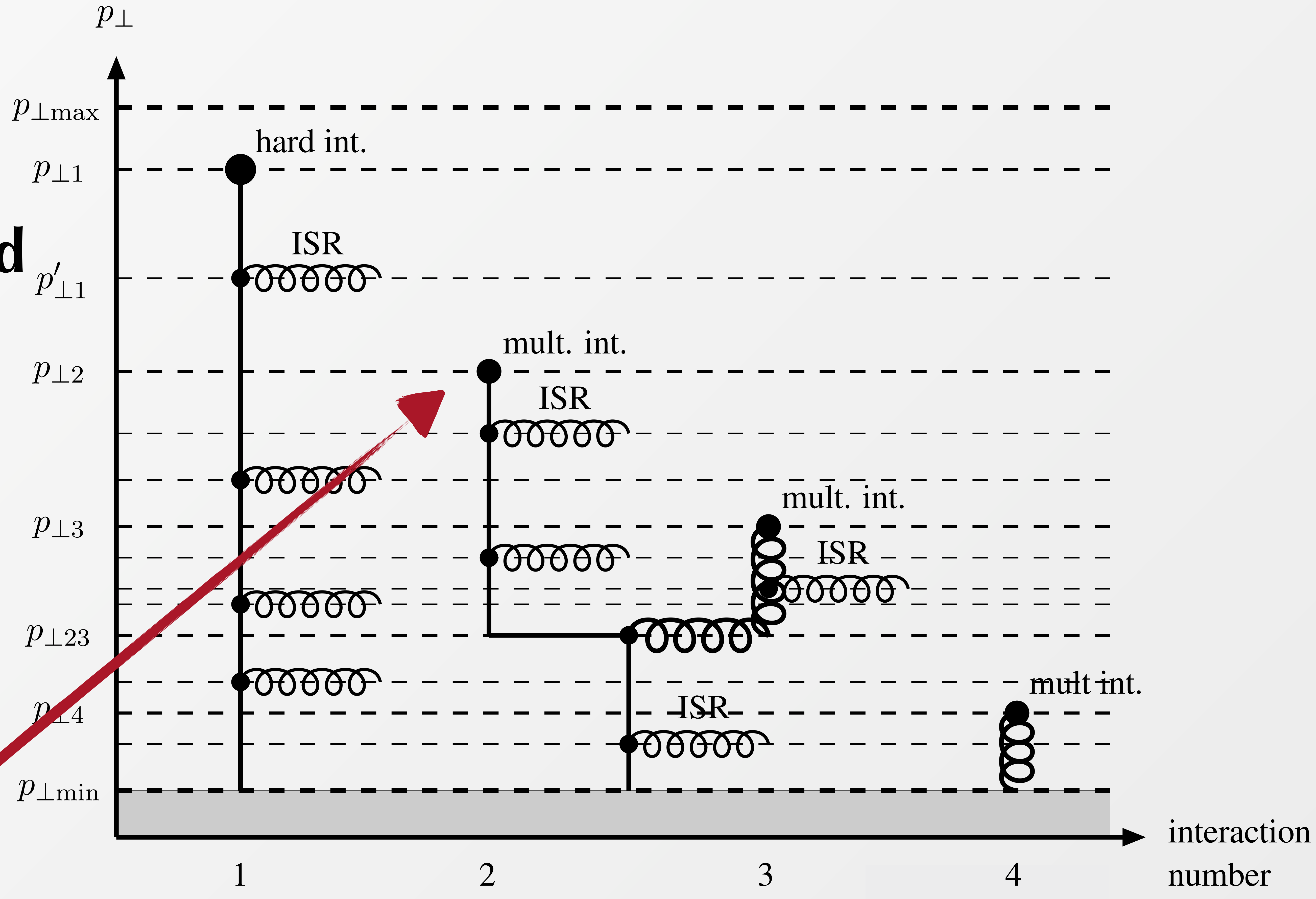
Solution to the problem comes by recasting MPI as a **Sudakov-style evolution**

Generation of MPI formulated as a **downward evolution in p_T** with probability

$$\frac{d\mathscr{P}}{dp_{T,i}} = \frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dp_{T,i}} \exp\left(- \int_{p_{T,i}}^{p_{T,i-i}} \frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dp_T} dp_T \right)$$

Interleave of MPI, ISR and FSR evolution in **one common sequence** in p_T

[Sjostrand, Skands 2005][Sjostrand, Cork 2011]

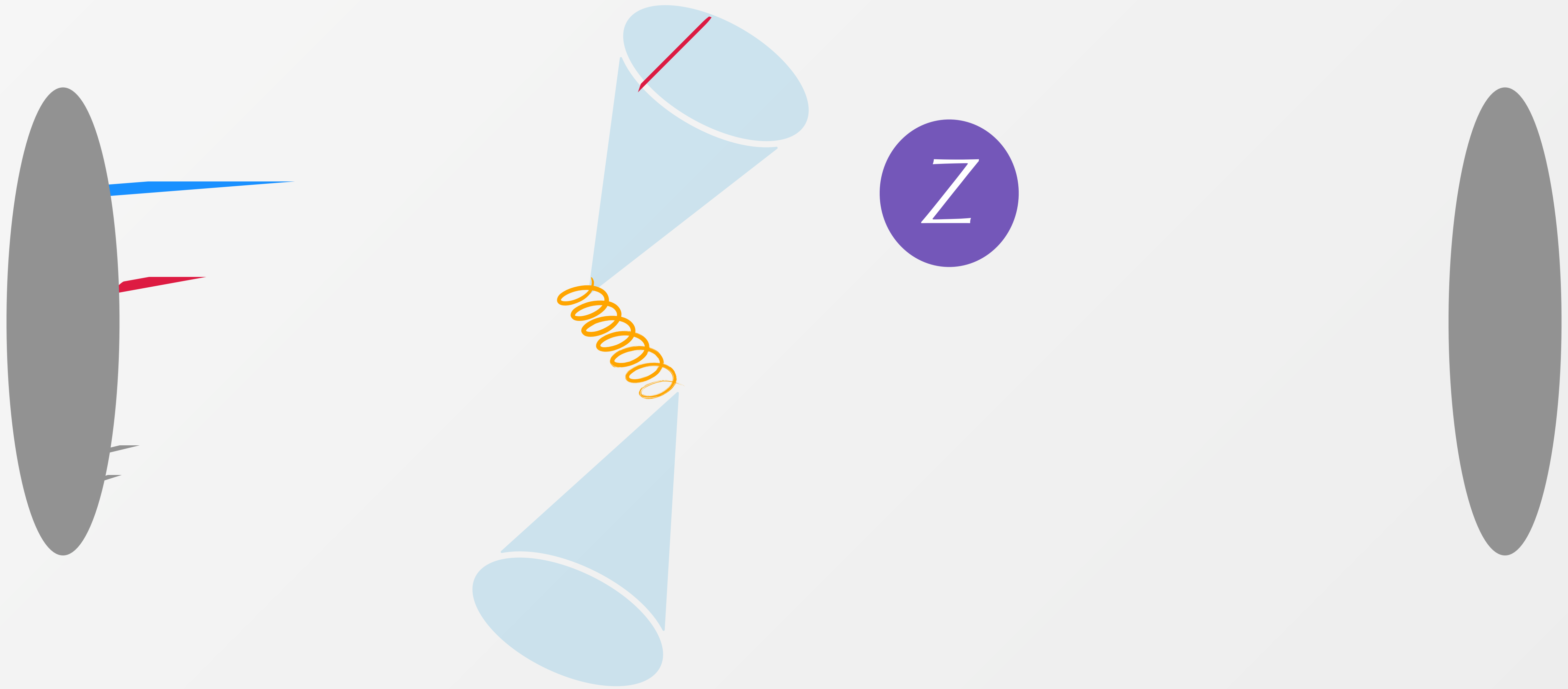


Soft MPI: summary

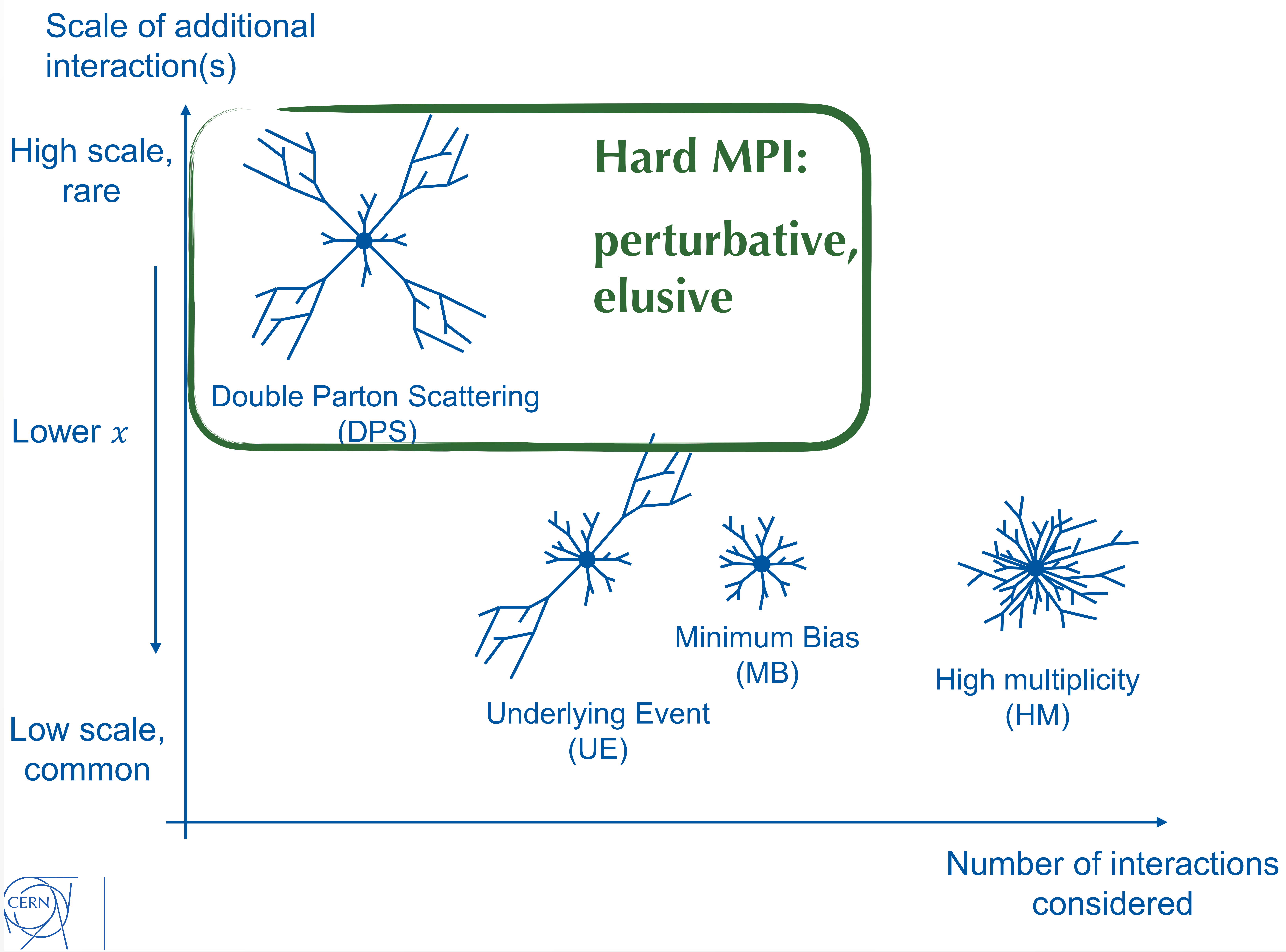
Monte Carlo models seek an unified description of hard jets, UE and MB

Although inspired by theoretical ideas, the development of MPI models is **rather decoupled from detailed theoretical calculations** due to the complexity of the environment with $\mathcal{O}(10)$ **soft** MPIs

An alternative approach to study MPI physics in a **much cleaner environment** is in events with two (or more) separate **hard processes** in an individual hadron-hadron collisions: double- or triple-parton scattering (DPS, TPS), more amenable to first-principle theoretical approaches



MPI ‘landscape’



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High scale, rare

Lower x

Low scale, common

Double Parton Scattering (DPS)

Hard MPI:

perturbative, elusive

Underlying Event (UE)

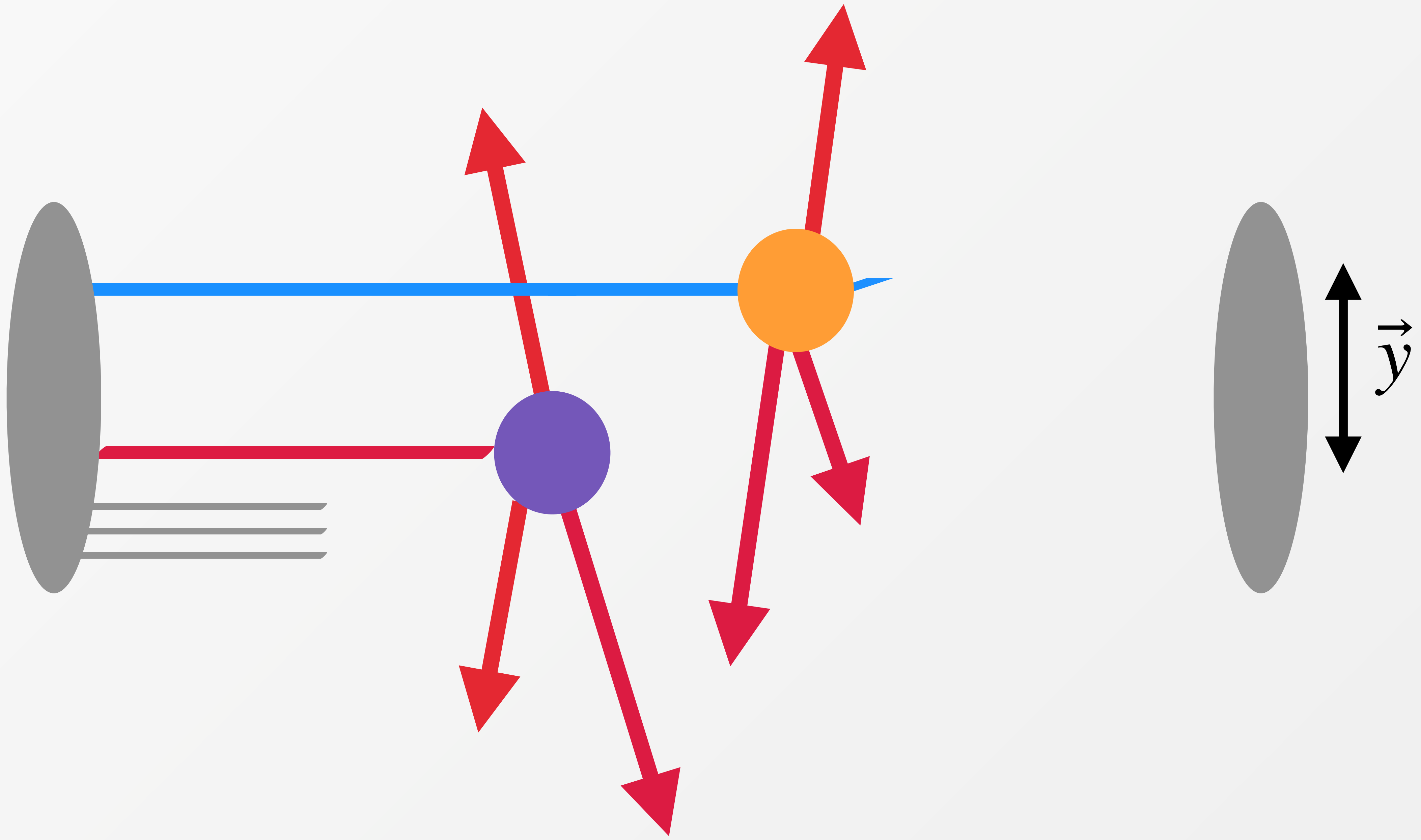
Minimum Bias (MB)

High multiplicity (HM)

Number of interactions considered

Gaunt, Bartalini MPI@LHC 2018

Double parton scattering: basic features



Two separate **hard interactions** in a **single** proton-proton collision

Postulated factorisation formula for integrated DPS cross section based on parton model / low order Feynman diagrams considerations

Parton level cross sections

$$\sigma^{A,B} = \frac{m}{2}\sum_{i,j,k,l}\int F_h^{ik}(x_1,x_2,\vec{y},Q_A,Q_B)F_h^{jl}(x_1',x_2',\vec{y},Q_A,Q_B)\times\hat{\sigma}_{ij}^{A}(x_1,x_1')\hat{\sigma}_{kl}^{B}(x_2,x_2')\,dx_1dx_1'dx_2dx_2'd^2\vec{y}$$

Collinear double-parton distributions

Double parton scattering: simplified assumptions

If **correlations between partons** are ignored

$$F_h^{ik}(x_1, x_2, \vec{y}) = \int d^2\vec{b} D^i(x_1, \vec{b}) D^j(x_1, \vec{b} + \vec{y})$$

D: impact-parameter dependent PDFs

Justified at small x given the large population of partons

Often the further approximation $D^i(x_1, \vec{b}) = D^i(x_1) G(\vec{b})$ is made, leading to the **pocket formula**

$$\sigma_{AB} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}}$$

where $\sigma_{\text{eff}} \sim O(20\,\text{mb})$ is a normalisation factor roughly connected with area over which partons are concentrated in the proton.

Challenges in hard MPI studies

Distinguishing two contributions:

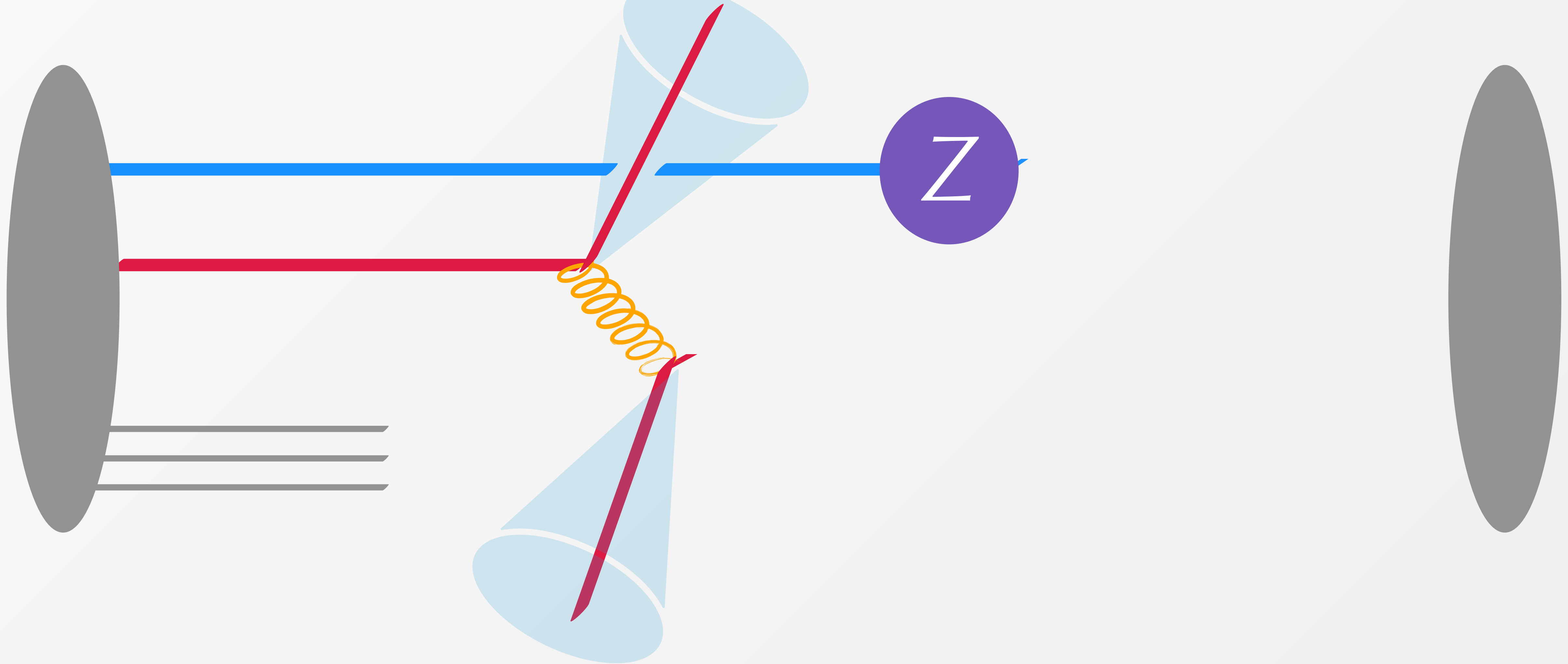
- two independent hard scatterings **(2HS)**
- a single hard scattering **(1HS)** with extra radiation

e.g. *Z* boson production: both contributions have experimental signature of *Z* boson (→ 2 leptons) + jets

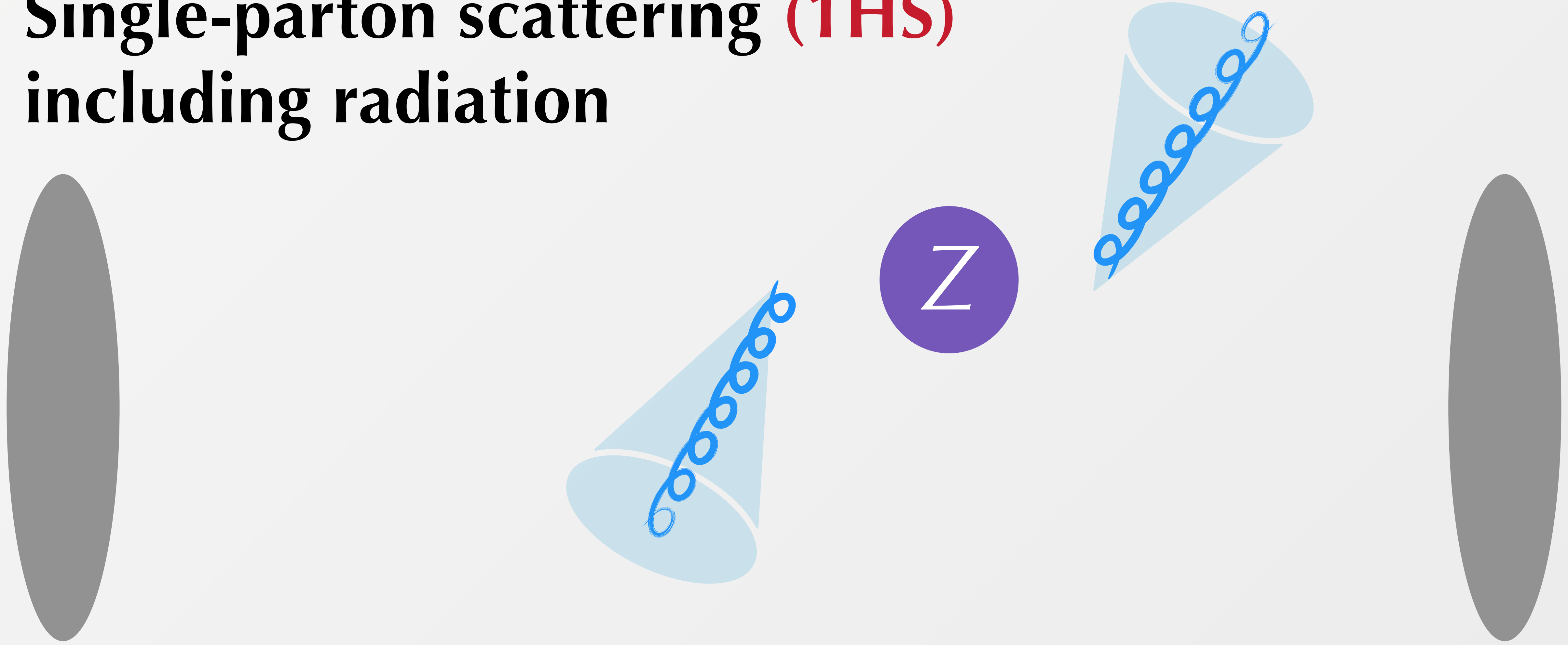
At the level of the total cross section, 2HS mechanism is power suppressed with respect to 1HS

$$\sigma_{2HS}/\sigma_{1HS} \sim \Lambda^2/Q^2$$

Double-parton scattering (2HS)



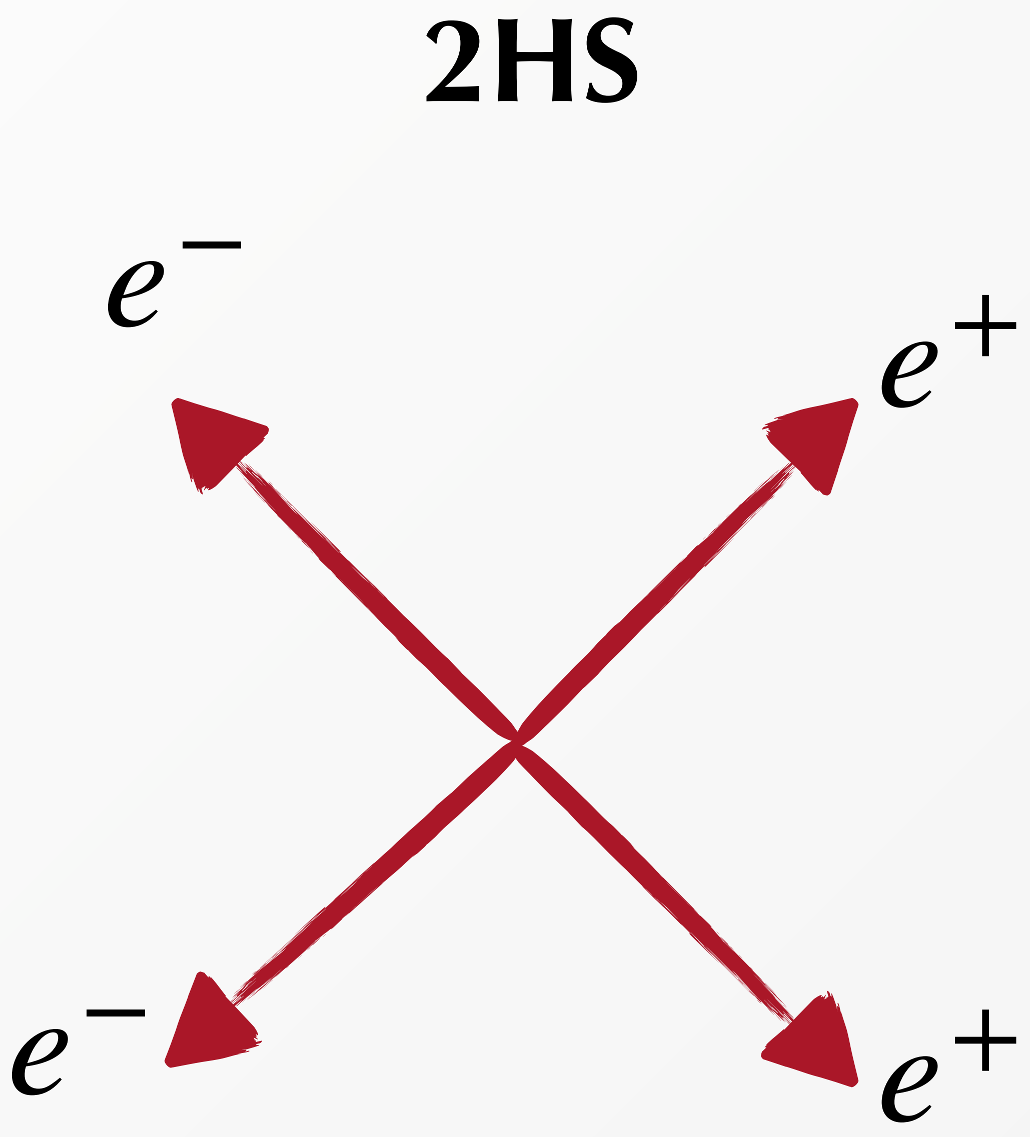
Background from Single-parton scattering (1HS) including radiation

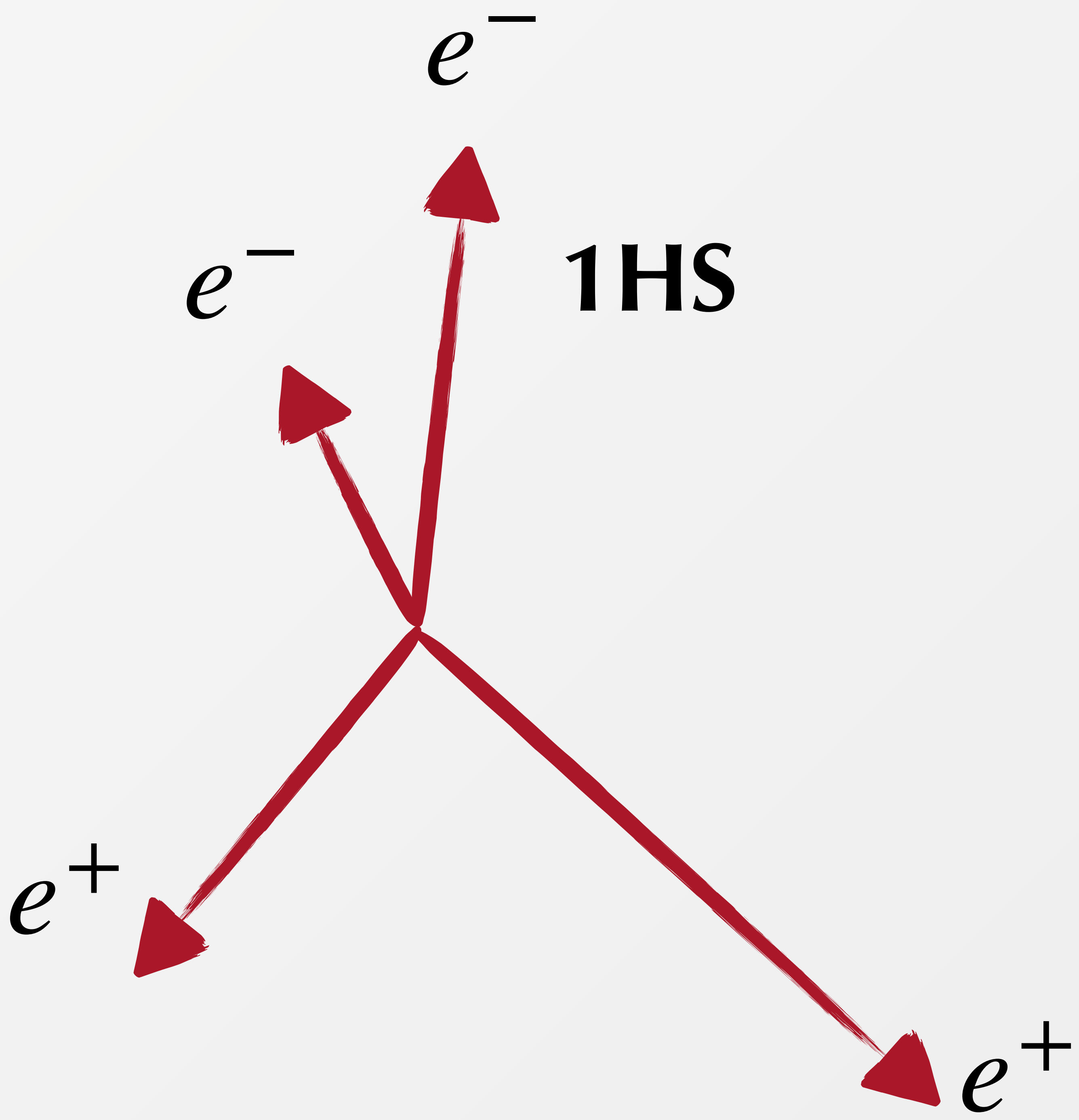


Strategy A

DPS populates the final state phase space in a different way from 1HS. In particular, for processes A and B it tends to populate the region of small $\vec{q}_A$, $\vec{q}_B$

$$pp \to e^+e^-e^+e^-$$



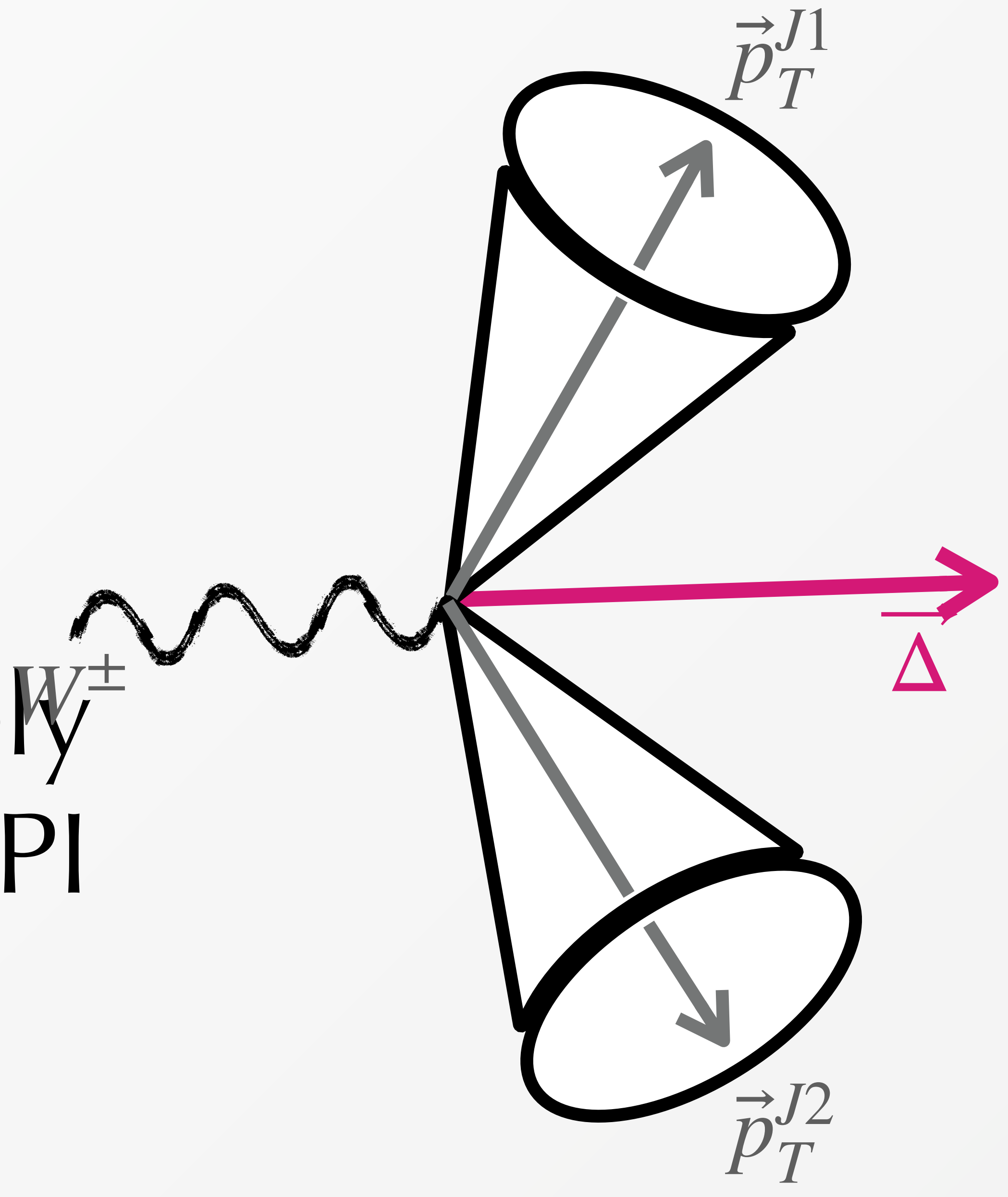


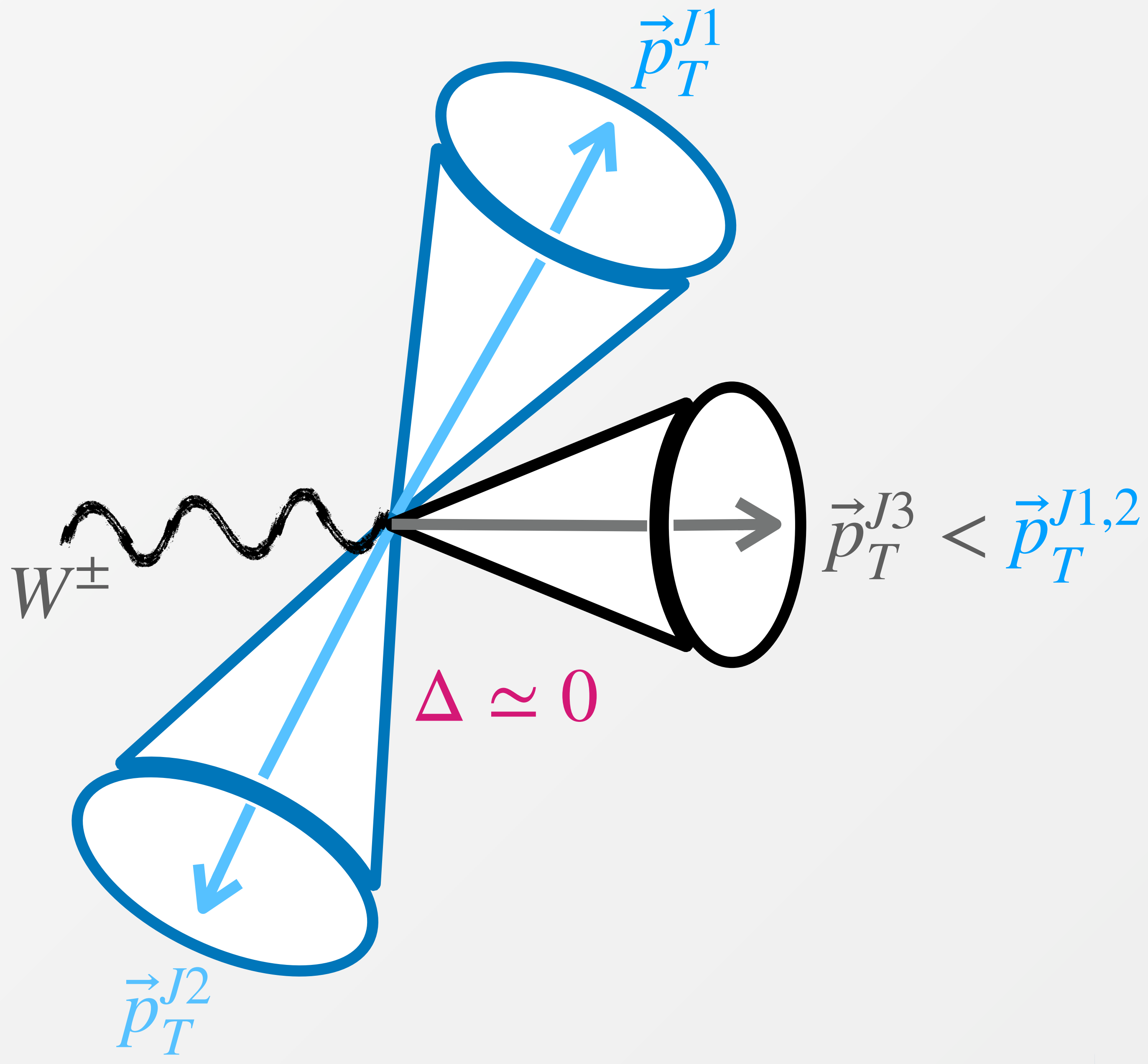
W+2-jets study

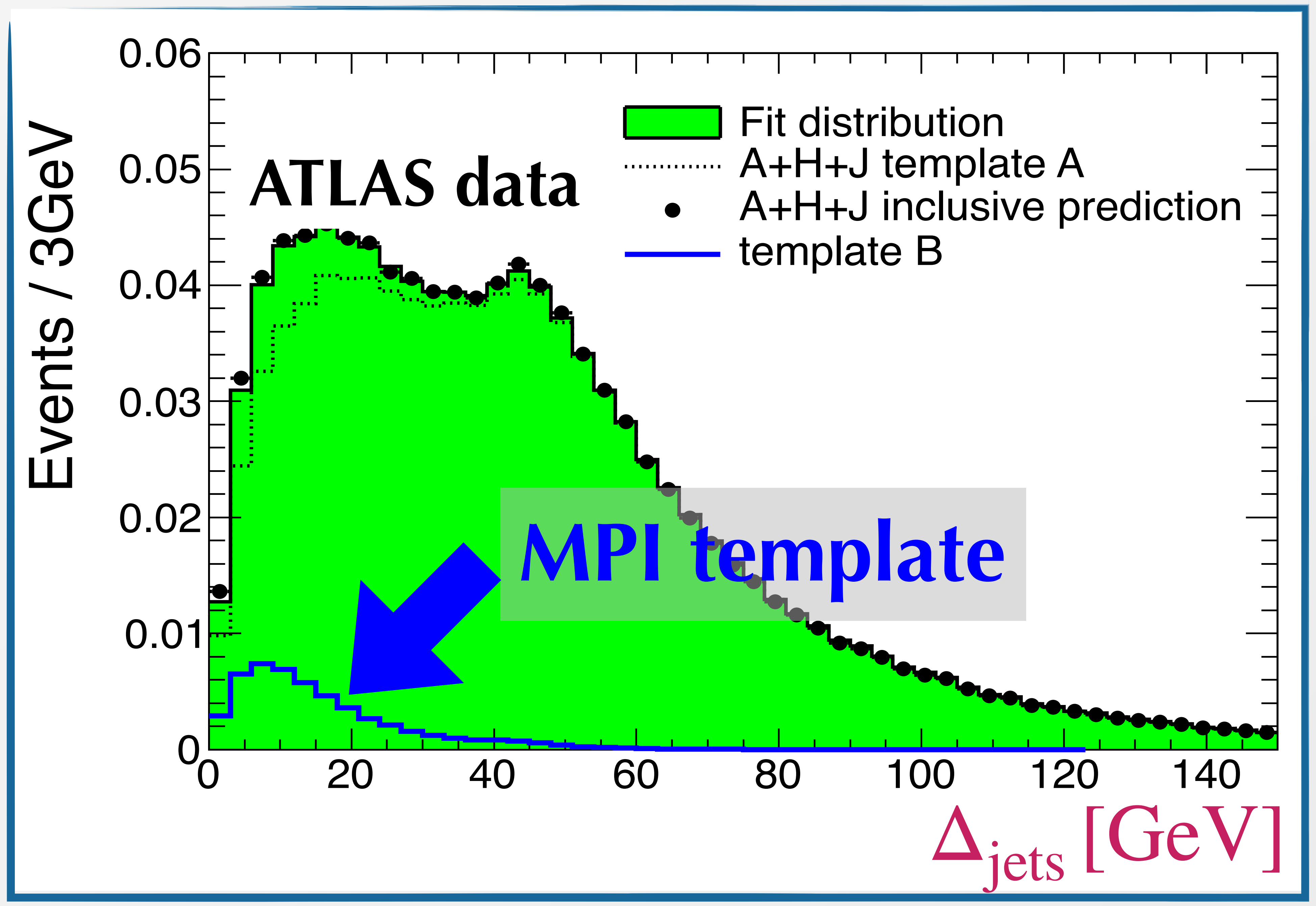
- E.g. ATLAS, $W \to \ell\nu$ + 2 jets 1301.6872
- Exploits fact that MPI jet-pair more likely to balance than radiation jet pair, so MPI should be enhanced for

$$\Delta_{\text{jets}} = \left|\vec{p}_T^{\,\text{J1}} + \vec{p}_T^{\,\text{J2}}\right| \to 0$$

- That works to some extent, but relative MPI (2HS) fraction is moderate ($\lesssim$ 10%)
- Quantitative analysis requires very good understanding of radiation in single hard scattering (1HS)



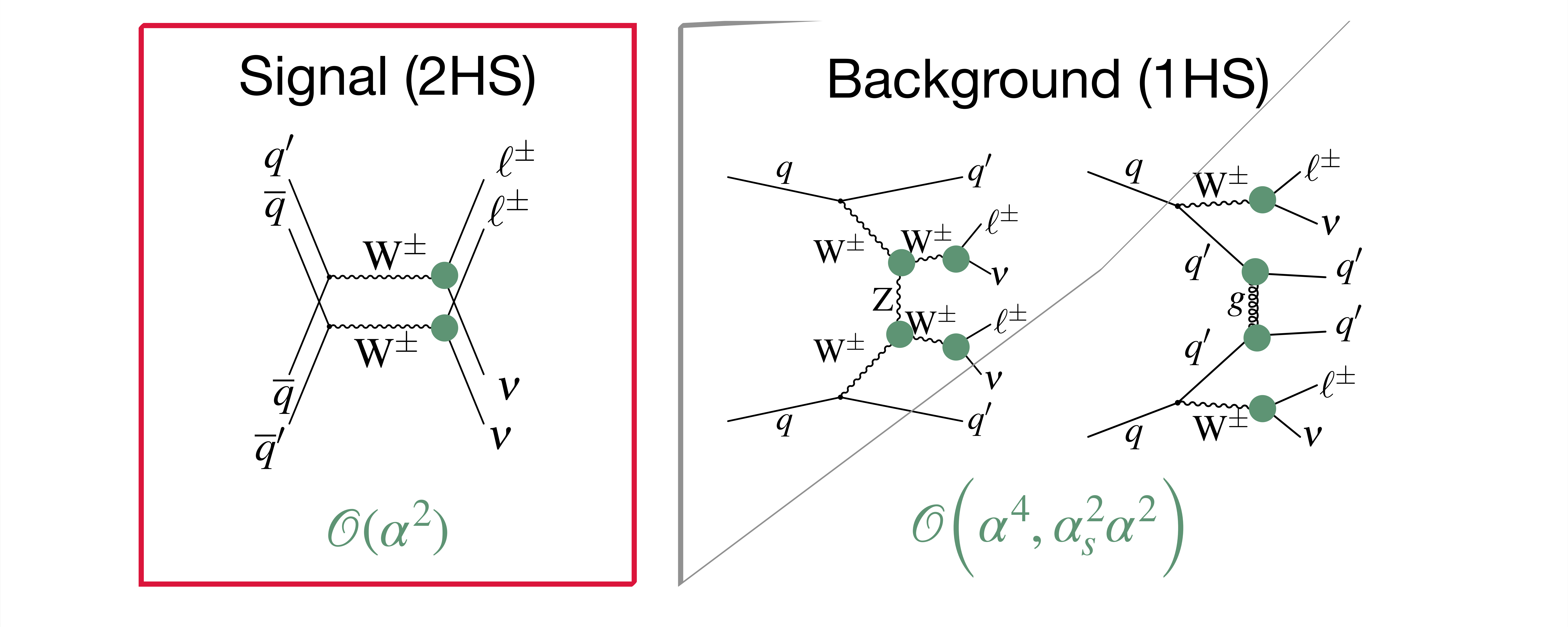




[ATLAS 1301.6872]

Strategy B

Identify processes where **1HS mechanism is suppressed** (e.g. small couplings)



Signal (2HS)

$\mathcal{O}(\alpha^2)$

Background (1HS)

$\mathcal{O}\left(\alpha^4, \alpha_s^2\alpha^2\right)$

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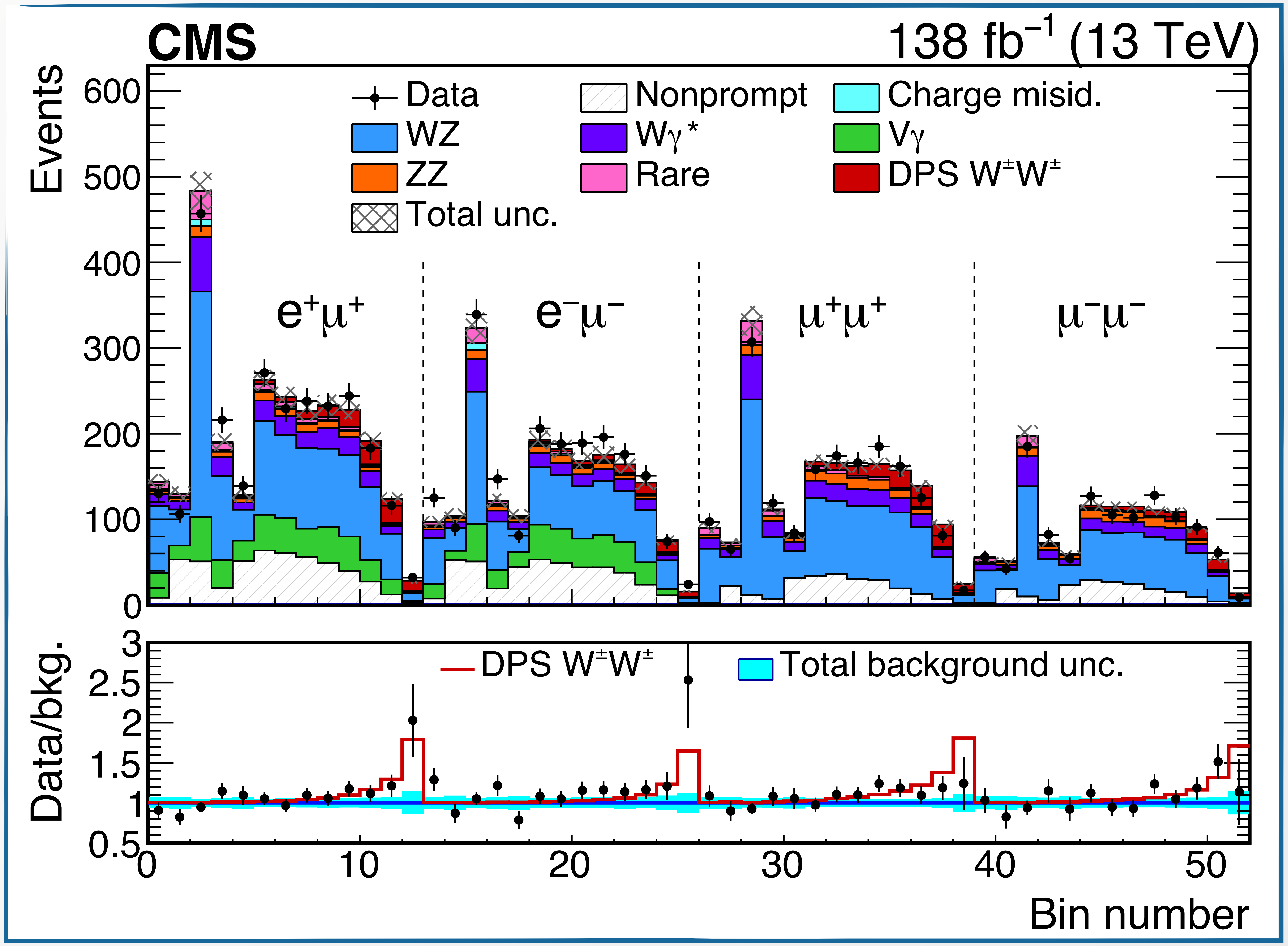
Avoid radiation issue: same-sign *WW*

- Here $W^{\pm}W^{\pm} \rightarrow$ same-sign leptons, CMS 2206.02681
- many other backgrounds: need for BDT makes it difficult to study MPI physics
- 6.2σ observation with full Run 2 dataset

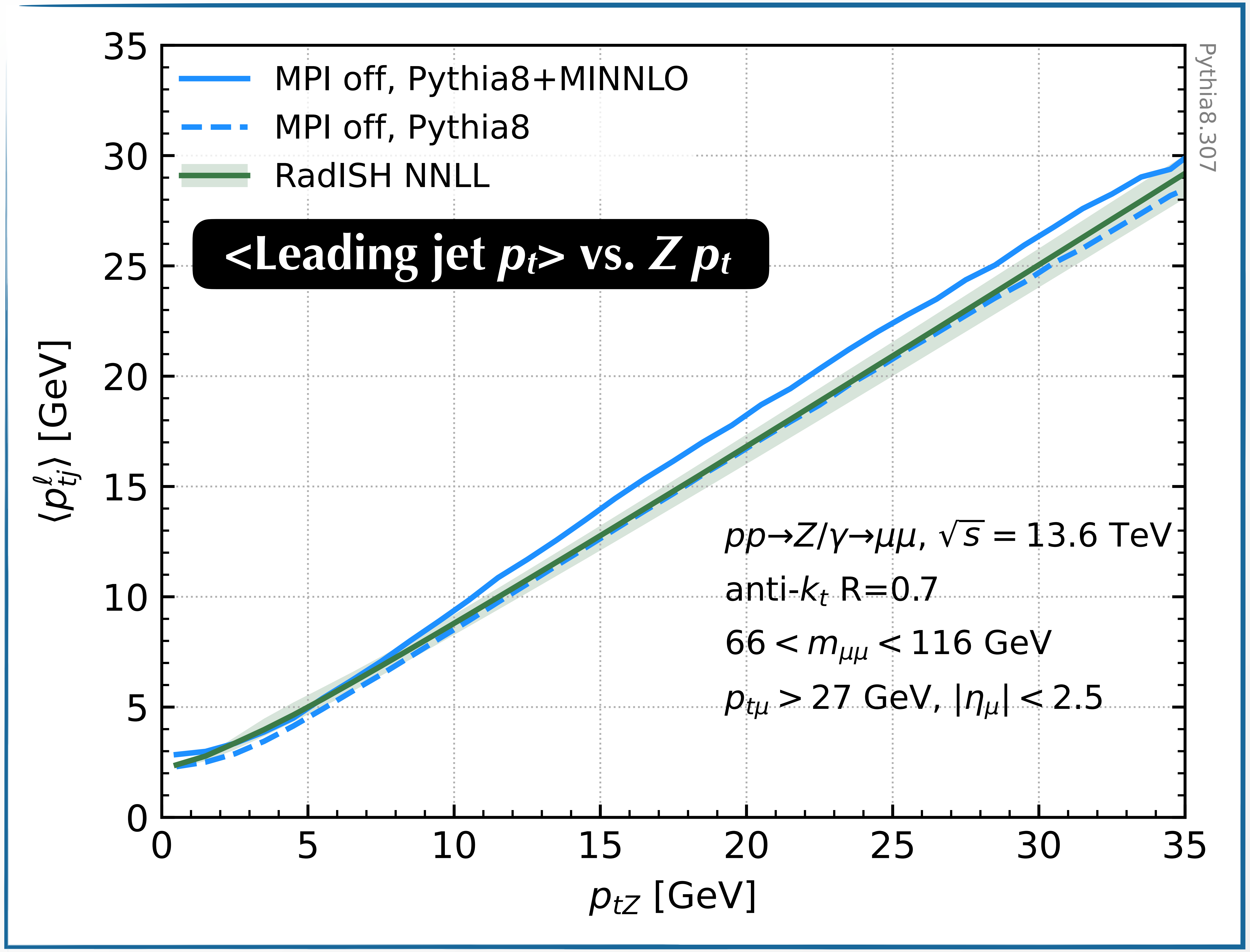
See also triple J/ψ production
[Shao, Zang 2019][CMS 2023]



[CMS 2206.02681]

Alternative route: can we study MPI in *Z* scattering?

[PRL 132 (2024) 4, 041901 Andersen, Monni, LR, Salam, Soto-Ontoso]



- Consider process with **MPI simulation turned off** (i.e. just 1HS)
- Look at avg. p_t of leading jet (p_{tj}^{ℓ}) as a function of Z p_t (p_{tZ})

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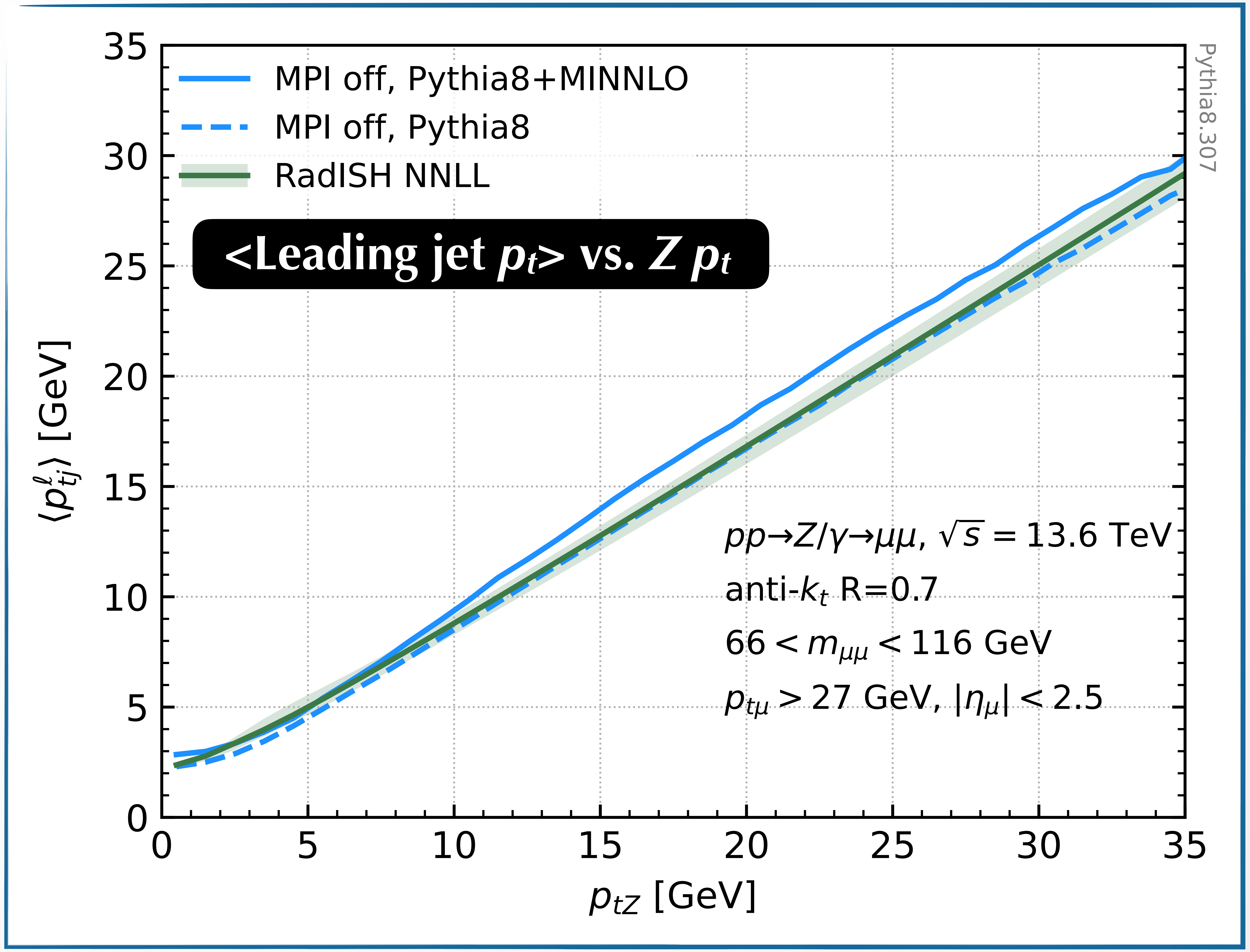

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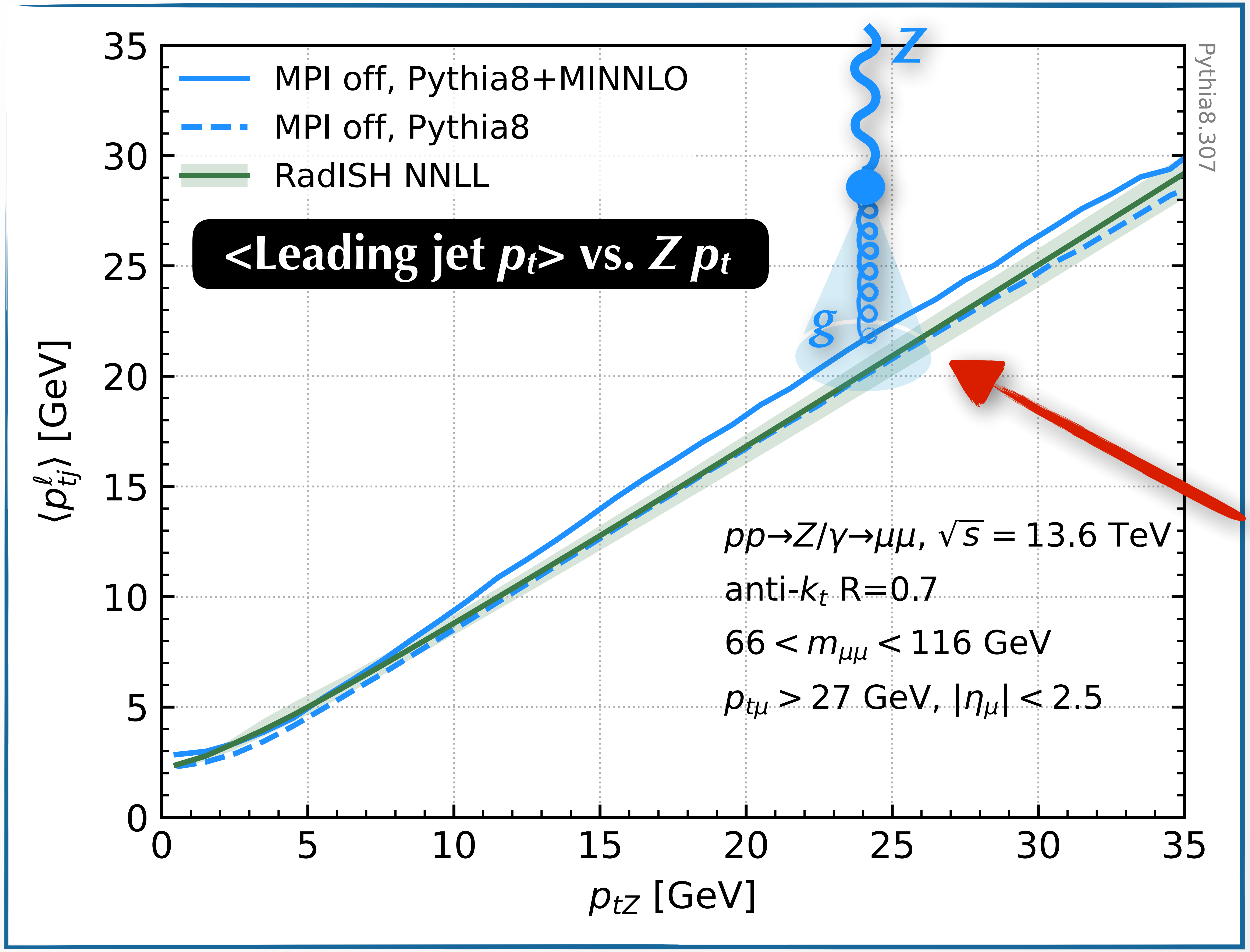

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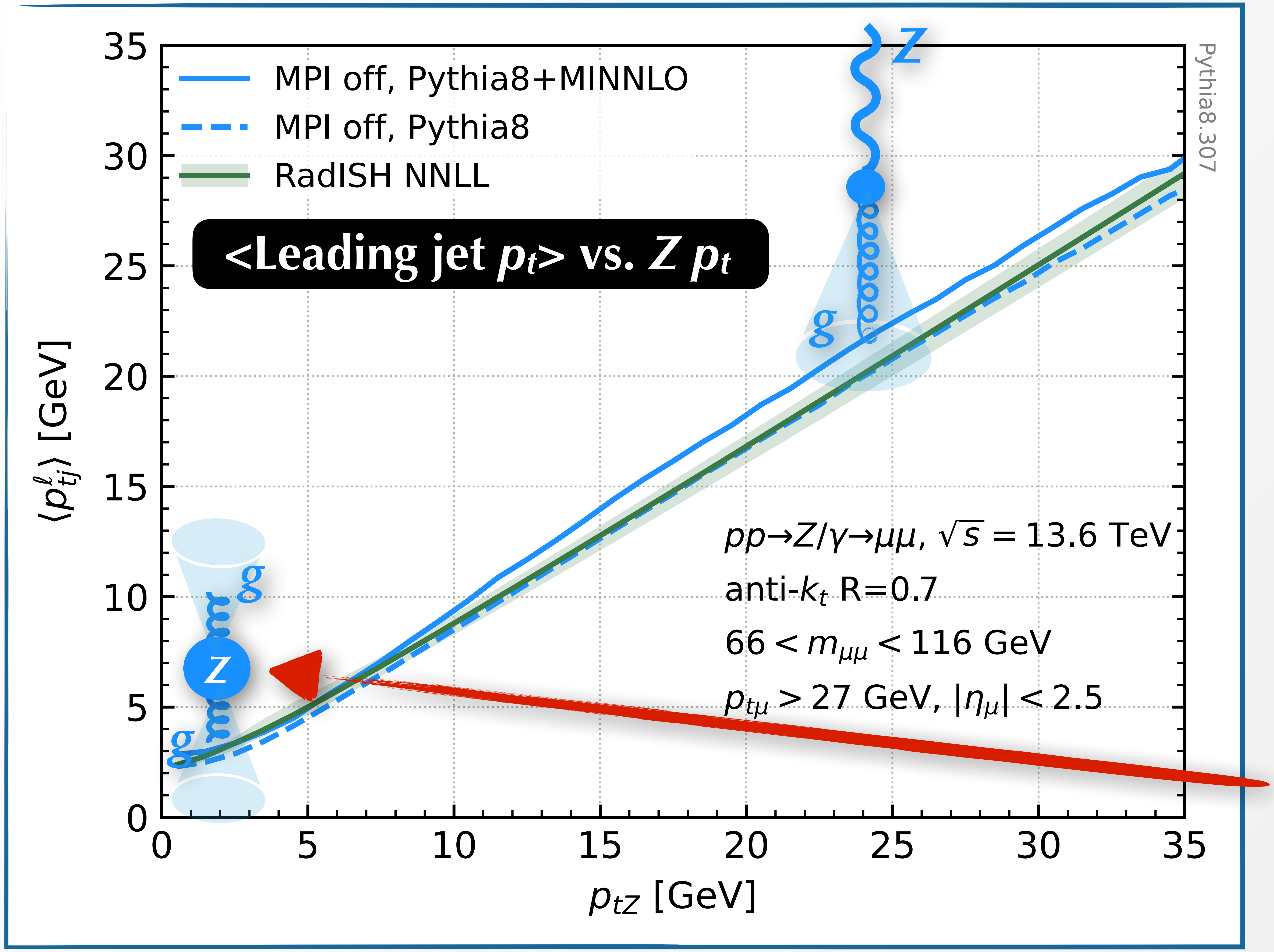

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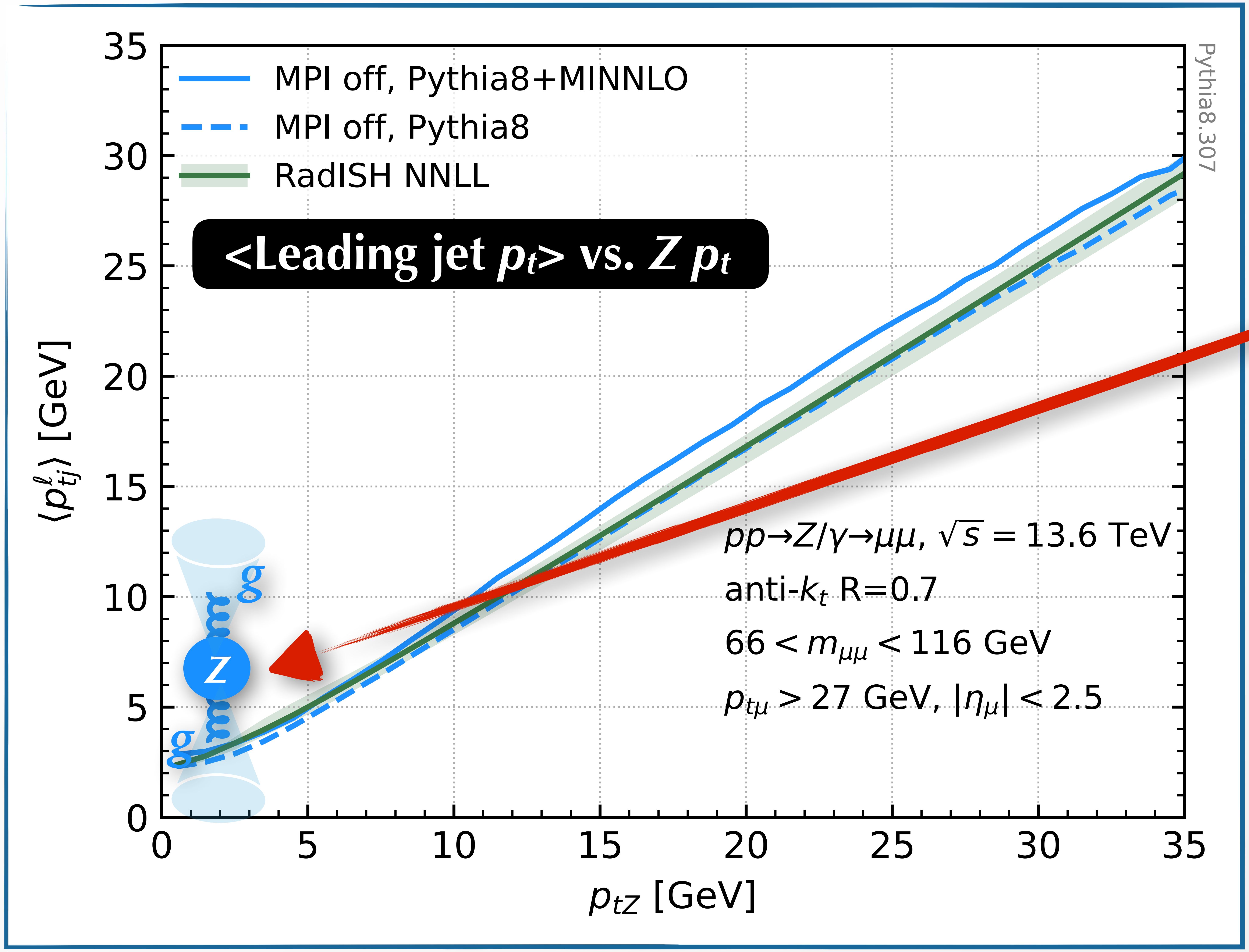
- **Most of p_{tZ} range:** almost perfect linear correlation, since **leading jet balances p_{tZ}**
- **For $p_{tZ} \to 0$:** $\langle p_{tj}^{\ell} \rangle$ saturates at about 2–3 GeV: **two (or more) soft jets balance each other**

Alternative route: can we study MPI in *Z* scattering?



Average leading jet p_T can be calculated from **resummation** in the limit $p_{tZ} \to 0$ **[Monni, LR, Torrielli '19]**

$$\langle p_{tj}^{\ell} \rangle_{p_{tZ}\to 0} \sim \Lambda_{\mathrm{QCD}} \left(\frac{m_Z}{\Lambda_{\mathrm{QCD}}} \right)^{\kappa \ln \frac{2+\kappa}{1+\kappa}}$$

$$\sim 2 - 3\,\mathrm{GeV} \qquad \kappa = \frac{2C_F}{\pi \beta_0}$$

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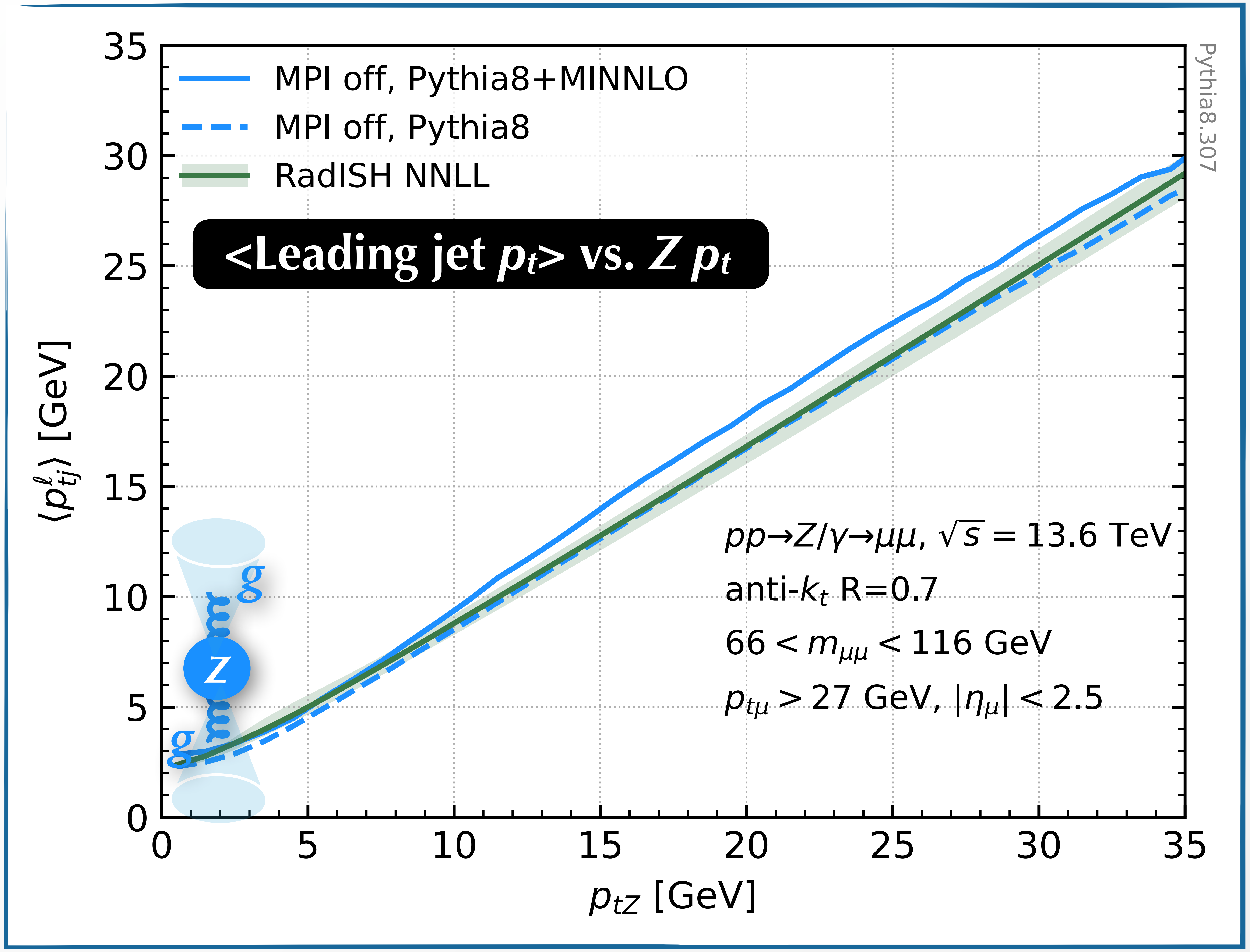

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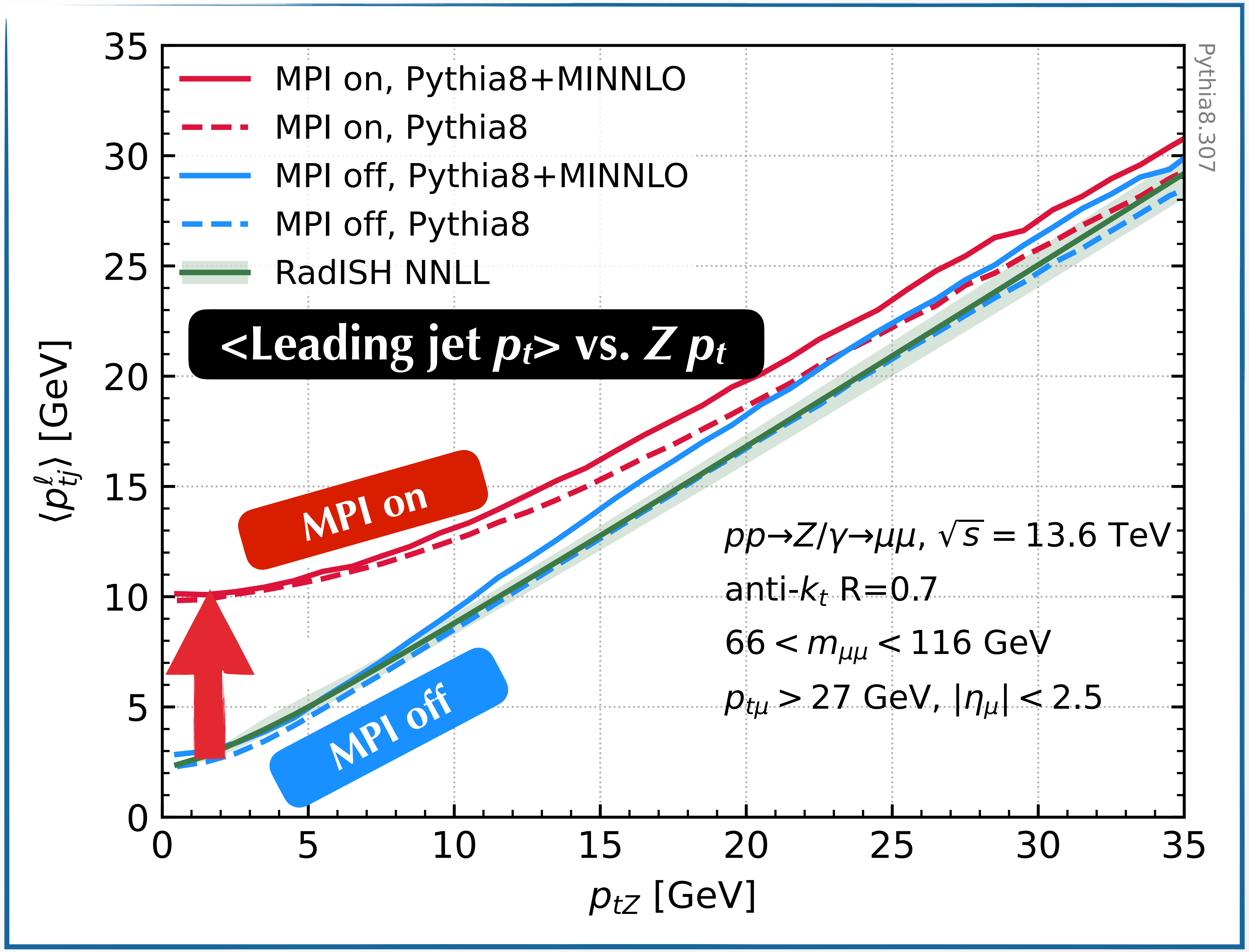
$$\sim 2-3\,\mathrm{GeV} \qquad \kappa = \frac{2C_F}{\pi\beta_0}$$

By **constraining** p_{tZ} we can **forbid most radiation** above this characteristic 2–3 GeV scale

[classic Parisi-Petronzio '79]

Alternative route: can we study MPI in Z scattering?



What happens when turning on MPI?

- for $p_{tZ} \to 0$, leading jet p_t is now ~10 GeV instead of 2–3 GeV
- Why? Because there is almost always an MPI jet that is much harder than the soft jets from Z-process cfr. current models in Pythia/Herwig/Sherpa which simulate MPI as semi-hard scatterings

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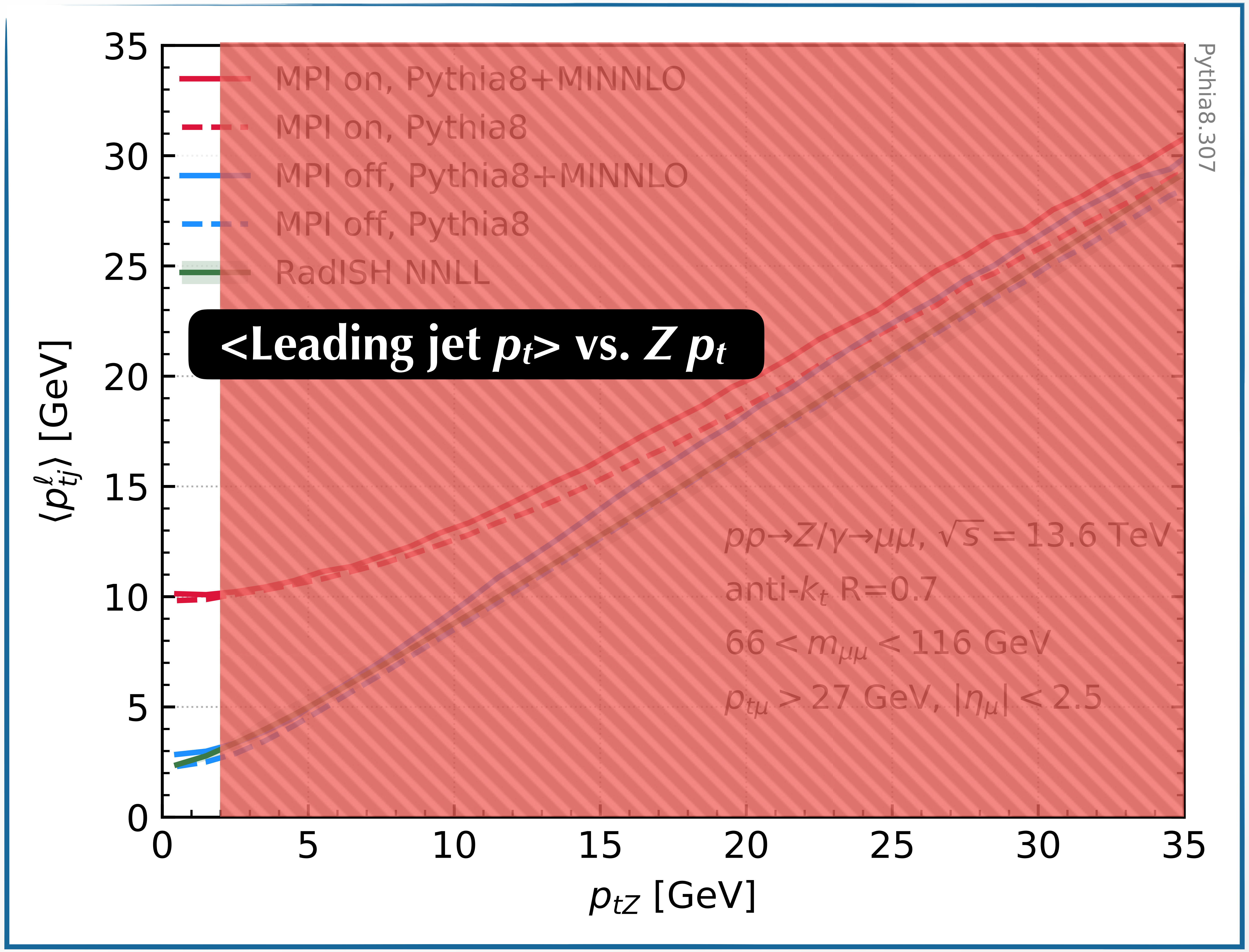
<Leading jet p_t> vs. Z p_t

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Suggests we should study MPI with **help of a tight cut on p_{tZ}**

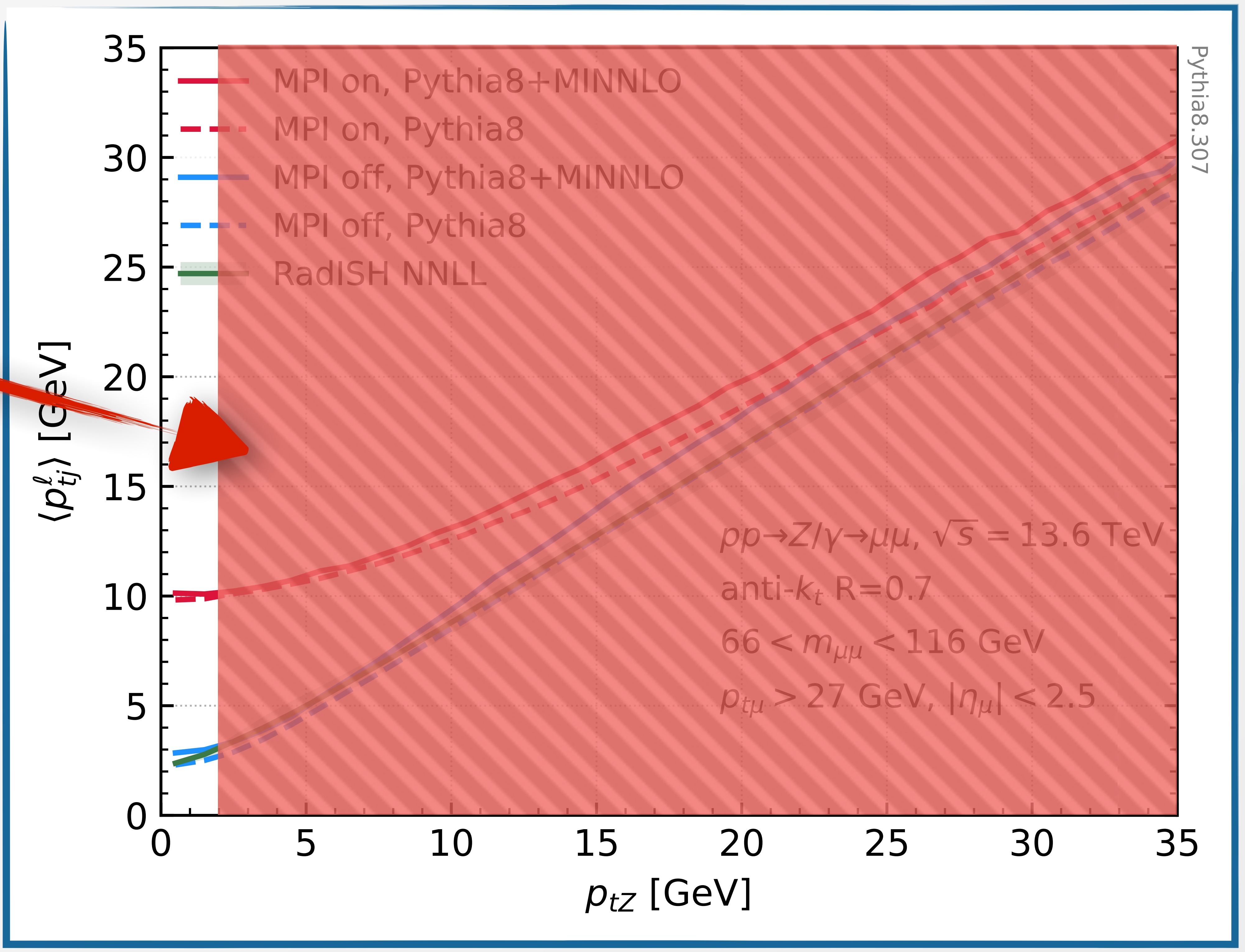
Our study: establish what cut to use, explore opportunities that open up

Need **balance** between

- **maximising statistics** (favours loose cut on Z)
- **minimising radiation** from Z hard system (favours tight cut on Z)

From $\langle p_{tj}^{\ell} \rangle$ vs. p_{tZ} plot optimum requirement is $p_{tZ} \lesssim 2\,\mathrm{GeV}$

- Smaller cut does not reduce scale of soft radiation from Z process and lower stats
- Higher cut increases average p_T of radiation
- Feasible given current experimental resolution



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$p_{tZ} < 2$ GeV cut retains 4 – 5% of Z-pole Drell-Yan events

For $Z \to \mu^+\mu^-$ residual cross section is ~40pb

~12 million events for 300fb^{-1} in Run 3

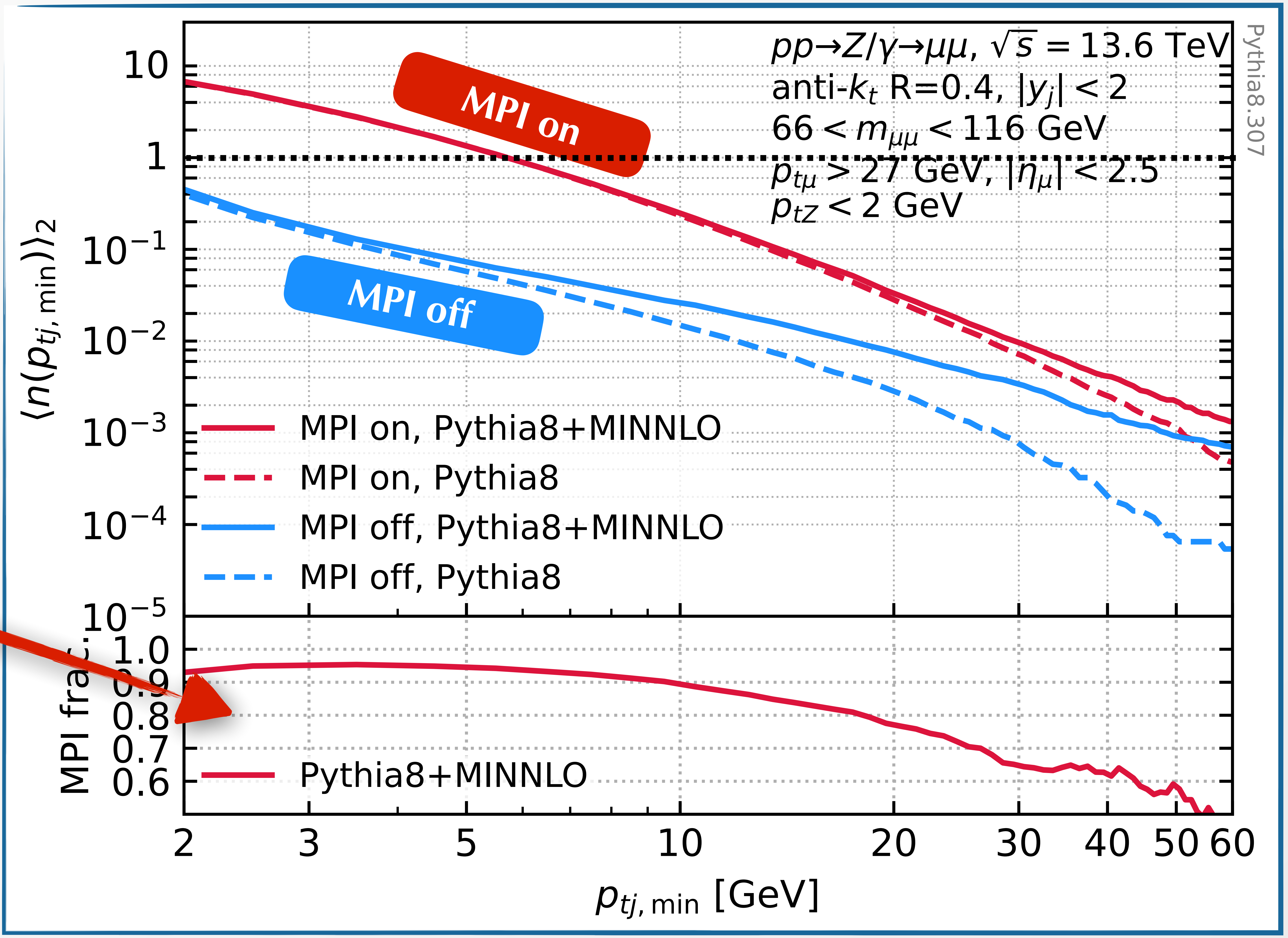
Simplest observable: cumulative inclusive jet spectrum for $p_{tZ} < 2$ GeV

Linear sum (for small jet radius) of

- **cumulative jet spectrum from 1HS process**
- **cumulative jet spectrum from any additional hard scatters (dominant!)**

MPI purity remains significant also at relatively high values of $p_{tj,\mathrm{min}}$

$p_{tj,\mathrm{min}}$	MPI purity
10 GeV	90%
20 GeV	78%
40 GeV	60%



Connection with "pocket formula" (σ effective)

$\langle n(p_{tj,\min})\rangle_{C_Z}$ = average number of jets above $p_{tj,\min}$ for a given cut C_Z on p_{tZ}

$$\langle n(p_{tj,\mathrm{min}})\rangle_{C_Z} = \frac{1}{\sigma(p_{tZ} < C_Z)} \int_{p_{tj,\mathrm{min}}} dp_{tj} \frac{d\sigma_{\mathrm{jet}}(p_{tZ} < C_Z)}{dp_{tj}}$$

Pure MPI part extracted by subtracting no-MPI calculation (thanks to linearity)

$$\langle n(p_{tj,\mathrm{min}})\rangle_{C_Z}^{\text{pure-MPI}} \equiv \langle n(p_{tj,\mathrm{min}})\rangle_{C_Z} - \langle n(p_{tj,\mathrm{min}})\rangle_{C_Z}^{\text{no-MPI}}$$

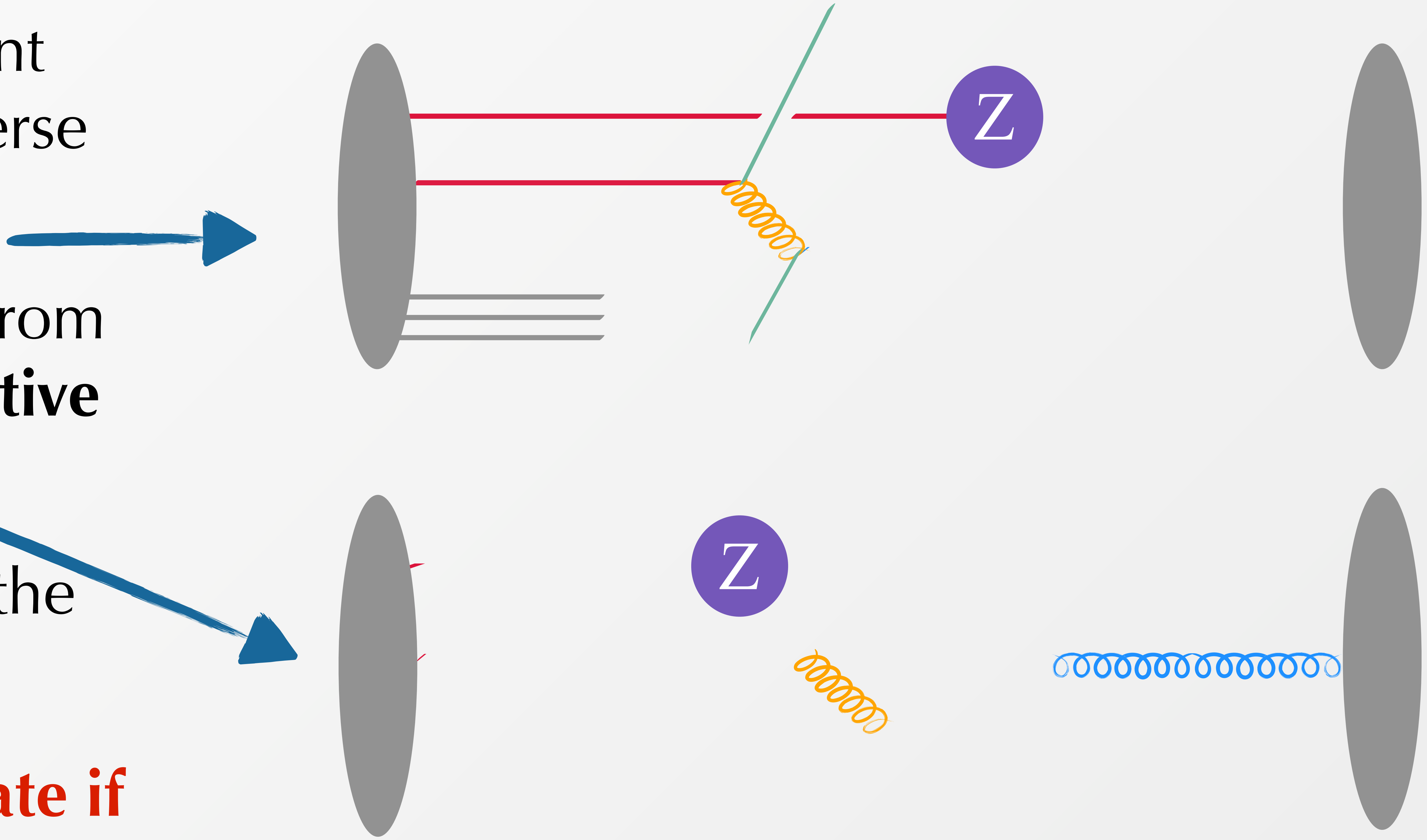
In σ_{eff} picture, pure-MPI part can be connected with jet rate in min-bias events (i.e. no Z)

NB: can be directly measured on data, identical systematics (e.g. with charge-track jets at low p_{tj})

$$\langle n(p_{tj,\mathrm{min}})\rangle_{C_Z}^{\text{pure-MPI}} \simeq \frac{1}{\sigma_{\mathrm{eff}}} \int_{p_{tj,\mathrm{min}}} dp_{tj} \frac{d\sigma_{\mathrm{jet}}^{\text{min-bias}}}{dp_{tj}}$$

Beyond the pocket formula

- Pocket formula is based on independent scatterings, with some effective transverse size over which partons are spread
- But we expect some partons to come from splitting of common parents,"**perturbative interconnection**"
- Such splittings tend to give more p_t to the partons → higher p_{tZ}
- **We should see an change of MPI jet rate if we relax the p_{tZ} cut**



Interconnection studies: Diehl & Schafer 1102.3081; Blok, Dokshitzer, Frankfurt & Strikman 1106.5533; Diehl, Gaunt & Schönwald, 1702.06486

Can one see effect of perturbative interconnection?

Measure cumulative jet rate with two p_{tZ} cuts:

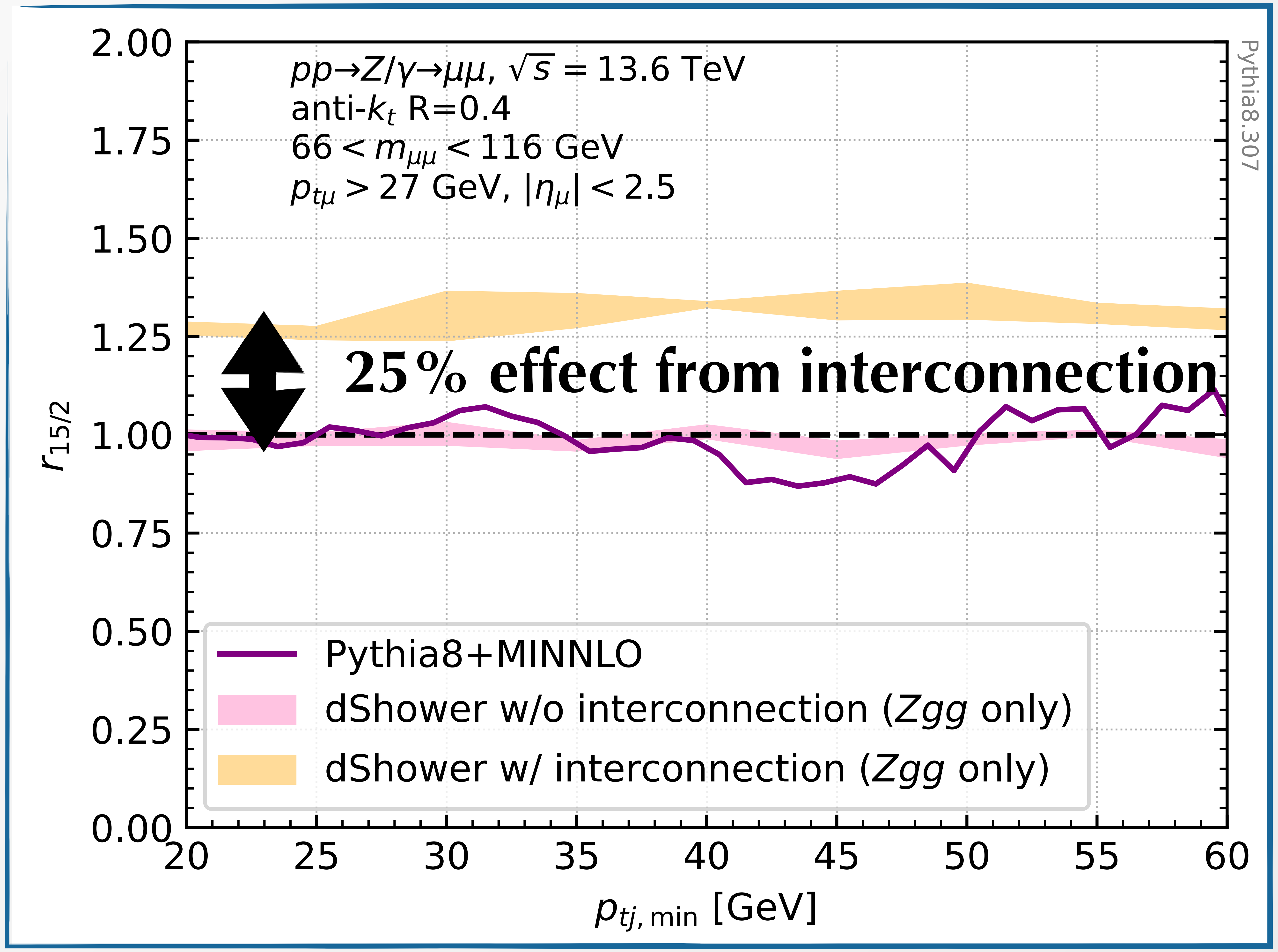
- tight (2 GeV)
- loose (15 GeV)

Take ratio of pure-MPI jet rates

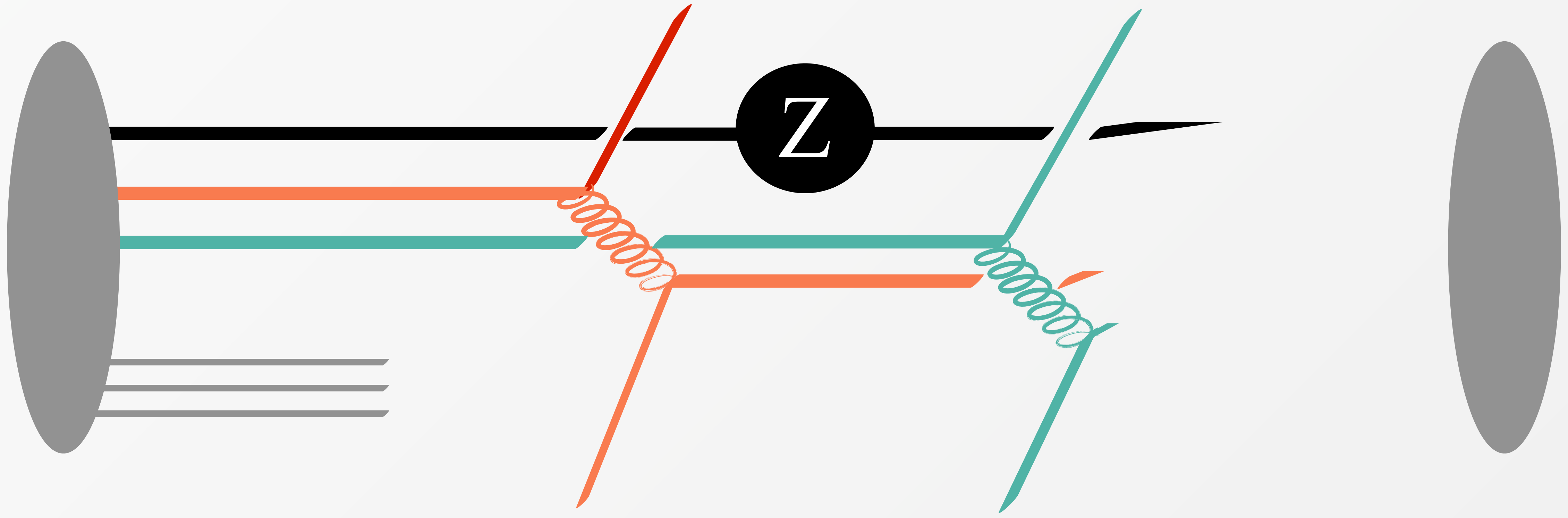
$$r_{15/2} = \frac{\langle n(p_{tj,\min}) \rangle_{15}^{\text{pure-MPI}}}{\langle n(p_{tj,\min}) \rangle_{2}^{\text{pure-MPI}}}$$

Compare to

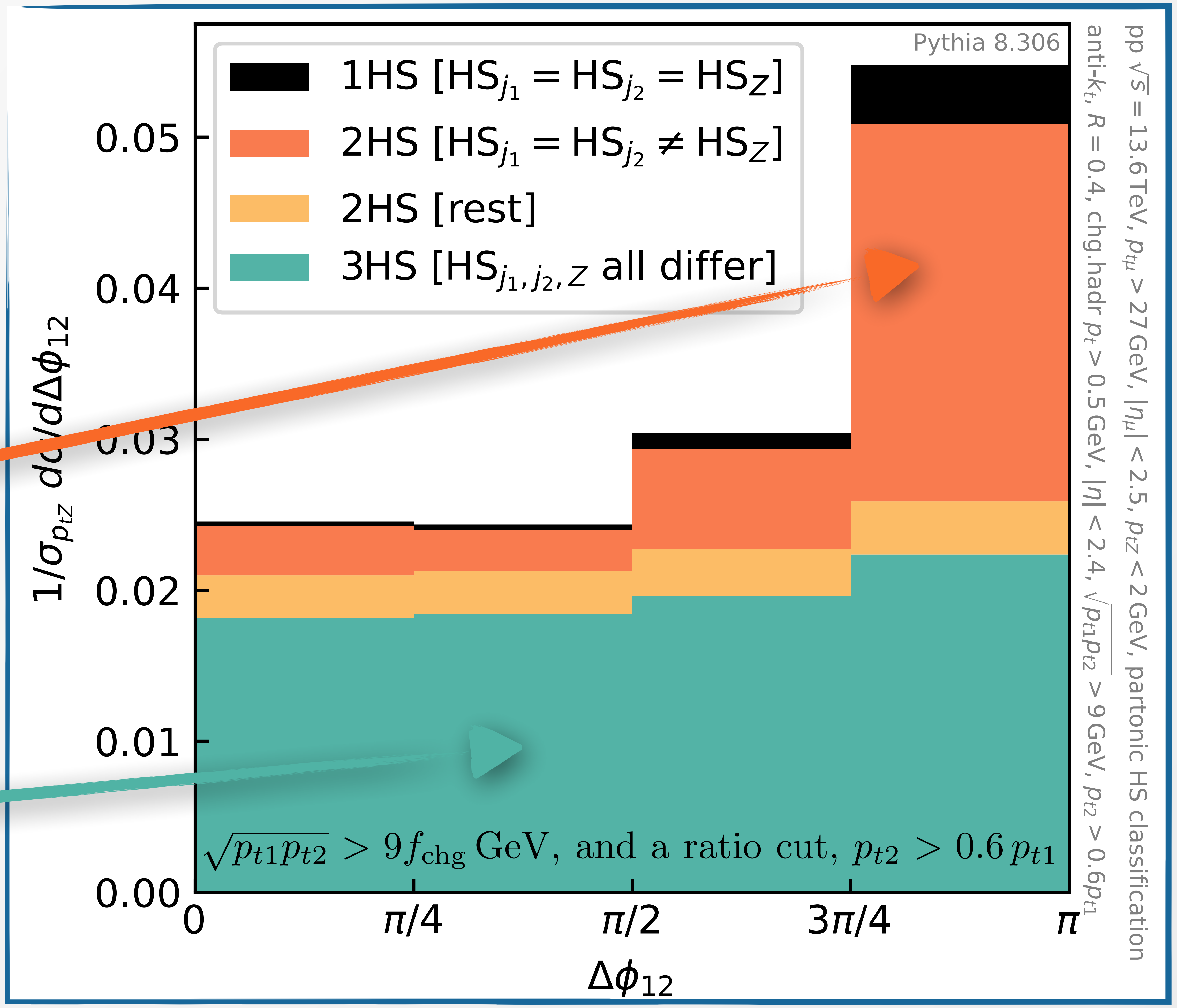
- **Pythia (+MiNNLO)**: no interconnection (expect $r = 1$)
- **dShower**: with option of interconnection
  [Cabouat, Gaunt, Ostrolenk, 1906.04669;
  Cabouat, Gaunt, 2008.01442]



Beyond 2HS

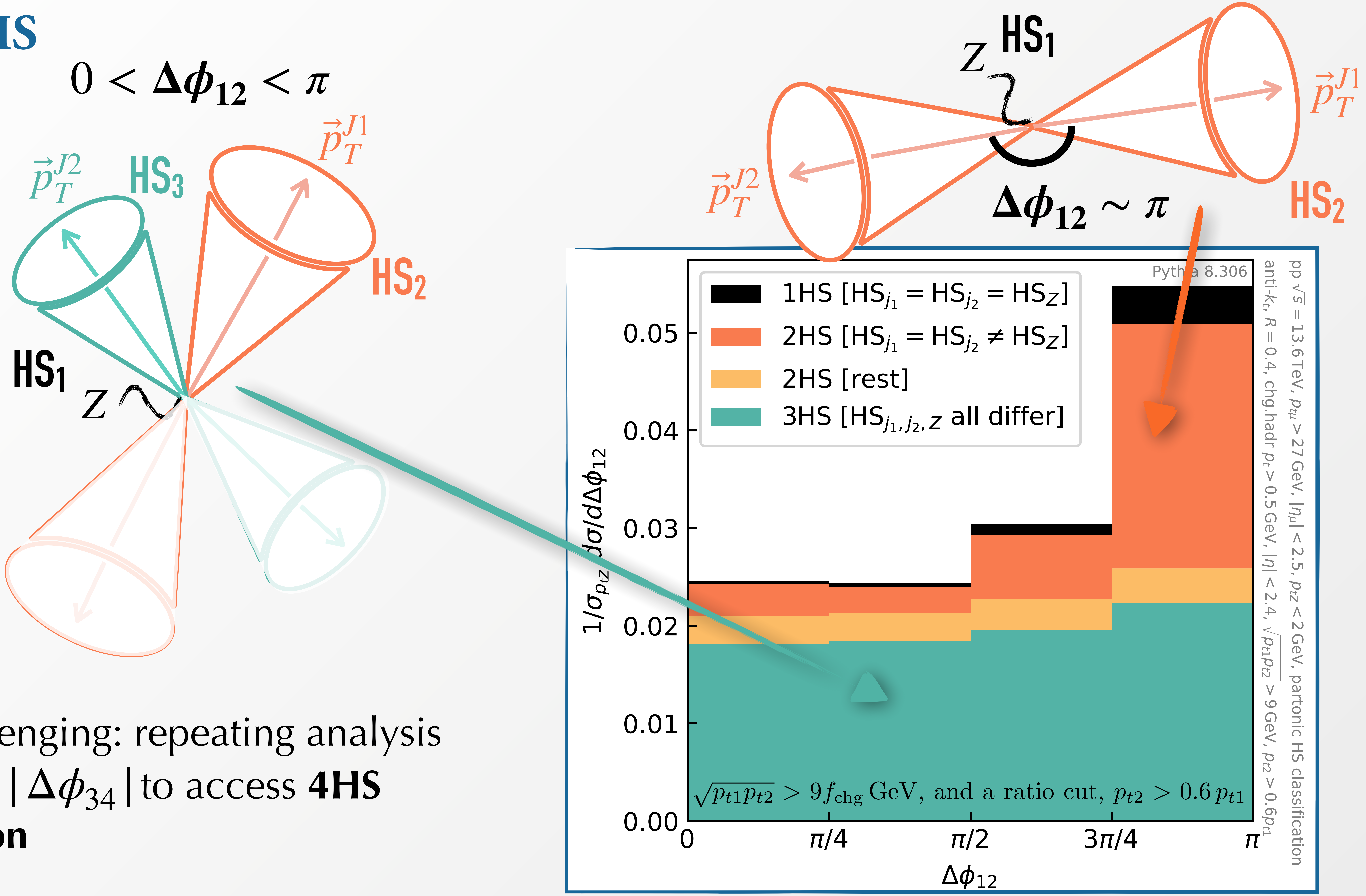


- Only measurements of 3HS are in J/ψ production, which is a difficult process to interpret even with just 1HS!
- Instead, put tight $p_{tZ} < 2\,\mathrm{GeV}$ cut and look at $\Delta\phi$ between two leading charged-track jets, with low p_{tj} cuts (~ 5 GeV on charged-track sum)
- gives clear **2HS** peak at $|\Delta\phi| \simeq \pi$
- gives distribution ~independent of $|\Delta\phi|$, when the Z and the 2 jets each come from different hard scatters **(total of 3HS)**



Beyond 2HS

$0 < \Delta\phi_{12} < \pi$



More challenging: repeating analysis examining $|\Delta\phi_{34}|$ to access **4HS contribution**

Conclusion

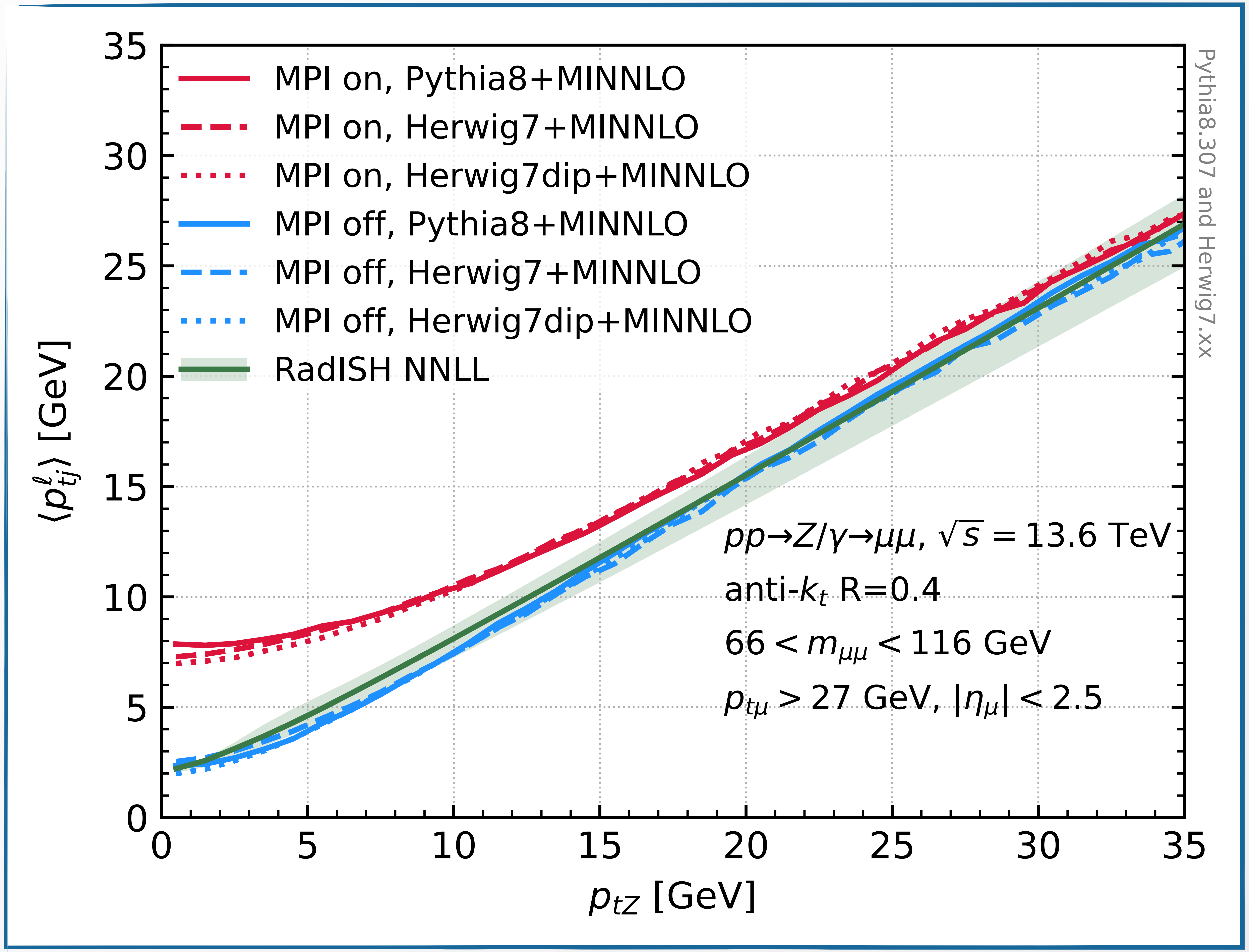
- MPI occur in almost all high-energy collisions at hadron colliders but their quantitative study remains challenging
- Modelling of (soft) MPI is an essential component of all major simulation tools
- Process with one (or more) hard extra interaction, 2HS/3HS etc, is interesting as a signal, and as a background to rare processes
- The insight gained by studying hard MPI may improve the modelling of soft MPI for MB/UE, since soft MPI models are conceived as extensions of the one used for harder MPI
- Study of Drell-Yan events with tight cut on p_{tZ} opens door to numerous new MPI studies (high-purity 2HS samples, interconnections, 3HS studies)

Overall

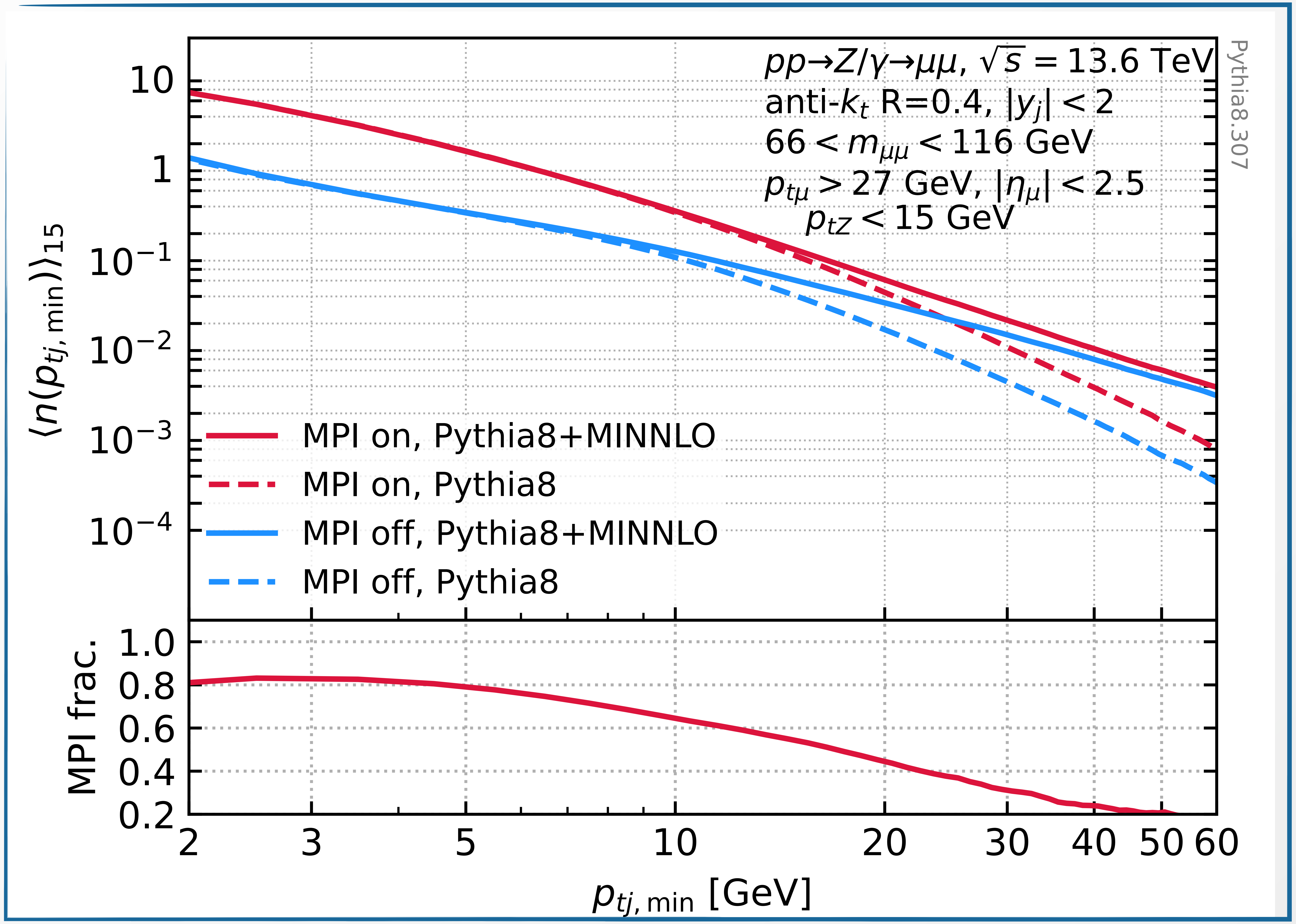
potential for significant impact on conceptual and quantitative understanding of MPI

Backup

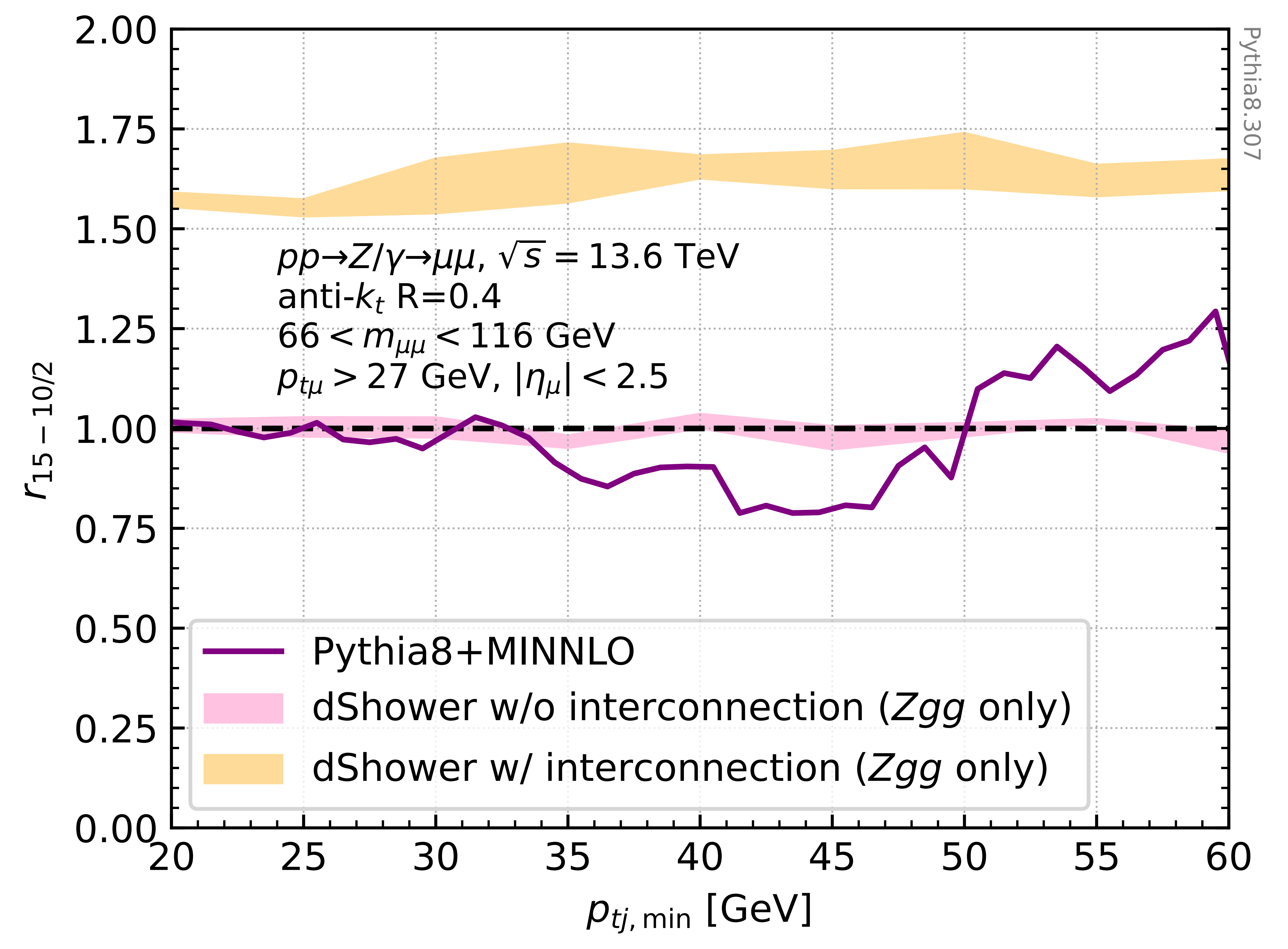
HERWIG results

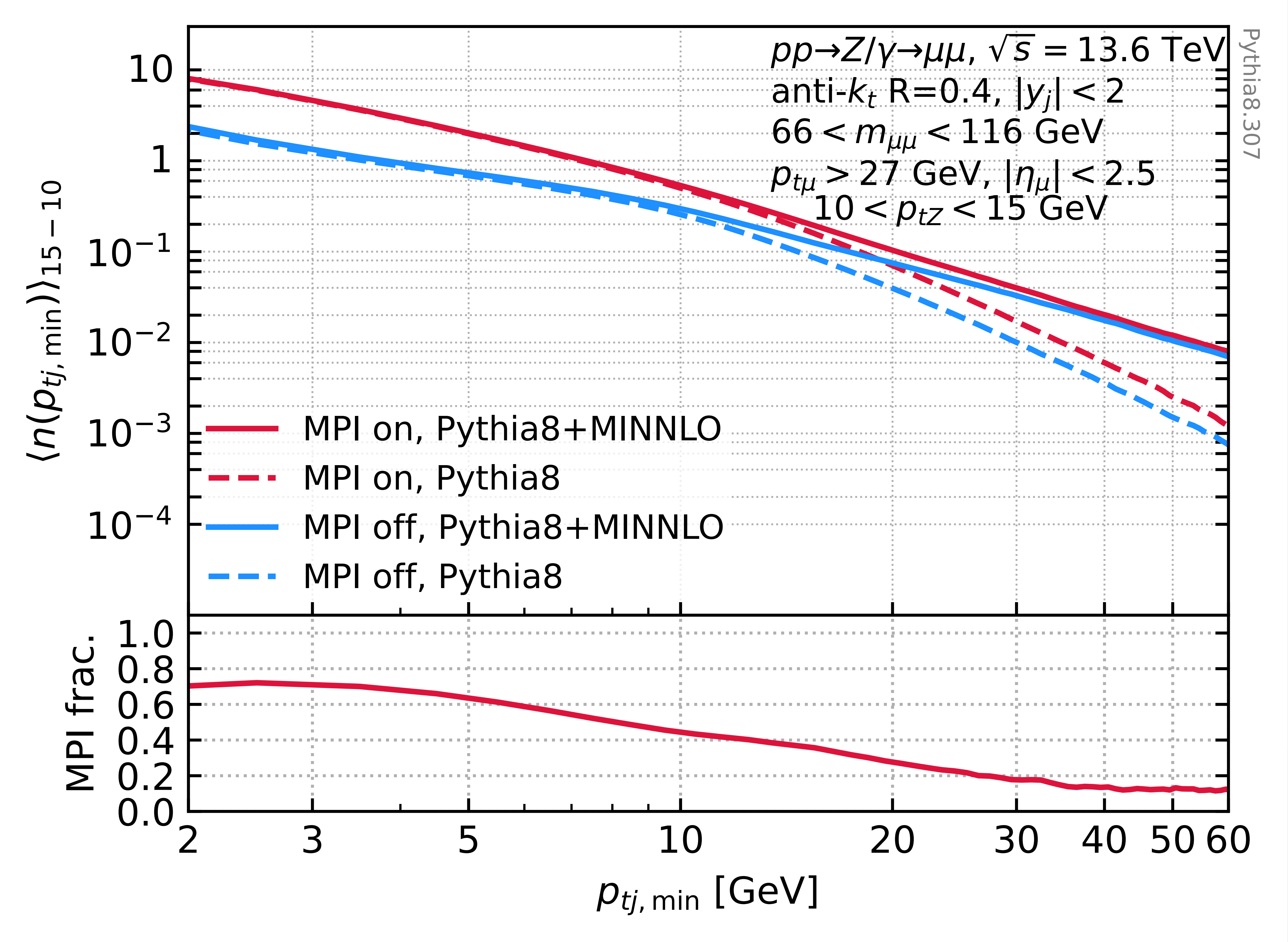


MPI purity with 15 GeV cut on p_{tZ}

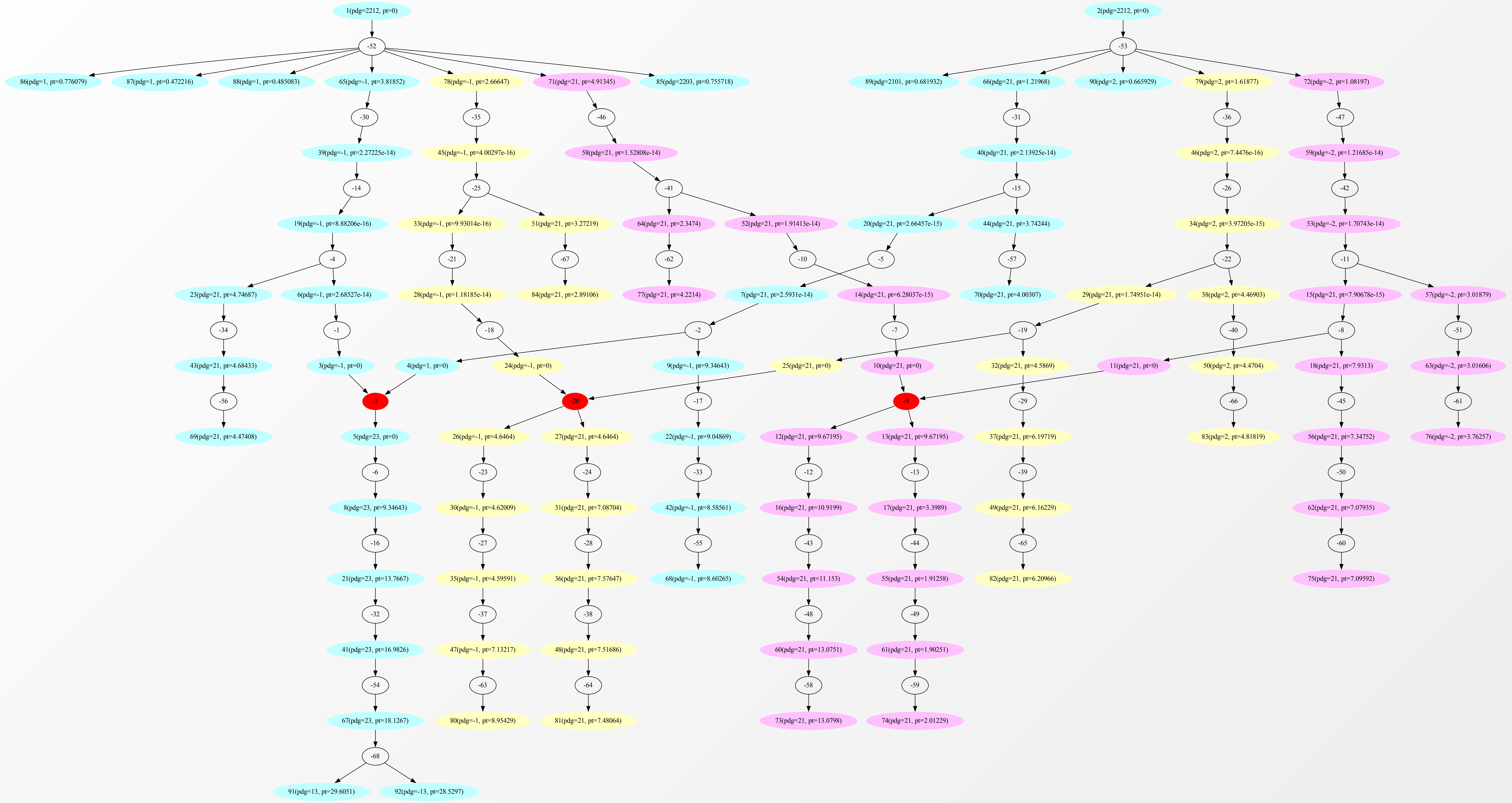


10 < p_{tZ} < 15 GeV for the loose sample: increases interconnection, reduces purity

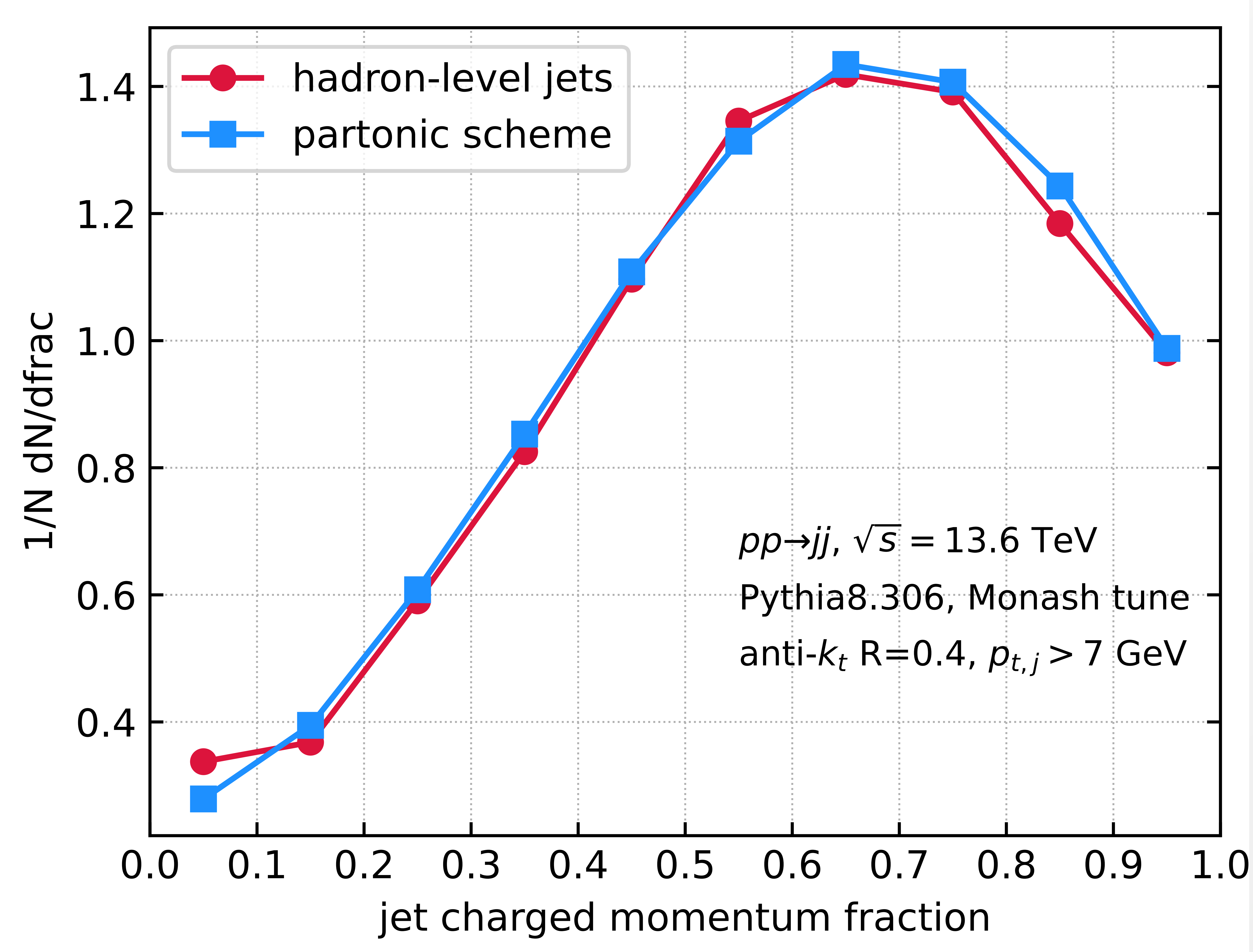




Extracting partonic hard-scattering classification from Pythia (via HepMC)



Validation of simple parton → charged hadron conversion for hard-scatter classification

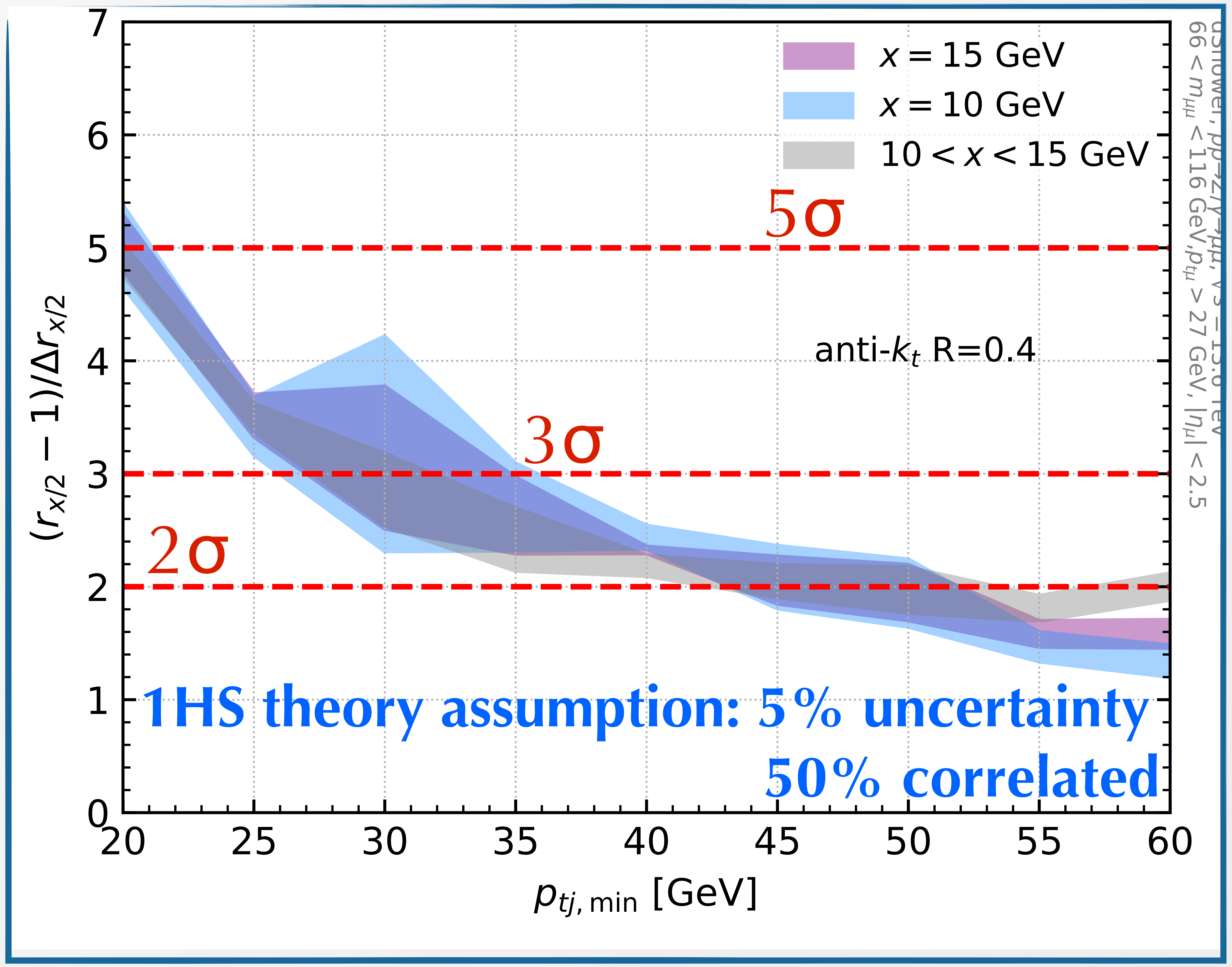


Interplay of significance with MPI purity

Significance of signal of perturbative interconnection in simulation, for dShower-sized effect vs. $p_{tj,\min}$ depends on assumptions for sizes of **theory uncertainties** on 1HS subtraction + **their correlation** between the two p_{tZ} cuts

- **Just barely feasible?**
- motivates NNLO (matched) *Z+2j* calculations to reduce current theory uncertainty (10-20%)

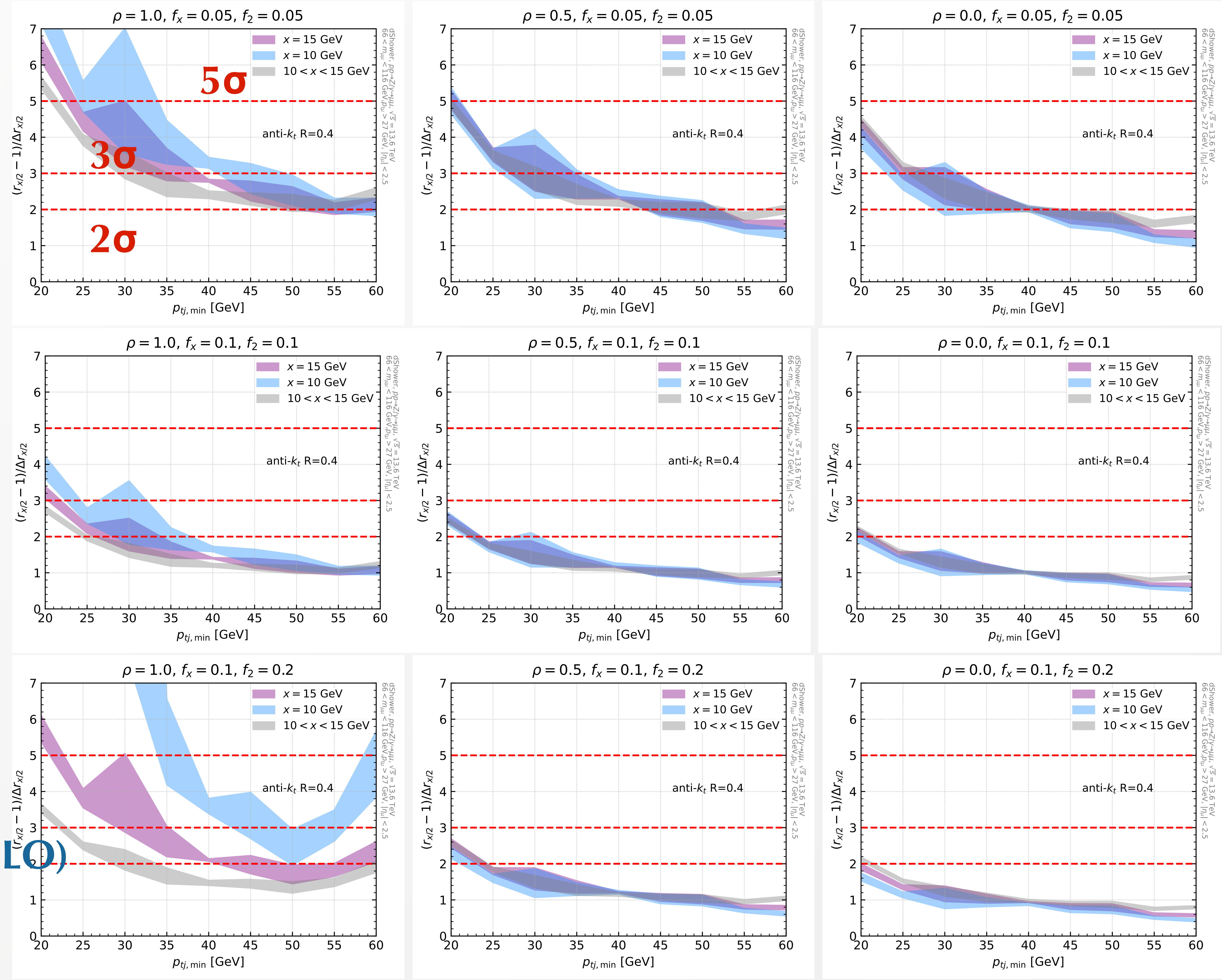
significance of signal
of perturbative interconnection



Interplay of significance with MPI purity in different scenarios

1HS Th. uncert. → **100% correlated** **50% correlated** **uncorrelated**

↓

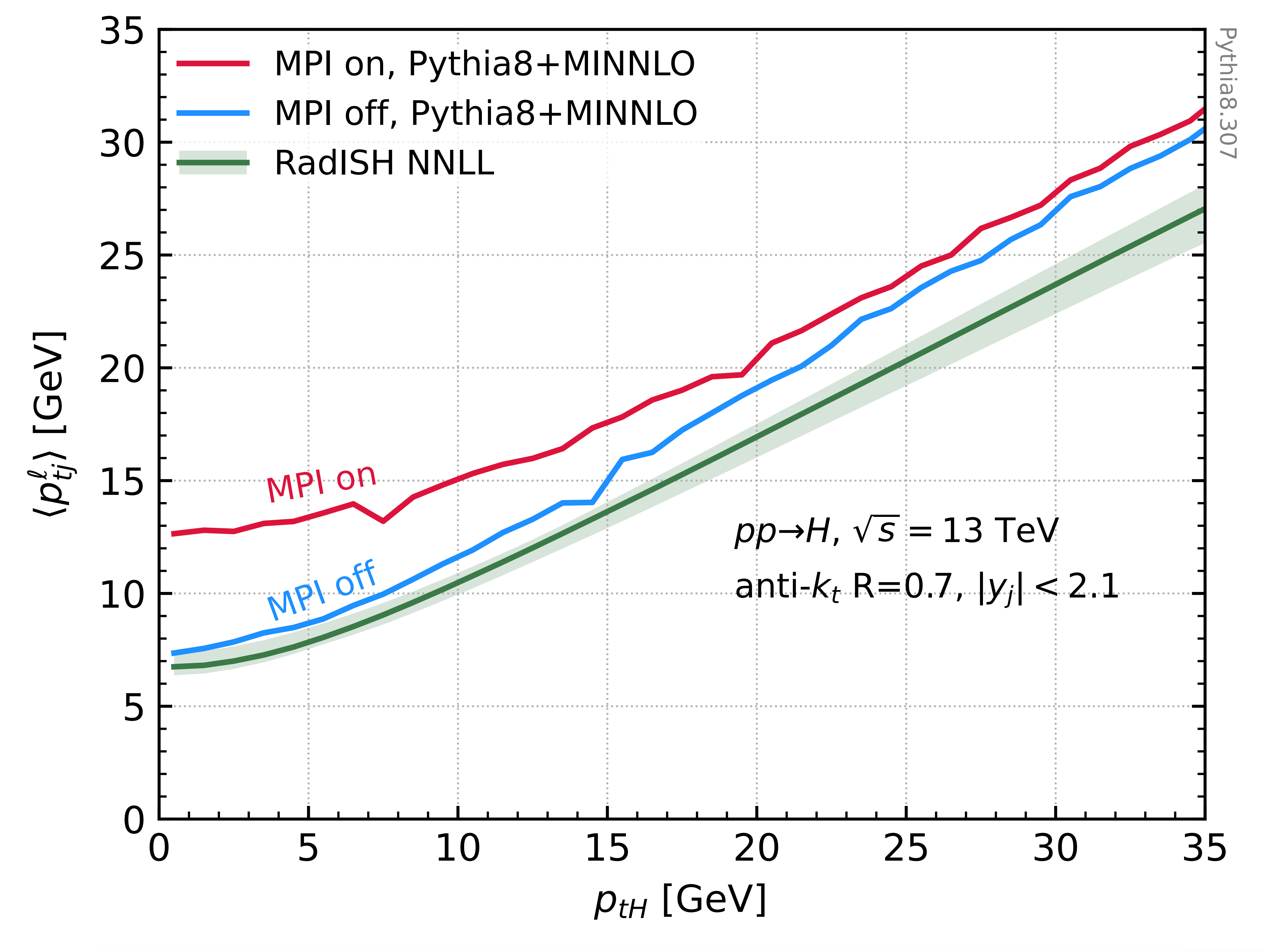


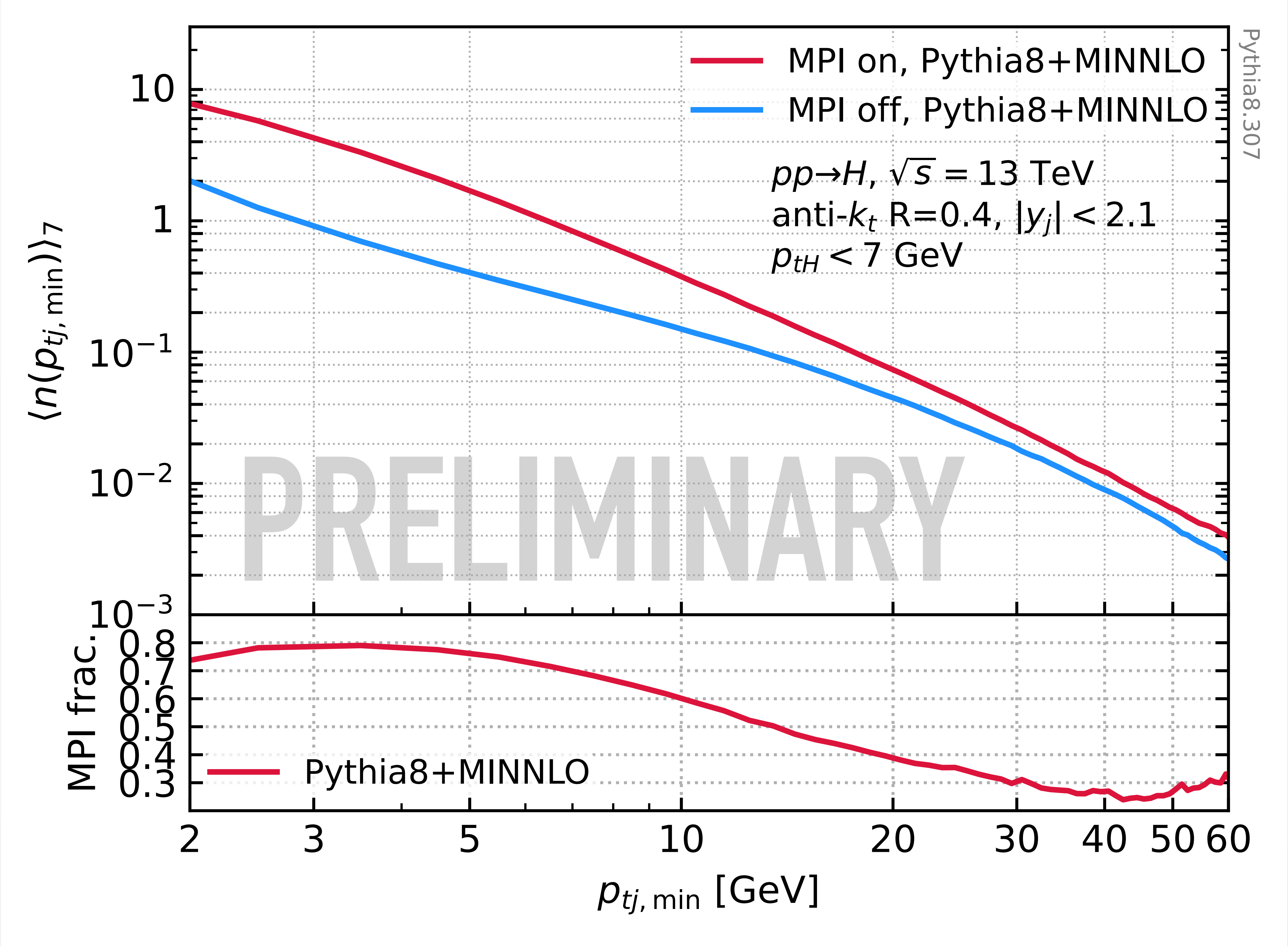
5%

10%

10%
/ 20%
(=MINNLO)

Higgs production (gg channel said to have smaller σeff, mainly from J/ψ)





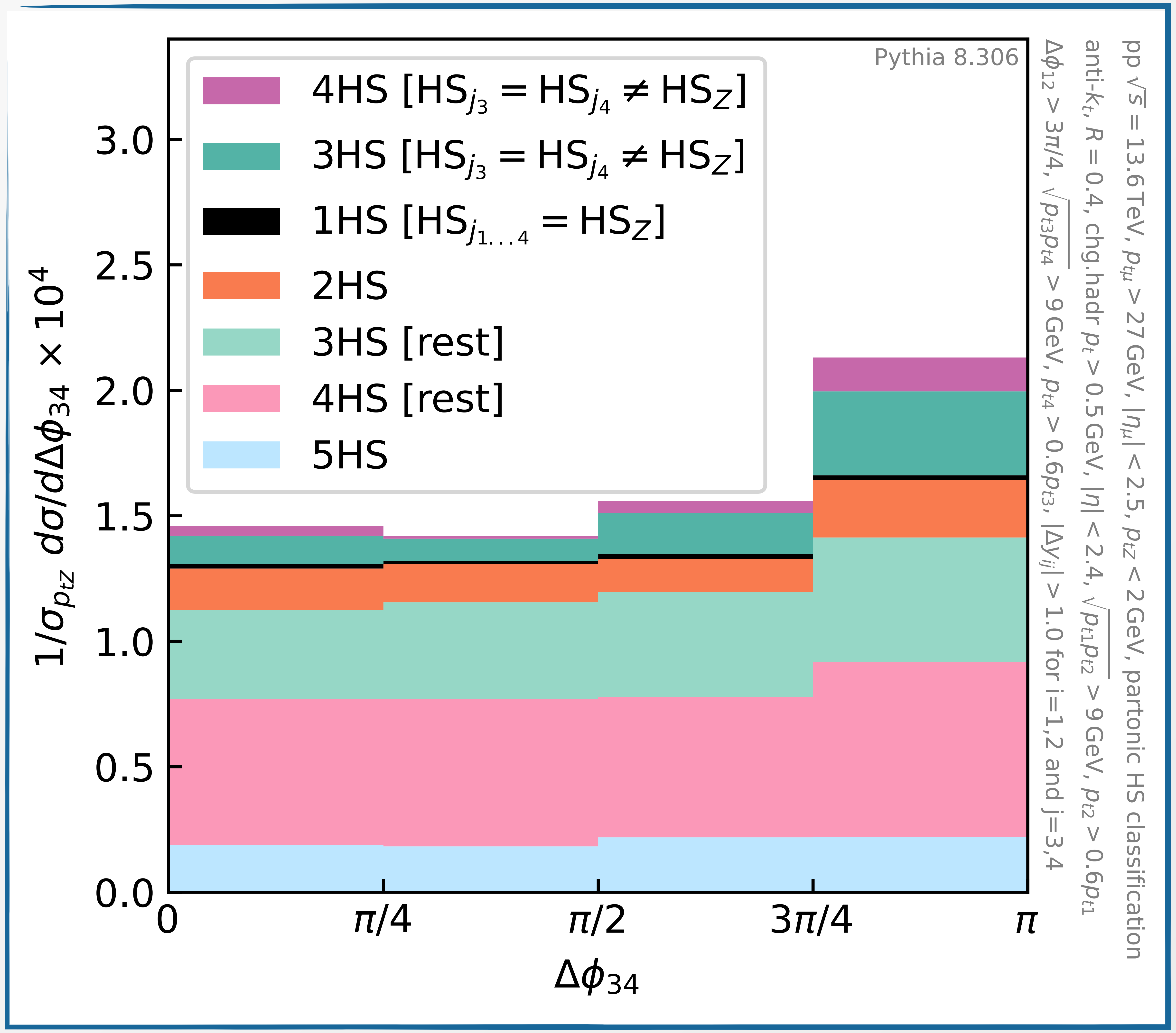
Optimal cut is $p_{tH} \lesssim 7$ [GeV]

~10% of events H events pass this cut

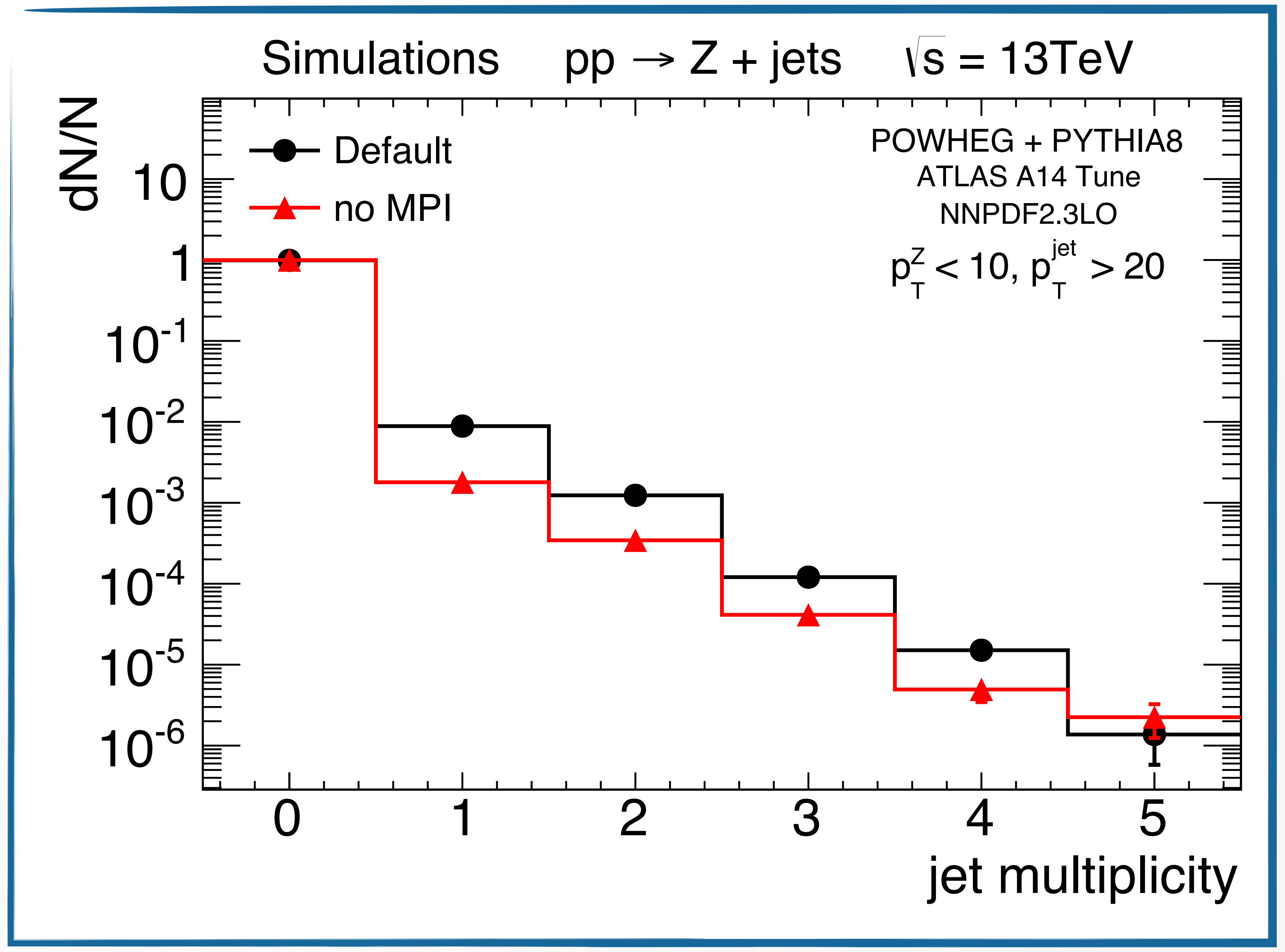
(with p_{tH} cut, full run 2+3 stats in $H\to ZZ^*\to 4\ell$ c. 50–100 events)

Beyond 3HS?

- Select four leading jets
- Pair them up (first two, next two)
- Require first two to be back-to-back
- Require $|\Delta y| > 1$ rapidity separations between first two and next two
- examine $|\Delta\phi_{34}|$
- see small peak around $|\Delta\phi_{34}| = \pi$ (3HS)
- **continuum includes substantial 4HS contribution!**

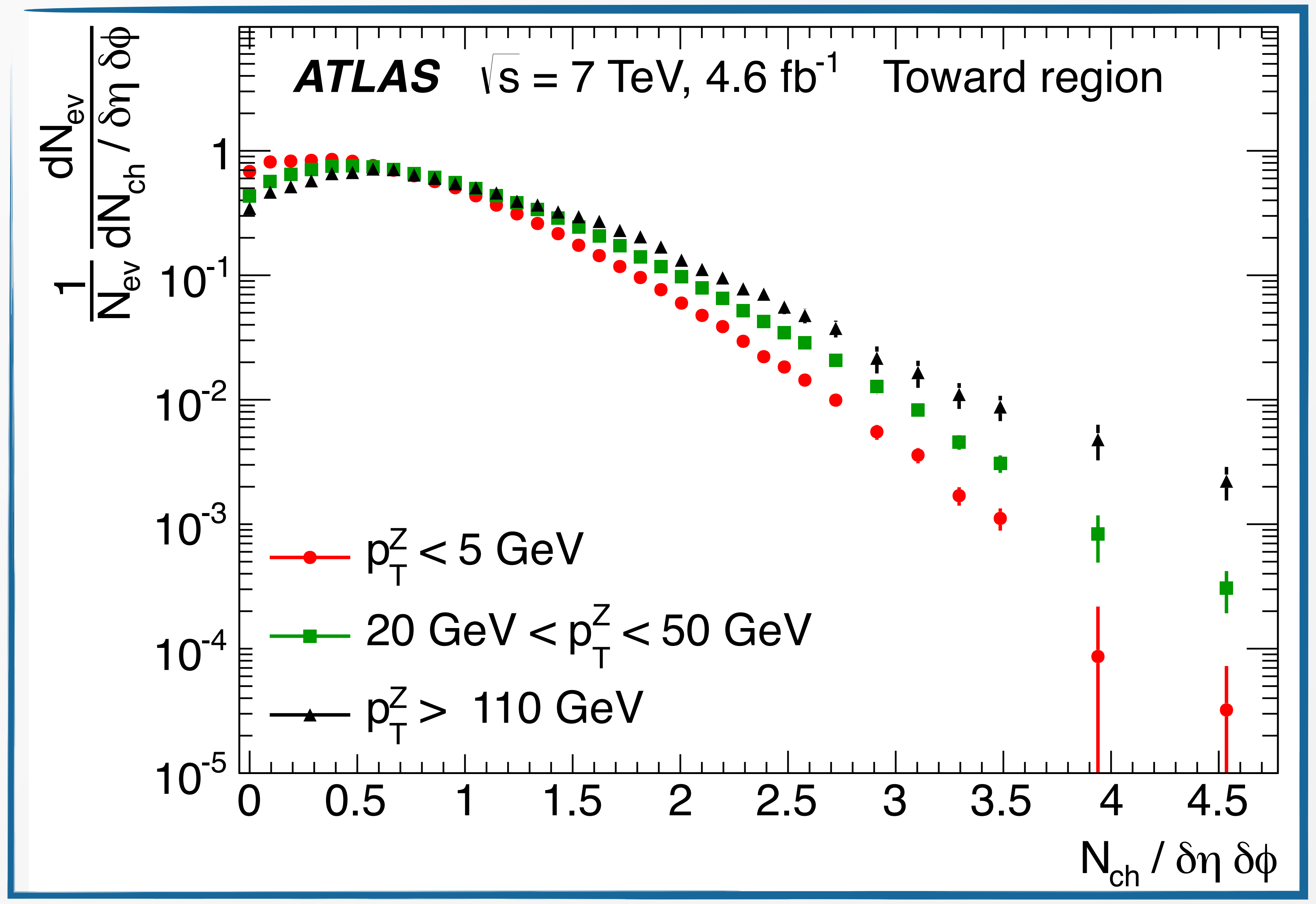


Past MPI studies with cuts on p_{tZ}

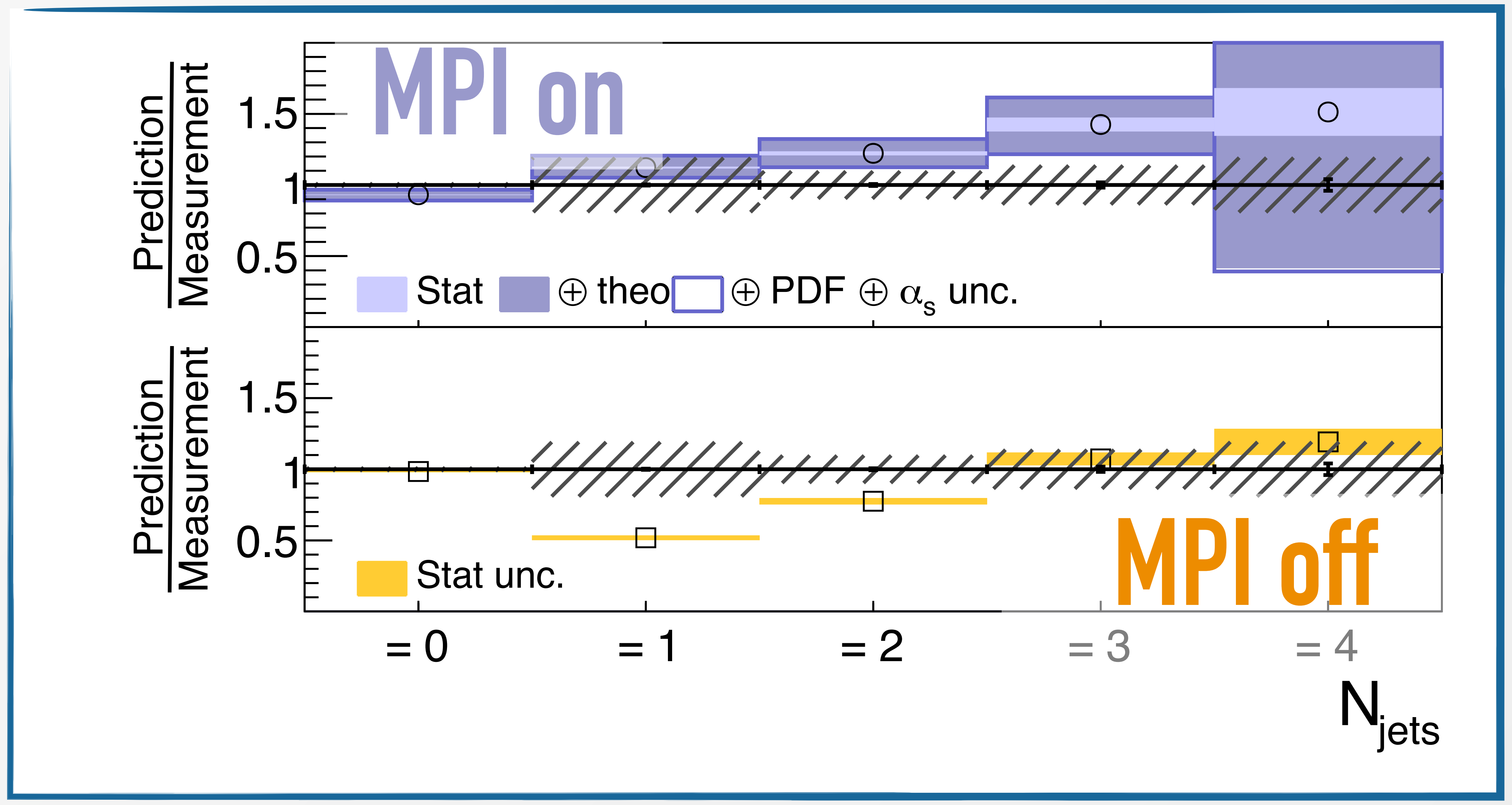


Bansal, Bansal, Kumar, Singh 1602.05392 suggested MPI studies with $p_{tZ} < 10\,\mathrm{GeV}$ for improved MPI purity

See also Alioli, Bauer, Guns, Tackmann, 1605.07192



ATLAS 1409.3433 mostly an underlying-event study, used $p_{tZ} < 5\,\mathrm{GeV}$

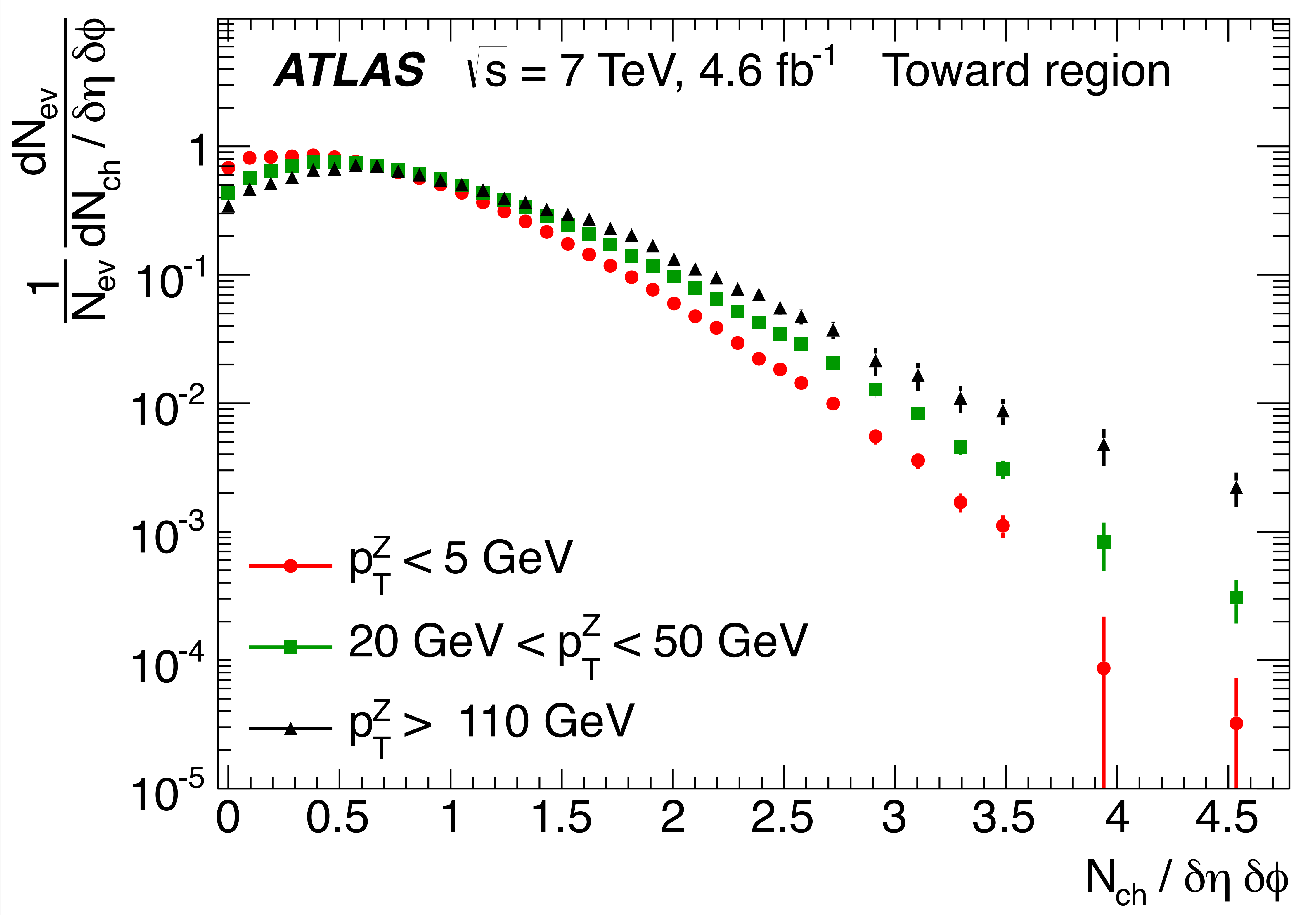


CMS 2210.16139:

results with $p_{tZ} < 10\,\mathrm{GeV}$, confirming some MPI enhancement

ATLAS 1409.3433

- mostly a UE study
- uses $p_T^Z < 5\,\mathrm{GeV}$



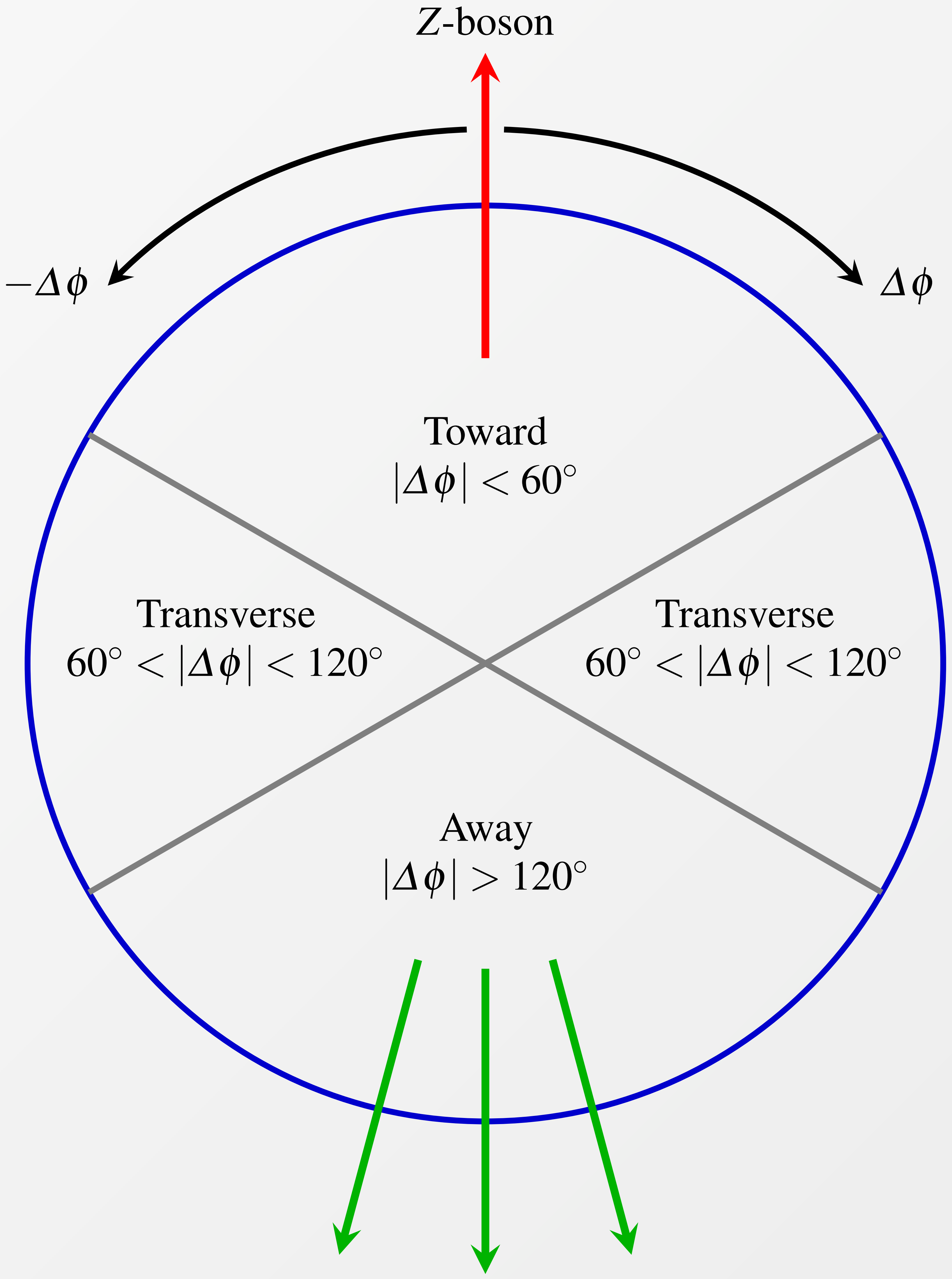
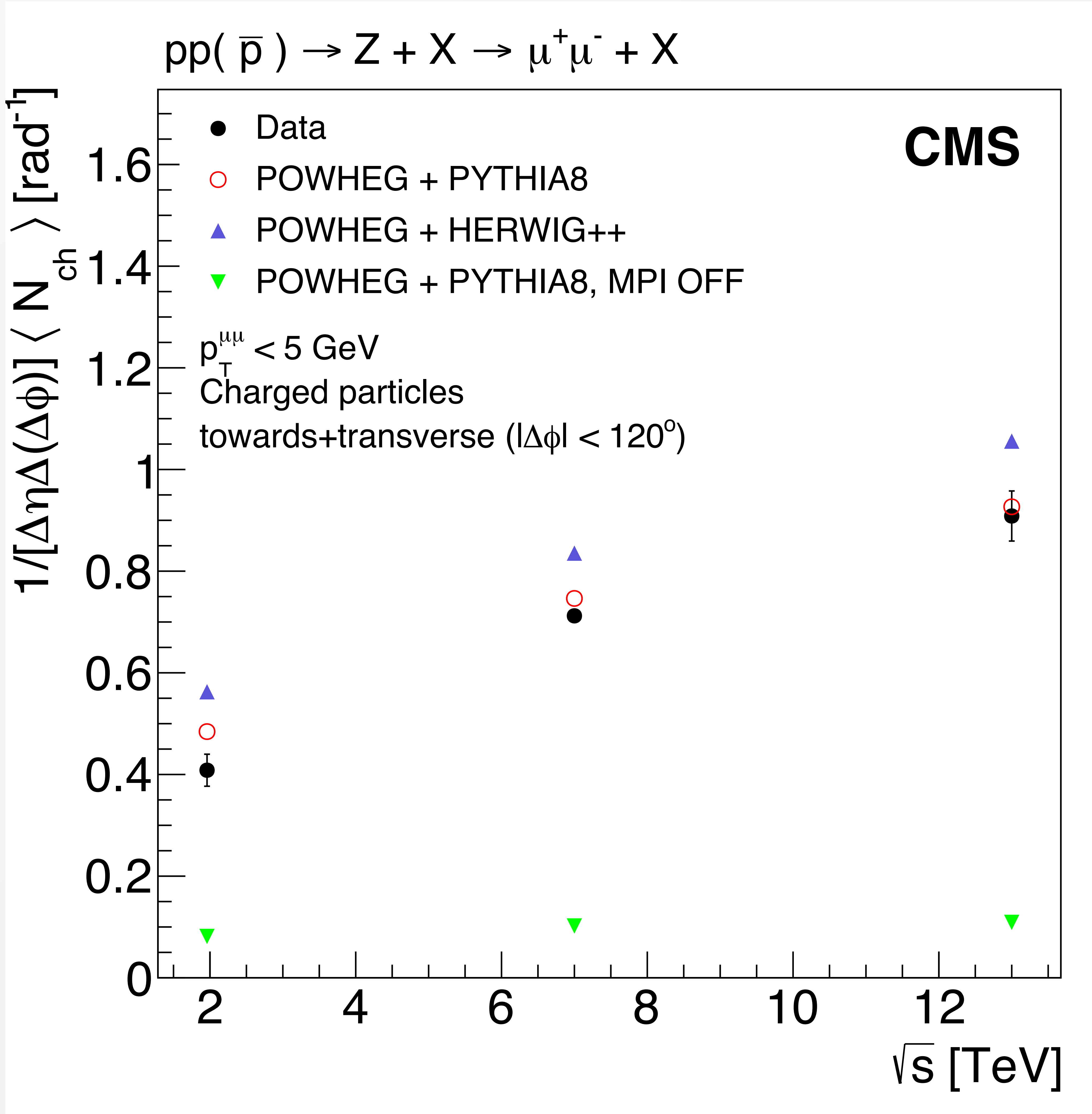


Fig. 1 Definition of UE regions as a function of the azimuthal angle with respect to the Z-boson.

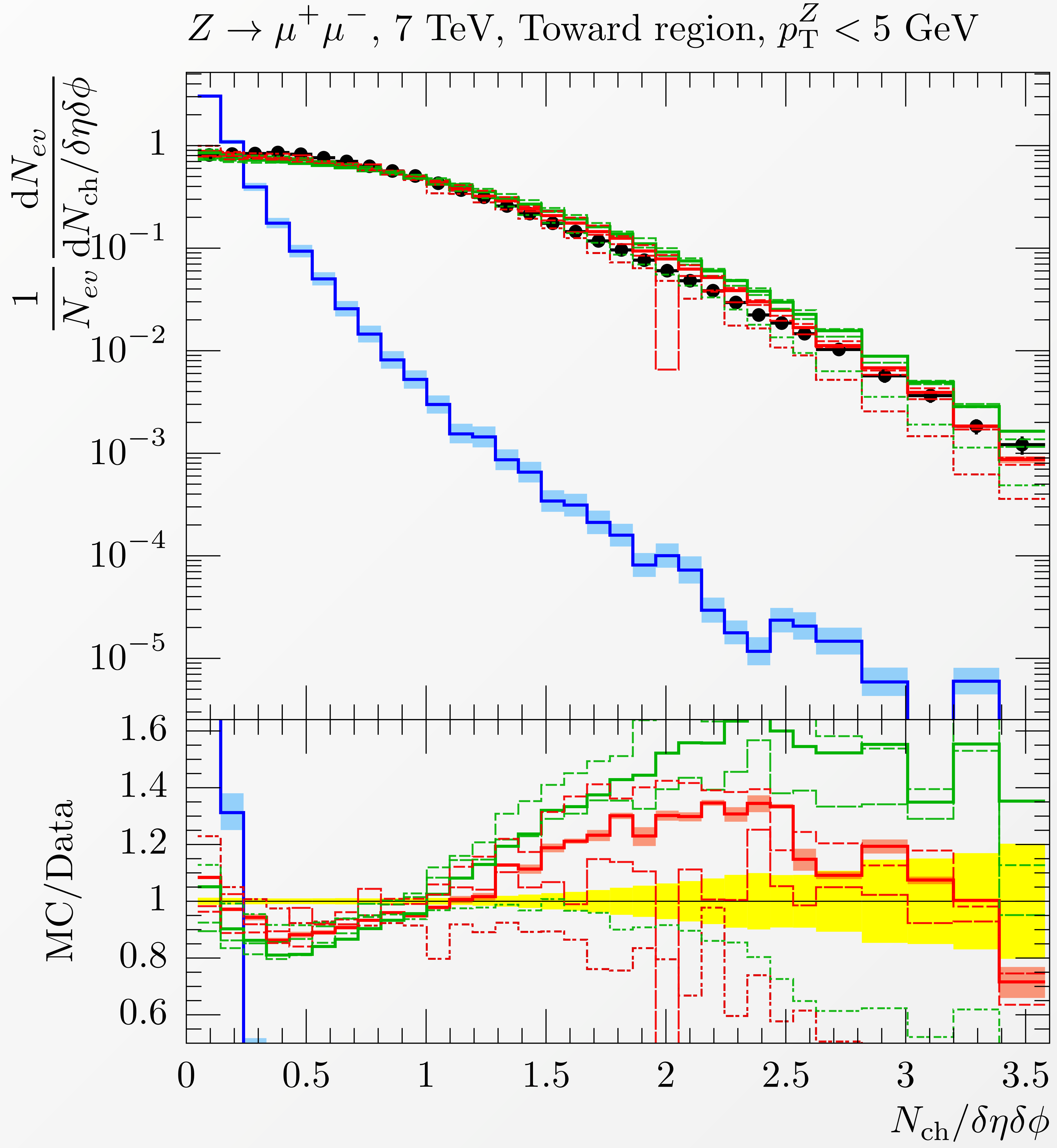
CMS 1711.04299

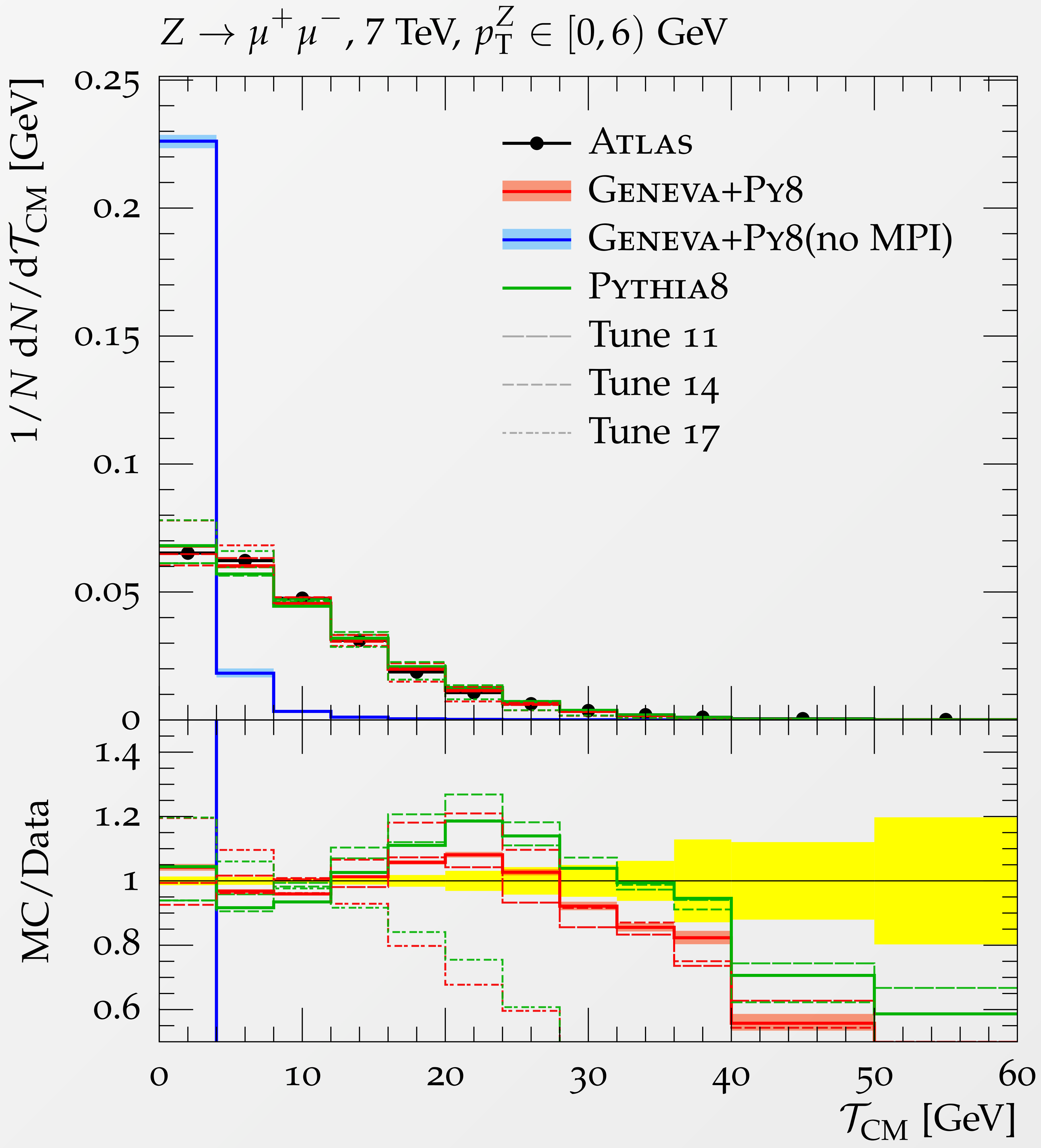
- mostly a UE study
- uses $p_T^Z < 5\,\mathrm{GeV}$



Alioli, Bauer, Guns, Tackmann, 1605.07192

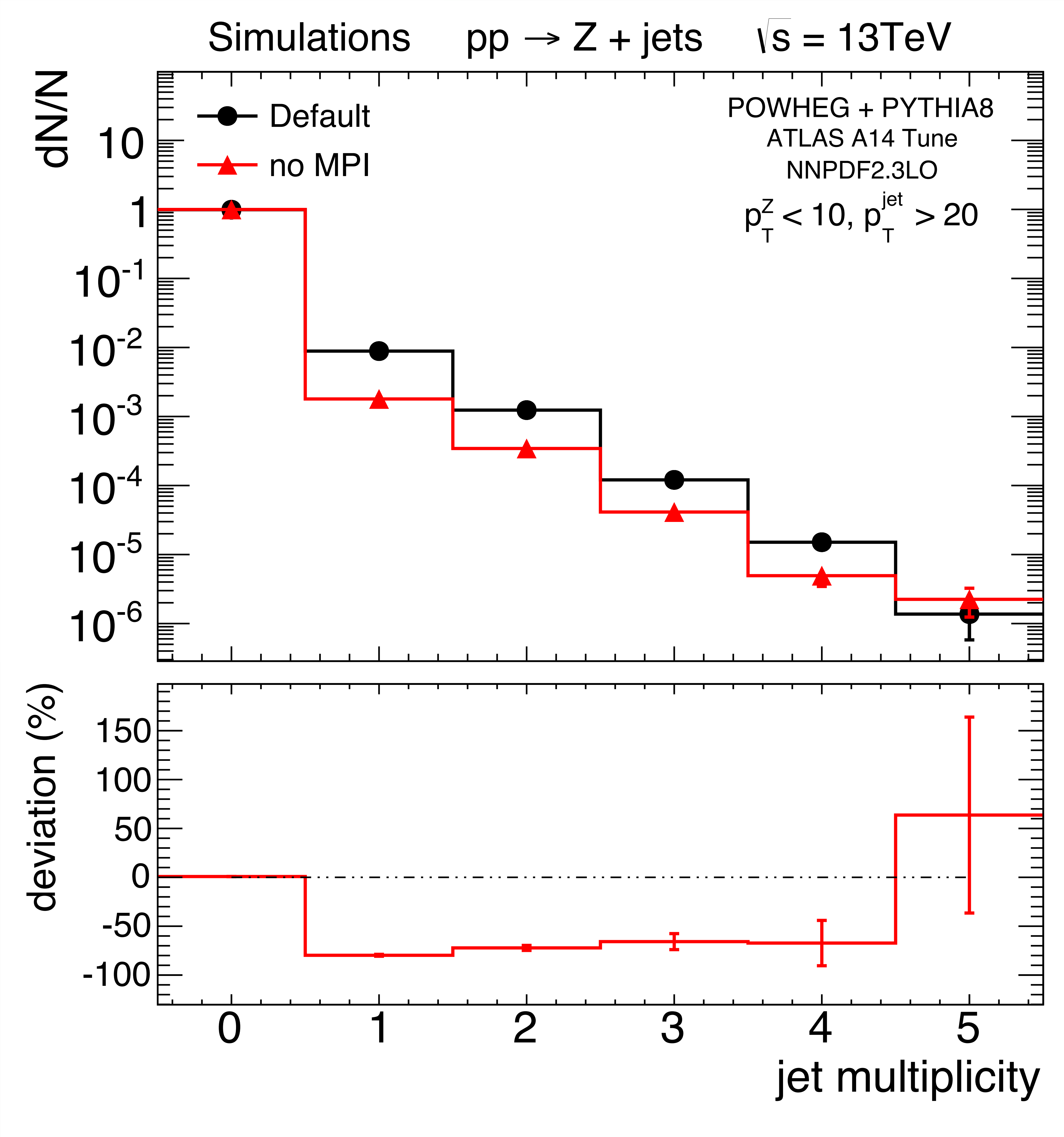
- explores $p_T^Z < 5\,\mathrm{GeV}$
- mainly a "UE" study





Bansal, Bansal, Kumar, Singh 1602.05392

- explores $p_T^Z < 10\,\mathrm{GeV}$ as central part of their study
- explores various jet cuts, including $p_T^{\mathrm{jet}} > 5\,\mathrm{GeV}$



CMS 2210.16139

- includes $p_T^Z < 10\,\mathrm{GeV}$ bin, with 25-50% MPI contribution for jets with $p_T^J > 30\,\mathrm{GeV}$
- includes $\Delta\phi_{j_1 j_2}$, though high p_T^J cut means only 2HS